

Memory Architecture II

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Lecture Topics

- Cache Memory
 - Direct Mapping
 - N-way Associative Set Mapping
 - Fully Associative Set Mapping
- Dependability
 - Parity Bits
 - Error Correction Code

Cache Memory

- **Cache memory** is SRAM in the processor that holds:
 - The most frequently used data
 - The most recently accessed data
- Processors will often have several levels of cache memory.
 - **Level 1 (L1) Cache** – smallest, fastest cache
 - **Level 2 (L2) Cache** – larger, slower than the L1 cache
 - **Level 3 (L3) Cache** – larger, slower than the L2 cache
 - And so on...

Cache Memory

- Cache memory holds copies of data from main memory in units called blocks
 - Each block has a fixed size of bytes
- For example, a cache memory could have 512 blocks that each store 128 bytes
 - $2^9 \text{ blocks} \times 2^7 \frac{\text{bytes}}{\text{block}} = 2^{16} \text{ bytes} = 64\text{KiB}$

Cache Memory – Direct Mapping

- In a **direct mapped** cache, each block of main memory is mapped to a block in cache memory.
- Main memory is much larger than the cache, so multiple blocks of main memory will map to the same block in the cache

Cache Memory – Direct Mapping

- To calculate the cache block that corresponds to a main memory block:

$$M_b \bmod B_c$$

- Where M_b is the block number of main memory block
- Where B_c is the total number of blocks in the cache

Cache Memory – Direct Mapping

- When a word is to be *read/accessed* from main memory, the CPU first checks the cache to see if the block that contains the word is already in the cache.
 - If so, it simply obtains that data (**Cache Hit**)
 - If not, then the block is copied from main memory to its appropriate cache block (**Cache Miss**)

Cache Memory – Direct Mapping

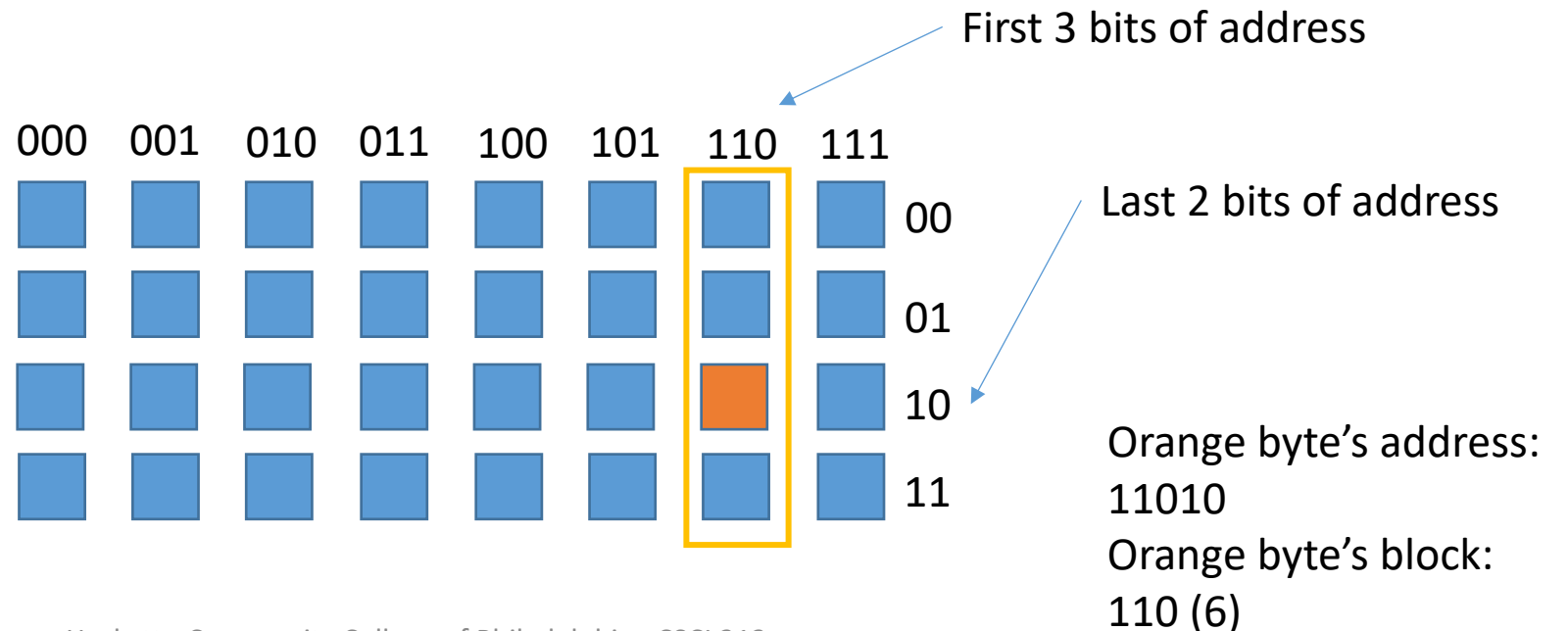
- When a word is to be *stored* to main memory, the CPU first checks the cache to see if that main memory address is already mapped in the cache.
 - If so, it simply stores that value in that block of the cache
 - If not, then the main memory block is copied to its appropriate cache block. The word is then written to that cache block.
- Either way, the data in the updated cache block must be copied back to main memory
 - Can be accomplished in one of two ways

Cache Memory – Direct Mapping

- One way is that whenever a write operation occurs, the new data in the cache is written back to its corresponding address in main memory
- Another way is the cache stores a **dirty bit** (or **valid bit**) for each cache block
 - When the content of the block is changed, the valid bit is set to 1
 - When a new main memory block is to be copied to cache, the valid bit of the corresponding cache block is checked.
 - If 1, it writes the cache block to its corresponding main memory block, then loads the new main memory block (and resets the valid bit to 0)

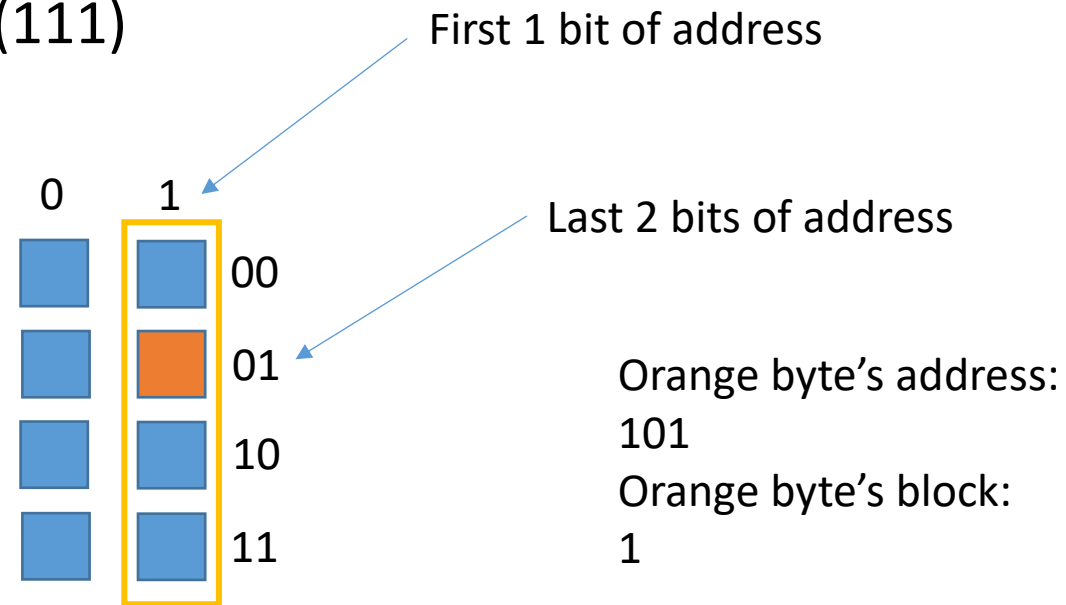
Cache Memory – Direct Mapping

- For illustration, here is a system with 32 bytes of main memory
 - Each square represents one byte
 - Each column is a block
 - Addresses 0 (00000) through 31 (111111)



Cache Memory – Direct Mapping

- We'll consider a simple CPU that has 16 bytes of cache memory
 - Each square represents one byte
 - Each column is a block
 - Addresses 0 (000) through 7 (111)



Cache Memory – Direct Mapping

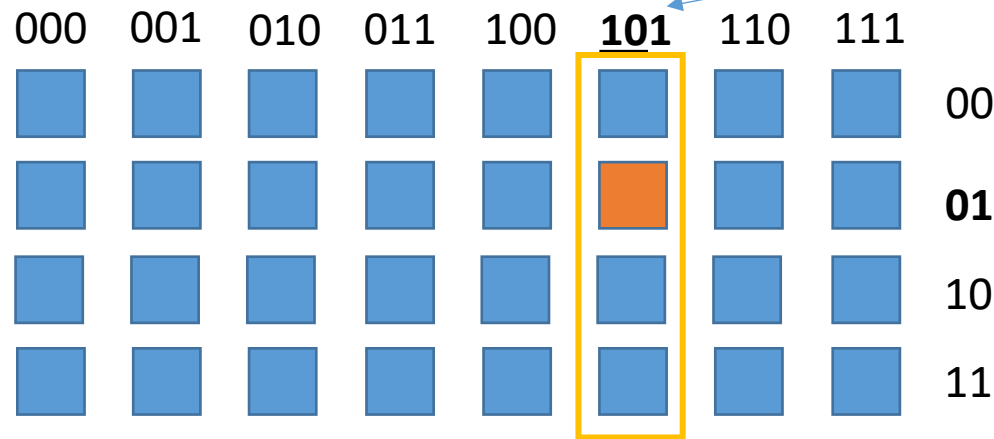
- Let's see how the main memory address 10101 maps to the cache

- 10 1 01**

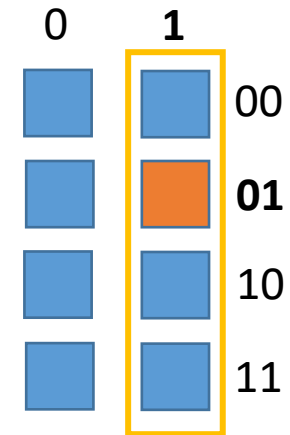
- 10 = **tag** or **main block**
- 1 = Cache block number
- 01 = Byte number in the cache block

tag	Cache block	Cache byte
10	1	01

RAM block



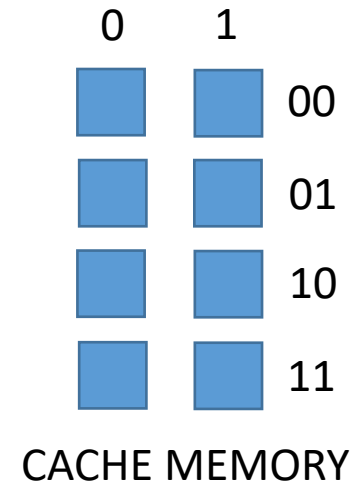
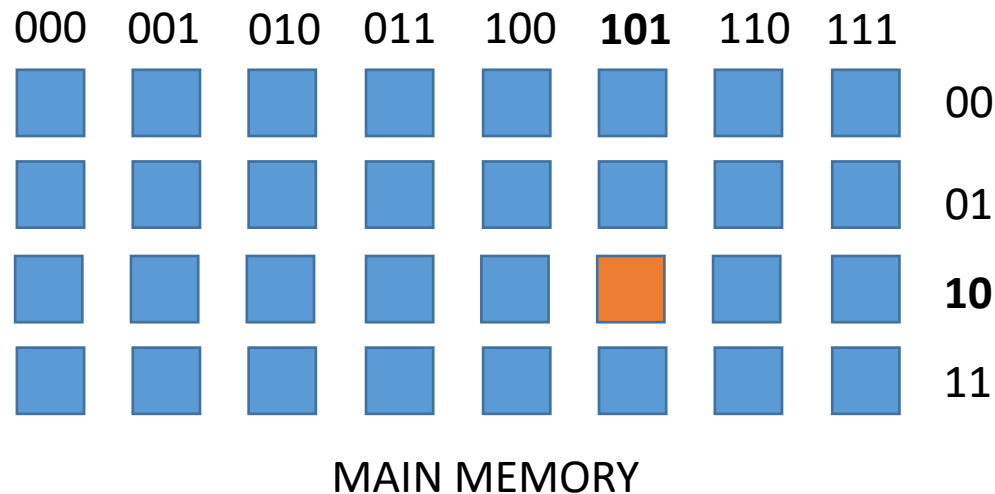
MAIN MEMORY



CACHE MEMORY

Cache Memory – Direct Mapping

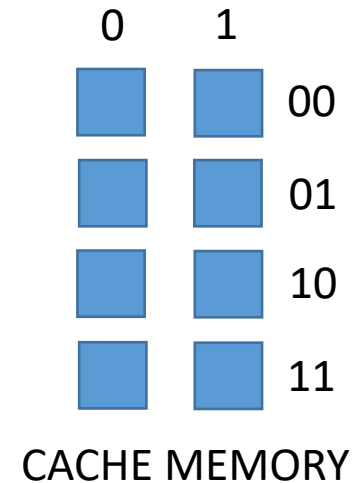
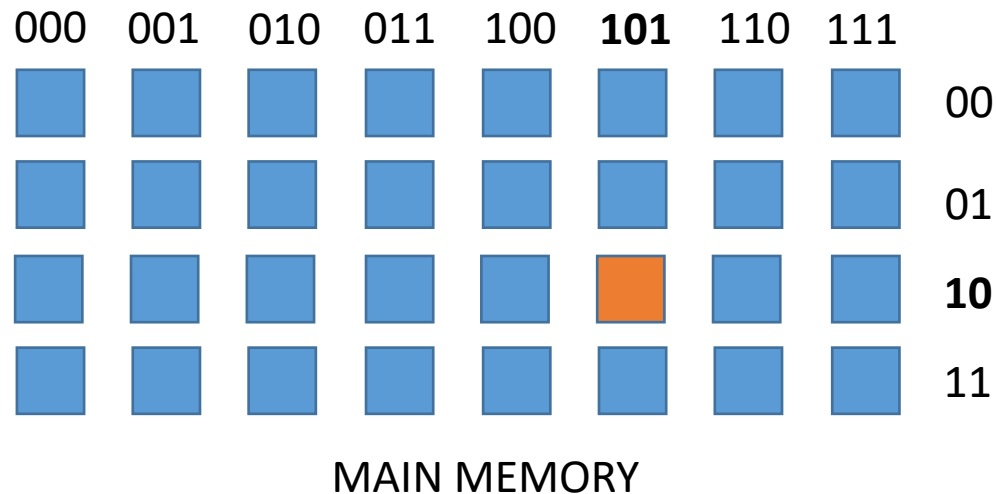
- The CPU is instructed to read the byte at address **10110** from main memory
 - The orange byte below



