> #

```
> running := proc(a, b, c, f, L, R)
  local i, n, x_arr, x, alpha_arr, beta_arr;
  n := numelems(a);
  alpha_arr := Array([-c[1]/b[1]]);
  beta_arr := Array([f[1]/b[1]]);
  f[1] := f[1] - a[1]·L;
  f[n] := f[n] - c[n]·R;
  a[1] := 0;
  c[n] := 0;
  for i from 2 to n do
   ArrayTools:-Append(alpha_arr, -c[i-1]/(a[i-1]·alpha_arr[i-1] + b[i-1]));
   ArrayTools:-Append(beta_arr, (f[i-1] - a[i-1]·beta_arr[i-1])/(a[i-1]·alpha_arr[i-1] + b[i-1]));
   end do;
  x_arr := Array([ ]);
  for i from 1 to n do
   ArrayTools:-Append(x_arr, cat(x, i));
  end do;
   x_arr[n] := (-a[n]·beta_arr[n] + f[n])/(a[n]·alpha_arr[n] + b[n]);
  for i from n - 1 to 1 by -1 do
   x_arr[i] := alpha_arr[i+1]·x_arr[i+1] + beta_arr[i+1];
   end do;
  return x_arr
  end proc:
```

$$alpha\_arr := Array\left(\left[-\frac{c[1]}{b[1]}\right]\right);\quad beta\_arr := Array\left(\left[\frac{f[1]}{b[1]}\right]\right);$$

>

> # y''+p(x)y'+q(x)y=f(x)

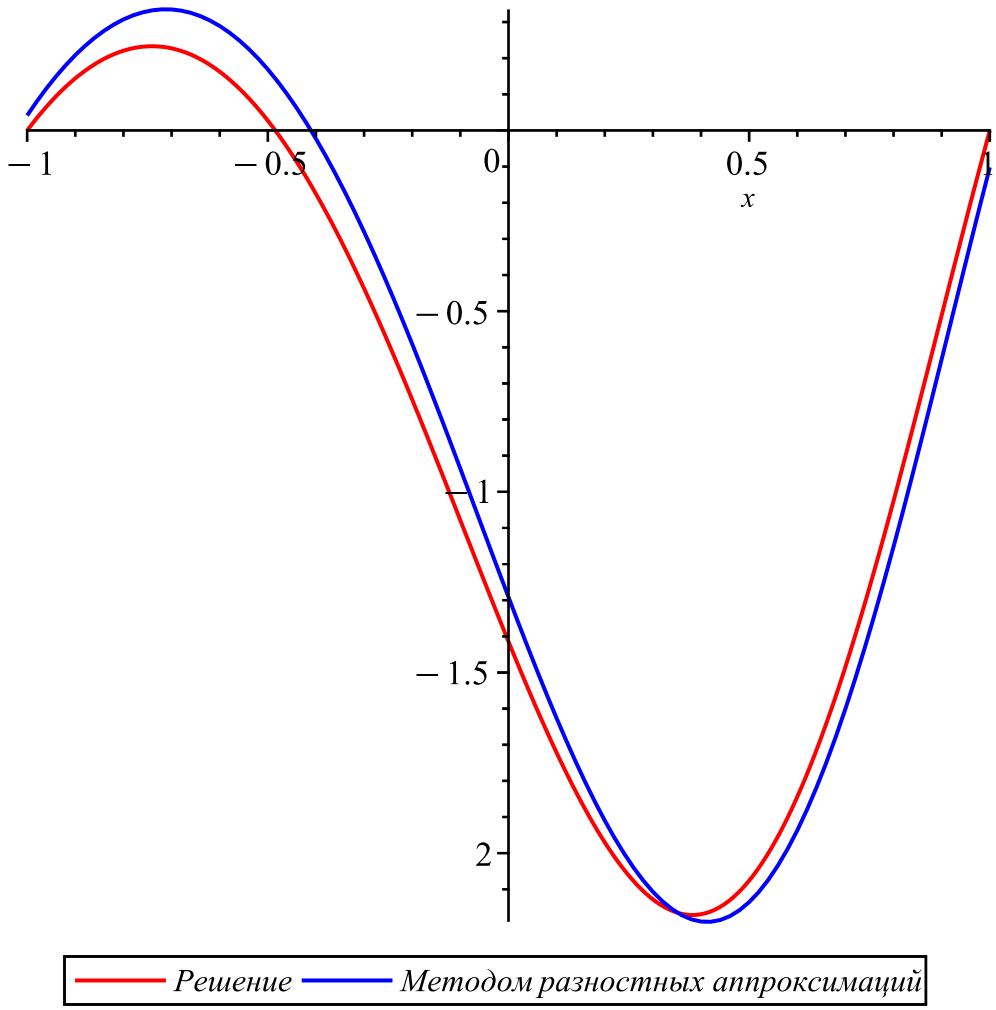
```
> difference_method := proc(p, q, f, h, l, r, L, R)
  local a, b, c, d, i, n, x_arr, xk;
   a := Array([ ]);
   b := Array([ ]);
   c := Array([ ]);
   d := Array([ ]);
   x_arr := Array([ ]);
   n := floor((r - l)/h);
   xk := l;
   for i from 1 to n + 1 do
    ArrayTools:-Append(a, 1 - p(xk)/2·h);
   ArrayTools:-Append(b, -2 + q(xk)·h^2);
   ArrayTools:-Append(c, 1 + p(xk)/2·h);
```

```
    ArrayTools:-Append(d, f(xk)·h^2);
    ArrayTools:-Append(x_arr, xk);
    xk := xk + h;
    end do;
    return [x_arr, running(a, b, c, d, L, R)]
   end proc:
>
> chebyshev_norm := proc(x_arr, y_arr, func)
    local res, i, temp;
    res := 0;
    for i from 1 to numelems(x_arr) do
     temp := abs(y_arr[i] − rhs(evalf(subs(x = x_arr[i], sol))));
     if temp > res then
     res := temp;
     end if
    end do;
    return res;
    end proc:
>
> evklid_norm := proc(x_arr, y_arr, func)
    local res, i;
    res := 0;
    for i from 1 to numelems(x_arr) do
     res := res + (y_arr[i] − rhs(evalf(subs(x = x_arr[i], sol))))^2
     end do;
    return sqrt(res)
   end proc:
>
>                                      #1
> k := 19 :
> alpha := sin(k) :
> beta := cos(k) :
> p(x) := 0 :
> q(x) := (1 + beta·x)/alpha :
> f(x) := − 1/alpha :
> ode := diff(y(x), x$2) + p(x)·diff(y(x), x) + q(x)·y(x) = f(x)
```

