Doubt Clearing Session

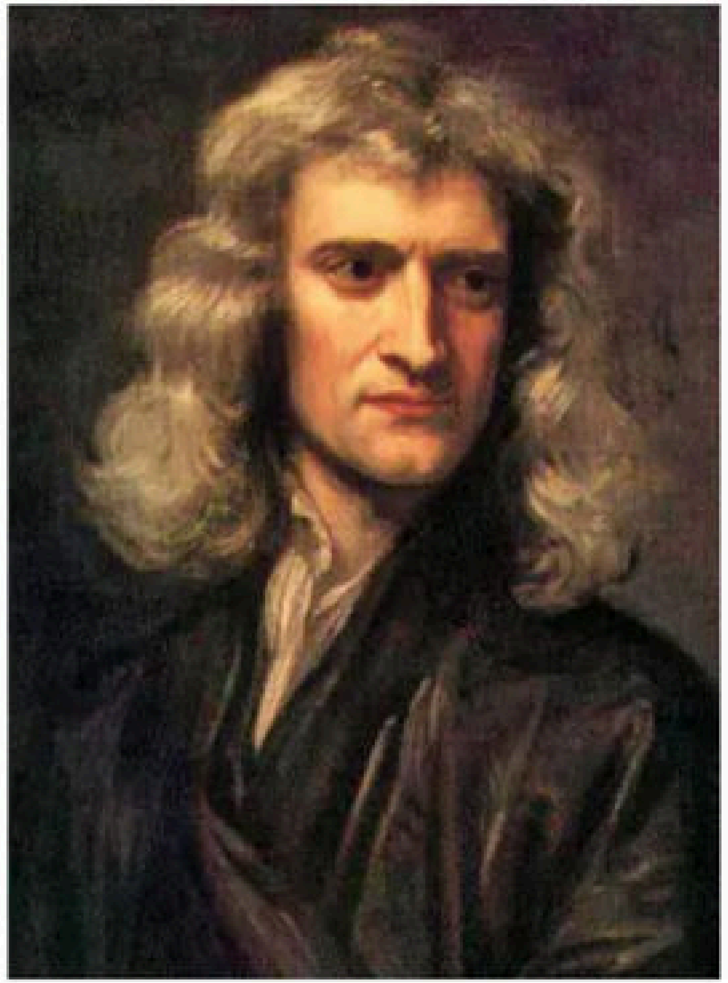
Course on Discrete Mathematics for GATE 2023

Sanchit Jain • Lesson 20 • Sept 23, 2022

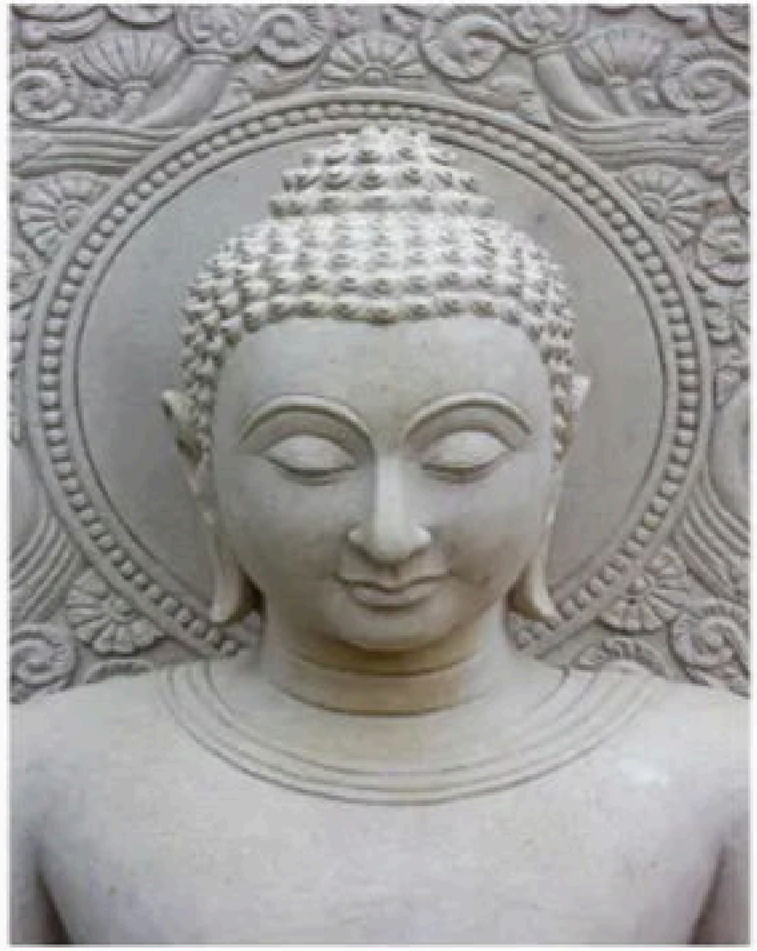
Proposition

- First we must look at the difference between Scientist and Philosopher.

- Philosopher give an idea or theory which may have different interpretation from person to person. It depends on the wisdom of a person.

