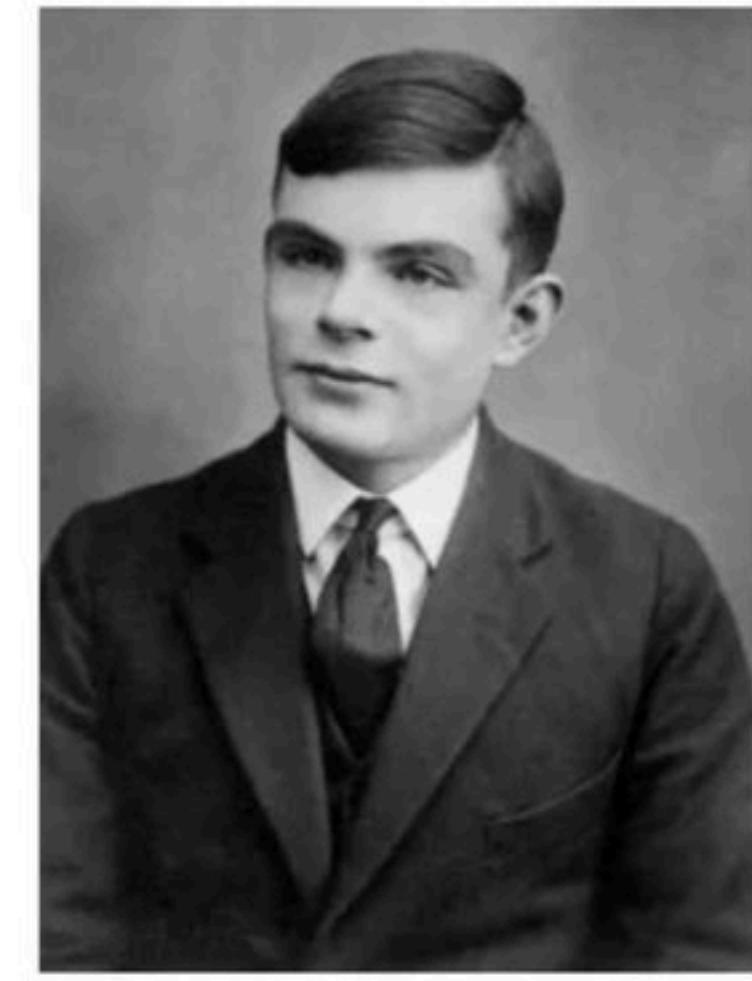
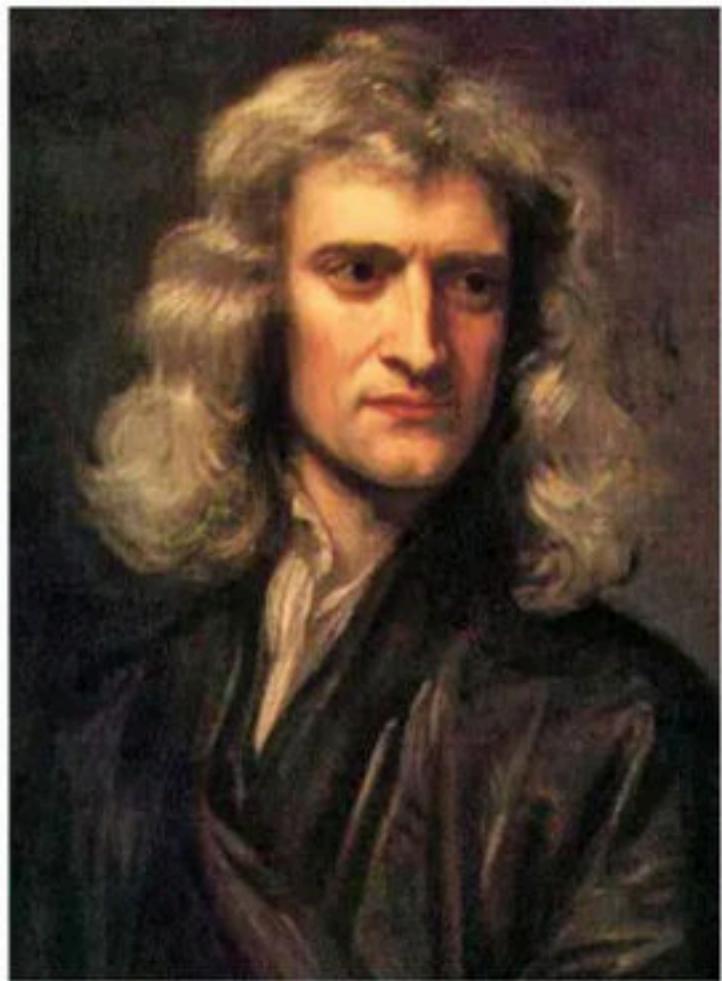


