

Repetitive Tracking Control of DC Motors Using a Fuzzy Iterative Learning Controller

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Abstract—In this paper, we first study the theoretical design of a fuzzy iterative learning controller for sampled-time linear time invariant systems. Then we investigate the digital circuit implementation of the proposed controller with application to repetitive position tracking control of DC servo motors. The stability and convergence of the learning system are analyzed under uncertainties of initial state errors, input disturbance and output measurement error. We show that the learning error will converge to a residual set whose level of magnitude will depend on the size of the uncertainties. The learning error will asymptotically converge to zero if all the uncertainties disappear. To improve the learning performance, a concept of fuzzy learning gain is introduced which is designed based on the tracking error in the current and past iterations. In addition to the theoretical analysis, the fuzzy iterative learning controller is realized by a digital circuit to prove its feasibility. VHDL is used as the circuit design tool. The designed circuit is then downloaded into an FPGA chip. The chip is then applied to a repetitive position tracking control of DC motors to verify the learning effect. According to the experimental results, we prove that the theoretical design and its digital circuit are correct and feasible.

Keywords—Iterative learning control, fuzzy learning gain, FPGA, VHDL, DC motor

I. INTRODUCTION

Iterative learning control (ILC) has been known to be one of the most effective control strategies for control systems in dealing with repeated tracking control or periodic disturbance rejection. In general, the ILC system improves its control performance by a simple self-tuning process without using accurate system model and can be applied to many practical applications. To begin with, the D-type, P-type, or PID-type iterative learning controllers [1]–[6] were developed. Basically, the control input is directly updated by a learning mechanism using the information of error and input in the previous iteration. However, for real implementation of iterative learning controller, it is necessary to store the control data in the memory. Therefore, it is more practical to design and analyze the iterative learning system in discrete-time domain [7]–[12]. In the field of discrete-time iterative learning control,

the plants considered in the above works are modeled as discrete-time systems. As most of the controlling plants are continuous-time systems, a more interesting issue called sampled-data iterative learning controller, i.e., the plant is modeled as a continuous-time system and the controller is implemented as a discrete-time algorithm, was studied in the recent years [13]–[15].

In this paper, we study the design and analysis of a sampled-data iterative learning controller for linear time-invariant systems. The proposed sampled-data iterative learning controller is a simple one-step-ahead D type ILC as follows:

$$u_{i+1}(n\Delta) = u_i(n\Delta) + \phi_i e_i((n+1)\Delta)$$

where i denotes the number of iteration, Δ denotes the sampling period and u_i, ϕ_i, e_i represent the control input, learning gain and tracking error at the i th trial, respectively. Similar to most of the related works, the closed loop stability and error convergence can be guaranteed if a sufficient condition of learning gain is satisfied even there exist initial state error, input disturbance and output measurement error. As the learning gain is in general dependent on the system model, it is difficult to design it if plant model is unknown. The main feature of this proposed iterative learning controller is that a fuzzy system is introduced to realize the learning gain such that it can be tuned between trials in order to overcome the design difficulty and increase the learning speed. A similar concept has been proposed in author's previous result [14]. However, it used a concept of model approximation for learning gain and needed an off-line training procedure for initial setting of the fuzzy system. In general it is difficult in real applications when considering the issue of controller implementation.

In this paper, a simple fuzzy system is constructed to realize the learning gain. Due to its simplicity, it is possible to implement the learning gain by using a digital circuit. The algorithm for the adaptation of learning gain is given as follows:

$$\phi_{i+1} = \phi_i + FL_i$$

where FL_i is a updating term to the learning gain ϕ_i for the next iteration provided by a fuzzy system. The main design challenge is how to choose suitable input variables and corresponding fuzzy rules for the fuzzy system. After designing the learning gain ϕ_i , we implement the proposed fuzzy iterative learning controller by utilizing a digital

integrated chip for real applications. The digital circuit in the experiment consists of three main parts including a fuzzy learning gain tuning circuit, an iterative learning control circuit and a hardware control circuit. We use VHDL as a design tool to realize the digital circuit. The circuit is then downloaded into an FPGA chip and an experiment of repetitive position tracking control of DC motors is executed. According to the experimental results, we prove that the theoretical design and the corresponding digital circuit for the proposed fuzzy iterative learning controller are correct and feasible.

