Odvodi

$f(x)$	$f'(x)$
x^n	nx^{n-1}
a^x	$a^x \ln(a)$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{sh} x$	$\operatorname{ch} x$
$\operatorname{ch} x$	$-\operatorname{sh} x$
$\operatorname{th} x$	$\frac{1}{\operatorname{ch}^2 x}$
$\operatorname{cth} x$	$-\frac{1}{\operatorname{sh}^2 x}$
$\operatorname{arsh} x$	$\frac{1}{\sqrt{x^2+1}}$
$\operatorname{arth} x$	$\frac{1}{1-x^2}$

Integrali

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}(n \neq -1)$
$\frac{1}{x}$	$\ln\|x\|$
e^x	e^x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\frac{1}{\cos^2 x}$	$\tan x$
$\frac{1}{\sin^2 x}$	$-\cot x$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$
$\frac{1}{\sqrt{1+x^2}}$	$\operatorname{arsh} x = \ln\left\|x+\sqrt{x^2+1}\right\|$
$\frac{1}{1+x^2}$	$\arctan x$

Per Partes

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$
$$\int udv = uv - \int vdu$$

Racionalne funkcije

$\int \frac{p(x)}{q(x)}dx, \qquad p(x),\ q(x)$ sta polinoma

1. Če je $st(p(x)) <= st(p(x))$ polinoma delimo
2. $q(x)$ razdelimo na linearne in kvadratne faktorje
3. Izraz pod integralom razcepimo na parcialne ulomke $\frac{p(x)}{q(x)} = \left[\frac{A_1}{x-a_1} + \cdots + \frac{A_{n_1}}{(x-a_1)^{n_1}}\right] + \cdots + \left[\frac{Z_1}{x-a_k} + \cdots + \frac{Z_{n_k}}{(x-a_k)^{n_k}}\right] + \left[\frac{\alpha_1 x+\beta_1}{x^2+b_1x+c_1} + \cdots + \frac{\alpha_{m_1}x+\beta_{m_1}}{(x^2+b_1x+c_1)^{n_1}}\right] + \cdots + \left[\frac{\varphi_1 x+\omega_1}{x^2+b_lx+c_l} + \cdots + \frac{\varphi_{m_l}x+\omega_{m_l}}{(x^2+b_lx+c_l)^{m_l}}\right]$
4. Integriramo vsakega zase

$$k \geq 2 \qquad st(p(x)) \leq 2k-1$$
$$st(q(x)) \leq 2k-3 \qquad (ax^2+bx+c) \qquad \text{nerazcepen v } \mathbb{R}$$
$$I = \int \frac{p(x)}{(ax^2+bx+c)^k} = \int \frac{Ax+B}{ax^2+bx+c} + \frac{q(x)}{(ax^2+bx+c)^{k-1}}$$

A,B, $q(x)$ poiščemo tako da enačbo odvajamo.

Korenske funkcije

1. $\int f(\sqrt{ax+b})dx \qquad t = \sqrt{ax+b}$
2. $\int f(\sqrt{ax^2+bx+c})dx$

   a $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ ga prevedemo na oblike:
   - Če je $a < 0 : \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$
   - Če je $a > 0 : \int \frac{dx}{\sqrt{x^2+c}} = \ln\left|x+\sqrt{x^2+c}\right|$

   b $\int \frac{p(x)}{\sqrt{ax^2+bx+c}} = q(x)\sqrt{ax^2+bx+c} + A\int \frac{dx}{\sqrt{ax^2+bx+c}}$
   $st(p(x)) - 1 = st(q(x))$ A, q(x) poiščemo z odvajanjem

Kotne funkcije

1.

$$\int \sin(ax)\sin(bx)dx = \int -\frac{1}{2}\left[\cos(a+b)x - \cos(a-b)x\right]dx =$$
$$= -\frac{1}{2}\left[\frac{sin(a-b)x}{(a-b)} - \frac{sin(a+b)x}{(a+b)}\right]$$
$$\int \cos(ax)\cos(bx)dx = \int \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]dx \ldots$$
$$\int \sin(ax)\cos(bx)dx = \int \frac{1}{2}\left[\sin(a+b)x + \sin(a-b)x\right]dx \ldots$$

2. $\int \cos^m x \sin^n x dx$

   (a) Eno od števil m,n je liho (npr. $m = 2k+1$)

   $$\int \cos^{2k} x \cos x \sin^n x dx = \int t^n(1-t^2)^k dt$$
   $$t = \sin x \quad dt = \cos x dx$$
   $$cos^{2k}x = (cos^2 x)^k = (1-t^2)^k$$

   (b) m, n sta oba soda, $m = 2m_1$,$n = 2n_1$

   $$\int \cos^{2m_1} x \sin^{2n_1} x dx = \int (\cos^2 x)^{m_1}(\sin^2 x)^{n_1} dx =$$
   $$= \int \left(\frac{1+\cos 2x}{2}\right)^{m_1}\left(\frac{1-\cos 2x}{2}\right)^{n_1} =$$
   $$= \text{vsota integralov oblike} \int \cos^k 2x dx$$

   kjer je $k \leq m_1 + n_1 = \frac{1}{2}(m+n) < m+1$
   Ce je k lih gremo po 1. točki
   Ce je k sod ponovimo postopek

   (c) $\int R(\cos x, \sin x)dx$ ($R\ldots$ racionalni izraz)

   $$t = \tan\frac{x}{2} \qquad \cos x = \frac{1-t^2}{t^2+1}$$
   $$\sin x = \frac{2t}{t^2+1} \qquad dx = \frac{2}{t^2+1}dt$$
   $$t = \tan x \qquad \cos x = \frac{1}{\sqrt{t^2+1}}$$
   $$\sin x = \frac{t}{\sqrt{t^2+1}} \qquad dx = \frac{dt}{t^2+1}$$

Uporaba integralov

1. Ploščina ravnisnkih likov
2. Dolžina krivulj

$$s = \int_b^a \sqrt{1 + f'(x)^2} dx$$

3. Prostornina vrtenine

$$V = \pi \int_b^a f(x)^2 dx$$

4. Površina vrtenine

$$S = 2\pi \int_b^a f(x)\sqrt{1 + f(x)^2} dx$$

Kotne funkcije

Adicijski izreki

$$\begin{aligned}
\sin x \pm y &= \sin x \cos y \pm \sin y \cos x \\
\cos x \pm y &= \cos x \cos y \mp \sin x \sin y \\
\tan x \pm y &= \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}
\end{aligned}$$

Faktorizacija

$$\begin{aligned}
\sin x + \sin y &= 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \\
\sin x - \sin y &= 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2} \\
\cos x + \cos y &= 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \\
\cos x - \cos y &= -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}
\end{aligned}$$

Razčlenjevanje

$$\begin{aligned}
\sin x \sin y &= -\frac{1}{2} \left(\cos (x+y) - \cos (x-y)\right) \\
\cos x \cos y &= \frac{1}{2} \left(\cos (x+y) + \cos (x-y)\right) \\
\sin x \cos y &= \frac{1}{2} \left(\sin (x+y) + \sin (x-y)\right)
\end{aligned}$$

$$\cos^2 x = \frac{\cos 2x + 1}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$