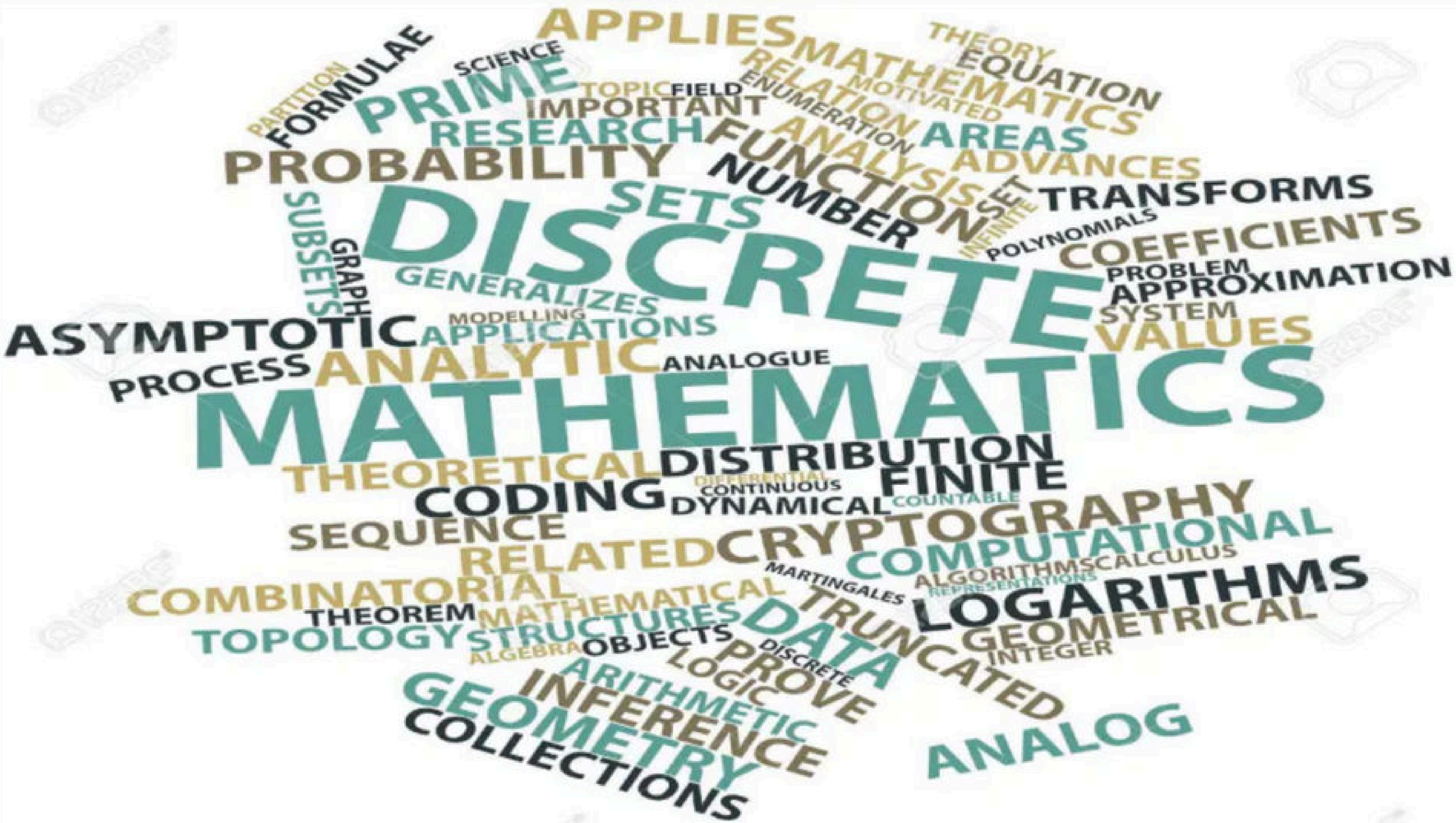
Set Theory - Part III

Course on Discrete Mathematics for GATE 2023

Sanchit Jain • Lesson 3 • Sept 3, 2022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ARITHMETIC
INFERENCE
GEOMETRY
COLLECTIONS
ANALOG

- Discrete mathematics is a core subject of theoretical computer science. It is not a directly application-oriented subject, but it provides tolls and mathematical models, which are applied to different areas in computer science.

- **GATE (7-9 MARKS)**

- Has a good weightage in general all objective and subjective examination.

- Will be asked in Interview for M.Tech, PhD or other government jobs. Not that important in software industry.

Syllabus (GATE)

- SET THEORY (1-2)
- RELATIONS (1-2)
- FUNCTIONS (1-2)
- GROUP THEORY (1-2)
- PROPOSITIONS & FIRST ORDER LOGIC (2-3)
- GRAPH THEORY (2-3)

Section 1: Engineering Mathematics

Discrete Mathematics: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

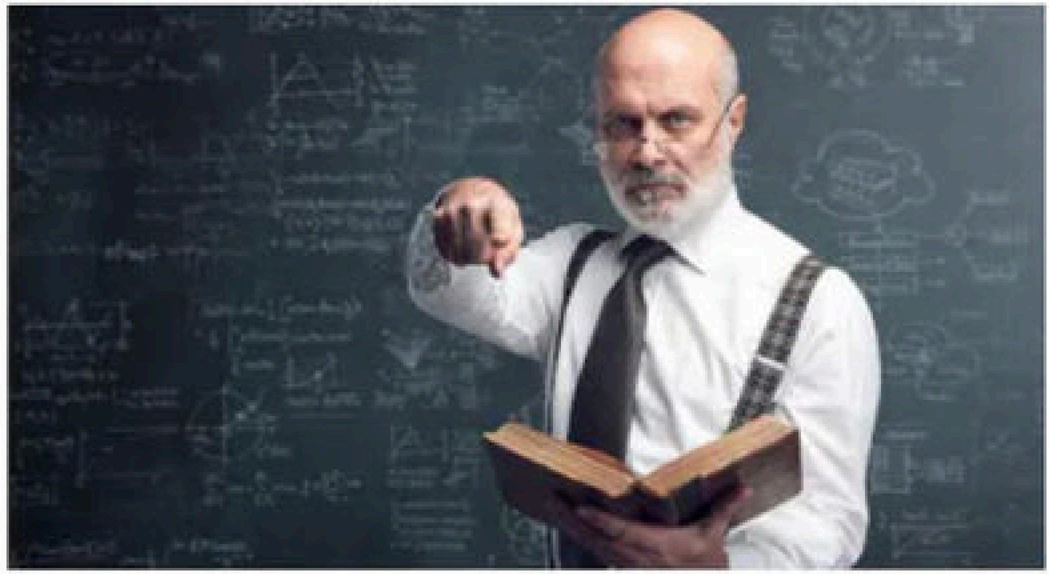
Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

What you can expect from me

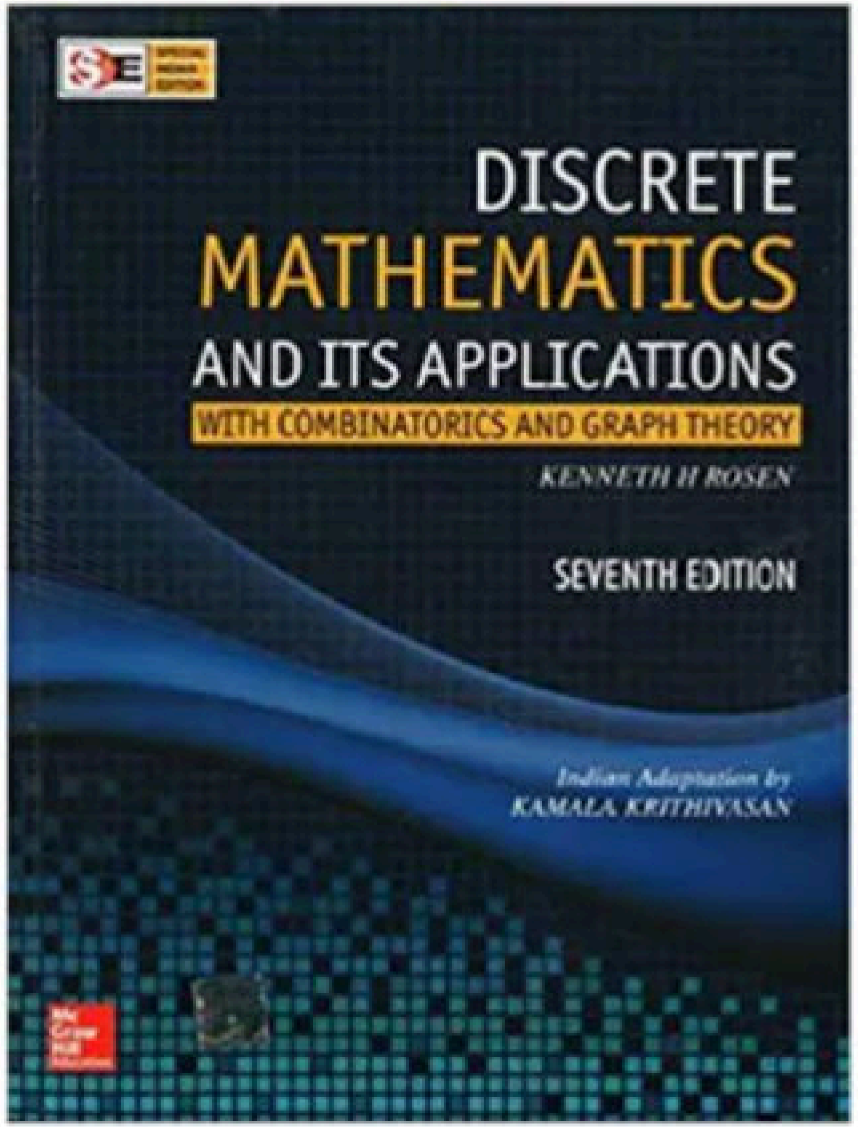
- Will take care of theory and numerical both
- Will give more weightage to the topics that are asked more frequently in GATE
- Will not emphasize more than required, on a topic
- Will provide PDF of related books and my PPT

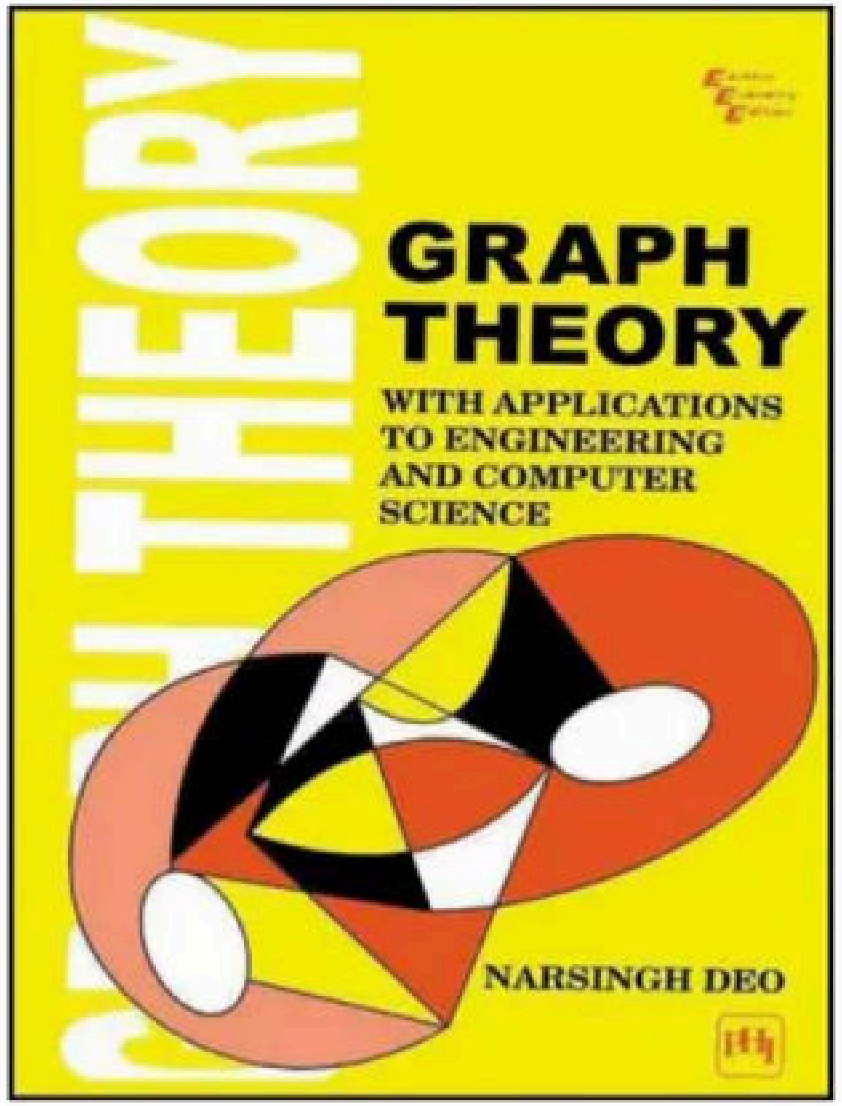


What i expect from you

- There is no hurry, feel free to ask questions any time through out the class, but first listen
- Please revise the entire lecture before and after the class
- Be regular, Consistency is most important
- More you practice, more clarity you will get
- If we follow all the above specified points Success in guaranteed