Proposition - Part V

Course on Discrete Mathematics for GATE 2023

Proposition

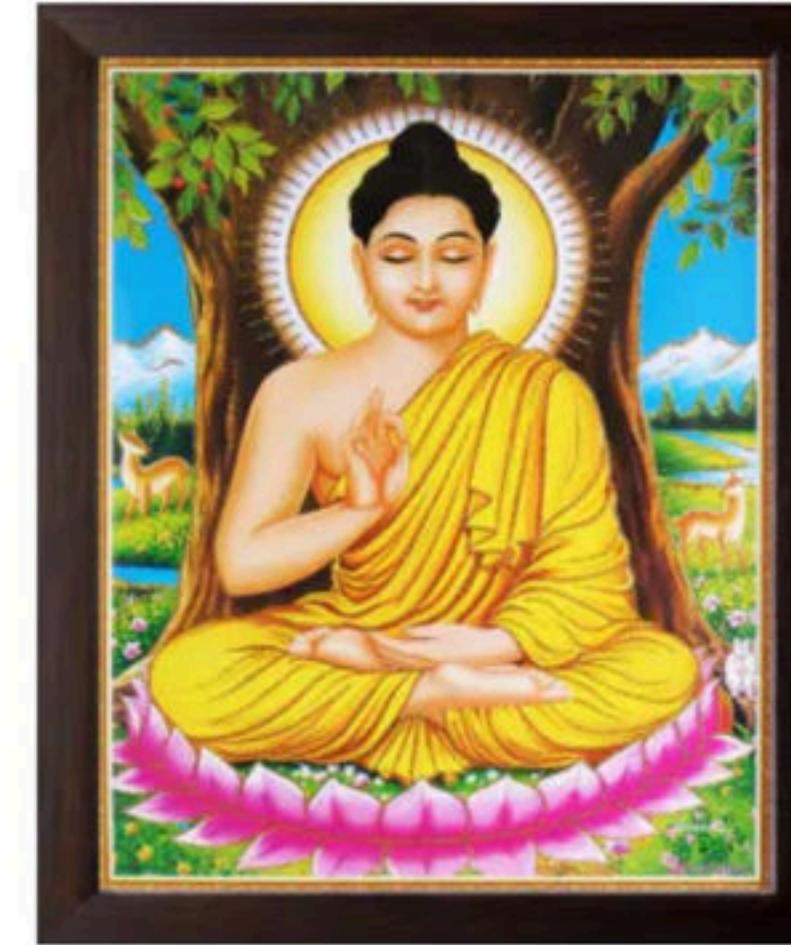
- First we must look at the difference between Scientist and Philosopher.
- Philosopher give an idea or theory which may have different interpretation from person to person. It depends on the wisdom of a person.



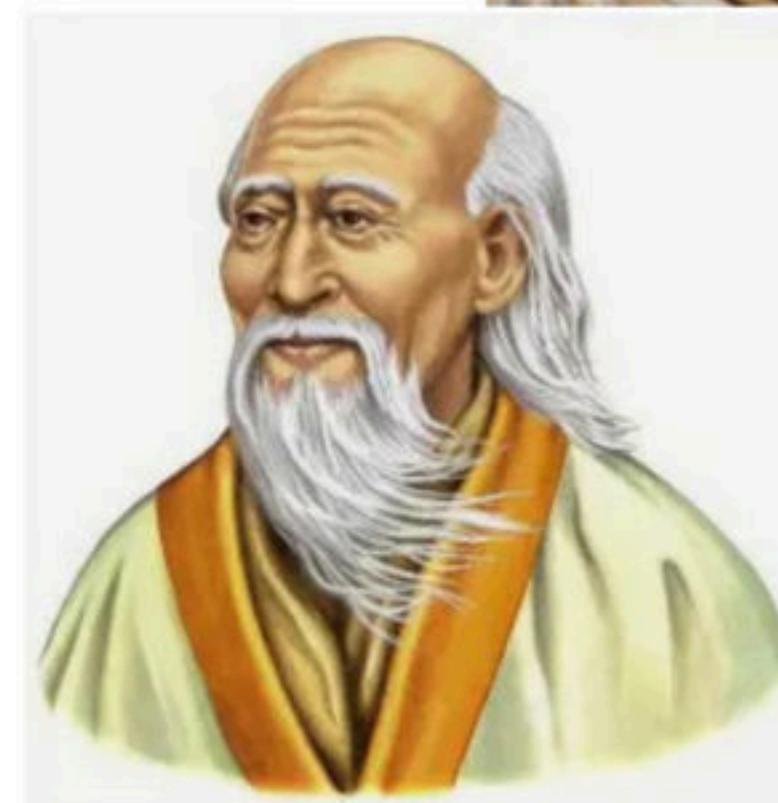
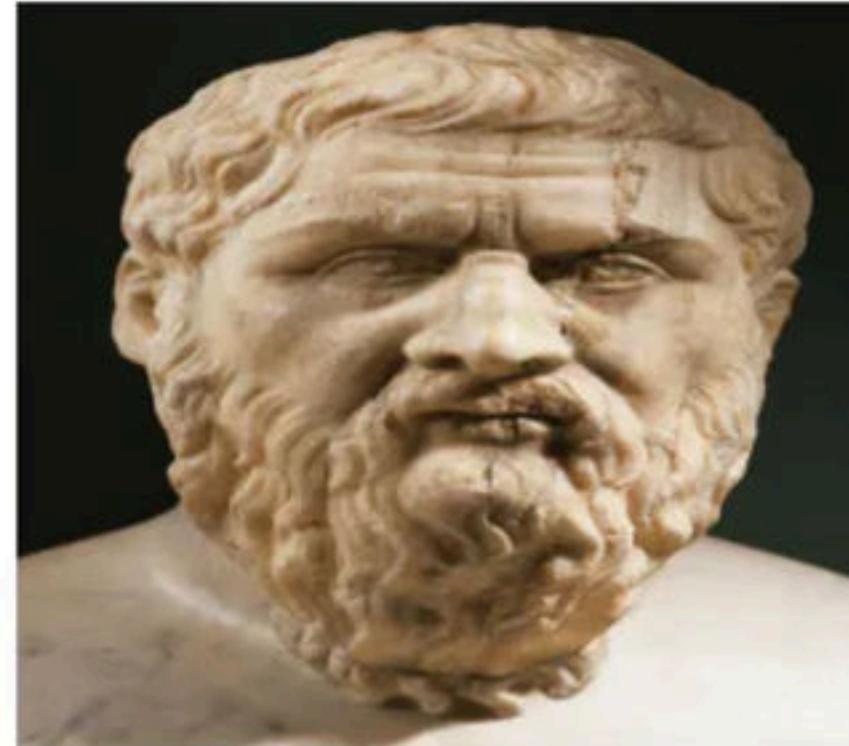
Mahaveer