$$ode := \frac{d^2}{dx^2}\, y(x) + \frac{(1 + \cos(19)\, x)\, y(x)}{\sin(19)} = -\frac{1}{\sin(19)} \tag{1}$$

```
> sol := dsolve({y(−1) = 0, y(1) = 0, ode}) :
> plot1 := plot(rhs(sol), x = −1 ..1) :
> res := difference_method(p, q, f, 0.02, −1, 1, 0, 0) :
> plot2 := plots:-pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1
        ..numelems(res[1]))], connect = true) :
```

> $plots[display](plot1, plot2, color = [red, blue], legend = [Решение, Методом\ разностных\ аппроксимаций])$



> $chebyshev\_norm(res[1], evalf(res[2]), sol)$

$$0.1568299875 \quad (2)$$

> **#2**

> $p(x) := x:$

> $q(x) := x:$

> $f(x) := x^2:$

> $l := 0:$

> $r := 1:$

> $L := 1:$

> $R := 1:$

> $h := 0.1:$

> $ode := diff(y(x), x\$2) + p(x) \cdot diff(y(x), x) + q(x) \cdot y(x) = f(x)$

$$ode := \frac{d^2}{dx^2}\, y(x) + x\left(\frac{d}{dx}\, y(x)\right) + x\, y(x) = x^2 \quad (3)$$