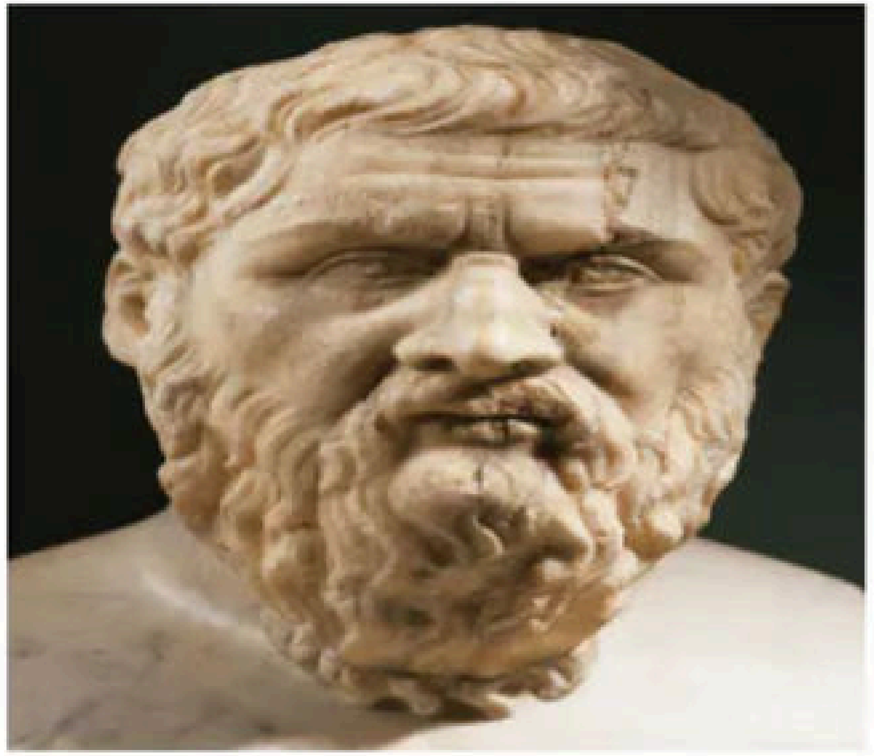


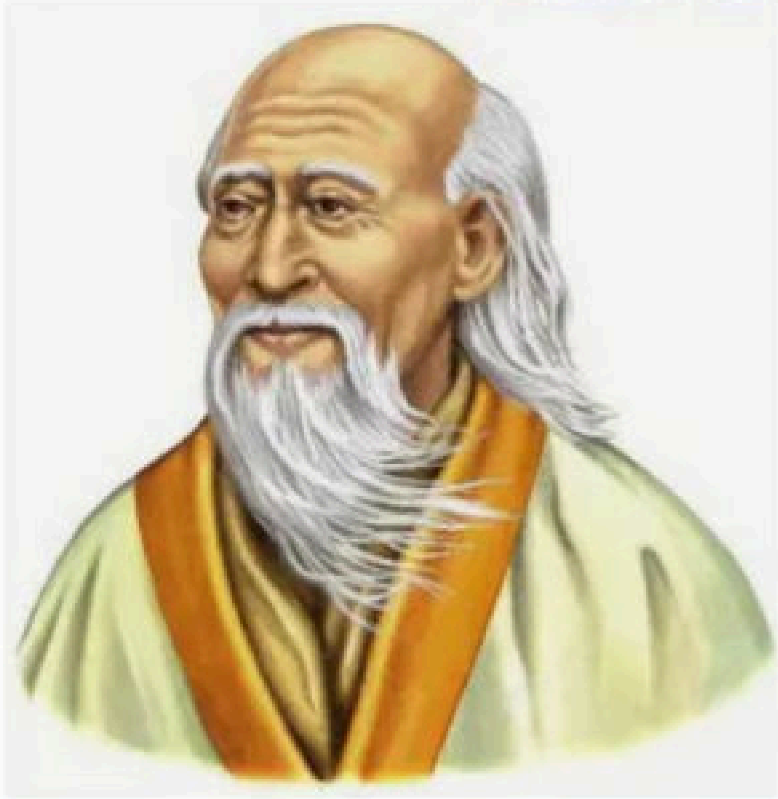
Mahaveer



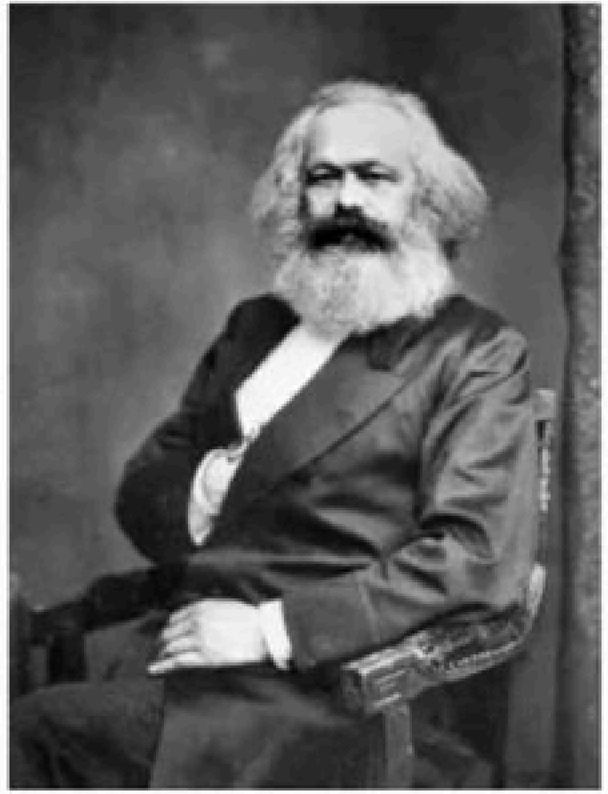
Gautam Buddha

- There are different philosophers in the world suggested different philosophy like, Chanakya Plato, Aristotle, Confucius, Laozi,

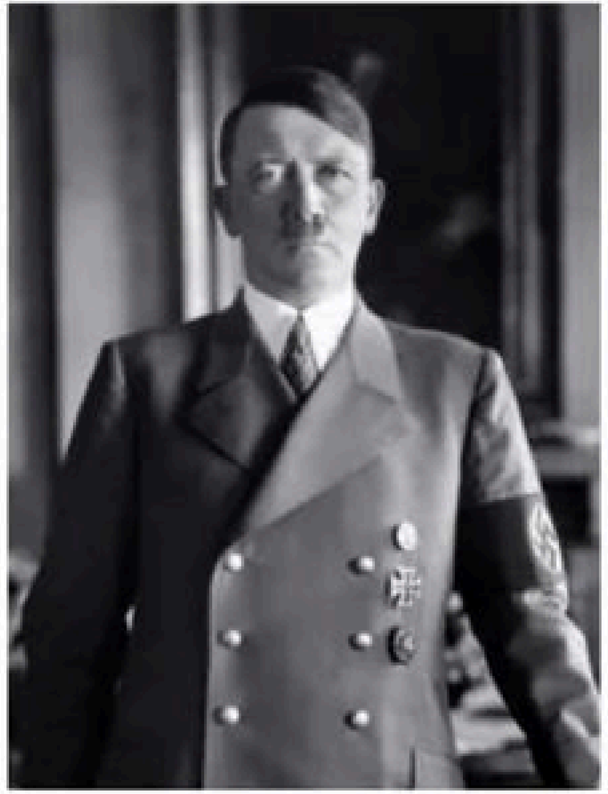




Karl Marx



Hitler



Mahatma Gandhi

Break

- Proposition with rules of logic actually is a method of reasoning (unambiguous, machinic, deterministic), given by Aristotle, who was the teacher of Alexander son of King Philip of Macedonia

- There may be different methods of reasoning for solving a problem apart from proposition.

- Proposition and rules of logic specify the meaning of mathematical statements. Logic is the basis of all mathematical reasoning, and of all automated reasoning.

- It has practical applications to the design of computing machines, to the specification of systems, to artificial intelligence, to computer programming, to programming languages, and to other areas of computer science, as well as to many other fields of study.

- To understand mathematics, we must understand what makes up a correct mathematical argument, that is, a proof.

- Once we prove a mathematical statement is true, we call it a theorem. A collection of theorems on a topic organize what we know about this topic.

- Everyone knows that proofs are important throughout mathematics, even in computer science. In fact, proofs are used to verify that computer programs produce the correct output for all possible input values, to show that algorithms always produce the correct result.

- The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

- The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic or predicate calculus(study of propositions)**.

- It was first developed systematically by the Greek philosopher Aristotle more than 2300 years ago.

- We now turn our attention to methods for producing new propositions from those that we already have. These methods were discussed by the English mathematician George Boole in 1854 in his book *The Laws of Thought.*

Break

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.
  1. Delhi is the capital of USA
  2. How are you doing
  3. 5<= 11
  4. Temperature is less than 10 C
  5. It is cold today
  6. Read this carefully
  7. X + y = z

1. Premises(proposition) is always considered to be true.

2. Premises is a statement that provides reason or support for the conclusion(proposition).

1. If a set of Premises(P) yield another proposition Q(Conclusion), then it is called an Argument.

2. An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

$\{P_1, P_2, P_3 \ldots, P_N\} \mid\!- Q$	P_1 P_2 P_3 . . . P_N …… Q …….	$\{P_1 \wedge P_2 \wedge P_3 \wedge \ldots. \wedge P_N\} \mid\!- Q$

Break

- **<u>Law of contradiction</u> -** the law of non-contradiction (LNC) (also known as the law of contradiction, principle of non-contradiction (PNC), or the principle of contradiction) states that

- Contradictory propositions cannot both be true in the same sense at the same time.
  - e.g. the two propositions "*A is B*" and "*A is not B*" are mutually exclusive.

- **Law of excluded middle** - the law of excluded middle (or the principle of excluded middle) states that for any proposition, either that proposition is true or its negation is true.

Break

Types of proposition

1. We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent propositions, just as letters are used to denote numerical variables.

2. The conventional letters used for propositional variables are *p, q, r, s*. The **truth value** of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

- Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Operators / Connectives

1. **Negation: -** let p be a proposition, then negation of p new proposition, denoted by $\neg p$, is the statement "it is not the case that p".

2. The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p. e.g. ¬"Michael's PC runs Linux" = "It is not the case that Michael's PC runs Linux." = "Michael's PC does not run Linux."

Negation	
P	¬P
F	
T	

Break

Conjunction

- Let p and q be propositions. The *conjunction* of p and q, denoted by $p \land q$, is the proposition "p and q."

- The conjunction $p \land q$ is true when both p and q are true and is false otherwise.

Conjunction		
p	q	p ∧ q
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \land q)$
Q	p

2	
P_1	P
Q	$p \land q$

3	
P_1	P
P_2	q
Q	$p \land q$

4	
P_1	$\neg(p \land q)$
P_2	P
Q	$\neg q$

5	
P_1	$\neg (p \land q)$
P_2	q
Q	$\neg p$

6	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	q

7	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	$\neg q$

Break

Disjunction

- Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition. "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Disjunction		
p	q	p ∨ q
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \wedge q)$
Q	$p \vee q$

2	
P_1	$p \vee q$
Q	$(p \wedge q)$

3	
P_1	$\neg(p \vee q)$
Q	$\neg p$

4	
P_1	$(p \vee q)$
Q	$\neg p$

5	
P_1	$(p \vee q)$
P_2	$\neg p$
Q	q

6	
P_1	$(p \vee q)$
P_2	$\neg q$
Q	p

7	
P_1	$(p \vee q)$
P_2	p
Q	$\neg q$

8	
P_1	$(p \vee q)$
P_2	p
Q	q

Break

Implication

1. Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q". The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

2. In conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion.

3. The statement $p \rightarrow q$ is called a conditional statement because $p \rightarrow q$ asserts that q is true on the condition that p holds.

Implication		
p	q	$p \rightarrow q$
F	F	
F	T	
T	F	
T	T	

- Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

1. "If Maria learns discrete mathematics, then she will find a good job."

2. "Maria will find a good job when she learns discrete mathematics."

3. "For Maria to get a good job, it is sufficient for her to learn discrete mathematics."

Implication		
p	q	$P \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$P \to q$	$\neg p$	$\neg q$	$\neg P \to \neg q$	$q \to p$	$\neg q \to \neg p$	$\neg p \lor q$
F	F							
F	T							
T	F							
T	T							

1. $p \rightarrow q$ *implication*

2. $q \rightarrow p$ *converse*

3. $\neg p \rightarrow \neg q$ *inverse*

4. $\neg q \rightarrow \neg p$ *contra positive*

1. $p \rightarrow q = \neg q \rightarrow \neg p$

2. $p \rightarrow q$ *will be true if either* p *is false or* q *is true,* $p \rightarrow q = \neg p \vee q$

Q consider the following arguments and find which of them are valid?

Modus Ponens	
P_1	$p \to q$
P_2	P
Q	Q

Modus Tollens	
P_1	$p \to q$
P_2	$\neg Q$
Q	$\neg p$

1	
P_1	$\neg p$
Q	$p \to q$

2	
P_1	q
Q	$p \to q$

3	
P_1	$\neg(p \to q)$
Q	$\neg q$

4	
P_1	$\neg(p \to q)$
Q	p

Break

Bi-conditional

- Let *p* and *q* be propositions. The bi*conditional statement* $p \leftrightarrow q$ is the proposition.
  - “ *p if and only* q”.

