

Proposition - Part I

Course on Discrete Mathematics for GATE 2023

Educator highlights

- Works at Knowledge Gate
- Studied at Delhi Technological University
- Have qualified gate Have experience of more than 8 years Have a YouTube channel with 5 lakh follower
- Lives in Ghaziabad, Uttar Pradesh, India
- Unacademy Educator since 19th July, 2019
- 844,388 live minutes taught in last 30 days
- Knows Punjabi, Hinglish, Hindi and English



Sanchit Jain

Legend in GATE - CS & IT

I am passionate for teaching computer science, having experience of more than 10 years. I teach all computer science subjects for GATE.

Follow

69M Watch mins

3M Watch mins (last 30 days)

30K Followers

3K Dedications



भाई में है दम
10 Subject खत्म

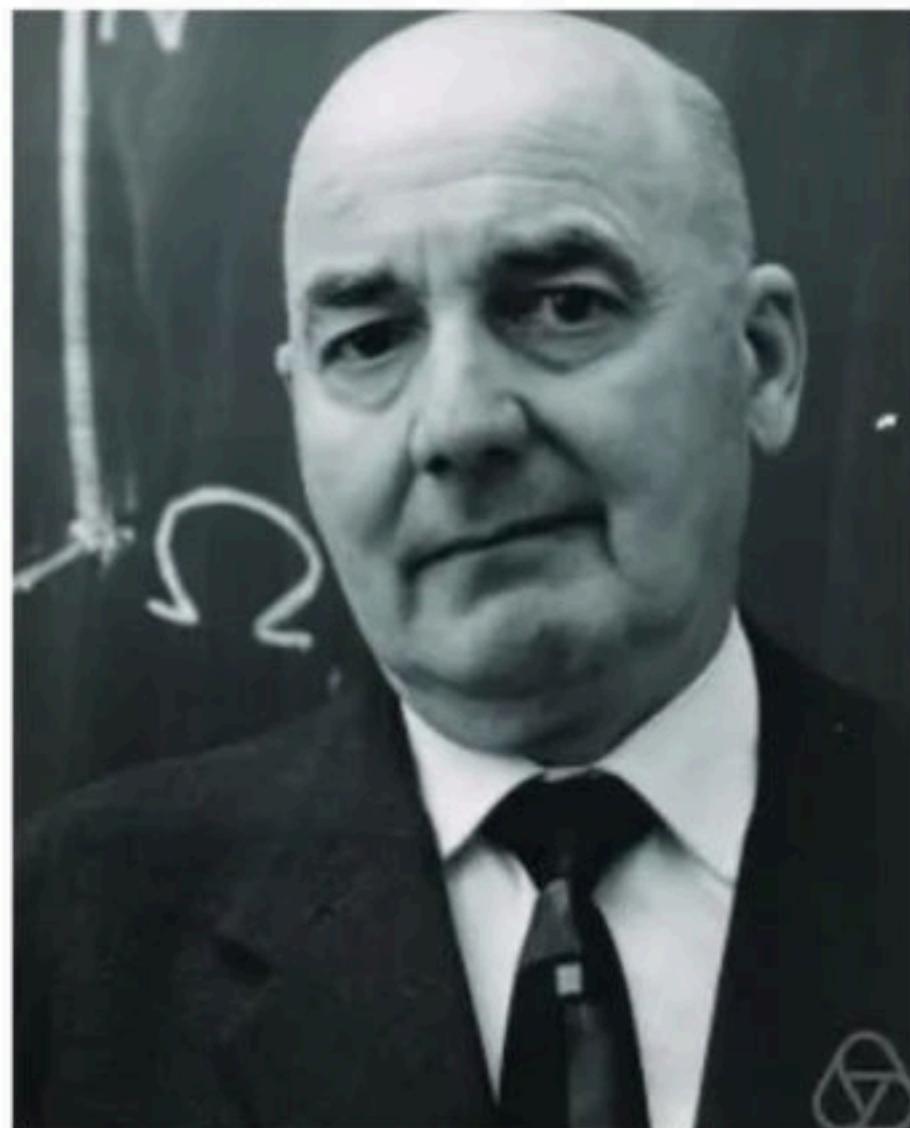
Months Wise Calendar for GATE-2023 (Unacademy Plus) (Sanchit Jain)

Month	Morning Class (6:30am – 8:00am)	Evening Class (6:30pm – 8:30pm)
Feb-2022		
Mar-2022	CN(16-Feb to 26-March) (54 Hours)	DM (2-Feb to 2-Mar) (50 Hours) DBMS (16-Mar to 2-May) (50 Hours) DE (4-May to 30-May) (40 Hours) COA (8-June to 19-July) (38 Hours) OS (21-July to 27-Aug) (58 Hours)
Apr-2022	TOC (6-Apr to 26-May) (58 Hours)	
May-2022	Compiler (1-June to 20-June) (34 Hours)	
Jun-2022	(DS & Programming) (22-June to 23-July) (56 Hours)	
Jul-2022	Algo (28-July to 25-Aug) (38 Hours)	
Aug-2022		
Sep-2022	DM (1-Sept to 10-Oct) (50 Hours)	CN (01-Sept to 12-Oct) (54 Hours)
Oct-2022	DBMS (13-Oct to 21-Nov) (50 Hours)	TOC (13-Oct to 30-Nov) (58 Hours)
Nov-2022	DE (24-Nov to 26-Dec) (40 Hours)	Compiler (1-Dec to 23-Dec) (34 Hours)
Dec-2022	COA (29-Dec to 30-Jan) (38 Hours)	DS & C (22-Dec to 2-Feb) (56 Hours)
Jan-2023		
Feb-2023		

Conversion of POSET into a Hasse Diagram

- If we want to study Partial order relation further then it will be better to convert it into more convenient notation so that it can be studied easily.
- This graphical representation is called Hasse Diagram

- In order theory, a Hasse diagram is a type of mathematical diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction.
- The diagrams are named after Helmut Hasse (1898–1979)



Steps to convert partial order relation into hasse diagram

- 1- Draw a vertex for each element in the Set
- 2- If $(a, b) \in R$ then draw an edge from a to b
- 3- Remove all Reflexive and Transitive edges
- 4- Remove the direction of edges and arrange them in the increasing order of heights.

Q Consider a Partial order relation and convert it into hasse diagram?

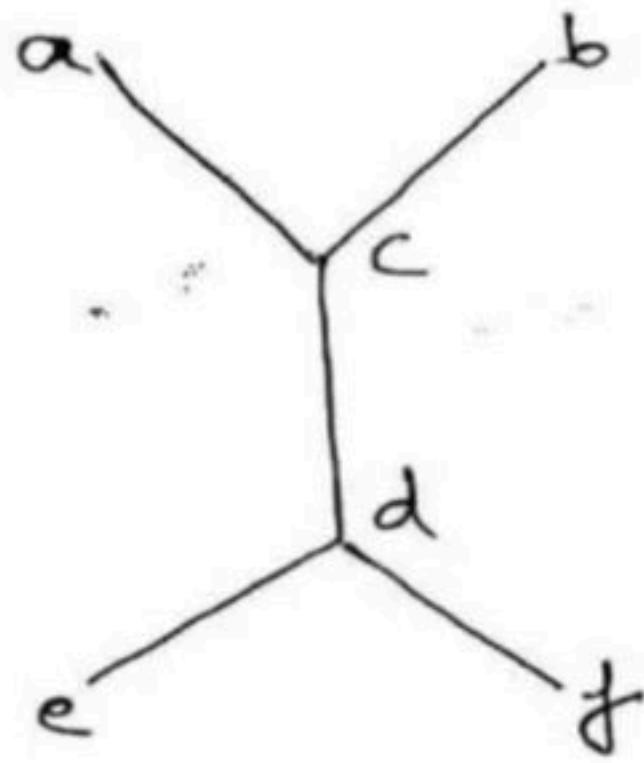
$$R = \{(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)\}$$

Q Consider a Partial order relation and convert it into hasse diagram?

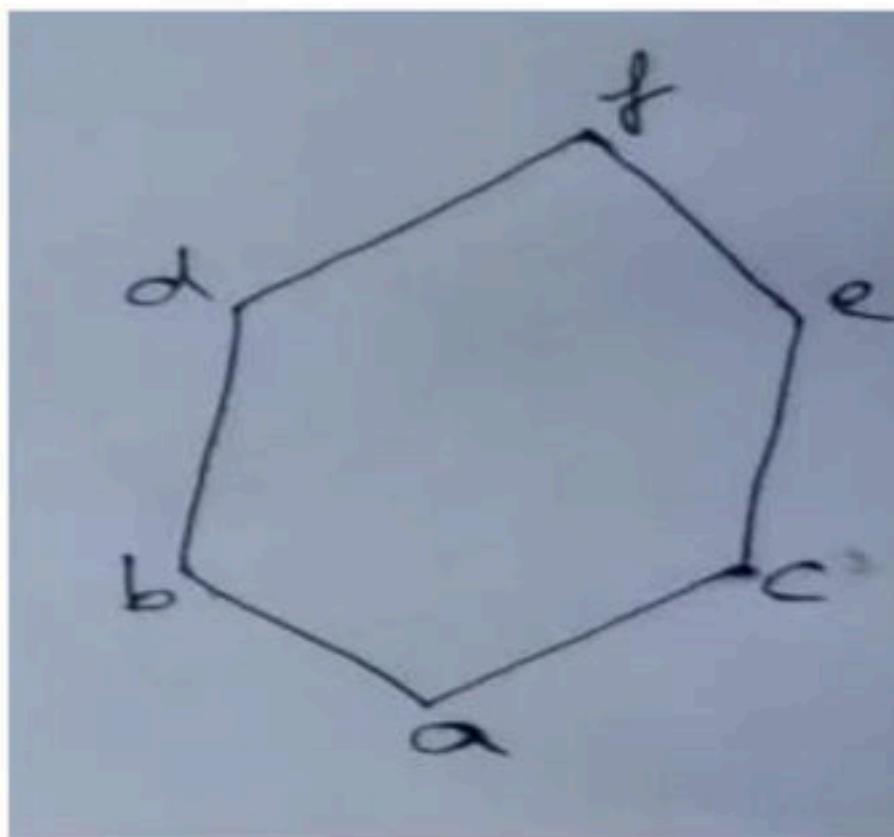
R = {(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)}

Break

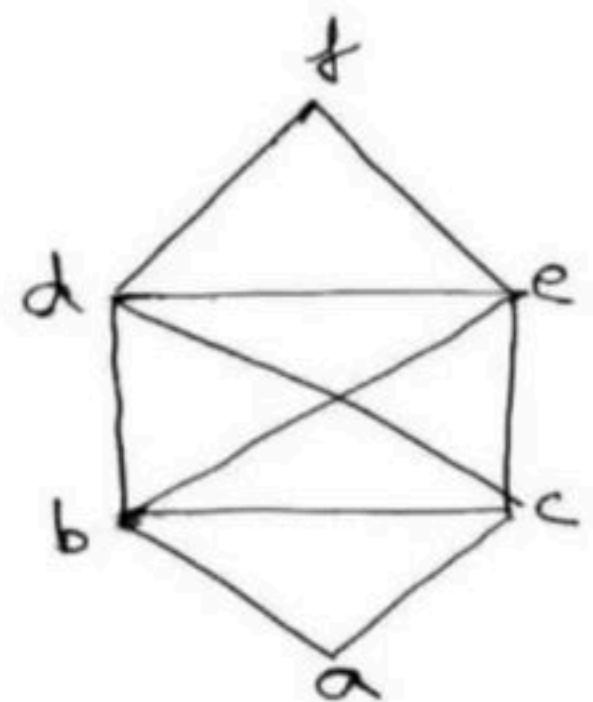
Q Study the following hasse diagram and find which of the following are valid?



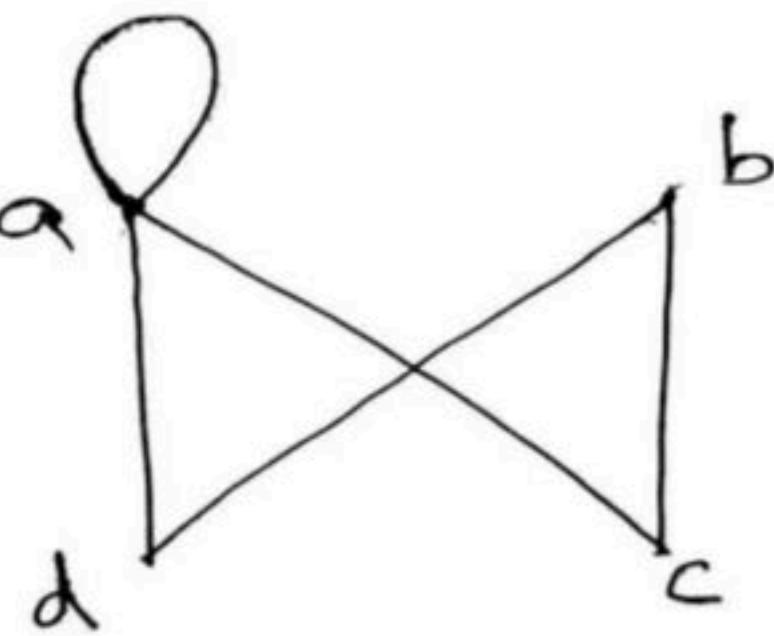
(1)



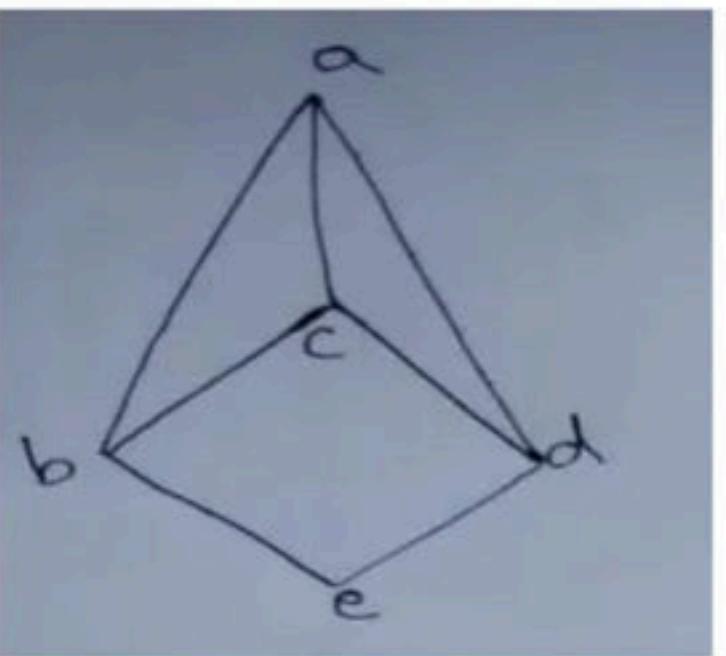
(2)



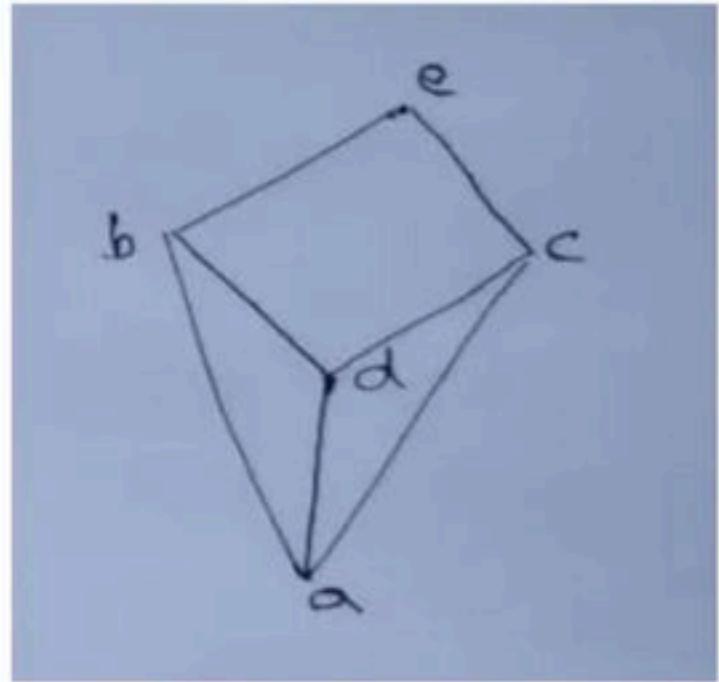
(3)



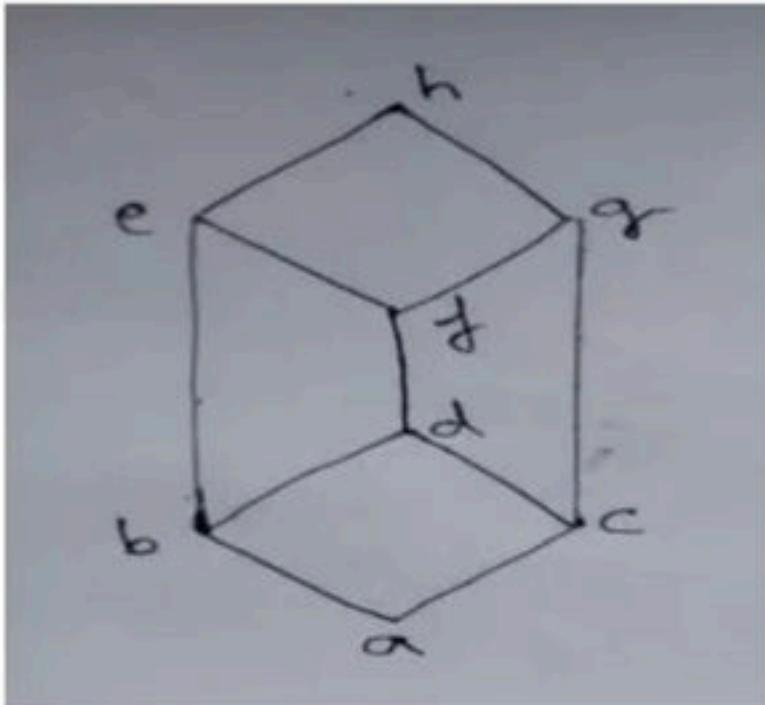
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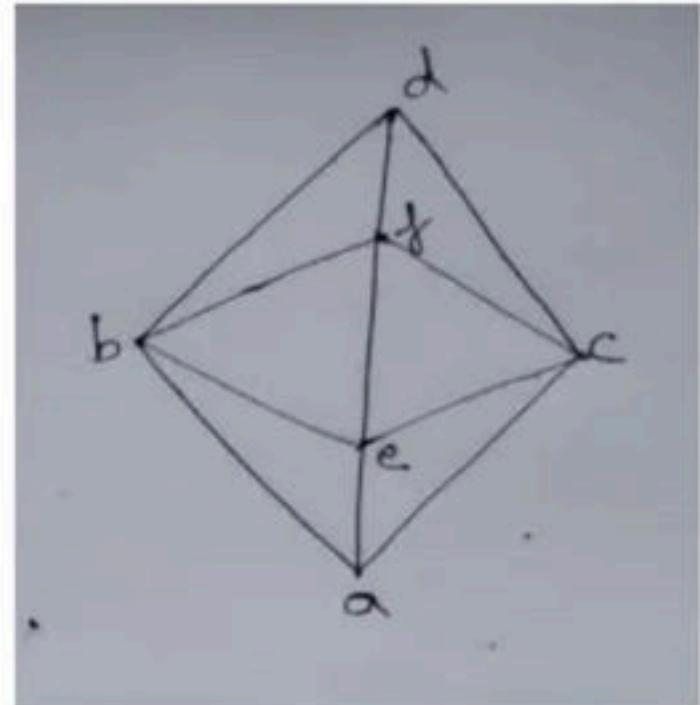
(5)



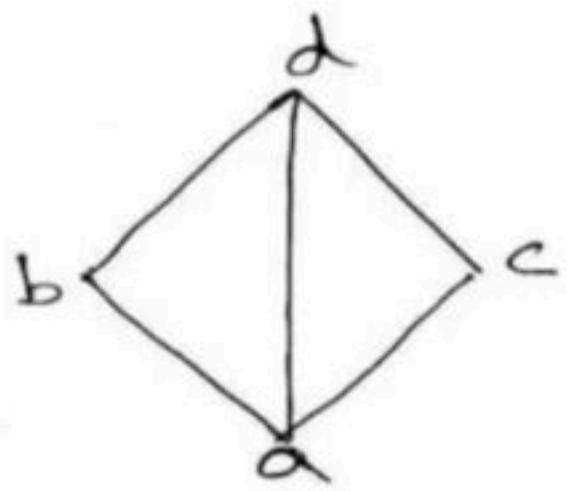
(5)



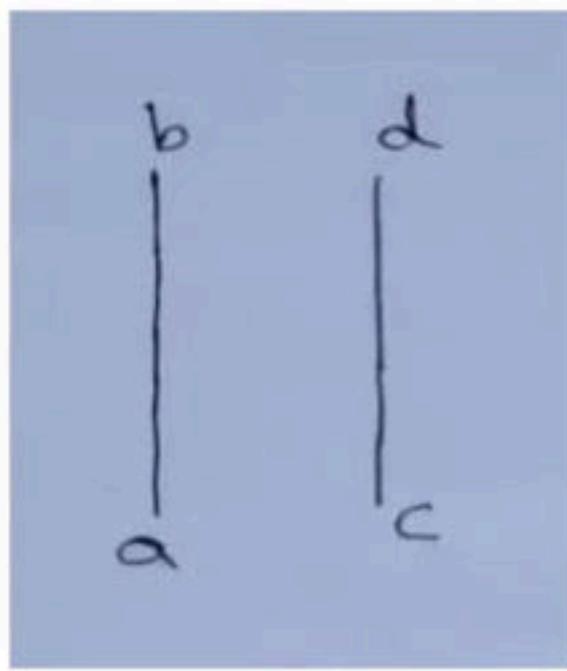
(6)



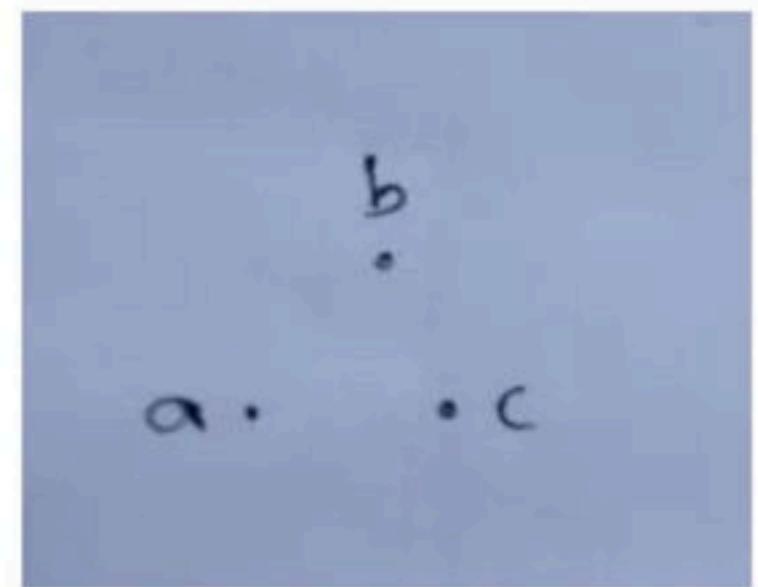
(7)



(8)



(9)



(10)

Conclusion

- We can not have a horizontal edge in a hasse diagram
- We can not have a reflexive and transitive edge in Hasse Diagram

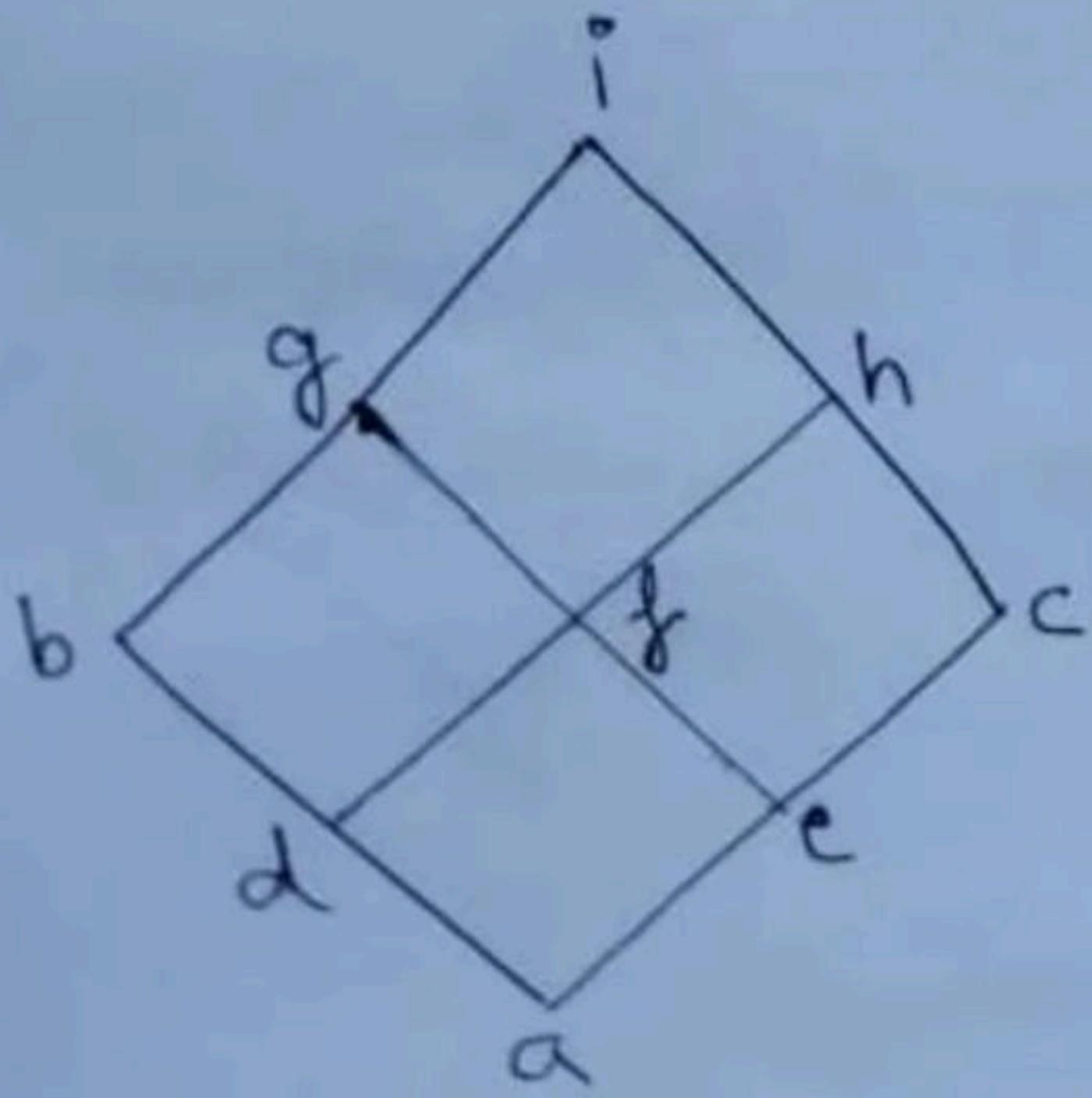
Q Let $X = \{2, 3, 6, 12, 24\}$, Let \leq be the partial order defined by $X \leq Y$ if x divides y . Number of edges as in the Hasse diagram of (X, \leq) is. **(GATE-1996) (1 Marks)**

- (a)** 3
- (b)** 4
- (c)** 9
- (d)** None of the above

Break

Elements of a Poset

1. **Maximal Element:** - An element is said to be maximal if it is not related to any other element in the Partial order relation.
2. **Minimal Element:** - An element is said to be minimal if no other element is related to it in the Partial order relation.



