

## Chapter 19

# Resonant Conversion

### Introduction

#### 19.1 Sinusoidal analysis of resonant converters

#### 19.2 Examples

Series resonant converter  
Parallel resonant converter

#### 19.3 Soft switching

Zero current switching  
Zero voltage switching

#### 19.4 Load-dependent properties of resonant converters

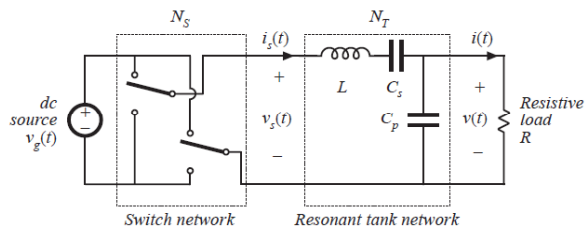
#### 19.5 Exact characteristics of the series and parallel resonant converters

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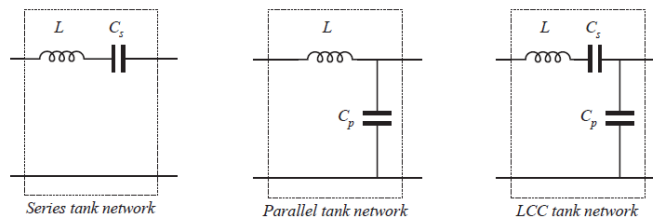
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## A class of resonant DC-to-AC inverters

Basic circuit



Several resonant tank networks

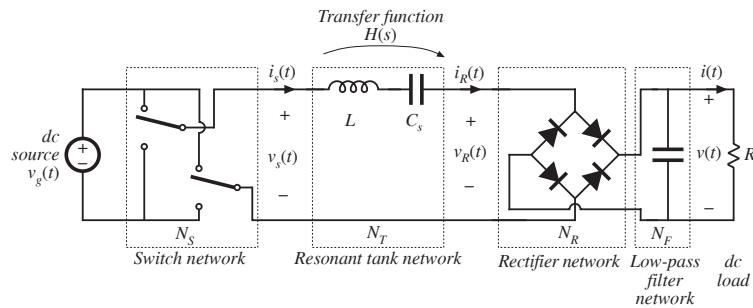


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## A resonant DC-DC converter

A resonant dc-dc converter:



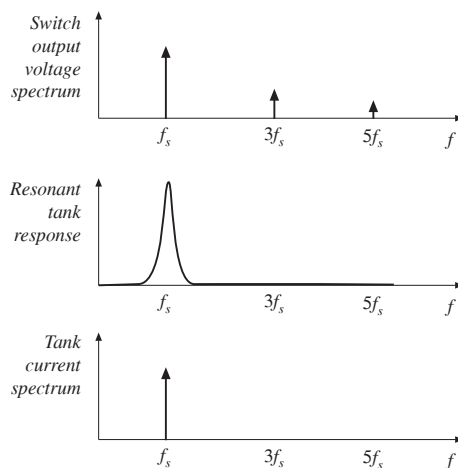
If tank responds primarily to fundamental component of switch network output voltage waveform, then harmonics can be neglected

### Section 19.1: modeling based on sinusoidal approximation

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## The sinusoidal approximation



Tank current and output voltage are essentially sinusoids at the switching frequency  $f_s$

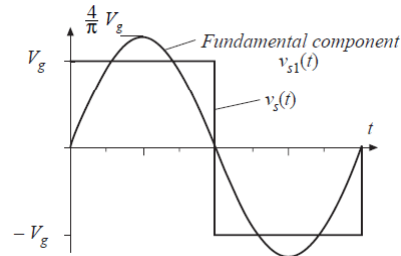
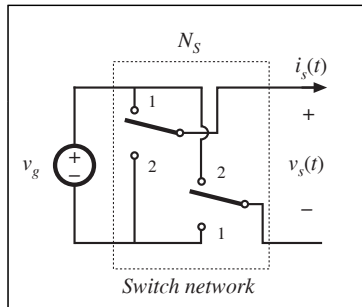
Neglect harmonics of switch output voltage waveform, and model only the fundamental component

