DC Motor Speed Regulation Using Repetitive Control

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Abstract—**Repetitive control was applied to DC motor speed regulation in a scan device to reject the motor cogging torque. A discrete time prototype repetitive controller was designed and integrated into an existing feedback loop using the plug-in concept to reject the approximately periodic cogging torque under a constant reference speed. System identification was conducted to obtain the linear plant model. With the implemented repetitive controller, the periodic speed error was attenuated by near 75%. Also, a robustness criterion was developed to guarantee successful implementation on real systems.**

Index Terms*—*Cogging Torque, DC Motors, Feedback Control, Motor Drives, Repetitive Control, Robustness, Velocity Regulation.

I. Introduction

DC motors are used as the actuators in many industrial control applications. A DC motor consists of magnetized or permanent magnet stators and laminated iron core armatures. Armature construction contains a number of windings and teeth. As the armature rotates, the output torques continues to strengthen or weaken. This torque variation due to motor construction and magnetic field interaction is the cogging torque. The cogging torque introduces torque ripples which generates speed ripples. The cogging torque is position dependent and its period is determined by the number of magnetic poles in the stator and the number of teeth in the armature. When a DC motor is rotating at high speed, the motor and load moment of inertia filters the cogging torque and reduces the cogging torque effects. At low speeds the torque ripples effect can be large where controllers or other techniques are required to reject the cogging torque. Improved velocity regulation is critical in these cases.

Low cost DC motors usually have a significant amount of cogging torque which is a major source of the speed error for high quality speed regulation. This paper considers an existing DC motor speed control system with a proportional and integral (PI) controller. During the constant speed portion of the speed profile, the speed output demonstrates a noticeable amount of speed error. In this speed error, a periodic error component is identified and attributed to the motor cogging torque which has a periodic component. Thus it is clear that the existing PI controller is not sufficient for eliminating periodic speed variation caused by the nearly periodic cogging torque. In this situation, more advanced controllers are required to maintain acceptable level of performance by explicitly rejecting the periodic cogging torque component. Many other controllers may be designed for this purpose such as [9], however, the goal here is to apply a control technique, repetitive control, to reducing speed variations due to the periodic component in the cogging torque.

Repetitive control has shown to be an effective method to track or reject external periodic signals yielding low or no error in comparison with conventional controls. It has been successfully applied in disc drive servo systems [1], non-circular machining [5], motor engine control [3], etc. Repetitive control is based on the internal model principle. A well know example is to use the integral control to track step reference signal or reject step disturbance. In general, the dynamics of an external signal should be included in the controller for perfect tracking or rejection of the signal. Repetitive control may be applied directly in the controller design of a system, or it may be applied to an existing feedback control system to remove the periodic error. This allows the original controller to control aspects such as noise reduction and speed control, etc. with the addition of the repetitive control for periodic signals. This type of use is the plug-in concept.

For the speed regulation system considered here, a periodic component in the speed error exists under the PI control. This remaining periodic error is due to the motor cogging torque as alluded to earlier. The repetitive control is applied, with the goal to reduce this periodic speed error using the plug-in concept. The motor speed has two components under two respective inputs, one is the linear response of the motor under the control input, and the other is the response due to the motor cogging torque. The later component will be periodic

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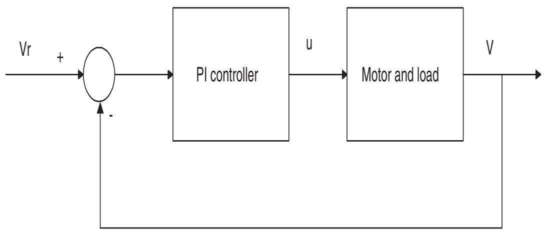

Fig. 1. PI Controller speed feedback loop

if the cogging torque is periodic. The former can be generated using a linear motor model. This linear motor model can be obtained through system identification tools using experimental data. With this treatment and modeling, the repetitive control is proved applicable and the plug-in repetitive controller is subsequently designed to minimize the periodic speed error portion while maintaining the stability of the feedback loop. The resulting controller consists of the PI controller and the added repetitive controller. This repetitive control improves the speed regulation and allows the use of less expensive motors for the cost reduction of the existing systems.

