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Recursive Search and Sort

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Lecture Topics

- Recursive Binary Search
- Recursive Bubble Sort
- Merge Sort
- Quick Sort

Using Recursion to perform a Binary Search

- You’ve seen the binary search implemented using an iterative algorithm.
- A binary search using a recursive algorithm is little easier to design.

Binary Search (Iterative Algorithm)

```
int binarySearch(int a[], int length, int searchValue) {
    int lowBoundary = 0;
    int highBoundary = length - 1;
    while(highBoundary >= lowBoundary) {
        int middle = (highBoundary + lowBoundary) / 2;
        if(searchValue == a[middle]) {
            return middle;
        }
        else if(searchValue > a[middle]) {
            lowBoundary = middle + 1;
        }
        else {
            highBoundary = middle - 1;
        }
    }
    return -1;
}
```

Using Recursion to perform a Binary Search

- Since any problem that can be solved iteratively can be solved recursively, we can design a recursive replacement for the iterative binary search.

- What is/are the base case(s) for a binary search?

- What is the recursive case?

Using Recursion to perform a Binary Search

- The base cases are
  - When the lower bound is greater than the higher bound (which means the value we are looking for was not found).
  - The value at middle index is the value we are looking for.

- The recursive case is to search the upper or lower half of the partition.

Binary Search (Recursive Algorithm)

```
int binarySearch(int a[], int start, int end, int searchValue) {
    if(start > end) {
        return -1;
    }

    int middle = (start + end) / 2;

    if(searchValue == a[middle]) {
        return middle;
    }
    else if(searchValue > a[middle]) {
        return binarySearch(a, middle+1, end, searchValue);
    }
    else {
        return binarySearch(a, start, middle-1, searchValue);
    }
}
```

Using Recursion to perform a Binary Search

- This recursive binary search algorithm, like the iterative version, also performs in logarithmic time.

- The depth of recursion will be the same as the number of iterations needed in the iterative version.

Using Recursion to perform a Bubble Sort

- You've seen the bubble sort implemented using an iterative algorithm.

- Since any problem that can be solved iteratively can be solved recursively, we can design a recursive replacement for the iterative bubble sort.

Bubble Sort (Iterative Algorithm)

```
void bubbleSort(int a[], int length) {
    for(int i = 0; i < length; i++) {
        for(int j = 1; j < length; j++) {
            if(a[j-1] > a[j]) {
                int temp = a[j-1];
                a[j-1] = a[j];
                a[j] = temp;
            }
        }
    }
}
```

Using Recursion to perform a Bubble Sort

- For an ascending sort, the first pass will move the largest value to the end of the array (length - 1).
  - The next pass will move the second largest value to index length - 2.
  - The next pass will move the third largest value to index length - 3.
  - And so on…

- The last pass of the iterative algorithm will ensure the smallest value is placed in index 0.
  - At this point, the smallest value is guaranteed to already be in index 0.

Using Recursion to perform a Bubble Sort

- The base case is
  - When the algorithm sorts for index 0.
  - Since all other values will have already been sorted in the array, it implies the value at index 0 is already in the correct position.

- The recursive case is for all sorting other indexes.

Bubble Sort (Recursive Algorithm)

```
void bubbleSort(int a[], int length) {
    if(length == 1) {
        return;
    }

    for(int i = 0; i < length-1; i++) {
        if(a[i] > a[i+1]) {
            int temp = a[i+1];
            a[i+1] = a[i];
            a[i] = temp;
        }
    }

    bubbleSort(a, length-1);
}
```

Using Recursion to perform a Bubble Sort

- This recursive bubble sort algorithm, like the iterative version, also performs in polynomial time.

- There are no advantages (speed/complexity) by using an a recursive bubble sort over an iterative bubble sort.
  - This is example is here mainly to show you the two different implementations (recursive vs iterative).

Recursive Sorting Algorithms

- We’ll now see a pair of sorting algorithms, Merge Sort and Quick Sort, that are typically implemented using recursion.
  - They can be done iteratively, but the code is much easier to read when implemented recursively.

- Both algorithms take a “divide and conquer” approach to sorting, much like the how the binary search algorithm performs its searches.