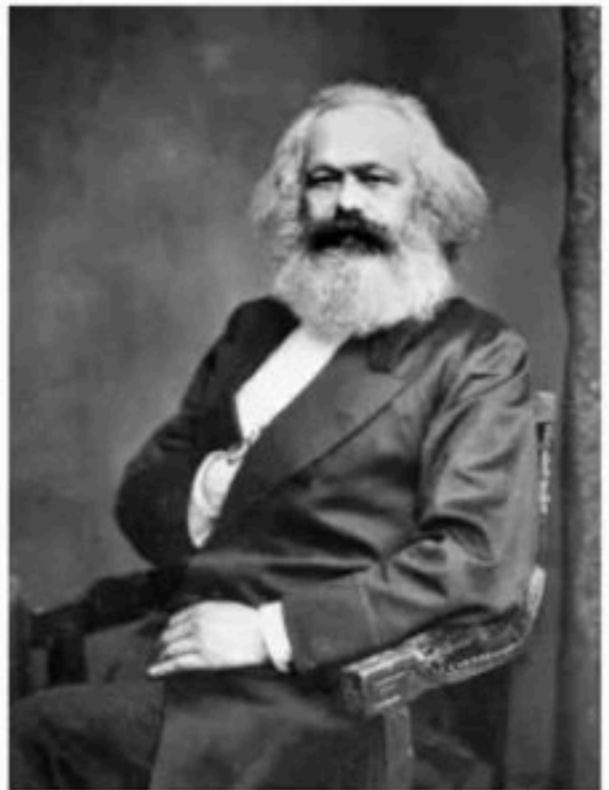
Gautam Buddha



- There are different philosophers in the world suggested different philosophy like, Chanakya, Plato, Aristotle, Confucius, Laozi,



Karl Marx



Hitler



Mahatma Gandhi



Break

- Proposition with rules of logic actually is a method of reasoning (unambiguous, machinic, deterministic), given by Aristotle, who was the teacher of Alexander son of King Philip of Macedonia
- There may be different methods of reasoning for solving a problem apart from proposition.

- Proposition and rules of logic specify the meaning of mathematical statements. Logic is the basis of all mathematical reasoning, and of all automated reasoning.
- It has practical applications to the design of computing machines, to the specification of systems, to artificial intelligence, to computer programming, to programming languages, and to other areas of computer science, as well as to many other fields of study.

- To understand mathematics, we must understand what makes up a correct mathematical argument, that is, a proof.
- Once we prove a mathematical statement is true, we call it a theorem. A collection of theorems on a topic organize what we know about this topic.

- Everyone knows that proofs are important throughout mathematics, even in computer science. In fact, proofs are used to verify that computer programs produce the correct output for all possible input values, to show that algorithms always produce the correct result.
- The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

- The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic** or **predicate calculus(study of propositions)**.
- It was first developed systematically by the Greek philosopher Aristotle more than 2300 years ago.
- We now turn our attention to methods for producing new propositions from those that we already have. These methods were discussed by the English mathematician George Boole in 1854 in his book *The Laws of Thought*.

Break

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

1. Delhi is the capital of USA
2. How are you doing
3. $5 \leq 11$
4. Temperature is less than 10 C
5. It is cold today
6. Read this carefully
7. $X + y = z$

1. Premises(proposition) is always considered to be true.
2. Premises is a statement that provides reason or support for the conclusion(proposition).

1. If a set of Premises(P) yield another proposition Q (Conclusion), then it is called an Argument.
2. An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

$\{P_1, P_2, P_3, \dots, P_N\} \vdash Q$	P_1 P_2 P_3 . . P_N Q	$\{P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_N\} \vdash Q$
--	---	--

Break

- **Law of contradiction** - the law of non-contradiction (LNC) (also known as the law of contradiction, principle of non-contradiction (PNC), or the principle of contradiction) states that
- Contradictory propositions cannot both be true in the same sense at the same time.
 - e.g. the two propositions "*A is B*" and "*A is not B*" are mutually exclusive.

- **Law of excluded middle** - the law of excluded middle (or the principle of excluded middle) states that for any proposition, either that proposition is true or its negation is true.

Break

Types of proposition

1. We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent propositions, just as letters are used to denote numerical variables.
2. The conventional letters used for propositional variables are p, q, r, s . The **truth value** of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

- Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Operators / Connectives

1. **Negation:** - let p be a proposition, then negation of p new proposition, denoted by $\neg p$, is the statement “it is not the case that p ”.
2. The truth value of the negation of p , $\neg p$, is the opposite of the truth value of p .
e.g. \neg “Michael’s PC runs Linux” = “It is not the case that Michael’s PC runs Linux.” = “Michael’s PC does not run Linux.”

