

1 Angle calculation

We calculate the angle by assuming that the ellipse (ear) we found was a circle in the xy-plane that was rotated about the y axis and then projected into the xy-plane again. The projection should theoretically be a proper camera projection but for simplification purposes we assume a parallel projection.

We can discard the translation and rotation of our ellipse. Our ellipse is the projection of a circle that was rotated about the y axis into the XY-plane.

We should be able to calculate the rotation angle from just four points at the ends of the major and minor axes respectively.

A rotation about the y axis only changes the x coordinate (and z, but we cannot determine the z coordinates from the image):

$$x = \cos \theta \cdot a \sin \alpha \tag{1}$$

$$x_1 = b = \cos \theta \cdot a \sin \frac{\pi}{2} \tag{2}$$

$$b = \cos \theta \cdot a \tag{3}$$

$$x_2 = -b = \cos \theta \cdot a \sin \frac{3\pi}{2} = \cos \theta \cdot (-a) \tag{4}$$

$$\theta = \arccos \frac{b}{a} \tag{5}$$