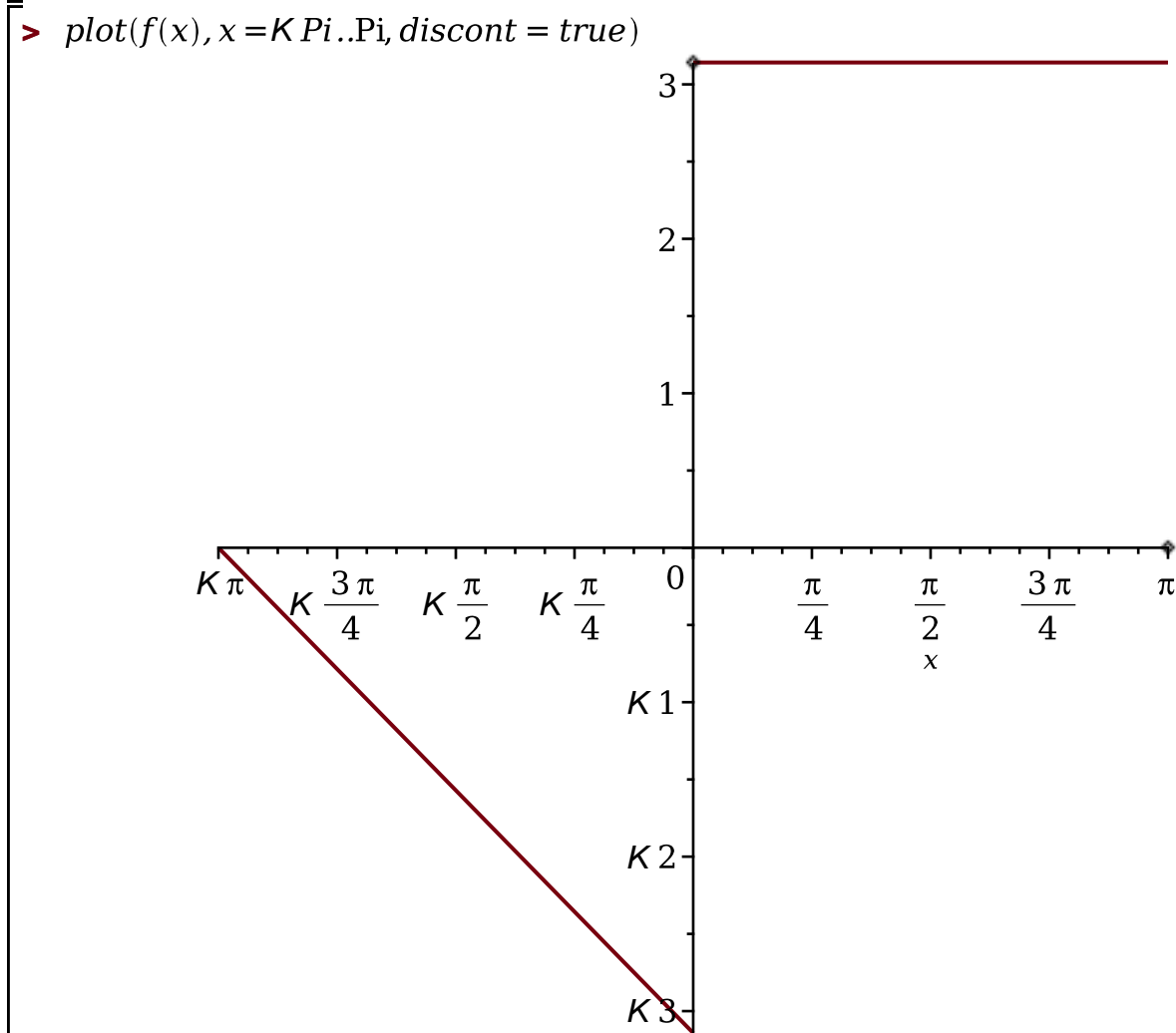


$$\begin{aligned}
 &> f := x \mapsto \text{piecewise}(K \pi \leq x < 0, K \pi K x, 0 \leq x < \pi, \pi) \\
 &\quad f := x \mapsto \begin{cases} K \pi K x & K \pi \leq x < 0 \\ \pi & 0 \leq x < \pi \end{cases}
 \end{aligned} \tag{1}$$