II. Problem Formulation

Consider the velocity control system in Fig. 1, where V is the speed of the load,V_r is the constant reference speed. It consists of a DC motor with a pulse width modulation (PWM) driver, a linear load, a digital encoder at the motor, and a PI controller. The digital encoder is used to generate quadrature signals representing shaft speed and position. The rotary motion of the motor shaft is translated into the linear motion of the load through a gear, pulley and belt transmission. With known gear ratio between the load and motor, the linear speed of the load can be calculated using the rotary encoder.

Consider the following DC motor governing equations

$$L\frac{di}{dt} + iR + k_b\omega = V_t, \tag{1}$$

$$J\frac{d\omega}{dt} = k_t i + T_d. \tag{2}$$

where ω is the motor speed, V_t is the motor terminal voltage, i is the winding current, k_b is the back-EMF constant of the motor, K_t is the torque constant, R is the terminal resistance, L is the terminal inductance, J is the motor and load inertia, and T_d is the disturbance torque. T_d consists of the cogging torque, T_{cog}, and the friction, T_f, $T_d = T_{cog} + T_f$. T_{cog} is position dependent. T_f is constant or slow varying.

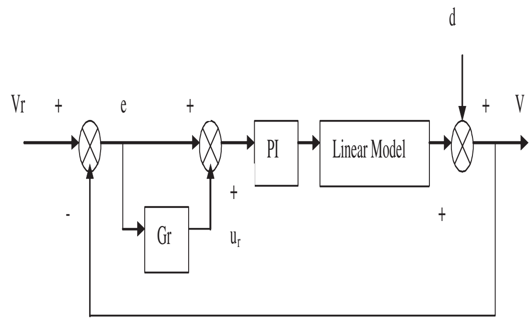

Fig. 2. Speed feedback loop with the repetitive controller

According to Eqs. 1 and 2, the velocity response in the Laplace domain is

$$\omega(s) = \frac{\frac{1}{k_b}}{t_m t_e s^2 + t_m s + 1}V_t(s) + \frac{\frac{1}{J}t_m(t_e s + 1)}{t_m t_e s^2 + t_m s + 1}T_d(s), \tag{3}$$

where $t_e = \frac{L}{R}$ is the electrical time constant, $t_m = \frac{RJ}{k_t k_b}$ is the mechanical time constant, and s is the Laplace variable. From Equation (3), the speed response has two sources, namely, the terminal voltage and the associated disturbance torque. The speed response due to the disturbance is the speed disturbance. The transfer function between the speed response and the terminal voltage can be obtained experimentally. To obtain this transfer function, large random voltage signals were generated and sent to the PWM drive to excite the motor, velocities were measured using the encoder. Based on the output velocities and respective input voltages, frequency response functions were calculated using the spectrum approach. The averaged frequency response function was used to obtain the system model for repetitive controller design through system identification. An accurate model is found crucial for the success of the controller design.

The existing velocity feedback is closed using a PI controller, Fig. 1. The linear speed of the load was regulated around constant reference speeds in real operation, with the speed disturbance, d, existing due to the disturbance T_d. Because of the feedback, the speed output of the closed-loop system stayed close to the constant reference speed. Thus, T_{cog} was nearly periodic and d contained a large periodic component in the steady state. A repetitive controller was inserted into the closed-loop system to reject this periodic component of d, shown in Fig. 2, where G_r is the repetitive controller.

