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Algorithm Complexity II

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Lecture Topics

- Jump Search
  - Square Root Time
- Cocktail Shaker Sort
- Fisher-Yates Shuffle
- Exponential Time
- Bogosort
  - Factorial Time

Jump Search

- Like Binary Search, the array must be pre-sorted.

- Begins searching at index 0 of an array.
  - Jumps *x*-number of indexes ahead.
  - If the value sought is larger, it jumps forward again.

  - If the value sought is smaller, it jumps back one step.
    - Performs a linear search from there, up to where it jumped back from.

Jump Search (Pseudocode)

Set *previous* to the first index (0)
Set *jump* to desired distance (eg. square root of array *a*'s length)

While *a*[min(*jump*, *a*'s length)-1] < value sought :
    previous = *jump*
    jump += *jump*
    If *jump* >= *a*'s length :
        break from the loop/we've jumped past the last element

While *a*[*previous*] < value sought :
    previous += 1
    If *previous* == min(*jump*, *a*'s length) :
        Value does not exist in the array

If *a*[*previous*] == the value sought :
    Value exists at index *previous*
Else:
    Value does not exist in the array

Jump Search (C++ Function)

```
int jumpSearch(int a[], int length, int searchValue) {
    int previous = 0;
    int jump = (int) sqrt(length);
    while(a[(jump < length ? jump : length-1)] < searchValue) {
        previous = jump;
        jump += jump;
        if(jump >= length) {
            break;
        }
    }
    while(a[previous] < searchValue) {
        previous += 1;
        if(previous == (jump < length ? jump : length)) {
            return -1;
        }
    }
    return a[previous] == searchValue ? previous : -1;
}
```

Keeps jumping forward

Chooses the smaller of the two

Prevents going out of bounds

Goes back and linear searches starting at the last jump point

Chooses the smaller of the two

Jump Search Time Cost

- The jump length will play a part in the time complexity of this algorithm.

- We chose the jump length to be the square root of the array's length.
  - This ensures the longest, yet most evenly distributed jumps possible.

Jump Search Time Cost

- We'll ignore the constant time operations of this algorithm and focus on the repetitive aspects.
  - We'll also use **n** to represent the length of the array being searched.

- The first while loop will iterate, at most, $\sqrt{\mathbf{n}}$ times.
  - If the jump length is the square root of the array length.

- The second while loop will iterate using a linear search.
  - We know the linear search performs in linear time.
  - However, this linear search will only iterate, at most, $\sqrt{\mathbf{n}}$ times.

