First order Predicate Logic

- Sometime propositional logic cannot derive any meaningful information even though, we as human can understand that argument is meaningful or not.

- P_1: Every Indian like cricket
- P_2: Sunny is an Indian
- Q: Sunny Likes cricket

- The reason propositional logic fails here because using only inference system we can not conclude Q from P_1 and P_2.

- In first order logic we understand, a new approach of subject and predicate to extract more information from a statement
  - 1 is a natural number (1 is subject, natural number is predicate)
  - we can write FOPL (short hand notation) for this as NatNo(1) = 1 is natural number
  - Similarly, we can understand the meaning of NatNo(2) as 2 is a natural number
  - NatNo(x): x is a natural number

- Sometime subject is not a single element but representing the entire group.
  - Every Indian like Cricket.
  - We can have a propositional function Cricket(x): x likes Cricket.
  - We can fix domain of discussion or universe of discourse as, x is an Indian.

- If i say four Indian are there I_1, I_2, I_3, I_4
- I_1 likes cricket $\wedge$ I_2 likes cricket $\wedge$ I_3 likes cricket $\wedge$ I_4 likes cricket
- $\text{Cricket}(I_1) \wedge \text{Cricket}(I_2) \wedge \text{Cricket}(I_3) \wedge \text{Cricket}(I_4)$
- But problem with this notation is as there is 130+ corers Indian this formula will become very long and in some case we actually do not know how many subjects are there in the universe of discourse. so, we again need a short hand formula.
- $\forall_x$ Cricket(x), if we confine x to be Indian then it means every x like cricket.

- **Universal quantifiers**: - The universal quantification of a propositional function is the proposition that asserts

- P(x) is true for all values of x in the universe of discourse.

- The universe of discourse specifies the possible value of x.

- $\forall_x$ P(x), i.e. for all value of a P(x) is true

Break

- Let try some other statement 'Some Indian like samosa'
  - if i say four Indian are there I_1, I_2, I_3, I_4
  - I_1 like samosa $\vee$ I_2 like samosa $\vee$ I_3 like samosa $\vee$ I_4 like samosa
  - $Samosa(I_1) \vee Samosa(I_2) \vee Samosa(I_3) \vee Samosa(I_4)$
  - $\exists_x$ Samosa(x), if we confine x to be Indian then it means some x likes samosa.

- **<u>Existential quantifiers</u>**: - with existential quantifier of a propositional that is true if and only if P(x) is true for at least one value of x in the universe of discourse.

- There exists an element x is the universe of discourse such that P(x) is true.

- $\exists_x P(x)$, i.e. for at least one value of a P(x) is true

Break

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes cricket
  - Indian(x): x is an Indian
  - Cricket(x): x likes Cricket

I_1, I_2, I_3, I_4

  - if I_1 is Indian then likes cricket ∧ if I_2 is Indian then likes cricket ∧ if I_3 is Indian then likes cricket ∧ if I_4 is Indian then likes cricket

  - $[\text{Indian}(I_1) \rightarrow \text{cricket}(I_1)] \wedge [\text{Indian}(I_2) \rightarrow \text{cricket}(I_2)] \wedge [\text{Indian}(I_3) \rightarrow \text{cricket}(I_3)] \wedge [\text{Indian}(I_4) \rightarrow \text{cricket}(I_4)]$

  - $\forall_x [\text{Indian}(x) \rightarrow \text{cricket}(x)]$

$q \rightarrow p$

$p \rightarrow q$

$\sim p \rightarrow \sim q$

$\sim q \rightarrow \sim p$

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes samosa
  - Indian(x): x is an Indian
  - Samosa(x): x likes Samosa

  - $\boxed{\text{if } I_1 \text{ is Indian then likes samosa}}$ $\vee$ $\boxed{\text{if } I_2 \text{ is Indian then likes samosa}}$ $\vee$ if I_3 is Indian then likes samosa $\vee$ if I_4 is Indian then likes samosa

