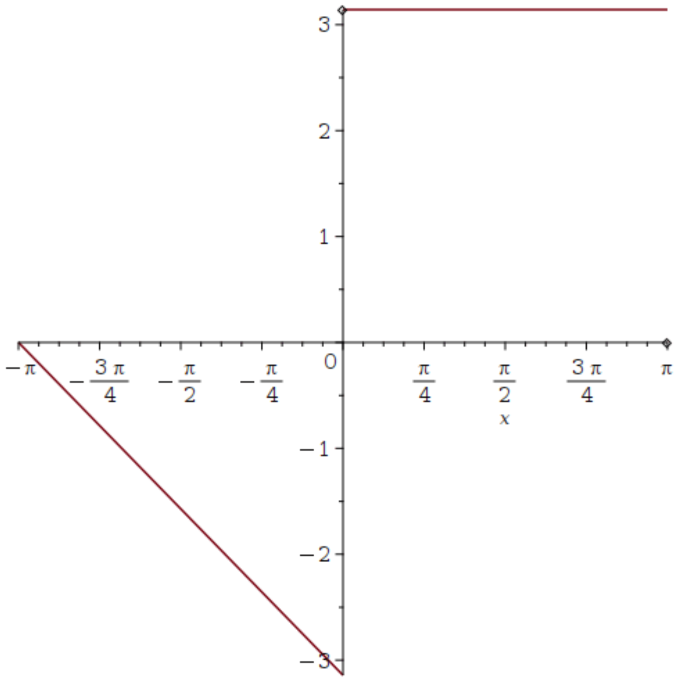
> $f := x \rightarrow piecewise(-\mathrm{Pi} \le x < 0, -Pi - x, 0 \le x < \mathrm{Pi}, \mathrm{Pi})$

$$f := x \mapsto \begin{cases} -\pi - x & -\pi \le x < 0 \\ \pi & 0 \le x < \pi \end{cases} \quad (1)$$

> $plot(f(x), x = -Pi..\mathrm{Pi}, discont = true)$



>

> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$

$$a0 := \frac{\pi}{2} \quad (2)$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint;$

$$an := \frac{(-1)^n - 1}{\pi n^2} \quad (3)$$

> $bn := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \sin(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint;$

$$bn := \frac{-(-1)^n + 2}{n} \quad (4)$$

```
> # Creating proc to get sum
```

```
> FurieSum :=proc(f, k)
  local a0, an, bn, n;
  description "return Furie Sum for -Pi .. Pi";
  a0 := simplify(int(f(x), x = - π..π) / π);
  assume(n::posint);
  an := simplify(int(f(x) * cos(n*x), x = - π..π) / π);
  bn := simplify(int(f(x) * sin(n*x), x = - π..π) / π);
  return 1/2*a0 + sum(an*cos(n*x) + bn*sin(n*x), n = 1 ..k)
  end proc:
```