Merge Sort Algorithm

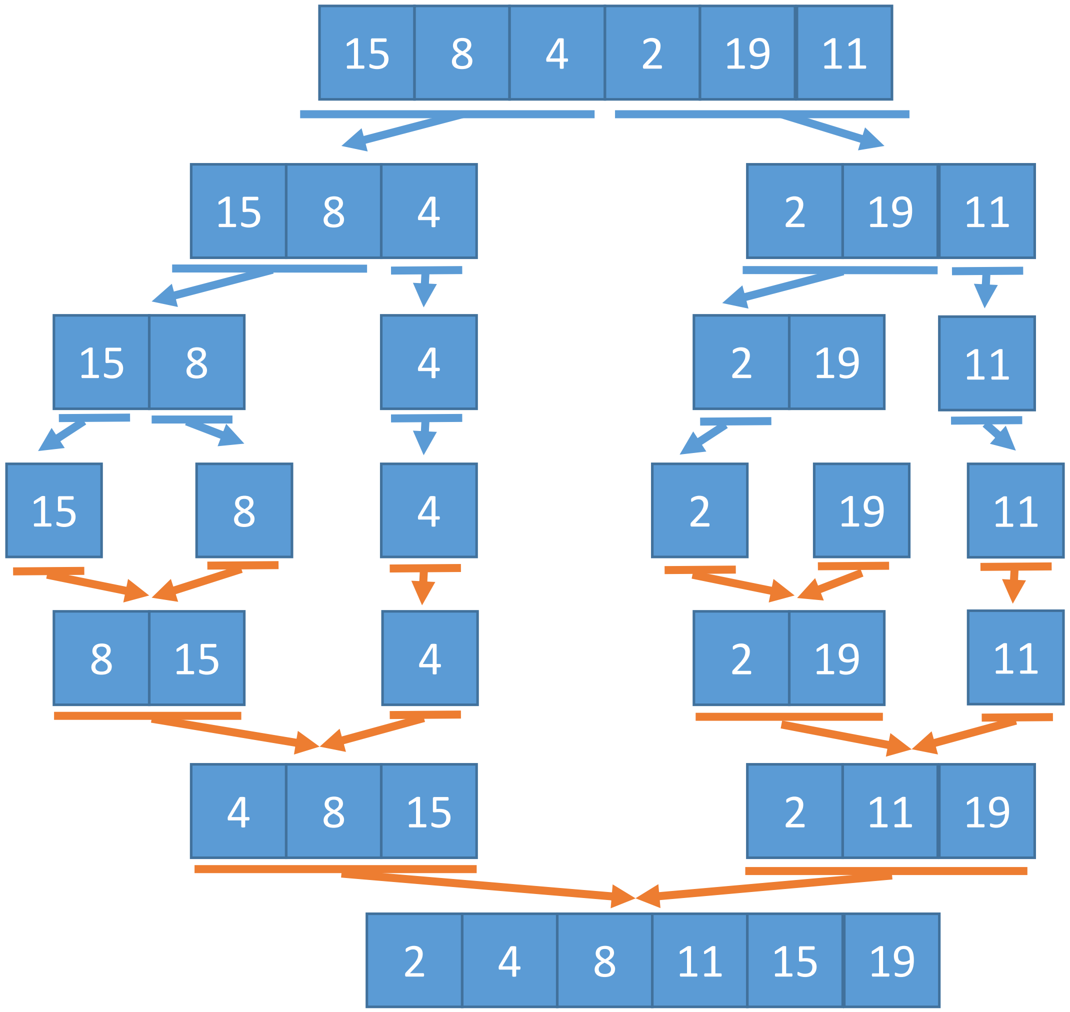
- In the Merge Sort algorithm, the array is repeatedly (recursively) split in half until it reaches halves that only contain one element.
  - At this point, the lowest depth has been reached.

- Then, working backwards, it sorts/merges the smaller arrays back together

Merge Sort Algorithm

- The image on the right gives the basic idea of how it works.
  - It divides up the array (Blue lines)
  - Then merges the array back together (Orange lines)

- Each merge involves two, sorted arrays.
  - Since the arrays to merge are in order, merging/sorting them is not computationally difficult.



