SPEEDAM 2010
International Symposium on Power Electronics,
Electrical Drives, Automation and Motion

A Simple Cuk Converter Derivated Two-Quadrant DC Motor Controller

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Abstract-- DC drives are still very important especially for low voltages and low power (e.g. in cars and robots). In this paper a Cuk-derived (one- and two-quadrant) converter is analyzed which makes it possible to control the voltage across the machine (and therefore the speed) from zero to three times of the input voltage. The model of the drive, the design of the devices, and some experimental results are given.

***Index Terms*—DC motor drives, power conversion, power electronics.**

I. INTRODUCTION

The basic converter is shown in Fig.1 [1] and consists of an inductor L, an active switch S, a capacitor C, and a passive switch D. The converter is a one-quadrant step-up step-down converter, which enables to drive a DC motor in one direction; controlled braking is not possible with this circuit. The positive pole of the output voltage is terminal 4. The positive pole of the voltage across the capacitor *C* is on the left side, the direction of the current through the inductor *L* is from left to right and for the armature inductor current from 3 to 4, which is also the direction of the source voltage of the machine. Neglecting the losses, with the duty ratio of the active switch *d,* and using the factthat the voltage-time areas across the inductors are zero in the steady-state case, the mean value of the output voltage $\bar{U}_2$ is

$$\bar{U}_2 = \frac{d}{1-d} \cdot U_1 \quad . \tag{1}$$

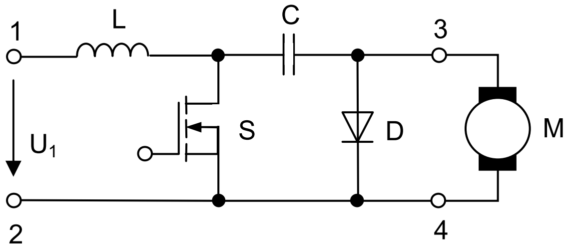

Fig. 1. One-quadrant step-up-down DC motor drive

Neglecting all losses, the source voltage of the machine in steady case is equal to the mean value of the output voltage. Therefore, the speed of the motor (with k_E as voltage constant of the machine) is

$$n_0 = \frac{1}{k_E} \cdot \frac{d}{1-d} \cdot U_1 \tag{2}$$

.

Figure 2 shows a two-quadrant converter derivated from Fig. 1. The converter consists now of a half-bridge with two active and two passive switches and again one inductor and one capacitor as storage elements. Now both current directions are possible and therefore controlled braking with feeding back energy into the input source is achieved.

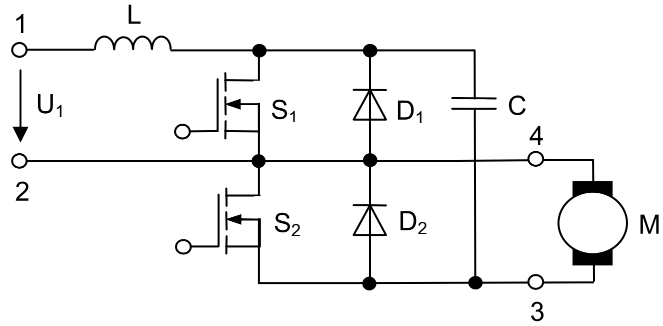

Fig. 2. Two-quadrant step-up-down DC motor drive

The equations for the output voltage and the speed are the same as in the case of the one-quadrant drive.

II. CONVERTER MODEL

For the DC motor the standard model based on [2] with a constant field flux which can be assumed if a permanent magnet or a constant extinction is used. For electrical machines also refer to [4]. R_A is the armature resistance, L_A the armature inductance, k_T and k_E the motor constants for torque and source voltage, J the inertia, and B the damping. The state variables are the inductor current i_L, the armature current i_{LA}, the capacitor voltage u_C, and the speed ω_M. The input variables are the input voltage u_1, and the work load (load torque) t_{WL}. The fixed forward voltage of the diode (the diode is modeled as a fixed forward voltage V_{FD2} and an additional voltage drop depending on the differential resistor of the diode) is included as an additional vector. The parasitic resistances are the on-resistance of the active switch R_S, the series resistance of the coil R_{SL}, the series resistor of the capacitor R_{SC}, and the differential resistor of the diode R_{SD2}.