$$\begin{aligned}
 &> a0 := \text{simplify}\left(\frac{1}{\pi} \cdot \text{int}(f(x), x = K \pi .. \pi)\right) \\
 &\quad a0 := \frac{\pi}{2}
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 &> an := \text{simplify}\left(\frac{1}{\pi} \cdot \text{int}(f(x) \cdot \cos(n \cdot x), x = K \pi .. \pi)\right) \text{ assuming } n :: \text{posint}; \\
 &\quad an := \frac{(K 1)^n K 1}{\pi n^2}
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 &> bn := \text{simplify}\left(\frac{1}{\pi} \cdot \text{int}(f(x) \cdot \sin(n \cdot x), x = K \pi .. \pi)\right) \text{ assuming } n :: \text{posint}; \\
 &\quad bn := \frac{K (K 1)^n + 2}{n}
 \end{aligned} \tag{4}$$

```
> # Creating proc to get sum
```

```
> FurieSum := proc(f, k)
  local a0, an, bn, n;
  description "return Furie Sum for -Pi .. Pi";
  a0 := simplify(int(f(x), x = K π..π) / π);
  assume(n::posint);
  an := simplify(int(f(x) * cos(n*x), x = K π..π) / π);
  bn := simplify(int(f(x) * sin(n*x), x = K π..π) / π);
  return 1/2*a0 + sum(an*cos(n*x) + bn*sin(n*x), n = 1..k)
end proc;
```

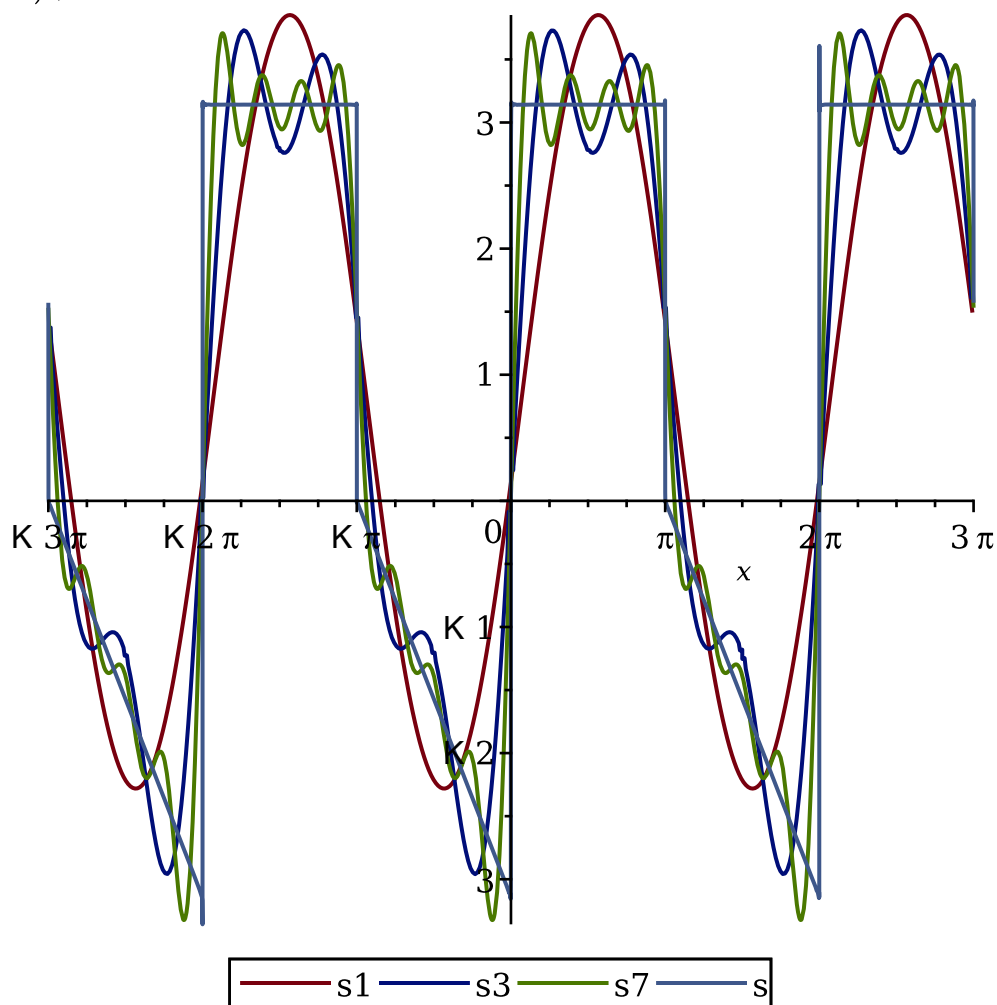
```
> S1 := FurieSum(f, 1):
```

```
> S3 := FurieSum(f, 3):
```