> $sol := dsolve(\{y(l) = L, y(r) = R, ode\})$

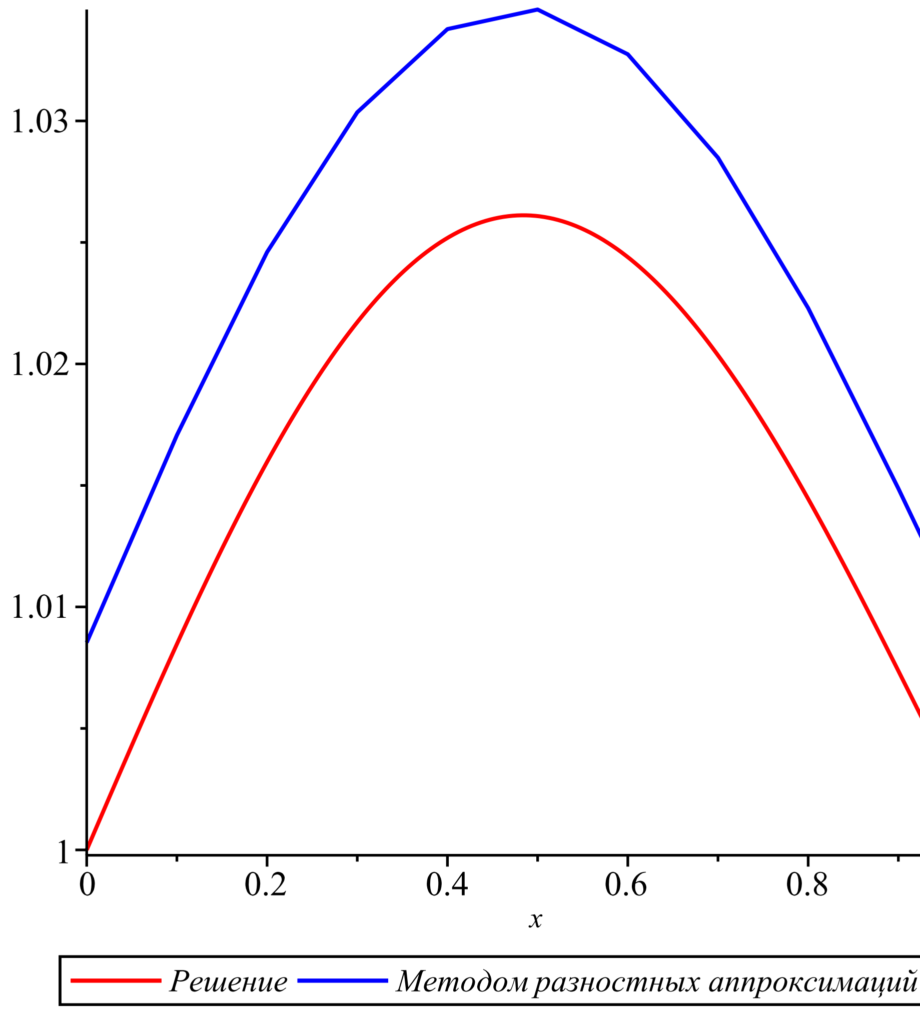
$$sol := y(x) = \frac{e^{-\frac{x(-2+x)}{2}} \left(2 e^{\frac{1}{2}} \operatorname{erfi}\left(\frac{\sqrt{2}}{2}\right) - \operatorname{erfi}(\sqrt{2})\right) e^{-\frac{1}{2}}}{\operatorname{erfi}\left(\frac{\sqrt{2}}{2}\right) - \operatorname{erfi}(\sqrt{2})} + \frac{I e^{x-\frac{1}{2}x^2} \operatorname{erf}\left(\frac{I}{2}\sqrt{2}(-2+x)\right)\left(-1+2 e^{\frac{1}{2}}\right) e^{-\frac{1}{2}}}{-\operatorname{erfi}\left(\frac{\sqrt{2}}{2}\right) + \operatorname{erfi}(\sqrt{2})} + x - 1 \quad (4)$$

> *plot1* := *plot*(*rhs*(*sol*), *x* = *l* ..*r*) :

> *res* := *difference_method*(*p*, *q*, *f*, *h*, *l*, *r*, *L*, *R*) :

> *plot2* := *plots*:-*pointplot*([*seq*([*convert*(*res*[1][*i*], *float*), *convert*(*res*[2][*i*], *float*)], *i* = 1 ..*numelems*(*res*[1]))], *connect* = *true*) :

> *plots*[*display*](*plot1*, *plot2*, *color* = [*red*, *blue*], *legend* = [*Решение*, *Методом разностных аппроксимаций*])



> *chebyshev_norm*(*res*[1], *evalf*(*res*[2]), *sol*)