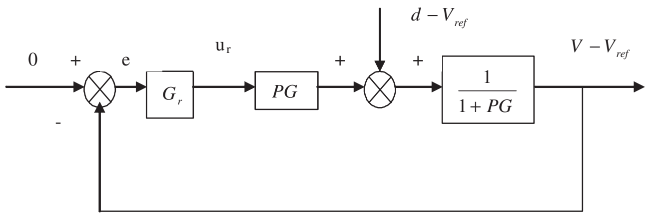

Fig. 3. Repetitive control feedback loop

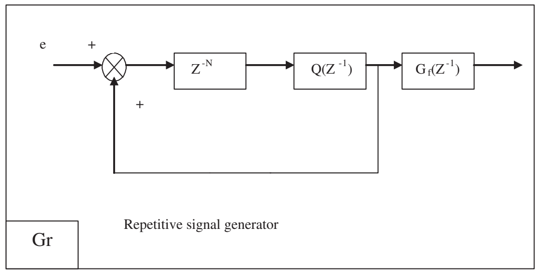

Fig. 4. Repetitive controller Structure

From Fig.2, we obtain the following equation

$$V - V_r = \frac{PG}{1+PG}u_r + \frac{1}{1+PG}(d - V_r) \qquad (4)$$

where u_r is the repetitive controller output, $P = \frac{\frac{1}{k_b}}{t_m t_e s^2 + t_m s + 1}$ is the transfer function of the motor from the terminal voltage to the motor speed, $G = k_p + k_i/s$ is the transfer function of the existing PI controller. From Eq. 4, the speed error comes from both d and V_r. Since V_r is constant and d is almost periodic, the repetitive controller G_r aims to reject the periodic component in d. Using Eq. 4, the repetitive controller design problem can be solved under the generalized repetitive control system structure [1], shown in Fig. 3.

III. Discrete-time Repetitive Controller

Fig. 4 shows the structure of the repetitive controller itself. It contains an internal positive feedback loop and a pre-filter $G_f(z^{-1})$. The positive feedback loop is a repetitive signal generator which produces a periodic signal having the same period as that of the periodic component in the speed disturbance d, the external periodic signal to be rejected, where N is the number of samples per period in Fig. 3. In the feed forward path of the positive feedback loop, there is a q-filter, $Q(z^{-1})$ which is included for robust stability of the feedback system [5],[3], [2]. Because, the identified system model was used for repetitive controller design, there is unmodeled dynamics which is not included in the model, especially in high frequency range. Besides, there are always nonlinear factors in the real system, like friction. This model and system mismatch may present a stability problem when the repetitive controller designed using a model is implemented. Usually, $Q(z^{-1})$ is a low-pass filter to accommodate the model uncertainty in mid to high frequency range. By doing so, the perfect tracking or rejection of a periodic signal is not available and the performance is traded for the stability. Note that the inclusion of the repetitive signal generator is based on internal model principle. The pre-filter $G_f(z^{-1})$ is used to ensure the nominal closed-loop system in Fig. 3 with the linear model is stable or has desired poles. It can be designed according to the rational transfer function approach by solving the Diophantine equation or using pole/zero cancellation and phase cancellation techniques such as the discrete-time prototype repetitive controller in [1]. In [1], $G_f(z^{-1})$ is a dynamic inversion of the system to be controlled by the repetitive controller.

A. *Prototype Repetitive Controller*

The discrete-time prototype repetitive controller gives $G_f(Z^{-1})$ based on pole/zero cancellation and phase cancellation proposed for zero phase error tracking controller (ZPETC) [6].

Let the plant for repetitive controller be

$$P(z^{-1}) = \frac{z^{-d_r}B^-(z^{-1})B^+(z^{-1})}{A(z^{-1})} \qquad (5)$$

where $B^-(z^{-1})$ contains zeros on or outside the unit circle and undesirable zeros in the unit circle, and $B^+(z^{-1})$ contains the rest of the zeros. All characteristic roots of $A(z^{-1}) = 0$ are inside the unit circle. Then, [1] proposes

$$G_f(z^{-1}) = k_r \frac{z^{d_r+nu}A(z^{-1})(z^{-nu}B^-(z))}{B^+(z^{-1})b}, \qquad (6)$$

where $b \geq \max_{\omega\in[0,\pi]}|B^-(e^{-j\omega})|^2$, $0 < k_r < 2$, nu is the order of $B^-(z^{-1})$, and $B^-(z)$ is obtained by replacing every z^{-1} in $B^-(z^{-1})$ by z.

B. *Closed-loop Robust Stability*

Normally, a linear plant model is used for the repetitive controller design. The linear model is only a linear approximation of the true plant. This leaves the time variability and the nonlinear behaviors of the plant uncounted. This mismatch between the plant model and the plant can cause instability of the control loop. $Q(z^{-1})$ is designed to improve the robust stability. The relationship between $Q(z^{-1})$ and the robust stability of the repetitive control structure used in [3] was given in Theorem 4.1 of [3]. Consider the repetitive controller structure in Fig. 4, and the repetitive control loop in Fig. 3, we developed a robust stability criterion similar to Theorem 4.1 in [3], which leads to the design of $Q(z^{-1})$. Based on Fig. 3, the equivalent plant for the repetitive controller is $P_r = \frac{PG}{1+PG}$. P_r has parametric uncertainties in R, L, k_t, etc. Cheap DC motor parameters normally have 10% to 15% tolerance specified in the motor manufacturer specifications.

