Partial Order Relation - Part III

Course on Discrete Mathematics for GATE 2023

Sanchit Jain • Lesson 11 • Sept 13, 2022

Cartesian Product

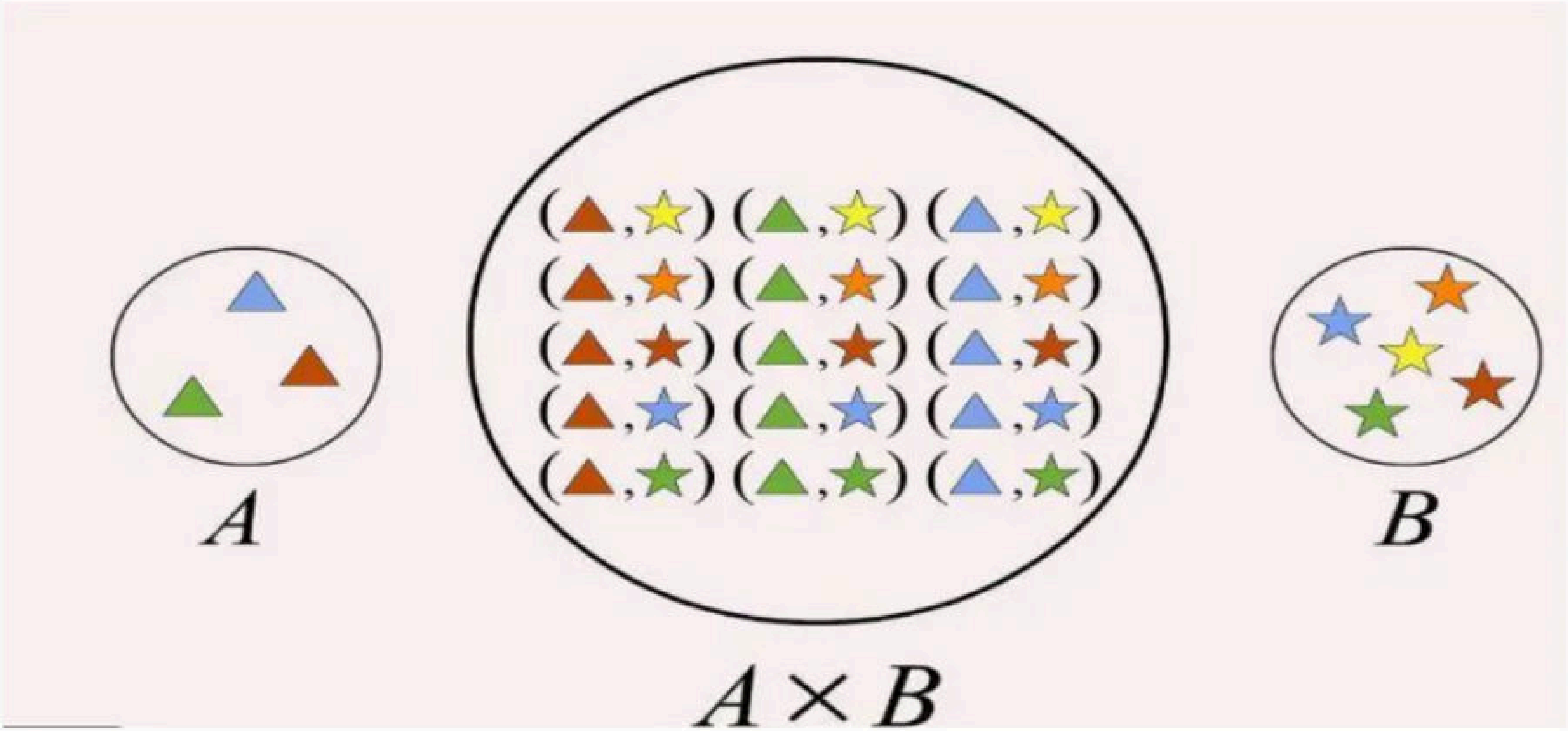
1. Cartesian Product of two sets A and B in the set of all ordered pairs, whose first member belongs to the first set and second member belongs to the second set, denoted by $A \times B$.

2. For E.g. if $A = \{a, b\}$, $B = \{1, 2, 3\}$

3. $A \times B = \{ \qquad \}$

- It is a kind of maximum relation possible, where every member of the first set belong to every member of the second set.

- $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$



1. In general, commutative law does not hold good $A\times B$!= $B\times A$

2. If $|A| = m$ and $|B| = n$ then $|A\times B| =$

Break

Relation

- **Relation**: - Let A and B are sets then every possible subset of 'A×B' is called a relation from A to B.

- If |A| =m and |B| = n then total no of element(pair) will be m*n, every element will have two choice weather to present or not present in the subset(relation), therefore the total number of relation possible is _________

For E.g. if $A = \{a, b\}$, $B = \{1, 2\}$

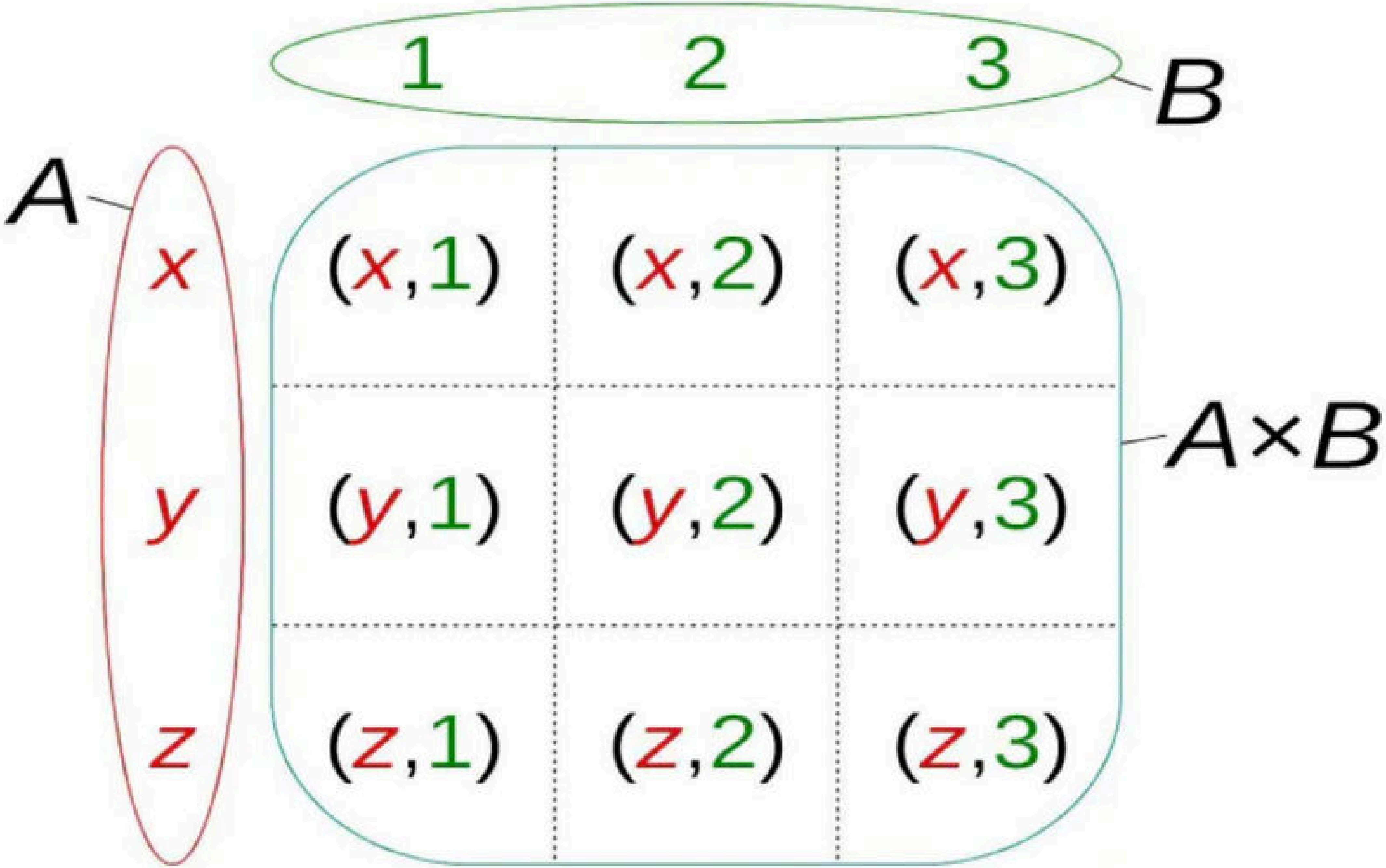
For E.g. if A = {a, b}, B = {1, 2}, A×B = {(a, 1), (a, 2), (b, 1), (b, 2)}

(a , 1)	(a , 2)	(b , 1)	(b , 2)	
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

For E.g. if A = {a, b}, B = {1, 2}

(a , 1)	(a , 2)	(b , 1)	(b , 2)	
0	0	0	0	{ }
0	0	0	1	{(b, 2)}
0	0	1	0	{(b, 1)}
0	0	1	1	{(b, 1), (b, 2)}
0	1	0	0	{(a, 2)}
0	1	0	1	{(a, 2), (b, 2)}
0	1	1	0	{(a, 2), (b, 1)}
0	1	1	1	{(a, 2), (b, 1), (b, 2)}
1	0	0	0	{(a, 1)}
1	0	0	1	{(a, 1), (b, 2)}
1	0	1	0	{(a, 1), (b, 1)}
1	0	1	1	{(a, 1), (b, 1), (b, 2)}
1	1	0	0	{(a, 1), (a, 2)}
1	1	0	1	{(a, 1), (a, 2), (b, 2)}
1	1	1	0	{(a, 1), (a, 2), (b, 1)}
1	1	1	1	{(a, 1), (a, 2), (b, 1), (b, 2)}

Matrix Representation



- Largest relation possible will be ________

- Smallest possible relation will be ________

Break

- **Complement of a relation**: - Let R be a relation from A to B, then the complement of relation will be denoted by R′, R^C or ~~R.~~

- $R' = \{(a, b) \mid (a, b) \in A \times B, (a, b) \in!\ R\}$

- $R' = (A \times B) - R$

- For E.g. if $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

- $R = \{(a, 1), (a, 3), (b, 2)\}$

- $R' = \{ \qquad \}$

$R \cup R' =$

$R \cap R' =$

Break

- **<u>Inverse of a relation</u>**: - Let R be a relation from A to B, then the inverse of relation will be a relation from B to A, denoted by R^{-1}.

- $R^{-1} = \{(b, a) \mid (a, b) \in R\}$

- $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

- $R = \{(a, 1), (a, 3), (b, 2)\}$

- $R^{-1} = \{ \qquad \}$

- $|R| \boxed{} |R^{-1}|$

- **Diagonal relation**: - A relation R on a set A is said to be diagonal relation if, R is a set of all ordered pair (x, x), for every $\forall x \in A$, sometimes it is also denoted by $\blacktriangle_A$

- $R = \{(x, x) \mid \forall\ x \in A\}$

	1	2	3
1	11		
2		22	
3			33

Q The number of binary relations on a set with n elements is: **(GATE-1999) (1 Marks)**

(A) n^2

(B) 2^n

(C) 2^{n^2}

(D) None of the above

Q Let A be a finite set of size n. The number of elements in the power set of A × A is: **(GATE-1993) (1 Marks)**

a) $2^{\wedge}(2^n)$

b) $2^{\wedge}(n^2)$

c) 2^n

d) None of the above

Break

Types of a Relation

- To further study types of relations, we consider a set A with n elements, then a cartesian product $A \times A$ will have n^2 elements(pairs). Therefore, total number of relation possible is $2^{n * n}$

- **Reflexive relation**: - A relation R on a set A is said to be reflexive,
- If $\forall x \in A$
  - $(x, x) \in R$

	1	**2**	**3**
1	11		
2		22	
3			33

Q consider a set A = {1,2,3}, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$		
2	ϕ		
3	{(1,1), (2,2), (3,3)}		
4	{(1,2), (2,3), (1,3)}		
5	{(1,1), (1,2), (2,1), (2,2)}		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}		
7	{(1,3), (2,1), (2,3), (3,2)}		

1. Smallest reflexive relation is ________

2. Largest reflexive relation is ________

3. Total number of reflexive relations will be _________

4. If two relations R_1 and R_2 are reflexive then their union and intersection will also be reflexive (T / F).

5. Any super set of reflexive relation will also be reflexive(T / F).

6. If a relation is reflexive then its inverse R^{-1} will also be reflexive (T / F).

1. Smallest reflexive relation is $\blacktriangle_A$

2. Largest reflexive relation is $A \times A$

3. Total number of reflexive relations will be $2^{n(n-1)}$

4. If two relations R_1 and R_2 are reflexive then their union and intersection will also be reflexive.

For E.g. if A = {a, b}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

Q What is the possible number of reflexive relations on a set of 5 elements? **(GATE-2010) (1 Marks)**

(A) 2^{10} **(B)** 2^{15} **(C)** 2^{20} **(D)** 2^{25}

Break

- **Irreflexive relation**: - A relation R on a set A is said to be Irreflexive,

1. If $\forall x \in A$
2. $(x, x) \notin R$

Q consider a set $A = \{1,2,3\}$, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$	Y	
2	ϕ	N	
3	$\{(1,1), (2,2), (3,3)\}$	Y	
4	$\{(1,2), (2,3), (1,3)\}$	N	
5	$\{(1,1), (1,2), (2,1), (2,2)\}$	N	
6	$\{(1,1), (2,2), (3,3), (1,3), (2,1)\}$	Y	
7	$\{(1,3), (2,1), (2,3), (3,2)\}$	N	

1. Smallest irreflexive relation is ________
2. Largest irreflexive relation is _______
3. Total number of irreflexive relation will be ________
4. If two relations R_1 and R_2 are Irreflexive then their union and intersection will also be Irreflexive (T / F).
5. If a relation R on a set A is reflexive, then R^C is Irreflexive, and vice versa (T / F).
6. Any sub set of irreflexive relation will also be irreflexive(T / F).
7. If a relation is irreflexive then its inverse R^{-1} will also be irreflexive (T / F).

For E.g. if $A = \{a, b\}$

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	
0	0	1	0	{(b, a)}
0	0	1	1	
0	1	0	0	{(a, b)}
0	1	0	1	
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

- Smallest irreflexive relation is ϕ
- Largest irreflexive relation is $(A \times A) - \blacktriangle_A$
- Total number of irreflexive relation will be $2^{n(n-1)}$
- If two relations R_1 and R_2 are Irreflexive then their union and intersection will also be Irreflexive.
- If a relation R on a set A is reflexive, then R^C is Irreflexive

Q Suppose that R_1 and R_2 are reflexive relations on a set A. Which of the following statements is correct? **(NET-July-2016)**

a) $R_1 \cap R_2$ is reflexive and $R_1 \cup R_2$ is irreflexive.

b) $R_1 \cap R_2$ is irreflexive and $R_1 \cup R_2$ is reflexive.

c) Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are reflexive.

d) Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are irreflexive

Break

- **<u>Symmetric relation</u>**: - A relation R on a set A is said to be Symmetric,

If $\forall\, a, b \in A$

$(a, b) \in R$

....

then $(b, a) \in R$

....

Q consider a set A = {1,2,3}, find which of the following relations are Symmetric, Anti-Symmetric and Asymmetric?

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	$A \times A$			
2	ϕ			
3	{(1,1), (2,2), (3,3)}			
4	{(1,2), (2,3), (1,3)}			
5	{(1,1), (1,2), (2,1), (2,2)}			
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}			
7	{(1,3), (2,1), (2,3), (3,2)}			

1. Smallest symmetric relation is ________
2. Largest symmetric relation is_______
3. Total number of symmetric relation will be _________
4. If a relation on a set A is symmetric then R __ R^{-1}
5. If two relations R_1 and R_2 are symmetric then their Union and Intersection will also be symmetric. (T / F)
6. If a relation is symmetric then its superset and subset will always be symmetric. (T / F)
7. If a relation is symmetric then its complement R^c will always be symmetric. (T / F)

For E.g. if $A = \{a, b\}$

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

1. Smallest symmetric relation is ϕ

2. Largest symmetric relation is $A \times A$

3. Total number of symmetric relation will be $2^{[n(n+1)]/2}$

4. If a relation on a set A is symmetric then $R = R^{-1}$

5. If two relations R1 and R2 are symmetric then their union and intersection will also be symmetric.

Q Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p-s = q-r$. Which one of the following is true about R? **(GATE-2015) (2 Marks)**

(A) Both reflexive and symmetric

(B) Reflexive but not symmetric

(C) Not reflexive but symmetric

(D) Neither reflexive nor symmetric

Q How many relations are there on a set with n elements that are symmetric and a set with n elements that are reflexive and symmetric? **(NET-Dec-2012)**

A) $2^{n(n+1)/2}$ and $2^n.3^{n(n-1)/2}$

B) $3^{n(n-1)/2}$ and $2^{n(n-1)}$

C) $2^{n(n+1)/2}$ and $3^{n(n-1)/2}$

D) $2^{n(n+1)/2}$ and $2^{n(n-1)/2}$

Break

- **Anti-Symmetric relation**: - A relation R on a set A with cartesian product A×A is said to be Anti-Symmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

(b, a) $\in$ R

....

a = b

....

Conclusion: Symmetry is not allowed but diagonal pairs are allowed

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	$A \times A$	Y		
2	ϕ	Y		
3	{(1,1), (2,2), (3,3)}	Y		
4	{(1,2), (2,3), (1,3)}	N		
5	{(1,1), (1,2), (2,1), (2,2)}	Y		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N		
7	{(1,3), (2,1), (2,3), (3,2)}	N		

1. Smallest Anti-symmetric relation is _________

2. Largest Anti-symmetric relation will contain ________elements

3. Total number of Anti-symmetric relation will be _________

4. A relation R on a set A is Anti-Symmetric if $(R \cap R^{-1}) \sqsubseteq \blacktriangle_A$ (T / F)

5. Sub set of a Anti-Symmetric will also be (T / F)

6. Super set of a Anti-Symmetric will also be (T / F)

7. If two relations R_1 and R_2 are Anti-symmetric then their _________need not to be Anti-symmetric but _________ will also be Anti-symmetric.

8. If a relation is Anti-symmetric then its complement R^c will always be Anti-symmetric. (T / F)

For E.g. if $A = \{a, b\}$

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	
0	1	1	1	
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	
1	1	1	1	

1. Smallest Anti-symmetric relation is ϕ
2. Largest Anti-symmetric relation will contain $n(n+1)/2$ elements
3. Total number of Anti-symmetric relation will be $2^n * 3^{[n(n-1)]/2}$
4. A relation R on a set A is Anti-Symmetric if $(R \cap R^{-1}) \sqsubseteq \blacktriangle_A$
5. Sub set of a Anti-Symmetric will also be Anti-Symmetric
6. If two relations R1 and R2 are Anti - symmetric then their union need not to be Anti-symmetric but intersection will also be Anti-symmetric.

Q Consider a set $A = \{a, b, c\}$ and R_1, R_2, R_3 and R_4 are relations on A which of the following is not true?

		Symmetric	Anti-Symmetric	True
1	$R_1 = \{(a, a), (c, c)\}$	Y	Y	
2	$R_2 = \{(a, b), (b, a), (a, c)\}$	N	N	
3	$R_3 = \{(a, b), (b, c), (a, c)\}$	N	Y	
4	$R_4 = \{(a, b), (b, a), (c, c)\}$	Y	N	

Q Consider the binary relation $R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$. Which one of the following is TRUE? **(GATE-2009) (1 Marks)**

(A) R is symmetric but NOT antisymmetric

(B) R is NOT symmetric but antisymmetric

(C) R is both symmetric and antisymmetric

(D) R is neither symmetric nor antisymmetric

Break

- **<u>Asymmetric relation</u>**: - A relation R on a set A is said to be Asymmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

………………………………

(b, a) $\notin$ R

………………………………

Conclusion: Symmetry is not allowed; even diagonal pairs are not allowed

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	$A \times A$	Y	N	
2	ϕ	Y	Y	
3	{(1,1), (2,2), (3,3)}	Y	Y	
4	{(1,2), (2,3), (1,3)}	N	Y	
5	{(1,1), (1,2), (2,1), (2,2)}	Y	N	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N	Y	
7	{(1,3), (2,1), (2,3), (3,2)}	N	N	

1. Smallest Asymmetric relation is ______

2. Largest Asymmetric relation will contain _________elements

3. Total number of Asymmetric relation will be ________

4. Every asymmetric relation is also anti-symmetric (T / F)

5. Sub set of a Asymmetric will also be Asymmetric (T / F)

6. Super set of a Asymmetric will also be Asymmetric(T / F)

7. If two relations R_1 and R_2 are Asymmetric then their Union will also be Asymmetric(T / F).

8. If two relations R_1 and R_2 are Asymmetric then their Intersection will also be Asymmetric(T / F).

9. If a relation is Asymmetric then its complement R^c will always be Asymmetric. (T / F)

For E.g. if $A = \{a, b\}$

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	
0	0	1	0	{(b, a)}
0	0	1	1	
0	1	0	0	{(a, b)}
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

1. Every asymmetric relation is also anti-symmetric
2. Smallest Asymmetric relation is ϕ
3. Largest Asymmetric relation will contain n(n-1)/2 elements
4. Total number of Asymmetric relation will be $3^{[n(n-1)]/2}$

Break

- **Transitive relation**: - A relation R on a set A is said to be Transitive,

If $\forall$ a, b, c $\in$ A

(a, b) $\in$ R

(b, c) $\in$ R

....

(a, c) $\in$ R

....

	Relation	Transitive
1	$A \times A$	
2	ϕ	
3	{(1,1), (2,2), (3,3)}	
4	{(1,2), (2,3), (1,3)}	
5	{(1,1), (1,2), (2,1), (2,2)}	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	
7	{(1,3), (2,1), (2,3), (3,2)}	
8	{(1,2)}	
9	{(1,3), (2,3)}	
10	{(1,2), (1,3)}	
11	{(2,3), (1,2)}	

1. Smallest Asymmetric relation is ______

2. Largest Asymmetric relation will contain __________ elements

3. If two relations R_1 and R_2 are Transitive then their _______ need not to be transitive but ___________ will also be transitive.

For E.g. if $A = \{a, b\}$

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	{(a, b), (b, a)}
0	1	1	1	{(a, b), (b, a), (b, b)}
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	{(a, a), (a, b), (b, a)}
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)	
0	0	0	0	{ }
0	0	0	1	{(b, b)}
0	0	1	0	{(b, a)}
0	0	1	1	{(b, a), (b, b)}
0	1	0	0	{(a, b)}
0	1	0	1	{(a, b), (b, b)}
0	1	1	0	
0	1	1	1	
1	0	0	0	{(a, a)}
1	0	0	1	{(a, a), (b, b)}
1	0	1	0	{(a, a), (b, a)}
1	0	1	1	{(a, a), (b, a), (b, b)}
1	1	0	0	{(a, a), (a, b)}
1	1	0	1	{(a, a), (a, b), (b, b)}
1	1	1	0	
1	1	1	1	{(a, a), (a, b), (b, a), (b, b)}

1. Smallest Asymmetric relation is ϕ

2. Largest Asymmetric relation will contain $A \times A$ elements

3. If two relations R_1 and R_2 are Transitive then their union need not to be transitive but intersection will also be transitive.

$\lvert A \rvert = n$	No of transitive relation
0	1
1	2
2	13
3	171
4	3994

Warshall's Algorithm: -

Q Consider a set A = {1,2,3} and a relation **R** = {(1,1), (1,3), (2,2), (3,1), (3,2)}?

	1	2	3
Column			
Row			

	1	2	3
1			
2			
3			

Q Consider a set A = {1,2,3,4} and a relation

R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}?

	1	2	3	4
Column				
Row				

	1	2	3	4
1				
2				
3				
4				

Warshall's Algorithm: -

Q Consider a set A = {1,2,3} and a relation **R** = {(1,1), (1,3), (2,2), (3,1), (3,2)}?

	1	2	3
Column			
Row			

	1	2	3
1			
2			
3			

Break

Q A binary relation R on N x N is defined as follows:

(a, b) R (c, d) if a <= c or b <= d

Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is TRUE? **(GATE- 2016) (2 Marks)**

(A) Both P and Q are true.

(B) P is true and Q is false.

(C) P is false and Q is true.

(D) Both P and Q are false.

Q Let R be the relation on the set of positive integers such that a R b if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is True? **(GATE-2015) (1 Marks)**

(A) R is symmetric and reflexive but not transitive

(B) R is reflexive but not symmetric and not transitive

(C) R is transitive but not reflexive and not symmetric

(D) R is symmetric but not reflexive and not transitive

Q The relation “divides” on a set of positive integers is _________. **(NET-June-2013)**

a) Symmetric and transitive

b) Anti symmetric and transitive

c) Symmetric only

d) Transitive only

Q A relation **R** in {1, 2,3,4,5,6} is given by {(1,2), (2,3), (3,4), (4,4), (4,5)}. This relation is: **(NET-Dec-2008)**

a) Reflexive **b)** Symmetric

c) Transitive **d)** not reflexive, not symmetric and not transitive

Q the transitive closure of a relation R on a set A whose relation matrix $\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$ is:

(NET-June-2005)

a) $\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$ **b)** $\begin{matrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{matrix}$ **c)** $\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$ **d)** $\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{matrix}$

Q The binary relation S = ϕ (empty set) on set A = {1,2,3} is **(GATE-2002) (2 Marks)**

(a) Neither reflexive nor symmetric

(b) Symmetric and reflexive

(c) Transitive and reflexive

(d) Transitive and symmetric

Q The binary relation R= {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)} on the set A = {1,2,3,4} is **(GATE-1998) (2 Marks)**

(a) reflexive, symmetric and transitive

(b) neither reflexive, nor irreflexive but transitive

(c) irreflexive, symmetric and transitive

(d) irreflexive and antisymmetric

Q The transitive closure of the relation {(1,2), (2,3), (3,4), (5,4)} on the set {1,2,3,4,5} is ____________. **(GATE-1989) (2 Marks)**

Break

- **<u>Equivalence Relation</u>**: - A relation R on a set A with cartesian product $A \times A$ is said to be Equivalence, if it is

1. **Reflexive**
2. **Symmetric**
3. **Transitive**

- If two relations R_1 and R_2 are Equivalence then their union need not to be equivalence but intersection will also be Equivalence.

R_1 : (a, b) iff (a + b) is even over the set of integers

R_2 : (a, b) iff (a + b) is odd over the set of integers

R_3 : (a, b) iff a x b > 0 over the set of non-zero rational numbers

R_4 : (a, b) iff $|a - b| \leq 2$ over the set of natural numbers

Break

Q Which of the relations on {0, 1, 2, 3} is an equivalence relation? **(NET-July-2018)**

a) {(0, 0) (0, 2) (2, 0) (2, 2) (2, 3) (3, 2) (3, 3)}

b) {(0, 0) (1, 1) (2, 2) (3, 3)}

c) {(0, 0) (0, 1) (0, 2) (1, 0) (1, 1) (1, 2) (2, 0)}

d) {(0, 0) (0, 2) (2, 3) (1, 1) (2, 2)}

Q Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are **(GATE-2007) (1 Marks)**

(A) n and n

(B) n^2 and n

(C) n^2 and 0

(D) n and 1

Q Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE? **(GATE-2005) (2 Marks)**

(A) $R \cup S$, $R \cap S$ are both equivalence relations

(B) $R \cup S$ is an equivalence relation

(C) $R \cap S$ is an equivalence relation

(D) Neither $R \cup S$ nor $R \cap S$ is an equivalence relation

Q Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is? **(GATE-1998) (1 Marks)**

(a) n **(b)** n^2 **(c)** 1 **(d)** $n + 1$

Q Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions **(GATE-1998) (1 Marks)**

(i) $R_1 \cup R_2$ is an equivalence relation

(ii) $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct?

a) Both assertions are true

b) Assertions (i) is true but assertions (ii) is not true

c) Assertions (ii) is true but assertions (i) is not true

d) Neither (i) nor (ii) is true

Q Let R be a non-empty relation on a collection of sets defined by A R B if and only if $A \cap B = \phi$. Then, (pick the true statement) **(GATE-1996) (2 Marks)**

(a) R is reflexive and transitive

(b) R is symmetric and not transitive

(c) R is an equivalence relation

(d) R is not reflexive and not symmetric

Break

1. In mathematics, when the elements of some set S have a notion of equivalence (formalized as an equivalence relation) defined on them, then one may naturally split the set S into **equivalence classes**.

2. These equivalence classes are constructed so that elements a and b belong to the same **equivalence class** if and only if they are equivalent.

- **Equivalence Class:** - of an element is denoted by [x].
  $[x] = \{y \mid y \in A \text{ and } (x, y) \in R\}$ for all $x \in A$

- We can have $[x] = [y]$, even if $x \neq y$

Q Consider A = {1, 2, 3, 4, 5} an equivalence relation R on A, R = {(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)} find the partition of a set A, defined by R.

[1] =

[2] =

[3] =

[4] =

[5] =

<u>Partitions of a Set</u>: - let A be a set, with n elements. Based on our understanding of equivalent classes, a subdivision of A into non-empty and non-overlapping subset is called a partition of A.

$A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots\ldots\ldots \cup A_n = A$

$A_1 \cap A_2 \cap A_3 \cap \ldots\ldots\ldots\ldots\ldots \cap A_n = \phi$

Q Consider $A = \{1, 2, 3, 4, 5\}$ an equivalence relation R on A, $R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)\}$ find the partition of a set A, defined by R.

$[1] = \{1, 4\}$

$[2] = \{2, 5\}$

$[3] = \{3\}$

$[4] = \{1,4\}$

$[5] = \{2, 5\}$

so we have partitions =

Q Let A = {1,2,3,4,5} is a set having partitions as {1, 4}, {2, 3, 5} , find the equivalence relation from which these partitions are created?

Q A relation R is defined on the set of integers as x Ry iff (x + y) is even. Which of the following statements is true? **(GATE-2000) (2 Marks)**

(a) R is not an equivalence relation

(b) R is an equivalence relation having 1 equivalence class

(c) R is an equivalence relation having 2 equivalence classes

(d) R is an equivalence relation having 3 equivalence classes

Break

Q How many different equivalence relations with exactly three different equivalence classes are there on a set with five elements? **(NET-July-2016)**

(A) 10 **(B)** 15 **(C)** 25 **(D)** 30

Q The number of equivalence relations of the set {1,2,3,4} is **(GATE-1997) (1 Marks)**

a) 15 **b)** 16 **c)** 24 **d)** 4

Break

- <u>**Partial Order Relation**</u>: - A relation R on a set A with cartesian product A×A is said to be partial order, if it is

1. **Reflexive**
2. **Anti - Symmetric**
3. **Transitive**

1. In mathematics, especially order theory, a **partially ordered set** (also **poset**) formalizes and generalizes the intuitive concept of an ordering, sequencing, or arrangement of the elements of a set.

2. A poset consists of a set together with a binary relation indicating that, for certain pairs of elements in the set, one of the elements precedes the other in the ordering.

3. The relation itself is called a "partial order." The word *partial* in the names "partial order" and "partially ordered set" is used as an indication that not every pair of elements needs to be comparable.

4. That is, there may be pairs of elements for which neither element precedes the other in the poset. Partial orders thus generalize total orders, in which every pair is comparable.

- **<u>Partial ordering set (Poset):</u>** - a set A with partial ordering relation R defined on A is called a POSET and is denoted by [A, R]

- For e.g. [A, /], [A, <=], [P(S), $\sqsubseteq$]

- **Total order relation:** - A Poset [A, R] is called a total order set, if every pair of elements are comparable i.e. either (a, b) or (b, a) $\in$ R, for $\forall$ a, b $\in$ A

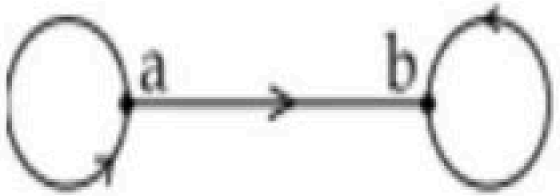
- For e.g. A = {1, 2, 3, 6}, then Poset [A,/] is not a total order relation but A = {1,2,4,8} will be

Q Which of the following statements is true? **(NET-July-2018)**

a) (Z, ≤) is not totally ordered —22 F

b) The set inclusion relation ⊆ is a partial ordering on the power set of a set S —49 T

c) ~~(Z, ≠) is a Poset~~ —16

d) The directed graph 

is not a partial order —14

(a,a) (b,b) (a,b)

$a \leq b$

$b \leq a$

$a \neq a$

Q A partial order P is defined on the set of natural numbers as follows. Here x/y denotes integer division. **(GATE-2007) (2 Marks)**

(1) $(0,0) \in P$ **(2)** $(a, b) \in P$ if and only if $a \% 10 <= b \% 10$ and $(a/10, b/10) \in P$.

Consider the following ordered pairs:

(i) (101,22) X **(ii)** (22,101) ✓ **(iii)** (145,265) **(iv)** (0,153)

~~**a)** I & iii~~ 1 33 ~~**b)** ii & iv~~ 18 ~~**c)** I & iv~~ 18 ✓ **d)** iii & iv 32

$(101,22) \rightarrow (10,2) \in P \rightarrow (1,0) \in P$

$1 \leq 2$ $0 \leq 2$ $1 \leq 0$ X

$(145,265) \rightarrow (14,26) \in P \rightarrow (1,2) \in P \rightarrow (0,0) \in P$

$5 \leq 5$ $4 \leq 6$ $1 \leq 2$

Q A relation R is defined on ordered pairs of integers as follows: (x, y) R (u, v) if x < u and y > v. Then R is **(GATR-2006) (1 Marks)**

~~(A)~~ Neither a Partial Order nor an Equivalence Relation — 75

(B) A Partial Order but not a Total Order — 12

(C) A Total Order — 6

(D) An Equivalence Relation — 6

((x, y), (u, v))

$x < u$ $y > v$

(5, 3) (5, 3)

Q let R_1 be a relation from A = {1,3,5,7} to B = {2,4,6,8} and R_2 be another relation from B to C = {1,2,3,4} as defined below **(GATE-2004) (1 Marks)**

(i) an element x in A is related to an element y in B if x + y is divisible by 3

(ii) an element x in B is related to an element y in C if x + y is even but not divisible by 3.

Which is the composite relation R_1R_2 from A to C?

29

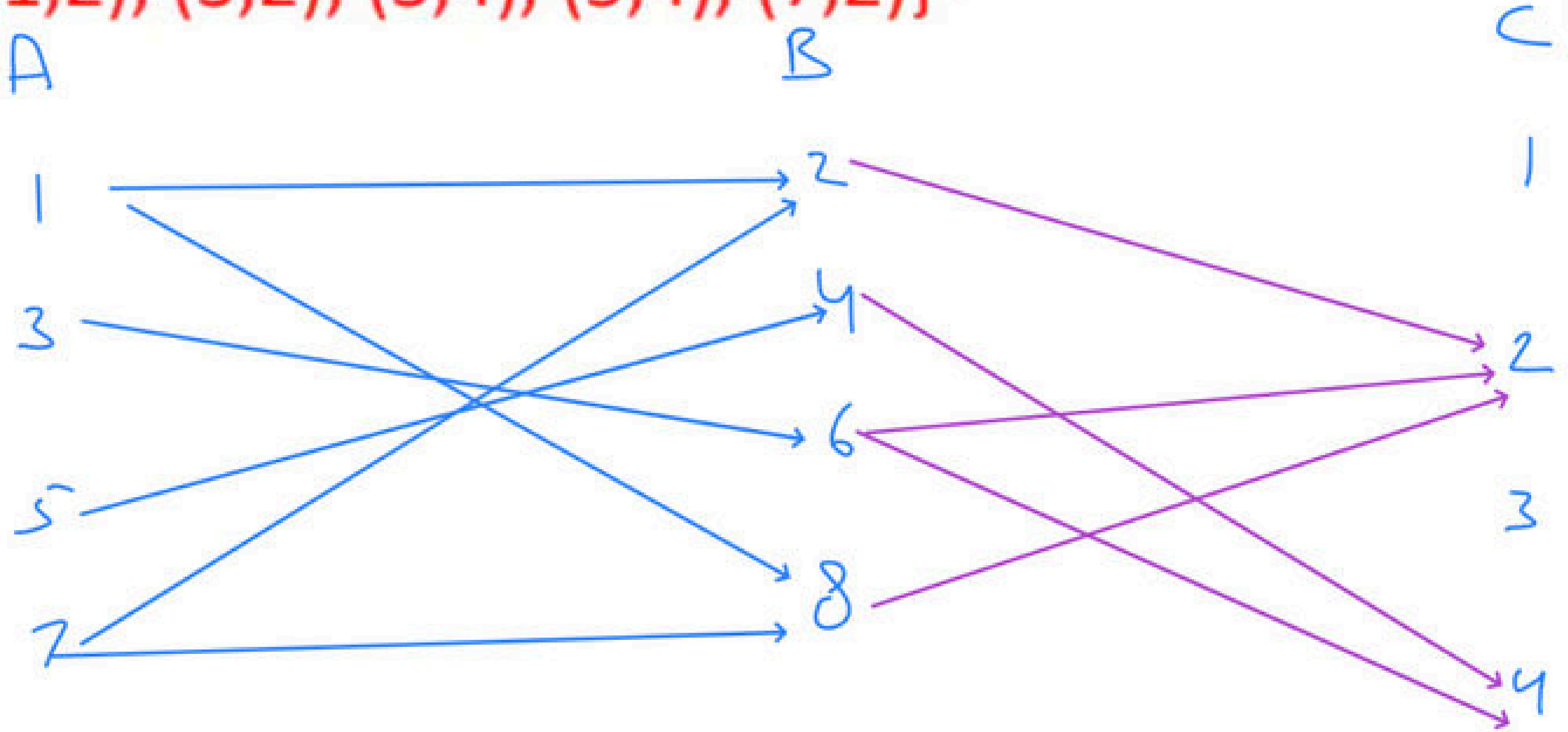
a) {(1,2), (1,4), (3,3), (5,4), (7,3)} – ''

b) {(1,2), (1,3), (3,2), (5,2), (7,3)}

~~**c)** {(1,2), (3,2), (3,4), (5,4), (7,2)}~~ – 50

d) {(3,2), (3,4), (5,1), (5,3), (7,1)} ''

(1,2) . (3,2) (3,4) (5,4), (7,2)



A relation R is said to be circular if $a\mathrm{R}b$ and $b\mathrm{R}c$ together imply $c\mathrm{R}a$. **(Gate-2021) (1 Marks)**

Which of the following options is/are correct?