Negation	
P	$\neg P$
F	
T	

Break

Conjunction

- Let p and q be propositions. The *conjunction* of p and q , denoted by $p \wedge q$, is the proposition “ p and q .”
- The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Conjunction		
p	q	$p \wedge q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \wedge q)$
Q	p

2	
P_1	P
Q	$p \wedge q$

3	
P_1	P
P_2	q
Q	$p \wedge q$

4	
P_1	$\neg(p \wedge q)$
P_2	P
Q	$\neg q$

5	
P_1	$\neg(p \wedge q)$
P_2	q
Q	$\neg p$

6	
P_1	$\neg(p \wedge q)$
P_2	$\neg p$
Q	q

7	
P_1	$\neg(p \wedge q)$
P_2	$\neg p$
Q	$\neg q$

Break

Disjunction

- Let p and q be propositions. The *disjunction* of p and q , denoted by $p \vee q$, is the proposition “ p or q .” The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Disjunction		
p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \wedge q)$
Q	$p \vee q$

2	
P_1	$p \vee q$
Q	$(p \wedge q)$

3	
P_1	$\neg(p \vee q)$
Q	$\neg p$

4	
P_1	$(p \vee q)$
Q	$\neg p$

5	
P_1	$(p \vee q)$
P_2	$\neg p$
Q	q

6	
P_1	$(p \vee q)$
P_2	$\neg q$
Q	p

7	
P_1	$(p \vee q)$
P_2	p
Q	$\neg q$

8	
P_1	$(p \vee q)$
P_2	p
Q	q

Break

Implication

1. Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition “if p , then q ”. The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.
2. In conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion.
3. The statement $p \rightarrow q$ is called a conditional statement because $p \rightarrow q$ asserts that q is true on the condition that p holds.

Implication		
p	q	$p \rightarrow q$
F	F	
F	T	
T	F	
T	T	

- Let p be the statement “Maria learns discrete mathematics” and q the statement “Maria will find a good job.” Express the statement $p \rightarrow q$ as a statement in English.

- “If Maria learns discrete mathematics, then she will find a good job.”
- “Maria will find a good job when she learns discrete mathematics.”
- “For Maria to get a good job, it is sufficient for her to learn discrete mathematics.”

Implication		
p	q	$P \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$P \rightarrow q$	$\neg p$	$\neg q$	$\neg P \rightarrow \neg q$	$q \rightarrow p$	$\neg q \rightarrow \neg p$	$\neg p \vee q$
F	F							
F	T							
T	F							
T	T							

1. $p \rightarrow q$ *implication*
2. $q \rightarrow p$ *converse*
3. $\neg p \rightarrow \neg q$ *inverse*
4. $\neg q \rightarrow \neg p$ *contra positive*

$$1. p \rightarrow q = \neg q \rightarrow \neg p$$

2. $p \rightarrow q$ will be true if either p is false or q is true, $p \rightarrow q = \neg p \vee q$

Q consider the following arguments and find which of them are valid?

Modus Ponens	
P_1	$p \rightarrow q$
P_2	p
Q	q

Modus Tollens	
P_1	$p \rightarrow q$
P_2	$\neg Q$
Q	$\neg p$

1	
P_1	$\neg p$
Q	$p \rightarrow q$

2	
P_1	q
Q	$p \rightarrow q$

3	
P_1	$\neg(p \rightarrow q)$
Q	$\neg q$

4	
P_1	$\neg(p \rightarrow q)$
Q	p

Break

Bi-conditional

- Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition.
 - “ p if and only q ”.
- The biconditional statement $p \leftrightarrow q$ is true when p and q have the same values, and false otherwise. Biconditional statements are also called bi-implications.
 - “ p is necessary and sufficient for q ”
 - “if p then q , and conversely”
 - “ p iff q .”
 - $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

Bi-conditional		
p	q	$P \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
Q	$p \rightarrow r$

2	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow r$
Q	r

3	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow s$
Q	$r \vee s$

4	
P_1	$p \rightarrow r$
P_2	$q \rightarrow s$
P_3	$\neg r \vee \neg s$
Q	$\neg p \vee \neg q$

Q consider the following arguments and find which of them are valid?

5	
P_1	p
P_2	q
Q	r

6	
P_1	p
P_2	$\neg p$
Q	q

7	
P_1	
Q	q

Break

Type of cases

- **Tautology/valid:** - A propositional function which is always having truth in the last column, is called tautology. E.g. $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
F	T	
T	F	

- **Contradiction/ Unsatisfiable:** - A propositional function which is always having false in the last column, is called Contradiction. E.g. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
F	T	
T	F	

- **Contingency**: - A propositional function which is neither a tautology nor a contradiction, is called Contingency. E.g. $p \vee q$

p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

- **Satisfiable:** - A propositional function which is not contradiction is satisfiable. i.e. it must have at least one truth value in the final column e.g. $p \vee q$

- **Functionality Complete Set:** - A set of connectives is said to be functionally complete if it is able to write any propositional function.
 - $\{\wedge, \neg\}$
 - $\{\vee, \neg\}$

Break

Let p and q be two propositions. Consider the following two formulae in propositional logic. **(Gate-2021) (1 Marks)**

- $S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$
- $S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

A. Both S_1 and S_2 are tautologies.

B. S_1 is a tautology but S_2 is not a tautology

C. S_1 is not a tautology but S_2 is a tautology

D. Neither S_1 nor S_2 is a tautology

$$\cancel{p} + (\cancel{v} + v)$$

$$p + 1$$

1

$$S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$$

$$[\cancel{p} \cdot (p + q)] \rightarrow q$$

$$\cancel{p} \cdot v \rightarrow q$$

$$(\cancel{p} \cdot v) + q$$

$$\cancel{p} + (\cancel{v} + v)$$

$$1 = T$$

$$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$$

$$q \rightarrow (\cancel{p} \cdot (p + q))$$

$$q \rightarrow (\cancel{p} \cdot q)$$

$$\cancel{q} + \cancel{p} \cdot q$$

$$\cancel{p} + \cancel{q}$$

$$\cancel{q} + \cancel{p} \cdot q$$

Choose the correct choice(s) regarding the following propositional logic assertion S : **(GATE- 2021)**

$$S : ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

- A. S is neither a tautology nor a contradiction
- B. S is a tautology
- C. S is a contradiction
- D. The antecedent of S is logically equivalent to the consequent of S

Q Let P, Q, R and S be Propositions.