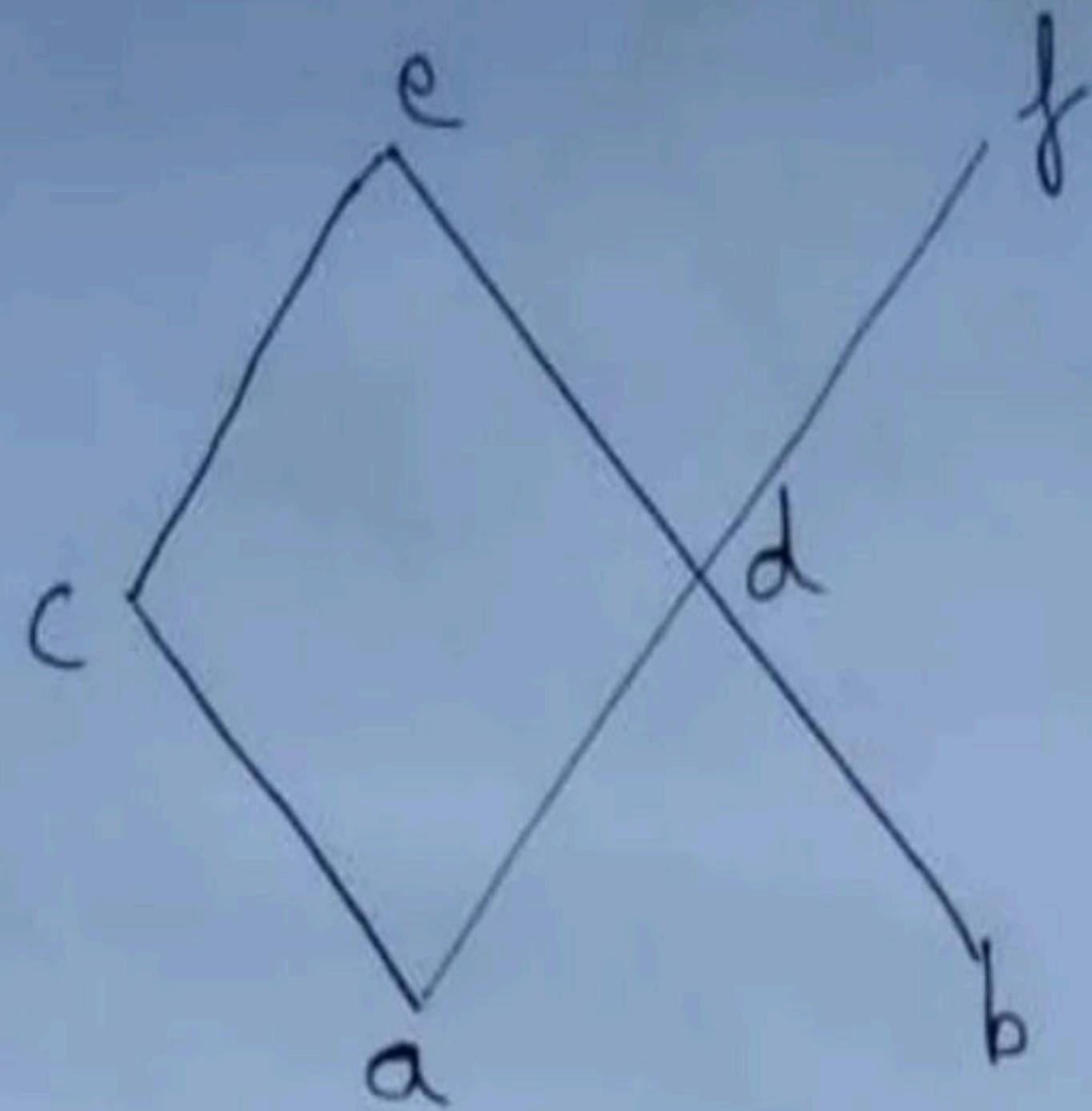
Minimal

Least

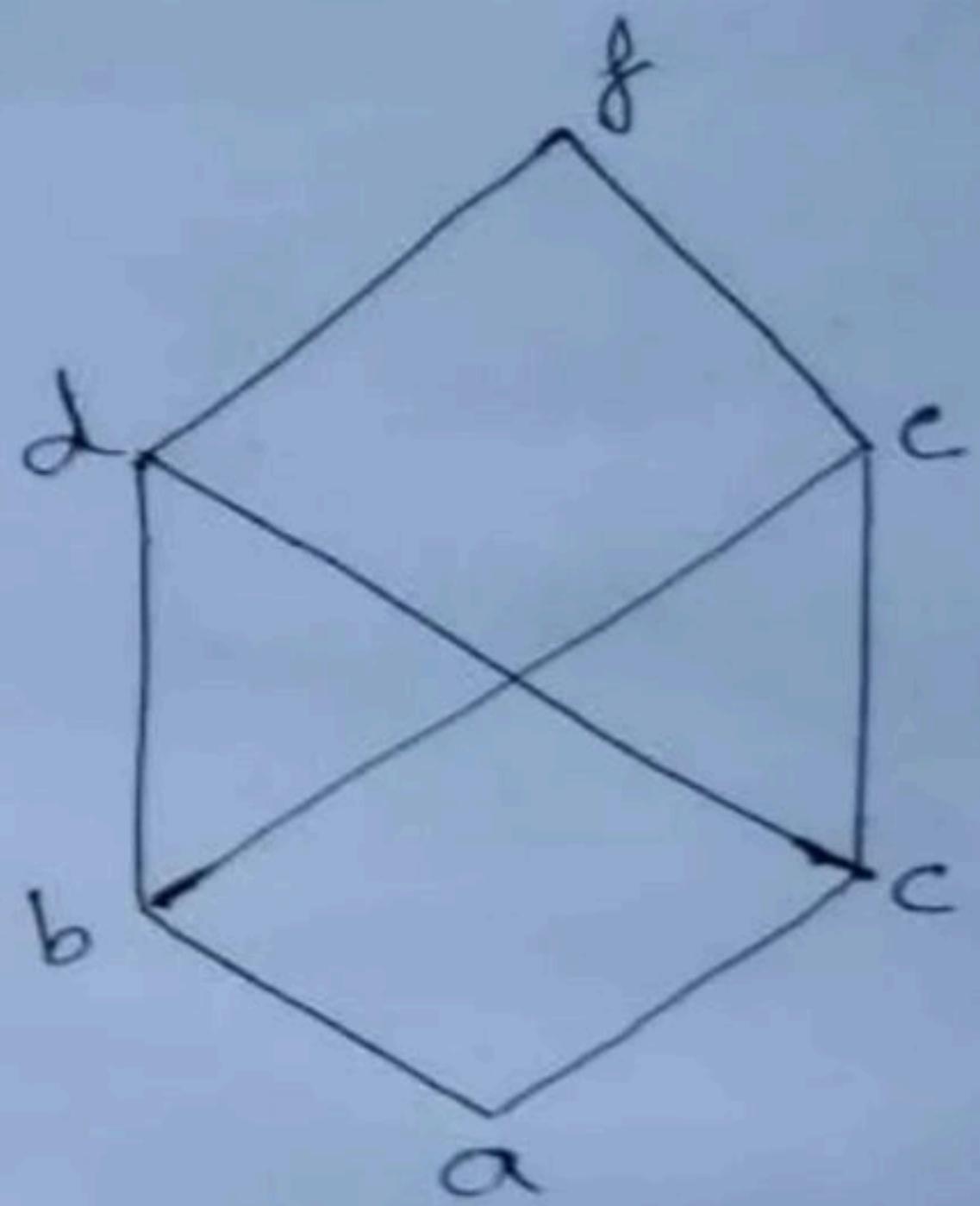
Maximal

Greatest

Elements



Elements
Minimal
Least
Maximal
Greatest



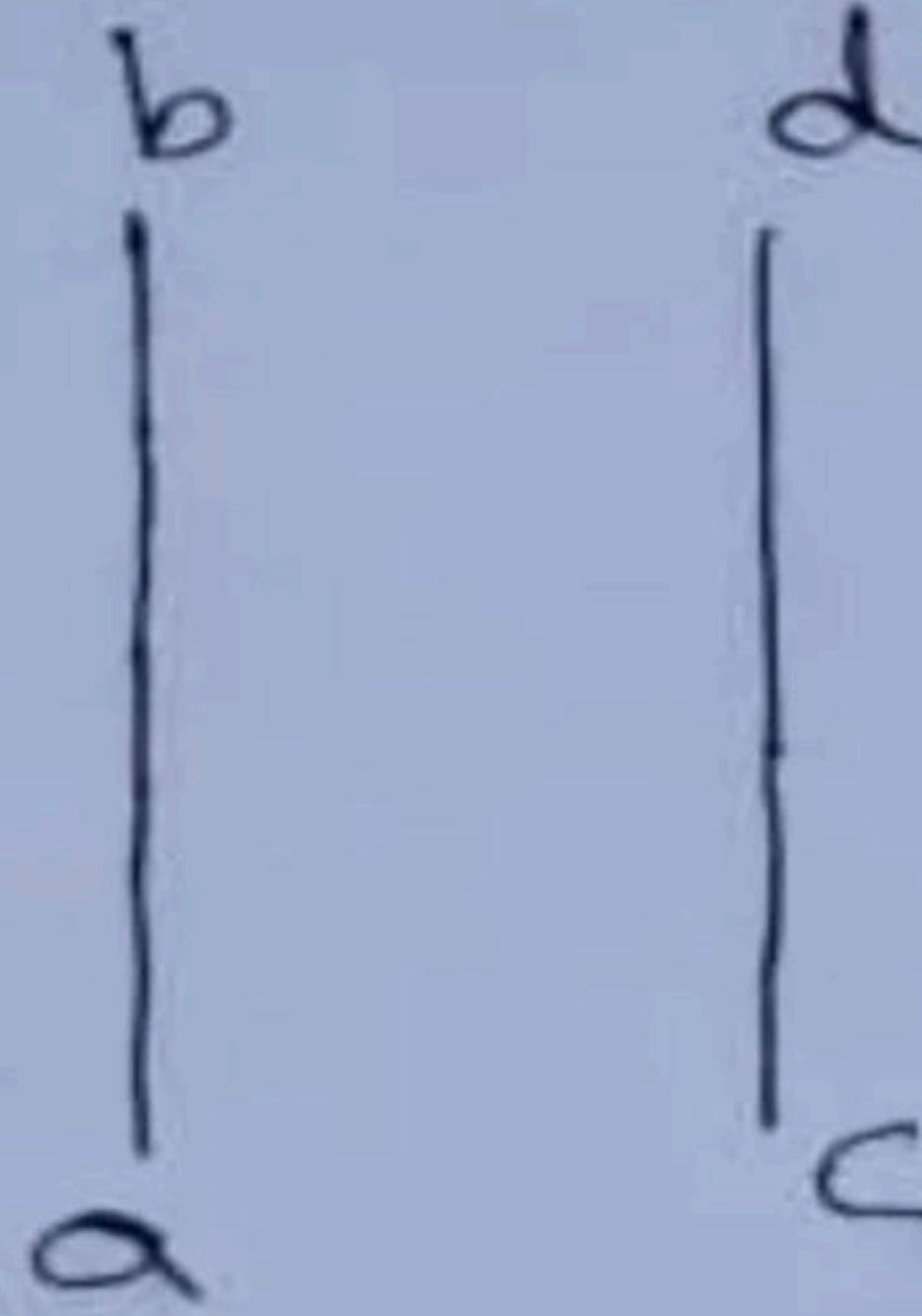
Minimal

Least

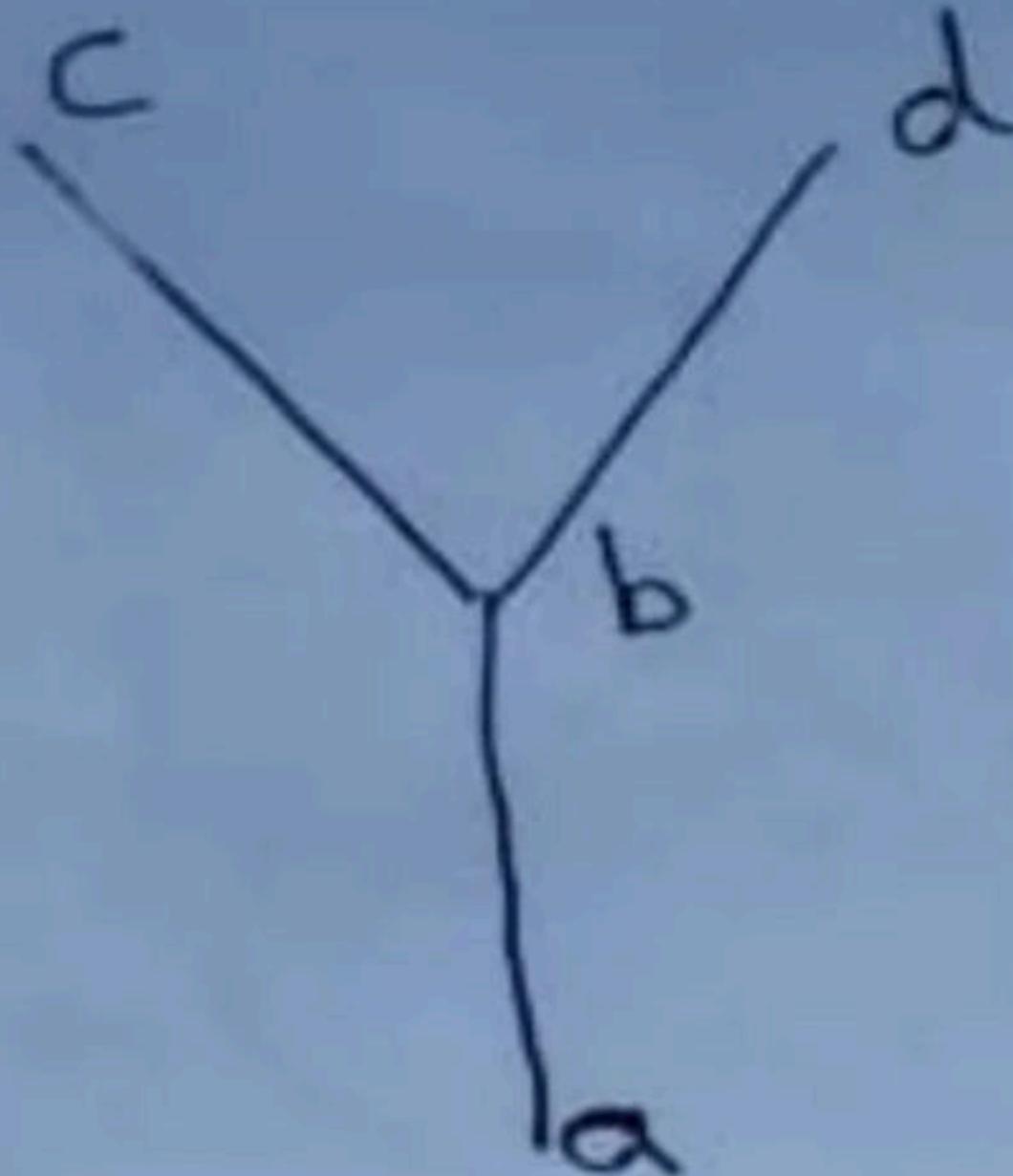
Maximal

Greatest

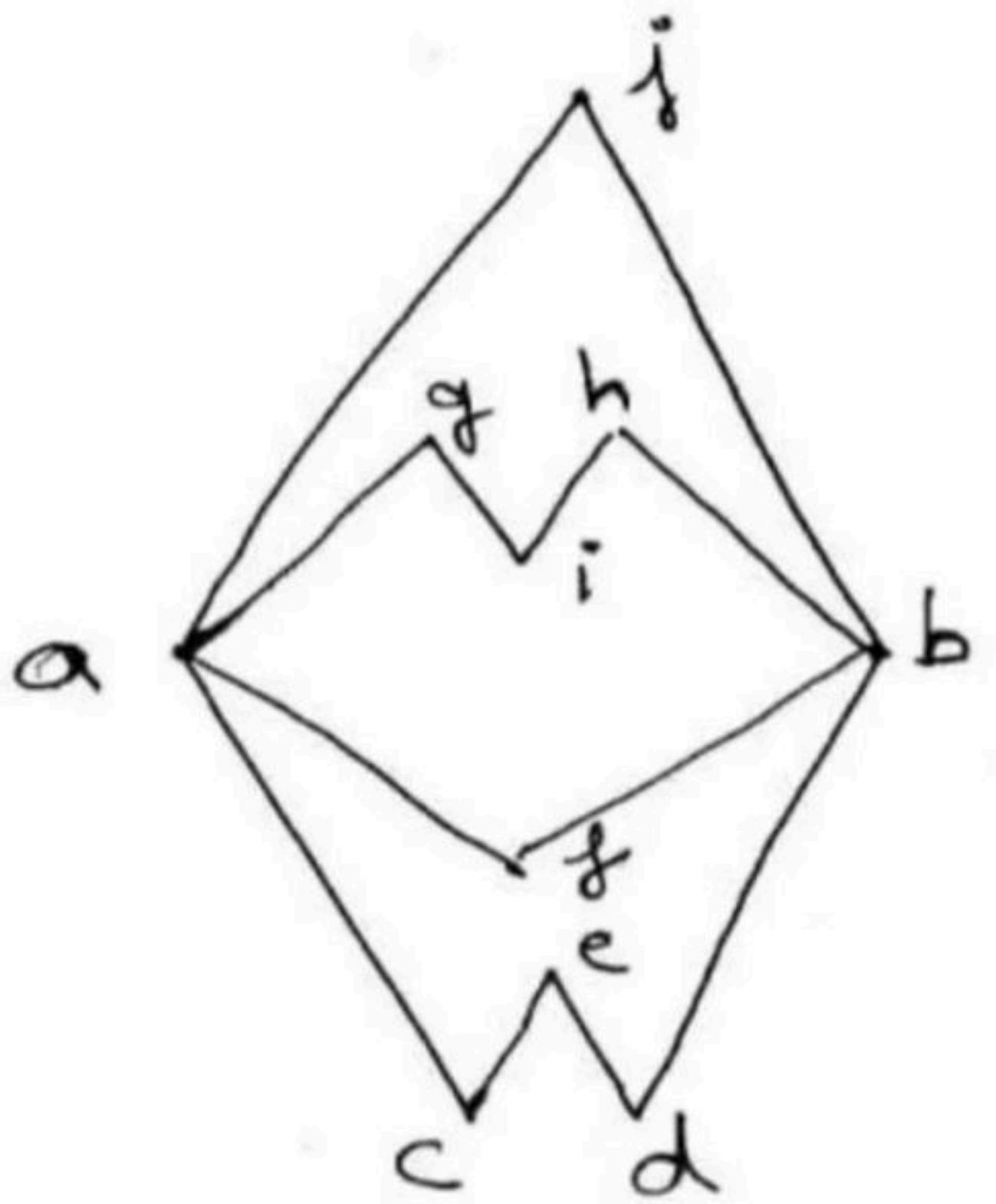
Elements



Elements
Minimal
Least
Maximal
Greatest



Elements	
Minimal	
Least	
Maximal	
Greatest	

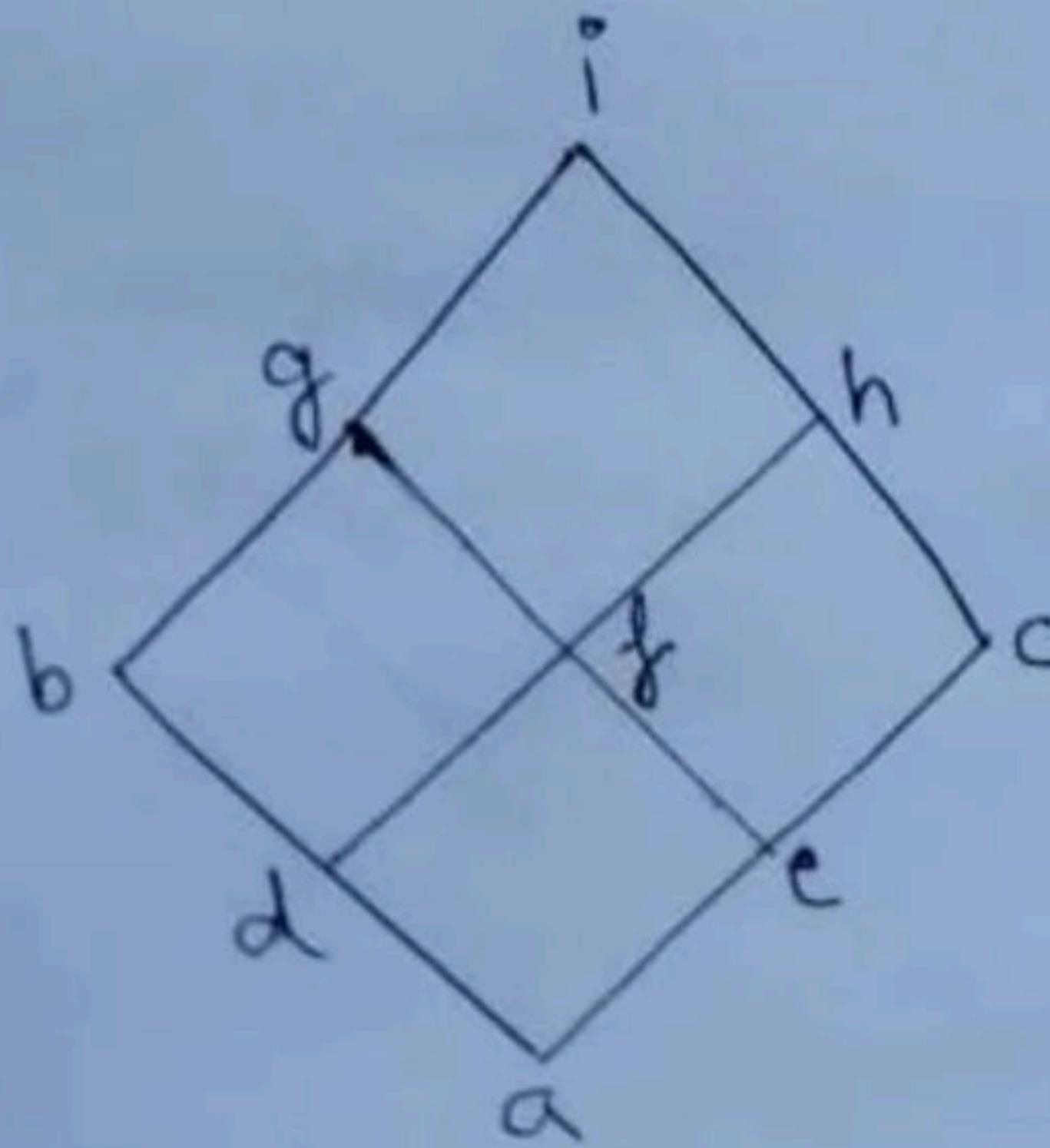


Elements
Minimal
Least
Maximal
Greatest

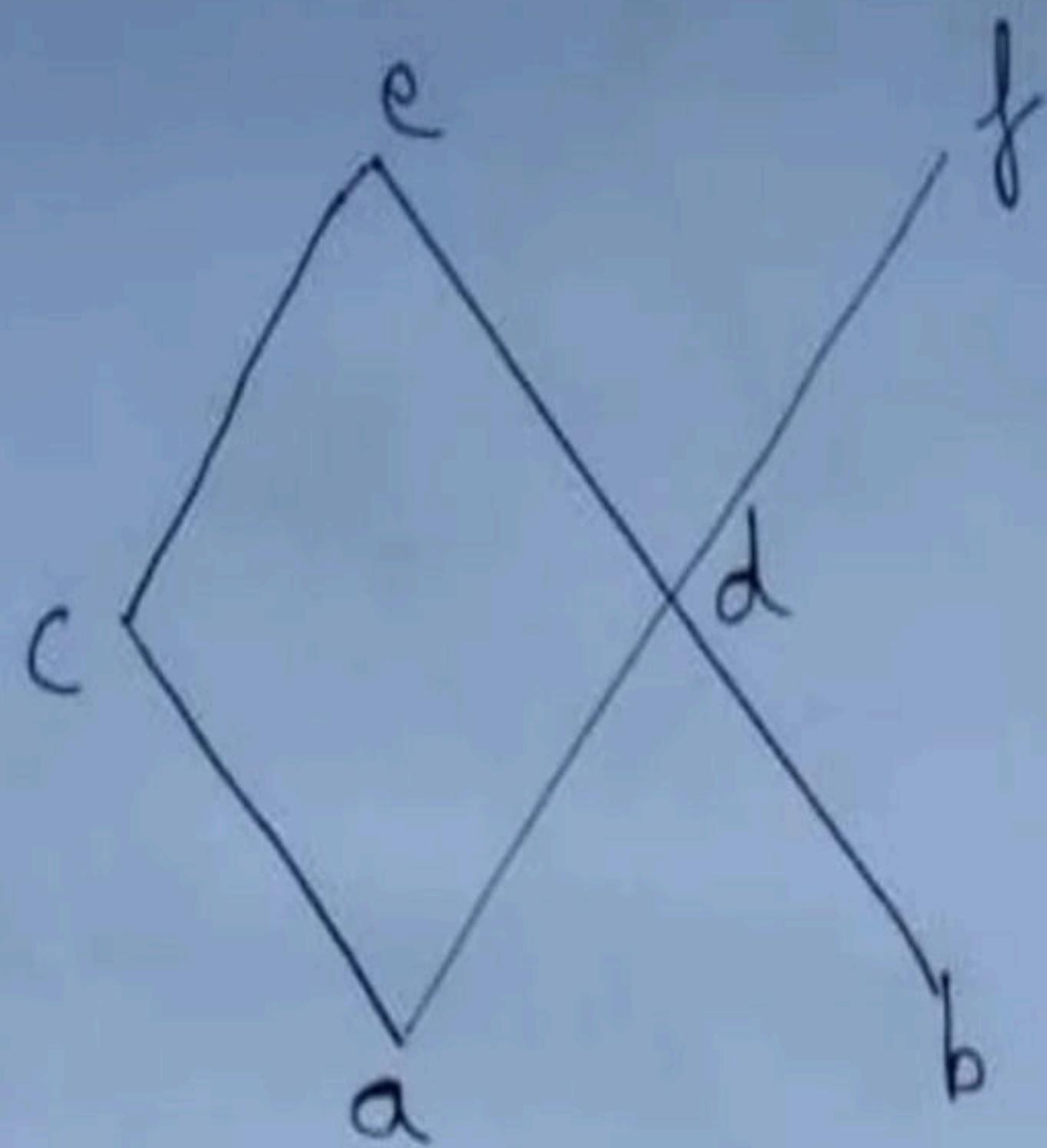
- Every hasse diagram will have at least one Maximal and Minimal element(one or more).

Break

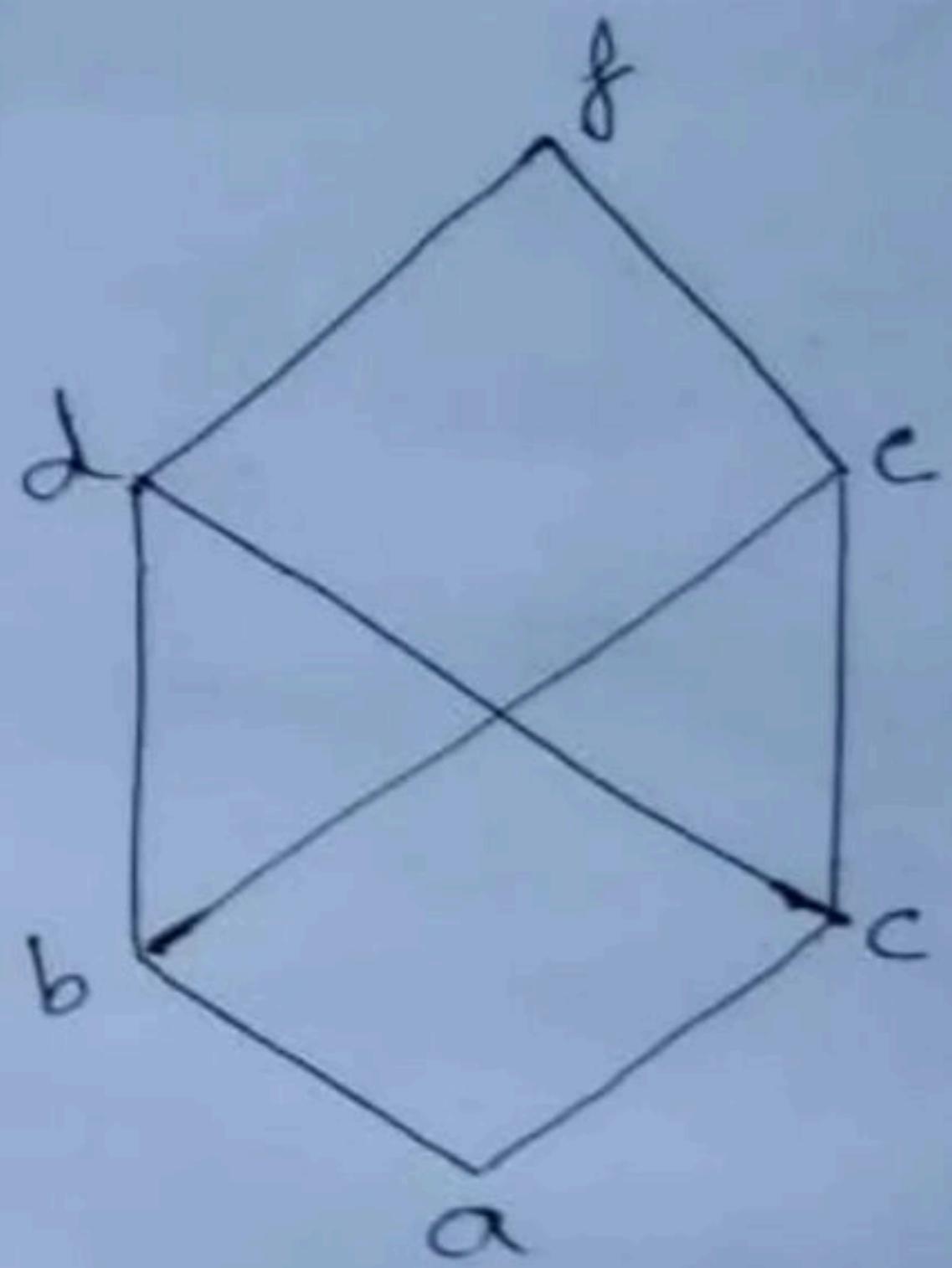
- **Greatest Element**: - An element is said to be Maximum/Greatest if it is not related to any other element but every element is related to it in Partial order relation. Or if a hasse diagram has only one Maximal element then it will also be Maximum/Greatest element.
- **Least Element**: - An element is said to be Minimum/Least if no other element is related to it but it is related to every element Partial order relation. Or if a hasse diagram has only one Minimal element then it will also be Minimum/Least element.



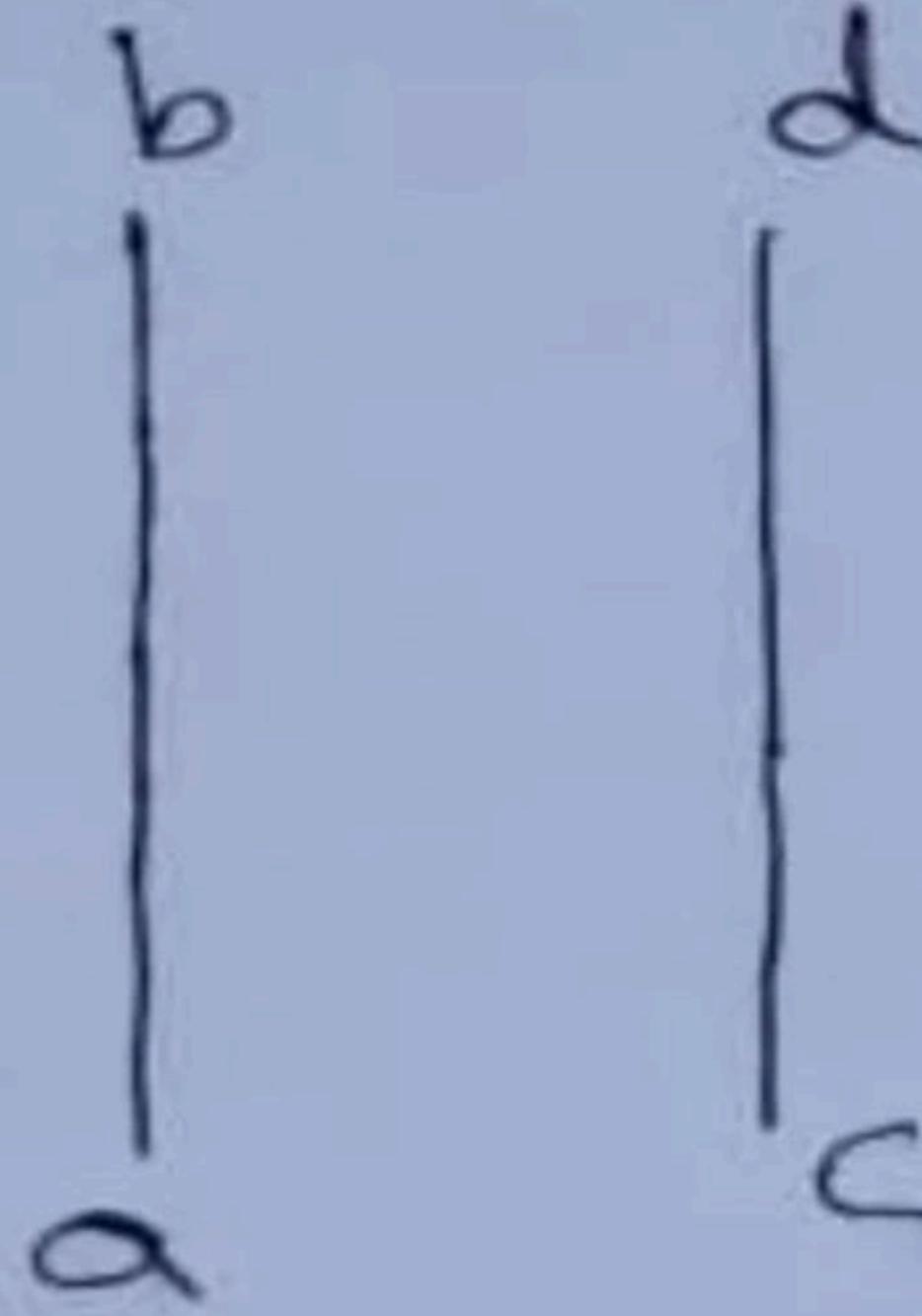
Elements
Minimal
Least
Maximal
Greatest



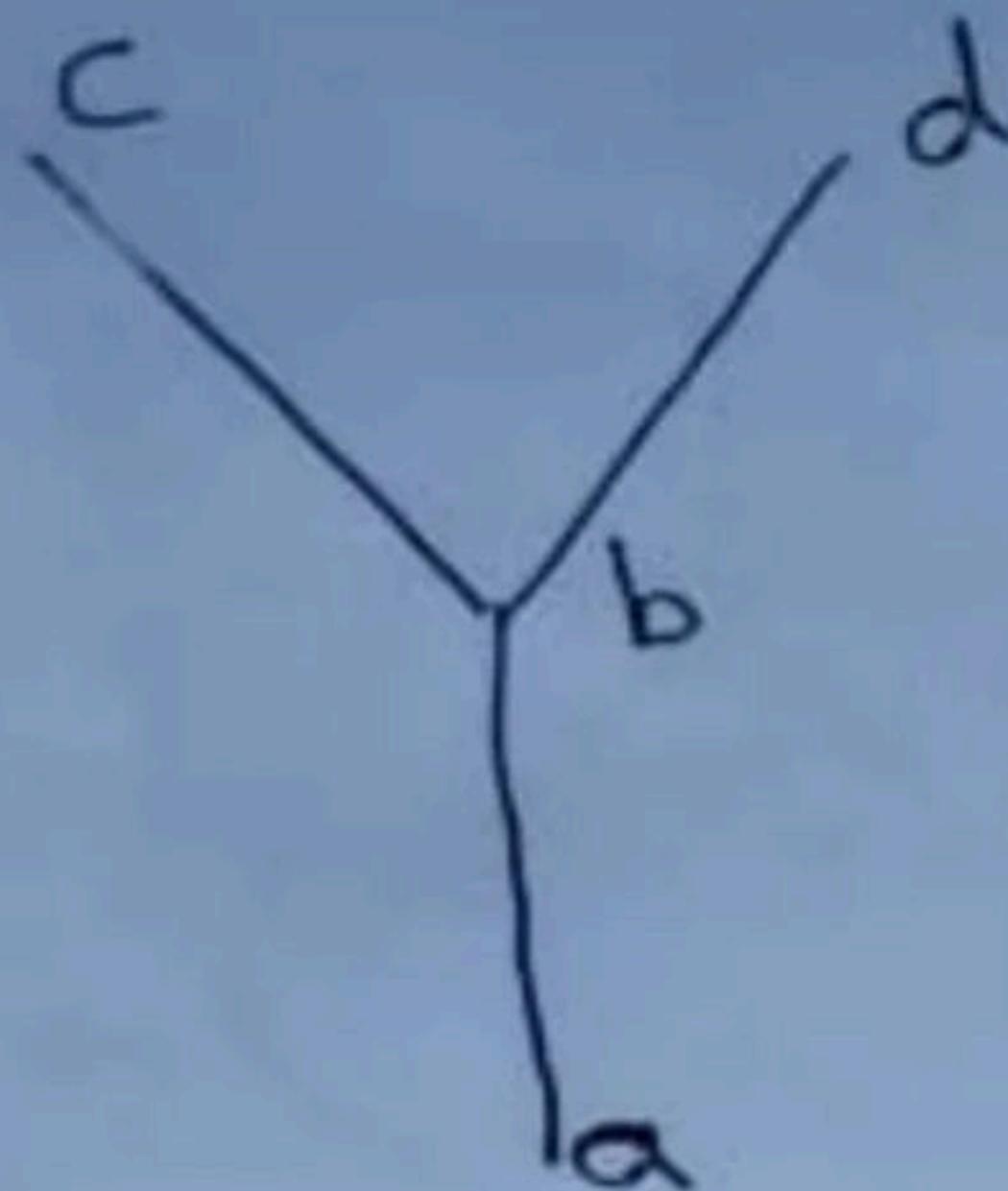
	Elements
Minimal	a, b
Least	
Maximal	e, f
Greatest	