A. If a relation S is reflexive and symmetric, then S is an equivalence relation.
B. If a relation S is circular and symmetric, then S is an equivalence relation.
C. If a relation S is reflexive and circular, then S is an equivalence relation.
D. If a relation S is transitive and circular, then S is an equivalence relation.

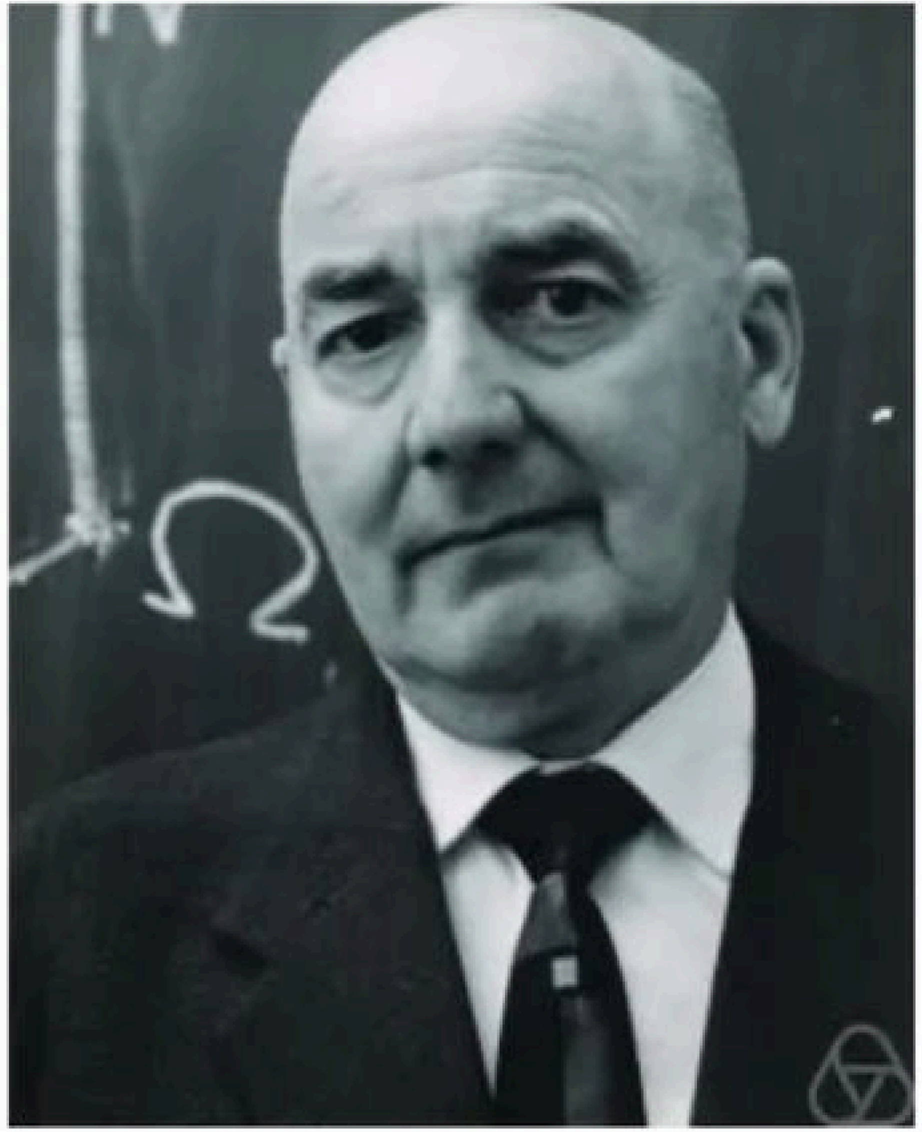
Conversion of POSET into a Hasse Diagram

- If we want to study Partial order relation further then it will be better to convert it into more convenient notation so that it can be studied easily.

- This graphical representation is called Hasse Diagram

- In order theory, a Hasse diagram is a type of mathematical diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction.

- The diagrams are named after Helmut Hasse (1898–1979)



Steps to convert partial order relation into hasse diagram

1- Draw a vertex for each element in the Set

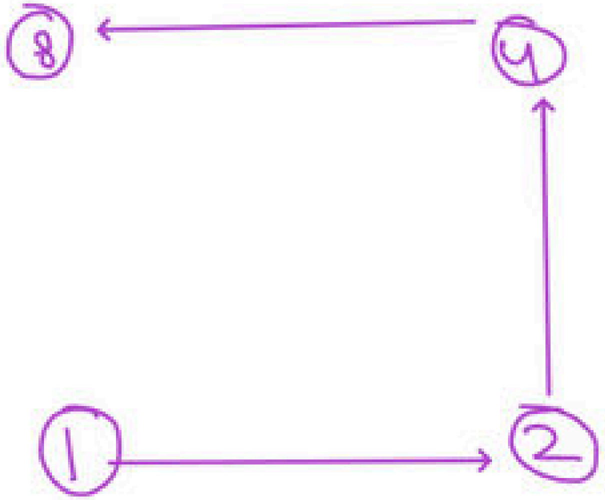
2- If (a, b) ϵ R then draw an edge from a to b

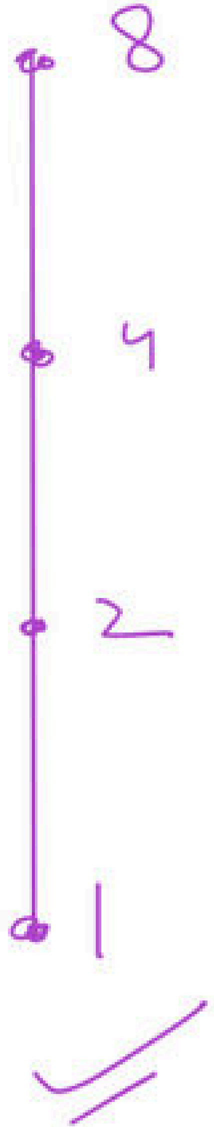
3- Remove all Reflexive and Transitive edges

4- Remove the direction of edges and arrange them in the increasing order of heights.

Q Consider a Partial order relation and convert it into hasse diagram?

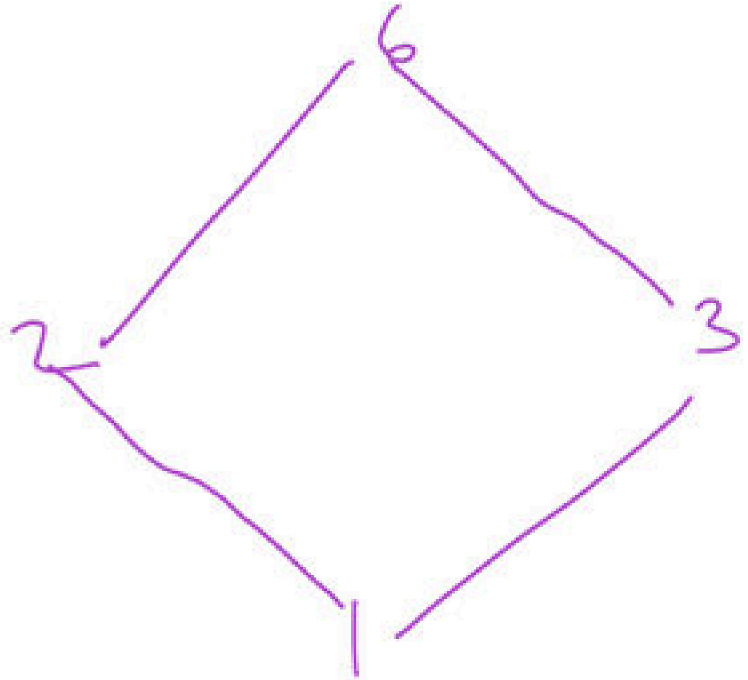
R = {(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)}





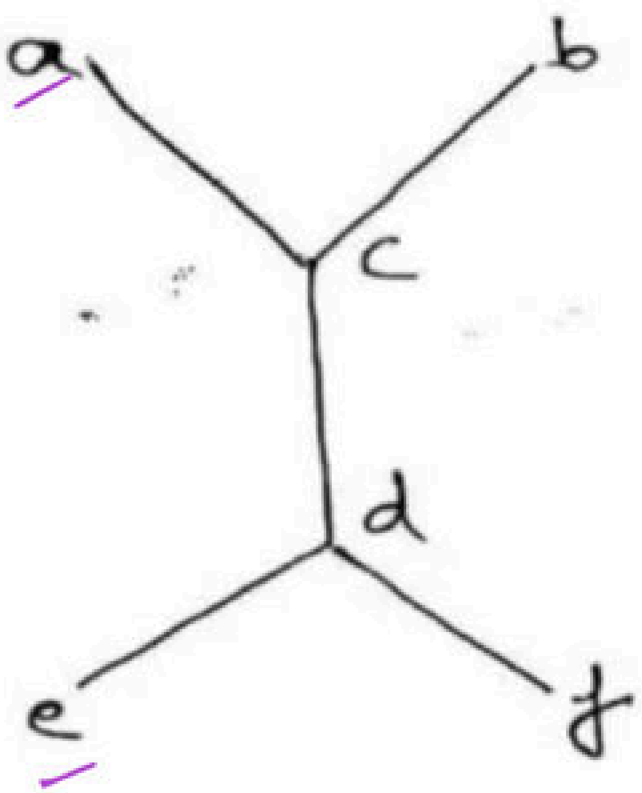
Q Consider a Partial order relation and convert it into hasse diagram?

R = {(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)}

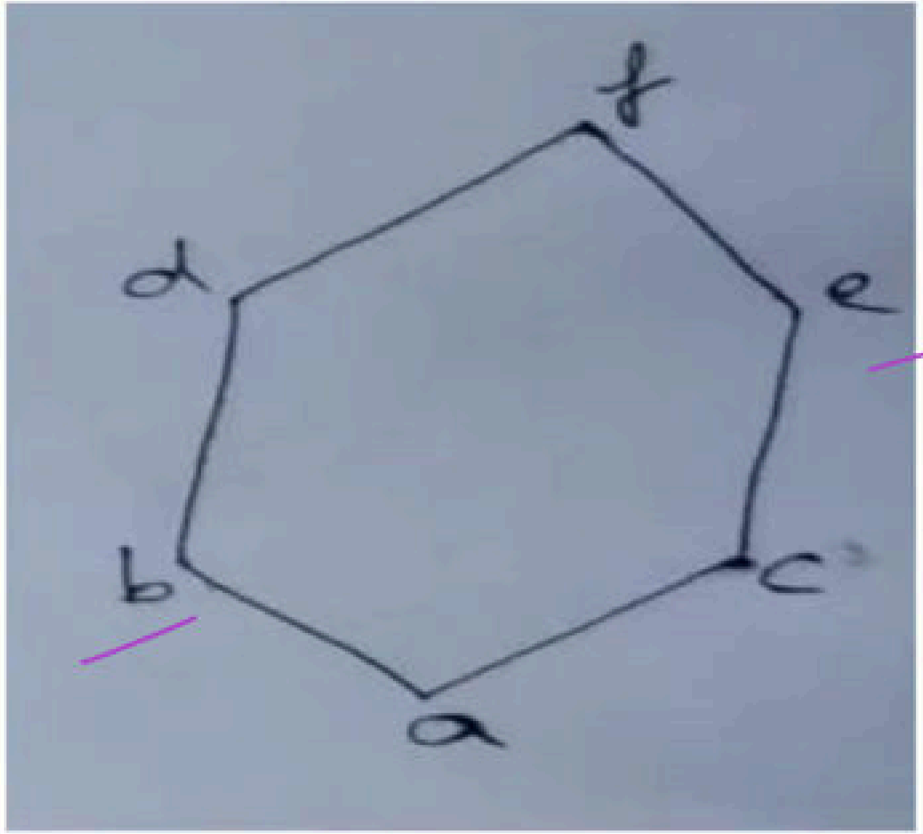


Break

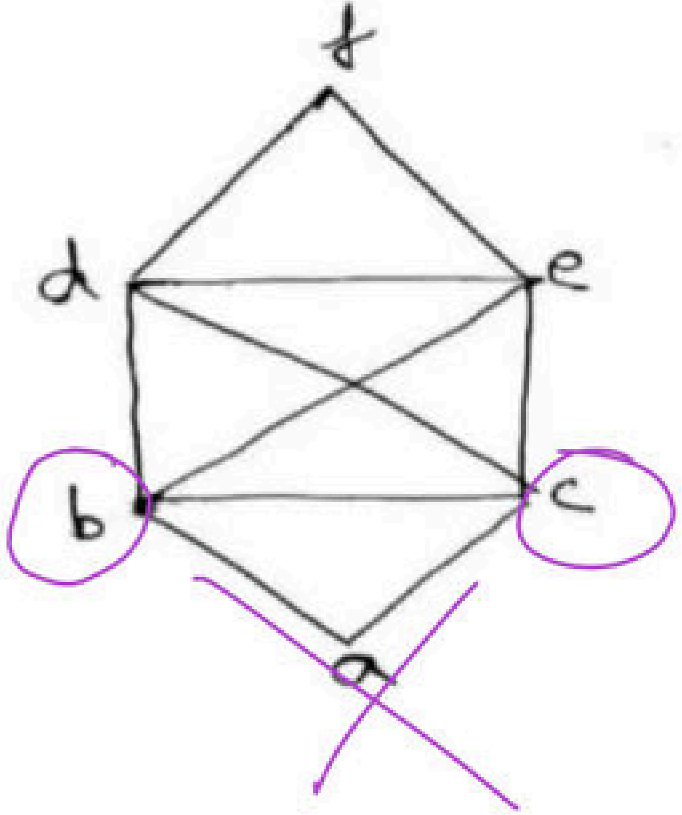
Q Study the following hasse diagram and find which of the following are valid?



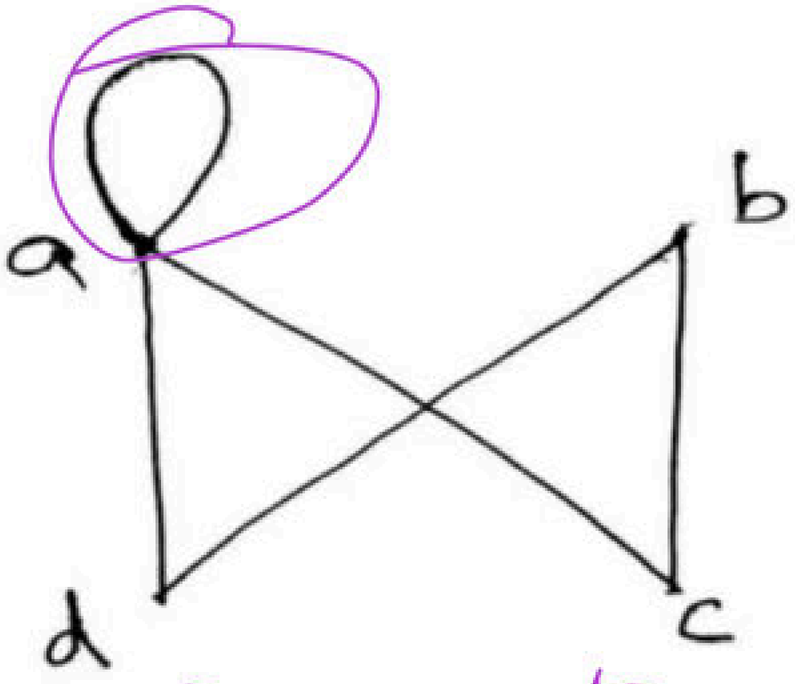
(1)



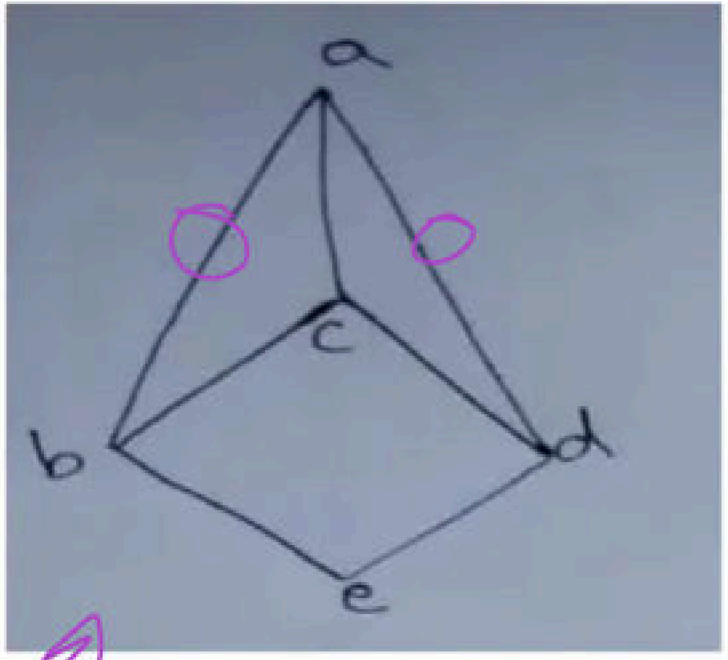
(2)



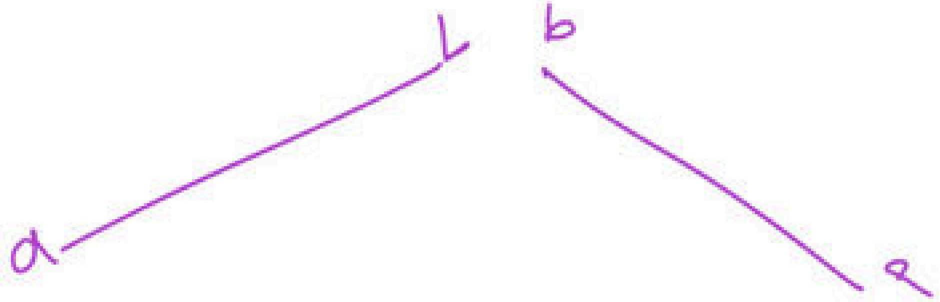
(3)

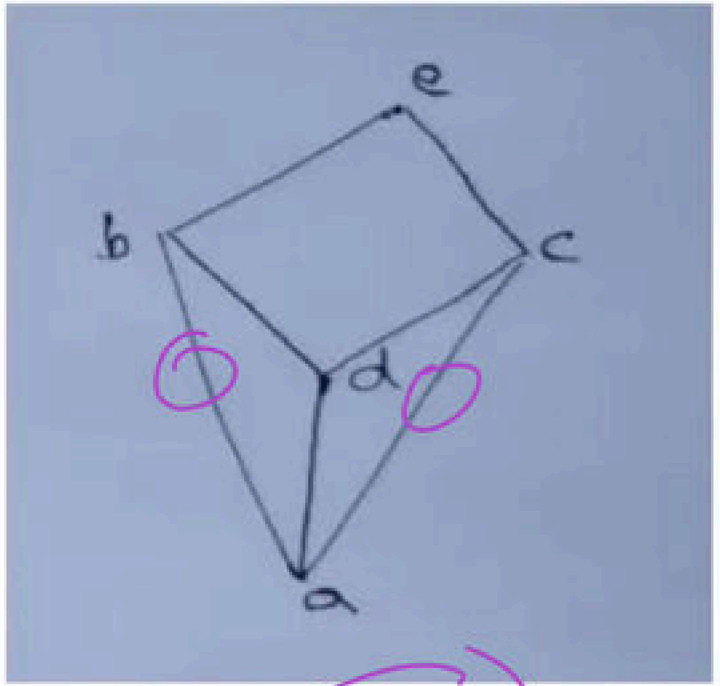


(4)

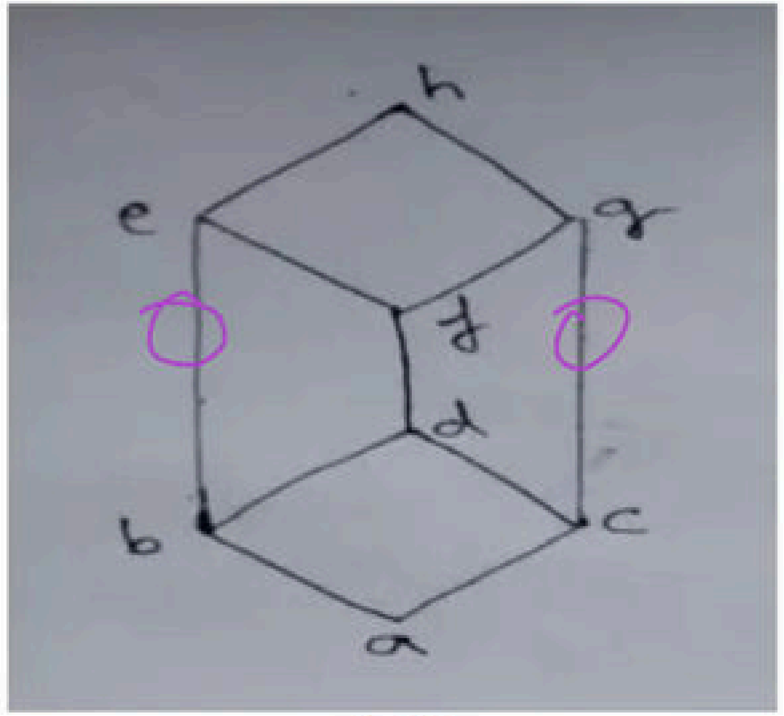


(5)

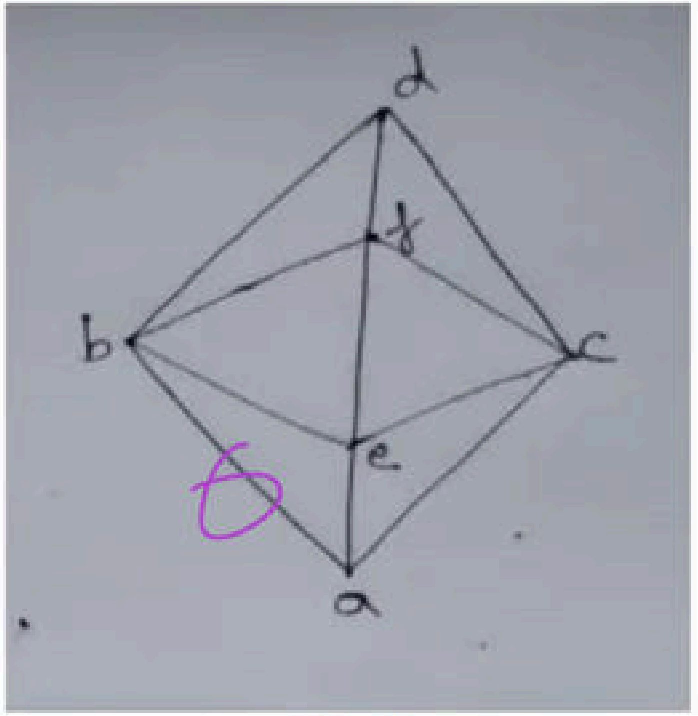




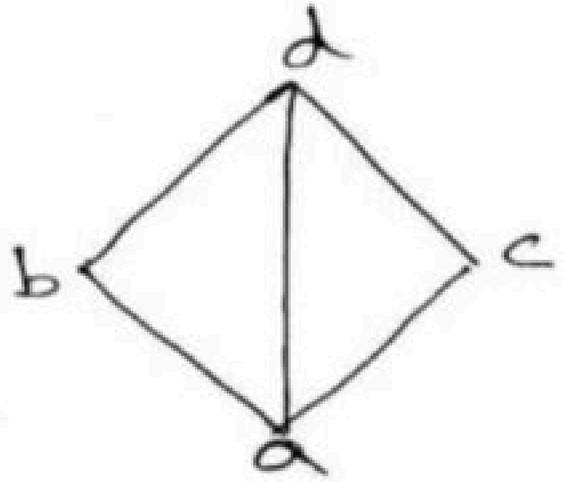
(5)



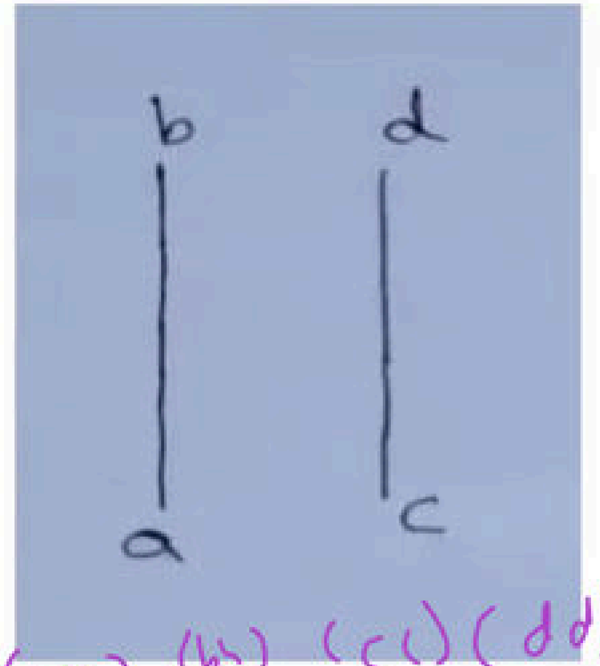
(6)



(7)



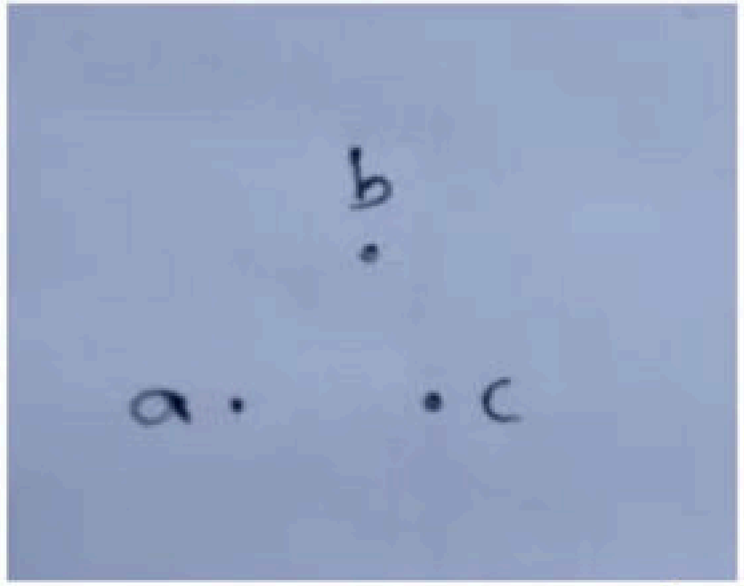
(8)



(aa) (bb) (cc) (dd)

(9)

(ab) (cd)



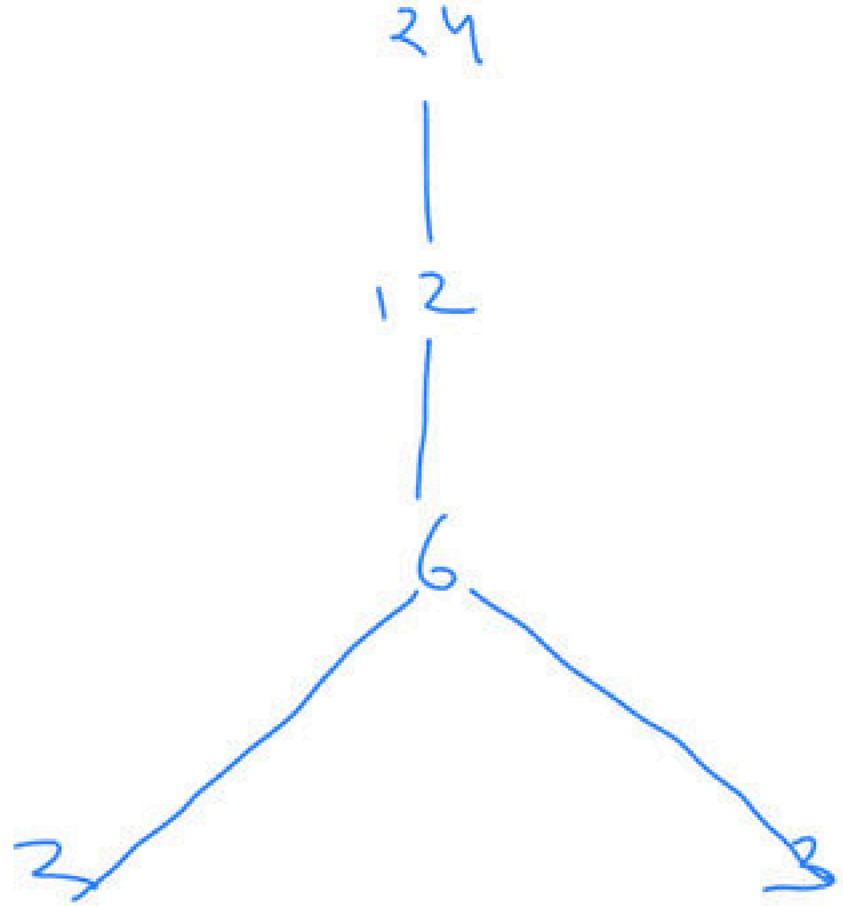
(10) (aa) (bb) (cc)

Conclusion

- We can not have a horizontal edge in a hasse diagram
- We can not have a reflexive and transitive edge in Hasse Diagram

Q Let X = {2,3,6,12,24}, Let ≤ be the partial order defined by X ≤ Y if x divides y. Number of edges as in the Hasse diagram of (X, ≤) is. **(GATE-1996) (1 Marks)**

(a) 3 — 1 **(b)** 4 — 77 **(c)** 9 — 13 **(d)** None of the above 9

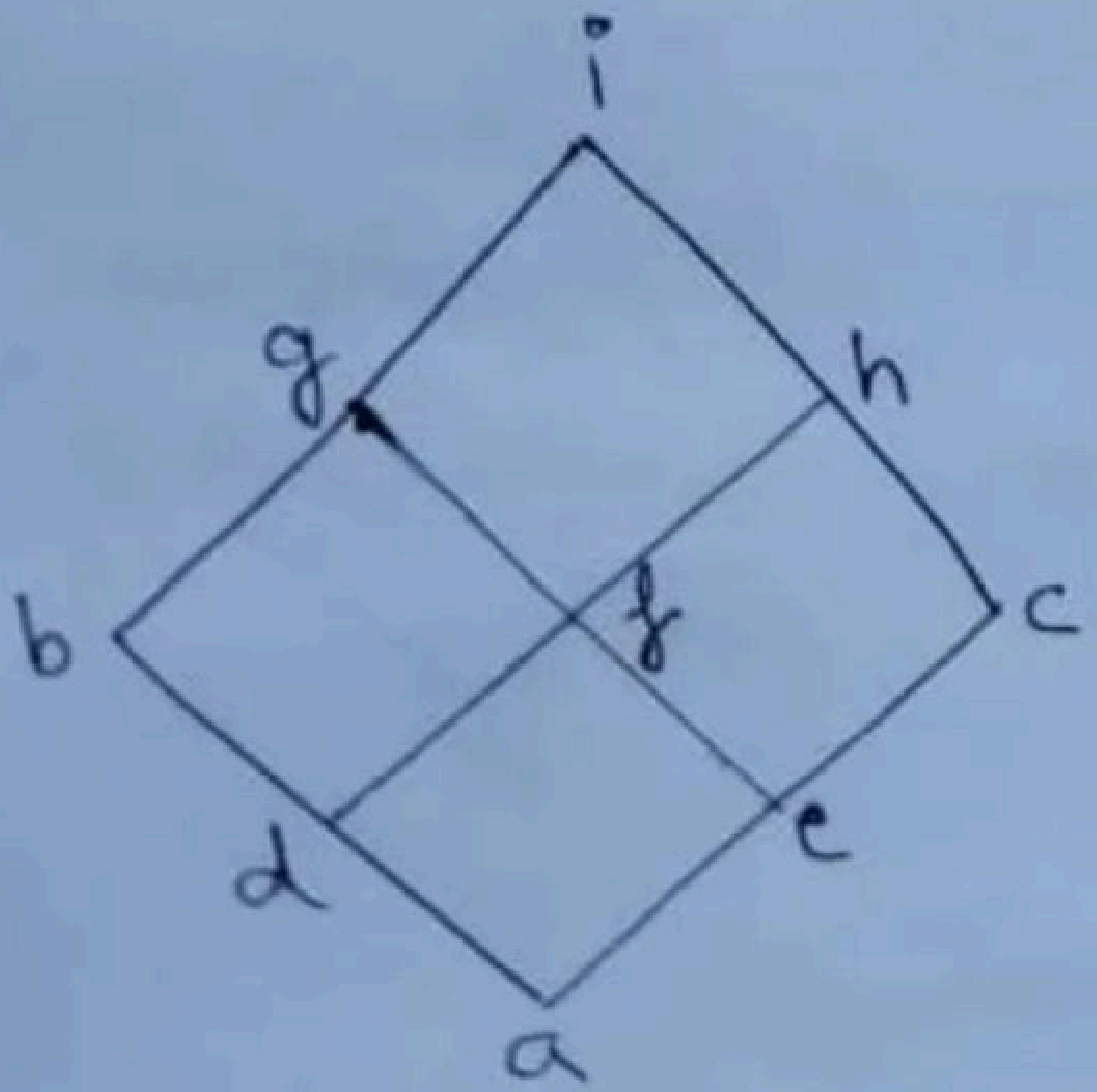


Break

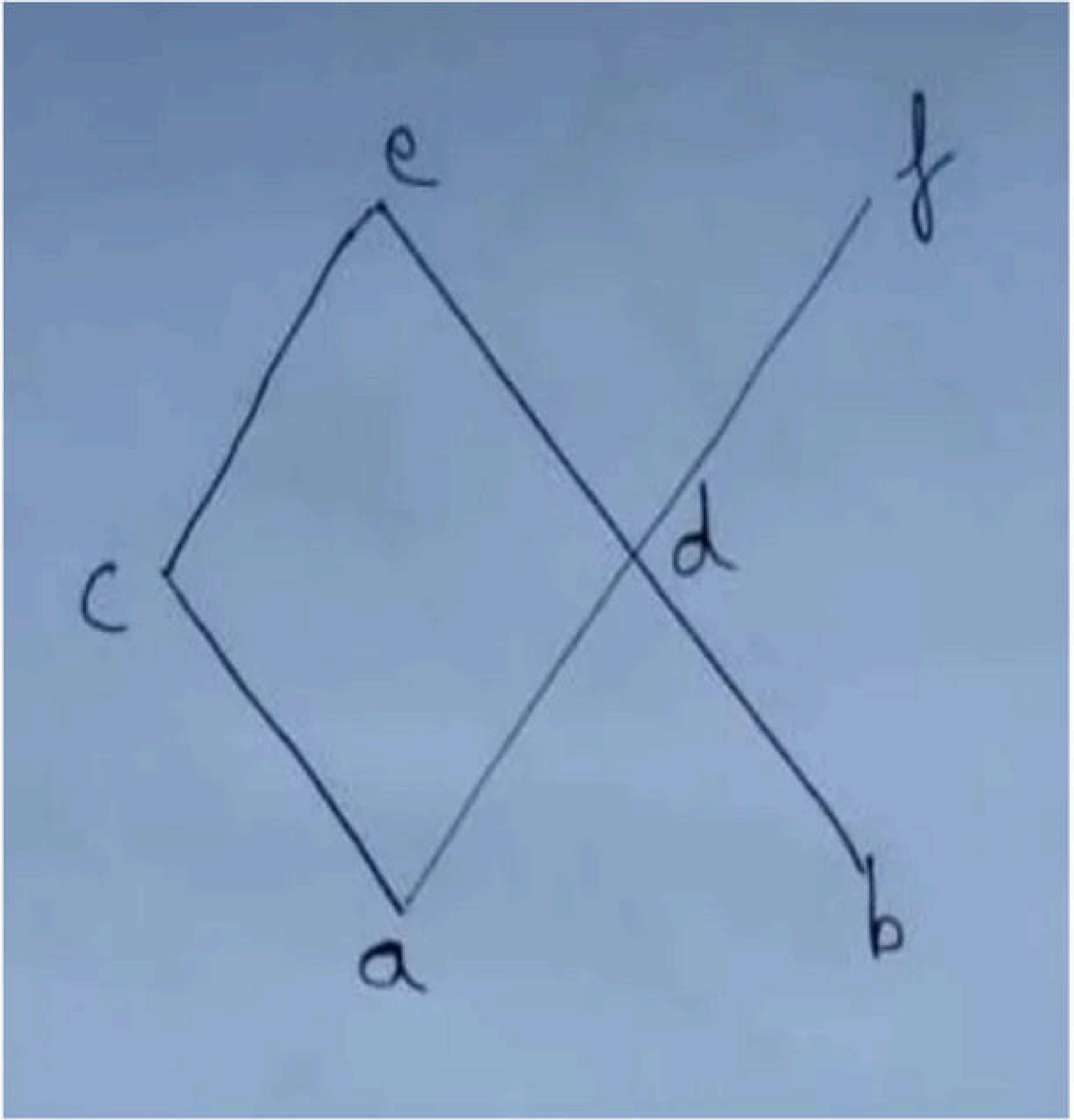
Elements of a Poset

1. **Maximal Element: -** An element is said to be maximal if it is not related to any other element in the Partial order relation.