This paper is organized as follows. In section II, the problem formulation and design of fuzzy iterative learning controller are presented. The circuit design of the controller and circuit block diagram are discussed in section III. The hardware setup and experiment results are shown in section IV. Finally, a conclusion is made in section V.

II. THE DESIGN AND ANALYSIS OF FUZZY ITERATIVE LEARNING CONTROLLER

In this paper, we consider the following class of continuous-time system with input disturbance and output measurement noise as

$$\begin{aligned}\dot{x}(t) &= A_c x(t) + B_c u(t) + w_c(t) \\ y(t) &= C_c x(t) + \xi_c(t)\end{aligned}\quad (1)$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the output vector, $u(t) \in R^m$ is the input vector, $w_c(t) \in R^n$ is the input disturbance vector and $\xi_c(t) \in R^m$ is the output noise vector. If the sampling period is Δ and the input between two successive sampling instants satisfies $u(t) = u(n\Delta)$, $n\Delta \leq t < (n+1)\Delta$, $n = 0, 1, 2, \dots$, then the continuous-time system (1) can be digitized into the following sampled-data system

$$\begin{aligned}x((n+1)\Delta) &= Ax(n\Delta) + Bu(n\Delta) + w(n\Delta) \\ y(n\Delta) &= Cx(n\Delta) + \xi(n\Delta)\end{aligned}\quad (2)$$

with suitably define $A, B, C, w(n\Delta), \xi(n\Delta)$. Now, we suppose that the control system can work repeatedly over a finite time interval $[0, T]$ and the sampling instants $n\Delta = 0, \Delta, 2\Delta, \dots$, $N\Delta$ will satisfy $N\Delta = T$. In order to describe the feature of iterative learning control system more precisely, we rewrite (2) as

$$\begin{aligned}x_i((n+1)\Delta) &= Ax_i(n\Delta) + Bu_i(n\Delta) + w_i(n\Delta) \\ y_i(n\Delta) &= Cx_i(n\Delta) + \xi_i(n\Delta)\end{aligned}\quad (3)$$

where i denotes the index of iteration. The control objective is to design a sampled-data fuzzy iterative learning controller such that the system output $y_i(n\Delta)$ will follow the desired output $y_d(n\Delta)$. The desired output $y_d(n\Delta)$ is generated by a desired input $u_d(n\Delta)$ through the dynamic system given as follows,

$$\begin{aligned}x_d((n+1)\Delta) &= Ax_d(n\Delta) + Bu_d(n\Delta) \\ y_d(n\Delta) &= Cx_d(n\Delta)\end{aligned}\quad (4)$$

To achieve the control objective, we need the following assumptions on (3) and (4).

- (A1) The input-output coupling matrix CB is nonsingular.
- (A2) The initial state error, input disturbance and output noise are assumed to satisfy $\|x_d(0) - x_i(0)\| \leq \varepsilon_1$, $\|w_i(n\Delta)\| \leq \varepsilon_2$, $\|\xi_i(n\Delta)\| \leq \varepsilon_3$ for all i, n and for some positive constants $\varepsilon_1, \varepsilon_2, \varepsilon_3$.
- (A3) The desired output $y_d(n\Delta)$ and desired input $u_d(n\Delta)$ are bounded.

Based on the assumptions, a sampled-data fuzzy iterative learning controller is proposed as follow:

$$u_{i+1}(n\Delta) = u_i(n\Delta) + \phi_i e_i((n+1)\Delta) \quad (4)$$

where ϕ_i is the learning gain and $e_i((n+1)\Delta) = y_d((n+1)\Delta) - y_i((n+1)\Delta)$. The following theorem states the theoretical result for this iterative learning control system.

