Module - 4

Push Down Automata:

Definition of Push Down Automata.

Definition

* A Push Down Automata (PDA) is a Seven Tuple such as

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

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Assistant Professor
Mob:8951150841

where

$Q \rightarrow$ is a Non-Empty Finite set of States.

$\Sigma \rightarrow$ is a Non-Empty finite set of input alphabets.

(Tape Symbol) $\Gamma \rightarrow$ is a Non-empty finite set of stack alphabets.

$\delta \rightarrow$ is a transition function from $Q \times (\Sigma \cup \varepsilon) \times \Gamma$ to finite sub set of $Q \times \Gamma^*$

$q_0 \in Q \rightarrow$ is the start state of machine

$Z_0 \in \Gamma \rightarrow$ is the initial symbol on the stack.

$F \subseteq Q \rightarrow$ is a set of Final states.

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* PDA is mainly used in order to overcome from the drawback of Finite Automata

* Finite Automata recognizes Regular languages, inorder to Recognize Context Free Languages (CFL) we make use of PDA.

F.A → RL
PDA → CFL.

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Assistant Professor
Mob:8951150841

* Problem with Finite Automata is that, F.A has Finite Amount of Memories and it can recognize only simple languages, Suppose if the language is Complex like $L=\{a^n b^n \mid n \geq 1\}$, F.A may not possible to use, because it requires More Memory to perform Comparisons and so on.

* Since Comparison Mechanisms in not their in F.A, we go for P.D.A to handle Complex languages.

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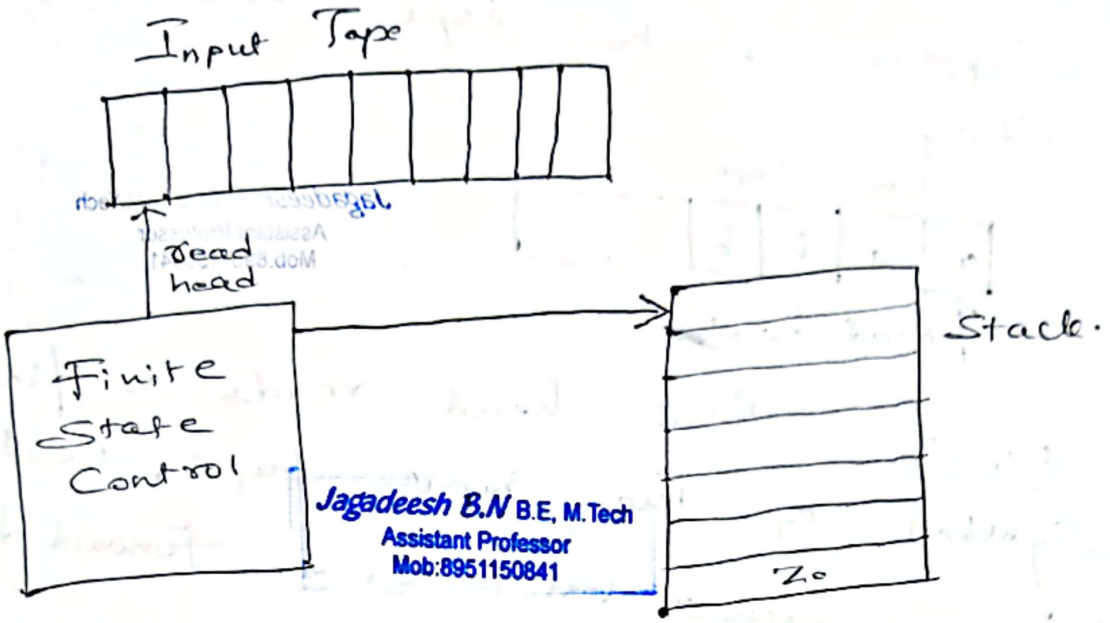
* Model / Block diagram of PDA as shown below,



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Assistant Professor
Mob:8951150841

Block Diagram of PDA

(i) Input Tape :-

* Input Tape is mainly used to store the input string.

* Here the input tape is divided into number of cells, where each cell can store one - symbol at a time.

(ii) Finite State Control :-

* FSC is also a Finite Automata, which contains Finite State Machines.

* Finite State Control maintains 2 heads. i.e

(a) Read head

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* Read head always points to the first symbol of the input tape.

* Suppose if the tape contains following string

Input Tape

a	a	b	b	

↑ Read head.

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Assistant Professor
Mob:8951150841

* Firstly Read head reads the first symbol of the input tape, Read head increments. (or) moves in forward direction.

* Therefore symbol-by-symbol data will be read from the input tape

(iii) Stack :-

* Normal Finite Automata contains only 2 components viz Finite State Control and Input tape, but here it contains another component called "Stack".

* Stack is a Datastructure, which is used for storing the elements.

* On stack we can perform 2 operations viz Push Operation and Pop Operation

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* Push Operation pushes the symbol onto the stack and Pop Operation removes the element from the stack.

* Push and Pop Operation will be performed on top of the stack.

* Second head always signifies the Top head.

* On the stack, initially top-most element of the stack contains Z_0.

* Stack produces either 2 states viz either Accept state (or) Reject state.

* Another advantage of using the stack is that the size of the stack is infinite/Unbounded i.e we can store any no of symbols onto the stack memory.

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Assistant Professor
Mob:8951150841

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The Languages of a PDA.

(1) Construct PDA for the language $L = \{ a^n b^n \mid n \geq 1 \}$.

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Assistant Professor
Mob:8951150841

(Ans)

Step1:- languages $L = \{ \overset{n=1}{ab}, \overset{n=2}{aabb}, \overset{n=3}{aaabbb} \ldots \}$ procedures are has follows viz,

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Assistant Professor
Mob 8951150841

(i) Initially push all the a's on to the stack.

(ii) Whenever b occurs change the state, and pop a from the stack.

(iii) Repeat step (ii) until the stack is empty.

Step2:- Below shows the transition function of PDA,

String:- | a | a | a | b | b | b | ∈ |

Transition Function

(i) $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

(ii) $\delta(q_0, a, a) = (q_0, aa)$

(iii) $\delta(q_0, b, a) = (q_1, \varepsilon)$ } Epsilon specifies pop

(iv) $\delta(q_1, b, a) = (q_1, \varepsilon)$ } Epsilon specifies pop

(v) $\delta(q_1, \varepsilon, Z_0) = (q_2, \varepsilon)$ } Epsilon specifies pop, here we have popped Z_0

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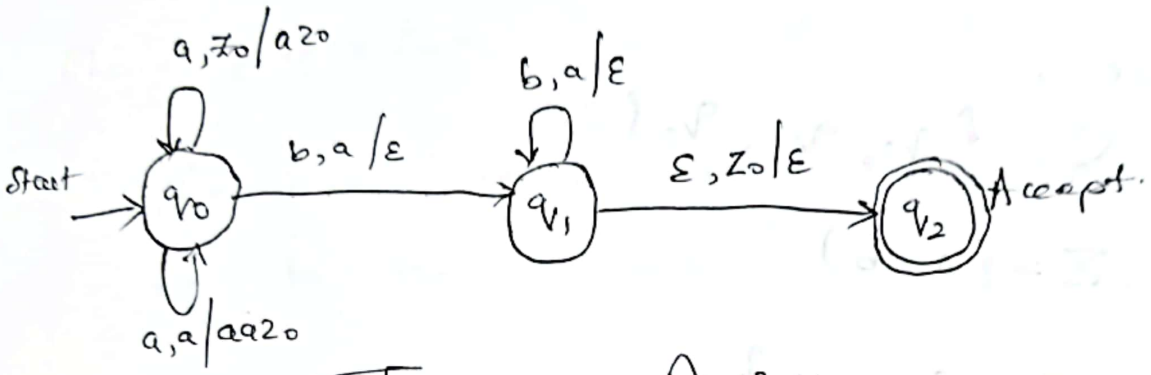
Step 3:- Next we shall write the transition diagram for the above transition functions,



Transition Diagram

Step 4:- Let us assume a string "aabb", whether it is correctly processed/NOT.

String :- aabbϵ

$\delta(q_0, aabb, Z_0) \vdash$ ~~(q_0, aZ_0)~~ (q_0, abb, aZ_0)

(fallen T) $\vdash$ (q_0, bb, aaZ_0)

$\vdash$ (q_1, b, aZ_0)

$\vdash$ (q_1, ε, Z_0)

$\vdash$ (q_2, ε)

q_2 → Final state; ε → Stack is Empty.

Hence the input string is completely accepted by this PDA.

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Assistant Professor
Mob:8951150841

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Steps:- Let us write / Substitute the 7-tuples of PDA has shown below,

$$A = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$$

Where,

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, Z_0\}$

δ = Transition Function.

$q_0 = \{q_0\}$ is the Start / initial State

$Z_0 = \{Z_0\}$ is the top-most symbol on the stack

$F = \{q_2\}$ is the Final State / Accept State.

Note:- * ~~Instantaneous~~ Instaneous Description, is used to describe the Configuration of PDA.

* ID is defined by triples (Q, w, γ) gamma

Where, $Q \rightarrow$ Represents Current State

$w \rightarrow$ Represents string,

$\gamma \rightarrow$ gamma represents Contents of a stack.

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2) Construct PDA for the language $L = \{a^n b^{2n}\}$ $n \geq 1\}$.

(Ans) Step1:- $L = \{\overset{n=1}{abb}, \overset{n=2}{aabbbb}, \overset{n=3}{aaabbbbbb} \ldots\}$

The procedures involved here are

<i> If we read a, push 2 a's on to the stack.

<ii> If we read b, pop the stack, and switch to new state

<iii> for the 1st pop operation change the state.

<iv> Repeat step <ii> until the stack is empty.

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Assistant Professor
Mob:8951150841

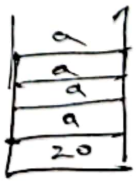
Step2:-

String :- | a | a | b | b | b | b | ∈ |

<i> $\delta(q_0, a, Z_0) = (q_0, aaZ_0) \rightarrow$

~~<ii> $\delta(q_0, a, a) = (q_0, aaZ_0)$~~

~~<iii> $\delta(q_0, b, a$~~

<ii> $\delta(q_0, a, a) = (q_0, aaa) \rightarrow$ 

<iii> $\delta(q_0, b, a) = (q_1, \epsilon) \rightarrow$

<iv> $\delta(q_1, b, a) = (q_1, \epsilon) \rightarrow$

<v> $\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon) \rightarrow$ Pop Z_0

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Step3:- Below shows the transition diagram for the above transition function.

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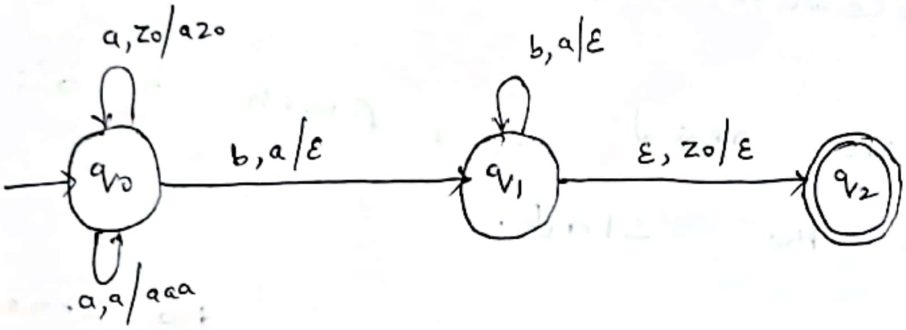
Assistant Professor
Mob:8951150841



Transition diagram

Step4:-

Accept string:-

$f(q_0, aabbbb, Z_0) \vdash (q_0, abbbb, aaZ_0)$

$\vdash (q_0, bbbb, aaaaZ_0)$

$\vdash (q_1, bbb, aaaZ_0)$

$\vdash (q_1, bb, aaZ_0)$

$\vdash (q_1, b, aZ_0)$

$\vdash (q_1, \varepsilon, Z_0)$

$\vdash (q_2, \varepsilon)$

Final state — Stack empty.