  - $[\text{Indian}(I_1) \wedge \text{samosa}(I_1)] \vee [\text{Indian}(I_2) \wedge \text{samosa}(I_2)] \vee [\text{Indian}(I_3) \wedge \text{samosa}(I_3)] \vee [\text{Indian}(I_4) \wedge \text{samosa}(I_4)]$

  - $\exists_x [\text{Indian}(x) \wedge \text{samosa}(x)]$

- let check validity of a statement “Some Indians like samosa” = $\boxed{\exists_x [Indian(x) \rightarrow samosa(x)]}$, x is human
- let human contains four elements I_1, I_2, I_3, I_4 out of which I_1, I_2 are Indian while I_3, I_4 are not Indian
- Suppose I_1, I_2, I_3 do not likes samosa
  - $[Indian(I_1) \rightarrow samosa(I_1)] \vee [Indian(I_2) \rightarrow samosa(I_2)] \vee [Indian(I_3) \rightarrow samosa(I_3)]$
  - $[T \rightarrow F] \vee [T \rightarrow F] \vee [F \rightarrow F]$
  - $[F] \vee [F] \vee [T]$
  - T

F → T

- conclusion $\exists_x$ is not used with $\rightarrow$

$$\frac{\exists \quad = \quad \wedge}{\forall \qquad\quad \rightarrow}$$

Break

Negation

- $\neg [\forall_x P(x)] = \exists_x \neg P(x)$
- $\neg [\exists_x P(x)] = \forall_x \neg P(x)$

Let L(x, y): x like y, which means x likes y or y is liked by x

1- $\forall_x \forall_y L(x, y)$

2- $\forall_y \forall_x L(x, y)$

3- $\exists_x \exists_y L(x, y)$

4- $\exists_y \exists_x L(x, y)$

5- $\forall_x \exists_y L(x, y)$

6- $\exists_y \forall_x L(x, y)$

7- $\forall_y \exists_x L(x, y)$

8- $\exists_x \forall_y L(x, y)$

Break

	1
P_1	$\exists_x P(x) \lor \exists_x Q(x)$
Q	$\exists_x (P(x) \lor Q(x))$

	2
P_1	$\exists_x (P(x) \lor Q(x))$
Q	$\exists_x P(x) \lor \exists_x Q(x)$

	3
P_1	$\exists_x P(x) \land \exists_x Q(x)$
Q	$\exists_x (P(x) \land Q(x))$

	4
P_1	$\exists_x (P(x) \land Q(x))$
Q	$\exists_x P(x) \land \exists_x Q(x)$

	1
P_1	$\forall_x P(x) \lor \forall_x Q(x)$
Q	$\forall_x (P(x) \lor Q(x))$

	2
P_1	$\forall_x (P(x) \lor Q(x))$
Q	$\forall_x P(x) \lor \forall_x Q(x)$

	3
P_1	$\forall_x P(x) \land \forall_x Q(x)$
Q	$\forall_x (P(x) \land Q(x))$

	4
P_1	$\forall_x (P(x) \land Q(x))$
Q	$\forall_x P(x) \land \forall_x Q(x)$

	1
P_1	$[\forall_x P(x) \rightarrow \forall_x Q(x)]$
Q	$\forall_x [P(x) \rightarrow Q(x)]$

	2
P_1	$\forall_x [P(x) \rightarrow Q(x)]$
Q	$[\forall_x P(x) \rightarrow \forall_x Q(x)]$

Break

Q consider the statement $\exists_x[P(x) \wedge \neg Q(x)]$, Which of the following is equivalent?

a) $\forall_x\, [P(x) \rightarrow Q(x)]$

b) $\forall_x\, [\neg P(x) \rightarrow Q(x)]$

c) $\neg\, \{\forall_x\, [P(x) \rightarrow Q(x)]\}$

d) $\neg\, \{\forall_x\, [\neg P(x) \rightarrow Q(x)]\}$

Q negation of the statement

$\exists_x \forall_y [F(x, y) \rightarrow \{G(x, y) \vee H(x, y)\}] = \forall_x \exists_y [F(x, y) \wedge \{\neg G(x, y) \wedge \neg H(x, y)\}]$?