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A. Converter States

In continuous inductor current mode there are two states. In state one the active switch is turned on (or the first active switch is turned on in case of the two-quadrant drive) and the passive switch is turned off leading to the state space description

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ \omega_M \end{pmatrix} = \begin{bmatrix} -\frac{R_{SL}+R_S}{L} & \frac{-R_S}{L} & 0 & 0 \\ -\frac{R_S}{L_A} & -\frac{R_{SC}+R_A+R_S}{L_A} & \frac{1}{L_A} & -\frac{k_E}{L_A} \\ 0 & -\frac{1}{C} & 0 & 0 \\ 0 & \frac{k_T}{J} & 0 & -\frac{B}{J} \end{bmatrix} \cdot \begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ \omega_M \end{pmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix}. \tag{3}$$

In state two the active switch is turned off (or the first active switch is turned off and the second switch is turned on in case of the two-quadrant drive) and the passive switch is turned off leading to the state space description

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ \omega_M \end{pmatrix} = \begin{bmatrix} -\frac{R_{SL}+R_{SC}+R_{SD2}}{L} & -\frac{R_{SD2}}{L} & -\frac{1}{L} & 0 \\ -\frac{R_{SD2}}{L_A} & -\frac{R_A+R_{SD2}}{L_A} & 0 & -\frac{k_E}{L_A} \\ \frac{1}{C} & 0 & 0 & 0 \\ 0 & \frac{k_T}{J} & 0 & -\frac{B}{J} \end{bmatrix} \cdot \begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ \omega_M \end{pmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix} + \begin{pmatrix} -\frac{1}{L} \\ -\frac{1}{L_A} \\ 0 \\ 0 \end{pmatrix} \cdot V_{FD2}. \tag{4}$$

When using two active switches in push-pull mode, the diode is shunted and V_{FD2} can be set to zero. R_{SD2} is then the on-resistance of the second switch.

B. State Space Averaging

Combining the two systems with the help of the state-space averaging method leads to a model which describes the drive in the mean

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_A \\ u_C \\ \omega_M \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} \end{bmatrix} \cdot \begin{pmatrix} i_L \\ i_A \\ u_C \\ \omega_M \end{pmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_{42} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix} + \begin{pmatrix} K_1 \\ K_2 \\ 0 \\ 0 \end{pmatrix} \cdot V_{FD2}. \tag{5}$$

For the elements of the matrixes one obtains

$$A_{11} = -\frac{R_{SL} + d \cdot R_S + (1-d) \cdot (R_{SC} + R_{SD2})}{L} \tag{6}$$

$$A_{12} = -\frac{d \cdot R_S + (1-d) \cdot R_{SD2}}{L} \tag{7}$$

$$A_{13} = \frac{d-1}{L} \tag{8}$$

$$A_{21} = -\frac{d \cdot R_S + (1-d) \cdot R_{SD2}}{L_A} \tag{9}$$

$$A_{22} = -\frac{R_A + d \cdot (R_S + R_{SC}) + (1-d) \cdot R_{SD2}}{L_A} \tag{10}$$

$$A_{23} = \frac{d}{L_A} \tag{11}$$

$$A_{24} = -\frac{k_E}{L_A} \tag{12}$$

$$A_{31} = \frac{1-d}{C} \tag{13}$$

$$A_{32} = -\frac{d}{C} \tag{14}$$

$$A_{42} = \frac{k_T}{J} \tag{15}$$

$$A_{44} = -\frac{B}{J} \tag{16}$$

$$B_{11} = \frac{1}{L} \tag{17}$$

$$B_{42} = -\frac{1}{J} \tag{18}$$

$$K_1 = \frac{d-1}{L} \tag{19}$$

$$K_2 = \frac{d-1}{L_A} \tag{20}$$

C. Linearization

Linearizing this system around the operating point enables us to calculate transfer functions for constructing Bode plots (cf. e.g. [3] which gives a detailed analysis of a different converter including signal flow graphs). The weighted matrix differential equation (5) representing the dynamic behavior of the converter is a nonlinear one. To use the possibilities of the linear control theory, a linearization is necessary. With capital letters for the operating point values and small letters for the disturbance around the operating point