Hence the input string is accepted by PDA.

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Reject String :-

$\delta(q_0, aabbbbb, z_0) \vdash (q_0, abbbbb, aaz_0)$

$\vdash (q_0, bbbbb, aaaaz_0)$

$\vdash (q_1, bbbb, aaaz_0)$

$\vdash (q_1, bbb, aaz_0)$

$\vdash (q_1, bb, az_0)$

$\vdash (q_1, b, z_0)$

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Assistant Professor
Mob:8951150841

Step 5 :- PDA Components viz

$A = (Q, \Sigma, \delta, q_0, F, \Gamma, Z_0)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

$\delta =$ Transition function of step (i)

$\Gamma = \{\epsilon, a, z_0\}$

$Z_0 = \{z_0\}$

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(3) Construct PDA for the language $L = \{wcw^R \mid w \in (a,b)^*\}$

(Ans) Step 1:- Let $L = \{ \underset{w\ c\ w^R}{aba\,c\,aba}, \underset{w\ c\ w^R}{abab\,c\,baba} \ldots \}$

Center Marker

* Here we need to push all the symbols on to the stack we get c.

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

* Suppose if we encounter the symbol "c", then don't perform any operation i.e keep the stack as it is.

* After Reading the symbol c, next top of the stack symbol is a/b, then pop the elements from the stack

Step 2:- Below shows the transition function for the following string :- | a | b | a | c | a | b | a | ε |

(i) $\delta(q_0, a, Z_0) = (q_0, aZ_0) \rightarrow \boxed{a \mid Z_0}$

~~(ii) $\delta(q_0, b, aZ_0) = (q_0, baZ_0)$~~

(ii) $\delta(q_0, b, Z_0) = (q_0, bZ_0) \rightarrow \boxed{b \mid Z_0}$

4 scenarios

(iii) $\delta(q_0, a, a) = (q_0, aa)$

(iv) $\delta(q_0, a, b) = (q_0, ab)$

(v) $\delta(q_0, b, a) = (q_0, ba)$

(vi) $\delta(q_0, b, b) = (q_0, bb)$

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(vii) $\delta(q_0, c, a) = (q_1, a)$
(viii) $\delta(q_0, c, b) = (q_1, b)$ } Center Marker and change the state.

(ix) $\delta(q_1, a, a) = (q_1, \varepsilon)$

(x) $\delta(q_1, b, b) = (q_1, \varepsilon)$

(xi) $\delta(q_1, \varepsilon, z_0) = (q_2, \varepsilon)$ } stack empty by popping z_0.

Step 3:- Below shows the transition diagram as shown below,

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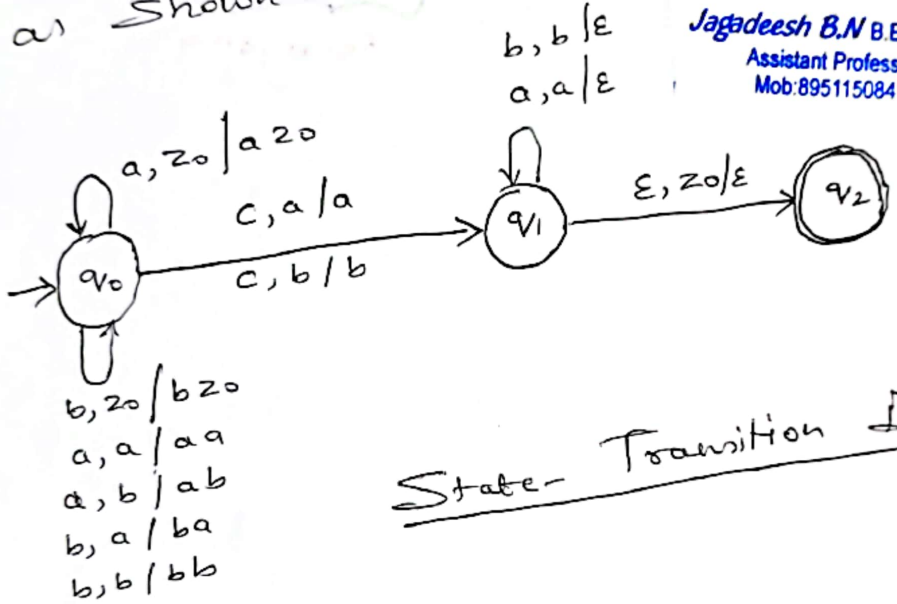
Assistant Professor
Mob:8951150841



State-Transition Diagram

Step 4:- Accept string

$\delta(q_0, abacaba, z_0)$ ~~$\vdash (q_0, abacaba$~~

$\vdash (q_0, bacaba, az_0)$

$\vdash (q_0, acaba, baz_0)$

$\vdash (q_0, caba, abaz_0)$

$\vdash (q_1, aba, abaz_0)$

$\vdash (q_1, ba, baz_0)$

$\vdash (q_1, a, az_0)$

$\vdash (q_1, \varepsilon, z_0)$

$\vdash (q_2, \varepsilon)$

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Steps:- Reject String

$\delta(q_0, abbcbbb) \vdash (q_0, bbcbbb, z_0)$

$\vdash (q_0, bcbbb, baz_0)$

$\vdash (q_0, cbbb, bbaz_0)$

$\vdash (q_1, bbb, bbaz_0)$

$\vdash (q_1, bb, baz_0)$

$\vdash (q_1, \underbrace{b, az_0}_{\text{Reject string.}})$

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Assistant Professor
Mob:8951150841

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(4) Design PDA for the language $L = \{ww^R \mid w \in (a+b)^*\}$

(Ans) Step1:- Let $L = \{abba, abaaba$

Step2:- Below shows the transition functions,

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Assistant Professor
Mob:8951150841

<i> $\delta(q_0, a, Z_0) = (q_0, aZ_0)$
<ii> $\delta(q_0, b, Z_0) = (q_0, bZ_0)$ } stack is empty push a/b

<iii> $\delta(q_0, a, b) = (q_0, ab)$
<iv> $\delta(q_0, b, a) = (q_0, ba)$ } Push either i/p a/b if stack top is a/b

<v> $\delta(q_0, a, a) = (q_0, aa)$
<vi> $\delta(q_0, b, b) = (q_0, bb)$ } Push, Remain in q_0

NPDA

<vii> $\delta(q_0, a, a) = (q_1, \varepsilon)$
<viii> $\delta(q_0, b, b) = (q_1, \varepsilon)$ } Pop, switch to q_1

~~<ix> $\delta(q_1, a, a) = (q_1, \varepsilon)$~~

<ix> $\delta(q_1, a, a) = (q_1, \varepsilon)$
<x> $\delta(q_1, b, b) = (q_1, \varepsilon)$ } Pop

<xi> $\delta(q_1, \varepsilon, Z_0) = (q_2, \varepsilon)$ } Pop Z_0

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Steps:- Below shows the transition diagram:

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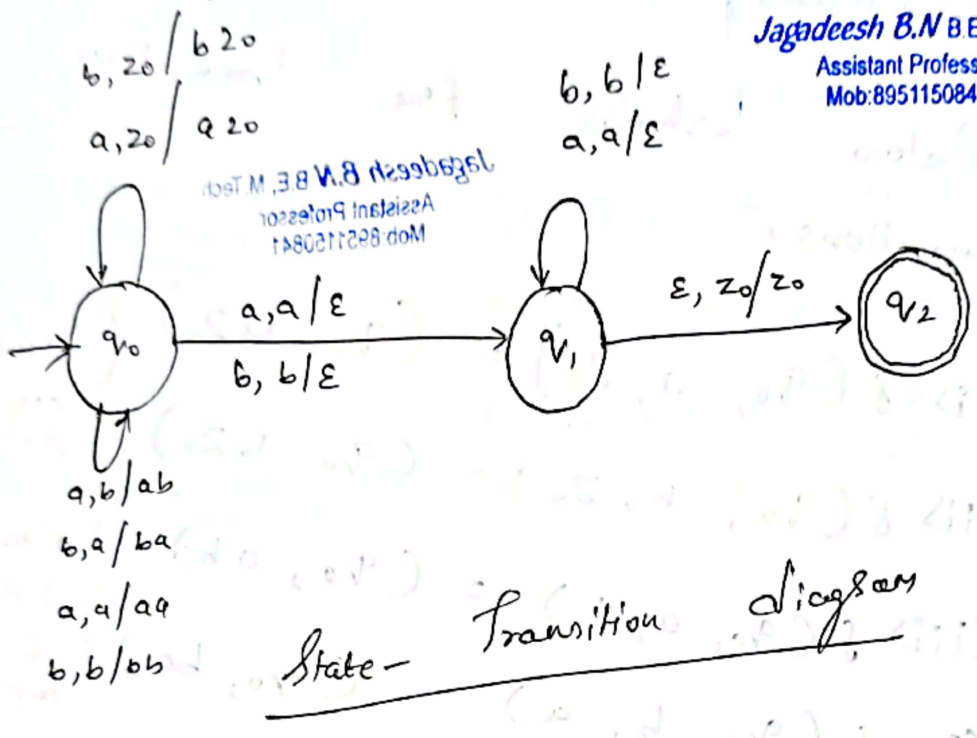
Assistant Professor
Mob:8951150841



State-Transition diagram

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(5) Design PDA for the language $L = \{ w / n_a(w) = n_b(w) \}$

(Ans) Step1:- Let us assume the language

$L = \{\varepsilon, abab, baba, bbaa, aabb \ldots\}$

Step2:- Let us write the transition functions for all the possibilities as shown below,

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Assistant Professor
Mob:8951150841

(i) $\delta(q_0, a, z_0) = (q_0, az_0)$

(ii) $\delta(q_0, b, z_0) = (q_0, bz_0)$

(iii) $\delta(q_0, a, a) = (q_0, aa)$

(iv) $\delta(q_0, b, b) = (q_0, bb)$

~~(v) $\delta(q_0, a, b) = (q_0, ab)$~~ (v) $\delta(q_0, a, b) = (q_0, ab)$

~~(vi) $\delta(q_0, b, a) = (q_0, ba)$~~ (vi) $\delta(q_0, b, a) = (q_0, ba)$

(vii) $\delta(q_0, a, b) = (q_1, \varepsilon)$

(viii) $\delta(q_0, b, a) = (q_1, \varepsilon)$

~~(ix) $\delta(q_1, a, b) = (q_1, \varepsilon)$~~

~~(x) $\delta(q_1, b, a) = (q_1, \varepsilon)$~~

(xi) $\delta(q_1, \varepsilon, z_0) = (q_2, \varepsilon)$

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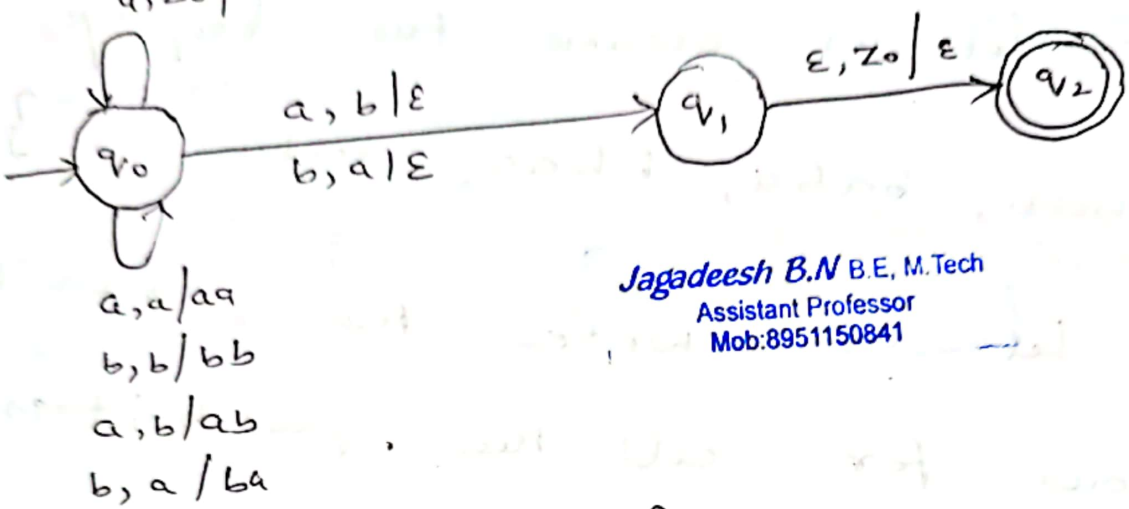
Step3:- Below shows the transition diagram.