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
	1 00	
	1 01	
	1 10	
	1 11	

Cache Memory – Direct Mapping

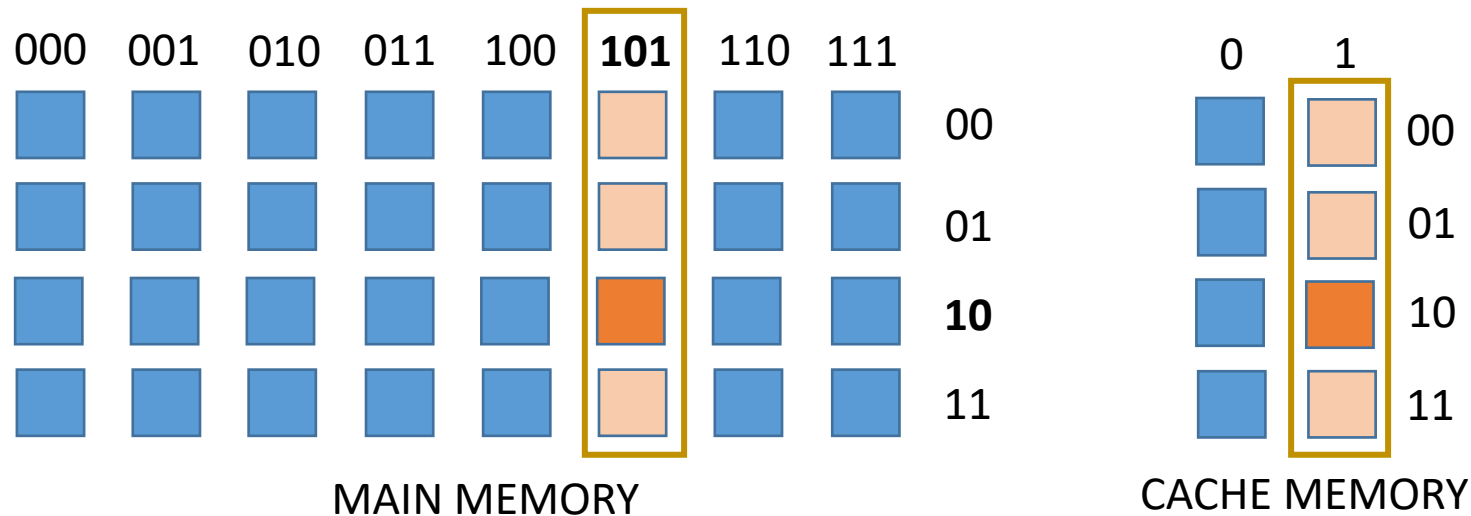
- First the CPU checks if the value is in the cache
 - **10 1 10**
 - Not present (*cache miss*)



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
	1 00	
	1 01	
	1 10	
	1 11	

Cache Memory – Direct Mapping

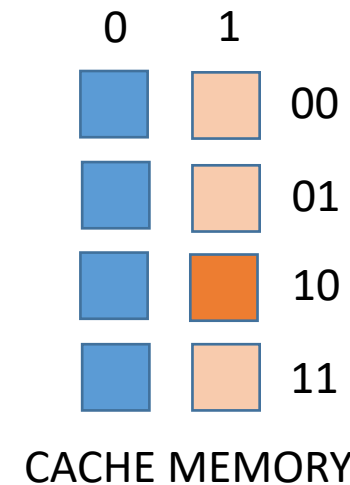
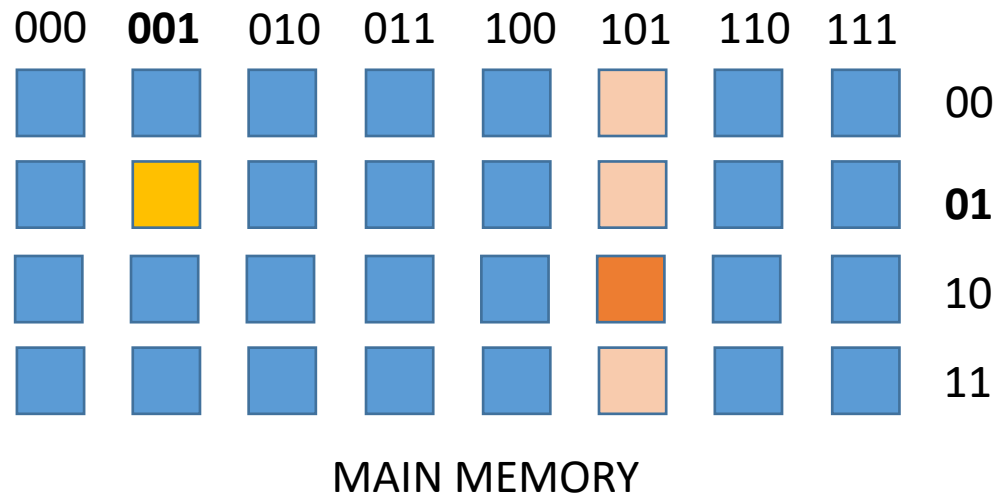
- The block (block 5) is loaded into cache
 - 10 1 00
 - 10 1 01
 - 10 1 10
 - 10 1 11



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
10	1 00	Blue 10100
10	1 01	Blue 10101
10	1 10	Orange 10110
10	1 11	Blue 10111

Cache Memory – Direct Mapping

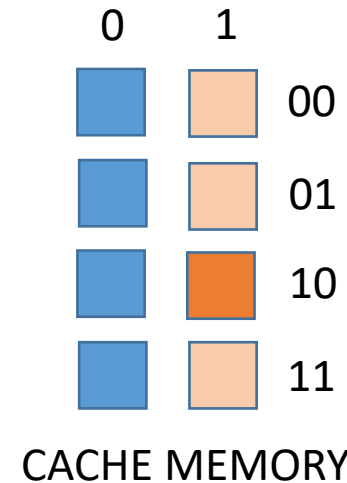
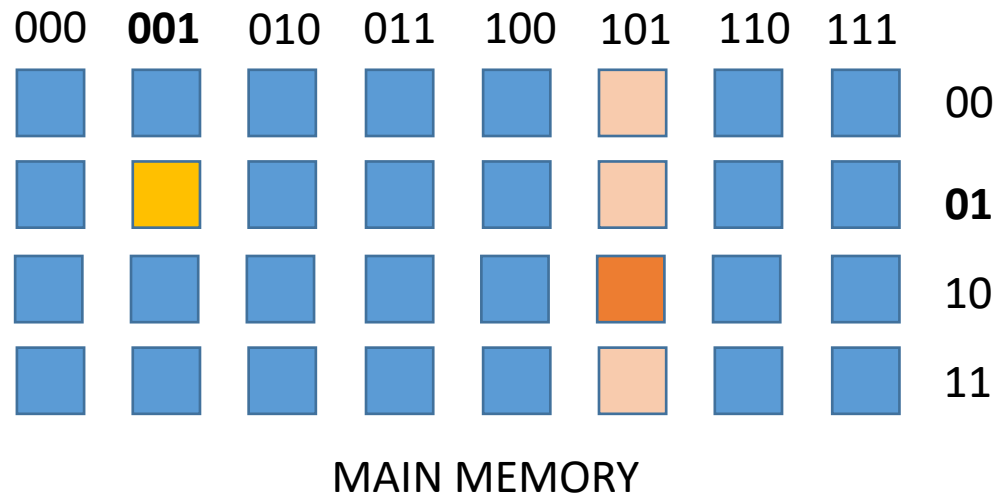
- The CPU is instructed to read the byte at address 00101 from main memory
 - The yellow byte below



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
10	1 00	Light Orange 10100
10	1 01	Light Orange 10101
10	1 10	Dark Orange 10110
10	1 11	Light Orange 10111

Cache Memory – Direct Mapping

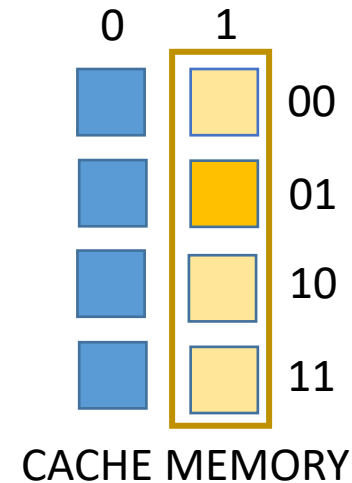
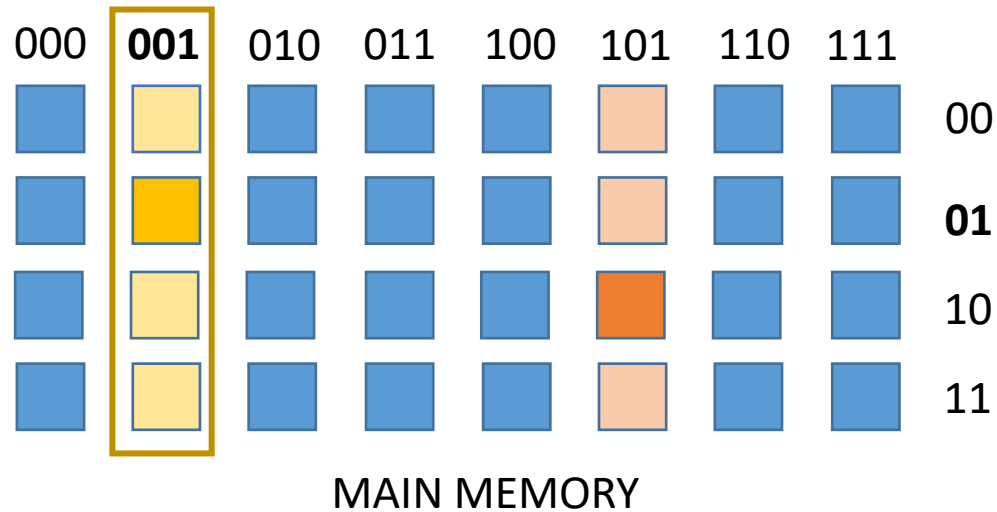
- First the CPU checks if the value is in the cache
 - **00 1 01**
 - Data in cache address 101, but wrong tag (*cache miss*)



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
10	1 00	Light Orange 10100
10	1 01	Light Orange 10101
10	1 10	Dark Orange 10110
10	1 11	Light Orange 10111

Cache Memory – Direct Mapping

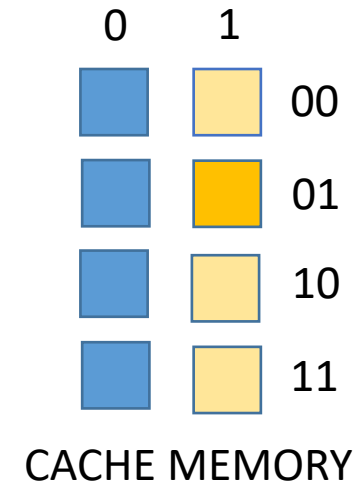
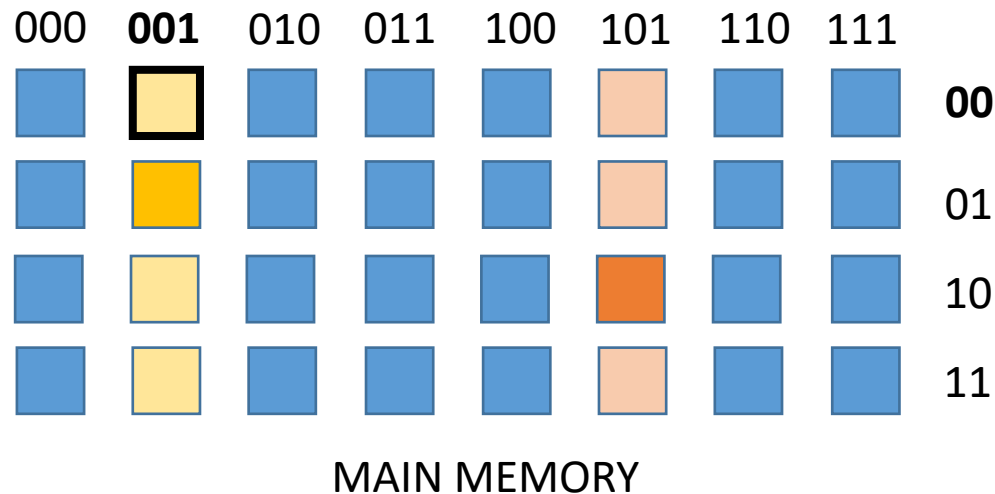
- The block (block 1) is loaded into cache
 - 00 1 00
 - 00 1 01
 - 00 1 10
 - 00 1 11



Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	<div></div> 00100
00	1 01	<div></div> 00101
00	1 10	<div></div> 00110
00	1 11	<div></div> 00111

Cache Memory – Direct Mapping

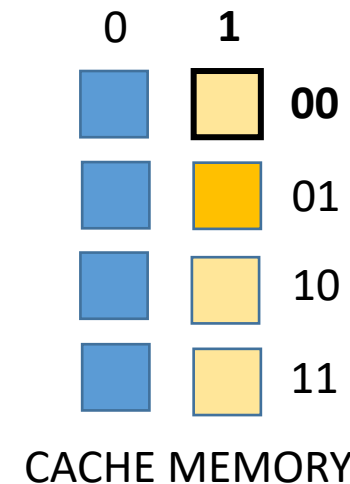
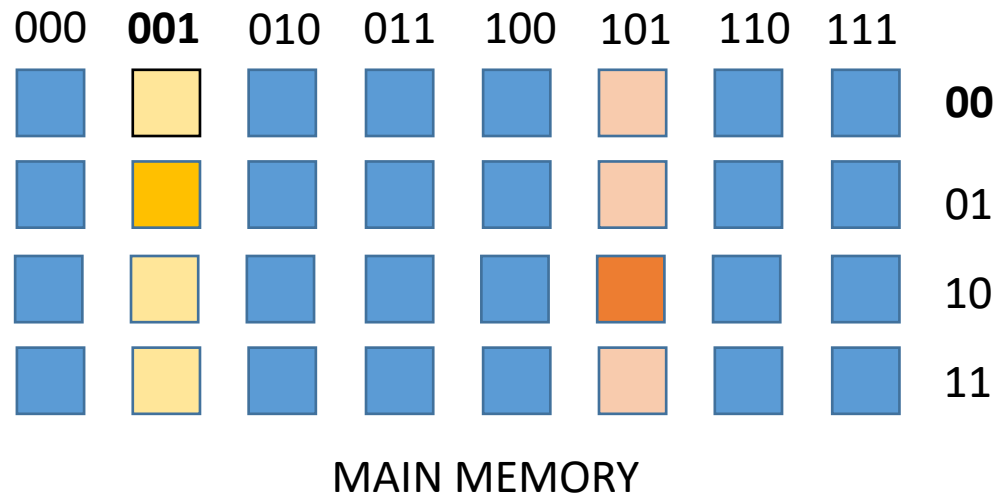
- The CPU is instructed to read the byte at address **00100** from main memory



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	Yellow 00100
00	1 01	Yellow 00101
00	1 10	Yellow 00110
00	1 11	Yellow 00111


