Module 5

Introduction to Turing Machine.

~~Problems that Computer Cannot Solve.~~

The Turing Machine, Problems

* We know that, the language Accepted by Finite Automata is called as a Regular Language,

$\boxed{FA \rightarrow RL}$

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The language Accepted by Push Down Automata is called as Context free Language

$\boxed{PDA \rightarrow CFL}$

The language Accepted by Turing Machine is called as Recursively Enumerable language.

$\boxed{TM \rightarrow REL}$

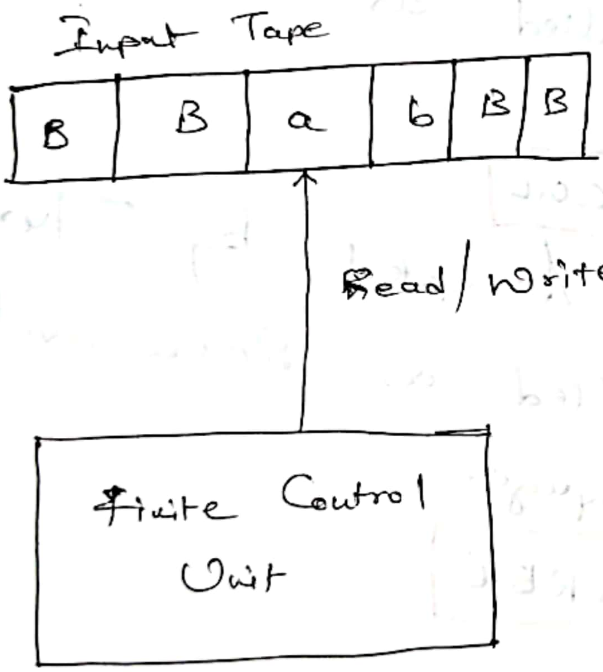
What is the need of TM?

* The problem with finite Automata, is that, finite Automata has a finite Memory.

* The problem with PDA is that, PDA can recognize only simple languages.

* Hence if we want to accept complex languages, then we need to go for Turing Machine.

* Below shows the Model of TM.



Model of Turing Machine.

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* Any Turing Machine Contains three Components viz
  - → Input tape
  - → Read/Write head
  - → Finite Control Unit

→ **Input Tape**

* Input Tape is divided into number of Cells, where each cell can store only one symbol at a time.
* The size of the input tape is infinite.

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→ **Read/Write head**

* By using Read/Write head we can perform either Read operation (or) Write Operation.
* At a time we can perform either Read operation / Write Operation.
* With the help of Read/Write head we can move either left-to-Right / Right-to-left.

→ **Finite Control Unit:-**

* It Contains all the state present in the Machine.

Formal Definition of Turing Machine

* A Turing Machine is defined with 7-tuples with

$A = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Where

$Q \rightarrow$ is the Non-Empty Finite set of States.

$\Sigma \rightarrow$ is the Non-Empty finite set of ~~Input~~ alphabets

$\Gamma \rightarrow$ is the ~~Stack~~ Input tape alphabet. $(\Sigma \subseteq \Gamma)$

$q_0 \rightarrow$ Initial State / Start state.

$F \rightarrow$ Final / Accept State.

$\delta \rightarrow$ is the transition Function

$$\boxed{Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}}$$

* Recursive Enumerable Language (REL) mainly Consists of 3 states viz, Accept & Halt, Reject & Halt, loop (or) Never halt

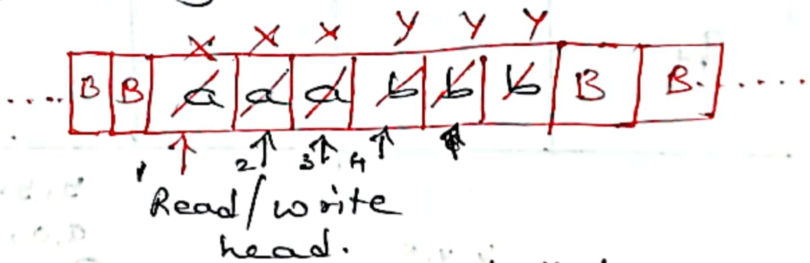
Problems on TM

(1) Design a Turing Machine for the Language $L = \{a^n b^n \mid n \geq 1\}$.