Merge Sort Algorithm

- The C++ functions are a bit too long to put here in their entirety.
  - See the Sample Code provided.
- The next slides explain, *very briefly*, what is needed.
  - Two functions:
    - One function for the algorithm
    - Another function that handles the merging process

Merge Sort

```
mergesort(array, left, right):
        If left boundary < right boundary :
                Find the middle, m
                mergesort(array, left, m)
                mergesort(array, m+1, right)

                merge(array, left, m, right)
```

Merge Sort

merge(*array*, *left*, *middle*, *right*) :
    leftArray[*middle* - *left* + 1]
    rightArray[*right* - *middle*]
    Copy left side of *array* to *leftArray*
    Copy right side of *array* to *rightArray*
    Put the values of *leftArray* and *rightArray* back into *array*, in the correct order
    Put the remaining value, leftover in either *leftArray* or *rightArray*, into *array*

Quick Sort Algorithm

- In the Quick Sort algorithm, the array is repeatedly (recursively) split into two smaller partitions, until the partitions only contain one element.
  - At this point, the lowest depth has been reached.
- The algorithm chooses a value in each partition, called the **pivot**.
  - One of the two partitions will contain any values less than the pivot.
  - The other partition will contain any values greater than the pivot.
- The process repeats recursively until partitions of length 1 are reached.
  - At which point, the array will have been sorted through the pivot processes.

Quick Sort Algorithm

- There are a few ways of selecting the pivot:
  - Always use the last element.
  - Always use the middle element.
  - Always use the first element.
  - Always use a randomly chosen element.

- The Sample Code provided uses the middle element as the selected pivot value.

- The next slides explain, *very briefly*, what is needed.

Quick Sort

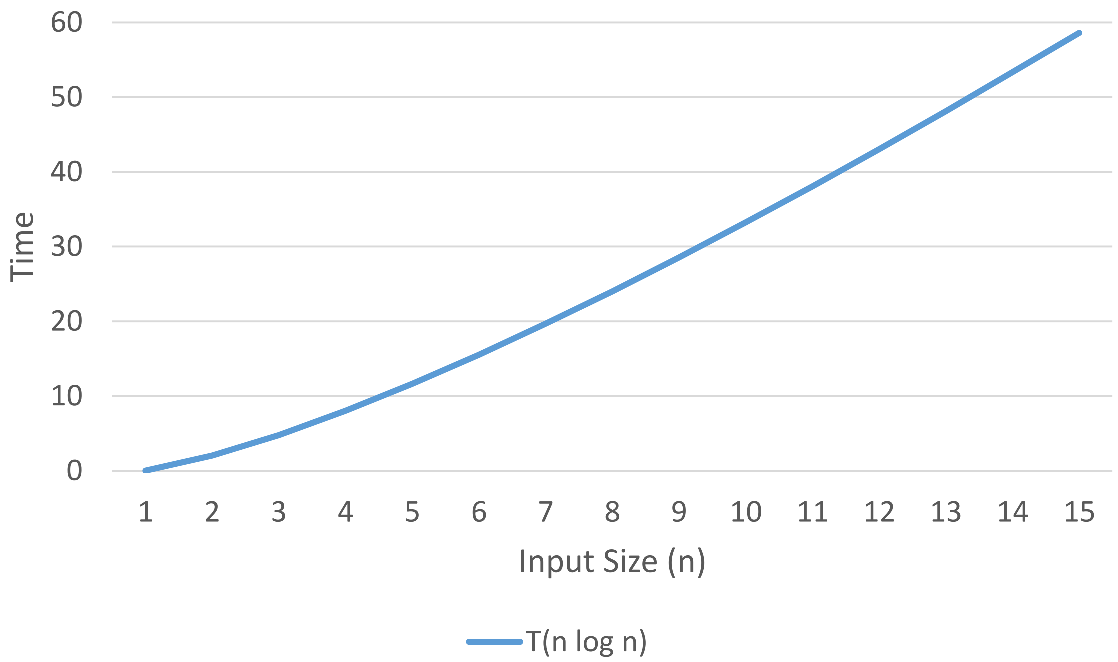
```
quicksort(array, start, end) :
        Select the pivot (the value in the middle)
        Partition the array
        quicksort(lower half)
        quicksort(upper half)
```

Merge Sort and Quick Sort

- Both algorithms use the divide and conquer process like a binary search.
  - Which we already determined performs in logarithmic time.
- Both algorithms, at one point or another, will have partitions with a length of 1.
  - The algorithms will perform the logarithmic divide and conquer operations for as many elements that exist in the array.
- Their time complexity is a mix of logarithmic and linear time.

