```
>                                    #
> running := proc(a, b, c, f, L, R)
    local i, n, x_arr, x, alpha_arr, beta_arr;
    n := numelems(a);
```

$$alpha\_arr := Array\left(\left[-\frac{c[1]}{b[1]}\right]\right);$$

$$beta\_arr := Array\left(\left[\frac{f[1]}{b[1]}\right]\right);$$

```
    f[1] := f[1] − a[1]· L;
    f[n] := f[n] − c[n]·R;
    a[1] := 0;
    c[n] := 0;
    for i from 2 to n do
```

$$ArrayTools\text{:-}Append\left(alpha\_arr, -\frac{c[i-1]}{a[i-1]\cdot alpha\_arr[i-1] + b[i-1]}\right);$$

$$ArrayTools\text{:-}Append\left(beta\_arr, \frac{f[i-1] - a[i-1]\cdot beta\_arr[i-1]}{a[i-1]\cdot alpha\_arr[i-1] + b[i-1]}\right);$$

```
    end do;
    x_arr := Array([ ]);
    for i from 1 to  n do
    ArrayTools:-Append(x_arr, cat(x, i));
    end do;
```

$$x\_arr[n] := \frac{-a[n]\cdot beta\_arr[n] + f[n]}{a[n]\cdot alpha\_arr[n] + b[n]};$$

```
    for i from n −1 to 1 by −1 do
    x_arr[i] := alpha_arr[i+1]·x_arr[i+1] + beta_arr[i+1];
    end do;
    return x_arr
    end proc:
>
>          #                                          y''+p(x)y'+q(x)y=f(x)
> difference_method := proc(p, q, f, h, l, r, L, R)
    local a, b, c, d, i, n, x_arr, xk;
    a := Array([ ]);
    b := Array([ ]);
    c := Array([ ]);
    d := Array([ ]);
    x_arr := Array([ ]);
```

$$n := \text{floor}\left(\frac{r-l}{h}\right);$$

```
    xk := l;
    for i from 1 to n + 1 do
```

$$ArrayTools\text{:-}Append\left(a, 1 - \frac{p(xk)}{2}\cdot h\right);$$

$$ArrayTools\text{:-}Append\left(b, -2 + q(xk)\cdot h^2 + \frac{p(xk)}{2}\cdot h\right);$$

```
    ArrayTools:-Append(c, 1);
```

```maple
      ArrayTools:-Append(d, f(xk)·h^2);
      ArrayTools:-Append(x_arr, xk);
      xk := xk + h;
      end do;
      return [x_arr, running(a, b, c, d, L, R)]
      end proc:
```

```maple
> chebyshev_norm := proc(x_arr, y_arr, func)
      local res, i, temp;
      res := 0;
      for i from 1 to numelems(x_arr) do
      temp := abs(y_arr[i] − rhs(evalf(subs(x = x_arr[i], sol))));
      if temp > res then
      res := temp;
      end if
      end do;
      return res;
      end proc:
```

```maple
> evklid_norm := proc(x_arr, y_arr, func)
      local res, i;
      res := 0;
      for i from 1 to numelems(x_arr) do
      res := res + (y_arr[i] − rhs(evalf(subs(x = x_arr[i], sol))))^2
      end do;
      return sqrt(res)
      end proc:
```

```maple
>
>                                    #1
> k := 19 :
> alpha := sin(k) :
> beta := cos(k) :
> p(x) := 0 :
```

$$> \quad q(x) := \frac{(1 + \text{beta}·x)}{\text{alpha}} :$$

$$> \quad f(x) := - \frac{1}{\text{alpha}} :$$

```maple
> ode := diff(y(x), x$2) + p(x)·diff(y(x), x) + q(x)·y(x) = f(x)
```