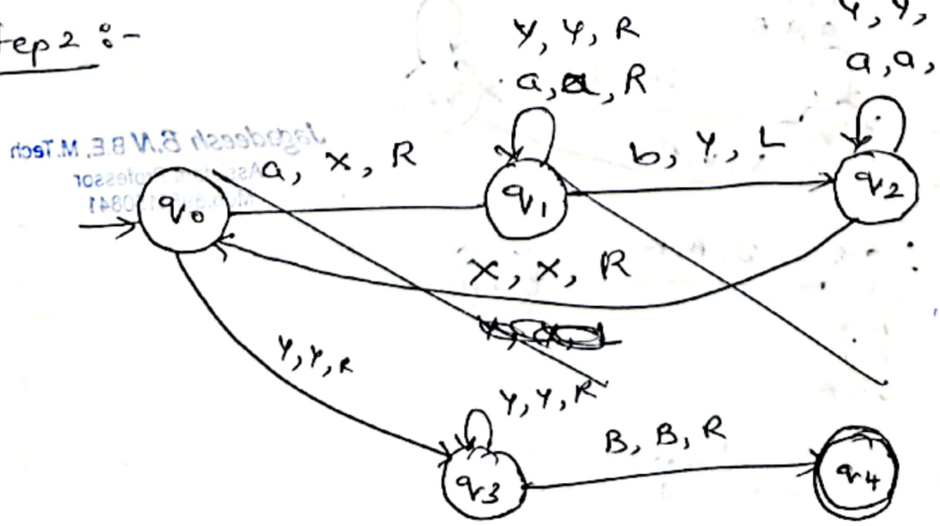
(Ans) Step 1: let $n = 2$, String = $\cancel{a}\cancel{a}\cancel{b}\cancel{b}$
X X Y Y
1 3 2 4

* Logic here is that if the i/p string a is read it will be replaced by X and it moves Right R until it finds b, if it finds b, replace it by Y and move towards left until it find X, then once again it moves left, and replace a by X and move towards Right, if it encounters Y, then move towards right, if it encounters b, then it is replaced by Y and so on.