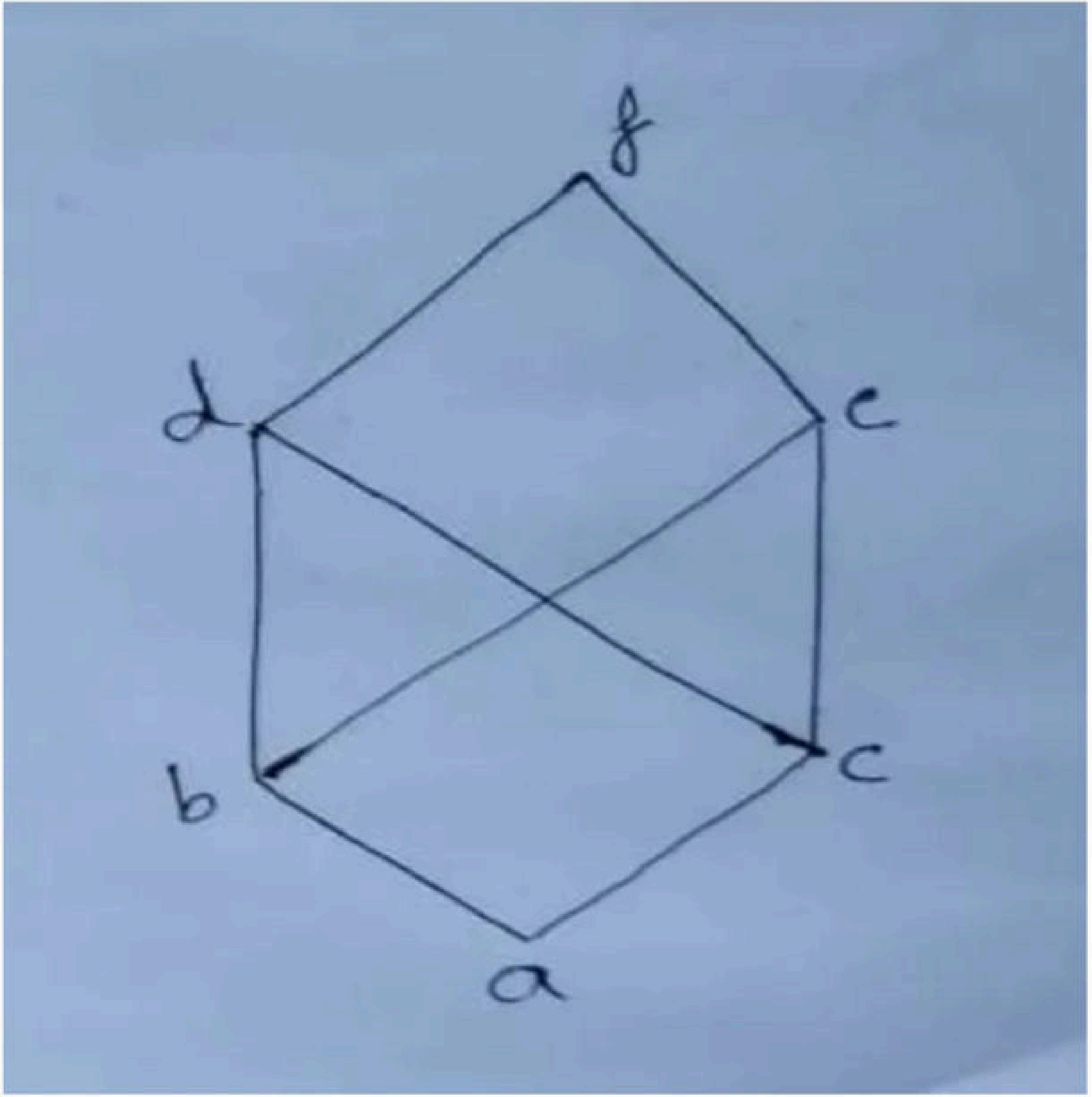
2. **Minimal Element: -** An element is said to be minimal if no other element is related to it in the Partial order relation.



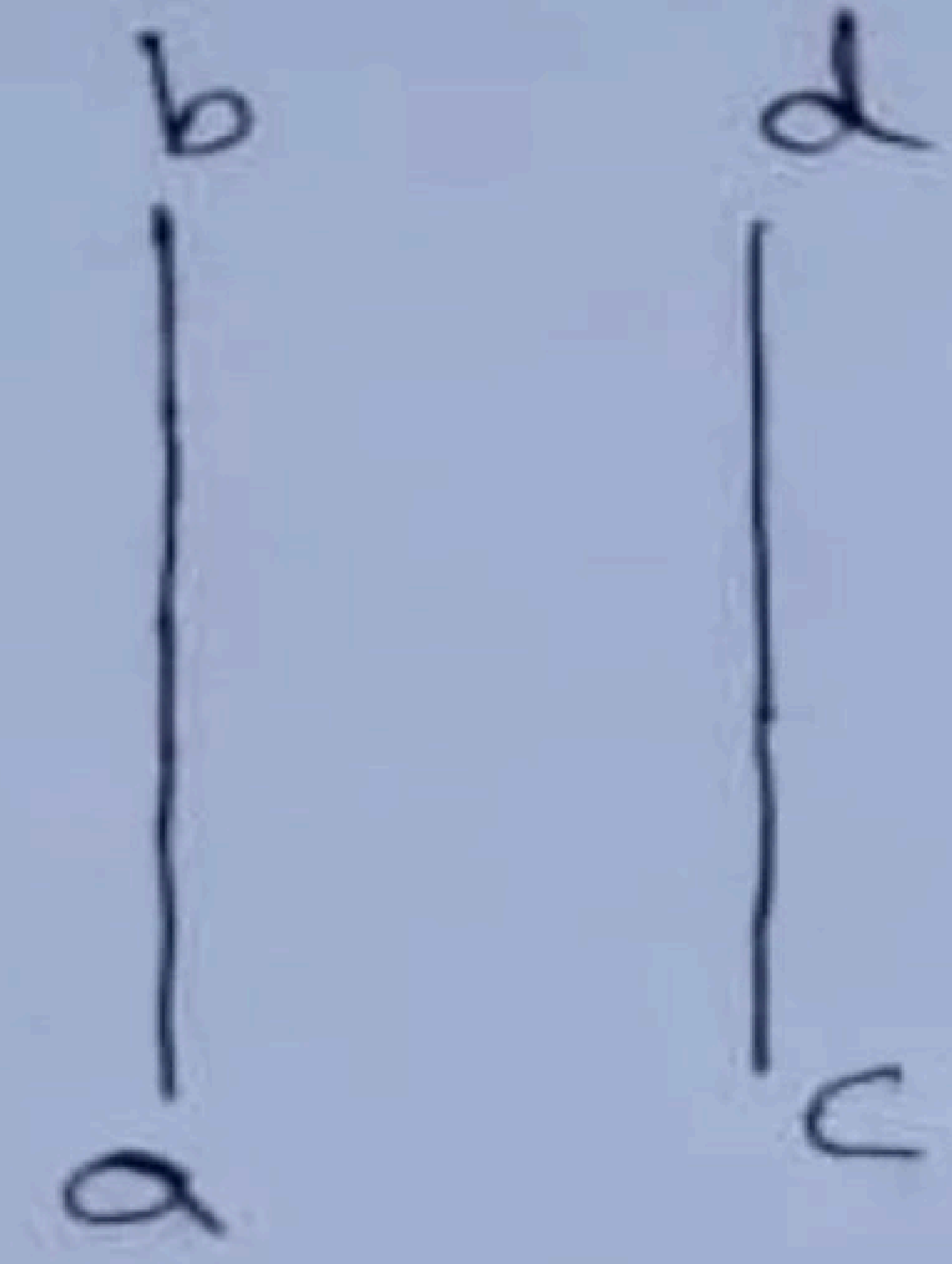
	Elements
Minimal	
Least	
Maximal	
Greatest	



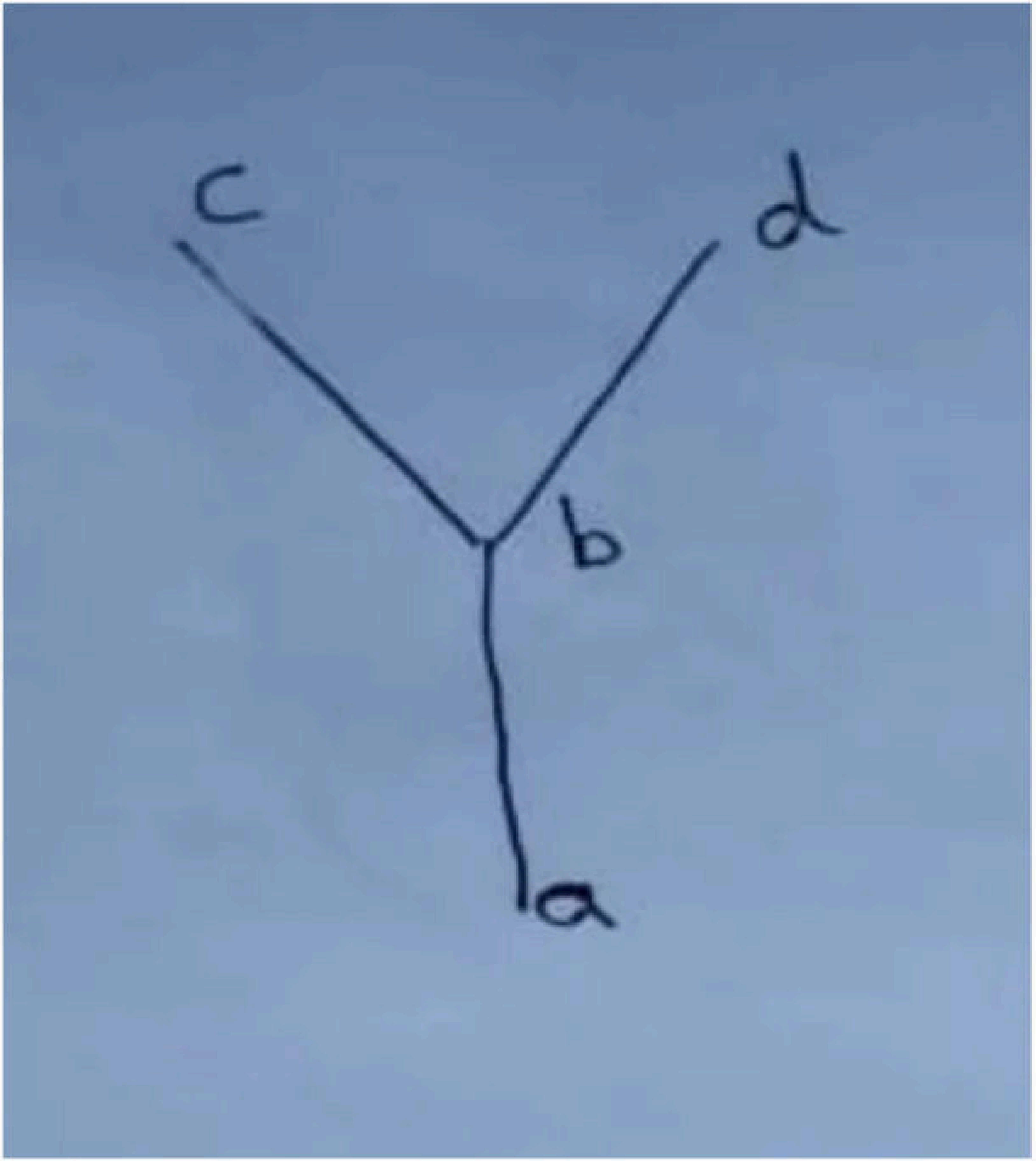
	Elements
Minimal	
Least	
Maximal	
Greatest	



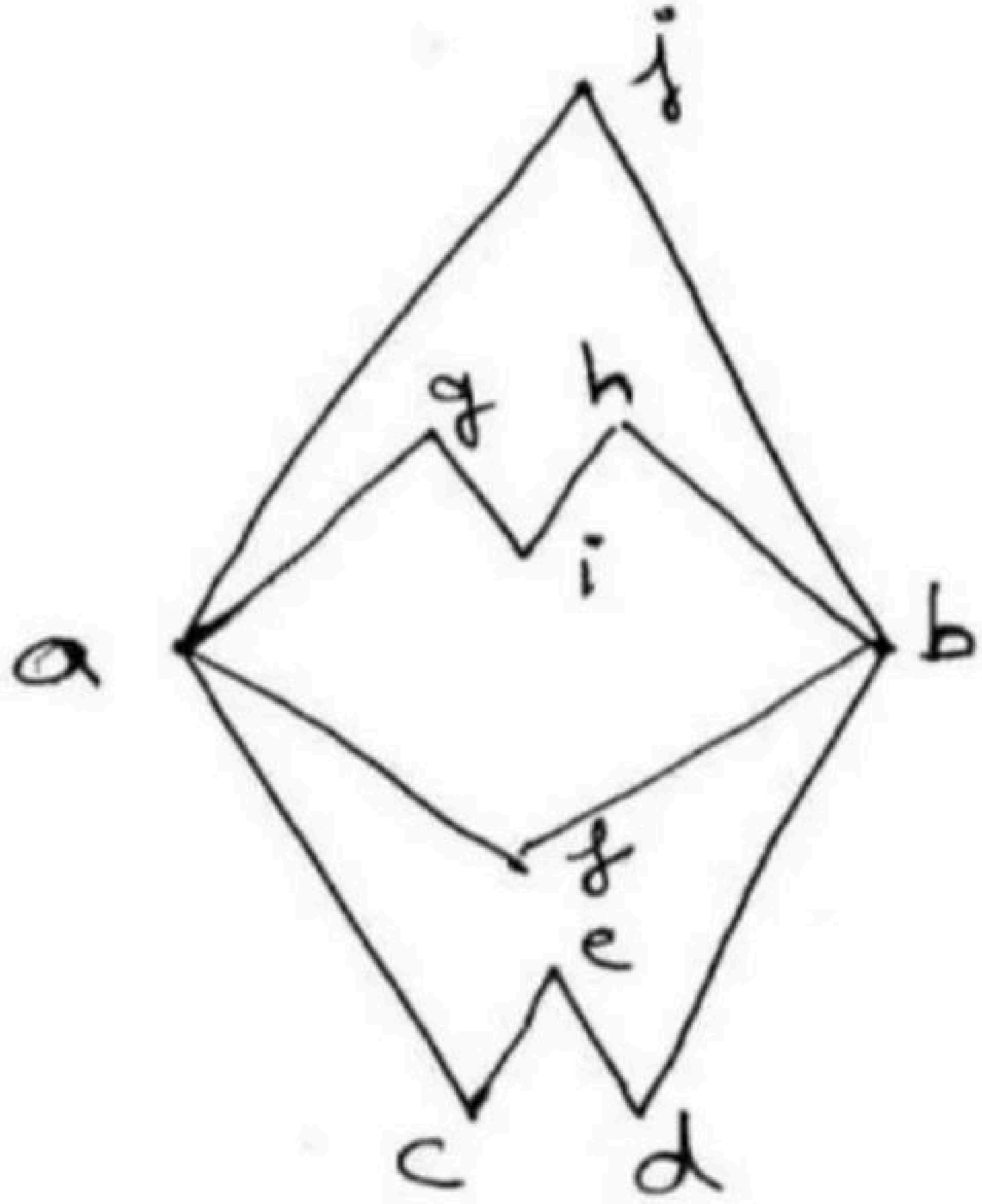
	Elements
Minimal	
Least	
Maximal	
Greatest	



	Elements
Minimal	
Least	
Maximal	
Greatest	



	Elements
Minimal	
Least	
Maximal	
Greatest	



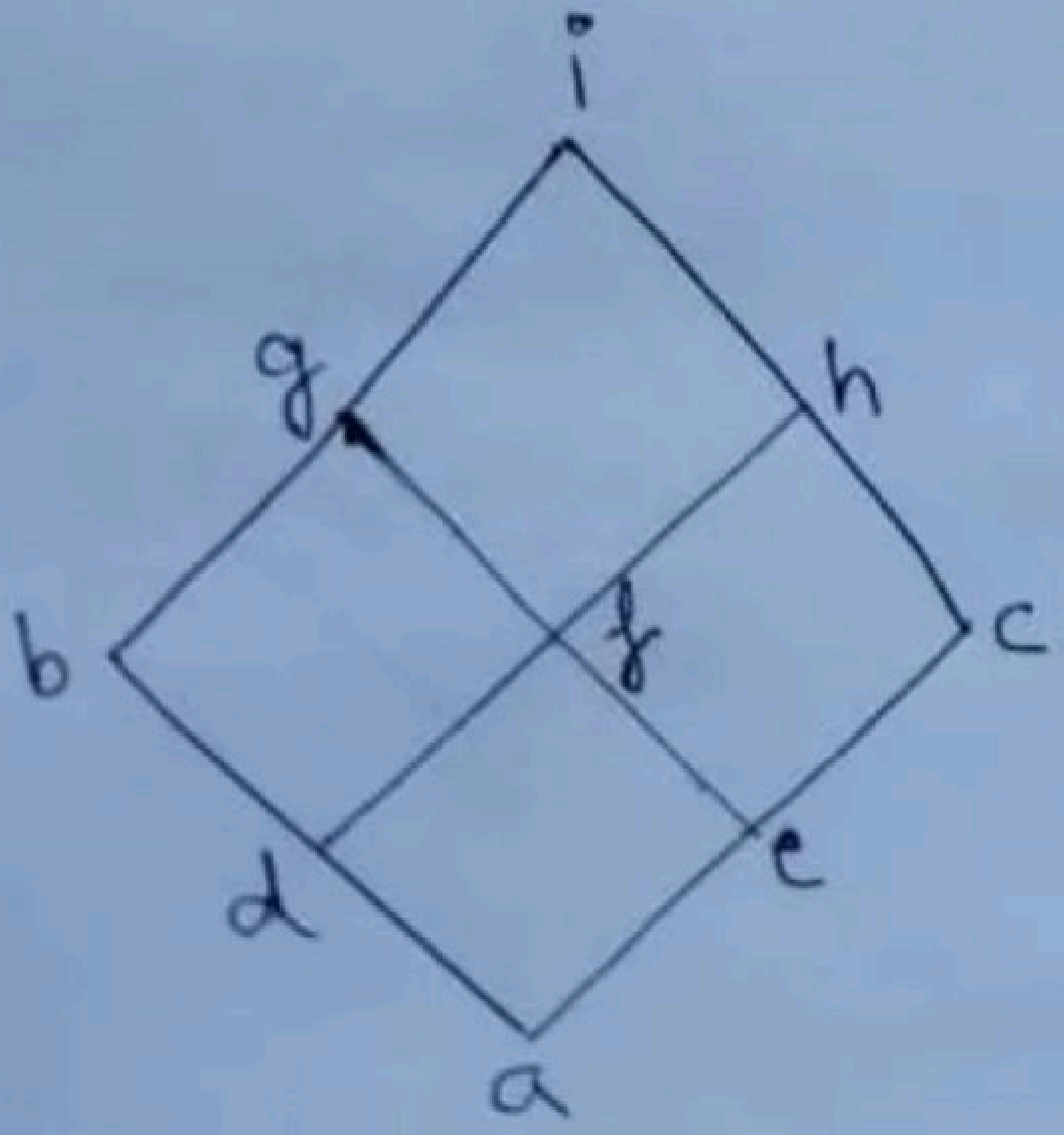
	Elements
Minimal	
Least	
Maximal	
Greatest	

- Every hasse diagram will have at least one Maximal and Minimal element(one or more).

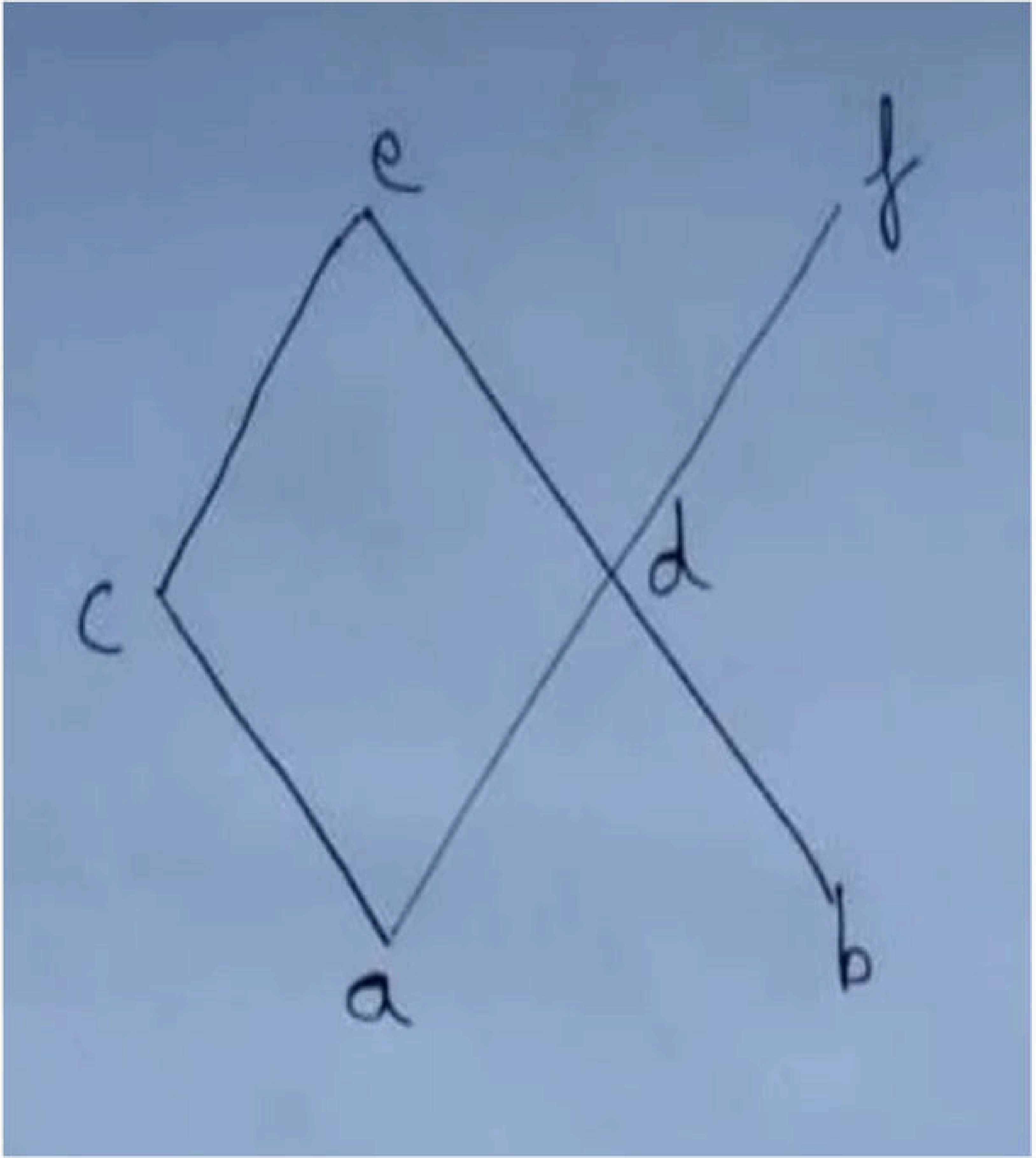
Break

- **<u>Greatest Element</u>: -** An element is said to be Maximum/Greatest if it is not related to any other element but every element is related to it in Partial order relation. Or if a hasse diagram has only one Maximal element then it will also be Maximum/Greatest element.

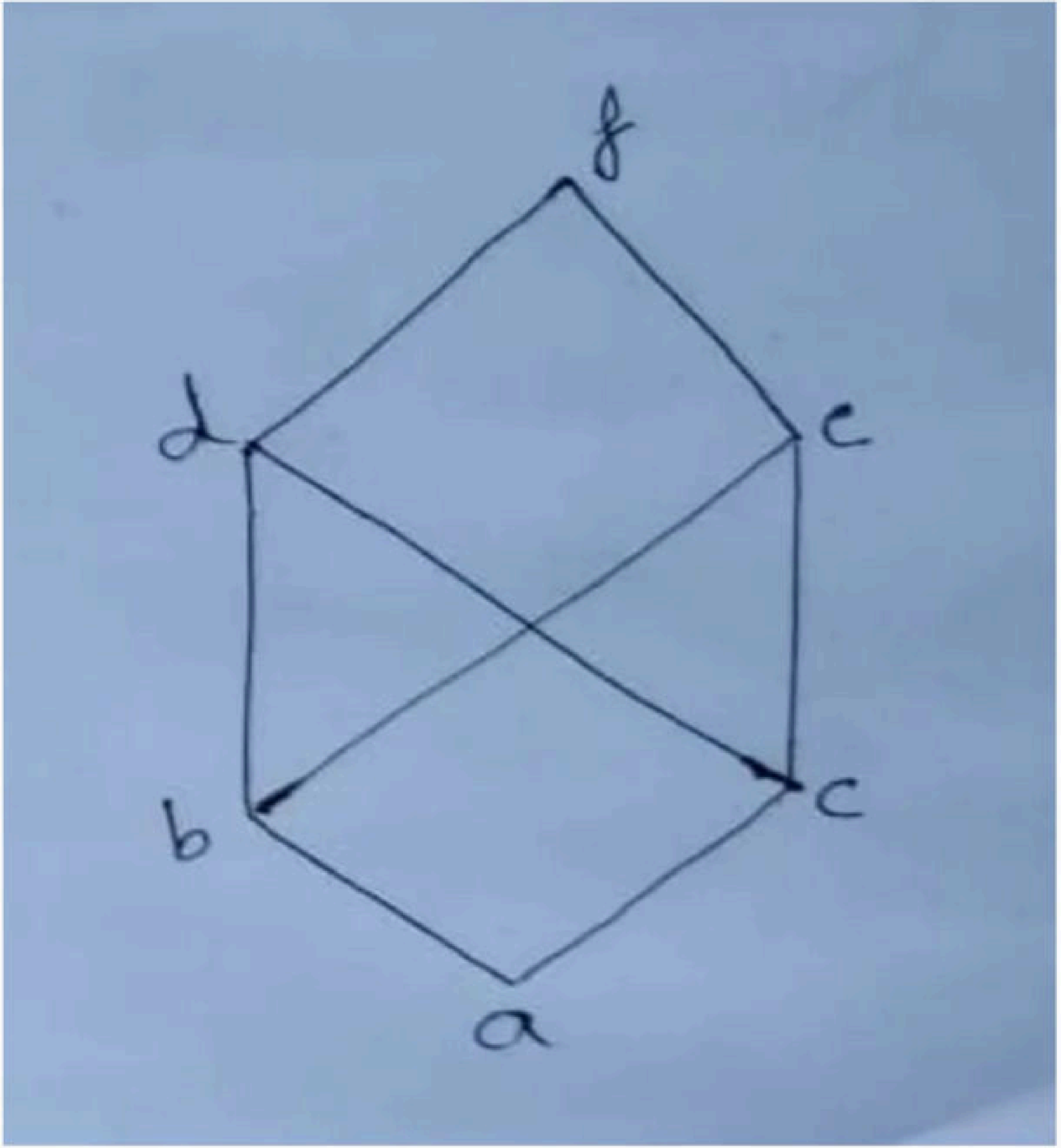
- **<u>Least Element</u>: -** An element is said to be Minimum/Least if no other element is related to it but it is related to every element Partial order relation. Or if a hasse diagram has only one Minimal element then it will also be Minimum/Least element.



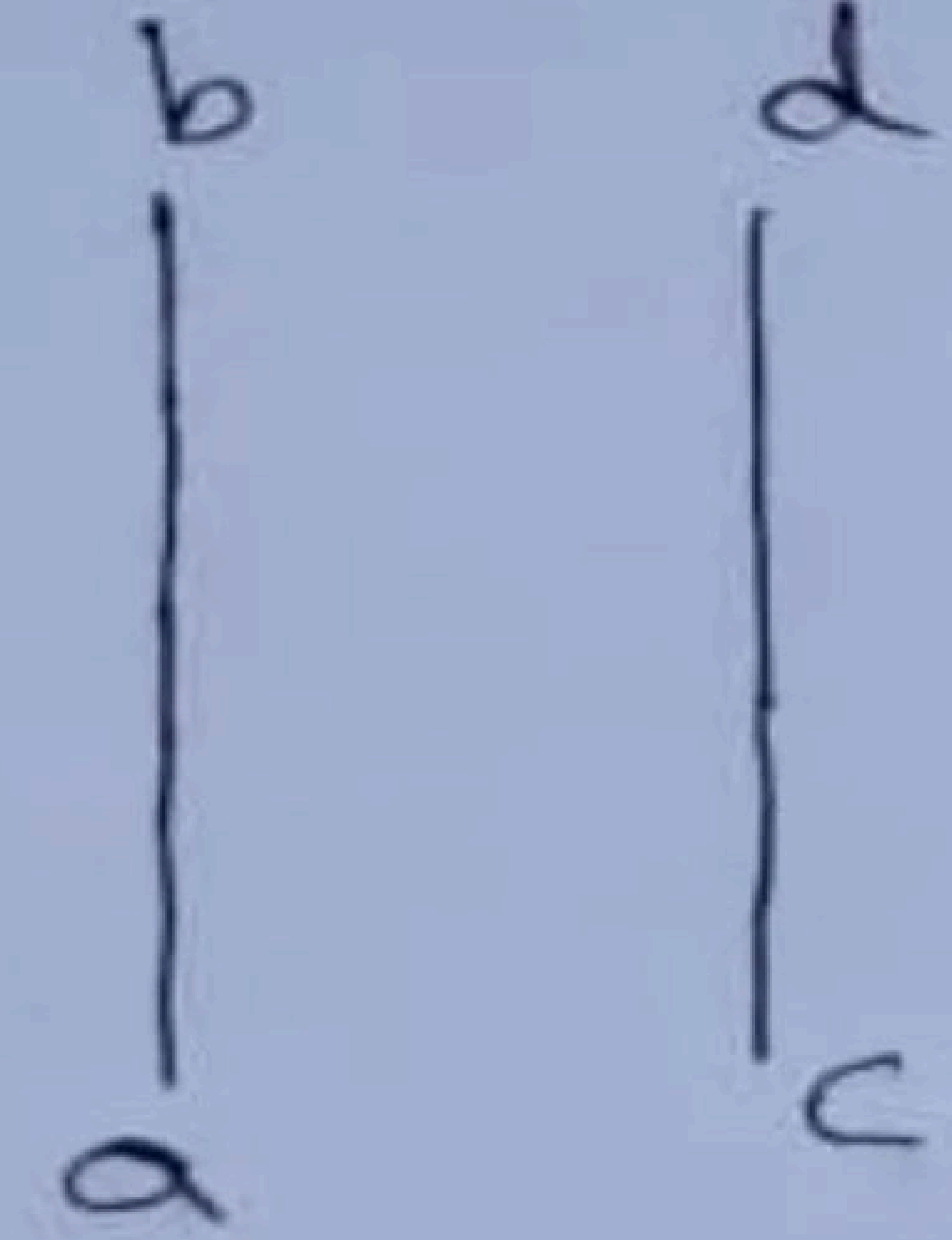
	Elements
Minimal	a
Least	
Maximal	i
Greatest	



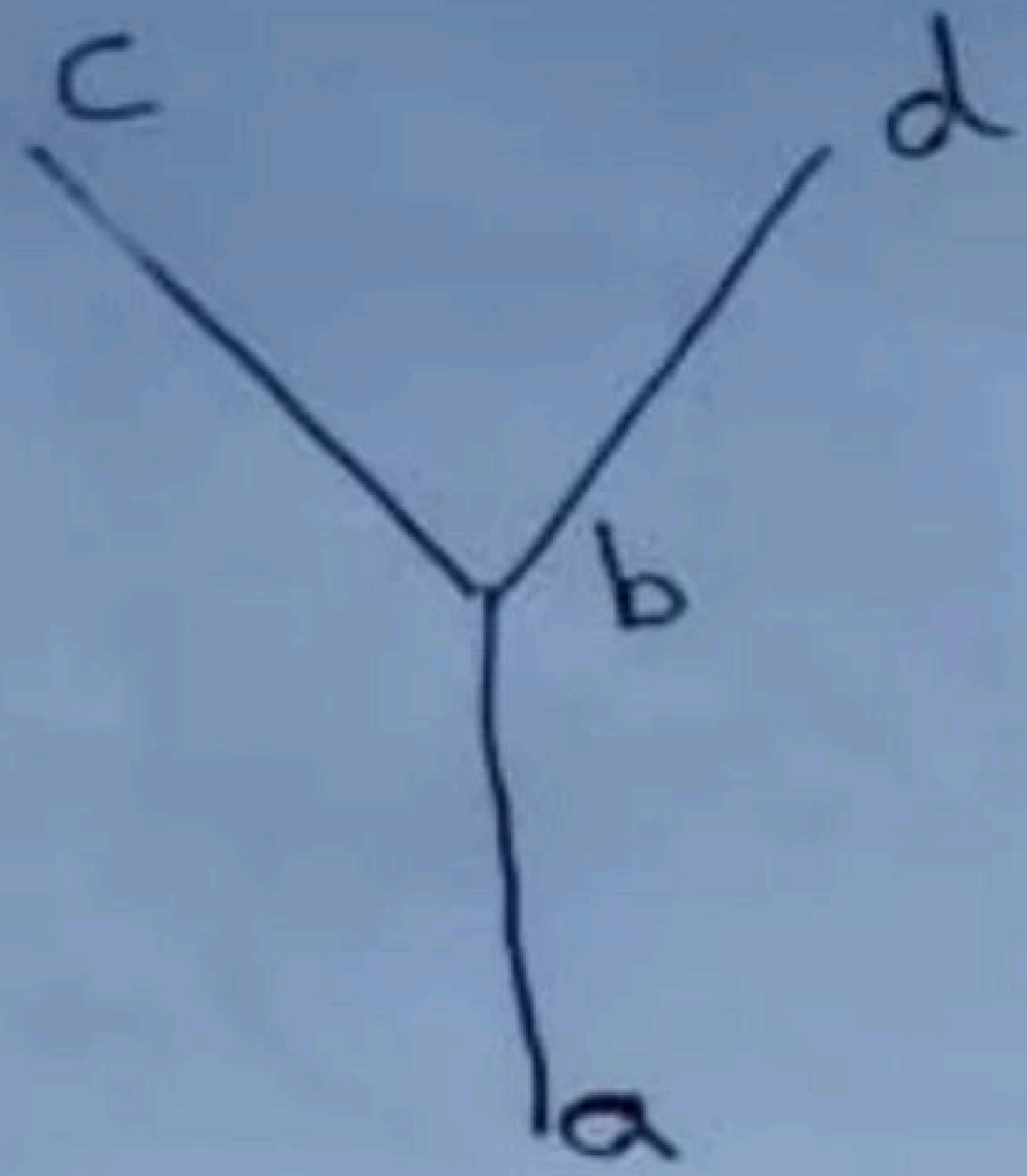
	Elements
Minimal	a, b
Least	
Maximal	e, f
Greatest	



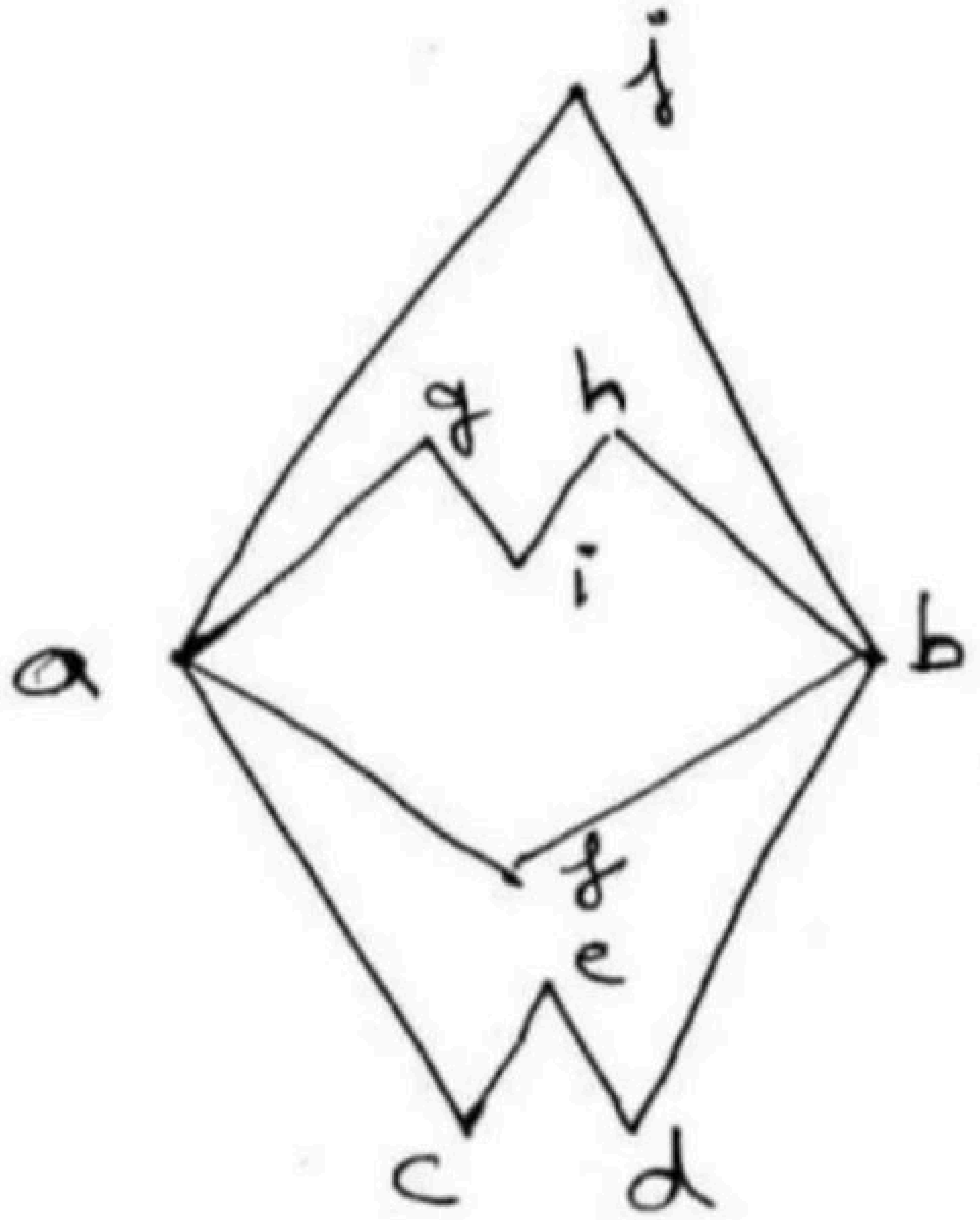
	Elements
Minimal	a
Least	
Maximal	f
Greatest	