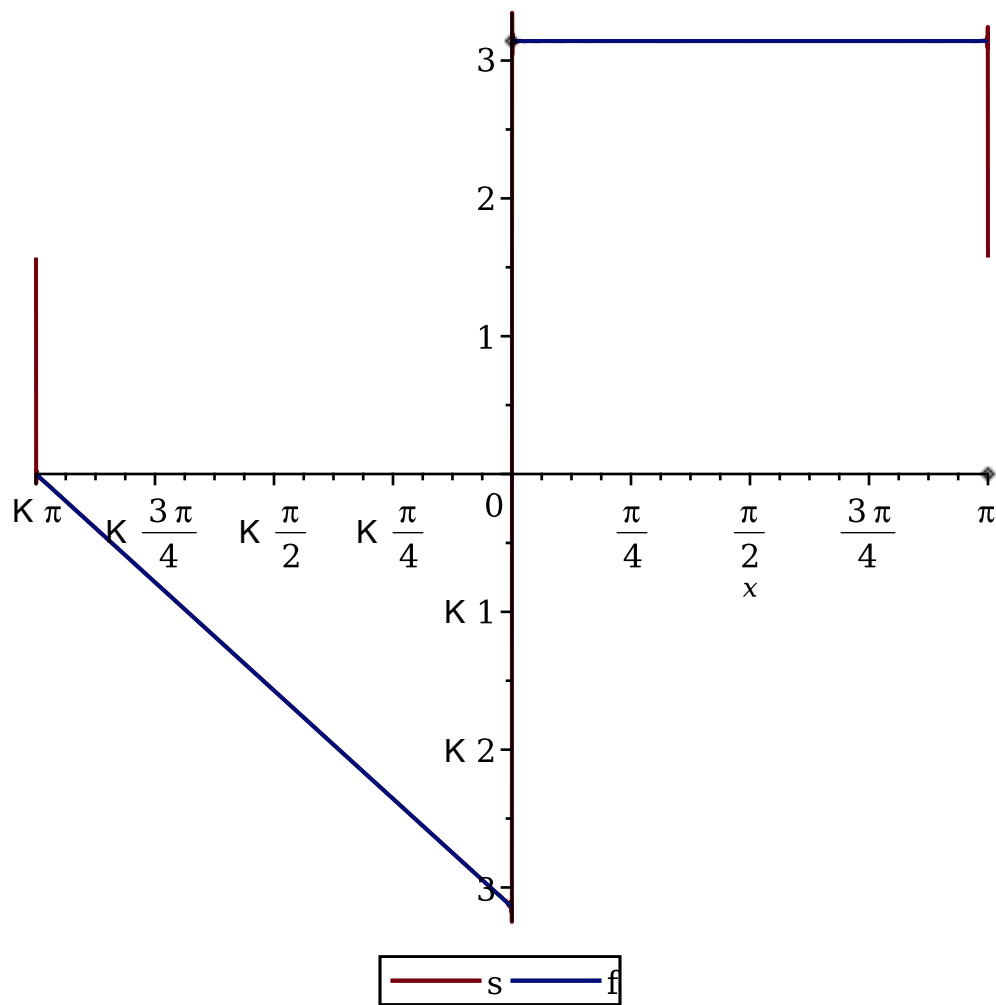
```
> S7 := FurieSum(f, 7):
```

```
> S := FurieSum(f, 10000):
```

```
> plot([S1, S3, S7, S], x = K 3π..3 π, legend = ["s1", "s3", "s7", "s"],
  = true);
```



```
> plot([S, f(x)], x = K π..π, legend = ["s", "f"],
  = true);
```



```
> #plots[animate]( plot, [FurieSum(f, k), x=K Pi..Pi], k = 1..5 );
```

