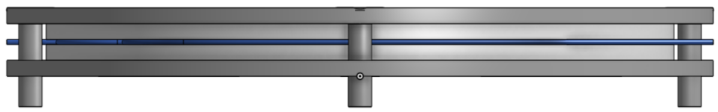
Critical Temperature Derivation

Radial Direction



The radial direction is simpler as it is unconstrained, so we can begin with it. The critical dimensions are the distance between the screws and the diameter of the ceramic plate. In the AlN system, so long as the distance between the screws remains larger than the diameter of the disk, there will not be stress concentration.

The screws are effectively moved by the Aluminum structural plates, so we can assume that the relevant quantaties here are the CTE of the AlN and Al. The CTE of Al ($23.5 \times 10^{-6} K^{-1}$) is greater than the CTE of AlN ($5.6 \times 10^{-6} K^{-1}$). This implies that we will not have stress buildup in the radial direction.

Unlike the new heater, the Boron Nitride disks were rigidly coupled to the screws via mounting holes. This indicates that stress concentrations can be tensile or compressive, again depending on the relevant CTEs. Again, the CTE of Al ($23.5 \times 10^{-6} K^{-1}$) is greater than the CTE of BN ($6.0 \times 10^{-6} K^{-1}$), so the rigid coupling implies tensile stress formation.

The strain on the BN disk is effectively dictated by the Al disk because of the larger magnitude CTE, and the rigid coupling implies that the strain on the Al will be equal to the strain on the BN. Therefore

$$\varepsilon_{BN} = \varepsilon_{Al} = \alpha_{Al} \Delta T$$

For some change in temperature ΔT, the stress induced in the BN will therefore be

$$\sigma = E_{BN} \Delta T (\alpha_{Al} - \alpha_{BN})$$

Which gives a critical temperature of

$$\Delta T = \frac{\sigma}{E_{BN}(\alpha_{Al} - \alpha_{BN})}$$

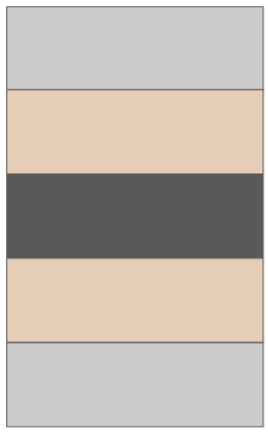
We can then substitute the actual values

α_{Al}	$23.5 \times 10^{-6} K^{-1}$
α_{BN}	$6.0 \times 10^{-6} K^{-1}$
E_{BN}	19.5×10^{9} Pa
$\sigma_{max,BN}$	83.3×10^{6} Pa

Which gives a critical temperature of $\Delta T = 244.1$ K. It is also worth noting that this is the absolute highest estimate of this critical temperature, as I used the highest available compressive strength and lowest available modulus from the data.

Axial Direction

The axial geometry is shown below. The upper and lower surfaces of the plates are secured using stainless steel screws, which effectively limits the axial expansion and causes a concentration of compressive stress in the AlN.



Assuming the screws have an initial length L_0^{screw}, a CTE of α^{screw}, and an initial stress of 0 Pa, application of some ΔT would result in a lengthening of the screws to:

$$L_T^{screw} = L_0^{screw} \alpha^{screw} \Delta T$$

Thus, the sum of the lengths of the other components must equal L_T^{screw}, that is:

$$L_T^{screw} = L_T^{Al} + L_T^{AlN} + L_T^{Ni}$$

We can also assume that each component will expand according to its coefficient of thermal expansion, so:

$$L_T^{screw} = L_0^{Al} + L_0^{AlN} + L_0^{Ni} + C(\alpha^{Al} + \alpha^{AlN} + \alpha^{Ni})$$

Where C is an "effective temperature" applied to the the system. Solving for C as a function of ΔT gives

$$C = \frac{L_T^{screw} - L_0^{screw}}{\alpha^{Al} + \alpha^{AlN} + \alpha^{Ni}} = \frac{\alpha^{screw}}{\alpha^{Al} + \alpha^{AlN} + \alpha^{Ni}} \Delta T$$

We can then use C to determine the stress in the Aluminum Nitride by first calculating the actual strain and the "desired" strain:

$$\varepsilon_{Actual}^{AlN} = \alpha^{AlN} C, \varepsilon_{Desired}^{AlN} = \alpha^{AlN} \Delta T$$

And subtracting them to calculate strain:

$$\sigma = E(\varepsilon_{Actual} - \varepsilon_{Desired}) = E_{AlN}\alpha_{AlN}\Delta T(\frac{\alpha^{screw}}{\alpha^{Al} + \alpha^{AlN} + \alpha^{Ni}} - 1)$$

Which finally gives a critical temperature of

$$\Delta T = \frac{\sigma}{E_{AlN}\alpha_{AlN}}(\frac{\alpha^{screw}}{\alpha^{Al} + \alpha^{AlN} + \alpha^{Ni}} - 1)^{-1}$$

We can then substitute in the actual values:

α_{Al}	$23.5 \times 10^{-6} K^{-1}$
α_{AlN}	$5.6 \times 10^{-6} K^{-1}$
α_{Ni}	$14.0 \times 10^{-6} K^{-1}$
α_{screw}	$18.4 \times 10^{-6} K^{-1}$
E_{AlN}	348×10^{9} Pa
$\sigma_{max,AlN}$	-1.97×10^{9} Pa (Compression)

To get $C = 0.427\Delta T$ and a critical temperature $\Delta T = 1763.92$ K