

Odводи

$f(x)$	$f'(x)$
x^n	nx^{n-1}
a^x	$a^x \ln(a)$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{sh} x$	$\operatorname{ch} x$
$\operatorname{ch} x$	$-\operatorname{sh} x$
$\operatorname{th} x$	$\frac{1}{\operatorname{ch}^2 x}$
$\operatorname{cth} x$	$-\frac{1}{\operatorname{sh}^2 x}$
$\operatorname{arsh} x$	$\frac{1}{\sqrt{x^2+1}}$
$\operatorname{arth} x$	$\frac{1}{1-x^2}$

Integrali

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} (n \neq -1)$
$\frac{1}{x}$	$\ln x $
e^x	e^x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\frac{1}{\cos^2 x}$	$\tan x$
$\frac{1}{\sin^2 x}$	$-\cot x$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$
$\frac{1}{\sqrt{1+x^2}}$	$\operatorname{arsh} x = \ln x + \sqrt{x^2 + 1} $
$\frac{1}{1+x^2}$	$\arctan x$

Per Partes

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$
$$\int u dv = uv - \int v du$$

Racionalne funkcije

$\int \frac{p(x)}{q(x)} dx,$ $p(x), q(x)$ sta polinoma

1. Če je $st(p(x)) \leq st(p(x))$ polinoma delimo

2. $q(x)$ razdelimo na linearne in kvadratne faktorje

3. Izraz pod integralom razcepimo na parcialne ulomke

$$\frac{p(x)}{q(x)} = \left[\frac{A_1}{x-a_1} + \dots + \frac{A_{n_1}}{(x-a_1)^{n_1}} \right] + \dots + \left[\frac{Z_1}{x-a_k} + \dots + \frac{Z_{n_k}}{(x-a_k)^{n_k}} \right] + \left[\frac{\alpha_1 x + \beta_1}{x^2 + b_1 x + c_1} + \dots + \frac{\alpha_{m_1} x + \beta_{m_1}}{(x^2 + b_1 x + c_1)^{m_1}} \right] + \dots + \left[\frac{\varphi_1 x + \omega_1}{x^2 + b_l x + c_l} + \dots + \frac{\varphi_{m_l} x + \omega_{m_l}}{(x^2 + b_l x + c_l)^{m_l}} \right]$$

4. Integriramo vsakega zase

$$k \geq 2 \qquad st(p(x)) \leq 2k - 1$$
$$st(q(x)) \leq 2k - 3 \qquad (ax^2 + bx + c) \qquad \text{nerazcepen v } \mathbb{R}$$
$$I = \int \frac{p(x)}{(ax^2 + bx + c)^k} = \int \frac{Ax + B}{ax^2 + bx + c} + \frac{q(x)}{(ax^2 + bx + c)^{k-1}}$$

A,B, $q(x)$ poiščemo tako da enačbo odvajamo.

Korenske funkcije

1. $\int f(\sqrt{ax+b})dx \qquad t = \sqrt{ax+b}$

2. $\int f(\sqrt{ax^2+bx+c})dx$
- a $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ ga prevedemo na oblike:
 - Če je $a < 0$: $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$
 - Če je $a > 0$: $\int \frac{dx}{\sqrt{x^2+c}} = \ln|x + \sqrt{x^2+c}|$

b $\int \frac{p(x)}{\sqrt{ax^2+bx+c}} = q(x)\sqrt{ax^2+bx+c} + A \int \frac{dx}{\sqrt{ax^2+bx+c}}$
 $st(p(x)) - 1 = st(q(x)) \quad A, q(x)$ poiščemo z odvajanjem

Kotne funkcije

1.

$$\int \sin(ax) \sin(bx)dx = \int -\frac{1}{2} [\cos(a+b)x - \cos(a-b)x] dx = -\frac{1}{2} \left[\frac{\sin(a-b)x}{(a-b)} - \frac{\sin(a+b)x}{(a+b)} \right]$$
$$\int \cos(ax) \cos(bx)dx = \int \frac{1}{2} [\cos(a+b)x + \cos(a-b)x] dx = \dots$$
$$\int \sin(ax) \cos(bx)dx = \int \frac{1}{2} [\sin(a+b)x + \sin(a-b)x] dx = \dots$$

2. $\int \cos^m x \sin^n x dx$

(a) Eno od števil m,n je liho (npr. $m = 2k + 1$)

$$\int \cos^{2k} x \cos x \sin^n x dx = \int t^n (1-t^2)^k dt$$
$$t = \sin x \quad dt = \cos x dx$$
$$\cos^{2k} x = (\cos^2 x)^k = (1-t^2)^k$$

- (b) m, n sta oba soda, $m = 2m_1, n = 2n_1$

$$\int \cos^{2m_1} x \sin^{2n_1} x dx = \int (\cos^2 x)^{m_1} (\sin^2 x)^{n_1} dx = \int \left(\frac{1 + \cos 2x}{2} \right)^{m_1} \left(\frac{1 - \cos 2x}{2} \right)^{n_1} dx =$$
$$= \text{vsota integralov oblike } \int \cos^k 2x dx$$

kjer je $k \leq m_1 + n_1 = \frac{1}{2}(m + n) < m + 1$
Ce je k lih gremo po 1. točki
Ce je k sod ponovimo postopek

- (c) $\int R(\cos x, \sin x) dx$ ($R \dots$ racionalni izraz)

$$t = \tan \frac{x}{2} \quad \cos x = \frac{1-t^2}{t^2+1}$$
$$\sin x = \frac{2t}{t^2+1} \quad dx = \frac{2}{t^2+1} dt$$
$$t = \tan x \quad \cos x = \frac{1}{\sqrt{t^2+1}}$$
$$\sin x = \frac{t}{\sqrt{t^2+1}} \quad dx = \frac{dt}{t^2+1}$$

Uporaba integralov

1. Ploščina ravninskih likov
2. Dolžina krivulj

$$s = \int_b^a \sqrt{1 + f'(x)^2} dx$$

3. Prostornina vrtenine

$$V = \pi \int_b^a f(x)^2 dx$$

4. Površina vrtenine

$$S = 2\pi \int_b^a f(x) \sqrt{1 + f'(x)^2} dx$$

Kotne funkcije

Adicijski izreki

$$\sin x \pm y = \sin x \cos y \pm \sin y \cos x$$

$$\cos x \pm y = \cos x \cos y \mp \sin x \sin y$$

$$\tan x \pm y = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Faktorizacija

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Razčlenjevanje

$$\sin x \sin y = -\frac{1}{2} (\cos (x+y) - \cos (x-y))$$

$$\cos x \cos y = \frac{1}{2} (\cos (x+y) + \cos (x-y))$$

$$\sin x \cos y = \frac{1}{2} (\sin (x+y) + \sin (x-y))$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$