Proposition - Part III

Course on Discrete Mathematics for GATE 2023

Sanchit Jain • Lesson 15 • Sept 17, 2022

Educator highlights

- Works at **Knowledge Gate**
- Studied at **Delhi Technological University**
- Have qualified gate Have experience of more than 8 years Have a YouTube channel with 5 lakh follower
- Lives in **Ghaziabad, Uttar Pradesh, India**
- Unacademy Educator since **19th July, 2019**
- **844,388 live minutes** taught in last 30 days
- Knows **Punjabi, Hinglish, Hindi and English**

Sanchit Jain

Follow

Legend in GATE - CS & IT

I am passionate for teaching computer science, having experience of more than 10 years. I teach all computer science subjects for GATE.

69M Watch mins | **3M** Watch mins (last 30 days) | **30K** Followers | **3K** Dedications

भाई में है दम
10 Subject ख़तम

Months Wise Calendar for GATE-2023 (Unacademy Plus) (Sanchit Jain)

Month	Morning Class (6:30am – 8:00am)	Evening Class (6:30pm – 8:30pm)
Feb-2022	CN(16-Feb to 26-March) (54 Hours)	DM (2-Feb to 2-Mar) (50 Hours)
Mar-2022		DBMS (16-Mar to 2-May) (50 Hours)
Apr-2022	TOC (6-Apr to 26-May) (58 Hours)	
May-2022		DE (4-May to 30-May) (40 Hours)
Jun-2022	Compiler (1-June to 20-June) (34 Hours) (DS & Programming) (22-June to 23-July) (56 Hours)	COA (8-June to 19-July) (38 Hours)
Jul-2022	Algo (28-July to 25-Aug) (38 Hours)	OS (21-July to 27-Aug) (58 Hours)
Aug-2022		
Sep-2022	**DM (1-Sept to 10-Oct) (50 Hours)**	**CN (01-Sept to 12-Oct) (54 Hours)**
Oct-2022	DBMS (13-Oct to 21-Nov) (50 Hours)	TOC (13-Oct to 30-Nov) (58 Hours)
Nov-2022	DE (24-Nov to 26-Dec) (40 Hours)	
Dec-2022	COA (29-Dec to 30-Jan) (38 Hours)	Compiler (1-Dec to 23-Dec) (34 Hours) DS & C (22-Dec to 2-Feb) (56 Hours)
Jan-2023		
Feb-2023		

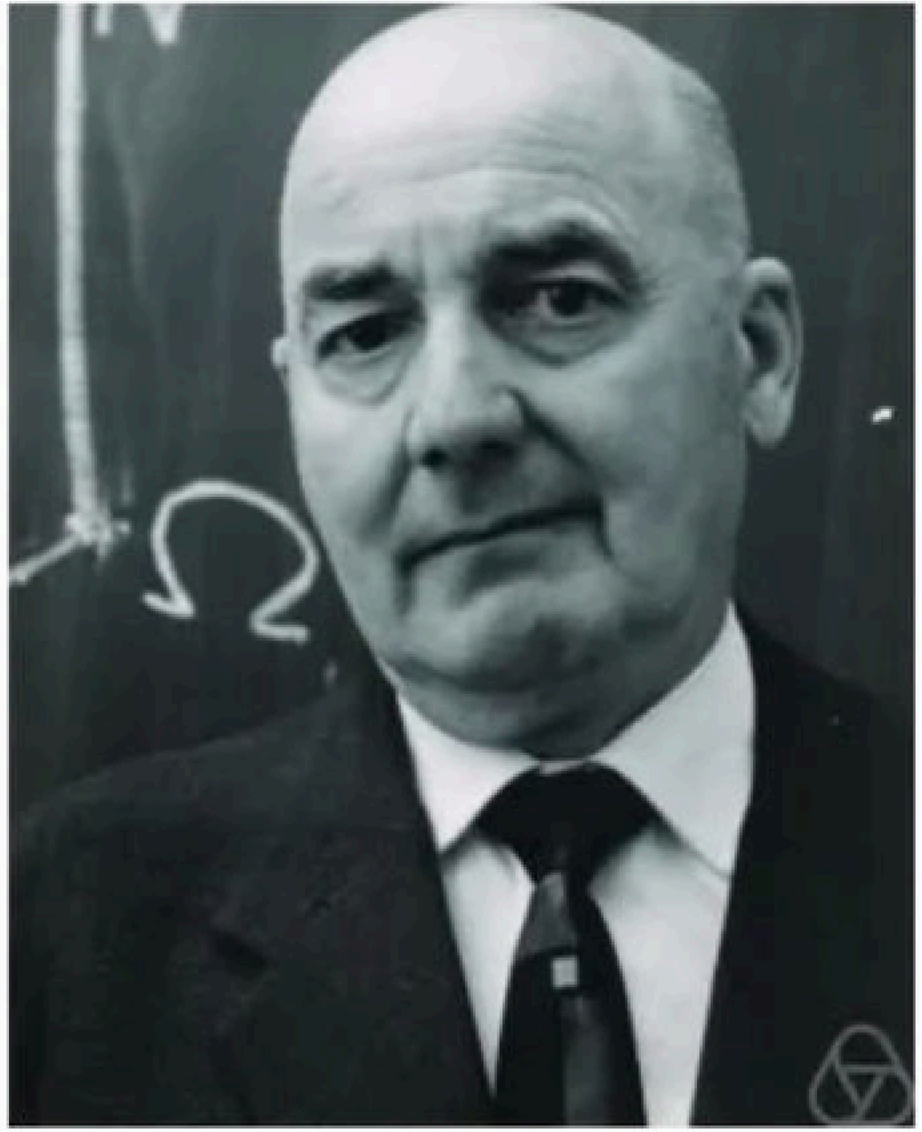
Conversion of POSET into a Hasse Diagram

- If we want to study Partial order relation further then it will be better to convert it into more convenient notation so that it can be studied easily.

- This graphical representation is called Hasse Diagram

- In order theory, a Hasse diagram is a type of mathematical diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction.

- The diagrams are named after Helmut Hasse (1898–1979)



Steps to convert partial order relation into hasse diagram

1- Draw a vertex for each element in the Set

2- If (a, b) ϵ R then draw an edge from a to b

3- Remove all Reflexive and Transitive edges

4- Remove the direction of edges and arrange them in the increasing order of heights.

Q Consider a Partial order relation and convert it into hasse diagram?

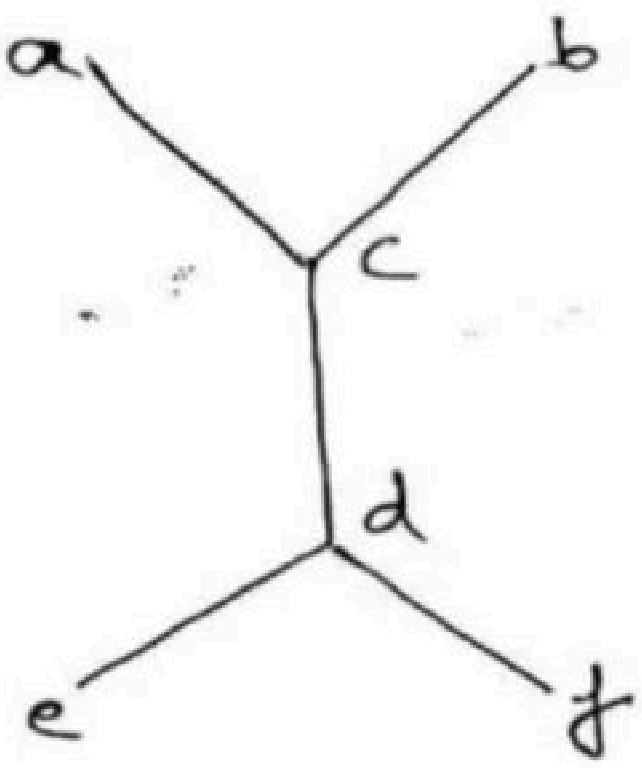
R = {(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)}

Q Consider a Partial order relation and convert it into hasse diagram?

R = {(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)}

Break

Q Study the following hasse diagram and find which of the following are valid?



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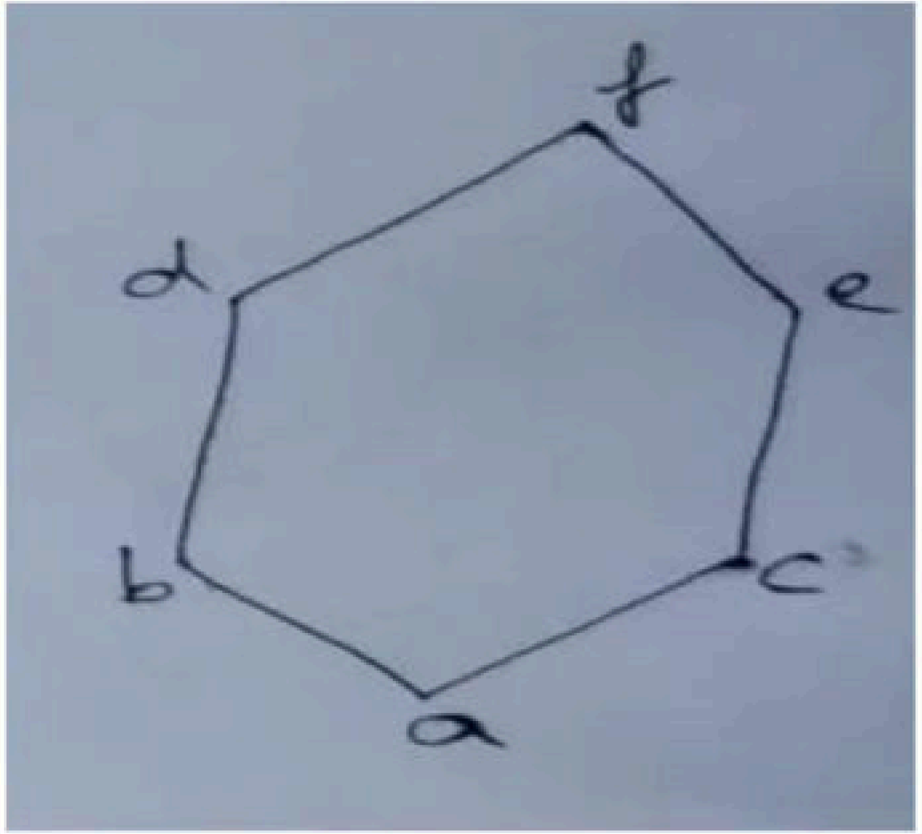

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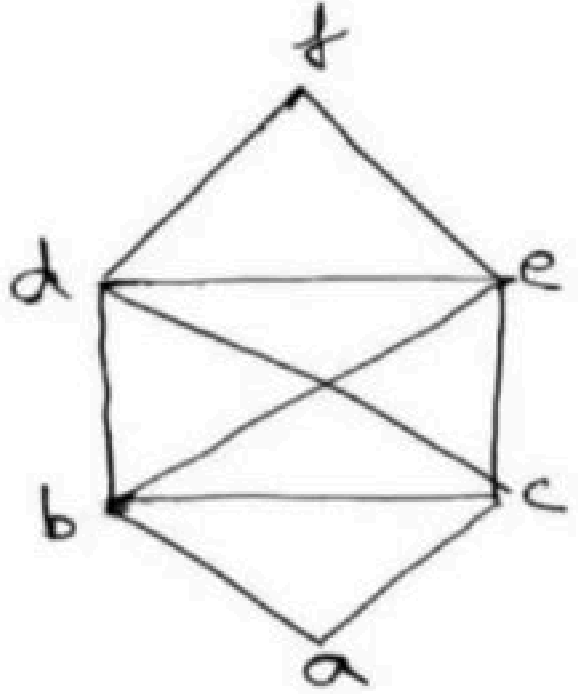

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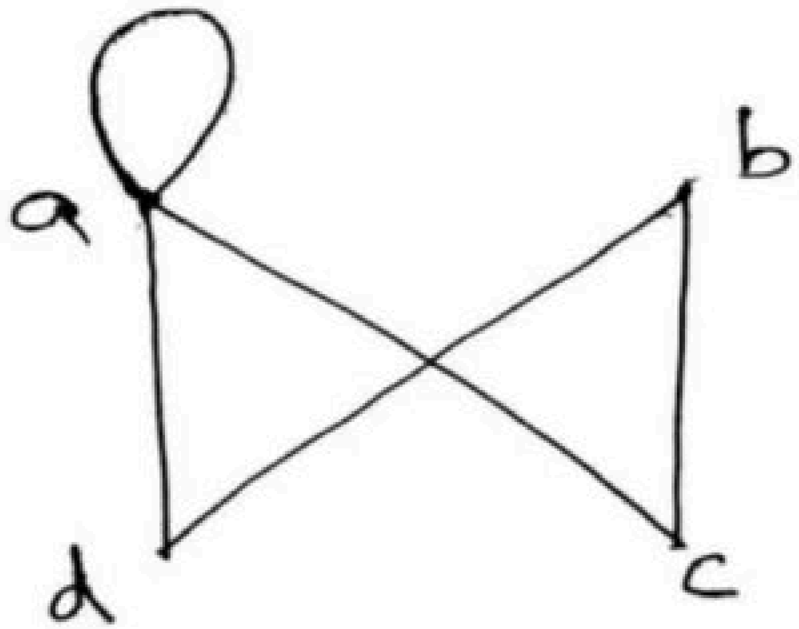

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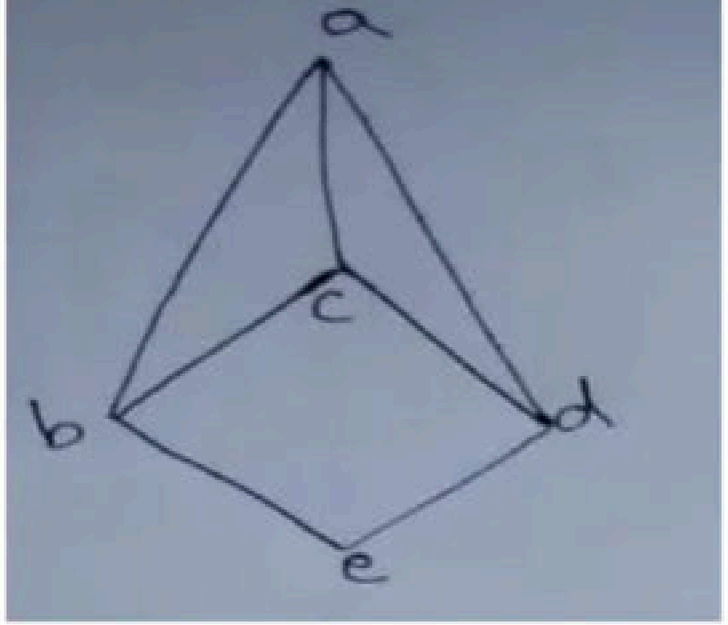

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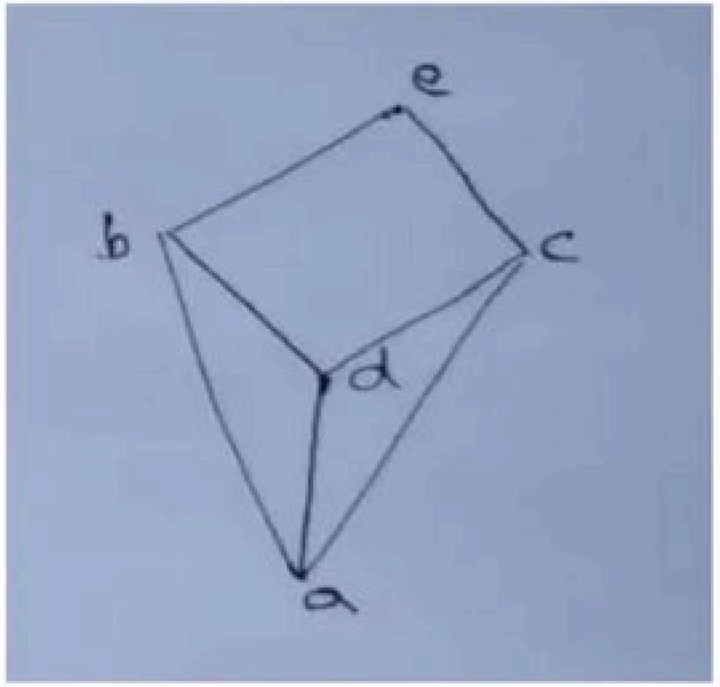

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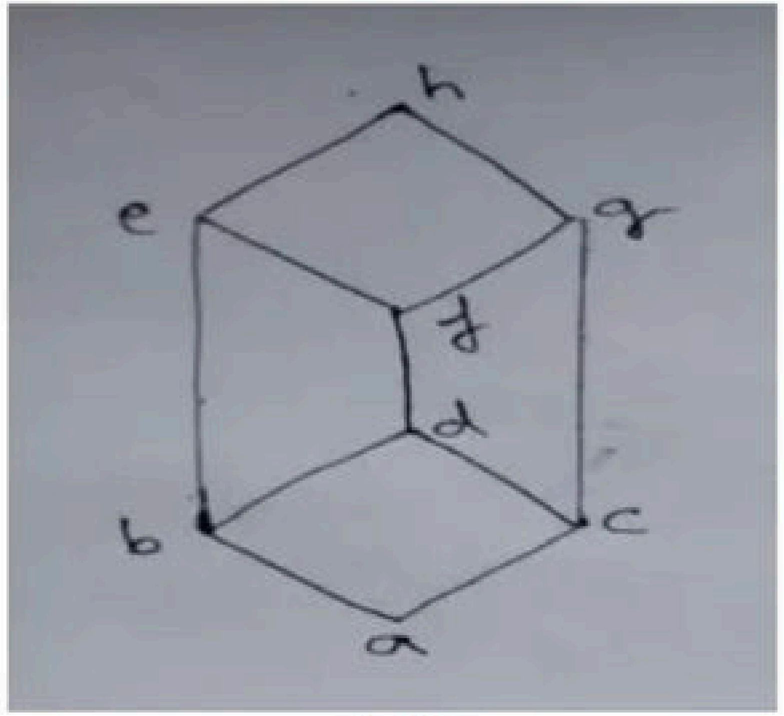

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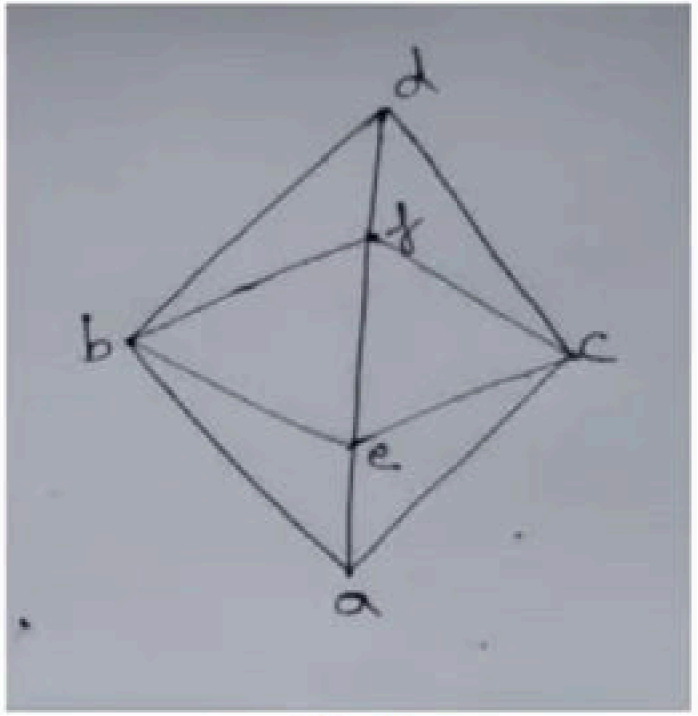

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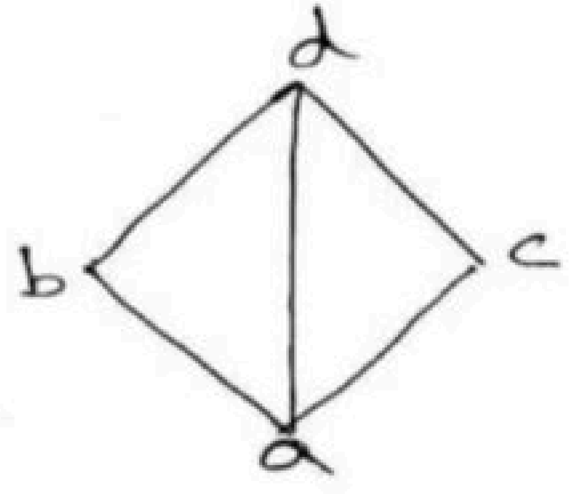

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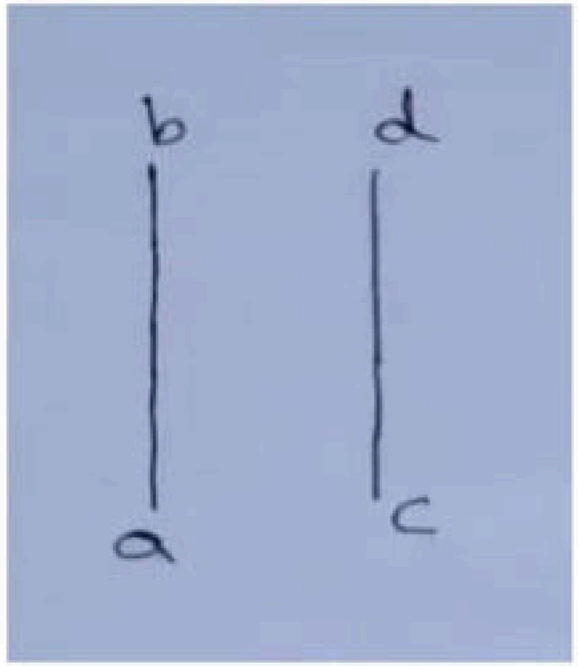

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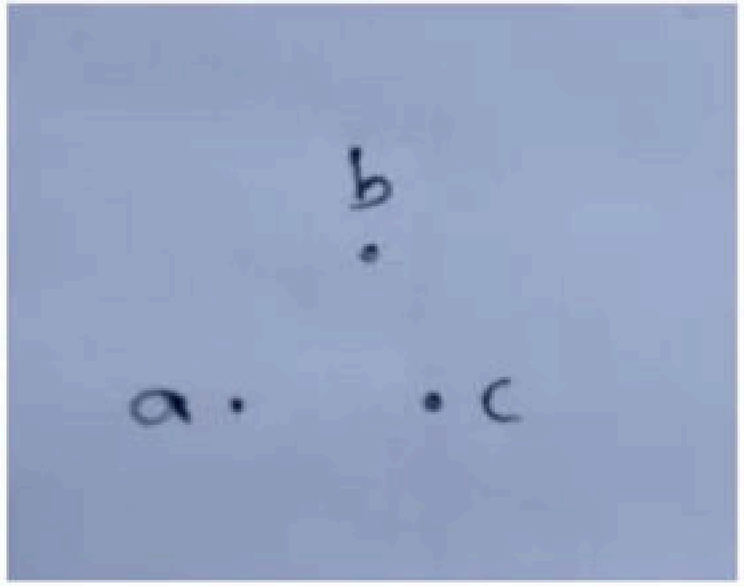


(10)

Conclusion

- We can not have a horizontal edge in a hasse diagram
- We can not have a reflexive and transitive edge in Hasse Diagram

Q Let X = {2,3,6,12,24}, Let ≤ be the partial order defined by X ≤ Y if x divides y. Number of edges as in the Hasse diagram of (X, ≤) is. **(GATE-1996) (1 Marks)**

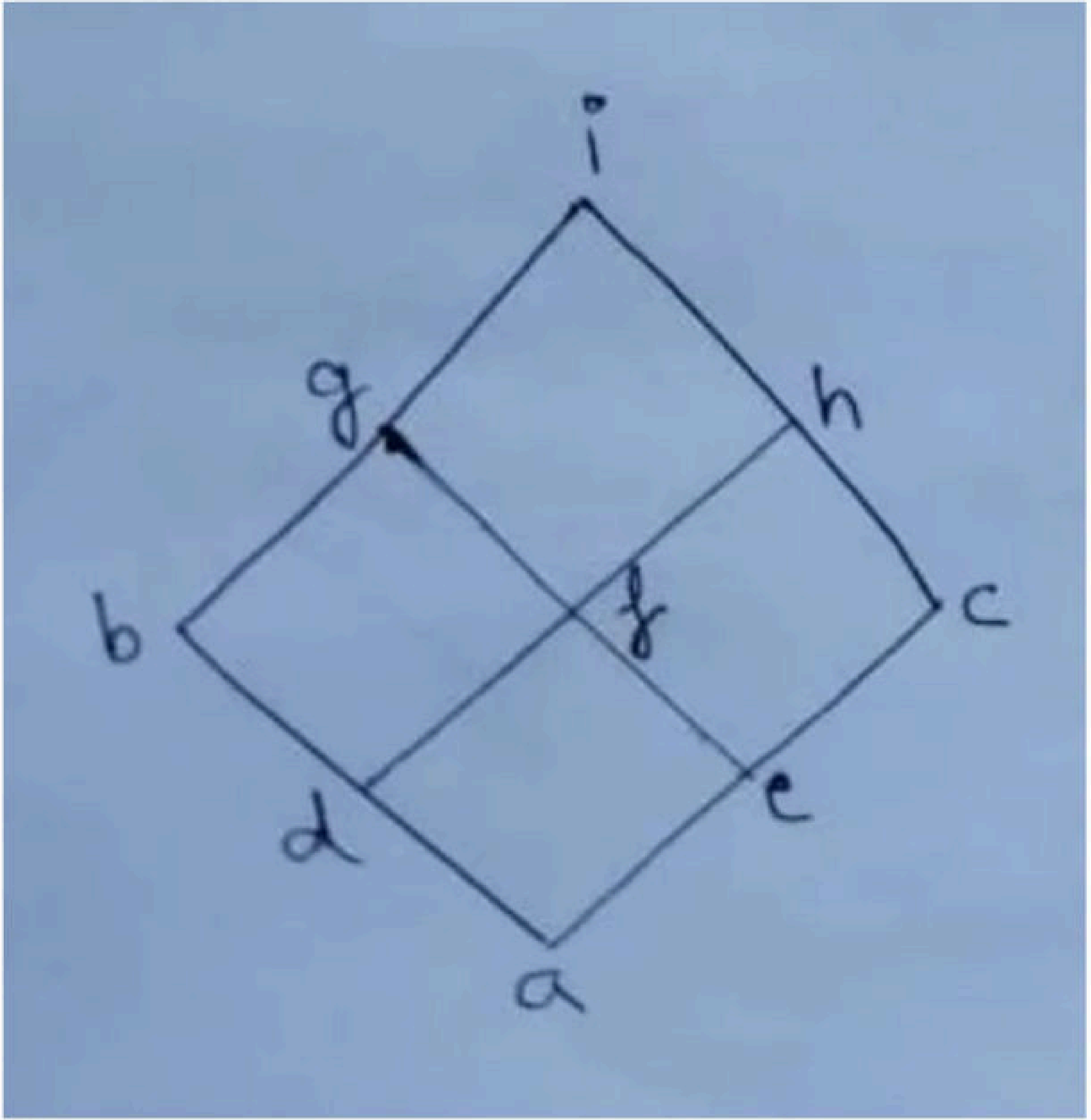
(a) 3 **(b)** 4 **(c)** 9 **(d)** None of the above

Break

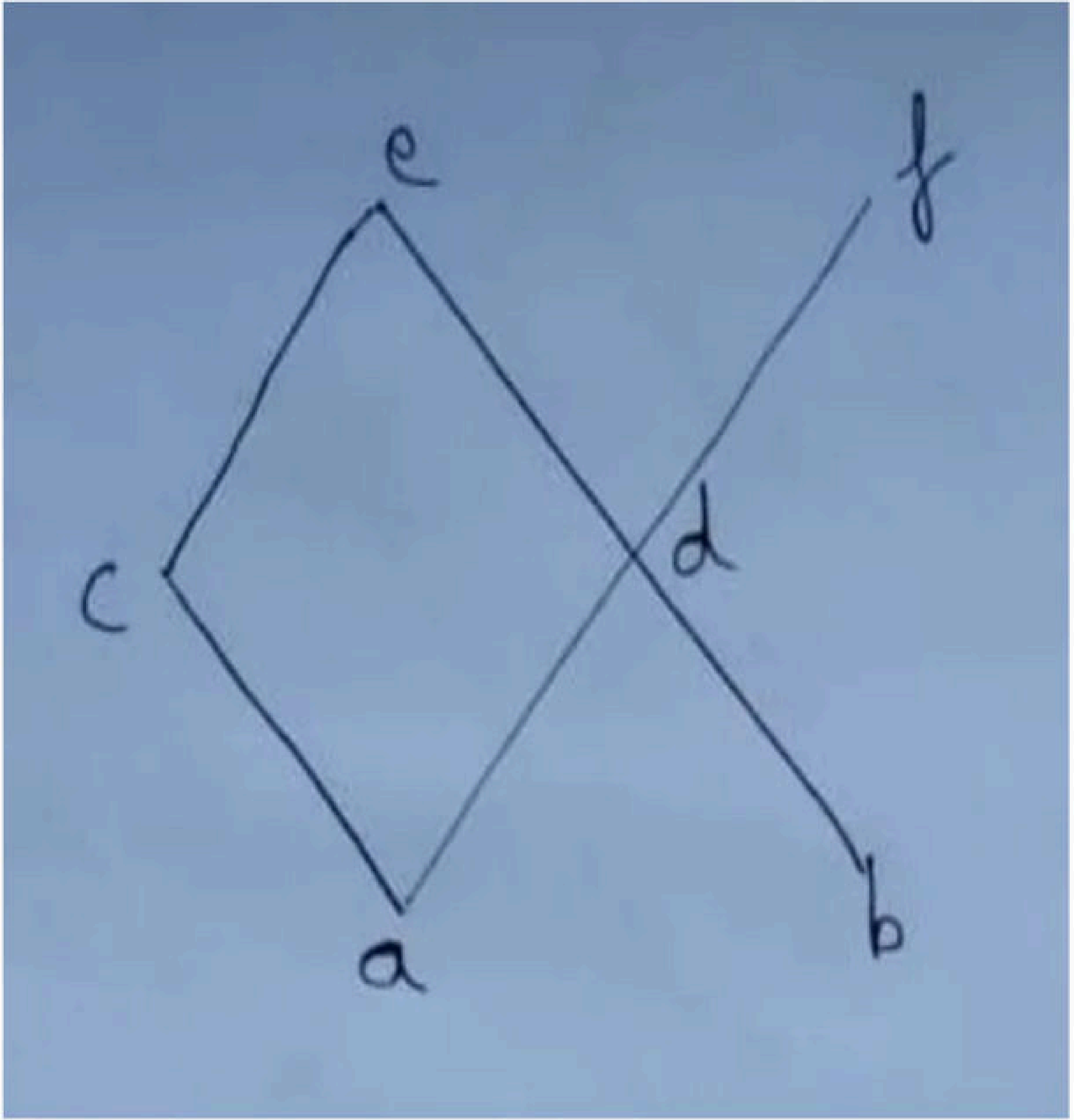
Elements of a Poset

1. **Maximal Element: -** An element is said to be maximal if it is not related to any other element in the Partial order relation.

2. **Minimal Element: -** An element is said to be minimal if no other element is related to it in the Partial order relation.



	Elements
Minimal	
Least	
Maximal	
Greatest	



	Elements
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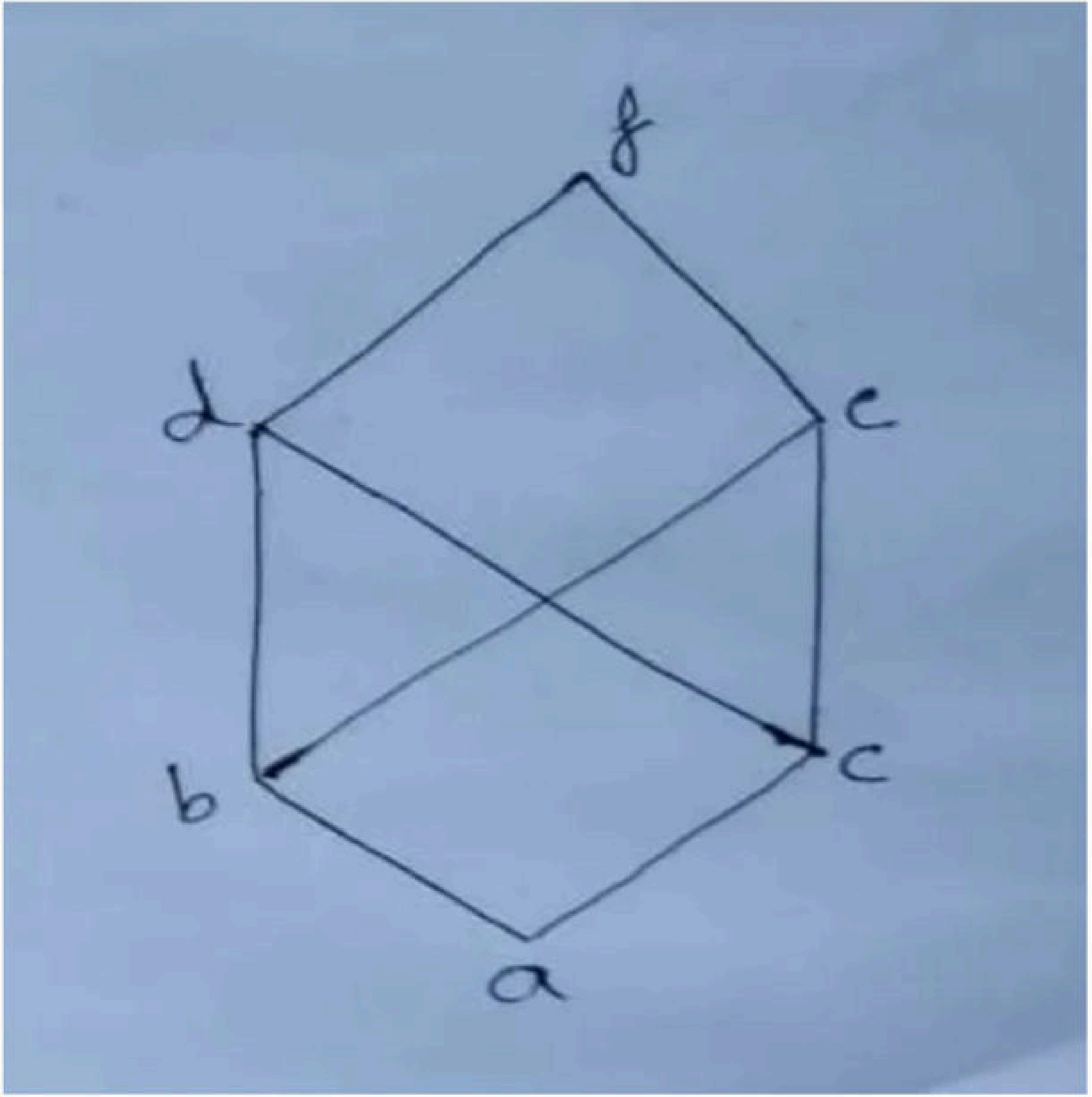
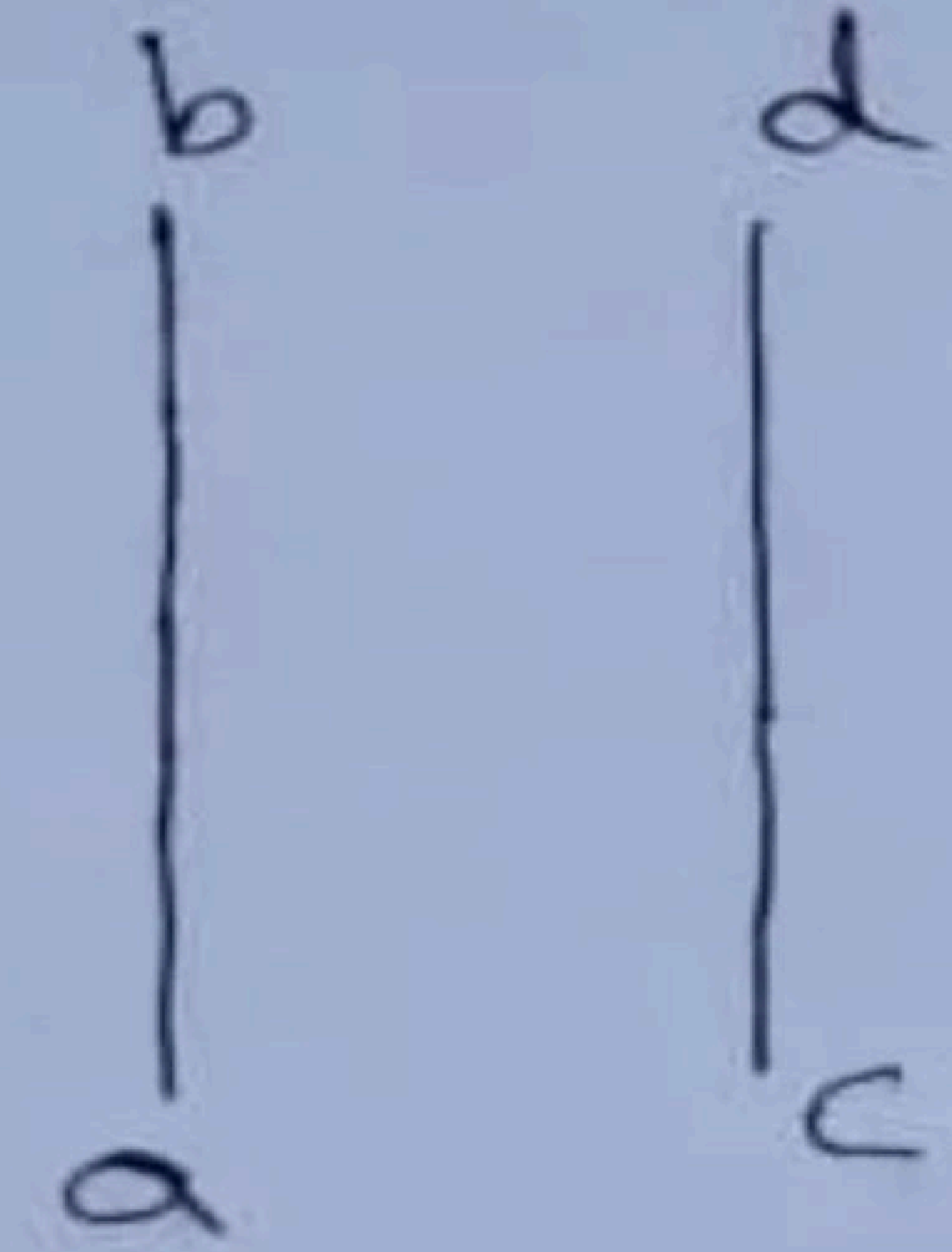
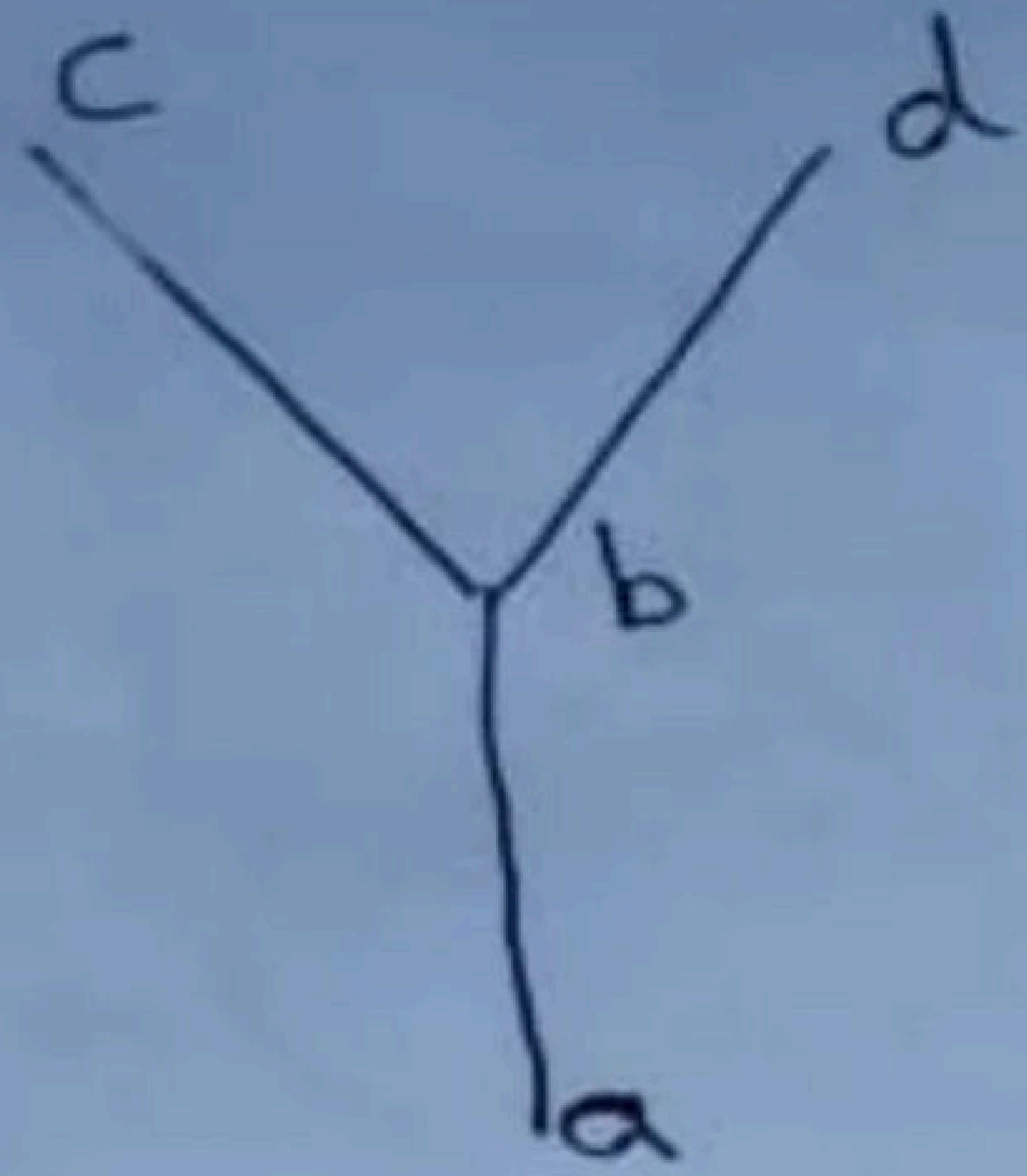
Least	
Maximal	
Greatest	