Books to be referred





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What is a SET

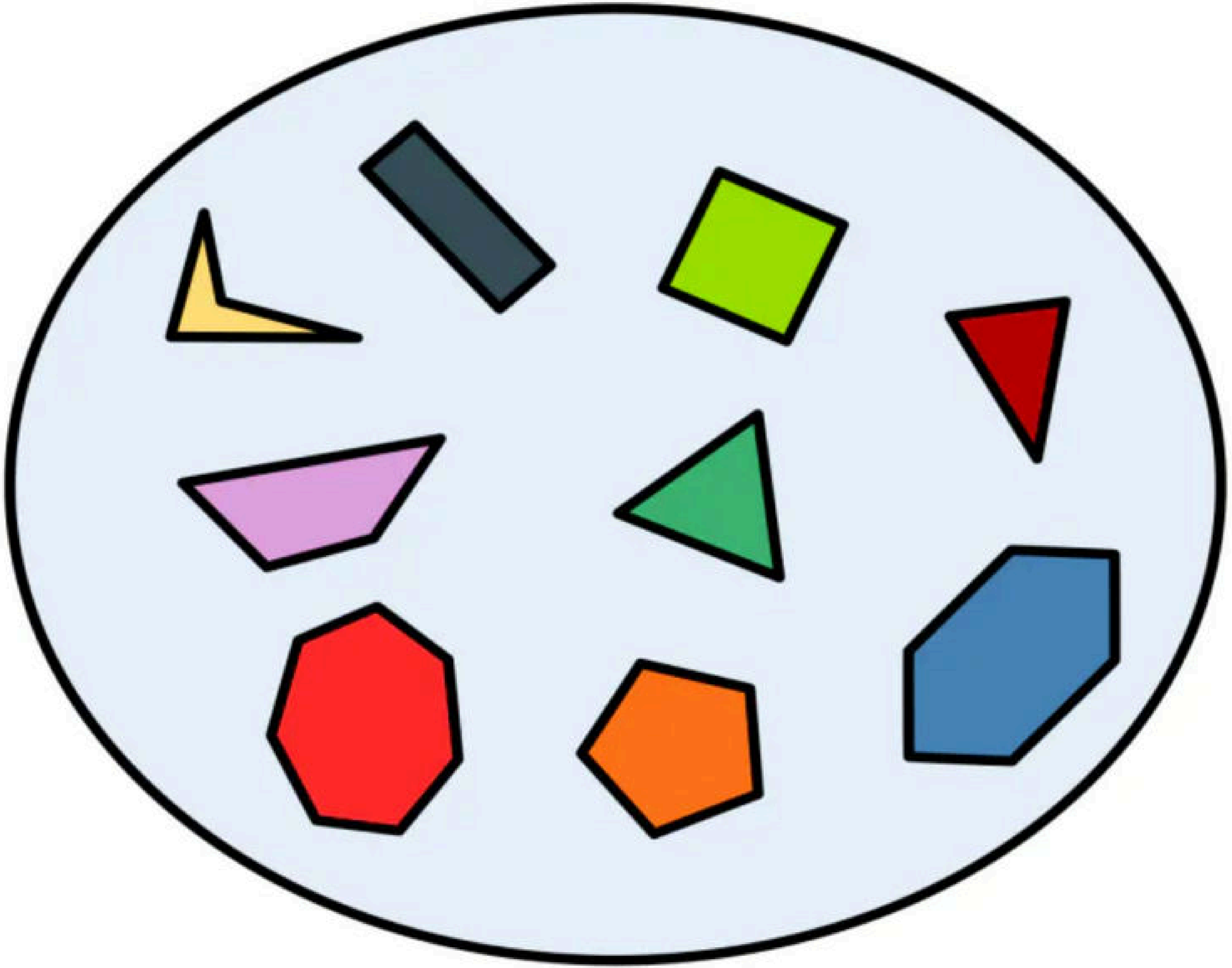
- Set are the fundamental discrete structures on which all the discrete structures are built. Sets are used to group objects together, formally speaking

- “An *unordered ,well-defined*, collection of *distinct* objects (Called elements or members of a set) of same type”. Here the type is defined by the one who is defining the set. For e.g.

- A = {0,2,4,6, ---}

- B = {1,3,5, ---}

- C= {x| x ∈ Natural number}



- A Set is generally denoted usually by capital letter. The objects of a set called the elements, or members of the set.

- A set is said to contain its elements.

- Lower case letters are generally used to denote the elements of the set.

- $x \in A$, means element x is a member of A

- $x \notin A$ means x is not a member of A

- **Cardinality of a set** — It is the number of elements present in a Set, denoted like $|A|$.

- For e.g. $A = \{0,2,4,6\}$, $|A| = 4$

Break

Representation of set

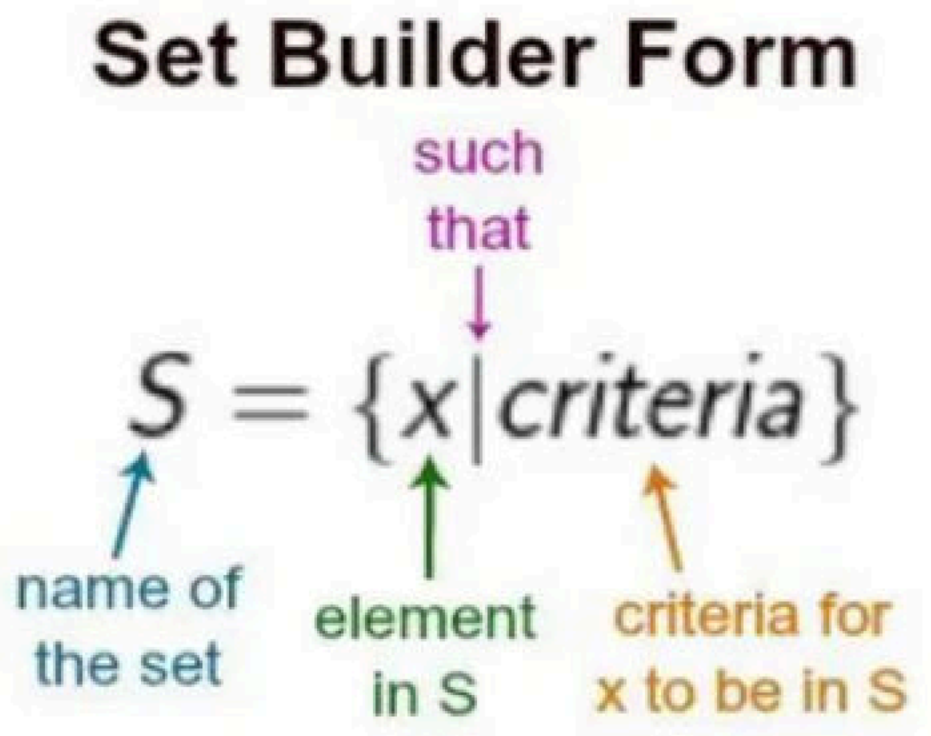
- **Tabular/Roster representation of set** - here a set is defined by actually listing its members. E.g.

- A = {a, e, i, o, u}

- B = {1, 2, 3, 4}

- C = {---- -4, -2, 0, 2, 4, -----}.

- **<u>Set Builder representations of set</u>**- here we specify the property which the elements of the set must satisfy. E.g.



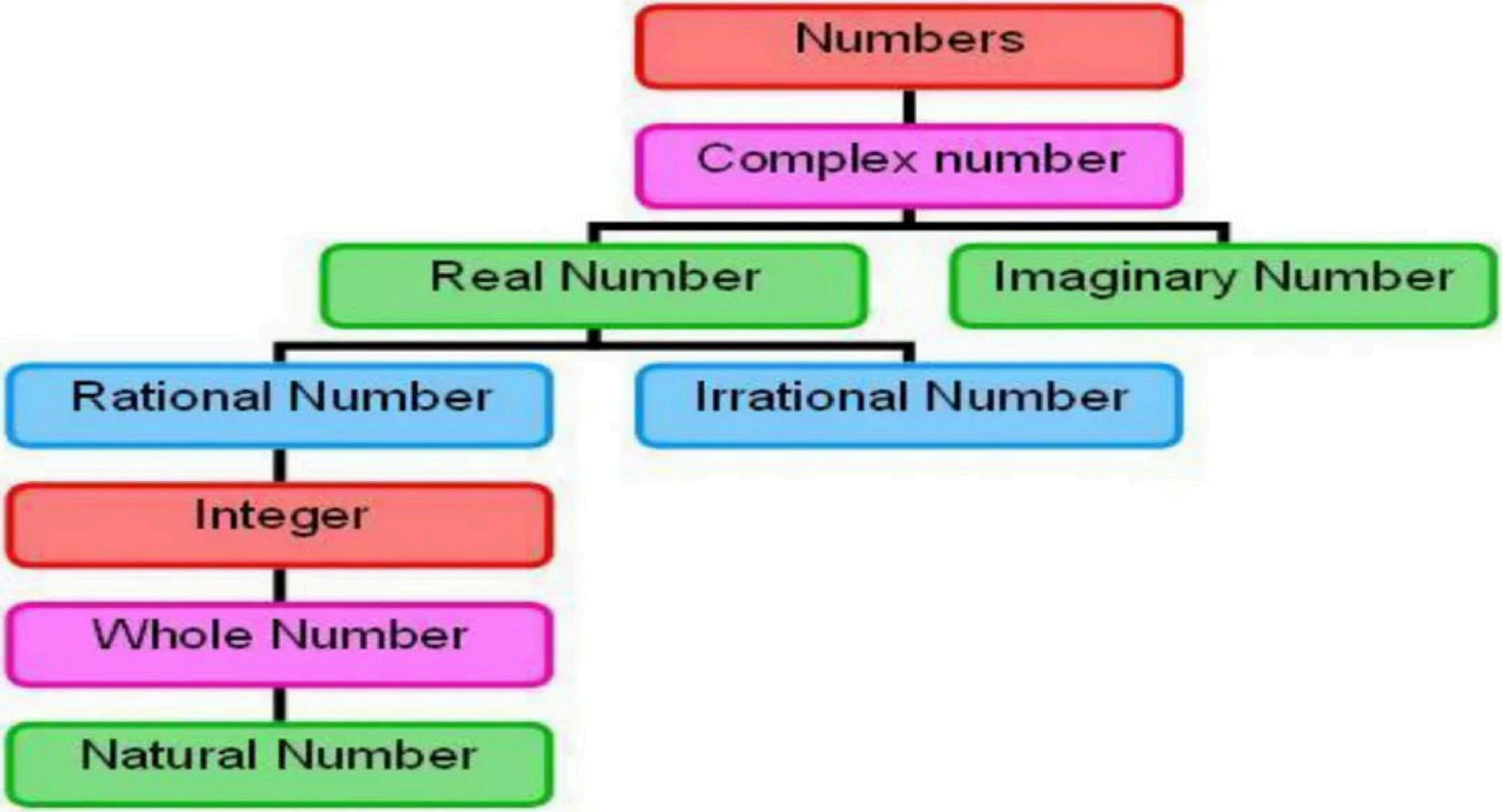
- A = {x| x is an odd positive number less than 10},