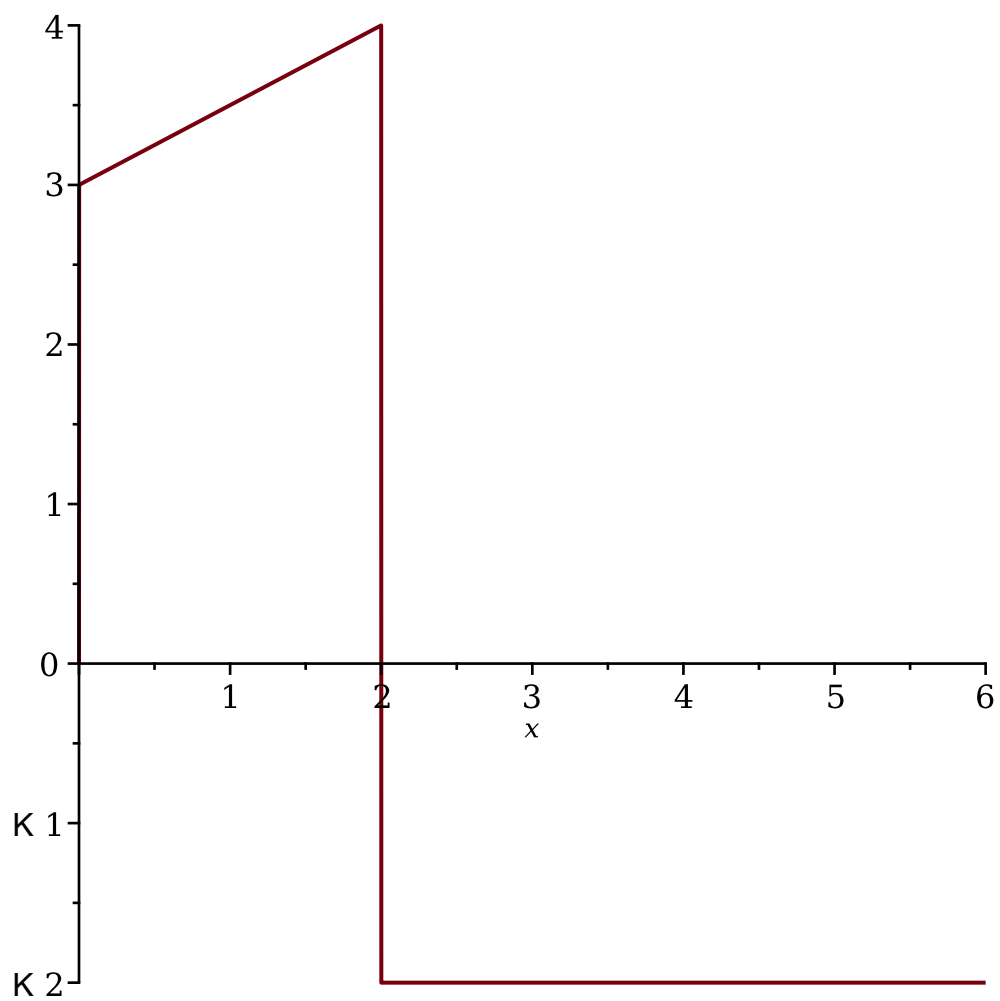
```
> #task 2 (variant 10)
```

```
> f := x -> piecewise(0 < x < 2, 0.5·x + 3, 2 ≤ x ≤ 6, K 2);
```

$$f := x \mapsto \begin{cases} 0.5 \cdot x + 3 & 0 < x < 2 \\ K 2 & 2 \leq x \leq 6 \end{cases}$$

(5)

```
> plot(f(x), x = 0..6);
```



```
> #Coefficients Fourier sum
```

```
> a0 := simplify( $\frac{1}{3} \cdot \text{int}(f(x), x = 0..6, \text{numeric} = \text{false}, \text{useunits} = \text{false})$ );
```

$a0 := K\ 0.3333333333$

(6)

$\triangleright an := \text{simplify}\left(\frac{1}{3} \cdot \text{int}\left(f(x) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{3}\right), x = 0..6\right)\right) \text{ assuming } n :: \text{posint};$

$an := \frac{1}{n^2} (1.909859317\ n \sin(2.094395102\ n)$

(7)

$+ 0.1519817755 \cos(2.094395102\ n) K\ 0.1519817755)$

$\triangleright bn := \text{simplify}\left(\frac{1}{3} \cdot \text{int}\left(f(x) \cdot \sin\left(\frac{n \cdot x \cdot \text{Pi}}{3}\right), x = 0..6\right)\right) \text{ assuming } n :: \text{posint};$

$$bn := \frac{1}{n^2} (K 1.909859317 n \cos(2.094395102 n) + 1.591549431 n + 0.1519817755 \sin(2.094395102 n)) \quad (8)$$

> **#Fourier sum procedure**

> *Furie* := **proc**(*f*, *k*, *l*)

**local** *a0*, *an*, *bn*;

$a0 := \frac{1}{l} \cdot \text{int}(f(x), x = 0..2 \cdot l);$

$an := \text{simplify}\left(\frac{1}{l} \cdot \text{int}\left(f(x) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{l}\right), x = 0..2 \cdot l\right) \text{ assuming } n :: \text{posint}\right);$

$bn := \text{simplify}\left(\left(\frac{1}{l} \cdot \text{int}\left(f(x) \cdot \sin\left(\frac{n \cdot x \cdot \text{Pi}}{l}\right), x = 0..2 \cdot l\right)\right) \text{ assuming } n :: \text{posint}\right);$

**return**  $1/2 \cdot a0 + \text{sum}\left(an \cdot \cos\left(n \cdot \frac{x \cdot \text{Pi}}{l}\right) + bn \cdot \sin\left(\frac{n \cdot x \cdot \text{Pi}}{l}\right), n = 1..k\right);$

**end proc**:

> *S1* := *Furie*(*f*, 1, 3) :

