Računski učinki

... Vse, kar odstopa od čistih izračunov vrednosti:
izjeme, nedeterminizem, pomnilnik, vhod/izhod, ...

Zaradi lepše semantike bomo izraze ločili na vrednosti in izračune
(fine-grain call-by-value)
← lenega izvajanja si v kombinaciji z učinki ne želimo

$A ::= unit \mid bool \mid int \mid A \to B$

vrednost
$v ::= x \mid () \mid true \mid false \mid n \mid \lambda x.c \mid rec\, f\, x.c$

izračun
$c ::= if\ v\ then\ c_1\ else\ c_2 \mid v_1 + v_2 \mid v_1 * v_2 \mid v_1 - v_2 \mid v_1 < v_2 \mid v_1 > v_2 \mid v_1 = v_2$
$\mid v_1\, v_2 \mid return\ v \mid do\ x \leftarrow c_1\ in\ c_2$
← izračunaj c_1 do končne vrednosti, nato jo shrani v x in nadaljuj s c_2.

~~$(1+2)+3$~~ $\quad do\ x \leftarrow 1+2\ in\ x+3$ ✓

~~if $x>0$ then 3 else 10~~ $\quad do\ y \leftarrow x>0\ in$ if y then (return 3) else (return 10) ✓

Operacijska semantika

$c \rightsquigarrow c'$ (relacija $v \rightsquigarrow v'$ nimamo)

$$\frac{}{if\ true\ then\ c_1\ else\ c_2 \rightsquigarrow c_1} \qquad \frac{}{if\ false\ then\ c_1\ else\ c_2 \rightsquigarrow c_2}$$ (tretjega pravila ni)

$$\frac{}{n_1 + n_2 \rightsquigarrow return(n_1 + n_2)}$$
$10 + 5 \rightsquigarrow return\ 15$

podobno za ostale

$$\frac{}{(\lambda x.c)\, v \rightsquigarrow c[v/x]}$$

$$\frac{}{(rec\, f\, x.c)\, v \rightsquigarrow c[v/x, (rec\, f\, x.c)/f]} \qquad \frac{c_1 \rightsquigarrow c_1'}{do\ x \leftarrow c_1\ in\ c_2 \rightsquigarrow do\ x \leftarrow c_1'\ in\ c_2} \qquad \frac{}{do\ x \leftarrow ret\ v\ in\ c \rightsquigarrow c[v/x]}$$

Primer

$(\lambda x.\, x*6)(3+4)$

do $y \leftarrow 3+4$ in $(\lambda x.\, x*6)\, y$

$\rightsquigarrow$ do $y \leftarrow$ (ret 7) in $(\lambda x.\, x*6)\, y$

$\rightsquigarrow (\lambda x.\, x*6)\, 7 \rightsquigarrow 7*6$

$\rightsquigarrow$ return 42.

Tipi

imamo dve relaciji : $\Gamma \vdash_v v : A$ in $\Gamma \vdash_c C : A$

$$\frac{\Gamma, x : A \vdash_c C : B}{\Gamma \vdash_v \lambda x.C : A \to B} \qquad \frac{\Gamma \vdash_v v_1 : A \to B \quad \Gamma \vdash_v v_2 : A}{\Gamma \vdash_c v_1\, v_2 : B}$$

$$\frac{\Gamma \vdash_v v : A}{\Gamma \vdash_c \text{return } v : A} \qquad \frac{\Gamma \vdash C_1 : A \quad \Gamma, x : A \vdash C_2 : B}{\Gamma \vdash \text{do } x \leftarrow C_1 \text{ in } C_2 : B}$$

ostalo doma.

Izjeme

Imejmo fiksno množico izjem $exc \in E$

$C ::= \ldots \mid \textbf{raise}\ exc \mid V_1\ /\!/\ V_2 \mid try\ C_1\ with\ C_2$

$$\frac{}{do\ x \leftarrow raise\ exc\ in\ C \rightsquigarrow raise\ exc} \qquad \frac{n_2 \neq 0}{n_1\ /\!/\ n_2 \rightsquigarrow ret(n_1\ /\!/\ n_2)} \qquad \frac{}{n\ /\!/\ 0 \rightsquigarrow raise\ Deljenje\check{z}Nic}$$

$$\frac{C_1 \rightsquigarrow C_1'}{try\ C_1\ with\ C_2 \rightsquigarrow try\ C_1'\ with\ C_2} \qquad \frac{}{try\ (ret\ v)\ with\ C \rightsquigarrow ret\ v} \qquad \frac{}{try\ (raise\ exc)\ with\ C \rightsquigarrow C}$$

$$\frac{}{\Gamma \vdash raise\ exc : A} \qquad \frac{\Gamma \vdash C_1 : A \qquad \Gamma \vdash C_2 : A}{\Gamma \vdash try\ C_1\ with\ C_2 : A}$$

Izrek o varnosti

Ohranitev: kot prej

Napredek Če $\vdash C : A$, tedaj:

- obstaja C', da velja $C \rightsquigarrow C'$
- obstaja v, da velja $C = return\ v$
- obstaja exc, da velja $C = raise\ exc$

Zadnjemu primeru se lahko izognemo s sistemom učinkov.