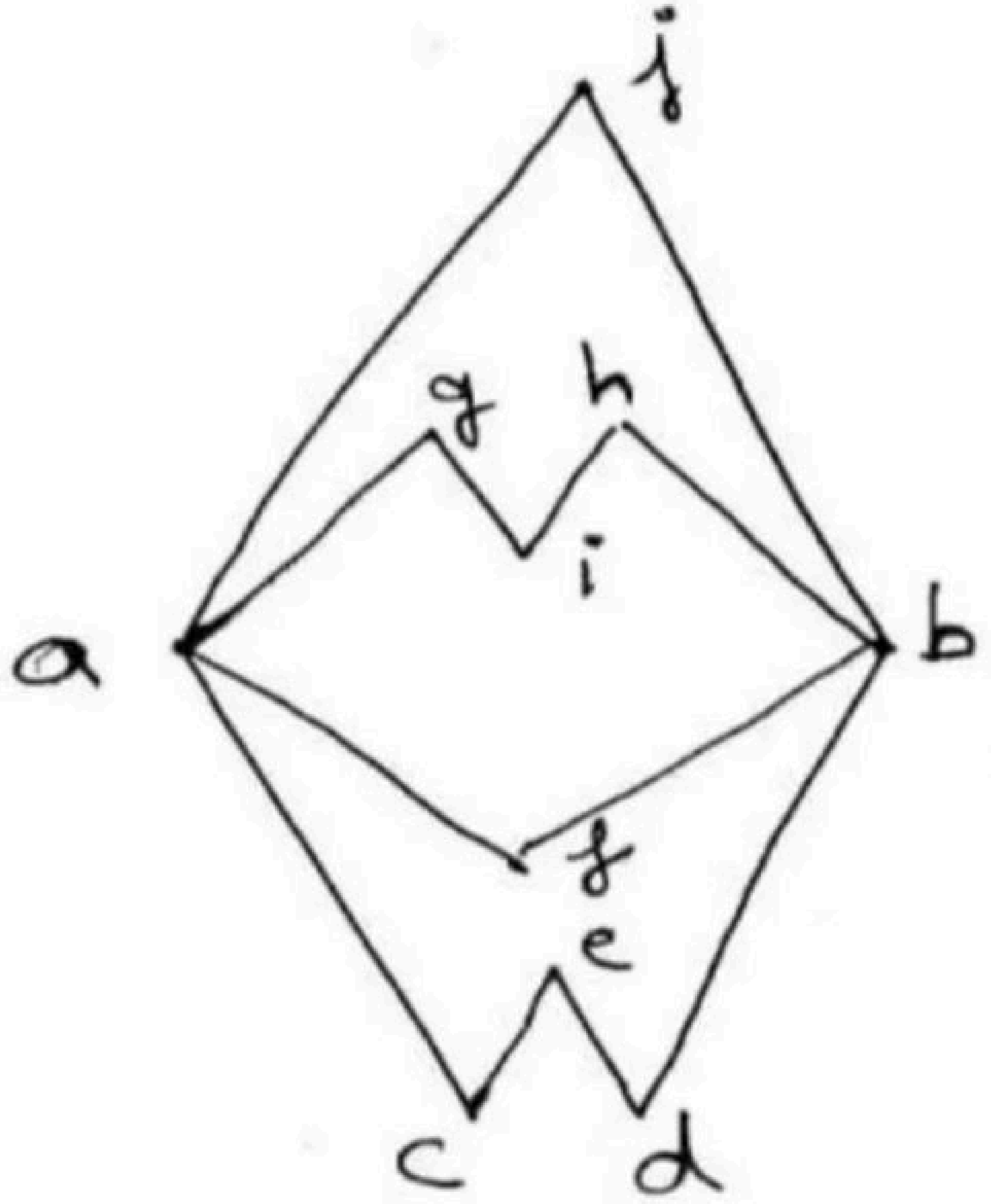
	Elements
Minimal	
Least	
Maximal	
Greatest	



	Elements
Minimal	
Least	
Maximal	
Greatest	



	Elements
Minimal	
Least	
Maximal	
Greatest	



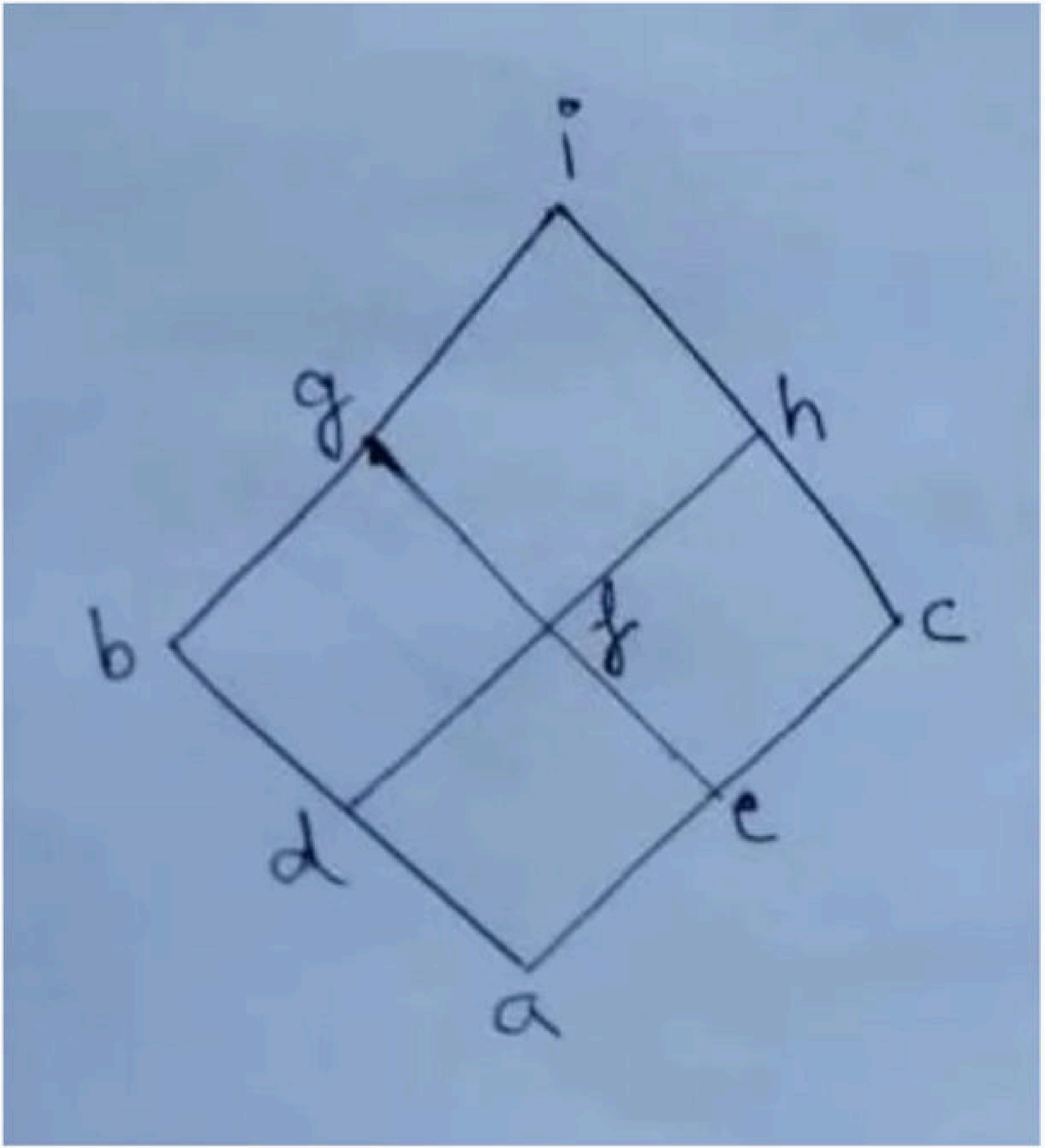
	Elements
Minimal	
Least	
Maximal	
Greatest	

- Every hasse diagram will have at least one Maximal and Minimal element(one or more).

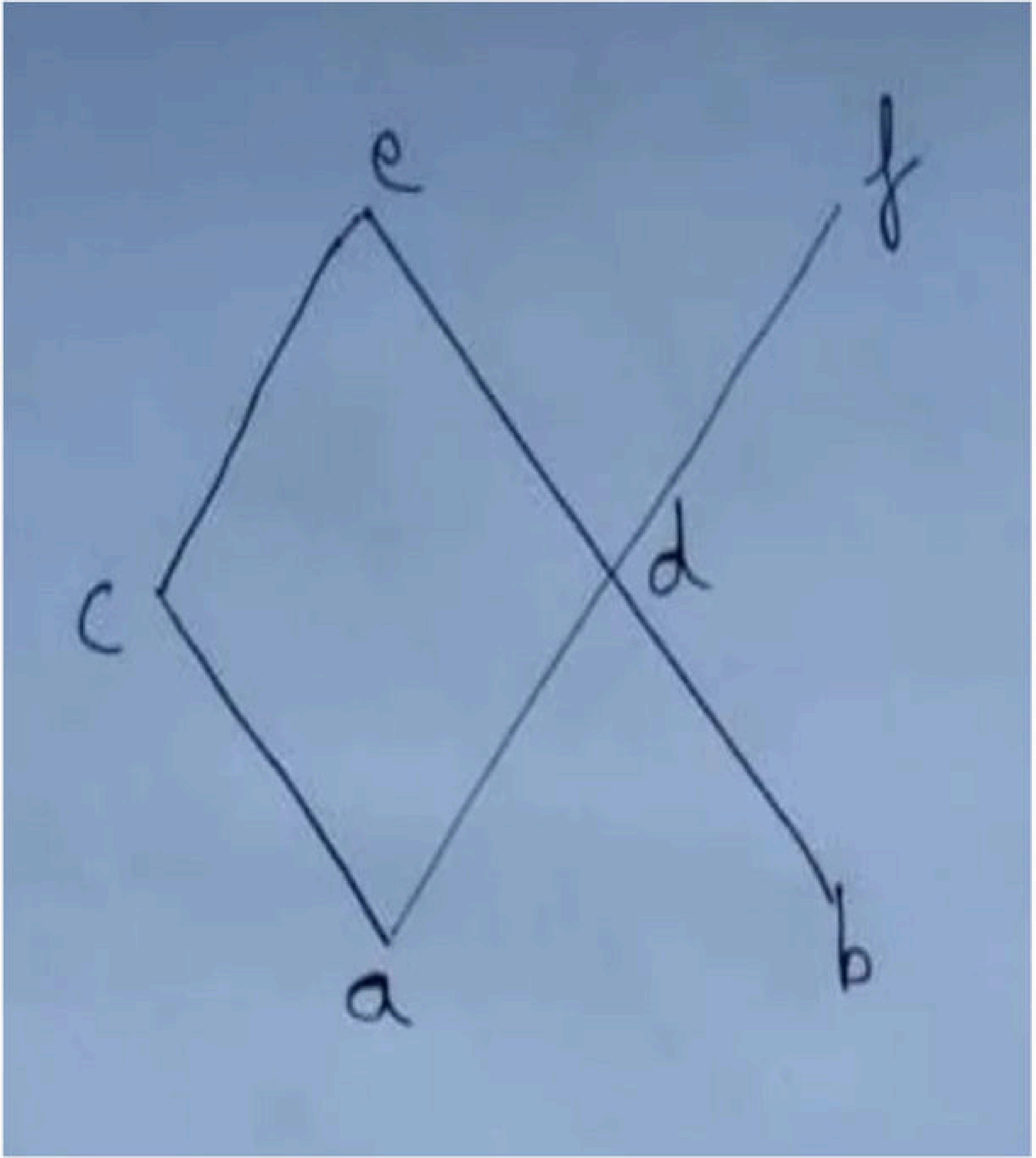
Break

- **<u>Greatest Element</u>: -** An element is said to be Maximum/Greatest if it is not related to any other element but every element is related to it in Partial order relation. Or if a hasse diagram has only one Maximal element then it will also be Maximum/Greatest element.

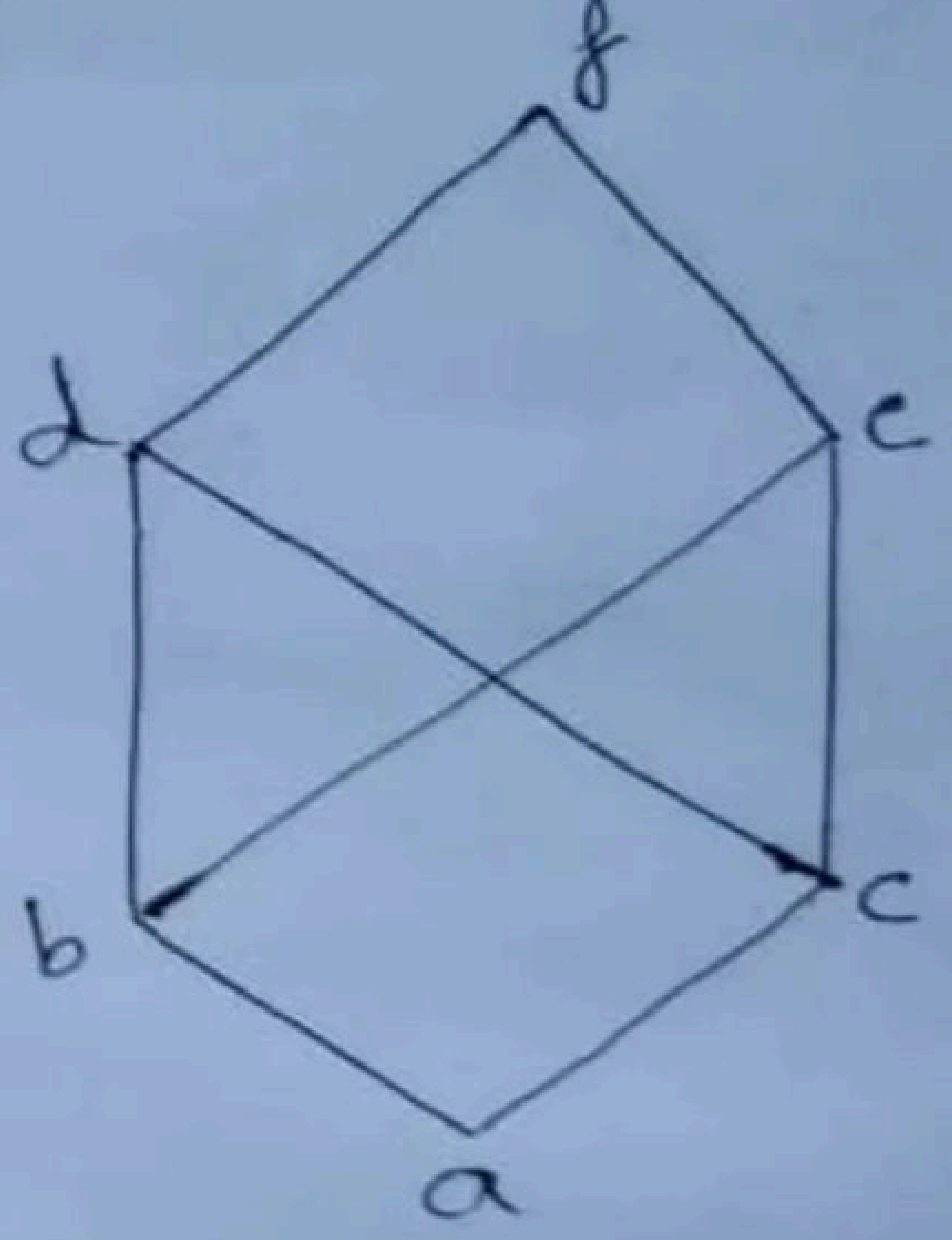
- **<u>Least Element</u>: -** An element is said to be Minimum/Least if no other element is related to it but it is related to every element Partial order relation. Or if a hasse diagram has only one Minimal element then it will also be Minimum/Least element.



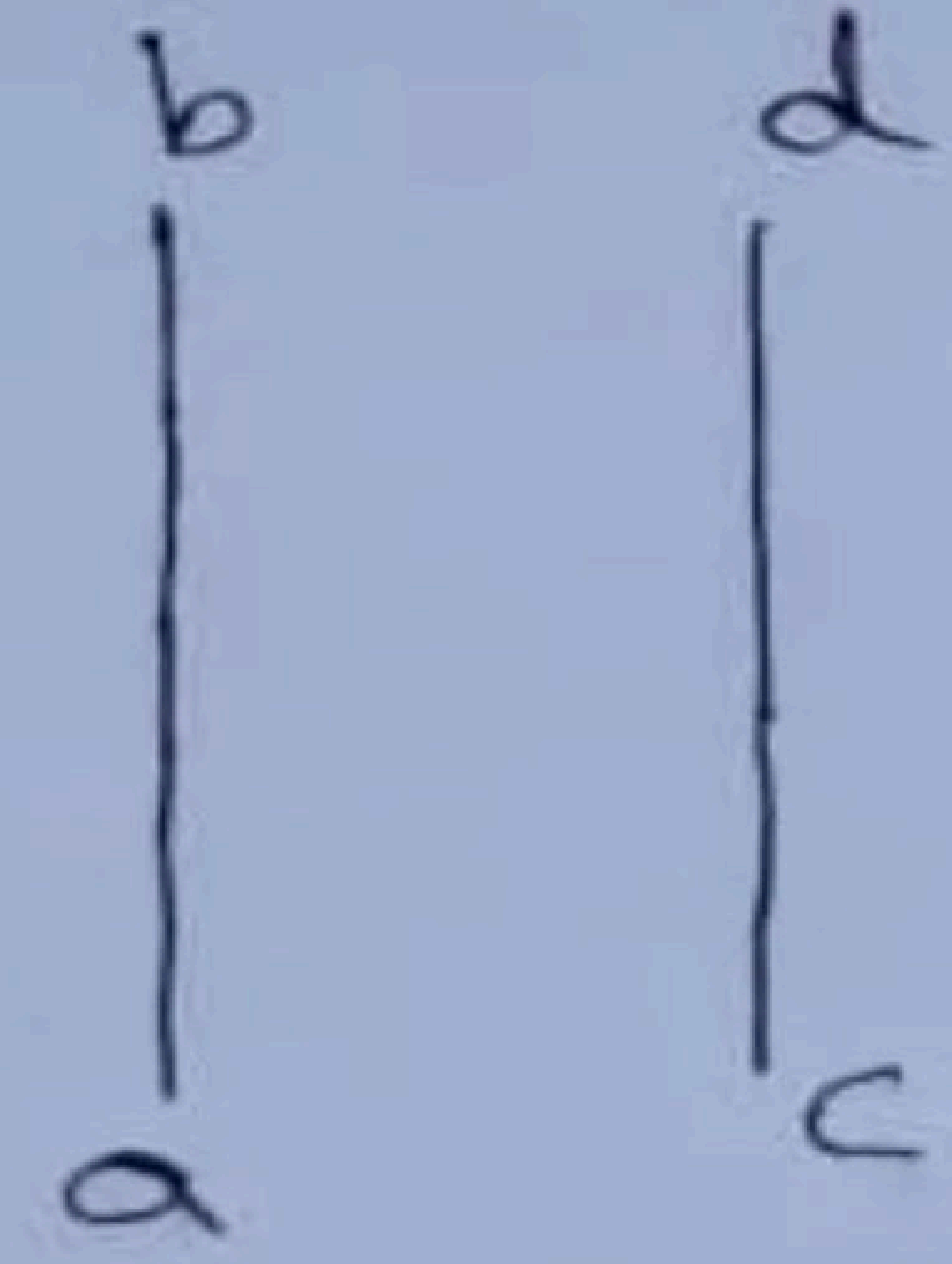
	Elements
Minimal	a
Least	
Maximal	i
Greatest	



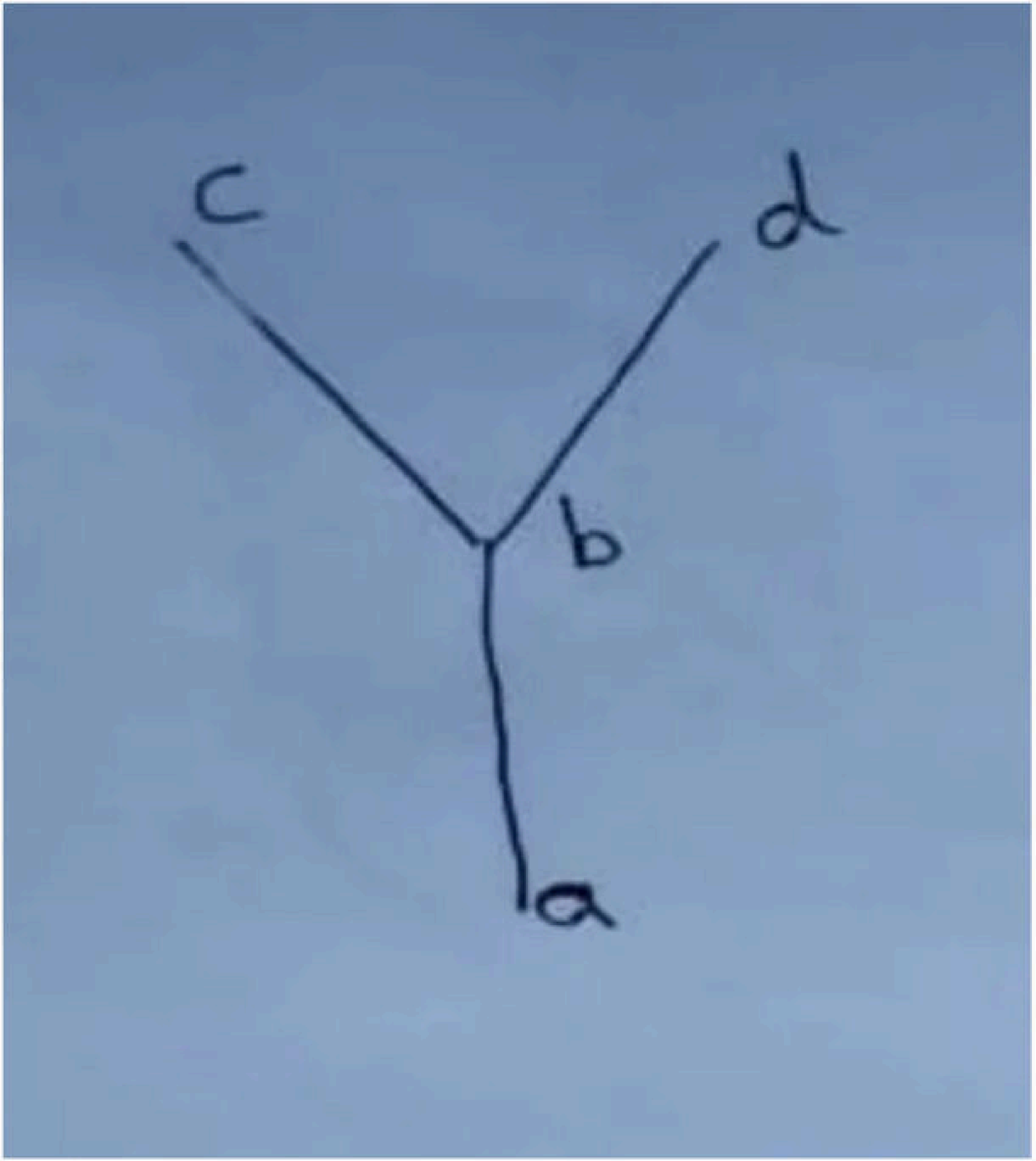
	Elements
Minimal	a, b
Least	
Maximal	e, f
Greatest	



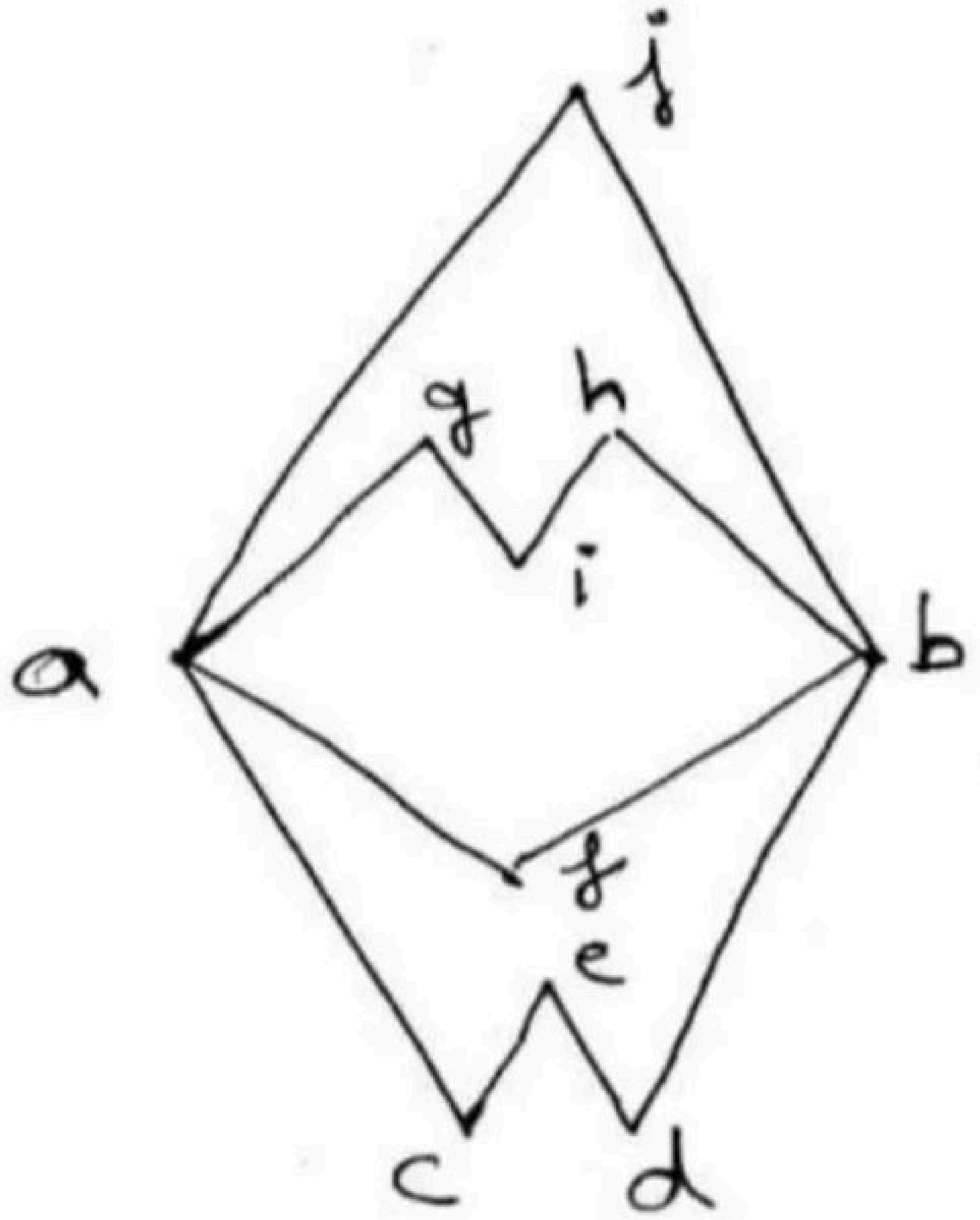
	Elements
Minimal	a
Least	
Maximal	f
Greatest	



	Elements
Minimal	a, c
Least	
Maximal	b, d
Greatest	



	Elements
Minimal	a
Least	
Maximal	c, d
Greatest	



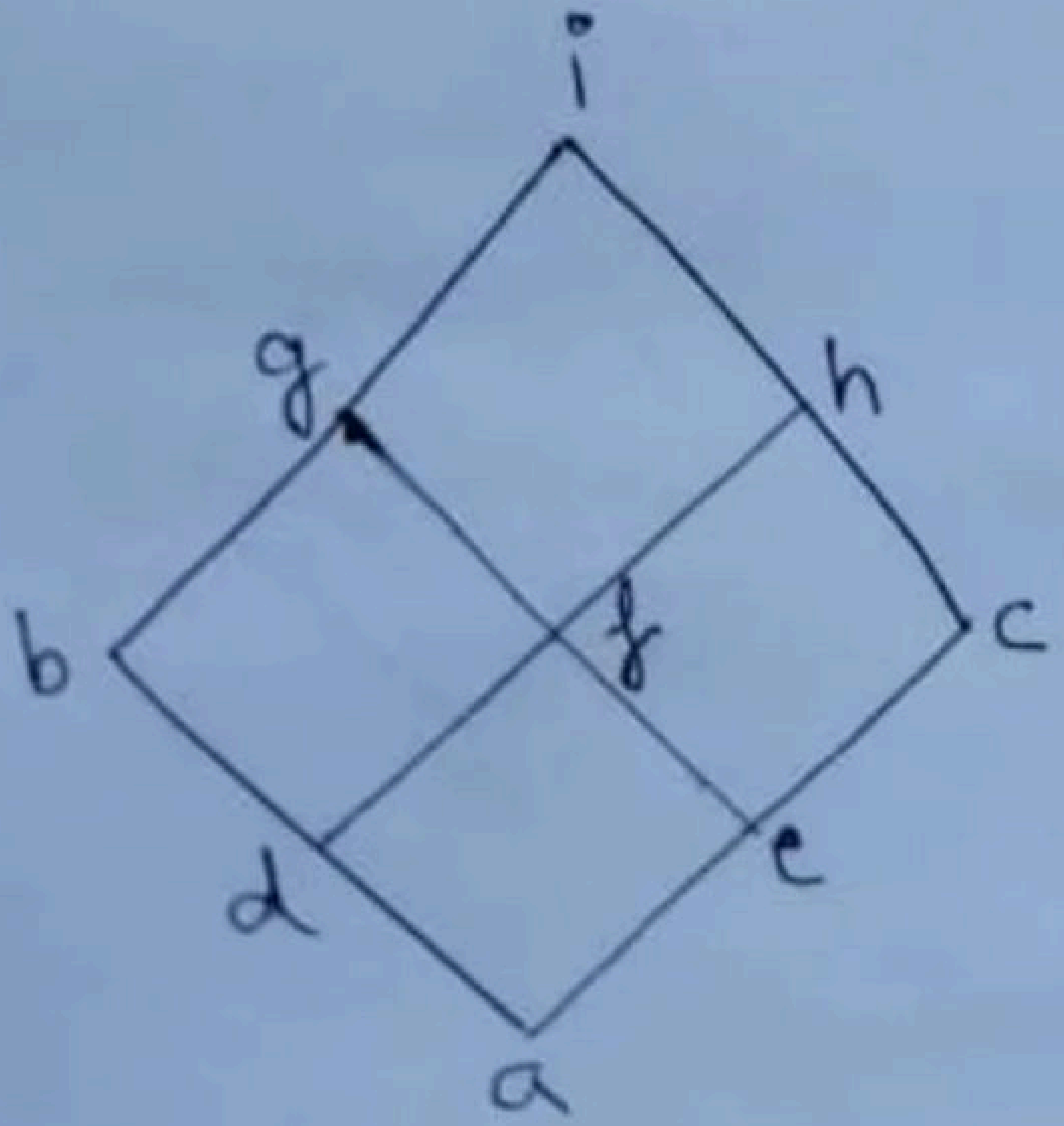
	Elements
Minimal	c, d, f, i
Least	
Maximal	j, g, h, e
Greatest	

1. Every hasse diagram will have at most one Greatest and Least element(zero or one) (T/F)
2. Every Greatest element is also Maximal (T/F)
3. Every Least element is also Minimal (T/F)
4. If there is only one Maximal element then it is called Greatest (T/F)
5. If there is only one Minimal element then it is called Least (T/F)

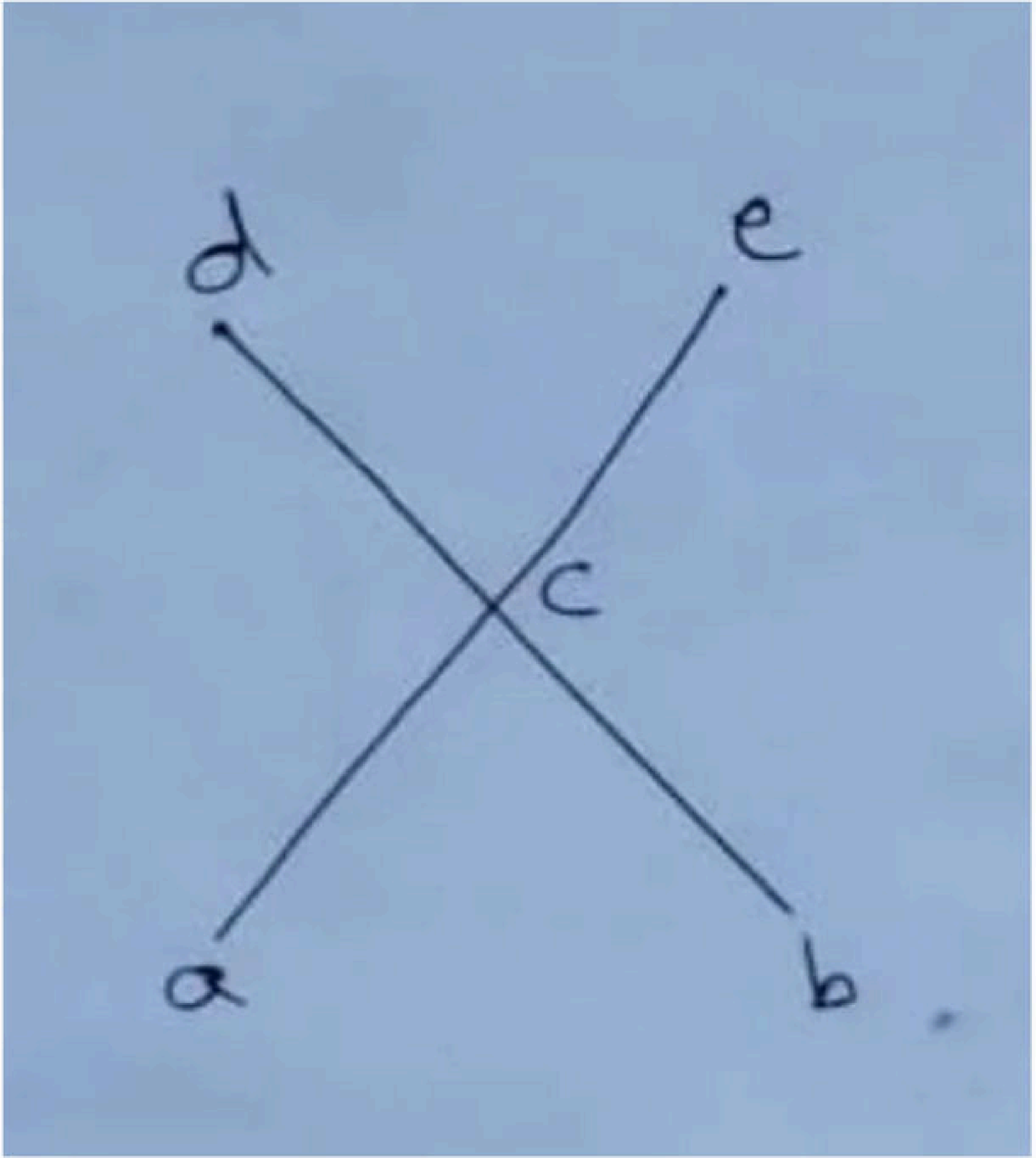
Break

- **Upper Bound: -** Upper bound of a subset B with respect to set A, will contain all those element to which all the elements of B is related.

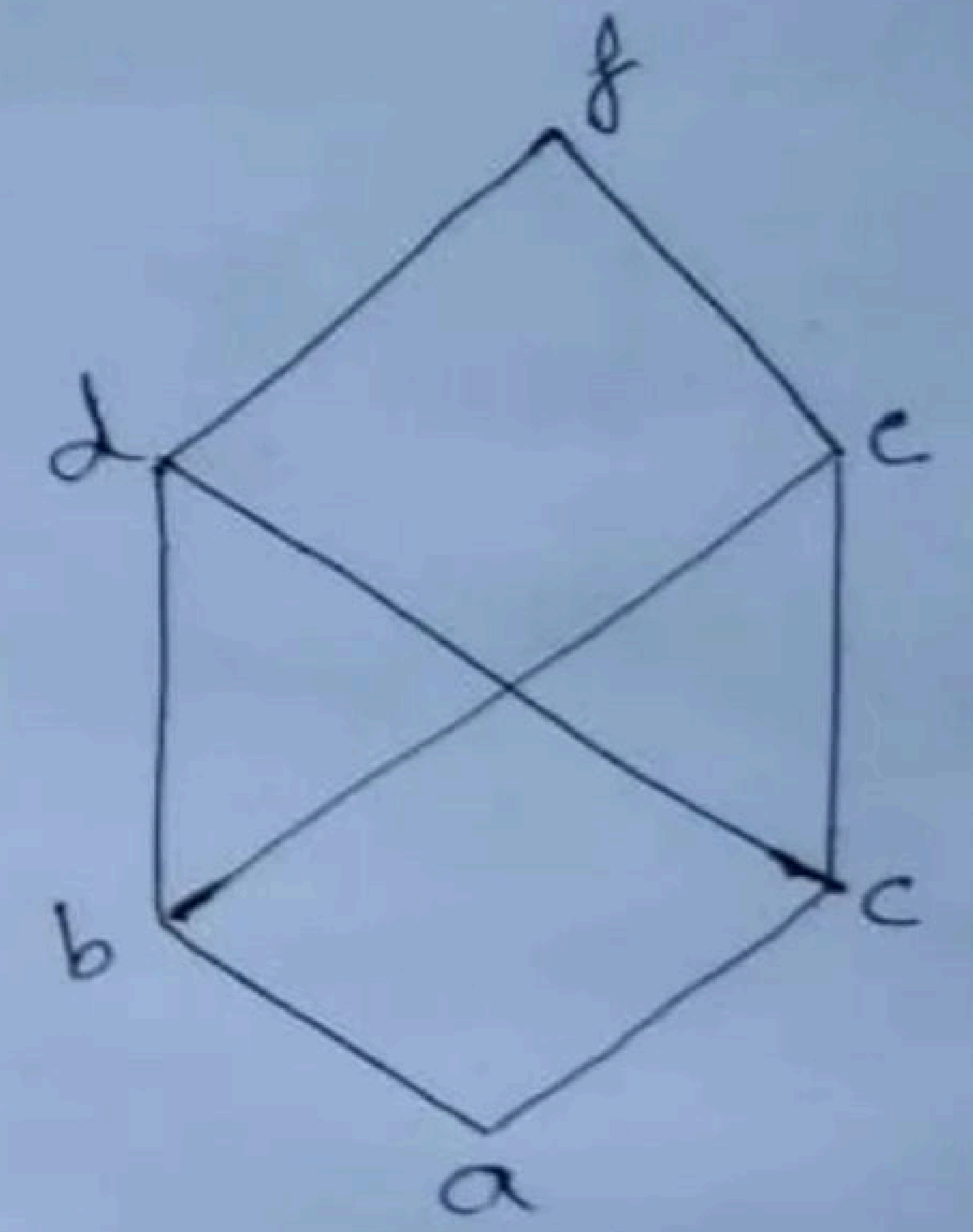
- **Lower Bound: -** lower bound of a subset B with respect to A, will contain all those elements which are related to every element of B.



