A note on load-balancing on Multiple-APs

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Sunday December 25 2016

-One AP
-One type

-set of flows A(t) arriving at system at each time slot and i.i.d(independent identically distributed) with mean λ and $|A(t)| \leq A_{max}$ $\forall$ t ≥ 0

-size of flow j at time t is denoted as $F_j(t)$ follows any distribution :
Pr{ $C_j(t)$ =C_k } = P_k ,k=1,2,. . .,k.
where $0 \leq c_1 \leq c_2 \leq c_k = c_{max}$

-$R_i(t)$:# of residulal packets of flow i at time t.

-N(t):the set of flows at time t.
($|N(t)|$):cardinality of set N(t), which denotes # of flows at time t

-We consider the polices under which the system evolves as a Markov Chain.We call the system stable if the underlying Markov Chain is positive recurrent.(the stability definition is quite stable. For simplicity,you can interpret it as the boundness of average # of flow in the system)

-under the stability definition, one can show that the stability region is
$\lambda \cdot \mathrm{IE}\left[\left\lceil\frac{F}{C_{max}}\right\rceil\right] \leq 1,$
where $\lceil x \rceil$ is the minimum integer that is greater than x.

-A scheduler is throughput-optimal if it achieves the network stability definition stability for any traffic that is strictly inside the caplicty region(i.e. $\exists \varepsilon > 0$ $s.t.$ $\lambda \cdot IE\left[\left\lceil\frac{F}{C_{max}}\right\rceil\right] + \varepsilon \leq 1$)
-Goal: develop an algorithm that acheives maximum throughput
-Algorithm : At each time t, maximumu channel among all flows ($C_i[t] = max C_j(t) \quad j \in N(t)$)
or $C_i[t] \geq R_i(t)$

-Proportion:the proposed algorithm is throughput-optimal.

Proof:
Let $W(t) = \sum_{j \in N(t)} \left\lceil\frac{R_j(t)}{C_{max}}\right\rceil$ be the workload at time t. $A = \sum_{j \in N(t)} \left\lceil\frac{F_j(t)}{C_{max}}\right\rceil$ amount of new workload injected to the system at time t.
$(IE[A(t)] = \lambda \cdot IE\left[\left\lceil\frac{F}{C_{max}}\right\rceil\right])$
$\mu(t)$:amount of workload decreasing at time t.

($\mu(t) = 1$ if the workload of flows decreases by one and $\mu(t)$ =0 otherwise) Then ,the evolution of flows can be described as
W(t+1)=W(t)+A(t)-$\mu(t)$
Choose lyapunov function:$V(t) = W^2(t)$
Then we have E[V(t+1)-V(t)—X(t)]
$= E[W^2(t+1) - W^2(t)|X(t)]$
$= E[W(t) + A(t) - \mu^2(t) - W^2(t)|X(t)]$
$= 2E[W(t)(A(t) - \mu(t)|X(t)] + E[(A(t) - \mu(t)^2|X(t)]$
$\leq 2\lambda IE\left[\left\lceil \frac{F}{C_{max}} \right\rceil\right] W(t) - 2IE[W(t)\mu(t)|X(t)] + B_1$
(We can show $IE[(A(t) - \mu(t)^2|X(t)] \leq B_1$ for some $B_1 > 0$)
$\leq 2(1-\varepsilon)W(t) - 2IE[W(t)\mu(t)|X(t)] + B_1$
Note that the probability that at least one flow whose channel rate is C_{max} is $1-(1-P_k)^{|N(t)|}$
Hence $E[W(t)\mu(t)|X(t)]$
$\geq (1-(1-P_k)^{|N(t)|})W(t)$
$\geq (1-\varepsilon/2)W(t) - (1-\varepsilon/2)W(t)$
$\{|N(t)| \leq \overline{N}\}$
$\geq (1-\varepsilon/2)W(t) - (1-\varepsilon/2)\overline{N}IE\left[\left\lceil \frac{F_{max}}{C_{max}} \right\rceil\right]$
$= (1-\varepsilon/2)W(t) - B_2$
where $B_2 \overset{\triangle}{=} (1-\varepsilon/2)\overline{N}IE\left[\left\lceil \frac{F_{max}}{C_{max}} \right\rceil\right] > 0$
Thus $\triangle V(t) \leq 2(1-\varepsilon)W(t) - 2(1-\varepsilon/2)W(t) + B_1 + B_2$
$= -\varepsilon W(t) + B_1 + B_2$