Cache Memory – Direct Mapping

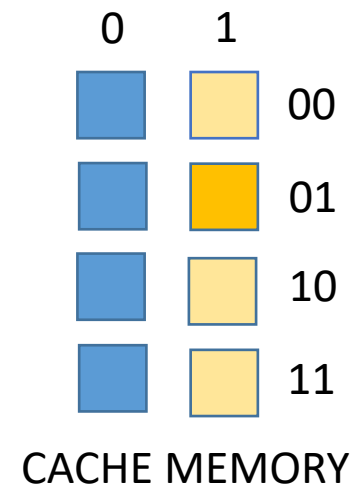
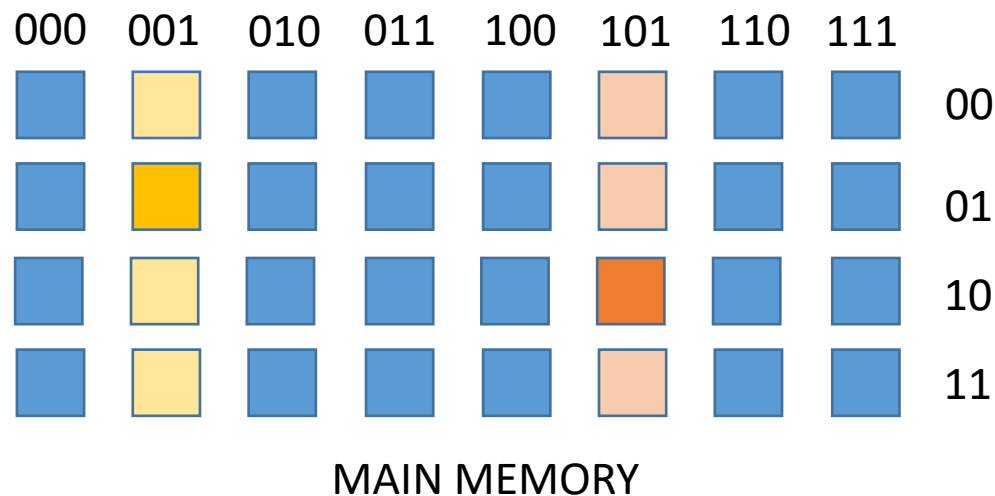
- First the CPU checks if the value is in the cache
 - **00 1 00**
 - Data in cache address 100 with correct tag (*cache hit*)
 - Does not retrieve from main memory







Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	00100
00	1 01	00101
00	1 10	00110
00	1 11	00111

Cache Memory – Direct Mapping

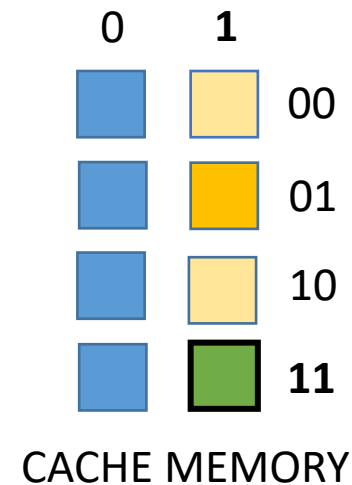
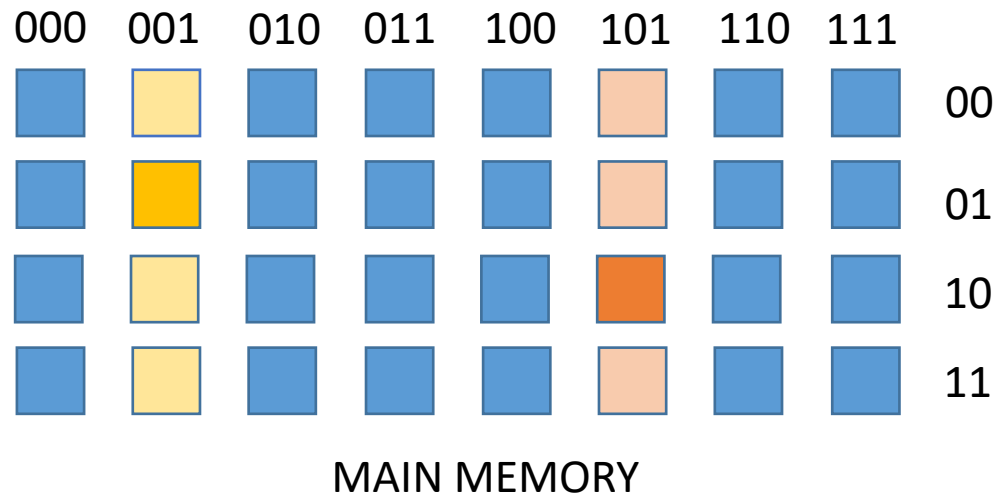
- The CPU is instructed to store a byte to address 00111 in main memory
 - Will be represented as 







Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	 00100
00	1 01	 00101
00	1 10	 00110
00	1 11	 00111

Cache Memory – Direct Mapping

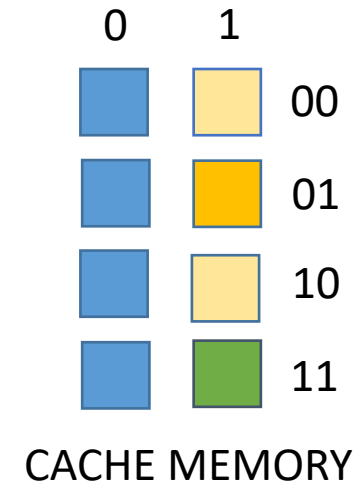
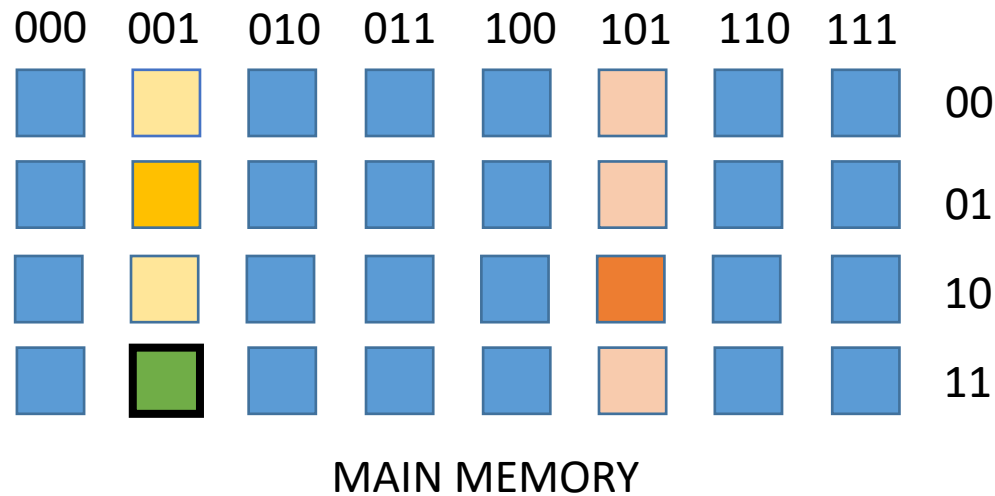
- The CPU writes the value to the cache
 - 00 1 11
 - Data in cache address 111 with correct tag (*cache hit*)
 - Data in cache is updated



Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	 00100
00	1 01	 00101
00	1 10	 00110
00	1 11	 00111


Cache Memory – Direct Mapping

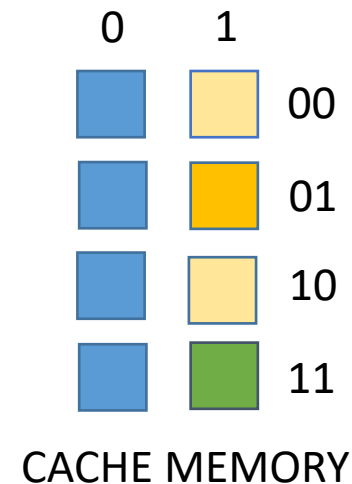
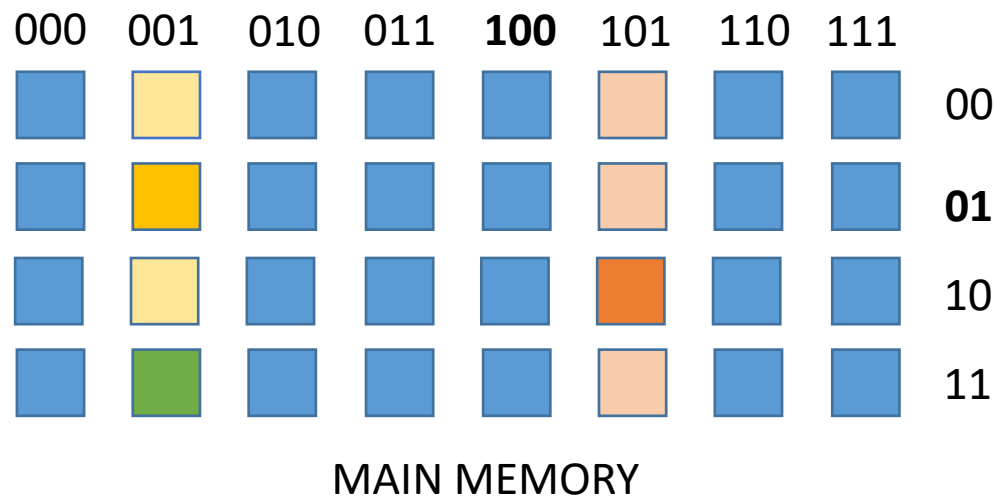
- Data is stored to main memory (or dirty bit is set)







Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	Yellow 00100
00	1 01	Yellow 00101
00	1 10	Yellow 00110
00	1 11	Green 00111

Cache Memory – Direct Mapping

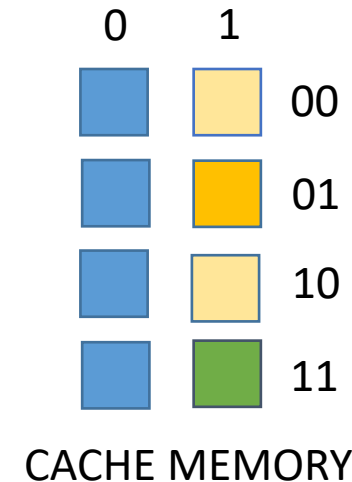
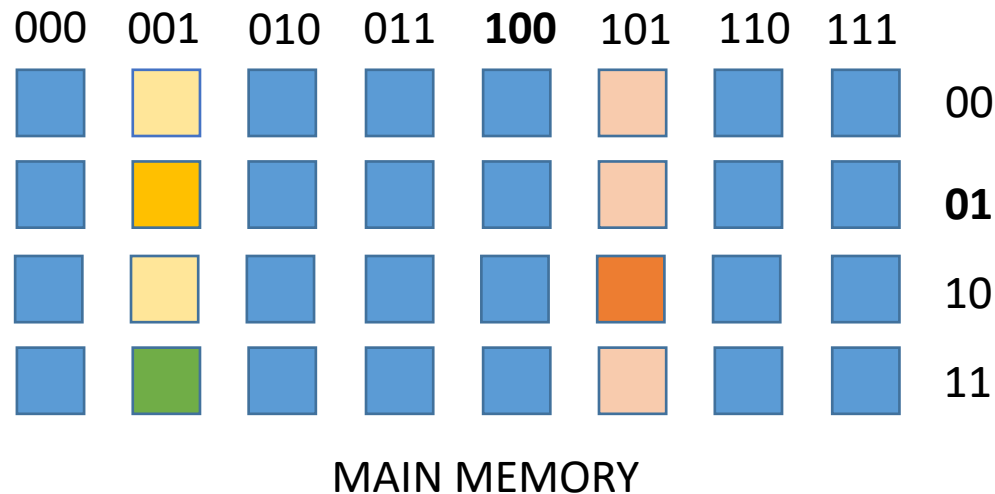
- The CPU is instructed to store a byte to address 10001 in main memory
 - Will be represented as 







Tag	Cache Address Set Byte	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	 00100
00	1 01	 00101
00	1 10	 00110
00	1 11	 00111

Cache Memory – Direct Mapping

- First the CPU checks if the value is in the cache
 - **10 0 01**
 - No data in cache address 001 (cache miss)

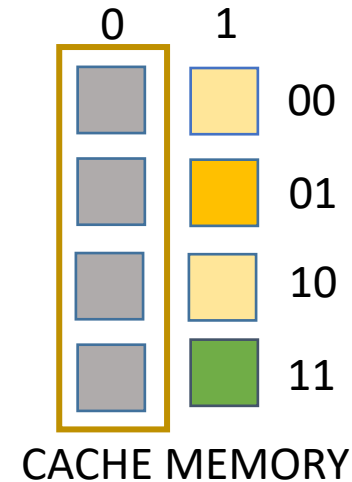
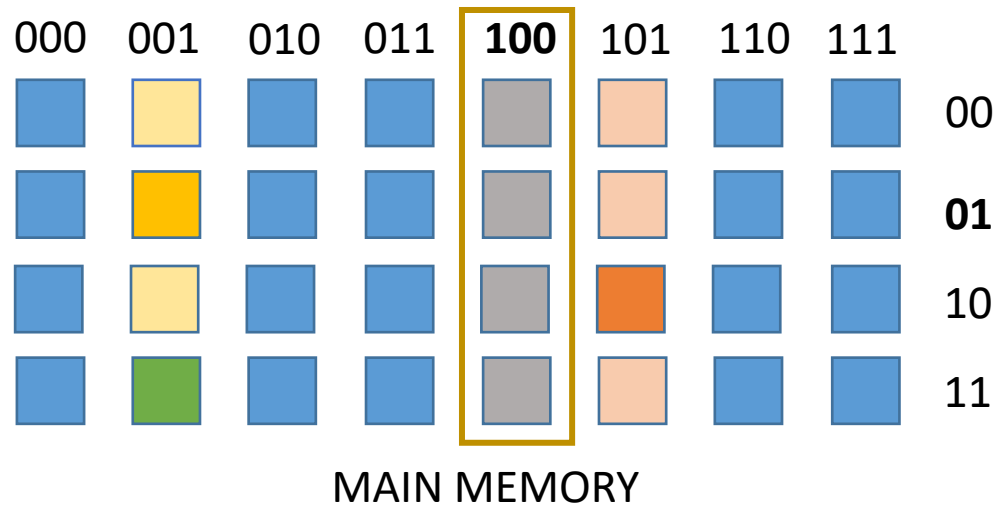


Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
	0 00	
	0 01	
	0 10	
	0 11	
00	1 00	 00100
00	1 01	 00101
00	1 10	 00110
00	1 11	 00111