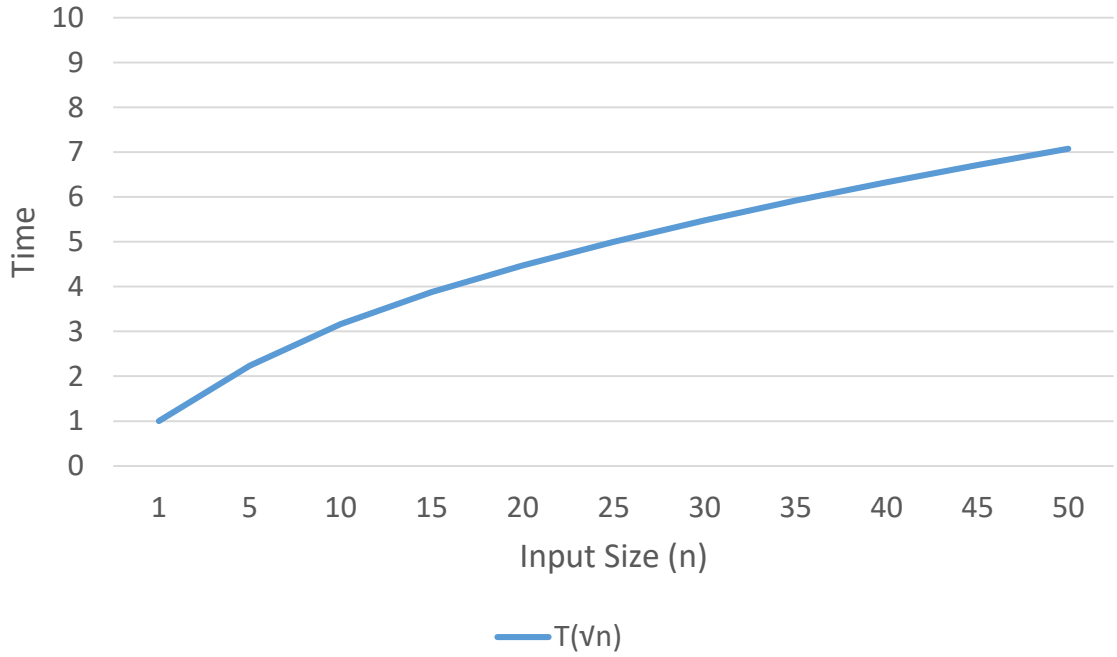
Jump Search Time Cost

- Again, ignoring the constant time operations of this algorithm, it's time cost can be estimated to be:
  - $T(n) = \sqrt{n} + \sqrt{n} = 2(\sqrt{n})$

Square Root Time

- **Square root time** is when an algorithm's running time is bounded by the square root of the input size.
  - $T(n) = \sqrt{n}$

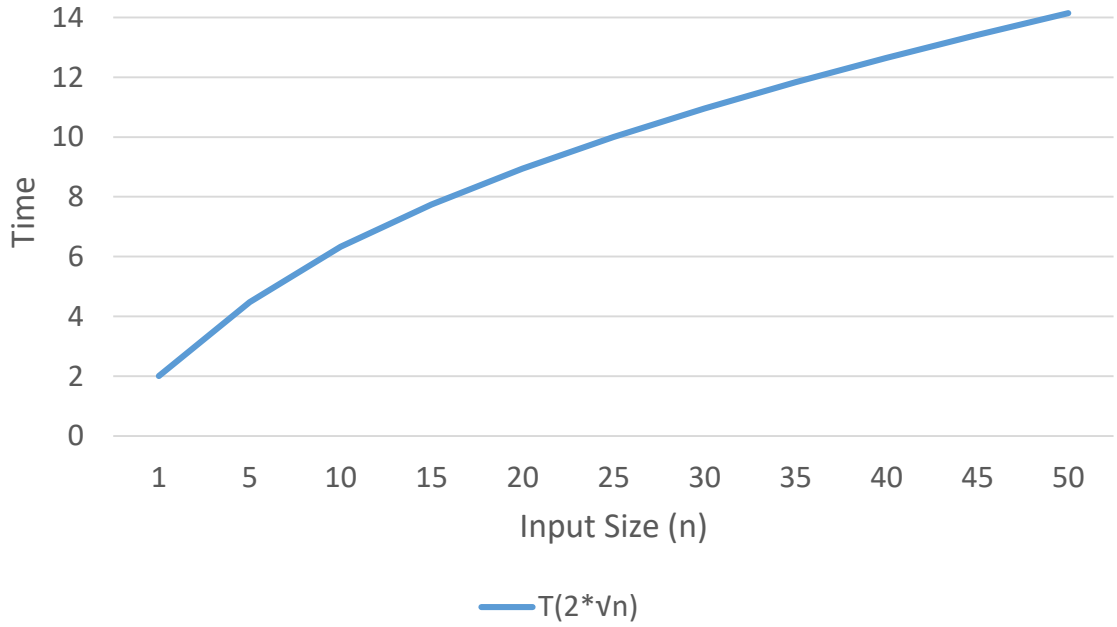
- $T(n) = \sqrt{n}$
  - $T(4) = 2$
  - $T(8) = 2.8$
  - $T(12) = 3.5$



Time Cost of the Jump Search

- The Jump Search performs in square root time.