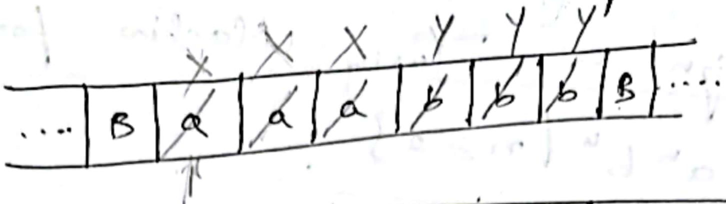
Let $n = 3$



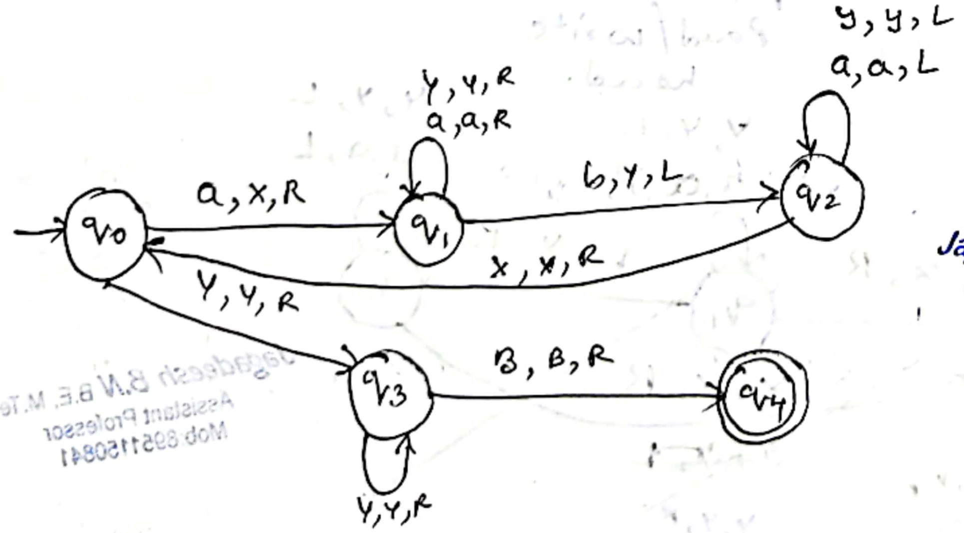
Step 2:-



Step2 :- Below shows the Turing Machine Table.



States	a	b	X	Y	B
q_0	(q_1, X, R)			(q_3, Y, R)	
q_1	(q_1, a, R)	(q_2, Y, L)		(q_1, Y, R)	
q_2	(q_2, a, L)		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, R)
q_4					



Transition diagram

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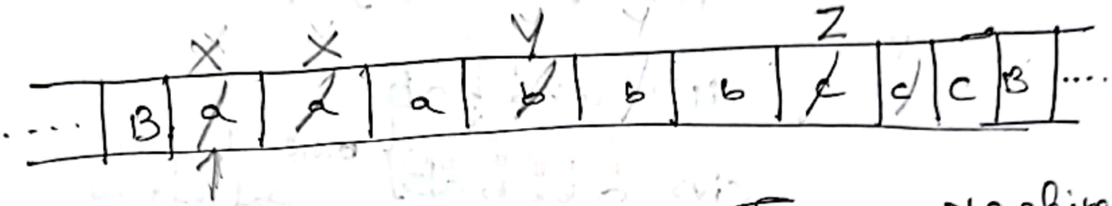
(2) Construct a Turing machine to accept $L = \{ a^n b^n c^n \mid n \geq 1 \}$

Step 1:- Consider a language

$L = \{ abc, aabbcc, aaabbbccc, \dots \}$



Step 2:- Construct the Turing machine table

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States	a	b	c	X	Y	Z	B
q_0	(q_1, X, R)				(q_4, Y, R)		
q_1	(q_1, a, R)	(q_2, Y, R)			(q_1, Y, R)		
q_2		(q_2, b, R)	(q_3, Z, L)			(q_2, Z, R)	
q_3	(q_3, a, L)	(q_3, b, L)		(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	
q_4					(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)
q_5							

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(3) Design a Turing Machine that accepts all palindromes over {a, b}

(Ans) ~~L = {abba~~

Step 1:-

<i> abbaB
<ii> abaabaB
Even length palindrome

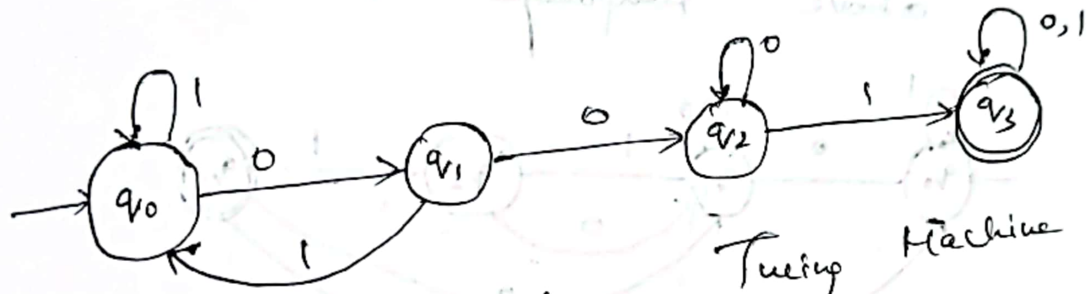
<iii> ababaB
<iv> abbbaB
odd length palindrome