	Elements
Minimal	a, c
Least	
Maximal	b, d
Greatest	



	Elements
Minimal	a
Least	
Maximal	c, d
Greatest	



	Elements
Minimal	c, d, f, i
Least	
Maximal	j, g, h, e
Greatest	

1. Every hasse diagram will have at most one Greatest and Least element(zero or one) (T/F)

2. Every Greatest element is also Maximal (T/F)

3. Every Least element is also Minimal (T/F)

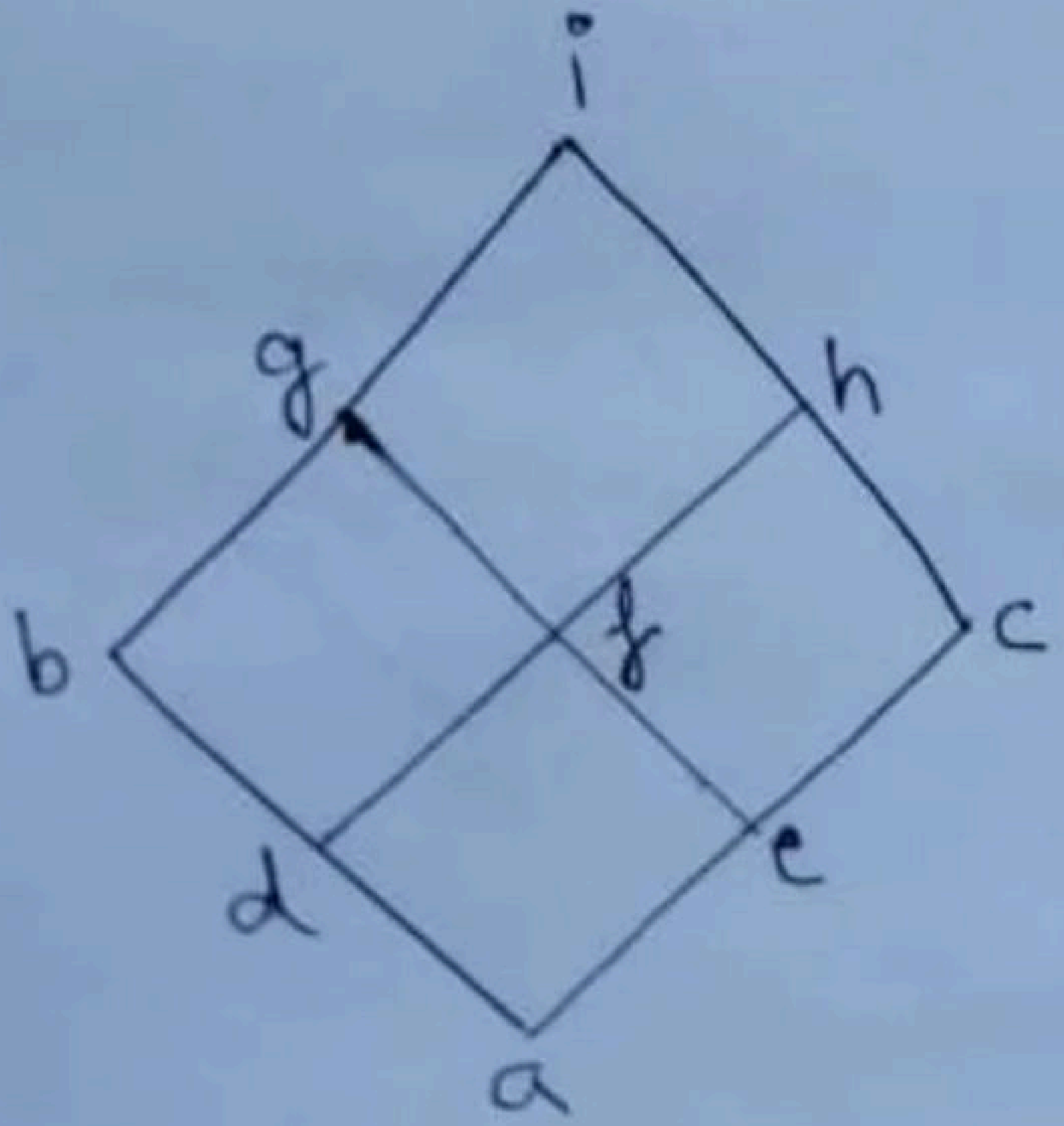
4. If there is only one Maximal element then it is called Greatest (T/F)

5. If there is only one Minimal element then it is called Least (T/F)

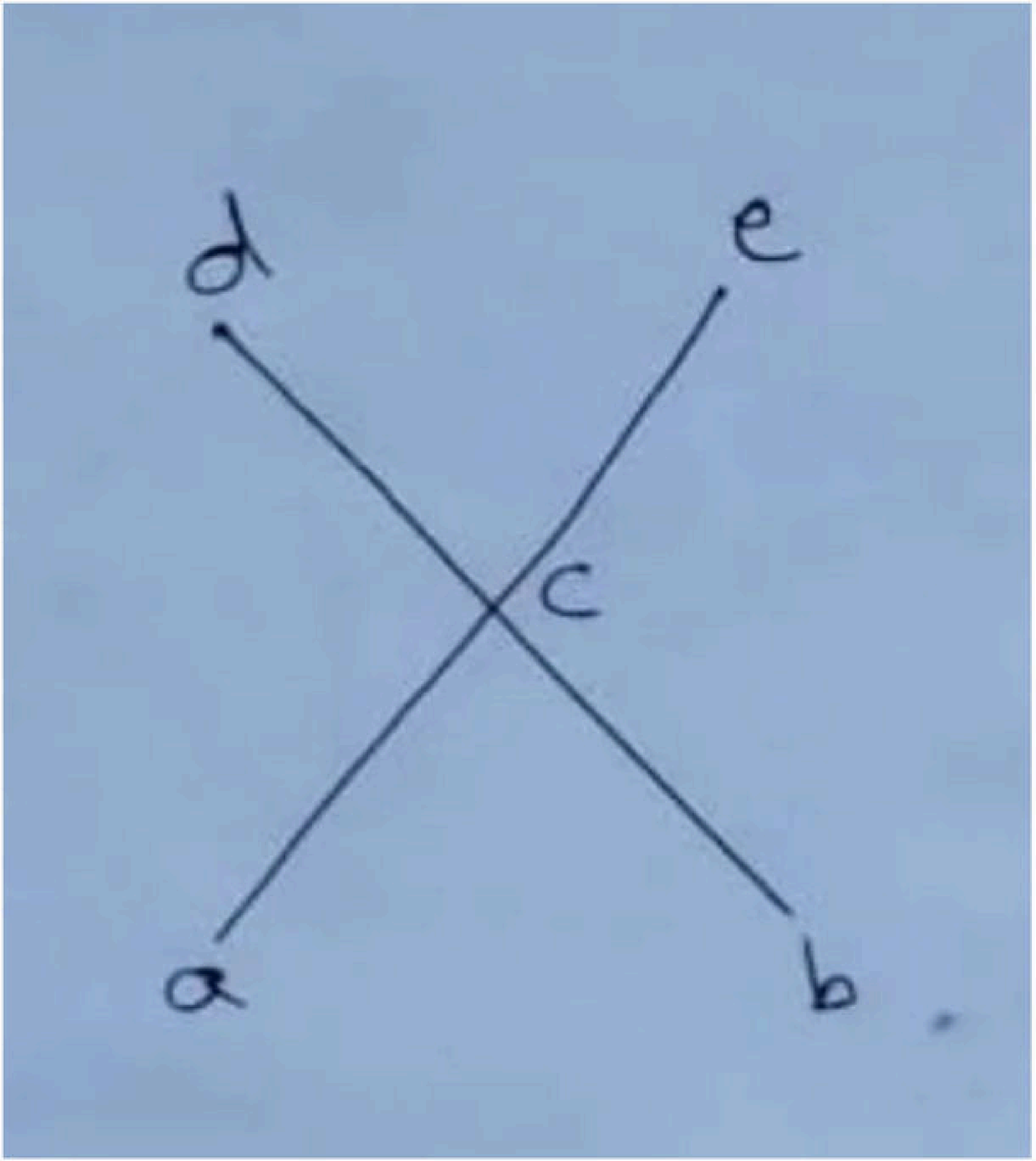
Break

- **Upper Bound: -** Upper bound of a subset B with respect to set A, will contain all those element to which all the elements of B is related.

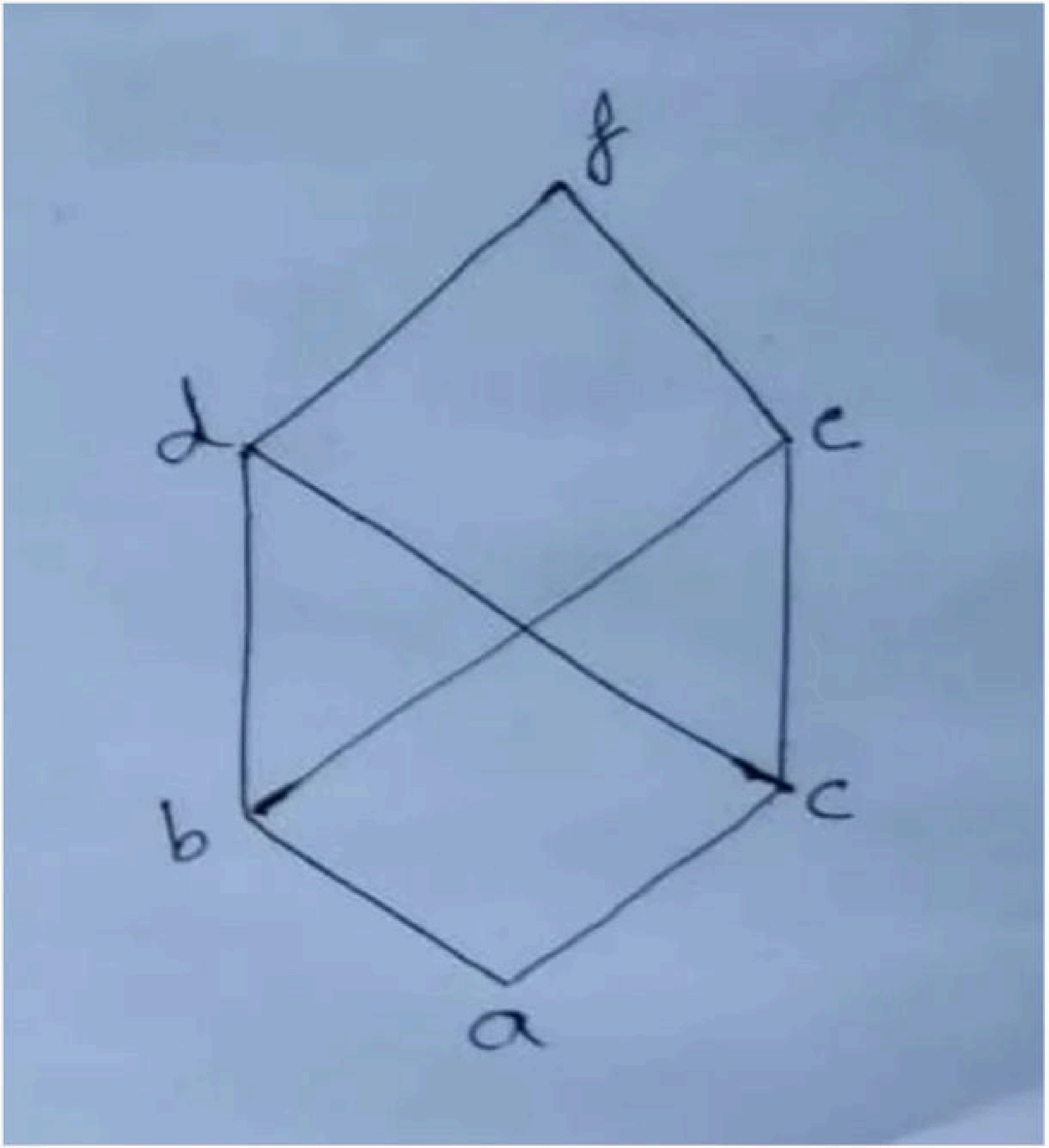
- **Lower Bound: -** lower bound of a subset B with respect to A, will contain all those elements which are related to every element of B.



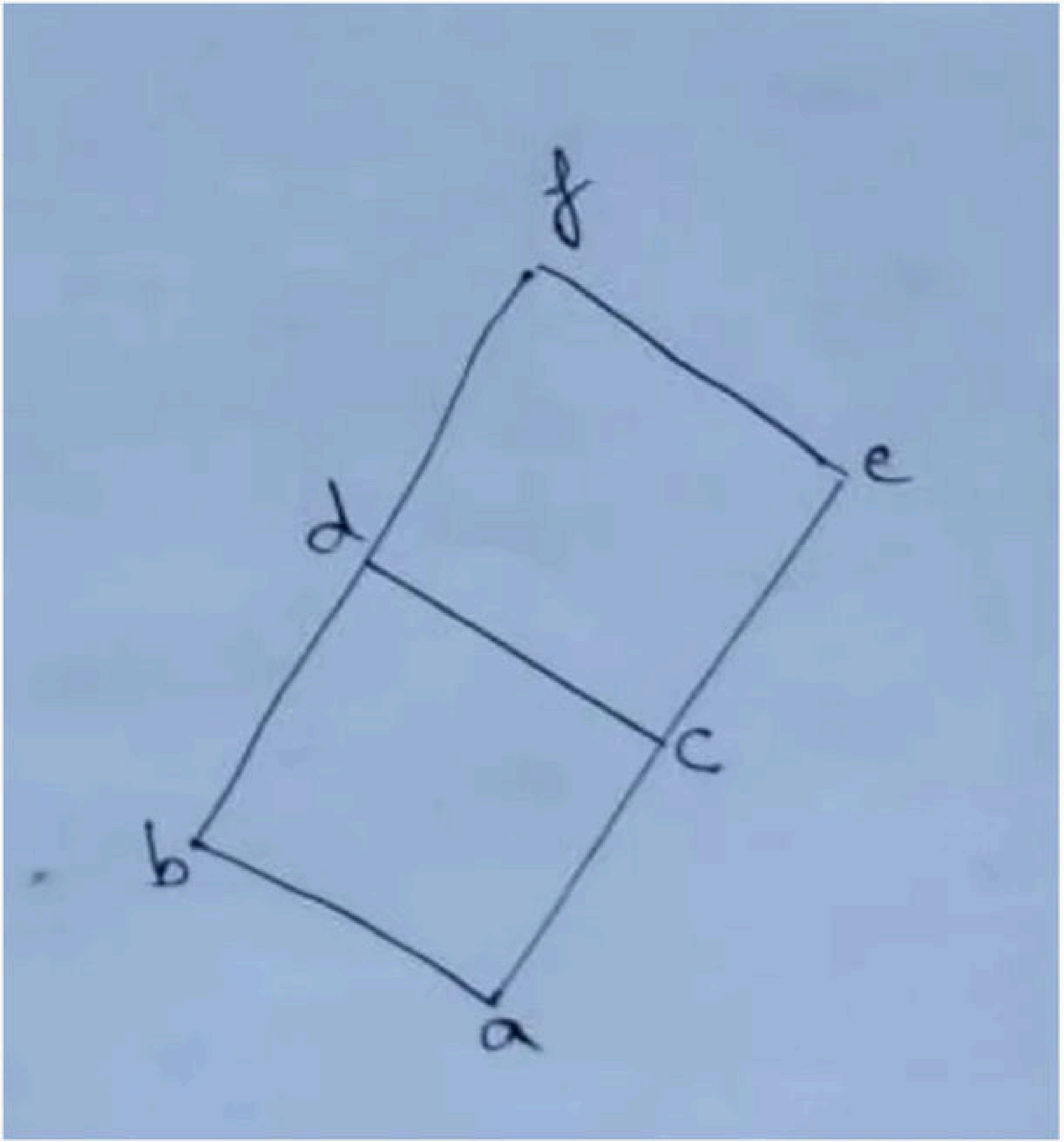
Elements	B = {g, f, e, c}	B = {d, f}
Upper Bound		
Lower Bound		



Elements	B = {d, e}	B = {c}
Upper Bound		
Lower Bound		



Elements	B = {d, e}	B = {b, c}
Upper Bound		
Lower Bound		



Elements	B = {d, c}	B = {b, e}
Upper Bound		
Lower Bound		

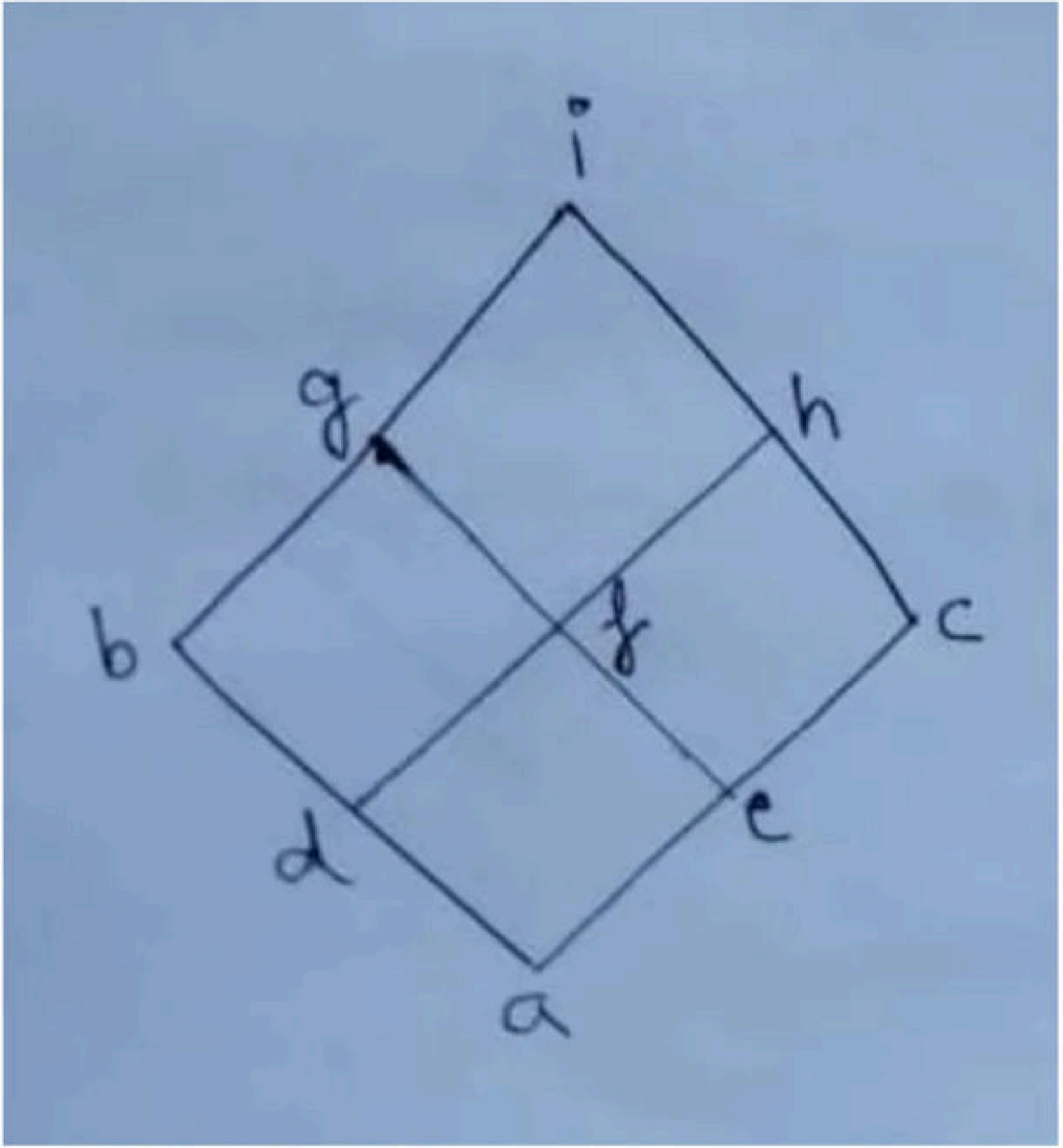
Break

Least Upper Bound / LUB / Join / Supremum / $\vee$

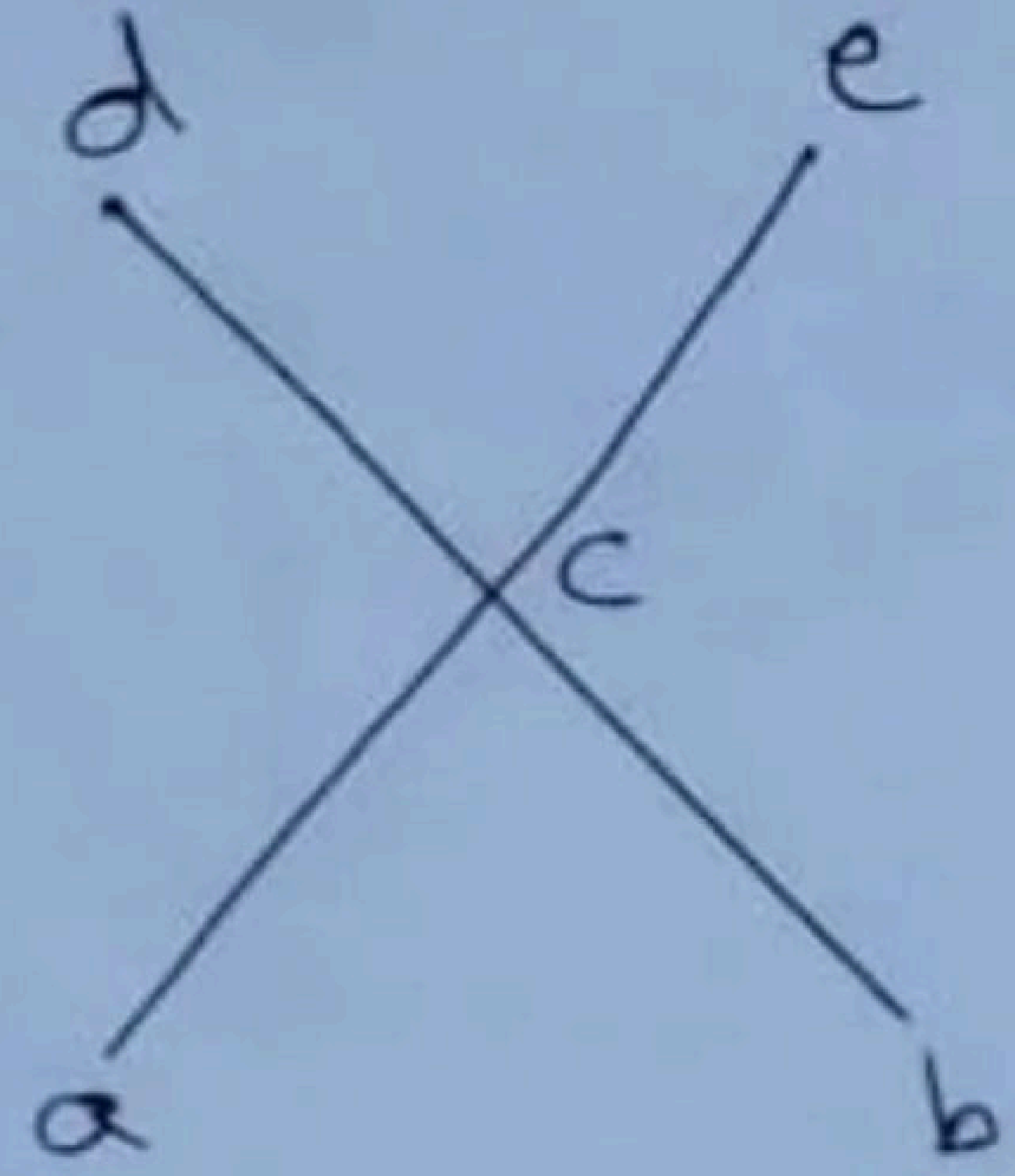
Least value in the upper bound

Greatest Lower Bound / GLB / Meet / Infimum / $\wedge$

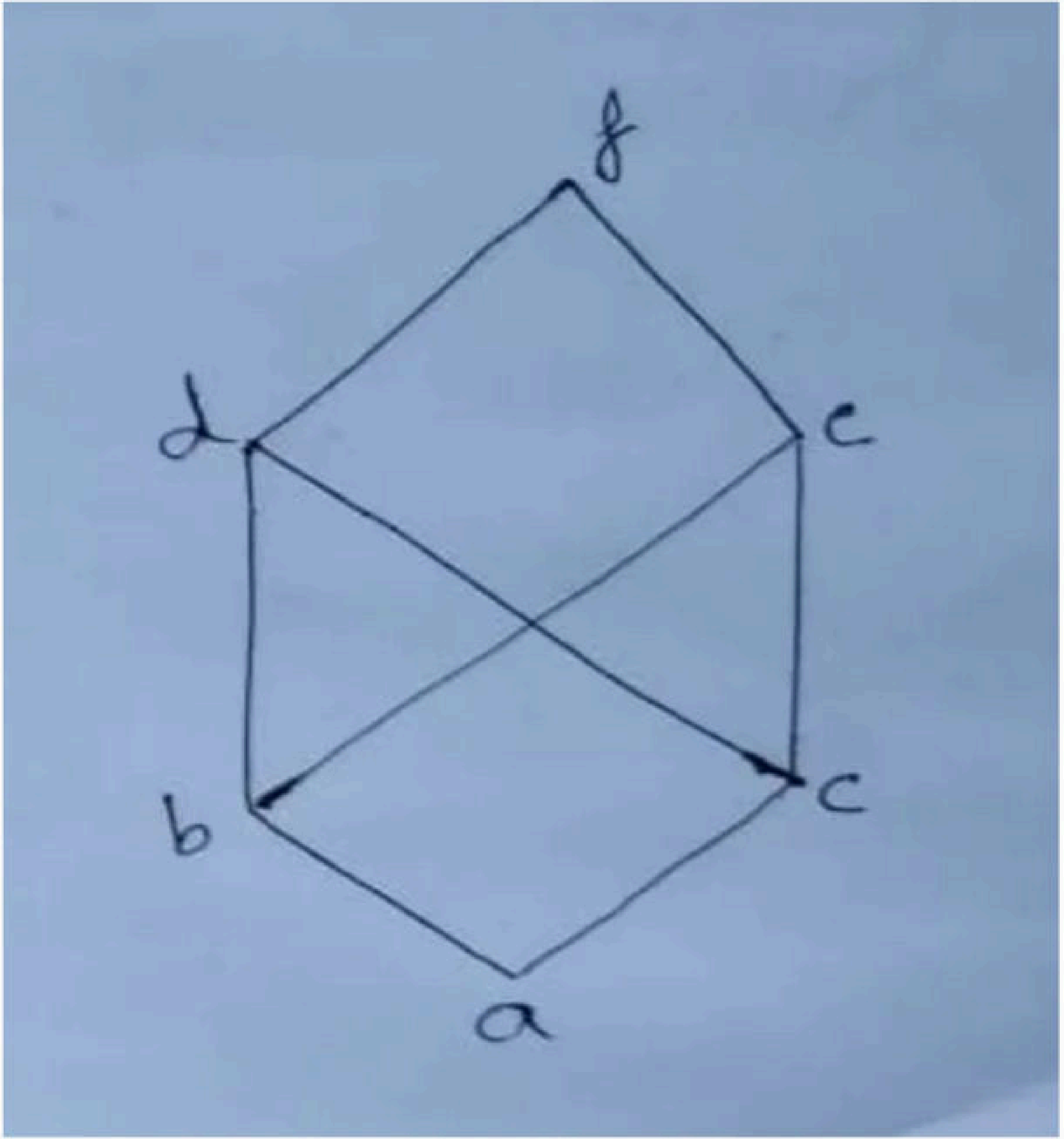
Greatest value in the lower bound



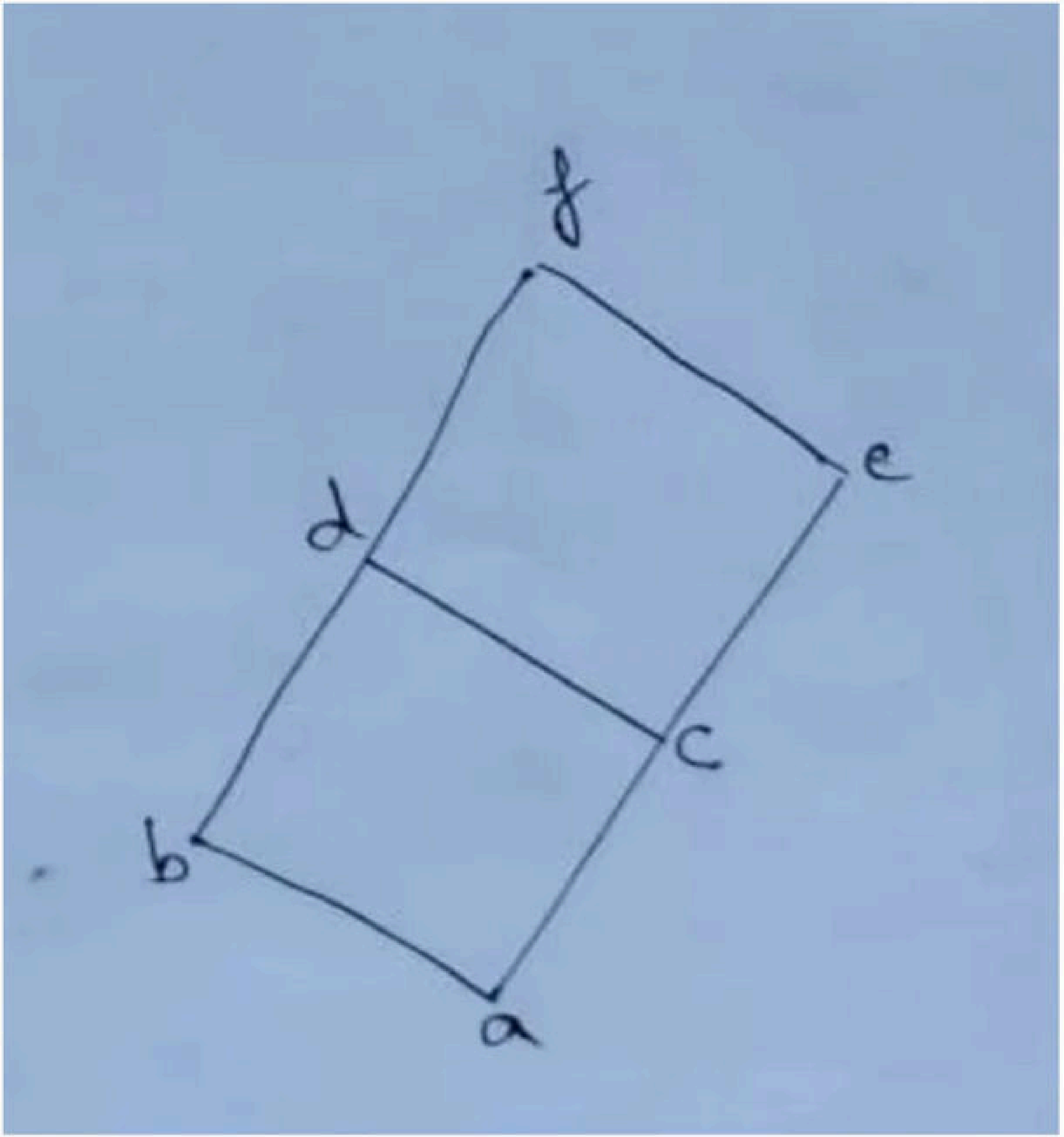
Elements	B = {g, f, e, c}	B = {d, f}
Upper Bound	{i}	{i, g, h, f}
Least Upper Bound		
Lower Bound	{a, e}	{a, d}
Greatest Lower Bound		



Elements	B = {d, e}	B = {c}
Upper Bound	{}	{d, e, c}
Least Upper Bound		
Lower Bound	{a, b, c}	{a, b, c}
Greatest Lower Bound		



Elements	B = {d, e}	B = {b, c}
Upper Bound	{f}	{d, e, f}
Least Upper Bound		
Lower Bound	{a, b, c}	{a}
Greatest Lower Bound		



Elements	B = {d, c}	B = {b, e}
Upper Bound	{f, d}	{f}
Least Upper Bound		
Lower Bound	{a, c}	{a}
Greatest Lower Bound		

Q Consider the Poset ({3,5,9,15,24,45}, /). Which of the following is correct for the given Poset? **(NET-JUNE-2019)**

a) There exist a greatest element and a least element

b) There exist a greatest element but not a least element

c) There exist a least element but not a greatest element

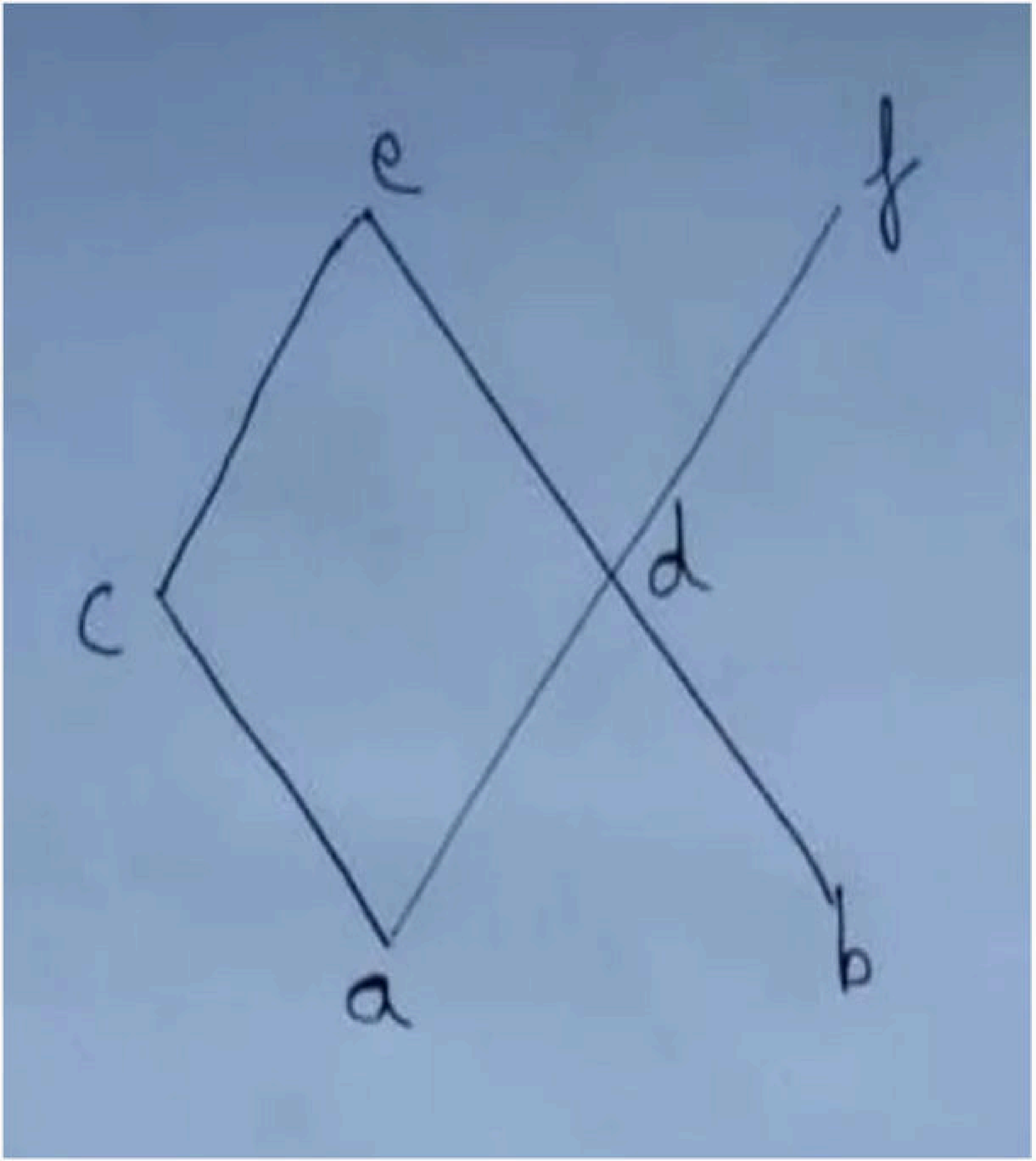
d) There does not exist a greatest element and a least element

Break

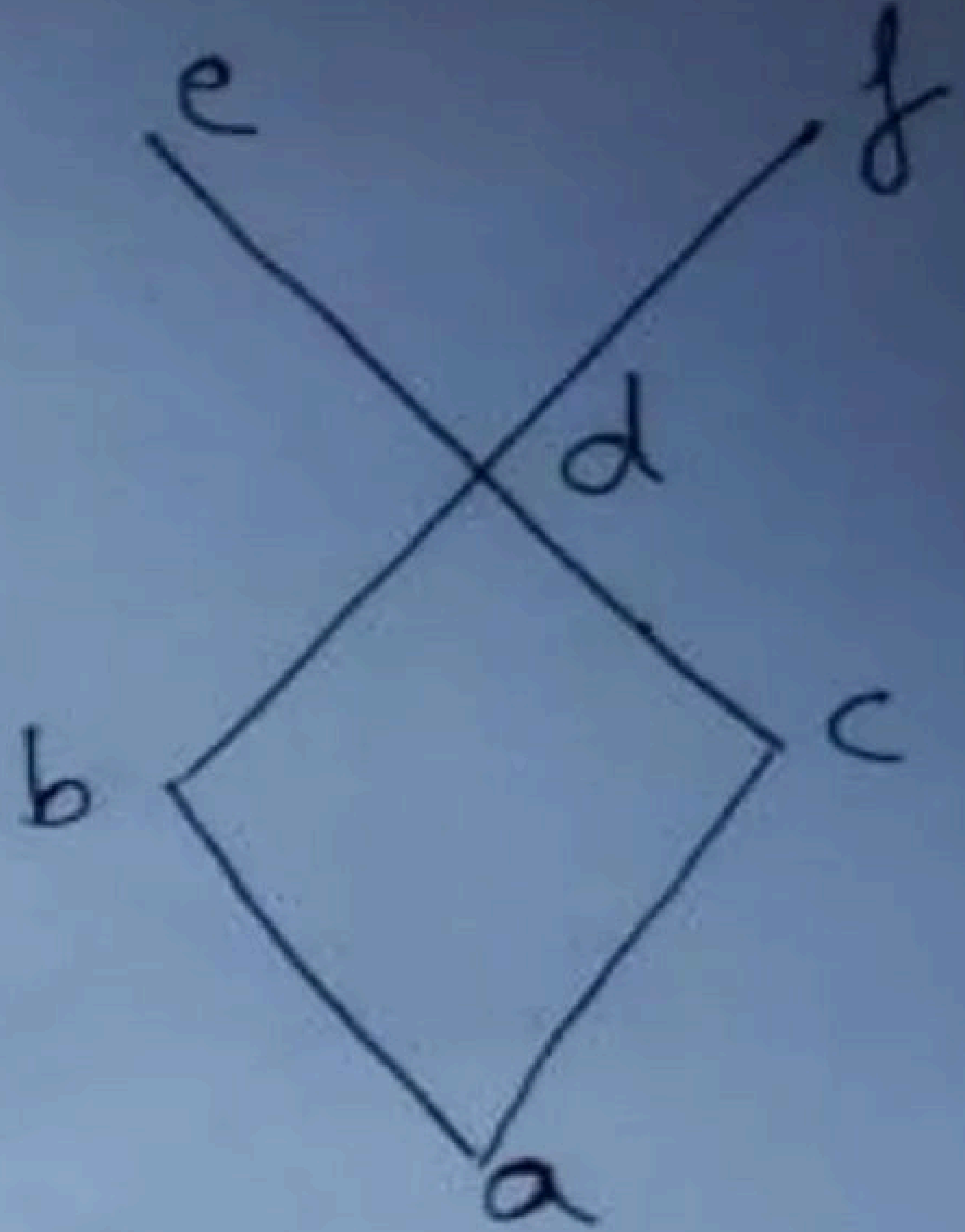
Join Semi Lattice :- A hasse diagram/Partial order relation is called Join Semi Lattice if for every elements their exists a Join.