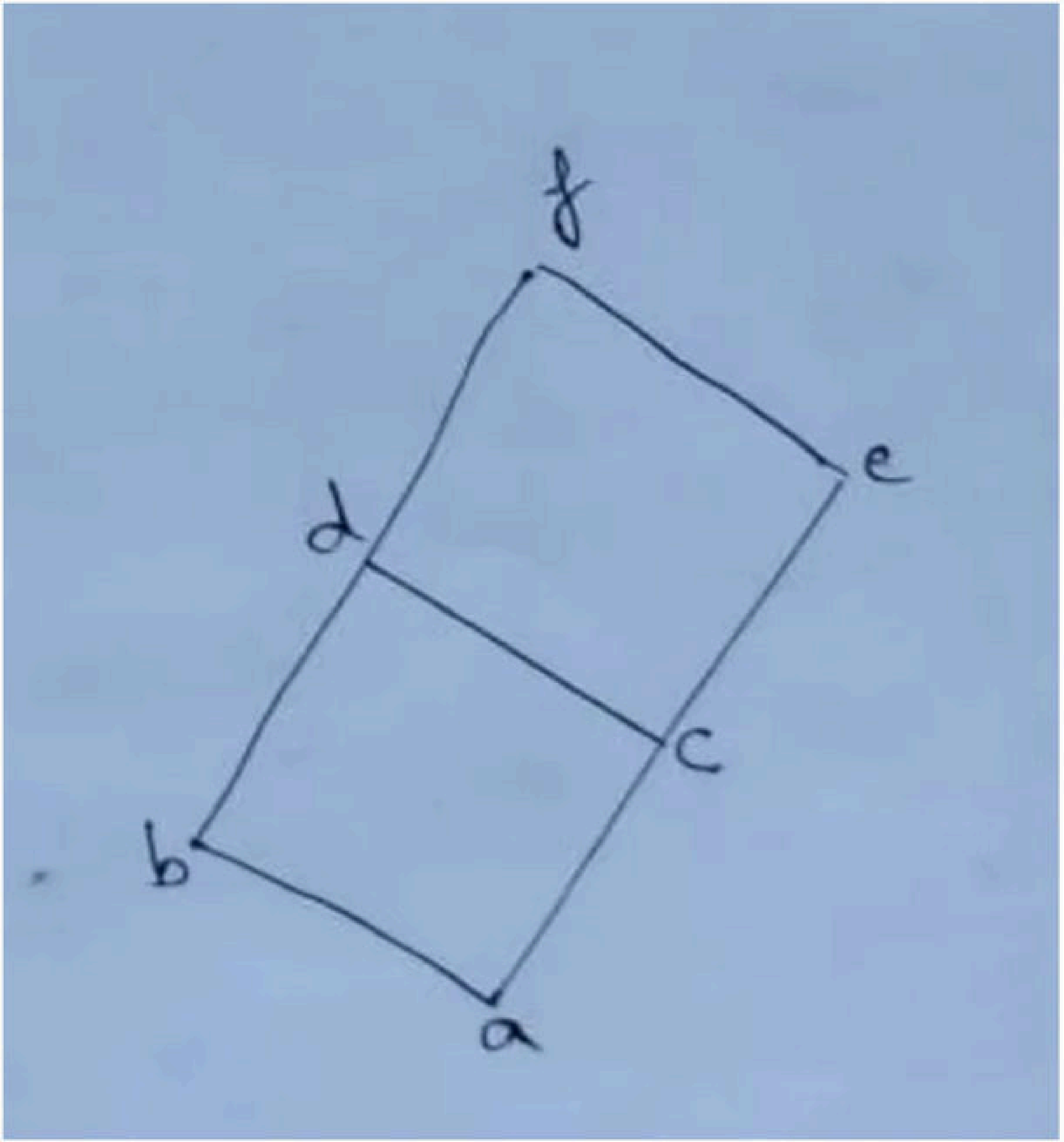
Elements	B = {g, f, e, c}	B = {d, f}
Upper Bound		
Lower Bound		



Elements	B = {d, e}	B = {c}
Upper Bound		
Lower Bound		



Elements	B = {d, e}	B = {b, c}
Upper Bound		
Lower Bound		



Elements	B = {d, c}	B = {b, e}
Upper Bound		
Lower Bound		

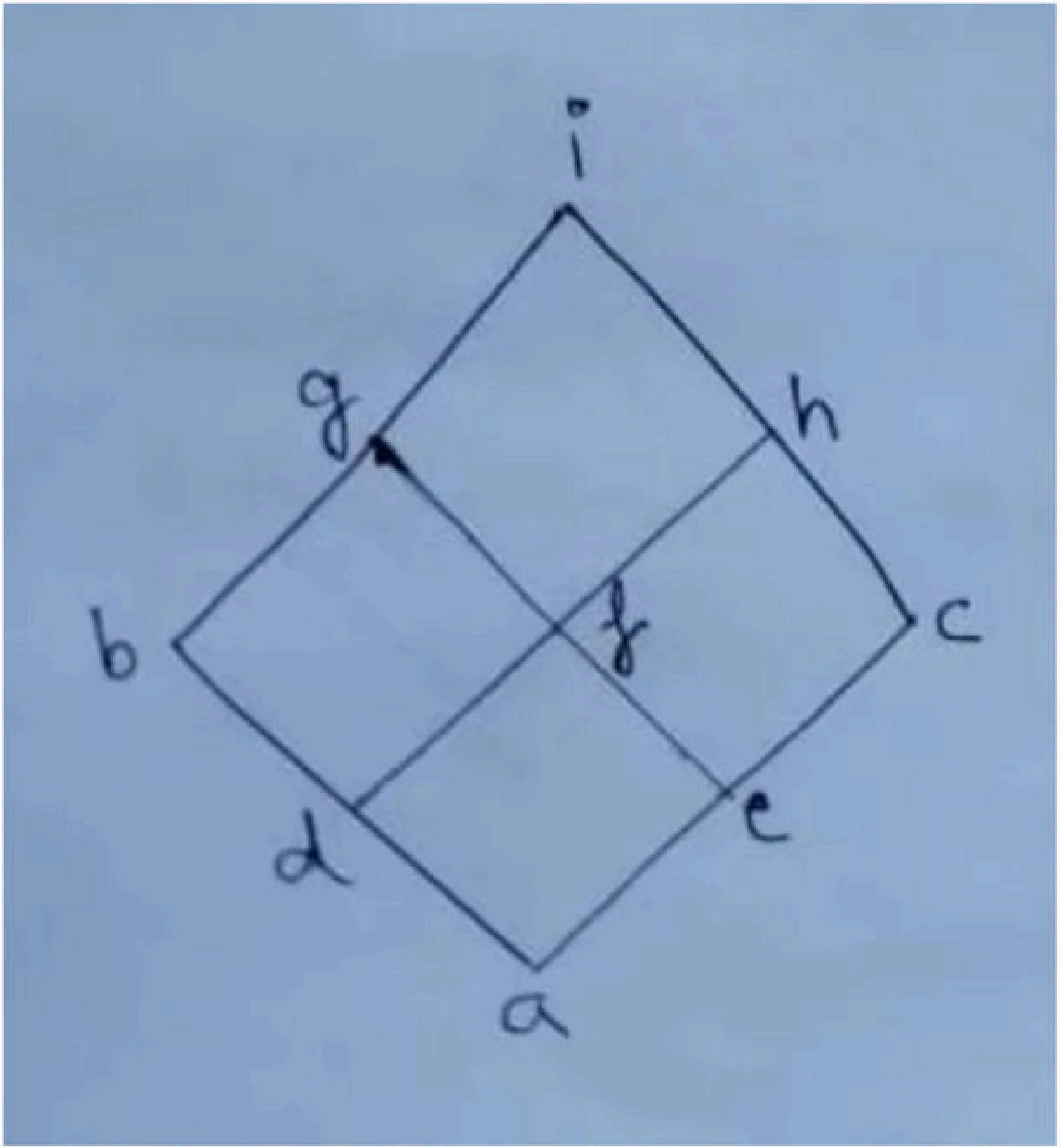
Break

Least Upper Bound / LUB / Join / Supremum / $\vee$

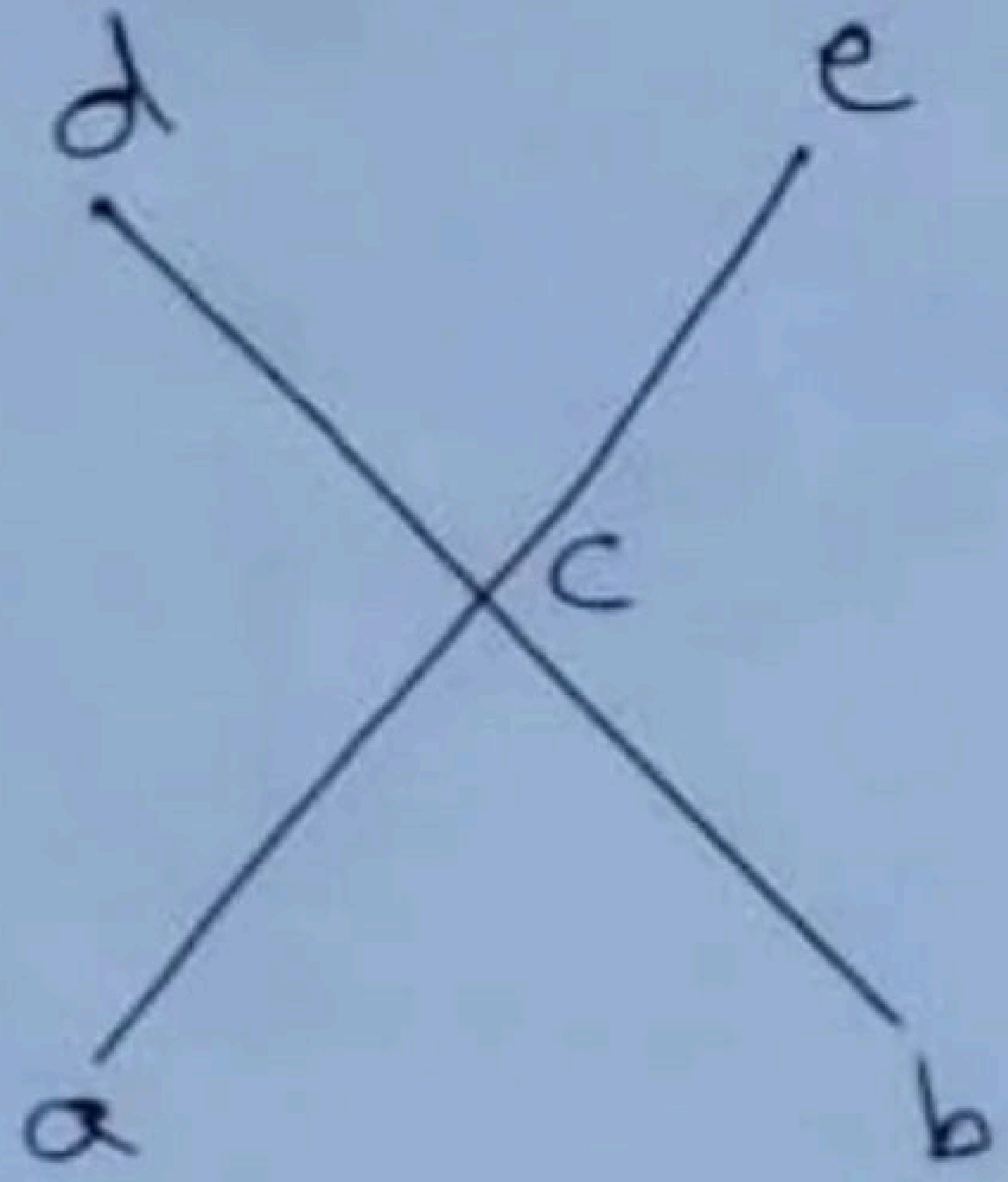
Least value in the upper bound

Greatest Lower Bound / GLB / Meet / Infimum / $\wedge$

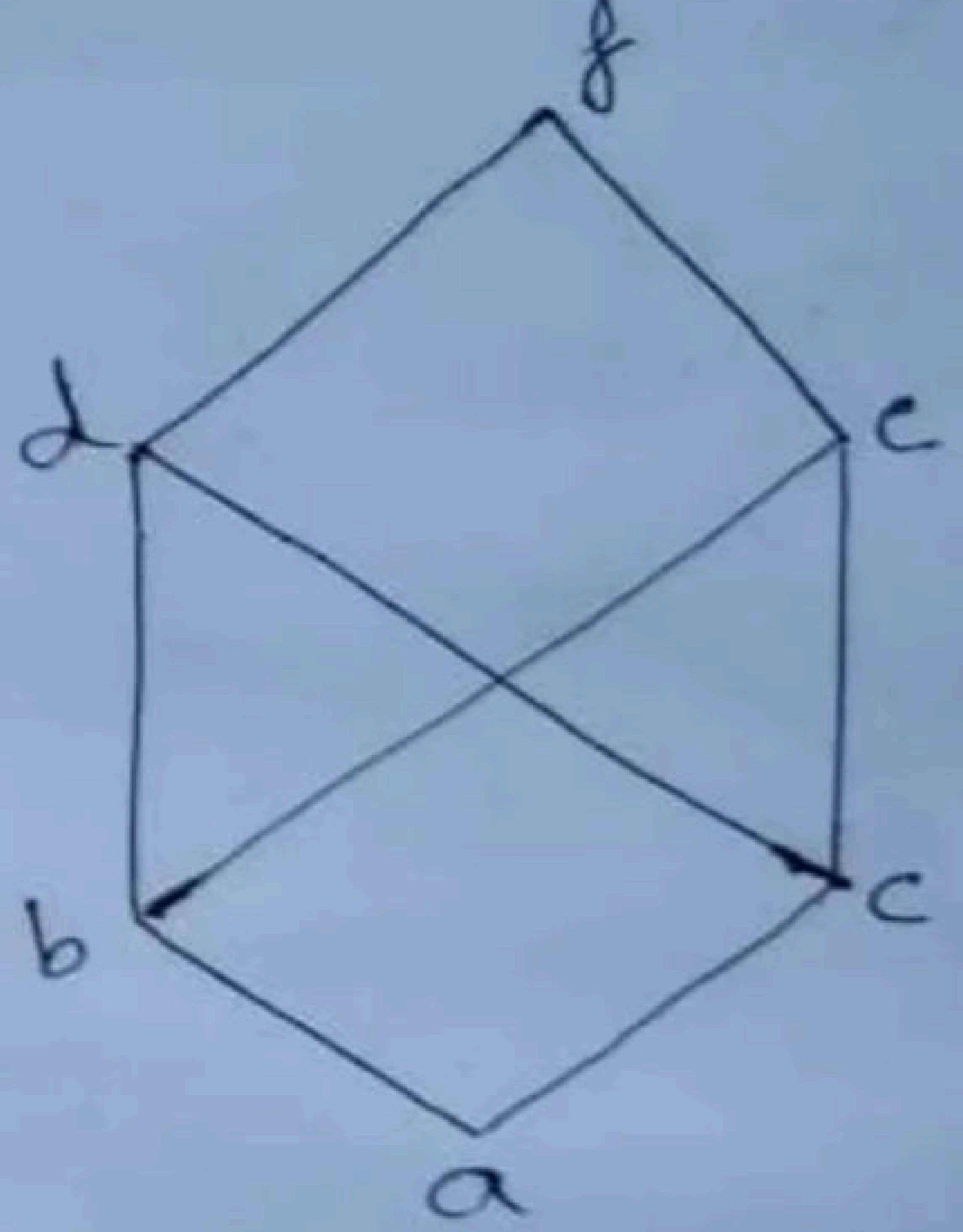
Greatest value in the lower bound



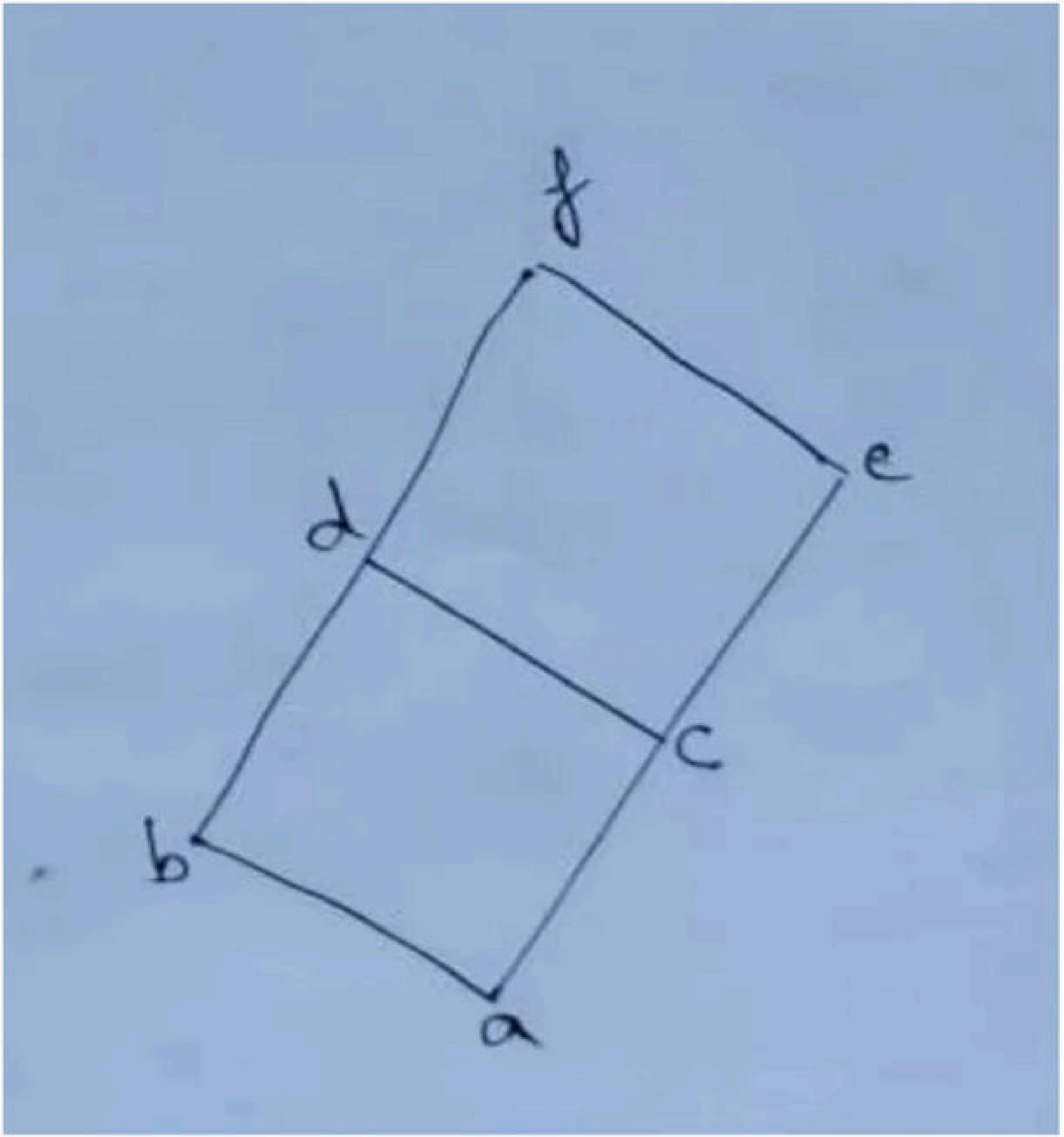
Elements	B = {g, f, e, c}	B = {d, f}
Upper Bound	{i}	{i, g, h, f}
Least Upper Bound		
Lower Bound	{a, e}	{a, d}
Greatest Lower Bound		



Elements	B = {d, e}	B = {c}
Upper Bound	{}	{d, e, c}
Least Upper Bound		
Lower Bound	{a, b, c}	{a, b, c}
Greatest Lower Bound		



Elements	B = {d, e}	B = {b, c}
Upper Bound	{f}	{d, e, f}
Least Upper Bound		
Lower Bound	{a, b, c}	{a}
Greatest Lower Bound		



Elements	B = {d, c}	B = {b, e}
Upper Bound	{f, d}	{f}
Least Upper Bound		
Lower Bound	{a, c}	{a}
Greatest Lower Bound		

Q Consider the Poset ({3,5,9,15,24,45}, /). Which of the following is correct for the given Poset? **(NET-JUNE-2019)**

a) There exist a greatest element and a least element

b) There exist a greatest element but not a least element

c) There exist a least element but not a greatest element

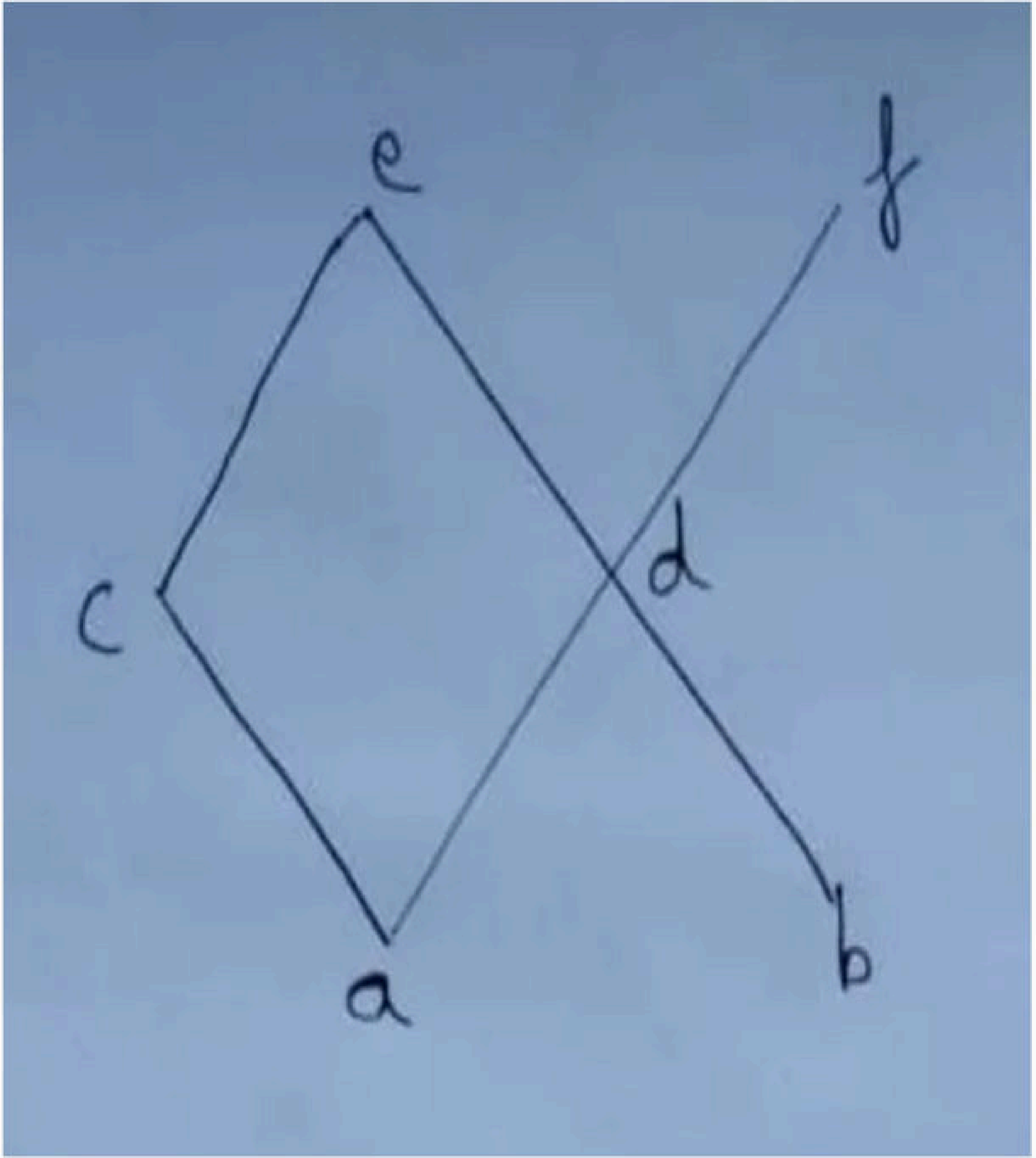
d) There does not exist a greatest element and a least element

Break

Join Semi Lattice :- A hasse diagram/Partial order relation is called Join Semi Lattice if for every elements their exists a Join.

<u>Meet Semi Lattice</u> :- A hasse diagram/Partial order relation is called Meet Semi Lattice if for every elements their exists a Meet.

Lattice :- A hasse diagram/Partial order relation is called Lattice if their exist a Join and Meet for every pair of element. Or A hasse diagram/Partial order relation is called Lattice if it is both Join Semi Lattice and Meet Semi Lattice.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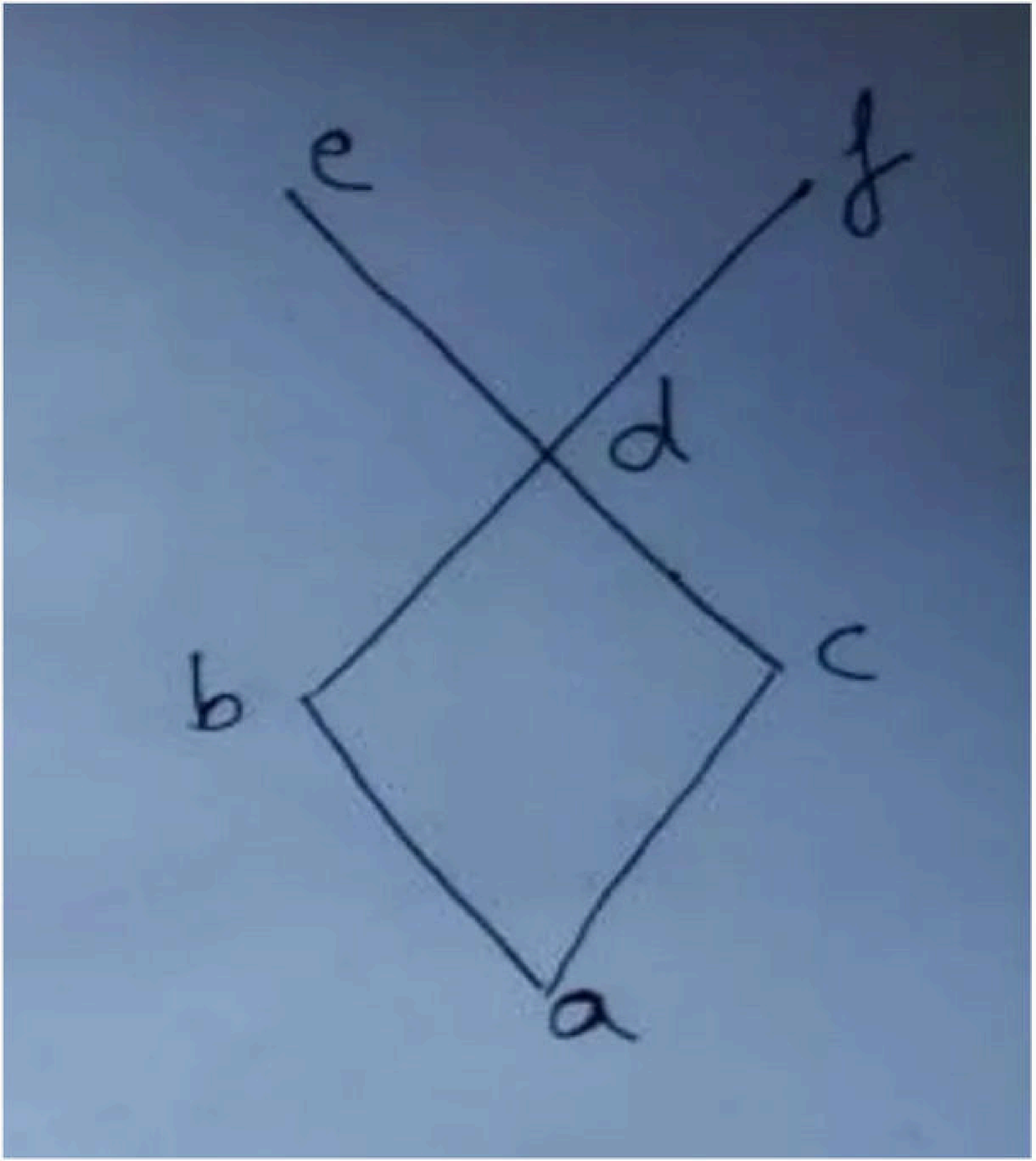
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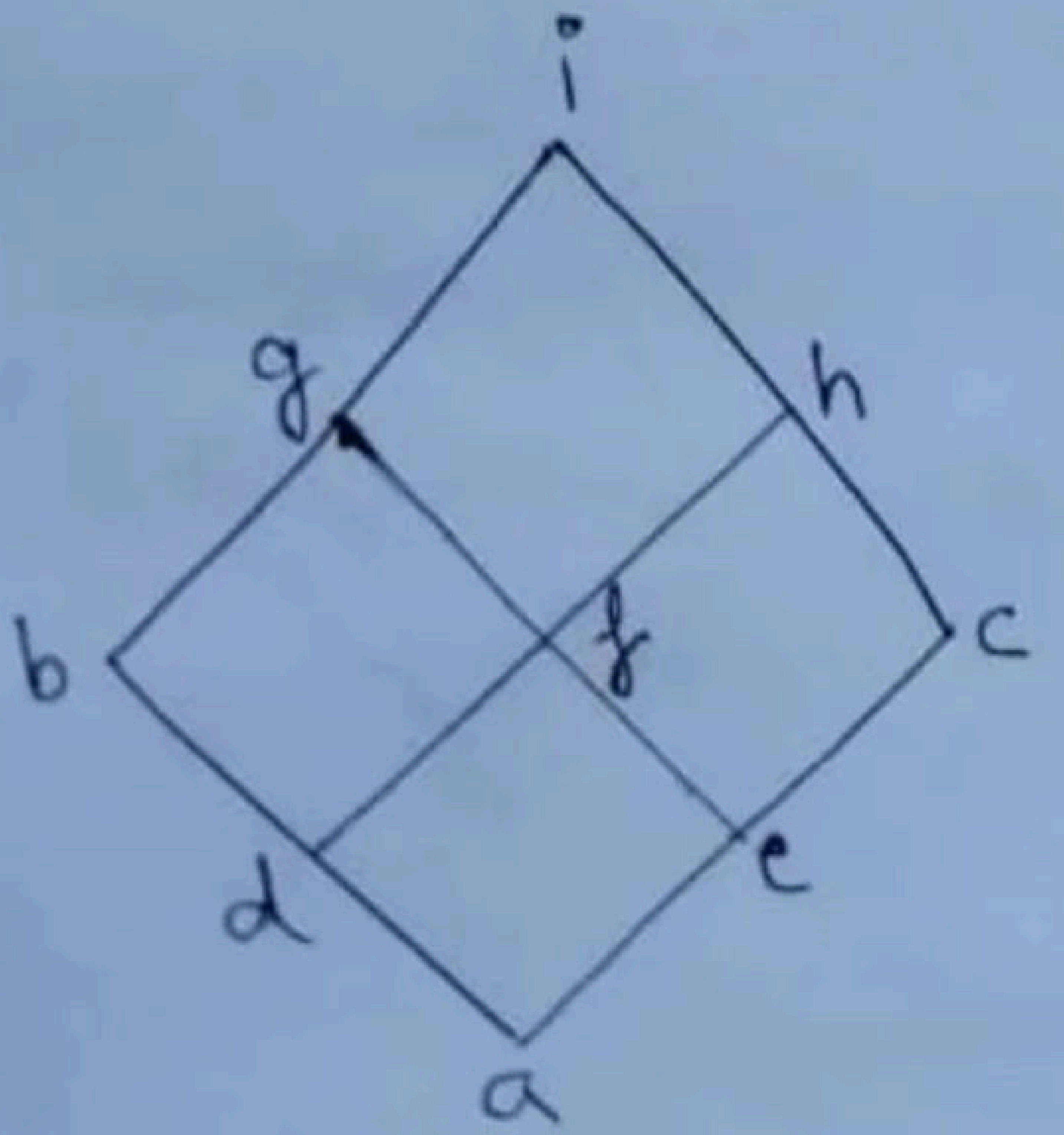
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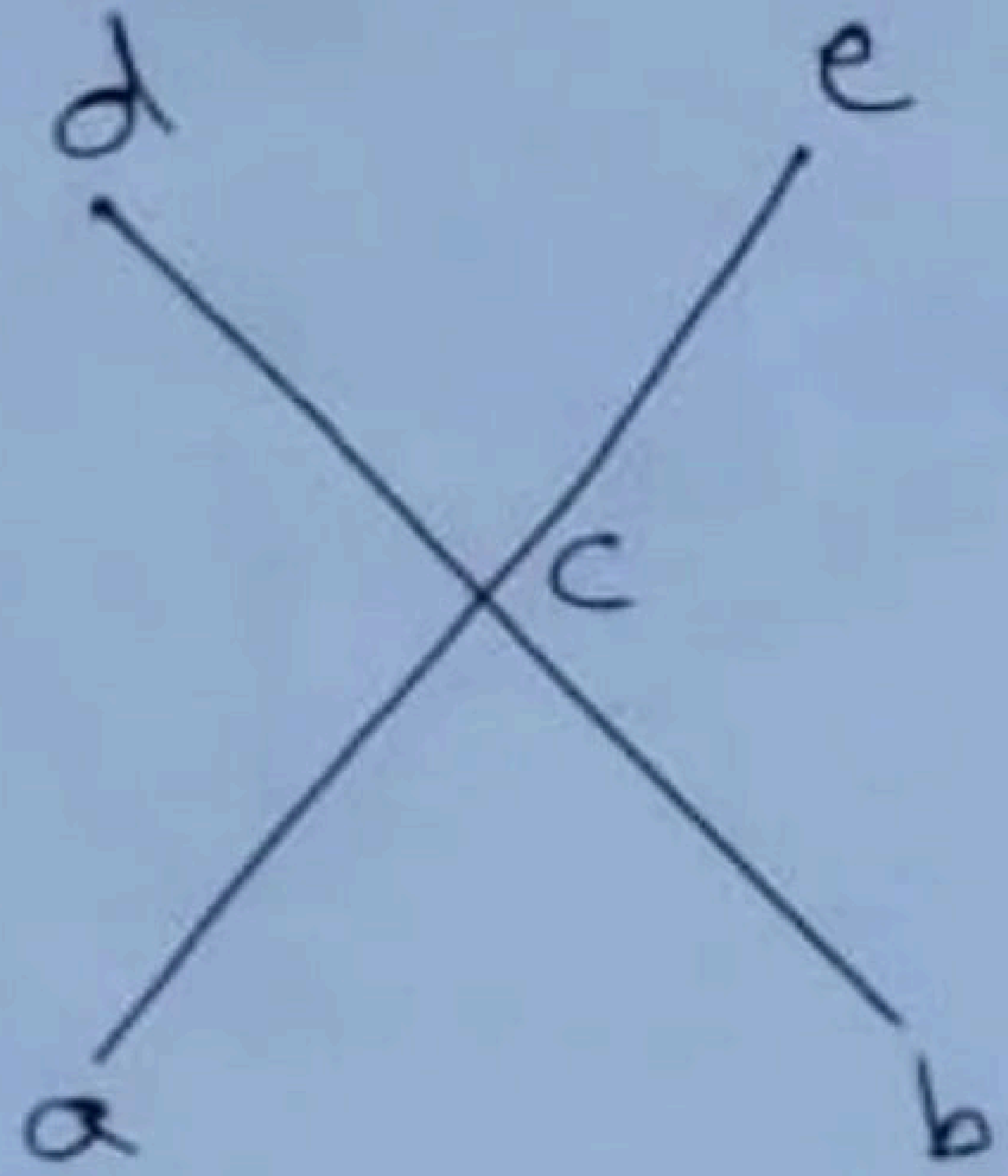
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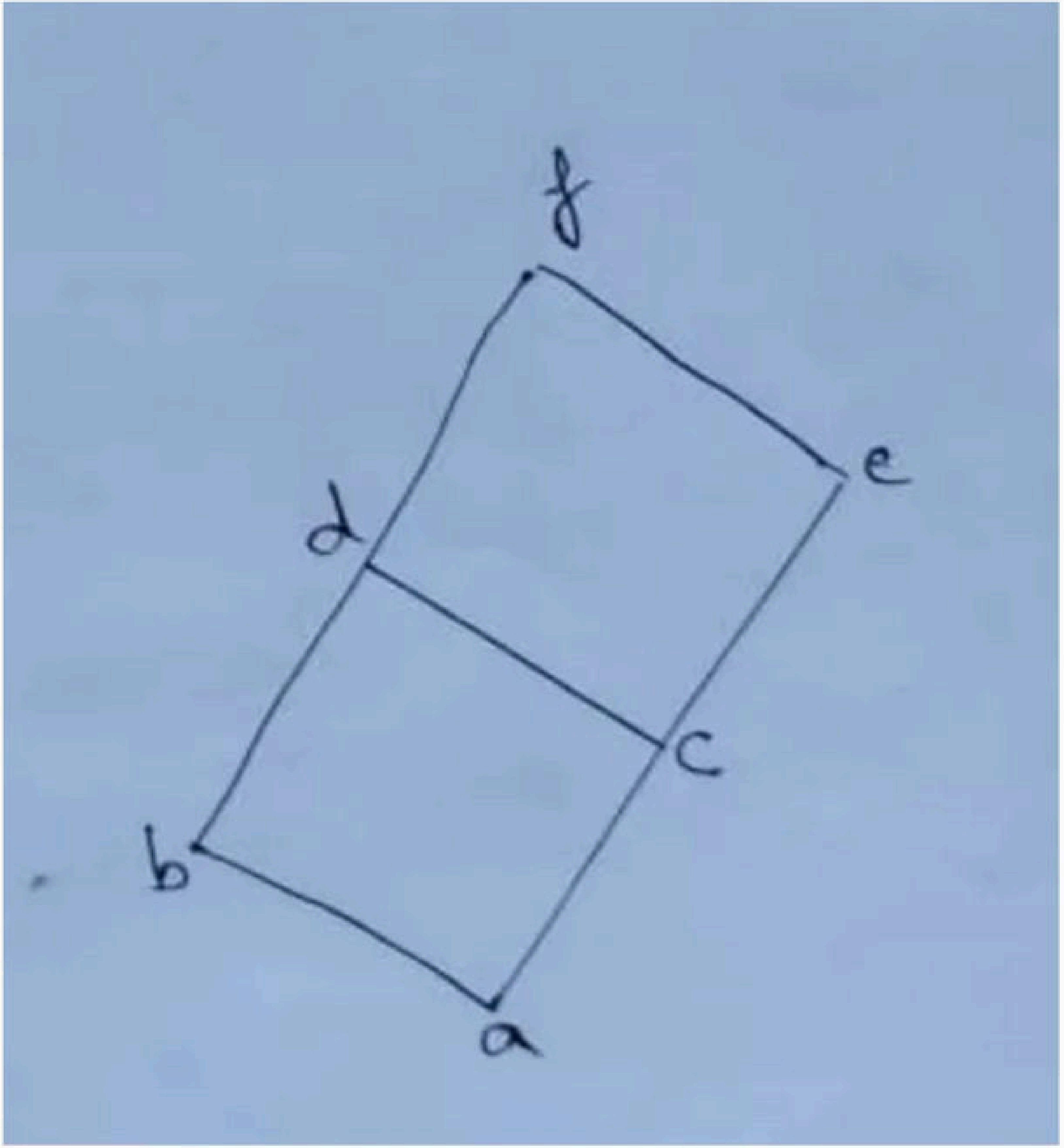
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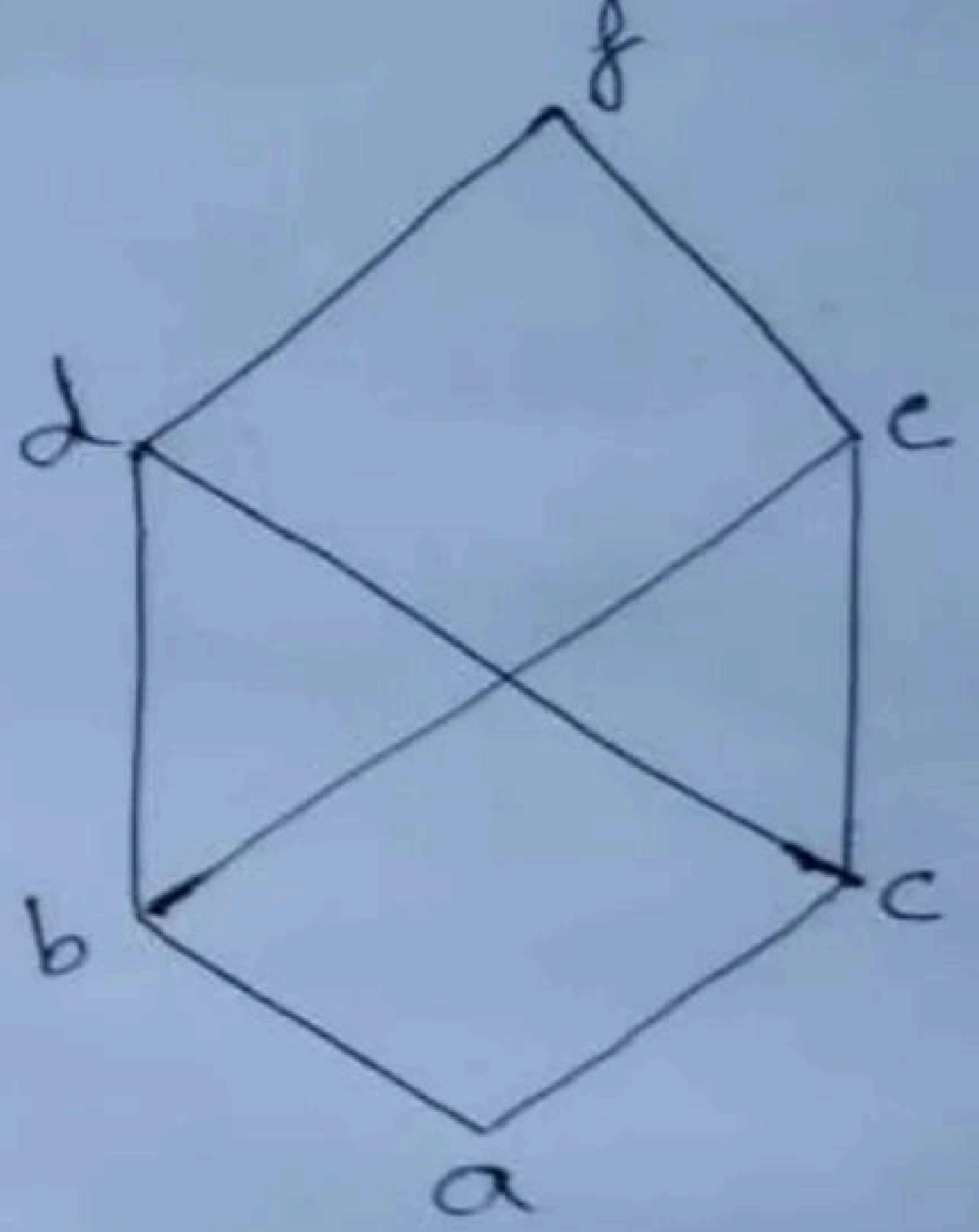