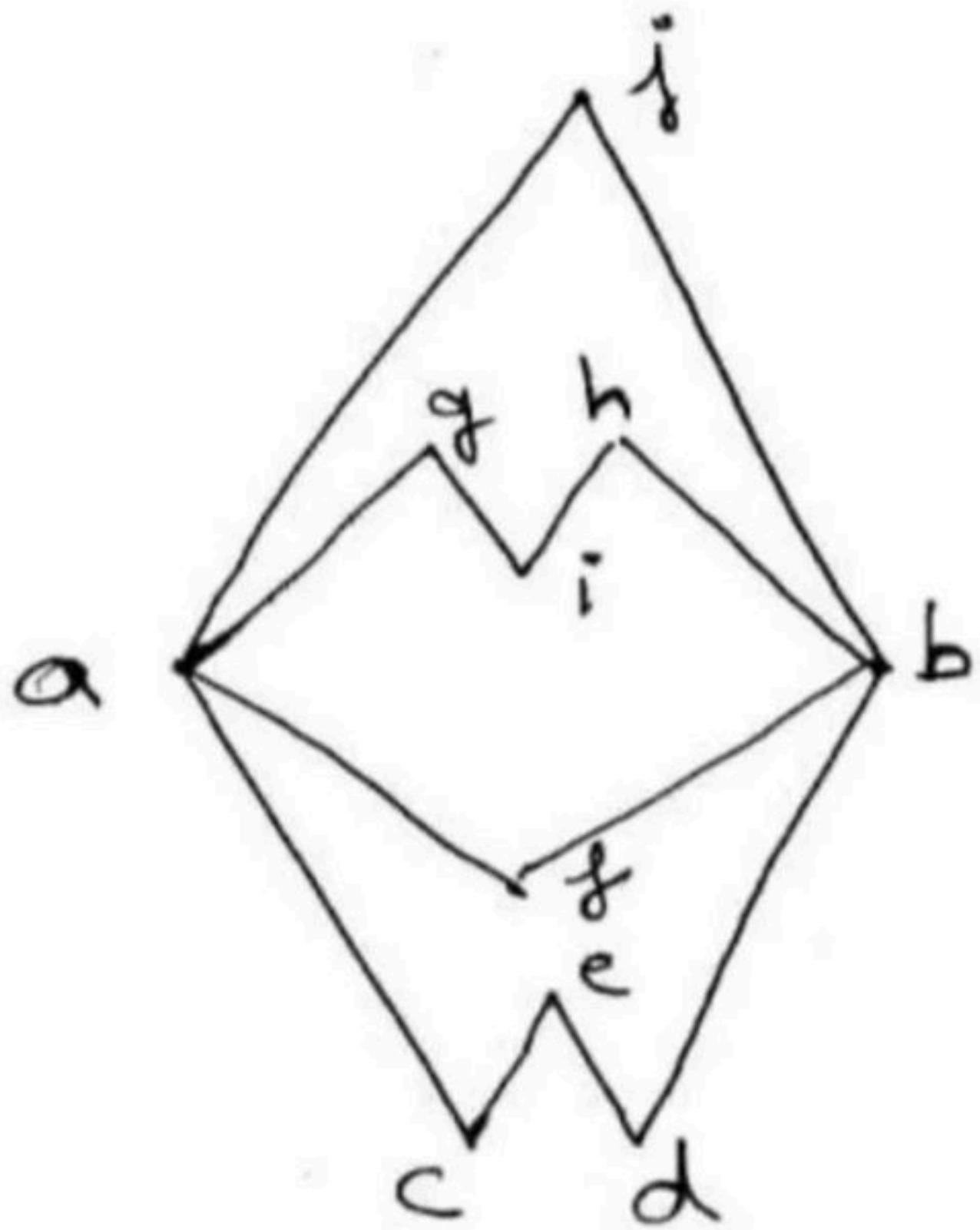
Elements
Minimal
Least
Maximal
Greatest



Elements	
Minimal	a, c
Least	
Maximal	b, d
Greatest	



Elements
Minimal
Least
Maximal
Greatest

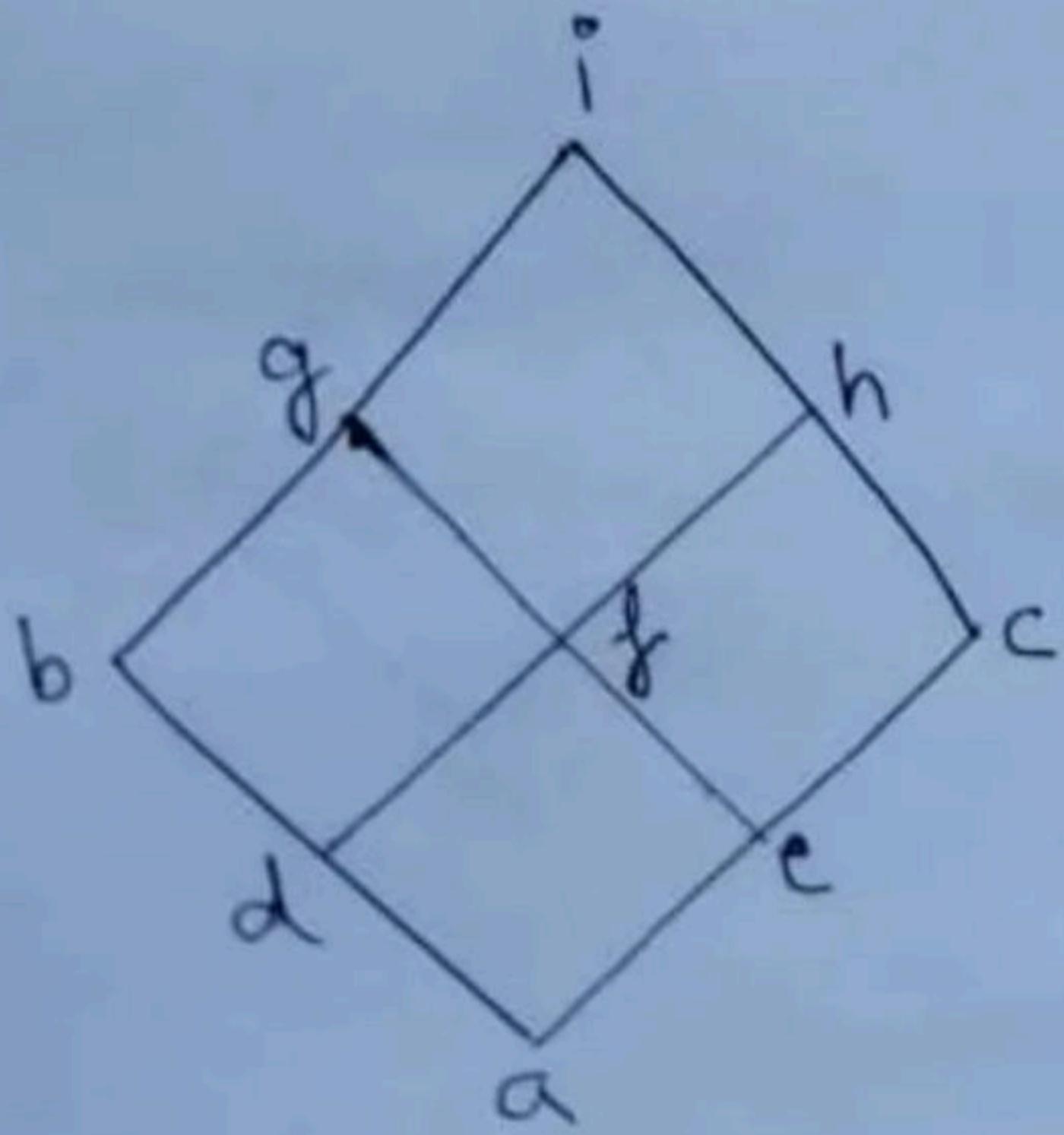


Elements	
Minimal	c, d, f, i
Least	
Maximal	j, g, h, e
Greatest	

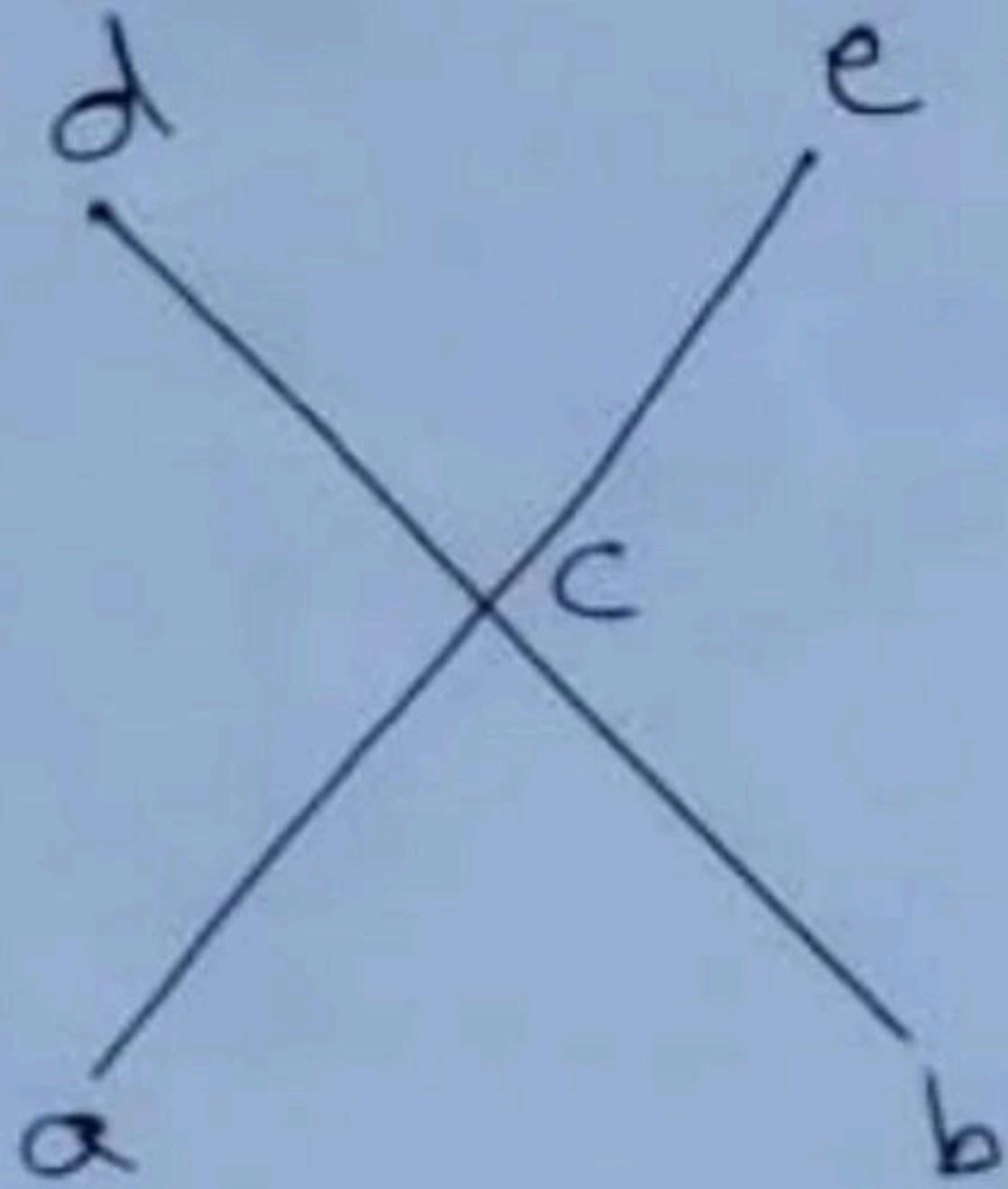
1. Every hasse diagram will have at most one Greatest and Least element(zero or one) (T/F)
2. Every Greatest element is also Maximal (T/F)
3. Every Least element is also Minimal (T/F)
4. If there is only one Maximal element then it is called Greatest (T/F)
5. If there is only one Minimal element then it is called Least (T/F)

Break

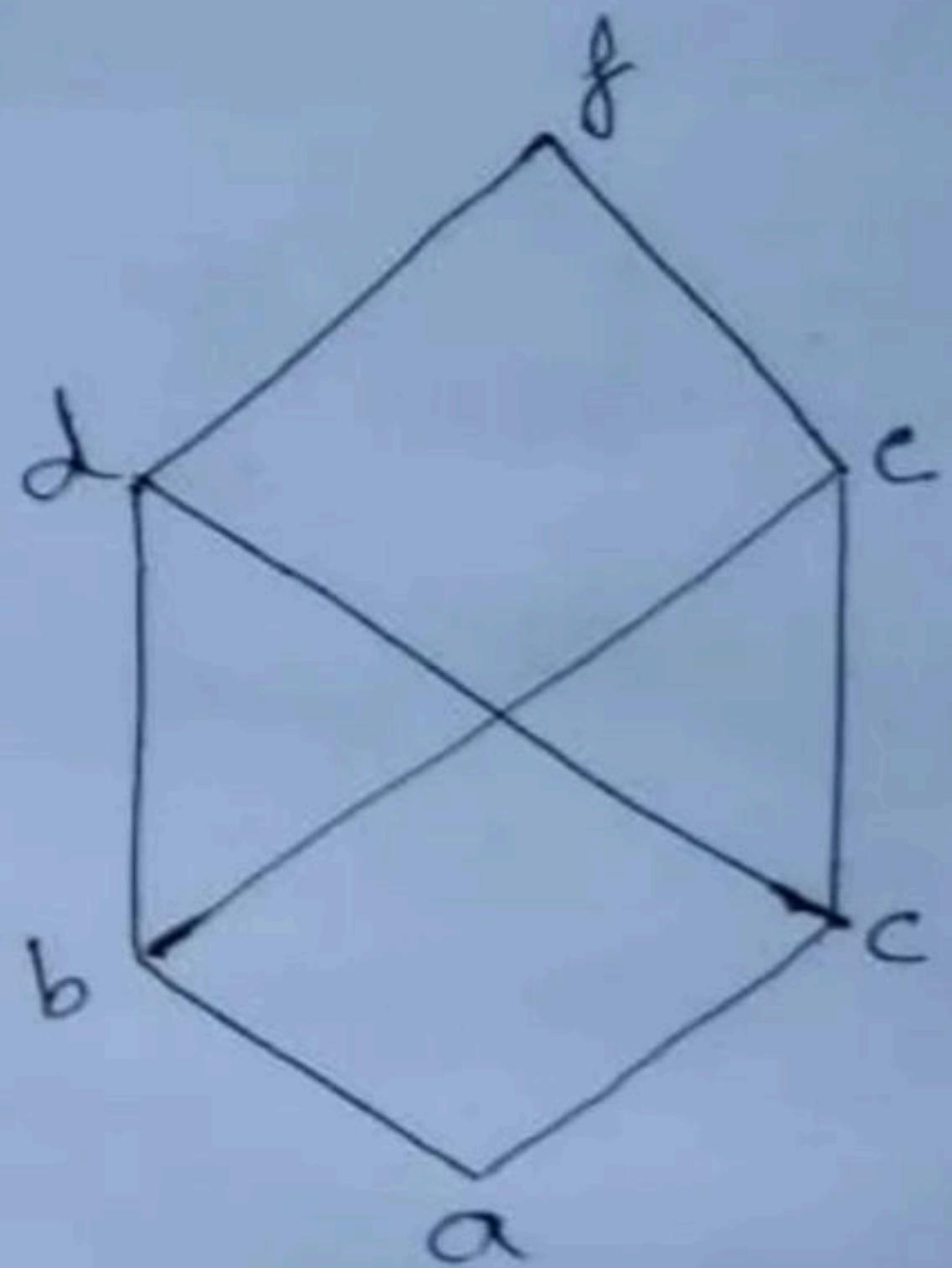
- **Upper Bound**: - Upper bound of a subset B with respect to set A, will contain all those element to which all the elements of B is related.
- **Lower Bound**: - lower bound of a subset B with respect to A, will contain all those elements which are related to every element of B.



Elements	$B = \{g, f, e, c\}$	$B = \{d, f\}$
Upper Bound		
Lower Bound		



Elements	$B = \{d, e\}$	$B = \{c\}$
Upper Bound		
Lower Bound		



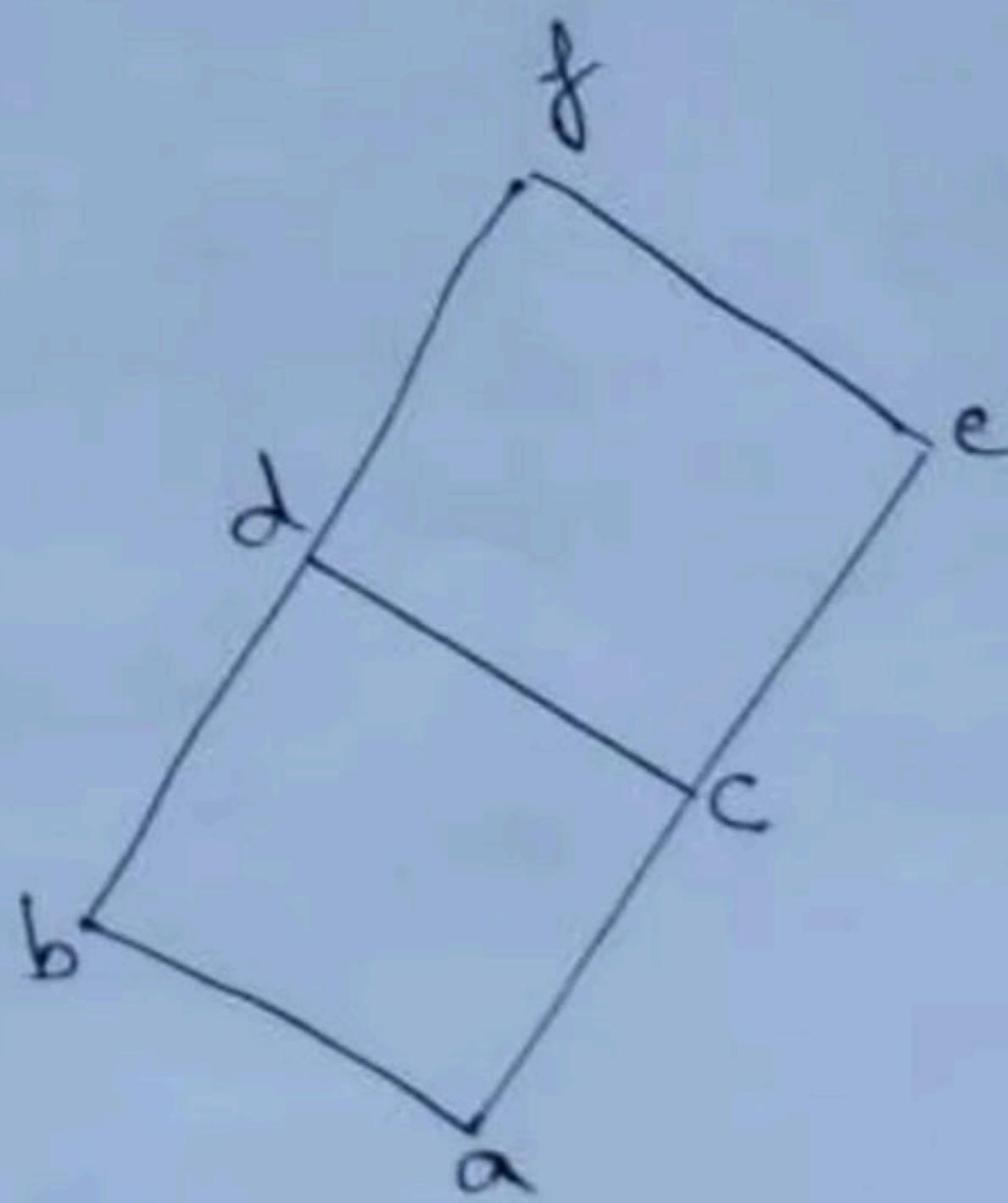
Elements

$B = \{d, e\}$

$B = \{b, c\}$

Upper
Bound

Lower
Bound



Elements	$B = \{d, c\}$	$B = \{b, e\}$
Upper Bound		
Lower Bound		

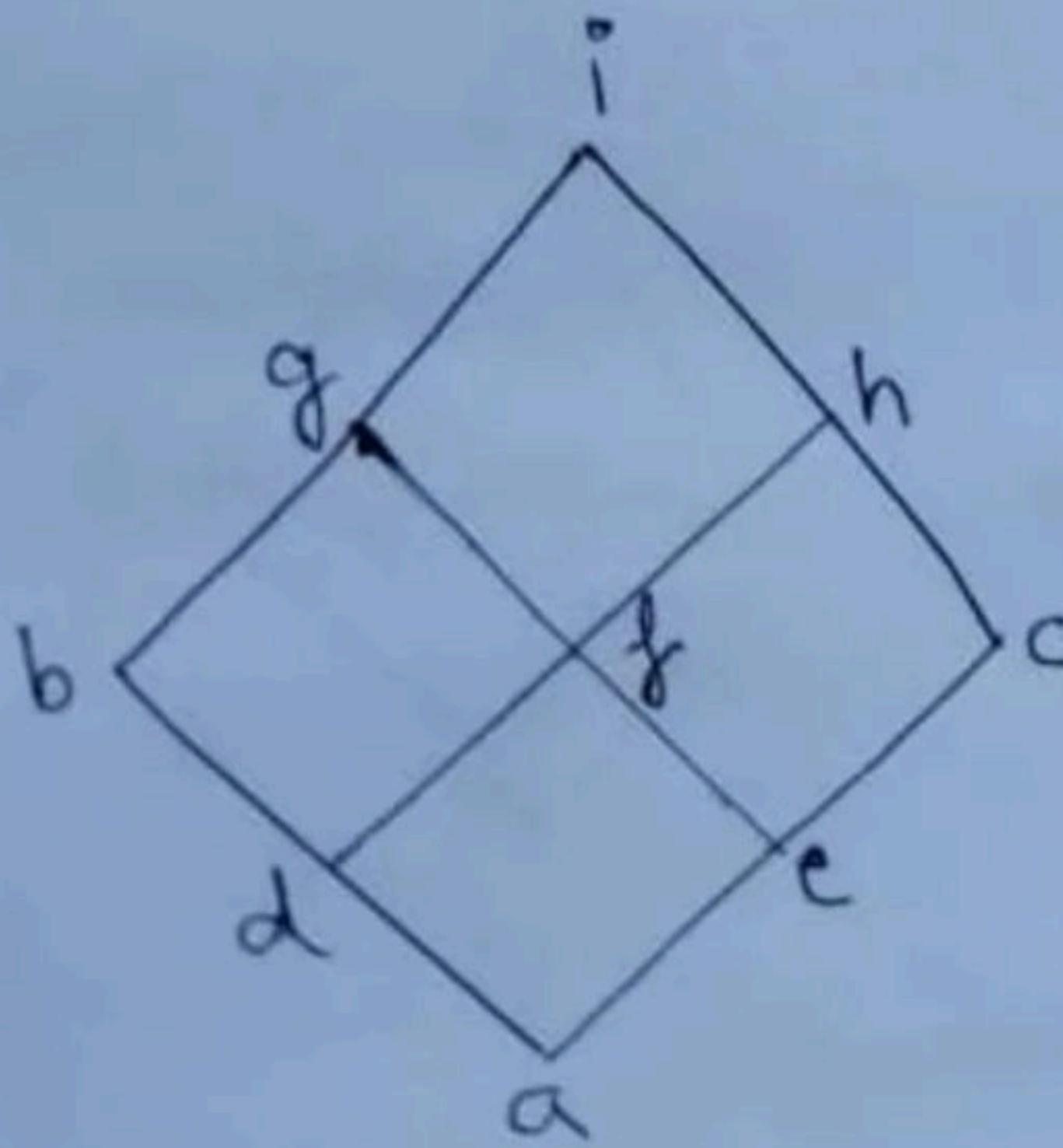
Break

Least Upper Bound / LUB / Join / Supremum / v

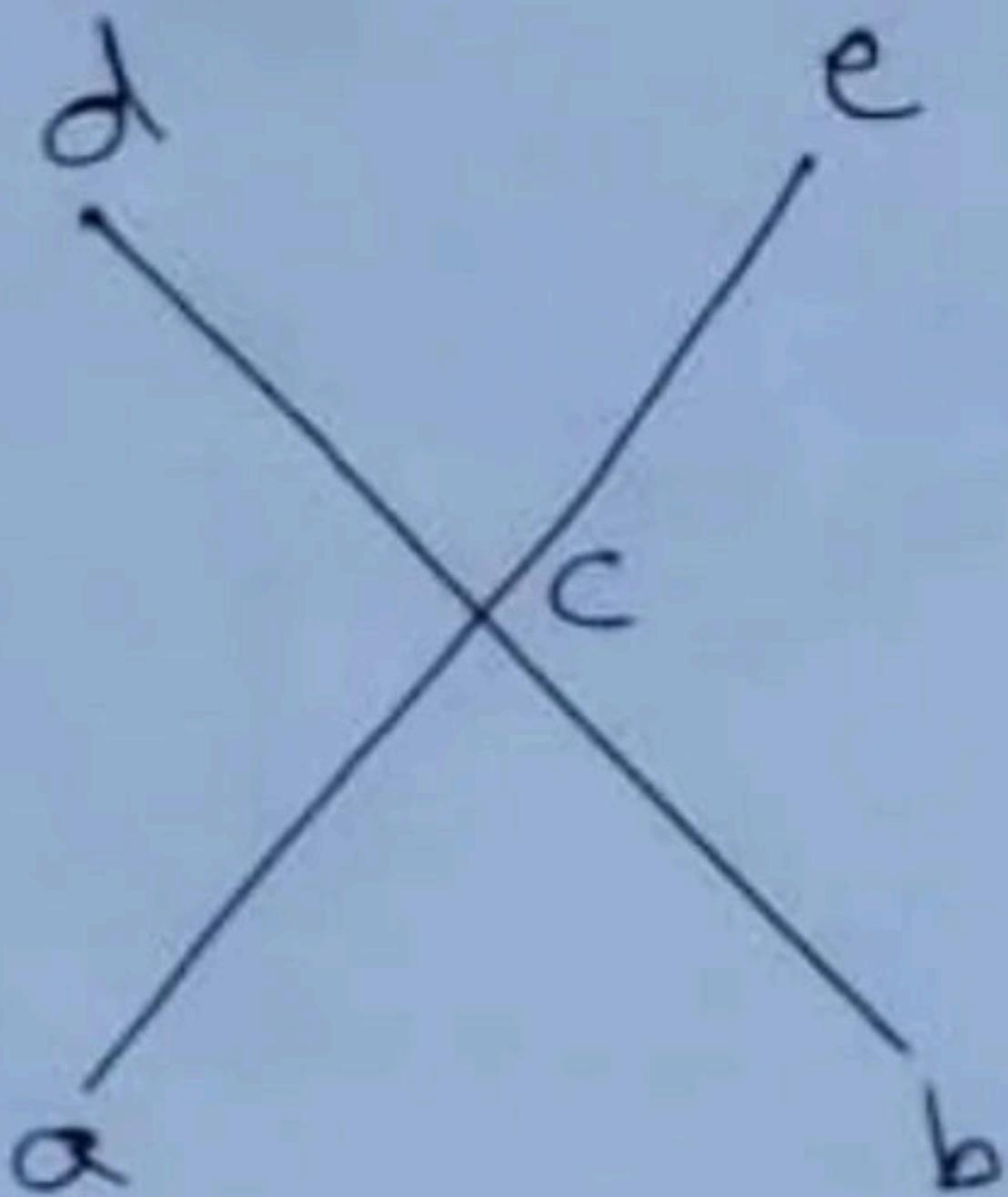
Least value in the upper bound

Greatest Lower Bound / GLB / Meet / Infimum / \wedge

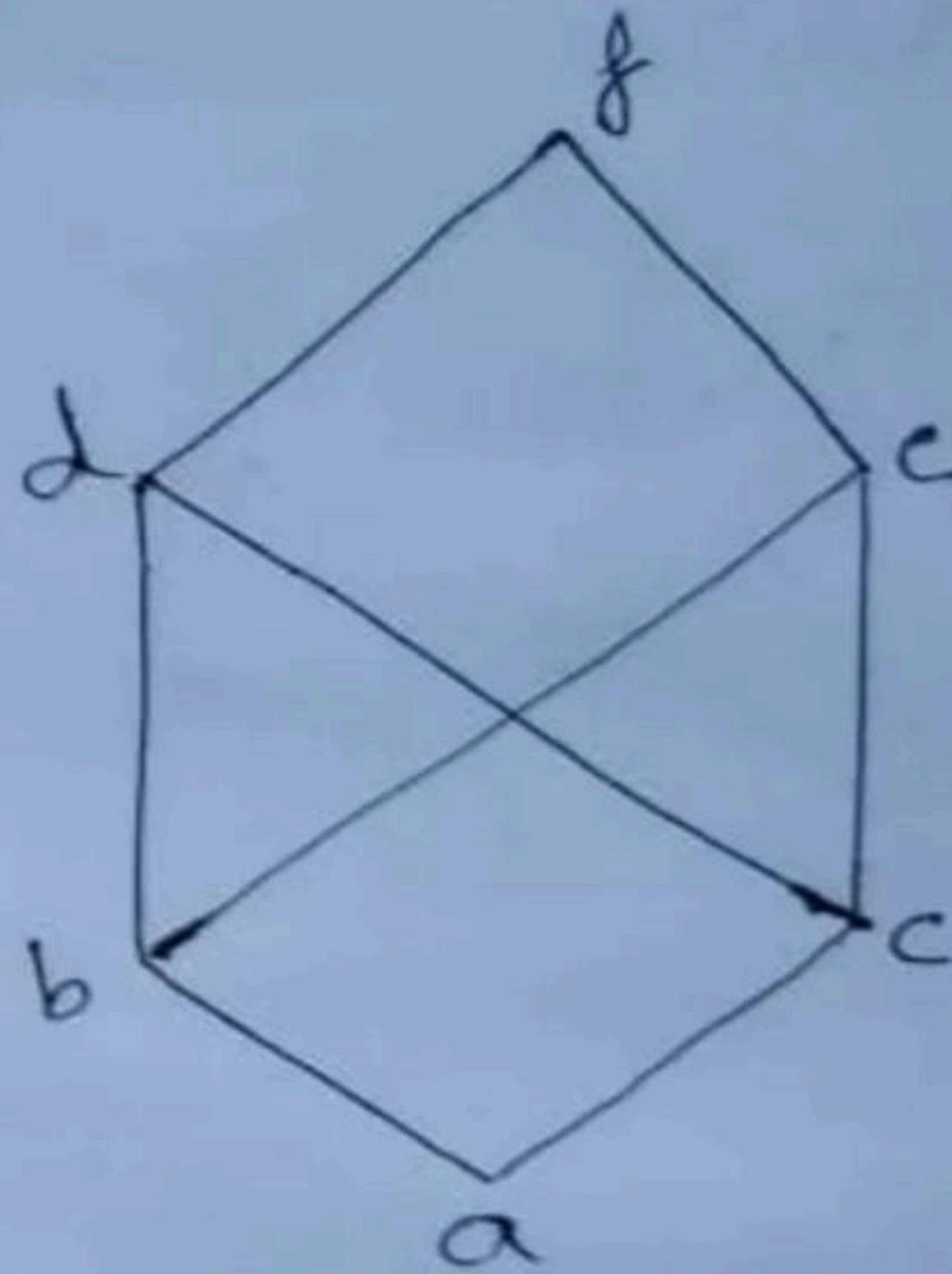
Greatest value in the lower bound



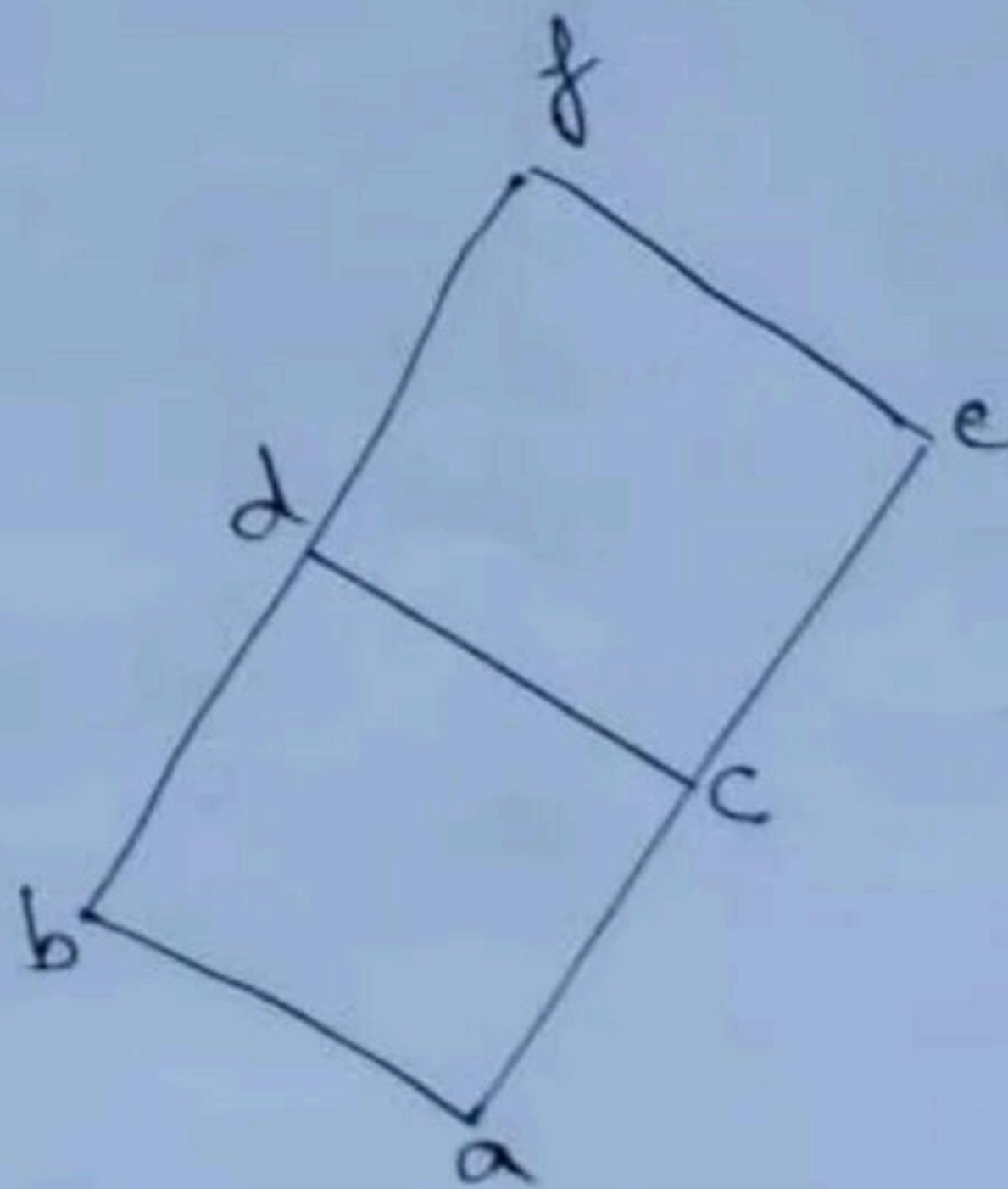
Elements	$B = \{g, f, e, c\}$	$B = \{d, f\}$
Upper Bound	{i}	{i, g, h, f}
Least Upper Bound		
Lower Bound	{a, e}	{a, d}
Greatest Lower Bound		