Remaining ac waveforms can be found via standard phasor analysis

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### 19.1.1 Controlled switch network model



Fourier series expansion of square-wave switch network output voltage  $v_s(t)$ :

$$v_s(t) = \frac{4V_g}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \sin(n\omega_s t)$$

The fundamental component is

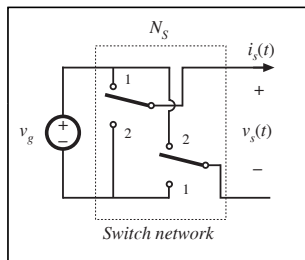
$$v_{s1}(t) = \frac{4V_g}{\pi} \sin(\omega_s t) = V_{s1} \sin(\omega_s t)$$

So model switch network output port with voltage source of value  $v_{s1}(t)$

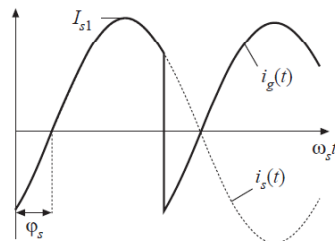
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### Model of switch network input port



Find dc (average) component of the switch network input current



Fundamental component of the output current:

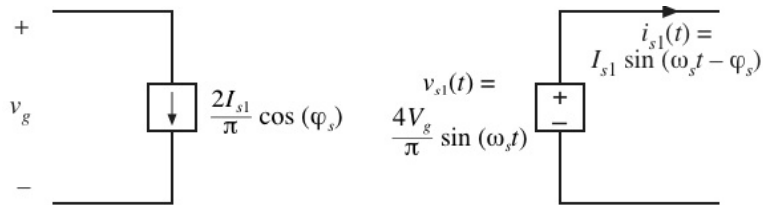
$$i_s(t) \approx I_{s1} \sin(\omega_s t - \phi_s)$$

$$\begin{aligned} \langle i_g(t) \rangle_{T_s} &= \frac{2}{T_s} \int_0^{T_s/2} i_g(\tau) d\tau \\ &\approx \frac{2}{T_s} \int_0^{T_s/2} I_{s1} \sin(\omega_s \tau - \phi_s) d\tau \\ &= \frac{2}{\pi} I_{s1} \cos(\phi_s) \end{aligned}$$

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## Switch network: equivalent circuit

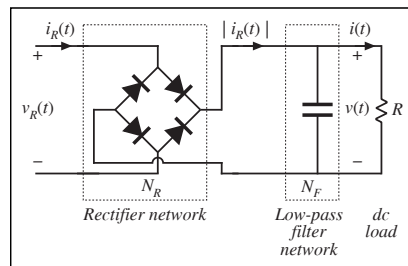


- Switch network converts dc to ac
- Dc components of input port waveforms are modeled
- Fundamental ac components of output port waveforms are modeled
- Model is power conservative: predicted **average** input and output powers are equal

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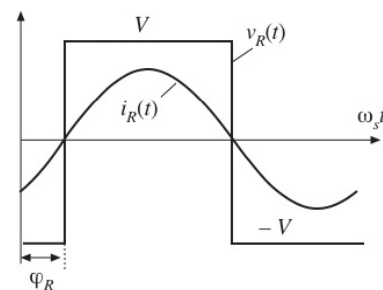
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## 19.1.2 Modeling the rectifier and capacitive filter networks



Assume large output filter capacitor, having small ripple.

$v_R(t)$  is a square wave, having zero crossings in phase with tank output current  $i_R(t)$ .