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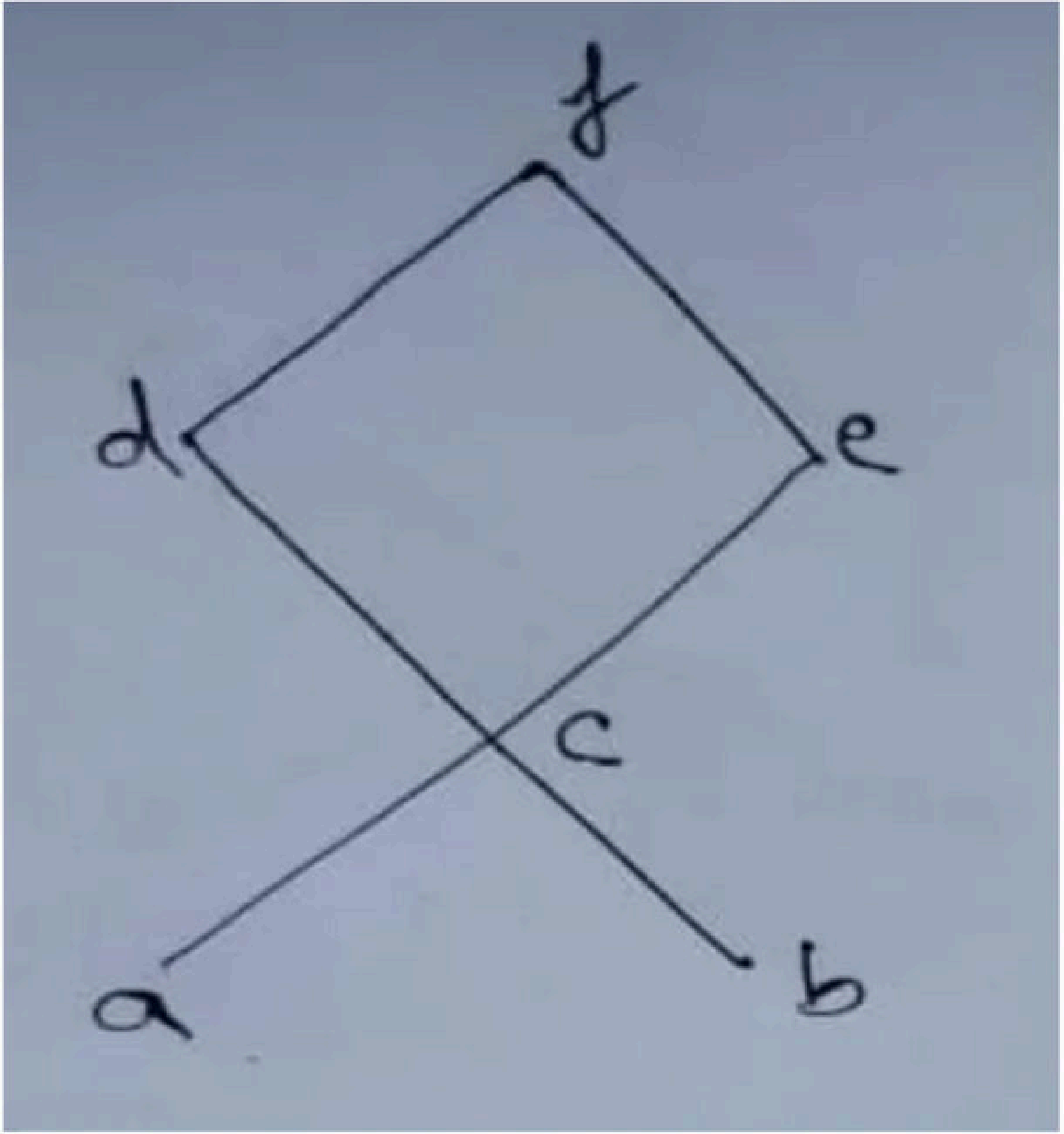

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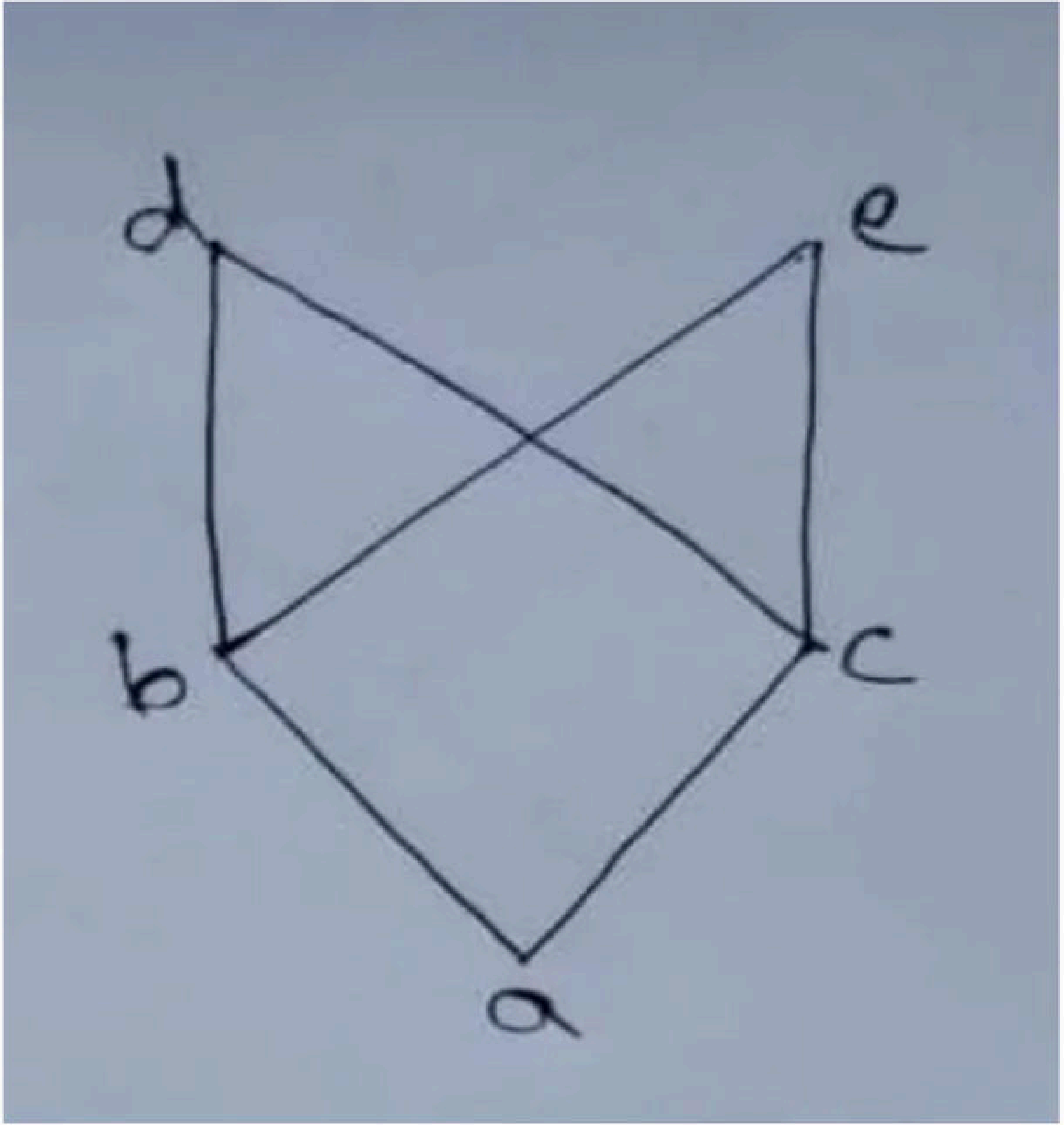
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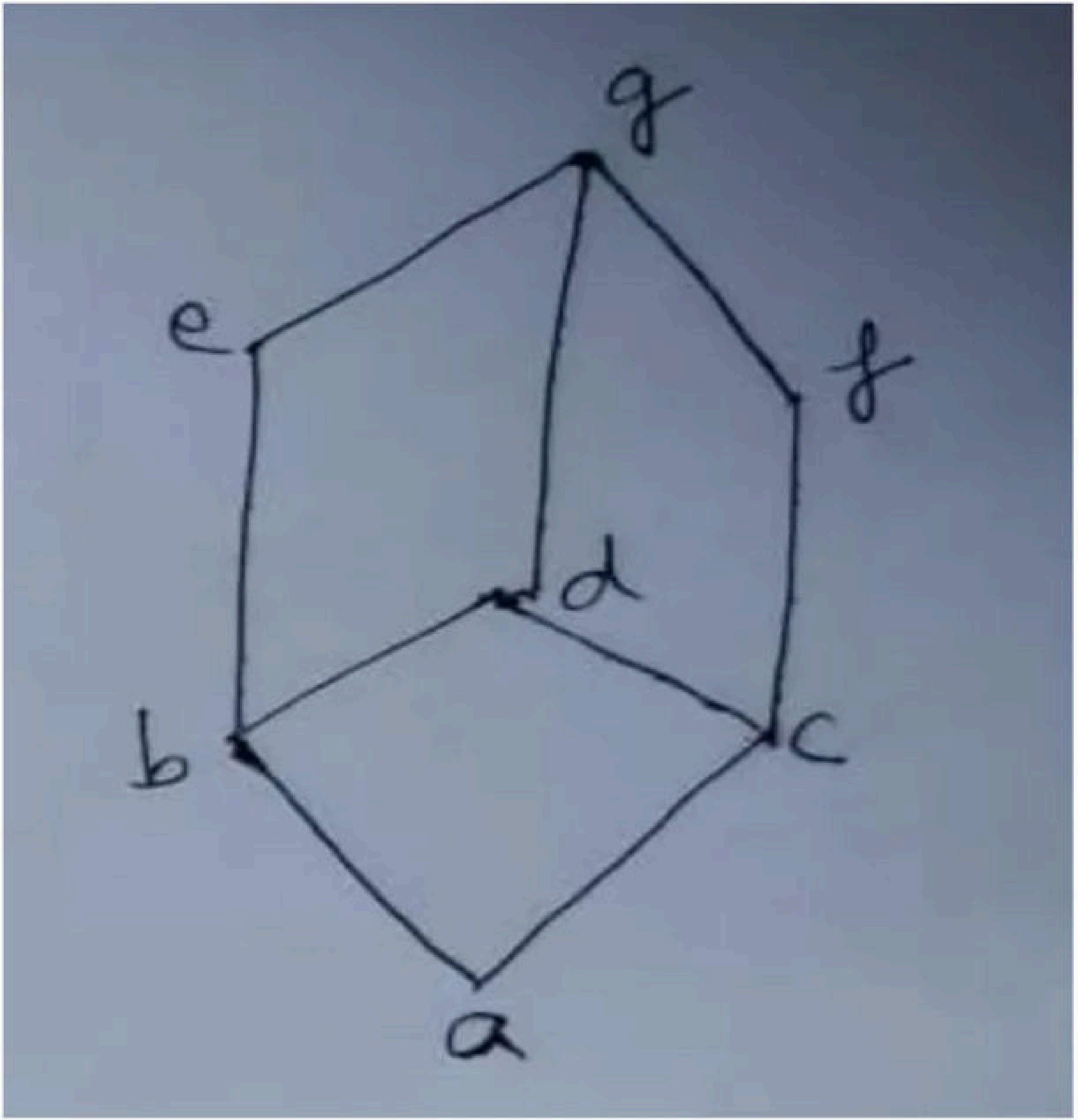

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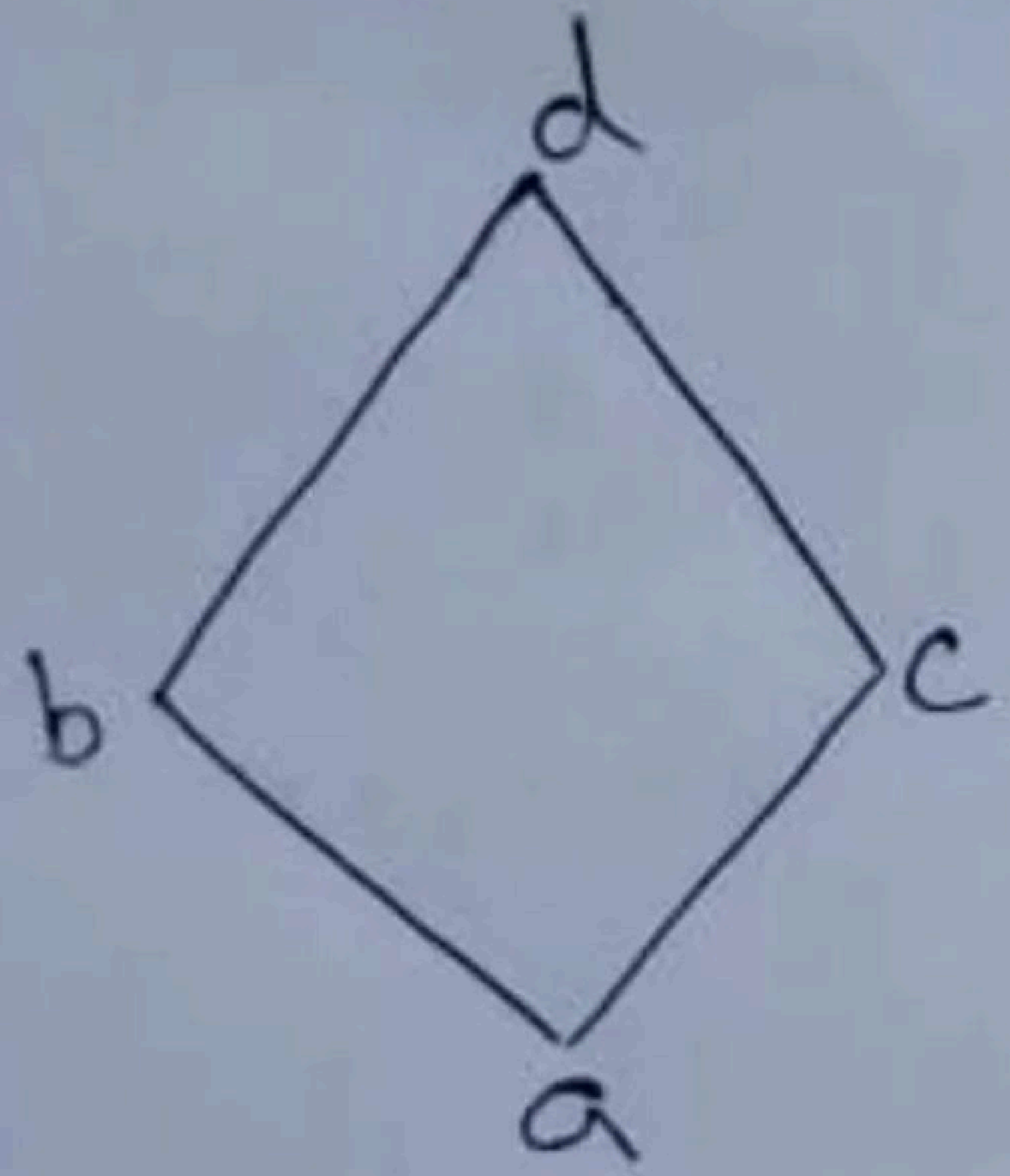

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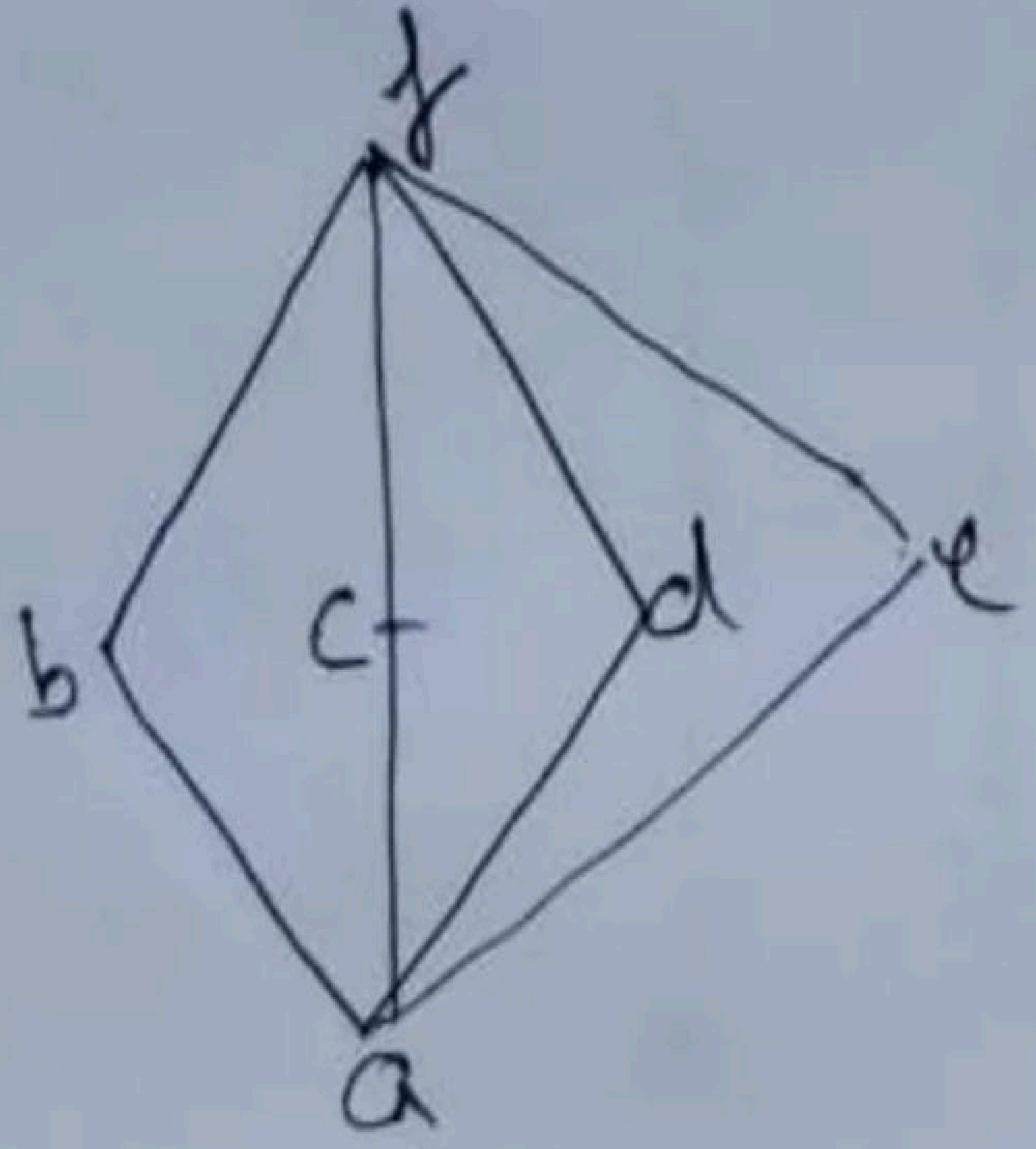
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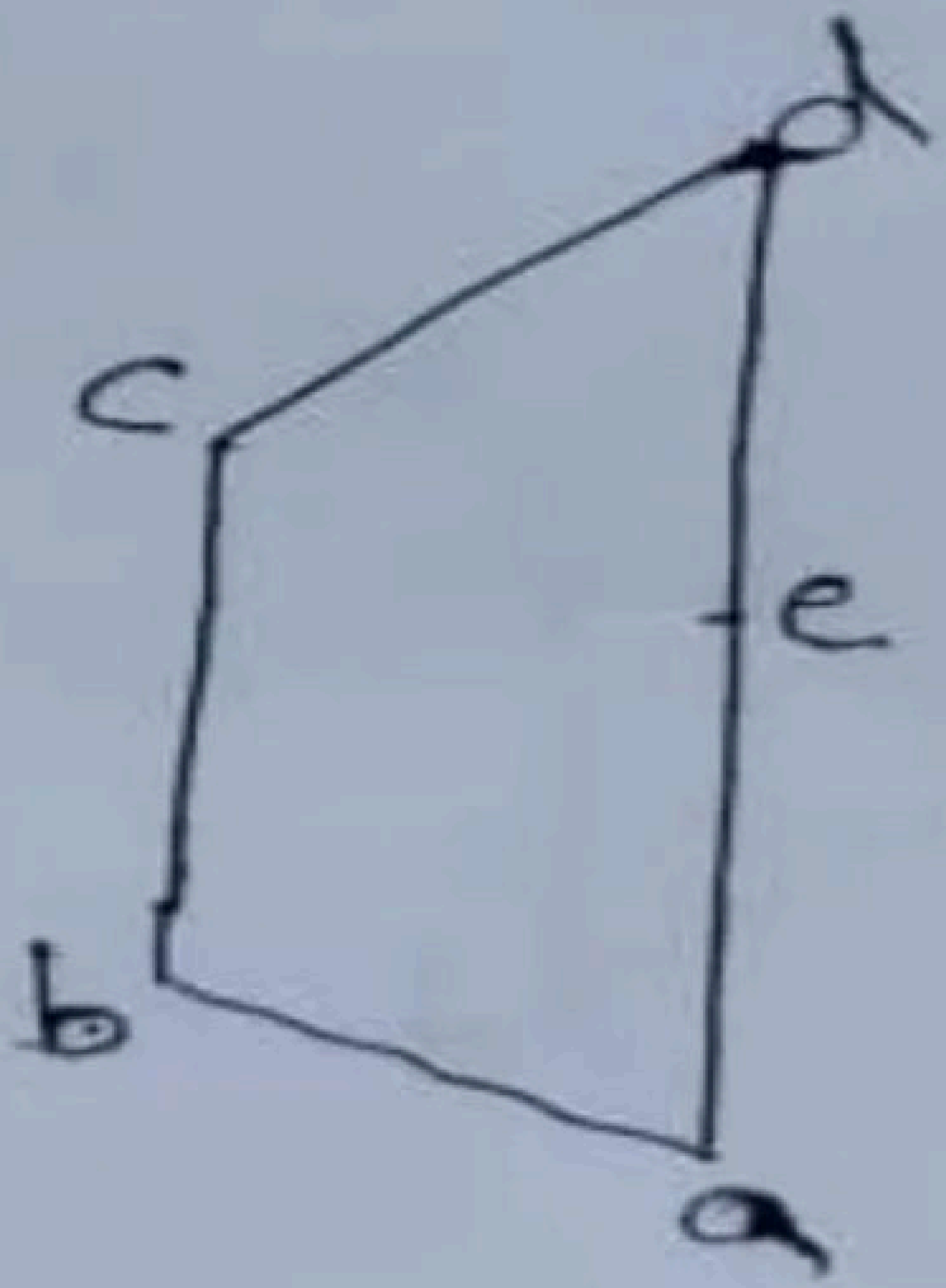
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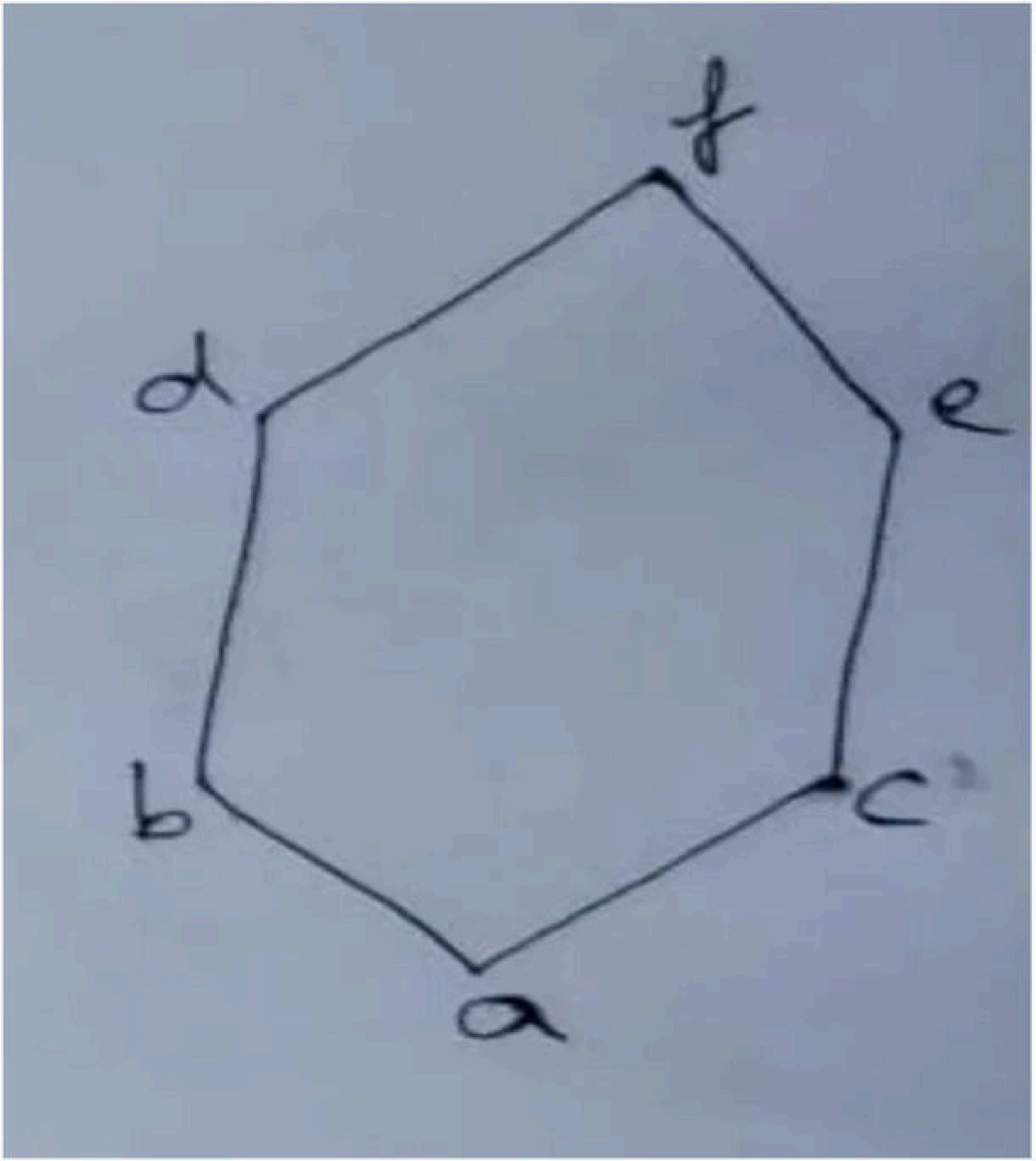
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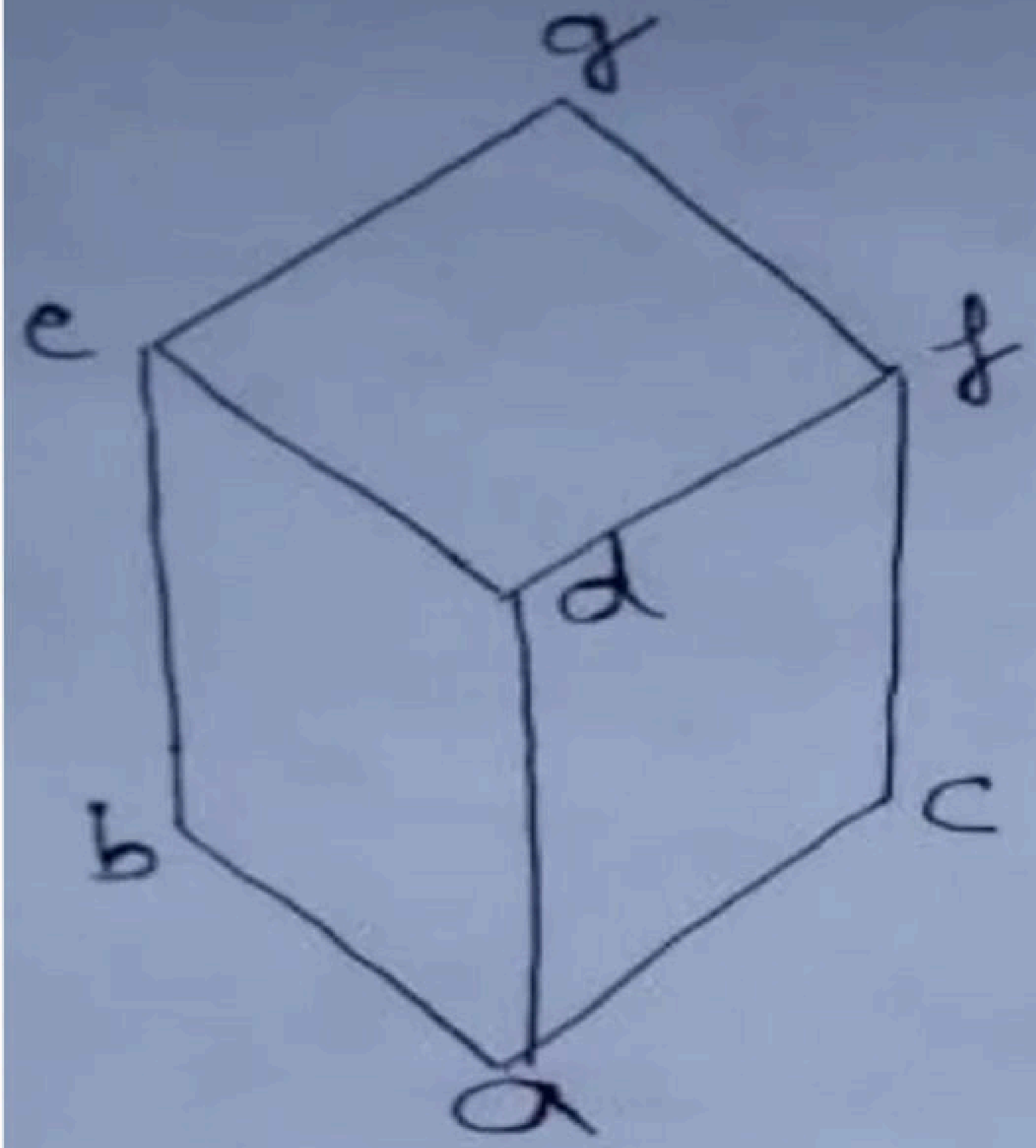
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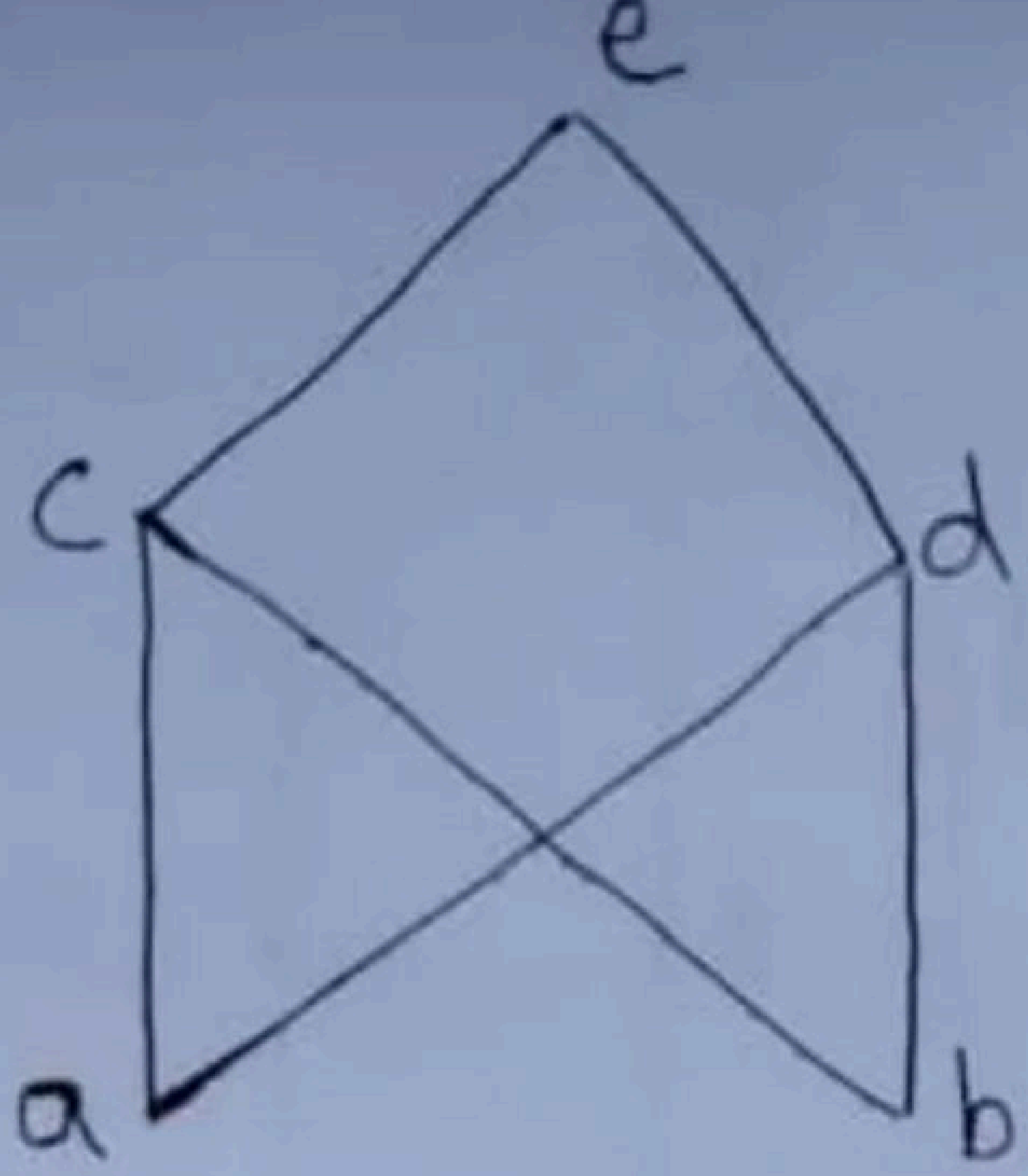
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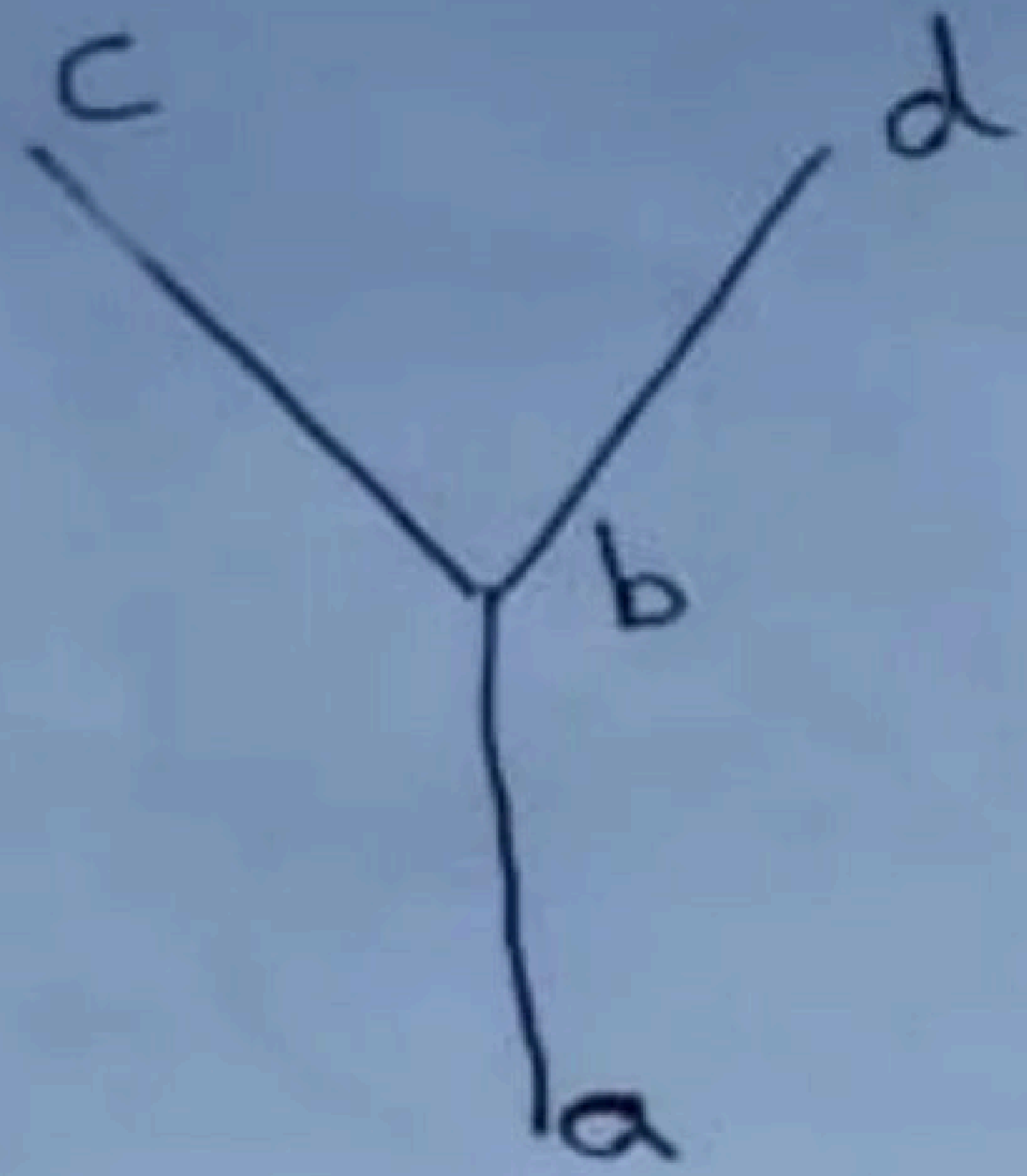
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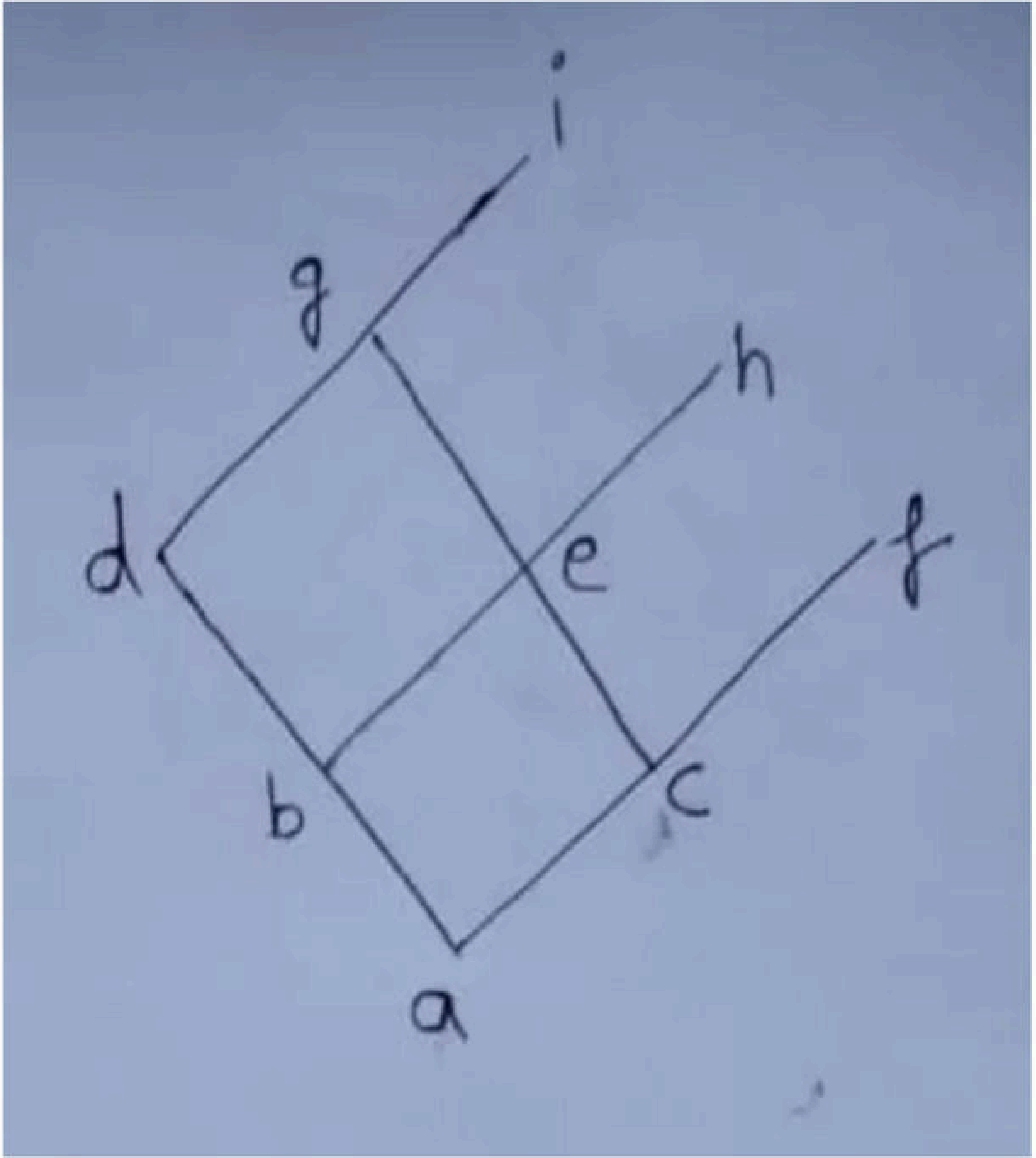