- The biconditional statement $p \leftrightarrow q$ *is true when p and q have the same values, and false otherwise. Biconditional statements are also called bi-implications.*
  - “*p* is necessary and sufficient for *q*”
  - “if *p* then *q*, and conversely”
  - “*p* iff *q*.”
  - $p \leftarrow\rightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

Bi-conditional		
p	q	$P \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
Q	$p \rightarrow r$

2	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow r$
Q	r

3	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow s$
Q	$r \vee s$

4	
P_1	$p \rightarrow r$
P_2	$q \rightarrow s$
P_3	$\neg r \vee \neg s$
Q	$\neg p \vee \neg q$

Q consider the following arguments and find which of them are valid?

5	
P_1	p
P_2	q
Q	r

6	
P_1	p
P_2	$\neg p$
Q	q

7	
P_1	
Q	q

Break

Type of cases

- **Tautology/valid: -** A propositional function which is always having truth in the last column, is called tautology. E.g. $p \lor \neg p$

p	¬ p	p ∨ ¬ p
F	T	
T	F	

- **<u>Contradiction/ Unsatisfiable</u>: -** A propositional function which is always having false in the last column, is called Contradiction. E.g. $\boldsymbol{p \land \neg p}$

p	¬ p	p ∧ ¬ p
F	T	
T	F	

- **Contingency:** - A propositional function which is neither a tautology nor a contradiction, is called Contingency. E.g. $\boldsymbol{p \vee q}$

p	q	p ∨ q
F	F	
F	T	
T	F	
T	T	

- **<u>Satisfiable</u>: -** A propositional function which is not contradiction is satisfiable. i.e. it must have at least one truth value in the final column e.g. $\boldsymbol{p} \vee \boldsymbol{q}$

- **<u>Functionality Complete Set</u>: -** A set of connectives is said to be functionally complete if it is able to write any propositional function.
  - $\{\wedge, \neg\}$
  - $\{\vee, \neg\}$

Break

Let p and q be two propositions. Consider the following two formulae in propositional logic. **(Gate-2021) (1 Marks)**

- $S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$
- $S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

A. Both S_1 and S_2 are tautologies.
B. S_1 is a tautology but S_2 is not a tautology
C. S_1 is not a tautology but S_2 is a tautology
D. Neither S_1 nor S_2 is a tautology

Choose the correct choice(s) regarding the following proportional logic assertion S: **(GATE- 2021)**

$$S : ((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$$

A. S is neither a tautology nor a contradiction

B. S is a tautology

C. S is a contradiction

D. The antecedent of S is logically equivalent to the consequent of S

Q Let P, Q, R and S be Propositions.
Assume that the equivalences $P \Leftrightarrow (Q \vee \neg Q)$ and $Q \Leftrightarrow R$ hold.
Then the truth value of the formula $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ is always:
(NET-Jan-2017)

A) True

B) False

C) Same as truth table of Q

D) Same as truth table of S

Q Consider two well-formed formulas in propositional logic

F_1: $P \Rightarrow \neg P$ **F_2**: $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which one of the following statements is correct? **(GATE-2001) (1 Marks)** **(NET-Jan-2017)**

A) F_1 is satisfiable, F_2 is valid

B) F_1 unsatisfiable, F_2 is satisfiable

C) F_1 is unsatisfiable, F_2 is valid

D) F_1 and F_2 are both satisfiable

Q Let p, q, and r be the propositions and the expression (p -> q) -> r be a contradiction. Then, the expression (r -> p)-> q is **(GATE-2017) (2 Marks)**

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

Q The first order logic (FOL) statement $((R \lor Q) \land (P \lor \neg Q))$ is equivalent to which of the following? **(NET-Jan-2017)**

A) $((R \lor \neg Q) \land (P \lor \neg Q) \land (R \lor P))$

B) $((R \lor Q) \land (P \lor \neg Q) \land (R \lor P))$

C) $((R \lor Q) \land (P \lor \neg Q) \land (R \lor \neg P))$

D) $((R \lor Q) \land (P \lor \neg Q) \land (\neg R \lor P))$

Q the statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below? **(GATE-2017) (1 Marks)**

1) $p \Rightarrow q$ **2)** $q \Rightarrow p$ **3)** $(\neg q) \vee (p)$ **4)** $(\neg p) \vee q$

a) 1 only **b)** 1 and 4 only **c)** 2 only **d)** 2 and 3 only

Q Let P and Q be two propositions, $\neg (P \leftrightarrow Q)$ is equivalent to: **(NET-Jan-2017)**

(I) $P \leftrightarrow \neg Q$ **(II)** $\neg P \leftrightarrow Q$ **(III)** $\neg P \leftrightarrow \neg Q$ (IV) $Q \rightarrow P$

A) Only (I) and (II) **B)** Only (II) and (III)

C) Only (III) and (IV) **D)** None of the above

Q In propositional logic if $(P \rightarrow Q) \wedge (R \rightarrow S)$ and $(P \vee R)$ are two premises such that

$(P \rightarrow Q) \wedge (R \rightarrow S)$

$P \vee R$

....

Y

....

is the premise: **(NET-Jan-2017)**

A) $P \vee R$ **B)** $P \vee S$

C) $Q \vee R$ **D)** $Q \vee S$

Q consider the following expression: **(GATE-2016) (1 Marks)**

i) false **ii)** Q **iii)** true **iv)** $P \lor Q$ **v)** $\neg Q \lor P$

The number of expressions given above that are logically implied by $P \land (P \Longrightarrow Q)$ is___

Q Consider the statement, "Either $-2 \le x \le -1$ or $1 \le x \le 2$". The negation of this statement is **(NET-July-2016)**

A) $x < -2$ or $2 < x$ or $-1 < x < 1$

B) $x < -2$ or $2 < x$

C) $-1 < x < 1$

D) $x \le -2$ or $2 < x$ or $-1 < x < 1$

Q The Boolean function $[\sim(\sim p \wedge q) \wedge \sim(\sim p \wedge \sim q)] \vee (p \wedge r)$ is equal to the Boolean function: **(NET-Aug-2016)**

a) q **b)** $p \wedge r$ **c)** $p \vee q$ **d)** p

Q Let p, q, r, s represents the following propositions. **(GATE-2016) (1 Marks)**

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ______________.

Q Consider the following logical inferences: **(NET-Aug-2016)**

I_1: If it is Sunday then school will not open. The school was open.
Inference: It was not Sunday.

I_2: If it is Sunday then school will not open. It was not Sunday.
Inference: The school was open.

Which of the following is correct?

A) Both I_1 and I_2 are correct inferences.

B) I_1 is correct but I_2 is not a correct inference.

C) I_1 is not correct but I_2 is a correct inference.

D) Both I_1 and I_2 are not correct inferences.

Q Consider the following two statements.

S_1: If a candidate is known to be corrupt, then he will not be elected.
S_2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?
(GATE-2015) (1 Marks)

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

Q Which of the following arguments are valid? **(NET-Dec-2015)**

(a) "If Gora gets the job and works hard, then he will be promoted. If Gora gets promotion, then he will be happy. He will not be happy, therefore, either he will not get the job or he will not work hard".

(b) "Either Puneet is not guilty or Pankaj is telling the truth. Pankaj is not telling the truth, therefore, Puneet is not guilty".

(c) If n is a real number such that $n>1$, then $n^2>1$. Suppose that $n^2>1$, then $n>1$.
Codes:

A) (a) and (c)
B) (b) and (c)
C) (a), (b) and (c)
D) (a) and (b)

Q "If my computations are correct and I pay the electric bill, then I will run out of money.

If I don't pay the electric bill, the power will be turned off.

Therefore, if I don't run out of money and the power is still on, then my computations are incorrect."

Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively. (Where ¬ means NOT) **(NET-June-2015)**

A) if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$
B) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(r \wedge p) \rightarrow c$
C) if $(c \wedge b) \rightarrow r$ and $\neg p \rightarrow b$, then $(\neg r \vee p) \rightarrow \neg C$
D) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

Q In Propositional Logic, given P and P → Q, we can infer ____________. **(NET-June-2015)**

a) ~ Q **b)** Q **c)** P ∧ Q **d)** ~ P ∧ Q

Q Which one of the following is NOT equivalent to $p \leftrightarrow q$? **(GATE-2015) (1 Marks)**

a) $(\neg p \vee q) \wedge (p \vee \neg q)$

b) $(\neg p \vee q) \wedge (q \rightarrow p)$

c) $(\neg p \wedge q) \vee (p \wedge \neg q)$

d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

Q In propositional logic $P \leftrightarrow Q$ is equivalent to (Where ~ denotes NOT) **(GATE-2015) (1 Marks)**

a) $\sim(P \vee Q) \wedge \sim(Q \vee P)$

b) $(\sim P \vee Q) \wedge (\sim Q \vee P)$

c) $(P \vee Q) \wedge (Q \vee P)$

d) $\sim(P \vee Q) \rightarrow \sim (Q \vee P)$

Q Consider the compound propositions given below as: **(NET-Dec-2015)**

(a) $p \vee \sim (p \wedge q)$ **(b)** $(p \wedge \sim q) \vee \sim (p \wedge q)$ **(c)** $p \wedge \quad (q \vee r)$

Which of the above propositions are tautologies?