$$i_L = I_{L0} + \hat{i}_L$$

$$i_{LA} = I_{LA0} + \hat{i}_{LA}$$

$$u_C = U_{C0} + \hat{u}_C$$

$$\omega = \Omega_0 + \hat{\omega}$$

$$u_1 = U_{10} + \hat{u}_1$$
$$t_{WL} = T_{WL0} + \hat{t}_{WL} \qquad (21)$$
$$d = D_0 + \hat{d}$$

one can calculate the linearized small signal model of the converter according to

$$\frac{d}{dt}\begin{pmatrix}\hat{i}_L\\ \hat{i}_{LA}\\ \hat{u}_C\\ \hat{\omega}_M\end{pmatrix} = \begin{bmatrix} A_{11L} & A_{12L} & A_{13L} & 0\\ A_{21L} & A_{22L} & A_{23L} & A_{24L}\\ A_{31L} & A_{32L} & 0 & 0\\ 0 & A_{42L} & 0 & A_{44L}\end{bmatrix}\cdot\begin{pmatrix}\hat{i}_L\\ \hat{i}_{LA}\\ \hat{u}_C\\ \hat{\omega}_M\end{pmatrix} +$$
$$+\begin{bmatrix} B_{11L} & 0 & B_{13L}\\ 0 & 0 & B_{23L}\\ 0 & 0 & B_{33L}\\ 0 & B_{42L} & 0\end{bmatrix}\cdot\begin{pmatrix}\hat{u}_1\\ \hat{t}_{WL}\\ \hat{d}\end{pmatrix} + \begin{pmatrix}K_{1L}\\ K_{2L}\\ 0\\ 0\end{pmatrix}\cdot V_{FD2} \qquad (22)$$

For the elements of the linearized matrixes one obtains

$$A_{11L} = -\frac{R_{SL} + D_0\cdot R_S + (1-D_0)\cdot(R_{SC}+R_{SD2})}{L} \qquad (23)$$

$$A_{12L} = -\frac{D_0\cdot R_S + (1-D_0)\cdot R_{SD2}}{L} \qquad (24)$$

$$A_{13L} = \frac{D_0 - 1}{L} \qquad (25)$$

$$A_{21L} = -\frac{D_0\cdot R_S + (1-D_0)\cdot R_{SD2}}{L_A} \qquad (26)$$

$$A_{22L} = -\frac{R_A + D_0\cdot(R_S + R_{SC}) + (1-D_0)\cdot R_{SD2}}{L_A} \qquad (27)$$

$$A_{23L} = \frac{D_0}{L_A} \qquad (28)$$

$$A_{24L} = -\frac{k_E}{L_A} \qquad (29)$$

$$A_{31L} = \frac{1-D_0}{C} \qquad (30)$$

$$A_{32L} = -\frac{D_0}{C} \qquad (31)$$

$$A_{42L} = \frac{k_T}{J} \qquad (32)$$

$$A_{44L} = -\frac{B}{J} \qquad (33)$$

$$B_{11L} = \frac{1}{L} \qquad (34)$$

$$B_{42L} = -\frac{1}{J} \qquad (35)$$

$$B_{13L} = -\frac{(-R_S + R_{SC} + R_{SD2})\cdot I_{L0} + (-R_S + R_{SD2})\cdot I_{LA0} + U_{C0}}{L} \qquad (36)$$

$$B_{23L} = -\frac{(-R_S + R_{SD2})\cdot I_{L0} + (-R_S - R_{SC} + R_{SD2})\cdot I_{LA0} + U_{C0}}{L_A} \qquad (37)$$