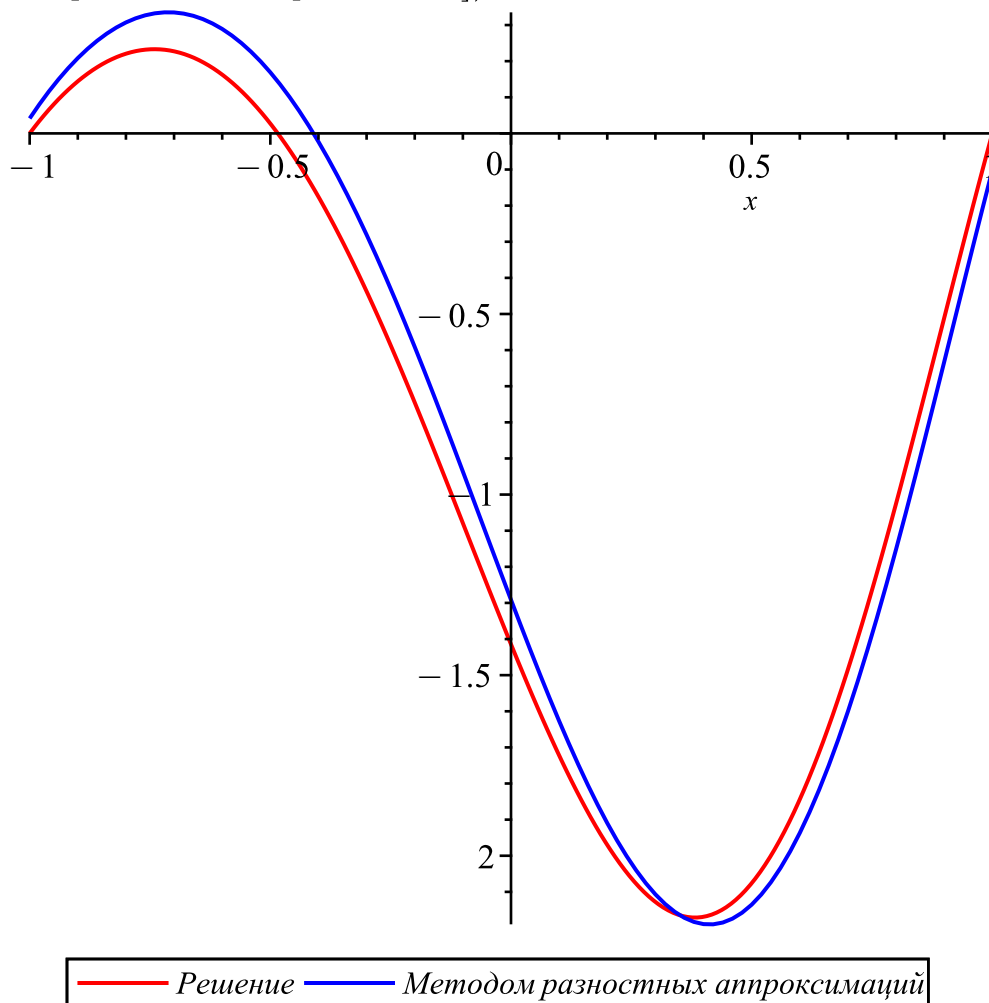$$ode := \frac{d^2}{dx^2} y(x) + \frac{(1 + \cos(19) x) y(x)}{\sin(19)} = -\frac{1}{\sin(19)} \tag{1}$$

```maple
> sol := dsolve({y(−1) = 0, y(1) = 0, ode}) :
> plot1 := plot(rhs(sol), x = −1 ..1) :
> res := difference_method(p, q, f, 0.02, −1, 1, 0, 0) :
> plot2 := plots:-pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1
      ..numelems(res[1]))], connect = true) :
```

> $plots[display](plot1, plot2, color = [red, blue], legend = [Решение,$
  $Методом\ разностных\ аппроксимаций])$