Quasi-Linear Time

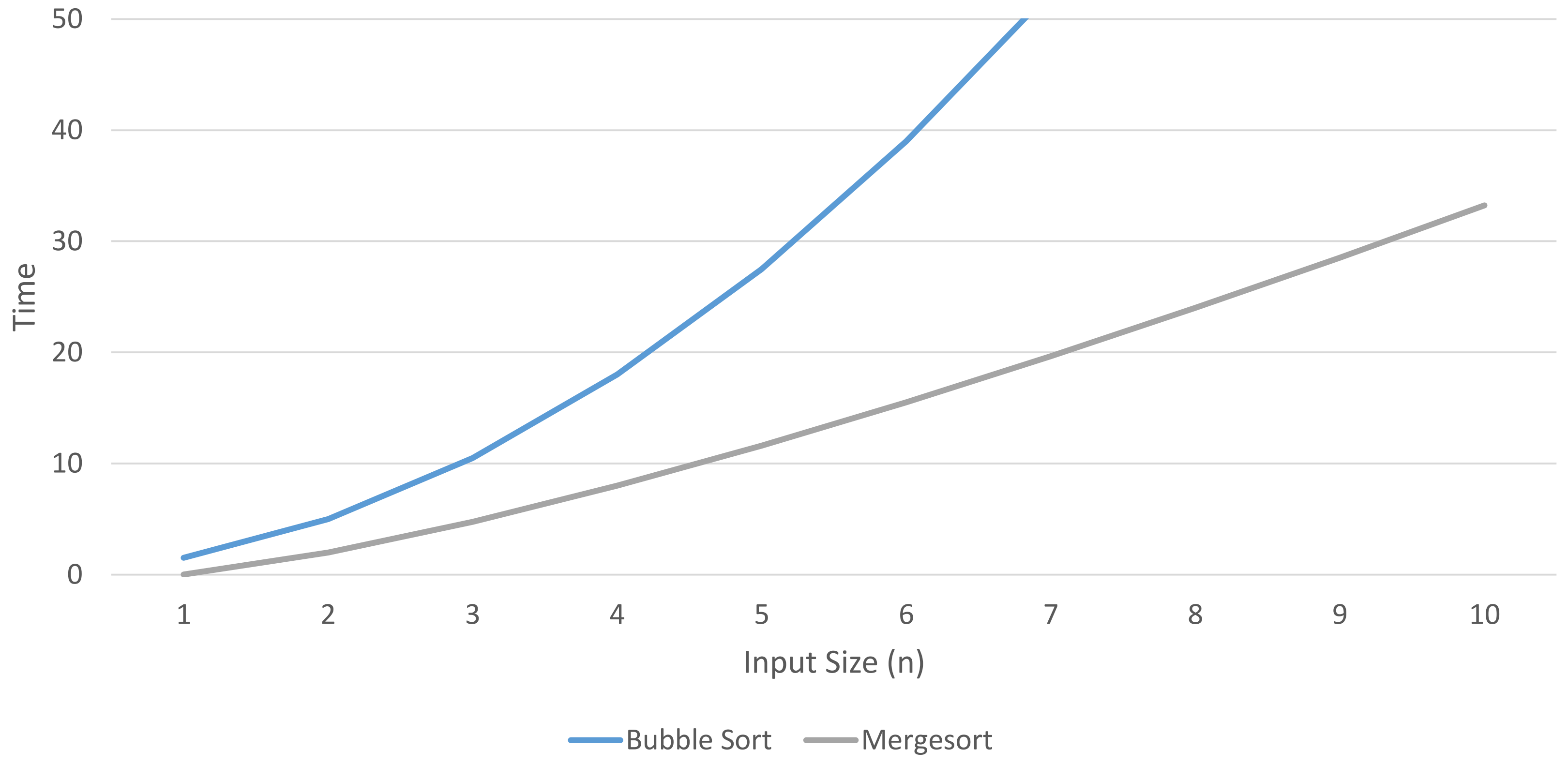
- Quasi-Linear (or Log-Linear) time is when an algorithm executes n-number of operations, where each operation performs in logarithmic time.
  - $T(n) = n \log_2 n$