A) (a) and (c) **B)** (b) and (c) **C)** (a) and (b) **D)** only (a)

Q In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following

"The result of the toss is head if and only if I am telling the truth"

Which of the following options is correct? **(Gate-2015)(2 Marks)**

a) The result is head

b) The result is tail

c) If the person is of Type 2, then the result is tail

d) If the person is of Type 1, then the result is tail

Q Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT? **(GATE-2014) (1 Marks)**

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

Q Which one of the following Boolean expressions is NOT a tautology? **(GATE-2014) (2 Marks)**

A) $((a \rightarrow b) \land (b \rightarrow c)) \rightarrow (a \rightarrow c)$

B) $(a \rightarrow c) \rightarrow (\sim b \rightarrow (a \land c))$

C) $(a \land b \land c) \rightarrow (c \lor a)$

D) $a \rightarrow (b \rightarrow a)$

Q Which one of the following propositional logic formulas is TRUE only when exactly two of p, q and r are TRUE? **(GATE-2014) (2 Marks)**

a) $((p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$

b) $(\sim (p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$

c) $((p \rightarrow q) \land r) \lor (p \land q \land \sim r)$

d) $(\sim (p \leftrightarrow q) \land r) \land (p \land q \land \sim r)$

Q Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**? **(GATE-2012) (1 Marks)**

(A) Both I_1 and I_2 are correct inferences
(B) I_1 is correct but I_2 is not a correct inference
(C) I_1 is not correct but I_2 is a correct inference
(D) Both I_1 and I_2 are not correct inferences

Q The proposition $\sim p \vee q$ is equivalent to **(NET-Dec-2011)**

(A) $p \to q$ **(B)** $q \to p$ **(C)** $p \leftrightarrow q$ **(D)** $p \vee q$

Q The binary operation $\odot$ is defined as follows

Which one of the following is equivalent to P$\vee$Q? **(GATE-2009) (2 Marks)**

a) $(\sim Q \odot \sim P)$

b) $(P \odot \sim Q)$

c) $(\sim P \odot Q)$

d) $(\sim P \odot \sim Q)$

P	Q	$P \odot Q$
T	T	T
T	F	T
F	T	F
F	F	T

Q An example of a Tautology is: **(NET-June-2008)**

a) $x \lor y$

b) $x \lor \neg y$

c) $x \lor \neg x$

d) $(x \rightarrow y) \land (y \rightarrow x)$

Q P and Q are two propositions. Which of the following logical expressions are equivalent?

(GATE-2008) (2 Marks)

I) $P \vee \neg Q$

II) $\neg(\neg P \wedge Q)$

III) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

IV) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

(A) Only I and II

(B) Only I, II and III

(C) Only I, II and IV

(D) All of I, II, III and IV

Q the Preposition $(p \to q) \wedge (\neg q \vee p)$ is equivalent to: **(NET-June-2006)**

a) $q \to p$

b) $p \to q$

c) $(q \to p) \wedge (p \to q)$

d) $(p \to q) \vee (q \to p)$

Q Consider the following propositional statements: **(GATE-2006) (2 Marks)**

$\mathbf{P_1}$: $((A \land B) \rightarrow C)) \equiv ((A \rightarrow C) \land (B \rightarrow C))$

$\mathbf{P_2}$: $((A \lor B) \rightarrow C)) \equiv ((A \rightarrow C) \lor (B \rightarrow C))$

Which one of the following is true?

(A) P_1 is a tautology, but not P_2

(B) P_2 is a tautology, but not P_1

(C) P_1 and P_2 are both tautologies

(D) Both P_1 and P_2 are not tautologies

Q A logical binary relation $\odot$, is defined as follows: **(GATE-2006) (2 Marks)**

Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$.

Which one of the following is equivalent to $A \wedge B$?

a) $(\sim A \odot B)$ **b)** $\sim(A \odot \sim B)$ **c)** $\sim(\sim A \odot \sim B)$ **d)** $\sim(\sim A \odot B)$

A	**B**	**$A \odot B$**
T	T	T
T	F	T
F	T	F
F	F	T

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is **(NET-dec-2005)**

a) True

b) Multiple Values

c) False

d) Cannot be determined

Q Let P, Q and R be three atomic prepositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology? **(GATE-2005) (2 Marks)**

(A) $X \equiv Y$ **(B)** $X \rightarrow Y$ **(C)** $Y \rightarrow X$ **(D)** $\neg Y \rightarrow X$

Q The following propositional statement is **(GATE-2004) (2 Marks)**

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

(A) satisfiable but not valid

(B) valid

(C) a contradiction

(D) none of the above

Q Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\neg p \lor q) \land (r \rightarrow s) \land (p \lor r)] \rightarrow (\neg s \rightarrow q)$

Q: $[(\neg p \land q) \land [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$

R: $[[(q \land r) \rightarrow p] \land (\neg q \lor p)] \rightarrow r$

S: $[p \land (p \rightarrow r) \land (q \lor \neg r)] \rightarrow q$

Which of the above arguments are valid? **(GATE-2004) (2 Marks)**

a) P and Q only

b) P and R only

c) P and S only

d) P, Q, R and S

Q Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \lor \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \land b) \rightarrow (a \land c) \lor d$ is always. **(GATE-2003) (2 Marks)**

(A) True

(B) False

(C) Same as the truth value of b

(D) Same as the truth value of d

Q The following resolution rule is used in logic programming:
Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg R)$
Which of the following statements related to this rule is FALSE? **(GATE-2003) (2 Marks)**

(A) $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid

(B) $(P \vee Q) \Rightarrow ((P \vee R) \wedge (Q \vee \neg R))$ is logically valid

(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable

(D) $(P \vee Q) \Rightarrow \text{FALSE}$ if and only if both P and Q are unsatisfiable

Q "If X, then Y unless Z" is represented by which of the following formulae in propositional logic? **(GATE-2002) (1 Marks)**

(A) $(X \wedge \neg Z) \rightarrow Y$

(B) $(X \wedge Y) \rightarrow \neg Z$

(C) $(X \rightarrow (Y \wedge \neg Z)$

(D) $(X \rightarrow Y (\wedge \neg Z)$

Q Which of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\neg$ as NOT, $\rightarrow$ as one-way implication and $\leftrightarrow$ as two-way implication **(GATE-1996) (2 Marks)**

a) $((x \rightarrow y) \wedge x) \rightarrow y$

b) $((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x$

c) $(x \rightarrow (x \vee y))$

d) $((x \vee y) \leftrightarrow (\neg x \rightarrow \neg y))$

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is **(GATE-1995) (2 Marks)**

a) True

b) Multiple Values

c) False

d) Cannot be determined

Q The proposition $p \land (\sim p \lor q)$ is: **(GATE-1993) (1 Marks)**

a) a tautology

b) logically equivalent to $p \land q$

c) logically equivalent to $p \lor q$

d) a contradiction

e) none of the above

Q Which of the following is/are a tautology? **(GATE-1992) (1 Marks)**

a) $a \vee b \rightarrow b \wedge c$

b) $a \wedge b \rightarrow b \vee c$

c) $a \vee b \rightarrow (b \rightarrow c)$

d) $a \rightarrow b \rightarrow (b \rightarrow c)$

Q Indicate which of the following well-formed formulae are valid: **(GATE-1990) (2 Marks)**

a) $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$

b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

d) $(P \Rightarrow R) \vee (Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)$

Break

Q consider the following argument

I_1: if today is Gandhi ji's birthday, then today is oct 2^{nd}

I_2: today is oct 2^{nd}

C: today is Gandhi ji's birthday

Q consider the following argument

I_1: if Canada is a country, then London is a city

I_2: London is not a city

C: Canada is not a country

Q find which of the following arguments are valid?

1) $((p \vee q) \vee \neg p) = T$

2) $\neg (p \vee q) \vee (\neg p \wedge q) \vee p = T$

3) $((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)) \wedge r = r$

4) $(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r)) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r) = T$

5) $(p \vee \neg(p \wedge q)) = T$

6) $(p \wedge q) \wedge (\neg p \vee \neg q) = F$

7) $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r$

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
P_3	$\neg r$
Q	$\neg p$

2	
P_1	$r \rightarrow s$
P_2	$p \rightarrow q$
P_3	$r \vee p$
Q	$s \vee q$

3	
P_1	$(p \rightarrow (q \rightarrow s))$
P_2	$\neg r \vee p$
P_3	q
P_4	p
Q	s

4	
P_1	$(p \rightarrow (r \rightarrow s))$
P_2	$\neg r \rightarrow \neg p$
P_3	p
Q	s

5	
P_1	$\neg p \rightarrow \neg r$
P_2	$\neg S$
P_3	$P \rightarrow w$
P_4	$R \vee s$
Q	W

6	
P_1	$\neg x \rightarrow y$
P_2	$\neg x \wedge \neg y$
Q	x

First order Predicate Logic

- Sometime propositional logic cannot derive any meaningful information even though, we as human can understand that argument is meaningful or not.