$$B_{33L} = -\frac{I_{L0} + I_{LA0}}{C} \qquad (38)$$

$$K_{1L} = \frac{D_0 - 1}{L} \qquad (39)$$

$$K_{2L} = \frac{D_0 - 1}{L_A} \qquad (40)$$

D. Stationary Equations

The stationary equations of the drive are described now. For the capacitor voltage one gets

$$U_{C0} = \frac{1}{1-D_0}\cdot\left\{\begin{array}{l} U_{10} - \left[R_S + (1-D_0)\cdot(R_{SC}+R_{SD2})\right]\cdot\\ \cdot(I_{L0} + I_{LA0}) - R_{SL}\cdot I_{L0} - (1-D_0)\cdot V_{FD2}\end{array}\right\}. \qquad (41)$$

If the converter is ideal (no losses in the devices) it works as a boost converter for the voltage across the capacitor with the well known equation

$$U_{C0} = \frac{U_{10}}{1-D_0}. \qquad (42)$$

From the second row of (5) an equation for the stationary angular speed can be derived according

$$\Omega_0 = \frac{D_0\cdot U_{C0} - R_A\cdot I_{LA0}}{k_E}$$
$$-\frac{1}{k_E}\cdot\left\{\begin{array}{l}\left[D_0\cdot R_S + (1-D_0)\cdot R_{SD2}\right]\cdot I_{L0} +\\ +\left[D_0\cdot(R_S + R_{SC}) + (1-D_0)\cdot R_{SD2}\right]\cdot I_{LA0} +\\ +(1-D_0)\cdot V_{FD2}\end{array}\right\}. \qquad (43)$$

Due to the fact that the mean value of the voltage across the armature means is

$$\bar{U}_2 = D_0\cdot U_{C0}\ , \qquad (44)$$

for ideal devices with U_{C0i} as the voltage across the capacitor the output voltage

$$\bar{U}_2 = D_0\cdot U_{C0i} = \frac{D_0}{1-D_0}\cdot U_1 \qquad (45)$$

can be achieved. The speed is now described by the well-known equation of the DC machine

$$\Omega_0 = \frac{\bar{U}_2 - R_A\cdot I_{LA0}}{k_E}\ . \qquad (46)$$

From the third row of (5) connection of the stationary armature current and the mean value of the inductor current can be expressed

$$I_{L0} = \frac{D_0}{1-D_0}\cdot I_{LA0}\ . \qquad (47)$$

From the fourth row of (5) one gets with the stationary work load T_{WL0} the mechanical equation

$$\Omega_0 = \frac{k_T\cdot I_{LA0} - T_{WL0}}{B}\ . \qquad (48)$$

III. DIMENSIONING

The values of the devices can be calculated according to the steady-state case in such a way that the converter works e.g. always in continuous inductor current mode and that the capacitor voltages change within a limited range Δu_C during one switching period. With

$$d = \frac{\bar{U}_2}{U_1 + \bar{U}_2} \quad . \tag{49}$$

and taking into account, that the armature current is nearly constant during the on time of the active switch $d \cdot T$ the value of the capacitor is

$$C = \frac{\bar{U}_2}{U_1 + \bar{U}_2} \cdot \frac{I_{LA0}}{\Delta u_C \cdot f} \quad . \tag{50}$$

Assuming a current ripple of ΔI the inductor must be

$$L = \frac{U_1 \cdot \bar{U}_2}{U_1 + \bar{U}_2} \cdot \frac{1}{\Delta I \cdot f} \quad . \tag{51}$$

For the dimensioning of the circuit the voltage rates for the semiconductors are important. The maximum voltage across the semiconductors is

$$U_{S,\max} = U_1 + \bar{U}_2 \quad . \tag{52}$$

IV. RESULTS

The DC motor is model MY1016 from Unite Motor. The parameters of the motor have been obtained by measurements and are

$R_A = 0{,}6\ \Omega$ (measured DC resistance)

$L_A = 16\ mH$ (measured at 50 Hz)

$i_{L0} = 1{,}22\ A$ (measured with no load)