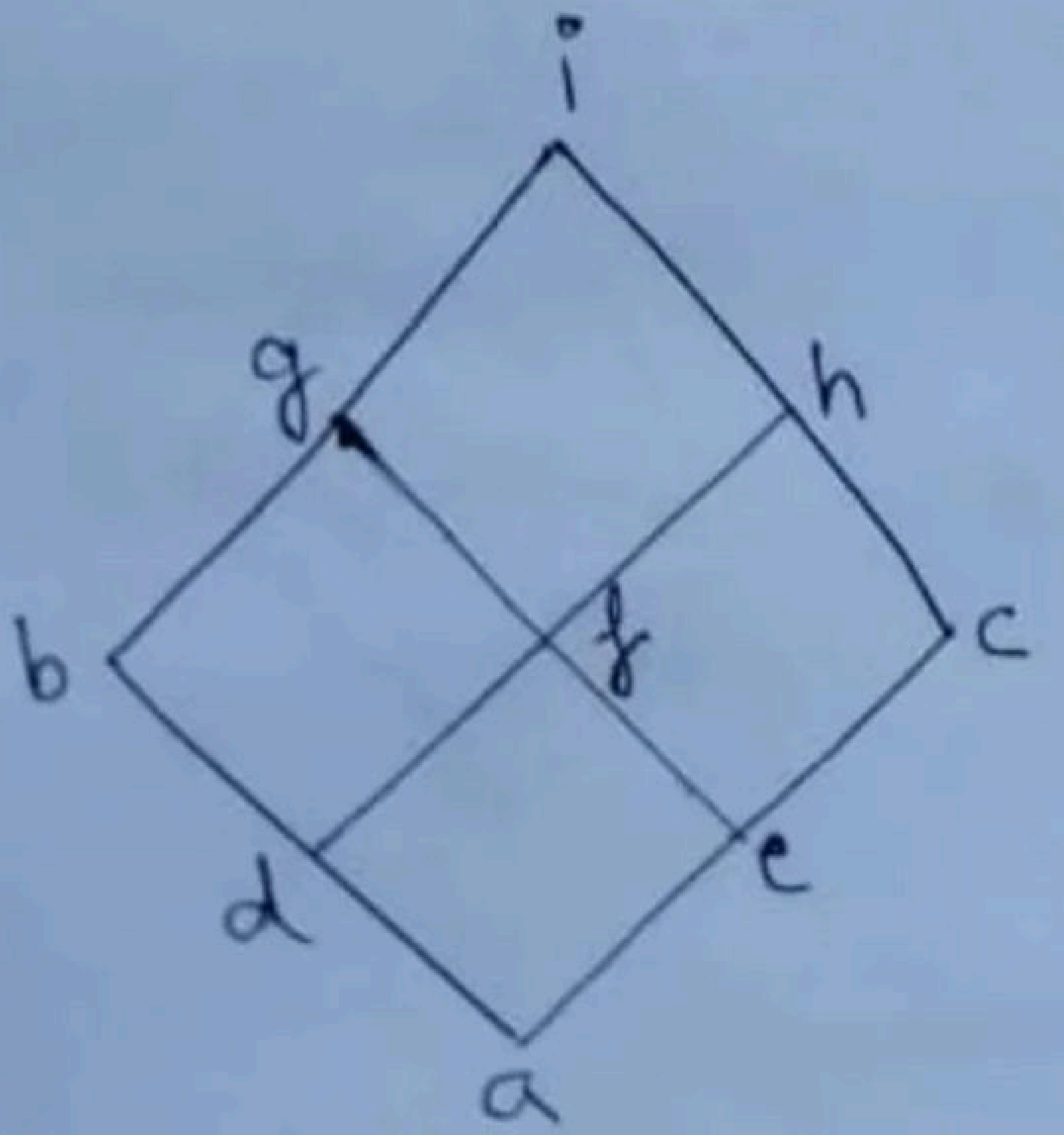
<u>Meet Semi Lattice</u> :- A hasse diagram/Partial order relation is called Meet Semi Lattice if for every elements their exists a Meet.

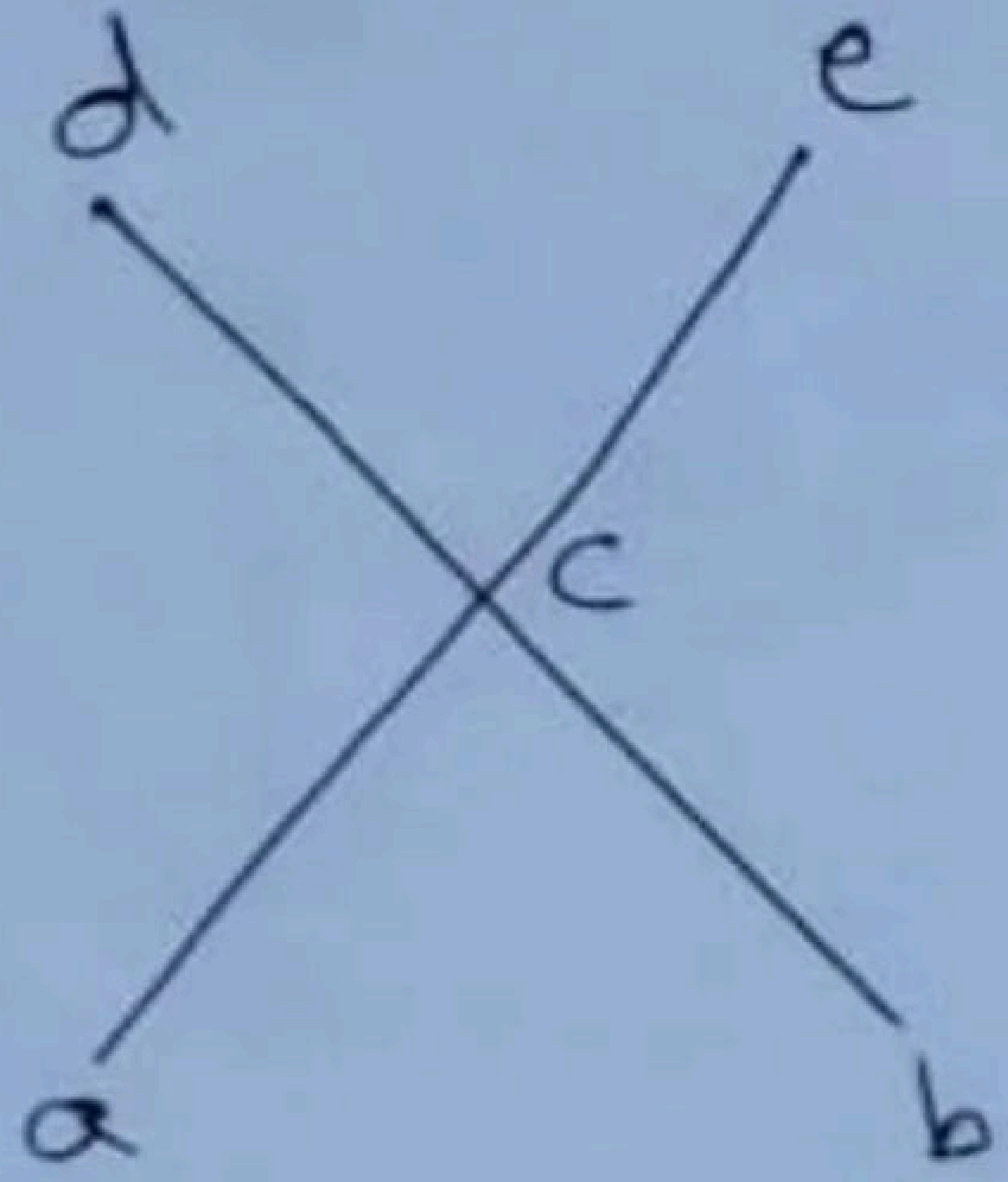
Lattice :- A hasse diagram/Partial order relation is called Lattice if their exist a Join and Meet for every pair of element. Or A hasse diagram/Partial order relation is called Lattice if it is both Join Semi Lattice and Meet Semi Lattice.