Cache Memory – Direct Mapping

- The block (block 4) is loaded into cache

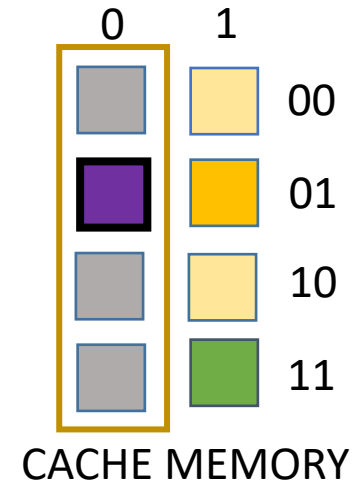
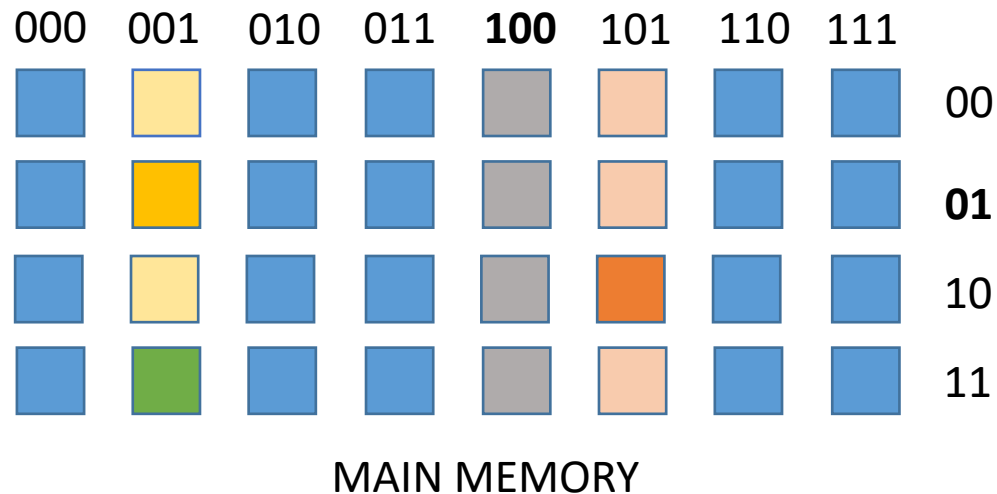
- 10 0 00
- 10 0 01
- 10 0 10
- 10 0 11



Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
10	0 00	10000
10	0 01	10001
10	0 10	10010
10	0 11	10011
00	1 00	00100
00	1 01	00101
00	1 10	00110
00	1 11	00111

Cache Memory – Direct Mapping

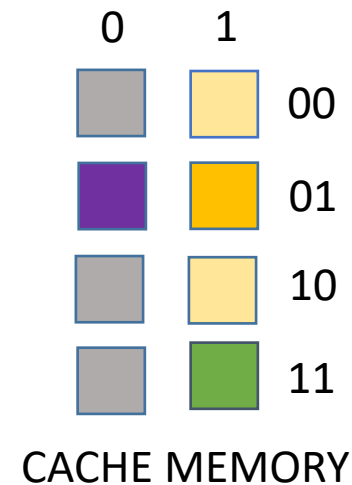
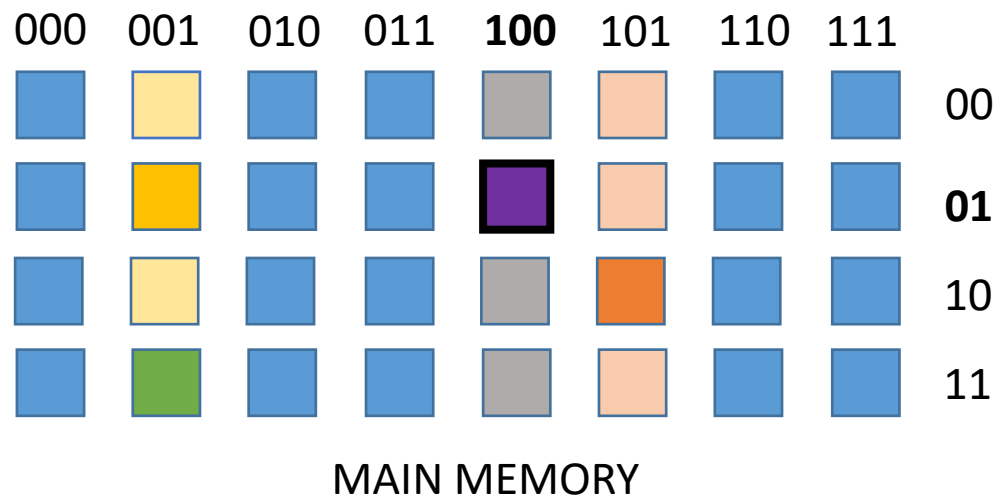
- The CPU writes the value to the cache
 - **10 0 01**
 - Data in cache is updated



Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
10	0 00	10000
10	0 01	10001
10	0 10	10010
10	0 11	10011
00	1 00	00100
00	1 01	00101
00	1 10	00110
00	1 11	00111

Cache Memory – Direct Mapping


- Data is stored to main memory (or dirty bit is set)



























Tag	Cache Address <i>Set Byte</i>	Data/Main Memory Address
10	0 00	10000
10	0 01	10001
10	0 10	10010
10	0 11	10011
00	1 00	00100
00	1 01	00101
00	1 10	00110
00	1 11	00111

Cache Memory – Direct Mapping

- Each row in the table below represents the read/write operations illustrated on the previous slides



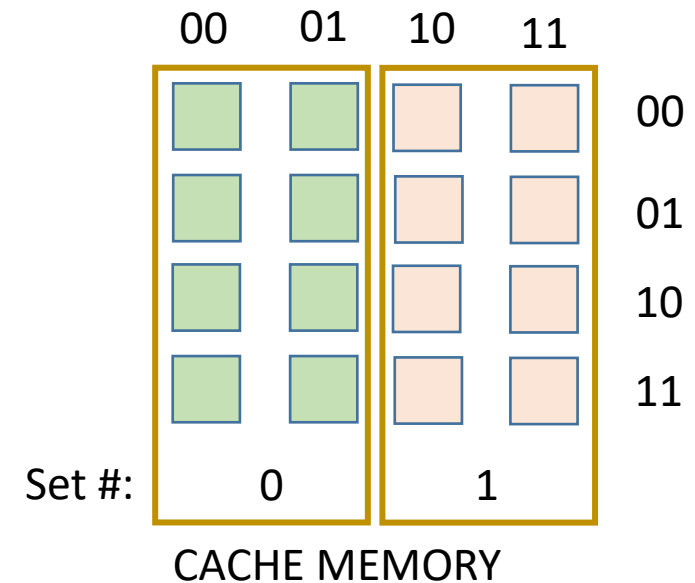
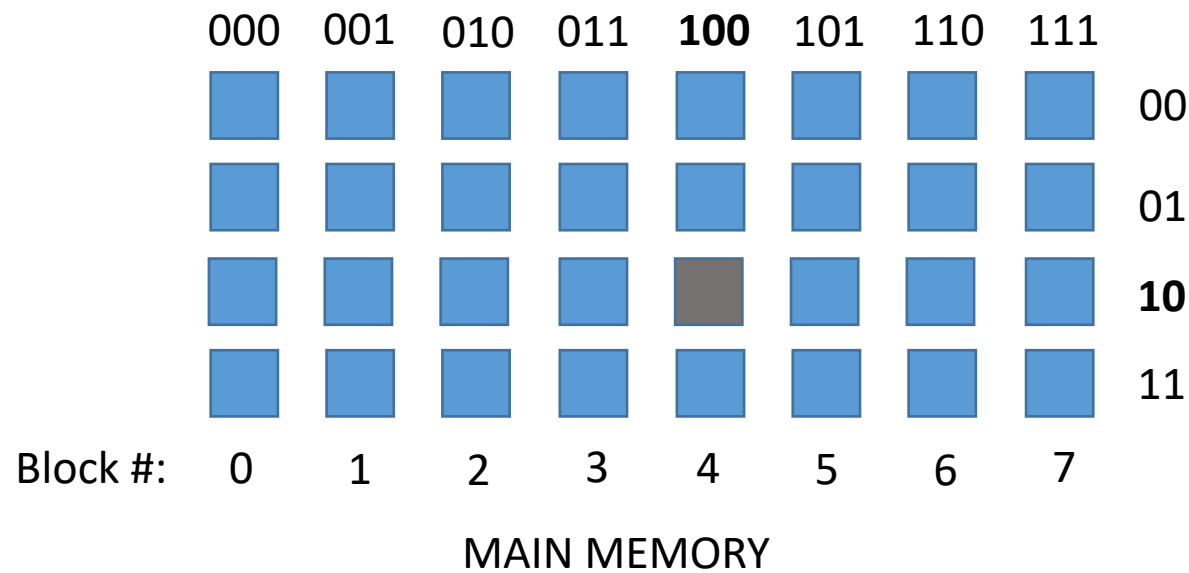
Memory Address	Hit or Miss	Cache Contents							
		0	1	2	3	4	5	6	7
10110	M								
00101	M								
00100	H								
00111	H								
10001	M								

Cache Memory – Associative Set Mapping

- In an **associative mapped** cache, each block of main memory is mapped to a *set of blocks* in cache memory.
- When a block is copied from RAM to the cache, the block can be stored in any one of the blocks that belong to the set.
- An ***n-way set associative cache*** indicates that each cache set contains n blocks per set.
 - A 3-way set associative cache has three blocks per set
 - A 1-way set associative cache has one block per set
 - A 1-way set associative cache *is* direct mapping

Cache Memory – Associative Set Mapping

- 2-way associative cache
- Consider the gray square/byte with the address **10010**



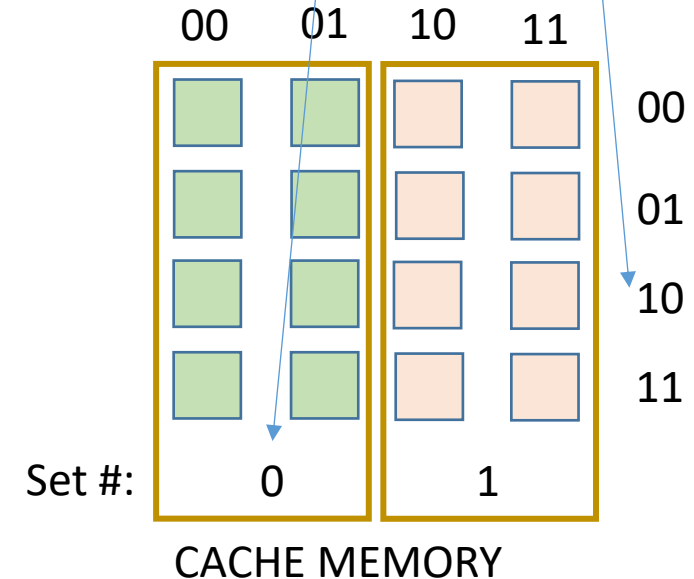
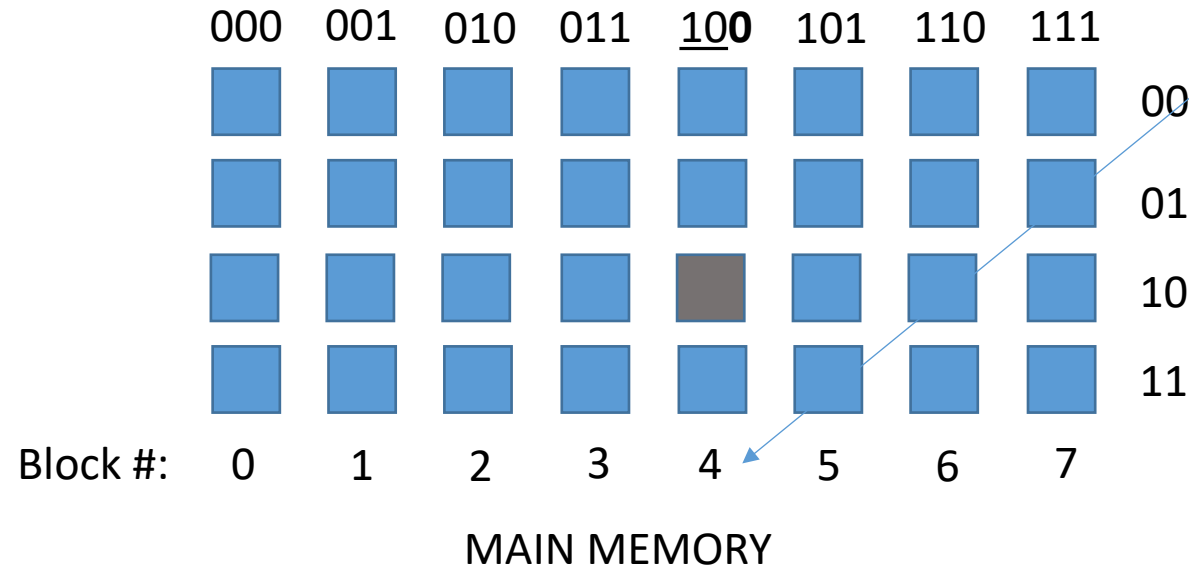
Cache Memory – Associative Set Mapping

- **10 0 10**

- 10 = **tag** or **main block**
- **0** = Cache set number
- 10 = Byte number in the cache block

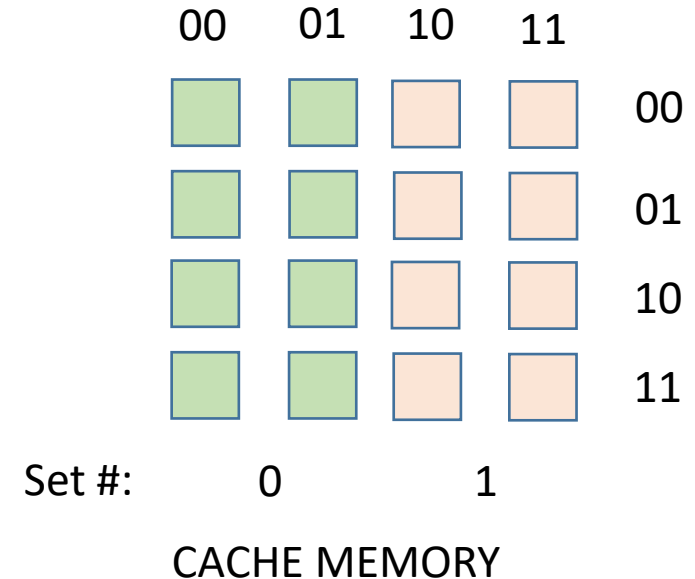
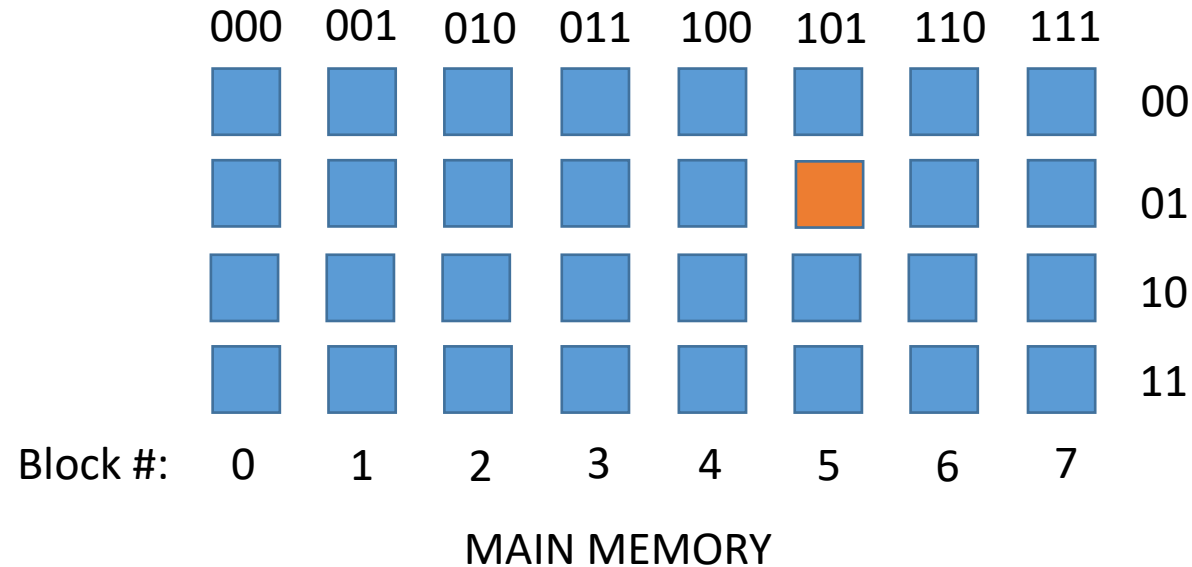
tag	Cache set	Cache byte
10	0	10