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Assistant Professor
Mob:8951150841

Transition Diagram.

(6) Design PDA for the language Balanced Parenthesis

(Ans) Step1:- Usually there are 3 symbols available in the Parenthesis viz,

( ) → Left & Right Parenthesis.

{ } → Left & Right Curley Braces.

[ ] → Left & Right Square Braces.

Step2:- Below shows the transition function.

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Assistant Professor
Mob:8951150841

<i> $\delta(q_0, (, Z_0) = (q_0, (Z_0)$

<ii> $\delta(q_0, \{, Z_0) = (q_0, \{Z_0)$

<iii> $\delta(q_0, [, Z_0) = (q_0, [Z_0)$

<iv> $\delta(q_0, (, () = (q_0, ((Z_0)$

<v> $\delta(q_0, \{, \{) = (q_0, \{\{Z_0)$

<vi> $\delta(q_0, [, [) = (q_0, [[Z_0)$

<vii> $\delta(q_0, (, [) = (q_0, ([Z_0)$

<viii> $\delta(q_0, [, () = (q_0, [(Z_0)$

<ix> $\delta(q_0, \{, () = (q_0, \{(Z_0)$

<x> $\delta(q_0, (, \{) = (q_0, (\{Z_0)$

<xi> $\delta(q_0, [, \{) = (q_0, [\{Z_0)$

<xii> $\delta(q_0, \{, [) = (q_0, \{[Z_0)$

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(xiii) $\delta(q_0, \}, \{) = (q_1, \varepsilon)$

(xiv) $\delta(q_0,), ( ) = (q_1, \varepsilon)$

(xv) $\delta(q_0,], [) = (q_1, \varepsilon)$

(xvi) $\delta(q_1, \varepsilon, z_0) = (q_2, \varepsilon)$

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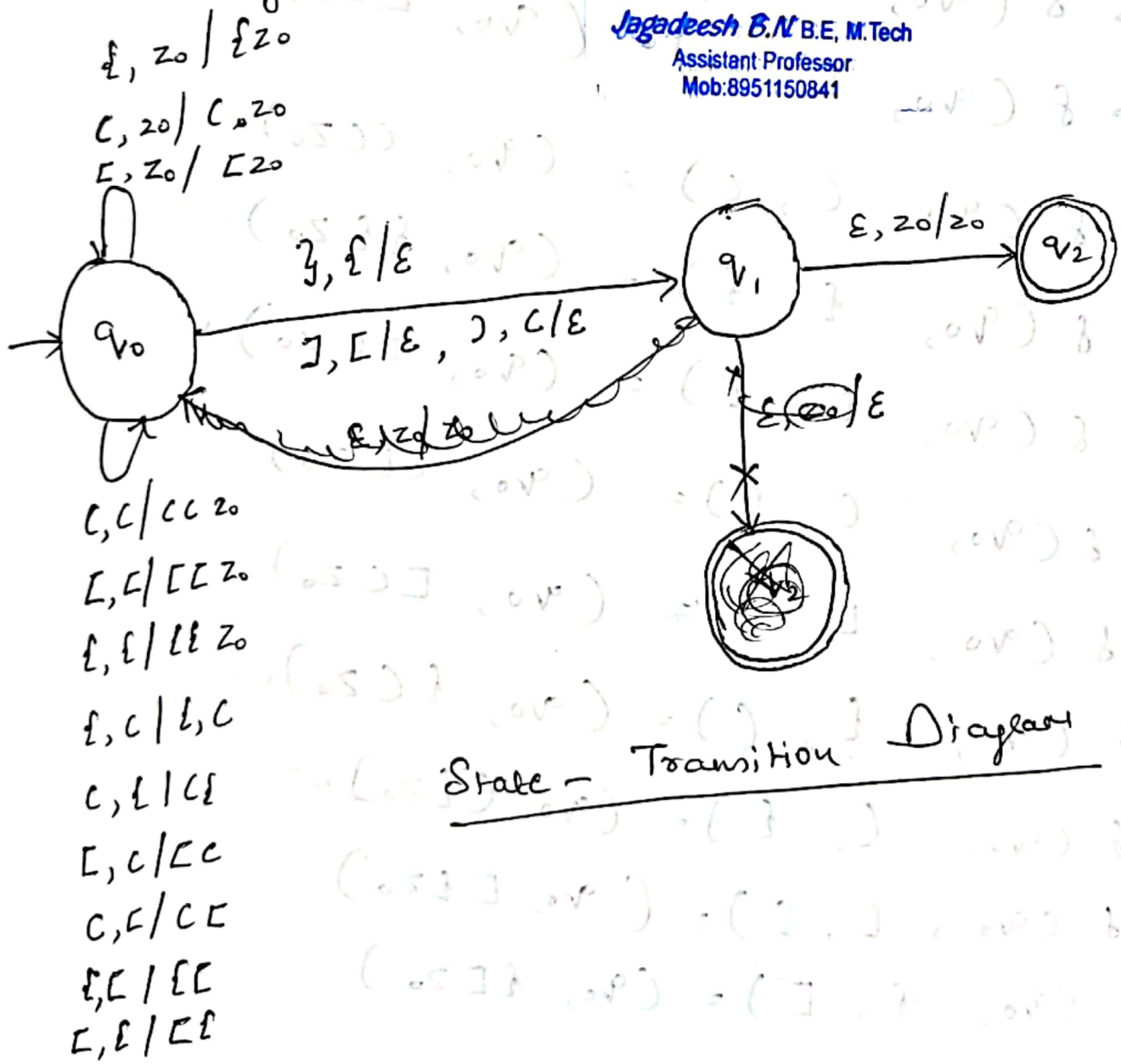
Assistant Professor
Mob:8951150841

Step 3:- Below shows the transition diagram.



State - Transition Diagram

(7) Design PDA for the language $L = \{ n_a(w) > n_b(w)$ where $w \in (a+b)^* \}$

(Ans) Step 1:- Let $L = \{ aab, baaba, ababaa \ldots \}$

Step 2:- * Here if the input symbol is a & stack topmost element is b

(or) if the input symbol is b & stack topmost element is a, perform pop operation.

* Secondly if the i/p symbol is a & stack topmost element is also a

(or) if the i/p symbol is b & stack topmost symbol is also b, then push the element on to the stack.

Step 3:- Below shows the transition function.

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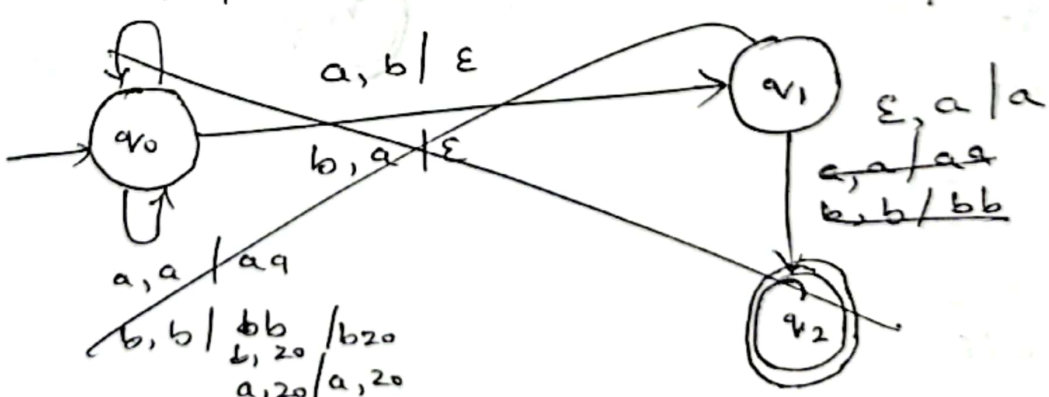
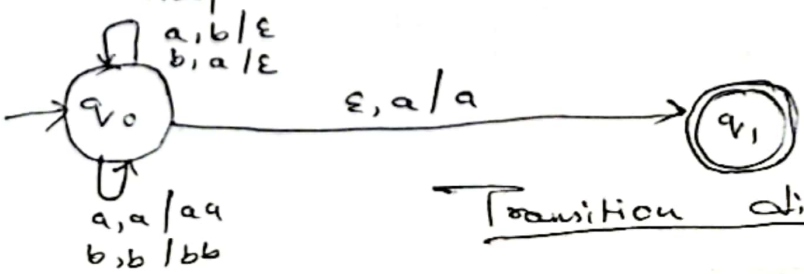
Assistant Professor
Mob:8951150841





Transition diagram

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(8) Design PDA for the language $L = \{ n_a(w) < n_b(w) \mid w \in (a+b)^* \}$

(Ans) Step 1:- Consider a language

$L = \{ abb, babba, \ldots \}$

Step 2:-

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, b, b) = (q_0, bb)$

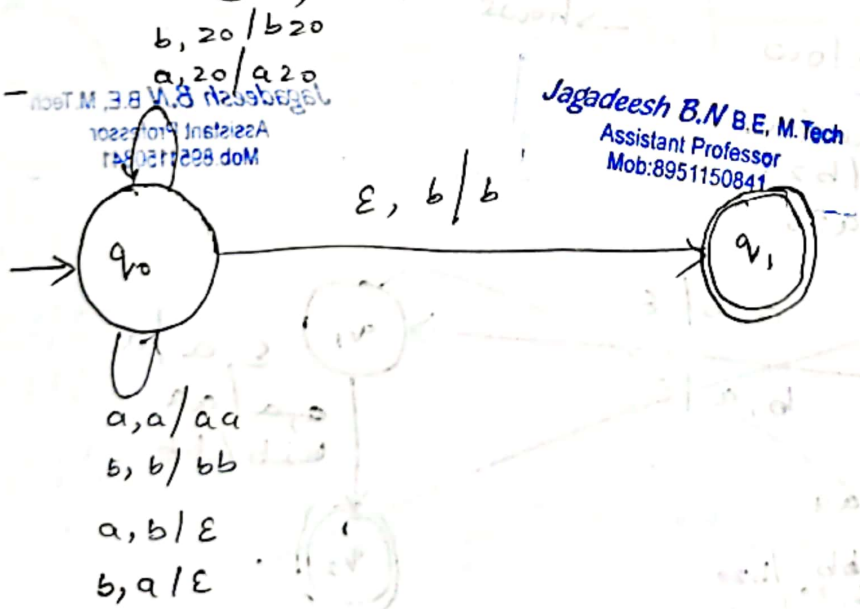
$\delta(q_0, a, b) = (q_0, \varepsilon)$

$\delta(q_0, b, a) = (q_0, \varepsilon)$

$\delta(q_0, \varepsilon, b) = (q_1, b)$

ab
aab

Step 3:-



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Assistant Professor
Mob:8951150841

Transition diagram

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Syntax Analysis Phase of Compilers : Part 2

Bottom-up Parsing

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

(i) Reductions
(ii) Handle Pruning
(iii) Shift-Reduce Parsing
(iv) Conflicts during Shift-Reduce Parsing.

What is Bottom-Up Parsing?

* Parser is also called as Syntax Analyzer.
* Parser accepts the tokens as input and checks for the correctness of the token.
* If the token syntax checked by the parser is correct, then parser generates Parse tree.
* Suppose if the token syntax checked by the parser is incorrect, then parser generates error message and reports it to the user.
* Parsing Techniques are of 2 types viz
  Top-Down Parsing technique.
  Bottom-Up Parsing technique.

