

Algorithms Lab HS24
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cadmo.ethz.ch/education/lectures/HS24/algolab

Exercise – Rapunzel

"Rapunzel, Rapunzel, laß dein Haar herunter!"

"Rapunzel! Rapunzel! Let down your hair that I may climb thy golden stair!"

Kinder- und Hausmärchen by Jacob Grimm (1785-1863) and Wilhelm Grimm (1786-1859)

Once upon a time there was a beautiful girl called Rapunzel with splendid long hair, golden as the sun. Alas, she was prisoner to an evil sorceress, who had locked her in a high tower with no doors or stairway, only a small window at the top.

One day, a king's son happened to be passing by the tower and heard Rapunzel's angelic singing. His heart touched by the girl's magical voice, he wanted to climb up the tower and meet her. He observed that when the sorceress wanted to go up, she would ask Rapunzel to let her hair down. The girl would then wind it around a window hook and let the rest of it down out of the window. Her hair was so long that it reached all the way down to the ground and the sorceress could climb up, using it as a ladder. So after waiting for the sorceress to go away, the prince asked Rapunzel to let her hair down so he could climb up to her.

The girl's hair is braided in a complicated hairstyle, using n hair ties. The hair ties are indexed $0,1,\ldots,n-1$, where hair tie 0 is the topmost one, that is, the one closest to Rapunzel's head. Some pairs of hair ties are connected by a strand of hair. We say that there is a strand of hair from hair tie u to hair tie v if u and v are connected by a strand of hair in such a way that u is closer to Rapunzel's head than v. A sequence $s_0,\ldots,s_{\ell-1}$ of hair ties such that there is a strand of hair from s_{j-1} to s_j , for all $j \in \{1,\ldots,\ell-1\}$, is called a rope of length ℓ starting at hair tie s_0 and ending at hair tie $s_{\ell-1}$. For each hair tie u, there is precisely one rope starting at 0 and ending at u.

Rapunzel's plan is to find an appropriate rope s_0, \ldots, s_{m-1} of length m and then wind hair tie s_0 around a window hook and let the rest of the rope down out of the window, so the prince can climb up. The girl has chosen m carefully, so that any rope of length m would reach the ground exactly. However, there is an additional difficulty: the prince is very sensitive to bright light and so the golden shining of the girl's magnificent hair could blind him if they are not careful. The hair around each hair tie $i \in \{0, \ldots, n-1\}$ has some brightness $h_i \geq 0$. The prince explains that the important quantity is the contrast of a rope, that is, the difference between the maximum and the minimum brightness of the hair around all hair ties in that rope. If that difference is too large, the prince's eyes could not adapt to the change. Thus, a rope is safe for the prince if it has contrast at most k.

A rope is considered by the prince and Rapunzel to be *climbable* if it is safe and it has length exactly m. Since Rapunzel wants to know her options, her task is then to find all hair ties at which some climbable rope starts.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

• It starts with a line that contains three integers n m k, separated by a space. They denote

- n, the number of hair ties $(1 \le n \le 10^5)$;
- m, the desired length of the rope $(2 \le m \le 10^4)$;
- k, the maximum contrast that is considered safe for the prince $(0 \le k \le 10^4)$.
- The following line defines the brightness of the hair around the hair ties. It contains n integers $h_0 \ldots h_{n-1}$, separated by a space, and such that $0 \le h_i < 2^{31}$, for $i \in \{0,\ldots,n-1\}$. Here h_i denotes the brightness around the i-th hair tie.
- The following n-1 lines define the strands of hair. Each line consists of two integers u v, separated by a space, and such that $u, v \in \{0, \dots, n-1\}$. This means that there exists a strand of hair from hair tie u to hair tie v.

Output For each test case output a single line with a sequence of all hair ties, separated by a space and listed in increasing order, for which there exists a climbable rope starting there. If there is no such hair tie, output a single line with 'Abort mission'.

Points There are four groups of test sets. Overall, you can achieve 100 points.

- 1. For the first group of test sets, worth 20 points, you may assume that for every hair tie, there is at most one rope starting from it. Furthermore, $n \le 10^3$ and $m, k \le 10^2$.
- 2. For the second group of test sets, worth 20 points, you may assume that for every hair tie, there is at most one rope starting from it.
- 3. For the third group of test sets, worth 20 points, you may assume that $n \leq 10^3$ and $m, k \leq 10^2$.
- 4. For the fourth group of test sets, worth 40 points, there are no additional assumptions.

Corresponding sample test sets are contained in testi.in/out, for $i \in \{1, 2, 3, 4\}$.

Sample Input Sample Output 1 5 3 2 4 2 5 3 6 7 2 0 0 1 1 2 3 2 2 Abort mission 0 1 4 7 5 8 6 3 1 1 2 2 3 3 4 5 3 2 3 1 1 4 5 5 6 9 3 4 5 3 2 3 0 2 1 7 2 3 0 1 0 2