- Using our example:
  - $T(n) = 2(\sqrt{n})$

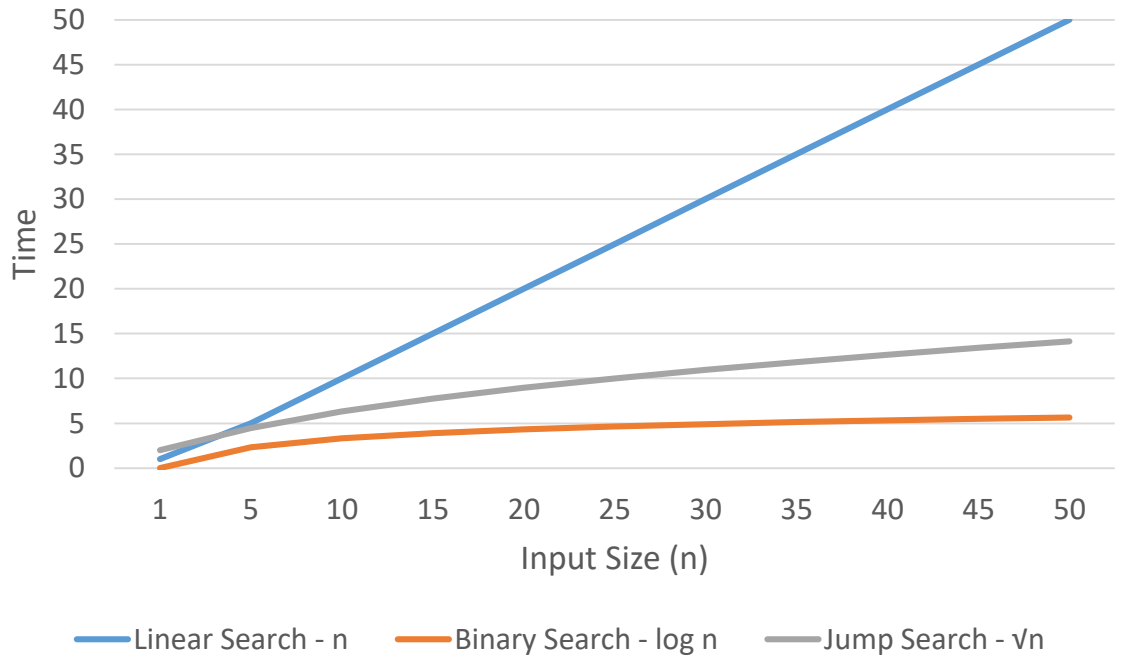


Linear vs Square Root vs Logarithmic

- We've seen three search algorithms with different time complexity.
  - Linear Search – Linear
  - Binary Search – Logarithmic
  - Jump Search – Square Root

- How do they compare?
  - We'll use the following estimates:
  - Linear Search – $T(n) = n$
  - Binary Search – $T(n) = \log_2 n$
  - Jump Search – $T(n) = \sqrt{n}$

Linear vs Logarithmic vs Square Root Time



- Square root time does better than linear time, but it's not as fast as logarithmic time.

Cocktail Shaker Sort

- The Cocktail Shaker Sort works in a similar fashion to the Bubble Sort algorithm.
- Neighboring pairs of values in an array are compared and then swapped, moving the largest values to the right. (Ascending order)
- But then it goes backwards, moving the smallest values to the left.

Cocktail Shaker Sort (Pseudocode)

```
Do :
        sorted = true
        For i in indexes 1 through length-1 :
                If a[ i-1 ] > a[ i ] :
                        Swap a[ i-1 ] and a[ i ]
                        sorted = false
        If sorted == true :
                break
        For j in indexes length-1 through 1 :
                If a[ j ] < a[ j-1 ] :
                        Swap a[ j ] and a[ j-1 ]
                        sorted = false
While sorted == false
```