$\Gamma \vdash V : A \qquad \Gamma \vdash C : A\,!\,E$

$$\frac{\Gamma \vdash V : A}{\Gamma \vdash ret\ v : A\,!\,\varnothing} \qquad \frac{\Gamma \vdash C_1 : A\,!\,E_1 \quad \Gamma, x:A \vdash C_2 : B\,!\,E_2}{\Gamma \vdash do\ x \leftarrow C_1\ in\ C_2 : B\,!\,E_1 \cup E_2} \qquad \frac{\Gamma, x:A \vdash C : B\,!\,E}{\Gamma \vdash \lambda x.C : A \xrightarrow{E} B}$$

$$\frac{\Gamma \vdash V_1 : A \xrightarrow{E} B \quad \Gamma \vdash V_2 : A}{\Gamma \vdash V_1\ V_2 : B\,!\,E} \qquad \frac{}{\Gamma \vdash raise\ exc : A\,!\,\{exc\}} \qquad \frac{\Gamma \vdash C_1 : A\,!\,E_1 \quad \Gamma \vdash C_2 : A\,!\,E_2}{\Gamma \vdash try\ C_1\ with\ C_2 : A\,!\,E_2}$$

Napredek $\vdash C : A\,!\,E$, tedaj:

- ✓
- ✓
- obstaja $exc \in E$

Nedeterminizem

$C ::= \dots \mid C_1 \oplus C_2$

$$\frac{}{C_1 \oplus C_2 \rightsquigarrow C_1} \qquad \frac{}{C_1 \oplus C_2 \rightsquigarrow C_2} \qquad \frac{\Gamma \vdash C_1 : A \quad \Gamma \vdash C_2 : A}{\Gamma \vdash C_1 \oplus C_2 : A}$$

$(\text{ret } 1 \oplus \text{ret } 2) \oplus \text{ret } 3 \rightsquigarrow \text{ret } 1 \oplus \text{ret } 2 \rightsquigarrow \text{ret } 1$, $\rightsquigarrow \text{ret } 2$, $\rightsquigarrow \text{ret } 3$

$do\ x \leftarrow C_1 \oplus C_2\ in\ C \rightsquigarrow (do\ x \leftarrow C_1\ in\ C) \oplus (do\ x \leftarrow C_2\ in\ C)$

$do\ x \leftarrow (\text{ret } 5) \oplus (\text{ret } 7)\ in\ (x * 6)$

$\rightsquigarrow$

$(do\ x \leftarrow \text{ret } 5\ in\ x*6) \oplus (do\ x \leftarrow \text{ret } 7\ in\ x*6)$

$do\ x \leftarrow \text{ret } 5\ in\ x*6$ $\rightsquigarrow$ $5*6$ $\rightsquigarrow$ $\text{ret } 30$

$do\ x \leftarrow \text{ret } 7\ in\ x*6$ $\rightsquigarrow$ $7*6$ $\rightsquigarrow$ $\text{ret } 42$

$v_1 \oplus v_2 \rightsquigarrow \text{ret } v_1$, $\rightsquigarrow \text{ret } v$

Nedeterminizem v.2.

$C ::= \dots \mid amb$

$amb \rightsquigarrow \text{return true}$ $\qquad$ $amb \rightsquigarrow \text{return false}$ $\qquad$ $\dfrac{}{\Gamma \vdash amb : bool}$

$amb := \text{return true} \oplus \text{return false}$

$C_1 \oplus C_2 := do\ x \leftarrow amb\ in\ (\text{if } x \text{ then } C_1 \text{ else } C_2)$

Pomnilnik

Vzemimo množico lokacij $\mathbb{L}$ in stanj $(\mathbb{Z}_\perp)^{\mathbb{L}}$ kot v IMPu.

$c ::= \ldots \mid \ell := v \mid !\ell$

$!\ell \rightsquigarrow ???$

Operacijska semantika spremenimo v $S, c \rightsquigarrow S', c'$

Pravila dopolnimo npr. $\dfrac{}{S, m_1 + m_2 \rightsquigarrow S, \mathrm{ret}(n_1 + n_2)}$ $\dfrac{}{S, (\lambda x.c)v \rightsquigarrow S, c[v/x]}$.

$$\frac{S, c_1 \rightsquigarrow S', c_1'}{S, \mathrm{do}\, x \leftarrow c_1 \,\mathrm{in}\, c_2 \rightsquigarrow S', \mathrm{do}\, x \leftarrow c_1' \,\mathrm{in}\, c_2} \qquad \frac{}{S, \ell := m \rightsquigarrow S[\ell \mapsto m], \mathrm{return}\ ()}$$

$$\frac{(\ell \mapsto m) \in S}{S, !\ell \rightsquigarrow S, \mathrm{return}\ m} \qquad \frac{\Gamma \vdash v : \mathrm{int}}{\Gamma \vdash \ell := v : \mathrm{unit}} \qquad \frac{}{\Gamma \vdash !\ell : \mathrm{int}}$$