- A = {x | x ϵ English alphabet && x is vowel}

- B = {x | x ϵ N && x <5}

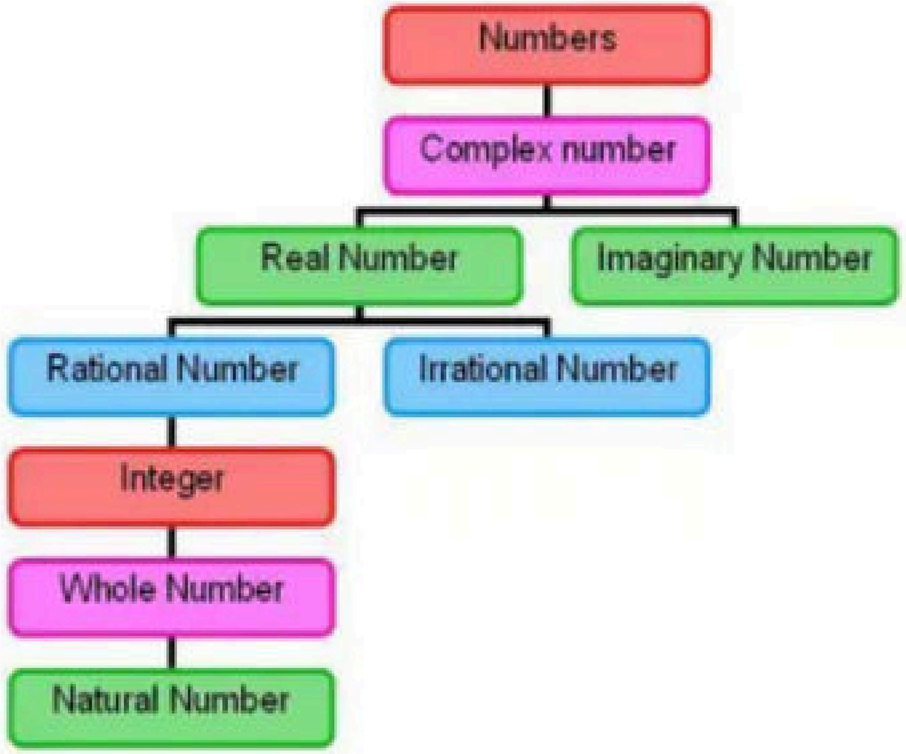
- C = {x | x ϵ Z && x%2 = 0 }

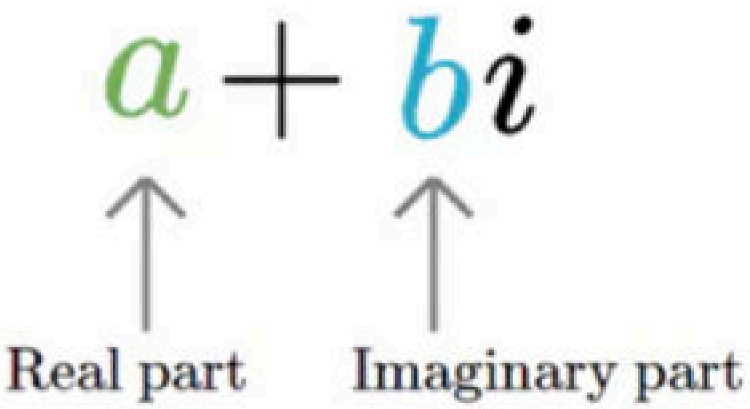
Break



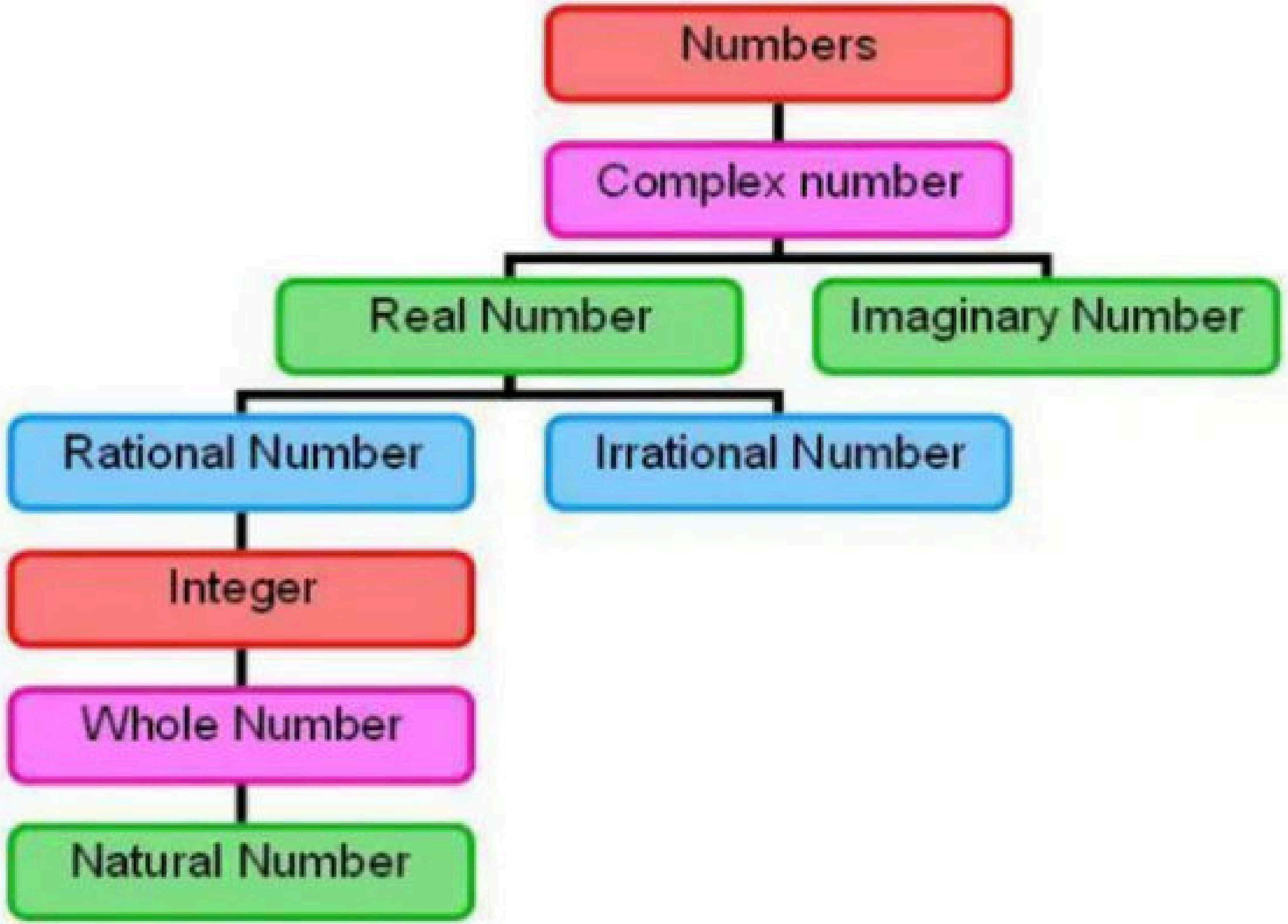
Numbers
Complex number
Real Number
Imaginary Number
Rational Number
Irrational Number
Integer
Whole Number
Natural Number

- **<u>Set of all Complex number(C)</u>** - A complex number is a number that can be expressed in the form 'a + bi', where 'a' and 'b' are real numbers and 'i' is the imaginary unit, that satisfies the equation $i^2 = -1$. In this expression, 'a' is the real part and 'b' is the imaginary part of the complex number.

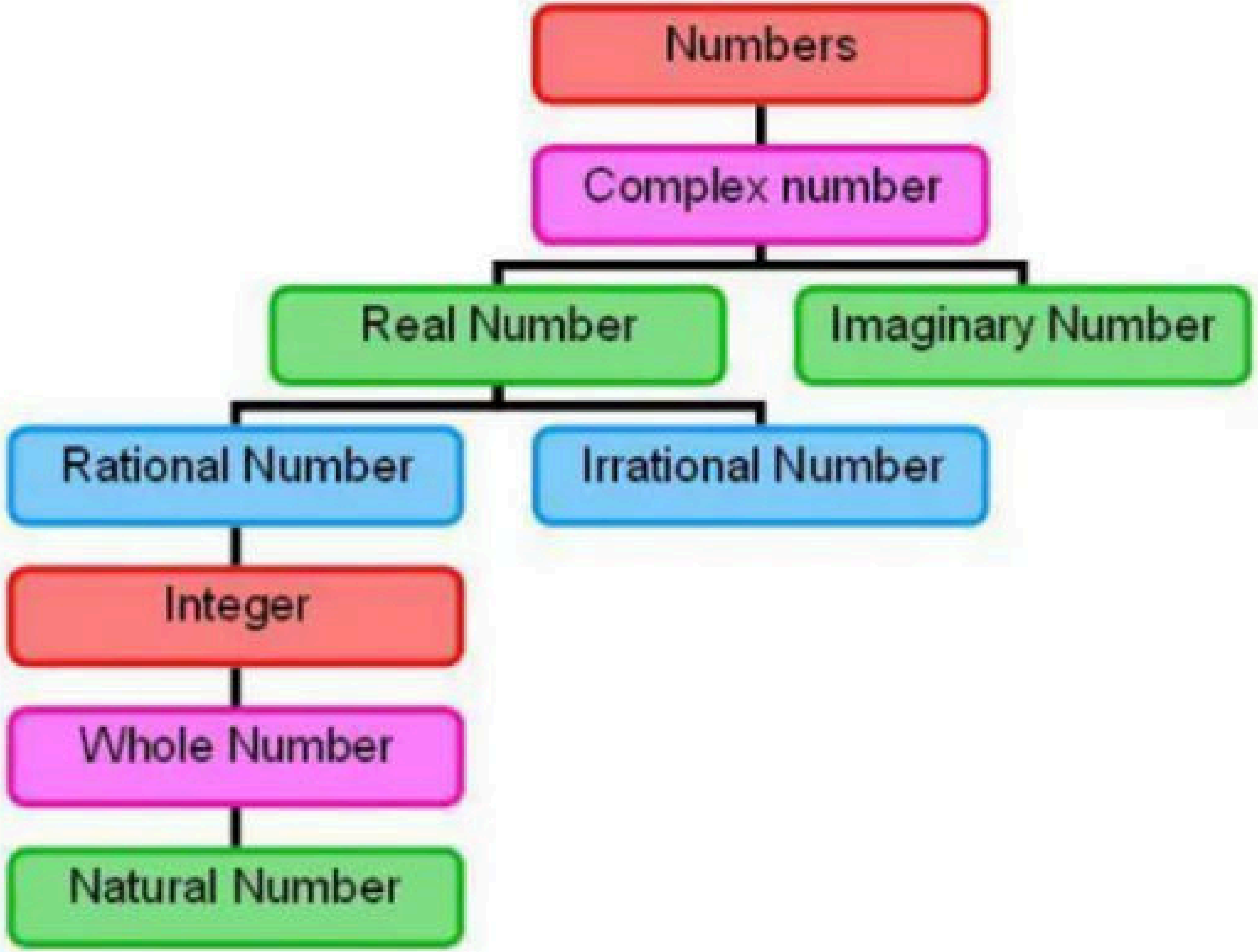




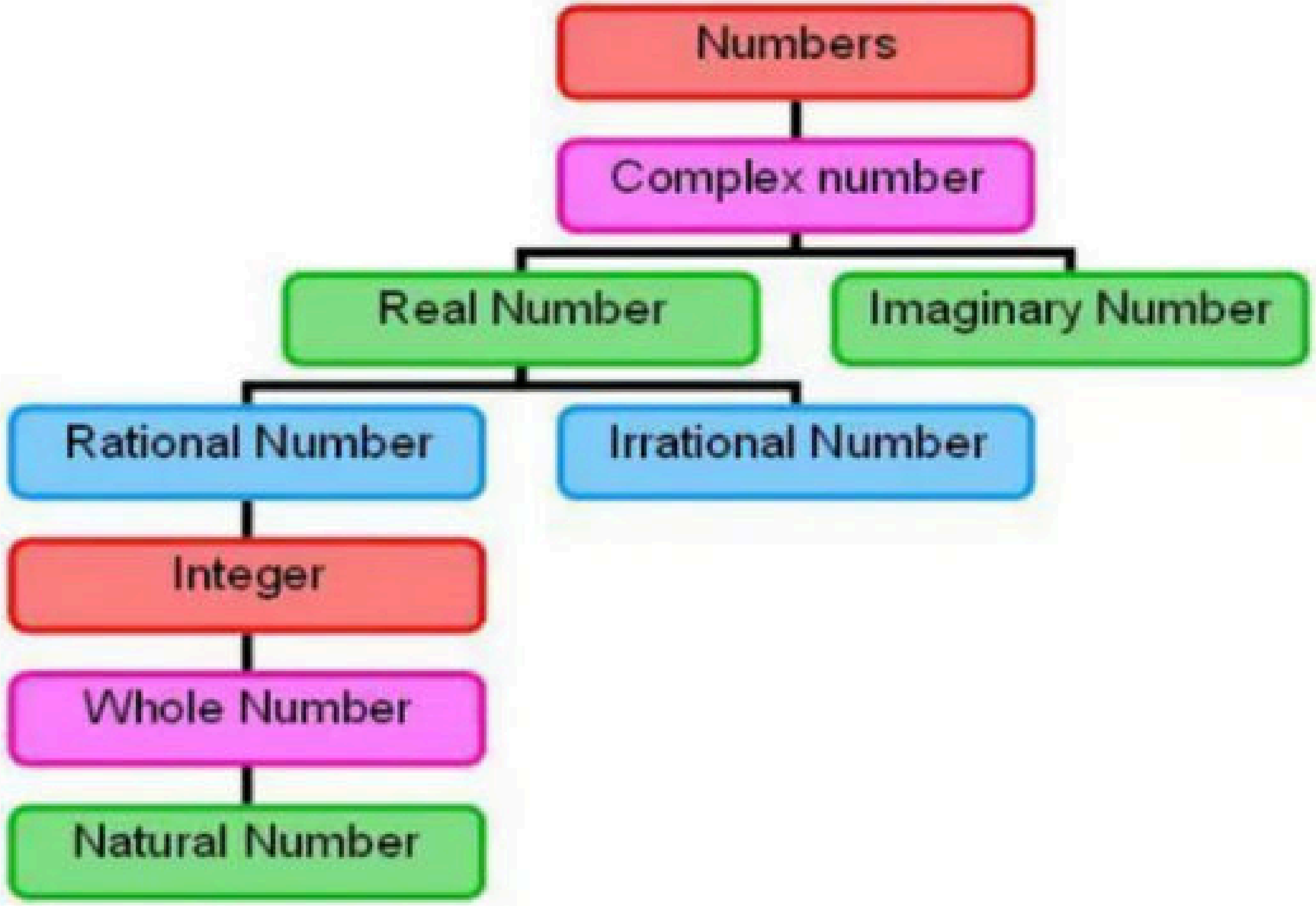
- **<u>Set of all Real number(R)</u> -** A real number is a value that represents a quantity along a continuous line, containing all of the rational numbers and all of the irrational numbers.