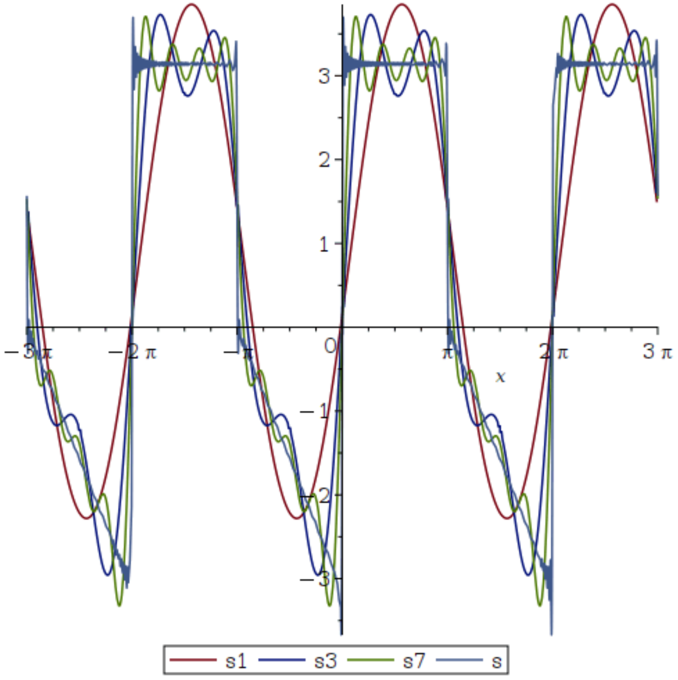
> $S1 := FurieSum(f, 1):$

> $S3 := FurieSum(f, 3):$

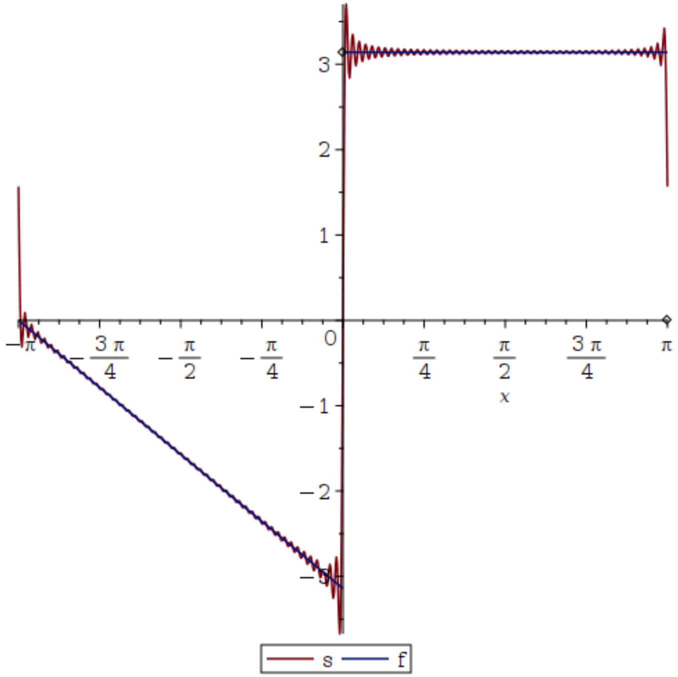
> $S7 := FurieSum(f, 7):$

> $S := FurieSum(f, 100):$

> $plot([S1, S3, S7, S], x = -3\pi..3\pi, legend = [\text{"s1"}, \text{"s3"}, \text{"s7"}, \text{"s"}], discont = true);$



> plot([S, f(x)], x = −π..π, legend = ["s", "f"], discont = true)



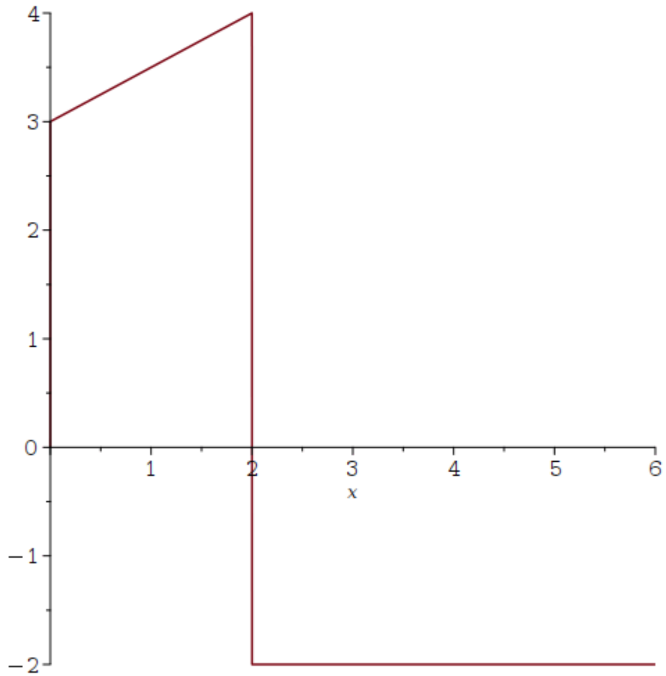
> #plots[animate](plot, [FurieSum(f, k), x = −Pi..Pi], k = 1 ..5);

> #task 2 (variant 10)

> $f := x \to piecewise(0 < x < 2, 0.5 \cdot x + 3, 2 \le x \le 6, -2);$

$$f := x \mapsto \begin{cases} 0.5 \cdot x + 3 & 0 < x < 2 \\ -2 & 2 \le x \le 6 \end{cases} \tag{5}$$

> plot(f(x), x = 0..6);



> #Coefficients Fourier sum

> $a0 := simplify\left(\frac{1}{3} \cdot int(f(x), x = 0..6, numeric = false, useunits = false)\right);$

$$a0 := -0.3333333333 \tag{6}$$

> $an := simplify\left(\frac{1}{3}\cdot int\left(f(x)\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot x}{3}\right), x = 0..6\right)\right)$ assuming $n :: posint;$

$$an := \frac{1.909859317\, n \sin(2.094395102\, n) + 0.1519817755 \cos(2.094395102\, n) - 0.1519817755}{n^2} \quad (7)$$