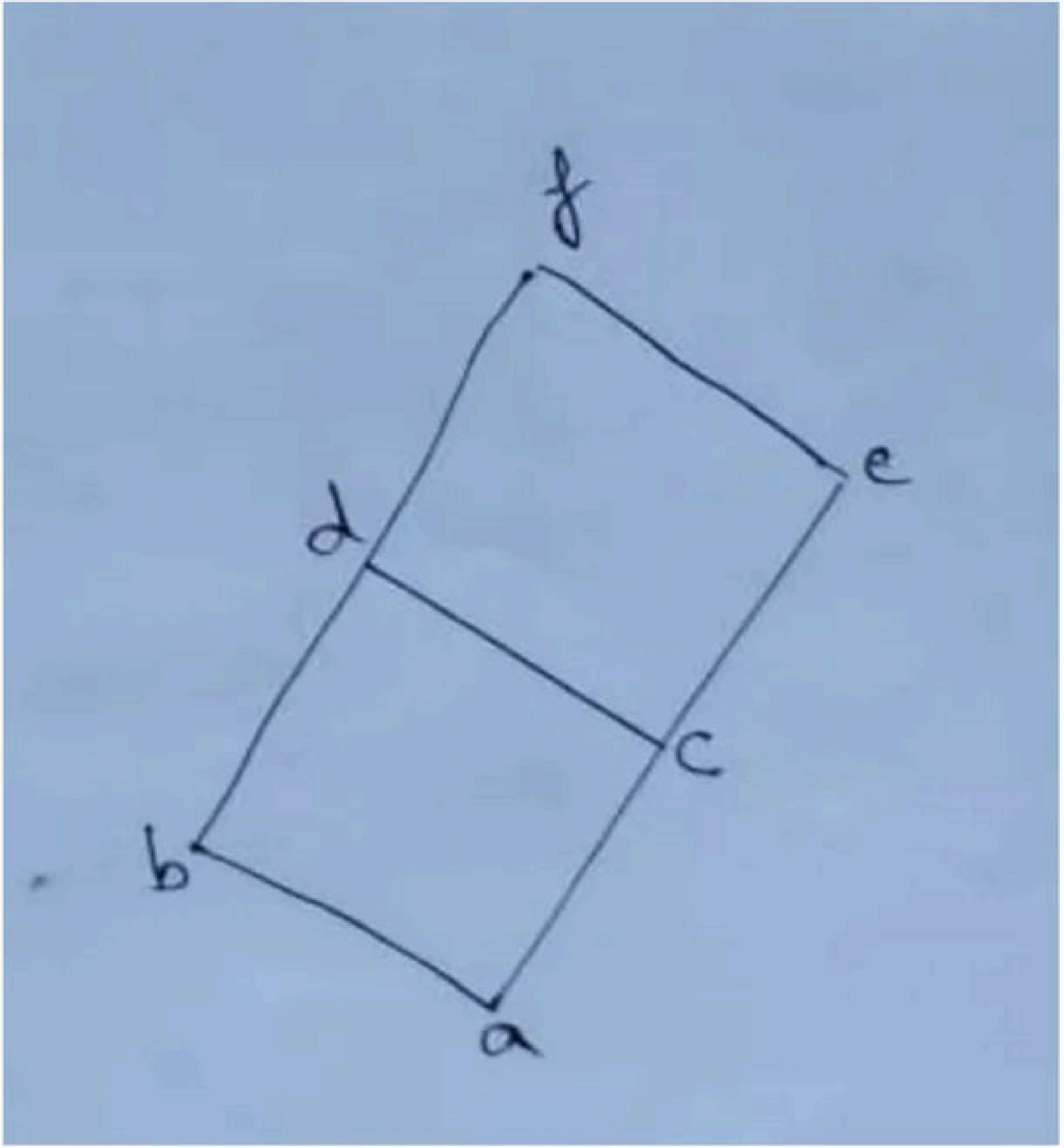


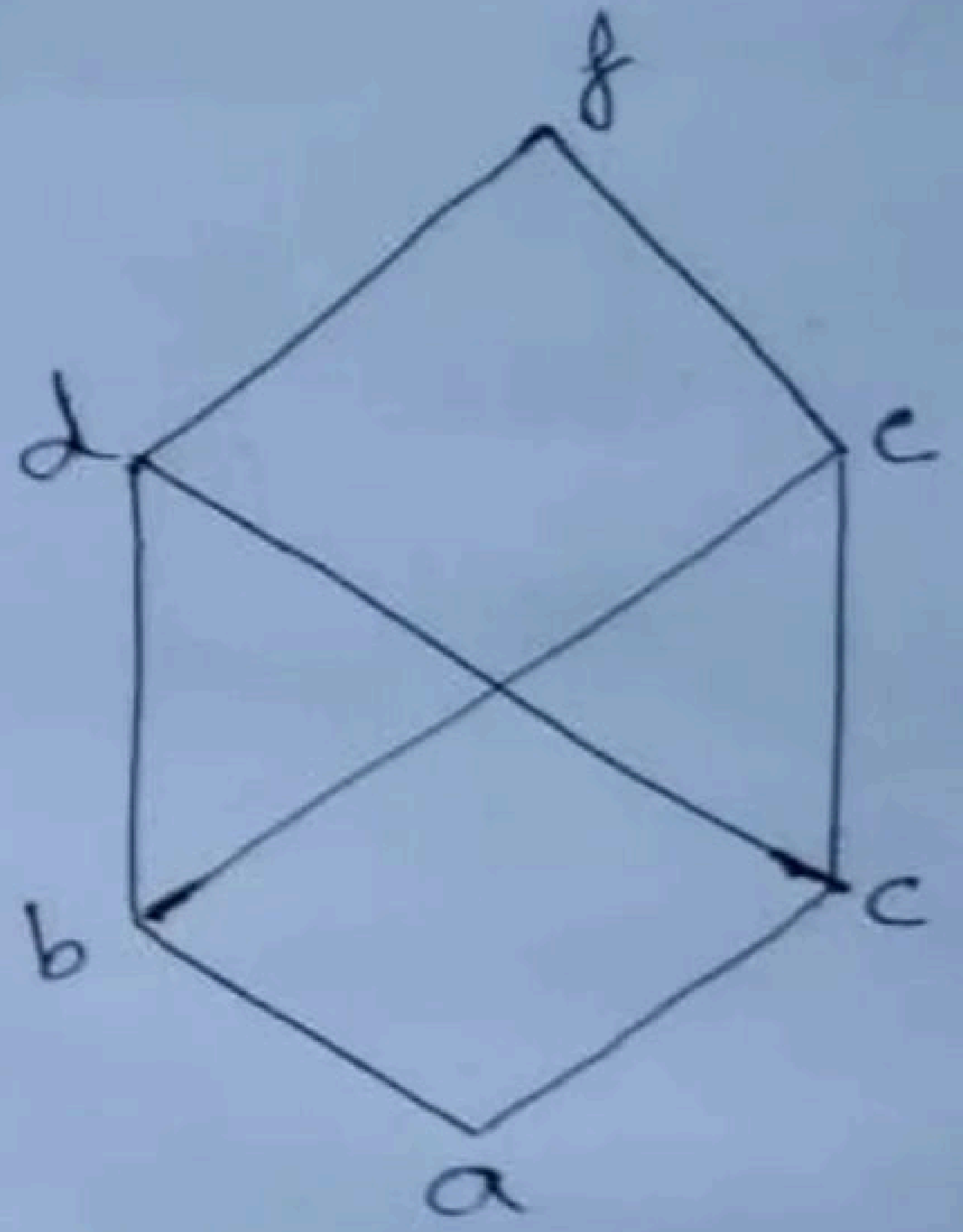
e
f
c
d
a
b

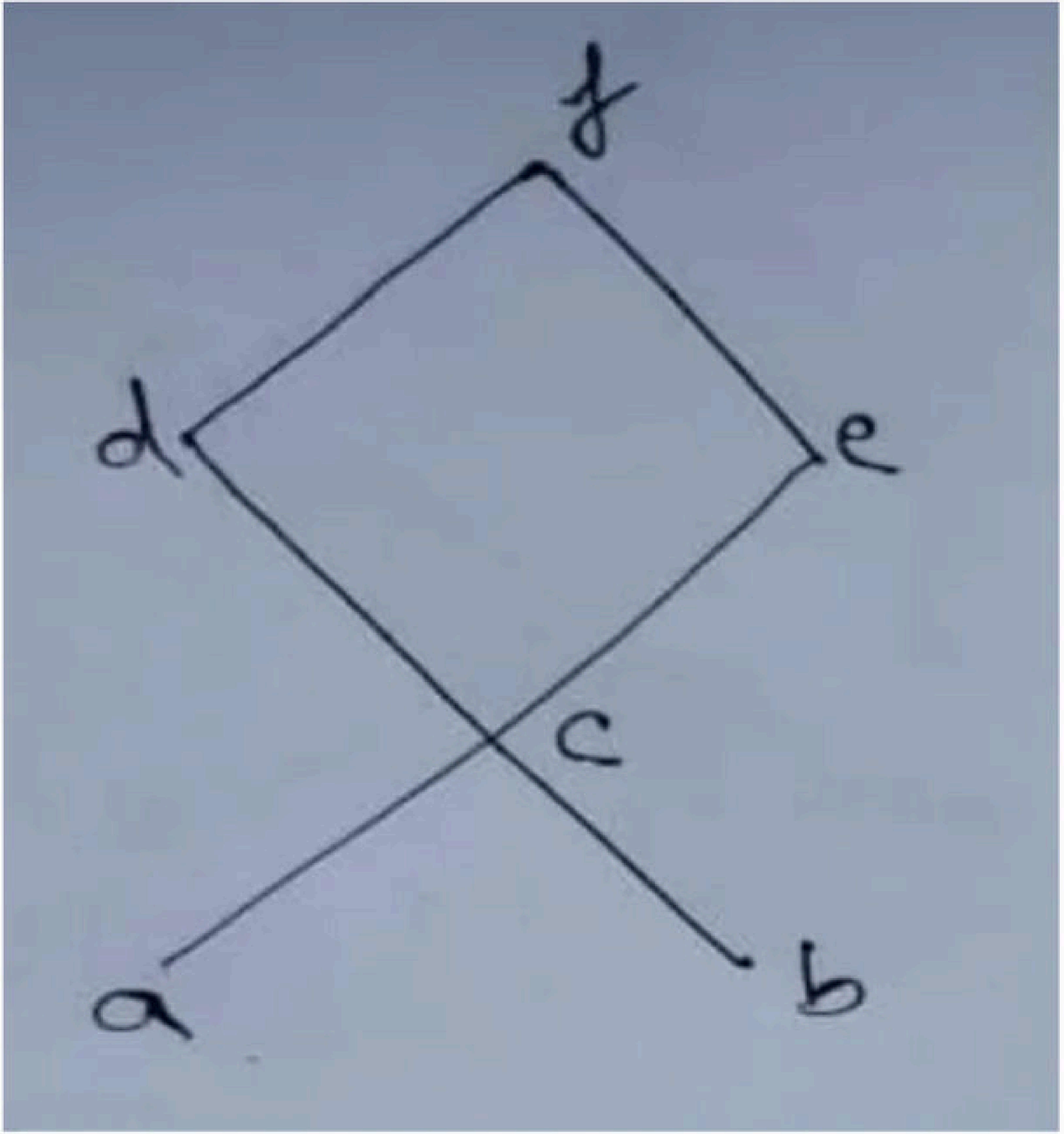

e
f
d
b
c
a

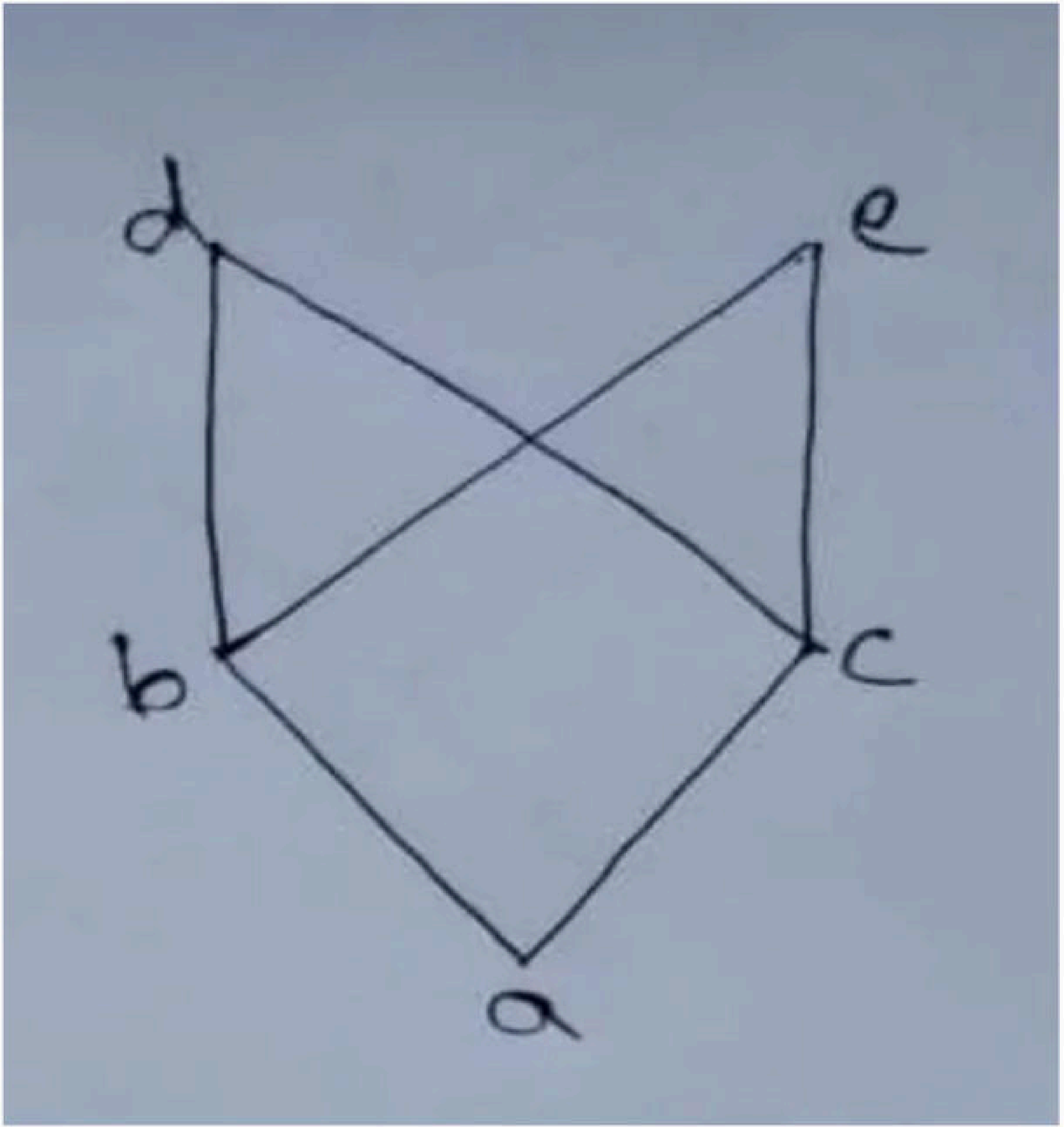

i
g
h
b
f
c
d
e
a

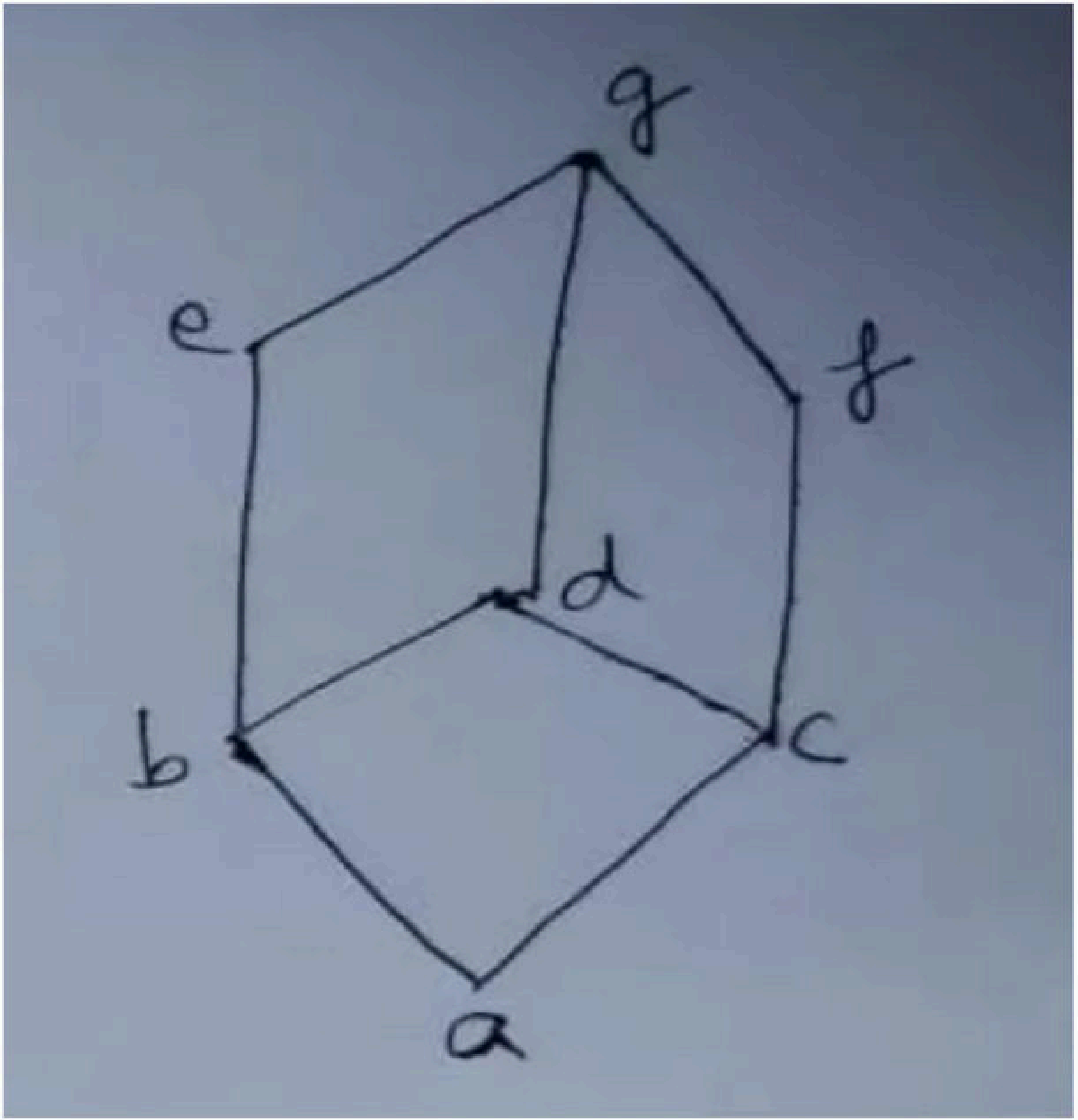

d
e
c
a
b

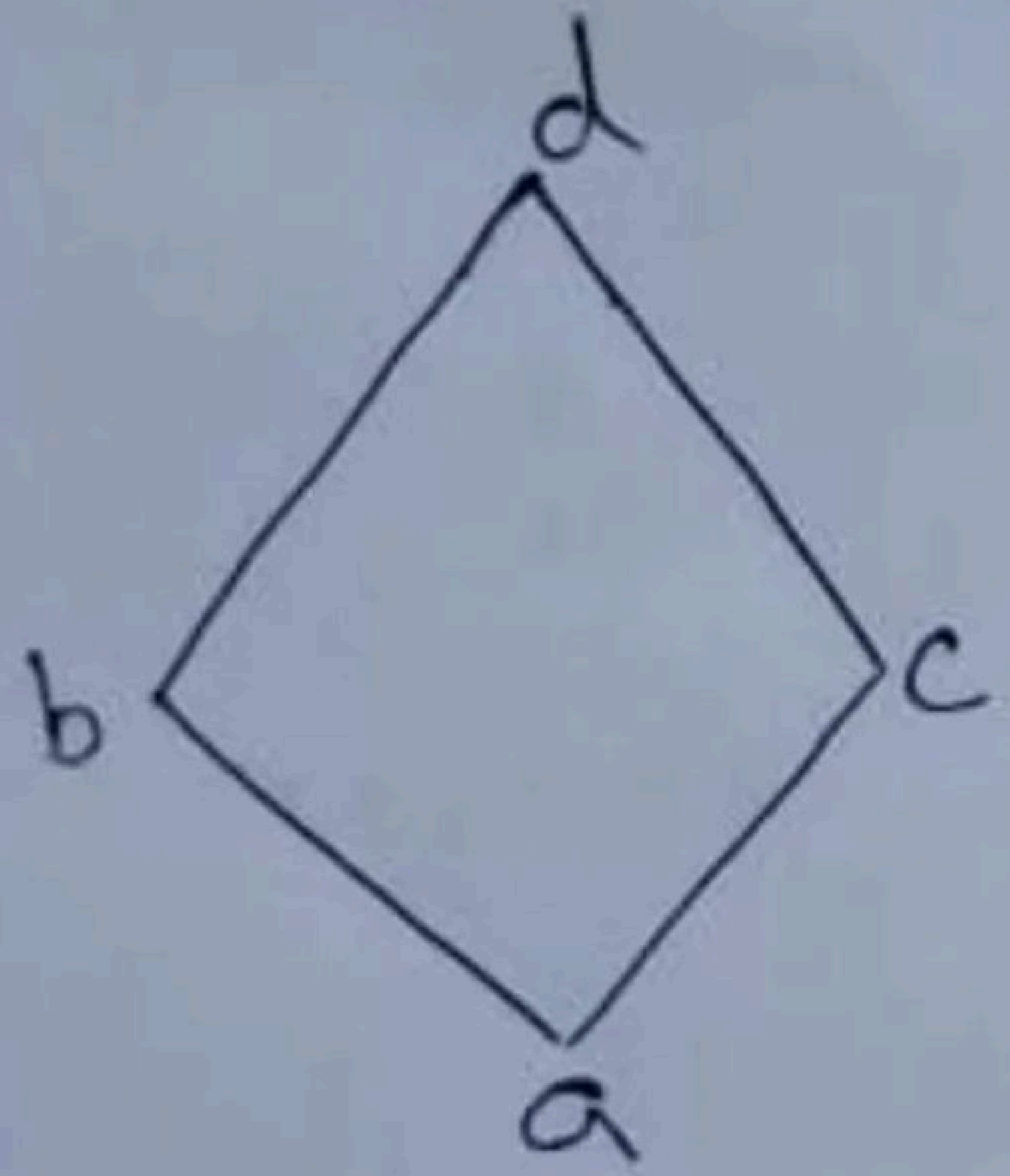

f
e
d
c
b
a

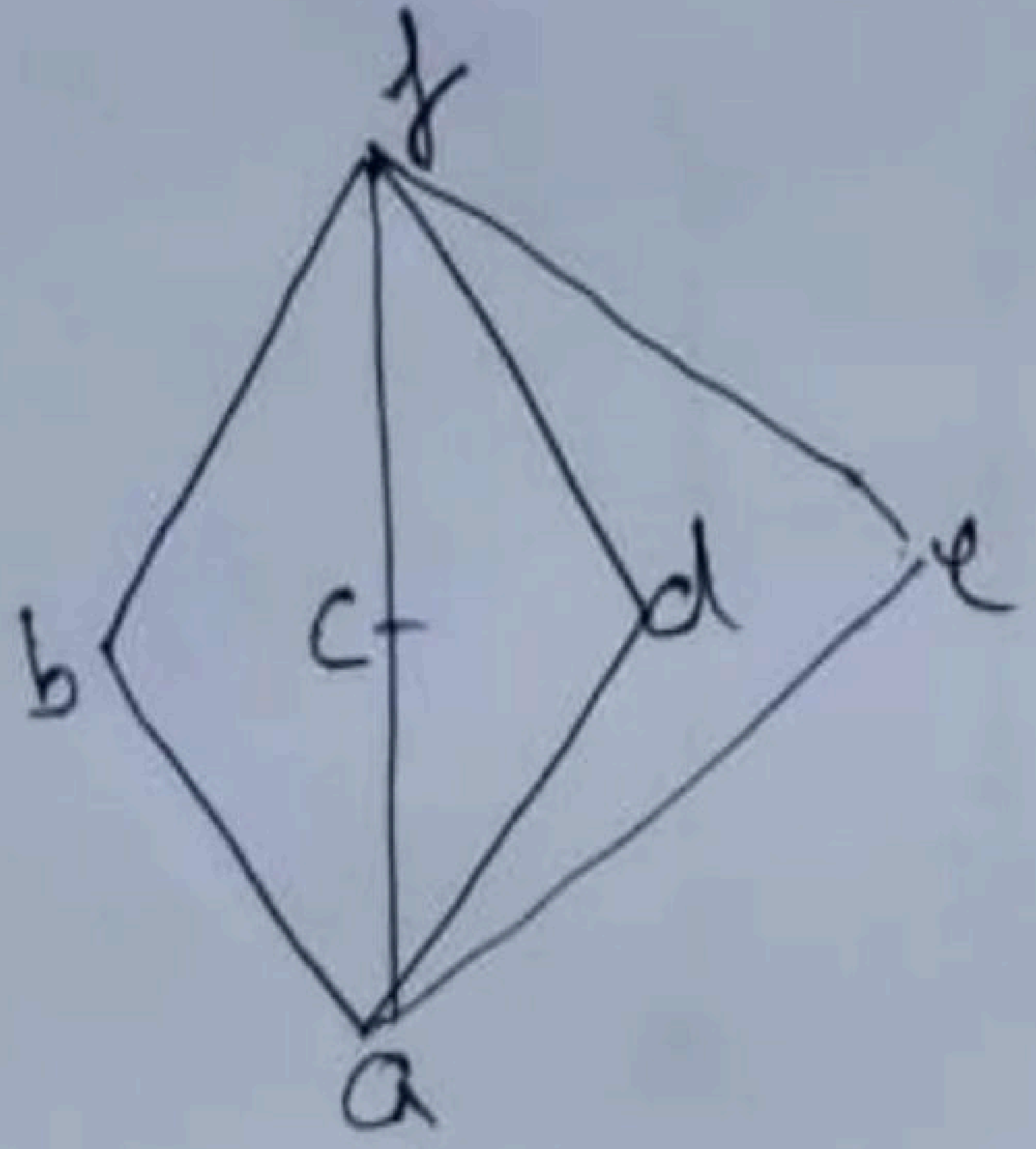

f
d
e
b
c
a

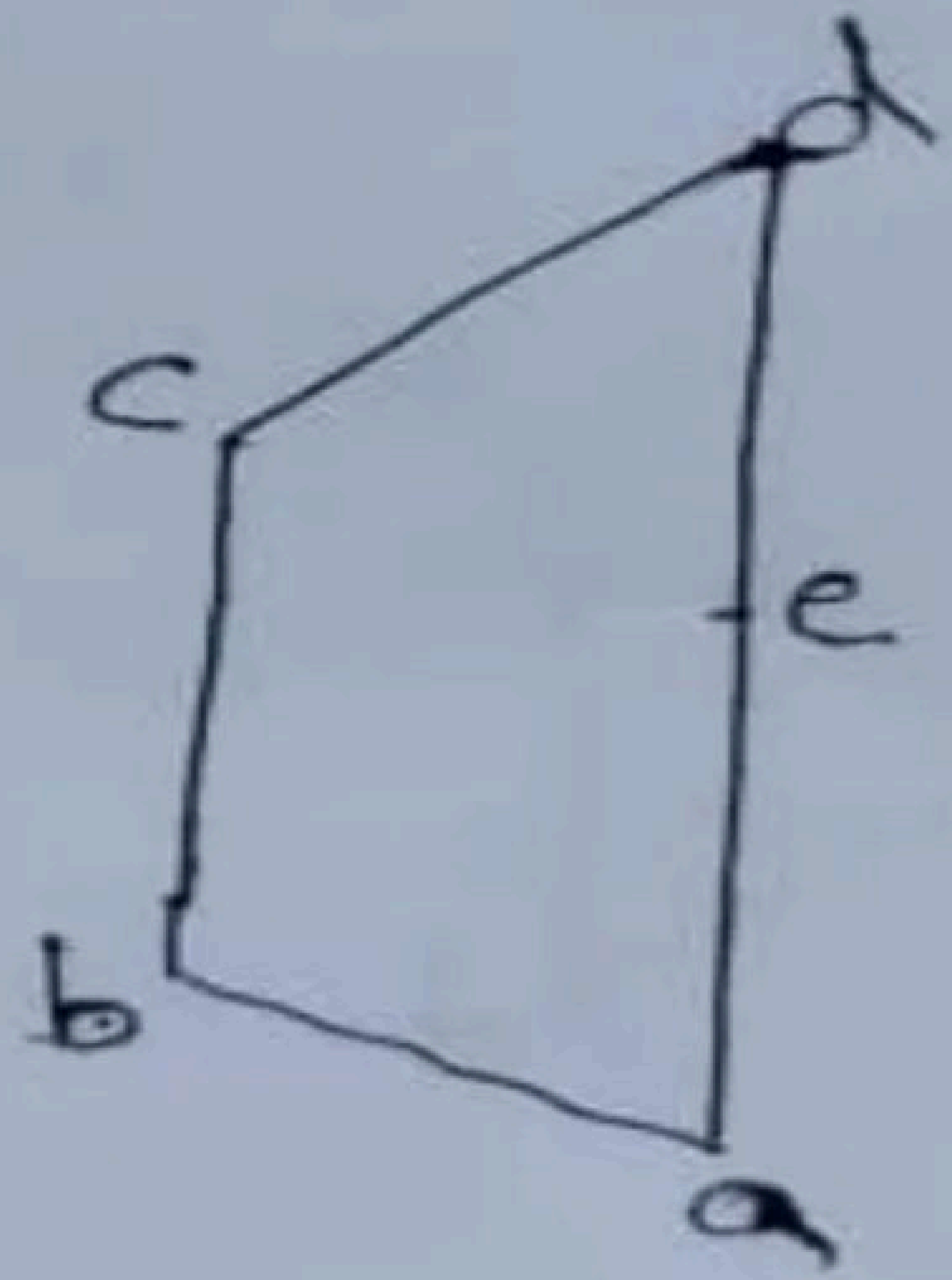

f
d
e
c
a
b


d
e
b
c
a


g
e
f
d
b
c
a

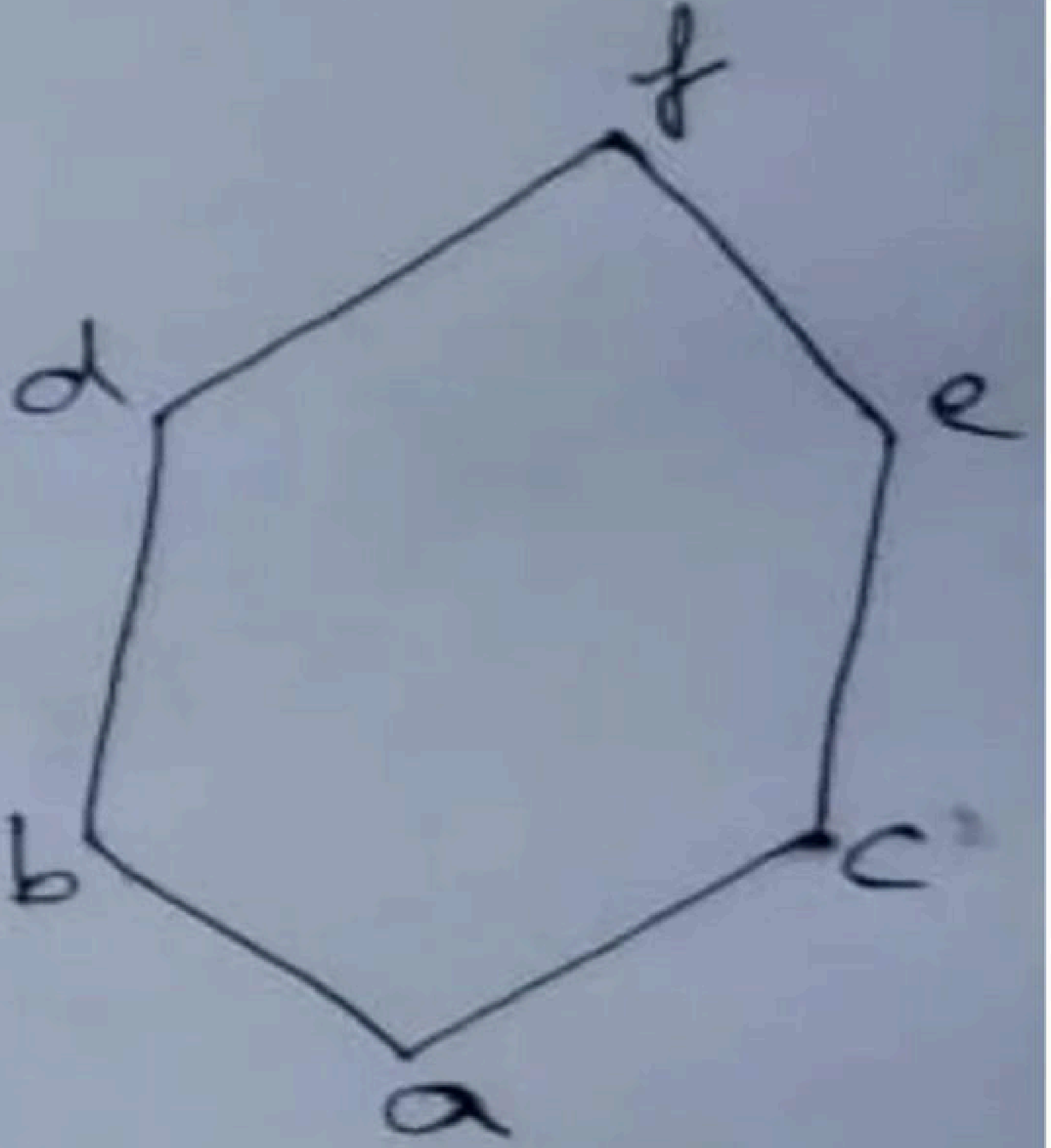

d
b
c
a

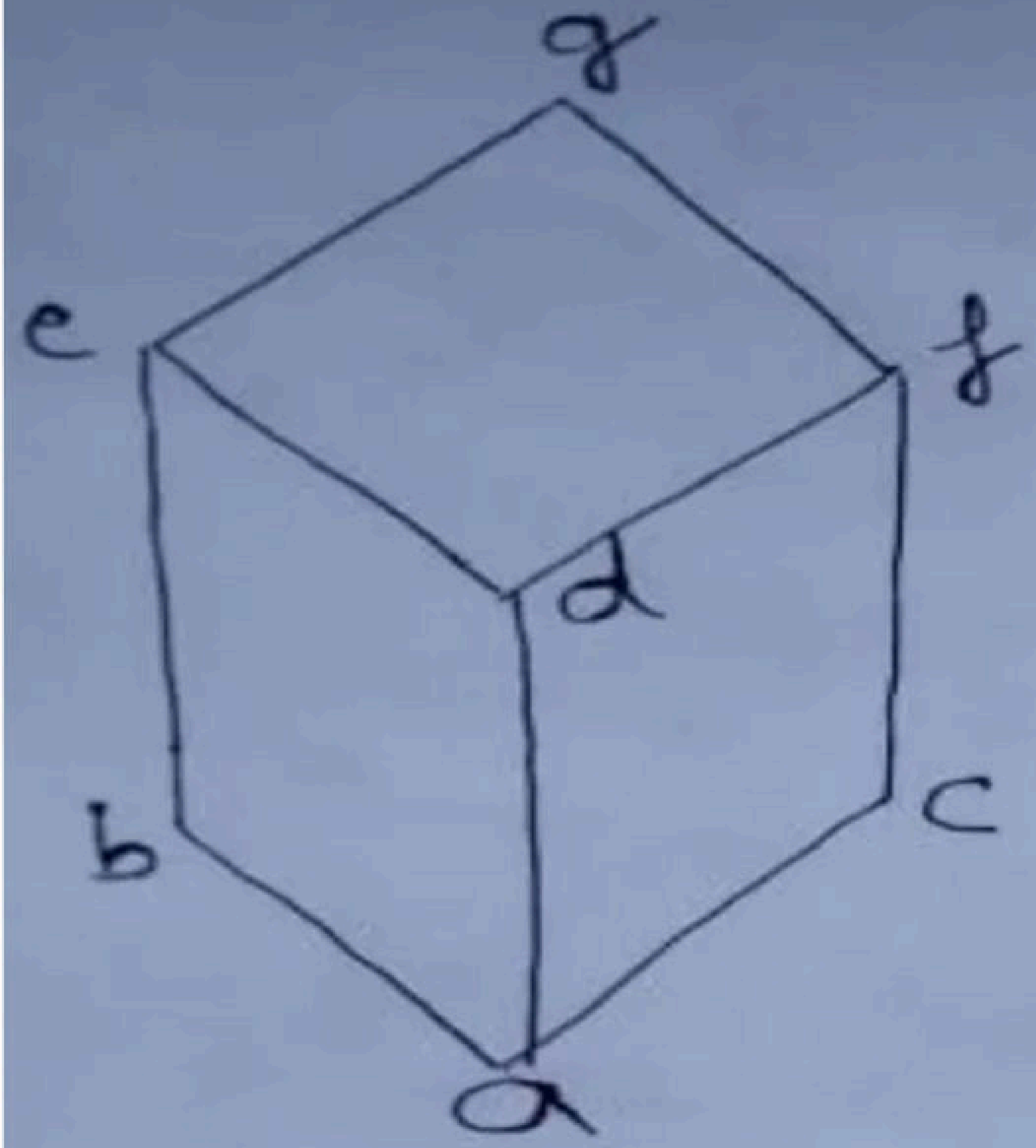

f
b
c
d
e
a

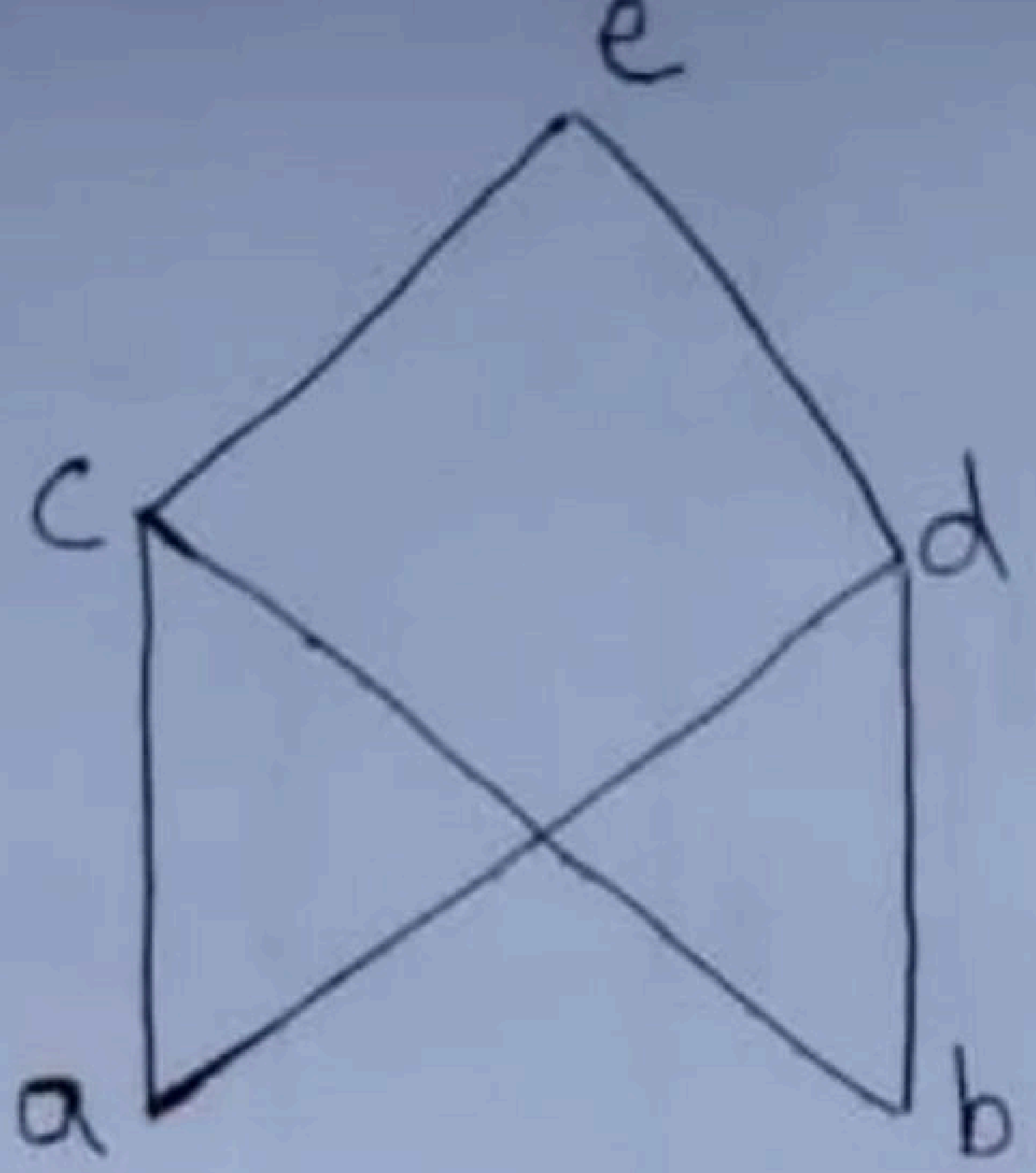


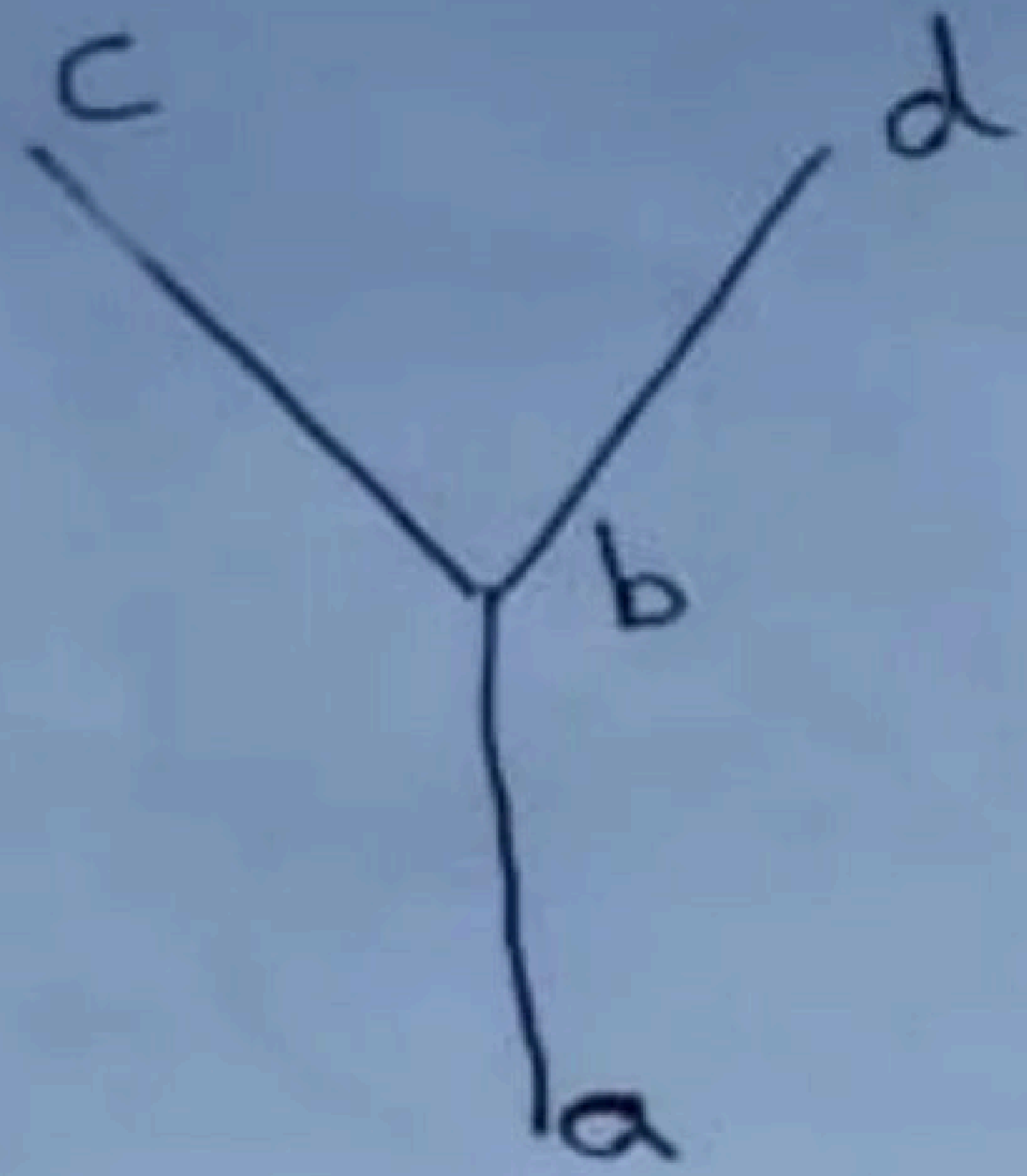
d
c
e
b
a

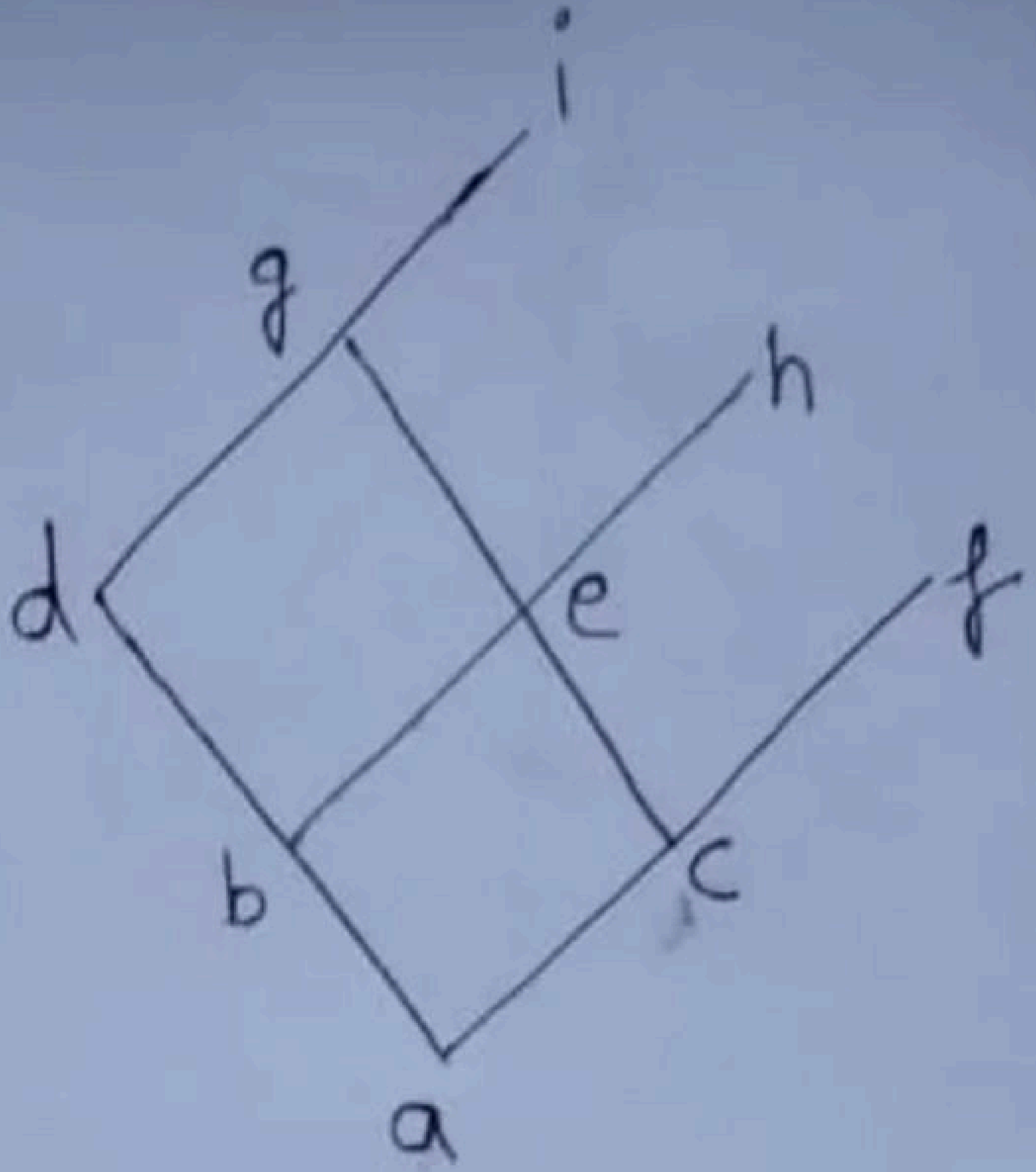
Break

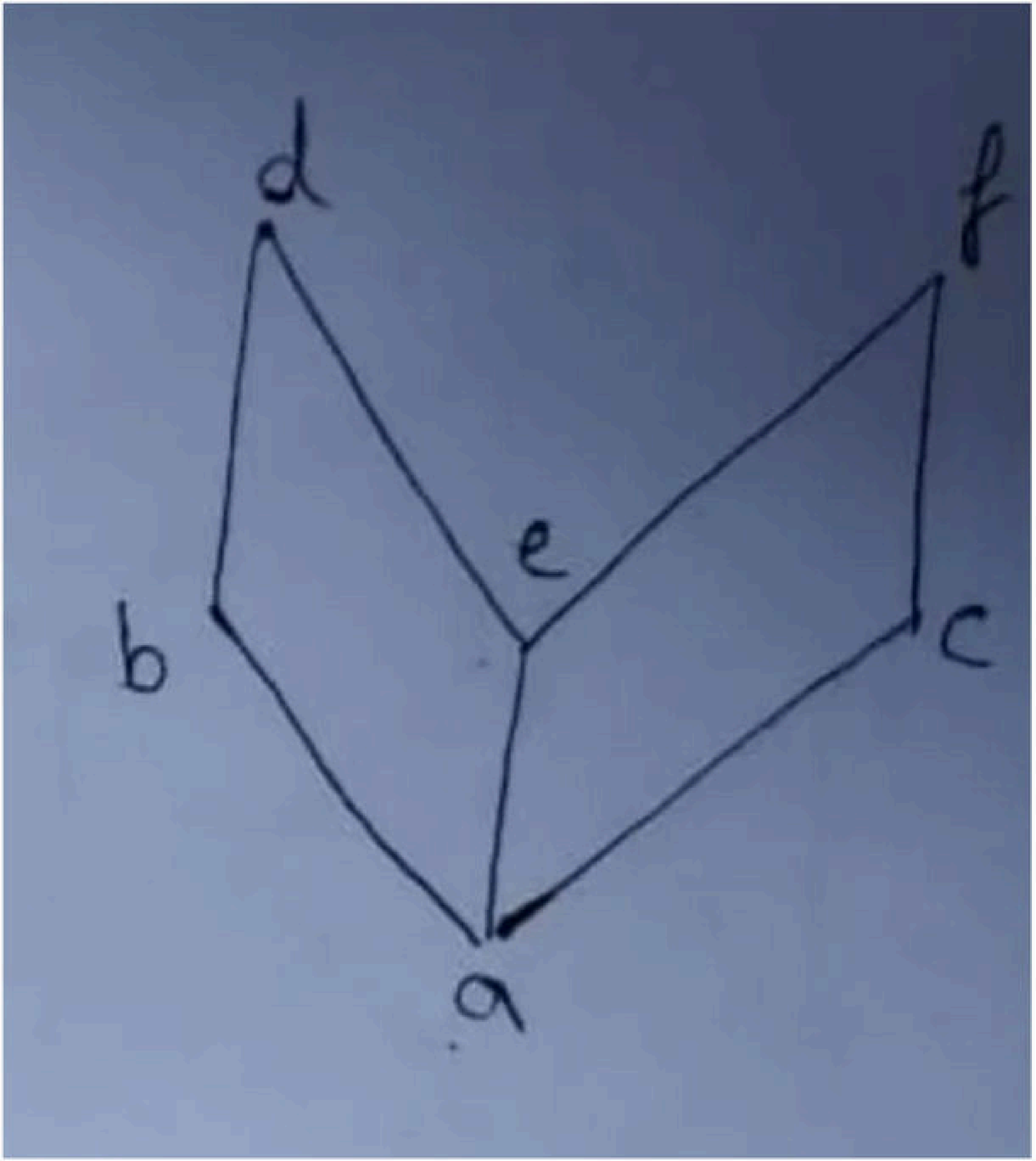

f
d
e
b
c
a


g
e
f
d
b
c
a

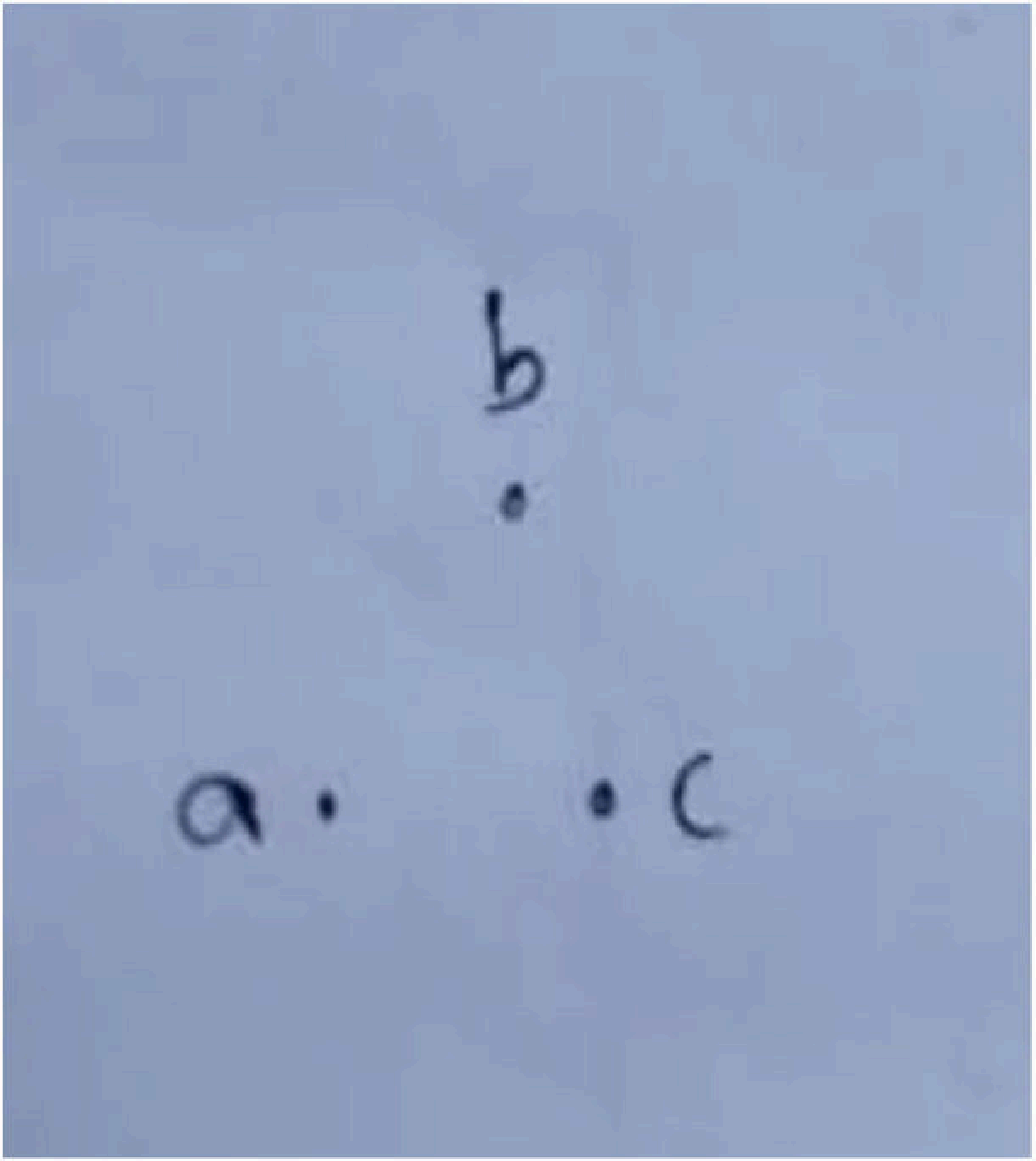

e
c
d
a
b


c
d
b
a

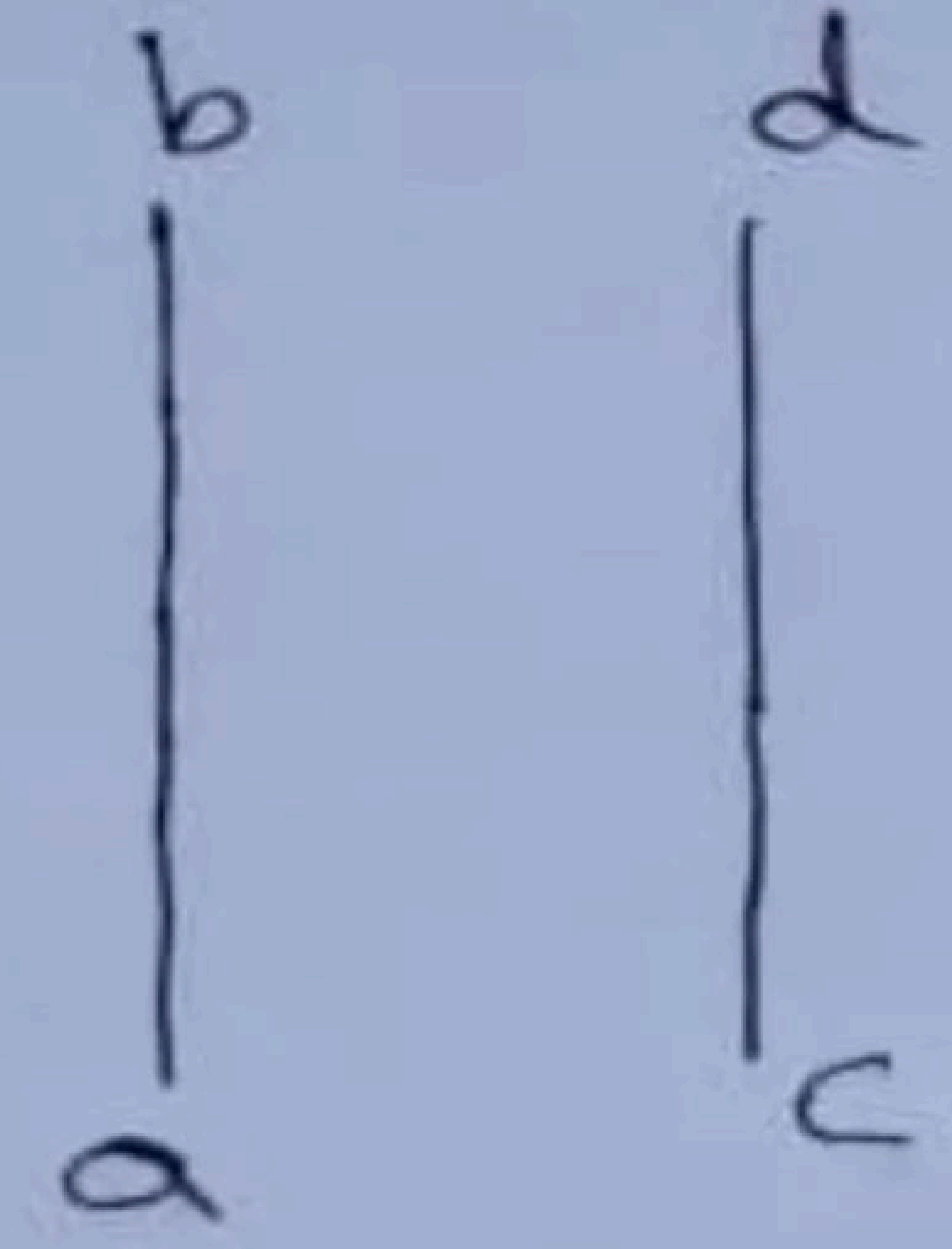

i
g
h
d
e
f
b
c
a

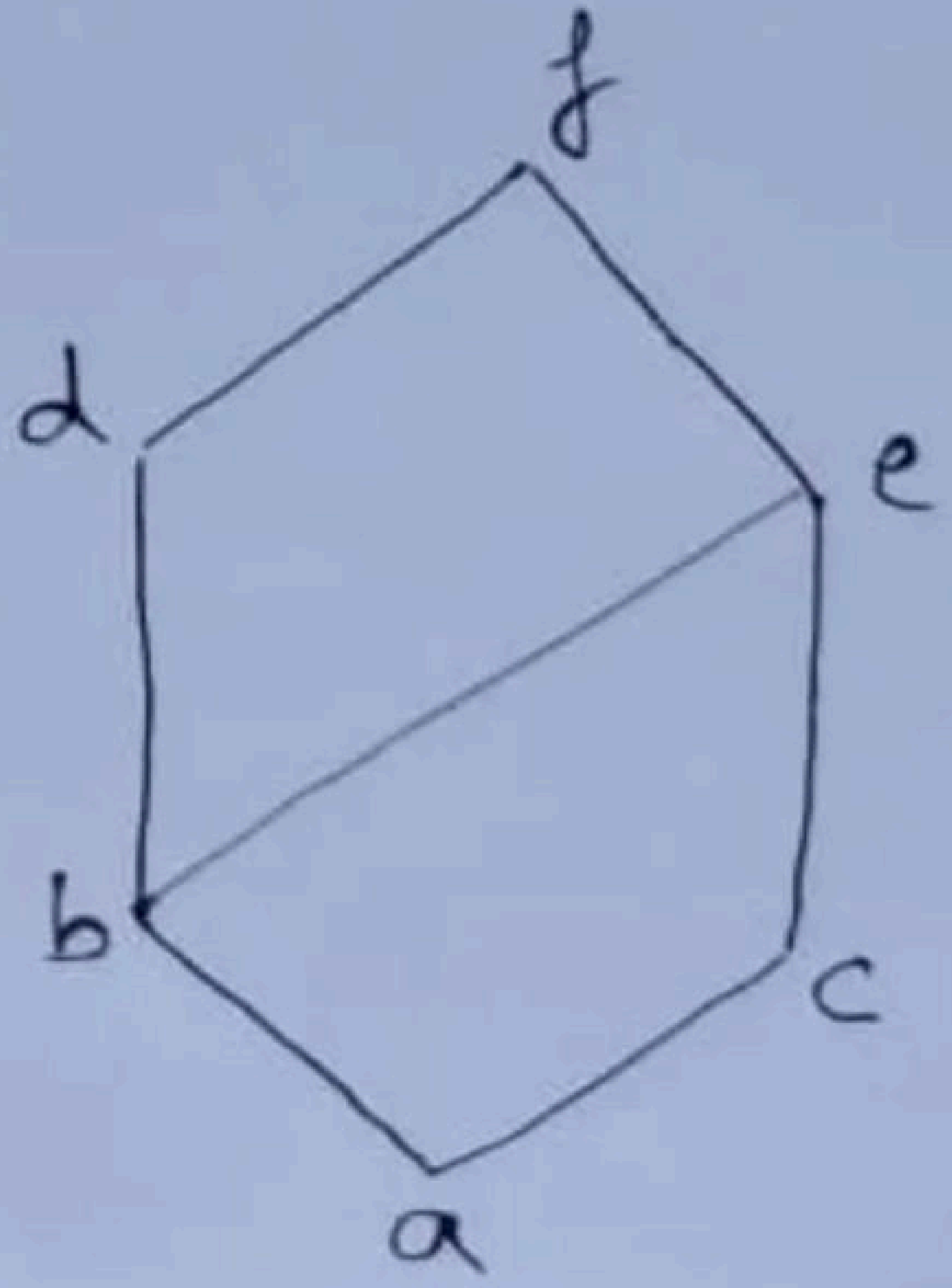

d
f
e
b
c
a

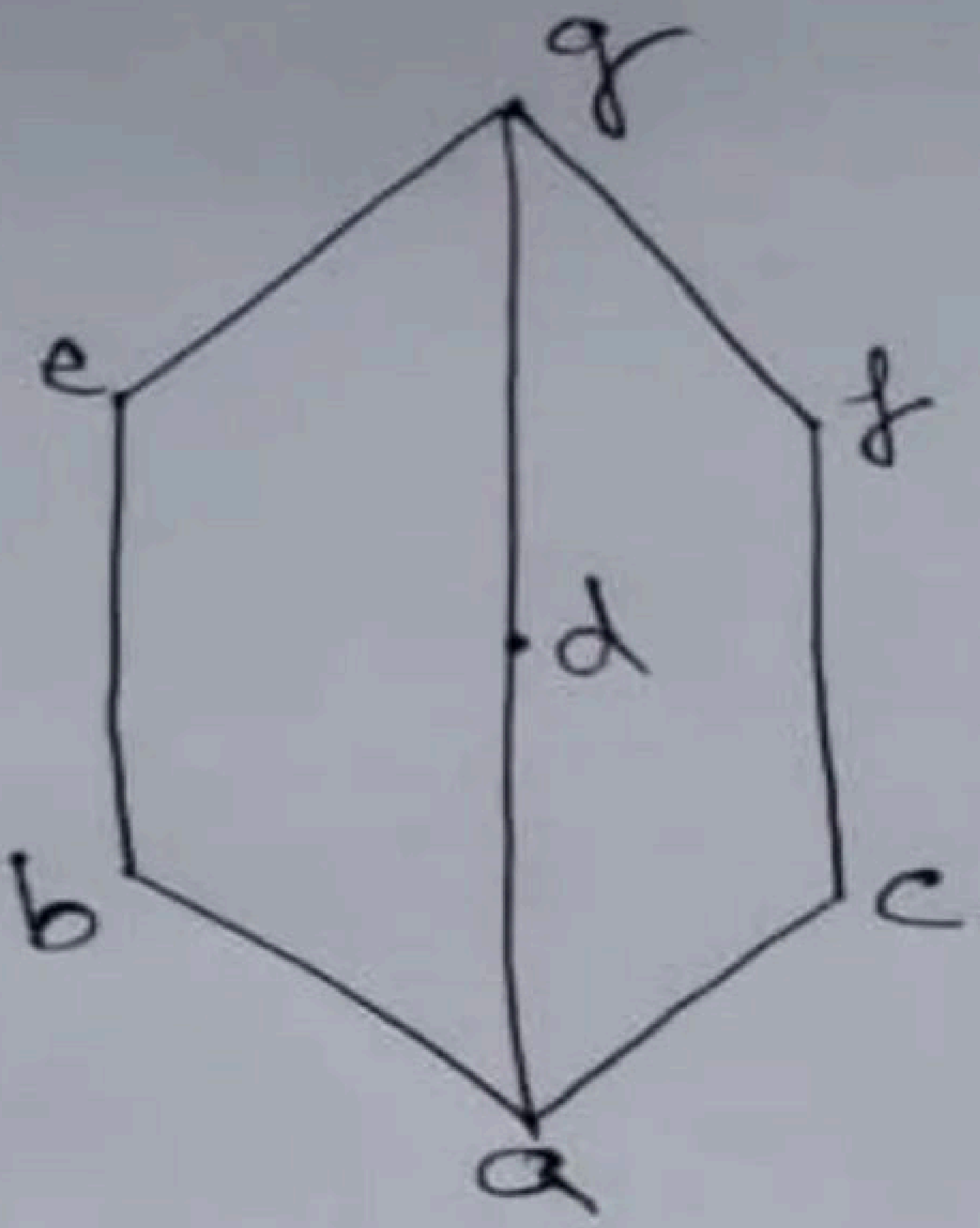

d
c
b
a

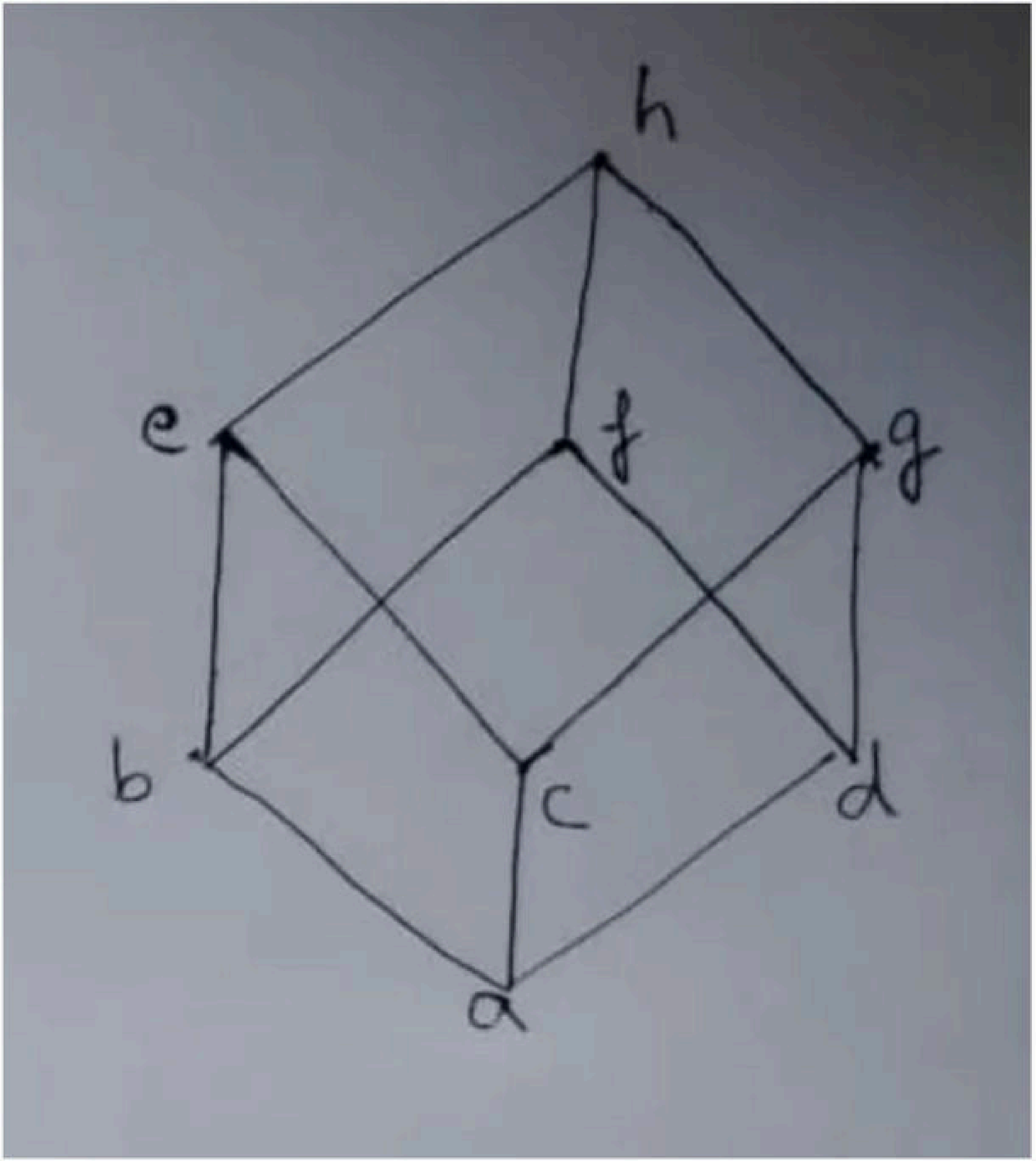

b
a
c


a


b
d
a
c

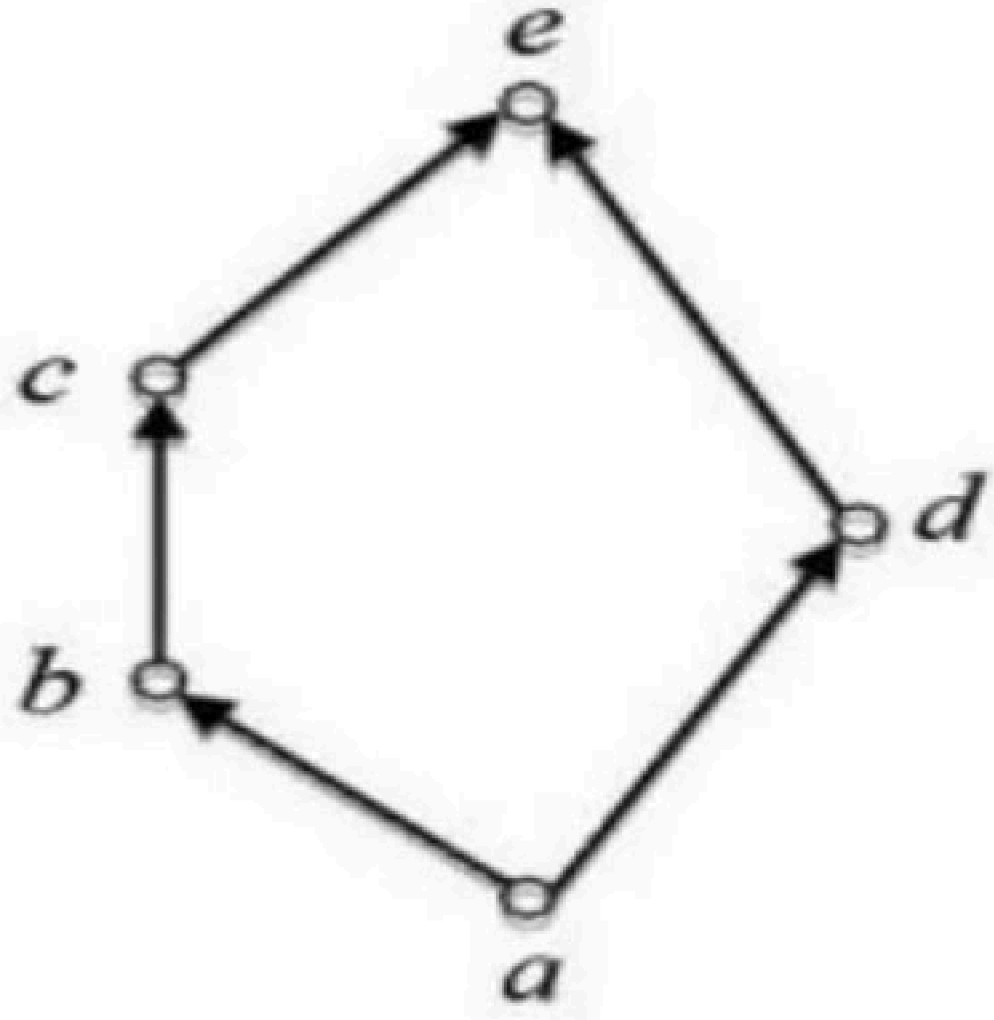

f
d
e
b
c
a


g
e
f
d
b
c
a


h
e
f
g
b
c
d
a

Break

Q Consider the set **X**={a, b, c, d, e} under partial ordering **R**={(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)}. The Hasse diagram of the partial order (X, R) is shown below. The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is ______ **(GATE-2017) (1 Marks)**



Q A partially ordered set is said to be a lattice if every two elements in the set have **(NET-Dec-2010)**

a) a unique least upper bound

b) a unique greatest lower bound

c) both (A) and (B)

d) none of the above

Q Consider the following Hasse diagrams