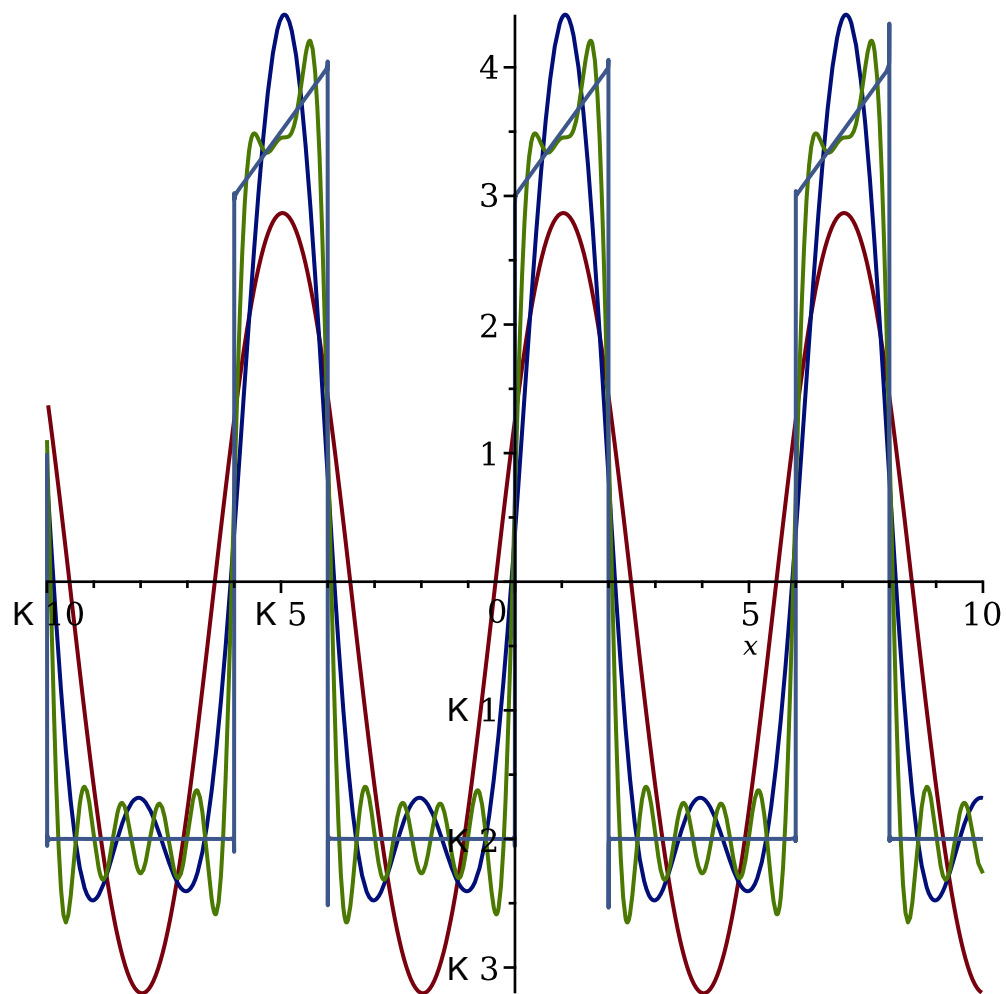
> *S3* := *Furie*(*f*, 3, 3) :

> *S5* := *Furie*(*f*, 5, 3) :

