HS 2020

Modeling Techniques: Problem Set 3

Thermal diffusion

Solve the two-dimensional heat equation

$$a\partial_t u(x,y,t) - (\partial_{xx} + \partial_{yy})u(x,y,t) = f(x,y,t) \tag{1}$$

Such situation is found, e.g., when a rectangular plane has an initial temperature distribution that starts to evolve over time. Choose the boundary conditions wisely as the problem lives in a finite space!

1. Solve the one-dimensional problem for $f(x,t) = 0$ using different a specific explicit or implicit finite difference scheme. Perform a stability analysis. Use an initial distribution $u(x,0) = u_0 \exp(-x^2/\sigma_x^2)$ and compare the numerical to the analytic result.

2. Once you have verified the accuracy of your code select, explore the effect of different boundary value problems at $x = -a$ and $x = a$ (Dirichlet, von Neuman, Cauchy).

3. Solve for $u(x,0) = u_0$ and

$$f(x,t) = f_0 \exp(-t^2/\tau^2)\sin^2\omega_0 t\ \delta(x) \tag{2}$$

4. Let the heat source move towards positive x values with a specific velocity v, i.e.

$$f(x,t) = f_0 \exp(-t^2/\tau^2)\sin^2\omega_0 t\ \delta(x - vt) \tag{3}$$

5. Repeat for two dimensions with different heat sources.