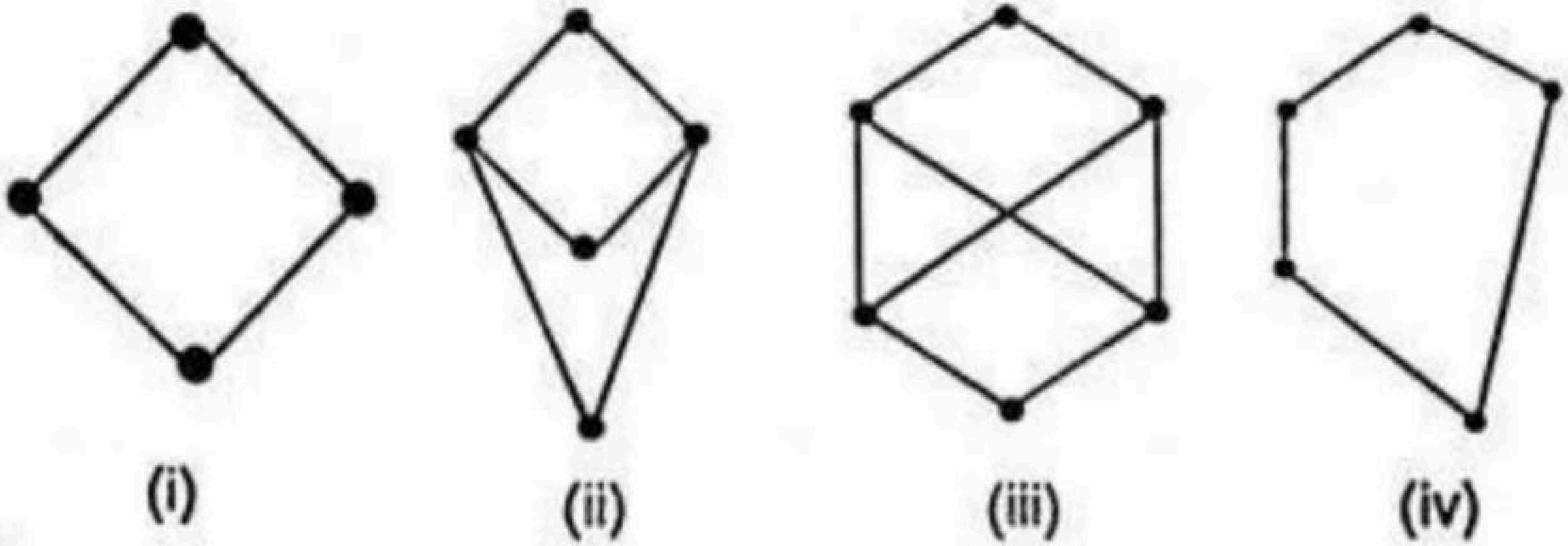
Which all of the above represent a lattice? **(GATE-2008) (2 Marks)**

(A) (i) and (iv) only

(B) (ii) and (iii) only

(C) (iii) only

(D) (i), (ii) and (iv) only



Q the inclusion of which of the following set into S = {{1,2}, {1,2,3}, {1,3,5}, {1,2,4}, {1,2,3,4,5}} is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?
(GATE-2004) (2 Marks)

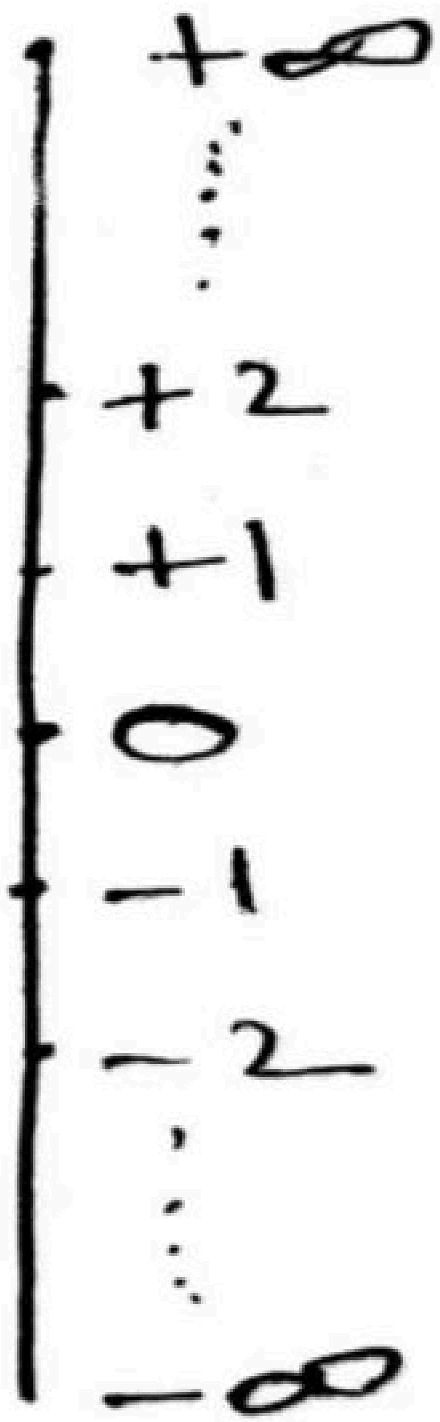
a) {1}

b) {1}, {2,3}

c) {1}, {1,3}

d) {1}, {1,3}, {1,2,3,4}, {1,2,3,5}

Break



Boolean algebra

- **Unbounded Lattice :-** If a lattice has infinite of elements then it is called Unbounded Lattice.

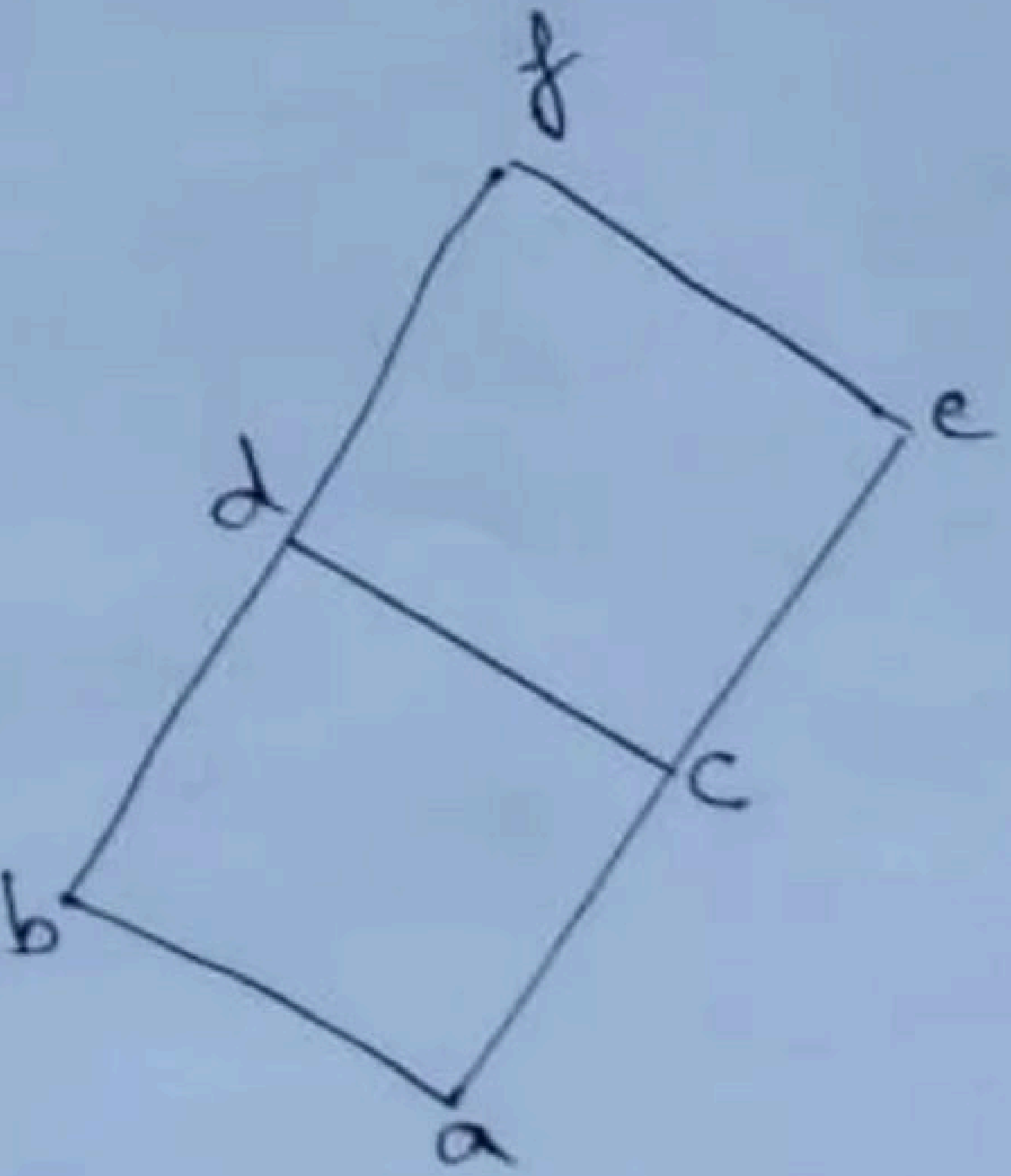
- **Bounded Lattice :-** If a lattice has finite number of elements then it is called Bounded lattice, there will be upper and lower bound in lattice.

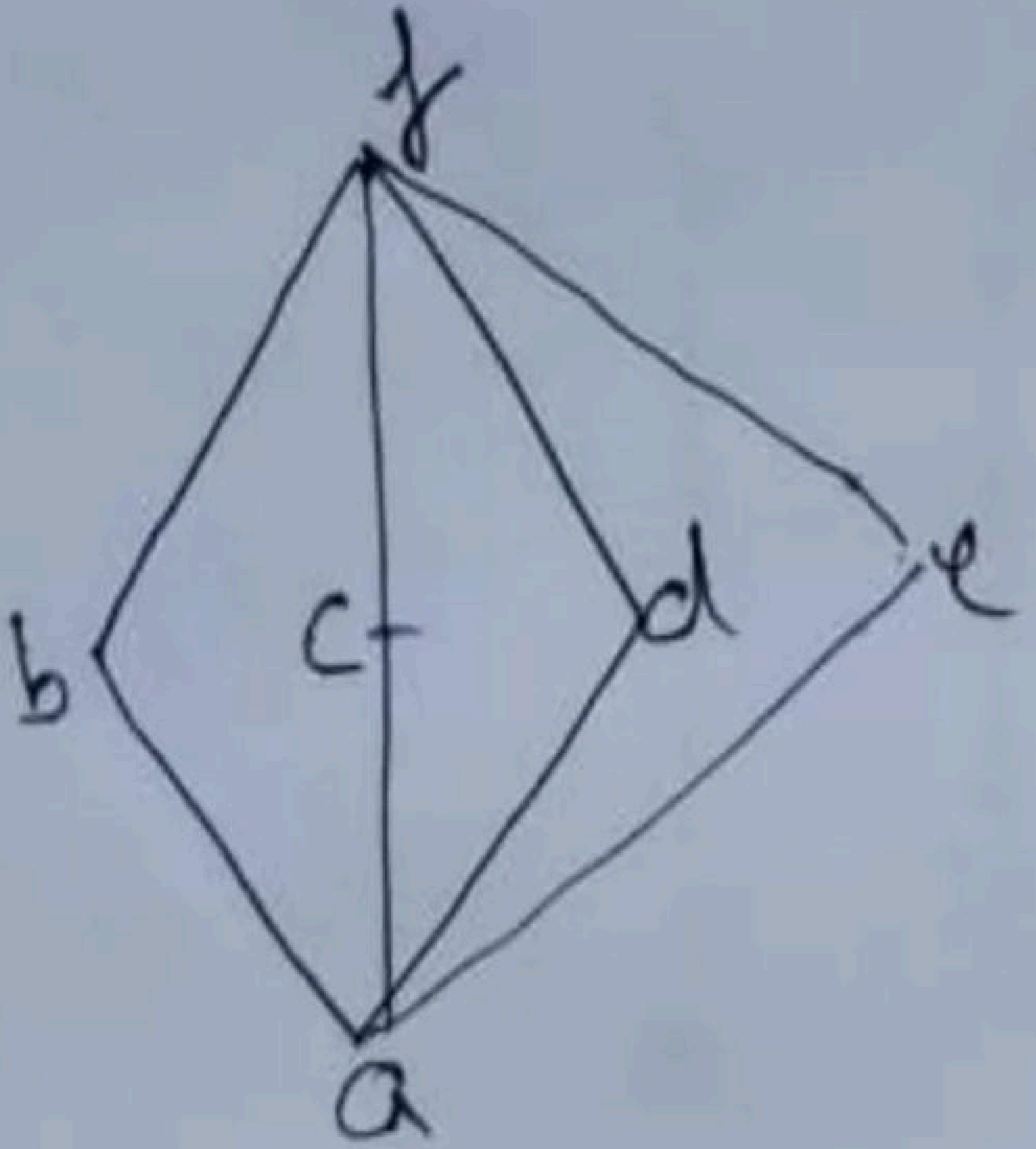
- **<u>Complement of an element in a Lattice</u> :-** If two elements a and a^c, are complement of each other, then the following equations must always holds good.
  $a \lor a^c$ = Upper bound of lattice
  $a \land a^c$ = Lower bound of lattice

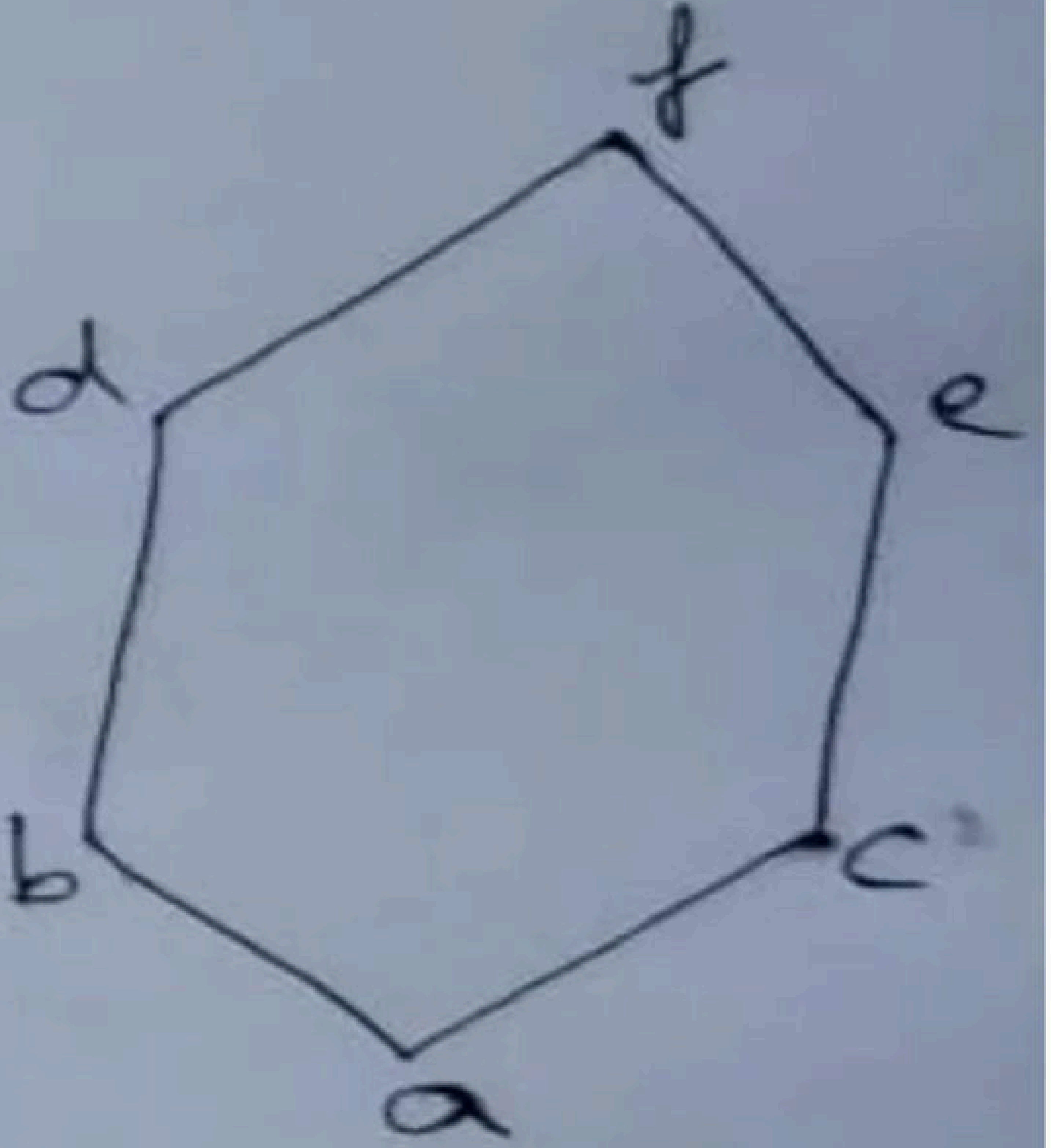
- **<u>Distributive Lattice</u> :-** A lattice is said to be distributed lattice. if for every element their exist at most one completemt(zero or one).

- **Complement Lattice :-** A Lattice is said to be Complement lattice. if for every element their exist at least one complement(one or more).

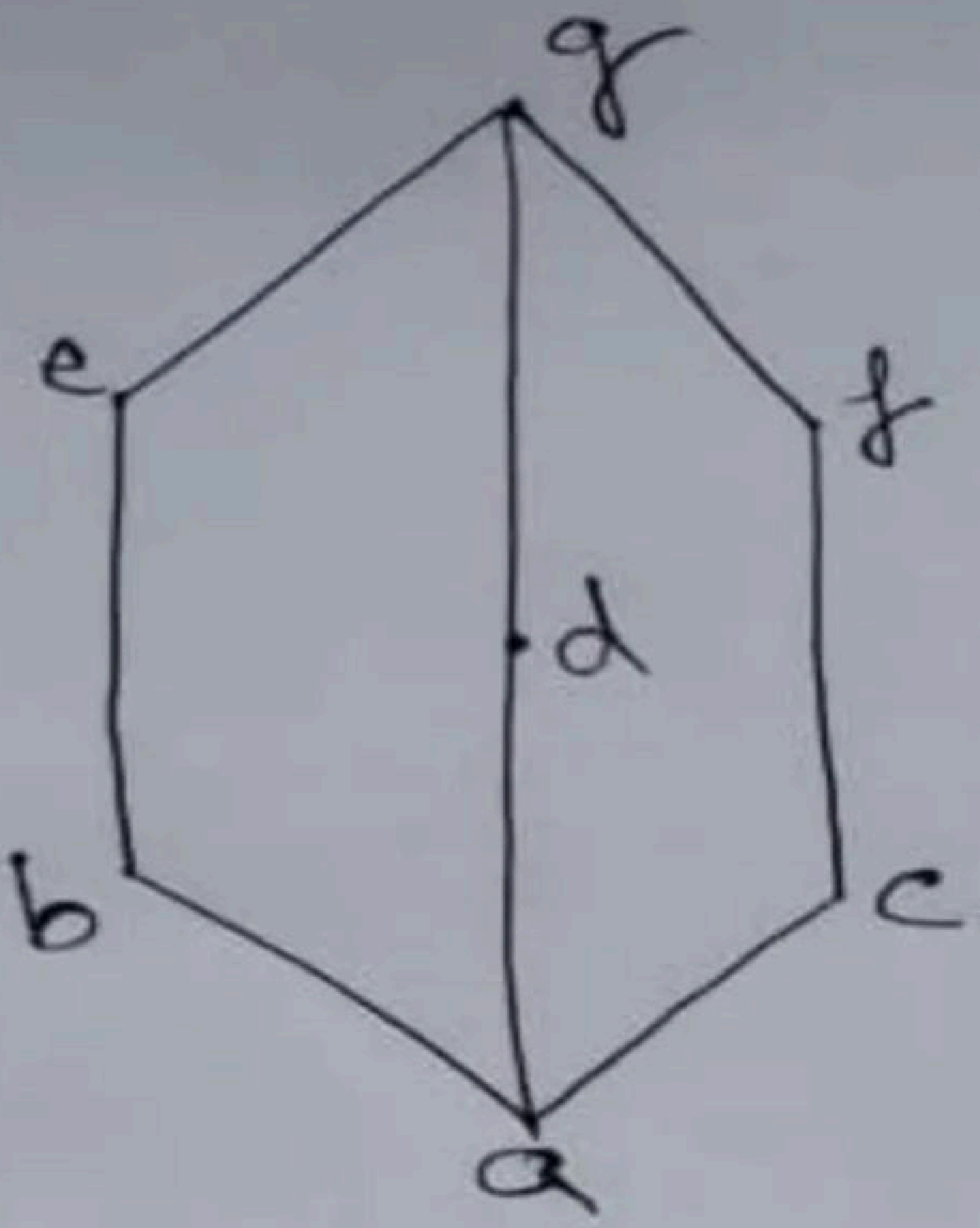
- **<u>Boolean Algebra</u> :-** A Lattice is said to be Boolean Algebra, if for every element their exist exactly one complement. Or if a lattice is both complemented and distributed then it is called Boolean Algebra.