Elements	$B = \{d, e\}$	$B = \{c\}$
Upper Bound	{}	{d, e, c}
Least Upper Bound		
Lower Bound	{a, b, c}	{a, b, c}
Greatest Lower Bound		



Elements	$B = \{d, e\}$	$B = \{b, c\}$
Upper Bound	{f}	{d, e, f}
Least Upper Bound		
Lower Bound	{a, b, c}	{a}
Greatest Lower Bound		



Elements	$B = \{d, c\}$	$B = \{b, e\}$
Upper Bound	$\{f, d\}$	$\{f\}$
Least Upper Bound		
Lower Bound	$\{a, c\}$	$\{a\}$
Greatest Lower Bound		

Q Consider the Poset $(\{3, 5, 9, 15, 24, 45\}, /)$. Which of the following is correct for the given Poset?
(NET-JUNE-2019)

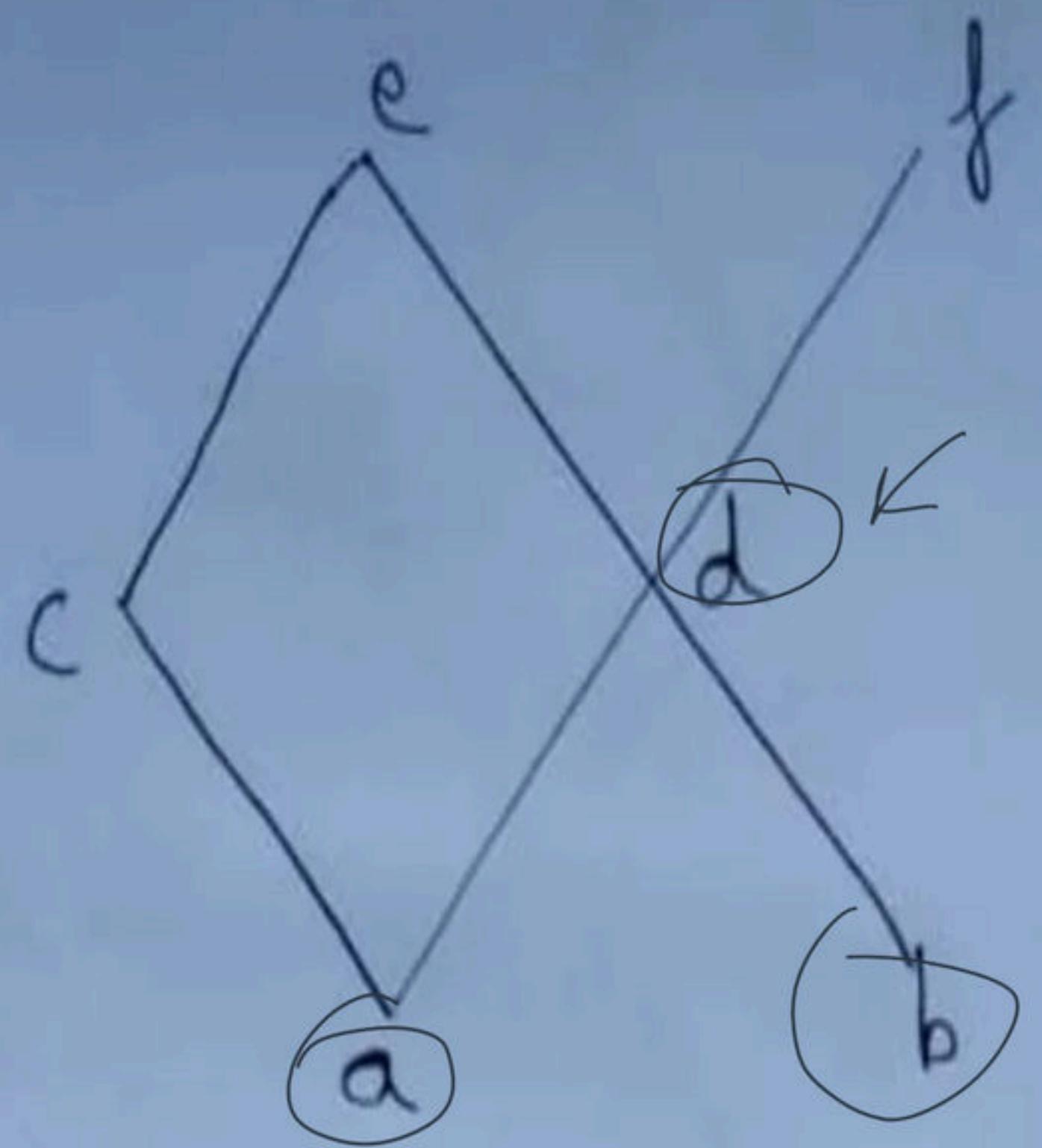
- a)** There exist a greatest element and a least element
- b)** There exist a greatest element but not a least element
- c)** There exist a least element but not a greatest element
- d)** There does not exist a greatest element and a least element

Break

Join Semi Lattice :- A hasse diagram/Partial order relation is called Join Semi Lattice if for every elements their exists a Join.

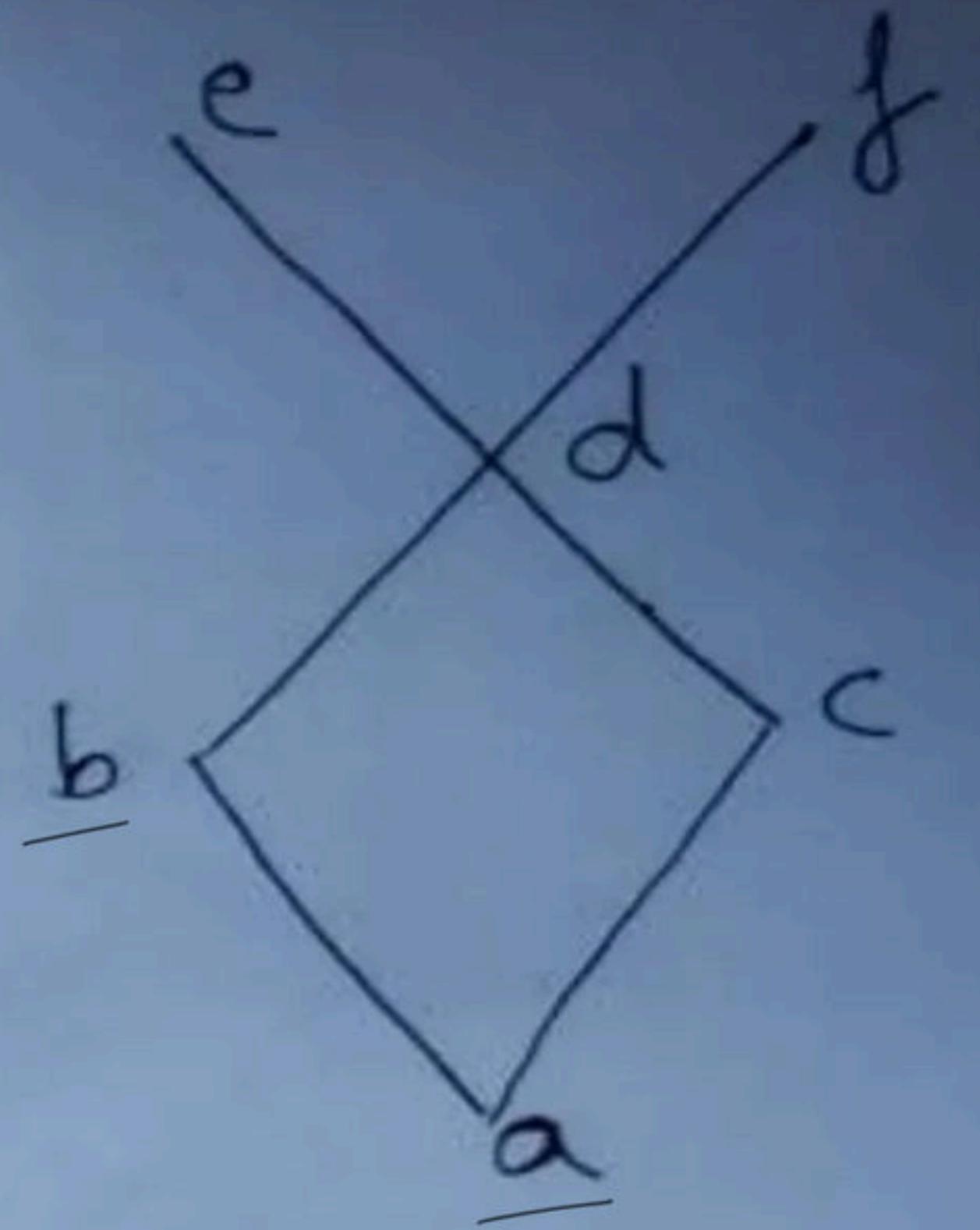
Meet Semi Lattice :- A hasse diagram/Partial order relation is called Meet Semi Lattice if for every elements their exists a Meet.

Lattice :- A hasse diagram/Partial order relation is called Lattice if there exist a Join and Meet for every pair of element. Or A hasse diagram/Partial order relation is called Lattice if it is both Join Semi Lattice and Meet Semi Lattice.