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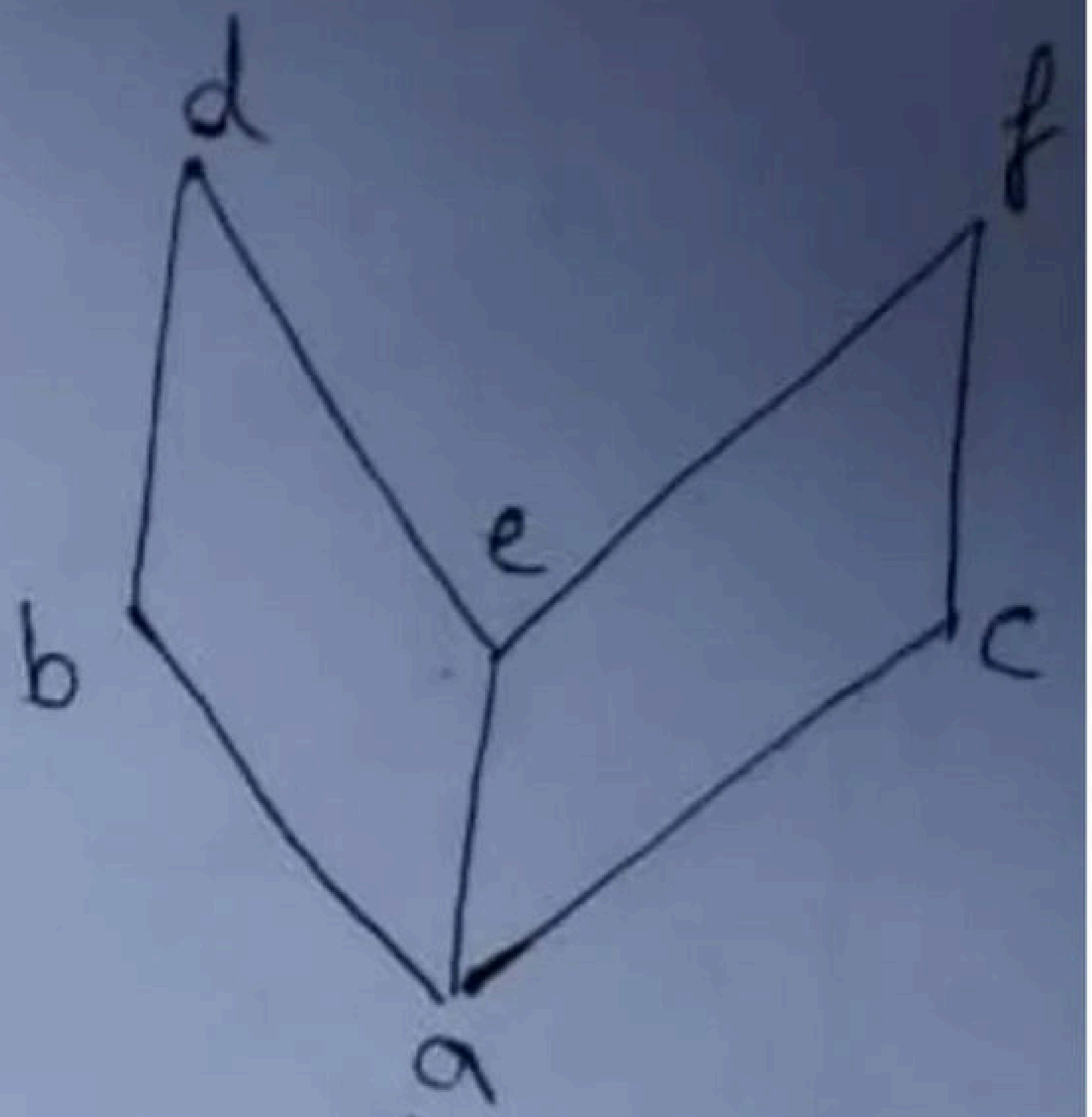
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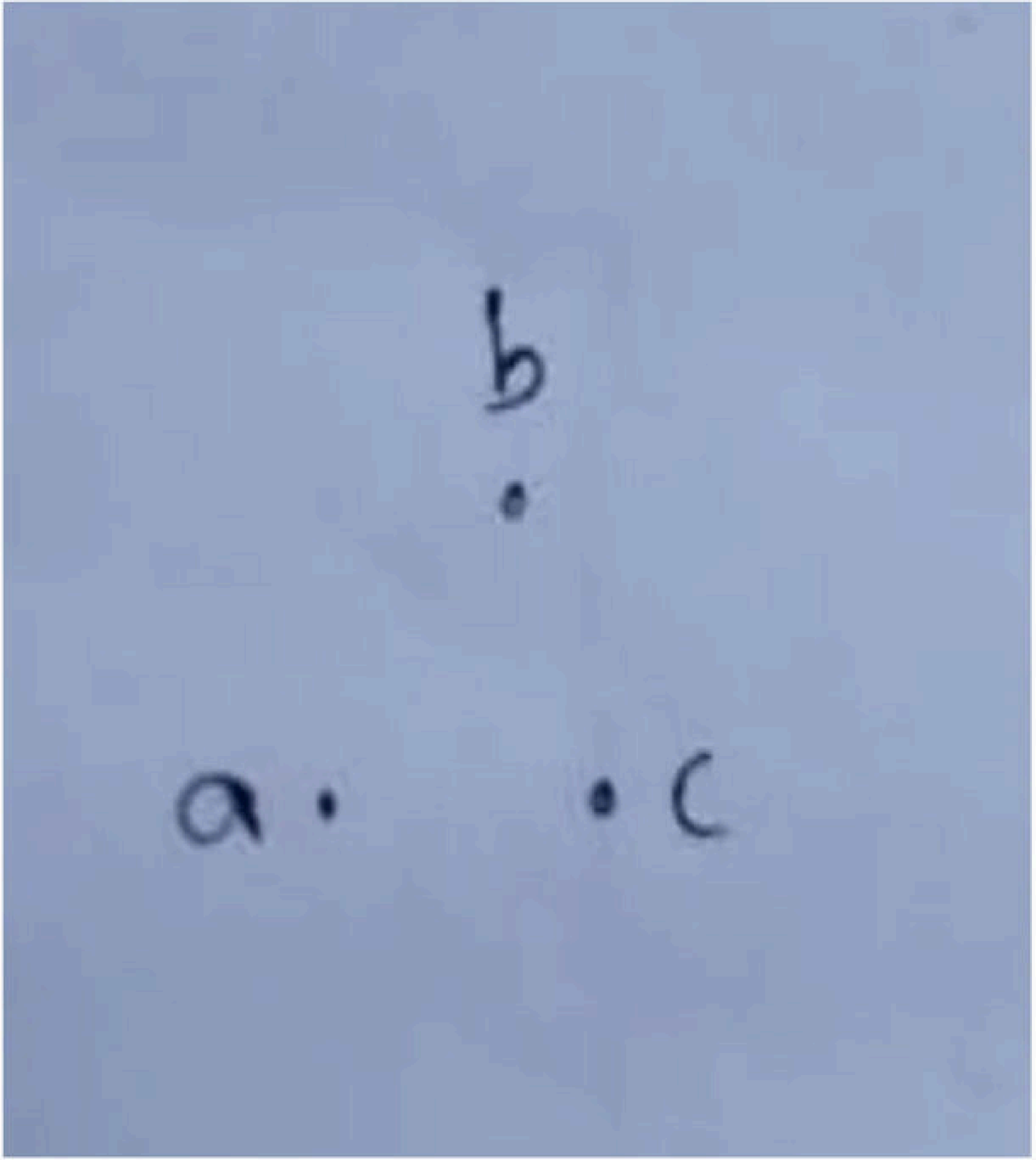
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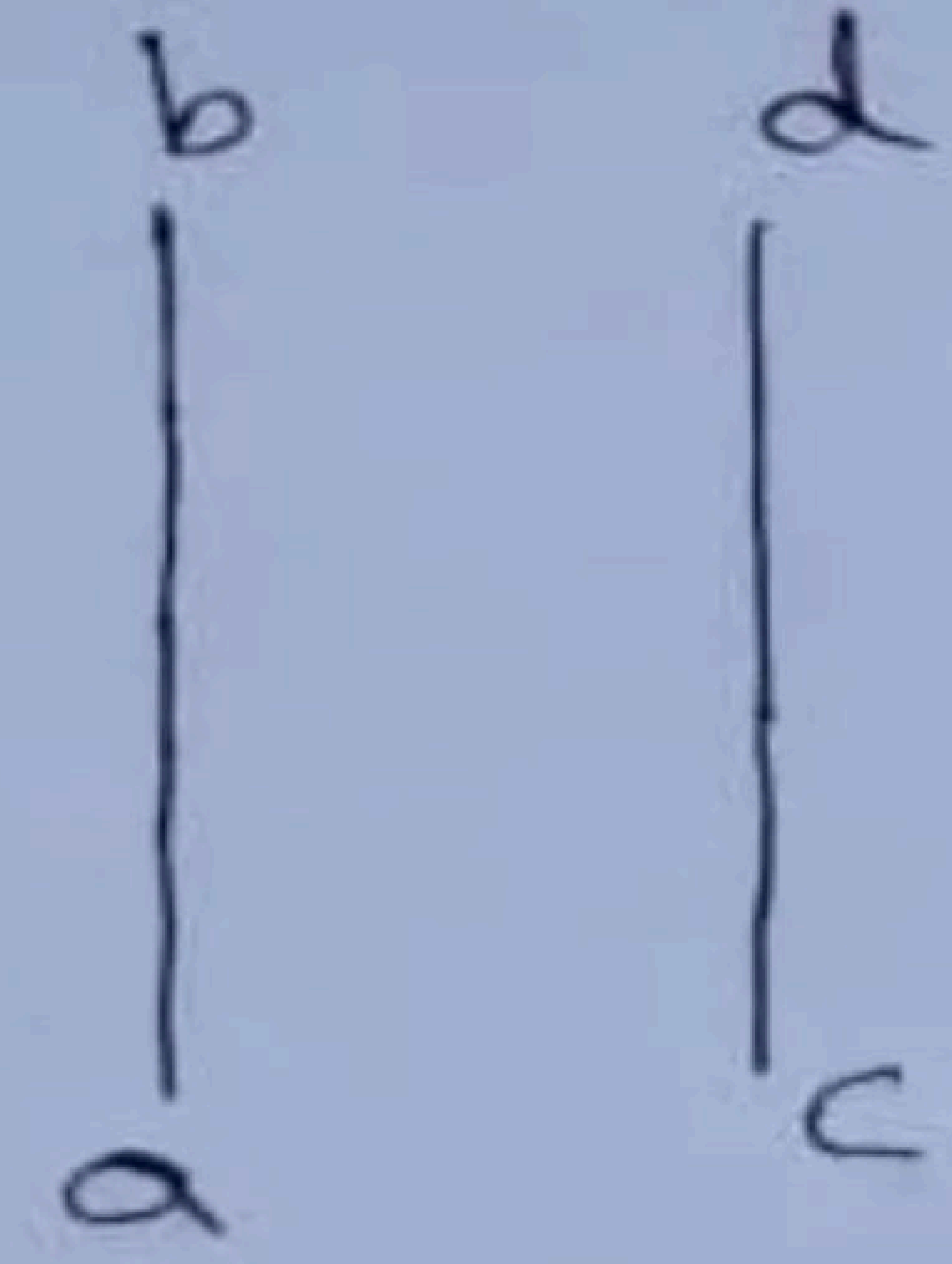
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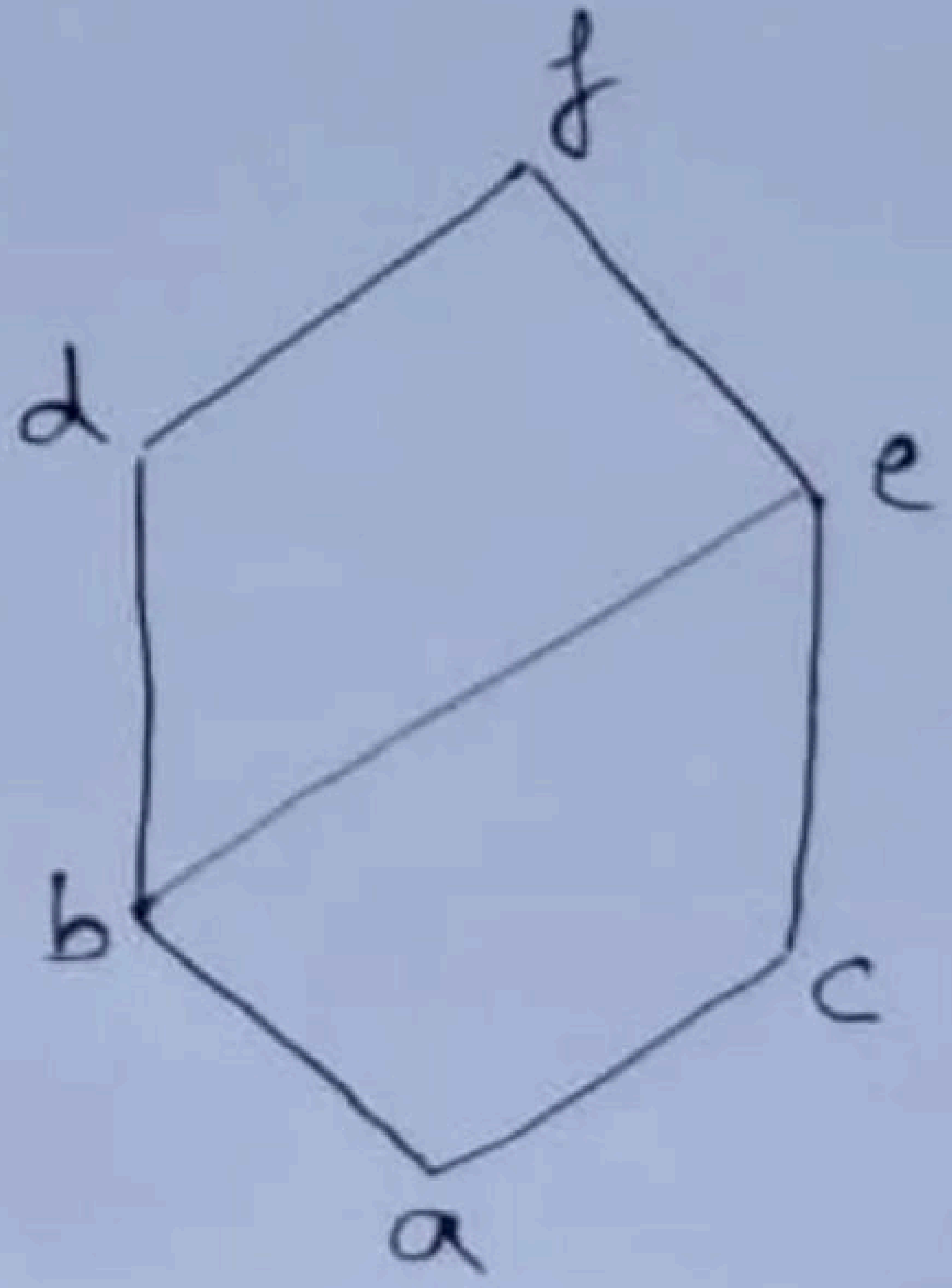
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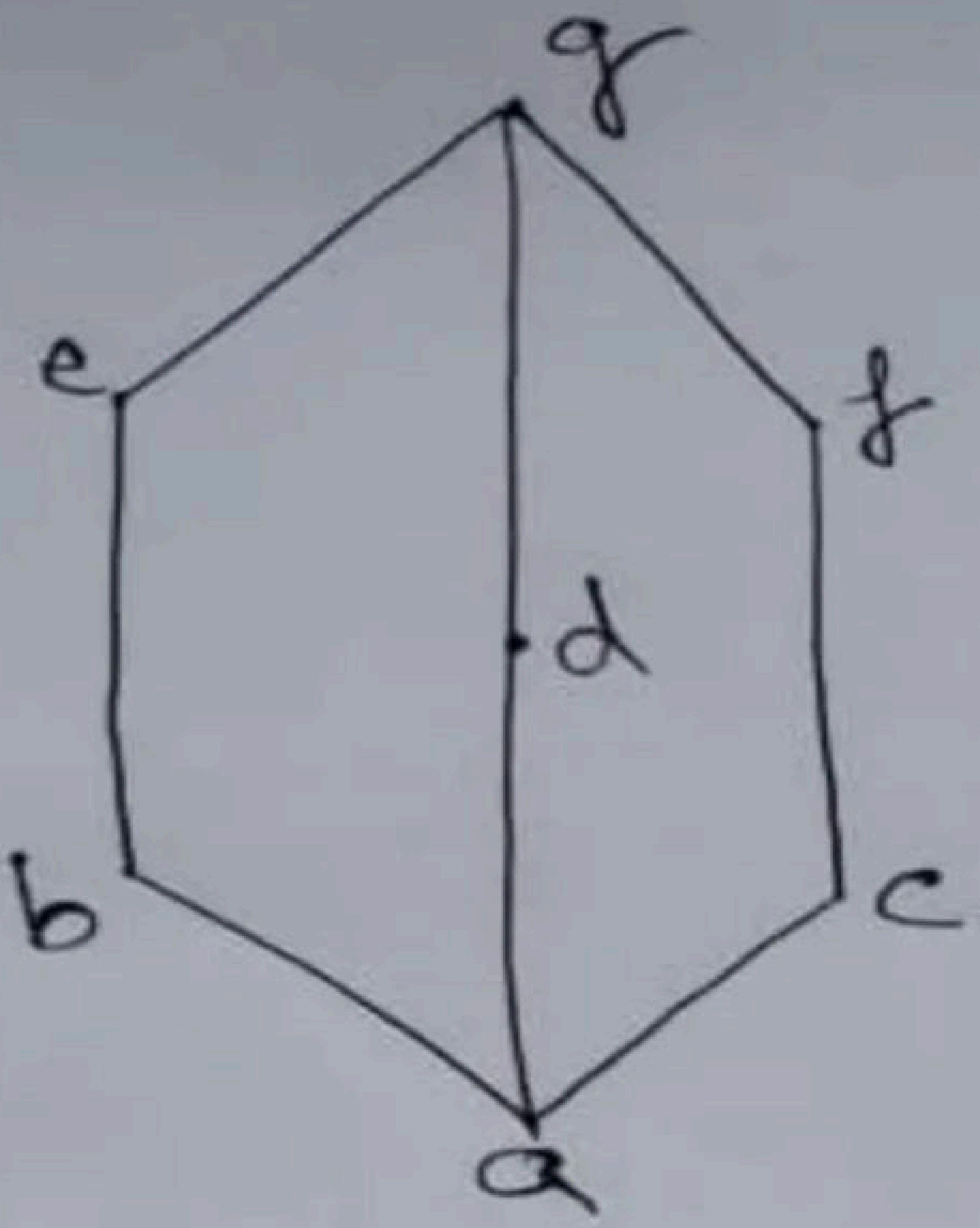
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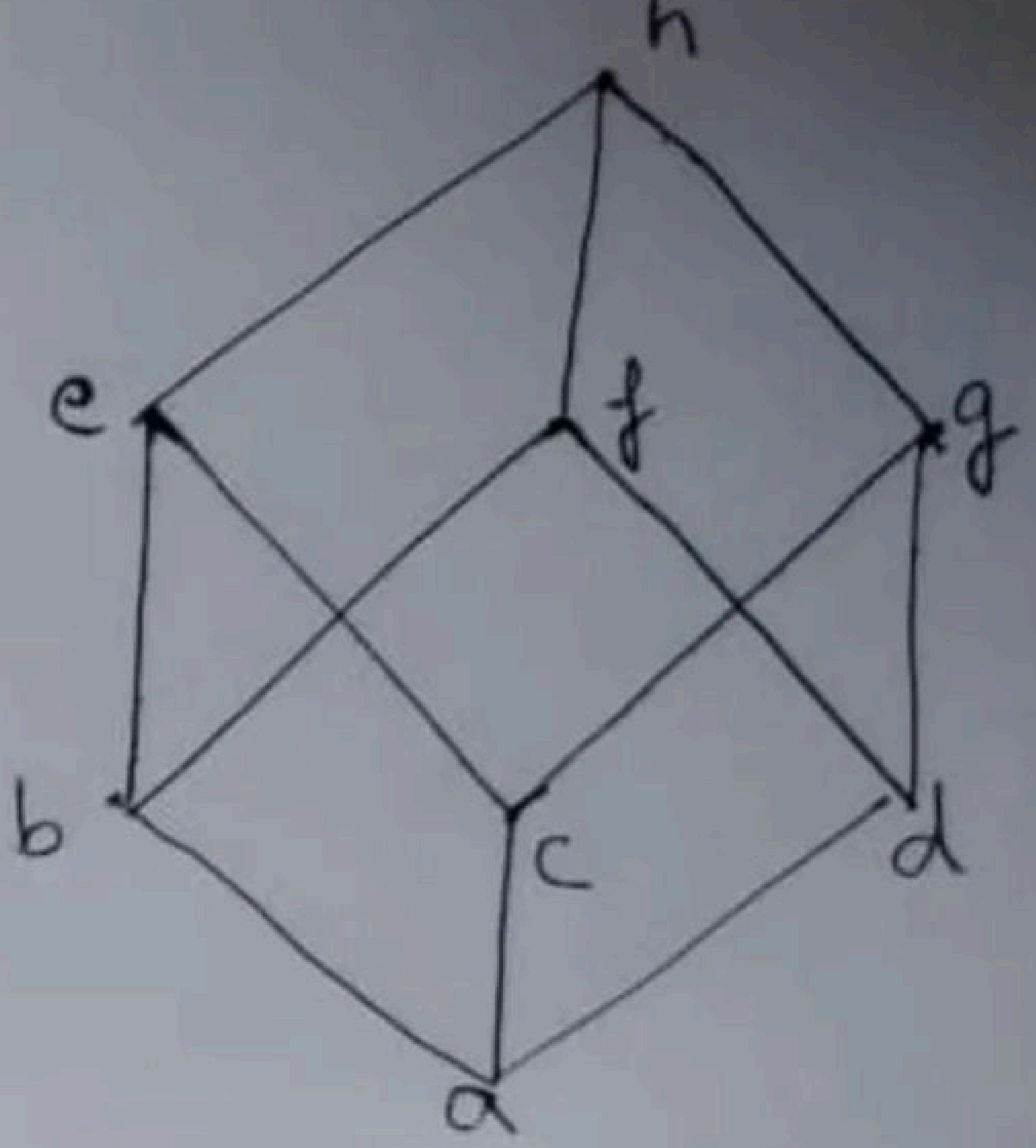
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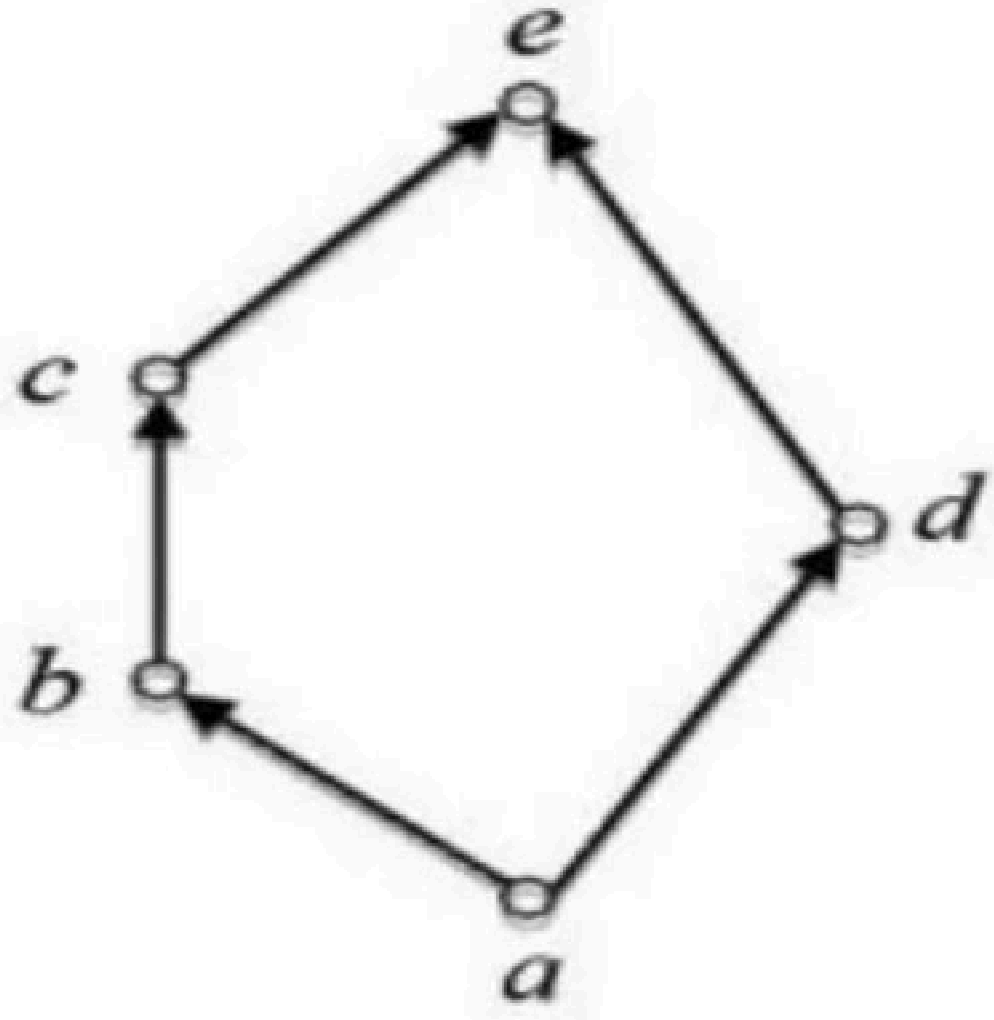
Break

Q Consider the set **X**={a, b, c, d, e} under partial ordering **R**={(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)}. The Hasse diagram of the partial order (X, R) is shown below. The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is ______ **(GATE-2017) (1 Marks)**



Q A partially ordered set is said to be a lattice if every two elements in the set have **(NET-Dec-2010)**

a) a unique least upper bound

b) a unique greatest lower bound

c) both (A) and (B)

d) none of the above

Q Consider the following Hasse diagrams

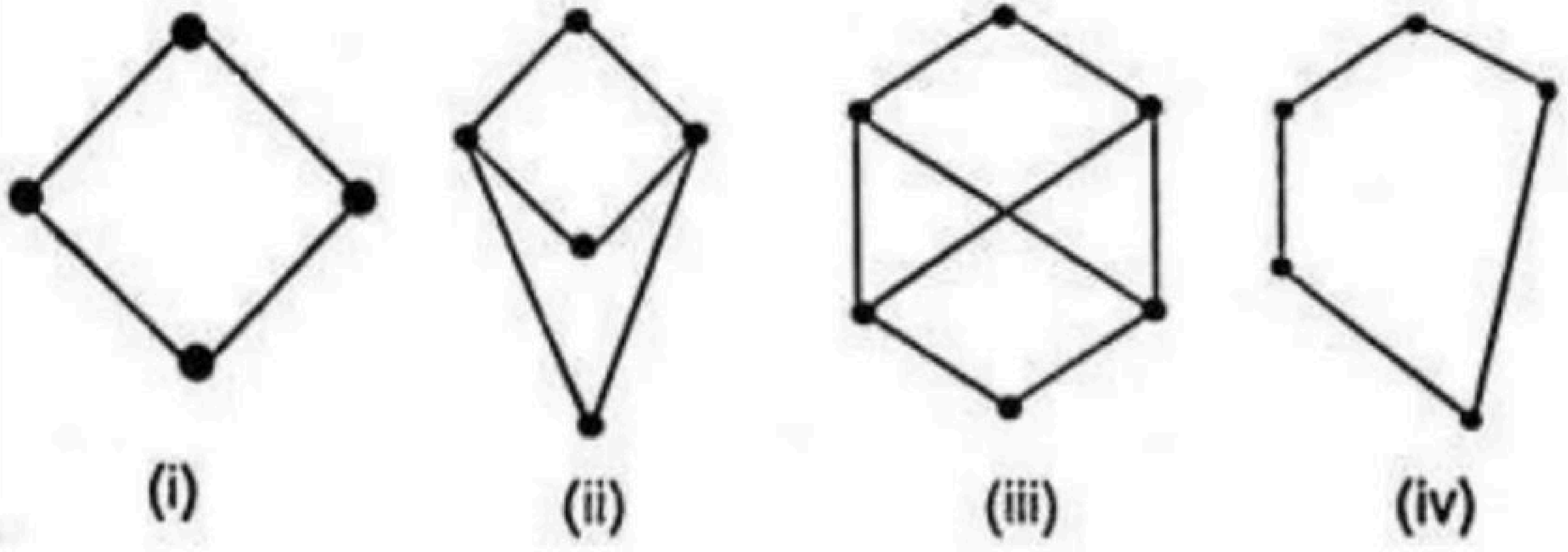
Which all of the above represent a lattice? **(GATE-2008) (2 Marks)**

(A) (i) and (iv) only

(B) (ii) and (iii) only

(C) (iii) only

(D) (i), (ii) and (iv) only



Q the inclusion of which of the following set into S = {{1,2}, {1,2,3}, {1,3,5}, {1,2,4}, {1,2,3,4,5}} is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?
(GATE-2004) (2 Marks)

a) {1}

b) {1}, {2,3}

c) {1}, {1,3}

d) {1}, {1,3}, {1,2,3,4}, {1,2,3,5}

Break

Boolean algebra

- **Unbounded Lattice :-** If a lattice has infinite of elements then it is called Unbounded Lattice.

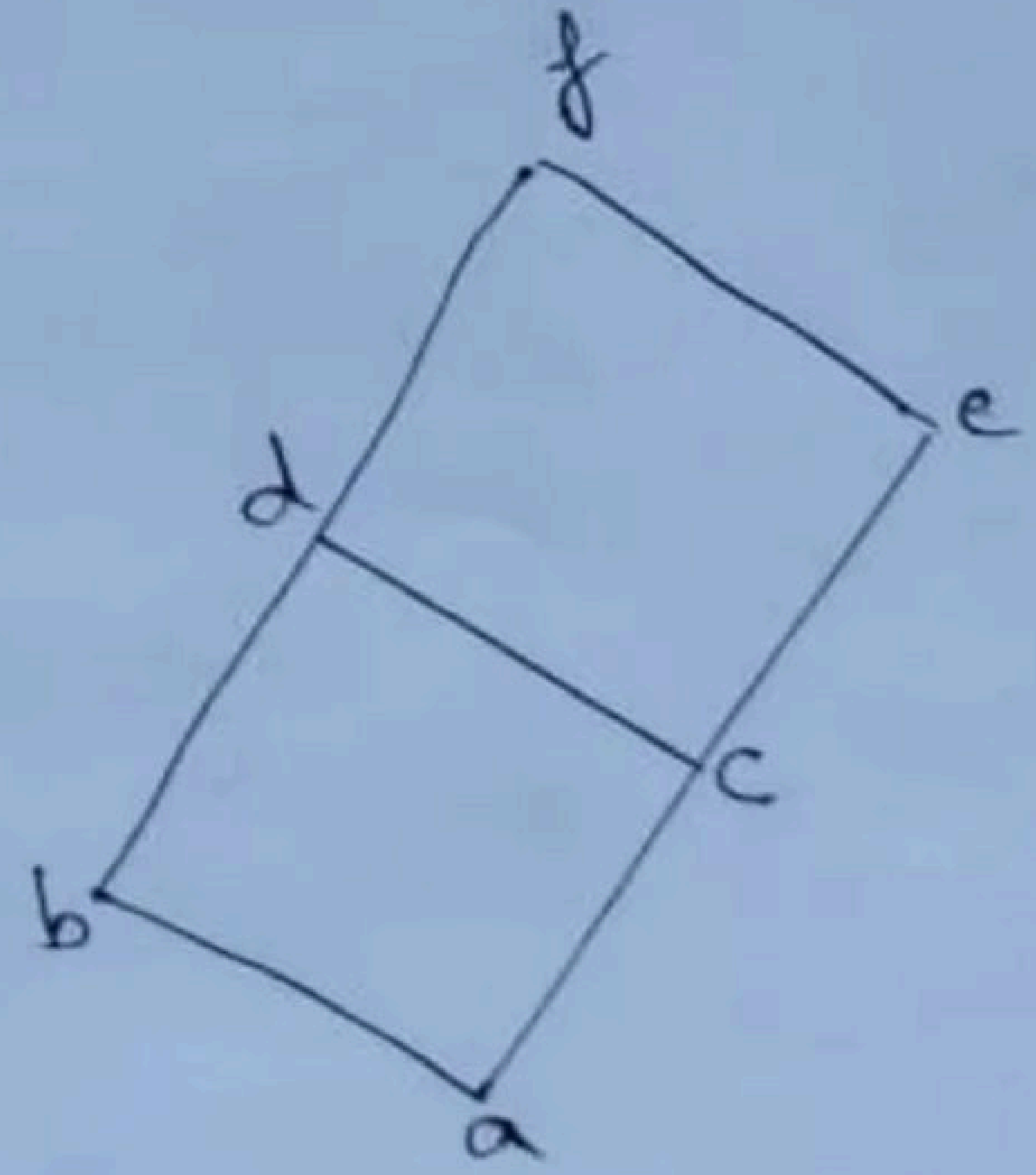
- **<u>Bounded Lattice</u> :-** If a lattice has finite number of elements then it is called Bounded lattice, there will be upper and lower bound in lattice.

- **<u>Complement of an element in a Lattice</u> :-** If two elements a and a^c, are complement of each other, then the following equations must always holds good.
  $a \vee a^c$ = Upper bound of lattice
  $a \wedge a^c$ = Lower bound of lattice

- **<u>Distributive Lattice</u> :-** A lattice is said to be distributed lattice. if for every element their exist at most one completemt(zero or one).

- **<u>Complement Lattice</u> :-** A Lattice is said to be Complement lattice. if for every element their exist at least one complement(one or more).