RAM block



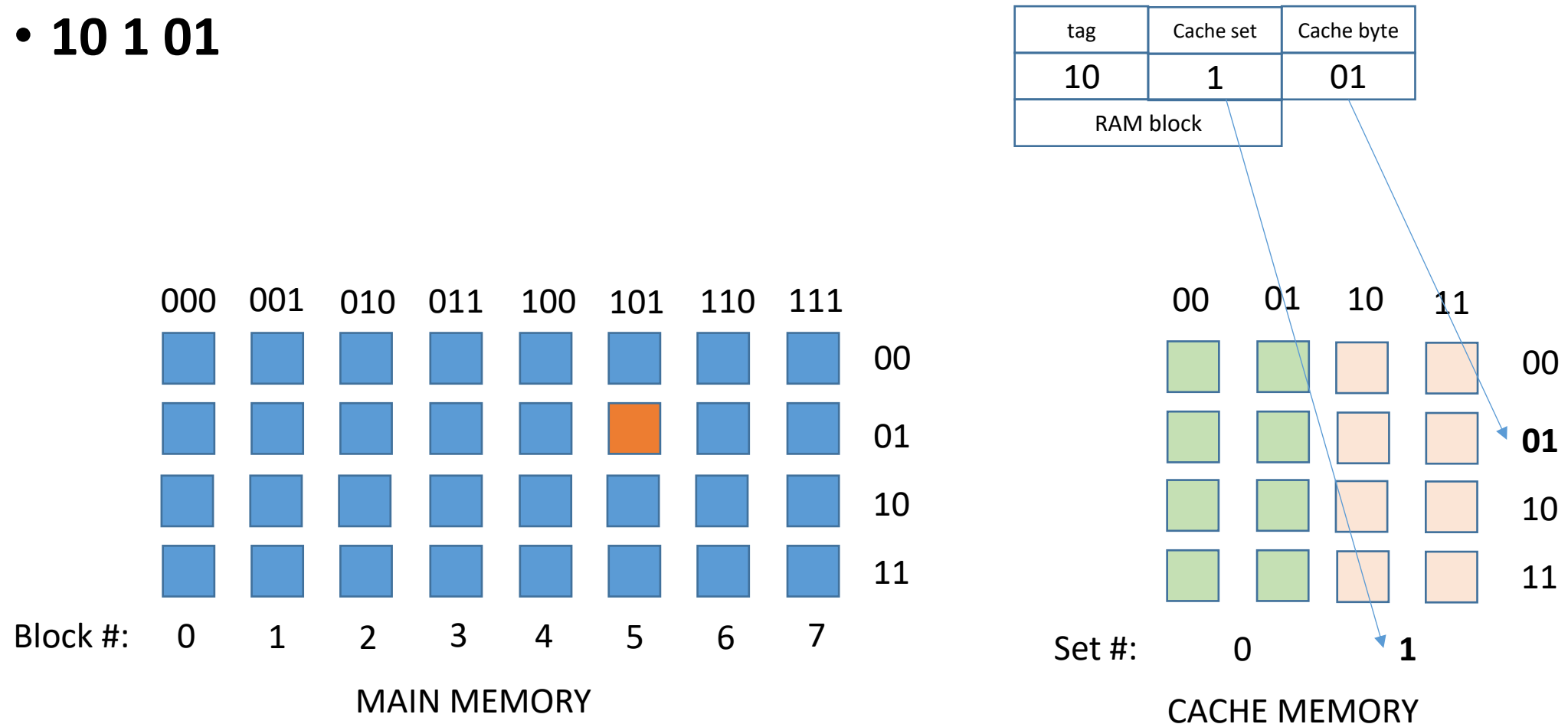
Cache Memory – Associative Set Mapping

- The CPU is instructed to read the byte at address **10101** from main memory
 - The orange byte below



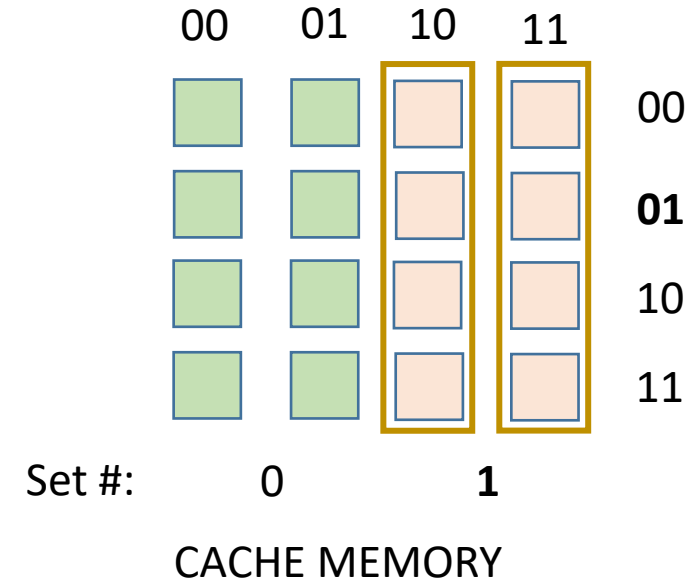
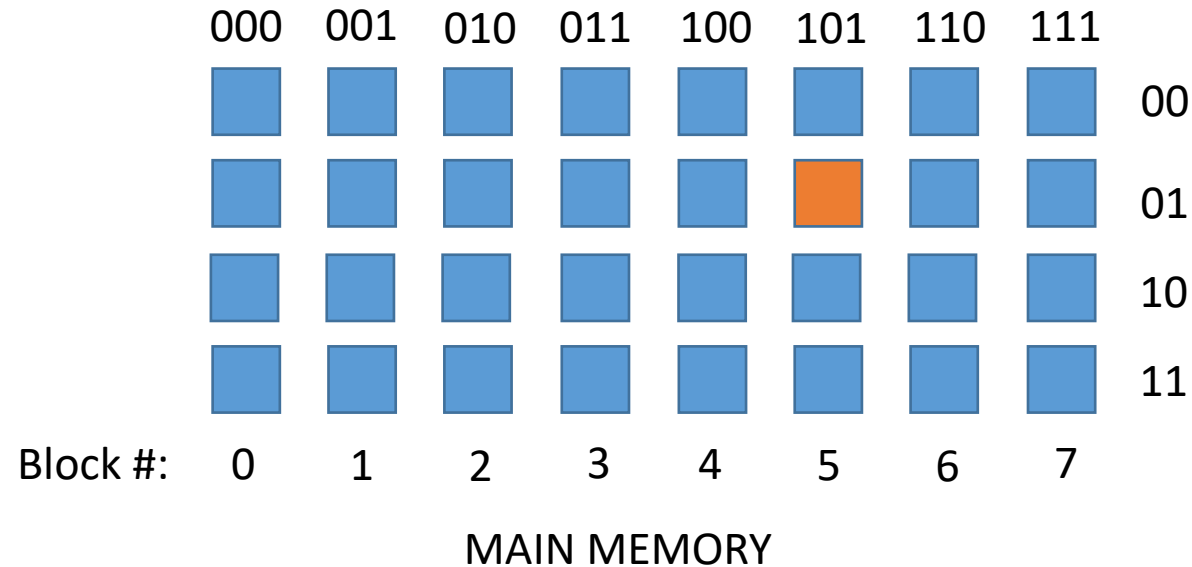
Cache Memory – Associative Set Mapping

- **10 1 01**



Cache Memory – Associative Set Mapping

- Within this set, there are two blocks to choose from (blocks 2 and 3).
 - Which block is chosen will depend on the *block replacement strategy* implemented

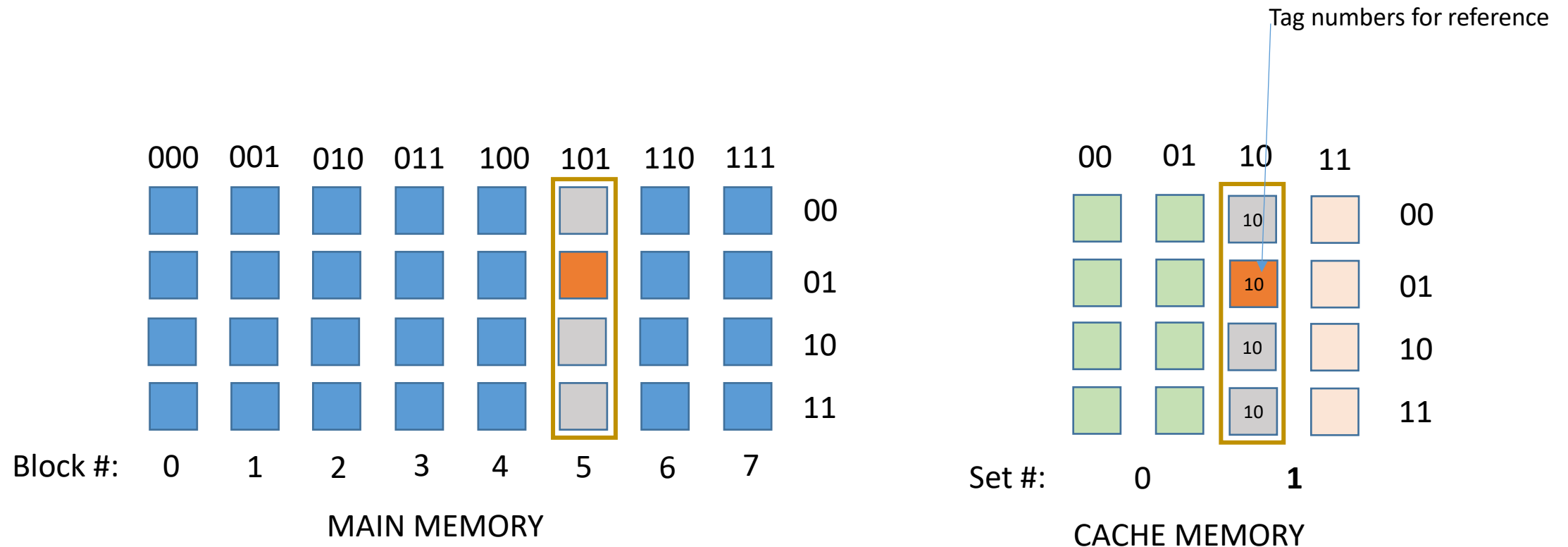


Cache Memory – Associative Set Mapping

- One such strategy is called **Least Recently Used (LRU)**
 - The block to be replaced in the set is the block that has been sitting, unused, for the longest time.
- Another block replacement strategy is called **First In, First Out (FIFO)**
 - The block to be replaced in the set is the block that has been in the cache for the longest time.
- A third strategy is to randomly choose a block for replacement
- Each strategy has their pros and cons
 - No perfect/optimal block replacement strategy

Cache Memory – Associative Set Mapping

- We'll assume LRU.
 - Block 2 is chosen

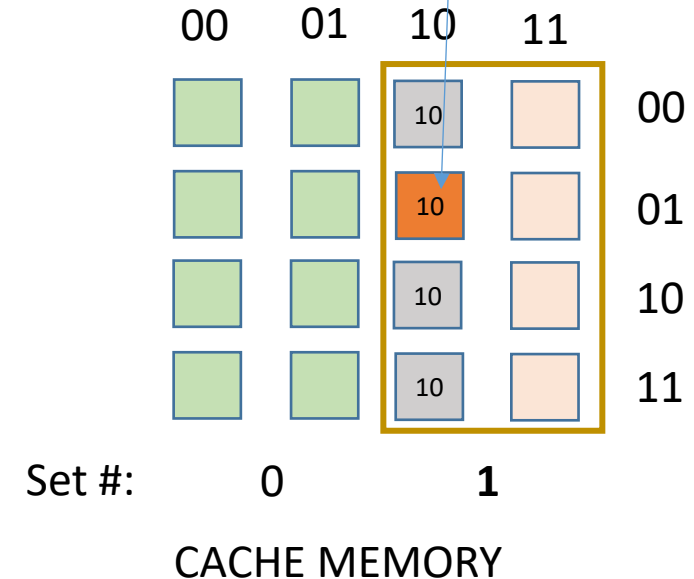
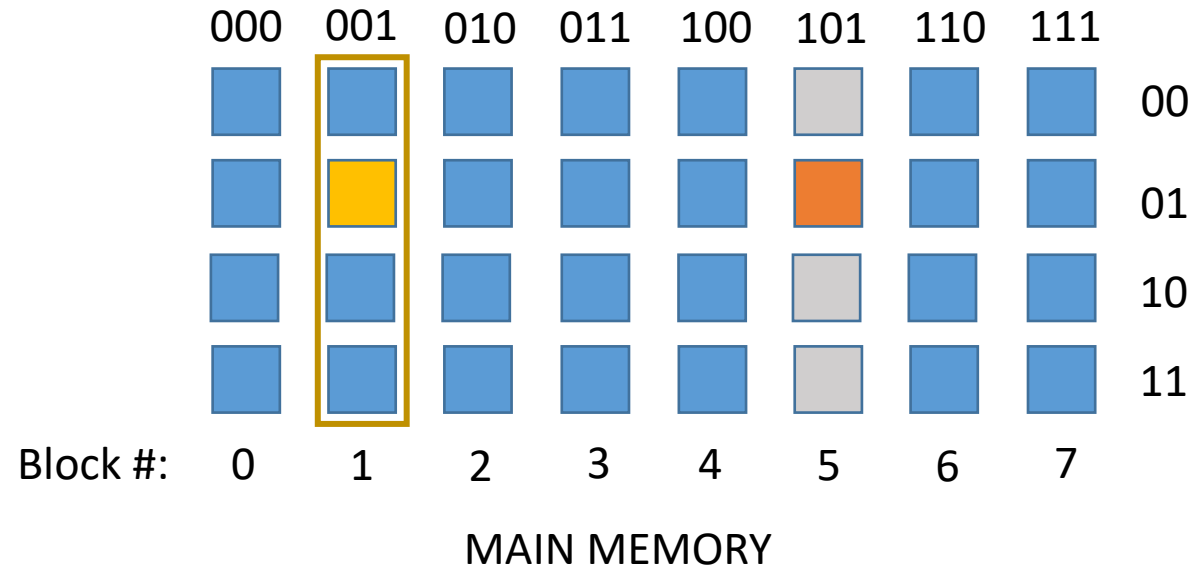