0.008637004 **(5)**

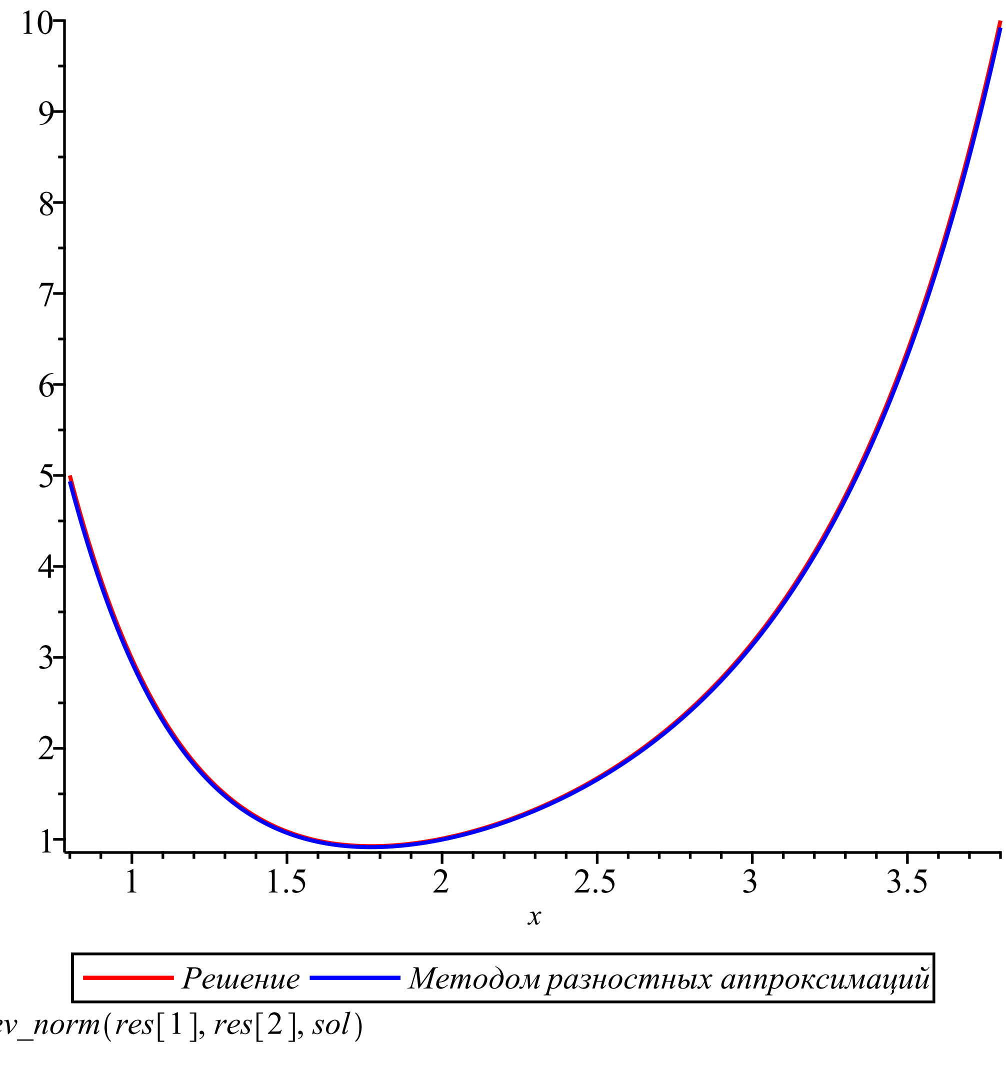
> **#3**

> $p(x) := 2$:

```
> q(x) := -1.5 x :
> f(x) := 2/x :
> l := 0.8 :
> r := 3.8 :
> L := 5 :
> R := 10 :
> h := 0.005 :
> ode := diff(y(x), x$2) + p(x)·diff(y(x), x) + q(x)·y(x) = f(x)
```

$$ode := \frac{d^2}{dx^2}\, y(x) + 2\, \frac{d}{dx}\, y(x) - 1.5\, x\, y(x) = \frac{2}{x} \qquad \textbf{(6)}$$

```
> sol := dsolve({y(l) = L, y(r) = R, ode}) :
> plot1 := plot(rhs(sol), x = l..r) :
> res := difference_method(p, q, f, h, l, r, L, R) :
> plot2 := plots:-pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1
      ..numelems(res[1]))], connect = true) :
> plots[display](plot1, plot2, color = [red, blue], legend = [Решение,
      Методом разностных аппроксимаций])
```




```
> chebyshev_norm(res[1], res[2], sol)
```