Q let in a set of all integers

G (x, y): x is greater than y

"for any given positive integer, there is a greater positive integer"

a) $\forall_x \exists_y G(x, y)$

b) $\exists_y \forall_x G(x, y)$

c) $\forall_y \exists_x G(x, y)$

d) $\exists_x \forall_y G(x, y)$

Q let in a set of all humans

L (x, y): x likes y

"there is someone, whom no one like"

a) $\forall_x \exists_y \{\neg L(x, y)\}$

b) $\{\neg \forall_x \exists_y L(x, y)\}$

c) $\neg \{\forall_y \exists_x L(x, y)\}$

d) $\neg \{\exists_Y \forall_X L(x, y)\}$

Break

Q Negation of the proposition $\exists_x H(x)$ **(NET-Jan-2017)**

(A) $\exists_x \neg H(x)$ **(B)** $\forall_x \neg H(x)$ **(C)** $\forall_x H(x)$ **(D)** $\neg x\ H(x)$

4 90 4 1

$\sim \exists x\, H(x)$

$\forall x \sim H(x)$

Q Consider the first-order logic sentence

F: $\forall_x (\exists_y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F? **(GATE-2017) (1 Marks)**

I. $\exists_y (\exists_x R(x, y))$

II. $\exists_y (\forall_x R(x, y))$

III. $\forall_y (\exists_x R(x, y))$

IV. $\sim\exists_x (\forall_y \sim R(x, y))$

(A) IV only

(B) I and IV only

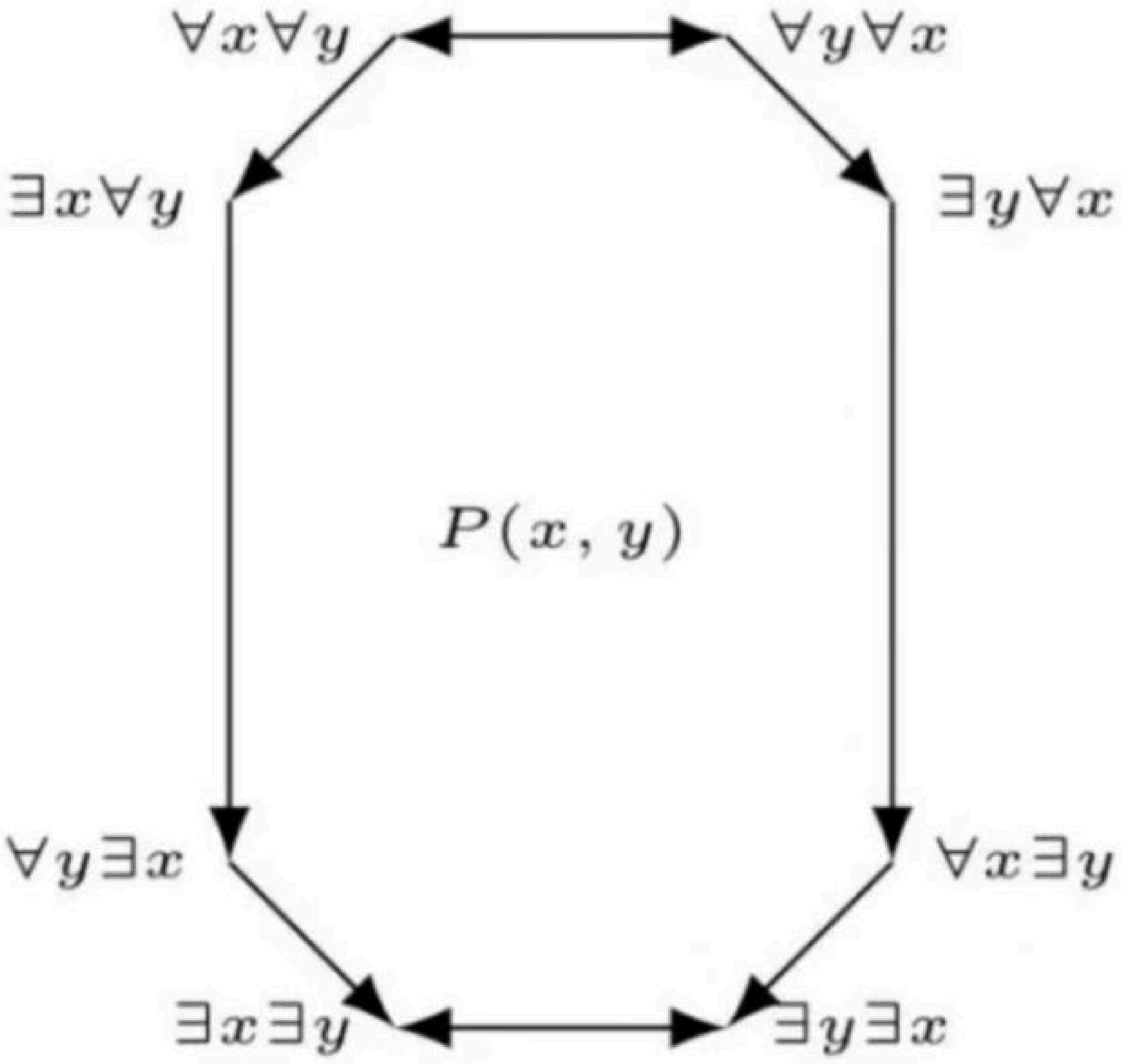
(C) II only

(D) II and III only

Q Consider the first-order logic sentence

F: $\forall_x (\exists_y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F? **(GATE-2017) (1 Marks)**

I. $\exists_y (\exists_x R(x, y))$ II. $\exists_y (\forall_x R(x, y))$ III. $\forall_y (\exists_x R(x, y))$ IV. $\sim\exists_x (\forall_y \sim R(x, y))$



(A) IV only

(B) I and IV only

(C) II only

(D) II and III only