▬▬ Решение ▬▬ Методом разностных аппроксимаций

> $chebyshev\_norm(res[1], evalf(res[2]), sol)$

$$0.1568299875 \qquad \textbf{(2)}$$

#2

> $p(x) := x :$

> $q(x) := x :$

> $f(x) := x^2 :$

> $l := 0 :$

> $r := 1 :$

> $L := 1 :$

> $R := 1 :$

> $h := 0.1 :$

> $ode := diff(y(x), x\$2) + p(x) \cdot diff(y(x), x) + q(x) \cdot y(x) = f(x)$

$$ode := \frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + x\left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) + x\, y(x) = x^2 \qquad \textbf{(3)}$$
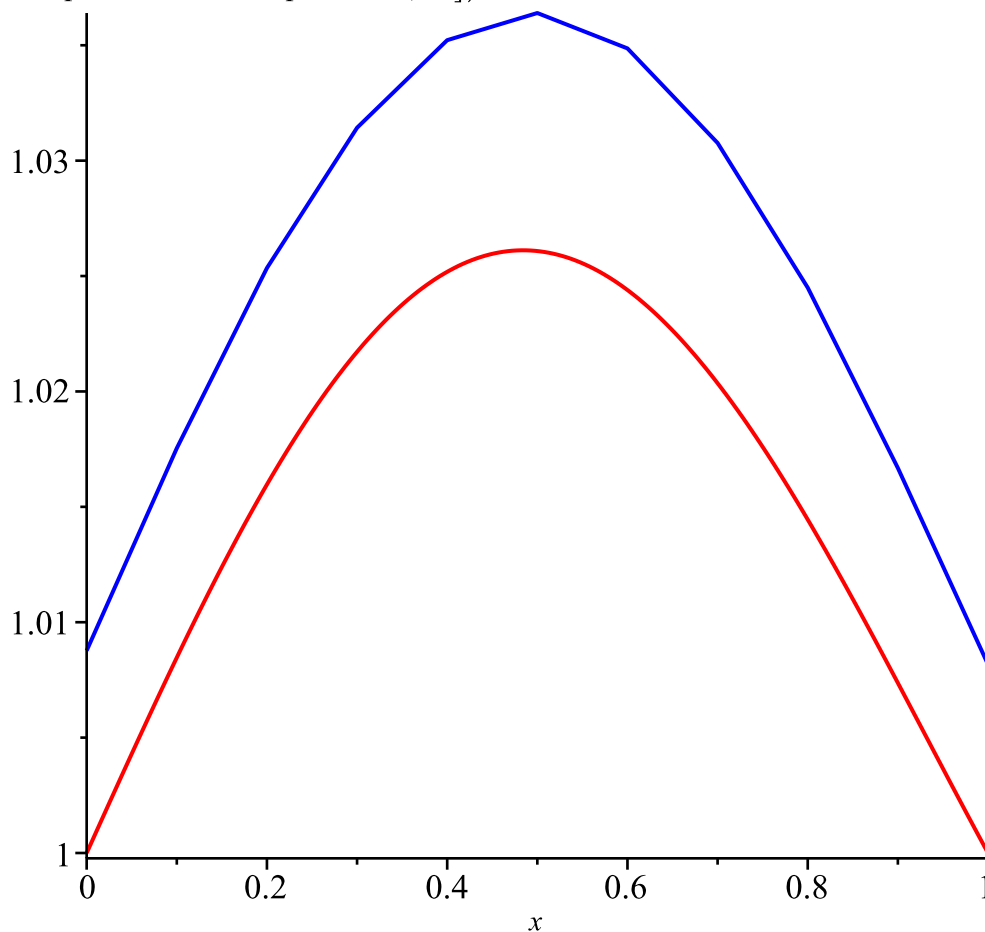
> $sol := dsolve(\{y(l) = L, y(r) = R, ode\})$

$$sol := y(x) = \frac{e^{-\frac{x(-2+x)}{2}}\left(2\,e^{\frac{1}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}}{2}\right) - \mathrm{erfi}\left(\sqrt{2}\right)\right)e^{-\frac{1}{2}}}{\mathrm{erfi}\left(\frac{\sqrt{2}}{2}\right) - \mathrm{erfi}\left(\sqrt{2}\right)}$$

$$+ \frac{I\,e^{x-\frac{1}{2}x^2}\,\mathrm{erf}\left(\frac{I}{2}\sqrt{2}\,(-2+x)\right)\left(-1+2\,e^{\frac{1}{2}}\right)e^{-\frac{1}{2}}}{-\mathrm{erfi}\left(\frac{\sqrt{2}}{2}\right) + \mathrm{erfi}\left(\sqrt{2}\right)} + x - 1$$

(4)

```
> plot1 := plot(rhs(sol), x = l..r) :
> res := difference_method(p, q, f, h, l, r, L, R) :
> plot2 := plots:-pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1
      ..numelems(res[1]))], connect = true) :
> plots[display](plot1, plot2, color = [red, blue], legend = [Решение,
      Методом разностных аппроксимаций])
```


| Решение | Методом разностных аппроксимаций |

```
> chebyshev_norm(res[1], evalf(res[2]), sol)
```
$$0.010468563$$