- P_1: Every Indian like cricket
- P_2: Sunny is an Indian
- Q: Sunny Likes cricket

- The reason propositional logic fails here because using only inference system we can not conclude Q from P_1 and P_2.

- In first order logic we understand, a new approach of subject and predicate to extract more information from a statement
  - 1 is a natural number (1 is subject, natural number is predicate)
  - we can write FOPL (short hand notation) for this as NatNo(1) = 1 is natural number
  - Similarly, we can understand the meaning of NatNo(2) as 2 is a natural number
  - NatNo(x): x is a natural number

$x \geq 5$

- Sometime subject is not a single element but representing the entire group.
  - Every Indian like Cricket.
  - We can have a propositional function $\boxed{\text{Cricket(x): x likes Cricket.}}$
  - We can fix domain of discussion or universe of discourse as, x is an Indian.

Cricket(x)

- If i say four Indian are there I_1, I_2, I_3, I_4
- I_1 likes cricket $\wedge$ I_2 likes cricket $\wedge$ I_3 likes cricket $\wedge$ I_4 likes cricket
- $\boxed{\text{Cricket}(I_1) \wedge \text{Cricket}(I_2) \wedge \text{Cricket}(I_3) \wedge \text{Cricket}(I_4)}$ ---- 140cr
- But problem with this notation is as there is 130+ corers Indian this formula will become very long and in some case we actually do not know how many subjects are there in the universe of discourse. so, we again need a short hand formula.
- $\forall_x$ Cricket(x), if we confine x to be Indian then it means every x like cricket.

$\forall x$ Indian(x)

Cricket(x) → Cricket(Atul)

Cricket(x) → $\forall x$ Indian(x)

- **<u>Universal quantifiers</u>**: - The universal quantification of a propositional function is the proposition that asserts

- P(x) is true for all values of x in the universe of discourse.

- The universe of discourse specifies the possible value of x.

- $\forall_x$ P(x), i.e. for all value of a P(x) is true

Break

- Let try some other statement 'Some Indian like samosa'

Samosa(x)

  - if i say four Indian are there I_1, I_2, I_3, I_4
  - I_1 like samosa $\vee$ I_2 like samosa $\vee$ I_3 like samosa $\vee$ I_4 like samosa
  - $Samosa(I_1) \vee Samosa(I_2) \vee Samosa(I_3) \vee Samosa(I_4)$
  - $\exists_x$ Samosa(x), if we confine x to be Indian then it means some x likes samosa.

$\forall$ $\exists$

- **Existential quantifiers**: - with existential quantifier of a propositional that is true if and only if P(x) is true for at least one value of x in the universe of discourse.

- There exists an element x is the universe of discourse such that P(x) is true.

- $\exists_x P(x)$, i.e. for at least one value of a P(x) is true

Break

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes cricket
  - Indian(x): x is an Indian
  - Cricket(x): x likes Cricket

  - if I_1 is Indian then likes cricket $\wedge$ if I_2 is Indian then likes cricket $\wedge$ if I_3 is Indian then likes cricket $\wedge$ if I_4 is Indian then likes cricket

  - $[\text{Indian}(I_1) \rightarrow \text{cricket}(I_1)] \wedge [\text{Indian}(I_2) \rightarrow \text{cricket}(I_2)] \wedge [\text{Indian}(I_3) \rightarrow \text{cricket}(I_3)] \wedge [\text{Indian}(I_4) \rightarrow \text{cricket}(I_4)]$

  - $\forall_x [\text{Indian}(x) \rightarrow \text{cricket}(x)]$

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes samosa
  - Indian(x): x is an Indian
  - Samosa(x): x likes Samosa

  - if I_1 is Indian then likes samosa $\vee$ if I_2 is Indian then likes samosa $\vee$ if I_3 is Indian then likes samosa $\vee$ if I_4 is Indian then likes samosa

  - $[\text{Indian}(I_1) \wedge \text{samosa}(I_1)] \vee [\text{Indian}(I_2) \wedge \text{samosa}(I_2)] \vee [\text{Indian}(I_3) \wedge \text{samosa}(I_3)] \vee [\text{Indian}(I_4) \wedge \text{samosa}(I_4)]$

  - $\exists_x [\text{Indian}(x) \wedge \text{samosa}(x)]$

- let check validity of a statement “Some Indians like samosa” = $\exists_x$ [Indian(x) $\rightarrow$samosa(x)], x is human
- let human contains four elements I_1, I_2, I_3, I_4 out of which I_1, I_2 are Indian while I_3, I_4 are not Indian
- Suppose I_1, I_2, I_3 do not likes samosa
  - [Indian(I_1) $\rightarrow$ samosa(I_1)] $\vee$ [Indian(I_2) $\rightarrow$ samosa(I_2)] $\vee$ [Indian(I_3) $\rightarrow$ samosa(I_3)]
  - [T $\rightarrow$ F] $\vee$ [T $\rightarrow$ F] $\vee$ [F $\rightarrow$ F]
  - [F] $\vee$ [F] $\vee$ [T]
  - T
- conclusion $\exists_x$ is not used with $\rightarrow$

Break

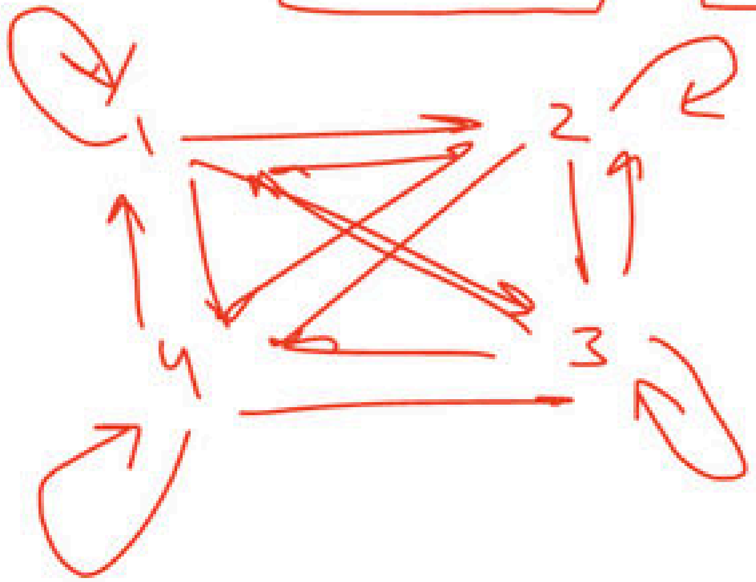
Negation

- $\neg\,[\cancel{\forall_x}\ P(x)] = \exists_x\ \neg P(x)$

- $\neg\,[\cancel{\exists_x}\ \cancel{P(x)}] = \forall_x\ \cancel{\neg P(x)}$

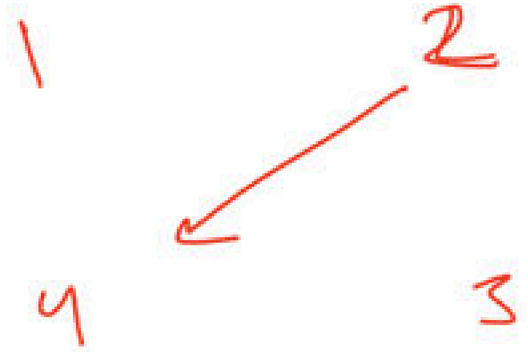
Let $L(x, y)$: x like y, which means x likes y or y is liked by x