- **Boolean Algebra :-** A Lattice is said to be Boolean Algebra, if for every element their exist exactly one complement. Or if a lattice is both complemented and distributed then it is called Boolean Algebra.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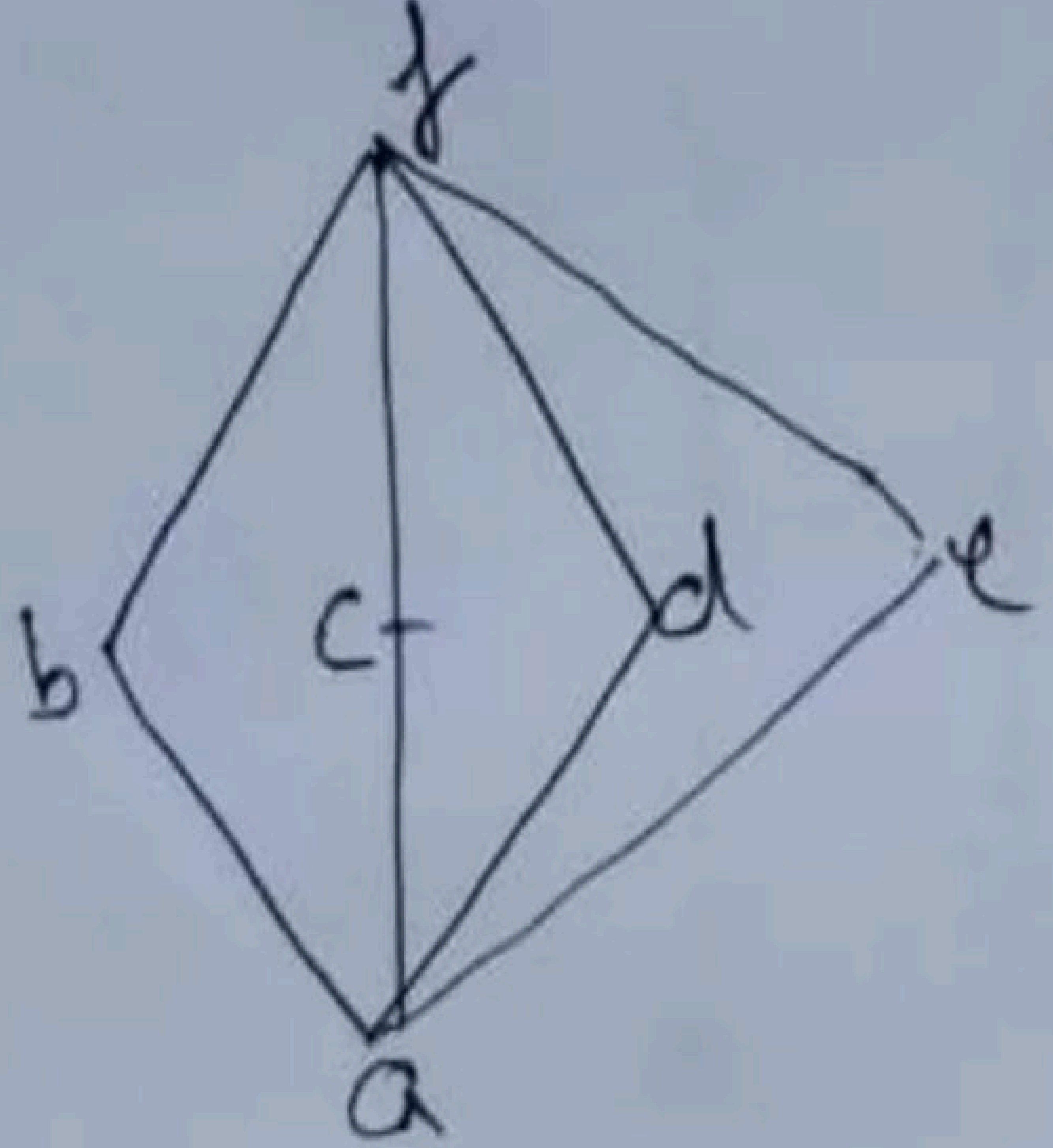
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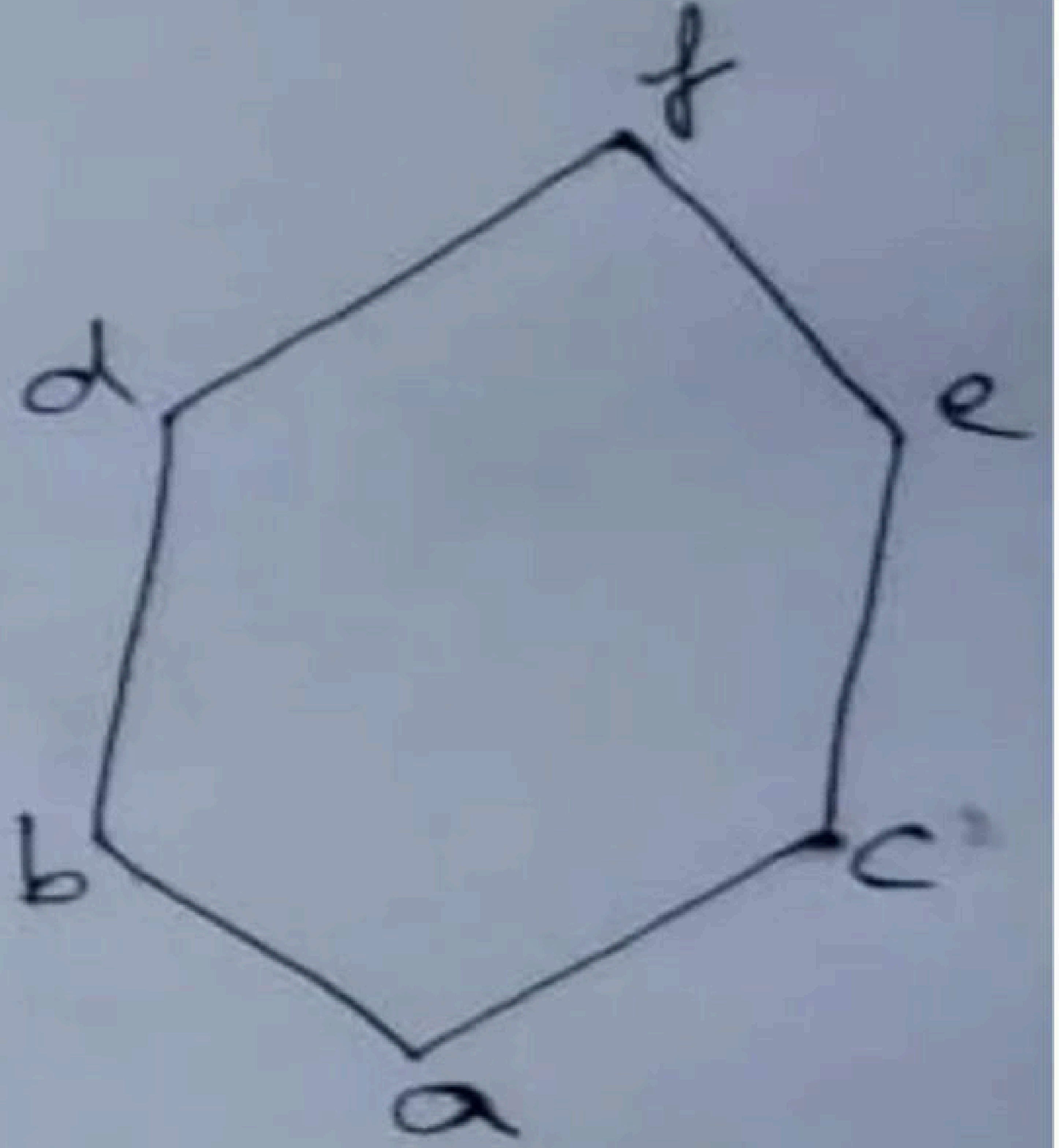
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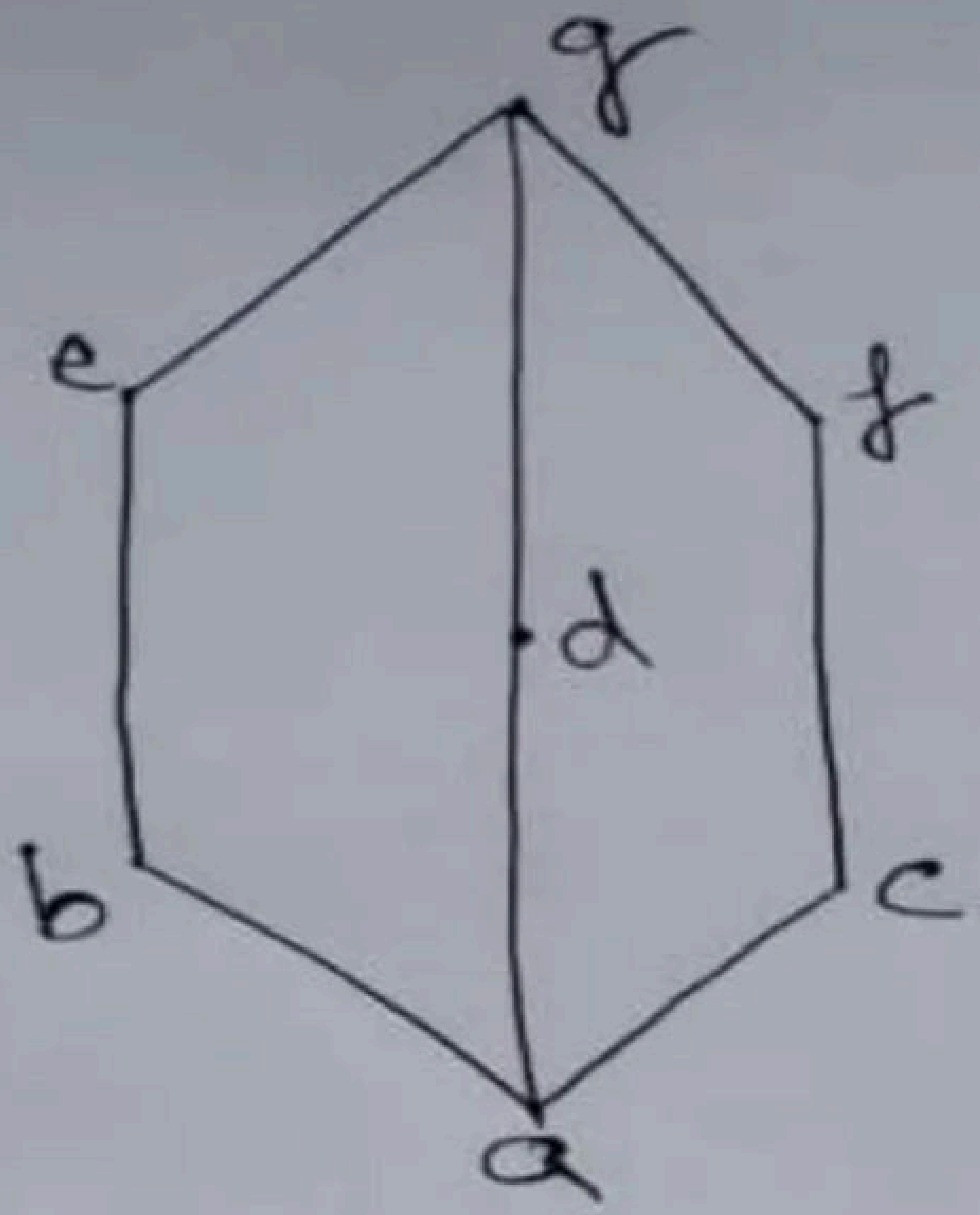
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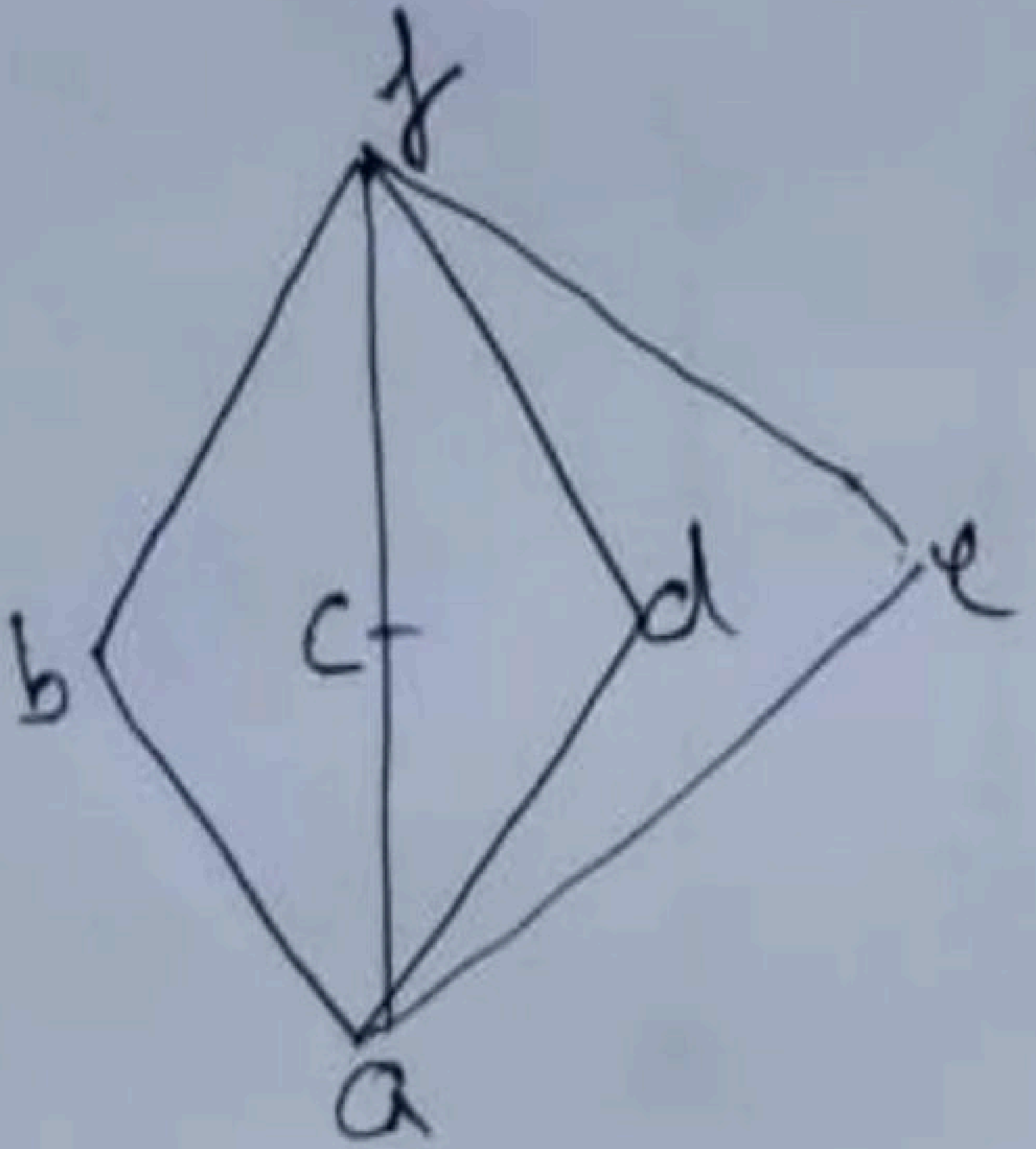
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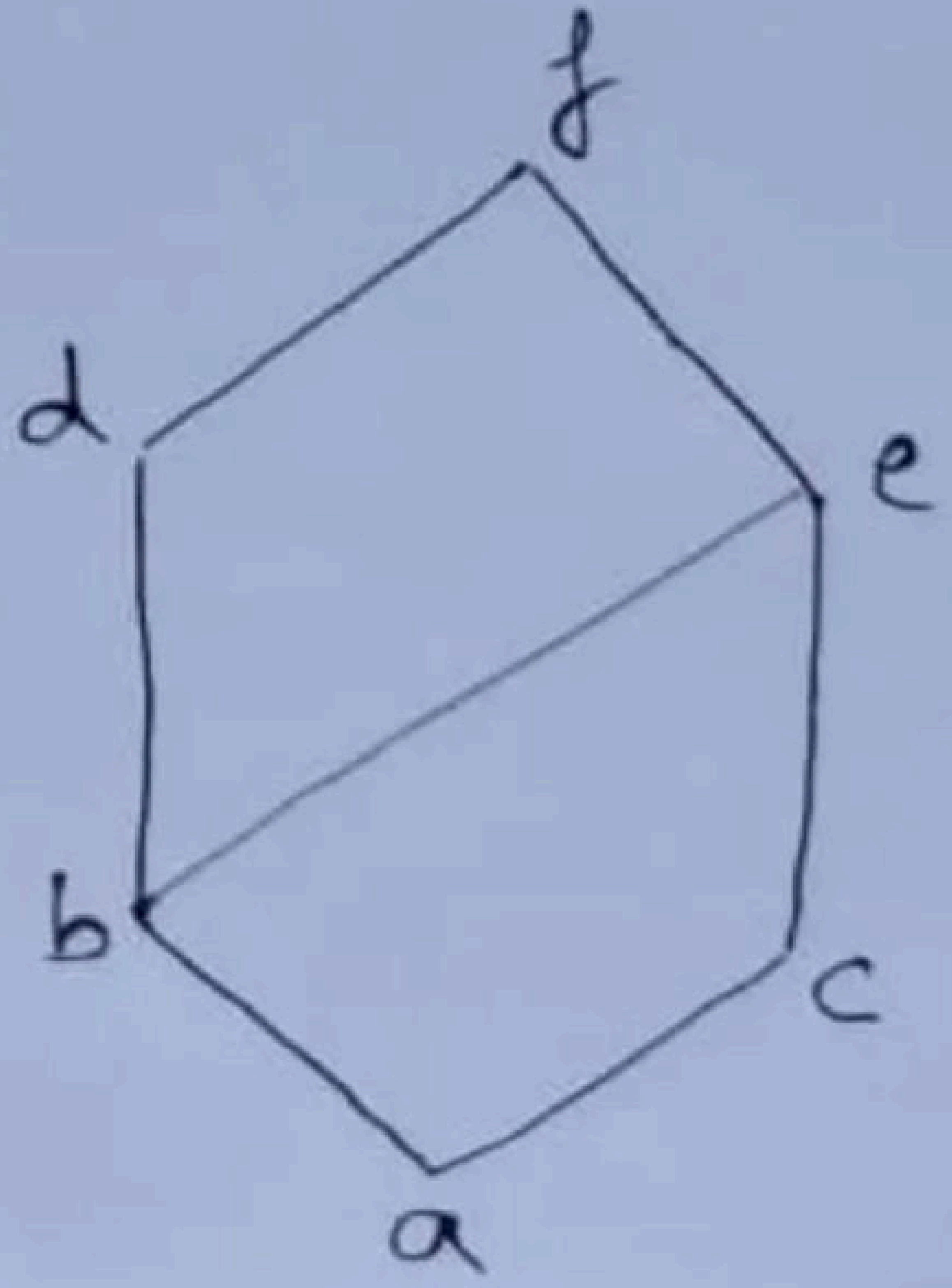
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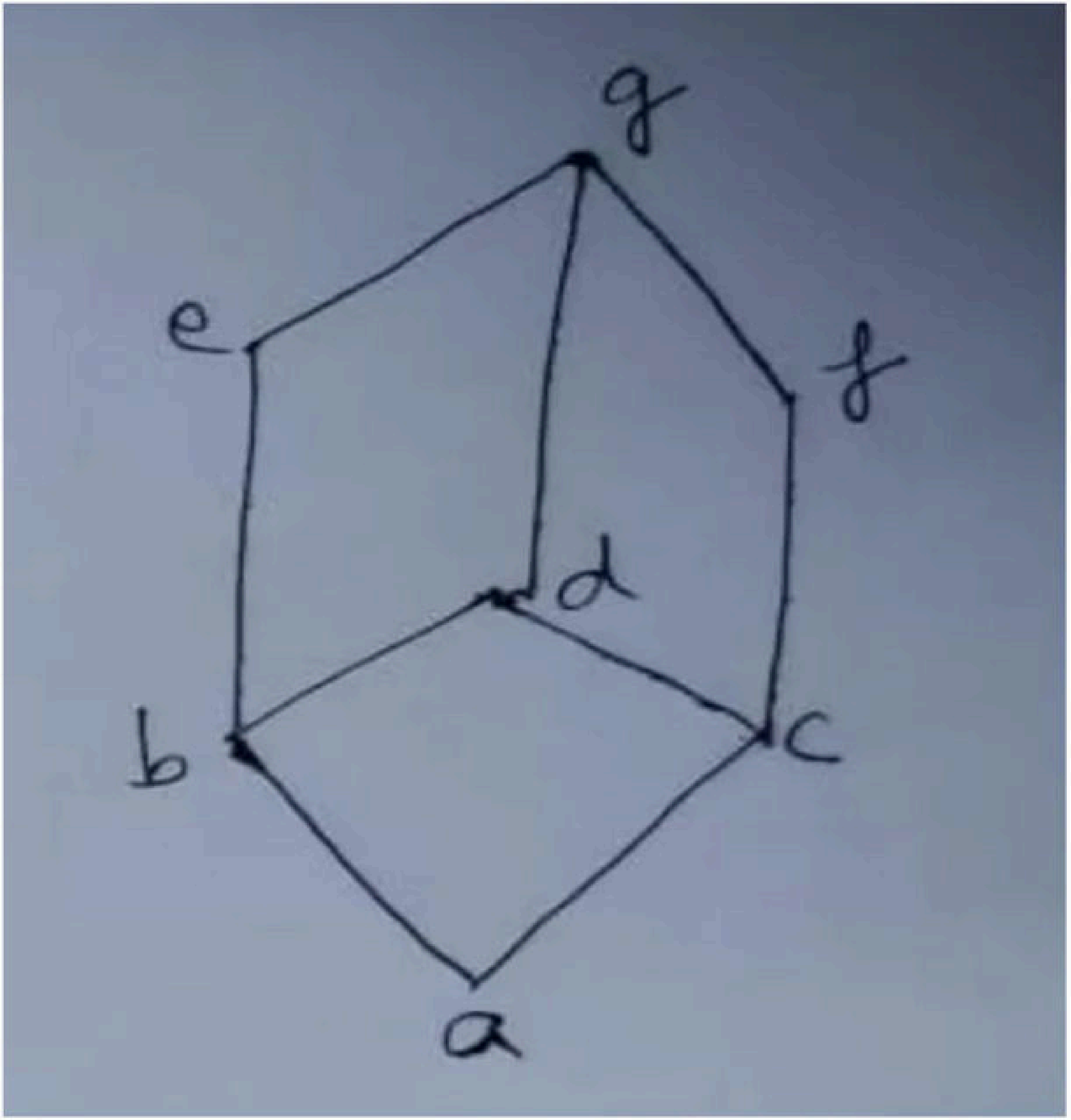
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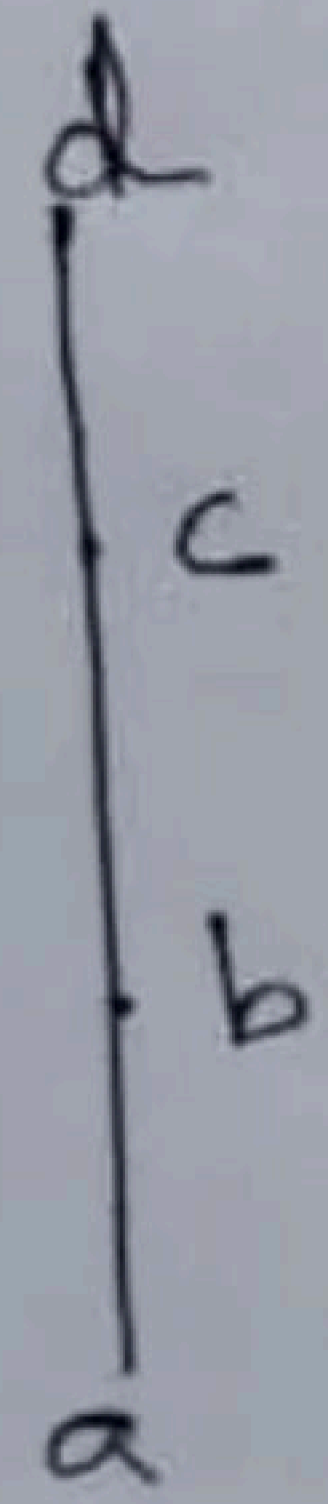
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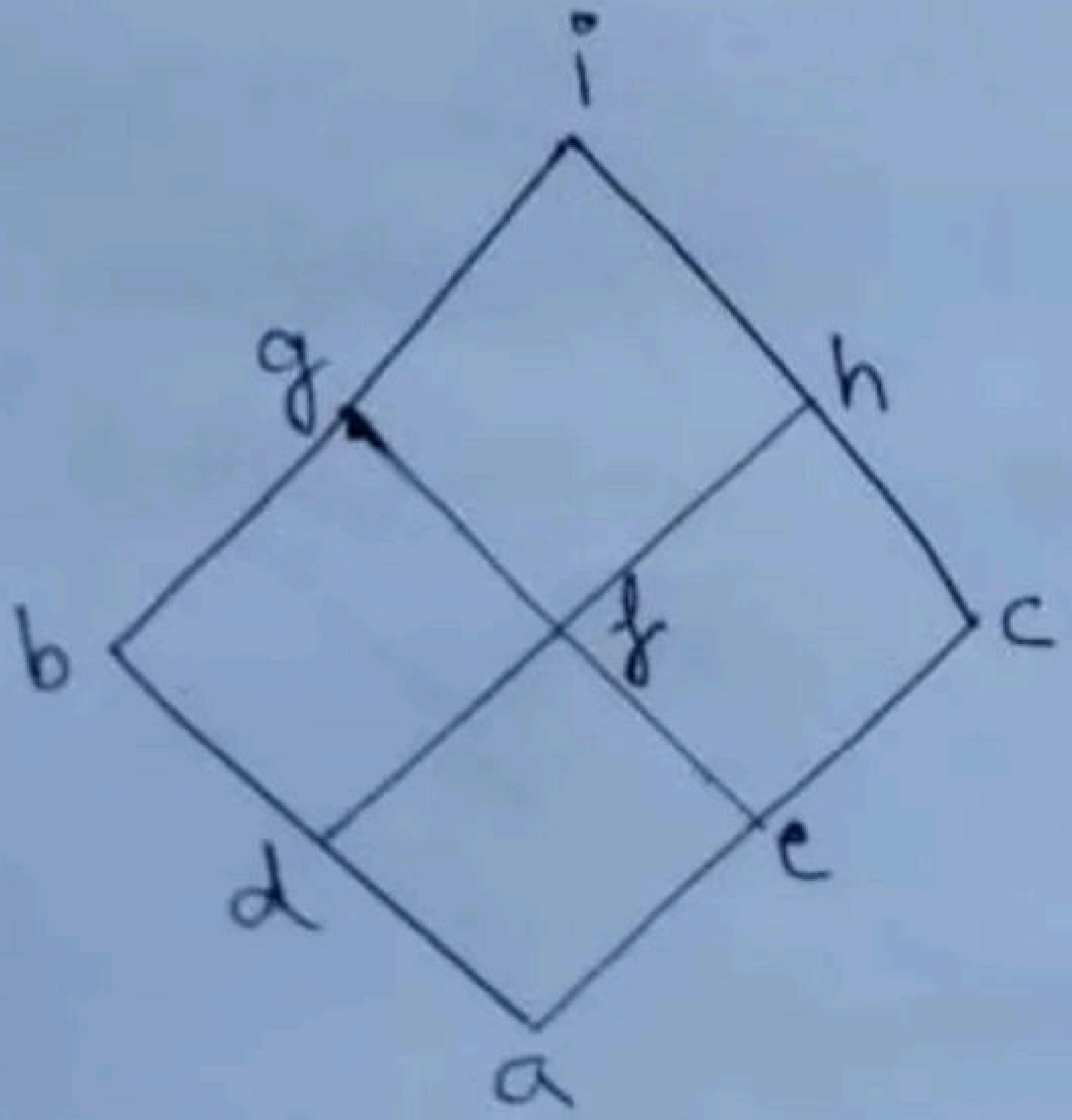

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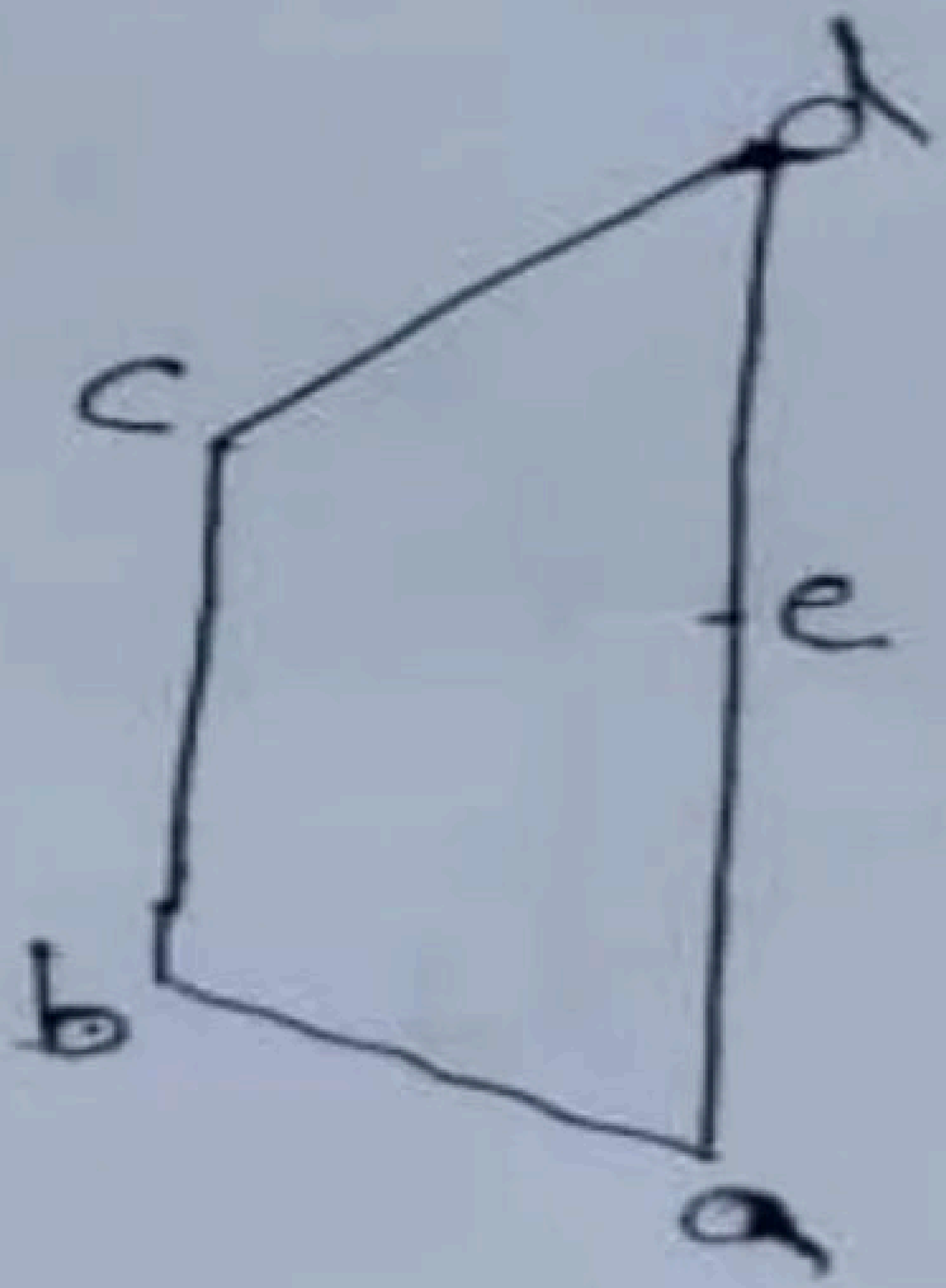
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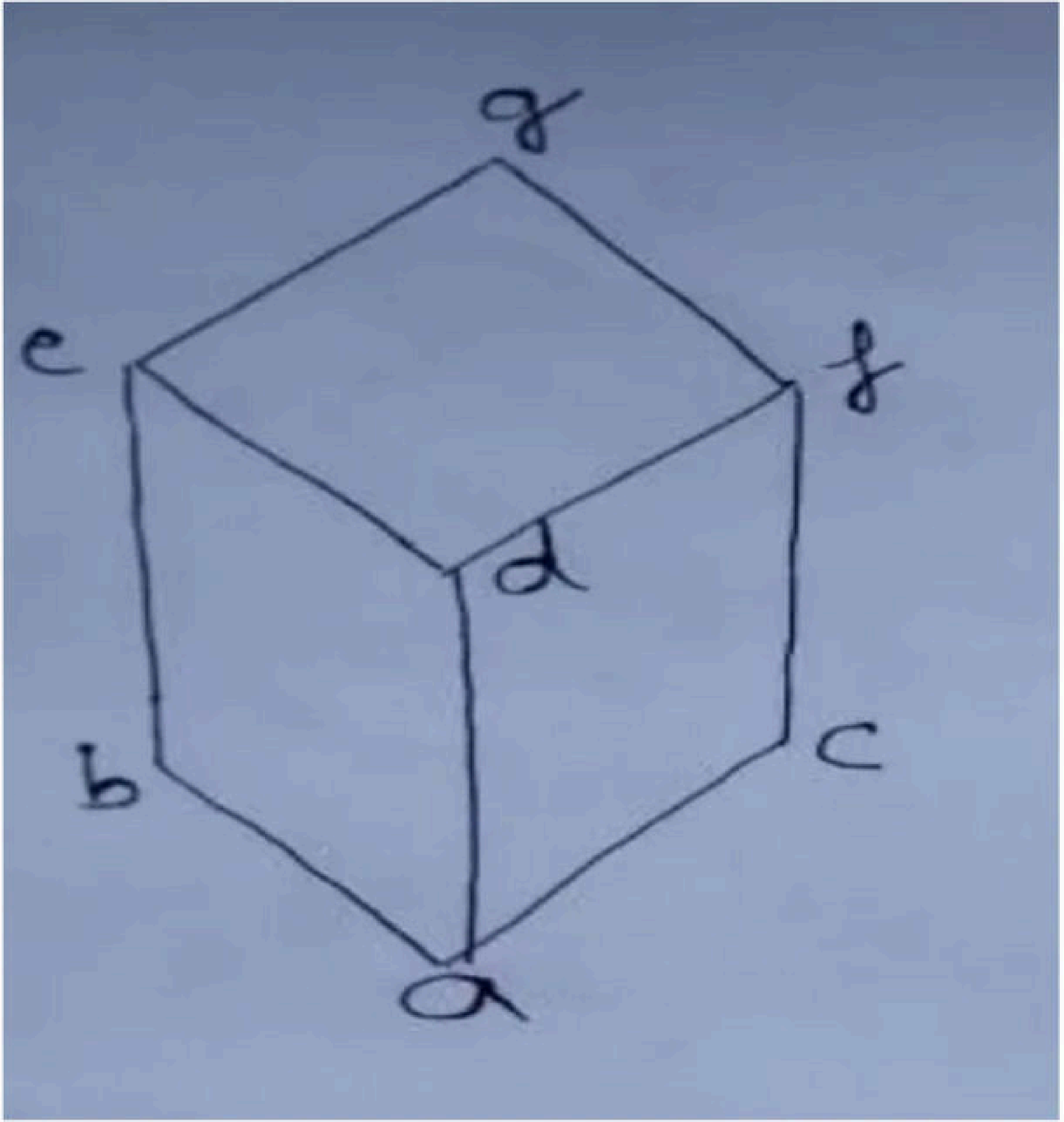
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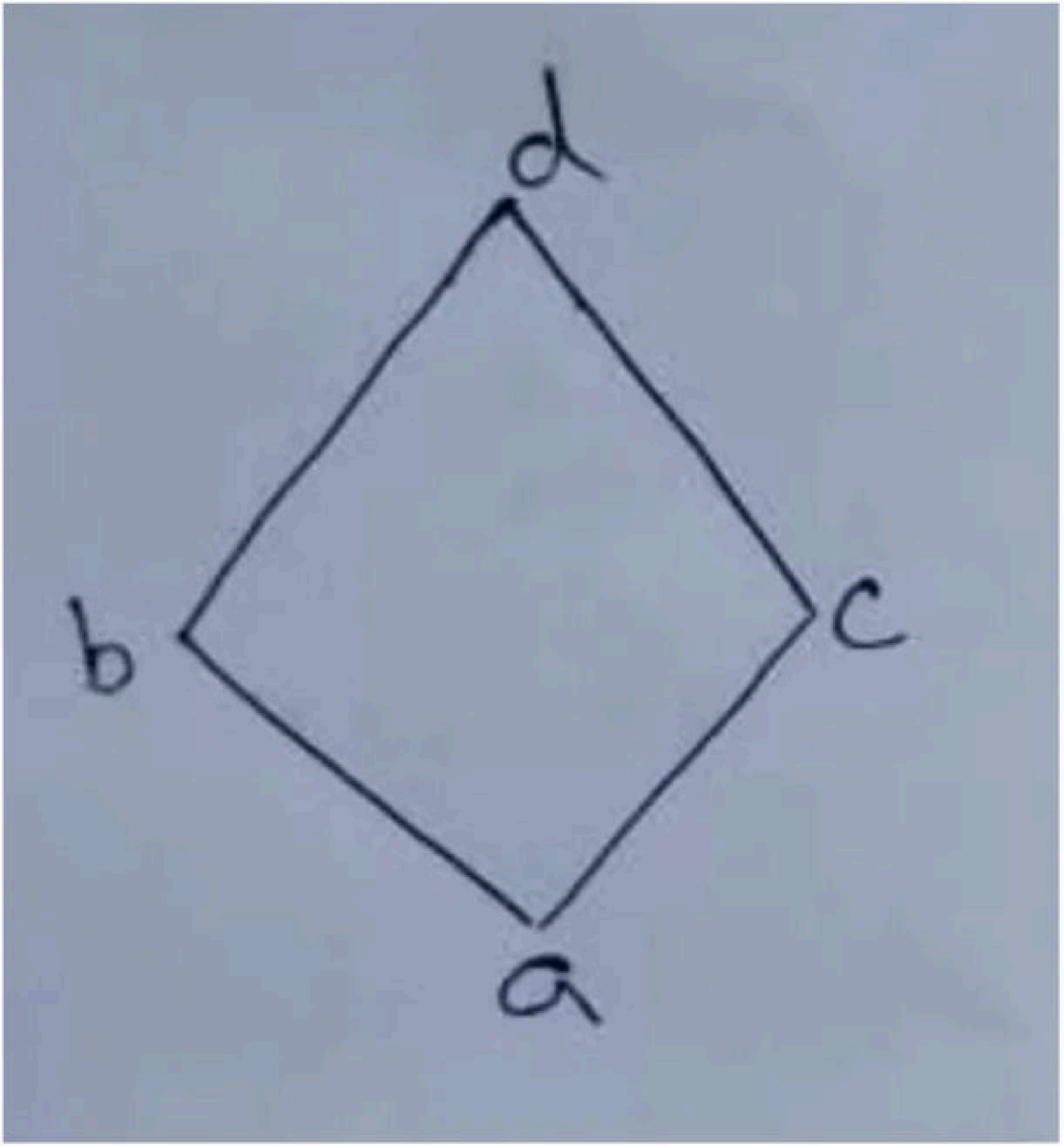
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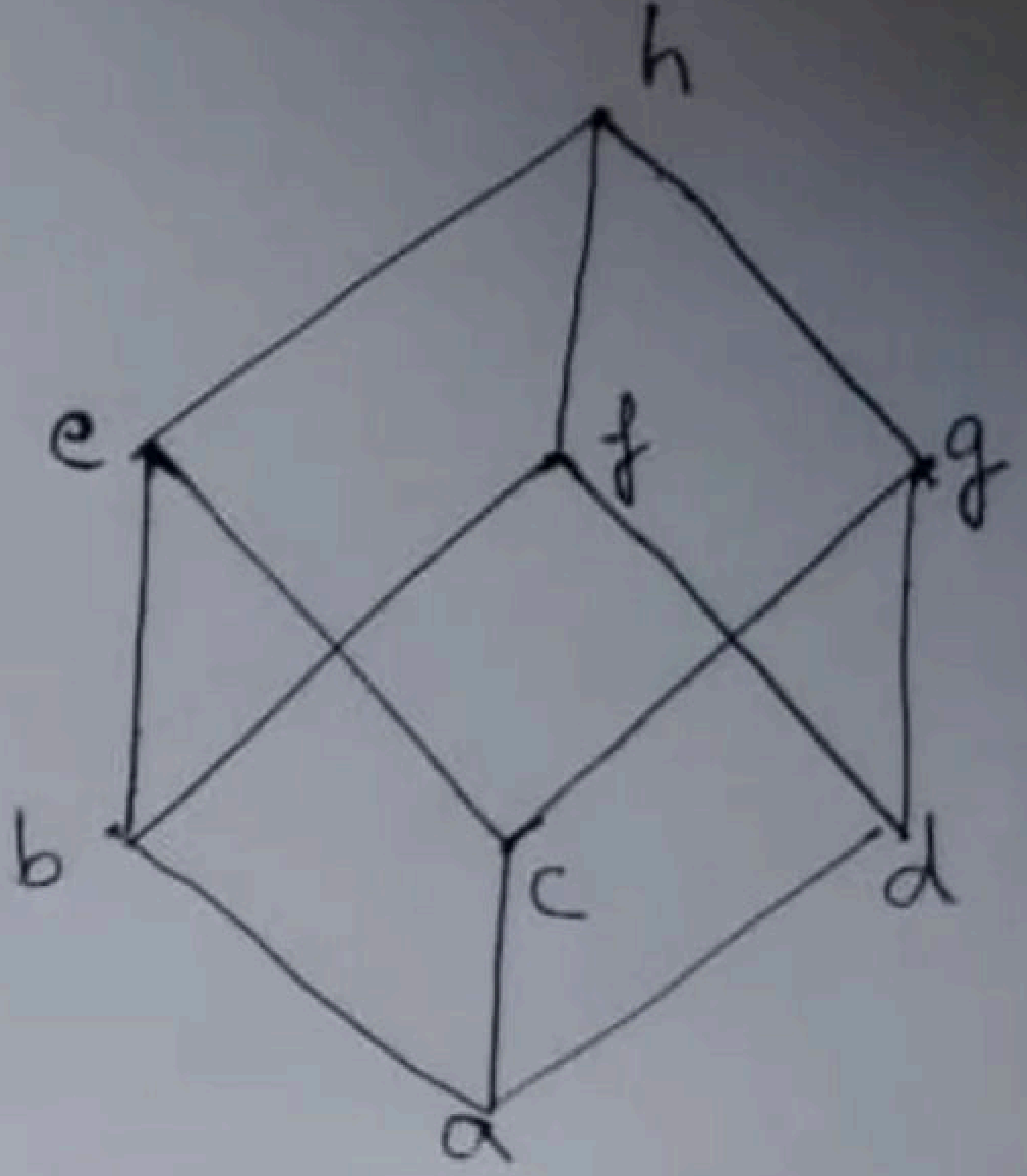
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Break