(5)

```
> 
```
#3
```
> p(x) := 2 :
```

> $q(x) := -1.5\,x :$

> $f(x) := \dfrac{2}{x} :$

> $l := 0.8 :$

> $r := 3.8 :$

> $L := 5 :$

> $R := 10 :$

> $h := 0.005 :$

> $ode := diff(y(x), x\$2) + p(x) \cdot diff(y(x), x) + q(x) \cdot y(x) = f(x)$

$$ode := \frac{d^2}{dx^2}\,y(x) + 2\,\frac{d}{dx}\,y(x) - 1.5\,x\,y(x) = \frac{2}{x} \qquad (6)$$

> $sol := dsolve(\{y(l) = L, y(r) = R, ode\}) :$

> $plot1 := plot(rhs(sol), x = l..r) :$

> $res := difference\_method(p, q, f, h, l, r, L, R) :$

> $plot2 := plots\text{:-}pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1$
$..numelems(res[1]))], connect = true) :$

> $plots[display](plot1, plot2, color = [red, blue], legend = [\text{Решение},$
$\text{Методом разностных аппроксимаций}])$
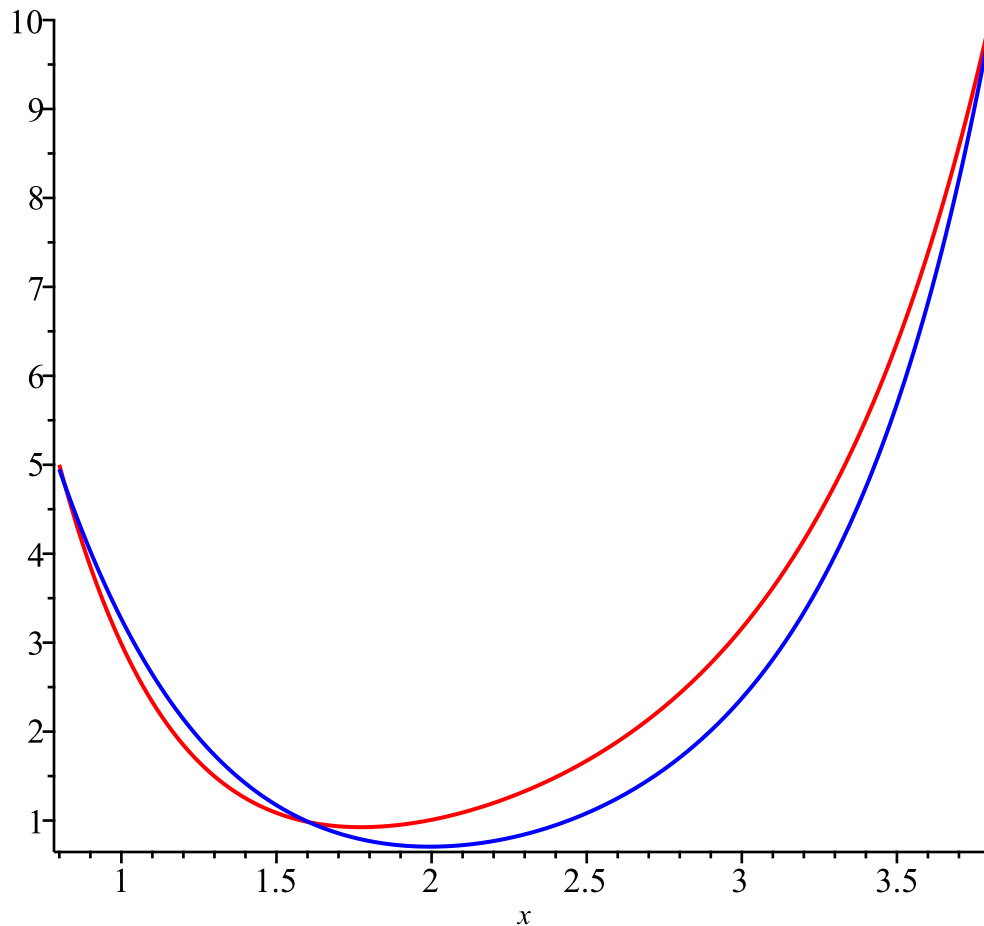
> $chebyshev\_norm(res[1], res[2], sol)$