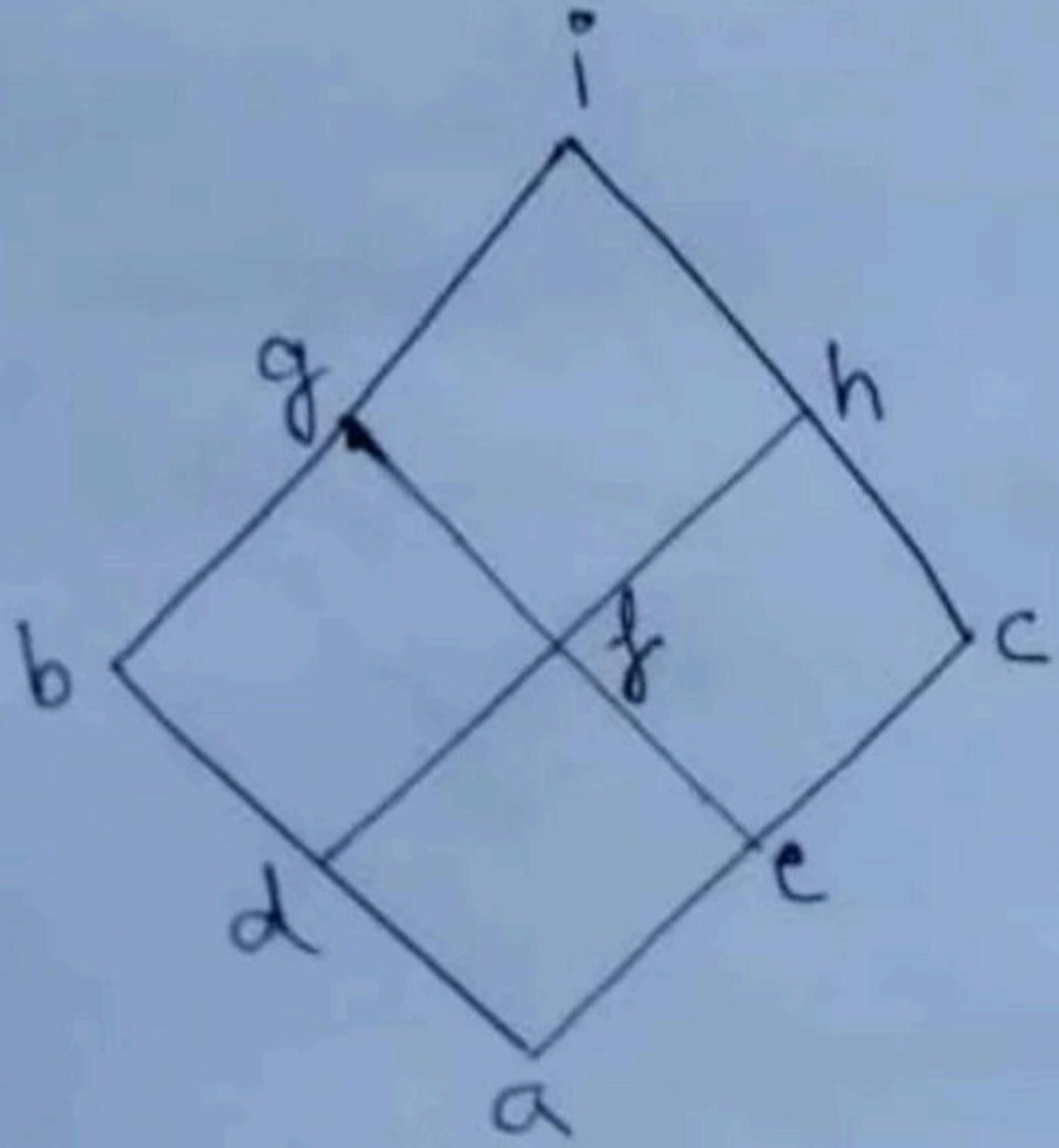
$$e \vee f = ?$$

$$a \wedge b = ?$$



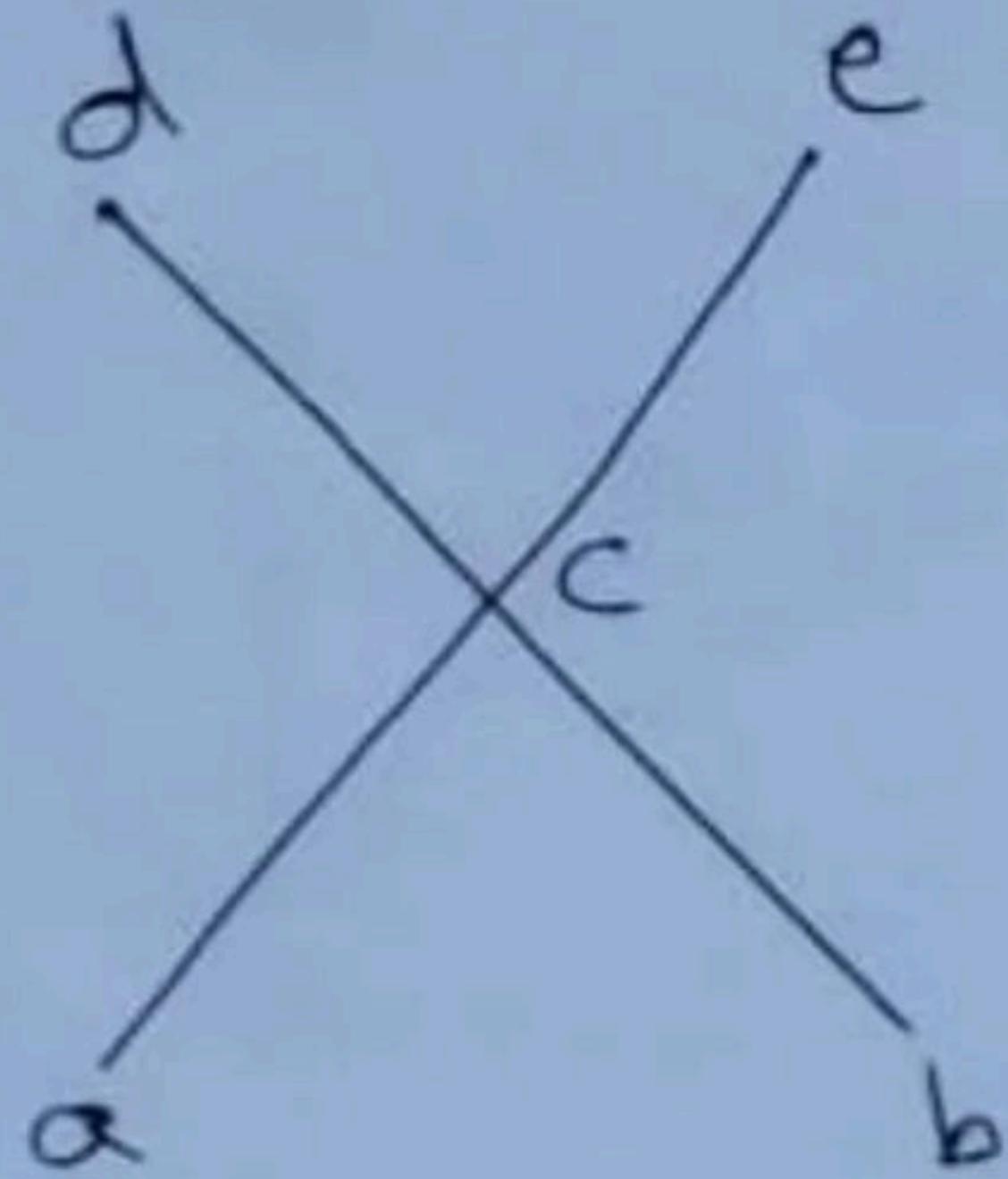
$e \vee f = ?$

MSL ✓



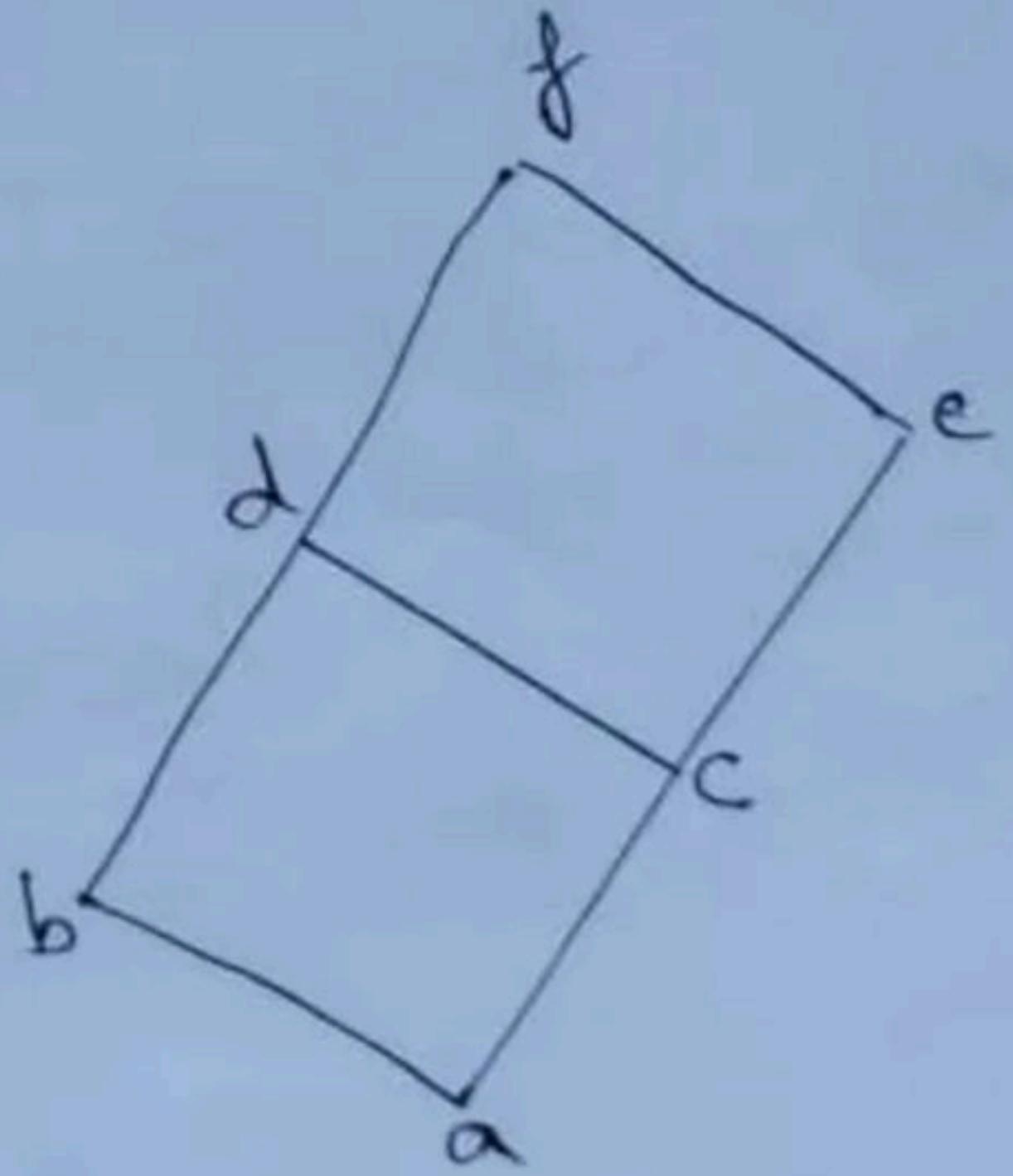
join = \vee

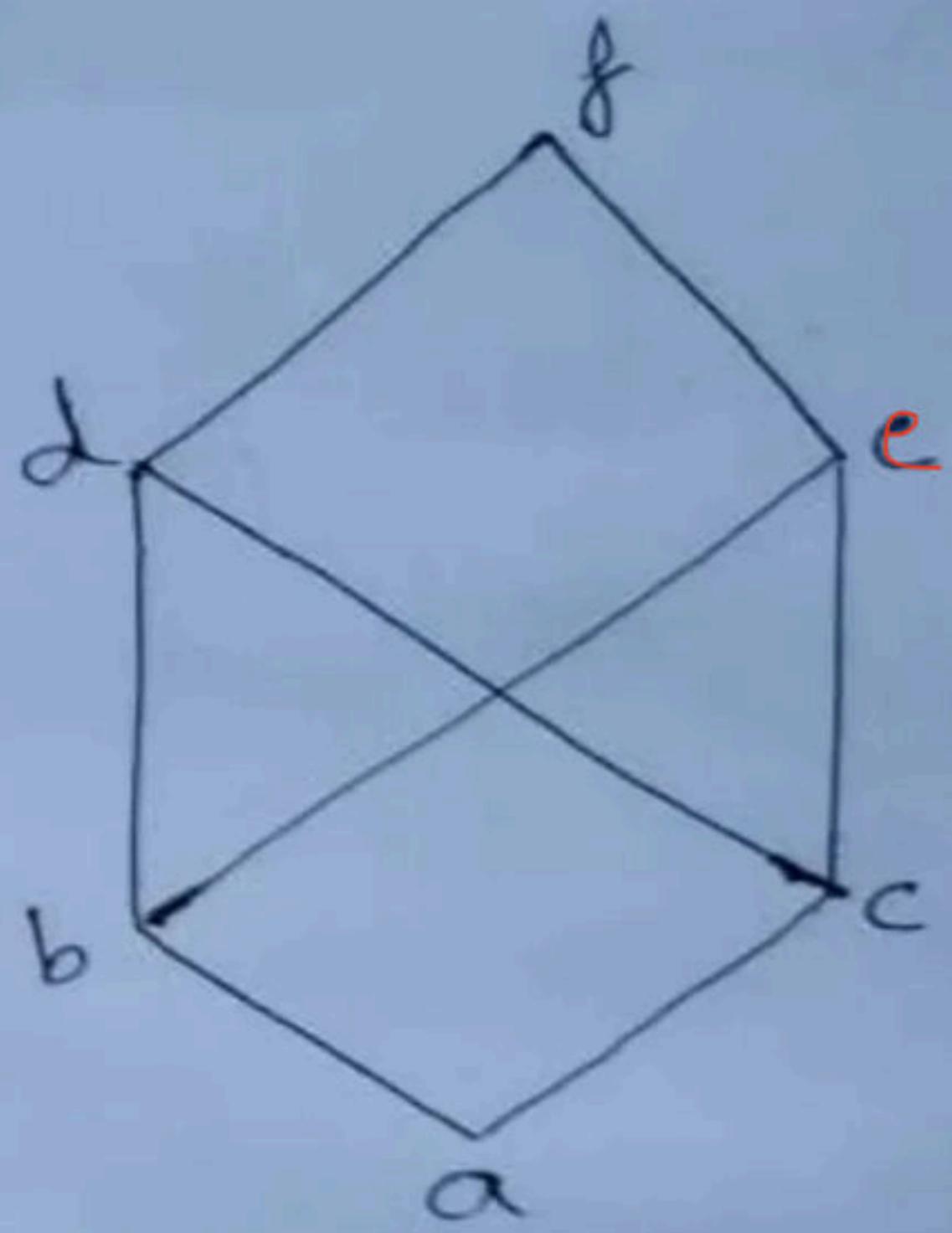
meet = \wedge



$$d \vee e = ?$$

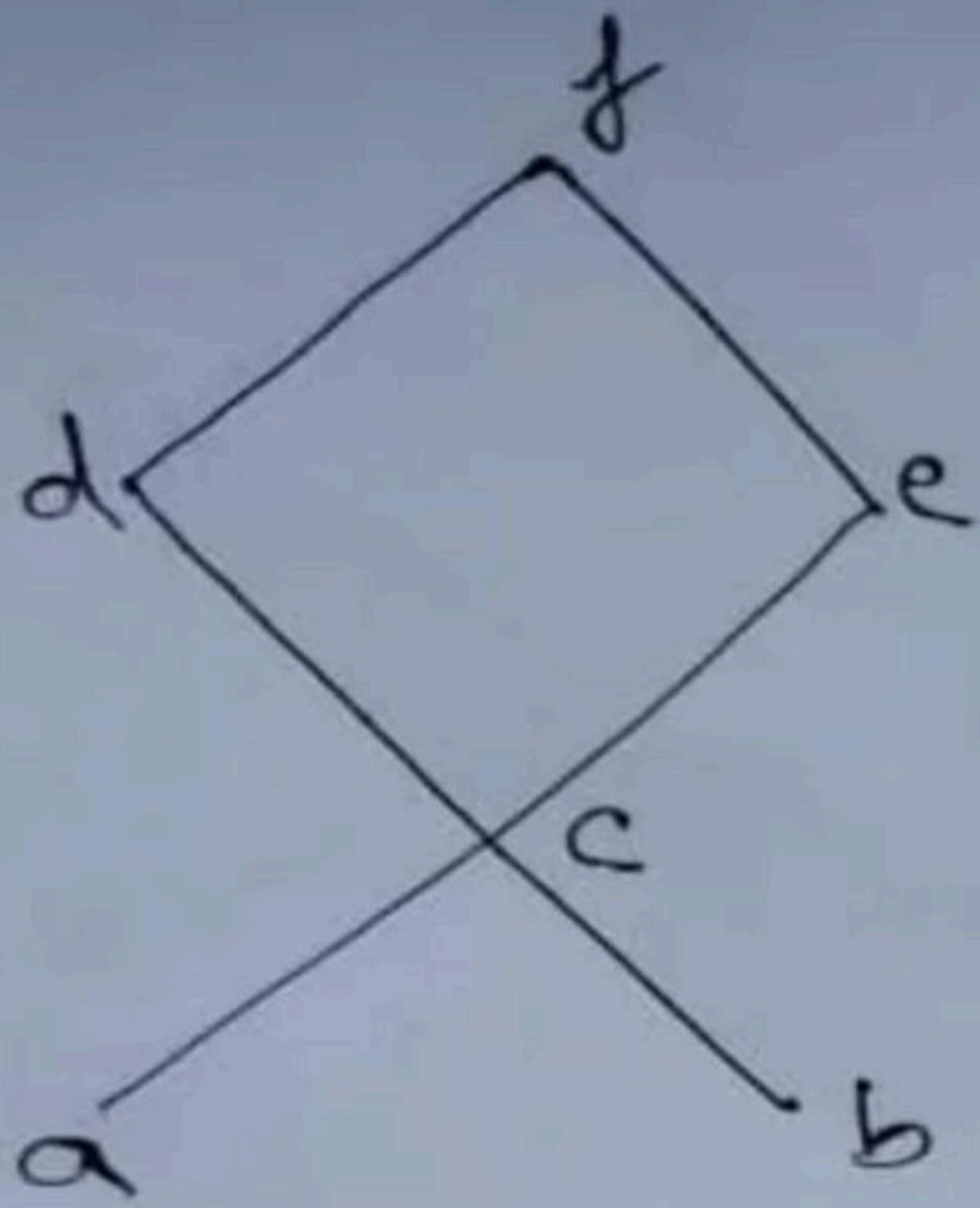
$$a \wedge b = ?$$





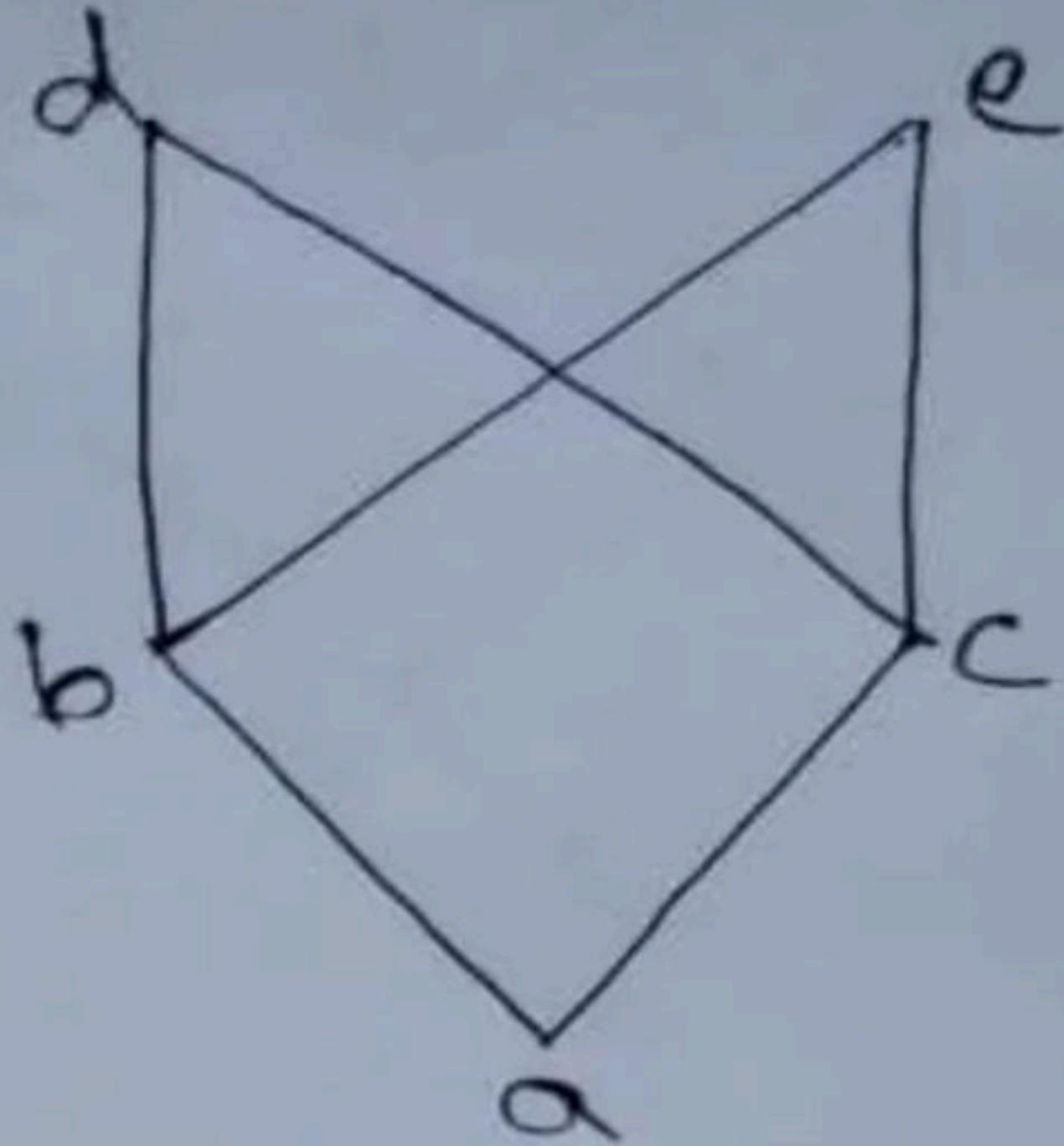
$$d \wedge e = ?$$

$$b \vee c = ?$$



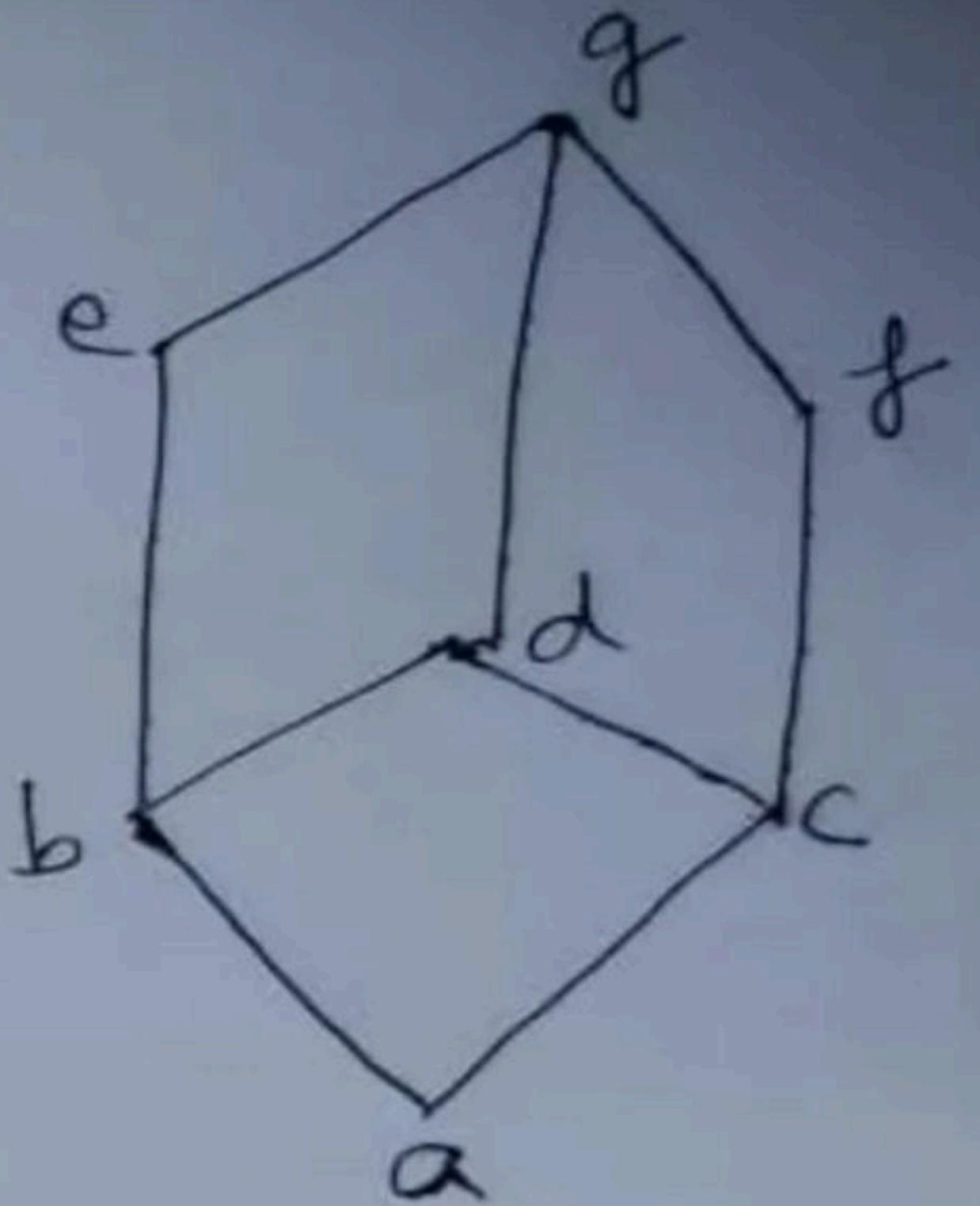
$$a \wedge b = ?$$

JSL

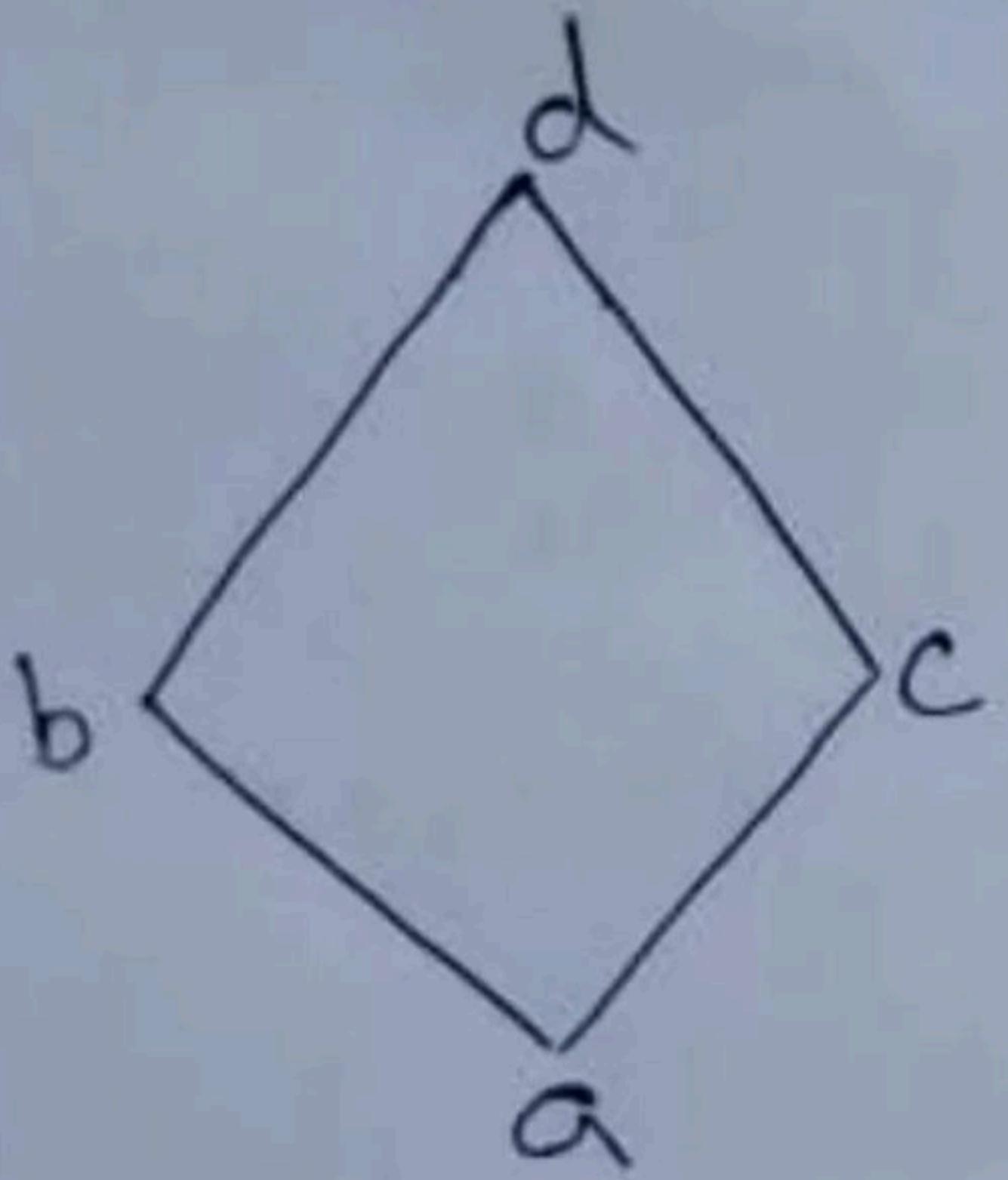


$$d \wedge e = ?$$

$$d \vee e = ?$$

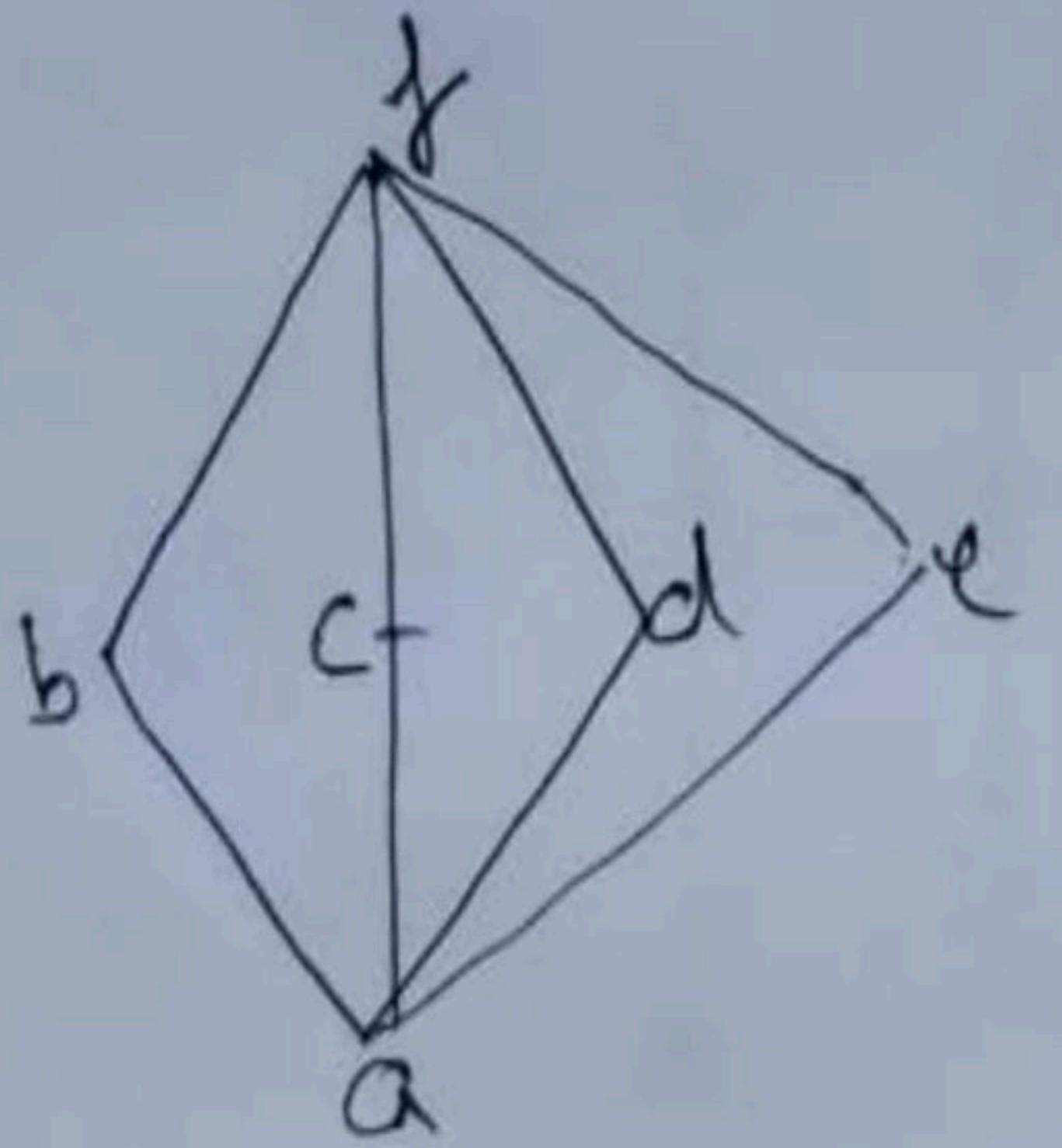


✓
✓



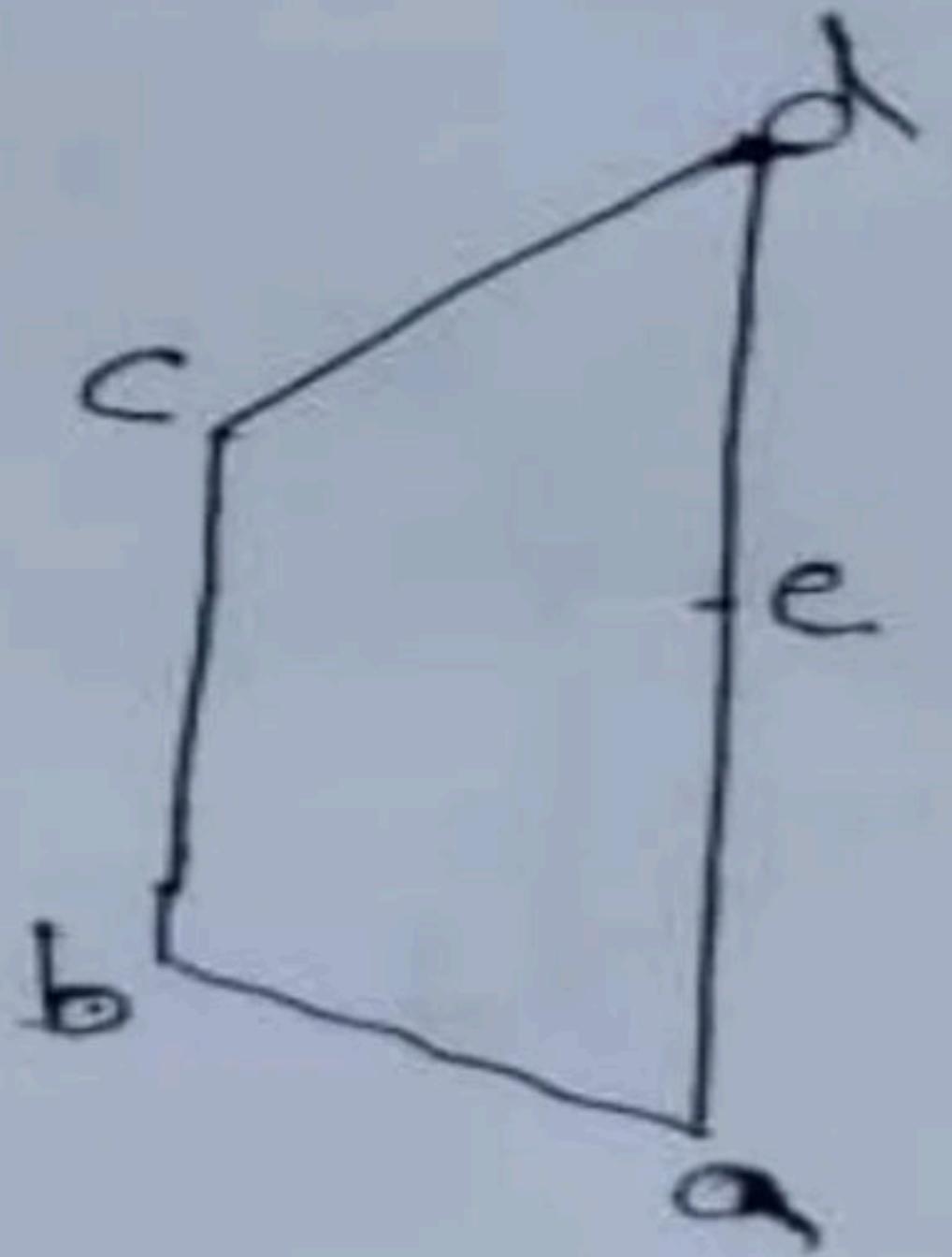
L





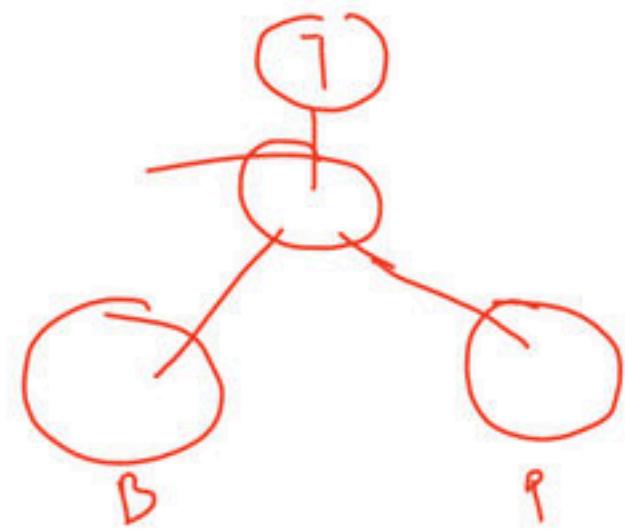
L

✓

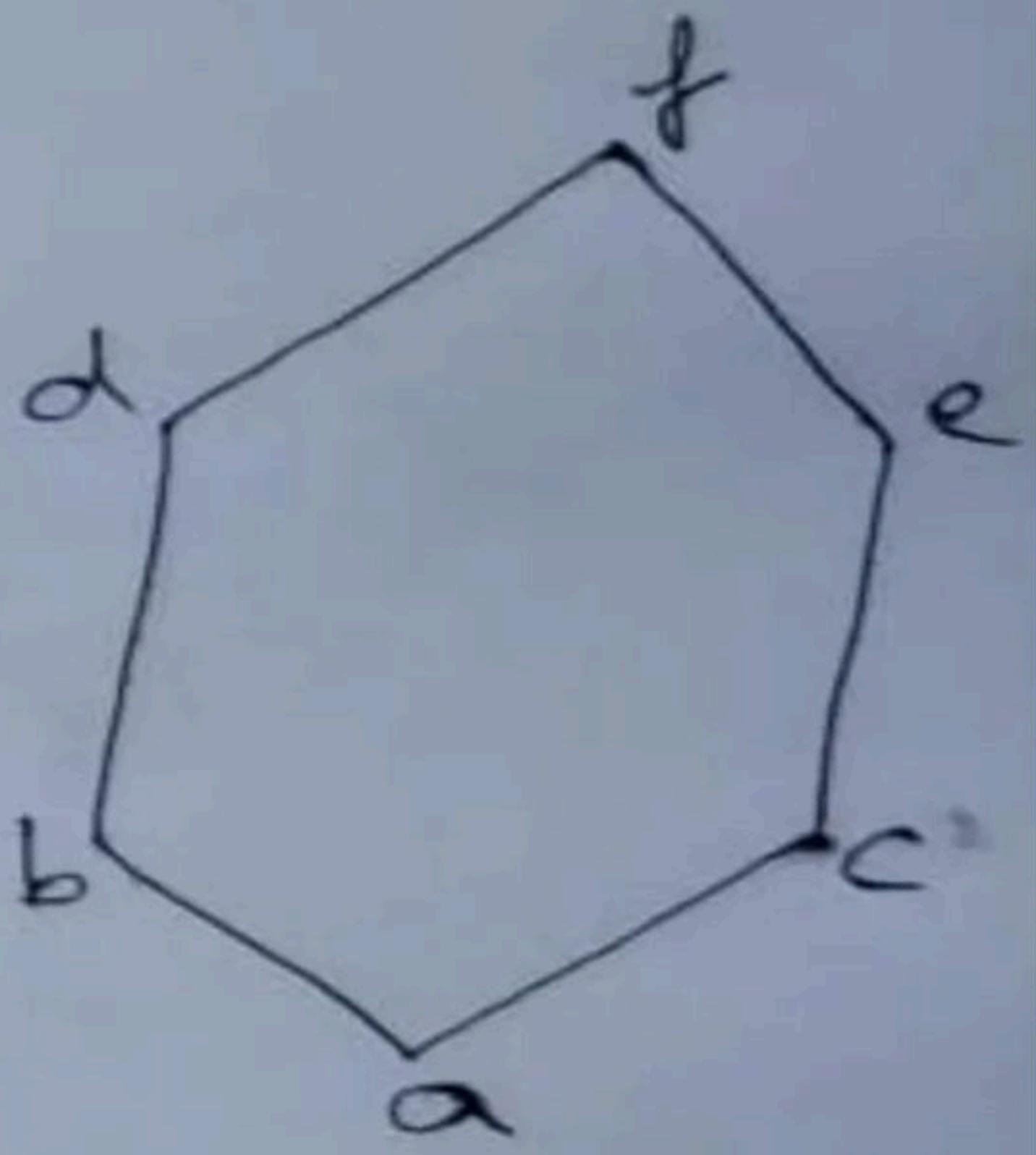


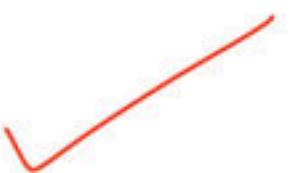
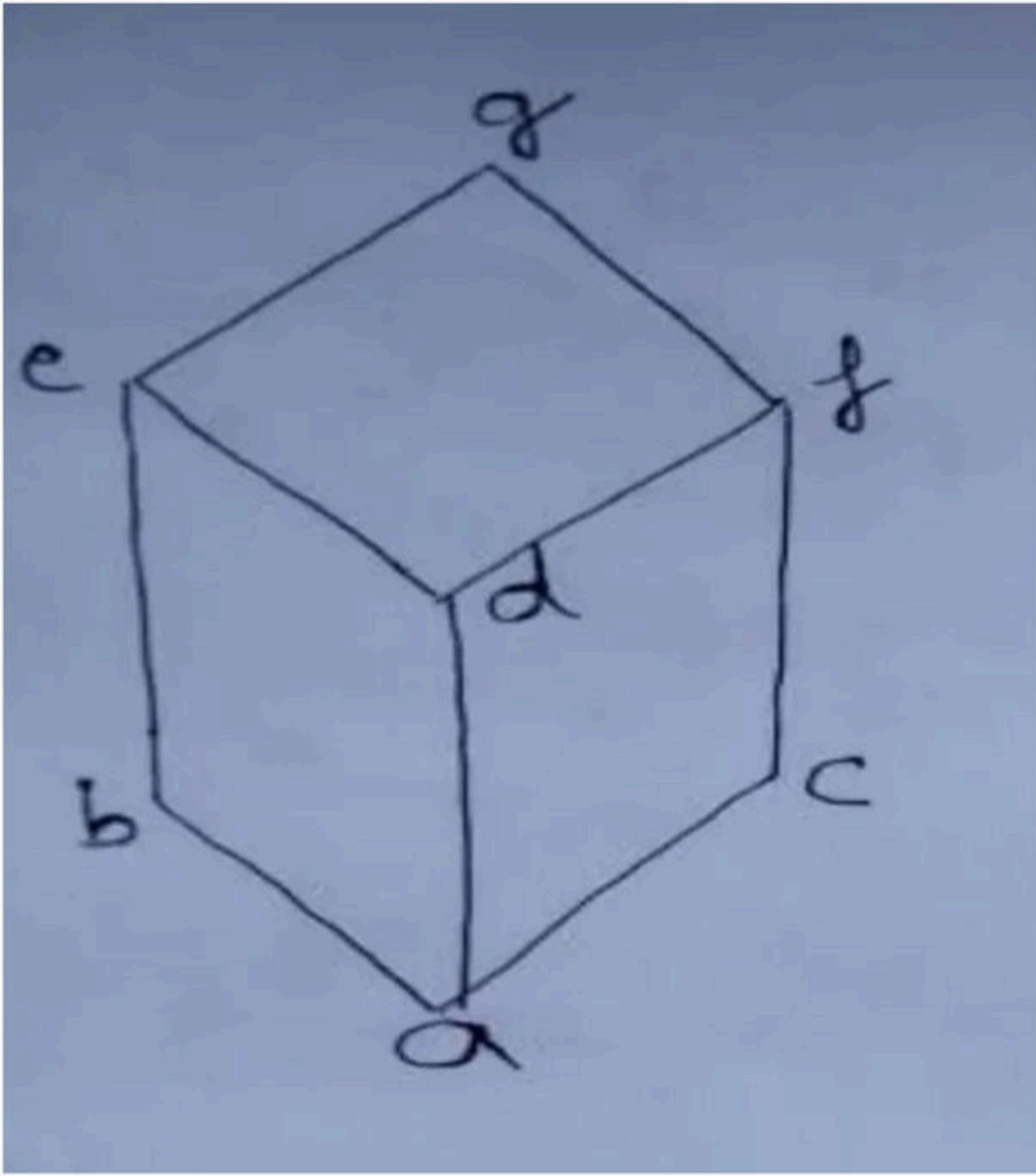
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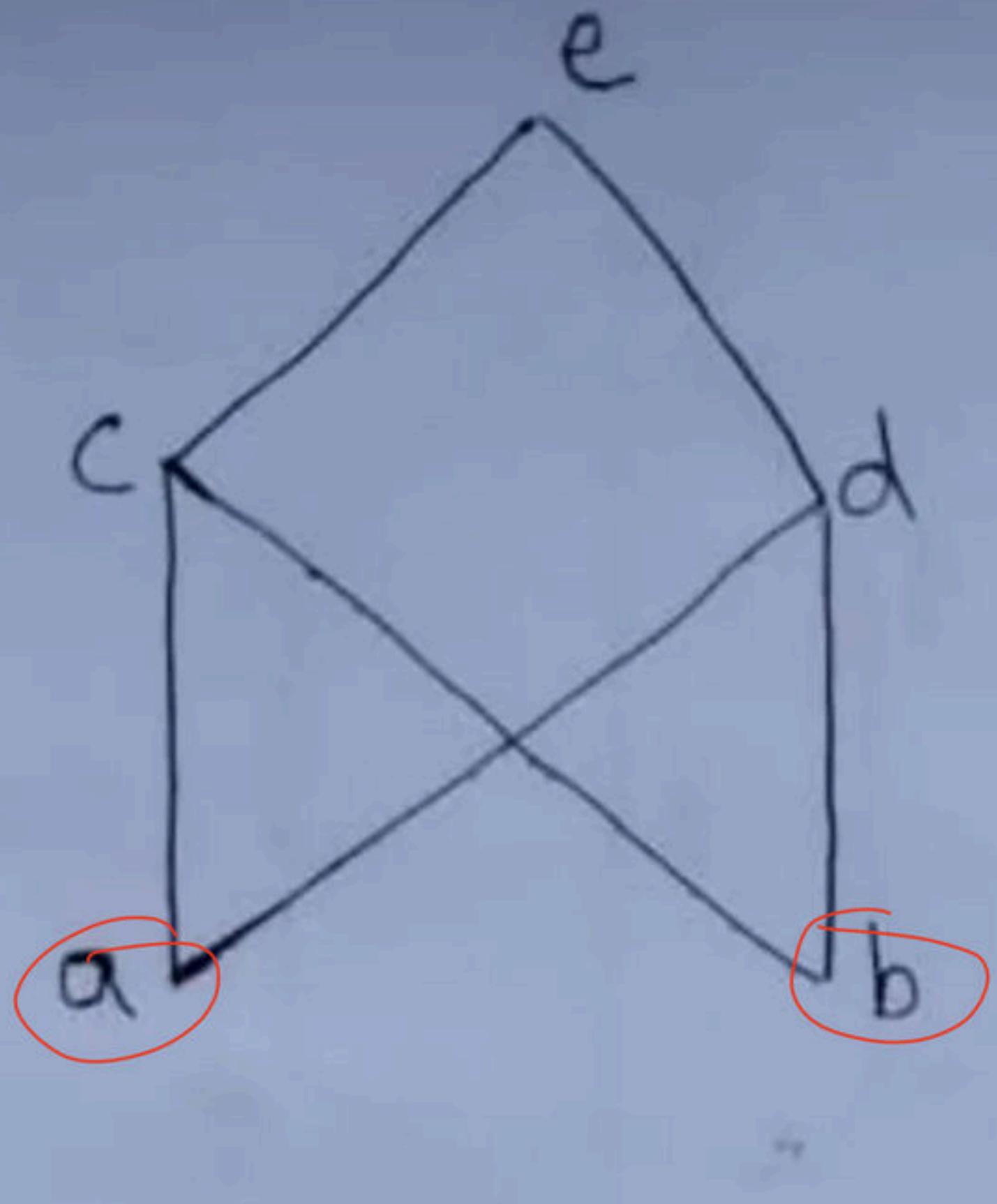
✓