Assume that the equivalences $P \Leftrightarrow (Q \vee \neg Q)$ and $Q \Leftrightarrow R$ hold.

Then the truth value of the formula $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ is always:

(NET-Jan-2017)

A) True $\cancel{-57}$

$$P \Leftrightarrow (Q \vee \neg Q)$$
$$P = T$$
$$\textcircled{Q = R}$$

B) False $\cancel{-1}$

$$(P \wedge Q) \rightarrow (R \wedge \neg R) \vee S$$
$$(T \wedge Q) \rightarrow (T \wedge \neg R) \vee S$$

C) Same as truth table of Q $\cancel{-20}$

$$Q \rightarrow R \vee S$$

D) Same as truth table of S $\cancel{-15}$

$$\boxed{Q \rightarrow Q \vee S}$$

$$\cancel{\neg R + (R \vee S)}$$
$$\text{m}_1$$

Q Consider two well-formed formulas in propositional logic

$$F_1: P \Rightarrow \neg P$$

$$F_2: (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$$

Which one of the following statements is correct? **(GATE-2001) (1 Marks) (NET-Jan-2017)**

A) F_1 is satisfiable, F_2 is valid

B) F_1 unsatisfiable, F_2 is satisfiable

C) F_1 is unsatisfiable, F_2 is valid

D) F_1 and F_2 are both satisfiable

Q Let p, q, and r be the propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is **(GATE-2017) (2 Marks)**

(A) a tautology

$$\stackrel{\text{∴}}{(p \rightarrow q) \rightarrow r = \text{F}}$$

$$(p \rightarrow q) \rightarrow r = \text{F}$$

$$\overline{p+q} + r = 0$$

(B) a contradiction

$$\stackrel{\text{∴}}{p \cdot \bar{q} + r = 0}$$
$$\Rightarrow \boxed{p \cdot \bar{q} = 0} \quad | \quad r = 0$$

$$(q \rightarrow p) \rightarrow q$$

$$(\bar{p} \rightarrow p) \rightarrow q$$

$$T \rightarrow q$$

$$\bar{T} \neq q$$

$$= \boxed{q}$$

(C) always TRUE when p is FALSE

\hookrightarrow 8

(D) always TRUE when q is TRUE

Q The first order logic (FOL) statement $((R \vee Q) \wedge (P \vee \neg Q))$ is equivalent to which of the following? **(NET-Jan-2017)**

A) $((R \vee \neg Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$

~~GJ B) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$~~

$$(r_1 + v) \cdot (p + \bar{q})$$

$$\cancel{(r_1 p + r_1 \bar{v} + v q)}$$

$$\cancel{(r_1 + v) \cdot (p + \bar{q}) \cdot (r_1 + p)}$$

$$\cancel{(r_1 p + r_1 \bar{v} + p v)} \quad (r_1 + p)$$

$$\cancel{r_1 p + r_1 \bar{v} + \cancel{p \bar{v}} + \cancel{p v} + p \bar{v}}$$

C) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee \neg P))$

D) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (\neg R \vee P))$

$$\cancel{(r_1 + p v)}$$

$$pv \cancel{(r_1 + p v)}$$

|

$$\cancel{p v}$$

Q the statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below? **(GATE-2017) (1 Marks)**

- 1)** $p \Rightarrow q$
 - 2)** $q \Rightarrow p$
 - 3)** $(\neg q) \vee (p)$
 - 4)** $(\neg p) \vee q$
-
- a)** 1 only
 - b)** 1 and 4 only
 - c)** 2 only
 - d)** 2 and 3 only

Q Let P and Q be two propositions, $\neg(P \leftrightarrow Q)$ is equivalent to: (NET-Jan-2017)

~~(I) $P \leftrightarrow \neg Q$~~

$$P = \neg Q$$

~~(II) $\neg P \leftrightarrow Q$~~

$$\neg P = Q$$

~~(III) $\neg P \leftrightarrow \neg Q$~~

$$\neg P = \neg Q$$

~~(IV) $Q \rightarrow P$~~

$$P \neq Q$$

A) Only (I) and (II)³⁷

B) Only (II) and (III)¹⁷

C) Only (III) and (IV)³⁷

D) None of the above⁹

$$\cancel{(P \not\leftrightarrow Q)} = (P \rightarrow Q) \wedge (Q \rightarrow P)$$

$$F$$

$$F$$

$$T$$

$$F$$

$$F$$

$$\sim(P \leftrightarrow Q)$$



$$P \cdot \bar{Q} + \bar{P} \cdot Q$$



$$\sim P \leftrightarrow \sim Q$$

$$\not\models \not P \cdot \bar{Q}$$

$$+ \bar{P} \cdot \bar{Q}$$

$$P \cdot \bar{Q} + \bar{P} \cdot \bar{Q}$$

$$P \leftrightarrow Q = \bar{P} \cdot \bar{Q} + P \cdot Q$$

$$\sim(P \leftrightarrow Q) = P \cdot \bar{Q} + \bar{P} \cdot Q$$

Q In propositional logic if $(P \rightarrow Q) \wedge (R \rightarrow S)$ and $(P \vee R)$ are two premises such that

$$(P \rightarrow Q) \wedge (R \rightarrow S)$$

$$P \vee R$$

.....