Step 2:- Construct the Turing Machine table,

States	a	b	B
q_0	(q_1, B, R)	(q_4, B, R)	(q_6, B, R)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)
q_2	(q_3, B, L)		(q_6, B, R)
q_3	(q_3, a, L)	(q_3, b, L)	(q_0, B, R)
q_4	(q_4, a, R)	(q_4, b, R)	(q_5, B, L)
q_5		(q_3, B, L)	(q_6, B, R)
q_6			

(7) Obtain a TM to accept the language $L = \{w \mid w \in (0+1)^*\}$ Containing the substring 001

(Ans) **Step 1:-** Construction of Automata for the above language L.

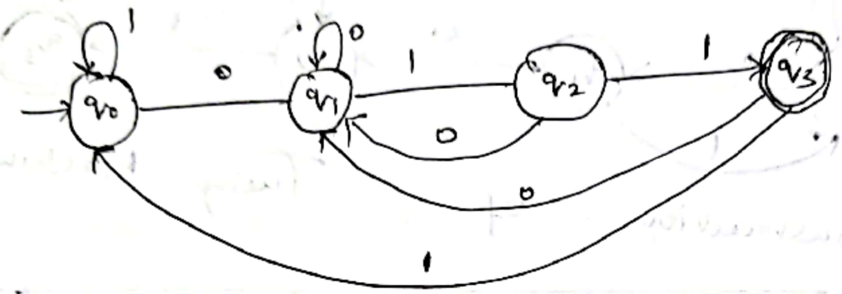


Step 2:- Construction of Turing Machine table,

δ(state)	0	1	B
q_0	$(q_1, 0, R)$	$(q_0, 1, R)$	
q_1	$(q_2, 0, R)$	$(q_0, 1, R)$	
q_2	$(q_2, 0, R)$	$(q_3, 1, R)$	
q_3	$(q_3, 0, R)$	$(q_3, 1, R)$	(q_4, B, R)
q_4			

(G) Obtain a Turing Machine to accept the language containing strings of 0's and 1's ending with 011

(Ans) Step 1:- Construction of Automata for the above language L i.e.



Step 2:- Construction of TM table has shown below,

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states	0	1	B
q_0	$(q_1, 0, R)$	$(q_0, 1, R)$	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_1, 0, R)$	$(q_3, 1, R)$	
q_3	$(q_1, 0, R)$	$(q_0, 1, R)$	(q_4, B, R)
q_4			

(6) Obtain a TM to accept the language $L = \{w \mid w$ is even and $\Sigma = \{a, b\}\}$.

(Ans) Step 1:- Construction of Automata for the above language L i.e.

Step 2:- Construction of TM table has shown below,

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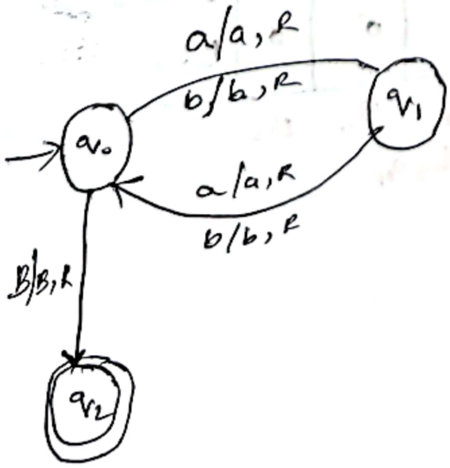
States	a	b	B
q_0	(q_1, a, R)	(q_1, a, R)	(q_2, B, R)
q_1	(q_0, a, R)	(q_0, a, R)	
q_2			



(7) Construct a Turing Machine to perform proper Subtraction.