$$0.811331070 \tag{7}$$

> $p(x) := 0 :$

> $q(x) := -8 :$

> $f(x) := 20\,x \cdot (1.5 - 0.5 \cdot x^2) :$

> $l := 2 :$

> $r := 3 :$

> $L := 2 :$

> $R := 3 :$

> $h := 0.000001 :$

> $ode := diff(y(x), x\$2) + q(x) \cdot y(x) = f(x)$

$$ode := \frac{d^2}{dx^2}\,y(x) - 8\,y(x) = 20\,x\,(1.5 - 0.5\,x^2) \tag{8}$$

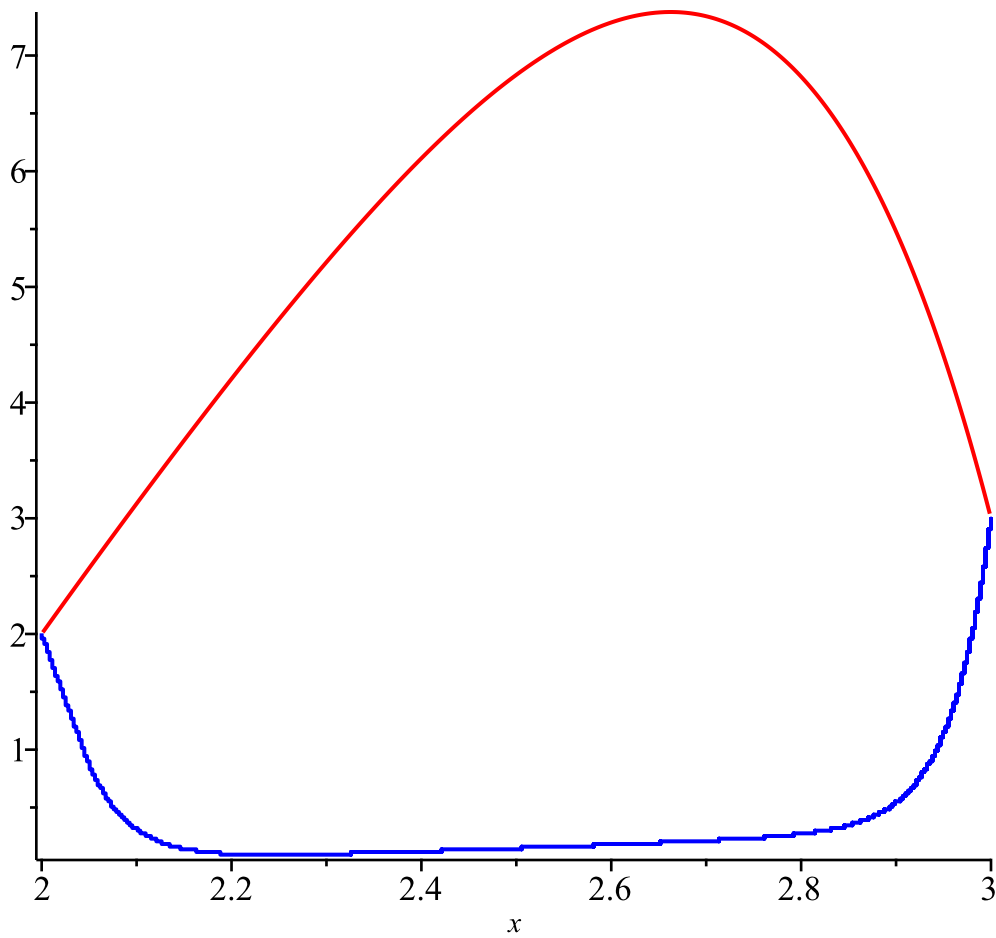> $sol := dsolve(\{y(l) = L, y(r) = R, ode\})$

$$sol := y(x) = \frac{e^{2\sqrt{2}\,x}\left(38 - 357\,e^{2\sqrt{2}}\right)e^{-4\sqrt{2}}}{16\,e^{4\sqrt{2}} - 16} - \frac{e^{-2\sqrt{2}\,x}\,e^{6\sqrt{2}}\left(38\,e^{2\sqrt{2}} - 357\right)}{16\,e^{4\sqrt{2}} - 16} + \frac{5\,x^3}{4} \tag{9}$$

$$- \frac{45\,x}{16}$$

> $plot1 := plot(rhs(sol), x = l..r) :$

> $res := difference\_method(p, q, f, h, l, r, L, R) :$

> $plot2 := plots:\text{-}pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1$
   $..numelems(res[1]))], connect = true) :$