Cache Memory – Associative Set Mapping

- **00101**

tag	Cache set	Cache byte
00	1	01
RAM block		

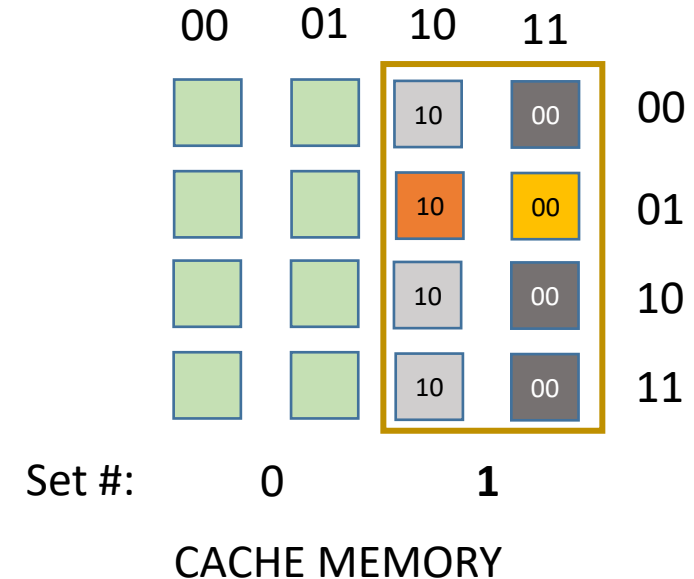
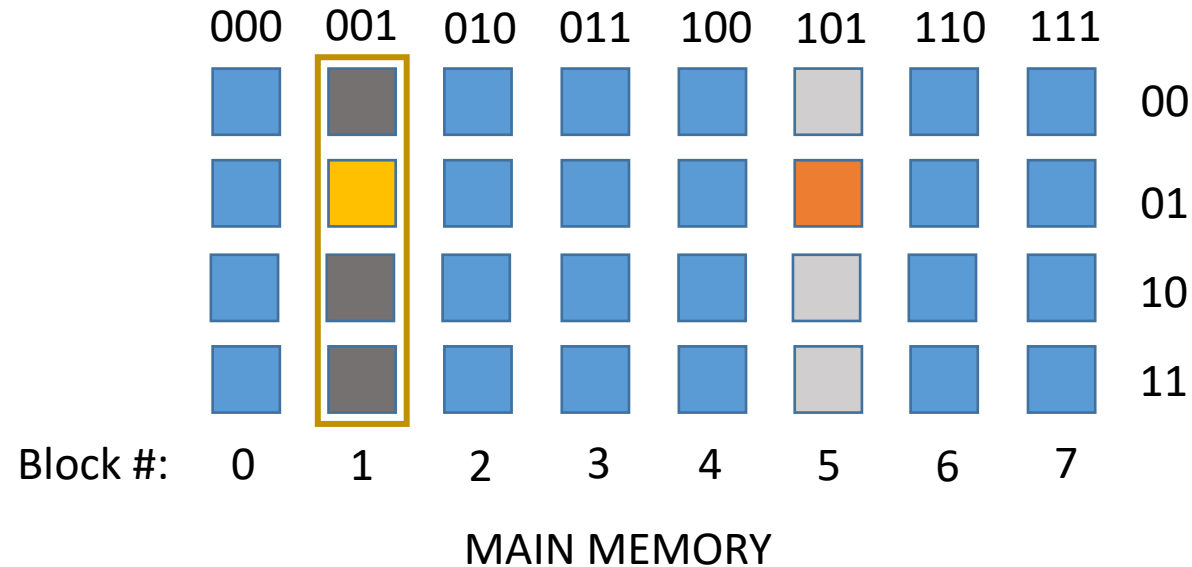
Tags don't match (miss)



Cache Memory – Associative Set Mapping

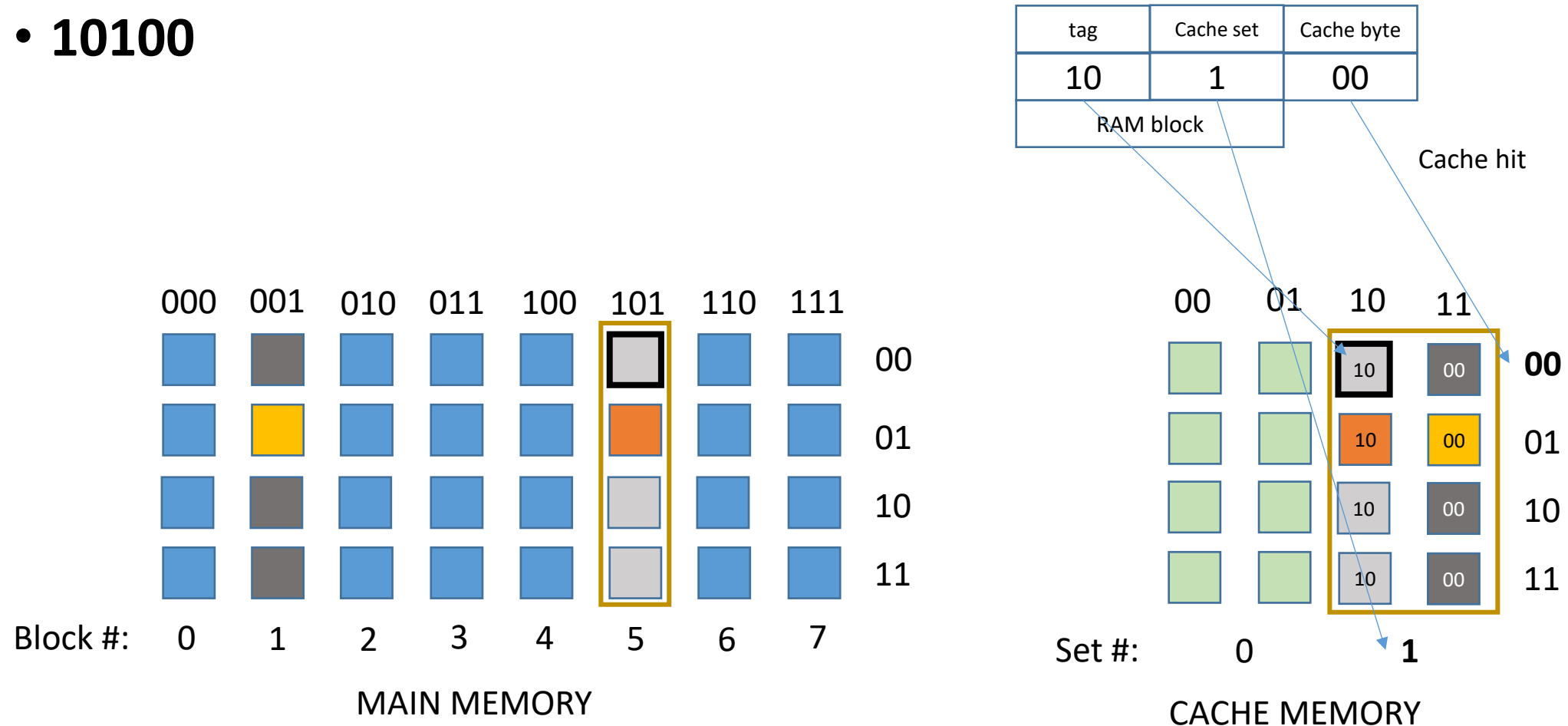
- **00101**

tag	Cache set	Cache byte
00	1	01
RAM block		



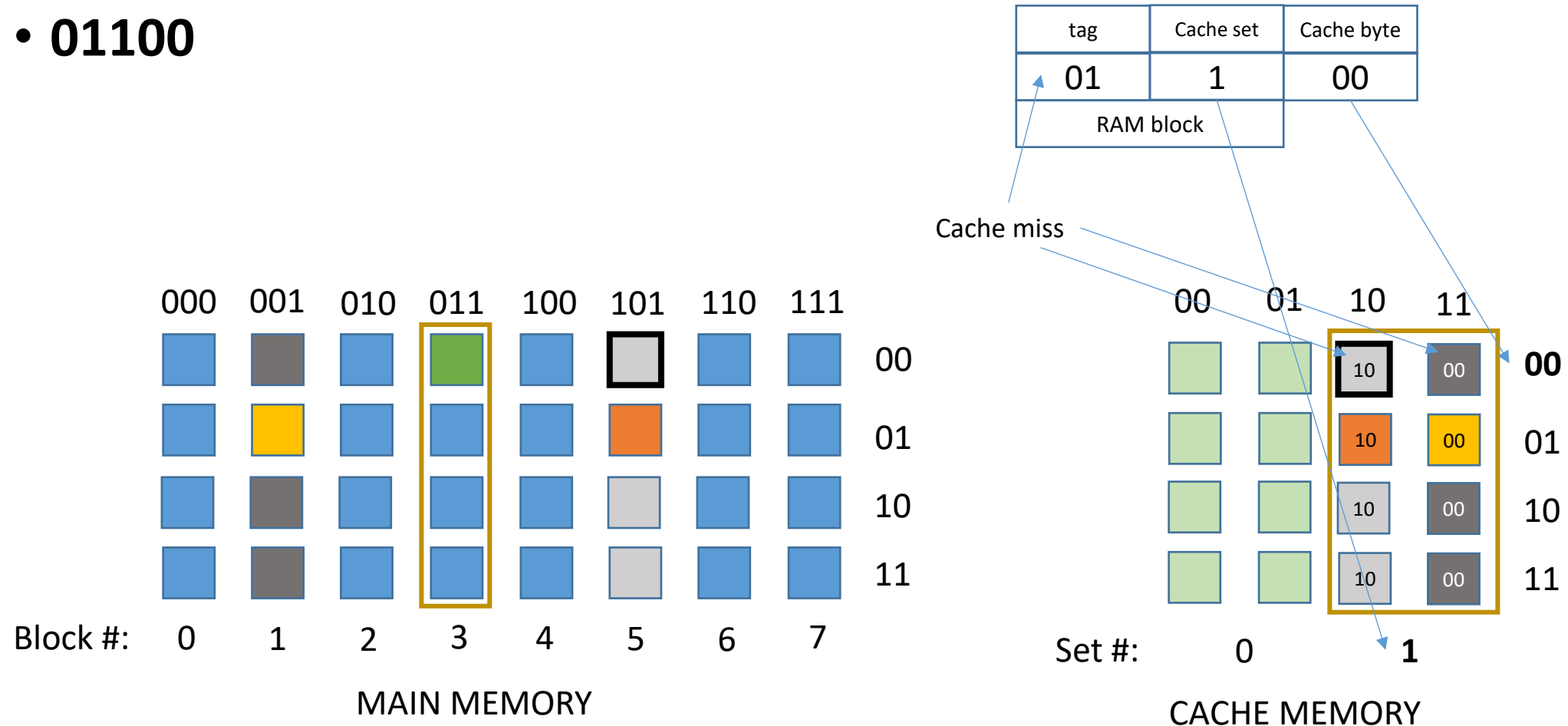
Cache Memory – Associative Set Mapping

- **10100**



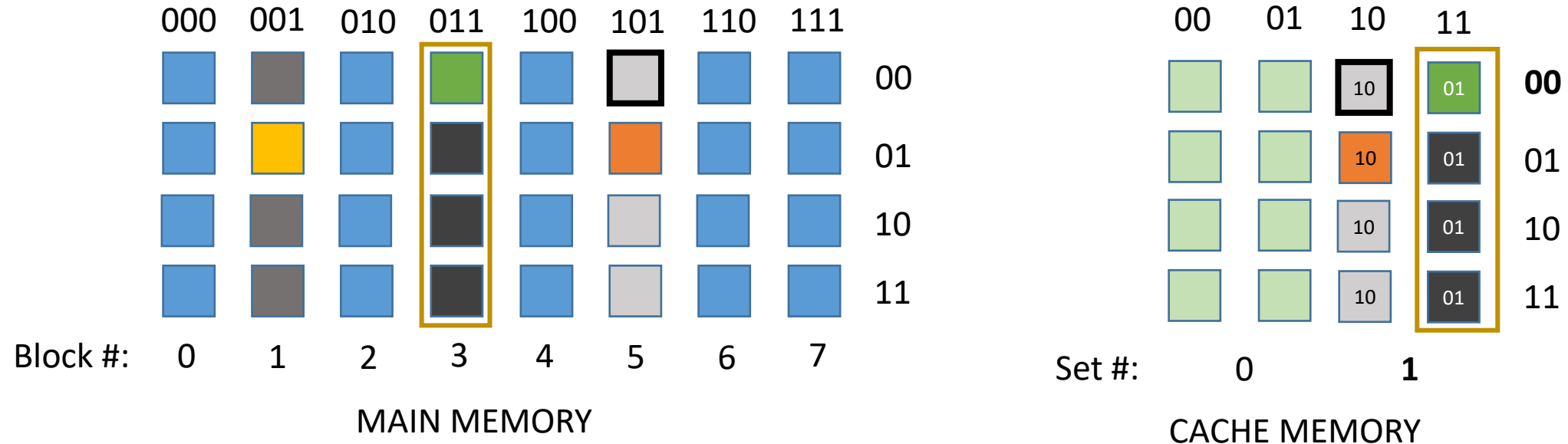
Cache Memory – Associative Set Mapping

- **01100**




Cache Memory – Associative Set Mapping

- Since we are using LRU, we will replace block 3 (since we just used block 2)
 - Had we been using FIFO, then block 2 would have been replaced because that has been in the cache longer than the data in block 3