$k_T = \frac{1{,}22\ Nm}{12{,}8\ A} = 0{,}095 \frac{Nm}{A}$ (specification and measured)

$k_E = \frac{36 - 1{,}22\ A \cdot 0{,}6\ \Omega}{\frac{3350 \frac{\mathrm{Rev}}{Min}}{60} \cdot 2 \cdot \pi} = 0{,}1 \frac{V \cdot s}{rad}$ (specification and measured)

$\mathrm{P}_0 = 36\ V \cdot 1{,}22\ A = 43{,}92\ W$

$P_{Mech} = P_0 - (1{,}22A)^2 \cdot 0{,}6\Omega = 43\ W$

$B = \frac{P_{Mech}}{\omega^2} = 0{,}00035 \frac{Nm \cdot s}{rad}$ (conversation of energy)

$J \approx \frac{B \cdot t_{Off}}{3} = 0{,}00073 \frac{Nm \cdot s^2}{rad}$ (measured full speed to stop).

The DC motor operates within the following operating limits:

$U_1 = 24\ V$

$f_S = 50\ kHz$ (*switching frequency*)

$n = 0 - n_{MAX} = 0 - 3350 \frac{\mathrm{Rev}}{\min}$

$T_{WL} \leq 1\ Nm$ (below motor rating).

Two coils PCV-2-104-10L from Coilcraft giving a total inductance of $50\mu H$ with an ESR of $16m\Omega$ and a current capacity of >20 A are used. As capacitor two FFB44E0476K $47\mu F/6A$ MKP capacitors from AVX are used giving a total capacitance of 94 μF with an ESR of $3.4 m\Omega$. The transistor is a STP40NF10 from ST microelectronics, which can carry a current of 50 A by a maximum drain source voltage of 100 V. The on-resistance is $28\ m\Omega$. Diodes STTH30L06D were chosen from ST. They have a current rating of 30 A, an on-resistance of $10\ m\Omega$, and a forward voltage of $750\ mV$.

Using a self-built test bench consisting of two DC motors measurements were performed. A schematic drawing of the test bench is shown in Fig. 3.

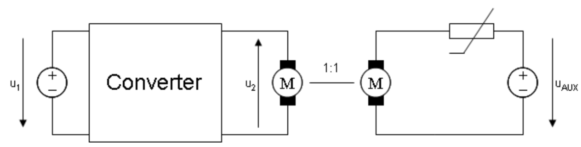

Fig. 3. Test bench for converter

The test bench consists of a primary power supply u_1, two DC motor which are mechanically coupled, a variable load resistor to control the torque of the second motor and an auxiliary power supply for the second DC motor. Please note that in our setup it is important to use power supplies which can source and sink current. A normal laboratory power supply would be destroyed, because in this setup energy can be fed back to the source. A good choice are lead-acid batteries which were used here.

Figure 4 and Fig. 5 show the output voltage of the converter and the input current of the converter (which is the same as the current through the converter inductor) in motor (the current is positive) and in generator mode (the current is negative, feeding energy back to the source) of the DC machine.

The current through the machine is nearly constant when observed at the oscilloscope, due to the much larger armature inductor compared to the converter inductor. Constant motor current leads to constant torque and to smooth speed of the machine.

Figur 6 and Fig. 7 show the armature current (first = upper trace), the inductor current (second trace), the armature voltage (fourth trace), and the voltage across the active switch (third trace) for the continuous and the discontinuous inductor current mode.