1- $\forall_x \forall_y L(x, y)$



2- $\forall_y \forall_x L(x, y)$

3- $\exists_x \exists_y L(x, y)$



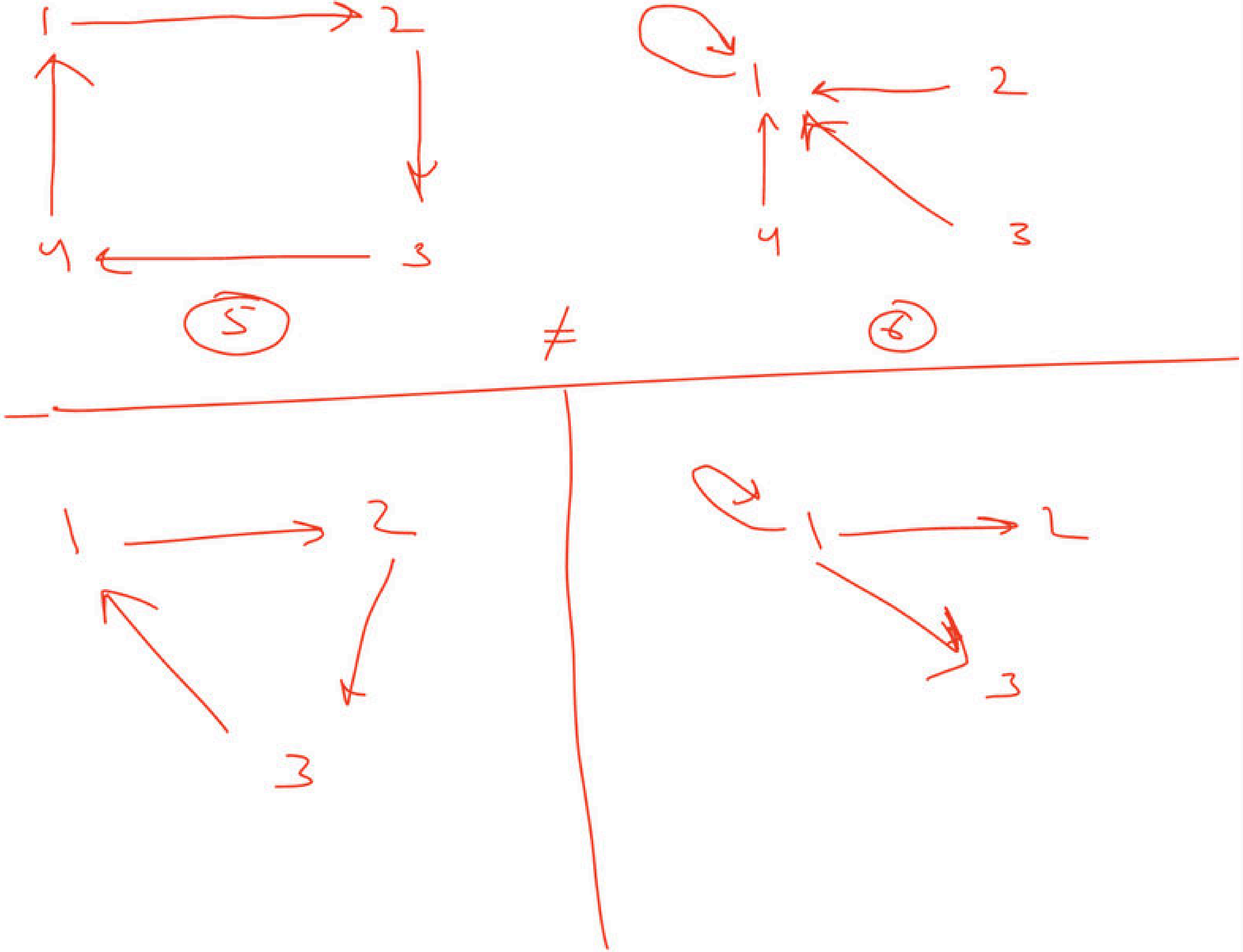
4- $\exists_y \exists_x L(x, y)$

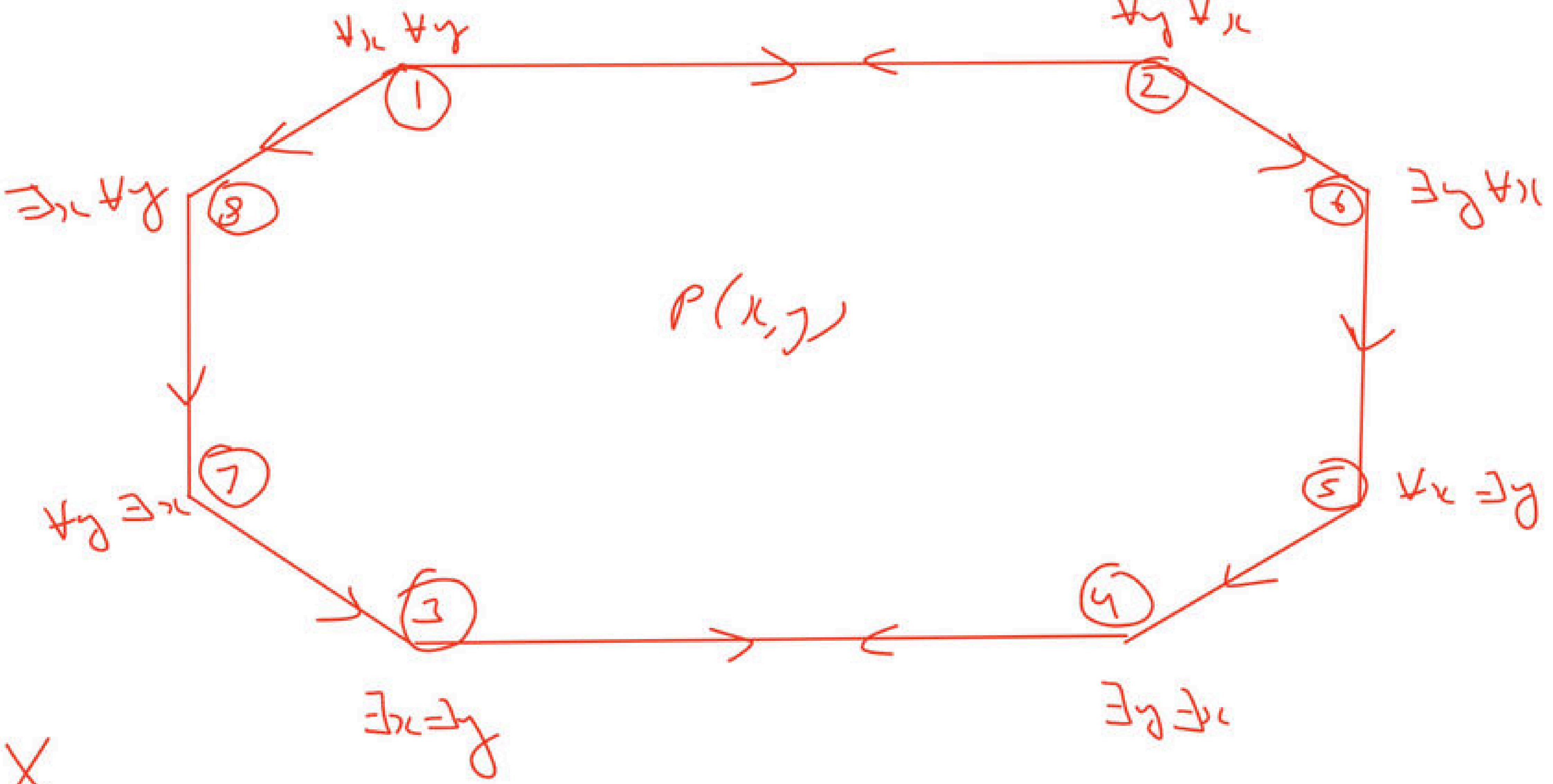
5- $\forall_x \exists_y L(x, y)$

6- $\exists_y \forall_x L(x, y)$

7- $\forall_y \exists_x L(x, y)$

8- $\exists_x \forall_y L(x, y)$





∃x ∀y

∀x ∃y

X

Break

	1
P_1	$\exists_x P(x) \vee \exists_x Q(x)$
Q	$\exists_x (P(x) \vee Q(x))$

	2
P_1	$\exists_x (P(x) \vee Q(x))$
Q	$\exists_x P(x) \vee \exists_x Q(x)$

	3
P_1	$\exists_x P(x) \wedge \exists_x Q(x)$
Q	$\exists_x (P(x) \wedge Q(x))$

	4
P_1	$\exists_x (P(x) \wedge Q(x))$
Q	$\exists_x P(x) \wedge \exists_x Q(x)$

	1
P_1	$\forall_x P(x) \vee \forall_x Q(x)$
Q	$\forall_x (P(x) \vee Q(x))$

	2
P_1	$\forall_x (P(x) \vee Q(x))$
Q	$\forall_x P(x) \vee \forall_x Q(x)$

	3
P_1	$\forall_x P(x) \wedge \forall_x Q(x)$
Q	$\forall_x (P(x) \wedge Q(x))$

	4
P_1	$\forall_x (P(x) \wedge Q(x))$
Q	$\forall_x P(x) \wedge \forall_x Q(x)$

	1
P_1	$[\forall_x P(x) \rightarrow \forall_x Q(x)]$
Q	$\forall_x [P(x) \rightarrow Q(x)]$

	2
P_1	$\forall_x [P(x) \rightarrow Q(x)]$
Q	$[\forall_x P(x) \rightarrow \forall_x Q(x)]$

Break

Q consider the statement $\exists_x[P(x) \wedge \neg Q(x)]$, Which of the following is equivalent?

a) $\forall_x\ [P(x) \rightarrow Q(x)]$

b) $\forall_x\ [\neg P(x) \rightarrow Q(x)]$

c) $\neg\ \{\forall_x\ [P(x) \rightarrow Q(x)]\}$

d) $\neg\ \{\forall_x\ [\neg P(x) \rightarrow Q(x)]\}$

Q negation of the statement

$\exists_x \forall_y [F(x, y) \rightarrow \{G(x, y) \vee H(x, y)\}] = \forall_x \exists_y [F(x, y) \wedge \{\neg G(x, y) \wedge \neg H(x, y)\}]$?