> $bn := simplify\left(\frac{1}{3}\cdot int\left(f(x)\cdot\sin\left(\frac{n\cdot x\cdot \mathrm{Pi}}{3}\right), x = 0..6\right)\right)$ assuming $n :: posint;$

$$bn := \frac{-1.909859317\, n \cos(2.094395102\, n) + 1.591549431\, n + 0.1519817755 \sin(2.094395102\, n)}{n^2} \quad (8)$$

> **#Fourier sum procedure**

> $Furie :=$ **proc**(f, k, l)
local $a0, an, bn;$
$a0 := \frac{1}{l}\cdot int(f(x), x = 0..2\cdot l);$
$an := simplify\left(\frac{1}{l}\cdot int\left(f(x)\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot x}{l}\right), x = 0..2\cdot l\right) \text{ assuming } n :: posint\right);$
$bn := simplify\left(\left(\frac{1}{l}\cdot int\left(f(x)\cdot\sin\left(\frac{n\cdot x\cdot \mathrm{Pi}}{l}\right), x = 0..2\cdot l\right)\right) \text{ assuming } n :: posint\right);$
return $1/2\cdot a0 + sum\left(an\cdot\cos\left(n\cdot\frac{x\cdot \mathrm{Pi}}{l}\right) + bn\cdot\sin\left(\frac{n\cdot x\cdot \mathrm{Pi}}{l}\right), n = 1..k\right);$
end proc:

> $S1 := Furie(f, 1, 3):$

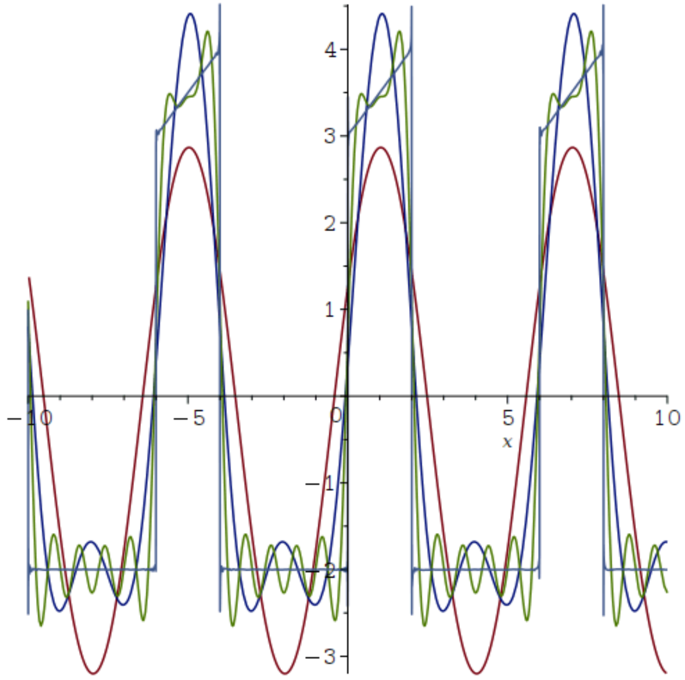
> $S3 := Furie(f, 3, 3):$

> $S5 := Furie(f, 5, 3):$

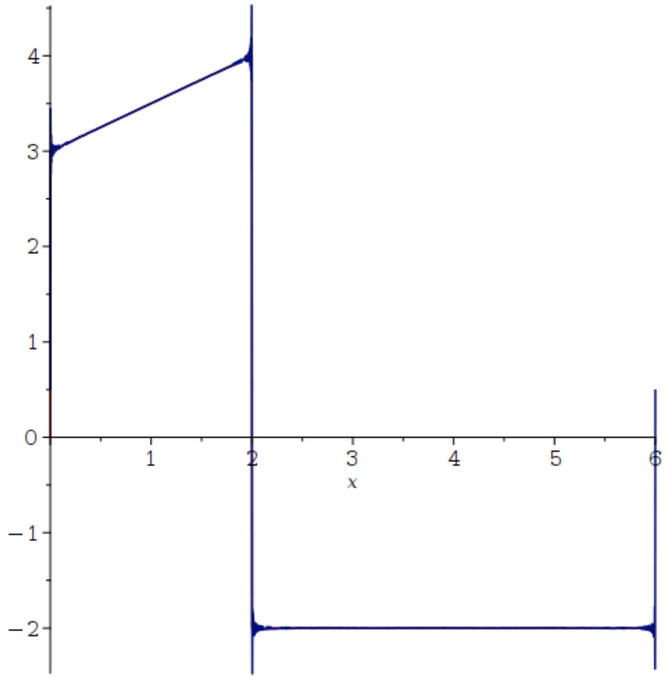
> $S7 := Furie(f, 7, 3):$

> $S := Furie(f, 1000, 3):$

> $plot([S1, S3, S7, S], x = -10..10);$



> $plot([f(x), S], x = 0..6);$



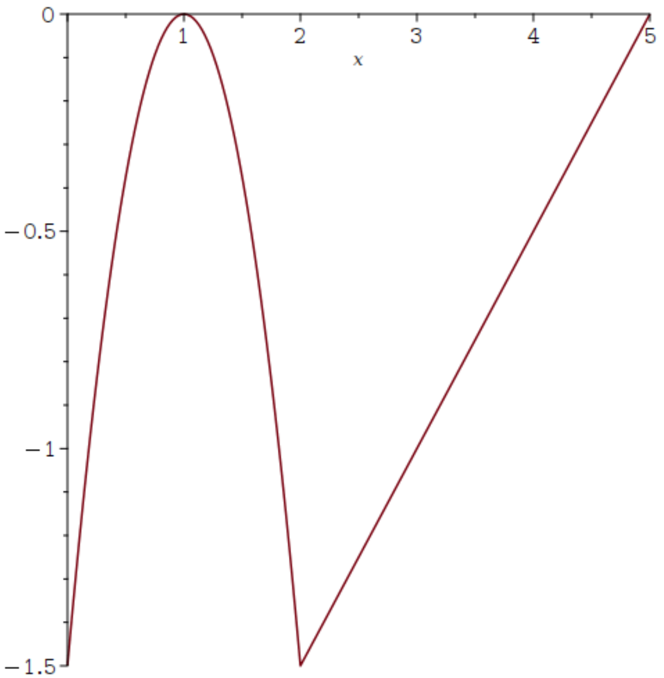
>

> # task 3 (variant 10)

> $f3 := x \rightarrow piecewise\left(x < 2, -\frac{3\cdot(x-1)^2}{2}, x \geq 2, \frac{(x-5)}{2}\right);\ l := 5$

$$f3 := x \mapsto \begin{cases} -\frac{3\cdot(x-1)^2}{2} & x < 2 \\ \frac{x}{2} - \frac{5}{2} & 2 \leq x \end{cases}$$
$$l := 5 \tag{9}$$