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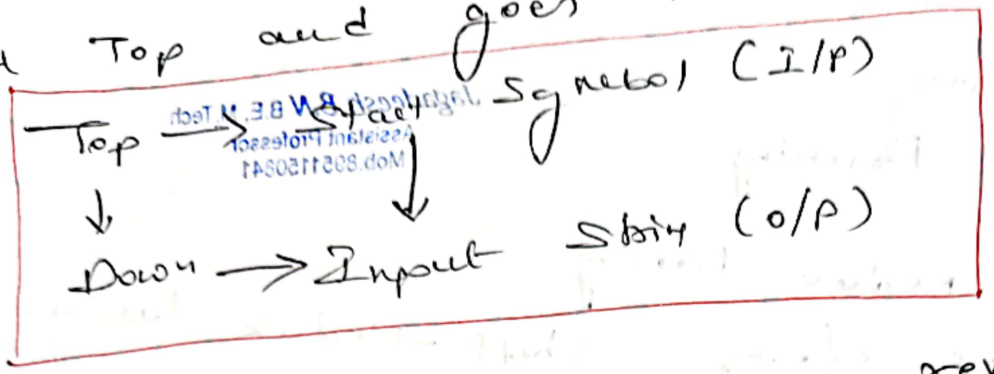
* In Top-Down parsing, the parser will construct parse tree begining from Top and goes towards down.



* In Bottom-up parsing is reverse to the Top-Down parsing, parsing will be generated from the Bottom (leaf node/ Input String) and goes towards start Symbol (Root node)

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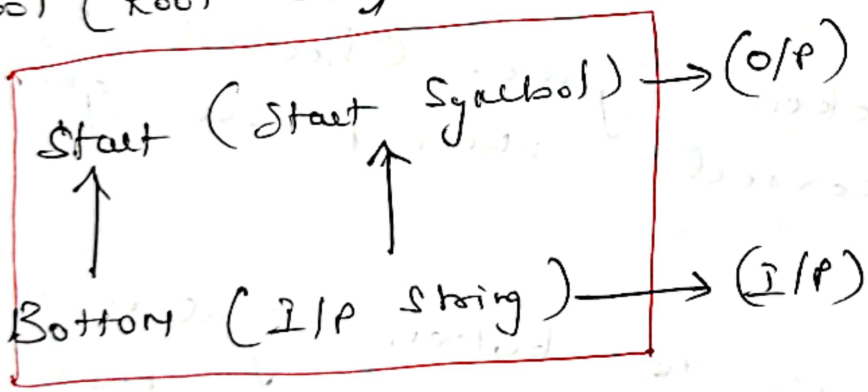
Assistant Professor
Mob:8951150841



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(i) Reductions

(1) $S \rightarrow aABe$

$A \rightarrow Abc \mid b$

$B \rightarrow d$

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Assistant Professor
Mob:8951150841

Input String :- "abbcde".

(Ans) Step1:- * Here we have to take a substring, and then check whether that substring matches to the production at the right side.

* If the substring matches, then substitute that substring with the corresponding left side of the production (Non-Terminal)

Step2:-

String :- abbcde → Input

(Called as Handle, and this process is called as Reduction)

abbcde (A→b)

aAbcde (A→Abc)

aAde (B→d)

aABe (S→aABe)

S → output

This process is said to be Bottom-Up Parsing

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Step 3:- In the above Step 2, the one inside the bracket is said to be a right Most derivation in reverse order.

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Assistant Professor
Mob:8951150841

S

$S \to aABe$

$S \to aAde$

$S \to aAbcde$

$S \to aabcde$

} Right-Most Derivation in Reverse Order (Bottom-to-top)

(2) $E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid id$

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Assistant Professor
Mob:8951150841

Input string :- id*id

(Ans) Step 1 :-

id*id (F→id) → Input

F*id (T→F)

~~T*id (E→T)~~

T*id (F→id)

T*F (T→T*F)

T (E→T)

E → Output

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(ii) Handle and Handle Pruning:-

* Handle means a substring which matches to the right side of the production, if it matches, then it is replaced with a corresponding left-side of the production.

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Assistant Professor
Mob:8951150841

Example:- $S \rightarrow aABe$

$A \rightarrow Abc \mid b$

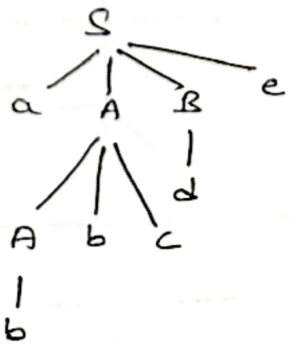
$B \rightarrow d$

String:- abbcde

Step 1:- (Ans)

Right Sentential Form	Handle	Reducing Production
a b b c d e	b	$A \rightarrow b$
a A b c d e	A b c	$A \rightarrow Abc$
a A d e	B d	$B \rightarrow d$
a A B e	a A B e	$S \rightarrow aABe$
S		

Step 2:- Bottom-up Parser generates Right most derivation in reverse order.



RMD Parse tree

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RMD

$S \to aABe$
$S \to aAde$
$S \to aAbcde$
$S \to abbcde$

Reverse of Reduction production in the previous table column generates RMD.

* Handle Pruning Means produces the Right Most Derivation in Reverse Order.

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Assistant Professor
Mob:8951150841

* "Handle" Means its a Substring which matches with the right side of the production.

Example :

$E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid id$

String : id*id

(Ans)

Right ~~Sential~~ Sentential Form	Handle	Reducing Production
id*id	id	$F \to id$
F*id	F	$T \to F$
T*id	id	$F \to id$
T*F	T*F	$T \to T*F$
T	T	$E \to T$
E		
Handle Pruning		

Obtaining Right Most Derivation in Reverse Order is said to be

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* The process of obtaining Right Most derivation in reverse order is called <u>Handle Pruning.</u>

* E

E -> T

E -> T * F

E -> T * id

E -> F * id

E -> id * id

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Assistant Professor
Mob:8951150841

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Continuation of PDA problems.

(1) Design a PDA for the language $L = \{ a^n b^n \mid n \geq 0 \}$

(Ans) Step 1:- The language $L = \{ \varepsilon, ab, aabb, aaabbb \ldots \}$

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Step 2:- Below shows the transition function for the PDA.

$$\delta(q_0, a, Z_0) = (q_1, aZ_0)$$

$$\delta(q_1, a, a) = (q_1, aa)$$

$$\delta(q_1, b, a) = (q_2, \varepsilon)$$

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Assistant Professor
Mob:8951150841

$$\delta(q_2, b, a) = (q_2, \varepsilon)$$

$$\delta(q_2, \varepsilon, Z_0) = (q_f, Z_0)$$

$$\delta(q_0, \varepsilon, Z_0) = (q_f, Z_0)$$

Step 3:- Below shows the Transition diagram for the above step 2.

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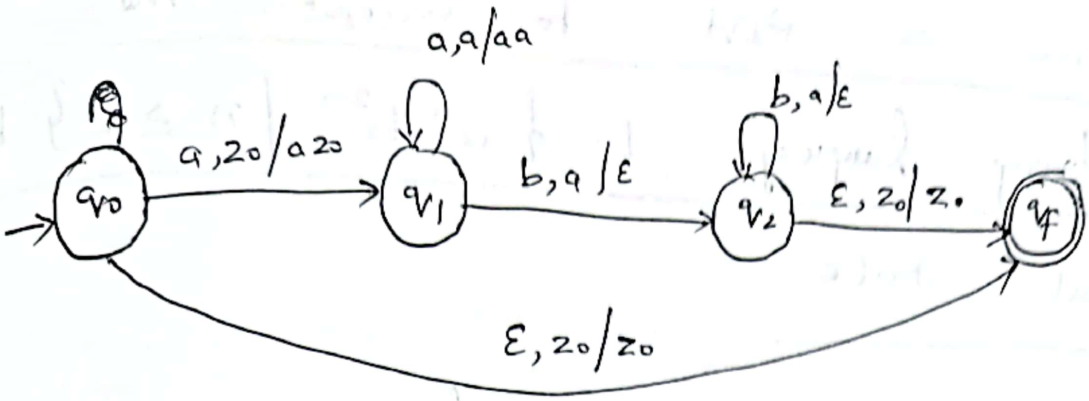

Transition diagram.

Step 4:- Components of PDA i.e

$A = (Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$

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Where,

$Q = \{q_0, q_1, q_2, q_f\}$

$\Sigma = \{a, b\}$

$\delta =$ Transition function of Step2.

$q_0 = \{q_0\}$ start state / Initial state

$F = \{q_f\}$ is an accepting state / final state.

$Z_0 = \{Z_0\}$ Initial symbol on to the stack.

$\Gamma = \{a, z_0\}$

Step 5:- Instantaneous Description (Q, w, γ)

$(\cancel{q_0, w}, \; (q_0, aaabbb, Z_0) \vdash (q_1, aabbb, aZ_0)$

$\vdash (q_1, abbb, aaZ_0)$

$\vdash (q_1, bbb, aaaZ_0)$

$\vdash (q_2, bb, aaZ_0)$

$\vdash (q_2, b, aZ_0)$

$\vdash (q_f, \varepsilon, Z_0)$

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(2) Give a PDA to accept the following language $L = \{ a^n b^{2n} \mid n \geq 1 \}$ by final state

(Ans) Step 1:- Let the language

$L = \{ abb, aabbbb, aaabbbbbb \ldots \}$

Step 2:- The transition function for the above PDA is written by,

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Assistant Professor
Mob:8951150841

$\delta(q_0, a, Z_0) = (q_1, aZ_0)$

$\delta(q_0, a, a) = (q_1, aa)$

$\delta(q_1, b, a) = (q_2, a)$

$\delta(q_2, b, a) = (q_3, \varepsilon)$

$\delta(q_3, b, a) = (q_2, a)$

$\delta(q_3, \varepsilon, Z_0) = (q_f, Z_0)$

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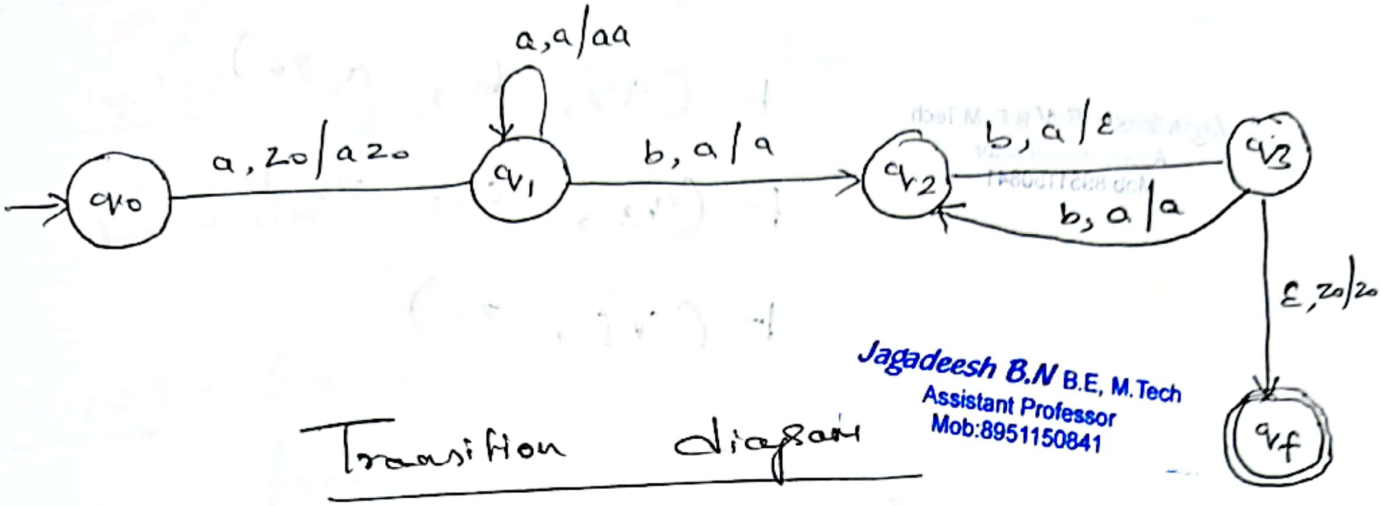
Step3:- Below shows the transition diagram



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Assistant Professor
Mob:8951150841

Transition diagram

Step4:- Components of PDA

$A = (Q, \Sigma, \delta, q_0, F, Z_0, I)$

Where.