Y

.....

is the premise: **(NET-Jan-2017)**

A) $P \vee R \neg \perp$

B) $P \vee S \neg \perp$

C) $Q \vee R \neg \perp$

D) $Q \vee S \neg \perp$

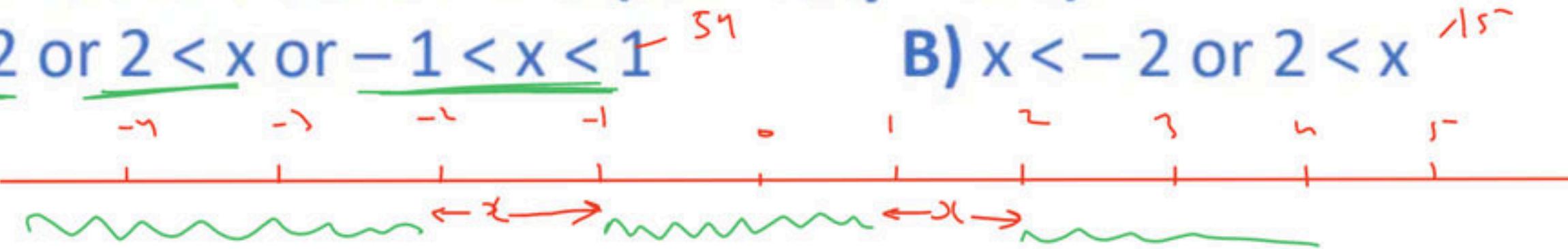
$$\begin{array}{c} P \rightarrow Q \\ R \rightarrow S \\ \hline \underline{P \vee R} \\ \text{---} \\ \text{Q} \vee S \end{array}$$

Q consider the following expression: (GATE-2016) (1 Marks)

- i) false
- ii) Q
- iii) true
- iv) $P \vee Q$
- v) $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is _____

Q Consider the statement, "Either $-2 \leq x \leq -1$ or $1 \leq x \leq 2$ ". The **negation** of this statement is (NET-July-2016)

- ~~A) $x < -2$ or $2 < x$ or $-1 < x < 1$~~ ⁵⁴
- ~~B) $x < -2$ or $2 < x$~~ ¹⁵
- 
- ~~C) $-1 < x < 1$~~ ¹⁷
- ~~D) $x \leq -2$ or $2 < x$ or $-1 < x < 1$~~ ¹⁷

Q The Boolean function $[\sim(\sim p \wedge q) \wedge \sim(\sim p \wedge \sim q)] \vee (p \wedge r)$ is equal to the Boolean function: **(NET-Aug-2016)**

- a) q
- b) $p \wedge r$
- c) $p \vee q$
- d) p

Q Let p, q, r, s represents the following propositions. (GATE-2016) (1 Marks)

$p: x \in \{8, 9, 10, 11, 12\}$

$q: x$ is a composite number

$r: x$ is a perfect square

$s: x$ is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is _____.

Q Consider the following logical inferences: (NET-Aug-2016)

$$\begin{array}{c} \cancel{S \rightarrow \sim S} / S \rightarrow \sim O \\ \cancel{S} \\ \hline \cancel{\sim S} \quad \checkmark \end{array}$$

I₁: If it is Sunday then school will not open. The school was open.
Inference: It was not Sunday.

I₂: If it is Sunday then school will not open. It was not Sunday.
Inference: The school was open.

$$\begin{array}{c} S \rightarrow \sim O \\ \sim S \\ \hline \cancel{\text{O}} \end{array}$$

Which of the following is correct?

- A) Both I₁ and I₂ are correct inferences. ³⁰ ↗
- ~~B) I₁ is correct but I₂ is not a correct inference.~~ ⁵⁵
- ~~C) I₁ is not correct but I₂ is a correct inference.~~ ⁸
- ~~D) Both I₁ and I₂ are not correct inferences.~~ ⁸

Q Consider the following two statements.

S₁: If a candidate is known to be corrupt, then he will not be elected.

S₂: If a candidate is kind, he will be elected.

$$\begin{array}{l} C \rightarrow \neg E \\ K \rightarrow E / \neg E \rightarrow \neg K \\ \boxed{C \rightarrow \neg K} \quad \cancel{K \rightarrow \neg L} \end{array}$$

Which one of the following statements follows from S₁ and S₂ as per sound inference rules of logic?
(GATE-2015) (1 Marks)

(A) If a person is known to be corrupt, he is kind $\xrightarrow{6} C \rightarrow K$

(B) If a person is not known to be corrupt, he is not kind $\xrightarrow{21} \neg C \rightarrow \neg K$

(C) If a person is kind, he is not known to be corrupt $\xrightarrow{68} K \rightarrow \neg C$

(D) If a person is not kind, he is not known to be corrupt $\xrightarrow{5} \neg K \rightarrow \neg C$

Q Which of the following arguments are valid? (NET-Dec-2015)

(a) "If Gora gets the job and works hard, then he will be promoted. If Gora gets promotion, then he will be happy. He will not be happy, therefore, either he will not get the job or he will not work hard".

(b) "Either Puneet is not guilty or Pankaj is telling the truth. Pankaj is not telling the truth, therefore, Puneet is not guilty".

(c) ~~If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$, then $n > 1$.~~

Codes:

A) (a) and (c) ²²

B) (b) and (c) ²³

C) (a), (b) and (c) ⁸

D) (a) and (b) ⁻¹

$$\frac{\cancel{P \rightarrow T} \rightarrow \cancel{\sim P \rightarrow g} / P \rightarrow g \rightarrow P \rightarrow T}{\sim P \rightarrow g \vee P \rightarrow T}$$

$$\frac{\cancel{\sim P \rightarrow T}}{\cancel{\sim P \rightarrow g} \vee \cancel{P \rightarrow T}}$$

$$\frac{(J \wedge H \wedge W) \rightarrow P // \sim P \rightarrow \sim(J \wedge H \wedge W)}{\sim J \vee \sim H \vee \sim W}$$

$$\frac{P \rightarrow H // \cancel{P \rightarrow \sim P} // \sim P}{\sim H}$$

$$\frac{\cancel{\sim H}}{\sim J \vee \sim H \sim W}$$

Q "If my computations are correct and I pay the electric bill, then I will run out of money.