If  $i_R(t)$  is a sinusoid:

$$i_R(t) = I_{R1} \sin(\omega_s t - \varphi_R)$$

Then  $v_R(t)$  has the following Fourier series:

$$v_R(t) = \frac{4V}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin(n\omega_s t - \varphi_R)$$

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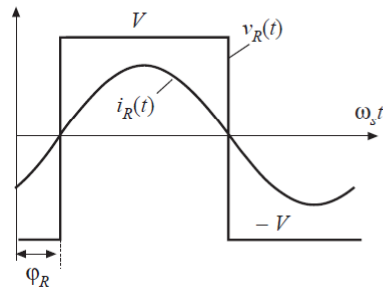
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## Sinusoidal approximation: rectifier

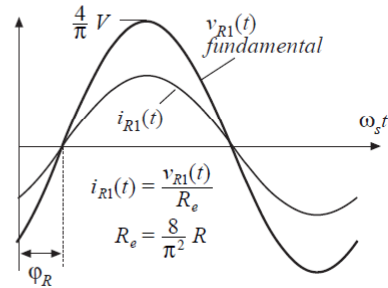
Again, since tank responds only to fundamental components of applied waveforms, harmonics in  $v_R(t)$  can be neglected.  $v_R(t)$  becomes

$$v_{R1}(t) = \frac{4V}{\pi} \sin(\omega_s t - \phi_R) = V_{R1} \sin(\omega_s t - \phi_R)$$

Actual waveforms



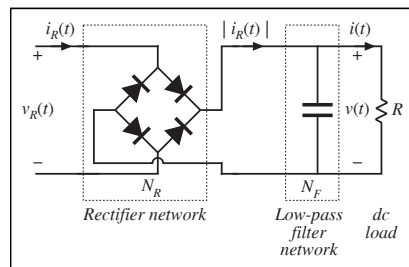
with harmonics ignored



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## Rectifier dc output port model

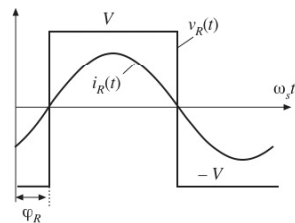


Output capacitor charge balance: dc load current is equal to average rectified tank output current

$$\langle |i_R(t)| \rangle_{T_s} = I$$

Hence

$$\begin{aligned} I &= \frac{2}{T_s} \int_0^{T_s/2} I_{R1} |\sin(\omega_s t - \phi_R)| dt \\ &= \frac{2}{\pi} I_{R1} \end{aligned}$$



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## Equivalent circuit of rectifier

Rectifier input port:

Fundamental components of current and voltage are sinusoids that are in phase

Hence rectifier presents a resistive load to tank network

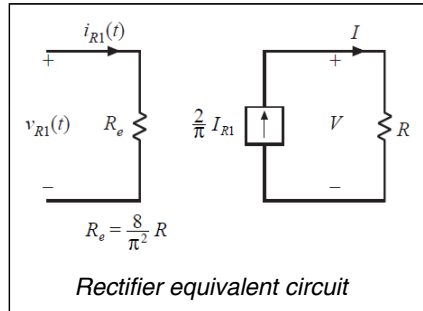
Effective resistance  $R_e$  is

$$R_e = \frac{v_{R1}(t)}{i_R(t)} = \frac{8}{\pi^2} \frac{V}{I}$$

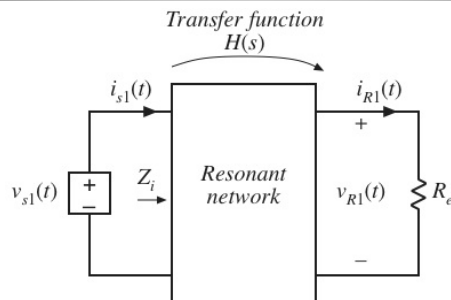
With a resistive load  $R$ , this becomes

$$R_e = \frac{8}{\pi^2} R = 0.8106R$$

Loss free resistor: all power absorbed by  $R_e$  is transferred to the output port



## 19.1.3 Resonant tank network



Model of ac waveforms is now reduced to a linear circuit. Tank network is excited by effective sinusoidal voltage (switch network output port), and is load by effective resistive load (rectifier input port)

Can solve for transfer function via conventional linear circuit analysis

## Solution of tank network waveforms

Transfer function:

$$\frac{v_{R1}(s)}{v_{s1}(s)} = H(s)$$

Ratio of peak values of input and output voltages:

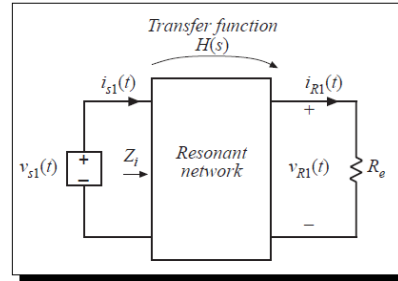
$$\frac{V_{R1}}{V_{s1}} = \|H(s)\|_{s=j\omega_s}$$

Solution for tank output current:

$$i_R(s) = \frac{v_{R1}(s)}{R_e} = \frac{H(s)}{R_e} v_{s1}(s)$$

which has peak magnitude

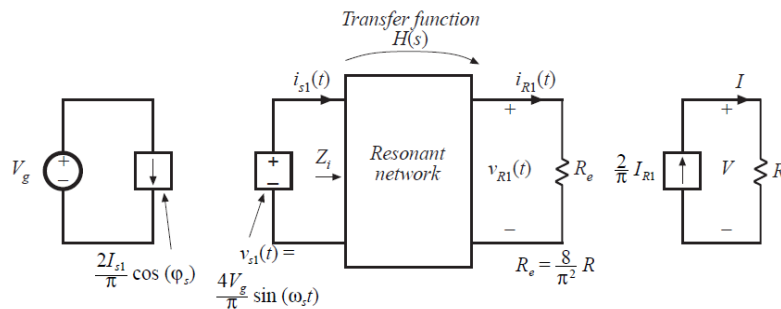
$$I_{R1} = \frac{\|H(s)\|_{s=j\omega_s}}{R_e} V_{s1}$$



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## 19.1.4 Solution of converter voltage conversion ratio $M = V/V_g$



$$M = \frac{V}{V_g} = \underbrace{\left(\frac{R}{V_g}\right)}_{\left(\frac{V}{I}\right)} \underbrace{\left(\frac{2}{\pi}\right)}_{\left(\frac{I}{I_{R1}}\right)} \underbrace{\left(\frac{1}{R_e}\right)}_{\left(\frac{I_{R1}}{V_{R1}}\right)} \underbrace{\left(\|H(s)\|_{s=j\omega_s}\right)}_{\left(\frac{V_{R1}}{V_{s1}}\right)} \underbrace{\left(\frac{4}{\pi}\right)}_{\left(\frac{V_{s1}}{V_g}\right)}$$

Eliminate  $R_e$ :

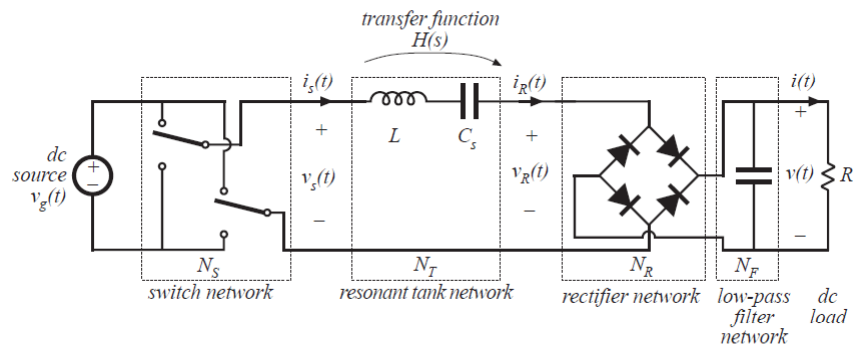
$$\frac{V}{V_g} = \|H(s)\|_{s=j\omega_s}$$