Theorem 1. *Consider the repetitive control loop in Fig. 3 with the repetitive controller shown in Fig. 4. The pre-filter $G_f(z^{-1})$ and the plant $P_r(z^{-1})$ are asymptotically stable. The closed-loop system in Fig. 3 is robustly stable if and only*

$$|Q(z^{-1})(1-G_f(z^{-1})P_r(z^{-1}))| < 1, \textit{where } |z| = 1 \quad (7)$$

Proof: The proof is straight forward based on the small gain theorem [7]. Readers may refer to the proof of Theorem 4.1 in [3]. ■

IV. Design and Implementation

Now we present the design of the repetitive controller for the existing DC motor speed loop with a PI controller. The DC motor was a Mabuchi RK-370 motor which has a three pole armature and one pair of magnetic poles. The measured frequency response of the DC motor with PWM driver was obtained through the spectral analysis using a large random voltage sent into the PWM driver and the speed response of the motor measured by the optical encoder. The random voltage was bounded by $20volts$ and the speed response was sampled with a $1kHz$ sample rate. The measured frequency response can then be used to obtain a linear model of the motor and PWM driver using Matlab system identification routines. Fig. 5 shows the measured frequency response and the frequency response of an identified linear model. However, we did not directly use this model to design the repetitive controller.

The existing speed loop was regulated around $V_r = -0.04ips$. Fig. 6 shows a portion of the speed at the steady state. The magnitude of the fourier transform of the speed is shown in Fig. 7, where the low frequency oscillations are caused by the periodic component of d and the oscillations in the mid frequency range are caused by the feedback (around bandwidth, about $45Hz$)

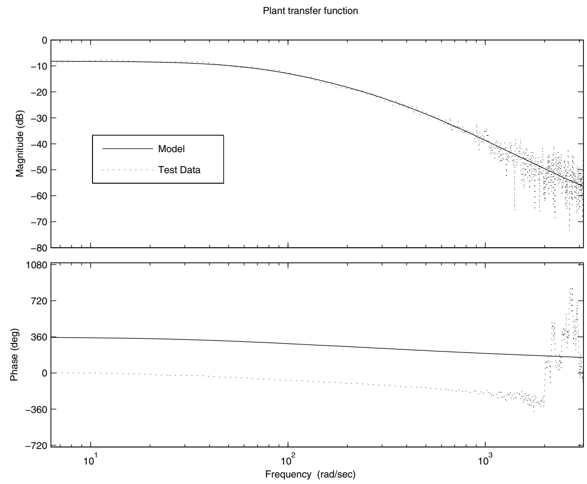

Fig. 5. Motor and PWM transfer functions: measured frequency response, dotted; model, solid.

and the non-periodic disturbances in the system. Fig. 8 shows those components under $10Hz$ in detail. The periodic speed error has the fundamental frequency of $1.285Hz$, or the smallest period of $0.778sec$. Because of $T_s = 0.001sec$, $N = 778$.

The second order q-filter used in [5] and [2] caused the closed-loop system to be unstable. It motivated the development of the stability theorem in this paper. According to Theorem 1, the q-filter was chosen as $Q(s) = \frac{1}{s/\omega_c+1}$. The cut-off frequency was chosen around five times the fundamental frequency by trila and error, $\omega_c = 40Rad/sec$. The high frequency roll-off in the closed-loop introduced by the q-filter also improves the robust stability against the unstructured uncertainty and the unmodelled dynamics at high frequencies.

The effective plant for the repetitive controller is $P_r = \frac{PG}{1+PG}$. A linear model of $P_r = \frac{PG}{1+PG}$ was identified in Matlab based on the product of the measured frequency response of P and that of the PI controller $G(s) = 10 + 500/s$. This approach worked better than calculating the model using the identified motor model, shown in Fig. 5, and the PI controller. The z-domain transfer function of P_r with the sample period $T_s = 0.001sec$ is

$$P_r(z^{-1}) = \frac{z^{-1}(0.01082 + 0.05065z^{-1} + 0.03443z^{-2})}{1 - 1.669z^{-1} + 0.8592z^{-2} - 0.09119z^{-3}}, \quad (8)$$

where the delay $d_r = 1$. There is one zero outside the unit circle. Therefore, $nu = 1$. $B^+(z^{-1}) = 0.01082(1 + 0.8254z^{-1})$, $B^-(z^{-1}) = 1 + 3.855z^{-1}$, $B^+(z) = 1 + 3.855z$, $b = 23.5710$. Let $k_r = 1$. The digital prototype repetitive controller, according to [1], is