$$0.077020123 \tag{7}$$

> **#4**

> $p(x) := 0:$

> $q(x) := -8:$

> $f(x) := 20\,x\cdot(1.5 - 0.5\cdot x^2):$

> $l := 2:$

> $r := 3:$

> $L := 2:$

> $R := 3:$

> $h := 0.000001:$

> $ode := diff(y(x), x\$2) + q(x)\cdot y(x) = f(x)$

$$ode := \frac{d^2}{dx^2}\,y(x) - 8\,y(x) = 20\,x\,(1.5 - 0.5\,x^2) \tag{8}$$

> $sol := dsolve(\{y(l) = L, y(r) = R, ode\})$

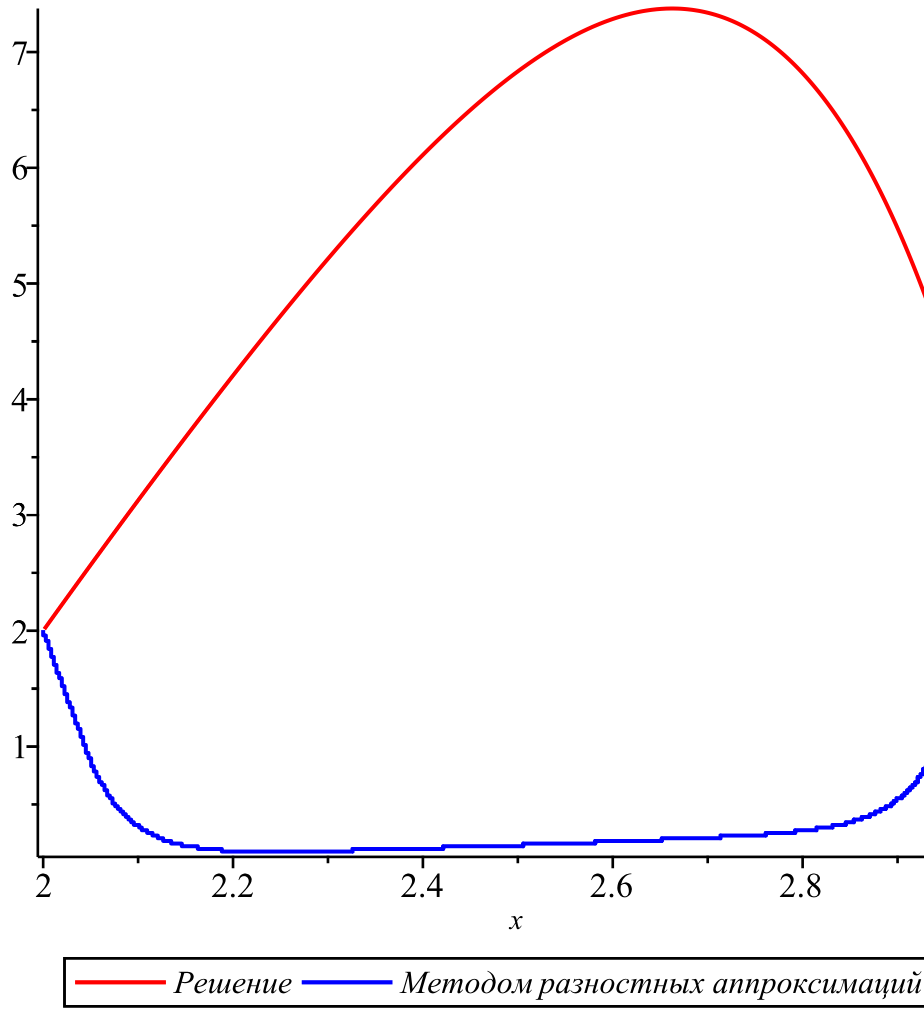
$$sol := y(x) = \frac{e^{2\sqrt{2}\,x}\left(38 - 357\,e^{2\sqrt{2}}\right)e^{-4\sqrt{2}}}{16\,e^{4\sqrt{2}} - 16} - \frac{e^{-2\sqrt{2}\,x}\,e^{6\sqrt{2}}\left(38\,e^{2\sqrt{2}} - 357\right)}{16\,e^{4\sqrt{2}} - 16} + \frac{5\,x^3}{4} - \frac{45\,x}{16} \tag{9}$$