Cocktail Shaker Sort (C++ Function)

```
void cocktailSort(int a[], int length) {
     bool sorted;
     do {
          sorted = true;
          for(int i = 1; i < length; i++) {
               if(a[i-1] > a[i]) {
                    int temp = a[i-1];
                    a[i-1] = a[i];
                    a[i] = temp;
                    sorted = false;
               }
          }
          if(sorted) {
               break;
          }
          for(int j = length-1; j > 0; j--) {
               if(a[j] < a[j-1]) {
                    int temp = a[k-1];
                    a[k-1] = a[k];
                    a[k] = temp;
                    sorted = false;
               }
          }
     } while(!sorted);
}
```

Forward

Reverse

Cocktail Shaker Sort Time Cost

- We'll ignore the constant time operations of this algorithm and focus on the repetitive aspects (its loops).
  - We'll also use **n** to represent the length of the array being searched.

- Both inner for loops iterate n times.
  - **2n** total

- At most, the outer while loop will iterate n/2 times.
  - Since we swap in both directions for each iteration of the outer loop.

Cocktail Shaker Sort Time Cost

- Again, ignoring the constant time operations of this algorithm, it's time cost can be estimated to be:
  - $T(n) = (n/2)(2n) = n^2$
- It also performs in polynomial time, like the other algorithms (bubble, insertion, and selection sorts).
- Typically, it will require less time than the bubble sort shown in the previous lecture slides.
  - This algorithm's outer loop iterates *at most* n/2 times.
  - Alternative bubble sort's outer loop iterates *at most* n times.
  - Original bubble sort's outer loop *always* iterates n times.
  - All three perform in polynomial time, but some generally perform faster than others.

Fisher-Yates Shuffle

- Nearly all of the algorithms we have seen solve the problem of sorting.

- The **Fisher-Yates Algorithm** is an algorithm that shuffles/randomizes the order of values in an array.
  - Performs in linear time.

Fisher-Yates Shuffle (Pseudocode)

Set seed value for the generator
For *i* in indexes length-1 of array *a* through 1 :
    j = A random number <= *i* but >= 0
    Swap *a*[*i*] and *a*[*j*]

Fisher-Yates Shuffle (C++ Function)

```
void shuffle(int a[], int length) {
    srand((int)time(0));
    for(int i = 1; i < length; i++) {
        int j = rand() % (i+1);
        int temp = a[j];
        a[j] = a[i];
        a[i] = temp;
    }
}
```

Sets seed to current time
(seconds since midnight, 1/1/1970)

Random number between 0 (included) and i+1 (excluded)

- Requires **`#include<ctime>`** for the time function.