$Q = \{q_0, q_1, q_2, q_3, q_f\}$

$\Sigma = \{a, b\}$

$\delta = \{$ Transition Function of Step2 $\}$

$q_0 = \{q_0\}$ Start / Initial State.

$F = \{q_f\}$ Final / Accept State.

$Z_0 = \{Z_0\}$ Initial symbol on to the stack.

$I = \{a, Z_0\}$

Step5:- Instantaneous Description

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$$\delta(q_0, abb, Z_0) \vdash (q_1, bb, aZ_0)$$
$$\vdash (q_2, bb, aZ_0)$$
$$\vdash (q_2, b, aZ_0)$$
$$\vdash (q_3, \varepsilon, Z_0)$$
$$\vdash (q_f, Z_0)$$

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Assistant Professor
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(3) Construct PDA for the language

$L = \{ a^n b^m c^n \mid n \geq 1, m \geq 1 \}$

(Ans) Step 1:- Consider the language

$L = \{ aabbbcc \ldots \}$

Step 2:- Below shows the transition function.

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$\delta(q_0, a, z_0) = (q_1, a z_0)$

$\delta(q_1, a, a) = (q_1, aa)$

$\delta(q_1, b, a) = (q_2, a)$

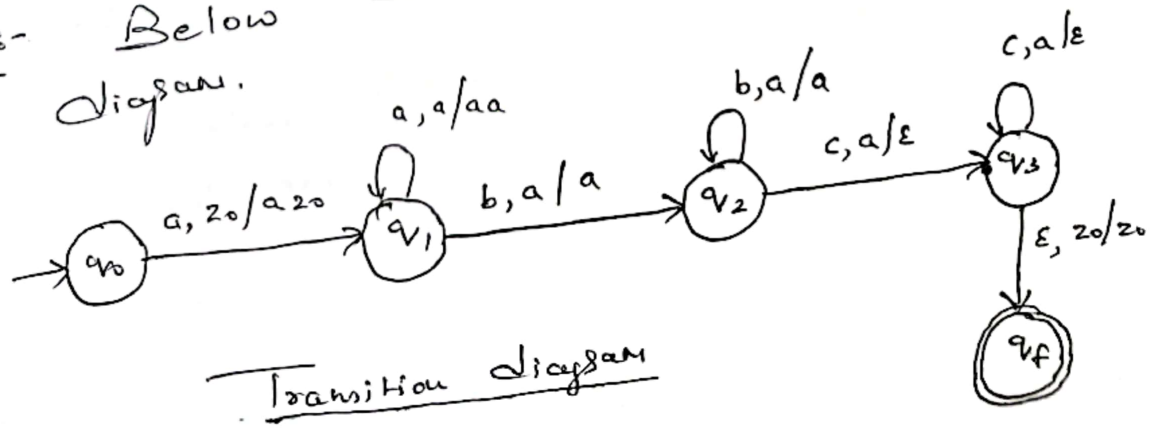
$\delta(q_2, b, a) = (q_2, a)$

$\delta(q_2, c, a) = (q_3, \varepsilon)$

$\delta(q_3, c, a) = (q_3, \varepsilon)$

$\delta(q_3, \varepsilon, z_0) = (q_f, z_0)$

Step 3:- Below shows the transition diagram.



Transition diagram

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Step4:- Instantaneous Description.

~~$(q_0, aabbbcc) \vdash (q_1, abbb$~~

$(q_0, aabbbcc, z_0) \vdash (q_1, abbbcc, az_0)$

$\vdash (q_1, bbbcc, aaz_0)$

$\vdash (q_2, bbcc, aaz_0)$

$\vdash (q_2, bcc, aaz_0)$

$\vdash (q_2, cc, aaz_0)$

$\vdash (q_3, c, az_0)$

$\vdash (q_3, \varepsilon, z_0)$

$\vdash (q_f, z_0)$

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Assistant Professor
Mob:8951150841

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(4) Obtain PDA to accept the language $L(M) = \{ wcw^R \mid w \in (a+b)^* \}$ where w^R is reverse of w by a final state.

(Ans) Step 1:- Let Language $L = \{ abacaba, baacaab \ldots \}$

Step 2:- Obtain the transition function for the following :-

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Assistant Professor
Mob:8951150841

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_f, \varepsilon)$

$\delta(q_1, b, b) = (q_f, \varepsilon)$

$\delta(q_f, \varepsilon, z_0) = (q_f, z_0)$

Step 3:- Below shows the transition diagram.

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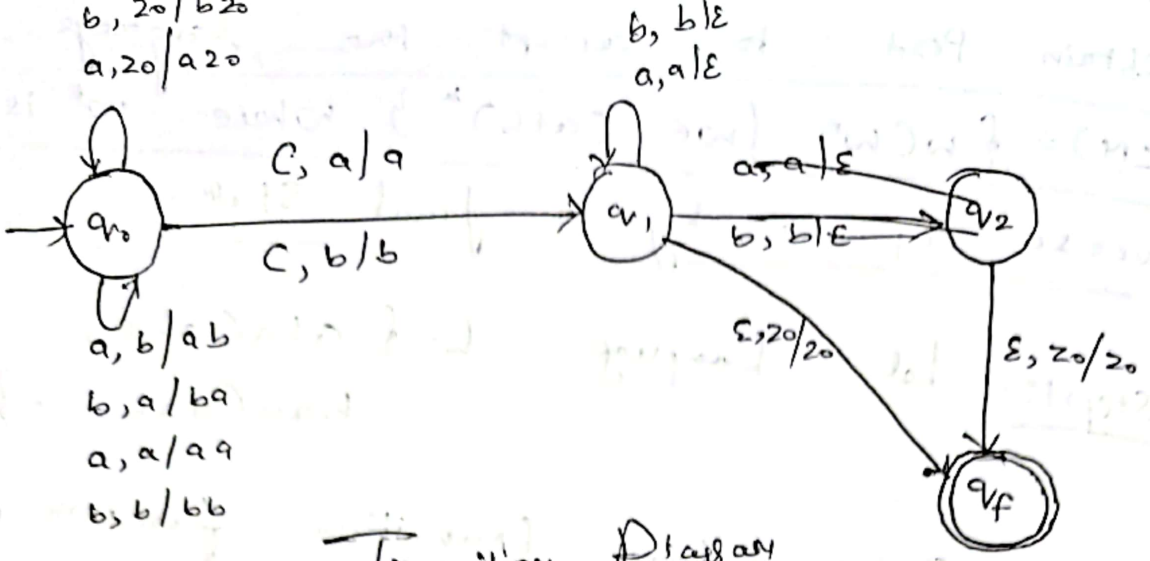

Tramition Diagram

Step1:- Instantaneous Description.

$\delta(q_0, aba(aba, Z_0) \vdash (q_0, ba(aba, aZ_0)$

$\vdash (q_0, a(aba, baZ_0)$

$\vdash (q_0, (aba, abaZ_0)$

$\vdash (q_1, aba, abaZ_0)$

$\vdash (q_1, ba, baZ_0)$

$\vdash (q_1, a, aZ_0)$

$\vdash (q_1, \varepsilon, Z_0)$

$\vdash (q_2, Z_0)$

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Assistant Professor
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(5) Obtain a PDA for the language $L = \{ w \mid w \in (a+b)^* \text{ and } n_a(w) = n_b(w) \}$ by a final state

(Ans) Step1:- Let the language $L = \{aabb, abab, baba, bbaa \ldots\}$

Step2:- Transition Function for the above language L is,

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$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

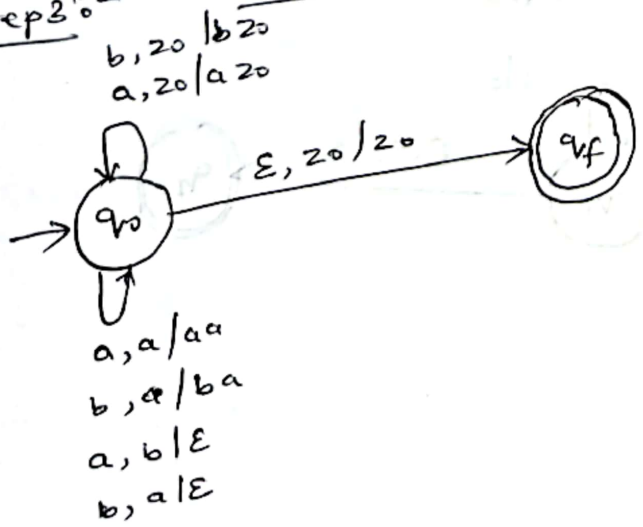
$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \varepsilon)$

$\delta(q_0, b, a) = (q_0, \varepsilon)$

$\delta(q_0, \varepsilon, z_0) = (q_f, z_0)$

Step3:- Transition diagram



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Another Solution

Obtain PDA for the language

$L = \{ a^n b^{2n} \mid n \geq 1 \}$

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(Ans) **Step 1:-** Consider a string for the language

$L = \{abb, aabbbb, aaabbbbbb \ldots\}$

Step 2:- Transition function

$\delta(q_0, a, z_0) = (q_0, aaz_0)$

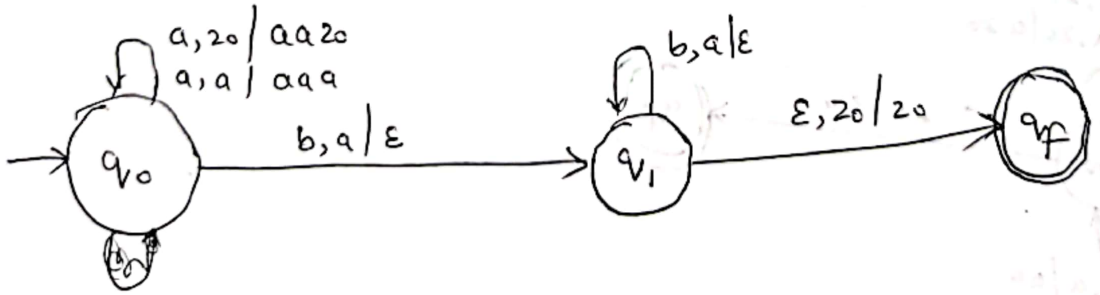
$\delta(q_0, a, a) = (q_0, aaa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z_0) = (q_f, z_0)$

Step 3:- Transition diagram



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(6) Obtain PDA for the language $L = \{ ww^R \mid w \in (a+b)^* \}$ by a final state

(Ans) Step1:- Let us consider a string for the language $L = \{ abaaba, baaaab, babaabab \}$

Step2:- Transition Function

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$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, b, z_0) = (q_0, b z_0)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, a) = (q_1, \varepsilon)$

$\delta(q_0, b, b) = (q_1, \varepsilon)$

$\delta(q_0, \varepsilon, z_0) = (q_f, z_0)$

$\delta(q_1, a, a) = (q_1, \varepsilon)$

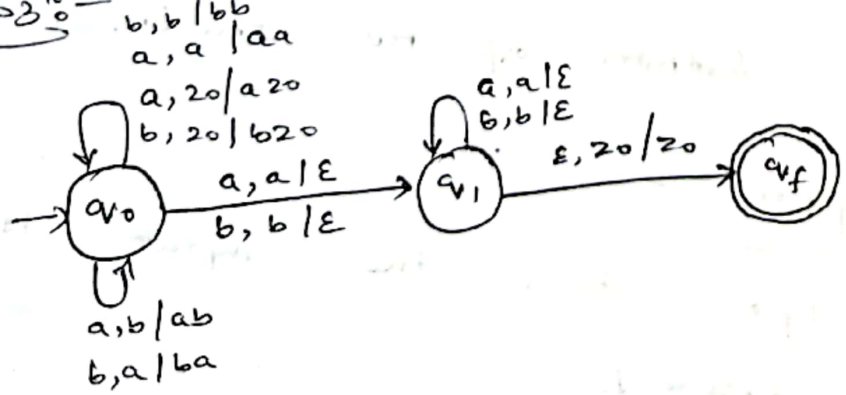
$\delta(q_1, b, b) = (q_1, \varepsilon)$

~~Step3:- Transition diagram~~

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

Step3:- Transition diagram



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Continuation of Bottom-Up Parsing

Shift Reduce Parsing

* Shift-Reduce Parser is a Bottom-Up parsing technique.
* Bottom-Up Parsing Means Constructs a parse tree beginning at the input string which is a leaf nodes and goes towards Up i.e the Root Node which is the start symbol of the grammar.
* In Shift Reduce parser we mainly use 2 Data-Structures viz
  (i) stack
  (ii) Input Buffer.