> $plot1 := plot(rhs(sol), x = l..r):$

> $res := difference\_method(p, q, f, h, l, r, L, R):$

> $plot2 := plots\text{:-}pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1..numelems(res[1]))], connect = true):$

> $plots[display](plot1, plot2, color = [red, blue], legend = [\text{Решение}, \text{Методом разностных аппроксимаций}])$



> $chebyshev\_norm(res[1], res[2], sol)$

$$7.178039157 \tag{10}$$

>

> **#5(1)**

> $p(x) := sign(x):$

> $q(x) := sign(-x):$

> $f(x) := 0:$

> $l := -1:$

> $r := 1:$

> $L := -10:$

> $R := 10:$

> $h := 0.00001:$

> $ode := diff(y(x), x\$2) + p(x) \cdot diff(y(x), x) + q(x) \cdot y(x) = f(x)$

$$ode := \frac{d^2}{dx^2}\, y(x) + \frac{d}{dx}\, y(x) - y(x) = 0 \tag{11}$$

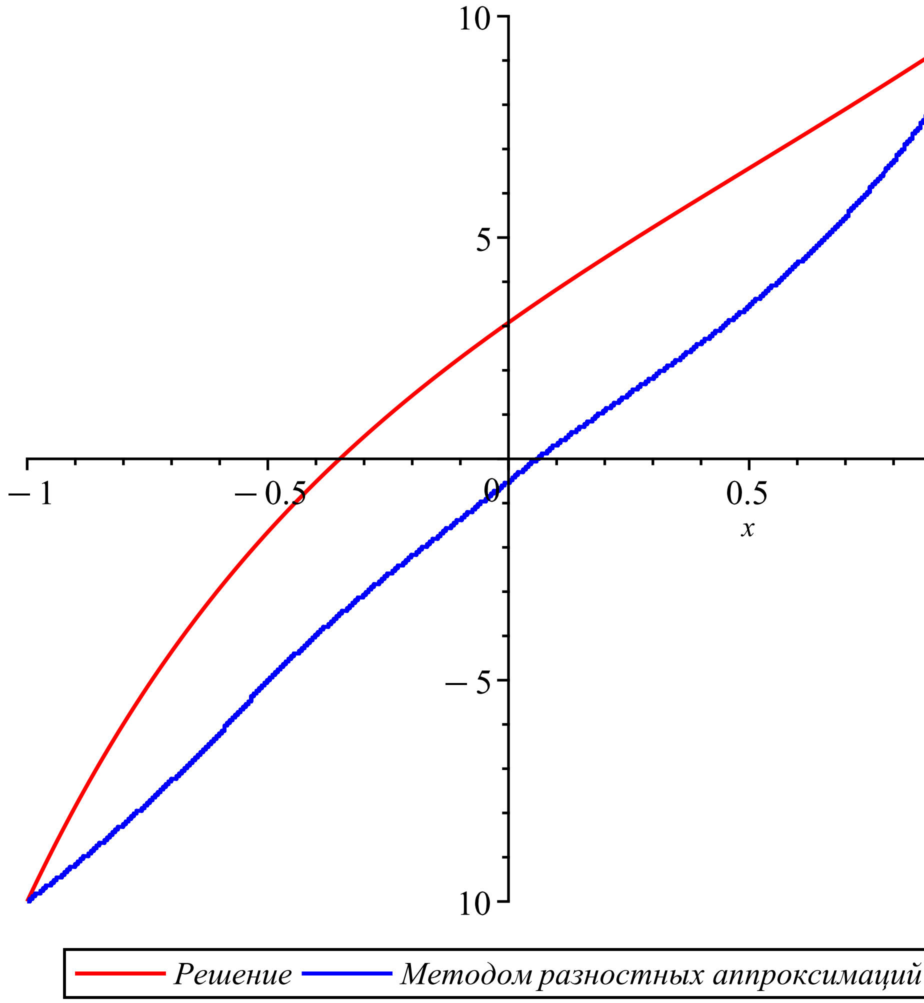
> $sol := dsolve(\{y(l) = L, y(r) = R, ode\}):$