Exponential Time

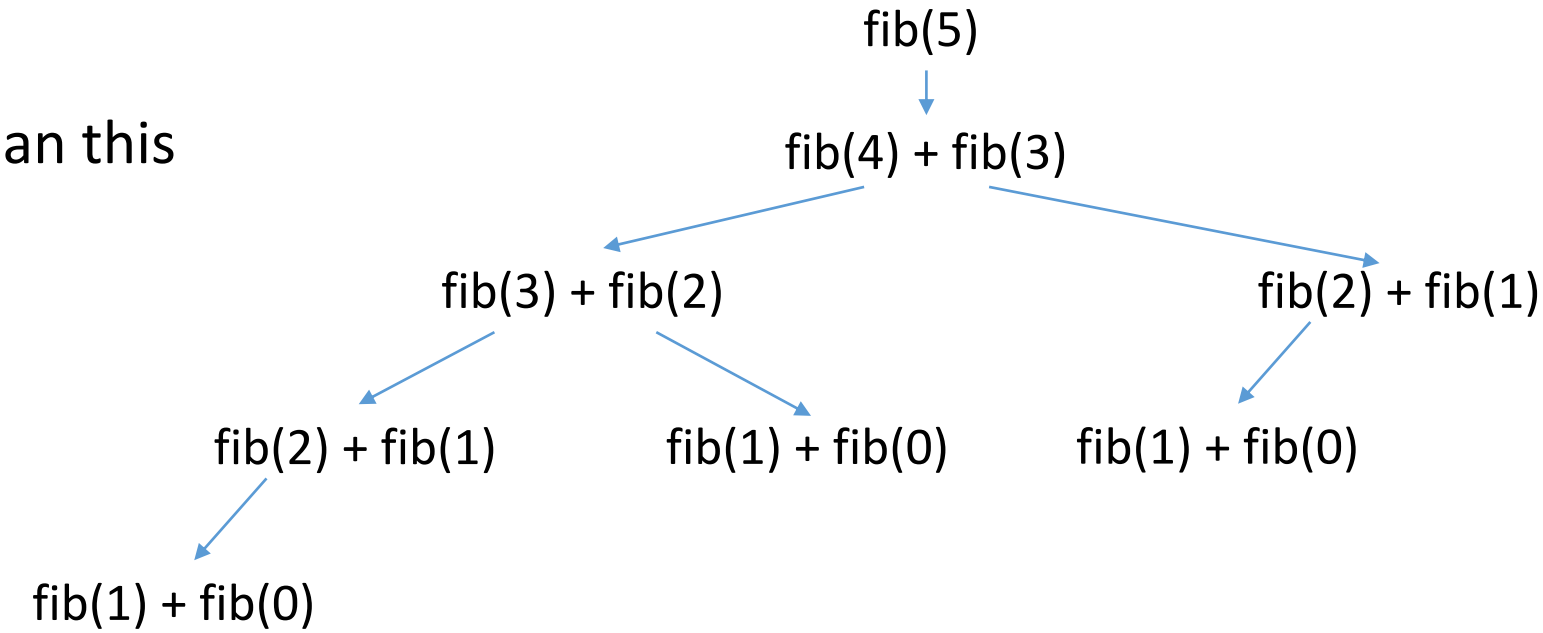
- Many recursive algorithms run in exponential time.

- The recursive solution to solving a Fibonacci series runs in exponential time.
  - Though, the iterative solution runs in linear time.

```
int fib(int n) {
    if (n == 0 || n == 1) {
        return 1;
    }
    else {
        return fib(n - 1) + fib(n - 2);
    }
}
```

Exponential Time

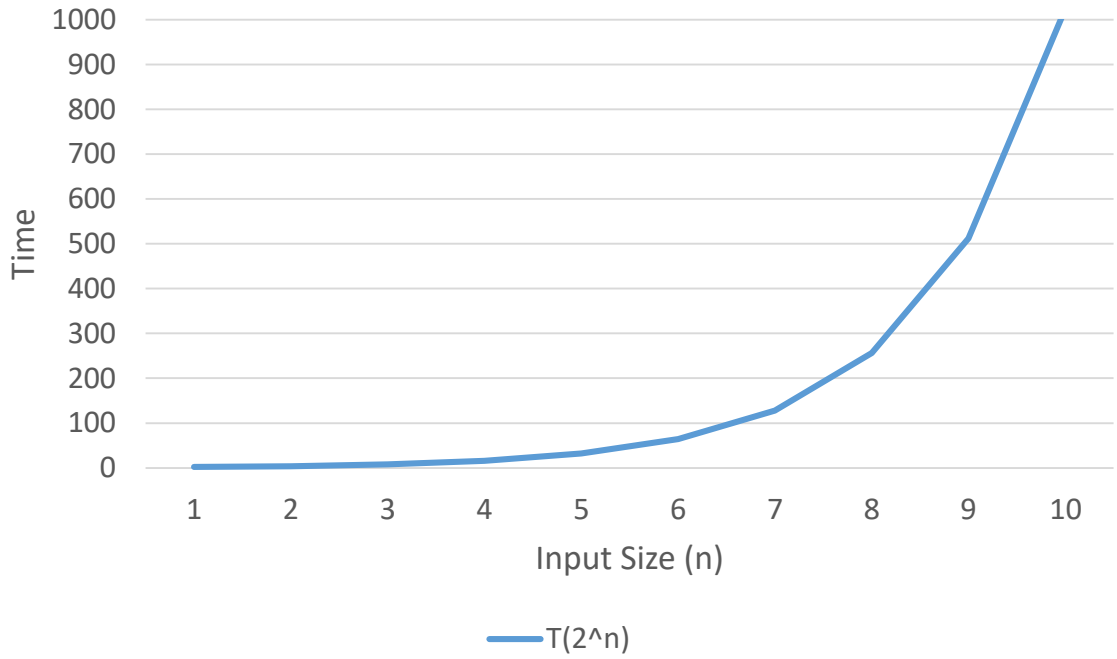
- Almost every function call splits into 2 more function calls.
- The depth is n levels
- $T(n) = 2^n$
  - Does a bit better than this