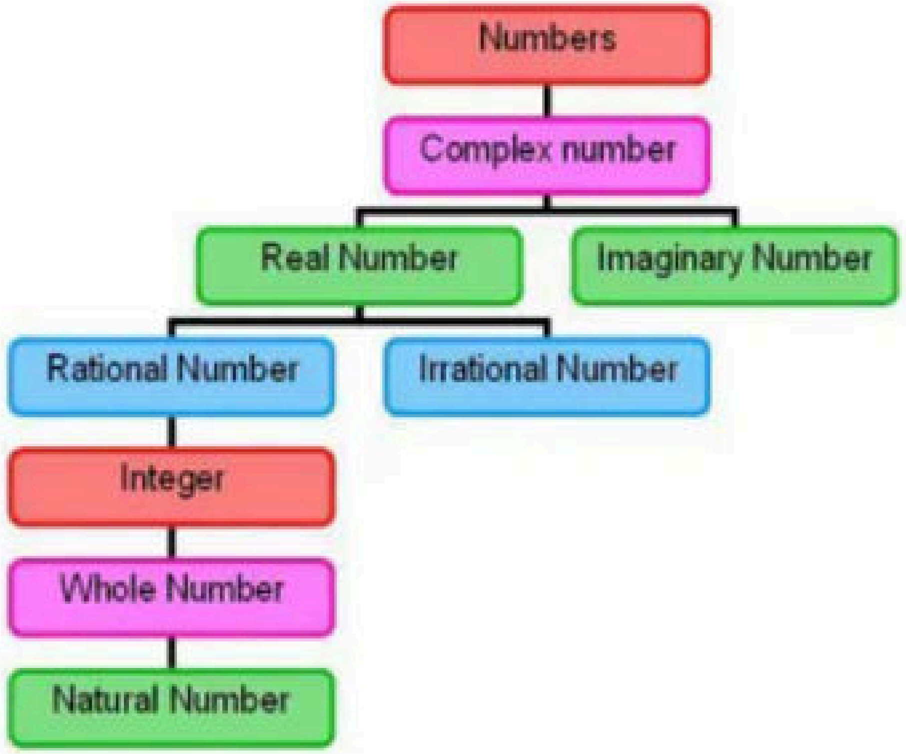
- **Set of all Rational number (Q)** - A rational number is any number that can be expressed as a fraction P/Q of two integers, a numerator P and a non-zero denominator Q.

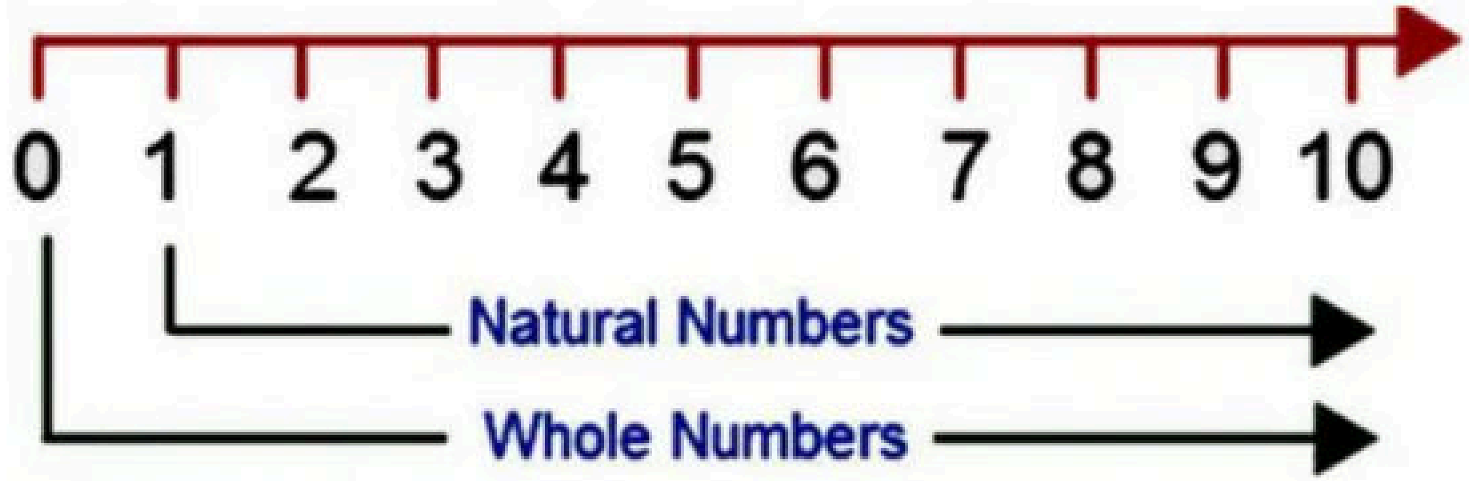


- **<u>Set of all Irrational number (R-Q or R/Q or P)</u>** - An irrational number is a real number that cannot be expressed as a fraction i.e. as a ratio of integers. Therefore, irrational numbers, when written as decimal numbers, do not terminate, nor do they repeat. E.g. root2.

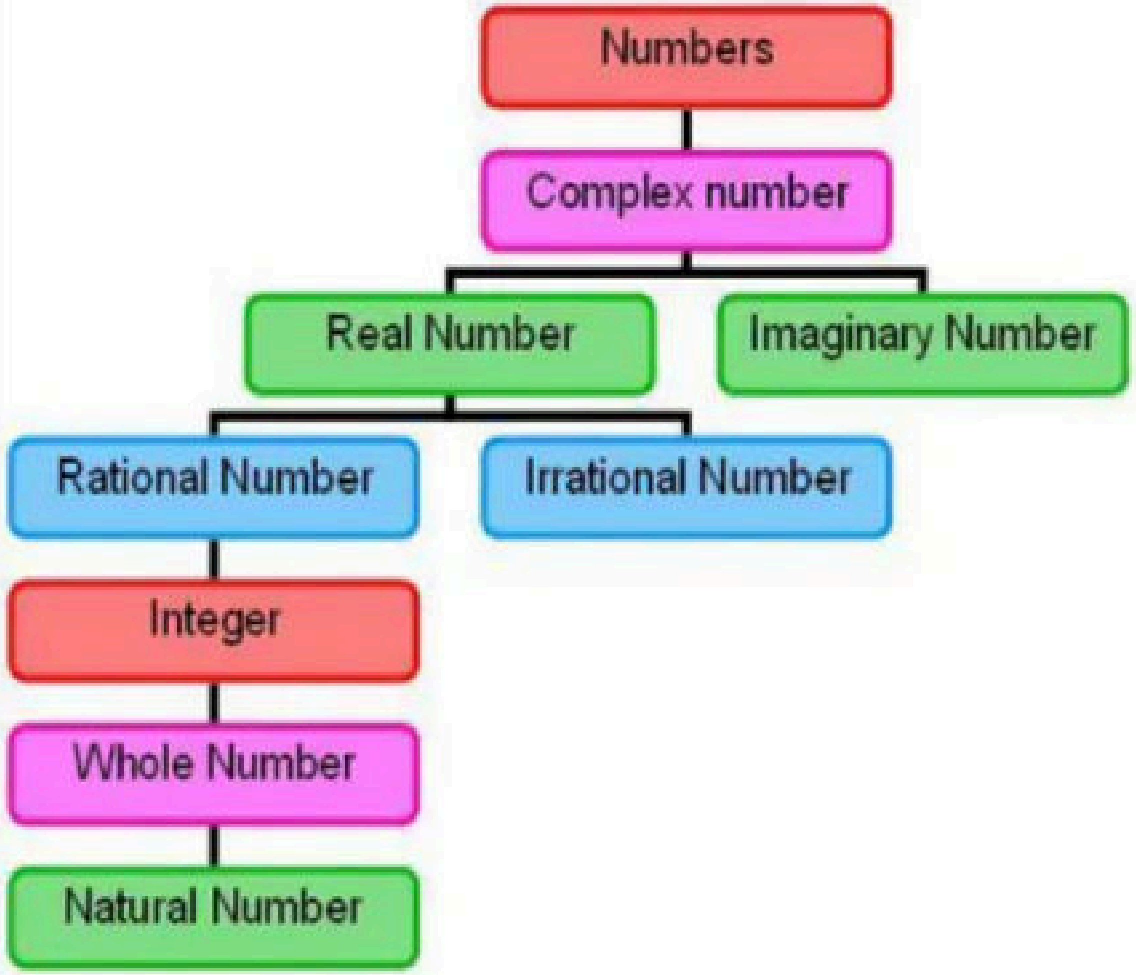


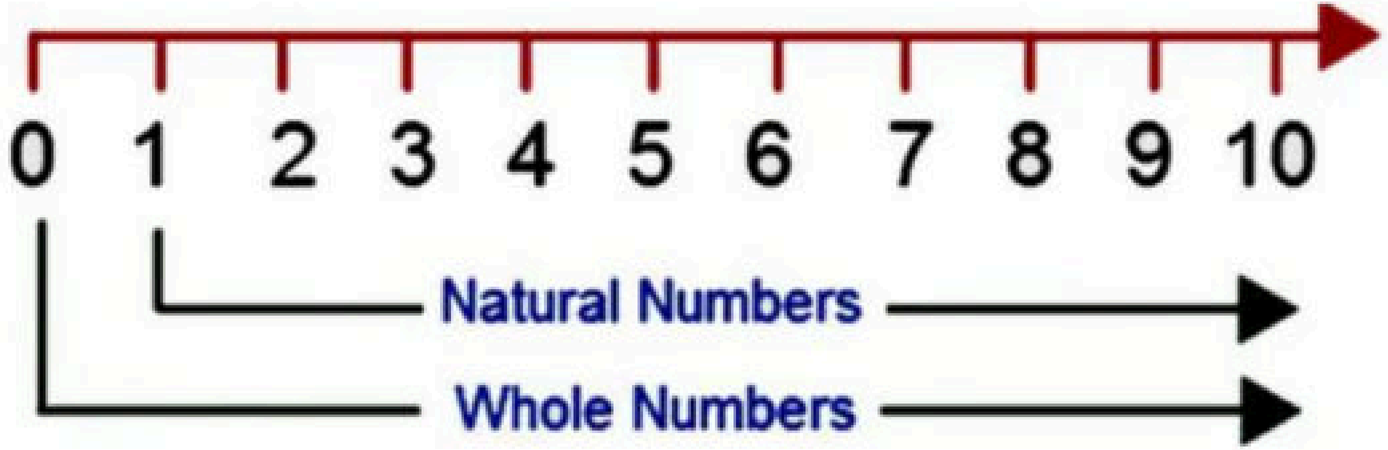
- **<u>Set of all Integer(Z)</u>** - An integer is a number that can be written without a fractional component.
- **<u>Set of all Whole number(W)</u>** - A whole number is a science expanded natural number. Set of natural number and zero





- **<u>Set of all-Natural number(N)</u>** - A natural number is a number that occurs commonly and obviously in nature. The set of natural numbers, can be defined as N = {1,2,3, 4.... ∞}



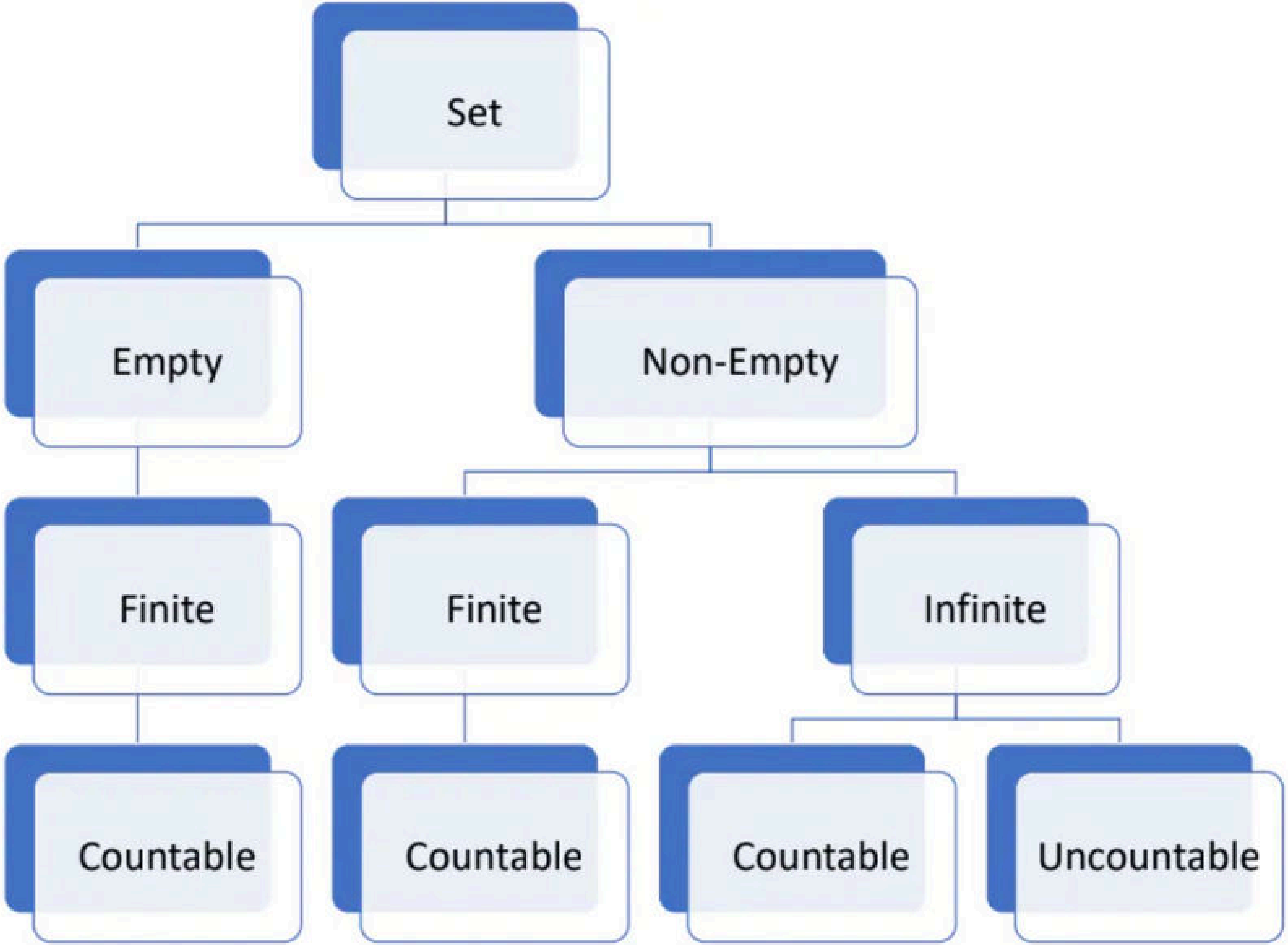


Break

- **<u>Finite set</u>** - If there are exactly '*n*' elements in *S* where '*n*' is a nonnegative integer, we say that *S* is a *finite set.*

- *i.e. if a* set contain specific or finite number of elements is called is called finite set. For e.g. A = {1,2,3,4}

- **Infinite set** – A set contain infinite number of elements is called infinite set, if the counting of different elements of the set does not come to an end. For e.g. a set of natural numbers.