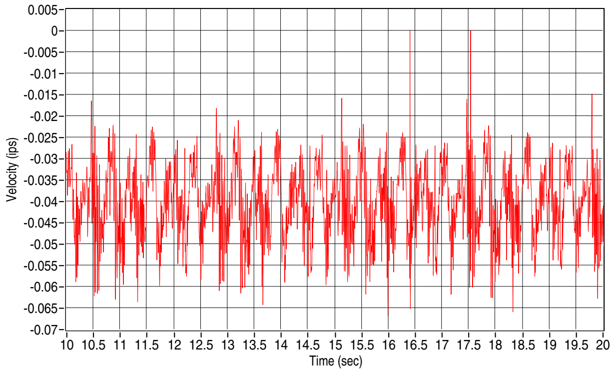

Fig. 6. Speed response of the existing speed loop

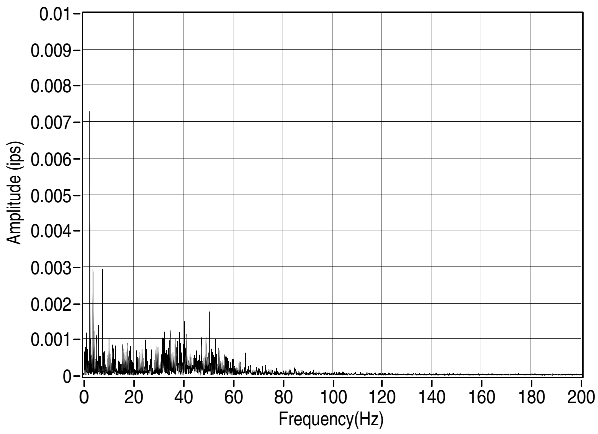

Fig. 7. FFT of the speed of the existing loop

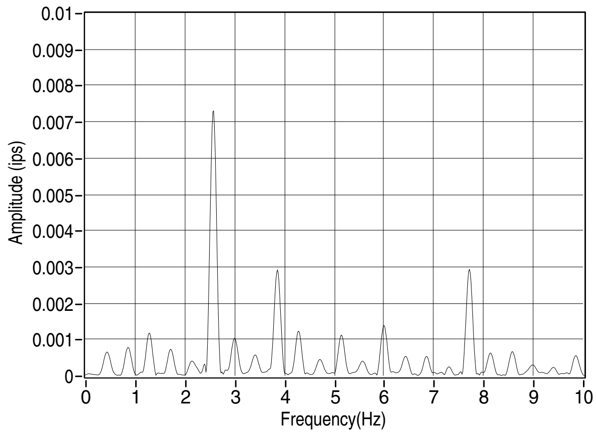

Fig. 8. Detailed speed frequency spectrum of existing speed loop

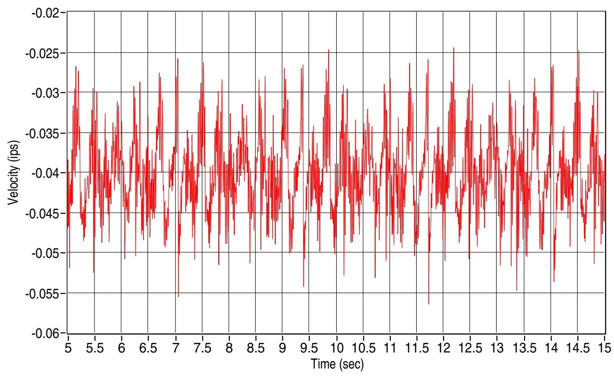

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Fig. 9. Speed response of the existing speed loop with repetitive control

$$G_f(z^{-1}) = \frac{15.1154z^2(1-1.4096z^{-1}+0.4263z^{-2}+0.1317z-3-0.0237z^{-4})}{1+0.8254z^{-1}} \quad (9)$$