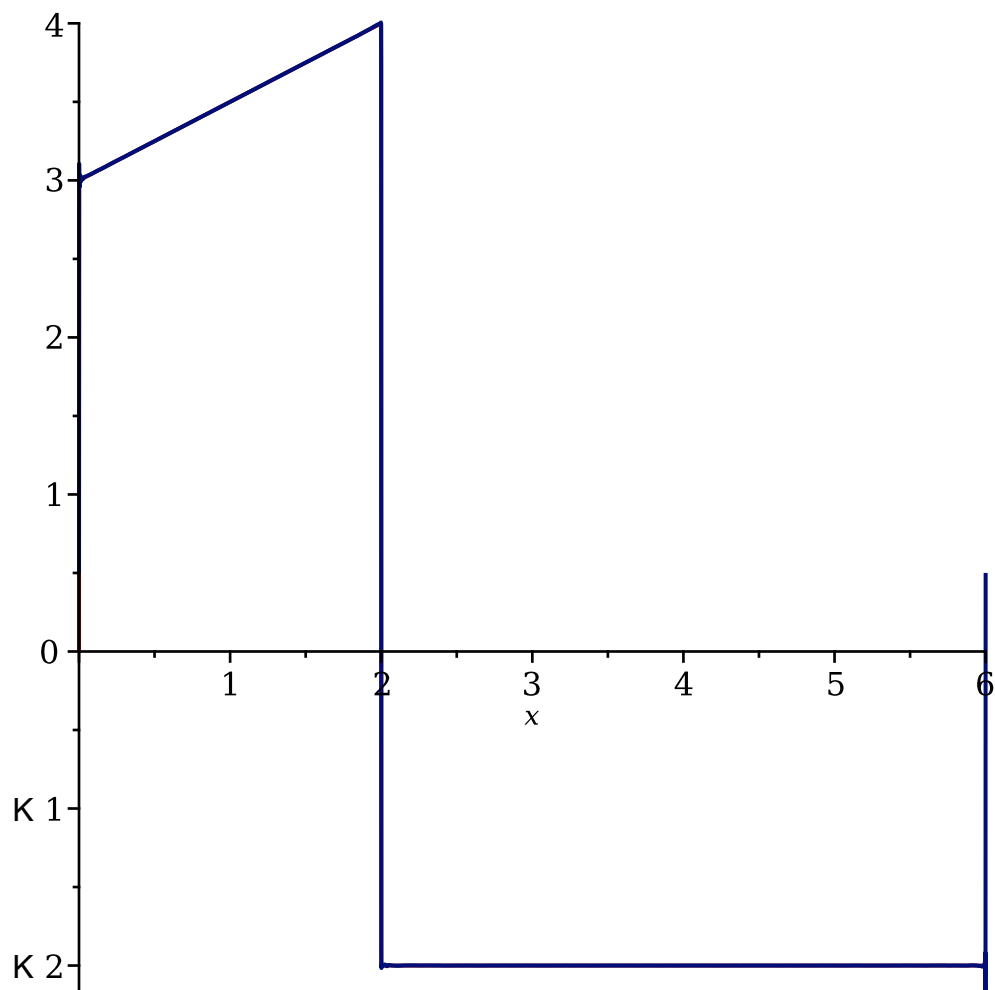
> *S7* := *Furie*(*f*, 7, 3) :

> *S* := *Furie*(*f*, 10000, 3) :

> *plot*([*S1*, *S3*, *S7*, *S*], *x* = K 10..10);



```
> plot([f(x), S], x = 0..6);
```



>

> # task 3 (variant 10)

>  $f_3 := x \mapsto \text{piecewise}\left(x < 2, K \frac{3 \cdot (x - 1)^2}{2}, x \geq 2, \frac{(x - 5)}{2}\right);$

$l := 5$

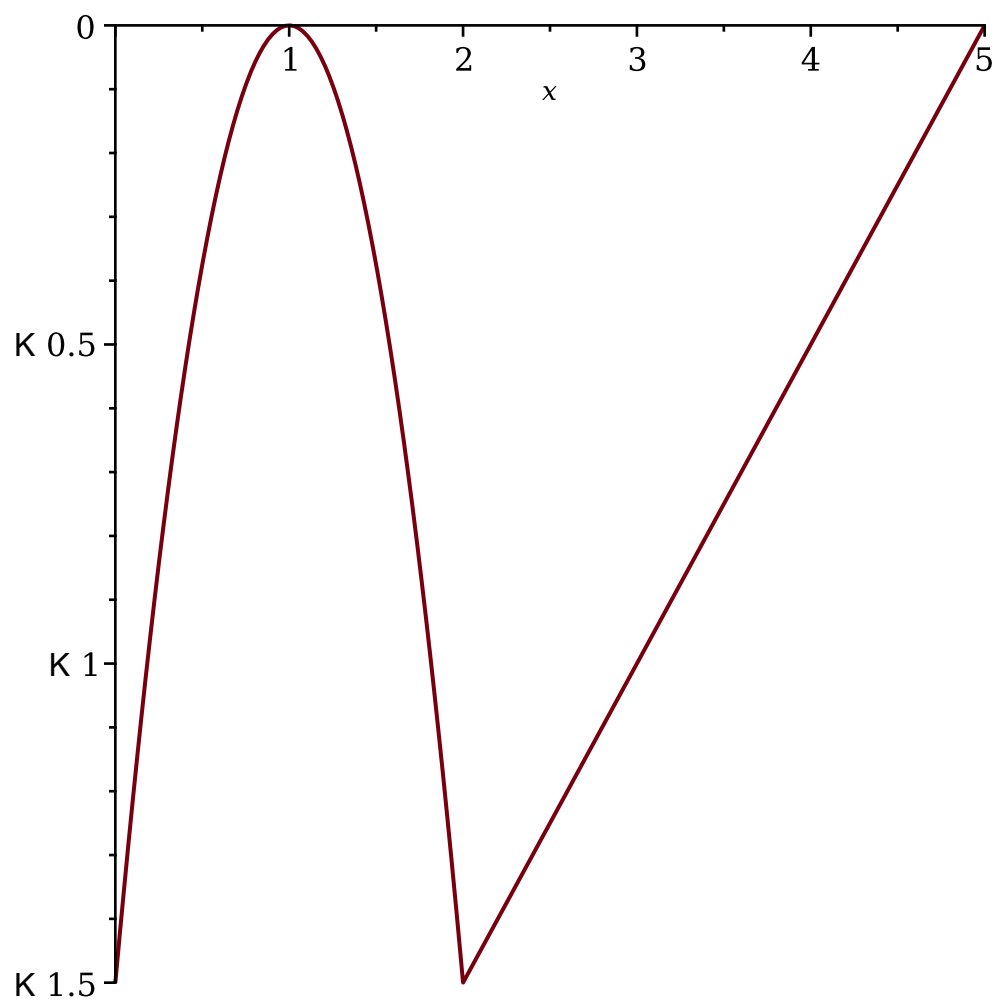
$$f_3 := x \mapsto \begin{cases} K \frac{3 \cdot (x - 1)^2}{2} & x < 2 \\ \frac{x}{2} - K \frac{5}{2} & 2 \leq x \end{cases}$$