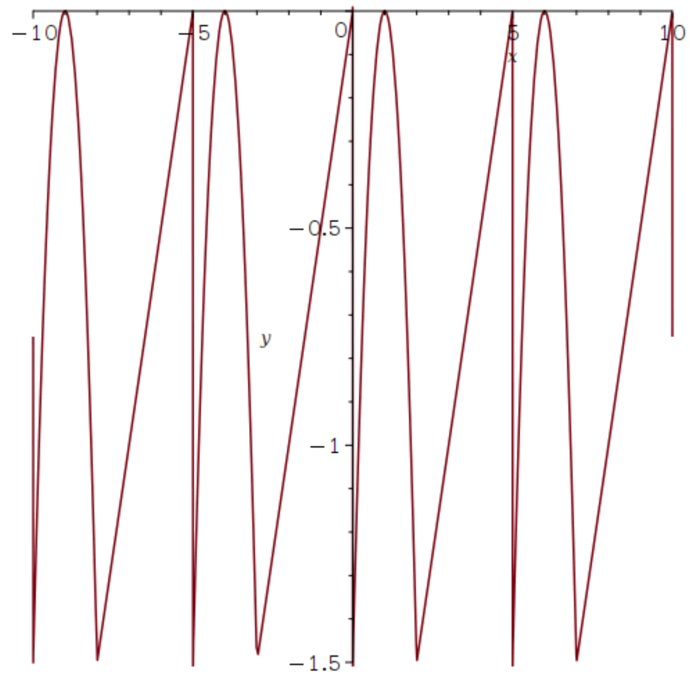
> $plot(f3(x), x = 0..5);$



>

> $S := Furie(f3, 10000, 2.5):$

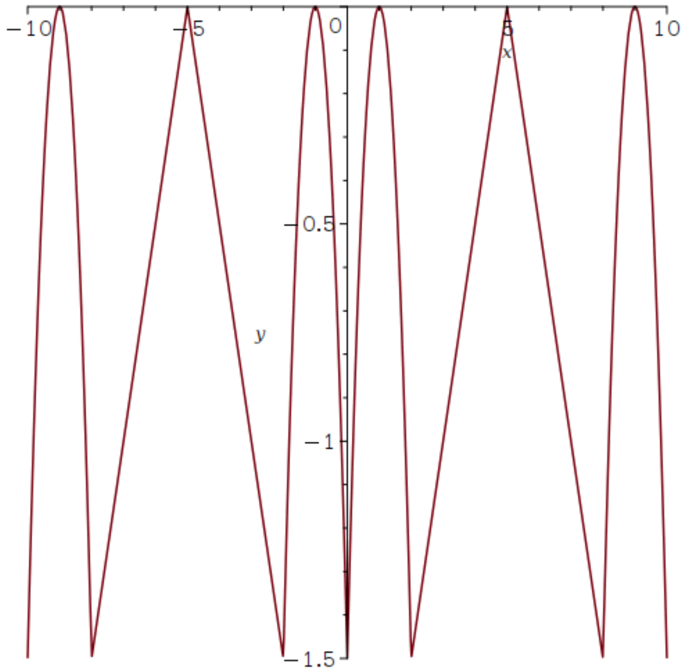
> $plot([S], x = -10..10, y = -1.5..0);$

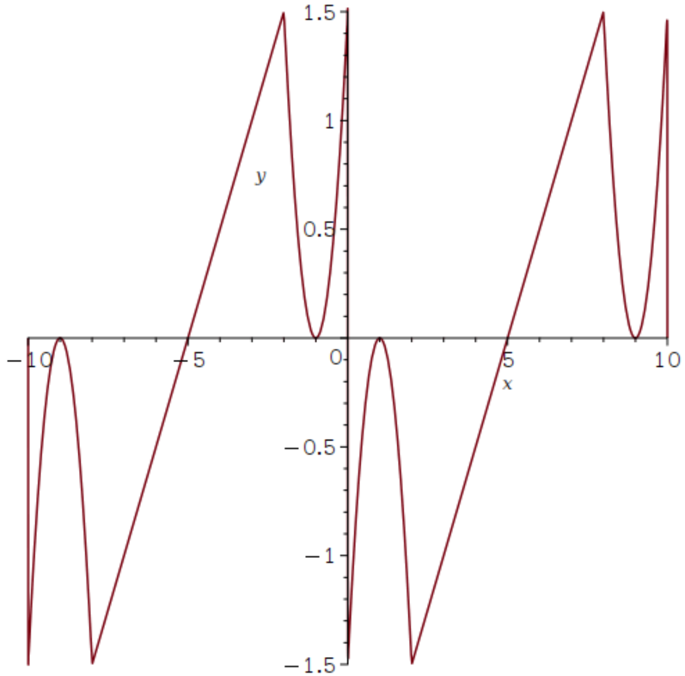


> $Furie := \textbf{proc}(f, k, l)$
local $a0, an, bn;$
$a0 := \frac{2}{l}\cdot int(f(x), x = 0..l);$
$an := simplify\left(\frac{2}{l}\cdot int\left(f(x)\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot x}{l}\right), x = 0..l\right) \text{ assuming } n :: posint\right);$
return $1/2\cdot a0 + sum\left(an\cdot\cos\left(n\cdot\frac{x\cdot \mathrm{Pi}}{l}\right), n = 1..k\right);$
end proc:

> $S := Furie(f3, 10000, 5):$

> $plot([S], x = -10..10, y = -1.5..0);$





> $Furie := \mathbf{proc}(f, k, l)$
local $a0, an, bn;$
$bn := simplify\left(\left(\frac{2}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{n \cdot x \cdot \mathrm{Pi}}{l}\right), x = 0..l\right)\right) \text{ assuming } n :: posint\right);$
return $sum\left(bn \cdot \sin\left(n \cdot \frac{x \cdot \mathrm{Pi}}{l}\right), n = 1..k\right);$
end proc:

>

>

> $S := Furie(f3, 10000, 5) :$

> $plot([S], x = -10..10, y = -1.5..1.5);$