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University Roll No..2115800023

Mid-Term Examination, Odd Semester 2021-22

B. Tech. (Hons.) CS / B. Tech. VLSI, I-Year, I-Semester

BMAS 0104 : Engineering Calculus

Time: 2 Hours

Maximum Marks: 15

Section- A

Note: Attempt All Three Questions.

3 x 1 = 3 Marks

Q. 1 If we change the coordinate system in a triple integral from Cartesian coordinate to Spherical polar coordinate system, we use the relation $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$. In this case small volume $dx\,dy\,dz=|J|\,dr\,d\theta\,d\phi$ where $J=\dfrac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$.

Calculate J.

Q. 2 Find all the 1st order partial derivatives of the following function.

$$f(x,z)=e^{-x}\sqrt{z^4+x^2}-\frac{2x+3z}{4z-7x}$$

Q. 3 Find the n[th] derivative of sin6x cos4x.

Section- B

Note: Attempt All Three Questions. $3 \times 2 = 6$ Marks

Q. 1 Use the composite function for $u = f(2x-3y, 3y-4z, 4z-2x)$,

compute the value $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$.

Q. 2 If u, v, w are roots of the cubic polynomial

$(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ in x, find the Jacobian $\frac{\partial(u, v, w)}{\partial(a, b, c)}$.

Q. 3 By using Lagrange's method, divide 24 into three positive numbers such that continued product of the first, square of second and the cube of the third may be maximum.

Section – C

Note: Attempt Any Two Questions. $2 \times 3 = 6$ Marks

Q.1 A rectangular box, which is open at the top, has a capacity of 256 cubic feet. Determine the dimensions of the box such that the least material is required for the construction. Use the Lagrange method of multipliers to obtain the solution.

Q. 2 Apply the Euler's theorem and its deductions, show that for

$$u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right),$$

(i) $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

(ii) $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u.\cos 2u}{4\cos^3 u}$$

Q. 3 Apply the Leibnitz's theorem for the function $y = \left(\sin^{-1} x\right)^2$, show that its n[th] derivative at $x = 0$ is

$$y_n(0) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 2.\ 2^2.4^2.\ 6^2.....(n-2)^2, & \text{if } n \text{ is even} \end{cases}$$