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Mob:8951150841

(i) Stack :-
* Stack stores Symbols of the grammar.
* Initially the stack contains $, which specifies the bottom of the stack.

(ii) Input Buffer :-
* Input Buffer stores the input string that to be parsed.
* End of the input buffer in a string is indicated by a $.

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* Here the Symbol $ specifies the bottom of the stack and end of the input string.

Actions of Shift-Reduce Parser

* There are 4 - Actions that can be performed on to the Shift-Reduce parser.

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(i) Shift
(ii) Reduce
(iii) Accept
(iv) Error

} 4 - Actions.

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(i) Shift :-

* Shift Means, it is a PUSH operations, which shifts the i/p symbol on to the top of the stack.
* Initially top of the stack contains $, on top of the $, the i/p symbol is pushed.

(ii) Reduce :-

* If top of the stack matches with the right side of the production, then it is reduced to the corresponding left ~~side~~ hand side Non-Terminal.

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<iii> Accept:-

* After Parsing the entire input string, the top of the stack contains $ and start symbol S, then the corresponding input string is accepted by the parser.

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<iv> Error:-

* If there is an syntax errors in the grammar (or) if the input string does not belong to the language of the grammar, then the parser generates the error messages.

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Example 1 :-

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<D> $E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

String is generated is id * id

(Ans) Step1:- Divide into 3 columns

STACK Operation	INPUT BUFFER	ACTION.
$	id*id $	Shift
$id	*id $	Reduce by F->id
$F	*id $	Reduce T->F
$T	*id $	Shift
$T*	id $	Shift
$T*id	$	Reduce F->id
$T*F	$	Reduce T->T*F
$T	$	Reduce E->T
$E	$	Accepted by the grammar.

Conflicts

Shift-Reduce

Reduce Reduce Conflicts

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Example 2:-

Consider the following grammar

$S \rightarrow (L) \mid a$

$L \rightarrow L, S \mid S$

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Parse the input string (a,(a,a)) using Shift Reduce Parser

(for) Step 1:- Divide into 3 columns.

Stack operation	Input Buffer	Actions
$	(a,(a,a))$	Shift
$(	a,(a,a))$	Shift a
$(a	,(a,a))$	Reduce S->a
$(S	,(a,a))$	Reduce L->S
$(L	,(a,a))$	Shift ,
$(L,	(a,a))$	Shift (
$(L,(	a,a))$	Shift a
$(L,(a	,a))$	Reduce S->a
$(L,(S	,a))$	Reduce L->S
$(L,(L	,a))$	Shift ,

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$ (L, (L,	a))$	shift a
$ (L, (L, a	))$	Reduce S→a
$ (L, (L, S	))$	Reduce L→L,S
$ (L, (L	))$	shift)
$ (L, (L)	)$	Reduce S→(L)
$ (L, S	)$	Reduce L→L,S
$ (L	)$	shift)
$ (L)	$	~~shift~~ Reduce S→(L)
$ S	$	Accepted by the parser.

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(3) Consider the grammar

$S \to S+S$

$S \to S-S$

$S \to (S)$

$S \to a$

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Input string : a - (a+a)

Stack operation	Input Buffer	Action
$	a - (a+a) $	Shift a
$ a	- (a+a) $	Reduce by S→a
$ S	- (a+a) $	Shift -
$ S -	(a+a) $	Shift (
$ S - (	a+a) $	Shift a
$ S - (a	+a) $	Reduce by S→a
$ S - (S	+a) $	Shift +
$ S - (S +	a) $	Shift a
$ S - (S + a	) $	Reduce by S→a
$ S - (S + S	) $	Reduce by S→S+S
$ S - (S	) $	Shift)
$ S - (S)	$	Reduce by S→(S)
$ S - S	$	Reduce S→S-S
$ S	$	Accepted by the String

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(4) Consider the grammar.

$E \rightarrow 2E2$

$E \rightarrow 3E3$

$E \rightarrow 4$

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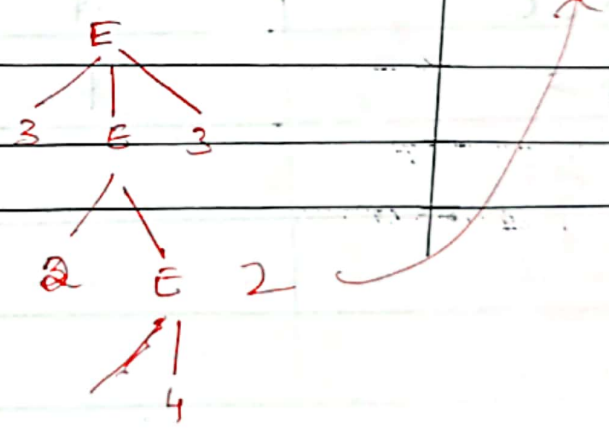
Input string 32423

Stack Operation	Input Buffer	Action
$	32423 $	Shift 3
$3	2423 $	Shift 2
$32	423 $	Shift 4
$324	23 $	Reduce by E→4
$32E	23 $	Shift 2
$32E2	3 $	Reduce E→2E2
$3E	3 $	Shift 3
$3E3	$	Reduce by E→3E3
$E	$	Accepted by the Parser.



Parse Tree

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(4) Consider the grammar

$S \rightarrow CC$

$C \rightarrow cC \mid d$

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Perform Shift Reduce Parsing for the input string w = cdcd

Stack Operation	Input Buffer	Action
$	cdcd$	Shift c
$c	dcd$	Shift d
$cd	cd$	Reduce C→d
$cC	cd$	Reduce C→cC
$C	cd$	shift c
$Cc	d$	shift d
$Ccd	$	Reduce C→d
$CcC	$	Reduce C→cC
$CC	$	Reduce S→CC
$S	$	Accepted by the parser

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(5) Construct Shift Reduce parser for the given grammar and Explain the Conflicts in Shift-Reduce-Parsing

$S \rightarrow aABe$

$A \rightarrow Abc \mid b$

$B \rightarrow d$

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String "abbcde"

(Ans)

Stack Operation	Input Buffer	Action
$	abbcde $	Shift a
$a	bbcde $	Shift b
$ab	bcde $	Reduce A->b
$aA	bcde $	Shift b
$aAb	cde $	Shift c
$aAbc	de $	Reduce A->Abc
$aA	de $	Shift d
$aAd	e $	Reduce B->d
$aAB	e $	Shift e
$aABe	$	Reduce S->aABe
$S	$	Accepted by the parser.

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(6) Consider the following grammar

$S \rightarrow TL$

$T \rightarrow int \mid float$

$L \rightarrow L, id \mid id$

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Assistant Professor
Mob:8951150841

Parse the input string int id, id; using Shift-Reduce parser.

(7) Consider string "10201", design Shift-Reduce parser for the following grammar,

$S \rightarrow 0S0 \mid 1S1 \mid 2$

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Introduction to LR Parsing

* LR means L stands for scanning the input symbol from left-to-Right and R stands for obtaining RMD in reverse order.

* LR parser is an efficient bottom-up parsing technique that can be used to parse large classes of Context free grammar called LR(0) parsing.

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

* L → stands for scanning symbols from left-to-Right

R → stands for RMD in reverse.

0 → stands for no of i/p symbols of lookahead.

* Types of parsing methods are

(i) SLR

(ii) CLR

(iii) LALR

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SLR

* SLR stands for Simple LR.

* SLR is the smallest class of grammar having few number of states.

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Assistant Professor
Mob:8951150841

* LR(0) and SLR is similar, the only difference is that, in LR(0) parsing, there is a chance of Shift-Reduce conflicts, to overcome from this drawback we go for SLR(1).

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Assistant Professor
Mob:8951150841

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(1) Construct SLR parsing table for the following grammar.

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

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Assistant Professor
Mob:8951150841

Also parse the input string id* id + id.

(Ans) Step 1:- Write the Augmented grammar i.e, for the above grammar.

$E' \rightarrow E$

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

→ ~~Augmented~~ Grammar.

$E' \rightarrow E$

$E \rightarrow E+T$

$E \rightarrow T$

$T \rightarrow T*F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow id$

⇒ Augmented Grammar.

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Step 2:- Calculate canonical collection of LR(0) items.

$I_0 \rightarrow$

0 $E' \rightarrow .E$

1 $E \rightarrow .E+T$

2 $E \rightarrow .T$

3 $T \rightarrow .T*F$

4 $T \rightarrow .F$

5 $F \rightarrow .(E)$

6 $F \rightarrow .id$

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Assistant Professor
Mob:8951150841

goto (I_0, E)

I_1 : $E' \rightarrow E.$

$E \rightarrow E.+T$

goto (I_0, T)

I_2 : $E \rightarrow T.$

$T \rightarrow T.*F$

goto (I_0, F)

I_3 : $T \rightarrow F.$

goto (I_0, ( )

I_4 : $F \rightarrow (.E)$

$E \rightarrow .E+T$

$E \rightarrow .T$

$T \rightarrow .T*F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

goto (I_0, id)

I_5 : $F \rightarrow id.$

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I_6: goto (I_1, +)

$E \to E + \cdot T$

$T \to \cdot T * F$

$T \to \cdot F$

$F \to \cdot (E)$

$F \to \cdot id$

I_7: goto (I_2, *)

$T \to T * \cdot F$

$F \to \cdot (E)$

$F \to \cdot id$

I_8: goto (I_4, E)

$F \to (E \cdot)$

$E \to E \cdot + T$

goto (I_4, T)

I_2: $\{ E \to T \cdot$, $T \to T \cdot * F$

goto (I_4, F)

I_3 $\{ T \to F \cdot$

goto (I_4, (

I_4 $\{ F \to (\cdot E)$, $E \to \cdot E + T$, $E \to \cdot T$, $T \to \cdot T * F$, $T \to \cdot F$, $F \to \cdot (E)$ $\}$

goto (I_4, id)

I_5 $\{ F \to id \cdot$

goto (I_6, T)

I_9: $E \to E + T \cdot$

$T \to T \cdot * F$

goto (I_6, F)

I_3 $\{ T \to F \cdot$

goto (I_6, (

I_4 $\{ F \to (\cdot E)$

goto (I_6, id)

I_5 $\{ F \to id \cdot$

goto (I_7, F)

I_{10}: $T \to T * F \cdot$

goto (I_7, (

I_4 $\{ F \to (\cdot E)$

goto (I_7, id)

I_5 $\{ F \to id \cdot$

goto (I_8,))

I_{11}: $F \to (E) \cdot$

goto (I_8, +)

I_6 $\{ E \to E + \cdot T$, $T \to \cdot T * F$ $\}$

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Assistant Professor
Mob:8951150841