Q Which one of the following well-formed formulae in predicate calculus is **NOT** valid? **(GATE-2016) (2 Marks)**

a) $(\forall_x p(x) \Longrightarrow \forall_x q(x)) \Longrightarrow (\exists_x \neg p(x) \vee \forall_x q(x))$

b) $(\exists_x p(x) \vee \exists_x q(x)) \Longrightarrow \exists_x (p(x) \vee q(x))$

c) $\exists_x (p(x) \wedge q(x)) \Longrightarrow (\exists_x p(x) \wedge \exists_x q(x))$

d) $\forall_x (p(x) \vee q(x)) \Longrightarrow (\forall_x p(x) \vee \forall_x q(x))$

Q Let P(m, n) be the statement **"m divides n"** where the Universe of discourse for both the variables is the set of positive integers. Determine the truth values of the following propositions. **(NET-Dec-2015)**

(a) $\exists m \, \forall n \, P(m, n)$ **(b)** $\forall n \, P(1, n)$ **(c)** $\forall m \, \forall n \, P(m, n)$

Codes:

A) (a) - True; (b) - True; (c) – False

B) (a) - True; (b) - False; (c) – False

C) (a) - False; (b) - False; (c) – False

D) (a) - True; (b) - True; (c) – True

Q Which one of the following well-formed formulae is a tautology? **(GATE-2015) (2 Marks)**

a) $\forall_x \exists_y R(x, y) \leftrightarrow \exists_y \forall_x R(x, y)$

b) $(\forall_x [\exists_y R(x, y) \rightarrow S(x, y)]) \rightarrow \forall_x \exists_y S(x, y)$

c) $[\forall_x \exists_y (P(x, y) \rightarrow R(x, y))] \leftrightarrow [\forall_x \exists_y (\neg P(x, y) \vee R(x, y))]$

d) $\forall_x \forall_y P(x, y) \rightarrow \forall_x \forall_y P(y, x)$

Q The CORRECT formula for the sentence, **"not all rainy days are cold"** is **(GATE-2014) (2 Marks)**