Cache Memory – Associative Set Mapping

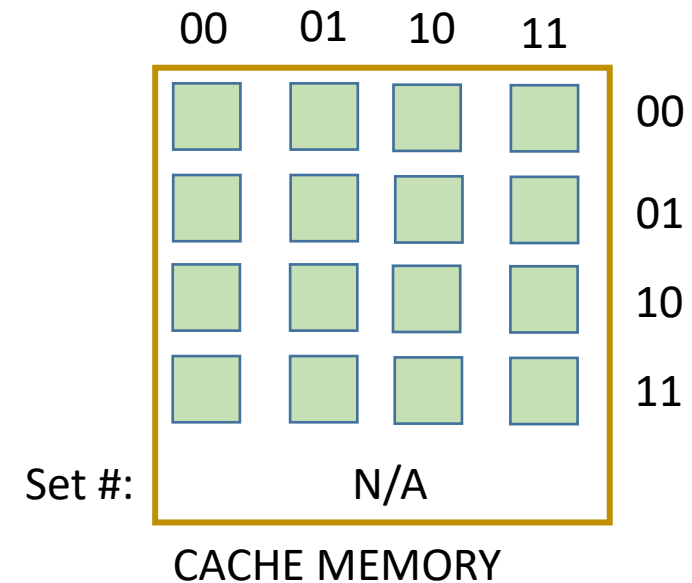
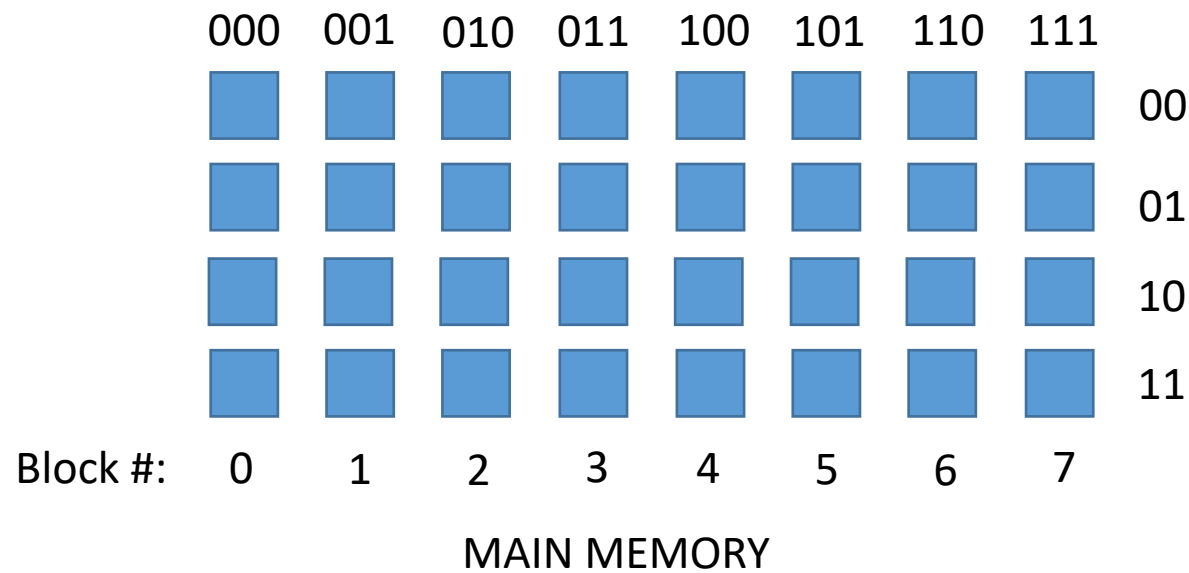
- Each row in the table below represents the read/write operations illustrated on the previous (associative mapping) slides



Memory Address	Hit or Miss	Cache Contents							
		Set 0 (Block 0)	Set 0 (Block 1)	Set 1 (Block 2)		Set 1 (Block 3)			
10101	M			10	10	10	10		
00101	M			10	10	10	10	00	00
10100	H			10	10	10	10	00	00
01100	M			10	10	10	10	01	01

Cache Memory – Fully Associative Set Mapping

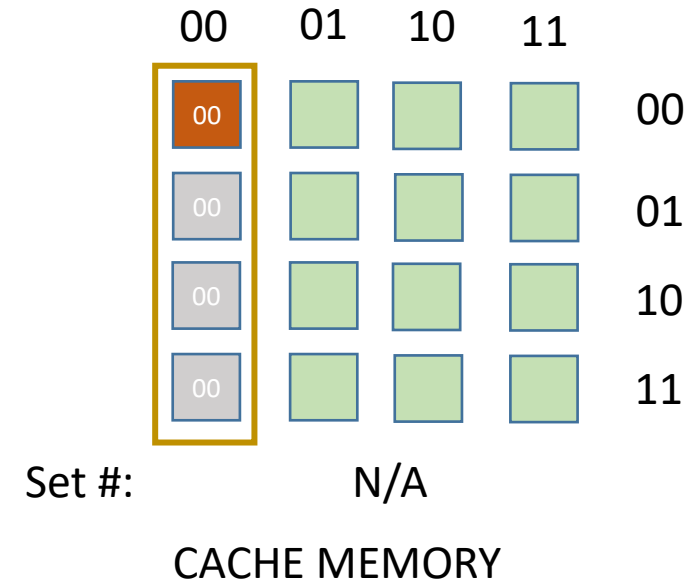
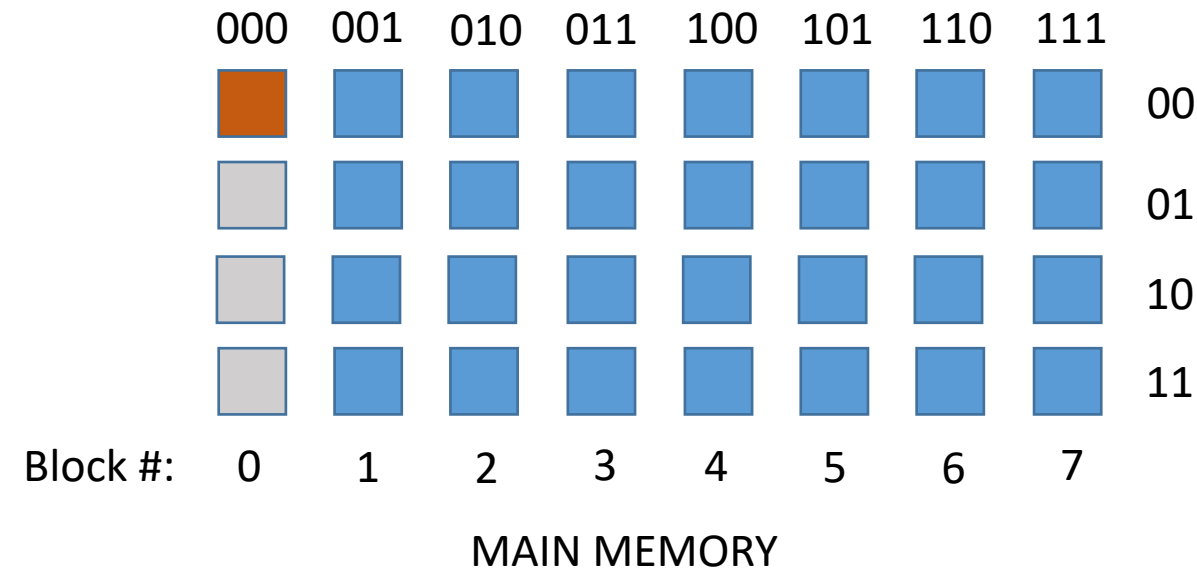
- A **fully associative mapped** cache consists of one set that contains every block of cache memory.



Cache Memory – Fully Associative Set Mapping

• **00000**

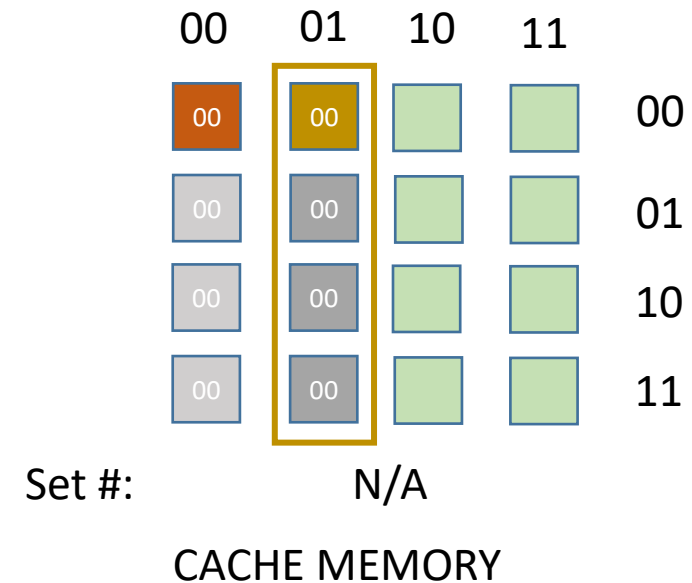
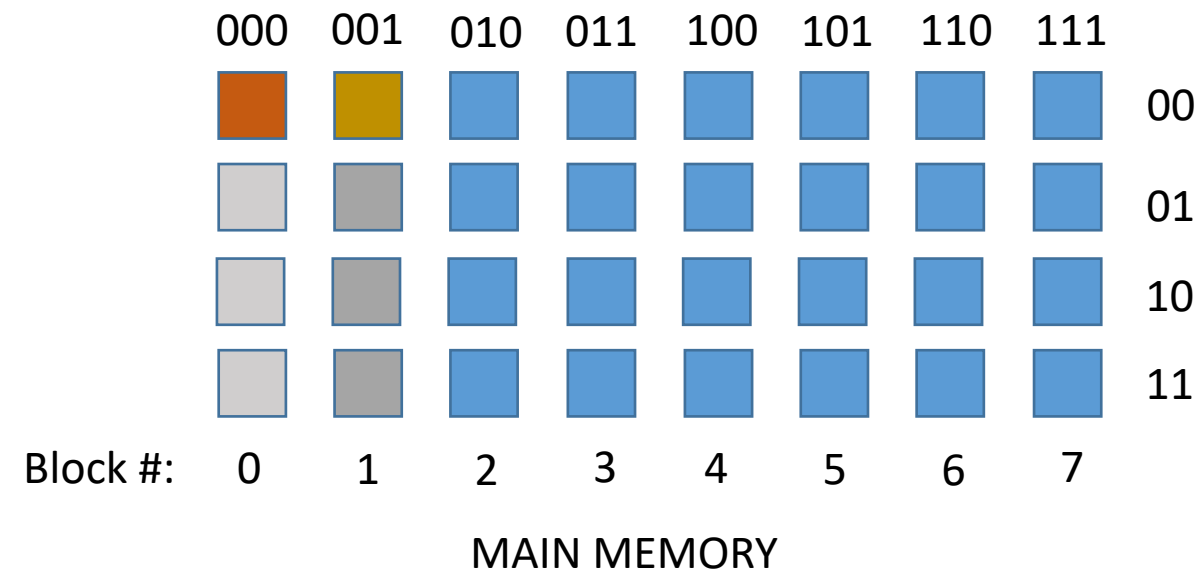
tag	Cache set	Cache byte
00	0	00
RAM block		



Cache Memory – Fully Associative Set Mapping

• **00100**

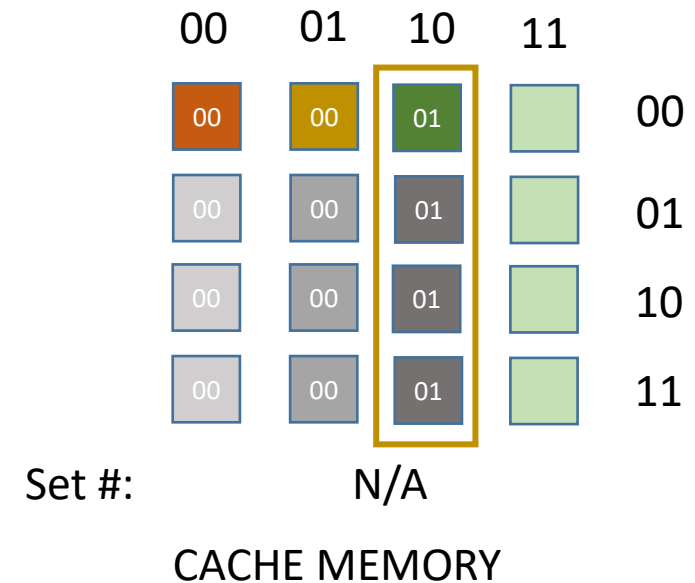
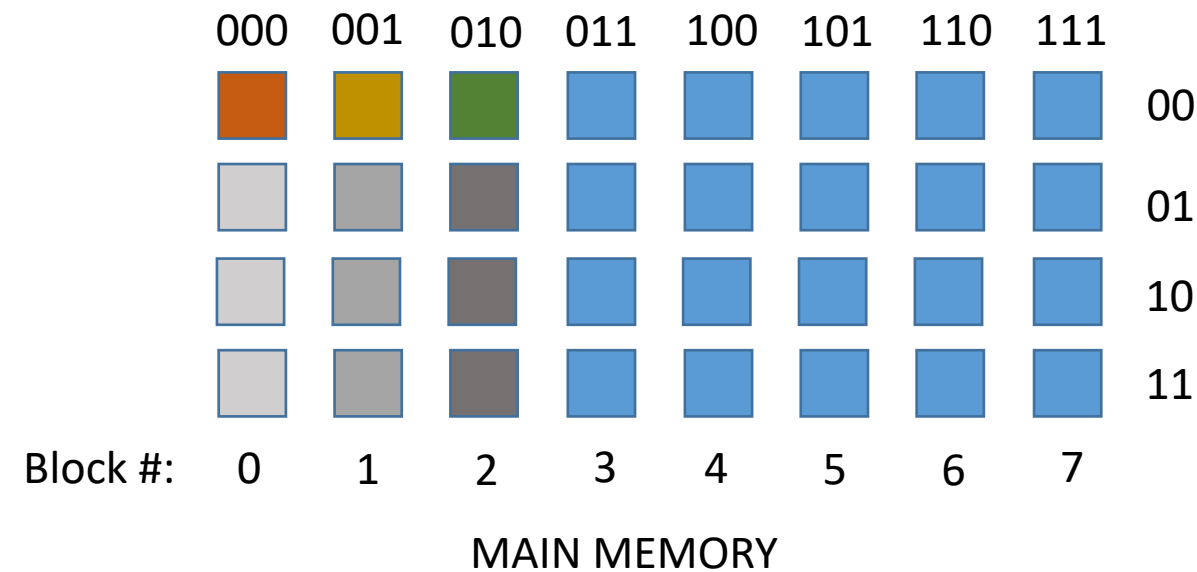
tag	Cache set	Cache byte
00	1	00
RAM block		