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goto(I_9, *)

$$I_7 \begin{cases} T \to T * . F \\ F \to . (E) \\ F \to . id \end{cases}$$

goto

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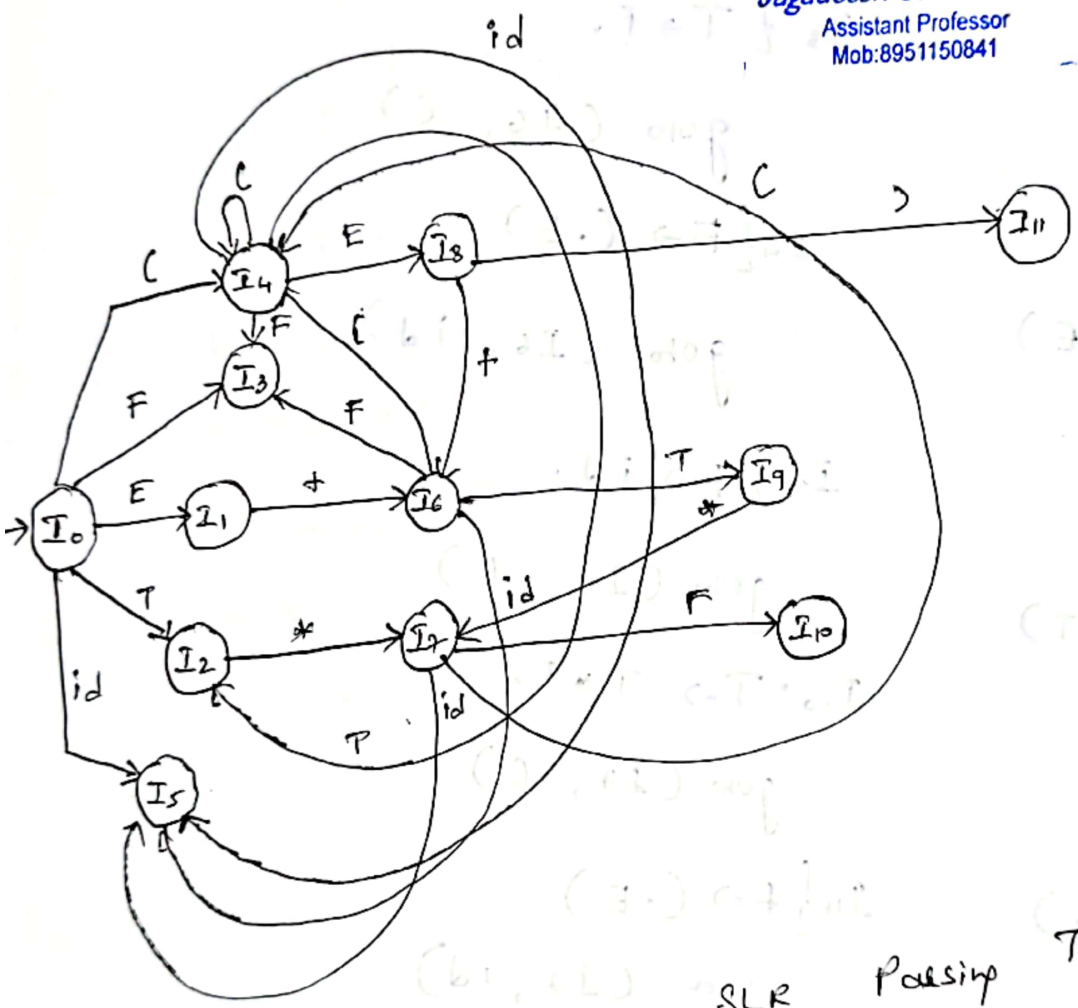
Assistant Professor
Mob:8951150841

Step 2:- Designing of Finite Automata



Step 3:- Construction of SLR Parsing Table, before to that we must find the Follow of the grammar, here to find Follow no need to Eliminate left Recursion / left Factoring.

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0 E → E+T

1 E → T

2 T → T*F

3 T → F

4 F → (E)

5 F → id

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FOLLOW (E) = { $, +,) }

FOLLOW (T) = { $, +,), * }

FOLLOW (F) = { $, +,), * }

	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	S_5			S_4			1	2	3
1		S_6				Accept			
2		r_2	S_7		r_2	r_2			
3		r_4	r_4		r_4	r_4			
4	S_5			S_4			8	2	3
5		r_6	r_6		r_6	r_6			
6	S_5			S_4				9	3
7	S_5			S_4					10
8		S_6			S_{11}				
9		r_1	S_7		r_1	r_1			
10		r_3	r_3		r_3	r_3			
11		r_5	r_5		r_5	r_5			

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Step 4:- Parse the input strings

Stack operations	Input Buffer	Action
$0	id * id + id $	Shift 5
$0id5	* id + id $	reduce F->id
$0F3	* id + id $	reduce T->F
$0T2	* id + id $	shift 7
$0T2*7	id + id $	Shift 5
$0T2*7id5	+ id $	reduce F->id
$0T2*7F10	+ id $	reduce T->T*F
$0T2	+ id $	reduce E->T
$0E1	+ id $	Shift 6
$0E1+6	id $	Shift 5
$0E1+6id5	$	reduce F->id
$0E1+6F3	$	reduce T->F
$0E1+6T9	$	reduce E->E+T
$0E1	$	Accept

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Assistant Professor
Mob:8951150841

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(2) Consider the following grammar

$S \rightarrow As \mid b$

$A \rightarrow SA \mid a$

Construct SLR parse table for the grammar. Show the Actions of the parser for the input string "abab".

(Ans) Step 1:- Augmented Grammar.

$S' \rightarrow S$

$S \rightarrow AS$

$S \rightarrow b$

$A \rightarrow SA$

$A \rightarrow a$

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Assistant Professor
Mob:8951150841

Step 2:- Calculate Canonical Collection of LR(0) items.

I_0

0 $S' \rightarrow .S$

1 $S \rightarrow .AS$

2 $S \rightarrow .b$

3 $A \rightarrow .SA$

4 $A \rightarrow .a$

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goto (I_0, S)

$S' \to S.$
$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
$S \to .b$

I_1

goto (I_0, A)

$S \to A.S$
$S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$

I_2

goto (I_0, b)

$S \to b.$

I_3

goto (I_0, a)

$A \to a.$

I_4

~~goto (I_1, A)~~

~~$A \to .SA$~~
~~$A \to .a$~~
~~$S \to .AS$~~
~~$S \to .b$~~

~~$A \to SA.$~~
~~$S \to A.S$~~
~~$S \to .AS$~~
~~$S \to .b$~~

goto (I_1, S)

$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
$S \to .b$

I_6

goto (I_1, a)

$A \to a.$

I_4

goto (I_1, b)

$S \to b.$

I_3

goto (I_2, S)

$S \to AS.$
$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
$S \to .b$

I_7

goto (I_1, A)

$A \to SA.$
$S \to A.S$
$S \to .AS$
$S \to .b$

I_5

~~$A \to .AS$~~
~~A~~

$A \to .SA$
$A \to .a$

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Assistant Professor
Mob:8951150841

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~~goto (I_1, a)~~

~~$A \to a.$ — I_6~~

~~goto (I_1, b)~~

~~$S \to b.$ — I_4~~

~~goto (I_2, S)~~

$S \to AS.$

goto (I_2, A)

$S \to A.S$
$S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$
— I_2

goto (I_2, b)

$S \to b.$ — I_3

goto (I_2, a)

$A \to a.$ — I_4

goto (I_2, S)

$S \to AS.$
$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
— I_7

goto (I_5, b)

$S \to b.$ — I_3

goto (I_5, a)

$A \to a.$ — I_4

goto (I_6, A)

$A \to SA.$
$S \to A.S$
$S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$
— I_5

goto (I_6, S)

$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
$S \to .b$
— I_6

goto (I_6, a)

$A \to a.$ — I_4

goto (I_6, b)

$S \to b.$ — I_3

goto (I_5, A)

$S \to A.S$
$S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$
— I_2

goto (I_7, A)

$A \to SA.$
$S \to A.S$
$S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$
— I_5

goto (I_7, S)

~~$A \to S.A$~~
~~$A \to .SA$~~
~~$A \to .a$~~

$A \to S.A$
$A \to .SA$
$A \to .a$
$S \to .AS$
$S \to .b$
— I_6

~~goto (I_7, A)~~

~~$A \to SA.$~~
~~$S \to A.S$~~
~~$S \to .AS$~~
~~$S \to .b$~~
~~$A \to .SA$~~
~~$A \to .b$~~
~~— I_5~~

goto (I_7, a)

$A \to a.$ — I_4

goto (I_7, b)

$S \to b.$ — I_3

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Assistant Professor
Mob 8951195841

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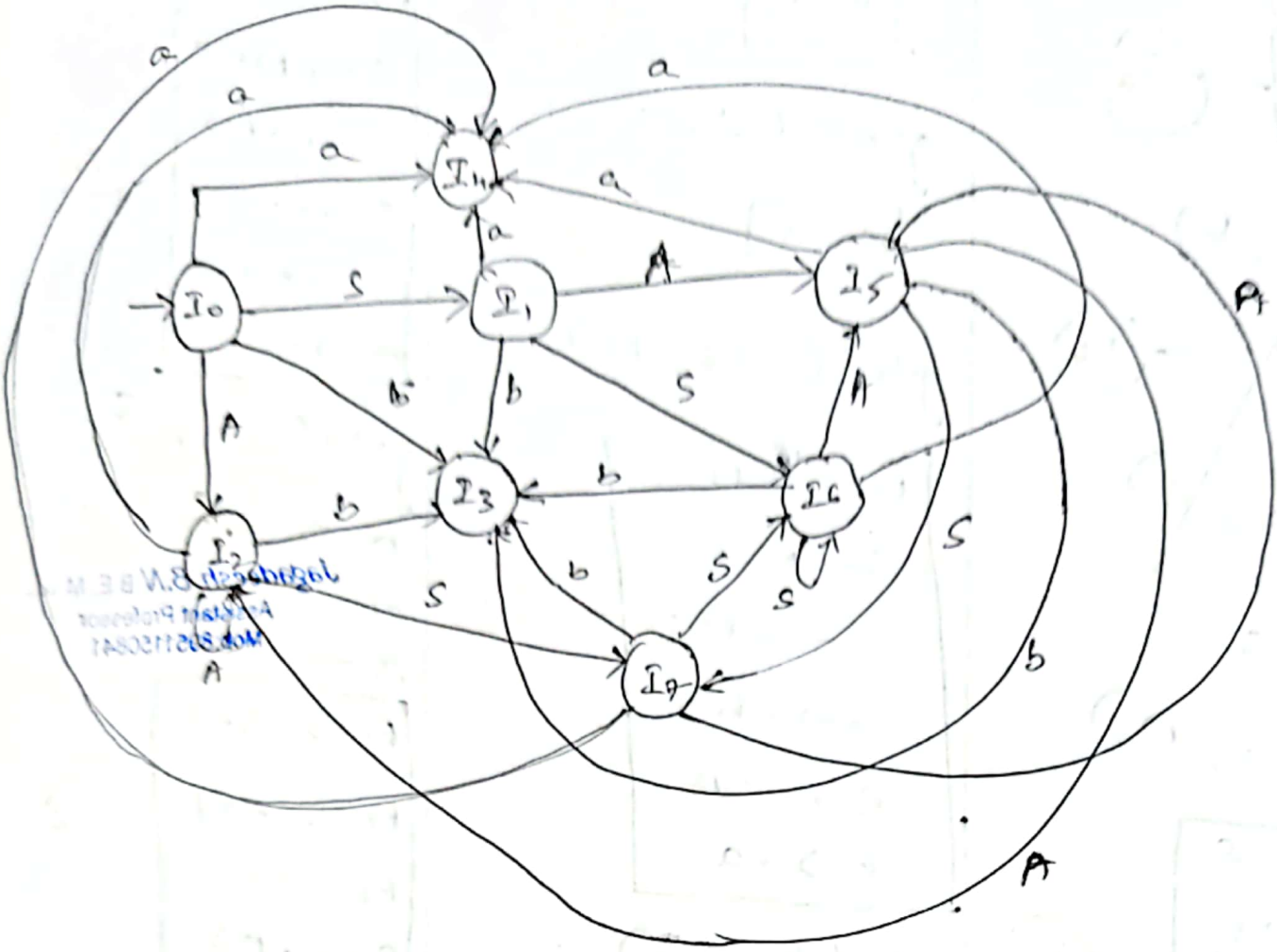
Step 2:- Designing of the finite Automata.