Exponential Time

- Exponential time is when an algorithm's running time is bounded by a constant raised to some power of n.
  - $T(n) = 2^n$

- $T(n) = 2^n$
  - T(4) = 16
  - T(8) = 256
  - T(12) = 4096



Bogosort

- Also called “Stupid Sort”, “Monkey Sort”, or “Shotgun Sort”

- Checks if the array is in order.
  - If it isn’t, it randomly shuffles the array.
  - Checks to see if it is in order.
  - If it isn’t, it shuffles it again and repeats.

- Infinite Monkey Theorem: Given enough time, a monkey in front of a typewriter randomly pressing keys would eventually produce the complete works of William Shakespeare.

Bogosort (Pseudocode)

```
Do:
        For i in indexes 1 through length-1 of array a:
                If a[ i-1 ] >= a[ i ] :
                        sorted = false
                        break

        If sorted == false :
                Shuffle a
                sorted = true

While sorted == true
```

Bogosort (C++ Function)

```
void bogosort(int a[], int length) {
     bool sorted;
     srand((int)time(0));
     do {
          sorted = true;
          for(int i = 1; i < length; i++) {
               if(a[i-1] > a[i]) {
                    sorted = false;
               }
          }
          if(!sorted) {
               for(int i = 1; i < length; i++) {
                    int j = rand() % (i+1);
                    int temp = a[j];
                    a[j] = a[i];
                    a[i] = temp;
               }
          }
     } while(!sorted);
}
```

Checks if the ordering is correct

Shuffles the array

Bogosort

- Obviously, this is a really bad sorting algorithm.

- It essentially tests different, possibly recurring permutations of the numbers in the array seeing if it was randomly placed in order.
  - The possibly recurring part is what makes it even worse.
  - It could “sort” the list in the same, wrong order multiple times.

Bogosort

- For an array with a length of 5, there are 120 possible permutations of the values.
  - 1 * 2 * 3 * 4 * 5 = 5! = 120