Q let in a set of all integers

G (x, y): x is greater than y

"for any given positive integer, there is a greater positive integer"

a) $\forall_x \exists_y G(x, y)$

b) $\exists_y \forall_x G(x, y)$

c) $\forall_y \exists_x G(x, y)$

d) $\exists_x \forall_y G(x, y)$

Q let in a set of all humans

L (x, y): x likes y

"there is someone, whom no one like"

a) $\forall_x \exists_y \{\neg L(x, y)\}$

b) $\{\neg \forall_x \exists_y L(x, y)\}$

c) $\neg \{\forall_y \exists_x L(x, y)\}$

d) $\neg \{\exists_Y \forall_X L(x, y)\}$

Break

Q Negation of the proposition $\exists_x H(x)$ **(NET-Jan-2017)**

(A) $\exists_x \neg H(x)$ **(B)** $\forall_x \neg H(x)$ **(C)** $\forall_x H(x)$ **(D)** $\neg x\ H(x)$

Q Consider the first-order logic sentence
F: $\forall_x (\exists_y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F? **(GATE-2017) (1 Marks)**

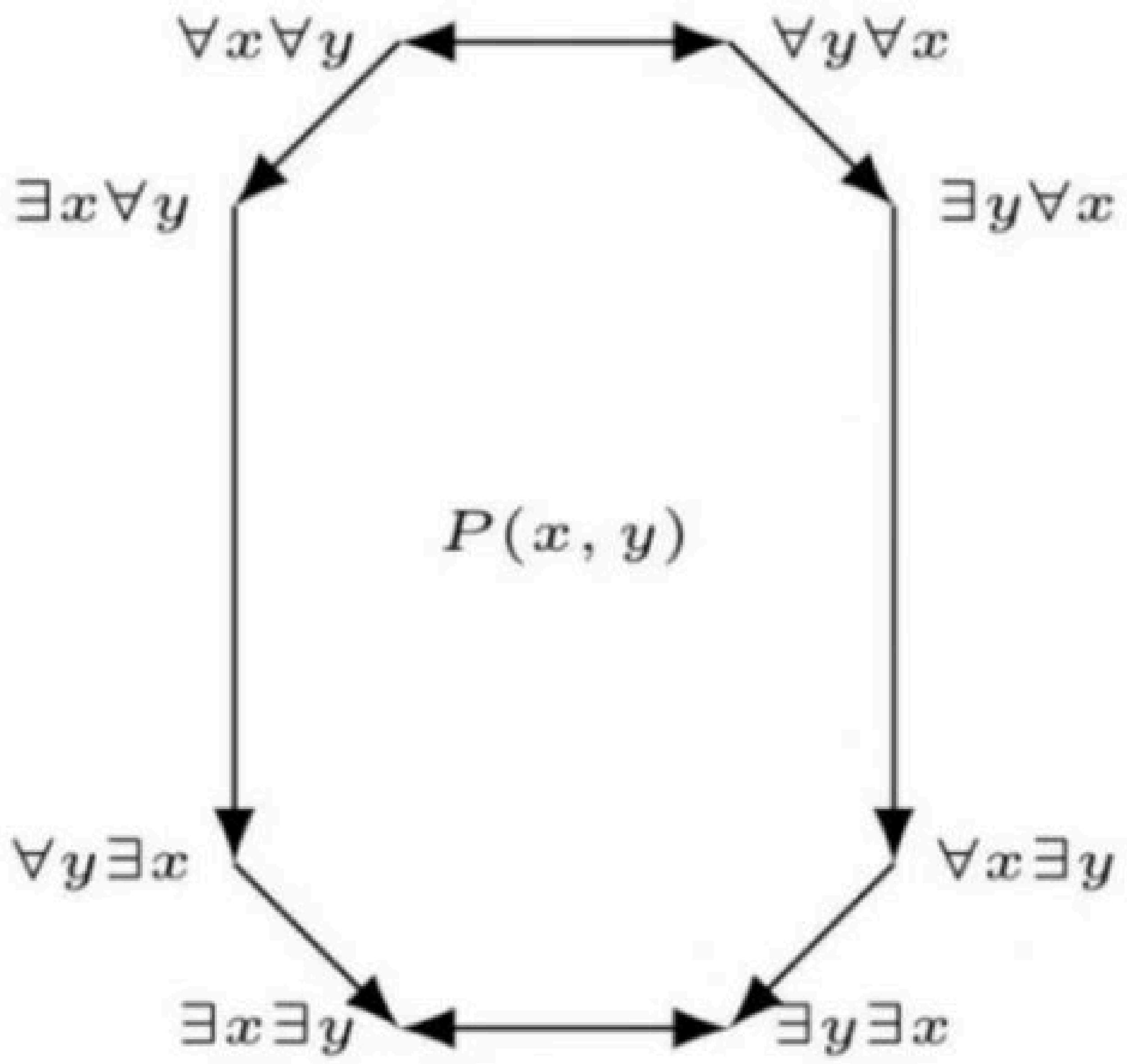
I. $\exists_y (\exists_x R(x, y))$
II. $\exists_y (\forall_x R(x, y))$
III. $\forall_y (\exists_x R(x, y))$
IV. $\sim\exists_x (\forall_y \sim R(x, y))$

(A) IV only
(B) I and IV only
(C) II only
(D) II and III only

Q Consider the first-order logic sentence

F: $\forall_x (\exists_y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F? **(GATE-2017) (1 Marks)**

I. $\exists_y (\exists_x R(x, y))$ II. $\exists_y (\forall_x R(x, y))$ III. $\forall_y (\exists_x R(x, y))$ IV. $\sim\exists_x (\forall_y \sim R(x, y))$



(A) IV only

(B) I and IV only

(C) II only

(D) II and III only

Q Which one of the following well-formed formulae in predicate calculus is **NOT** valid? **(GATE-2016) (2 Marks)**

a) $(\forall_x p(x) \Rightarrow \forall_x q(x)) \Rightarrow (\exists_x \neg p(x) \vee \forall_x q(x))$

b) $(\exists_x p(x) \vee \exists_x q(x)) \Rightarrow \exists_x (p(x) \vee q(x))$

c) $\exists_x (p(x) \wedge q(x)) \Rightarrow (\exists_x p(x) \wedge \exists_x q(x))$

d) $\forall_x (p(x) \vee q(x)) \Rightarrow (\forall_x p(x) \vee \forall_x q(x))$

Q Let P(m, n) be the statement **"m divides n"** where the Universe of discourse for both the variables is the set of positive integers. Determine the truth values of the following propositions. **(NET-Dec-2015)**

(a) $\exists m \, \forall n \, P(m, n)$ **(b)** $\forall n \, P(1, n)$ **(c)** $\forall m \, \forall n \, P(m, n)$

Codes:

A) (a) - True; (b) - True; (c) – False

B) (a) - True; (b) - False; (c) – False

C) (a) - False; (b) - False; (c) – False

D) (a) - True; (b) - True; (c) – True

Q Which one of the following well-formed formulae is a tautology? **(GATE-2015) (2 Marks)**

a) $\forall_x \exists_y R(x, y) \leftrightarrow \exists_y \forall_x R(x, y)$

b) $(\forall_x [\exists_y R(x, y) \rightarrow S(x, y)]) \rightarrow \forall_x \exists_y S(x, y)$

c) $[\forall_x \exists_y (P(x, y) \rightarrow R(x, y))] \leftrightarrow [\forall_x \exists_y (\neg P(x, y) \vee R(x, y))]$

d) $\forall_x \forall_y P(x, y) \rightarrow \forall_x \forall_y P(y, x)$

Q The CORRECT formula for the sentence, **“not all rainy days are cold”** is **(GATE-2014) (2 Marks)**

a) $\forall_d (Rainy(d) \land \sim Cold(d))$

b) $\forall_d (\sim Rainy(d) \rightarrow Cold(d))$

c) $\exists_d (\sim Rainy(d) \rightarrow Cold(d))$

d) $\exists_d (Rainy(d) \land \sim Cold(d))$

Q Consider the statement

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement? **(GATE-2014) (1 Marks)**

a) $\forall_x$: glitters(x) $\Rightarrow$ $\neg$gold(x)

b) $\forall_x$: gold(x) $\Rightarrow$ glitters(x)

c) $\exists_x$: gold(x) $\wedge$ $\neg$glitters(x)

d) $\exists_x$: glitters(x) $\wedge$ $\neg$gold(x)

Q The notation $\exists!_x P(x)$ denotes the proposition "there exists a unique x such that P(x) is true". Give the truth values of the following statements: **(NET-June-2014)**

I. $\exists!_x P(x) \rightarrow \exists x P(x)$

II. $\exists!_x \neg P(x) \rightarrow \neg \forall x P(x)$

(A) Both I & II are true.

(B) Both I & II are false.

(C) I – false, II – true

(D) I – true, II – false

Q Which one of the following is **NOT** logically equivalent to $\neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$? **(GATE-2013) (2 Marks)**

a) $\forall_x(\exists_z(\neg\beta) \rightarrow \forall_y(\alpha))$

b) $\forall_x(\forall_z(\beta) \rightarrow \exists_y(\neg\alpha))$

c) $\forall_x(\forall_y(\alpha) \rightarrow \exists_z(\neg\beta))$

d) $\forall_x(\exists_y(\neg\alpha) \rightarrow \exists_z(\neg\beta))$

Q What is the logical translation of the following lowing statement? **(GATE-2013) (2 Marks)**

"None of my friends are perfect."