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## 19.2 Examples

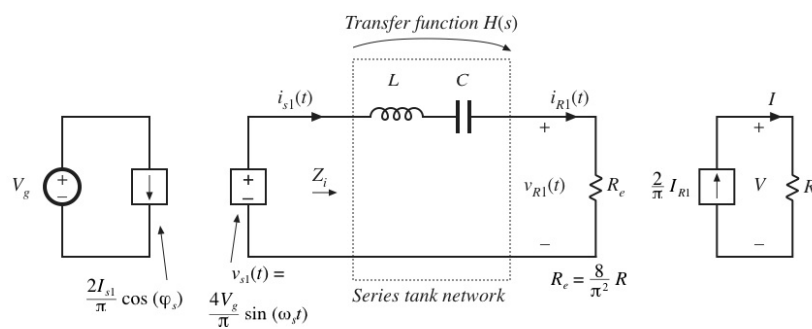
### 19.2.1 Series resonant converter



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### Model: series resonant converter



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Construction of  $Z_i$  – resonant (high Q) case  
 $C = 0.1 \mu\text{F}$ ,  $L = 1 \text{ mH}$ ,  $R_e = 10 \Omega$

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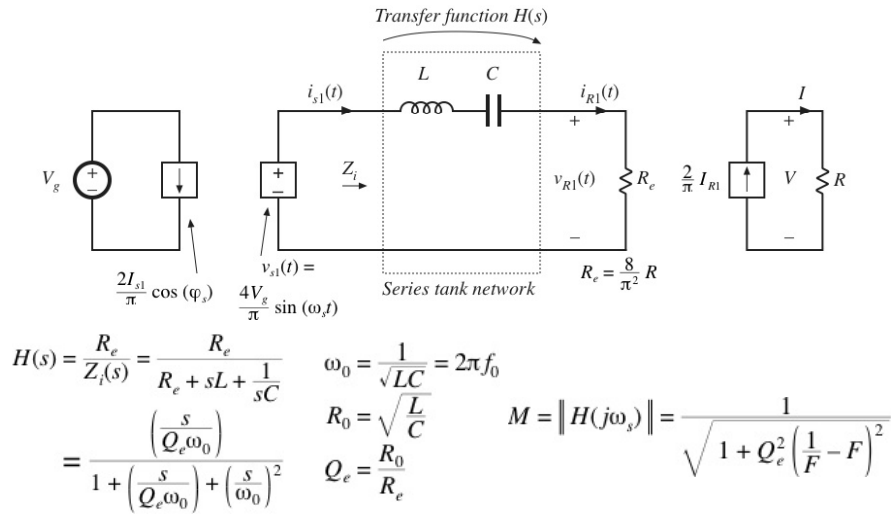
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Construction of  $H = V / V_g$  – resonant (high Q) case  
 $C = 0.1 \mu\text{F}$ ,  $L = 1 \text{ mH}$ ,  $R_e = 10 \Omega$

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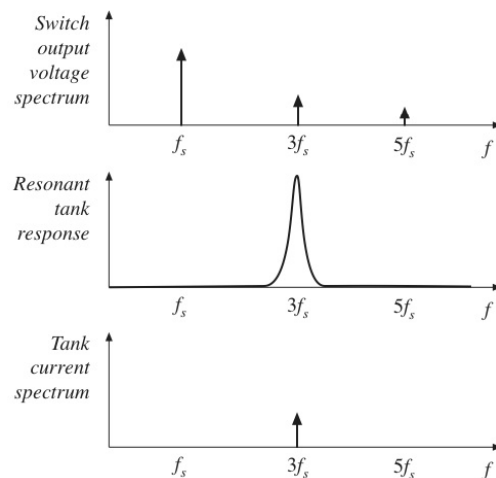
## Model: series resonant converter



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## 19.2.2 Subharmonic modes of the SRC



Example: excitation of tank by third harmonic of switching frequency

Can now approximate  $v_s(t)$  by its third harmonic:

$$v_s(t) \approx v_{sn}(t) = \frac{4V_g}{n\pi} \sin(n\omega_s t)$$

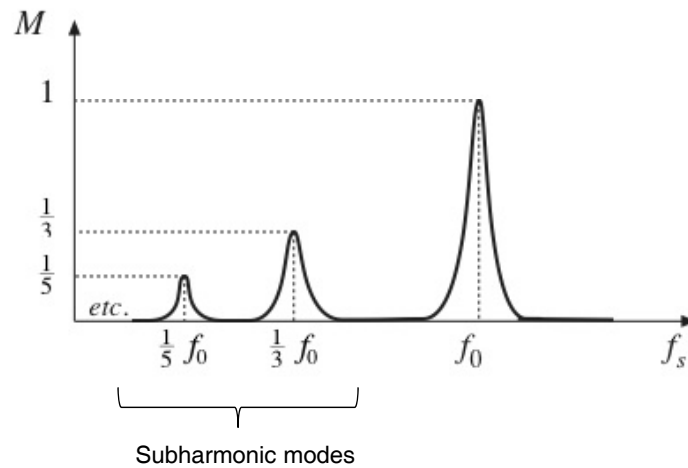
Result of analysis:

$$M = \frac{V}{V_g} = \frac{\|H(jn\omega_s)\|}{n}$$

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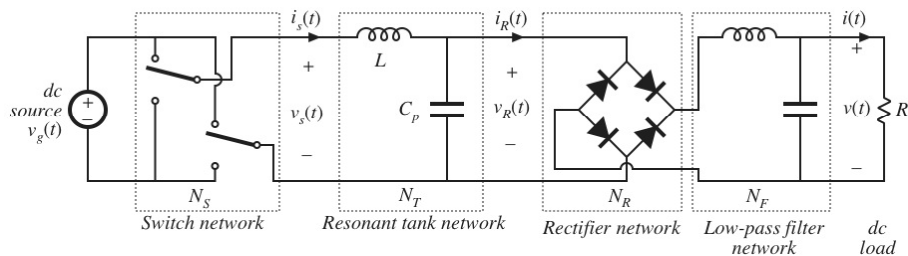
## SRC DC conversion ratio M



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## 19.2.3 Parallel resonant dc-dc converter



Differs from series resonant converter as follows:

- Different tank network

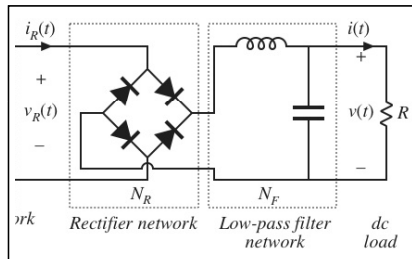
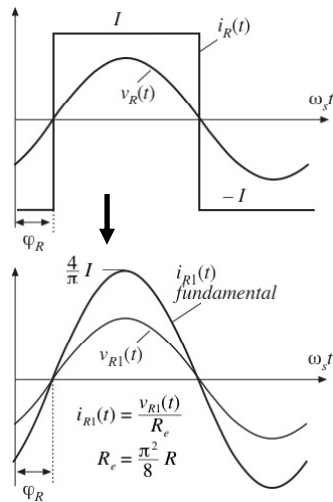
- Rectifier is driven by sinusoidal voltage, and is connected to inductive-input low-pass filter

- Need a new model for rectifier and filter networks

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## Model of uncontrolled rectifier with inductive filter network – input port



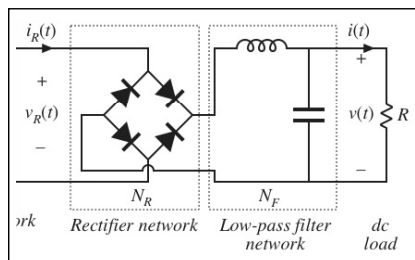
Fundamental component of  $i_R(t)$ :

$$i_{R1}(t) = \frac{4I}{\pi} \sin(\omega_s t - \varphi_R)$$

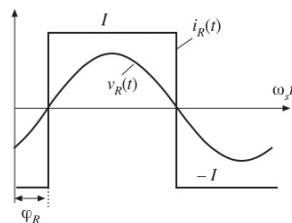
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## Model of uncontrolled rectifier with inductive filter network – output port



Output inductor volt second balance:  
dc voltage is equal to average  
rectified tank output voltage



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## Effective resistance $R_e$

