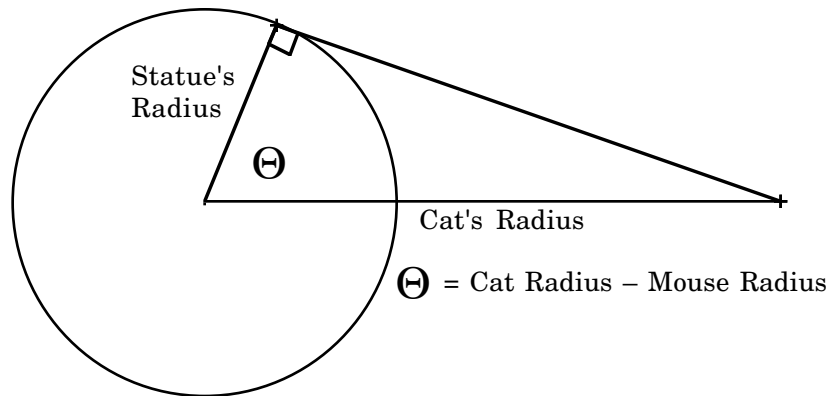


Derivation of formulas used in “Cat and Mouse”

Most of the mathematical expressions used in the program in fact capture very clear visual situations. Let’s start with the SEES relation. Our model of the cat assumes that it is looking straight at the statue, so the limit of the cat’s vision is given by the two rays that are tangent to the statue’s base. Let’s look at one; the other will be symmetric about the line between the cat and the statue. The following diagram has all the essential facts:



Since the line of sight is tangent to the circle, it forms a right angle with the radius. So from the definition of cosine we get

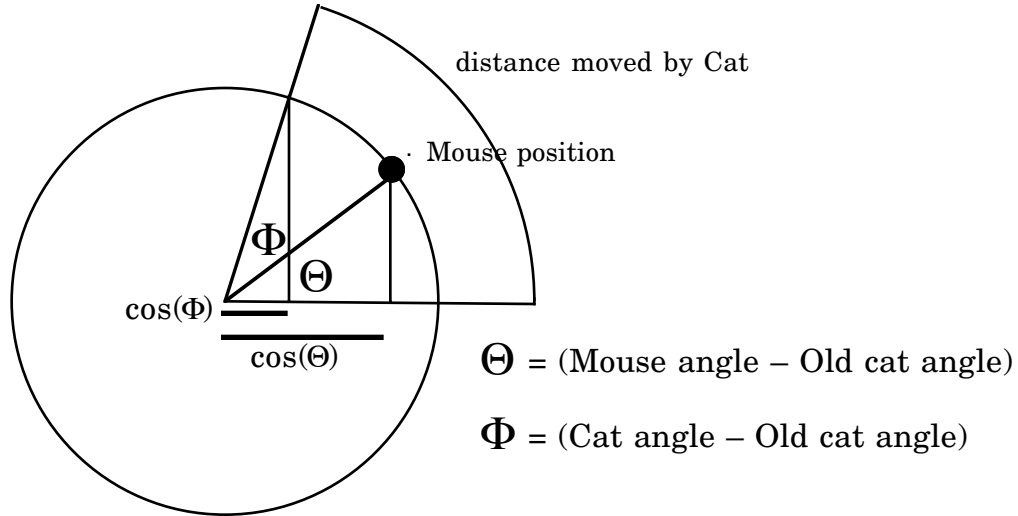
$$\cos(\Theta) = (\text{Statue's Radius}) / (\text{Cat's Radius})$$

Since the statue’s radius is by definition 1.0, we can insert that into the expression and multiply both sides by the cat’s radius:

$$(\text{Cat's Radius}) * \cos(\Theta) = 1.0$$

That gives us the limiting condition. To confirm the inequality one need only note that the cat can see the mouse for all angles less than Θ , and the cosine function increases for decreasing angles.

Finding the expression for BTWEEN is a bit more difficult because there are three angles: the previous cat angle, the current mouse angle, and the current cat angle. To solve this problem we will subtract away one angle to change to a more convenient frame of reference (as if the old cat angle was the zero position). The intuition behind the BTWEEN relation is to project the distance traveled by both the cat and mouse onto the line between the old cat position and the center of the statue. Those distances will be cosines of some angles shown on the following diagram:



From the diagram it is clear that if the mouse position is between the cat's old position and the cat's new position, the line traveled by the mouse must be longer than the line traveled by the cat (the cat has to get closer to the origin). So we get the first part of the BTWEEN relation:

$$\cos(\Theta) > \cos(\Phi)$$

The second condition is a check to make sure that the angles are in the same quadrant:

$$\cos(\Phi - \Theta) > \cos(\Phi)$$

will be true is if Φ is a positive angle (cosine of a smaller angle is larger), but will not be true if Φ is negative. Requiring both conditions makes sure that the above diagram is correct, and that the mouse has been indeed passed by the cat.