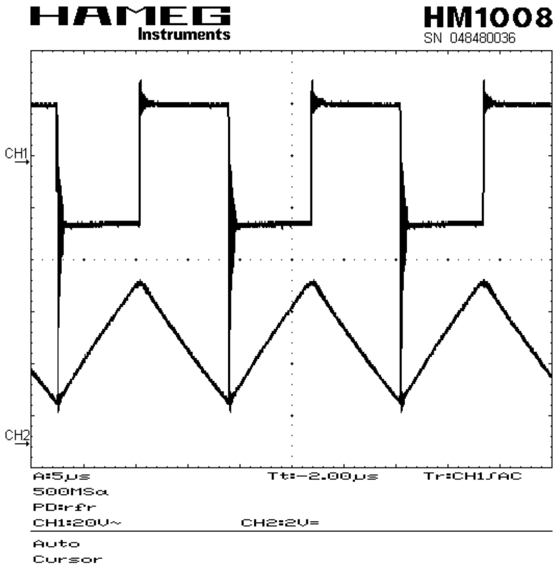

Fig. 4. Output Voltage u2 (CH1) and inductor current (1 V/A CH2), motor mode

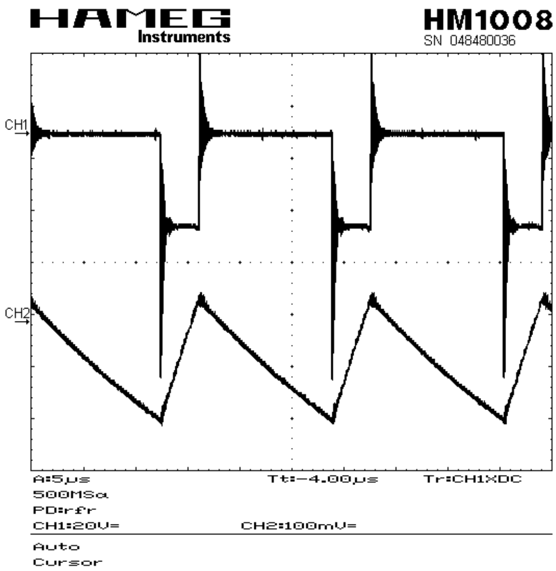

Fig. 5. Output Voltage u2 (CH1) and inductor current (1 V/A CH2), generator mode

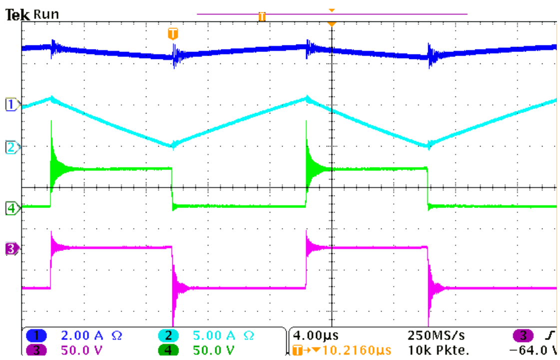

Fig. 6. Ch1: Armature current (first = upper trace), Ch 2: inductor current (second trace), Ch 3: armature voltage (fourth trace), Ch4: voltage across the active switch

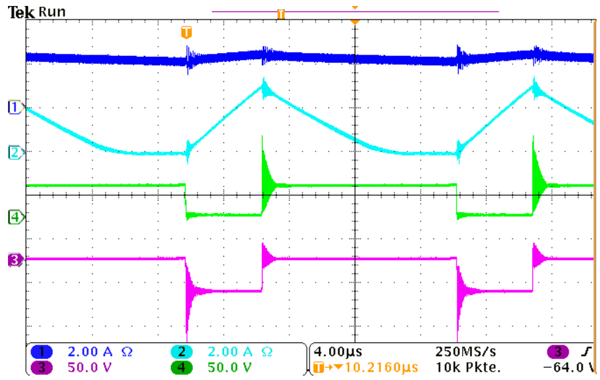

Fig. 7. Ch1: Armature current (first = upper trace), Ch 2: inductor current (second trace), Ch 3: armature voltage (fourth trace), Ch4: voltage across the active switch

V. Conclusions

A motor drive using a special buck-boost converter enabling lower and also higher voltage across the machine compared to the supply voltage was presented. The circuit for the two-quadrant drive can be implemented with a half-bridge and can be controlled by a half-bridge driver. It is therefore useful to drive the two switches in push-pull mode to achieve synchronous rectification and to achieve a better efficiency. The converter is especially useful for automotive applications [5] and small battery or solar voltaic fed drives.

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