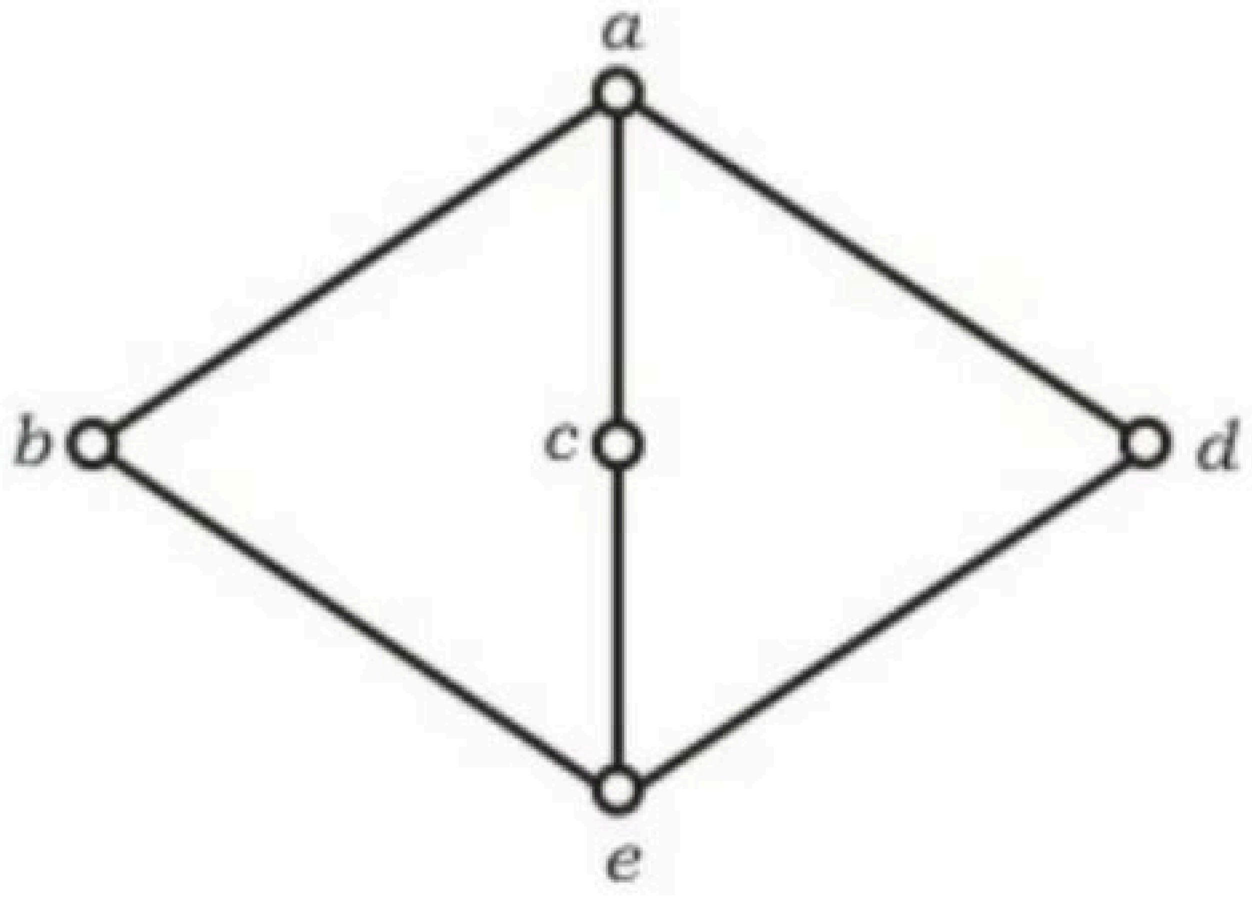
Q The following is the Hasse diagram of the Poset [{a, b, c, d, e}, ≤]
The Poset is **(GATE-2005) (1 Marks)**

(A) not a lattice

(B) a lattice but not a distributive lattice

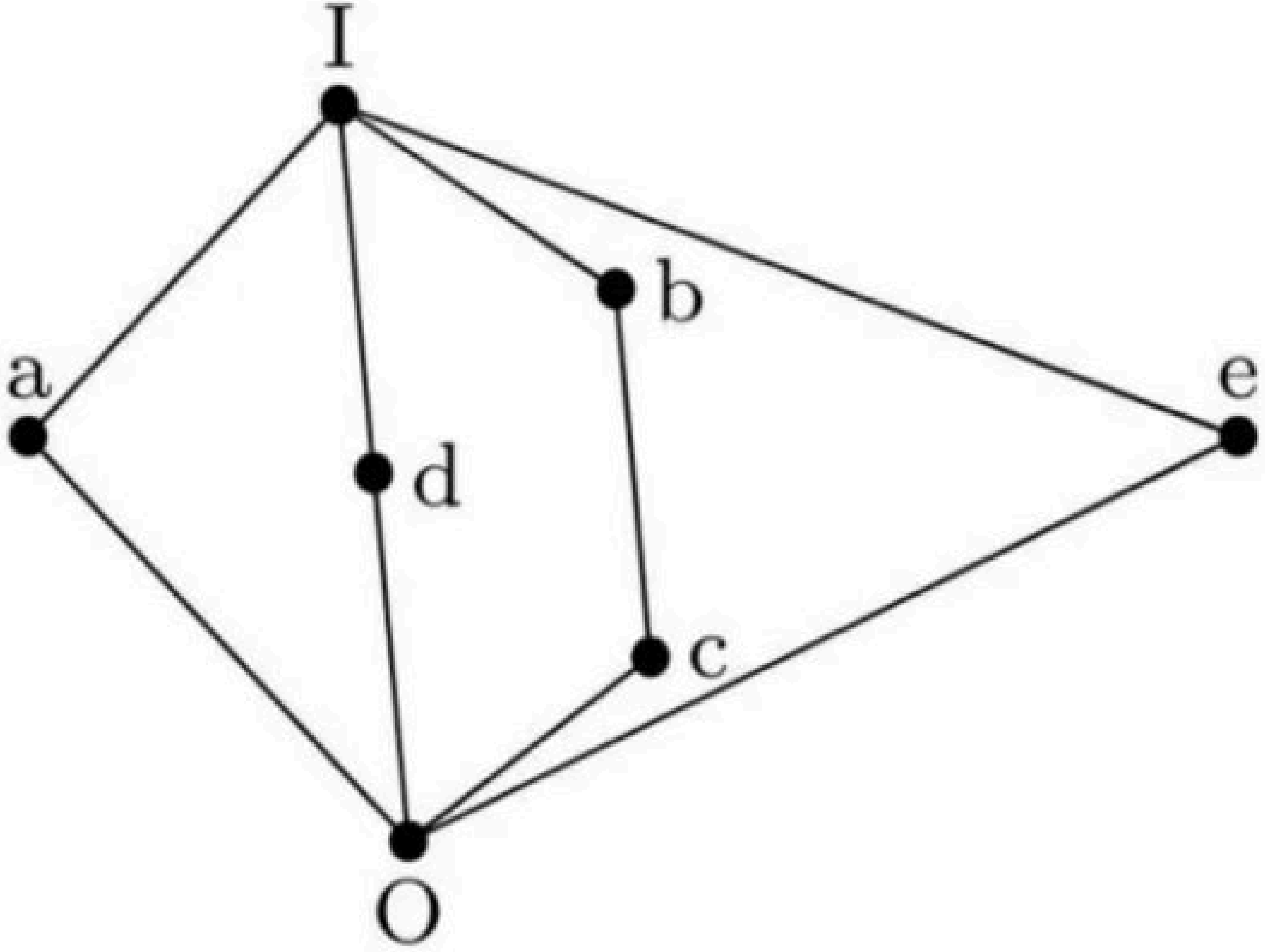
(C) a distributive lattice but not a Boolean algebra

(D) a Boolean algebra



Q The complement(s) of the element 'a' in the lattice shown in below figure is (are) ____

(GATE-1988) (2 Marks)



Q Find which of the following is a lattice and Boolean Algebra?

(1) $[D_{10}, /]$ = $[\{1, 2, 5, 10\}, /]$

(2) $[D_{12}, /]$ $\{1, 2, 3, 4, 6, 12\}$ X

(3) $[D_{30}, /]$ 1, 2, 3, 5, 6, 10, 15, 30
B.A

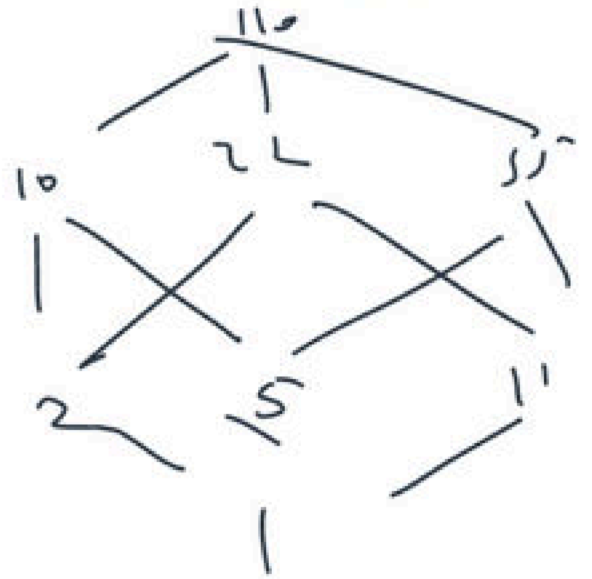
(4) $[D_{45}, /]$ 1, 3, 5, 9, 15, 45 X

(5) $[D_{64}, /]$ 1, 2, 4, 8, 16, 32, 64 X

(6) $[D_{81}, /]$ 1, 3, 9 . . .

(7) $[D_{91}, /]$ 1, 7, 13, 91
B.A

(8) $[D_{110}, /]$ 1, 2, 5, 10, 11, 22, 55, 110



Q Find which of the following is a lattice and Boolean Algebra?

(1) [{1,2,3,4,6,9}, /] X

(2) [{2,3,4,6,12}, /] X

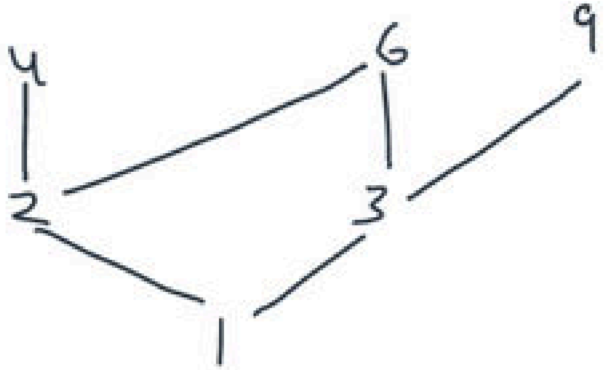
(3) [{1,2,3,5,30}, /] X CL ~~BA~~

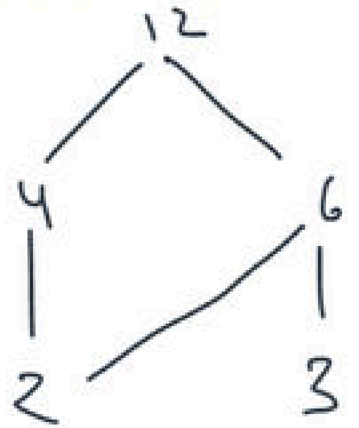
(4) [{1,2,3,6,9,18}, /] DL ~~BA~~

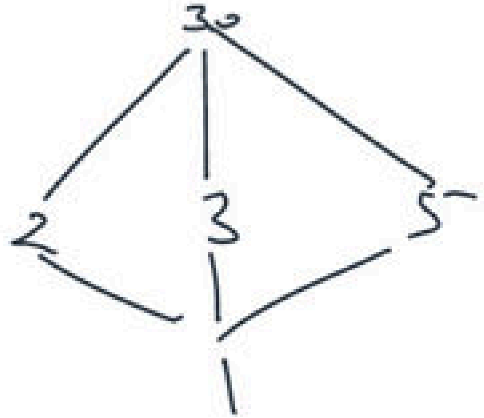
(5) [{2,3,4,9,12,18}, /] X

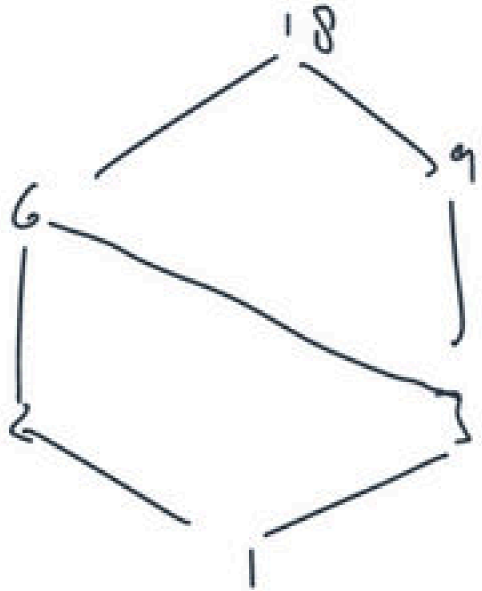
(6) [R, <=] – L

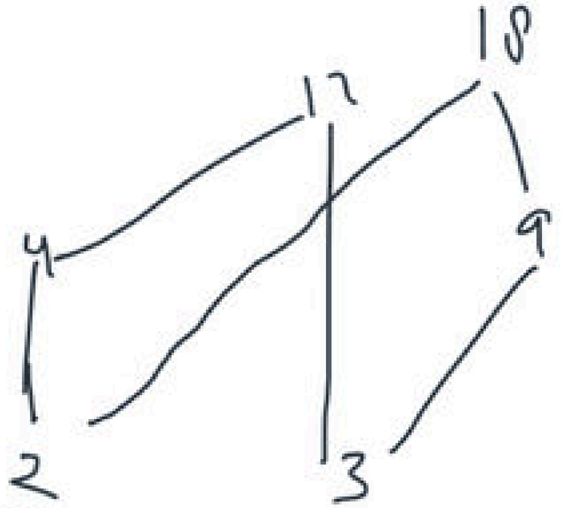
(7) [P(A), ⊑], A = {1,2,3}











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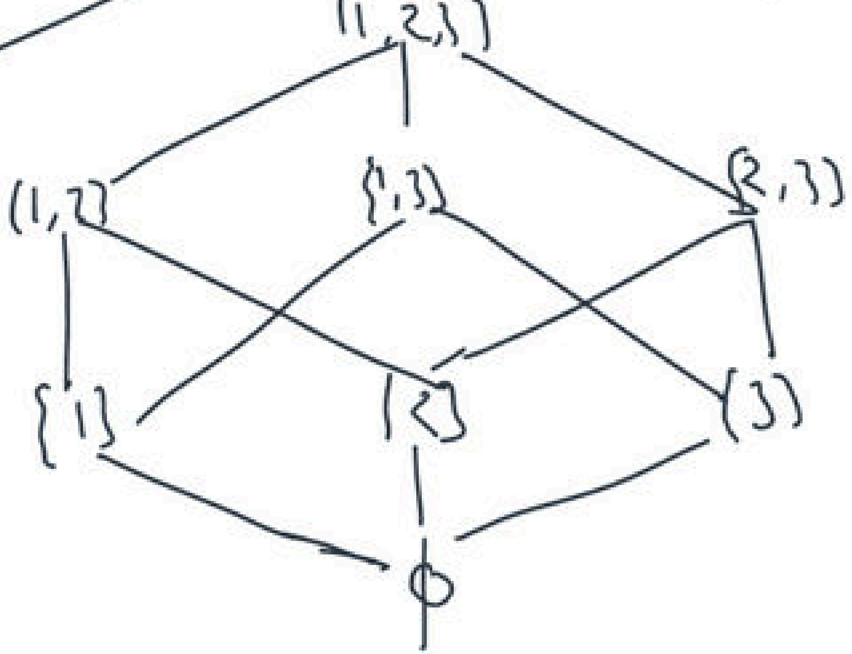
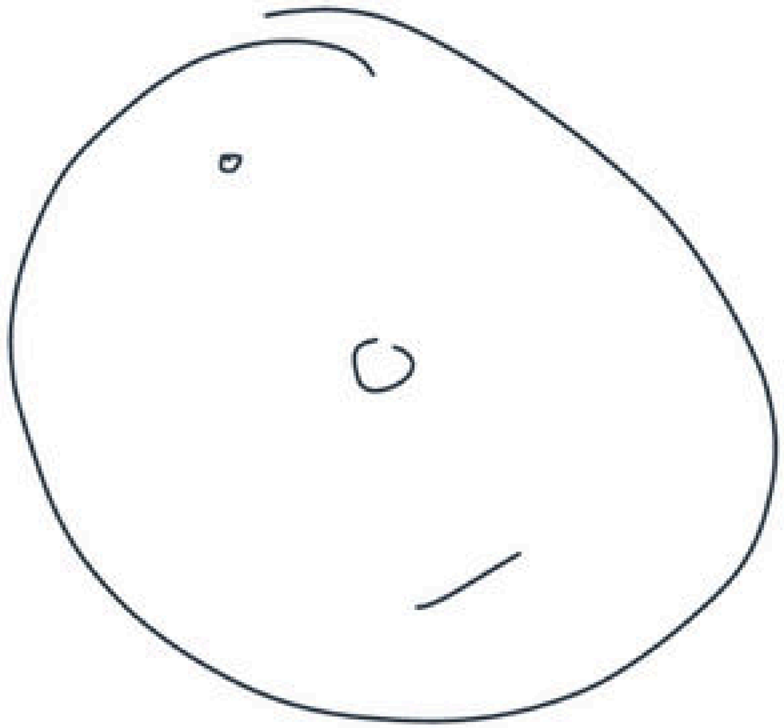
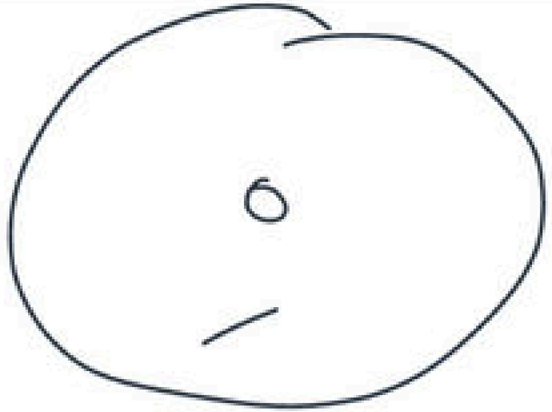
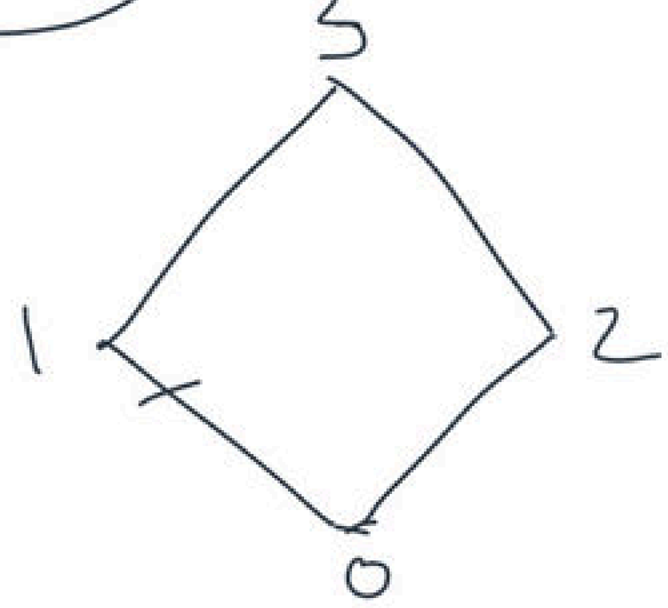


2, 4, 8, 16, 32, — —







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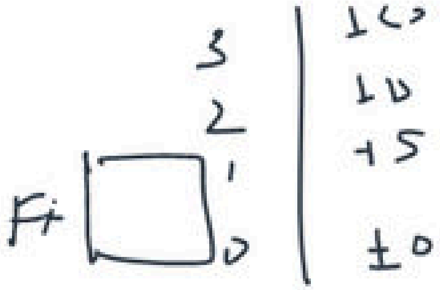
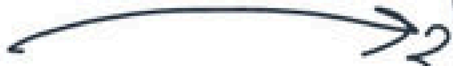
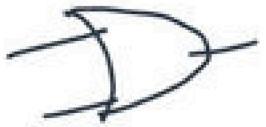
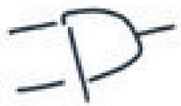




2^n / 4^n

a	b	or $a+b$	and $a \cdot b$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1



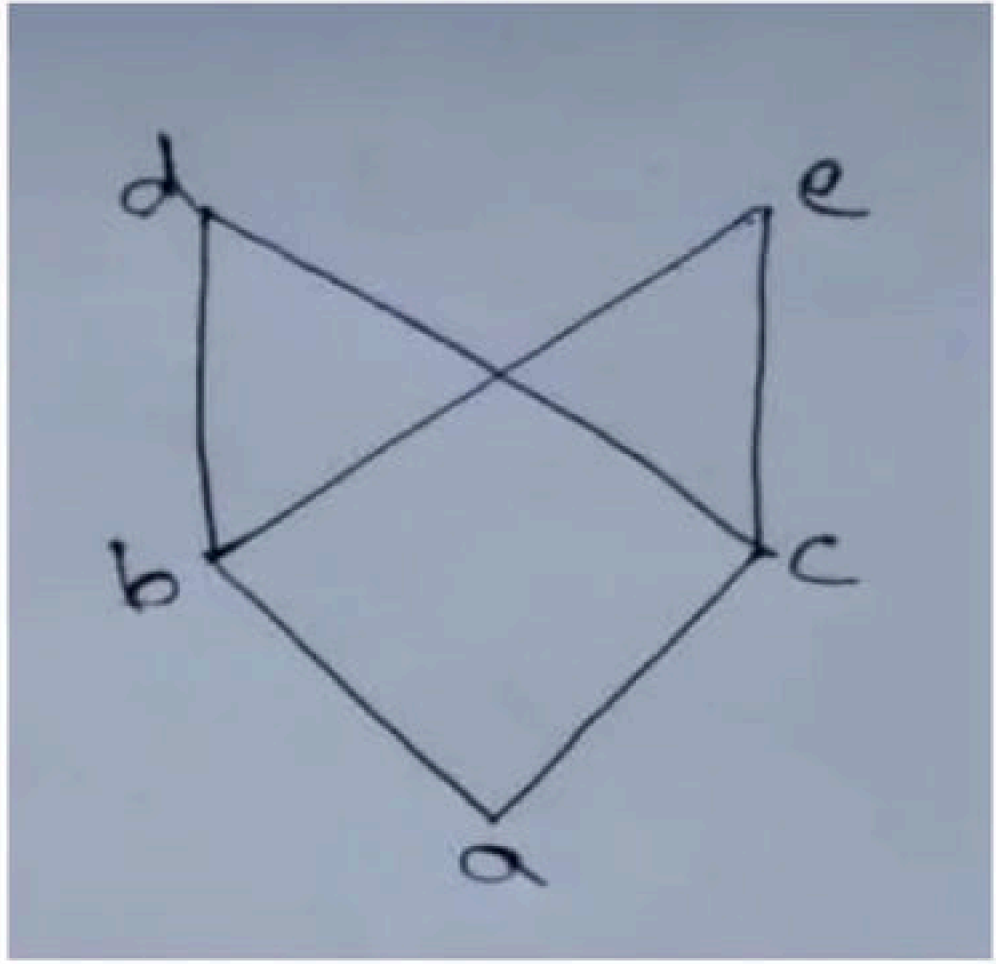


$\vee$ $\wedge$

1

0

Q Consider the following hasse diagram, find which of the following is true?

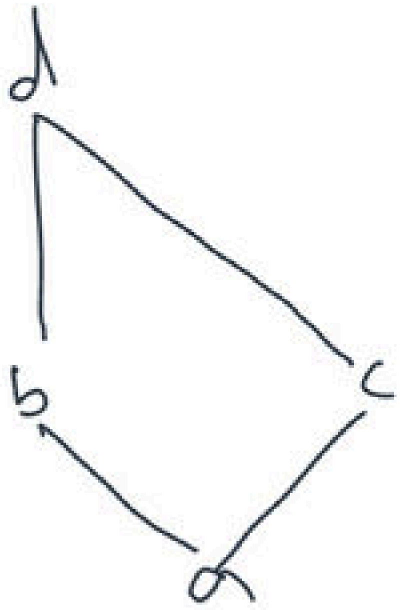


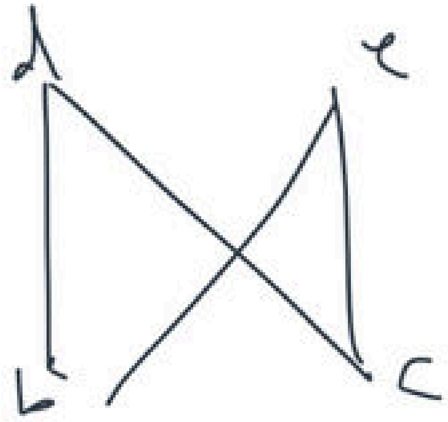
a) ~~it is a lattice~~

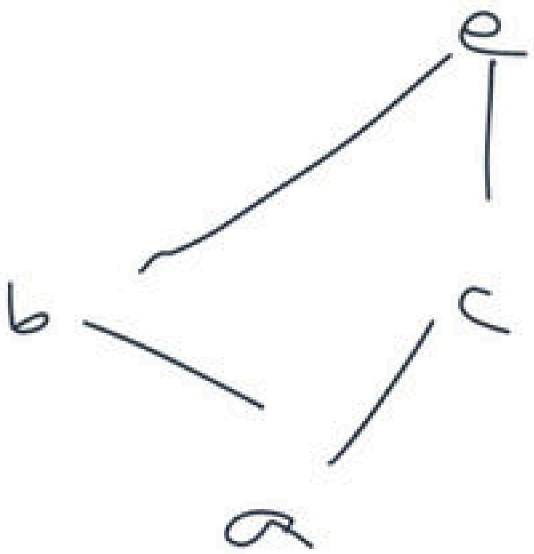
b) subset {a, b, c , d} is a lattice ×

c) subset {b, c, d, e} is a lattice ×

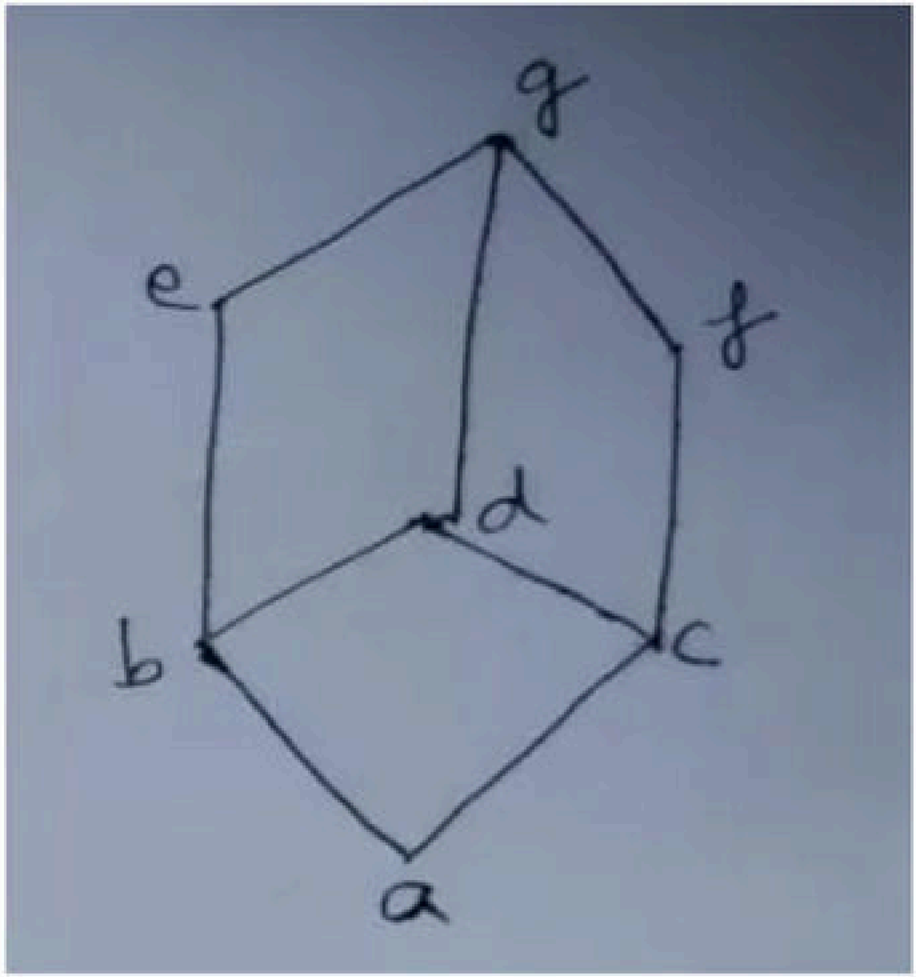
d) subset {a, b, c, e} is a lattice







Q Consider the following hasse diagram, find which of the following is true?

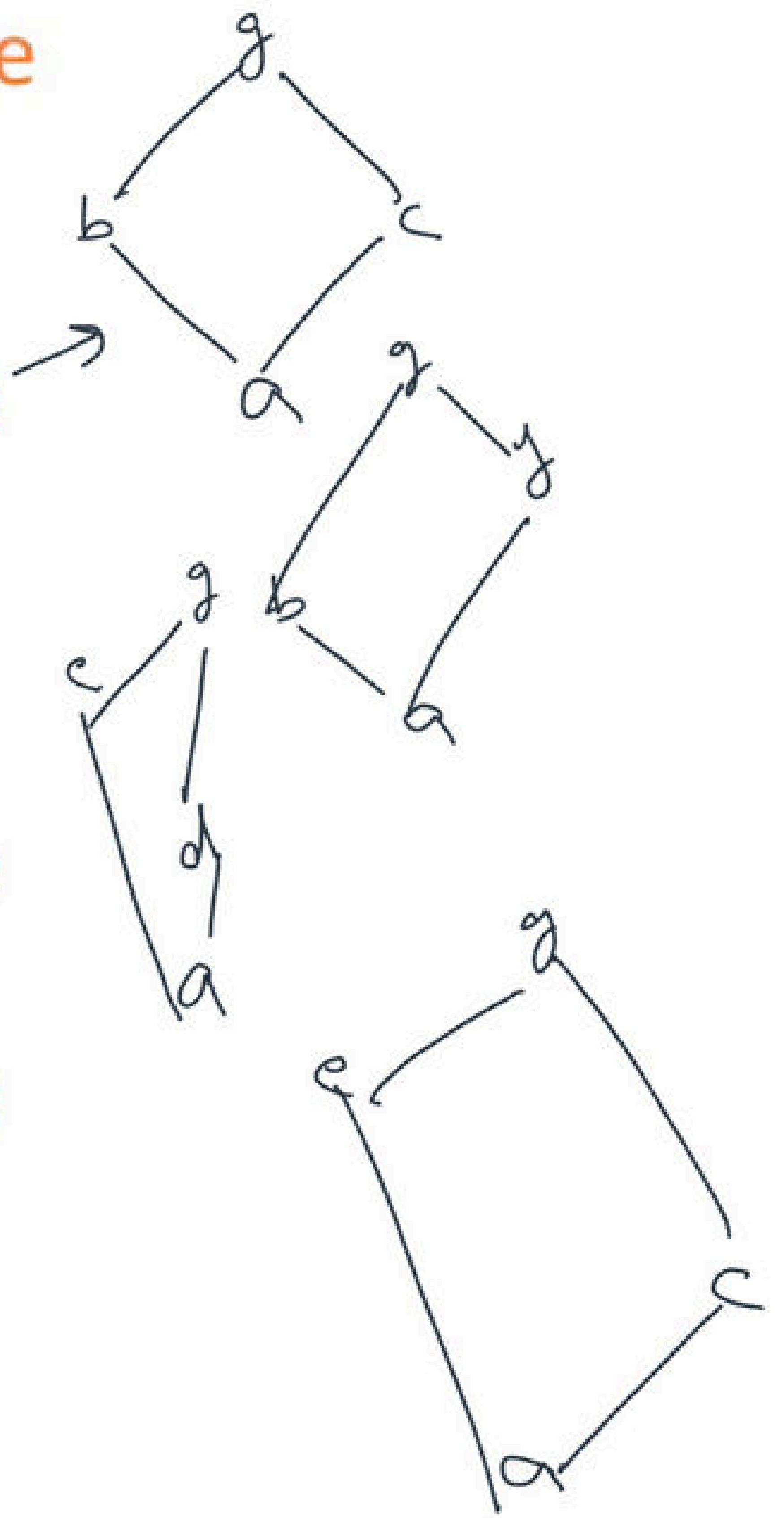


a) subset {a, b, c, g} is a lattice

b) subset {a, b, f, g} is a lattice

c) subset {a, d, e, g} is a lattice

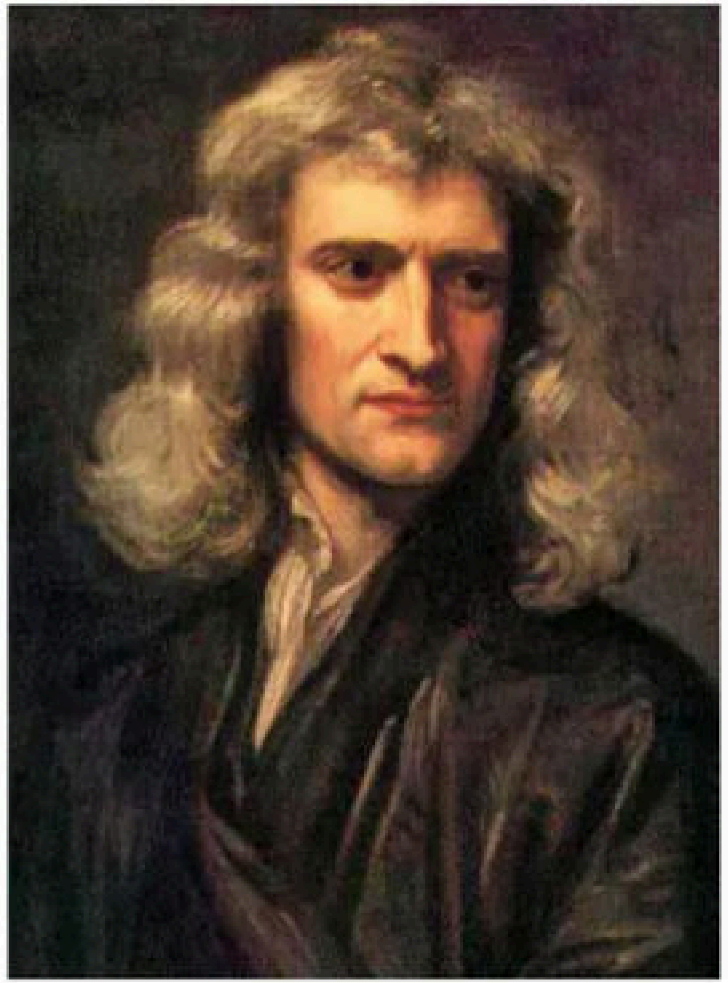
d) subset {a, c, e, g} is a lattice



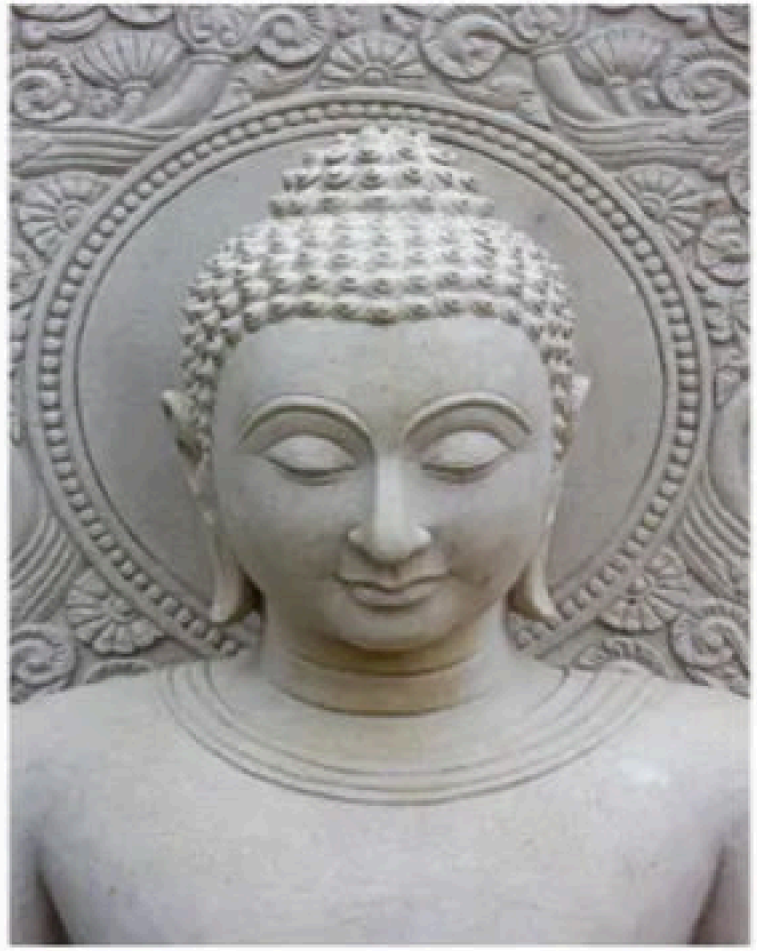
Proposition

- First we must look at the difference between Scientist and Philosopher.