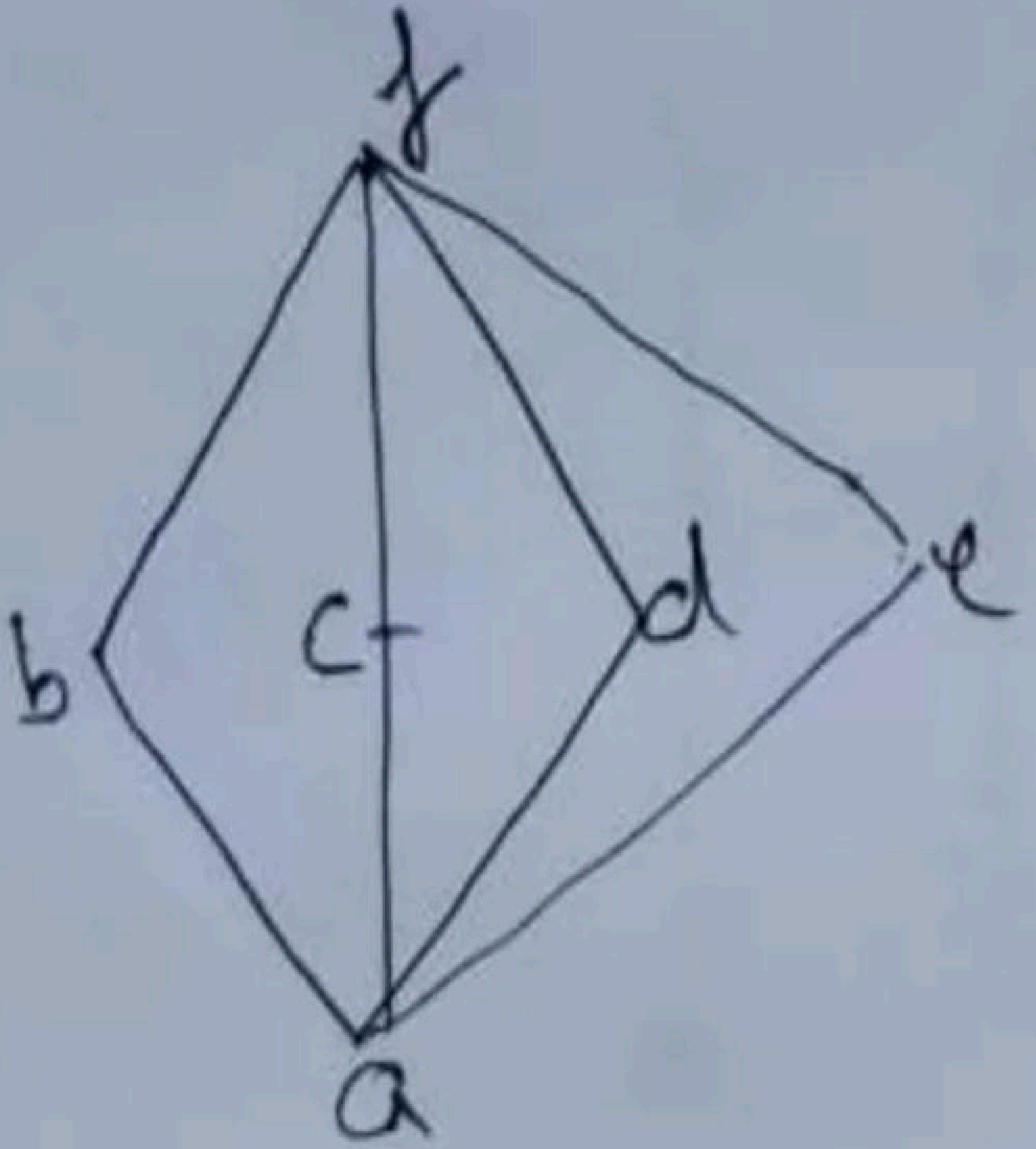

f
e
d
c
b
a


f
b
c
d
e
a

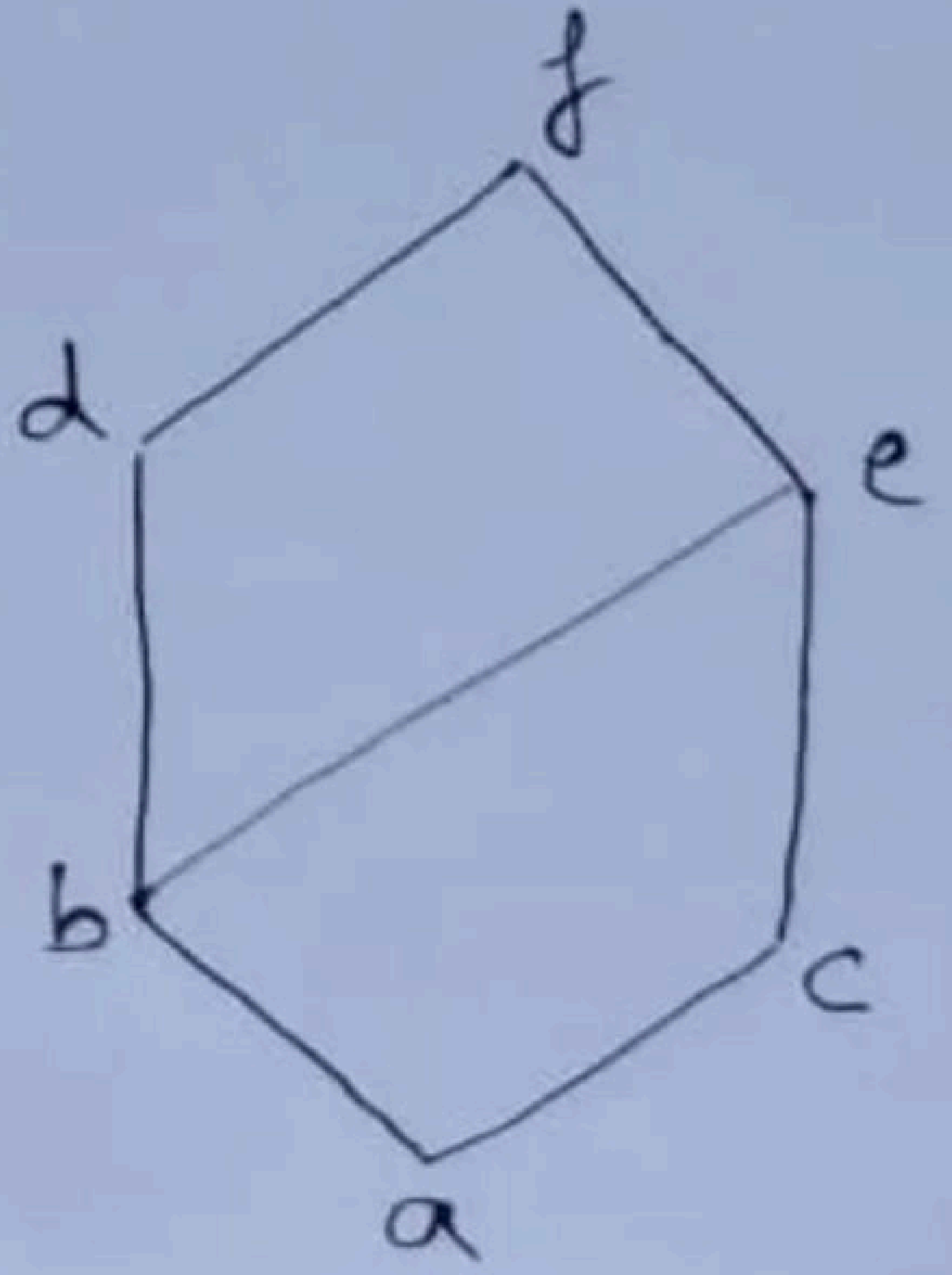


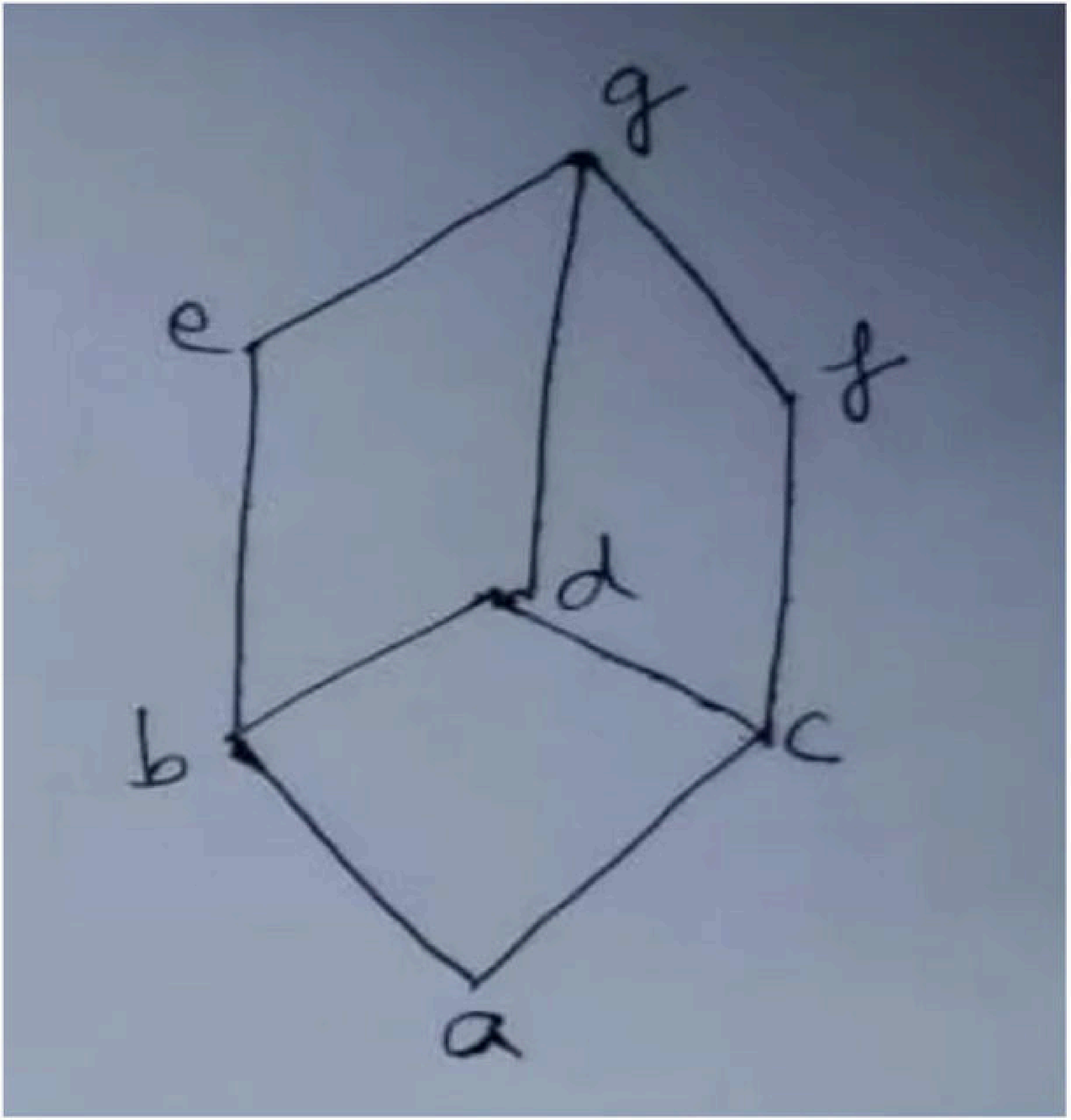
f
d
e
b
c
a


g
e
f
d
b
c
a



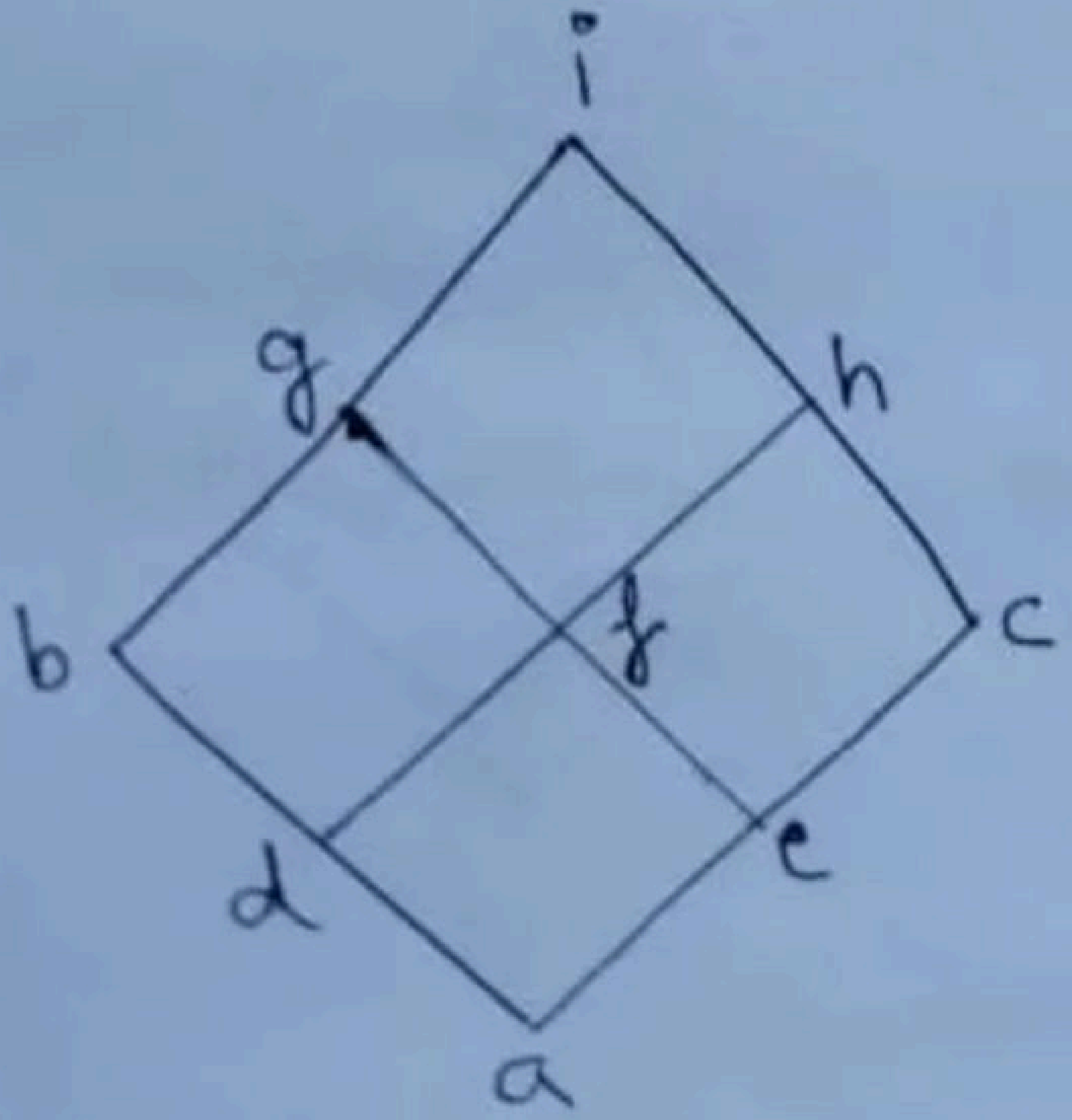
f
b
c
d
e
a


f
d
e
b
c
a

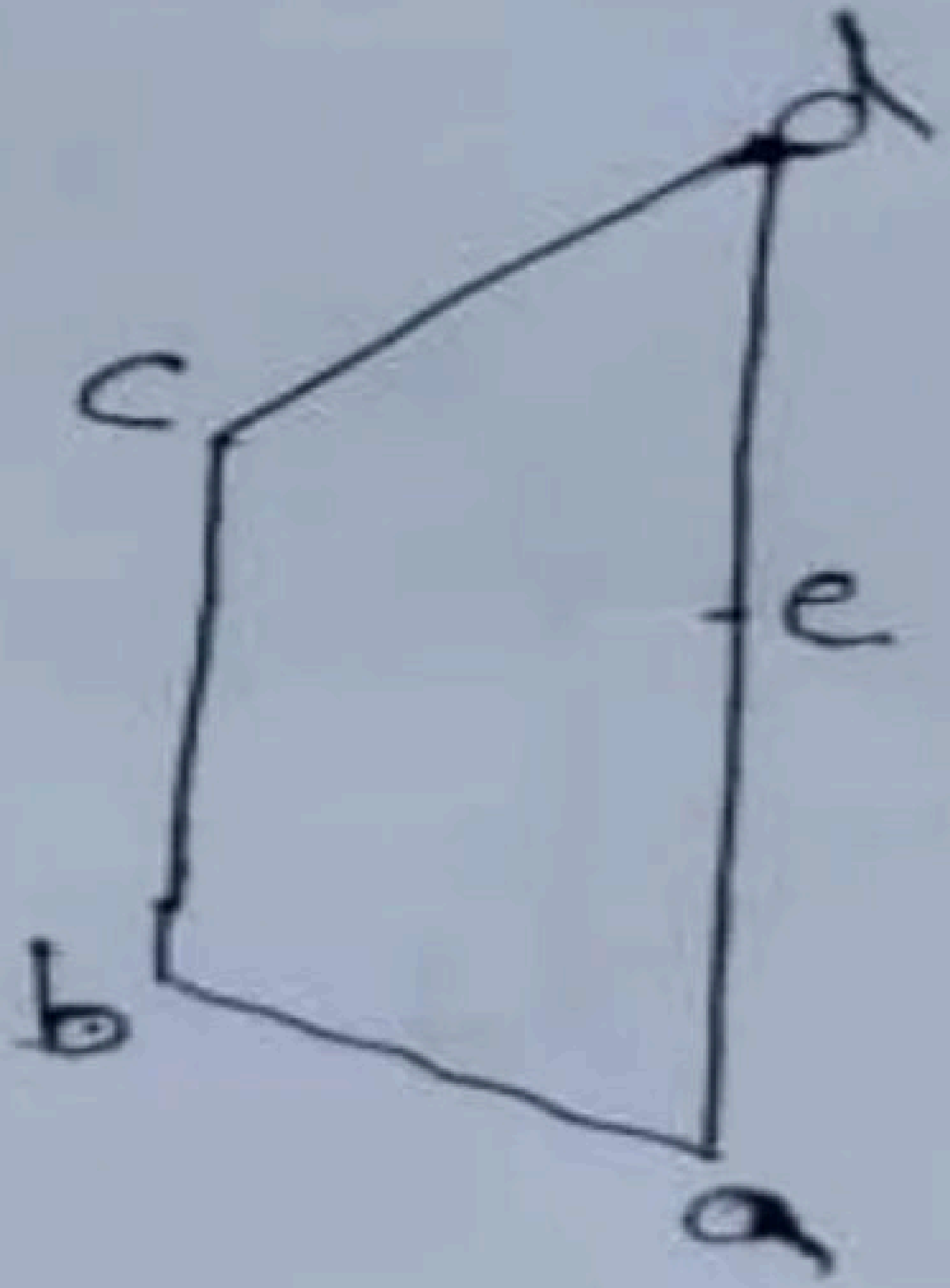

g
e
f
d
b
c
a

Break

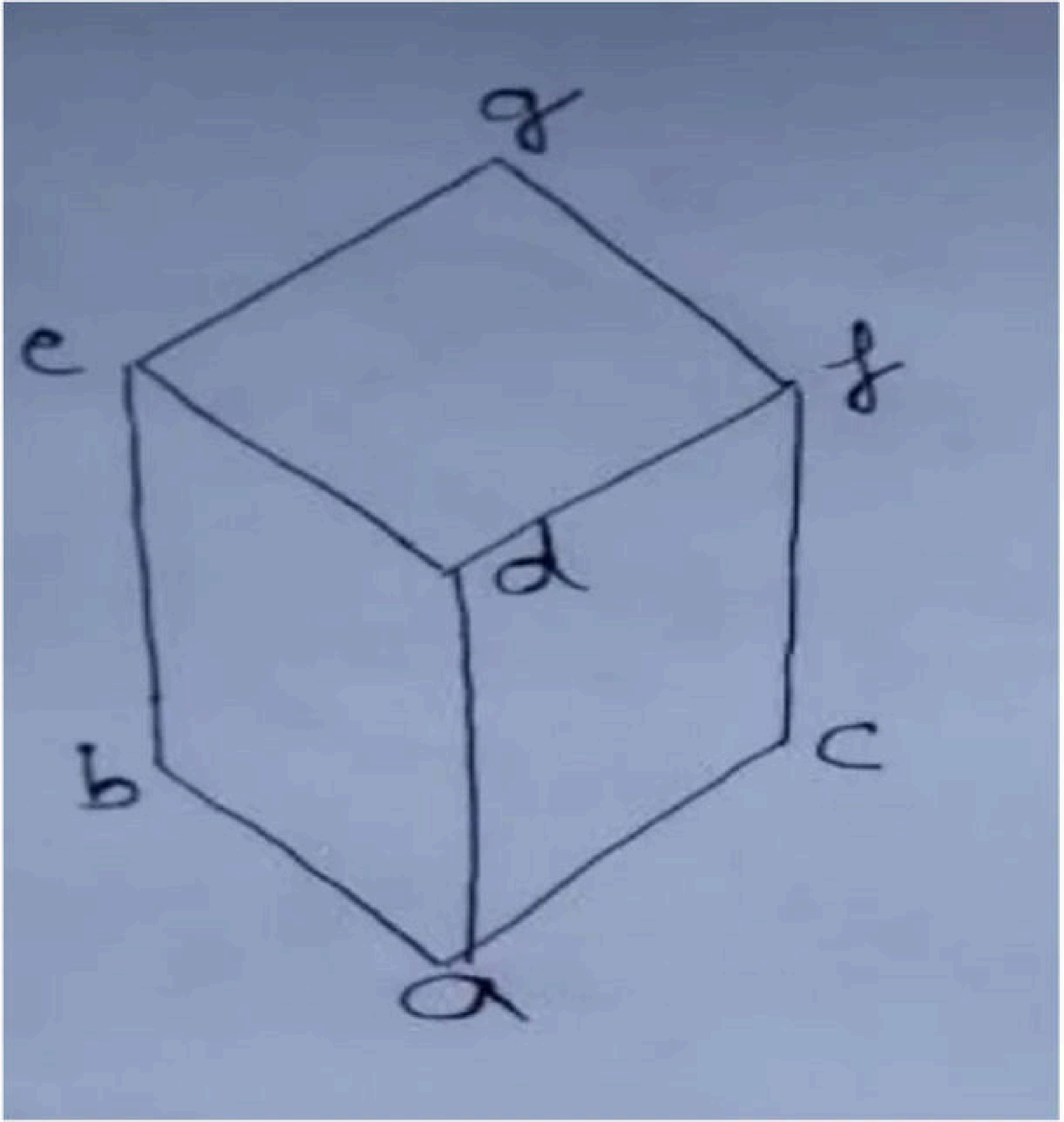

d
c
b
a

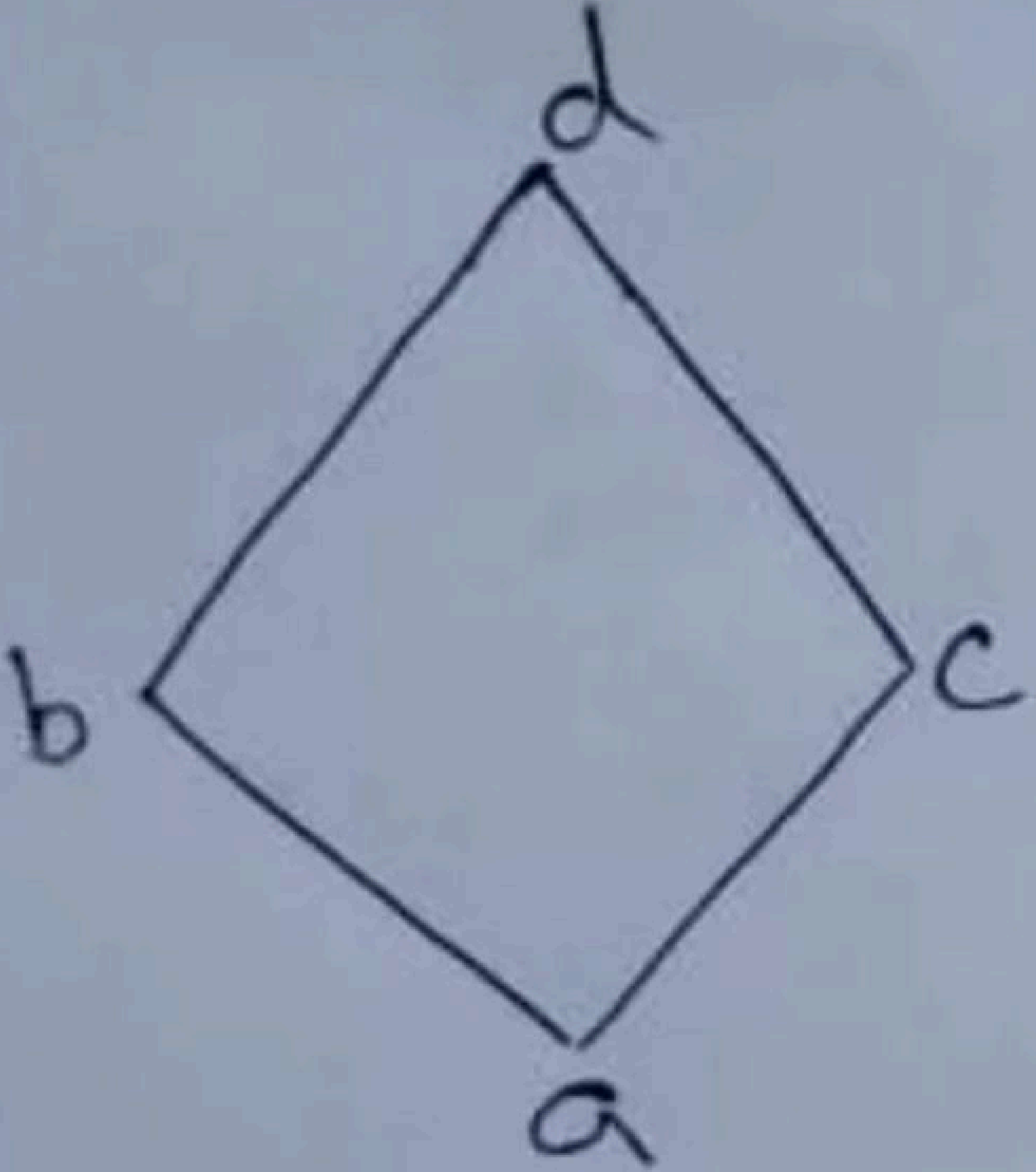


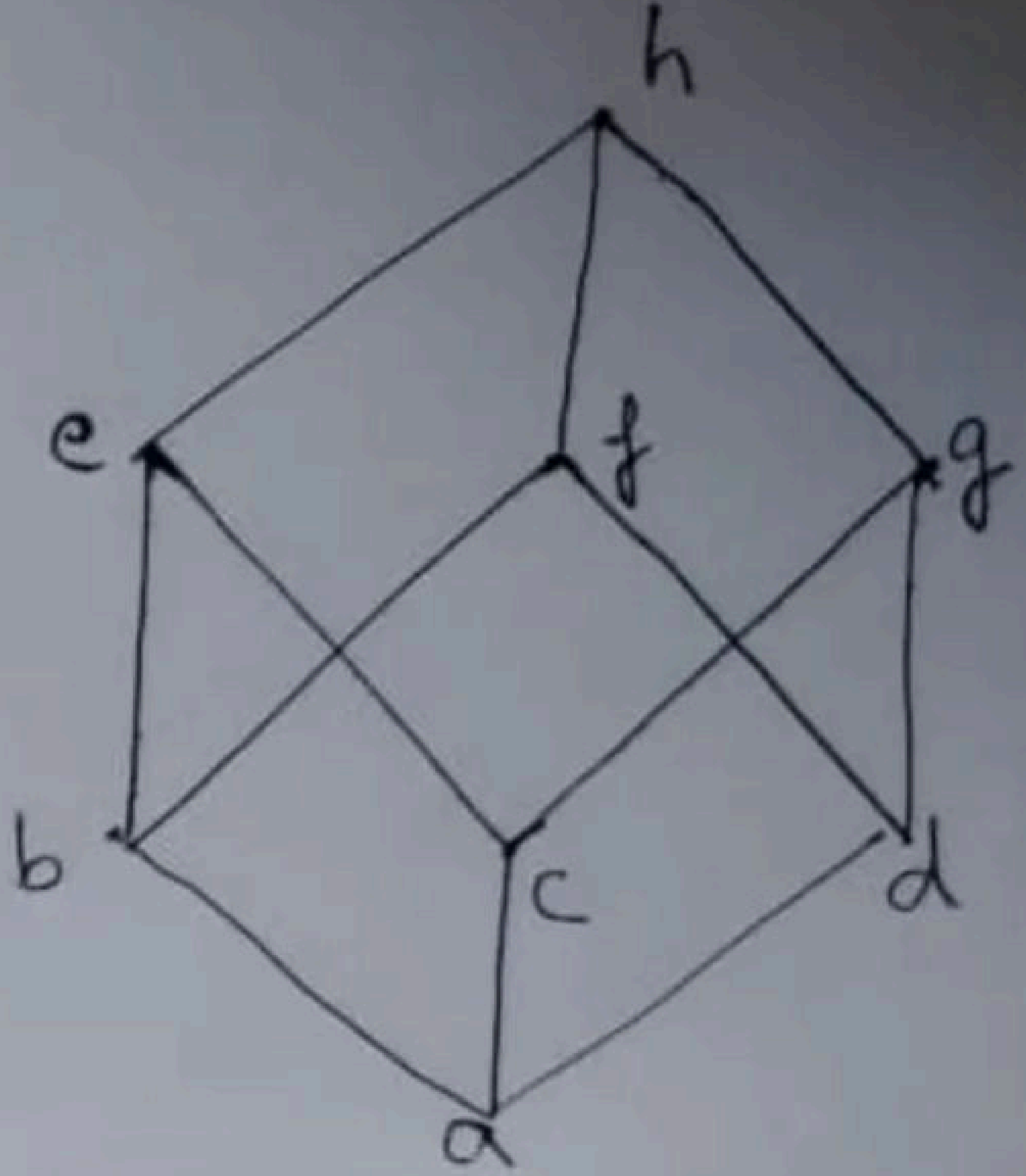
i
g
h
b
f
c
d
e
a



d
c
e
b
a


g
e
f
d
b
c
a


d
b
c
a


h
e
f
g
b
c
d
a

Break

Q The following is the Hasse diagram of the Poset [{a, b, c, d, e}, ≤]

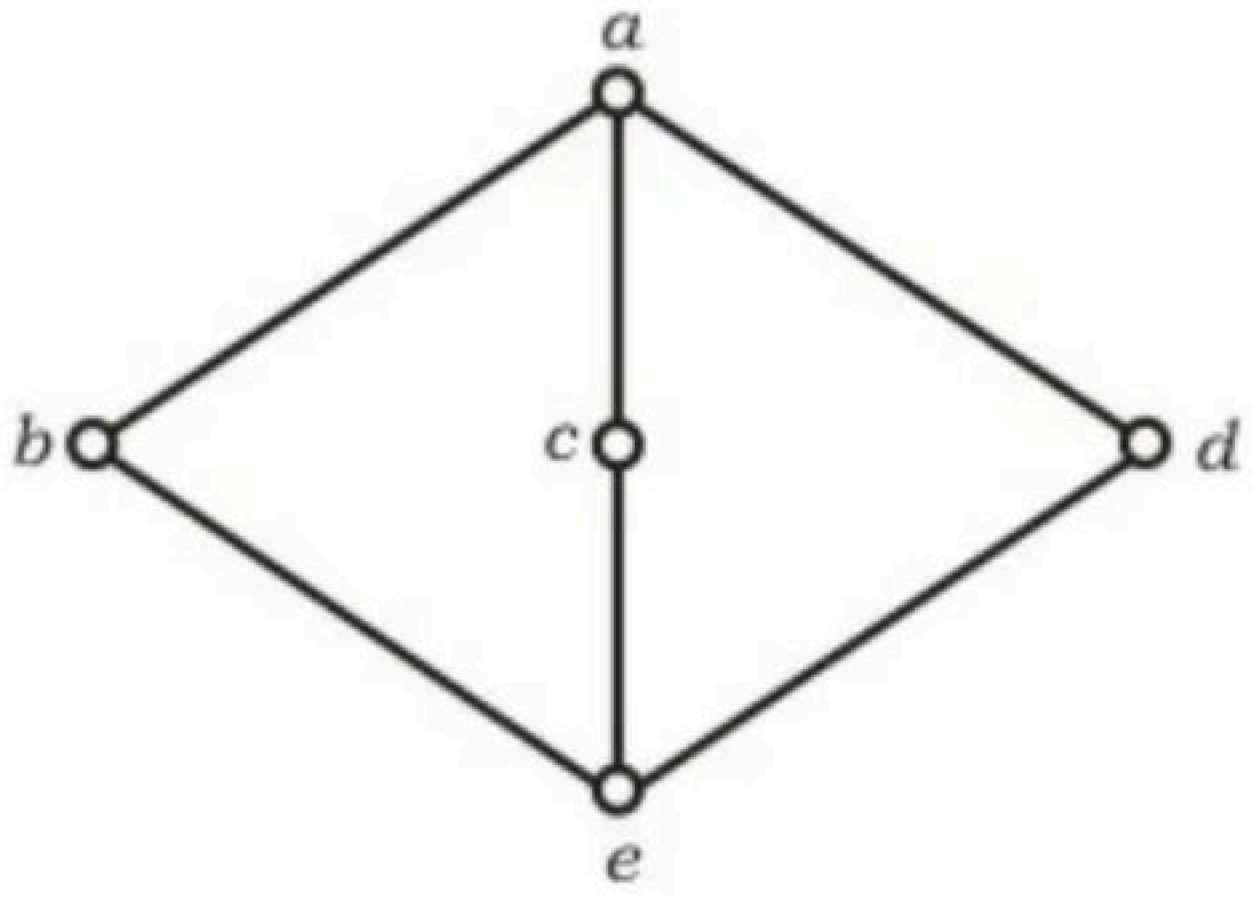
The Poset is **(GATE-2005) (1 Marks)**

(A) not a lattice

(B) a lattice but not a distributive lattice

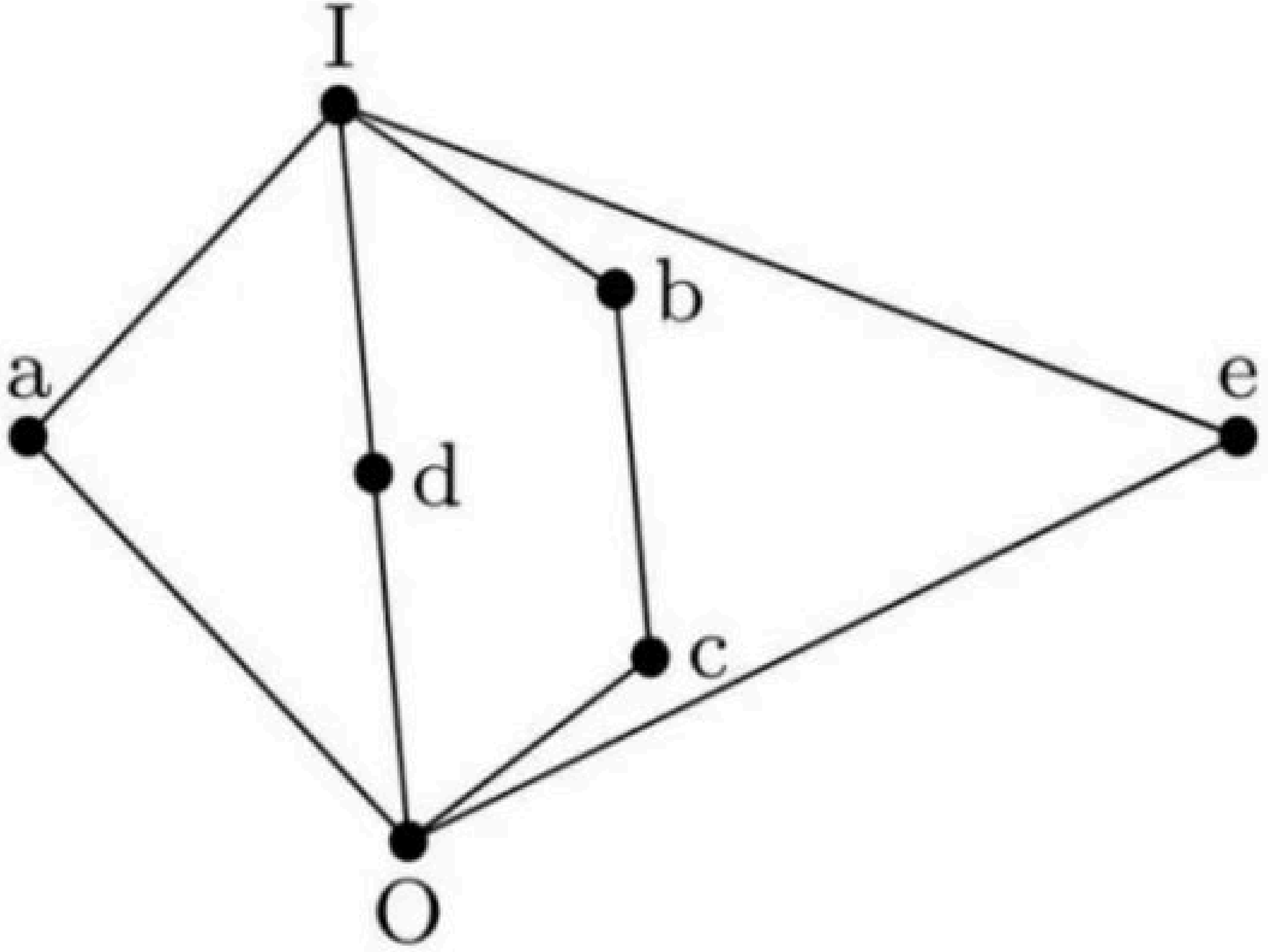
(C) a distributive lattice but not a Boolean algebra

(D) a Boolean algebra



Q The complement(s) of the element 'a' in the lattice shown in below figure is (are) ____

(GATE-1988) (2 Marks)



Q Find which of the following is a lattice and Boolean Algebra?

(1) $[D_{10}, /]$

(2) $[D_{12}, /]$

(3) $[D_{30}, /]$

(4) $[D_{45}, /]$

(5) $[D_{64}, /]$

(6) $[D_{81}, /]$

(7) $[D_{91}, /]$

(8) $[D_{110}, /]$

Q Find which of the following is a lattice and Boolean Algebra?

(1) [{1,2,3,4,6,9}, /]

(2) [{2,3,4,6,12}, /]

(3) [{1,2,3,5,30}, /]

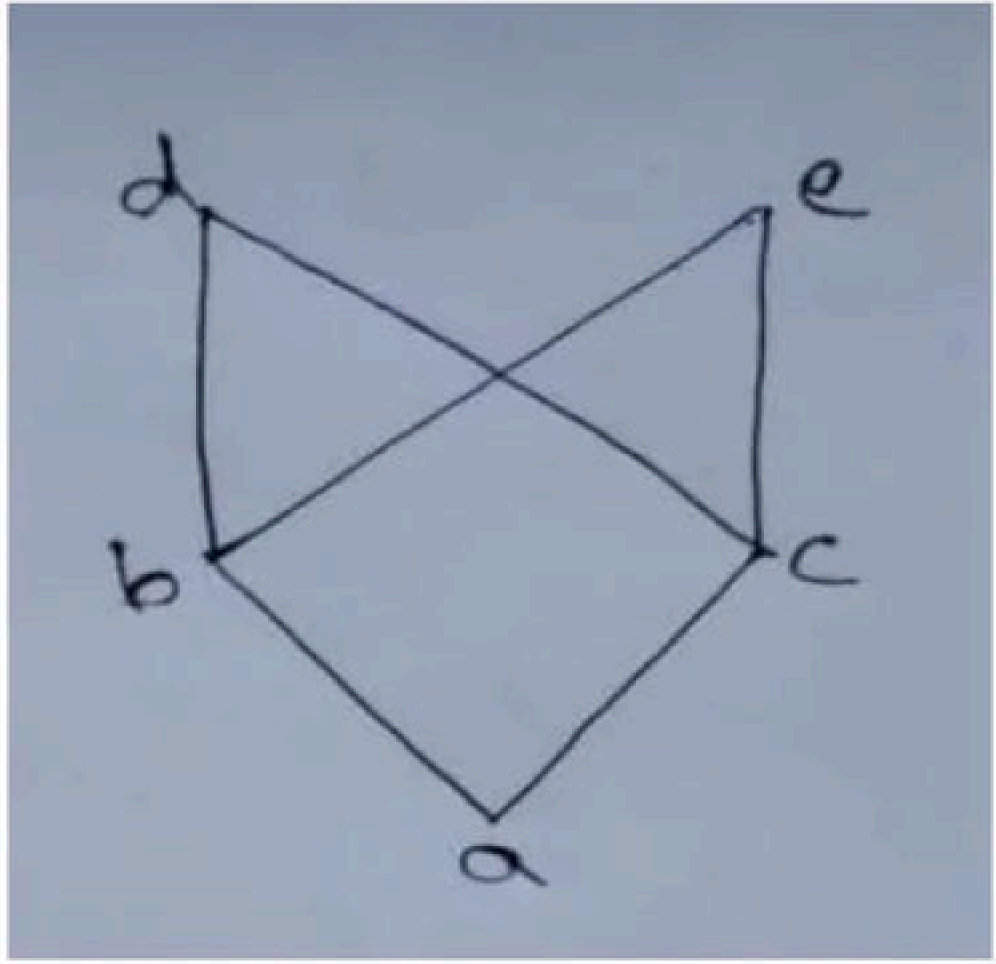
(4) [{1,2,3,6,9,18}, /]

(5) [{2,3,4,9,12,18}, /]

(6) [R, <=]

(7) [P(A), $\sqsubseteq$], A = {1,2,3}

Q Consider the following hasse diagram, find which of the following is true?



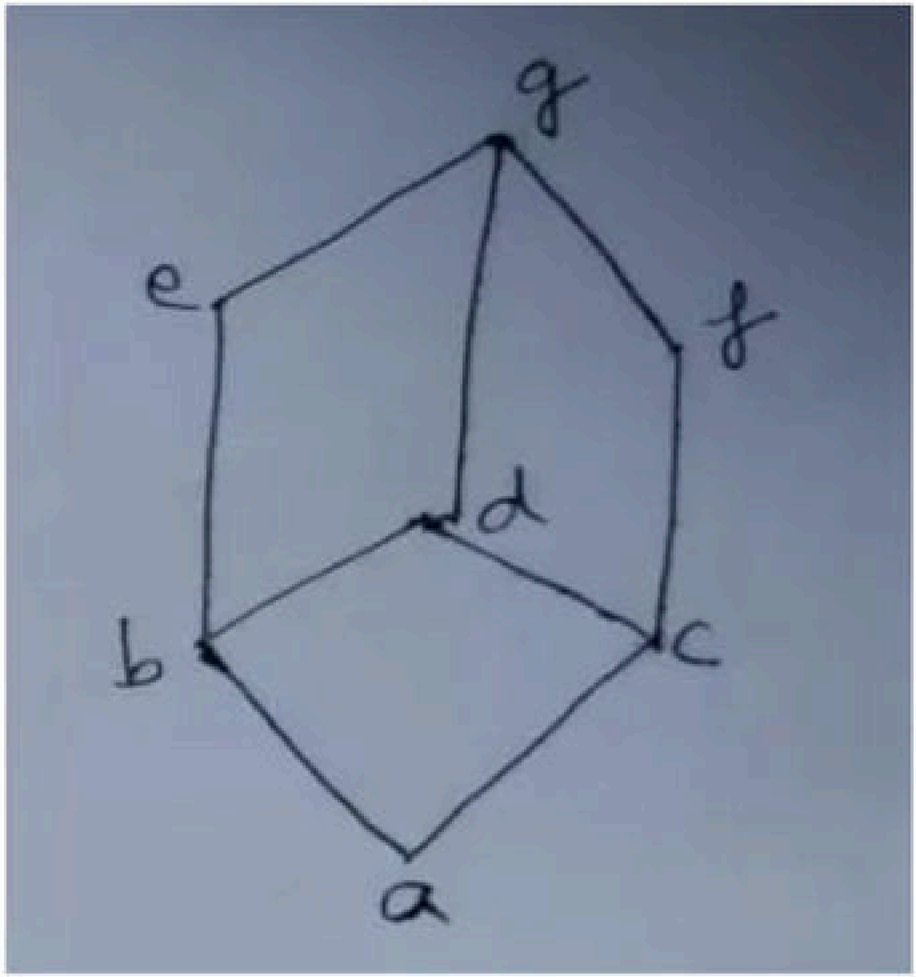
a) it is a lattice

b) subset {a, b, c , d} is a lattice

c) subset {b, c, d, e} is a lattice

d) subset {a, b, c, e} is a lattice

Q Consider the following hasse diagram, find which of the following is true?



a) subset {a, b, c, g} is a lattice

b) subset {a, b, f, g} is a lattice

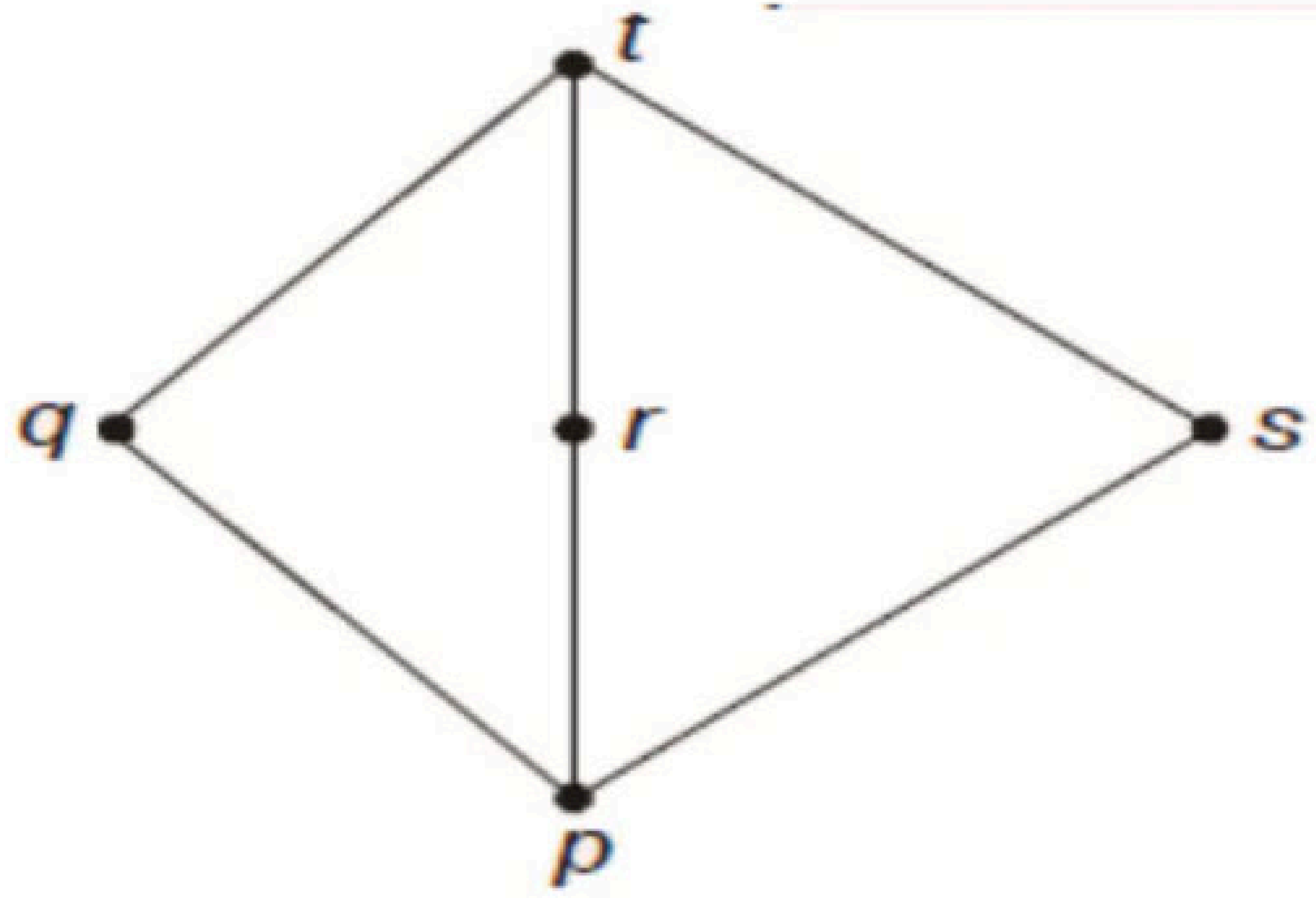
c) subset {a, d, e, g} is a lattice

d) subset {a, c, e, g} is a lattice

Break

Q Suppose L = {p, q, r, s, t} is a lattice represented by the following Hasse diagram: For any $x, y \in L$, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let $L^3 = \{(x, y, z): x, y, z \in L\}$ be the set of all ordered triplets of the elements of L. Let P_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then **(GATE-2015) (2 Marks)**

(A) $Pr = 0$ **(B)** $Pr = 1$ **(C)** $0 < Pr \leq 1/5$ **(D)** $1/5 < Pr < 1$



Q Consider the set S = {a, b, c, d}.
Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on
S: π_1={~~abcd~~}, π_2={~~ab~~, ~~cd~~}, π_3={~~abc~~, ~~d~~}, π_4={~~a~~, ~~b~~, ~~c~~, ~~d~~}
Let $\prec$ be the partial order on the set of partitions $S'=\{\pi_1,\pi_2,\pi_3,\pi_4\}$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The Poset diagram for $(S', \prec)$ is: **(GATE-2007) (2 Marks)**