- $T(n) = n \log_2 n$
  - $T(4) = 4 \log_2 4$
  - $T(8) = 8 \log_2 8$
  - $T(12) = 12 \log_2 12$

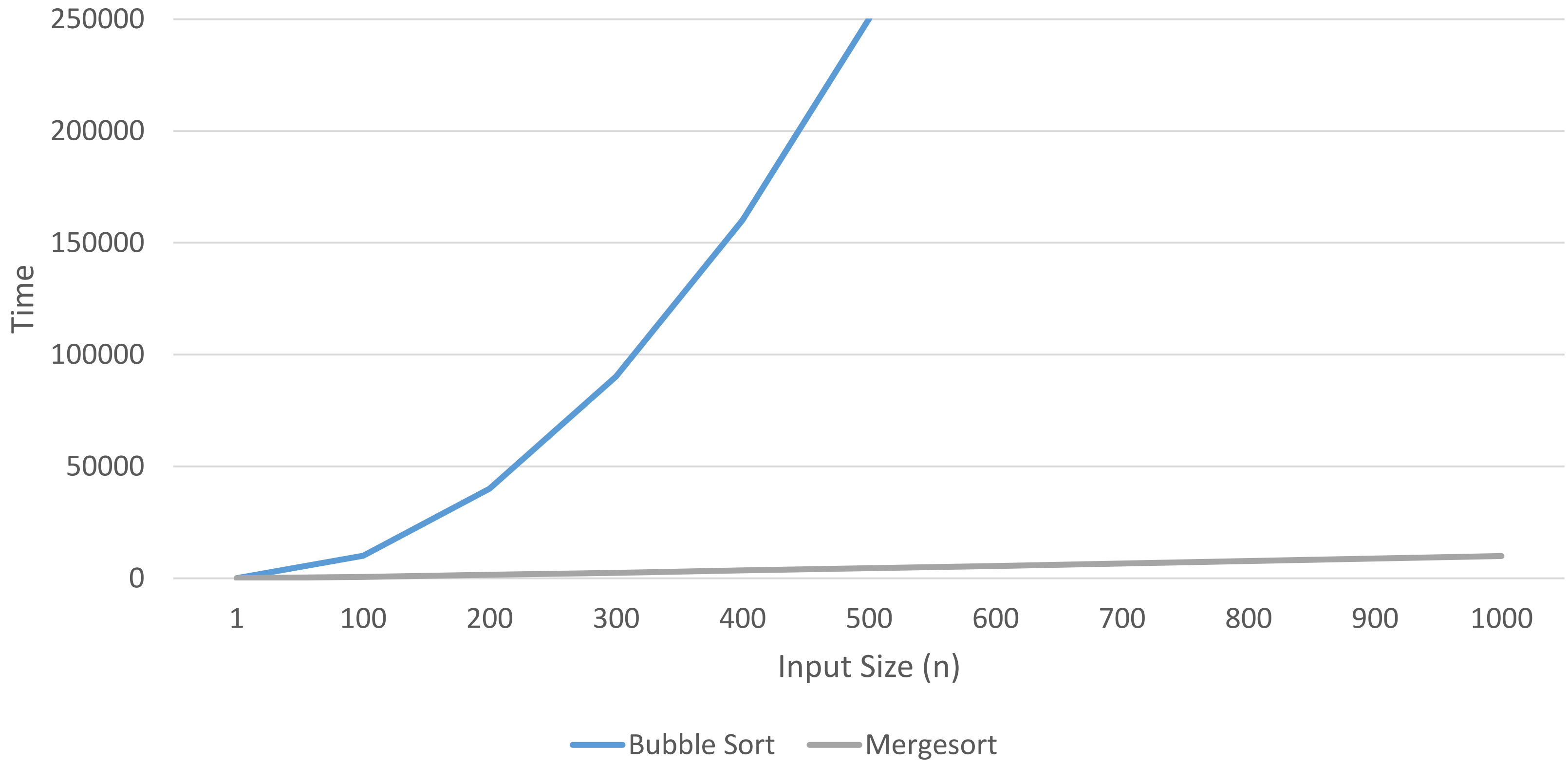
Bubble Sort vs Merge Sort