> $plot1 := plot(rhs(sol), x = l..r):$

> $res := difference\_method(p, q, f, h, l, r, L, R):$

> $plot2 := plots\text{:-}pointplot([seq([convert(res[1][i], float), convert(res[2][i], float)], i = 1..numelems(res[1]))], connect = true):$

> $plots[display](plot1, plot2, color = [red, blue], legend = [\text{Решение}, \text{Методом разностных аппроксимаций}])$



> $chebyshev\_norm(res[1], res[2], sol)$

$$3.648654868 \tag{12}$$

> $ode := diff(y(x), x\$2) + 123456 \cdot x \cdot y(x) = 999999$

$$ode := \frac{d^2}{dx^2}\, y(x) + 123456\, x\, y(x) = 999999 \tag{13}$$

> $f := dsolve(\{y(-1) = -1, y(1) = 1, ode\})$

$$f := y(x) = \Bigg(333333\,\pi\, 1929^{2/3}\, \mathrm{AiryBi}(-4\, 1929^{1/3})\, \big(\mathrm{AiryBi}(4\, 1929^{1/3})\, \mathrm{AiryAi}(-4\, 1929^{1/3} x) - \mathrm{AiryAi}(4\, 1929^{1/3})\, \mathrm{AiryBi}(-4\, 1929^{1/3} x)\big) \left(\int_{-1}^{1} \mathrm{AiryAi}(-4\, 1929^{1/3}\, \_z1)\, d\_z1\right) - 333333\, \mathrm{AiryBi}(-4\, 1929^{1/3} x)\, \pi\, 1929^{2/3}\, \big(\mathrm{AiryAi}(-4\, 1929^{1/3})\, \mathrm{AiryBi}(4\, 1929^{1/3}) - \mathrm{AiryBi}(-4\, 1929^{1/3})\, \mathrm{AiryAi}(4\, 1929^{1/3})\big) \Big(\tag{14}$$

$$\int_{-1}^{x} \mathrm{AiryAi}\left(-4\ 1929^{1/3}\ \_zI\right) \mathrm{d}\_zI\Bigg) - 333333\ \pi\ 1929^{2/3}\ \mathrm{AiryAi}\Big($$

$$-4\ 1929^{1/3}\Big) \Big(\mathrm{AiryBi}\big(4\ 1929^{1/3}\big)\ \mathrm{AiryAi}\big(-4\ 1929^{1/3}\ x\big) - \mathrm{AiryAi}\big(4\ 1929^{1/3}\big)\ \mathrm{AiryBi}\Big($$

$$-4\ 1929^{1/3}\ x\Big)\Big) \left(\int_{-1}^{1} \mathrm{AiryBi}\left(-4\ 1929^{1/3}\ \_zI\right) \mathrm{d}\_zI\right) + 333333\ \mathrm{AiryAi}\Big($$

$$-4\ 1929^{1/3}\ x\Big)\ \pi\ 1929^{2/3}\ \Big(\mathrm{AiryAi}\big(-4\ 1929^{1/3}\big)\ \mathrm{AiryBi}\big(4\ 1929^{1/3}\big) - \mathrm{AiryBi}\Big($$

$$-4\ 1929^{1/3}\Big)\ \mathrm{AiryAi}\big(4\ 1929^{1/3}\big)\Big) \left(\int_{-1}^{x} \mathrm{AiryBi}\left(-4\ 1929^{1/3}\ \_zI\right) \mathrm{d}\_zI\right) + \Big(2572\ \mathrm{AiryBi}\Big($$

$$-4\ 1929^{1/3}\Big) + 2572\ \mathrm{AiryBi}\big(4\ 1929^{1/3}\big)\Big)\ \mathrm{AiryAi}\big(-4\ 1929^{1/3}\ x\big) - 2572\ \mathrm{AiryBi}\Big($$

$$-4\ 1929^{1/3}\ x\Big)\ \Big(\mathrm{AiryAi}\big(4\ 1929^{1/3}\big) + \mathrm{AiryAi}\big(-4\ 1929^{1/3}\big)\Big)\Big) \Big/ \Big(2572\ \mathrm{AiryAi}\Big($$

$$-4\ 1929^{1/3}\Big)\ \mathrm{AiryBi}\big(4\ 1929^{1/3}\big) - 2572\ \mathrm{AiryBi}\big(-4\ 1929^{1/3}\big)\ \mathrm{AiryAi}\big(4\ 1929^{1/3}\big)\Big)$$

>