If I don't pay the electric bill, the power will be turned off.

Therefore, if I don't run out of money and the power is still on, then my computations are incorrect."

Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively. (Where \neg means NOT) (NET-June-2015)

- A) if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$
- B) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(r \wedge p) \rightarrow c$
- C) if $(c \wedge b) \rightarrow r$ and $\neg p \rightarrow b$, then $(\neg r \vee p) \rightarrow \neg c$
- D) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

Q In Propositional Logic, given P and $P \rightarrow Q$, we can infer _____.
(NET-June-2015)

- a) $\sim Q$
- b) Q
- c) $P \wedge Q$
- d) $\sim P \wedge Q$

Q Which one of the following is NOT equivalent to $p \leftrightarrow q$? **(GATE-2015)**
(1 Marks)

- a) $(\neg p \vee q) \wedge (p \vee \neg q)$
- b) $(\neg p \vee q) \wedge (q \rightarrow p)$
- c) $(\neg p \wedge q) \vee (p \wedge \neg q)$
- d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

Q In propositional logic $P \leftrightarrow Q$ is equivalent to (Where \sim denotes NOT)
(GATE-2015) (1 Marks)

- a) $\sim(P \vee Q) \wedge \sim(Q \vee P)$
- b) $(\sim P \vee Q) \wedge (\sim Q \vee P)$
- c) $(P \vee Q) \wedge (Q \vee P)$
- d) $\sim(P \vee Q) \rightarrow \sim(Q \vee P)$

Q Consider the compound propositions given below as: (NET-Dec-2015)

(a) $p \vee \sim(p \wedge q)$

(b) $(p \wedge \sim q) \vee \sim(p \wedge q)$

(c) $p \wedge (q \vee r)$

Which of the above propositions are tautologies?

A) (a) and (c)

B) (b) and (c)

C) (a) and (b)

D) only (a)

Q In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following

"The result of the toss is head if and only if I am telling the truth"

Which of the following options is correct? **(Gate-2015)(2 Marks)**

- a) The result is head
- b) The result is tail
- c) If the person is of Type 2, then the result is tail
- d) If the person is of Type 1, then the result is tail

Q Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT? **(GATE-2014) (1 Marks)**

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

Q Which one of the following Boolean expressions is NOT a tautology?
(GATE-2014) (2 Marks)

A) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

B) $(a \rightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

C) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

D) $a \rightarrow (b \rightarrow a)$

- Q** Which one of the following propositional logic formulas is TRUE only when exactly two of p, q and r are TRUE? **(GATE-2014) (2 Marks)**
- a)** $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$ **b)** $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
- c)** $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$ **d)** $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

Q Consider the following logical inferences.

I₁: If it rains then the cricket match will not be played.

The cricket match was played.

Inference: There was no rain.

I₂: If it rains then the cricket match will not be played.

It did not rain.

Inference: The cricket match was played.

Which of the following is **TRUE?** (GATE-2012) (1 Marks)

- (A) Both I₁ and I₂ are correct inferences
- (B) I₁ is correct but I₂ is not a correct inference
- (C) I₁ is not correct but I₂ is a correct inference
- (D) Both I₁ and I₂ are not correct inferences

Q The proposition $\sim p \vee q$ is equivalent to **(NET-Dec-2011)**

- (A)** $p \rightarrow q$ **(B)** $q \rightarrow p$ **(C)** $p \leftrightarrow q$ **(D)** $p \vee q$

Q The binary operation \odot is defined as follows

Which one of the following is equivalent to $P \vee Q$? **(GATE-2009) (2 Marks)**

a) $(\sim Q \odot \sim P)$

b) $(P \odot \sim Q)$

c) $(\sim P \odot Q)$

d) $(\sim P \odot \sim Q)$

P	Q	$P \odot Q$
T	T	T
T	F	T
F	T	F
F	F	T

Q An example of a Tautology is: (NET-June-2008)

a) $x \vee y$

b) $x \vee \neg y$

c) $x \vee \neg x$

d) $(x \rightarrow y) \wedge (y \rightarrow x)$

P and Q are two propositions. Which of the following logical expressions are equivalent?
(GATE-2008) (2 Marks)

I) $P \vee \neg Q$

II) $\neg(\neg P \wedge Q)$

III) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

IV) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

- (A) Only I and II
- (B) Only I, II and III
- (C) Only I, II and IV
- (D) All of I, II, III and IV

Q the Preposition $(p \rightarrow q) \wedge (\neg q \vee p)$ is equivalent to: **(NET-June-2006)**

a) $q \rightarrow p$

b) $p \rightarrow q$

c) $(q \rightarrow p) \wedge (p \rightarrow q)$

d) $(p \rightarrow q) \vee (q \rightarrow p)$

Q Consider the following propositional statements: (GATE-2006) (2 Marks)

$$P_1 : ((A \wedge B) \rightarrow C) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$$

$$P_2 : ((A \vee B) \rightarrow C) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$$

Which one of the following is true?

- (A) P_1 is a tautology, but not P_2
- (B) P_2 is a tautology, but not P_1
- (C) P_1 and P_2 are both tautologies
- (D) Both P_1 and P_2 are not tautologies

Q A logical binary relation \odot , is defined as follows: (GATE-2006) (2 Marks)

Let \sim be the unary negation (NOT) operator, with higher precedence than \odot .