- Max array length: 10

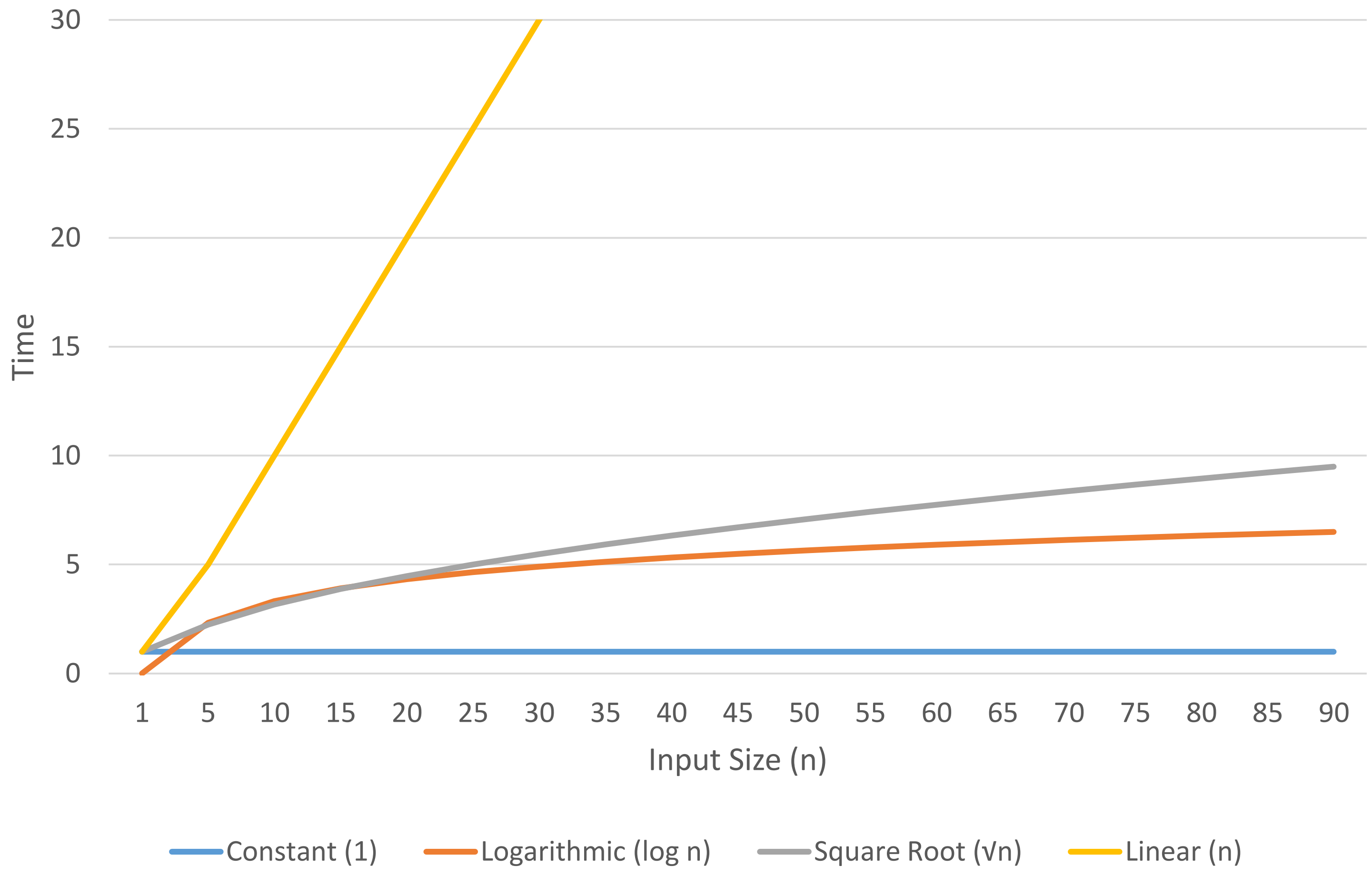


Bubble Sort vs Merge Sort

- Max array length: 1000



Complexities



Complexities