- Philosopher give an idea or theory which may have different interpretation from person to person. It depends on the wisdom of a person.

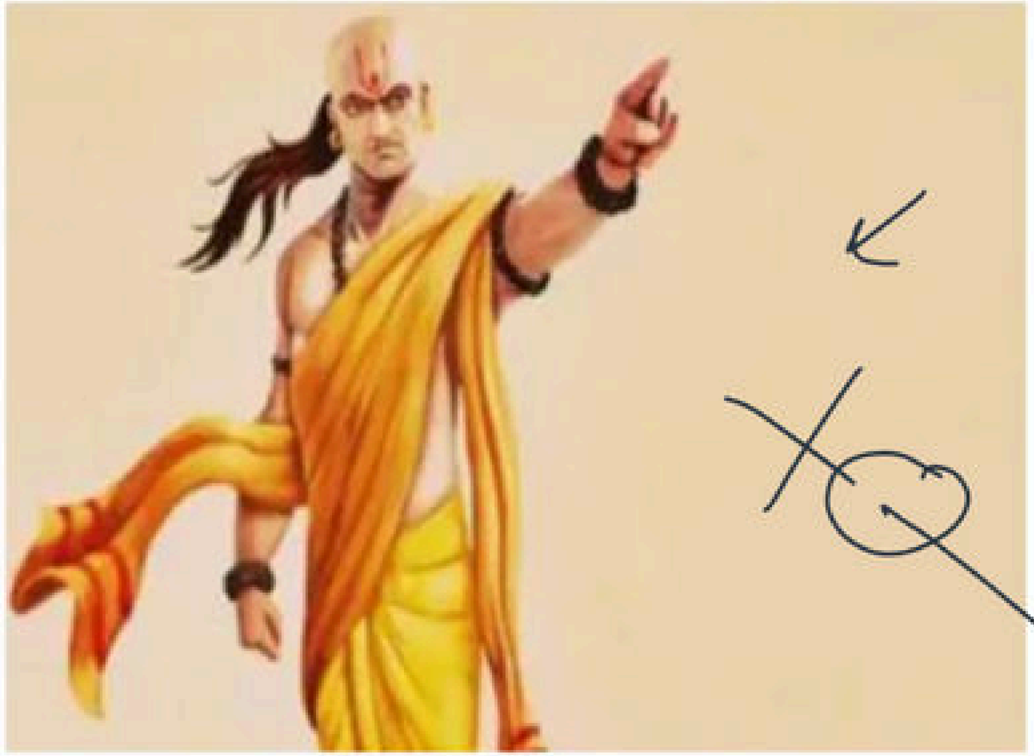


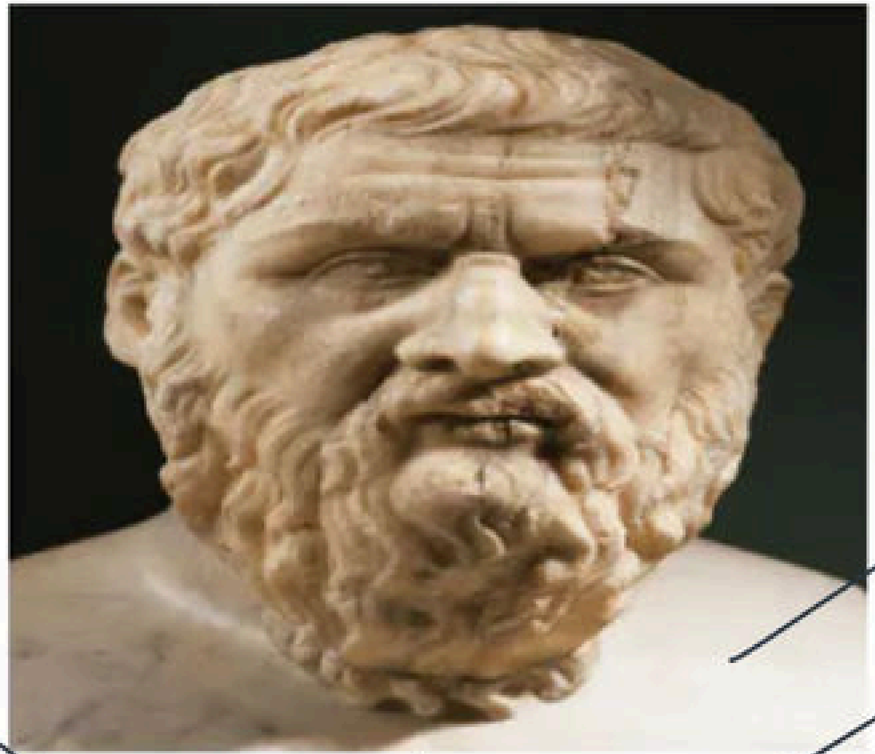
Mahaveer

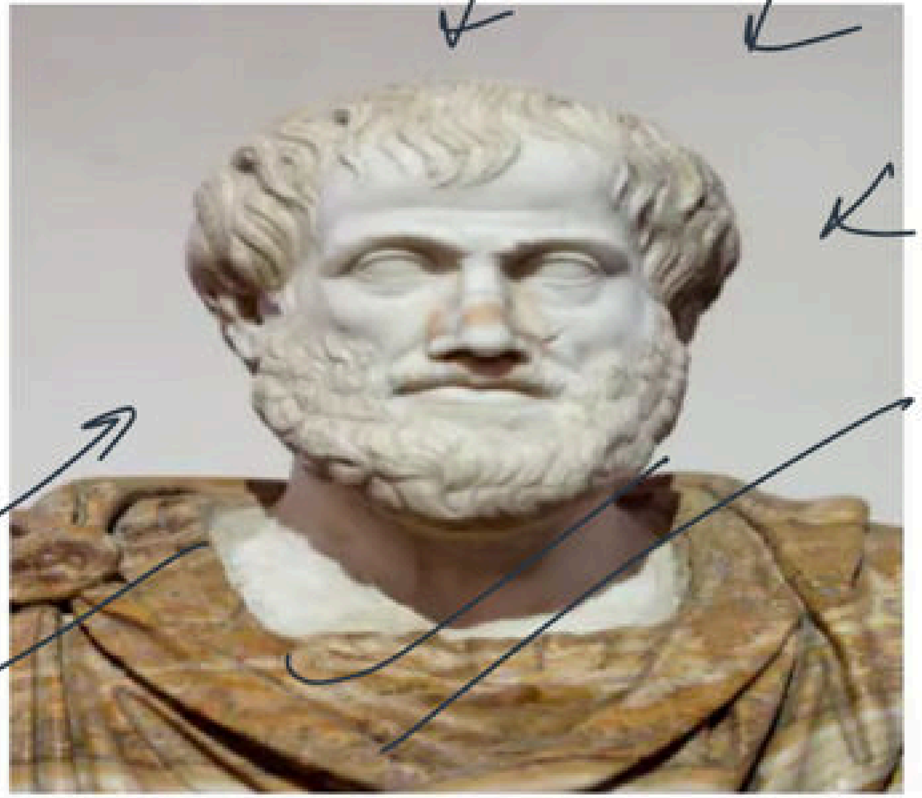


Gautam Buddha

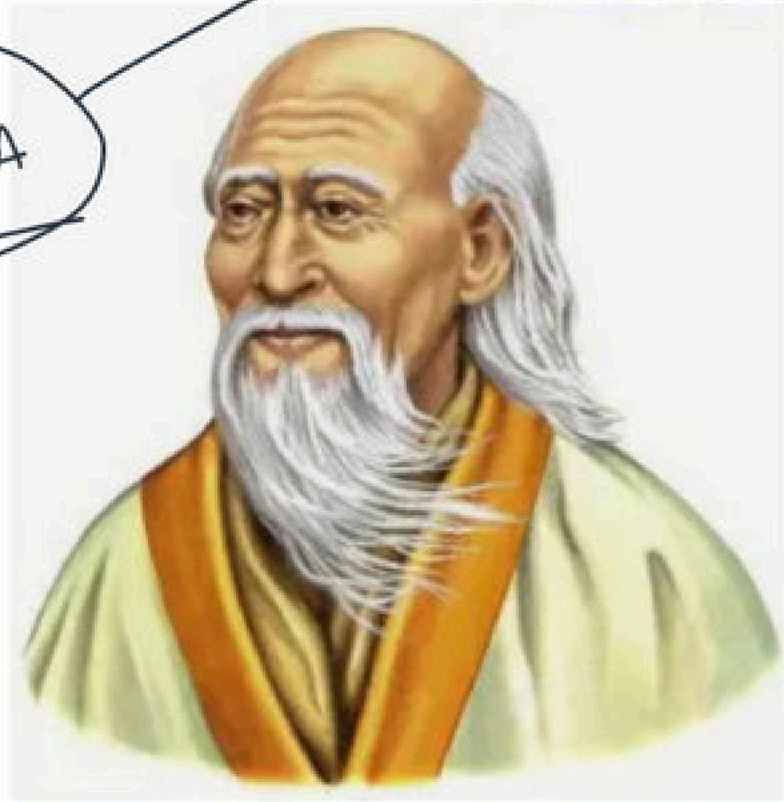
- There are different philosophers in the world suggested different philosophy like, Chanakya Plato, Aristotle, Confucius, Laozi,





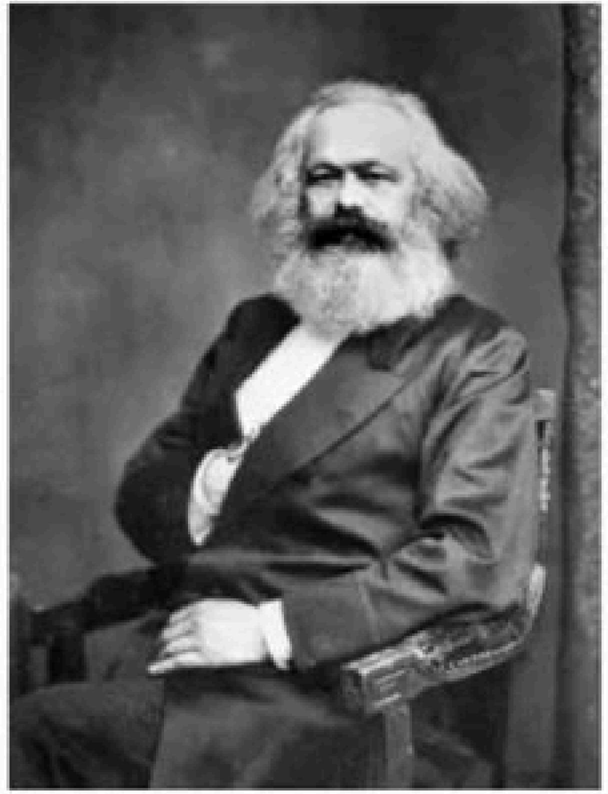


C.M A



TFI

Karl Marx



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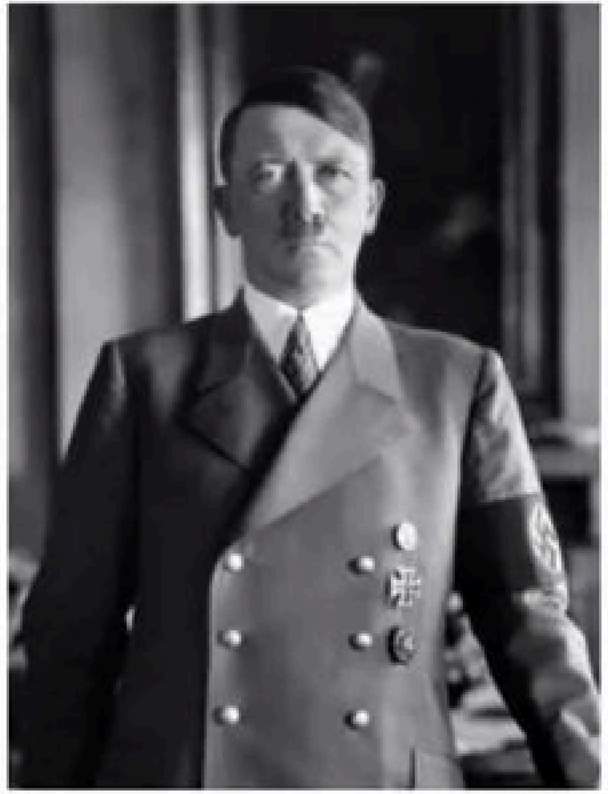


Mahatma Gandhi

Break

- Proposition with rules of logic actually is a method of reasoning (unambiguous, machinic, deterministic), given by Aristotle, who was the teacher of Alexander son of King Philip of Macedonia

- There may be different methods of reasoning for solving a problem apart from proposition.

- Proposition and rules of logic specify the meaning of mathematical statements. Logic is the basis of all mathematical reasoning, and of all automated reasoning.

- It has practical applications to the design of computing machines, to the specification of systems, to artificial intelligence, to computer programming, to programming languages, and to other areas of computer science, as well as to many other fields of study.

- To understand mathematics, we must understand what makes up a correct mathematical argument, that is, a proof.

- Once we prove a mathematical statement is true, we call it a theorem. A collection of theorems on a topic organize what we know about this topic.

- Everyone knows that proofs are important throughout mathematics, even in computer science. In fact, proofs are used to verify that computer programs produce the correct output for all possible input values, to show that algorithms always produce the correct result.

- The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

- The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic or predicate calculus(study of propositions)**.

- It was first developed systematically by the Greek philosopher Aristotle more than 2300 years ago.

- We now turn our attention to methods for producing new propositions from those that we already have. These methods were discussed by the English mathematician George Boole in 1854 in his book *The Laws of Thought.*

Break

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.
  1. Delhi is the capital of USA
  2. How are you doing
  3. 5<= 11
  4. Temperature is less than 10 C
  5. It is cold today
  6. Read this carefully
  7. X + y = z

1. Premises(proposition) is always considered to be true.

2. Premises is a statement that provides reason or support for the conclusion(proposition).

1. If a set of Premises(P) yield another proposition Q(Conclusion), then it is called an Argument.

2. An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

$\{P_1, P_2, P_3 \ldots, P_N\} \mid\!- Q$	P_1 P_2 P_3 . . . P_N …… Q …….	$\{P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_N\} \mid\!- Q$

Break

- **<u>Law of contradiction</u> -** the law of non-contradiction (LNC) (also known as the law of contradiction, principle of non-contradiction (PNC), or the principle of contradiction) states that

- Contradictory propositions cannot both be true in the same sense at the same time.
    - e.g. the two propositions "*A is B*" and "*A is not B*" are mutually exclusive.

- **<u>Law of excluded middle</u>** - the law of excluded middle (or the principle of excluded middle) states that for any proposition, either that proposition is true or its negation is true.

Break

<u>Types of proposition</u>

1. We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent propositions, just as letters are used to denote numerical variables.

2. The conventional letters used for propositional variables are p, q, r, s. The **truth value** of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

- Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Operators / Connectives

1. **Negation: -** let p be a proposition, then negation of p new proposition, denoted by $\neg p$, is the statement "it is not the case that p".

2. The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p. e.g. ¬"Michael's PC runs Linux" = "It is not the case that Michael's PC runs Linux." = "Michael's PC does not run Linux."

Negation	
P	¬P
F	
T	

Break

Conjunction

- Let p and q be propositions. The *conjunction* of p and q, denoted by $p \land q$, is the proposition "p and q."

- The conjunction $p \land q$ is true when both p and q are true and is false otherwise.

Conjunction		
p	q	p ∧ q
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \land q)$
Q	p

2	
P_1	P
Q	$p \land q$

3	
P_1	P
P_2	q
Q	$p \land q$

4	
P_1	$\neg(p \land q)$
P_2	P
Q	$\neg q$

5	
P_1	$\neg (p \land q)$
P_2	q
Q	$\neg p$

6	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	q

7	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	$\neg q$

Break

Disjunction

- Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition. "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Disjunction		
p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \wedge q)$
Q	$p \vee q$

2	
P_1	$p \vee q$
Q	$(p \wedge q)$

3	
P_1	$\neg(p \vee q)$
Q	$\neg p$

4	
P_1	$(p \vee q)$
Q	$\neg p$

5	
P_1	$(p \vee q)$
P_2	$\neg p$
Q	q

6	
P_1	$(p \vee q)$
P_2	$\neg q$
Q	p

7	
P_1	$(p \vee q)$
P_2	p
Q	$\neg q$

8	
P_1	$(p \vee q)$
P_2	p
Q	q

Break

Implication

1. Let p and q be propositions. The *conditional statement* $p \to q$ is the proposition "if p, then q". The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise.

2. In conditional statement $p \to q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion.

3. The statement $p \to q$ is called a conditional statement because $p \to q$ asserts that q is true on the condition that p holds.

Implication		
p	q	p → q
F	F	
F	T	
T	F	
T	T	

- Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

1. "If Maria learns discrete mathematics, then she will find a good job."

2. "Maria will find a good job when she learns discrete mathematics."

3. "For Maria to get a good job, it is sufficient for her to learn discrete mathematics."

Implication		
p	q	$P \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$P \to q$	$\neg p$	$\neg q$	$\neg P \to \neg q$	$q \to p$	$\neg q \to \neg p$	$\neg p \vee q$
F	F							
F	T							
T	F							
T	T							

1. $p \to q$ *implication*

2. $q \to p$ *converse*

3. $\neg p \to \neg q$ *inverse*

4. $\neg q \to \neg p$ *contra positive*

1. $p \rightarrow q = \neg q \rightarrow \neg p$

2. $p \rightarrow q$ will be true if either p is false or q is true, $p \rightarrow q = \neg p \vee q$

Q consider the following arguments and find which of them are valid?

Modus Ponens	
P_1	$p \to q$
P_2	P
Q	Q

Modus Tollens	
P_1	$p \to q$
P_2	$\neg Q$
Q	$\neg p$

1	
P_1	$\neg p$
Q	$p \to q$

2	
P_1	q
Q	$p \to q$

3	
P_1	$\neg(p \to q)$
Q	$\neg q$

4	
P_1	$\neg(p \to q)$
Q	p

Break

Bi-conditional

- Let *p* and *q* be propositions. The bi*conditional statement* $p \leftrightarrow q$ is the proposition.
  - " *p if and only* q".

- The biconditional statement $p \leftrightarrow q$ *is true when p and q have the same values, and false otherwise. Biconditional statements are also called bi-implications.*
  - "*p* is necessary and sufficient for *q*"
  - "if *p* then *q*, and conversely"
  - "*p* iff *q*."
  - $p \leftarrow\rightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

Bi-conditional		
p	q	$P \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
Q	$p \rightarrow r$

2	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow r$
Q	r

3	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow s$
Q	$r \vee s$

4	
P_1	$p \rightarrow r$
P_2	$q \rightarrow s$
P_3	$\neg r \vee \neg s$
Q	$\neg p \vee \neg q$

Q consider the following arguments and find which of them are valid?

5	
P_1	p
P_2	q
Q	r

6	
P_1	p
P_2	$\neg p$
Q	q

7	
P_1	
Q	q

Break

Type of cases

- **Tautology/valid: -** A propositional function which is always having truth in the last column, is called tautology. E.g. $\boldsymbol{p} \vee \neg \boldsymbol{p}$

p	¬ p	p ∨ ¬ p
F	T	
T	F	

- **<u>Contradiction/ Unsatisfiable</u>: -** A propositional function which is always having false in the last column, is called Contradiction. E.g. $\boldsymbol{p \land \neg p}$

p	¬ p	p ∧ ¬ p
F	T	
T	F	

- **<u>Contingency:</u>** - A propositional function which is neither a tautology nor a contradiction, is called Contingency. E.g. $\boldsymbol{p \vee q}$

p	q	p ∨ q
F	F	
F	T	
T	F	
T	T	

- <u>**Satisfiable**</u>**: -** A propositional function which is not contradiction is satisfiable. i.e. it must have at least one truth value in the final column e.g. $\boldsymbol{p} \vee \boldsymbol{q}$

- **Functionality Complete Set: -** A set of connectives is said to be functionally complete if it is able to write any propositional function.
  - $\{\wedge, \neg\}$
  - $\{\vee, \neg\}$

Break

Let p and q be two propositions. Consider the following two formulae in propositional logic. **(Gate-2021) (1 Marks)**

- $S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$
- $S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

A. Both S_1 and S_2 are tautologies.
B. S_1 is a tautology but S_2 is not a tautology
C. S_1 is not a tautology but S_2 is a tautology
D. Neither S_1 nor S_2 is a tautology

Choose the correct choice(s) regarding the following proportional logic assertion S: **(GATE- 2021)**

$$S : ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

A. S is neither a tautology nor a contradiction

B. S is a tautology

C. S is a contradiction

D. The antecedent of S is logically equivalent to the consequent of S

Q Let P, Q, R and S be Propositions.
Assume that the equivalences $P \Leftrightarrow (Q \lor \neg Q)$ and $Q \Leftrightarrow R$ hold.
Then the truth value of the formula $(P \land Q) \Rightarrow ((P \land R) \lor S)$ is always:
(NET-Jan-2017)

A) True

B) False

C) Same as truth table of Q

D) Same as truth table of S

Q Consider two well-formed formulas in propositional logic

$\mathbf{F_1}: P \Rightarrow \neg P$ $\mathbf{F_2}: (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which one of the following statements is correct? **(GATE-2001) (1 Marks)** **(NET-Jan-2017)**

A) F_1 is satisfiable, F_2 is valid

B) F_1 unsatisfiable, F_2 is satisfiable

C) F_1 is unsatisfiable, F_2 is valid

D) F_1 and F_2 are both satisfiable

Q Let p, q, and r be the propositions and the expression (p -> q) -> r be a contradiction. Then, the expression (r -> p)-> q is **(GATE-2017) (2 Marks)**

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

Q The first order logic (FOL) statement $((R \vee Q) \wedge (P \vee \neg Q))$ is equivalent to which of the following? **(NET-Jan-2017)**

A) $((R \vee \neg Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$

B) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$

C) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee \neg P))$

D) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (\neg R \vee P))$

Q the statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below? **(GATE-2017) (1 Marks)**

1) $p \Rightarrow q$ **2)** $q \Rightarrow p$ **3)** $(\neg q) \vee (p)$ **4)** $(\neg p) \vee q$

a) 1 only **b)** 1 and 4 only **c)** 2 only **d)** 2 and 3 only

Q Let P and Q be two propositions, $\neg (P \leftrightarrow Q)$ is equivalent to: **(NET-Jan-2017)**

(I) $P \leftrightarrow \neg Q$ **(II)** $\neg P \leftrightarrow Q$ **(III)** $\neg P \leftrightarrow \neg Q$ (IV) $Q \rightarrow P$

A) Only (I) and (II)

B) Only (II) and (III)

C) Only (III) and (IV)

D) None of the above

Q In propositional logic if $(P \to Q) \wedge (R \to S)$ and $(P \vee R)$ are two premises such that

$(P \to Q) \wedge (R \to S)$

$P \vee R$

....

Y

....

is the premise: **(NET-Jan-2017)**

A) $P \vee R$ **B)** $P \vee S$

C) $Q \vee R$ **D)** $Q \vee S$

Q consider the following expression: **(GATE-2016) (1 Marks)**

i) false **ii)** Q **iii)** true **iv)** $P \vee Q$ **v)** $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Longrightarrow Q)$ is___

Q Consider the statement, "Either $-2 \leq x \leq -1$ or $1 \leq x \leq 2$". The negation of this statement is **(NET-July-2016)**

A) $x < -2$ or $2 < x$ or $-1 < x < 1$

B) $x < -2$ or $2 < x$

C) $-1 < x < 1$

D) $x \leq -2$ or $2 < x$ or $-1 < x < 1$

Q The Boolean function $[\sim(\sim p \wedge q) \wedge \sim(\sim p \wedge \sim q)] \vee (p \wedge r)$ is equal to the Boolean function: **(NET-Aug-2016)**

a) q **b)** $p \wedge r$ **c)** $p \vee q$ **d)** p

Q Let p, q, r, s represents the following propositions. **(GATE-2016) (1 Marks)**

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ____________.

Q Consider the following logical inferences: **(NET-Aug-2016)**

I_1: If it is Sunday then school will not open. The school was open.
Inference: It was not Sunday.

I_2: If it is Sunday then school will not open. It was not Sunday.
Inference: The school was open.

Which of the following is correct?

A) Both I_1 and I_2 are correct inferences.
B) I_1 is correct but I_2 is not a correct inference.
C) I_1 is not correct but I_2 is a correct inference.
D) Both I_1 and I_2 are not correct inferences.

Q Consider the following two statements.

S_1: If a candidate is known to be corrupt, then he will not be elected.
S_2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?
(GATE-2015) (1 Marks)

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

Q Which of the following arguments are valid? **(NET-Dec-2015)**

(a) "If Gora gets the job and works hard, then he will be promoted. If Gora gets promotion, then he will be happy. He will not be happy, therefore, either he will not get the job or he will not work hard".

(b) "Either Puneet is not guilty or Pankaj is telling the truth. Pankaj is not telling the truth, therefore, Puneet is not guilty".

(c) If n is a real number such that $n>1$, then $n^2>1$. Suppose that $n^2>1$, then $n>1$.
Codes:

A) (a) and (c)
B) (b) and (c)
C) (a), (b) and (c)
D) (a) and (b)

Q “If my computations are correct and I pay the electric bill, then I will run out of money.

If I don’t pay the electric bill, the power will be turned off.

Therefore, if I don’t run out of money and the power is still on, then my computations are incorrect.”

Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively. (Where ¬ means NOT) **(NET-June-2015)**

A) if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$
B) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(r \wedge p) \rightarrow c$
C) if $(c \wedge b) \rightarrow r$ and $\neg p \rightarrow b$, then $(\neg r \vee p) \rightarrow \neg C$
D) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

Q In Propositional Logic, given P and P → Q, we can infer ____________. **(NET-June-2015)**

a) $\sim Q$ **b)** Q **c)** $P \wedge Q$ **d)** $\sim P \wedge Q$

Q Which one of the following is NOT equivalent to $p \leftrightarrow q$? **(GATE-2015) (1 Marks)**

a) $(\neg p \lor q) \land (p \lor \neg q)$

b) $(\neg p \lor q) \land (q \rightarrow p)$

c) $(\neg p \land q) \lor (p \land \neg q)$

d) $(\neg p \land \neg q) \lor (p \land q)$

Q In propositional logic $P \leftrightarrow Q$ is equivalent to (Where ~ denotes NOT) **(GATE-2015) (1 Marks)**

a) $\sim(P \vee Q) \wedge \sim(Q \vee P)$

b) $(\sim P \vee Q) \wedge (\sim Q \vee P)$

c) $(P \vee Q) \wedge (Q \vee P)$

d) $\sim(P \vee Q) \rightarrow \sim(Q \vee P)$

Q Consider the compound propositions given below as: **(NET-Dec-2015)**

(a) $p \vee \sim (p \wedge q)$ **(b)** $(p \wedge \sim q) \vee \sim (p \wedge q)$ **(c)** $p \wedge$ $(q \vee r)$

Which of the above propositions are tautologies?

A) (a) and (c) **B)** (b) and (c) **C)** (a) and (b) **D)** only (a)

Q In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following

"The result of the toss is head if and only if I am telling the truth"

Which of the following options is correct? **(Gate-2015)(2 Marks)**

a) The result is head

b) The result is tail

c) If the person is of Type 2, then the result is tail

d) If the person is of Type 1, then the result is tail

Q Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT? **(GATE-2014) (1 Marks)**

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

Q Which one of the following Boolean expressions is NOT a tautology? **(GATE-2014) (2 Marks)**

A) $((a \rightarrow b) \land (b \rightarrow c)) \rightarrow (a \rightarrow c)$

B) $(a \rightarrow c) \rightarrow (\sim b \rightarrow (a \land c))$

C) $(a \land b \land c) \rightarrow (c \lor a)$

D) $a \rightarrow (b \rightarrow a)$

Q Which one of the following propositional logic formulas is TRUE only when exactly two of p, q and r are TRUE? **(GATE-2014) (2 Marks)**

a) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

b) $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

c) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

d) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

Q Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**? **(GATE-2012) (1 Marks)**

(A) Both I_1 and I_2 are correct inferences
(B) I_1 is correct but I_2 is not a correct inference
(C) I_1 is not correct but I_2 is a correct inference
(D) Both I_1 and I_2 are not correct inferences

Q The proposition $\sim p \vee q$ is equivalent to **(NET-Dec-2011)**

(A) $p \rightarrow q$ **(B)** $q \rightarrow p$ **(C)** $p \leftrightarrow q$ **(D)** $p \vee q$

Q The binary operation $\odot$ is defined as follows

Which one of the following is equivalent to P∨Q? **(GATE-2009) (2 Marks)**

a) $(\sim Q \odot \sim P)$

b) $(P \odot \sim Q)$

c) $(\sim P \odot Q)$

d) $(\sim P \odot \sim Q)$

P	**Q**	**P$\odot$Q**
T	T	T
T	F	T
F	T	F
F	F	T

Q An example of a Tautology is: **(NET-June-2008)**

a) $x \vee y$

b) $x \vee \neg y$

c) $x \vee \neg x$

d) $(x \rightarrow y) \wedge (y \rightarrow x)$

Q P and Q are two propositions. Which of the following logical expressions are equivalent?

(GATE-2008) (2 Marks)

I) $P \vee \neg Q$

II) $\neg(\neg P \wedge Q)$

III) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

IV) $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

(A) Only I and II

(B) Only I, II and III

(C) Only I, II and IV

(D) All of I, II, III and IV

Q the Preposition $(p \rightarrow q) \wedge (\neg q \vee p)$ is equivalent to: **(NET-June-2006)**

a) $q \rightarrow p$

b) $p \rightarrow q$

c) $(q \rightarrow p) \wedge (p \rightarrow q)$

d) $(p \rightarrow q) \vee (q \rightarrow p)$

Q Consider the following propositional statements: **(GATE-2006) (2 Marks)**

$$P_1 : ((A \land B) \rightarrow C)) \equiv ((A \rightarrow C) \land (B \rightarrow C))$$

$$P_2 : ((A \lor B) \rightarrow C)) \equiv ((A \rightarrow C) \lor (B \rightarrow C))$$

Which one of the following is true?

(A) P_1 is a tautology, but not P_2

(B) P_2 is a tautology, but not P_1

(C) P_1 and P_2 are both tautologies

(D) Both P_1 and P_2 are not tautologies

Q A logical binary relation $\odot$, is defined as follows: **(GATE-2006) (2 Marks)**
Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$.
Which one of the following is equivalent to $A \wedge B$?

a) $(\sim A \odot B)$ **b)** $\sim(A \odot \sim B)$ **c)** $\sim(\sim A \odot \sim B)$ **d)** $\sim(\sim A \odot B)$

A	B	A $\odot$ B
T	T	T
T	F	T
F	T	F
F	F	T

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is **(NET-dec-2005)**

a) True

b) Multiple Values

c) False

d) Cannot be determined

Q Let P, Q and R be three atomic prepositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology? **(GATE-2005) (2 Marks)**

(A) $X \equiv Y$ **(B)** $X \rightarrow Y$ **(C)** $Y \rightarrow X$ **(D)** $\neg Y \rightarrow X$

Q The following propositional statement is **(GATE-2004) (2 Marks)**

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

(A) satisfiable but not valid

(B) valid

(C) a contradiction

(D) none of the above

Q Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\neg s \rightarrow q)$

Q: $[(\neg p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\neg q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \neg r)] \rightarrow q$

Which of the above arguments are valid? **(GATE-2004) (2 Marks)**

a) P and Q only

b) P and R only

c) P and S only

d) P, Q, R and S

Q Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \wedge b) \rightarrow (a \wedge c) \vee d$ is always. **(GATE-2003) (2 Marks)**

(A) True

(B) False

(C) Same as the truth value of b

(D) Same as the truth value of d

Q The following resolution rule is used in logic programming:
Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg R)$
Which of the following statements related to this rule is FALSE? **(GATE-2003) (2 Marks)**

(A) $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid

(B) $(P \vee Q) \Rightarrow ((P \vee R) \wedge (Q \vee \neg R))$ is logically valid

(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable

(D) $(P \vee Q) \Rightarrow$ FALSE if and only if both P and Q are unsatisfiable

Q "If X, then Y unless Z" is represented by which of the following formulae in propositional logic? **(GATE-2002) (1 Marks)**

(A) $(X \wedge \neg Z) \rightarrow Y$

(B) $(X \wedge Y) \rightarrow \neg Z$

(C) $(X \rightarrow (Y \wedge \neg Z)$

(D) $(X \rightarrow Y (\wedge \neg Z)$

Q Which of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\neg$ as NOT, $\rightarrow$ as one-way implication and $\leftrightarrow$ as two-way implication **(GATE-1996) (2 Marks)**

a) $((x \rightarrow y) \wedge x) \rightarrow y$

b) $((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x$

c) $(x \rightarrow (x \vee y))$

d) $((x \vee y) \leftrightarrow (\neg x \rightarrow \neg y))$

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is **(GATE-1995) (2 Marks)**

a) True

b) Multiple Values

c) False

d) Cannot be determined

Q The proposition $p \wedge (\sim p \vee q)$ is: **(GATE-1993) (1 Marks)**

a) a tautology

b) logically equivalent to $p \wedge q$

c) logically equivalent to $p \vee q$

d) a contradiction

e) none of the above

Q Which of the following is/are a tautology? **(GATE-1992) (1 Marks)**

a) $a \vee b \rightarrow b \wedge c$

b) $a \wedge b \rightarrow b \vee c$

c) $a \vee b \rightarrow (b \rightarrow c)$

d) $a \rightarrow b \rightarrow (b \rightarrow c)$

Q Indicate which of the following well-formed formulae are valid: **(GATE-1990) (2 Marks)**

a) $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$

b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

d) $(P \Rightarrow R) \vee (Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)$

Break

Q consider the following argument

I_1: if today is Gandhi ji's birthday, then today is oct 2^{nd}

I_2: today is oct 2^{nd}

C: today is Gandhi ji's birthday

Q consider the following argument

I_1: if Canada is a country, then London is a city

I_2: London is not a city

C: Canada is not a country

Q find which of the following arguments are valid?

1) $((p \vee q) \vee \neg p) = T$

2) $\neg (p \vee q) \vee (\neg p \wedge q) \vee p = T$

3) $((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)) \wedge r = r$

4) $(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r)) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r) = T$

5) $(p \vee \neg(p \wedge q)) = T$

6) $(p \wedge q) \wedge (\neg p \vee \neg q) = F$

7) $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r$

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
P_3	$\neg r$
Q	$\neg p$

2	
P_1	$r \rightarrow s$
P_2	$p \rightarrow q$
P_3	$r \vee p$
Q	$s \vee q$

3	
P_1	$(p \rightarrow (q \rightarrow s))$
P_2	$\neg r \vee p$
P_3	q
P_4	p
Q	s

4	
P_1	$(p \rightarrow (r \rightarrow s))$
P_2	$\neg r \rightarrow \neg p$
P_3	p
Q	s

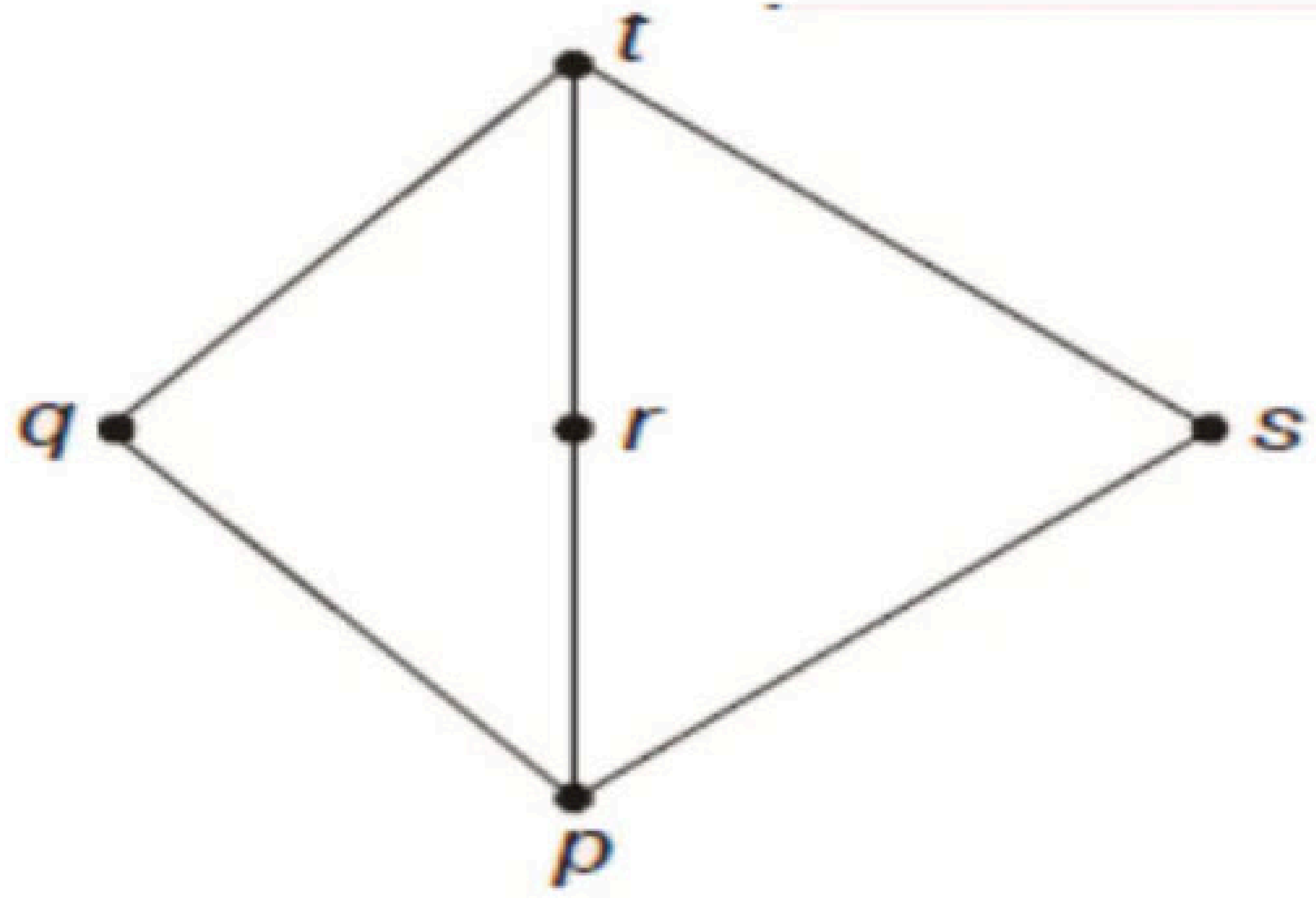
5	
P_1	$\neg p \rightarrow \neg r$
P_2	$\neg S$
P_3	$P \rightarrow w$
P_4	$R \vee s$
Q	W

6	
P_1	$\neg x \rightarrow y$
P_2	$\neg x \wedge \neg y$
Q	x

Break

Q Suppose $L = \{p, q, r, s, t\}$ is a lattice represented by the following Hasse diagram: For any $x, y \in L$, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let $L^3 = \{(x, y, z): x, y, z \in L\}$ be the set of all ordered triplets of the elements of L. Let P_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then **(GATE-2015) (2 Marks)**

(A) Pr = 0 **(B)** Pr = 1 **(C)** 0 < Pr ≤ 1/5 **(D)** 1/5 < Pr < 1



Q Consider the set S = {a, b, c, d}.
Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on
S: π_1={~~abcd~~}, π_2={~~ab~~, ~~cd~~}, π_3={~~abc~~, ~~d~~}, π_4={~~a~~, ~~b~~, ~~c~~, ~~d~~}
Let $\prec$ be the partial order on the set of partitions $S'=\{\pi_1,\pi_2,\pi_3,\pi_4\}$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The Poset diagram for $(S', \prec)$ is: **(GATE-2007) (2 Marks)**