Which one of the following is equivalent to $A \wedge B$?

a) $(\sim A \odot B)$

b) $\sim(A \odot \sim B)$

c) $\sim(\sim A \odot \sim B)$

d) $\sim(\sim A \odot B)$

A	B	$A \odot B$
T	T	T
T	F	T
F	T	F
F	F	T

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where \neg is negation, \vee is inclusive OR and \rightarrow is implication, is (NET-dec-2005)

- a) True
- b) Multiple Values
- c) False
- d) Cannot be determined

Q Let P, Q and R be three atomic prepositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology? **(GATE-2005) (2 Marks)**

- (A)** $X \equiv Y$ **(B)** $X \rightarrow Y$ **(C)** $Y \rightarrow X$ **(D)** $\neg Y \rightarrow X$

Q The following propositional statement is (GATE-2004) (2 Marks)

$$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$$

- (A)** satisfiable but not valid
- (B)** valid
- (C)** a contradiction
- (D)** none of the above

Q Let p, q, r and s be four primitive statements. Consider the following arguments:

$$P: [(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\neg s \rightarrow q)$$

$$Q: [(\neg p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$$

$$R: [[(q \wedge r) \rightarrow p] \wedge (\neg q \vee p)] \rightarrow r$$

$$S: [p \wedge (p \rightarrow r) \wedge (q \vee \neg r)] \rightarrow q$$

Which of the above arguments are valid? **(GATE-2004) (2 Marks)**

- a) P and Q only
- b) P and R only
- c) P and S only
- d) P, Q, R and S

Q Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \wedge b) \rightarrow (a \wedge c) \vee d$ is always. **(GATE-2003) (2 Marks)**

(A) True

(B) False

(C) Same as the truth value of b

(D) Same as the truth value of d

Q The following resolution rule is used in logic programming:

Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg R)$

Which of the following statements related to this rule is FALSE? **(GATE-2003) (2 Marks)**

(A) $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid

(B) $(P \vee Q) \Rightarrow ((P \vee R) \wedge (Q \vee \neg R))$ is logically valid

(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable

(D) $(P \vee Q) \Rightarrow \text{FALSE}$ if and only if both P and Q are unsatisfiable

Q “If X, then Y unless Z” is represented by which of the following formulae in propositional logic? **(GATE-2002) (1 Marks)**

- (A)** $(X \wedge \neg Z) \rightarrow Y$ **(B)** $(X \wedge Y) \rightarrow \neg Z$
- (C)** $(X \rightarrow (Y \wedge \neg Z))$ **(D)** $(X \rightarrow Y) \wedge \neg Z$

Q Which of the following is false? Read \wedge as AND, \vee as OR, \neg as NOT, \rightarrow as one-way implication and \leftrightarrow as two-way implication (**GATE-1996**) (2 Marks)

a) $((x \rightarrow y) \wedge x) \rightarrow y$

b) $((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x$

c) $(x \rightarrow (x \vee y))$

d) $((x \vee y) \leftrightarrow (\neg x \rightarrow \neg y))$

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where \neg is negation, \vee is inclusive OR and \rightarrow is implication, is **(GATE-1995) (2 Marks)**

- a)** True
- b)** Multiple Values
- c)** False
- d)** Cannot be determined

Q The proposition $p \wedge (\sim p \vee q)$ is: (GATE-1993) (1 Marks)

- a)** a tautology
- b)** logically equivalent to $p \wedge q$
- c)** logically equivalent to $p \vee q$
- d)** a contradiction
- e)** none of the above

Q Which of the following is/are a tautology? (GATE-1992) (1 Marks)

- a)** $a \vee b \rightarrow b \wedge c$
- b)** $a \wedge b \rightarrow b \vee c$
- c)** $a \vee b \rightarrow (b \rightarrow c)$
- d)** $a \rightarrow b \rightarrow (b \rightarrow c)$

Q Indicate which of the following well-formed formulae are valid: (GATE-1990) (2 Marks)

a) $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$

b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

d) $(P \Rightarrow R) \vee (Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)$

Break

Q consider the following argument

I₁: if today is Gandhi ji's birthday, then today is oct 2nd

I₂: today is oct 2nd

C: today is Gandhi ji's birthday

Q consider the following argument

I₁: if Canada is a country, then London is a city

I₂: London is not a city

C: Canada is not a country

Q find which of the following arguments are valid?

1) $((p \vee q) \vee \neg p) = T$

2) $\neg(p \vee q) \vee (\neg p \wedge q) \vee p = T$

3) $((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)) \wedge r = r$

$$4) (p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r)) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r) = T$$

$$5) (p \vee \neg(p \wedge q)) = T$$

$$6) (p \wedge q) \wedge (\neg p \vee \neg q) = F$$

$$7) (\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r$$

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
P_3	$\neg r$
Q	$\neg p$

2	
P_1	$r \rightarrow s$
P_2	$p \rightarrow q$
P_3	$r \vee p$
Q	$s \vee q$

3	
P_1	$(p \rightarrow (q \rightarrow s))$
P_2	$\neg r \vee p$
P_3	q
P_4	p
Q	s

4	
P_1	$(p \rightarrow (r \rightarrow s))$
P_2	$\neg r \rightarrow \neg p$
P_3	p
Q	s

5	
P_1	$\neg p \rightarrow \neg r$
P_2	$\neg S$
P_3	$P \rightarrow w$
P_4	$R \vee s$
Q	w

6	
P_1	$\neg x \rightarrow y$
P_2	$\neg x \wedge \neg y$
Q	x