Set
Empty
Non-Empty
Finite
Finite
Infinite
Countable
Countable
Countable
Uncountable

- **Countable set** – A set is said to be countable if there can be a one to one mapping between the elements of the set and natural numbers. E.g. Set of stars.

- **Uncountable set** – A set is said to be uncountable if there cannot be a one to one mapping between the elements of the set and natural numbers. E.g. Set of real numbers.

Q Which of the following is/are not true? **(NET-Dec-2015)**

(a) The set of negative integers is countable.

(b) The set of integers that are multiples of 7 is countable.

(c) The set of even integers is countable.

(d) The set of real numbers between 0 and ½ is countable.

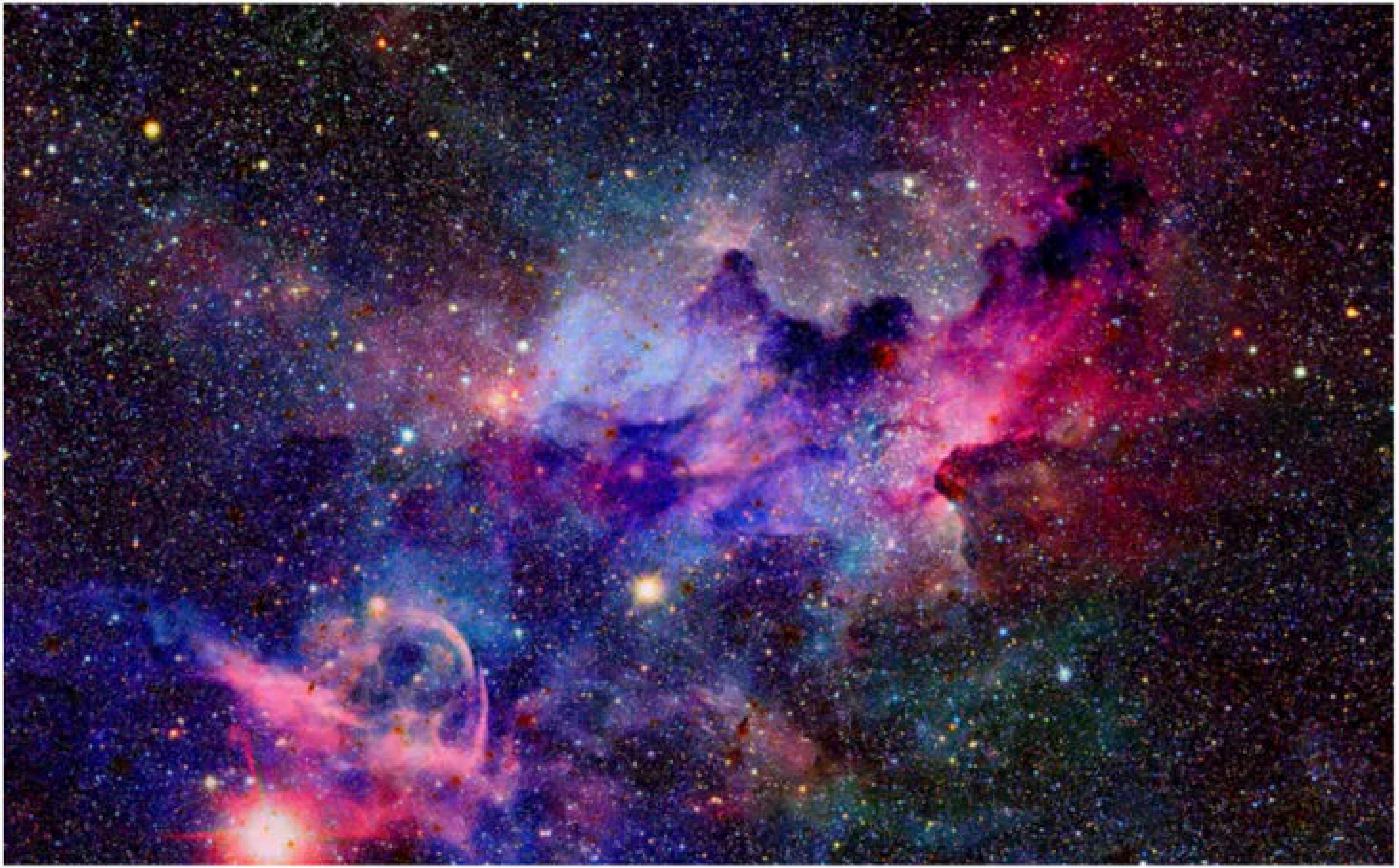
(A) (a) and (c) **(B)** (b) and (d)

(C) (b) only **(D)** (d) only

Break

- **Null set / empty set** - Is the unique set having no elements. its size or cardinality is zero i.e. $|\phi| = 0$. It is denoted by a symbol ϕ or $\{\}$. A set with one element is called singleton set.

- **Universal set** – if all the sets under investigation are subsets of a fixed set, i.e. the set containing all objects under investigation, in Venn diagram it is represented by a rectangle, and it is denoted by U.

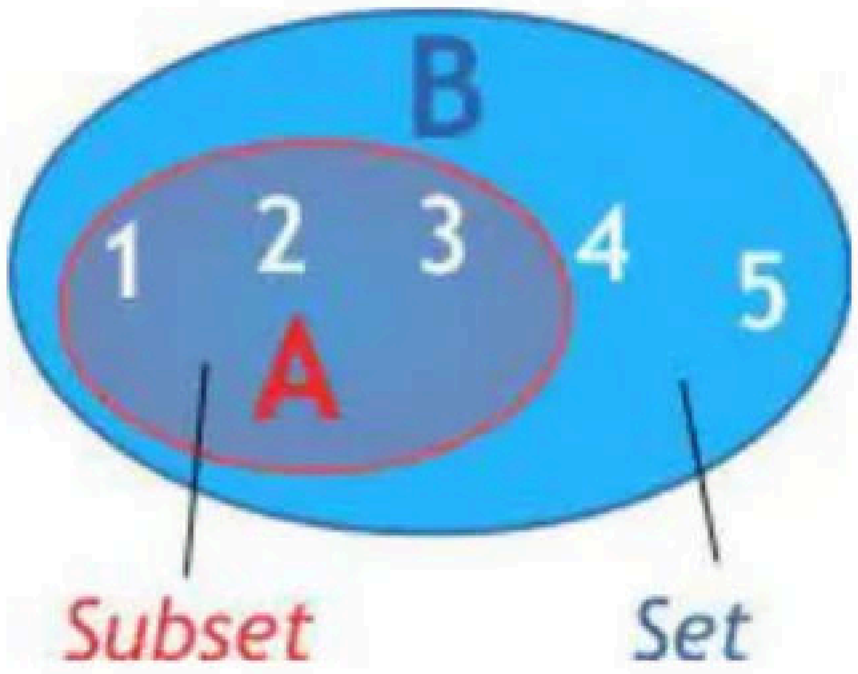


Break

- **<u>Subset of a set</u>** – If every element of set A is also an element of set B i.e.
- $\forall x(x \in A \rightarrow x \in B)$, then A is called subset of B and is written as $A \sqsubseteq B$. B is called the superset of A.

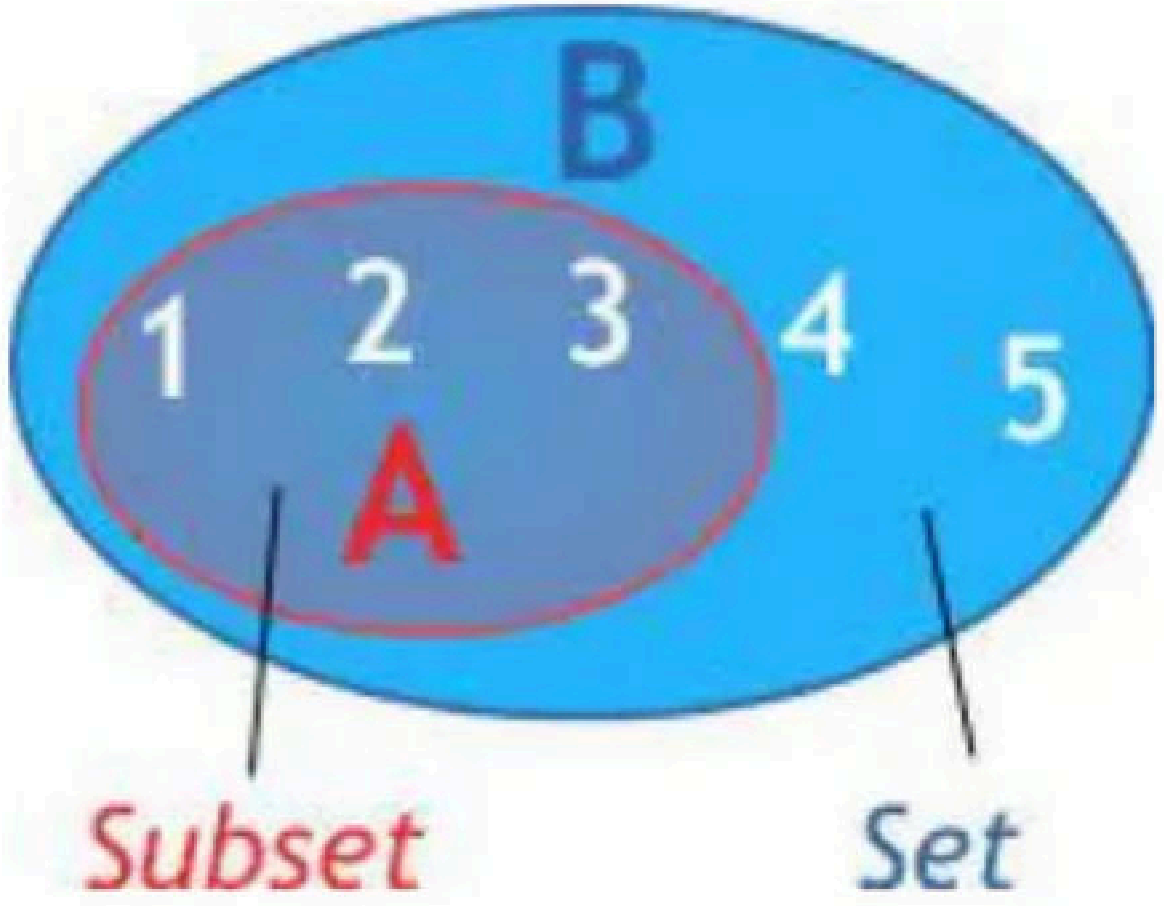
- E.g. A = {1,2,3} B = {1,2,3,4,5}

- Note that to show that *A* is not a subset of *B* we need only find one element $x \in A$ with $x \notin B$. To show that $A \sqsubseteq B$, show that if $x \in A$, then $x \in B$.