$l := 5$

>  $\text{plot}(f_3(x), x = 0..5);$

(9)

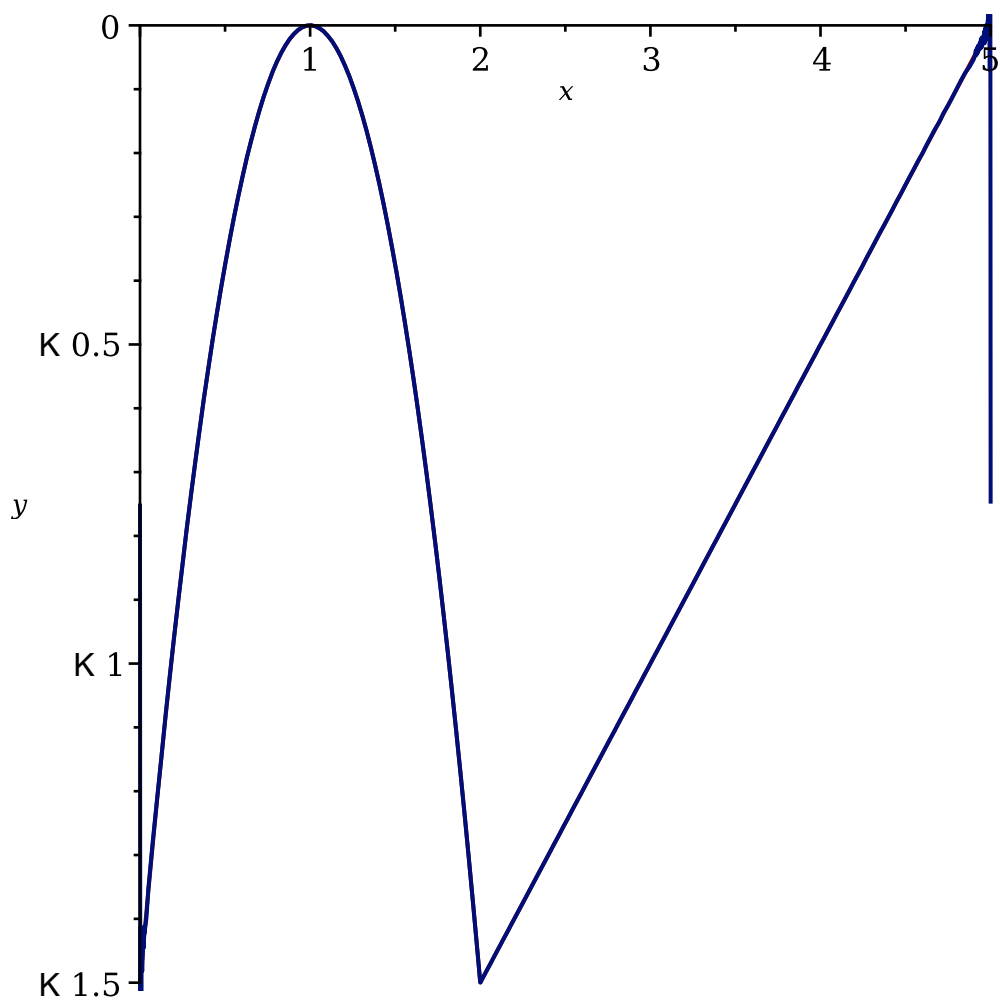




```

>
> S := Furie(f3, 1000, 2.5) :
> plot([f3(x), S], x = 0..5, y = K 1.5..0);

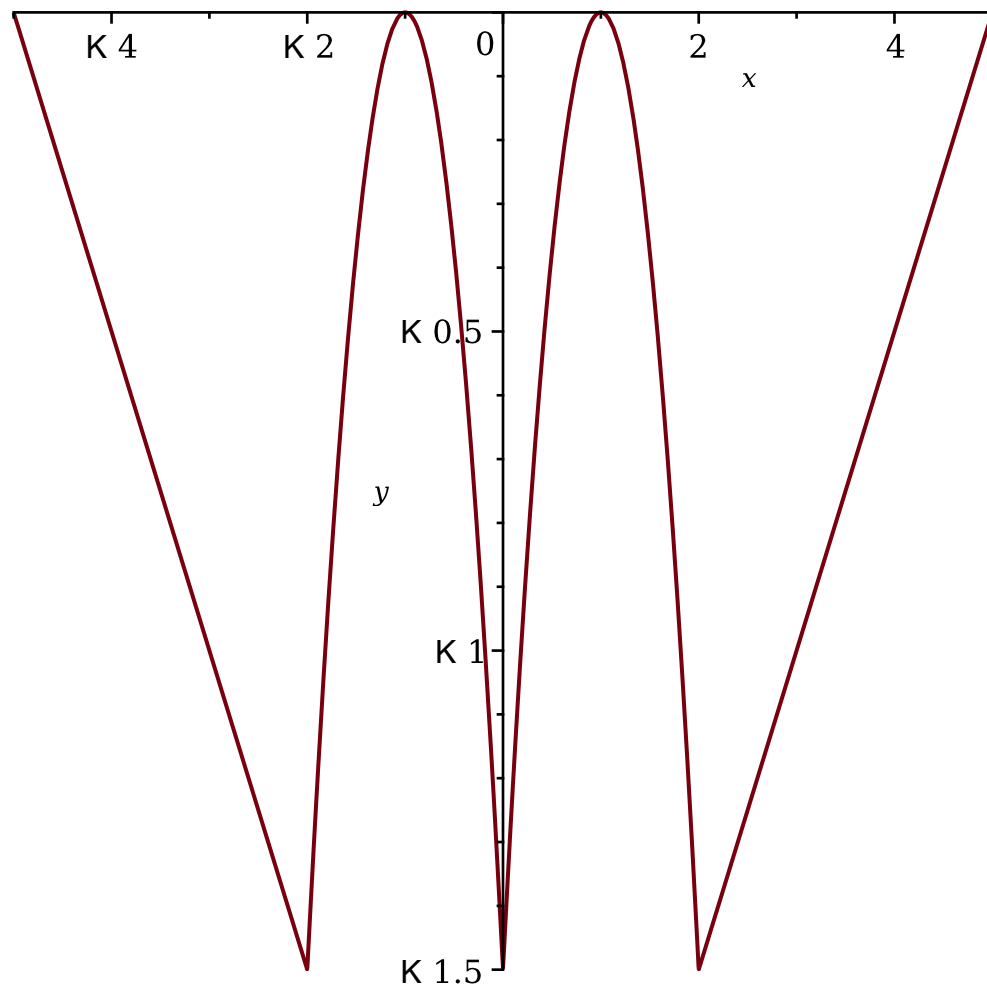
```



```

> Furie := proc (f, k, l)
  local a0, an, bn;
  a0 :=  $\frac{2}{l} \cdot \text{int}(f(x), x = 0..l);$ 
  an := simplify $\left(\frac{2}{l} \cdot \text{int}\left(f(x) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{l}\right), x = 0..l\right) \text{ assuming } n :: \text{posint}\right);$ 
  return  $1/2 \cdot a0 + \text{sum}\left(an \cdot \cos\left(n \cdot \frac{x \cdot \text{Pi}}{l}\right), n = 1..k\right);$ 
end proc;
> S := Furie(f3, 1000, 5) :
> plot([S], x = K 5..5, y = K 1.5..0);

```



```

> Furie := proc (f, k, l)
local a0, an, bn;
bn := simplify( $\left( \left( \frac{2}{l} \cdot \text{int} \left( f(x) \cdot \sin \left( \frac{n \cdot x \cdot \text{Pi}}{l} \right), x = 0..l \right) \right) \right)$  assuming  $n :: \text{posint}$ );
return sum( $bn \cdot \sin \left( n \cdot \frac{x \cdot \text{Pi}}{l} \right)$ ,  $n = 1..k$ );
end proc;
>
> S := Furie(f3, 1000, 5) :
> plot([S], x = K 5..5, y = K 1.5..1.5);

```