$$f(m,n) = \begin{cases} m-n, & \text{if } m > n \\ 0, & \text{if } m \leq n \end{cases}$$

(A) **Case 1:** $m = 4 \quad n = 2$

....B	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	0	$\cancel{1}$	$\cancel{0}$	$\cancel{0}$	B	B....
	B_1	B_3	B_5	0	$0_6 \rightarrow q_F$	B_4	B_2		

Case 2: $m = 2, \quad n = 2$

B	$\cancel{0}$	$\cancel{0}$	$\cancel{1}$	$\cancel{0}$	$\cancel{0}$	B....
	B_1	B_3	$B_5 \rightarrow q_F$	B_4	B_2	

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Case 3: $m = 2, \quad n = 4$

B	$\cancel{0}$	$\cancel{0}$	$\cancel{1}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	B...
	B_1	B_3	B_5	B_7	$B_6 \rightarrow q_F$	B_4	B_2	

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$m = 4, \; n = 2$

States	0	1	B
q_0	(q_1, B, R)	(q_4, B, R)	
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(q_2, B, L)
q_2	(q_3, B, L)	$(q_f, 0, R)$ ~~$(q_2, 1, L)$~~	~~(q_0, B, R)~~
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	(q_4, B, R)		(q_F, B, R)
q_F			

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(8) Construct Turing Machine to perform Multiplication operation.

$$f(m,n) = m * n$$

(Ans) Step 1:- if $m=3$, $n=2$, then $m*n=6$, we can perform multiplication operation using repeated addition i.e

$$\begin{array}{r} 2 \\ 2 \\ 2 \\ \hline 6 \end{array} \quad (or) \quad \begin{array}{r} 3 \\ 3 \\ \hline 6 \end{array}$$

…	B	~~0~~	0	0	1	0 0	1	B	…

B ~~0~~ 0 1 0 0 1 0 0

B B ~~0~~ 1 0 0 1 0 0 0 0

B B B ~~1~~ 0 0 1 0 0 0 0 0 0

B B B B ~~0~~ 0 1 0 0 0 0 0 0

B B B B B ~~0~~ 1 0 0 0 0 0 0

B B B B B B ~~1~~ 0 0 0 0 0 0

B B B B B B B $\underbrace{0\ 0\ 0\ 0\ 0\ 0}_{\text{6 zeros.}}$

Step 2:- Below shows a Turing Machine/Transition table,

States	0	1	X	B
q_0	(q_1, B, R)	(q_6, B, R)		
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$		
q_2	(q_3, X, R)	$(q_5, 1, L)$		
q_3	$(q_3, 0, R)$	$(q_3, 1, R)$		$(q_4, 0, L)$
q_4	$(q_4, 0, L)$	$(q_4, 1, L)$	(q_2, X, R)	
q_5	$(q_5, 0, L)$	$(q_5, 1, L)$	$(q_5, 0, L)$	(q_0, B, R)
q_6	(q_6, B, R)	(q_7, B, R)		
q_7				

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Design a Turing Machine for the language $L = \{a^{2n} b^n \mid n \geq 1\}$

(Ans) Step1 :- Consider the String. i.e,

....B	B	$\cancel{a}$	$\cancel{a}$	$\cancel{a}$	$\cancel{a}$	$\cancel{a}$	$\cancel{a}$	$\cancel{b}$	$\cancel{b}$	$\cancel{b}$	B	B.....
		X	X	X	X	X	X	Y	Y	Y		

Step2 :- Construct the Turing Machine Table / Transition Table.

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States	a	b	X	Y	B
q_0	(q_1, X, R)			(q_4, Y, R)	
q_1	(q_2, X, R)				
q_2	(q_2, a, R)	(q_3, Y, L)		(q_2, Y, R)	
q_3	(q_3, a, L)	$(q_3,$	(q_0, X, R)	(q_3, Y, L)	
q_4				(q_4, Y, R)	(q_5, B, R)
q_5					