Theorem: Consider the system (1) with the digitalized sampled-data system (3) satisfying assumptions (A1)~(A3). If we apply the sampled-data fuzzy iterative learning controller (4) to the system (3) and the learning gain satisfies

$$\sup_{i \in \{1, 2, \dots, \infty\}} \|I - \phi_i CB\| = \rho < 1, \quad (5)$$

then we can guarantee that there exists a positive constant σ depending on the magnitude of $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that

$$\lim_{i \rightarrow \infty} \|y_d(n\Delta) - y_i(n\Delta)\| \leq \sigma, \forall n \in \{0, 1, 2, \dots, N\}$$

Furthermore, if $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$, then we have

$$\lim_{i \rightarrow \infty} \|y_d(n\Delta) - y_i(n\Delta)\| = 0, \forall n \in \{0, 1, 2, \dots, N\}$$

Proof: Please see [16].

Based on the above theorem and the convergent condition (5), we know that the convergent speed of the learning error will depend on the value of ρ . How to design the learning gain ϕ_i to get a better learning performance becomes an important design issue. In this paper, we apply a fuzzy system to implement the learning gain for single input single output plants. The learning algorithm for adaptation of learning gain is designed as follows:

$$\phi_{i+1} = \phi_i + FL_i$$

where FL_i is a updating term to the learning gain ϕ_i for next iteration provided by a fuzzy system. By defining the maximum absolute error at i th iteration as

$$E_i = \max_{n \in \{0, 1, 2, \dots, N\}} \|e_i(n\Delta T)\|,$$

we use maximum absolute error E_i and change of maximum absolute error $dE_i = E_i - E_{i-1}$ as input variables of the fuzzy system. FL_i is the output variable used to determine the change of learning gain for the next iteration. More precisely, three fuzzy sets are designed for input variables E_i and dE_i as shown in Figure 1 and Figure 2 respectively.

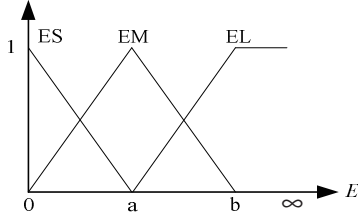


Figure 1: The fuzzy sets for input variable E_i

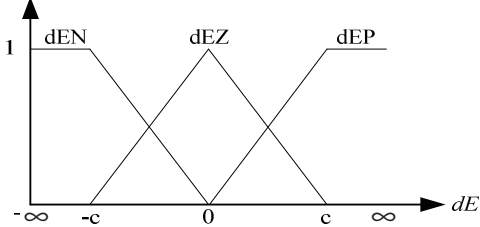


Figure 2: The fuzzy sets for input variable dE_i

In Figure 1, the fuzzy sets ES, EM and EL represent small, medium and large respectively. The parameters a, b are determined according to the specifications of real control problem. On the other hand, the fuzzy sets dEN, dEZ, dEP in Figure 2 represent negative, zero and positive respectively. The parameter c will be determined by trial and error. Finally, the consequent part of the fuzzy system is given as shown in Figure 3. All the values of NL, NM, ..., PL will also be determined by trial and error.

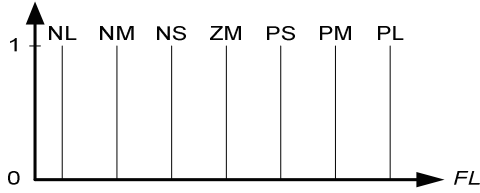


Figure 3: The singletons for output variable FL_i

The basic design concept of the rule base for fuzzy learning gain can be decided as follows. For example, If E_i is large, then we choose a positive FL_i to increase ϕ_i . If dE_i is a positive value, then we choose a negative FL_i to decrease the learning gain since the learning error along iteration axis is now divergent. If dE_i is a negative value which implies the learning error is convergent, then we should choose a positive FL_i to possibly increase the learning speed. By combining the above simple facts, it is easy to construct a suitable fuzzy rule base as follows:

- Rule 1: IF E is ES and dE is dEN, then FL is PS
- Rule 2: IF E is EM and dE is dEN, then FL is PM
- Rule 3: IF E is EL and dE is dEN, then FL is PL
- Rule 4: IF E is ES and dE is dEZ, then FL is ZM
- Rule 5: IF E is EM and dE is dEZ, then FL is PS
- Rule 6: IF E is EL and dE is dEZ, then FL is PM
- Rule 7: IF E is ES and dE is dEP, then FL is NS

Rule 8: IF E is EM and dE is dEP, then FL is NM

Rule 9: IF E is EL and dE is dEP, then FL is NL

All the values of NL, NM, ..., PL will be defined in the real implementation. However, in order to keep the learning gain to be bounded and away from zero, we set $0.01 \leq \phi_i \leq 1$ during the learning process.