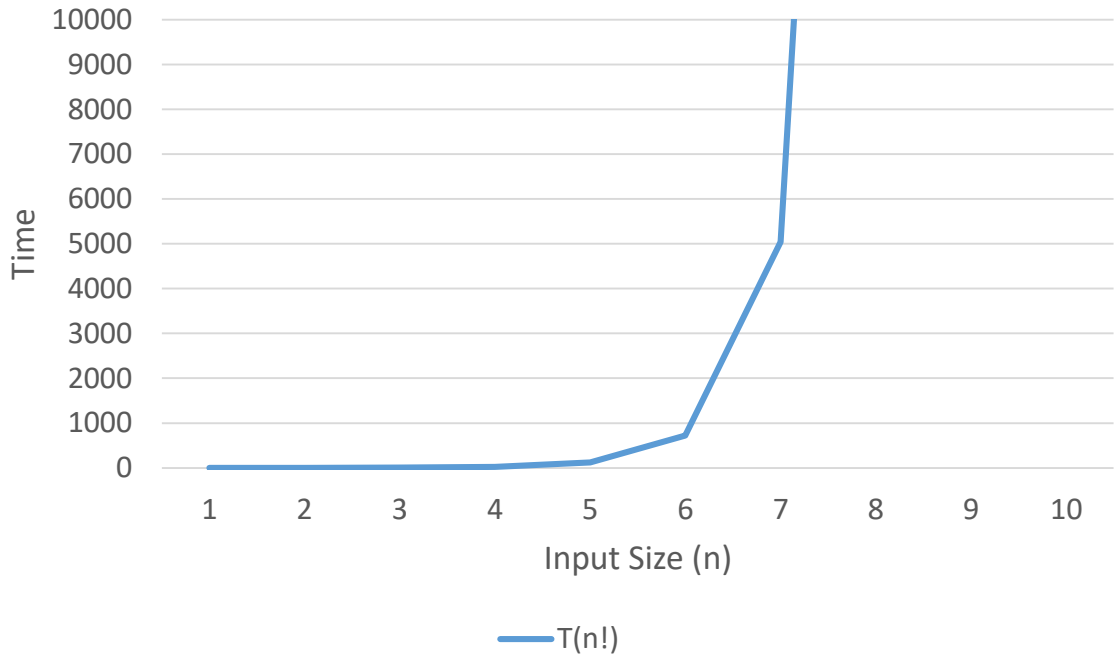
- For an array with a length of 100, there are $9.33*10^{157}$ possible permutations.
  - $100! = 9.33*10^{157}$

- For an array of length n, there are n! possible permutations.

Factorial Time

- **Factorial time** is when an algorithm's running time is bounded by a factorial of the input size.
  - T(n) = n!

- T(n) = n!
  - T(4) = 24
  - T(8) = 40320
  - T(12) = 479001600



Factorial Time

- Technically, the version of the Bogosort shown is unbounded to a time complexity since the same permutation might appear multiple times.
  - There are other versions where it doesn't get the same permutation more than once.

- Bogosort's only practical purpose is being a demonstration of an algorithm that performs in factorial time.
  - Don't actually use it in the real world.

Time and Space Complexity of These Algorithms

- Time Complexity
  - Jump Search – Square Root
  - Cocktail Shaker Sort – Polynomial
  - Fisher-Yates Shuffle – Linear
  - Bogosort – Factorial
- Space Complexity
  - Jump Search –Linear
  - Cocktail Shaker Sort –Linear
  - Fisher-Yates Shuffle –Linear
  - Bogosort –Linear

O-Notation

- O-notation for algorithms seen:
  - Linear Search O(n)
  - Binary Search O(log n)
  - Jump Search $O(\sqrt{n})$

  - Bubble Sort $O(n^2)$
  - Insertion Sort $O(n^2)$
  - Selection Sort $O(n^2)$
  - Cocktail Sort $O(n^2)$
  - Bogo Sort O(n!)

Ω-Notation

- Ω-notation for algorithms shown:
  - Linear Search $\Omega(1)$
  - Binary Search $\Omega(1)$
  - Jump Search $\Omega(1)$

  - Bubble Sort $\Omega(n)$
  - Insertion Sort $\Omega(n)$
  - Selection Sort $\Omega(n^2)$
  - Cocktail Sort $\Omega(n)$
  - Bogo Sort $\Omega(n)$

Θ-Notation

- Θ-notation for algorithms shown:
  - Linear Search $\Theta(n)$
  - Binary Search $\Theta(\log n)$
  - Jump Search $\Theta(\sqrt{n})$