Again define

$$R_e = \frac{v_{R1}(t)}{i_{R1}(t)} = \frac{\pi V_{R1}}{4I}$$

In steady state, the dc output voltage  $V$  is equal to the average value of  $|v_R|$ :

$$V = \frac{2}{T_s} \int_0^{T_s/2} V_{R1} \left| \sin(\omega_s t - \phi_R) \right| dt = \frac{2}{\pi} V_{R1}$$

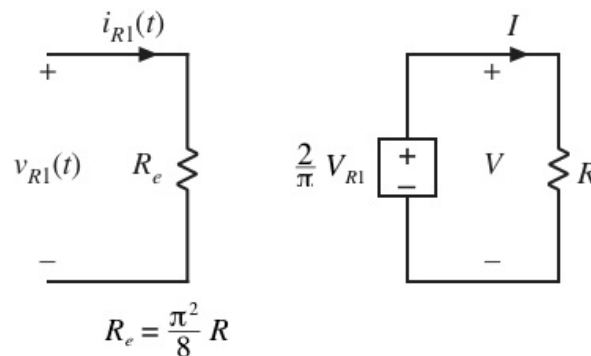
For a resistive load,  $V = IR$ . The effective resistance  $R_e$  can then be expressed

$$R_e = \frac{\pi^2}{8} R = 1.2337R$$

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## Equivalent circuit model of uncontrolled rectifier with inductive filter network

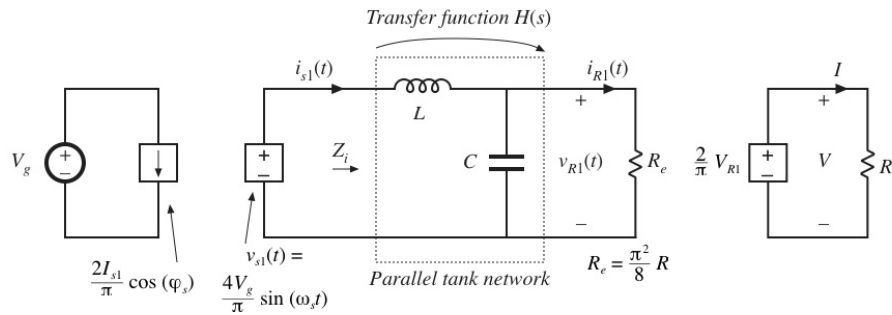


Output port modeled as a dependent voltage source based on rectified tank voltage, in contrast to SRC where output port is modeled as dependent current source based on rectified tank current

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## Equivalent circuit model Parallel resonant dc-dc converter



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## Ways to construct transfer function $H$ in terms of impedances

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Construction of  $Z_o$  – resonant (high Q) case  
 $C = 0.1 \mu\text{F}$ ,  $L = 1 \text{ mH}$ ,  $R_e = 1 \text{ k}\Omega$

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Construction of  $H = V / V_g$  – resonant (high Q) case  
 $C = 0.1 \mu\text{F}$ ,  $L = 1 \text{ mH}$ ,  $R_e = 1 \text{ k}\Omega$

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## Construction of $H$

## Dc conversion ratio of the PRC

$$\begin{aligned} M &= \frac{8}{\pi^2} \left\| \frac{Z_o(s)}{sL} \right\|_{s=j\omega_s} = \frac{8}{\pi^2} \left\| \frac{1}{1 + \frac{s}{Q_e \omega_0} + \left(\frac{s}{\omega_0}\right)^2} \right\|_{s=j\omega_s} \\ &= \frac{8}{\pi^2} \frac{1}{\sqrt{(1 - F^2)^2 + \left(\frac{F}{Q_e}\right)^2}} \end{aligned}$$

At resonance, this becomes  $M = \frac{8}{\pi^2} \frac{R_e}{R_0} = \frac{R}{R_0}$

- PRC can step up the voltage, provided  $R > R_0$
- PRC can produce  $M$  approaching infinity, provided output current is limited to value less than  $V_g / R_0$

## Comparison of approximate and exact PRC characteristics

*Exact equation:*

solid lines

*Sinusoidal approximation:*

shaded lines