Cache Memory – Fully Associative Set Mapping

- **01000**

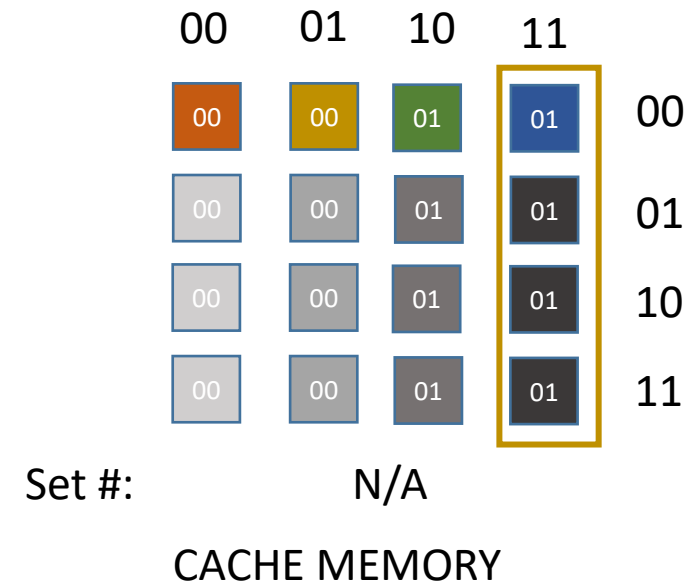
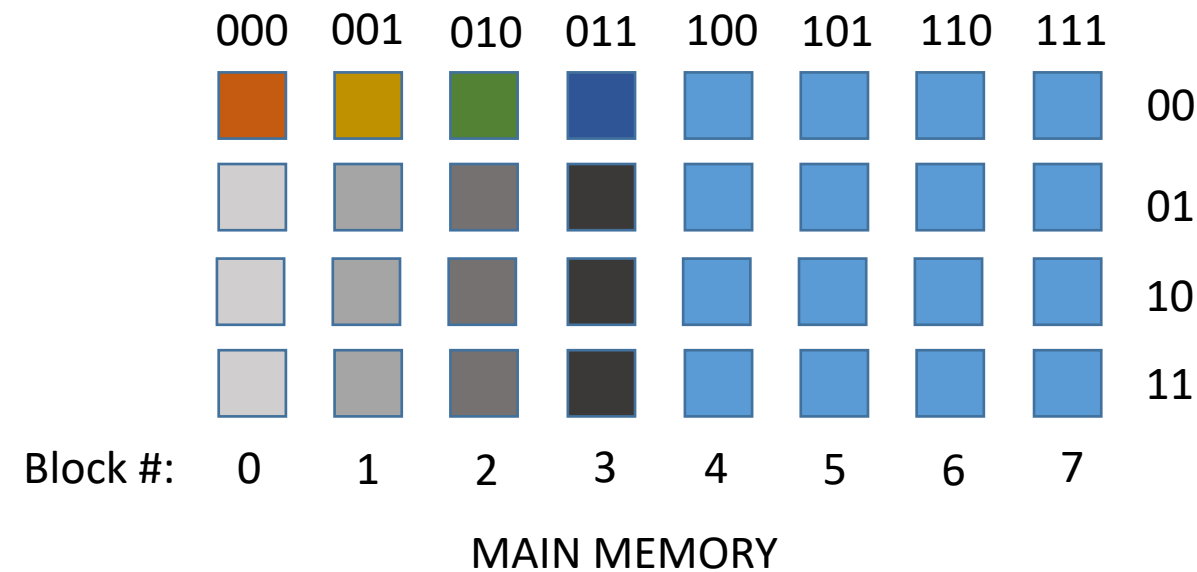
tag	Cache set	Cache byte
01	0	00
RAM block		



Cache Memory – Fully Associative Set Mapping

• **01100**

tag	Cache set	Cache byte
01	1	00
RAM block		

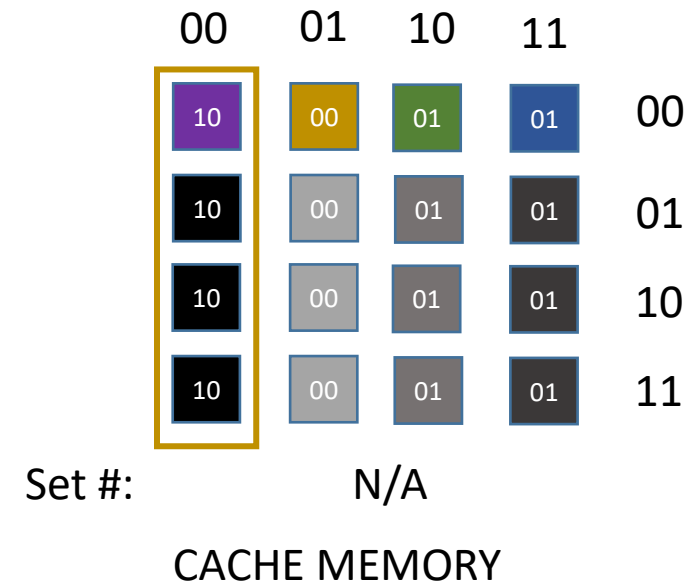
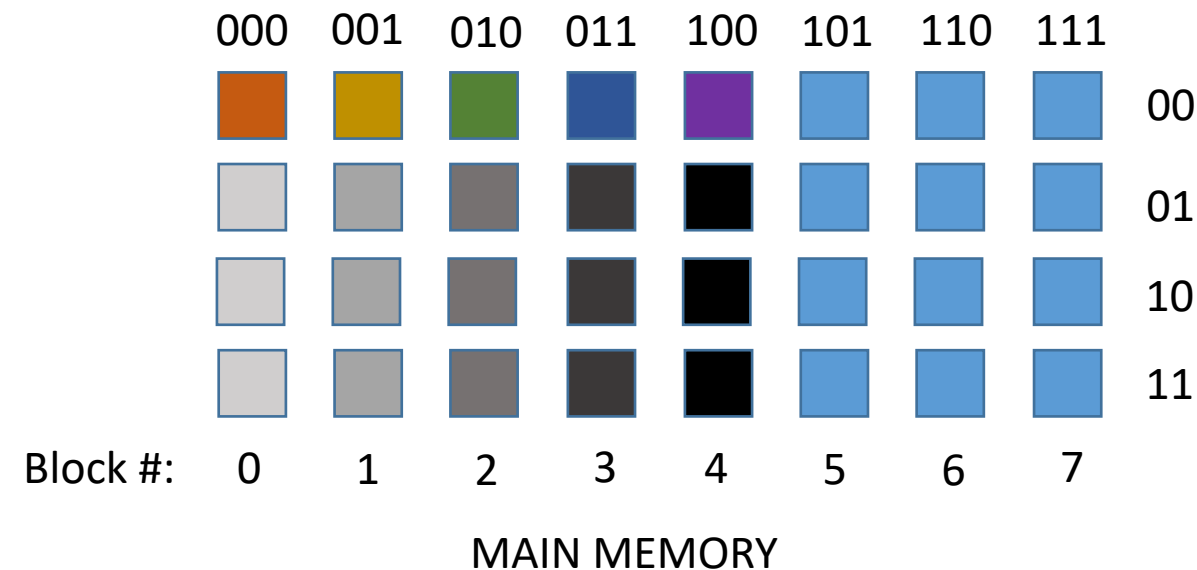


Cache Memory – Fully Associative Set Mapping

- **10000**


- Out of space
- The block to replace (LRU or FIFO in this case) is block 0

tag	Cache set	Cache byte
10	0	00
RAM block		



Cache Memory – Associative Set Mapping

- Each row in the table below represents the read/write operations illustrated on the previous (associative mapping) slides



Memory Address	Hit or Miss	Cache Contents															
		Block 0				Block 1				Block 2				Block 3			
00000	M	00	00	00	00												
00100	M	00	00	00	00	00	00	00	00								
01000	M	00	00	00	00	00	00	00	00	01	01	01	01				
01100	M	00	00	00	00	00	00	00	00	01	01	01	01	01	01	01	
10000	M	10	10	10	10	00	00	00	00	01	01	01	01	01	01	01	

Dependability

- A device (such as a hard drive) alternates between two states of service
- **Service Accomplishment**
 - The device is working (delivering service) as designed
- **Service Interruption**
 - The device is not working (not delivering service) as designed
- A **failure** causes the device to go from Service Accomplishment to Service Interruption
- A **restoration** causes the device to go from Service Interruption to Service Accomplishment

Dependability

- The measure of continuous, uninterrupted Service Accomplishment until the device experiences a failure its **reliability**
 - Two such measures of reliability are:
 - **Mean Time to Failure (MTTF)**
 - The average time it takes until a device fails
 - **Annual Failure Rate (AFR)**
 - The percentage of devices expected to fail in a year within a given MTTF

Dependability

- As an example, we'll say a server farm has 25,000 servers.
 - Each server has 3 hard disks (all the same model)
 - $3 \times 25000 = 75000 \text{ disks}$
 - The MTTF of these disks are said to be 1,000,000 hours

$$\text{One year} = 365 \text{ days} \times 24 \frac{\text{hours}}{\text{day}} = 8760 \text{ hours}$$

$$\text{AFR} = \frac{8760 \text{ hours}}{1000000 \text{ hours}} = .00876 = .876\%$$

- The AFR of each disk is 0.876%

$$75000 \text{ disks} \times .00876 = 657$$

- 657 disks are expected to fail annually
 - $\frac{657 \text{ disks}}{365 \text{ days}} = 1.8 \sim 2 \text{ disks per day}$

Dependability

- Service Interruption is measured by **Mean Time to Repair (MTTR)**
- The **Mean Time Between Failures (MTBF)** is the sum of the MTTF and MTTR
- Availability is the measure of Service Accomplishment (MTTF) with respect to the alternation between Service Accomplishment and Service Interruption (MTBF)

$$Availability = \frac{MTTF}{(MTTF + MTTR)}$$

Dependability

- Using the previous example, we'll say MTTR for each disk in the server farm is 2 hours.
 - Each server has 3 hard disks (all the same model)
 - $3 \times 25000 = 75000$ *disks*
 - The MTTF of these disks are said to be 1,000,000 hours
 - 657 disks are expected to fail annually (calculated on a previous slide)

$$Availability = \frac{1000000 \text{ hours}}{(1000000 \text{ hours} + (2 \text{ hours} \times 657 \text{ disks}))} \sim .9986 \sim 99.86\%$$

Dependability

- MTTF can be increased through improving the quality of components or designing them to continue operating in the event of failures
 - **Fault avoidance**
 - Preventing faults in the design of the device/components
 - **Fault tolerance**
 - Using redundancy to continue functioning in the event of faults
 - **Fault forecasting**
 - Predicting faults so that devices/components may be replaced before a fault occurs

Parity Bits

- **Parity bits** are used as a way of validating that binary data makes it from source to destination intact.
 - A type of *error detection code*
- Before a binary word is sent through the system's data path (we'll use 8-bit words for this example), the total number of 1's in the word are counted.
 - If there are an even number of 1's, a 0 is appended to the number
 - If there are an odd number of 1's, a 1 is appended to the number
- Either way, parity bits ensure an even number of 1's in the $n+1$ bit word.
- If the received binary word (plus the parity bit) contain an odd number of 1's, it indicates there was a problem with the integrity of the data, and the word should be re-sent.

Parity Bits

- 01101100

- Even number of 1's

- **0**01101100

Parity bit

Bit gets corrupted in transit

0011**1**1100

Destination

001111100

- Not an even number of 1's
- Data must have been corrupted.

- 01001100

- Odd number of 1's

- **1**01001100

Parity bit

Parity Bits

- Parity bits alone are only able to *detect* errors.
- It is possible that an error will not be detected:

- 01101100

- Even number of 1's

- 001101100

← Parity bit

Bits get corrupted in transit

001111101

Destination

001111101

- Even number of 1's
 - No error detected

Error Correction Code

- **Error Correction Code** (ECC or Hamming Code, named after its creator Richard Hamming) uses multiple parity bits that can detect and *correct* errors in a binary word.
- A simple algorithm is followed to insert parity bits into the binary data.

Error Correction Code

1. Number the bits starting at 1 from the left side:

1	2	3	4	5	6	7	8
0	1	1	1	1	1	0	1

2. Mark the positions that are powers of 2 (1, 2, 4, 8, 16, and so on) as the parity bits

1	2	3	4	5	6	7	8
0	1	1	1	1	1	0	1

Error Correction Code

3. All bit positions that are not powers of two will be used for the actual data:

1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8	9	10	11	12
0	1	1	1	1	1	0	1	→	_	_	0	_	1	1	1	_	1	1	0	1

Error Correction Code

4. The position of a parity bit determines which bits it checks:
 - Bit 1 checks positions 1, 3, 5, 7, 9, and so on
 - Bit 2 checks positions 2, 3, 6, 7, 10, 11, 14, 15, and so on
 - Bit 4 checks positions 4-7, 12-15, 20-23, and so on
 - Bit 8 checks positions 8-15, 24-31, 40-47, and so on
- Each bit of the actual data is checked by two or more parity bits

Error Correction Code

- Bit 1 checks positions 1, 3, 5, 7, 9, and so on:

1	2	3	4	5	6	7	8	9	10	11	12
_	_	0	_	1	1	1	_	1	1	0	1

- Odd number of 1's

1	2	3	4	5	6	7	8	9	10	11	12
1	_	0	_	1	1	1	_	1	1	0	1

Error Correction Code

- Bit 2 checks positions 2, 3, 6, 7, 10, 11, 14, 15, and so on :

1	2	3	4	5	6	7	8	9	10	11	12
1	_	0	_	1	1	1	_	1	1	0	1

- Odd number of 1's

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	_	1	1	1	_	1	1	0	1

Error Correction Code

- Bit 4 checks positions 4-7, 12-15, 20-23, and so on :

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	_	1	1	1	_	1	1	0	1

- Even number of 1's

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	0	1	1	1	_	1	1	0	1

Error Correction Code

- Bit 8 checks positions 8-15, 24-31, 40-47, and so on :

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	0	1	1	1	_	1	1	0	1

- Odd number of 1's

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	0	1	1	1	1	1	1	0	1

Error Correction Code

- We'll say these bits are received with an error:

1 1 0 0 1 1 1 1 1 1 0 1



1 1 0 0 0 1 1 1 1 1 0 1

Error Correction Code

- Bit 1 gets checked (positions 1, 3, 5, 7, 9, and so on):
 - Odd parity; **Error detected**

1 1 0 0 0 1 1 1 1 0 1

- Bit 2 gets checked (positions 2, 3, 6, 7, 10, 11, 14, 15, and so on):
 - Even parity; **No error detected**

1 1 0 0 0 1 1 1 1 0 1

Error Correction Code

- Bit 4 gets checked (positions 4-7, 12-15, 20-23, and so on):
 - Odd parity; **Error detected**

1 1 0 0 0 1 1 1 1 0 1

- Bit 8 gets checked (positions 8-15, 24-31, 40-47, and so on):
 - Even parity; **No error detected**

1 1 0 0 0 1 1 1 1 0 1

Error Correction Code

- **Bit 1; Error detected** 1 1 0 0 0 1 1 1 1 1 1
- **Bit 2; No error detected** 1 1 0 0 0 1 1 1 1 1 1
- **Bit 4; Error detected** 1 1 0 0 0 1 1 1 1 1 1
- **Bit 8; No error detected** 1 1 0 0 0 1 1 1 1 1 1

- **Bit 1 + Bit 4 = Bit 5**
 - **Bit 5 is wrong**

1 1 0 0 1 1 1 1 1 1 1