Course Name: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Course Outcome

CO 1: Know the rank of a matrix and its applications in solving systems of linear equations
CO 2: Find the Eigen values and Eigen vectors of a square matrix
CO 3: Solve ordinary and partial differential equations of higher orders
CO 4: Classify the linear partial differential equations as elliptic, parabolic, and hyperbolic
CO 5: Expand a function in half range Fourier sine and cosine series
CO 6: Apply the method of separation of variables to solve wave and heat flow equations of one dimension

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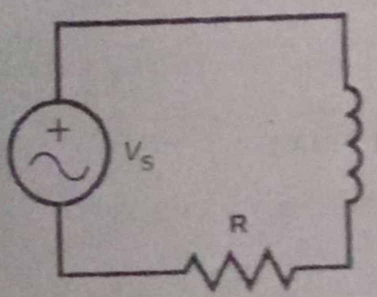
End Term Examination, Even Semester 2021-22
B. Tech. (H) CS and EC (VLSI), I Year, II Semester
BMAS 0105 LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

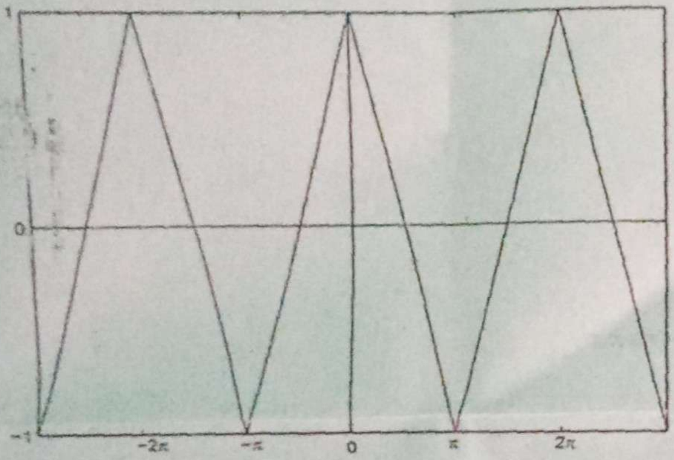
Time: 3 Hours **Maximum Marks: 45**

Instructions. (1) Attempt all the sections.
(2) Marks of the questions and internal choice are indicated in each section.

Section – A

Attempt All Questions. $4 \times 5 = 20$ Marks

No.	Detail of Question	Marks	CO	BL	KL
1	Solve the following system of linear equations: $x-2y+3z=6; 3x-y=1, 4y-2z=2$	4	1	E	P
2	Find the inverse of the following matrix using Cayley-Hamilton's theorem for square matrix: $A=\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix}.$ **OR** An electric circuit (as shown in Figure 1) has in series an electromotive force given by, $V_s = 50\sin 20t\, V$, a resistor of 5Ω, and an inductor of $0.4H$. If the initial current is 0, find the current at time $t > 0$.  Figure 1	4	2/3	A/C	C/M

3	Solve the partial differential equation: $$\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y$$	4	3	E	P
4	The power output of a DC to AC converter/ transformer is noted as triangle wave function depicted in figure 2 and represented as $$f(x)=1-\frac{2x}{\pi};\ 0<x<\pi$$ If the output is represented as half range cosine-series as $a_o+\sum_n a_n \cos nx,$ show that the DC part of the output (a_o) is zero. Also create the expression for a_n. (See Fig. 2)  **Figure 2.**	4	5	C	M
5	Use separation of variables method to find a model solution of the partial differential equation: $$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2},$$ assuming that u has periodic nature with respect to both the space variables, the time t and distance x.	4	6	U	F

Section – B

Attempt All Questions $3 \times 5 = 15$ Marks

No.	Detail of Question	Marks	CO	BL	KL
1	With the help of Fourier Series for the function given in question 4 of Section A, show that $$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$$	3	5	An	C

2	Find the Fourier series of the piecewise smooth function $f(x)=\begin{cases}-x, & -2<x<0,\\ \frac{1}{2}, & -0<x<2\end{cases}$	3	5	A	P
3	Classify the following partial differential equations in the region indicated regions: [a] **Region :** $(1,\infty)\times\mathcal{R}$ $(x^2+2x+1)\frac{\partial^2 z}{\partial x^2}+(x^2-2x-3)\frac{\partial^2 z}{\partial x\partial y}-(x-2)\frac{\partial^2 z}{\partial y^2}+(9+12x-2x^2y-4x^3+y^2x^4)\frac{\partial z}{\partial x}=x^2y-2xy^2+6$ [b] **Region :** $\mathcal{R}\times\left(\sqrt{\frac{7}{6}},\infty\right)$ $(2-xy)\frac{\partial^2 z}{\partial x^2}+(x^2-2xy-3)\frac{\partial^2 z}{\partial x\partial y}+(x^2+2xy+1)\frac{\partial^2 z}{\partial y^2}=0.$ *Note*: $\mathcal{R}$ denote real line	3	4	R	F
4	Solve the non-homogenous linear partial differential equation: $s+ap+bq+abz=e^{mx+ny}$ where the terms have their usual meanings and $m\neq -b, n\neq -a$.	3	3	U	C
5	Find an explicit solution of the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2},$ under the following boundary conditions: $u(0,t)=u(L,t)=0,$ $u(x,0)=0$ for $0<x<L,$ $u_t(x,0)=x$ for $0<x<L,$	3	6	E	P

Section – C

Attempt All Questions. $5 \times 2 = 10$ Marks

S.No	Detail of Question	Marks	CO	BL	KL
1	Find all the Eigen value and Eigen Vectors of the matrix: $A = \begin{pmatrix} 7 & 1 & 3 \\ -3 & 2 & -3 \\ -3 & -2 & -1 \end{pmatrix}$,	5	2	U	C
2	Solve the following problem of one-dimensional heat flow: $\frac{\partial u}{\partial t} = 0.003 \frac{\partial^2 u}{\partial x^2}$ $u(0,t) = u(1,t) = 0$ $u(x,0) = 50x(1-x)$ for $0 < x < 1$. OR, Solve the one dimensional heat flow equation, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with the following conditions: $u(0,t) = 0, u(L,t) = 0$ and, $u(x,0) = f(x)$ for $0 < x < L$.	5	6	An	P

CO – Course Outcome, BL – Abbreviation for Bloom's Taxonomy Level (R-Remember, U-Understand, A-Apply, An-Analyze, E-Evaluate, C-Create), KL – Abbreviation for Knowledge Level (F-Factual, C-Conceptual, P-Procedural, M-Metacognitive)