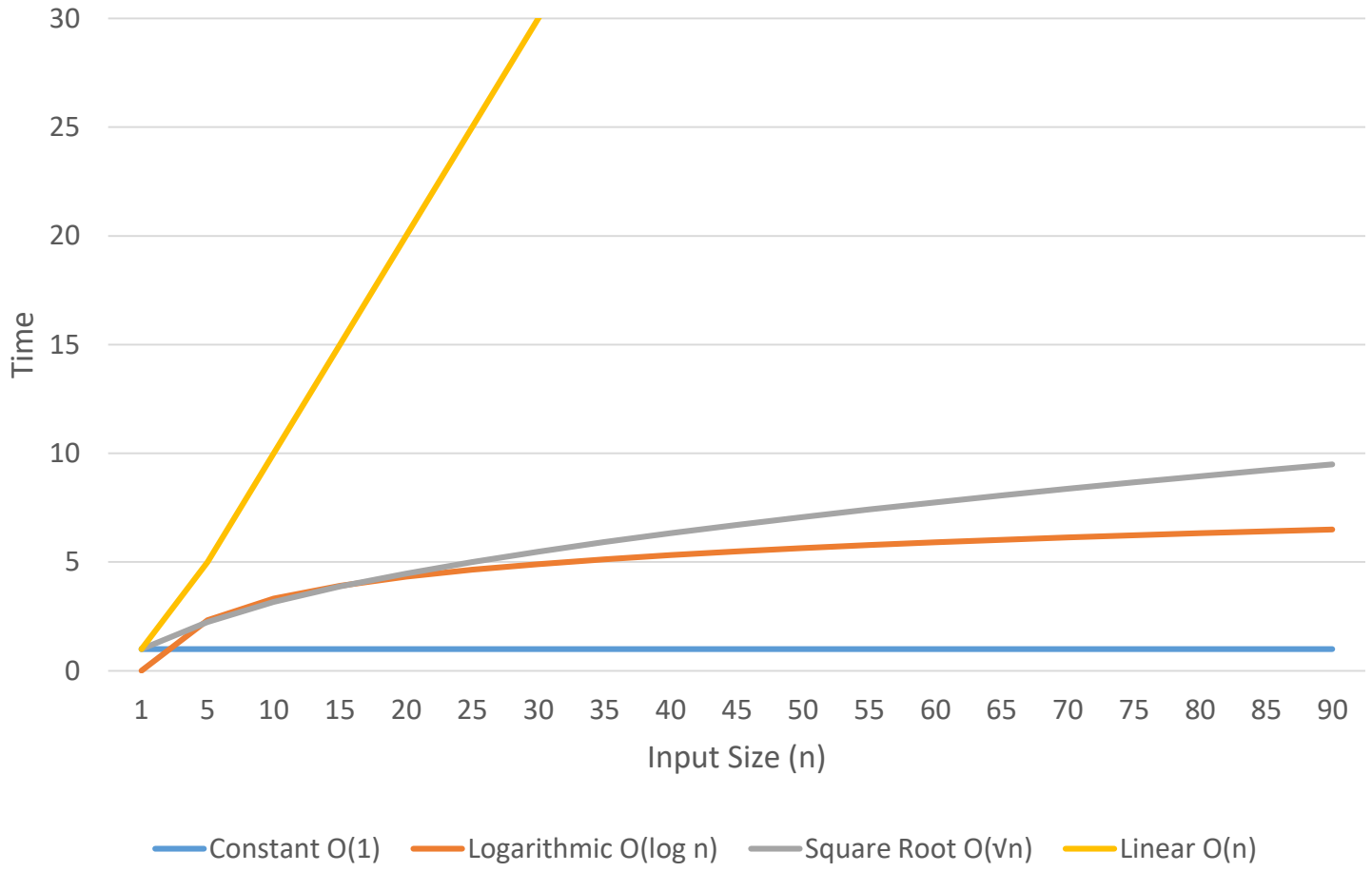
  - Bubble Sort $\Theta(n^2)$
  - Insertion Sort $\Theta(n^2)$
  - Selection Sort $\Theta(n^2)$
  - Cocktail Sort $\Theta(n^2)$
  - Bogo Sort $\Theta(n!)$

Space Complexities

- O-notation for algorithms shown (Auxiliary space only):
  - Linear Search O(1)
  - Binary Search O(1)
  - Jump Search O(1)

  - Bubble Sort O(1)
  - Insertion Sort O(1)
  - Selection Sort O(1)
  - Cocktail Sort O(1)
  - Bogo Sort O(1)

Complexities



Complexities