a) $\forall_d (Rainy(d) \land \sim Cold(d))$

b) $\forall_d (\sim Rainy(d) \rightarrow Cold(d))$

c) $\exists_d (\sim Rainy(d) \rightarrow Cold(d))$

d) $\exists_d (Rainy(d) \land \sim Cold(d))$

Q Consider the statement

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement? **(GATE-2014) (1 Marks)**

a) $\forall_x$: glitters(x) $\Rightarrow$ ¬gold(x) — 10

$\sim\sim[\exists x\ [gl(x) \wedge \sim go(x)]]$

b) $\forall_x$: gold(x) $\Rightarrow$ glitters(x) — 14

$\sim[\forall x\ [\sim gl(x) \vee go(x)]]$

c) $\exists_x$: gold(x) $\wedge$ ¬glitters(x) — 7

$\sim[\forall x[gl(x) \rightarrow go(x)]]$

d) $\exists_x$: glitters(x) $\wedge$ ¬gold(x) — 67

Q The notation $\exists!_x P(x)$ denotes the proposition "there exists a unique x such that P(x) is true". Give the truth values of the following statements: **(NET-June-2014)**

I. $\exists!_x P(x) \rightarrow \exists x P(x)$

II. $\exists!_x \neg P(x) \rightarrow \neg\forall x P(x)$

(A) Both I & II are true.

(B) Both I & II are false.

(C) I – false, II – true

(D) I – true, II – false

Q Which one of the following is **NOT** logically equivalent to $\neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$? **(GATE-2013) (2 Marks)**

a) $\forall_x(\exists_z(\neg\beta) \rightarrow \forall_y(\alpha))$

b) $\forall_x(\forall_z(\beta) \rightarrow \exists_y(\neg\alpha))$

c) $\forall_x(\forall_y(\alpha) \rightarrow \exists_z(\neg\beta))$

d) $\forall_x(\exists_y(\neg\alpha) \rightarrow \exists_z(\neg\beta))$

Q What is the logical translation of the following lowing statement? **(GATE-2013) (2 Marks)**

"None of my friends are perfect."

A) $\cancel{\exists_x (F(x) \land \neg P(x))}$ 28

joy

B) $\cancel{\exists_x (\neg F(x) \land P(x))}$ 5

$$\forall x [F(x) \land \sim P(x)]$$

C) $\cancel{\exists_x (\neg F(x) \land \neg P(x))}$ 10

D) $\neg \boxed{\exists_x (F(x) \land P(x))}$ 57

$$\forall x [\sim F(x) \lor \sim P(x)]$$

$$\forall x [F(x) \rightarrow \sim P(x)]$$

Q The truth value of the statements :

$\exists!_x P(x) \rightarrow \exists_x P(x)$ and $\exists!_x \sim P(x) \rightarrow \sim\forall_x P(x)$, (where the notation $\exists!_x P(x)$ denotes the proposition "There exists a unique x such that P(x) is true") are : **(NET-Dec-2013)**

(A) True and False

(B) False and True

(C) False and False

(D) True and True

Q Let Q (x, y) denote **"x + y = 0"** and let there be two quantifications given as

(i) $\exists_y \forall_x Q(x, y)$ → f

(ii) $\forall_x \exists_y Q(x, y)$

−32 + 32

47 + (−47)

Where, x and y are real numbers. Then which of the following is valid? **(NET-Dec-2012)**

~~**(a)** I is true and II is false~~ – 6

(b) I is false and II is true – 44

~~**(c)** I is false and II is also false~~ –16

~~**(d)** both I and II are true~~ –34

Q What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational" (GATE-2012) (1 Marks)

a) $\exists_x$ (real(x) $\vee$ rational(x))

b) $\forall_x$ (real(x) $\rightarrow$ rational(x))

c) $\exists_x$ (real(x) $\wedge$ rational(x))

d) $\exists_x$ (rational(x) $\rightarrow$ real(x))

Q Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate? **(GATE-2011) (2 Marks)**

$$P(x) = \boxed{\neg(x=1)} \wedge \boxed{\forall_y(\exists_z(x=y*z) \Rightarrow (y=x) \vee (y=1))}$$