Step 3:- Construction of SLR parsing table, before construction of SLR parsing table we must find the FIRST & FOLLOW of LHS of Non-terminals.

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

~~Follow~~

FIRST(S) = {b, a}

FIRST(A) = {a, b}

FOLLOW(S) = {$, a, b}

FOLLOW(A) = {b, a}

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	ACTION			GOTO	
	a	b	$	S	A
0	S4	S3		1	2
1	S4	S3	Accept	6	5
2	S4	S3		7	2
3	r2	r2	r2		
4	r4	r4			
5	S4/r3	S3/r3		7	2
6	S4	S3		6	5
7	S4/r1	S3/r1	r1	6	5

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Assistant Professor
Mob:8951150841

In the above parsing table, each and every cell must contain only one entry, but if we consider state (5, a) = S4/r3 and also in state (5, b) = (S3/r3) and (7, b) = (S3/r1) hence (7, a) = (S4/r1) and there is a shift-Reduce Conflicts.

Step4:- Let us check whether the string will be accepted by the parser/not.

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STACK OPERATION	INPUT BUFFER	ACTION
$0	abab$	Shift 4
$0a4	bab$	reduce A->a
$0A2	bab$	Shift 3
$0A2b3	ab$	reduce S->b
$0A2S7	ab$	reduce S->AS
$0S1	ab$	shift 4
$0S1a4	b$	reduce A->a
$0S1A5	b$	reduce A->SA
$0A2	b$	Shift S3
$0A2b3	$	reduce S->b
$0A2S7	$	reduce S->AS

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

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$0 S1	$	Accept

(3) Construct Canonical LR(0) items for the grammar,

$S \rightarrow L=R \mid R$

$L \rightarrow *R \mid id$

$R \rightarrow L$

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

Also Construct SLR parsing table and parse the Input String *id=id

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More Powerful LR Parsers

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

CLR Parser

* CLR stands for Canonical LR parser where L means Scanning the i/p Symbol Left-to-Right and R means obtaining the RMD in reverse order.

* The difference ~~betw~~ between SLR and CLR parser is that in SLR we calculate LR(0) items and in CLR we calculate LR(1) items.

* LR(1) items means LR(0) + lookahead Symbol.

* Procedures is Similar to rest of SLR parser.

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(3) Construct CLR parse table for the following grammar

$S \to CC$

$C \to cC \mid d$

and parse the input string "dd".

(Ans) Step1:-

$S' \to S$
$S \to CC$
$C \to cC$
$C \to d$

→ Augmented Grammar.

Step2:- Calculating LR(1) items.

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

~~$S' \to .S, \$$~~
~~$S \to .CC$~~
~~$C \to .cC$~~
~~$C \to .d$~~

~~$S' \to .S, \$$~~
~~$S \to .CC,$~~

$S' \to .S, \$$ ⇒ first$(\varepsilon . \$) = \$$

$S \to .CC, \$$ ⇒ first$(C . \$) =$ first$(C) = \{c, d\} \Rightarrow c/d$

$C \to .cC, c/d$

$C \to .d, c/d$

I_0

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goto (I_0, S)

$S' \to S. , \$$ — I_1

goto (I_0, C)

first $(\varepsilon . \$) = \$$

$S \to C . C , \$$

$C \to . cC , \$$

$C \to . d , \$$

— I_2

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Assistant Professor
Mob:8951150841

goto (I_2, c)

$C \to c . C , c/d$

$C \to . cC , c/d$

$C \to . d , c/d$

— I_3

goto (I_2, d)

$C \to d. , \$$ — I_7

goto (I_3, C)

$C \to cC. , c/d$ — I_8

goto (I_3, c)

$C \to c . C , c/d$

$C \to . cC , c/d$

$C \to . d , c/d$

— I_3

goto (I_3, d)

$C \to d. , c/d$ — I_4

goto (I_0, d)

$C \to d. , c/d$ — I_4

goto (I_2, C)

$S \to CC. , \$$ — I_5

goto (I_2, c)

$C \to c . C , \$$

$C \to . cC , \$$

$C \to . d , \$$

— I_6

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goto (I6, c)

$C \to cC.$, $\$$ — I_9

goto (I6, c)

$C \to c.C$, $\$$

$C \to .cC$, $\$$

$C \to .d$, $\$$ — I_6

goto (I6, d)

$C \to d.$, $\$$ — I_7

0 $S' \to .S$

1 $S \to .CC$

2 $C \to .cC$

3 $C \to .d$

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Assistant Professor
Mob 8951195841

Step 3:- Construction of CLR parse table

	ACTION			GOTO	
	c	d	$	S	C
0	S3	S4		1	2
1			ACCEPT		
2	S6	S7			5
3	S3	S4			8
4	r3	r3			
5			r1		
6	S6	S7			9
7			r3		
8	r2	r2			
9			r2		

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Step4:- Parse the input string

STACK OPERATION	INPUT BUFFER	Action
$0	dd $	Shift 4
$0d4	d $	reduce r3 C->d
$0C2	d $	Shift 7
$0C2d7	$	reduce r3 C->d
$0C2C5	$	reduce r1 S->CC
$0S1	$	ACCEPT

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Assistant Professor
Mob:8951150841

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Construct CLR parser for the grammar

$S \rightarrow L=R \mid R$

$L \rightarrow *R \mid id$

$R \rightarrow L$ and also parse the string id = id.

(Ans) Step 1:- Write the Augmented grammar.

$S' \rightarrow S$

$S \rightarrow L=R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow id$

$R \rightarrow L$

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Assistant Professor
Mob:8951150841

Step 2:- Calculate Canonical collection of LR(1) items.

I_0

$S' \rightarrow .S, \$$

$S \rightarrow .L=R, \$$

$S \rightarrow .R, \$$

$L \rightarrow .*R, =/\$$

$L \rightarrow .id, =/\$$

$R \rightarrow .L, \$$

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goto (I_0, S)

$S' \to S.$, \$ — I_1

goto (I_0, L)

$S \to L. = R$, \$

$R \to L.$, \$ — I_2

goto (I_0, R)

$S \to R.$, \$ — I_3

goto (I_0, *)

$L \to *.R$, =/\$

$R \to .L$, =/\$

$L \to .*R$, =/\$

$L \to .id$, =/\$ — I_4

goto (I_0, id)

$L \to id.$, =/\$ — I_5

goto (I_2, =)

$S \to L = .R$, \$

$R \to .L$, \$

$L \to .*R$, \$

$L \to .id$, \$ — I_6

goto (I_4, R)

$L \to *R.$, =/\$ — I_7

goto (I_4, L)

$R \to L.$, =/\$ — I_8

goto (I_4, *)

$L \to *.R$, =/\$

$R \to .L$, =/\$

$L \to .*R$, =/\$

$L \to .id$, =/\$ — I_4

goto (I_4, id)

$L \to id.$, =/\$ — I_5

goto (I_6, R)

$S \to L = R.$, \$ — I_9

goto (I_6, L)

$R \to L.$, \$ — I_{10}

goto (I_6, *)

$L \to *.R$, \$

$R \to .L$, \$

$L \to .*R$, \$

$L \to .id$, \$ — I_{11}

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Assistant Professor
Mob:8951150841

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goto (I_6 , id)

$L \to id \cdot , \$$ — I_{10}

goto (I_{11} , R)

$L \to * \cdot R \cdot , \$$ — I_{13}

goto (I_{11} , L)

$R \to L \cdot , \$$ — I_{10}

goto (I_{11} , *)

$L \to * \cdot R , \$$

$R \to \cdot L , \$$

$L \to \cdot * R , \$$

$L \to \cdot id , \$$ — I_N

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Assistant Professor
Mob:8951150841

goto (I_{11} , id)

$L \to id \cdot , \$$ — I_{12}

0 $S' \to \cdot S$

1 $S \to \cdot L = R$

2 $S \to \cdot R$

3 $L \to \cdot * R$

4 $L \to \cdot id$

5 $R \to \cdot L$

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Step3:- Construction of CLR parsing table.

	ACTION				GOTO		
	id	=	*	$	S	L	R
0	S_5		S_4		1	2	3
1				Accept			
2		S_6		r_5			
3				r_2			
4	S_5		S_4			8	7
5		r_4		r_4			
6	S_{12}		S_{11}			10	9
7		r_3		r_3			
8		r_5		r_5			
9				r_1			
10				r_5			
11	S_{12}		S_{11}			10	13
12				r_4			
13				r_3			

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Assistant Professor
Mob:8951150841

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Step 4:- Parse the input String

Stack Operation	Input Buffer	Action
$0	id = id $	shift 5
$0 id 5	= id $	reduce r4 L->id
$0 L 2	= id $	shift 6
$0 L 2 = 6	id $	Shift 12
$0 L 2 = 6 id 12	$	reduce r4 L->id
$0 L 2 = 6 L 10	$	reduce r5 R->L
$0 L 2 = 6 R 9	$	reduce r1 S->L=R
~~$0 L 2 = 6~~ $0 S 1	$	ACCEPT

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Assistant Professor
Mob:8951150841

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* LALR stands for Look - Ahead, L Scan the i/p symbol from left to Right R Means obtain the RMD in reverse order.

Construct LALR parser for the grammar

$S \to CC$

$C \to aC \mid b$

parse the string bb

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

(Ans) Step 1 :- Write the Augmented grammar

0 $S' \to S$

1 $S \to CC$

2 $C \to aC$

3 $C \to b$

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Step2 :- Calculate LR(1) items.

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob.8951150841

I_0:
$S' \to .S, \$$
$S \to .CC, \$$
$C \to .aC, a/b$
$C \to .b, a/b$

goto (I_0, S)
I_1:
$S' \to S., \$$

goto (I_0, C)
I_2:
$S \to C.C, \$$
$C \to .aC, \$$
$C \to .b, \$$

goto (I_0, a)
I_3:
$C \to a.C, a/b$
$C \to .aC, a/b$
$C \to .b, a/b$

goto (I_0, b)
I_4:
$C \to b., a/b$

goto (I_2, C)
I_5:
$S \to CC., \$$

goto (I_2, a)
I_6:
$C \to a.C, \$$
$C \to .aC, \$$
$C \to .b, \$$

goto (I_2, b)
I_7:
$C \to b., \$$

goto (I_3, C)
I_8:
$C \to aC., a/b$

goto (I_3, a)
I_3:
$C \to a.C, a/b$
$C \to .aC, a/b$
$C \to .b, a/b$

goto (I_3, b)
I_9:
$C \to b., \$$

goto (I_6, C)
I_9:
$C \to aC., \$$

goto (I_6, a)
I_6:
$C \to a.C, \$$
$C \to .aC, \$$
$C \to .b, \$$

goto (I_6, b)
I_7:
$C \to b., \$$

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Next if we consider the state I_3 & I_6 productions are same, only its look-Ahead symbol is different.

I_{36}

$C \to a.C,\ a/b/\$$
$C \to .aC,\ a/b/\$$
$C \to .b,\ a/b/\$$

I_{36}

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

$C \to b.,\ a/b/\$$

I_{47}

$C \to aC.,\ a/b/\$$

I_{89}

Step3:- Construction of LALR parse table

	ACTION			GOTO	
	a	b	$	S	C
0	S_{36}	S_{47}		1	2
1			ACCEPT		
2	S_{36}	S_{47}			5
36	S_{36}	S_{47}			89
47	r_3	r_3	r_3		
5			r_1		
89	r_2	r_2	r_2		

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Step4:- Let us see how to parse the input string.

STACK OPERATION	INPUT BUFFER	ACTION
$0	bb $	shift 47
$0b47	b $	reduce r3 C->b
$0C2	b $	shift 47
$0C2b47	$	reduce r3 C->b
$0C2C5	$	reduce r1 S->CC
$01	$	ACCEPT

Jagadeesh B.N B.E, M.Tech
Assistant Professor
Mob:8951150841

(2) Construct LALR parser for the grammar:

S -> L=R | R

L -> *R | id

R -> L and also parse the string id=id.

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