ETH

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Exercise – *Astérix and the Tour of Gaul*

After a short period of peace and quiet time in the village of the indomitable Gauls, inspector general Overanxius arrived in the Roman camp of Compendium. He was on a mission from Julius Caesar to lead an attack against the Gauls. Of course, the mission was a spectacular failure. To avoid a complete defeat, Overanxius gave orders to build a palisade around the village to prevent the villagers from spreading their dangerous ideas through Gaul.

Astérix and Obélix decide to escape from the entrapment and go on a tour of Gaul with the goal of sharing and spreading as many of the culinary traditions as possible around the country. To this end, they plan to buy typical dishes and drinks along the way and transport those culinary items for a few days to bring them to new cities and regions.

Astérix has mapped out a route with n stops. The route starts at stop s_0, continues through s_1, s_2, and so on, ending at stop s_{n-1}. Obélix has a large bag to transport the food. However, there are limitations on how many food items they can transport without attracting too much attention and ending up getting caught by the Romans. Thus for every $i \in \{0, \ldots, n-2\}$, they decide on a maximum number c_i of food items that can be transported between stops s_i and s_{i+1}.

To be fully prepared for the trip, Astérix has also written down a long list of all the food items and cities where they could potentially buy and distribute them: Ham from Lutetia would be well received in Lugdunum, Champagne from Durocortorum would delight the people in Aginum, the salad from Nicae would enrich the menues in Massilia, and so on. Some food items are of larger cultural significance than other food items. Therefore, every food item has an assigned *significance*.

Your task is to come up with a transportation plan by selecting a subset of the food items to be transported. Astérix and Obélix can transport at most one item of each food item on their list. The goal is to *maximize the sum of significances* of the transported food items, while respecting the specified limits on the maximum number of food items in their bag for each leg of the trip. (At a stop, food items brought there are unloaded from the bag before any new items are loaded.)

Input The first line of the input contains the number $t \leqslant 30$ of test cases. Each of the t test cases is described as follows:

- It starts with a line that contains two integers n m, separated by a space. They denote
  - n, the number of stops along the tour $s_0, s_1, \ldots, s_{n-1}$ ($2 \leqslant n \leqslant 3 \cdot 10^2$);
  - m, the number of food items on Astérix's list ($1 \leqslant m \leqslant 10^5$).
- The second line contains $n-1$ integers $c_0 \ldots c_{n-2}$, separated by a space, where c_i denotes the maximum number of food items that can be in the bag between stop s_i and s_{i+1} ($0 \leqslant c_i \leqslant 10^2$).

- The following m lines describe the food items. Each food item is described by three integers a b d, separated by a space, and such that $0 \leqslant a < b \leqslant n - 1$ and $1 \leqslant d \leqslant 2^7$. This means that this food item would need to be bought at stop s_a and brought to stop s_b and that this item has a significance of d.

Output For each test case output a line with one integer x that is the maximum *achievable significance*, defined as follows. A significance x is *achievable*, if there is a set of food items whose significances sum up to x and such that all the food items in the set can be transported between their corresponding stops without transporting more than c_i food items between stop s_i and stop s_{i+1}.

Points There are five groups of test sets, each of which is worth 20 points. So you can achieve 100 points in total.

1. For the first group of test sets, you may assume that all c_i values are the same and that there are no more than $3 \cdot 10^2$ food items ($m \leqslant 3 \cdot 10^2$).

2. For the second group of test sets, you may assume that all c_i values are the same and that all food items are of same unit significance ($d = 1$).

3. For the third group of test sets, you may assume that all c_i values are the same.

4. For the fourth group of test sets, you may assume that there are no more than 50 stops along the tour and no more than 10^4 food items ($n \leqslant 50$, $m \leqslant 10^4$).

5. For the fifth group of test sets, there are no additional assumptions.

Corresponding sample test sets are contained in `testi.in/out`, for $i \in \{1, 2, 3, 4, 5\}$.

Sample Input	Sample Output
3	3
6 4	94
2 2 2 2 2	86
0 3 1	
1 5 1	
3 4 1	
0 3 1	
6 4	
2 2 2 2 2	
0 3 32	
1 5 40	
3 4 16	
0 3 38	
6 4	
2 2 2 1 0	
0 3 32	
1 5 40	
3 4 16	
0 3 38	