(A) P(x) being true means that x is a prime number — 38

(B) ~~P(x) being true means that x is a number other than 1~~ — 26

(C) ~~P(x) is always true irrespective of the value of x~~ — 13

(D) ~~P(x) being true means that x has exactly two factors other than 1 and x~~ 23

$x \neq 1$

1 * x

x * 1

$x = y * z$

y = x (z = 1)

y = 1 (x = z)

6 = 2 * 3

7 = 1 × 7

7 × 1

$\forall x$ $\longrightarrow$

$\exists x$ $\wedge$

Q Suppose the predicate F (x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula $\forall_x \exists_y \exists_t (\neg F(x, y, t))$? **(GATE-2010) (2 Marks)**

(A) Everyone can fool some person at some time

(B) No one can fool everyone all the time

(C) Everyone cannot fool some person all the time

(D) No one can fool some person at some time

Q Consider the following well-formed formulae:

1) $\neg\forall x(P(x))$ **2)** $\neg\exists x(P(x))$ **3)** $\neg\exists x(\neg P(x))$ **4)** $\exists x(\neg P(x))$

Which of the above are equivalent? **(GATE-2009) (2 Marks)**

a) I and III

b) I and IV

c) II and III

d) II and IV

Q Which one of the following is the most appropriate logical formula to represent the statement? **"Gold and silver ornaments are precious".** The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious **(GATE-2009) (2 Marks)**

(A) $\forall_x (P(x) \rightarrow (G(x) \wedge S(x)))$ – 4

(B) $\forall_x ((G(x) \wedge S(x)) \rightarrow P(x))$ – 11

(C) $\exists_x ((G(x) \wedge S(x)) \rightarrow P(x)$ – 6

(D) $\forall_x ((G(x) \vee S(x)) \rightarrow P(x))$ – 79

Q Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statements represents the following. Each finite state automaton has an equivalent pushdown automaton. **(GATE-2008) (1 Marks)**

a) $(\forall_x \text{ fsa}(x)) \Rightarrow (\exists y \text{ pda}(y) \wedge \text{equivalent}(x, y))$

b) $\neg\forall_y(\exists x \text{ fsa}(x) \Rightarrow \text{pda}(y) \wedge \text{equivalent}(x, y))$

c) $\forall_x\exists_y (\text{fsa}(x) \wedge \text{pda}(y) \wedge \text{equivalent}(x, y))$

d) $\forall_x\exists_y(\text{fsa}(y) \wedge \text{pda}(x) \wedge \text{equivalent}(x, y))$

Q Which of the following first order formula is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable. **(GATE-2008) (2 Marks)**

(A) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$

(B) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$

(C) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

(D) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

Q Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x)be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: **"Not every graph is connected"** ? **(GATE-2007) (2 Marks)**

(A) $\neg\forall_x (Graph(x) \rightarrow Connected(x))$

(B) $\exists_x (Graph(x) \wedge \neg Connected(x))$

(C) $\neg\forall_x (\neg Graph(x) \vee Connected(x))$

(D) $\forall_x (Graph(x) \rightarrow \neg Connected(x))$

Q Which one of these first-order logic formulae is valid? **(GATE-2007) (2 Marks)**

(A) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow (\forall x P(x) \Rightarrow \forall x Q(x))$

(B) $\exists x(P(x) \vee Q(x)) \Rightarrow (\exists x P(x) \Rightarrow \exists x Q(x))$

(C) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow (\exists x P(x) \wedge \exists x Q(x))$

(D) $\forall x \exists y\, P(x, y) \Rightarrow \exists y \forall x\, P(x, y)$

Q Which one of the first order predicate calculus statements given below correctly express the following English statement? **(GATE-2006) (2 Marks)**

"Tigers and lions attack if they are hungry or threatened"

a) $\forall_x[(\text{tiger}(x) \wedge \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)]$ — 33

b) $\forall_x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \wedge \text{attacks}(x)]$ — 9

c) $\forall_x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \text{attacks}(x) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x))]$ — 18

d) $\forall_x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)]$ — 40

$p \rightarrow q$

$\exists$ $\wedge$

$\forall$ $\rightarrow$

Q What is the first order predicate calculus statement equivalent to the following?