> $plots[display](plot1, plot2, color = [red, blue], legend = [Решение,$
   $Методом\ разностных\ аппроксимаций])$

Решение — Методом разностных аппроксимаций

> $chebyshev\_norm(res[1], res[2], sol)$

$$7.178039157 \qquad \textbf{(10)}$$

>

> #5(1)

> $p(x) := sign(x):$

> $q(x) := sign(-x):$

> $f(x) := 0:$

> $l := -1:$

> $r := 1:$

> $L := -10:$

> $R := 10:$

> $h := 0.00001:$

> $ode := diff(y(x), x\$2) + p(x) \cdot diff(y(x), x) + q(x) \cdot y(x) = f(x)$

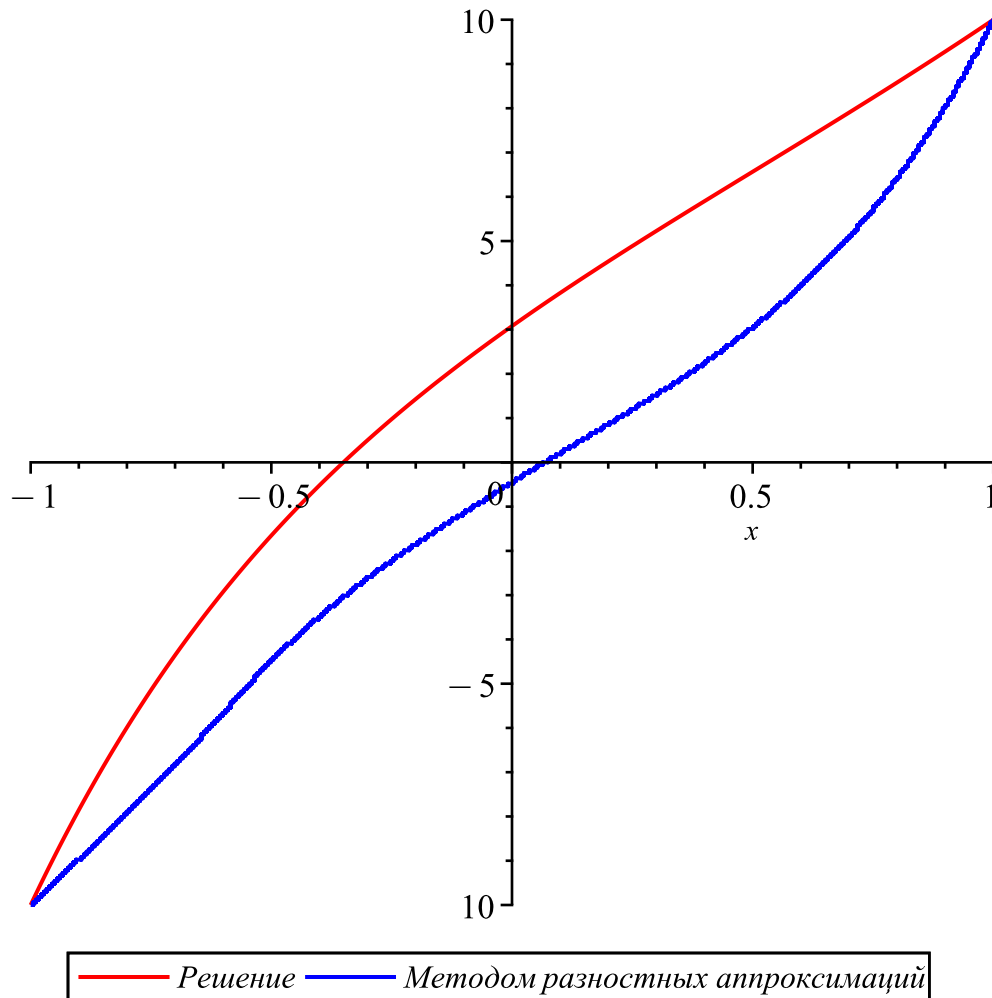$$ode := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) - y(x) = 0 \qquad \textbf{(11)}$$

> $sol := dsolve(\{y(l) = L, y(r) = R, ode\}):$

> $plot1 := plot(rhs(sol), x = l..r):$

> $res := difference\_method(p, q, f, h, l, r, L, R):$

> $plot2 := plots{:}\text{-}pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1 ..numelems(res[1]))], connect = true):$