III. THE CIRCUIT DESIGN OF FUZZY ITERATIVE LEARNING CONTROLLER

In this section, we are going to present a digital circuit for realization of the fuzzy iterative learning controller. The designed circuit is then applied to repetitive position tracking control of DC motors. The block diagram of fuzzy iterative learning control circuit and DC motor is shown in Figure 4. In this controller circuit, the block of *Hardware Control Circuit* is used to receive and decode the motor position from the motor encoder. The decoded motor position is then transmitted to the block of *Iterative Learning Control Circuit*. The block of *Iterative Learning Control Circuit* calculates E_i , dE_i and provides these data to the block of *Fuzzy Learning Gain Tuning Circuit* in order to determine FL_i . Based on FL_i , the iterative learning control algorithm in the block of *Iterative Learning Control Circuit* will calculate the control input $u_{i+1}(t)$ for next iteration. Finally, a PWM circuit inside the block of *Hardware Control Circuit* will send out the next control input $u_{i+1}(t)$ (or equivalently YA) to drive the DC motor in order to achieve the desired control objective. The final learning control result is also sent via a digital IO card (DIO card) to PC for the purpose of data storage.

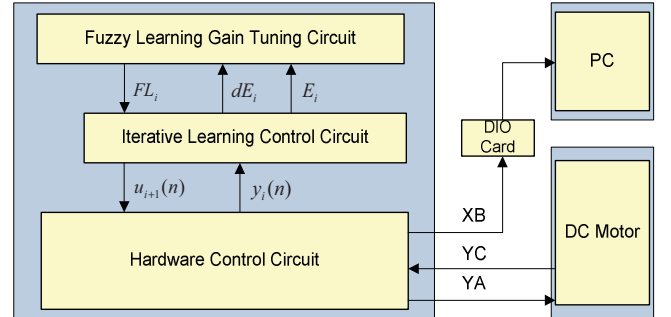


Figure 4: The block diagram of fuzzy iterative learning controller with application to repetitive position tracking control of DC motors.

Next, the detailed internal block diagrams for each circuit, i.e., *Hardware Control Circuit*, *Iterative Learning Control Circuit*, and *Fuzzy Learning Gain Tuning Circuit* are shown in Figure 5, Figure 6 and Figure 7 respectively. All the signals defined in the block diagrams of these Figures are utilized to write the VHDL code. We will not discuss these signal definitions here for the sake of simplicity.