Every teacher is liked by some student (GATE-2005) (2 Marks)

(A) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

(B) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) ^ likes (y, x)]]

(C) $\exists_{(y)}$ $\forall_{(x)}$ [teacher (x) $\rightarrow$ [student (y) ^ likes (y, x)]]

(D) $\forall_{(x)}$ [teacher (x) ^ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

Q Let P(x) and Q(x) be arbitrary predicates. Which of the following statements is always TRUE? **(GATE-2005) (2 Marks)**

(A) $((\forall x(P(x) \lor Q(x)))) \Longrightarrow ((\forall x P(x)) \lor (\forall x Q(x)))$

(B) $(\forall x(P(x) \Longrightarrow Q(x))) \Longrightarrow ((\forall x P(x)) \Longrightarrow (\forall x Q(x)))$

(C) $(\forall x(P(x)) \Longrightarrow \forall x(Q(x))) \Longrightarrow (\forall x(P(x) \Longrightarrow Q(x)))$

(D) $(\forall x(P(x)) \Leftrightarrow (\forall x(Q(x)))) \Longrightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

Q Let a(x, y), b(x, y,) and c(x, y) be three statements with variables x and y chosen from some universe. Consider the following statement: **(GATE-2004) (2 Marks)**

$(\exists x)(\forall y)\ [(a(x, y) \wedge b(x, y)) \wedge \neg c(x, y)]$

Which one of the following is its equivalent?

a) $(\forall x)(\exists y)\ [(a(x, y) \vee b(x, y)) \rightarrow c(x, y)]$

b) $(\exists x)(\forall y)[(a(x, y) \vee b(x, y)) \wedge \neg c(x, y)]$

c) $\neg(\forall x)(\exists y)[(a(x, y) \wedge b(x, y)) \rightarrow c(x, y)]$

d) $\neg(\forall x)(\exists y)[(a(x, y) \vee b(x, y)) \rightarrow c(x, y)]$

Q Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y. **(GATE-2004) (1 Marks)**

(A) $(\exists_x)$ (boy(x) → $(\forall_y)$ (girl(y) ^ taller (x, y)))

(B) $(\exists_x)$ (boy(x) ^ $(\forall_y)$ (girl(y) ^ taller (x, y)))

(C) $(\exists_x)$ (boy(x) → $(\forall_y)$ (girl(y) → taller (x, y)))

(D) $(\exists_x)$ (boy(x) ^ $(\forall_y)$ (girl(y) → taller (x, y)))

Q Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable) **(GATE-2003) (2 Marks)**

a) $\{(\forall x)[\alpha] \Rightarrow (\forall x)[\beta]\} \Rightarrow \{(\forall x)[\alpha \Rightarrow \beta]\}$

b) $(\forall x)[\alpha]\Rightarrow(\exists x)[\alpha\wedge\beta]$

c) $\{(\forall x)[\alpha \vee \beta]\} \Rightarrow \{(\exists x)[\alpha]) \Rightarrow (\forall x)[\alpha]\}$

d) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha]) \Rightarrow (\forall x)[\beta])$

Q Which of the following predicate calculus statements is/are valid? **(GATE-1992) (1 Marks)**

a) $(\forall(x))P(x) \lor (\forall(x))Q(x) \Longrightarrow (\forall(x))(P(x) \lor Q(x))$

b) $(\exists(x))P(x) \land (\exists(x))Q(x) \Longrightarrow (\exists(x))(P(x) \land Q(x))$

c) $(\forall(x))(P(x) \lor Q(x)) \Longrightarrow (\forall(x))P(x) \lor (\forall(x))Q(x)$

d) $(\exists(x))(P(x) \lor Q(x)) \Longrightarrow \sim(\forall(x))P(x) \lor (\exists(x))Q(x)$