Izpeljava tipov

Problem, ki ga rešujemo: (izpeljava tipa)

Če imam izraz e, ali obstaja A, da velja $\emptyset \vdash e : A$

Enostavnejši problem: (preverjanje tipa)

Če imam izraz e in tip A, ali velja $\emptyset \vdash e : A$.

Mi bomo reševali prvega s Hindley-Milnerjevim algoritmom.

Ideja: ko moramo ugoniti tip v izpeljavi, ustvarimo spremenljivko $\alpha, \beta, \ldots$

Če se morajo tipi ujemati, dodamo enačbo

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{}{f : \alpha \vdash f : \alpha} \qquad \dfrac{}{f : \alpha \vdash 10 : int}}{f : \alpha \vdash f\ 10 : \beta}
}{\emptyset \vdash \lambda f.\, f\ 10 : \alpha \to \beta}
\qquad
\dfrac{
\dfrac{\dfrac{}{x : \gamma \vdash x : \gamma} \qquad \dfrac{}{x : \gamma \vdash 3 : int}}{x : \gamma \vdash x > 3 : \delta}
}{\emptyset \vdash \lambda x.\, x > 3 : \alpha}
}{\emptyset \vdash (\lambda f.\, f\ 10)\,(\lambda x.\, x > 3) : \beta}
$$

$$
\begin{array}{l} \alpha = int \to \beta \\ \alpha = \gamma \to \delta \\ \gamma = int \\ \delta = bool \end{array}
\rightsquigarrow
\begin{array}{l} \alpha = int \to \beta \\ \alpha = int \to bool \\ \gamma = int \\ \delta = bool \end{array}
\rightsquigarrow
\begin{array}{l} \alpha = int \to bool \\ int \to bool = int \to \beta \\ \gamma = int \\ \delta = bool \end{array}
$$

$$
\rightsquigarrow
\begin{array}{l} \alpha = int \to bool \\ int = int \\ bool = \beta \\ \gamma = int \\ \delta = bool \end{array}
\rightsquigarrow
\begin{array}{l} \alpha = int \to bool \\ \beta = bool \\ \gamma = int \\ \delta = bool \end{array}
$$

$$A ::= \alpha \mid \cdots$$

$\Gamma \vdash e : A \mid \mathcal{E}$ ... v kontekstu Γ za e izpeljemo tip A ob omejitvah $\mathcal{E}$.

$$\frac{}{\Gamma \vdash n : \mathrm{int} \mid \emptyset} \qquad \frac{\Gamma \vdash e_1 : A_1 \mid \mathcal{E}_1 \qquad \Gamma \vdash e_2 : A_2 \mid \mathcal{E}_2}{\Gamma \vdash e_1 + e_2 : \mathrm{int} \mid \mathcal{E}_1, \mathcal{E}_2, A_1 = \mathrm{int}, A_2 = \mathrm{int}}$$ pod. za $-$, $*$

$$\frac{}{\Gamma \vdash \mathrm{true} : \mathrm{bool} \mid \emptyset} \qquad \frac{}{\Gamma \vdash \mathrm{false} : \mathrm{bool} \mid \emptyset}$$ $=, <, >$: D.N.

$$\frac{\Gamma \vdash e : A \mid \mathcal{E} \qquad \Gamma \vdash e_1 : A_1 \mid \mathcal{E}_1 \qquad \Gamma \vdash e_2 : A_2 \mid \mathcal{E}_2}{\Gamma \vdash \mathrm{if}\ e\ \mathrm{then}\ e_1\ \mathrm{else}\ e_2 : A_1 \mid \mathcal{E}, \mathcal{E}_1, \mathcal{E}_2, A = \mathrm{bool}, A_1 = A_2}$$

(se ne pojavlja v $\Gamma, B, \mathcal{E}$)

$$\frac{\Gamma, x : \alpha \vdash e : B \mid \mathcal{E} \qquad \alpha \text{ svež}}{\Gamma \vdash \lambda x. e : \alpha \to B \mid \mathcal{E}} \qquad \frac{\Gamma \vdash e_1 : A_1 \mid \mathcal{E}_1 \qquad \Gamma \vdash e_2 : A_2 \mid \mathcal{E}_2 \qquad \alpha \text{ svež}}{\Gamma \vdash e_1\, e_2 : \alpha \mid \mathcal{E}_1, \mathcal{E}_2, A_1 = A_2 \to \alpha}$$

<u>Primer</u>: $\lambda f. \lambda x.\ \mathrm{if}\ x\ \mathrm{then}\ f\,x\ \mathrm{else}\ f(f\,x)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \beta} \qquad \dfrac{\dfrac{}{\Gamma \vdash f : \alpha} \quad \dfrac{}{\Gamma \vdash x : \beta}}{\Gamma \vdash f\,x : \gamma} \qquad \dfrac{\dfrac{}{\Gamma \vdash f : \alpha} \quad \dfrac{\dfrac{}{\Gamma \vdash f : \alpha} \quad \dfrac{}{\Gamma \vdash x : \beta}}{\Gamma \vdash f\,x : \sigma}}{\Gamma \vdash f(f\,x) : \varphi}}{\overbrace{f : \alpha, x : \beta}^{\Gamma} \vdash \mathrm{if}\ x\ \mathrm{then}\ f\,x\ \mathrm{else}\ f(f\,x) : \gamma}}{f : \alpha \vdash \lambda x. \mathrm{if}\ x\ \mathrm{then}\ f\,x\ \mathrm{else}\ f(f\,x) : \beta \to \gamma}}{\emptyset \vdash \lambda f. \lambda x.\ \mathrm{if}\ x\ \mathrm{then}\ f\,x\ \mathrm{else}\ f(f\,x) : \alpha \to (\beta \to \gamma)}$$

$$(\mathrm{bool} \to \mathrm{bool}) \to (\mathrm{bool} \to \mathrm{bool})$$

$\alpha = \beta \to \gamma$
$\alpha = \beta \to \sigma$
$\alpha = \sigma \to \varphi$
$\varphi = \gamma$
$\beta = \mathrm{bool}$

rešimo $\leadsto$

$\beta = \sigma = \gamma = \varphi = \mathrm{bool}$
$\alpha = \mathrm{bool} \to \mathrm{bool}$