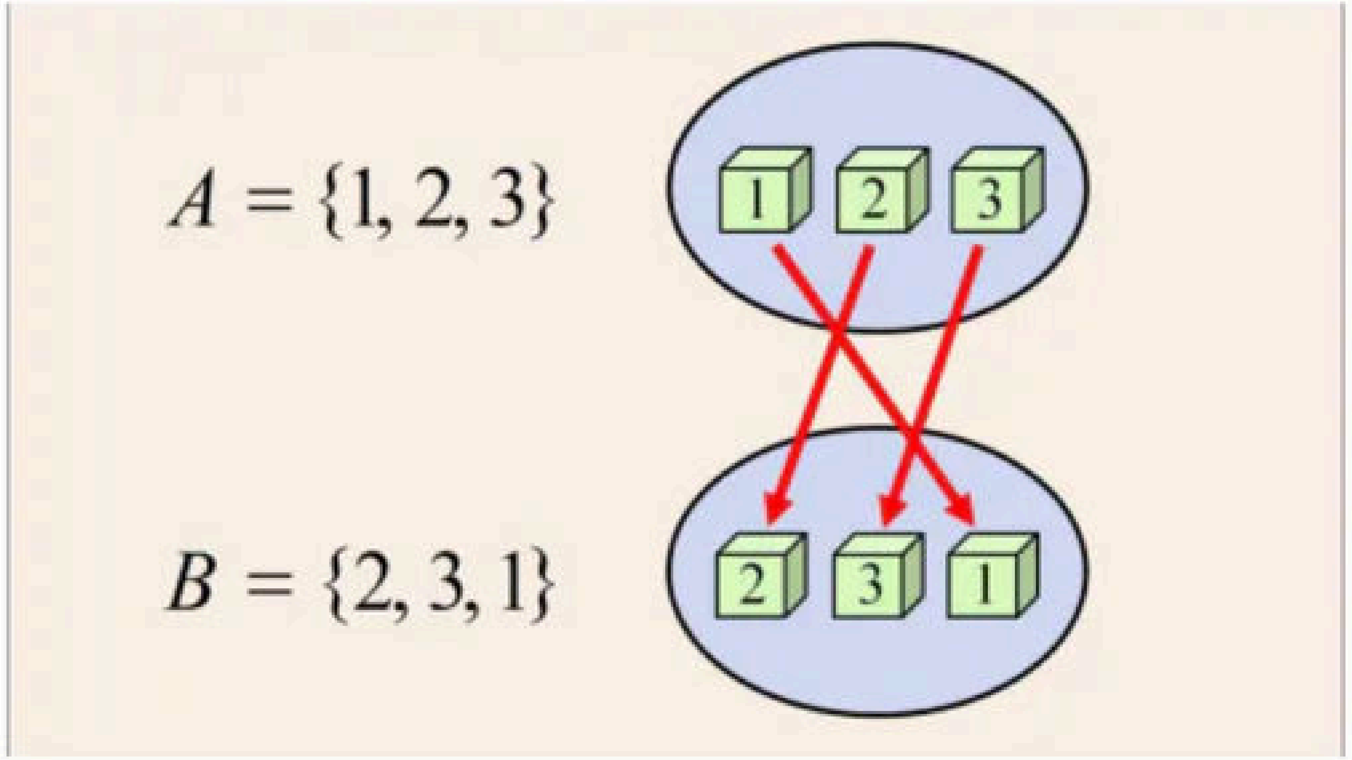


- $\phi \sqsubseteq A$, Empty Set ϕ is a subset for every set
- $A \sqsubseteq U$, Every Set is a subset of Universal set U
- $A \sqsubseteq A$, Every Set is a subset of itself.

- **<u>Proper subset</u> –** if A is a subset of B and $A \neq B$, then A is said to be a proper subset of B, i.e. there is at least one element in B which is not in A. denoted as $A \subset B$.



- **Equality of sets** – If two sets A and B have the same element and therefore every element of A also belong to B and every element of B also belong to A, then the set A and B are said to be equal and written as A=B.
- if A ⊑ B and B ⊑ A, then A=B
- $\forall x(x \in A \leftrightarrow x \in B)$



- **Power set** – let A be any set, then the set of all subsets of A is called power set of A and it is denoted by P(A) or 2^A.

- If A= {1,2,3}, then P(A) = {ϕ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3}}

- Cardinality of the power set of A is n, $|P(A)| = 2^n$

Venn Diagram

- Sets can be represented graphically using Venn diagrams, named after the English mathematician John Venn, who introduced their use in 1881. In Venn diagrams the **universal set** *U,* Contains all the objects under consideration, is represented by a rectangle.

- (Note that the universal set varies depending on which objects are of interest.) Inside this rectangle, circles or other geometrical figures are used to represent sets. Sometimes points are used to represent the particular elements of the set. Venn diagrams are often used to indicate the relationships between sets.

Break

Q For any Set A, which of the following are true?

1) $\phi \in A$ — F

$A \subseteq B$

$A = \{1, 2\}$

2) $\phi \sqsubseteq A$ — T

$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

3) $\phi \in 2^A$ — T

$1 \in P(A)$ X

$A = \{1, 2\}$

4) $\phi \sqsubseteq 2^A$ — T

$B = \{1, 2, 3\}$

5) $A \in 2^A$ — T

6) $A \sqsubseteq 2^A$ X

Q If ϕ is an empty set. Then $|P(P(P(\phi)))|$ = ______?

a) 1 **b)** 2 **c)** 4 **d)** none of above

$A = \{1\}$

$P(A) = \{\phi, \{1\}\}$

$P(P(A)) = \{\phi, \{\phi\}, \{\{1\}\}, \{\phi, \{1\}\}\}$

$\phi = \{\ \} = 0$

$P(\phi) = \{\phi\}$ $2^0 = 1$

$P(P(\phi)) = \{\phi, \{\phi\}\}$ $2^1 = 2$

$2^2 = 4$

$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Q The cardinality of the power set of {0, 1, 2 . . ., 10} is __________.

(GATE-2015) (1 Marks)

11

(A) 1024 **(B)** 1023 **(C)** 2048 **(D)** 2043

21 2 ~~77~~ 0

$2^{11} = \underline{\underline{2048}}$

$|A| = n$

$\downarrow$

$|2^A| = 2^n$

Q For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True. **(GATE-2015) (1 Marks)**

I) $\phi \in 2^A$

II) $\phi \subseteq 2^A$

III) $\{5, \{6\}\} \in 2^A$

IV) $\{5, \{6\}\} \subseteq 2^A$

(A) I and III only

(B) II and III only

(C) I, II and III only

(D) I, II and IV only

Break

Q The power set of the set $\{\Phi\}$ is: **(NET-Dec-2012)**

a) $\{\Phi\}$ — 6

$A = \{\phi\}$

$P(A) = \{\phi, \{\phi\}\}$

b) $\{\Phi, \{\Phi\}\}$ — 91

c) $\cancel{\{0\}}$ — 3

d) $\{0, \Phi, \{\Phi\}\}$ — 0

Q let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either S_1 is subset of S_2 or S_2 is subset of S_1. What is the maximum cardinality of C? **(GATE-2005) (2 Marks)**

a) n 13 3

b) n+1 50

c) $2^{n-1}+1$ 5 34

d) n! 4

$A = \{a, b, c\}$

$C = \{\phi, \cancel{\{a\}}, \{b\}, \cancel{\{c\}}, \{a, b\}, \cancel{\{b, c\}}, \cancel{\{a, c\}}, \{a, b, c\}\}$

1 2 3 4

$n = 3 \longrightarrow 4$

▲ 8 • Asked by Sunrit

Sir, isko leke kal hw mai confused thaa, ekbaar dekhenge, wahi wala ques jo abhi kiya

$$S = \{\phi\}$$

$$P(S) = \{\phi, \{\phi\}\}$$

$$P(P(S)) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$$

$$P(S) \cap P(P(S)) = \{\phi, \{\phi\}\} \neq \underline{\{\phi\}}$$

Q Let P(S) denotes the power set of set S. Which of the following is always true? **(GATE-2000) (2 Marks)**

a) ~~$P(P(S)) = P(S)$~~ -4

(b) $P(S) \cap P(P(S)) = \{\phi\}$ 74

$S = \{a\}$

$P(S) = \{ \phi, \{a\}\}$

$P(P(S)) = \{ \phi, \{\phi\}, \{\{a\}\}, \{\phi, \{a\}\}\}$

(c) ~~$P(S) \cap S = P(S)$~~ -10

(d) ~~$S \notin P(S)$~~ -11

$A = \{1, 2, \phi$

$B = \{3, 4, \phi\}$ $= \{\phi\}$

$A \cap B = \phi = \{\} =$

1 2 3

$$\_\_\_\_\_ \cap \_\_\_\_\_ = \{\ \} = \phi$$

$$\phi\,\_\_\_\_\_ \qquad \phi\,\_\_\_\_\_ \qquad = \{\phi\}$$

Q The number of elements in the power set P(S) of the set S= {{Ø},1, {2,3}} is: **(GATE-1995) (1 Mark)**

a) 2 **b)** 4 **c)** ~~8~~ **d)** None of the above

1 6 8 8 5

$P(S) = \{ \phi, \{\{\phi\}\}, \{1\}, \{\{2,3\}\}, \{\{\phi\},1\}, \{1,\{2,3\}\}, \{\{\phi\},\{2,3\}\}, \{\{\phi\},1,\{2,3\}\} \}$

Q.29 Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S, and $A \subseteq B$ is ________.

2011 n(2)

$3 \times 3 \times 3$ ———— $\times 3 = 3^{10} = 59{,}049$

$S = \{1, 2, 3, \ldots, 10\}$

$(2^{10} \times 2^{10}) = 2^{20}$

$A \subseteq B$

$A = (1, 2)$

$A = (1, 2, 3)$

x

	A	B
1	0	0
2	0	1
3	1	0
4	1	1

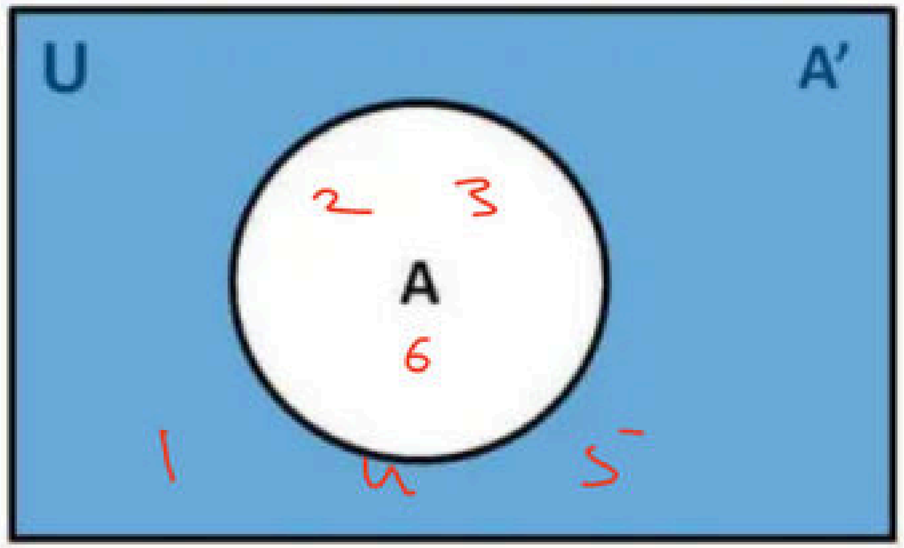
Break

Operation on sets

$\phi^c = U$

- **Complement of set** – Set of all x such that $x \notin A$, but $x \in U$.
- $A^c = \{x \mid x \notin A \;\&\; x \in U\}$

$U^c = \phi$



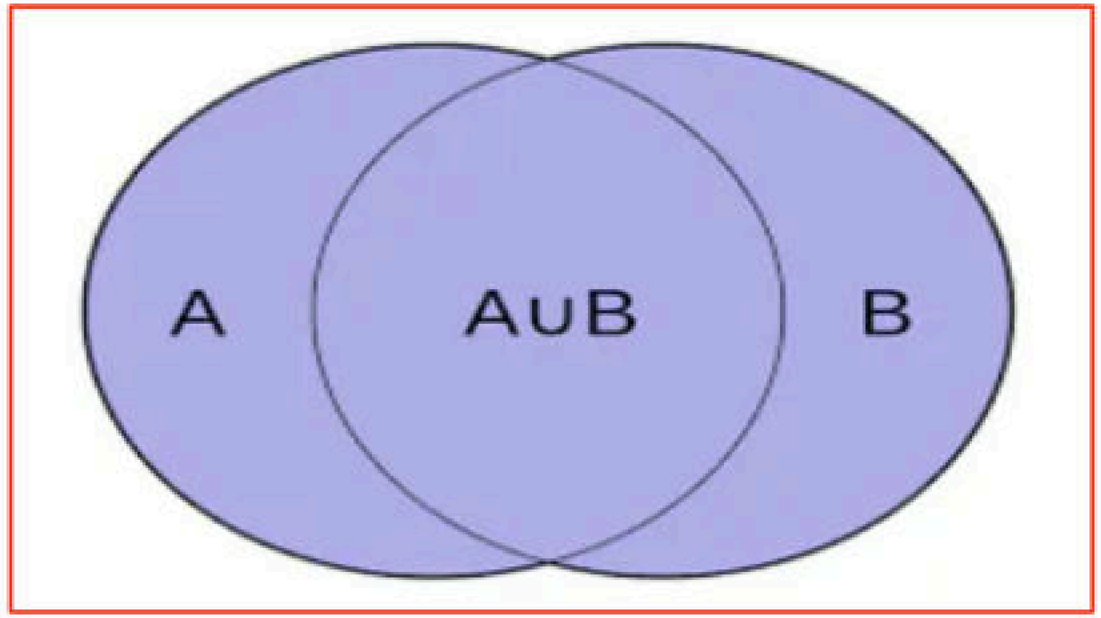
$U = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3, 6\}$

$A^c = \{1, 4, 5\}$

A^c $\bar{A}$

- **Union of sets** – Union of two sets A and B is a set of all those elements which either belong to A or B or both, it is denoted by A U B.
- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