Break

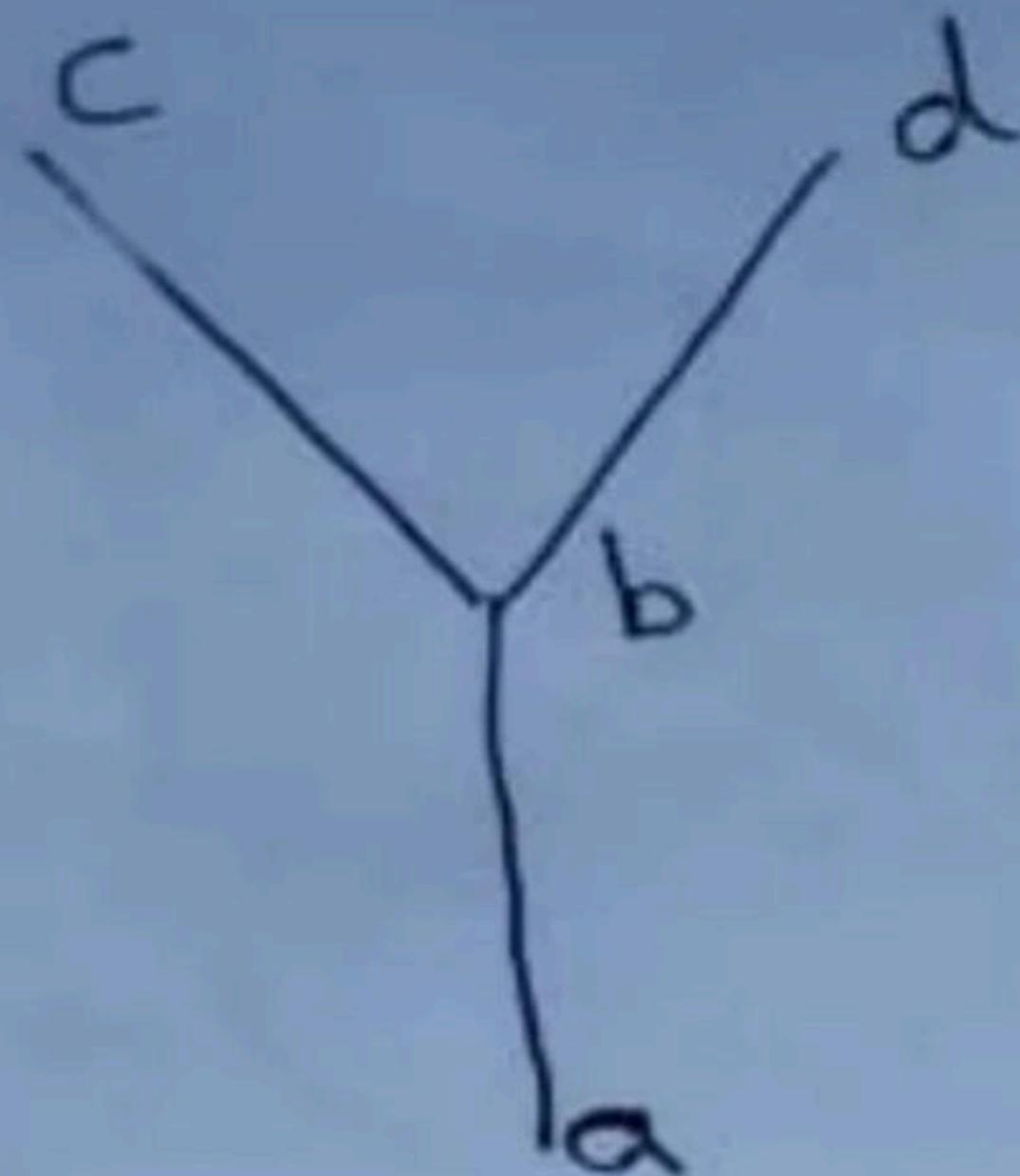






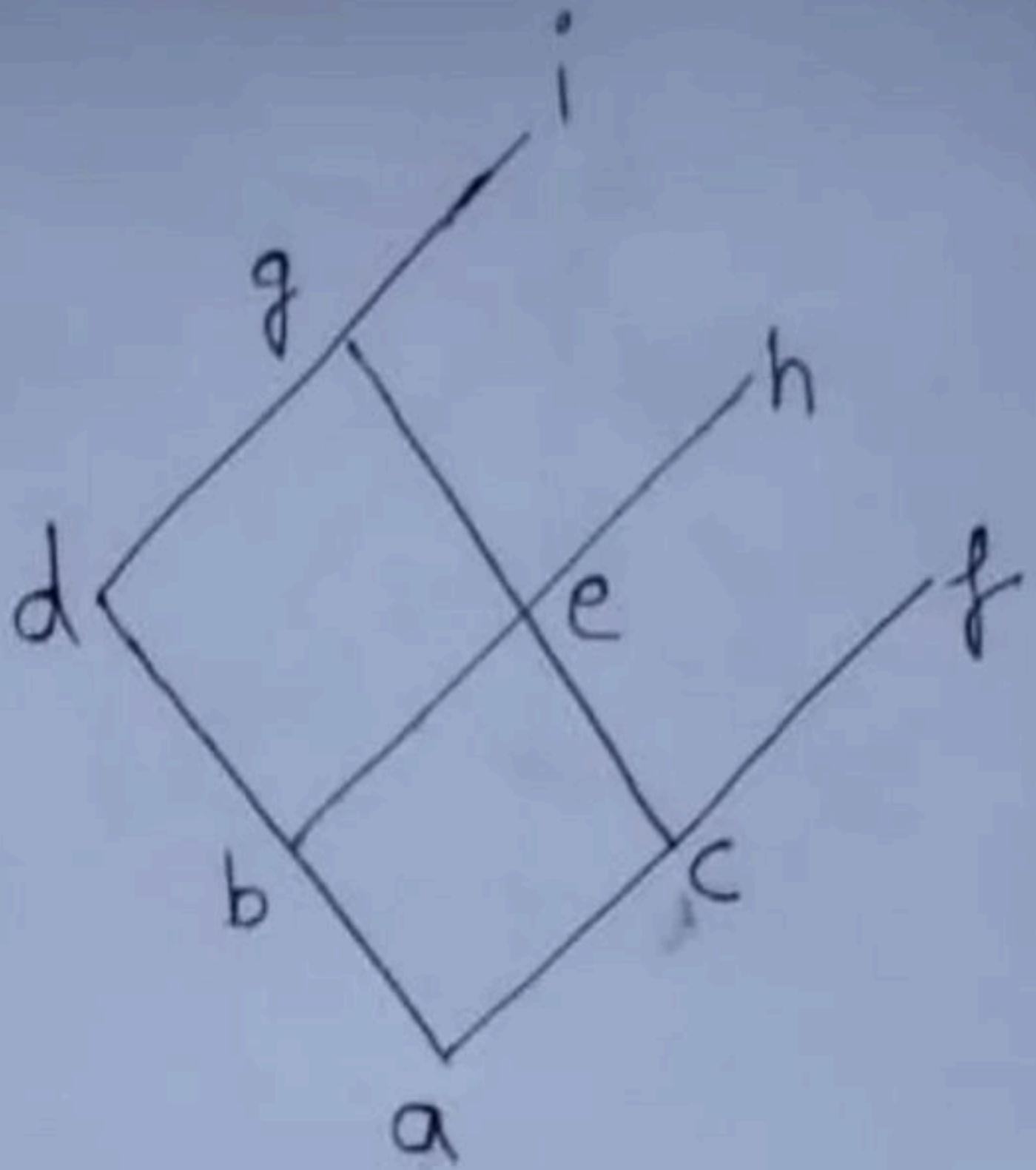
$$a \vee b = ?$$

$$a \wedge b = ?$$



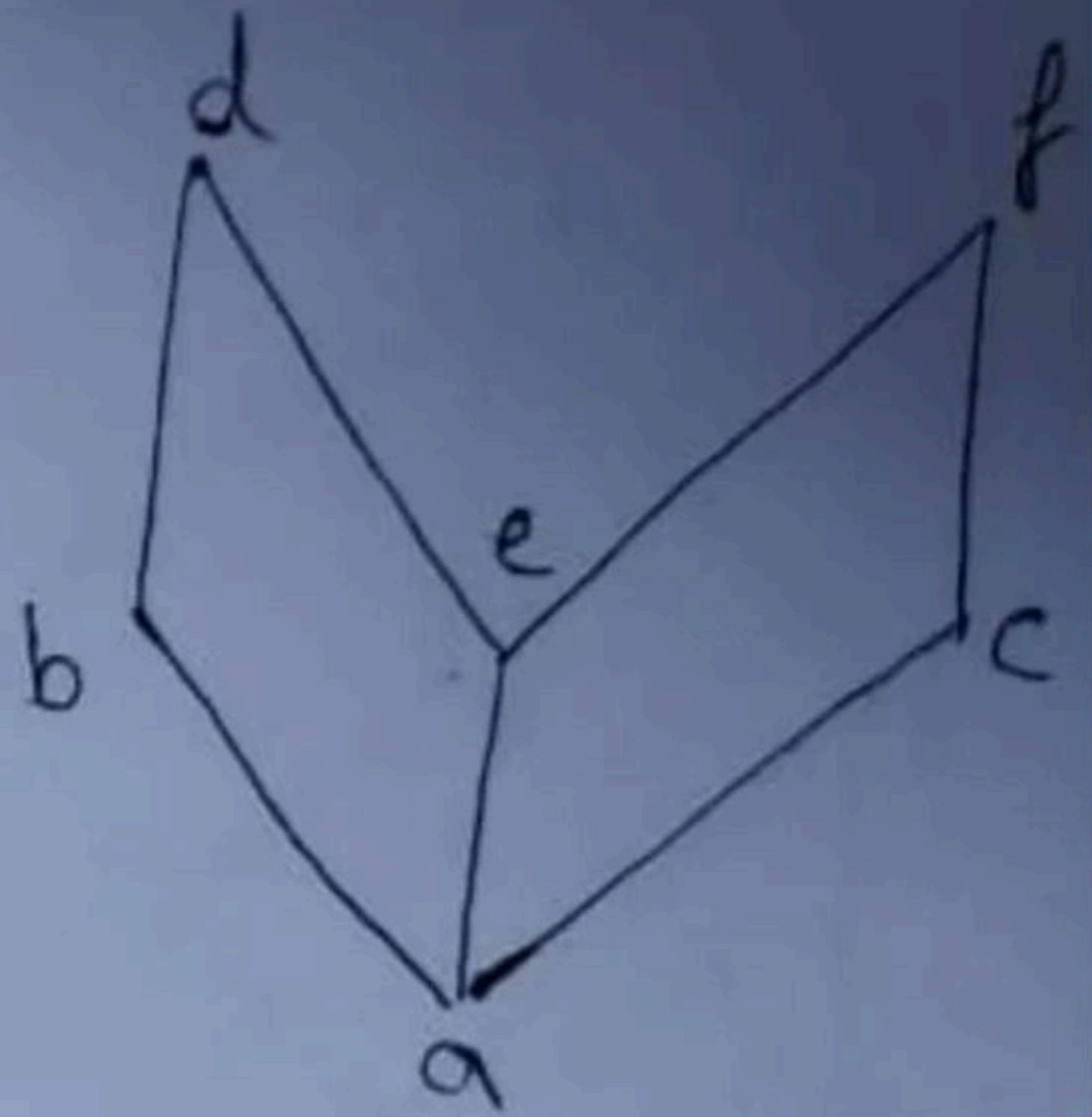
c v d = ?

MSL ✓



$$i \vee h = ?$$

MSL ✓



$$d \vee f = ?$$

M SL
— — —

a
b
c
d

L
✓

a . . c

b

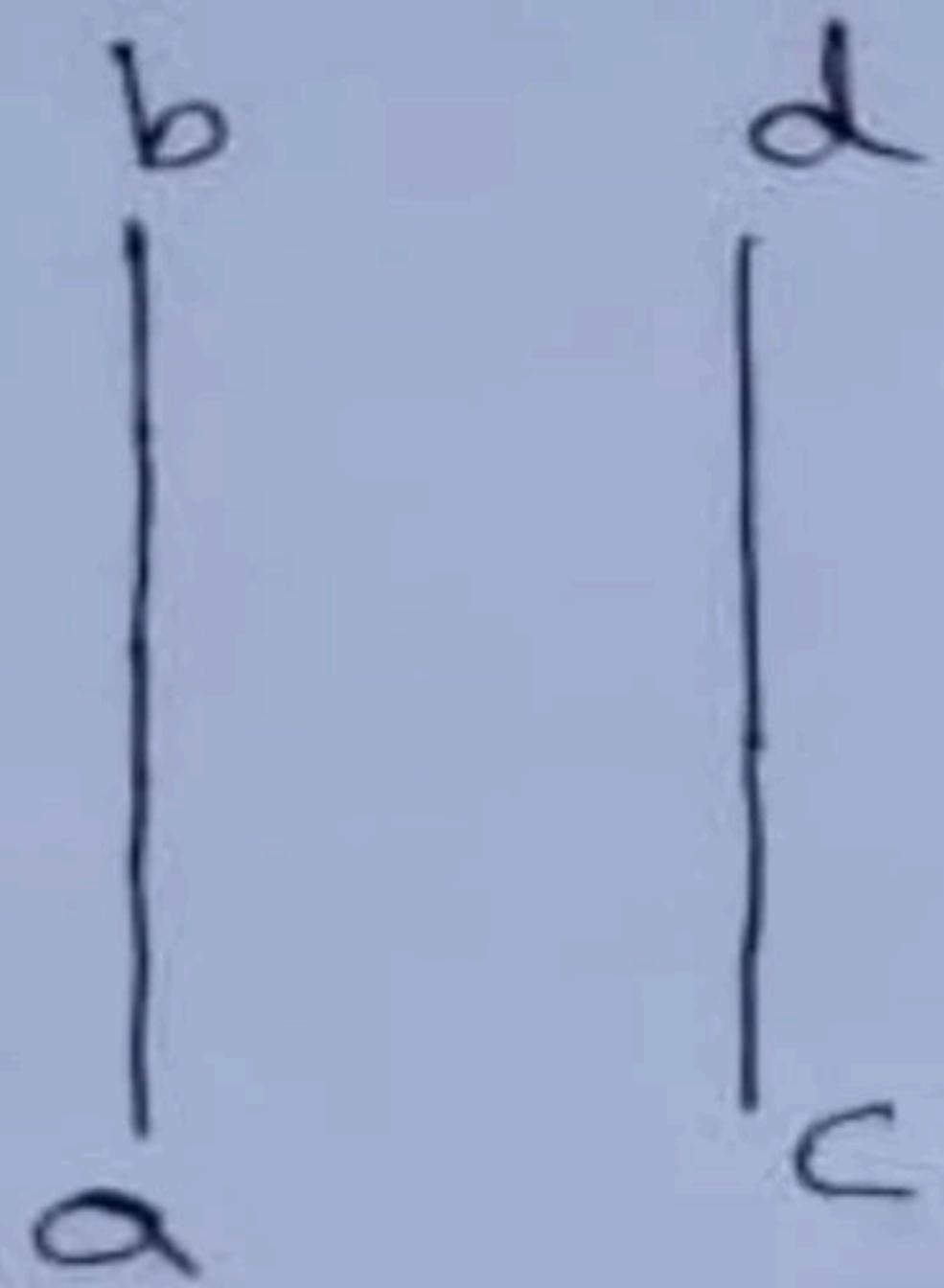
$$a \wedge b = ?$$

$$a \vee \perp = ?$$

$$\begin{array}{l} a \vee a = a \\ a \wedge a = a \end{array}$$

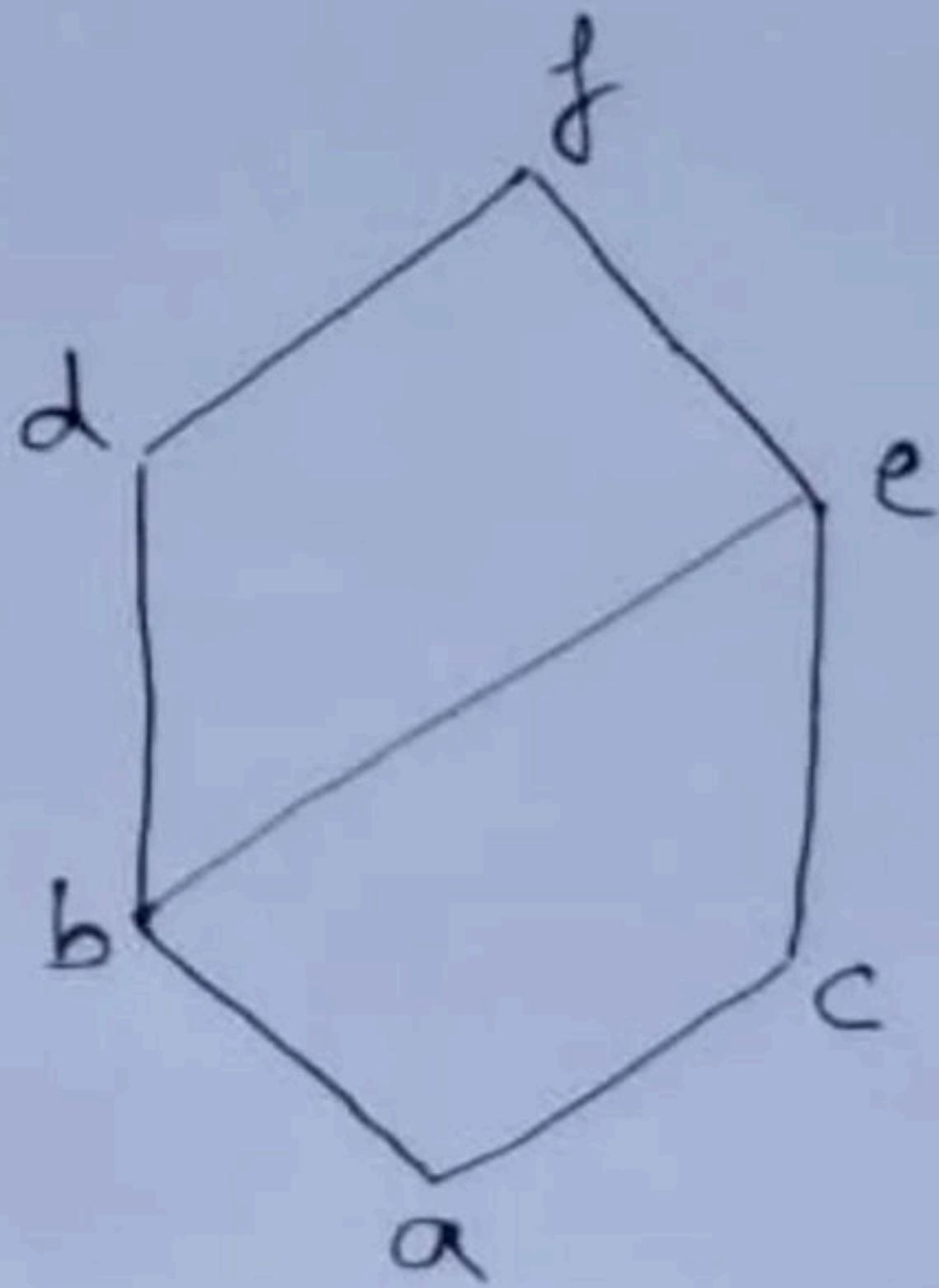
$$a \wedge a = a$$

$$a \vee a = a$$

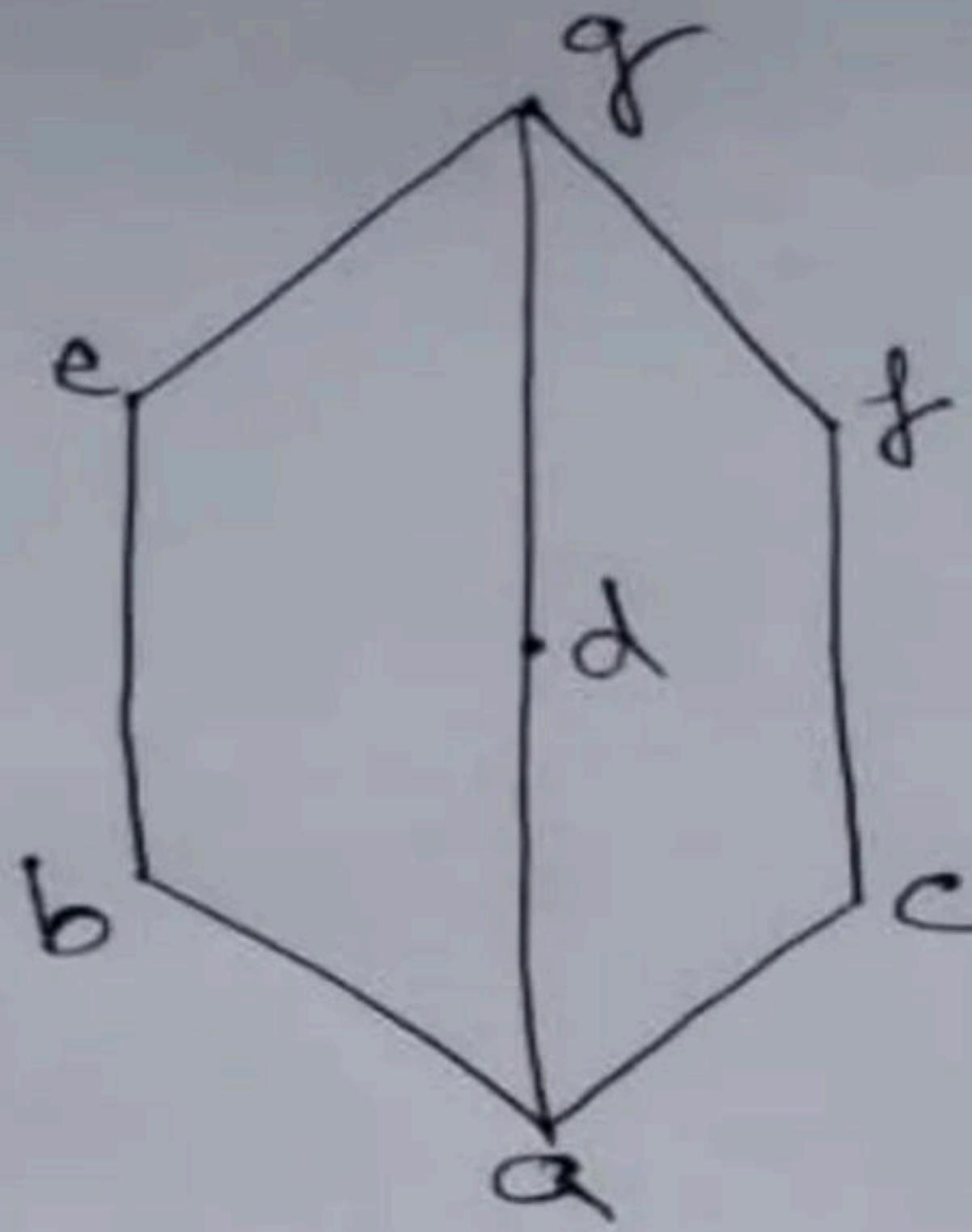


$$a \wedge d = 1$$

$$a \vee b = ?$$

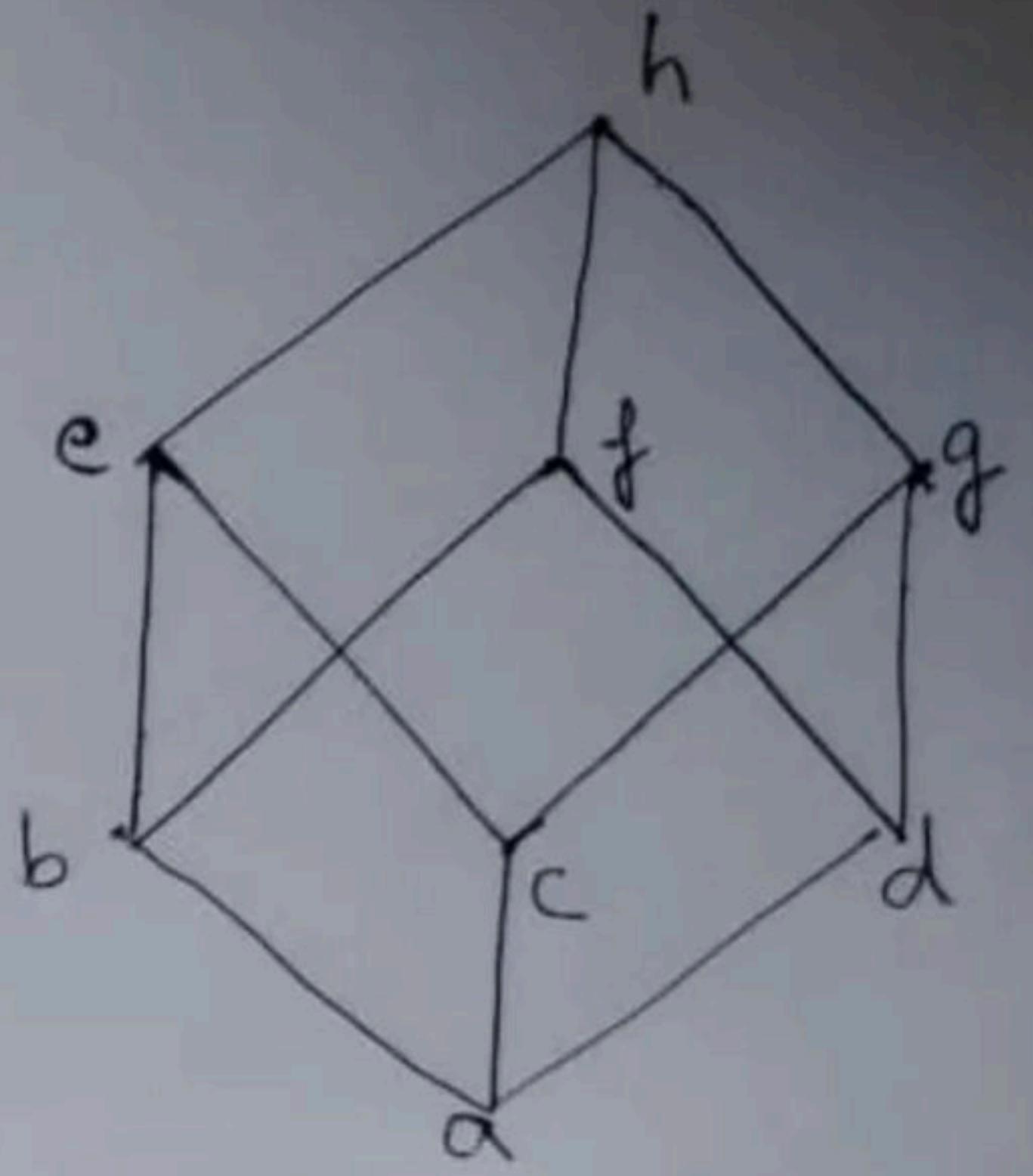


L ✓



L

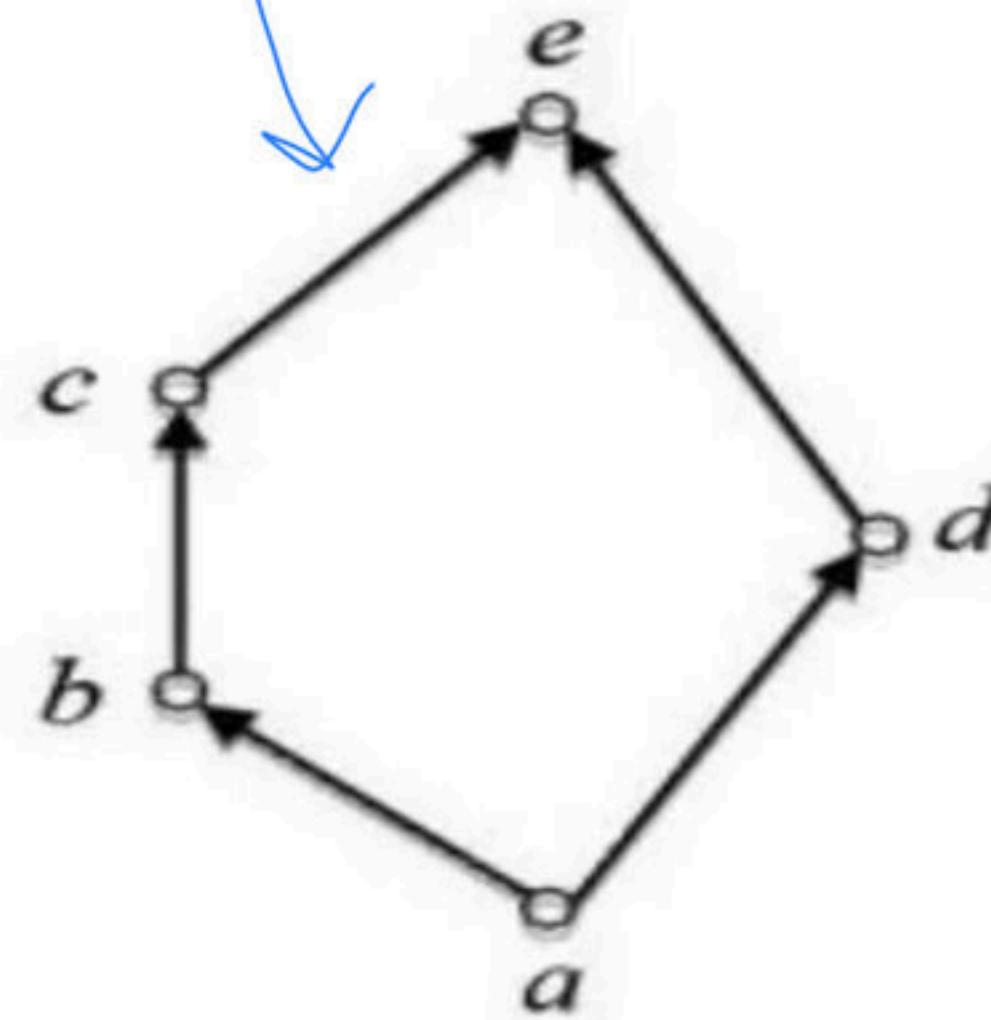
✓



L
✓

Break

Q Consider the set $X=\{a, b, c, d, e\}$ under partial ordering $R=\{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, \underline{c}), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)\}$. The Hasse diagram of the partial order (X, R) is shown below. The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is (GATE-2017) (1 Marks)



Q A partially ordered set is said to be a lattice if every two elements in the set have **(NET-Dec-2010)**

- a) a unique least upper bound
- b) a unique greatest lower bound
- c) both (A) and (B)
- d) none of the above

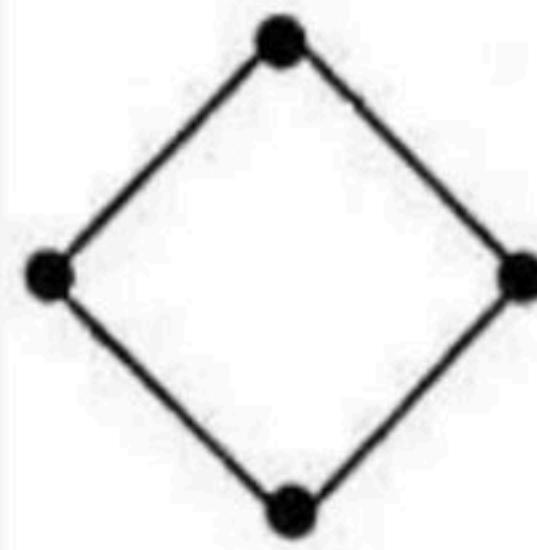
Q Consider the following Hasse diagrams

Which all of the above represent a lattice? **(GATE-2008) (2 Marks)**

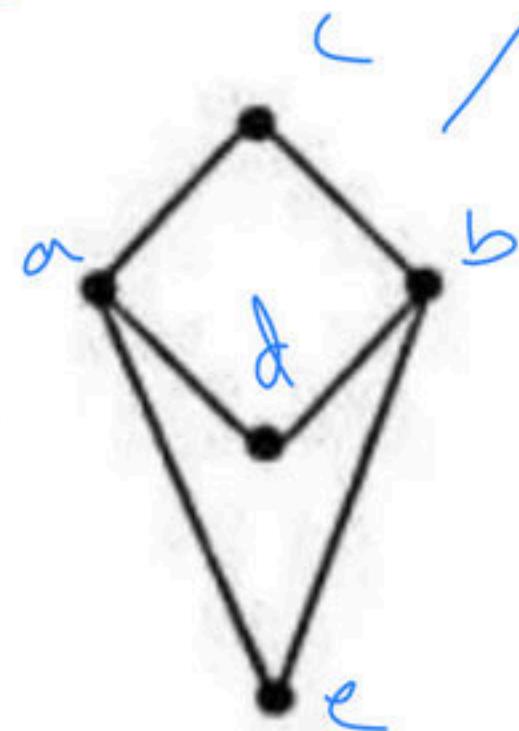
(A) (i) and (iv) only ⁴²

(B) (ii) and (iii) only ⁷

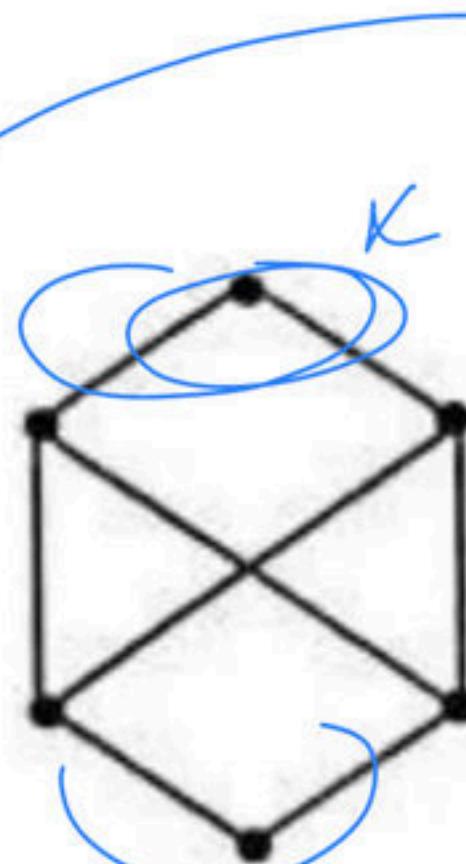
(C) (iii) only ²



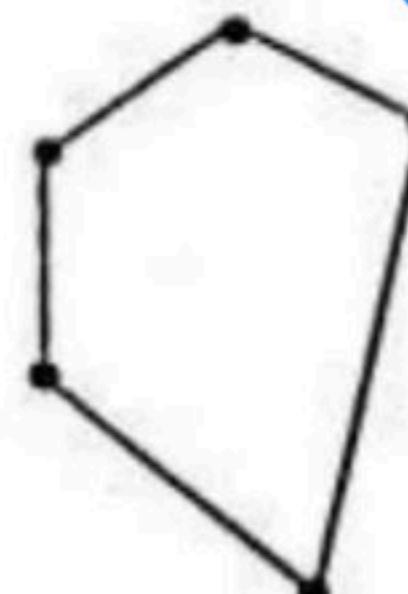
(i)



(ii)



(iii)

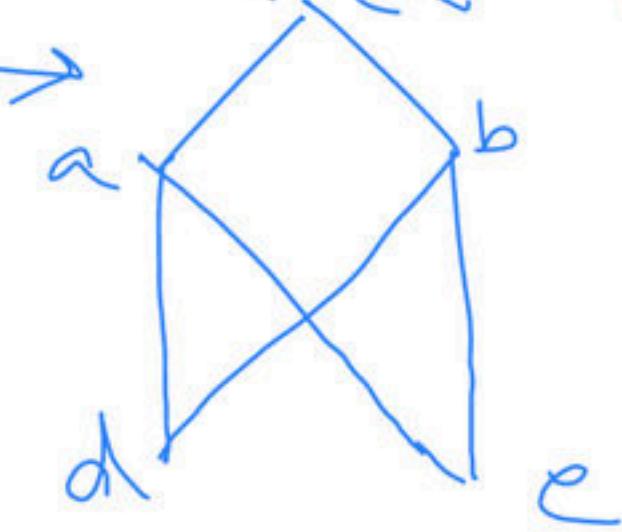


(iv)

$$d \wedge e = ?$$
$$d \vee e = ?$$

X

✓

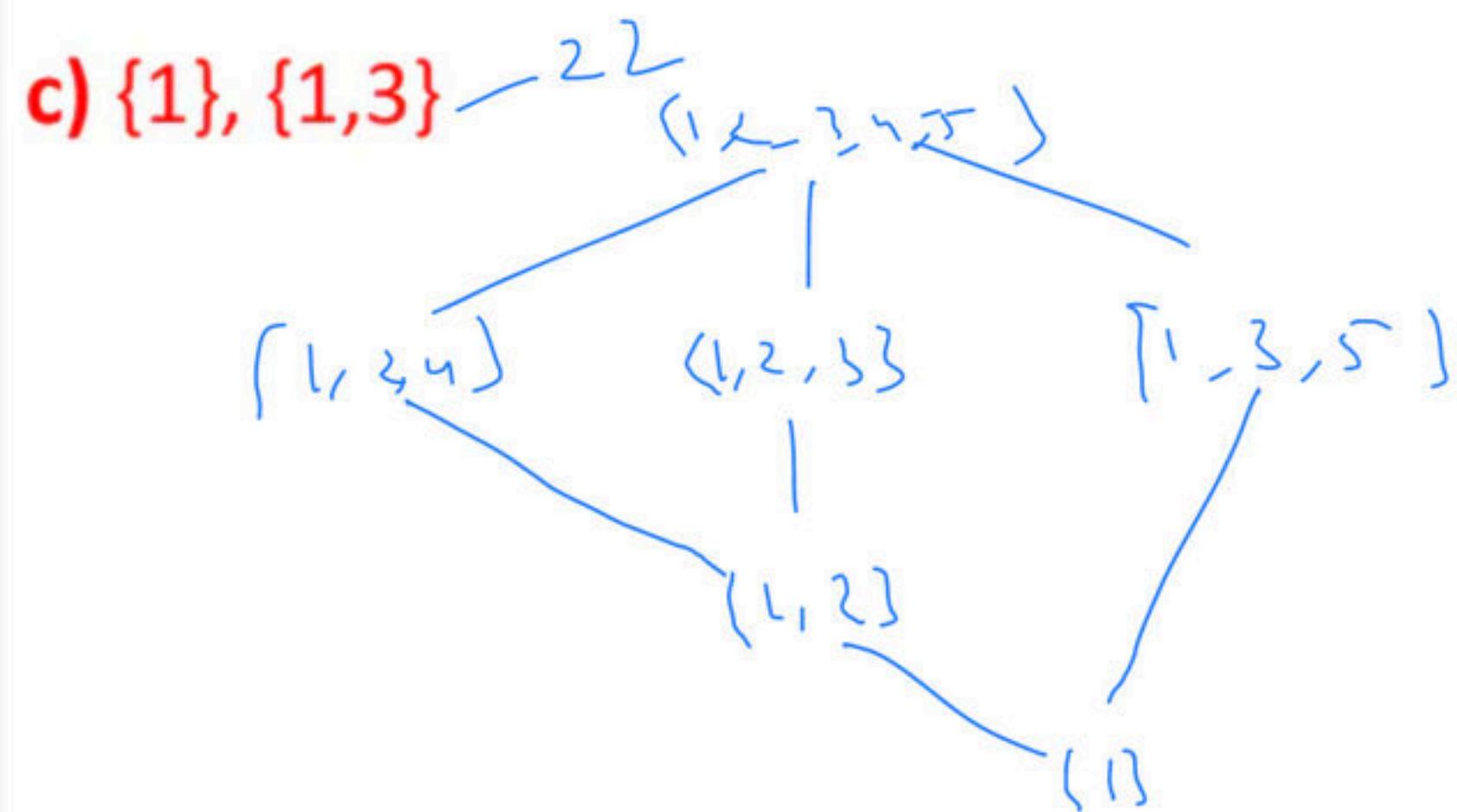


(D) (i), (ii) and (iv) only ¹⁹

Q the inclusion of which of the following set into $S = \{\{1,2\}, \{1,2,3\}, \{1,3,5\}, \{1,2,4\}, \{1,2,3,4,5\}\}$ is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?
(GATE-2004) (2 Marks)

a) {1} $\nearrow 3$

b) {1}, {2,3} $\nearrow 4$



d) {1}, {1,3}, {1,2,3,4}, {1,2,3,5} $\nearrow 1$

Break

Boolean algebra

- **Unbounded Lattice** :- If a lattice has infinite of elements then it is called Unbounded Lattice.