A) $\exists_x(F(x) \wedge \neg P(x))$

B) $\exists_x(\neg F(x) \wedge P(x))$

C) $\exists_x(\neg F(x) \wedge \neg P(x))$

D) $\neg\exists_x(F(x) \wedge P(x))$

Q The truth value of the statements :

$\exists!_x P(x) \rightarrow \exists_x P(x)$ and $\exists!_x \sim P(x) \rightarrow \sim\forall_x P(x)$, (where the notation $\exists!_x P(x)$ denotes the proposition "There exists a unique x such that P(x) is true") are : **(NET-Dec-2013)**

(A) True and False

(B) False and True

(C) False and False

(D) True and True

Q Let Q (x, y) denote **"$x + y = 0$"** and let there be two quantifications given as

(i) $\exists_y \forall_x Q(x, y)$

(ii) $\forall_x \exists_y Q(x, y)$

Where, x and y are real numbers. Then which of the following is valid? **(NET-Dec-2012)**

(a) I is true and II is false

(b) I is false and II is true

(c) I is false and II is also false

(d) both I and II are true

Q What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational" (GATE-2012) (1 Marks)

a) $\exists_x$ (real(x) $\vee$ rational(x))

b) $\forall_x$ (real(x) $\rightarrow$ rational(x))

c) $\exists_x$ (real(x) $\wedge$ rational(x))

d) $\exists_x$ (rational(x) $\rightarrow$ real(x))

Q Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate? **(GATE-2011) (2 Marks)**

$$P(x) = \neg(x=1) \land \forall_y(\exists_z(x=y*z) \Rightarrow (y=x) \lor (y=1))$$

(A) P(x) being true means that x is a prime number

(B) P(x) being true means that x is a number other than 1

(C) P(x) is always true irrespective of the value of x

(D) P(x) being true means that x has exactly two factors other than 1 and x

Q Suppose the predicate F (x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula $\forall_x \exists_y \exists_t (\neg F(x, y, t))$? **(GATE-2010) (2 Marks)**

(A) Everyone can fool some person at some time

(B) No one can fool everyone all the time

(C) Everyone cannot fool some person all the time

(D) No one can fool some person at some time

Q Consider the following well-formed formulae:

1) $\neg\forall x(P(x))$ **2)** $\neg\exists x(P(x))$ **3)** $\neg\exists x(\neg P(x))$ **4)** $\exists x(\neg P(x))$

Which of the above are equivalent? **(GATE-2009) (2 Marks)**

a) I and III

b) I and IV

c) II and III

d) II and IV

Q Which one of the following is the most appropriate logical formula to represent the statement? **"Gold and silver ornaments are precious".** The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious **(GATE-2009) (2 Marks)**

(A) $\forall_x (P(x) \rightarrow (G(x) \wedge S(x)))$

(B) $\forall_x ((G(x) \wedge S(x)) \rightarrow P(x))$

(C) $\exists_x ((G(x) \wedge S(x)) \rightarrow P(x)$

(D) $\forall_x ((G(x) \vee S(x)) \rightarrow P(x))$

Q Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statements represents the following. Each finite state automaton has an equivalent pushdown automaton. **(GATE-2008) (1 Marks)**

a) $(\forall_x \text{ fsa}(x)) \Rightarrow (\exists y \text{ pda}(y) \wedge \text{equivalent}(x, y))$

b) $\neg\forall_y(\exists x \text{ fsa}(x) \Rightarrow \text{pda}(y) \wedge \text{equivalent}(x, y))$

c) $\forall_x\exists_y (\text{fsa}(x) \wedge \text{pda}(y) \wedge \text{equivalent}(x, y))$

d) $\forall_x\exists_y(\text{fsa}(y) \wedge \text{pda}(x) \wedge \text{equivalent}(x, y))$

Q Which of the following first order formula is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable. **(GATE-2008) (2 Marks)**

(A) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$

(B) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$

(C) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

(D) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

Q Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x)be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: **"Not every graph is connected"** ? **(GATE-2007) (2 Marks)**

(A) $\neg\forall_x$ (Graph(x) $\rightarrow$ Connected(x))

(B) $\exists_x$ (Graph(x) $\wedge$ $\neg$Connected(x))

(C) $\neg\forall_x$ ($\neg$Graph(x) $\vee$ Connected(x))

(D) $\forall_x$ (Graph(x) $\rightarrow$ $\neg$Connected(x))

Q Which one of these first-order logic formulae is valid? **(GATE-2007) (2 Marks)**

(A) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow (\forall x P(x) \Rightarrow \forall x Q(x))$

(B) $\exists x(P(x) \vee Q(x)) \Rightarrow (\exists x P(x) \Rightarrow \exists x Q(x))$

(C) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow (\exists x P(x) \wedge \exists x Q(x))$

(D) $\forall x \exists y\, P(x, y) \Rightarrow \exists y \forall x\, P(x, y)$

Q Which one of the first order predicate calculus statements given below correctly express the following English statement? **(GATE-2006) (2 Marks)**

"Tigers and lions attack if they are hungry or threatened"

a) $\forall_x[(\text{tiger}(x) \land \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)]$

b) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \land \text{attacks}(x)]$

c) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow \text{attacks}(x) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x))]$

d) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)]$

Q What is the first order predicate calculus statement equivalent to the following?

Every teacher is liked by some student (GATE-2005) (2 Marks)

(A) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

(B) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) ^ likes (y, x)]]

(C) $\exists_{(y)}$ $\forall_{(x)}$ [teacher (x) $\rightarrow$ [student (y) ^ likes (y, x)]]

(D) $\forall_{(x)}$ [teacher (x) ^ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

Q Let P(x) and Q(x) be arbitrary predicates. Which of the following statements is always TRUE? **(GATE-2005) (2 Marks)**

(A) $((\forall x(P(x) \vee Q(x)))) \Longrightarrow ((\forall x P(x)) \vee (\forall x Q(x)))$

(B) $(\forall x(P(x) \Longrightarrow Q(x))) \Longrightarrow ((\forall x P(x)) \Longrightarrow (\forall x Q(x)))$

(C) $(\forall x(P(x)) \Longrightarrow \forall x(Q(x))) \Longrightarrow (\forall x(P(x) \Longrightarrow Q(x)))$

(D) $(\forall x(P(x)) \Leftrightarrow (\forall x(Q(x)))) \Longrightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

Q Let a(x, y), b(x, y,) and c(x, y) be three statements with variables x and y chosen from some universe. Consider the following statement: **(GATE-2004) (2 Marks)**

$(\exists x)(\forall y)\ [(a(x, y) \land b(x, y)) \land \neg c(x, y)]$

Which one of the following is its equivalent?

a) $(\forall x)(\exists y)\ [(a(x, y) \lor b(x, y)) \rightarrow c(x, y)]$

b) $(\exists x)(\forall y)[(a(x, y) \lor b(x, y)) \land \neg c(x, y)]$

c) $\neg(\forall x)(\exists y)[(a(x, y) \land b(x, y)) \rightarrow c(x, y)]$

d) $\neg(\forall x)(\exists y)[(a(x, y) \lor b(x, y)) \rightarrow c(x, y)]$

Q Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y. **(GATE-2004) (1 Marks)**

(A) $(\exists_x)\ (boy(x) \rightarrow (\forall_y)\ (girl(y)\ \wedge\ taller\ (x, y)))$

(B) $(\exists_x)\ (boy(x)\ \wedge\ (\forall_y)\ (girl(y)\ \wedge\ taller\ (x, y)))$

(C) $(\exists_x)\ (boy(x) \rightarrow (\forall_y)\ (girl(y) \rightarrow taller\ (x, y)))$

(D) $(\exists_x)\ (boy(x)\ \wedge\ (\forall_y)\ (girl(y) \rightarrow taller\ (x, y)))$

Q Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable) **(GATE-2003) (2 Marks)**

a) $\{(\forall x)[\alpha] \Rightarrow (\forall x)[\beta]\} \Rightarrow \{(\forall x)[\alpha \Rightarrow \beta]\}$

b) $(\forall x)[\alpha] \Rightarrow (\exists x)[\alpha \wedge \beta]$

c) $\{(\forall x)[\alpha \vee \beta]\} \Rightarrow \{(\exists x)[\alpha]) \Rightarrow (\forall x)[\alpha]\}$

d) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha]) \Rightarrow (\forall x)[\beta])$

Q Which of the following predicate calculus statements is/are valid? **(GATE-1992) (1 Marks)**

a) $(\forall(x))P(x) \vee (\forall(x))Q(x) \Longrightarrow (\forall(x))(P(x) \vee Q(x))$

b) $(\exists(x))P(x) \wedge (\exists(x))Q(x) \Longrightarrow (\exists(x))(P(x) \wedge Q(x))$

c) $(\forall(x))(P(x) \vee Q(x)) \Longrightarrow (\forall(x))P(x) \vee (\forall(x))Q(x)$

d) $(\exists(x))(P(x) \vee Q(x)) \Longrightarrow \sim(\forall(x))P(x) \vee (\exists(x))Q(x)$