$A = \{1, 2, 3, 4\}$

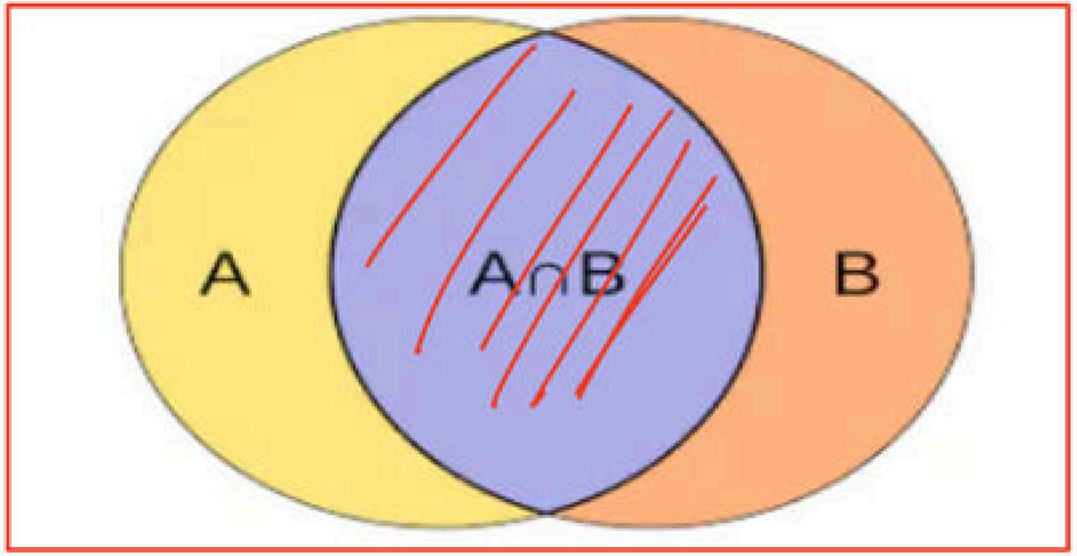
$B = \{3, 4, 5, 6\}$

$A \cup B = \{ 1, 2, 3, \cancel{4}, 4, \cancel{4}, 5, 6 \}$

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

$\underset{4}{|A|} + \underset{4}{|B|} = \underset{6}{|A \cup B|}$? $+ |A \cap B|$

- **Intersection of sets** - Intersection of two sets A and B is a set of all those elements which belong to both A and B, and is denoted by $A \cap B$.
- $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



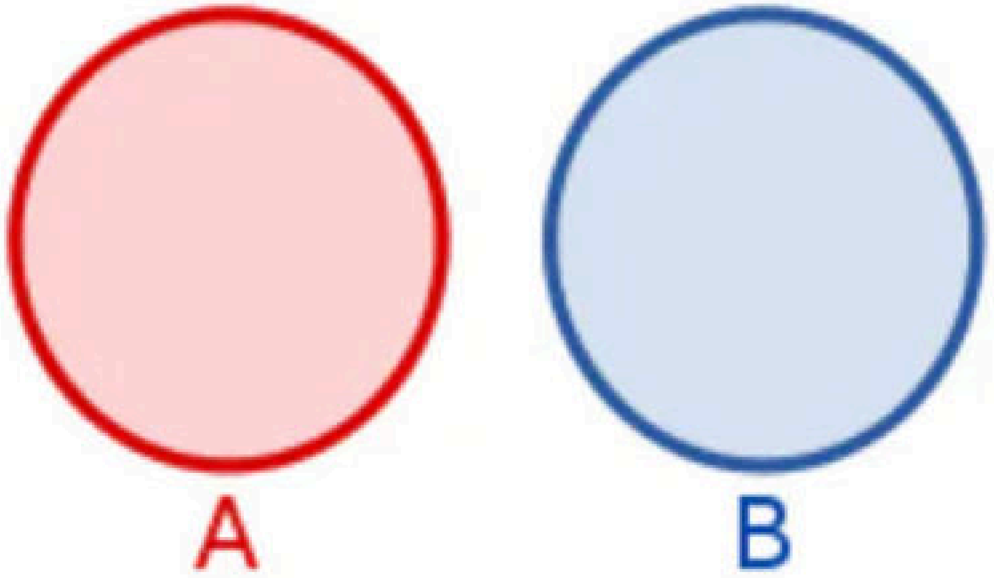
$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

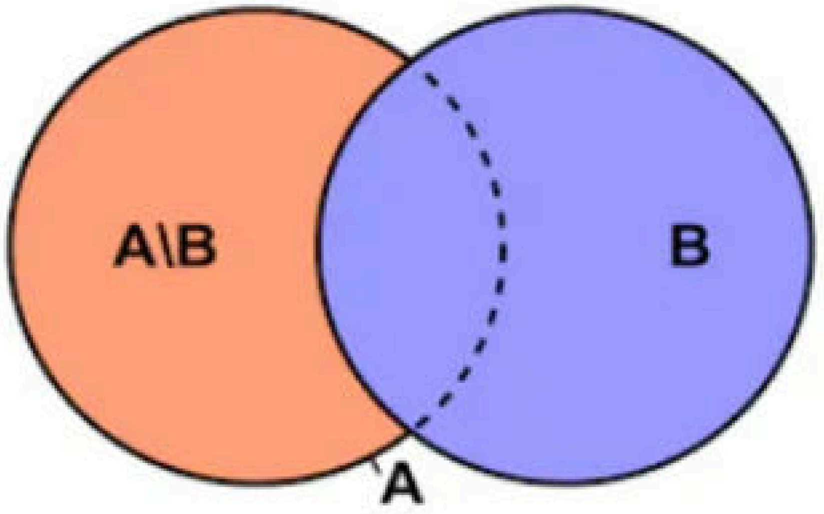
$A \cap B = \{\ 3,\ 4\ \}$

- **<u>Disjoint sets</u>** -- Two sets are said to be disjoint if they do not have a common element, i.e. no element in A is in B and no element in B is in A.

- $A \cap B = \phi$



- **Set difference** – the set difference of two sets A and B, is the set of all the elements which belongs to A but do not belong to B.
- $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

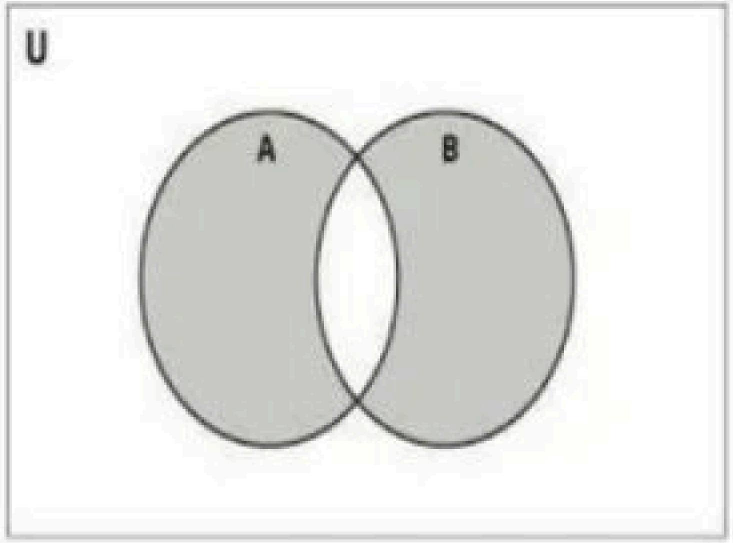


$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A - B = \{ \qquad\qquad\qquad \}$

Symmetric difference – the symmetric difference of two sets A and B is the set of all the elements that are in A or in B but not in both, denoted as. $A \oplus B = (A \cup B) - (A \cap B)$



$A \oplus B = \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

$A \oplus B = (A - B) \cup (B - A)$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \oplus B = \{ \qquad\qquad\qquad\qquad \}$

Q if $A_i = \{-i, ... -2, -1, 0, 1, 2, ... i\}$ **(NET-July-2018)**

then $\cup_{i=1}^{\infty} Ai$ is

a) Z

b) Q

c) R

d) C

Q The power set of A U B, where A = {2, 3, 5, 7} and B = {2, 5, 8, 9} is **(NET-Dec-2012)**

a) 256

b) 64

c) 16

d) 4

Break

Q Consider the following statements?

a) Finite union of finite sets is _________(finite/infinite)

b) Finite union of Infinite sets is _________(finite/infinite)

c) Infinite union of finite sets is _________(finite/infinite)

d) if after finite number of union result is infinite set, then at least of the input set is infinite (T / F)

e) if after finite number of union result is infinite set, then all of the input set is infinite (T / F)

f) Finite intersection of finite sets is _________(finite/infinite)

g) Finite intersection of Infinite sets is _________(finite/infinite)

h) If after finite number of intersection result is infinite set, then at least of the input set is infinite (T / F)

i) If after finite number of intersection result is infinite set, then all of the input set is infinite (T / F)

Q Let S be an infinite set S_1, S_2............, S_n be Sets such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$ Then, **(GATE-1993) (1 Marks)**

(a) At least one of the set S_i is a finite set

(b) Not more than one of the set S_i can be finite

(c) At least one of the sets S_i is an infinite set

(d) Not more than one of the sets S_i can be infinite

Break

Q which of the following is not true?

a) $A - B = A \cap B^c$

b) $A - (A - B) = A \cap B$

c) $A - (A \cap B) = A - B$

d) $A - (A - B) = B$

Q If $A \subset B$, then which of the following is not true?

(a) $A \cup B = B$

(b) $A \cap B = A$

(c) $B^C \subset A^C$

(d) $B - A = \phi$

Q Which the following in not true?

a) If $A \sqsubseteq \phi$, then $A = \phi$

b) $(A \cap B^C) \cup (A \cap B) = A$

c) $B \cup (A \cap B) = B$

d) $(A \cap B^C) \cup (A \cap B) \cup (A^C \cap B) \cup (A^C \cap B^C) = A \cap B$

Q Which of the following is true?

(i) $(A - B) - C = A - (C - B)$

(ii) $(A - B) - C = (A - C) - B$

(iii) $(A - B) - C = A - (B \cap C)$

(iv) $(A \cap B) - (B \cap C) = \{A - (A \cap C)\} - (A - B)$

a) i & iii

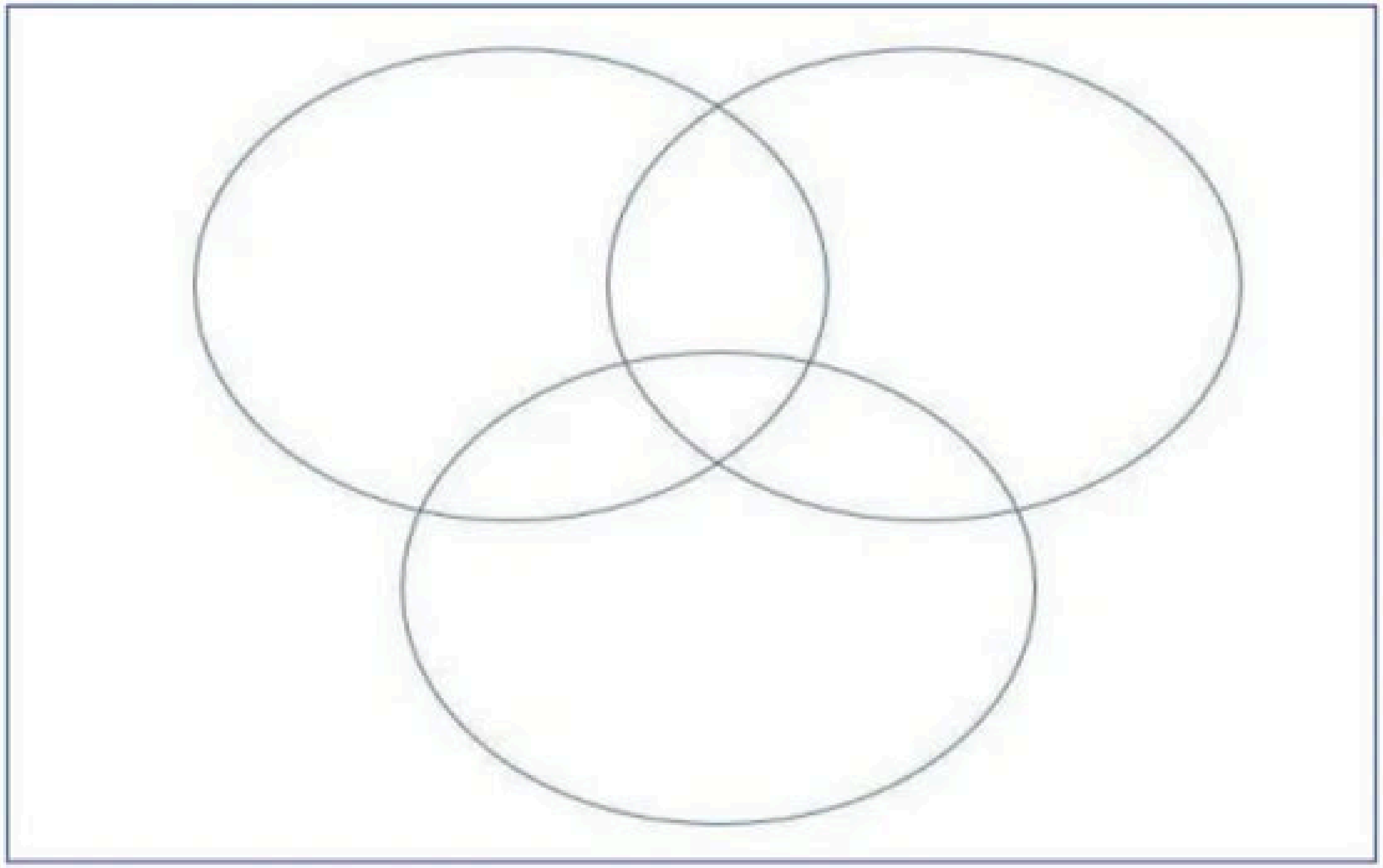
b) ii & iv

c) i, ii, iv

d) ii & iii

Break

Q In a college, there are three student clubs, Sixty students are only in the Drama club, 80 students are only in the Dance club, 30 students are only in Maths club, 40 students are in both Drama and Dance clubs, 12 students are in both Dance and Maths clubs, 7 students are in both Drama and Maths clubs, and 2 students are in all clubs. If 75% of the students in the college are not in any of these clubs, then the total number of students in the college is ______________. **(GATE-2019) (2 Mark)**



Q The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _______. **(GATE-2017) (1 Marks)**

Q How many multiples of 6 are there between the following pairs of numbers? **(NET-Jan-2017)**
0 and 100 and –6 and 34

a) 16 and 6

b) 17 and 6

c) 17 and 7

d) 16 and 7

Q Let A and B be sets in a finite universal set U. Given the following: $|A - B|$, $|A \oplus B|$, $|A| + |B|$ and $|A \cup B|$ Which of the following is in order of increasing size? **(NET-Dec-2016)**

a) $|A - B| < |A \oplus B| < |A| + |B| < |A \cup B|$

b) $|A \oplus B| < |A - B| < |A \cup B| < |A| + |B|$

c) $|A \oplus B| < |A| + |B| < |A - B| < |A \cup B|$

d) $|A - B| < |A \oplus B| < |A \cup B| < |A| + |B|$

Break

Q Suppose U is the power set of the set $S = \{1,2,3,4,5,6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T′ denote the complement of T. For any T, $R \in U$, let $T \setminus R$ be the set of all elements in T which are not in R.