- **Bounded Lattice** :- If a lattice has finite number of elements then it is called Bounded lattice, there will be upper and lower bound in lattice.

- **Complement of an element in a Lattice** :- If two elements a and a^c , are complement of each other, then the following equations must always hold good.

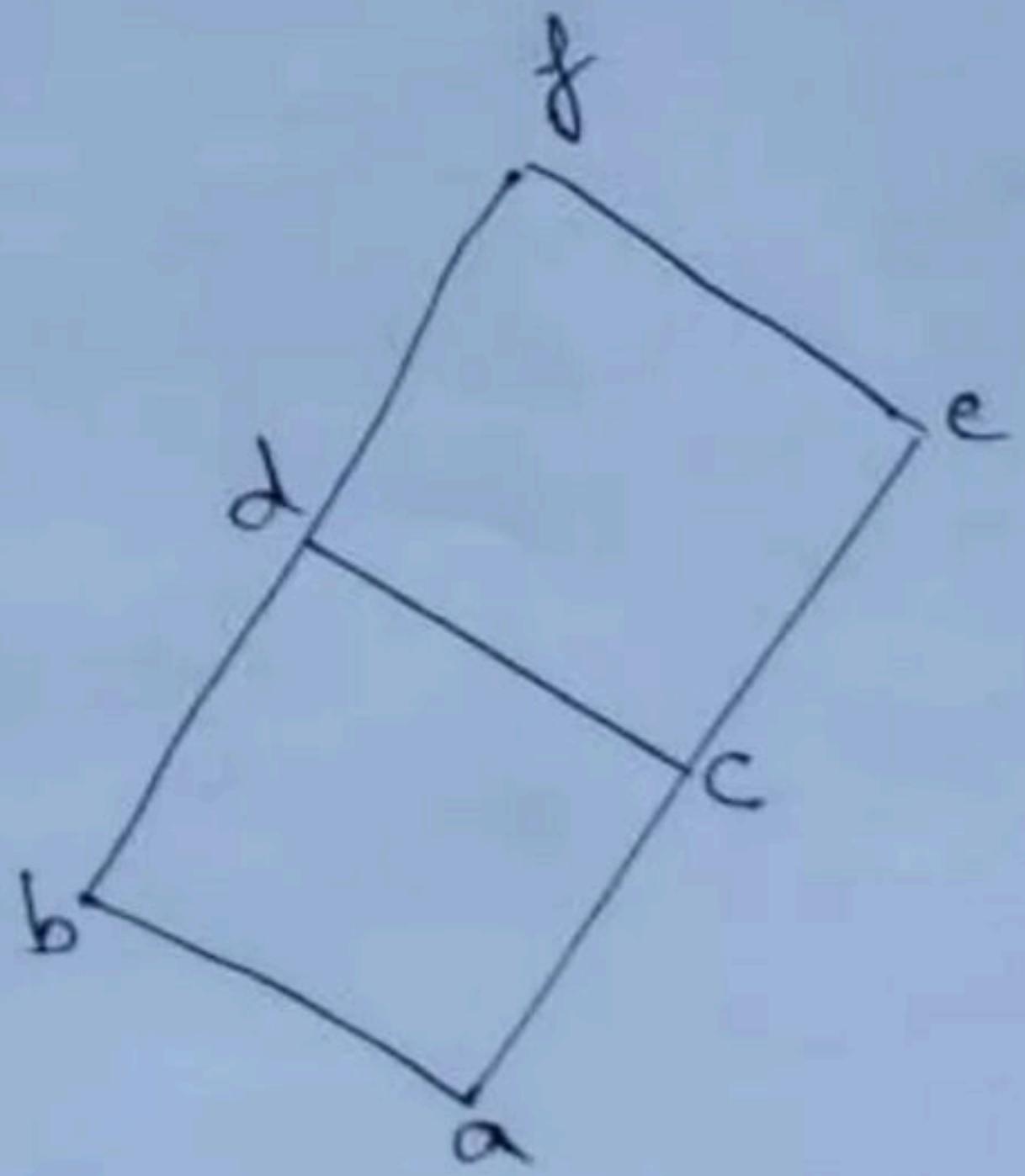
$a \vee a^c = \text{Upper bound of lattice}$

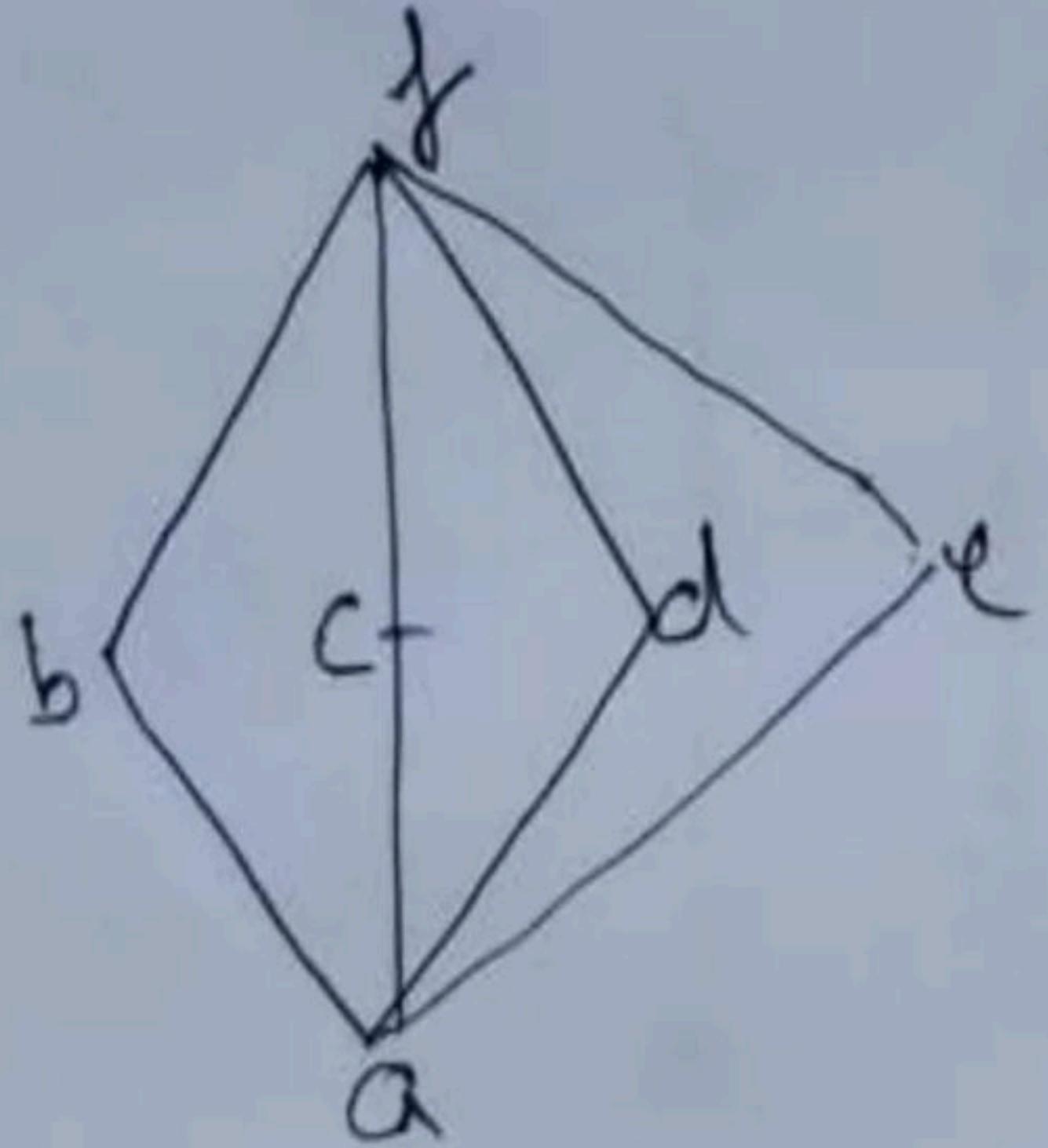
$a \wedge a^c = \text{Lower bound of lattice}$

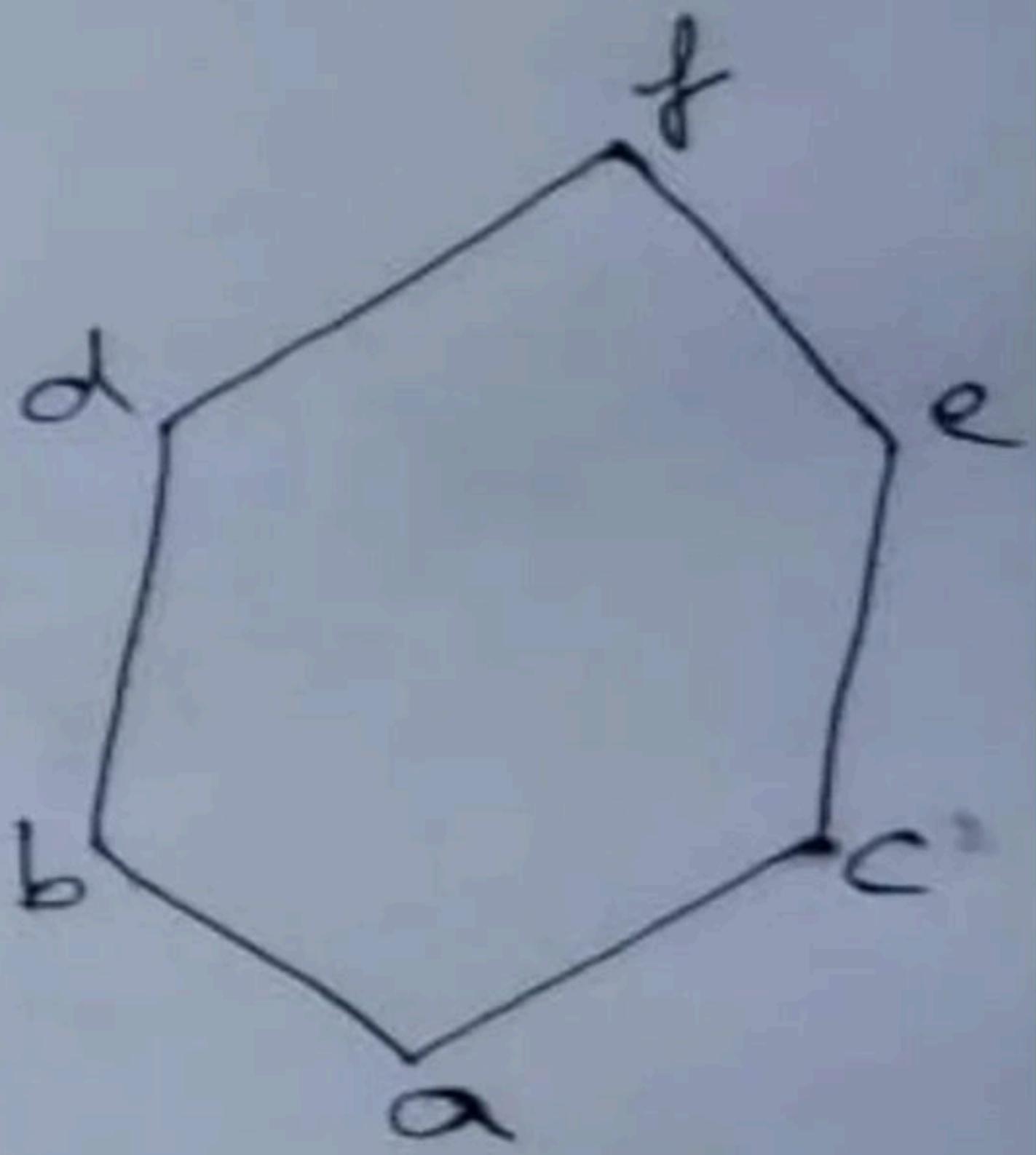
- **Distributive Lattice** :- A lattice is said to be distributed lattice. if for every element there exist at most one completemt(zero or one).

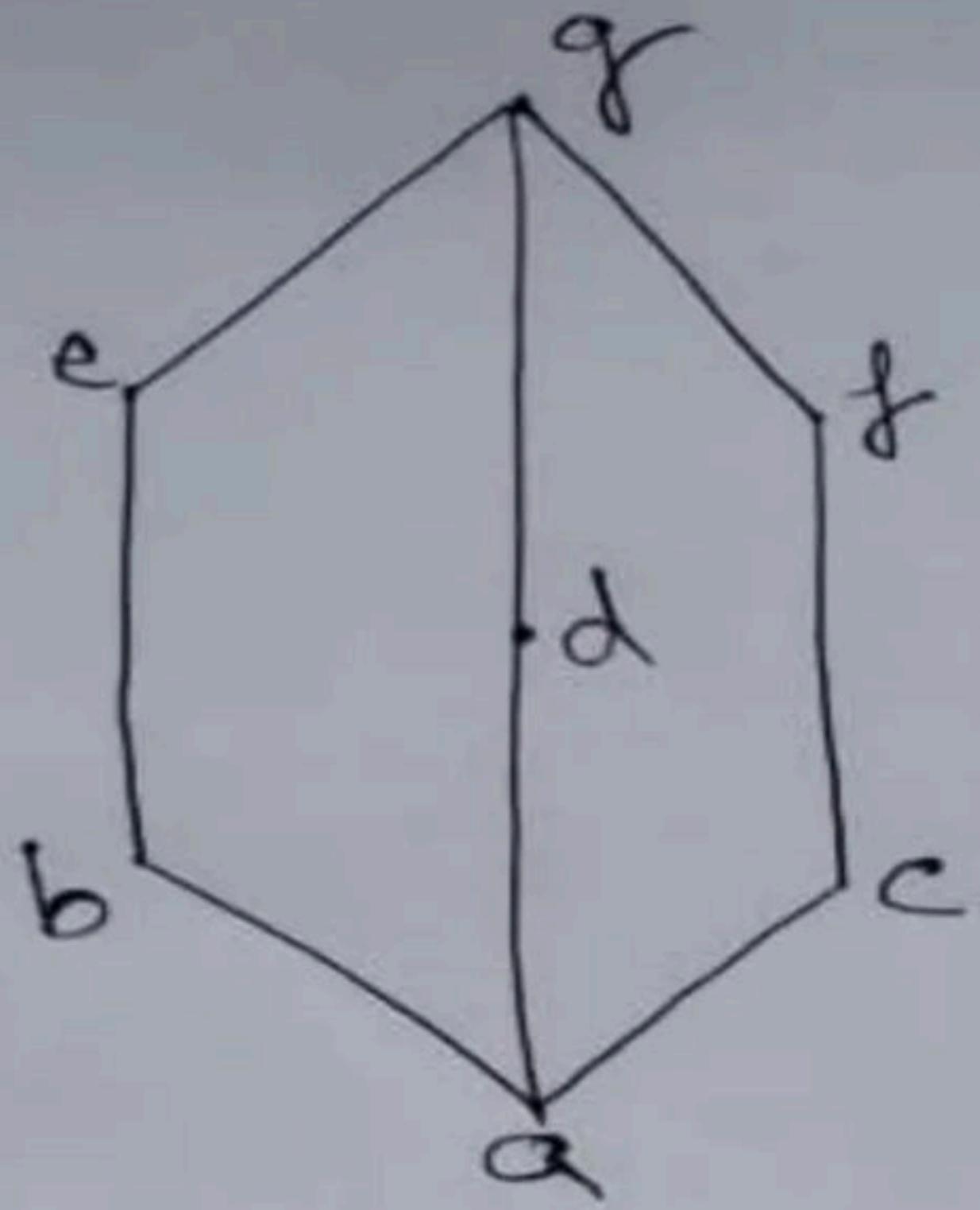
- **Complement Lattice** :- A Lattice is said to be Complement lattice. if for every element there exist at least one complement(one or more).

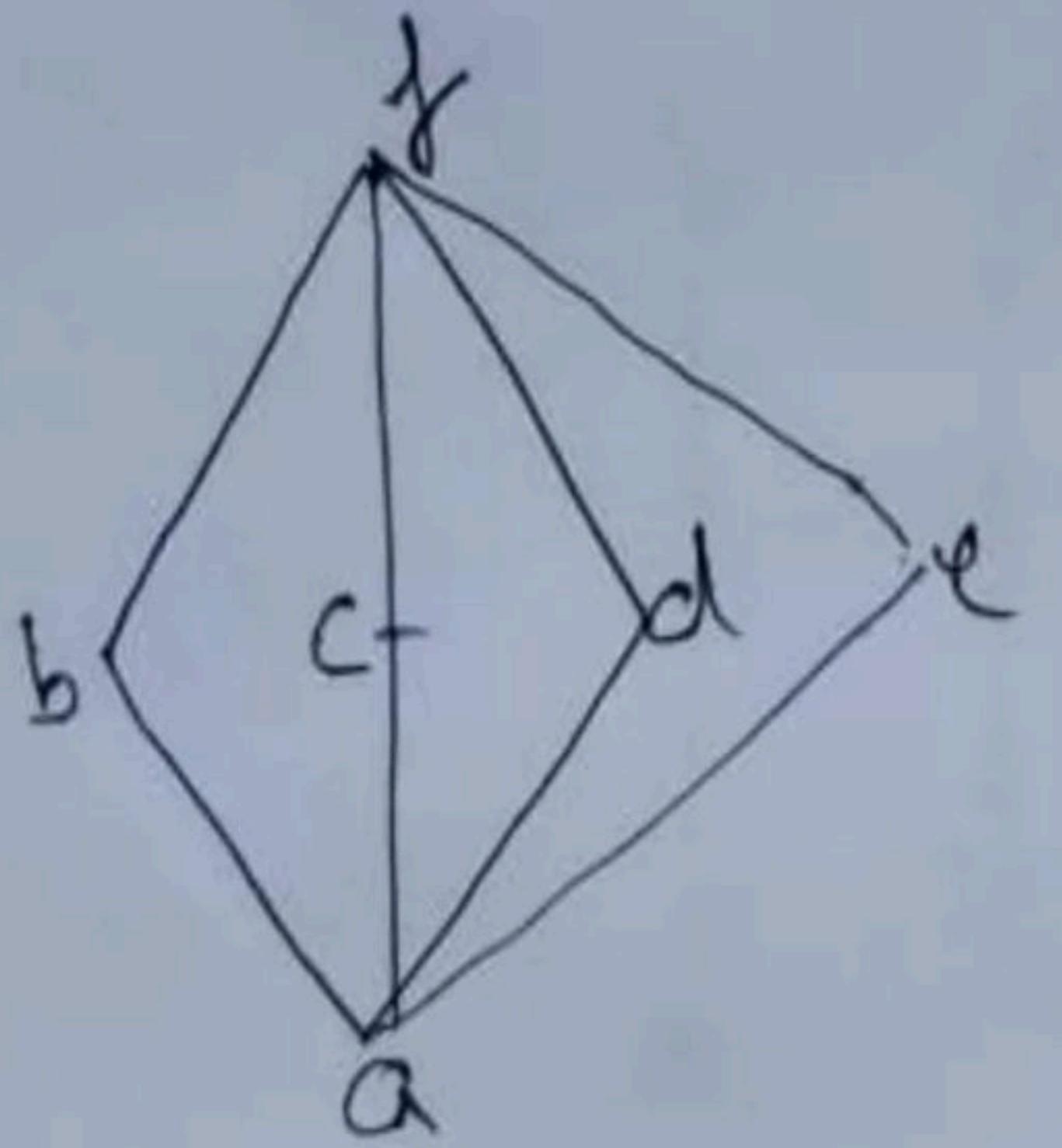
- **Boolean Algebra** :- A Lattice is said to be Boolean Algebra, if for every element there exist exactly one complement. Or if a lattice is both complemented and distributed then it is called Boolean Algebra.