The above designed repetitive controller was implemented digitally in LabVIEW with the real-time module and FPGA module. The speed response at the steady state is shown in Fig. 9, which shows a clear improvement. The standard deviation in ips went down from 0.0084 to 0.0048. The frequency spectrum of the speed response is shown in Fig. 10, which shows that the low frequency harmonics have been greatly suppressed and that the mid frequency range harmonics remain basically unchanged. Fig. 11 shows the low frequency harmonics in the speed response, i.e. $1.285Hz$, $2.57Hz$, $3.855Hz, \cdots$. Compare Fig. 7 to Fig. 10 and Fig. 8 to Fog. 11, it is clear that the periodic speed error has been significantly attenuated, however the non-periodic error has not been reduced. The introduction of the repetitive controller does not improve the rejection of non-periodic disturbances or improve the reference tracking (as a separated objective from the disturbance rejection). The sum of the amplitudes of the first six harmonics decreased from $0.0162ips$ to $0.00397ips$, a near 75% reduction of the periodic error is achieved by augmenting the repetitive controller into the existing loop.

The robustness of the repetitive control loop was examined based on Theorem 1. Bode plot of $Q(z^{-1})(1-G_f(z^{-1})P_r(z^{-1}))$ is shown in Fig. 12, where, based on Theorem 1, the repetitive control loop has a robustness gain margin over 3.

V. Conclusions

This paper presented an application of the repetitive control for DC motor speed regulation. In an existing DC motor speed loop, the motor cogging torque causes speed variations at the steady state. A periodic component exists in the speed error. A repetitive controller was designed based on the plug-in concept to attenuate the

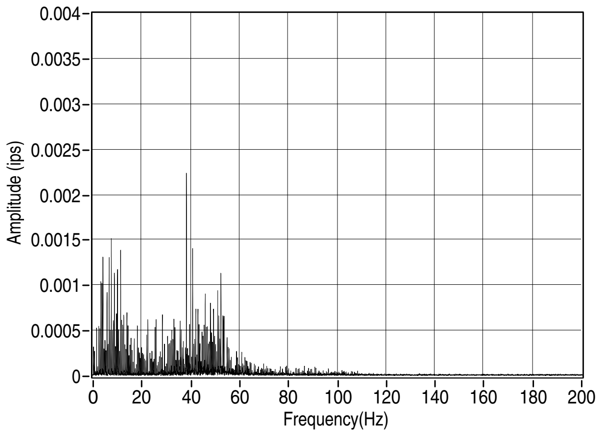

Fig. 10. FFT of the speed of the existing loop with repetitive control

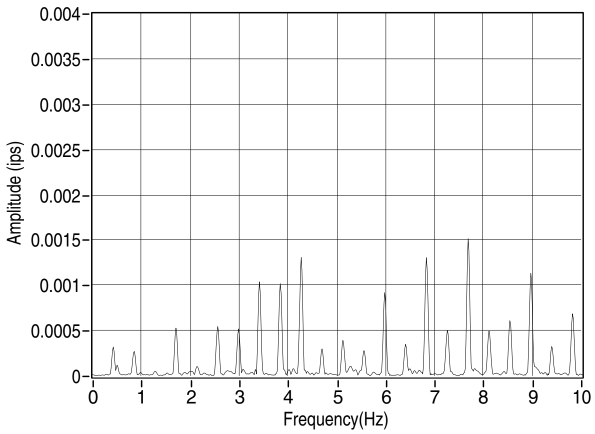

Fig. 11. Detailed speed frequency spectrum with repetitive control

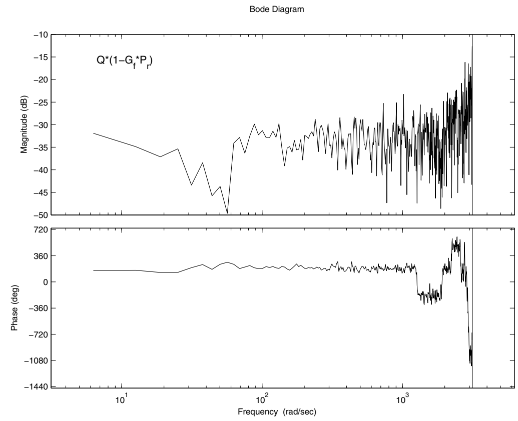

Fig. 12. Robustness of the repetitive control loop

periodic component of the speed error. For successful implementation, the modeling and robustness are the key factors. The measured frequency response was used to obtain the repetitive control plant model. A robustness criterion was developed to ensure the closed loop stability. Based on this criterion, a low-pass q-filter was designed to improve the stability robustness. After implementing the repetitive controller, a 75% periodic error attenuation was achieved on a real system.

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VI. BIOGRAPHIES

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