(GATE-2015) (2 Marks)

Which one of the following is true?

a) $\forall X \in U, (|X| = |X'|)$

B) $\exists X \in U, \exists Y \in U, (|X|=5, |Y|=5 \text{ and } X \cap Y = \varphi)$

C) $\forall X \in U, \forall Y \in U, (|X|=2, |Y|=3 \text{ and } X \setminus Y = \varphi)$

D) $\forall X \in U, \forall Y \in U, (X \setminus Y = Y' \setminus X')$

Q Consider a set A= {1,2, 3, ………,1000}. How many members of A shall be divisible by 3 or by 5 or by both 3 and 5? **(NET-Dec-2014)**

a) 533 **b)** 599 **c)** 467 **d)** 66

Q Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say U < V if the minimum element in the symmetric difference of the two sets is in U. Consider the following two statements: **(GATE-2014) (2 Marks)**

S_1: There is a subset of S that is larger than every other subset.

S_2: There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

(A) Both S_1 and S_2 are true

(B) S_1 is true and S_2 is false

(C) S_2 is true and S_1 is false

(D) Neither S_1 nor S_2 is true

Q If P, Q, R are subsets of the universal set U, then **(GATE-2008) (1 Marks)**

$$(P \cap Q \cap R) \cup (P^C \cap Q \cap R) \cup Q^C \cup R^C$$

(A) $Q^c \cup R^c$

(B) $P \cup Q^c \cup R^c$

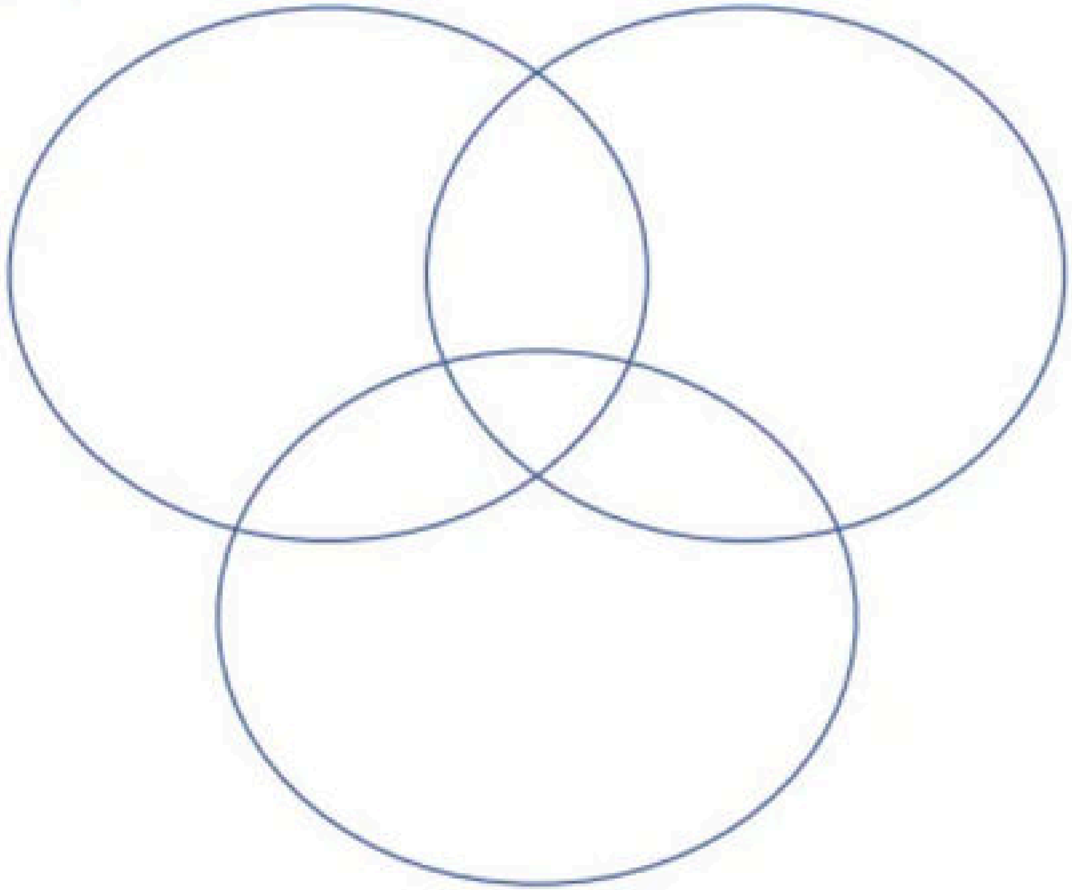
(C) $P^c \cup Q^c \cup R^c$

(D) U

Break

Q let p, q and r be sets let @ denotes the symmetric difference operator defined as

P @ q = (p ∪ q) – (p ∩ q)? **(GATE-2006) (2 Mark)**



I) p @ (q ∩ r) = (p @ q) ∩ (P @ r)

II) p∩ (q ∩ r) = (p ∩ q) @ (p @ r)

a) I only **b)** II only

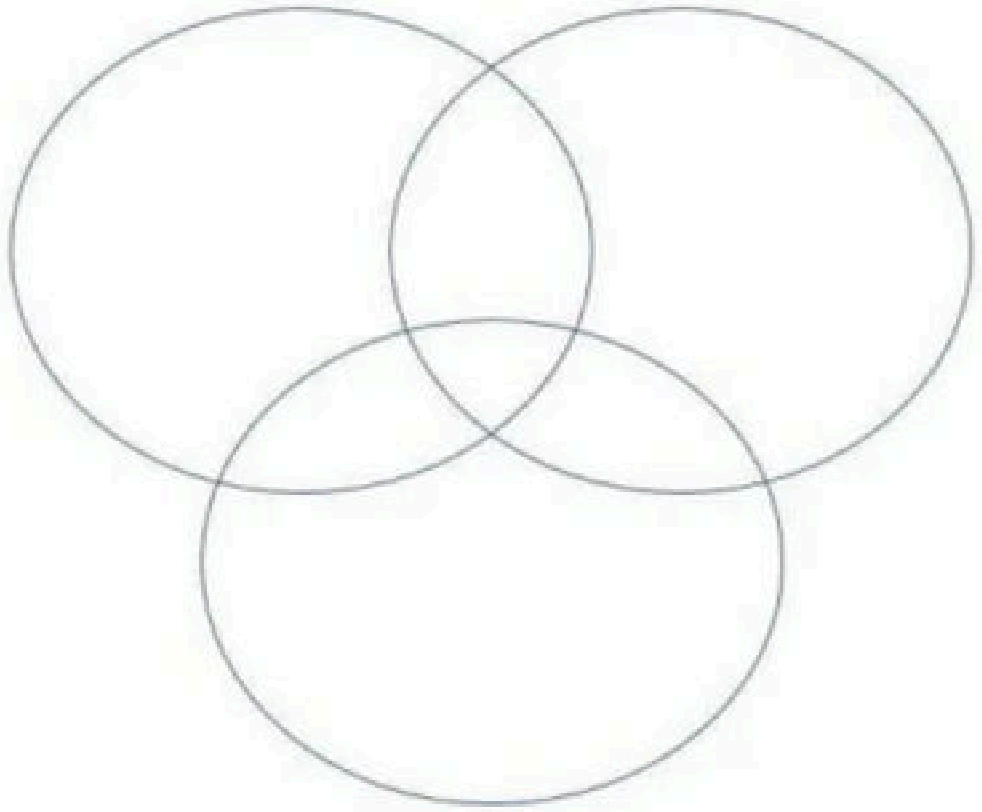
c) neither I nor II **d)** both I and II

Q Let E, F and G be finite sets.

Let $X = (E \cap F) - (F \cap G)$ and $Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true? **(GATE-2006) (2 Mark)**

(A) $X \subset Y$ **(B)** $X \supset Y$ **(C)** $X = Y$ **(D)** $X - Y \neq \phi$ and $Y - X \neq \phi$



Q what is the cardinality of the set of integers X defined below **(GATE-2006) (2 Mark)**

X = {n | 1<=n<=123, n is not divisible by 2,3 or 5}?

a)90

b)33

c)37

d)44

Q Let A, B and C be non-empty sets and let

$X=(A-B)-C$

$Y=(A-C)-(B-C)$

Which one of the following is TRUE? **(GATE-2005) (1 Marks)**

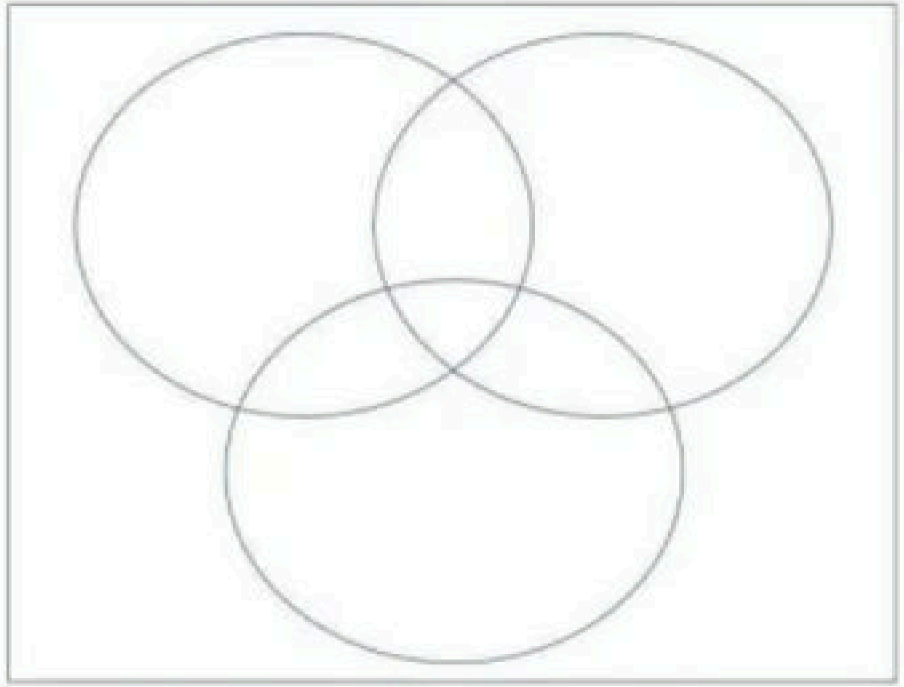
a) $X=Y$ **b)** $X \subset Y$ **c)** $Y \subset X$ **d)** None of these

Break

Q In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Programming Language and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses? **(GATE-2004) (1 Mark)**

(A) 175 **(B)** 20 **(C)** 25 **(D)** 35

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$



Q Consider the following statements:

S_1: There exists infinite sets A, B, C such that $A \cap (B \cup C)$ is finite.

S_2: There exists two irrational numbers x and y such that $(x + y)$ is rational.

Which of the following is true about S_1 and S_2? **(GATE-2001) (2 Mark)**

(a) Only S_1 is correct

(b) Only S_2 is correct

(c) Both S_1 and S_2 are correct

(d) None of S_1 and S_2 is correct

Q Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. the set $(a - b) \cup (b - a) \cup (a \cap b)$ is equal to. **(GATE- 1996) (1 Mark)**

(a) $A \cup B$ **(b)** $A^c \cup b^c$ **(c)** $A \cap B$ **(d)** $A^c \cap b^c$

Q The bit string for the sets, $A = \{1,3,5,7,9\}$ and $B = \{1,2,3,4,5\}$ are 1010101010 and 1111100000 respectively. If the universal set is $U = \{1, 2....,10\}$ is represented 1111111111 which of the following is false?

a) $A \cup B = 1111111010$

b) $A \cap B = 1010100000$

c) $A - B= 0000001010$

d) $A^C = 000011111$

Break

Idempotent law

- $A \cup A = A$
- $A \cap A = A$

Associative law

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Commutative law

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Distributive law

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law

- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$

Identity law

- $A \cup \phi = A$
- $A \cap \phi = \phi$
- $A \cup U = U$
- $A \cap U = A$

Complement law

- $A \cup A^C = U$
- $A \cap A^C = \phi$
- $U^C = \phi$
- $\phi^C = U$

Involution law

- $((A)^C)^C = A$