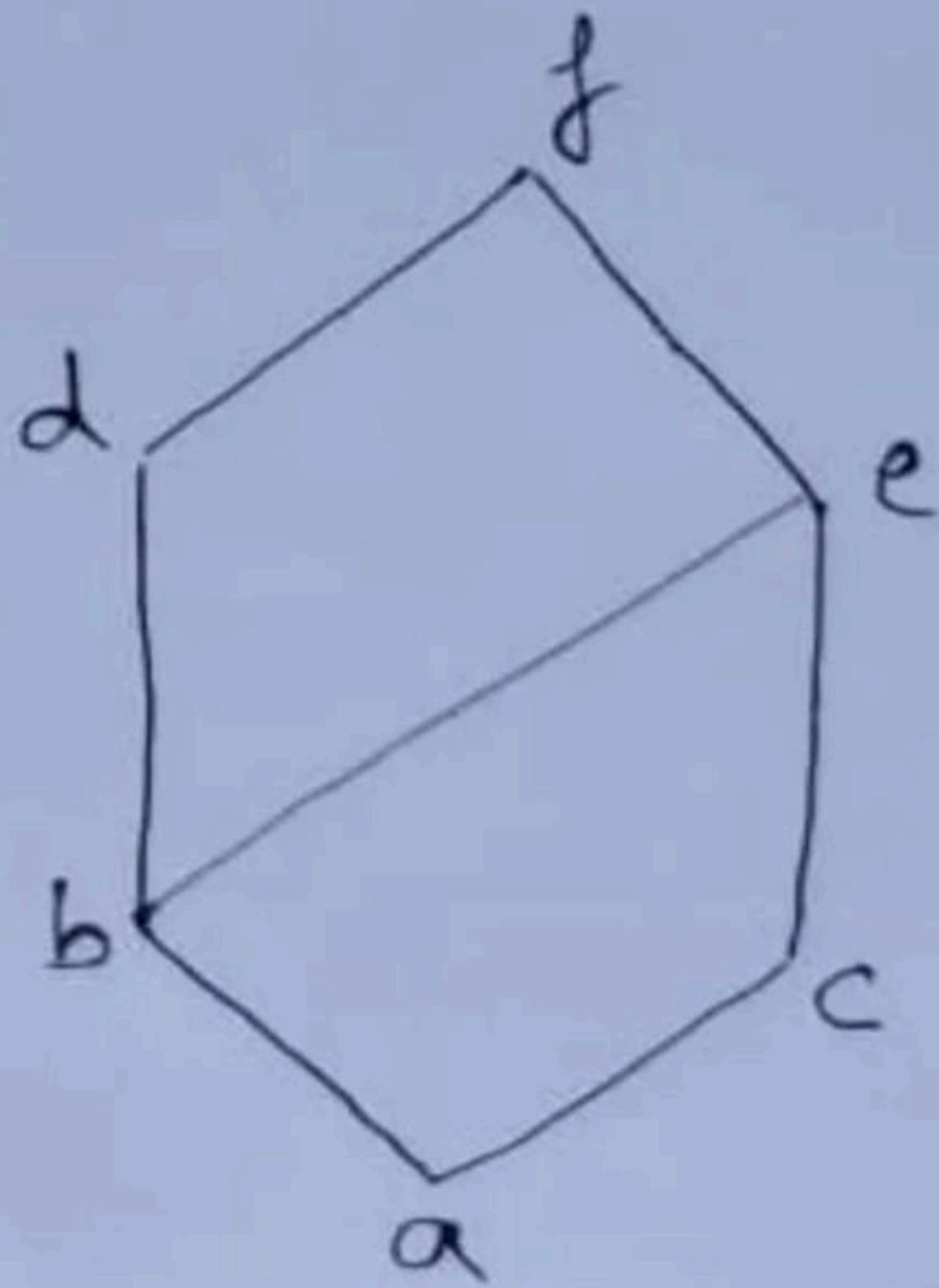


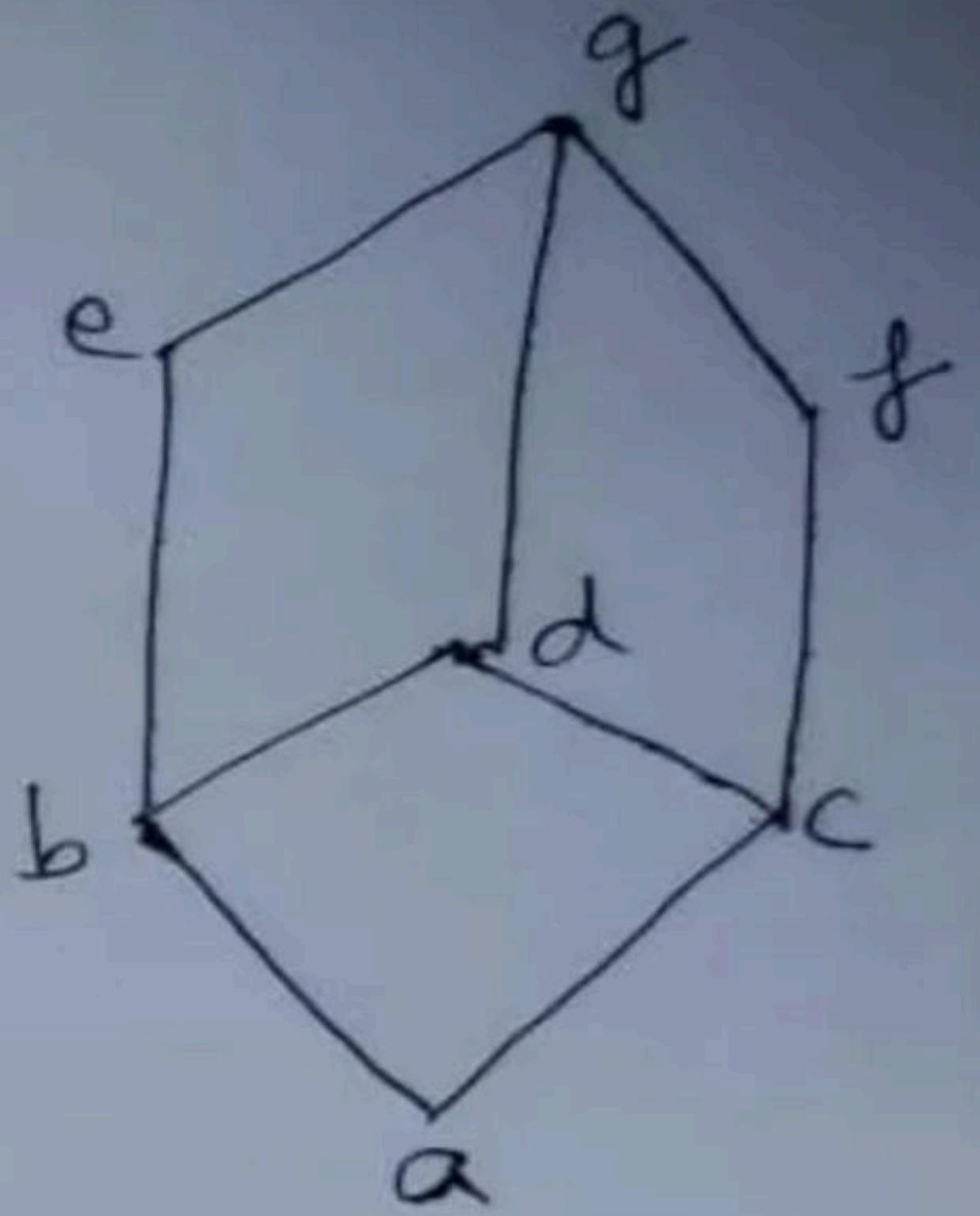












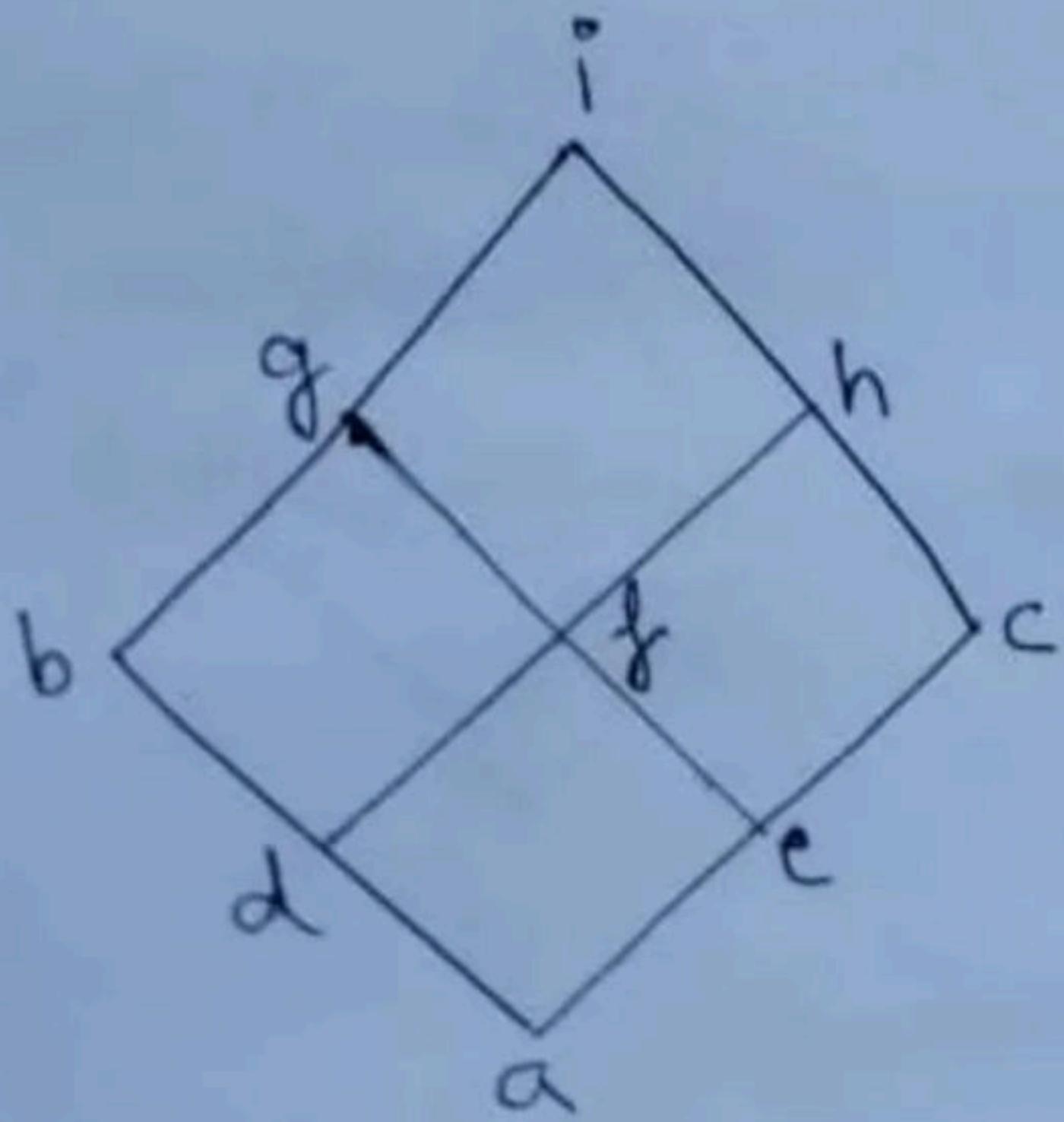
Break

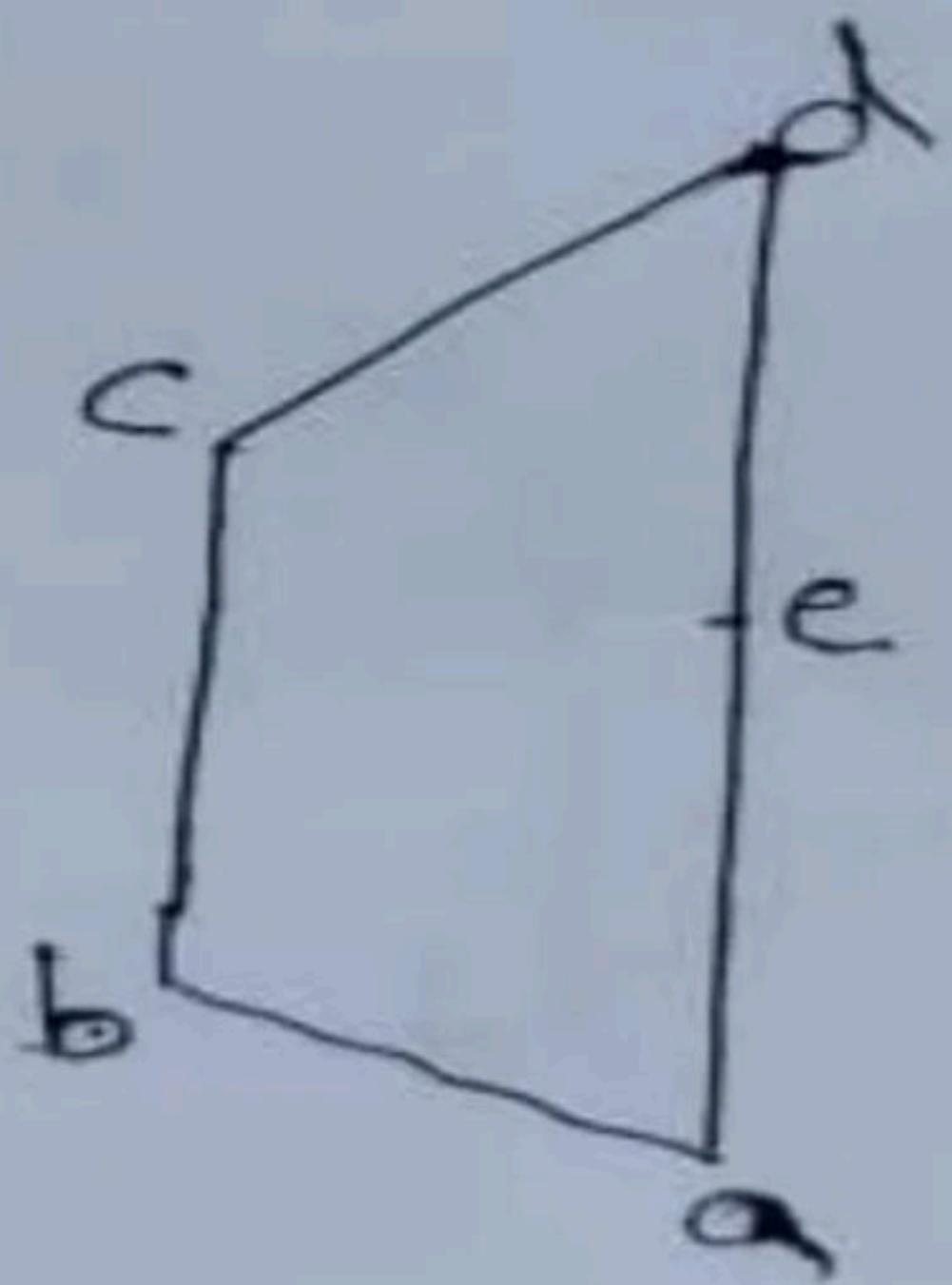
d

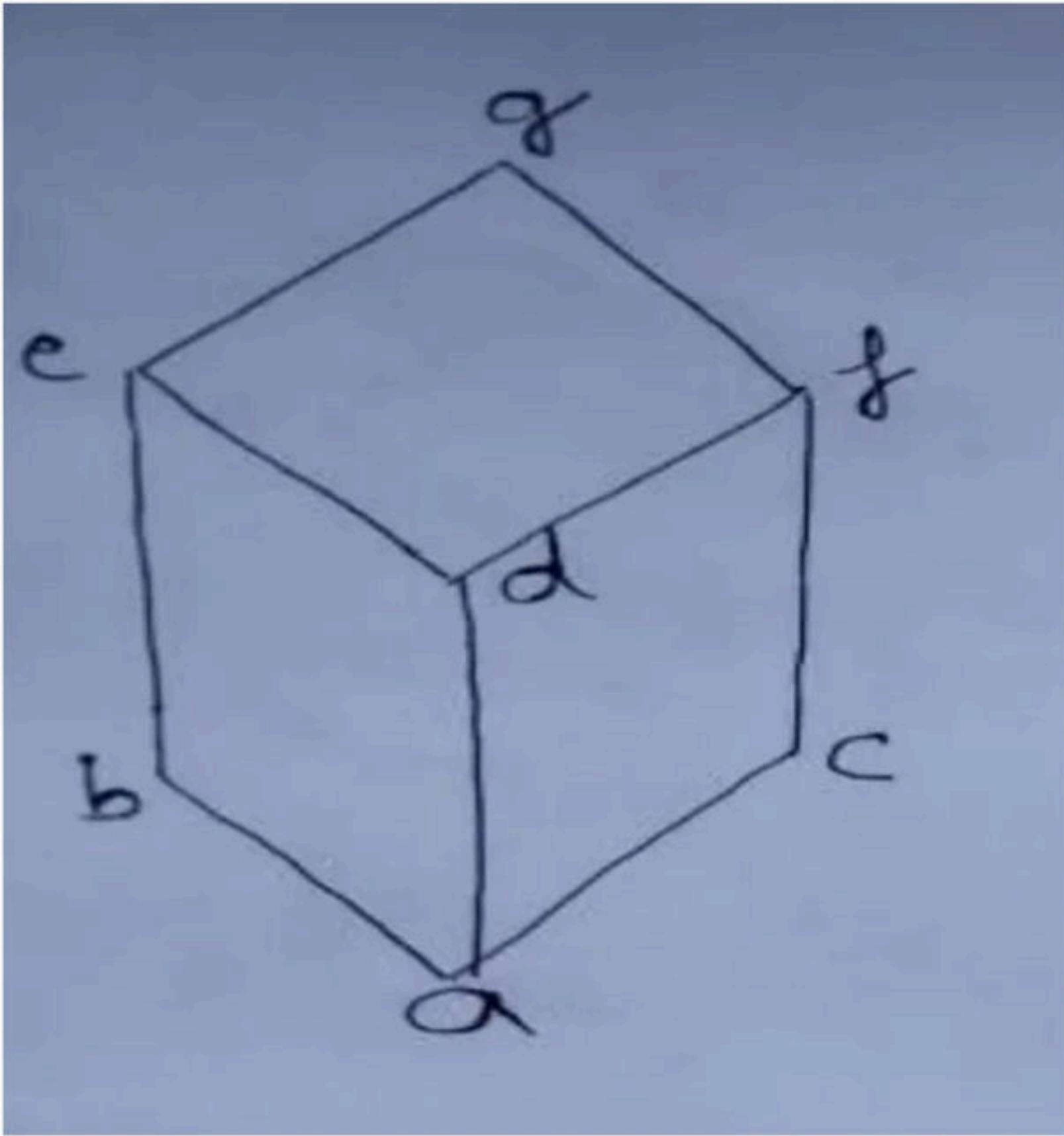
c

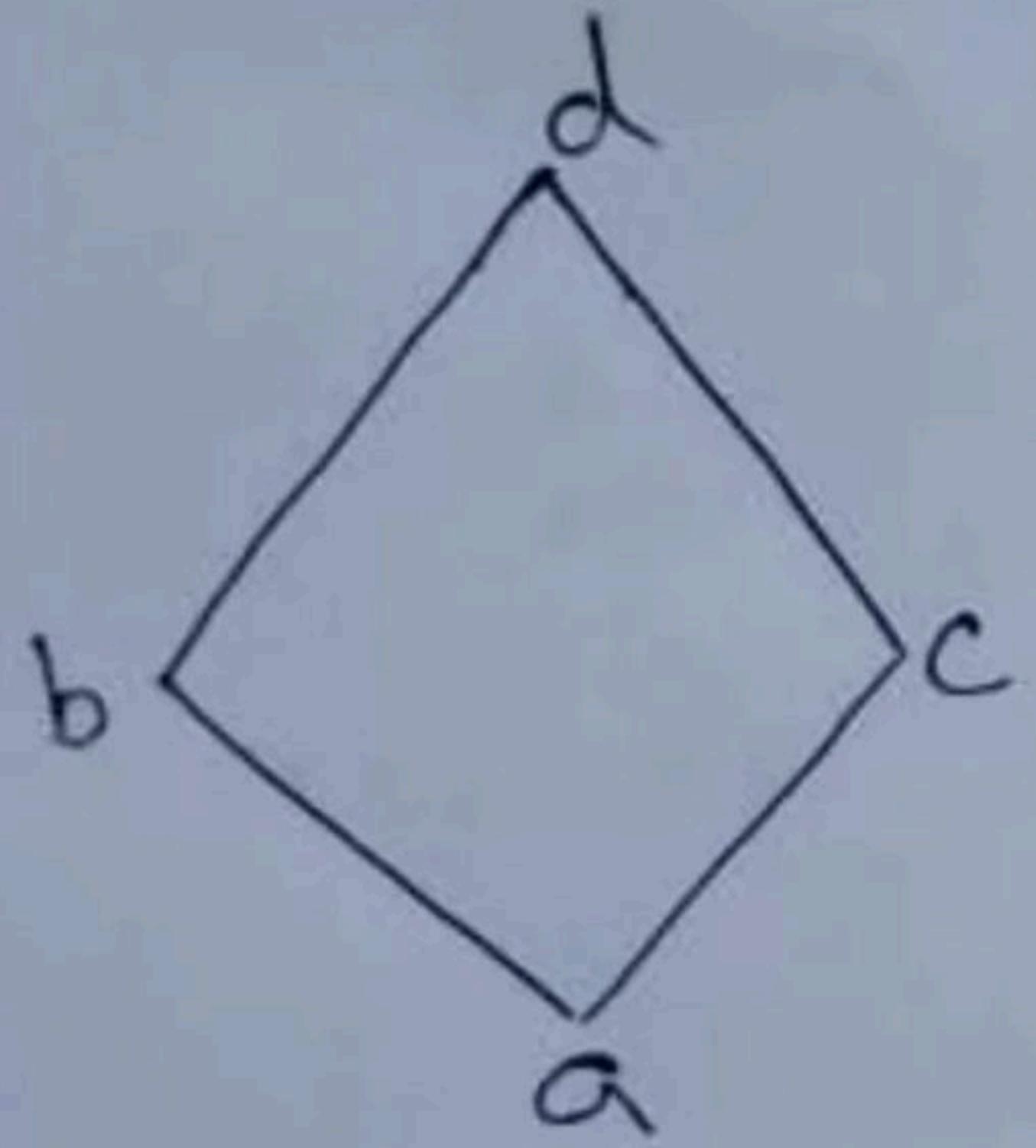
b

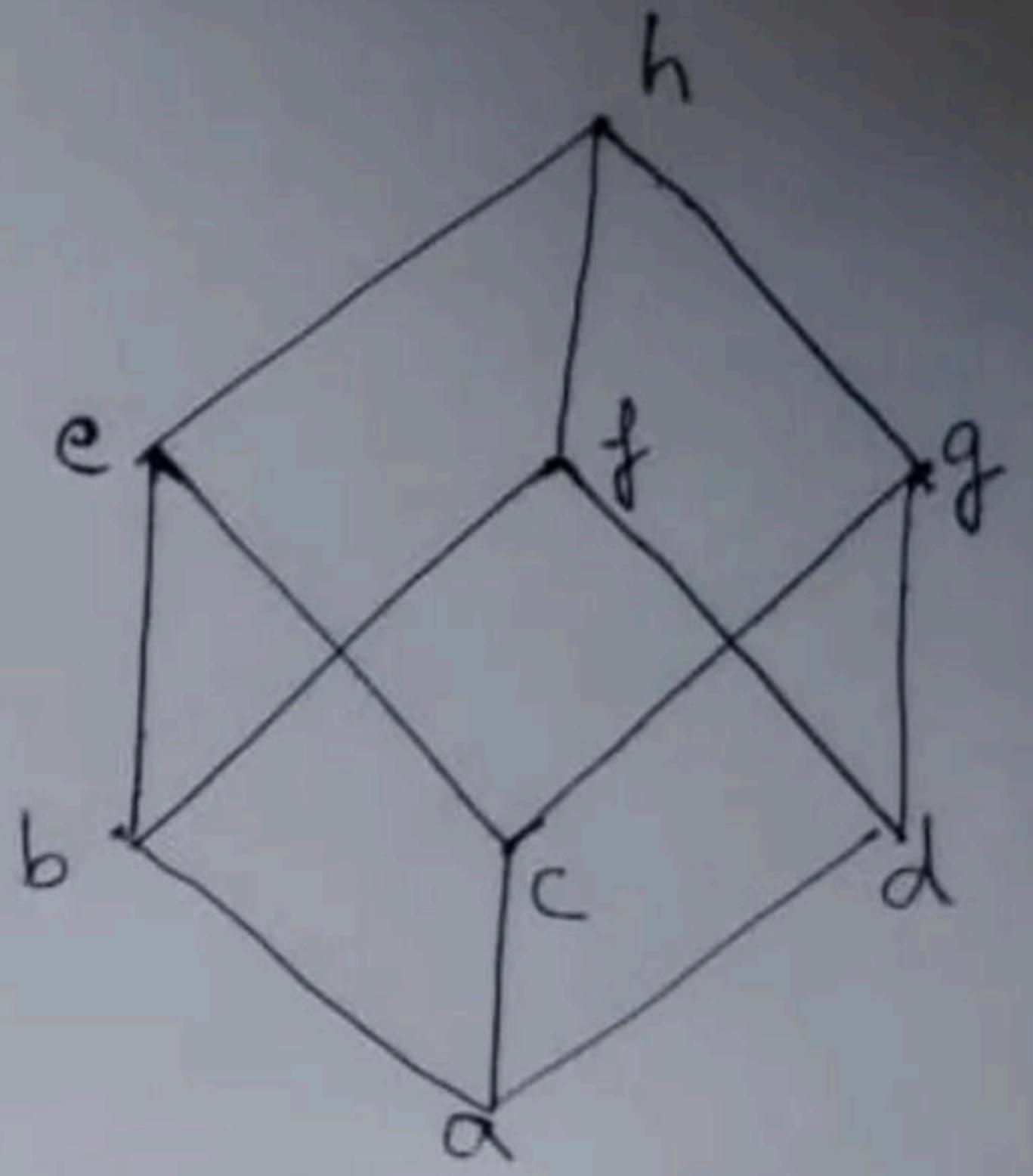
a









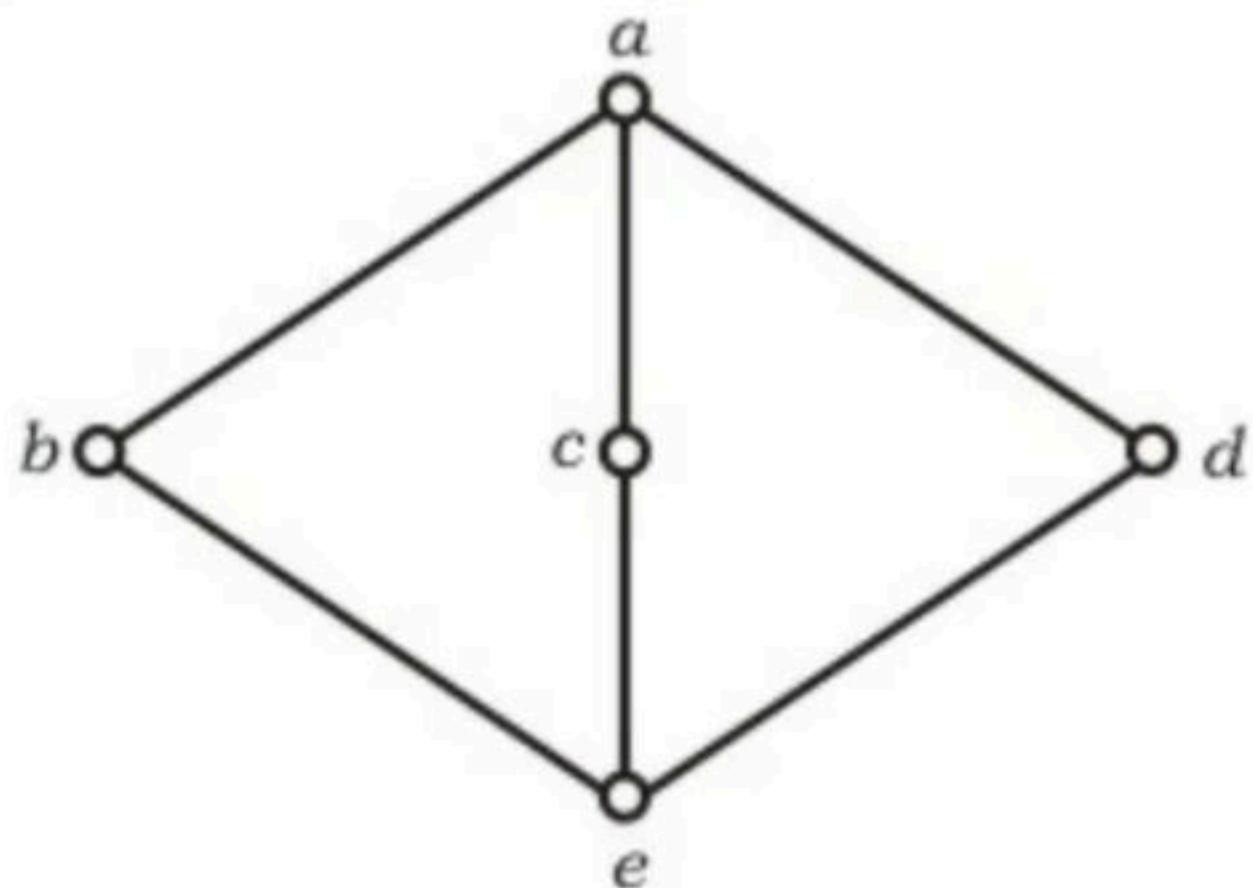


Break

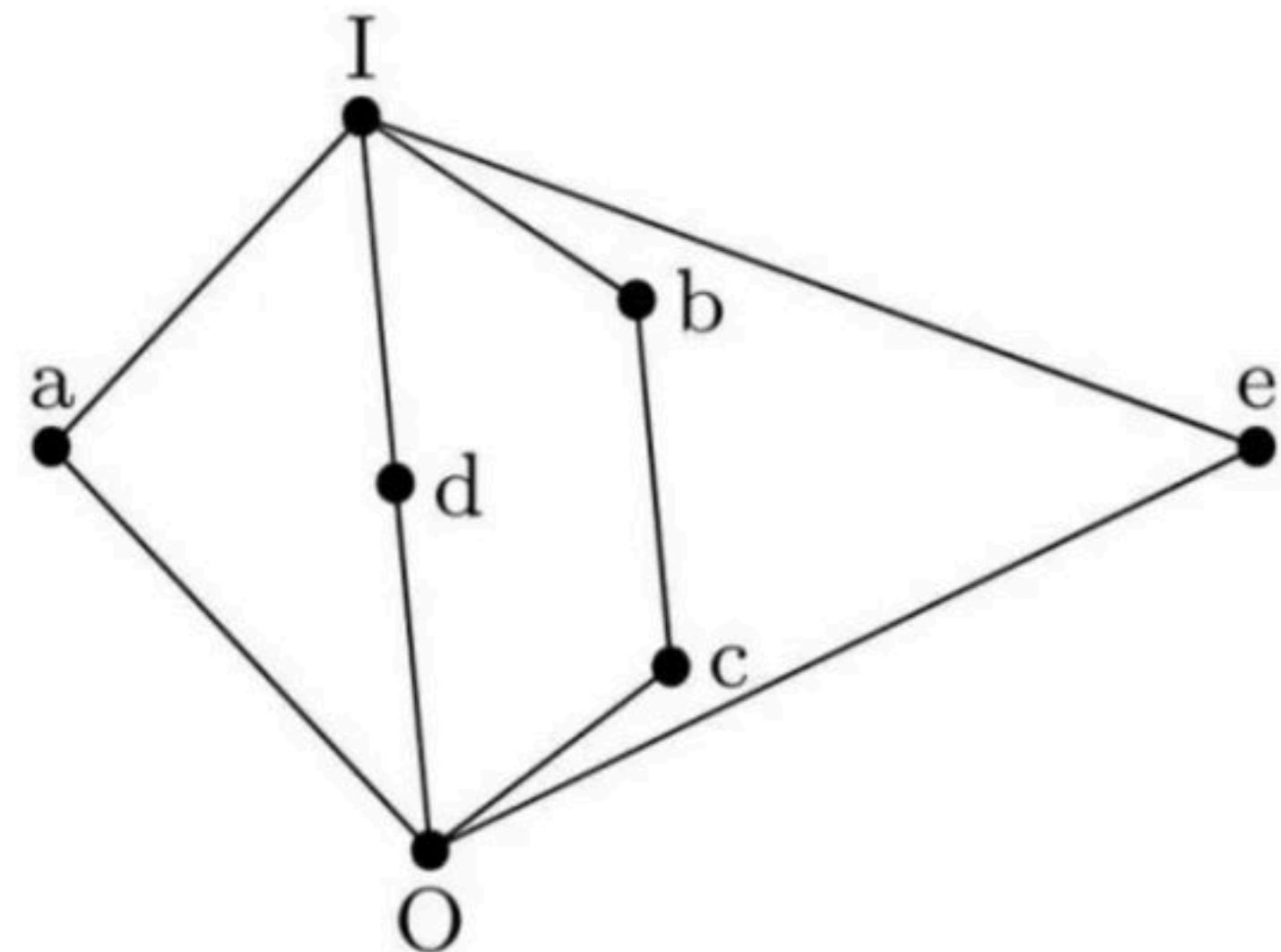
Q The following is the Hasse diagram of the Poset $\{\{a, b, c, d, e\}, \leq\}$

The Poset is **(GATE-2005) (1 Marks)**

- (A)** not a lattice
- (B)** a lattice but not a distributive lattice
- (C)** a distributive lattice but not a Boolean algebra
- (D)** a Boolean algebra



Q The complement(s) of the element 'a' in the lattice shown in below figure is (are) _____
(GATE-1988) (2 Marks)



Q Find which of the following is a lattice and Boolean Algebra?

(1) $[D_{10}, /]$

(2) $[D_{12}, /]$

(3) $[D_{30}, /]$

(4) $[D_{45}, /]$

(5) $[D_{64}, /]$

(6) $[D_{81}, /]$

(7) $[D_{91}, /]$

(8) $[D_{110}, /]$

Q Find which of the following is a lattice and Boolean Algebra?

(1) $[\{1,2,3,4,6,9\}, /]$

(2) $[\{2,3,4,6,12\}, /]$

(3) $[\{1,2,3,5,30\}, /]$

(4) $[\{1,2,3,6,9,18\}, /]$

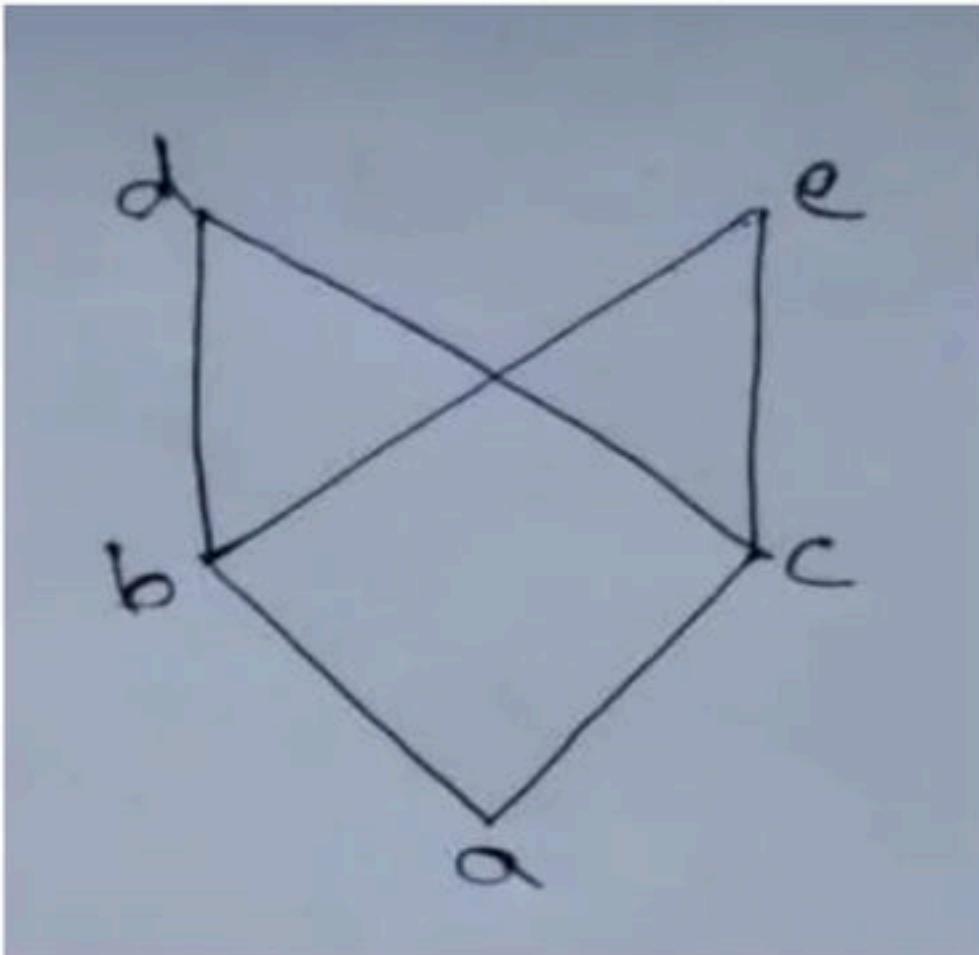
(5) $[\{2,3,4,9,12,18\}, /]$

(6) $[R, \leq]$

(7) $[P(A), \sqsubseteq], A = \{1,2,3\}$

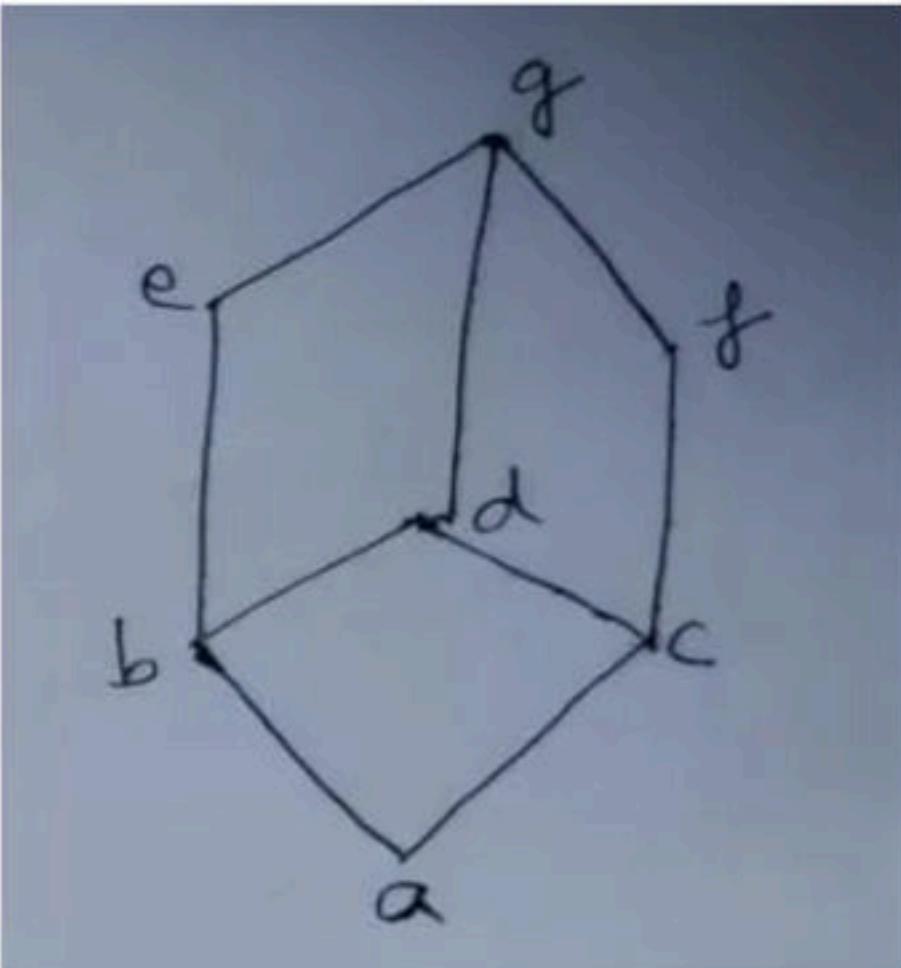
Q Consider the following hasse diagram,
find which of the following is true?

- a) it is a lattice
- b) subset {a, b, c , d} is a lattice
- c) subset {b, c, d, e} is a lattice
- d) subset {a, b, c, e} is a lattice



Q Consider the following hasse diagram, find which of the following is true?

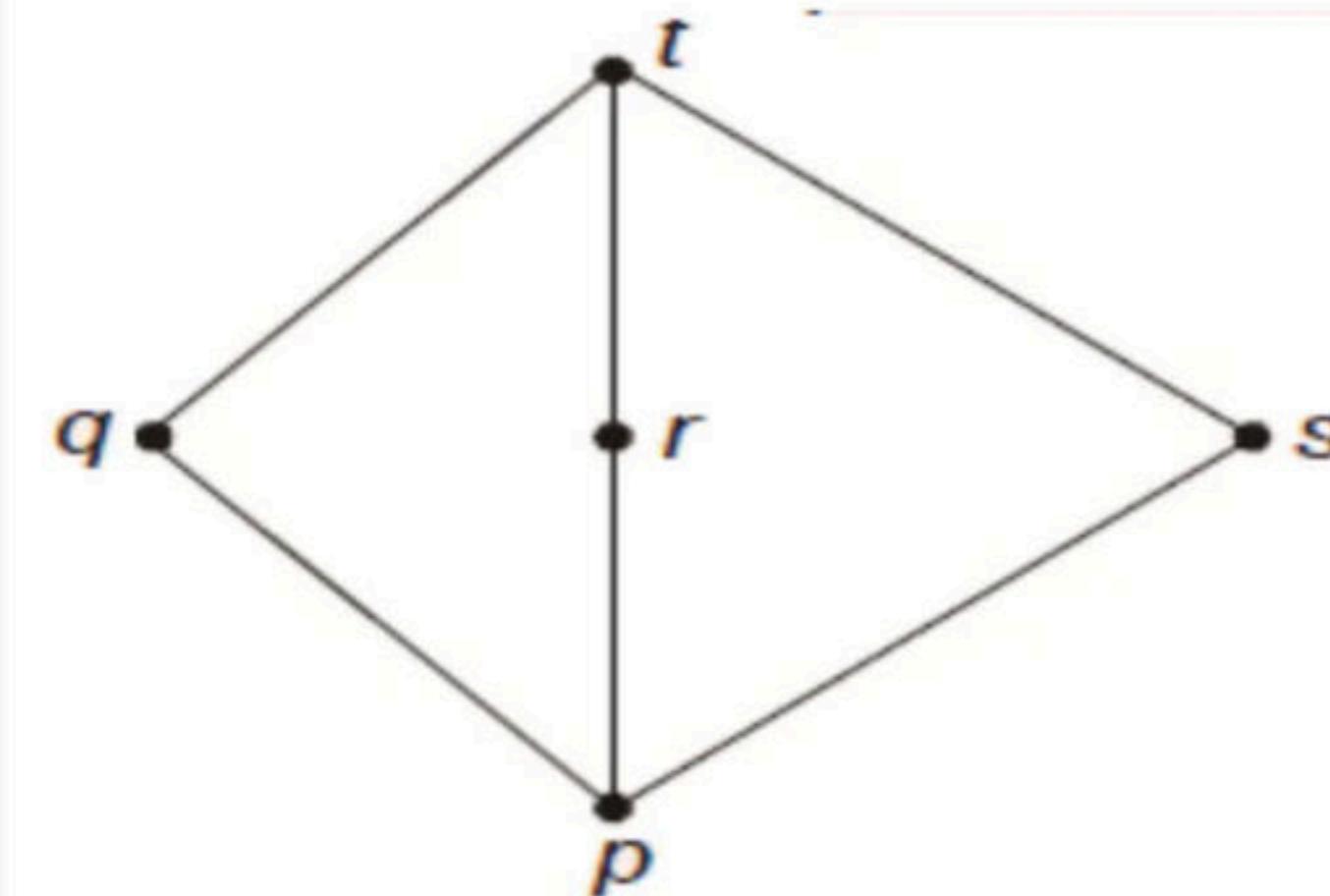
- a) subset $\{a, b, c, g\}$ is a lattice
- b) subset $\{a, b, f, g\}$ is a lattice
- c) subset $\{a, d, e, g\}$ is a lattice
- d) subset $\{a, c, e, g\}$ is a lattice



Break

Q Suppose $L = \{p, q, r, s, t\}$ is a lattice represented by the following Hasse diagram:
For any $x, y \in L$, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let $L^3 = \{(x, y, z) : x, y, z \in L\}$ be the set of all ordered triplets of the elements of L . Let P_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then **(GATE-2015) (2 Marks)**

- (A)** $P_r = 0$ **(B)** $P_r = 1$ **(C)** $0 < P_r \leq 1/5$ **(D)** $1/5 < P_r < 1$



Q Consider the set $S = \{a, b, c, d\}$.

Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on S :

$$\pi_1 = \{\overline{abcd}\}, \quad \pi_2 = \{\overline{ab}, \overline{cd}\}, \quad \pi_3 = \{\overline{abc}, \overline{d}\}, \quad \pi_4 = \{a, b, c, d\}$$

Let \prec be the partial order on the set of partitions $S' = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j . The Poset diagram for (S', \prec) is: **(GATE-2007) (2 Marks)**