> $plots[display](plot1, plot2, color = [red, blue], legend = [\textit{Решение}, \textit{Методом разностных аппроксимаций}])$

Legend: ▬ Решение   ▬ Методом разностных аппроксимаций

> $chebyshev\_norm(res[1], res[2], sol)$

$$3.699513425 \qquad (12)$$

> $ode := diff(y(x), x\$2) + 123456 \cdot x \cdot y(x) = 999999$

$$ode := \frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 123456\, x\, y(x) = 999999 \qquad (13)$$

> $f := dsolve(\{y(-1) = -1, y(1) = 1, ode\})$

$$f := y(x) = \left(333333\, \pi\, 1929^{2/3}\, \mathrm{AiryBi}(-4 \cdot 1929^{1/3})\, \big(\mathrm{AiryBi}(4 \cdot 1929^{1/3})\, \mathrm{AiryAi}(\right. \qquad (14)$$

$$-4 \cdot 1929^{1/3}\, x) - \mathrm{AiryAi}(4 \cdot 1929^{1/3})\, \mathrm{AiryBi}(-4 \cdot 1929^{1/3}\, x) \big) \left(\int_{-1}^{1} \mathrm{AiryAi}(\right.$$

$$-4 \cdot 1929^{1/3}\, \_z1)\, \mathrm{d}\_z1 \bigg) - 333333\, \mathrm{AiryBi}(-4 \cdot 1929^{1/3}\, x)\, \pi\, 1929^{2/3}\, \big(\mathrm{AiryAi}($$

$$-4 \cdot 1929^{1/3})\, \mathrm{AiryBi}(4 \cdot 1929^{1/3}) - \mathrm{AiryBi}(-4 \cdot 1929^{1/3})\, \mathrm{AiryAi}(4 \cdot 1929^{1/3}) \big) \Big($$

$$\int_{-1}^{x} \mathrm{AiryAi}\left(-4\ 1929^{1/3}\_z1\right)\,\mathrm{d}\_z1\Bigg) - 333333\,\pi\,1929^{2/3}\,\mathrm{AiryAi}\big($$

$$-4\ 1929^{1/3}\big)\left(\mathrm{AiryBi}\left(4\ 1929^{1/3}\right)\mathrm{AiryAi}\left(-4\ 1929^{1/3}\,x\right) - \mathrm{AiryAi}\left(4\ 1929^{1/3}\right)\mathrm{AiryBi}\big(\right.$$

$$-4\ 1929^{1/3}\,x\big)\big)\left(\int_{-1}^{1}\mathrm{AiryBi}\left(-4\ 1929^{1/3}\_z1\right)\,\mathrm{d}\_z1\right) + 333333\,\mathrm{AiryAi}\big($$

$$-4\ 1929^{1/3}\,x\big)\,\pi\,1929^{2/3}\left(\mathrm{AiryAi}\left(-4\ 1929^{1/3}\right)\mathrm{AiryBi}\left(4\ 1929^{1/3}\right) - \mathrm{AiryBi}\big(\right.$$

$$-4\ 1929^{1/3}\big)\,\mathrm{AiryAi}\left(4\ 1929^{1/3}\right)\big)\left(\int_{-1}^{x}\mathrm{AiryBi}\left(-4\ 1929^{1/3}\_z1\right)\,\mathrm{d}\_z1\right) + \left(2572\,\mathrm{AiryBi}\big(\right.$$

$$-4\ 1929^{1/3}\big) + 2572\,\mathrm{AiryBi}\left(4\ 1929^{1/3}\right)\big)\,\mathrm{AiryAi}\left(-4\ 1929^{1/3}\,x\right) - 2572\,\mathrm{AiryBi}\big($$

$$-4\ 1929^{1/3}\,x\big)\left(\mathrm{AiryAi}\left(4\ 1929^{1/3}\right) + \mathrm{AiryAi}\left(-4\ 1929^{1/3}\right)\right)\Bigg)\Bigg/\Big(2572\,\mathrm{AiryAi}\big($$

$$-4\ 1929^{1/3}\big)\,\mathrm{AiryBi}\left(4\ 1929^{1/3}\right) - 2572\,\mathrm{AiryBi}\left(-4\ 1929^{1/3}\right)\mathrm{AiryAi}\left(4\ 1929^{1/3}\right)\Big)$$

>