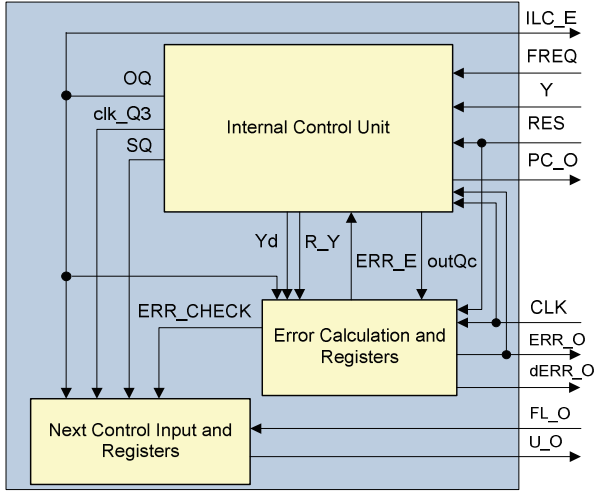


Figure 5: The internal block diagram of Iterative Learning Control Circuit

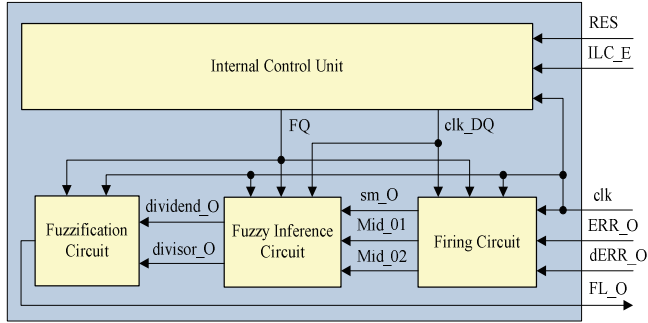


Figure 6: The internal block diagram of Fuzzy Learning Gain Tuning Circuit

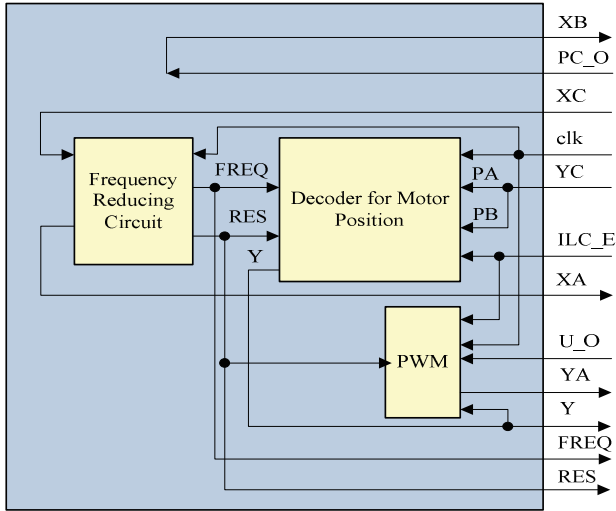


Figure 7: The internal block diagram of Hardware Control Circuit

IV. HARDWARE SETUP AND EXPERIMENT

In order to demonstrate the feasibility of the proposed fuzzy iterative learning control circuit, we use VHDL as a design tool to write the circuit code. The circuit code is compiled by using ISE and then downloaded into an FPGA chip to repetitively control the DC servo motor. The

experimental data is sent by an interface card to PC for storage. The hardware setup for the experiment is shown in Figure 7. It includes a DC servo motor, an FPGA control board and a PC.

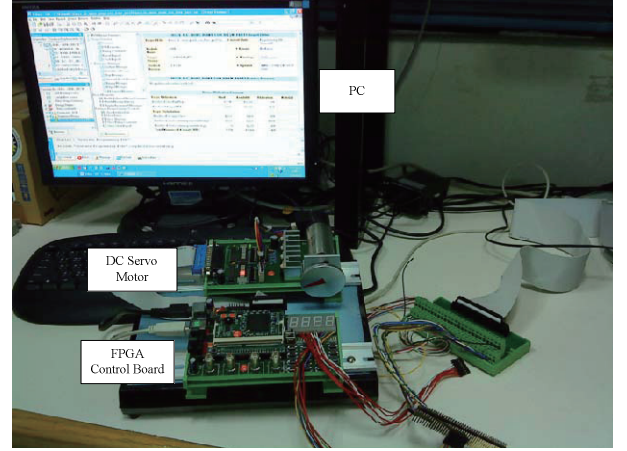


Figure 8: The hardware setup for experiment

In this experiment, the desired position trajectory is set to be a straight line, i.e., the motor speed is constant. The time interval for each trial is $[0, 0.42]$. There are 20 sampling instants during this learning time interval. The number of iteration for each experiment is 500. Figure 9 shows the maximum absolute error E_i versus the number of iteration. It is noted that the value of 1000 in the vertical axis of Figure 9 is equivalent to 160° . After about 120 trials of repetitive learning, E_i converges to a neighborhood of 50 which is equivalent to 8° . The sampled output $y_{500}(n\Delta)$ at the 500th iteration and the desired output $y_d(n\Delta)$ are shown in Figure 10. In order to make a comparison between fuzzy iterative learning controller and constant learning gain iterative learning controller, we fixed the initial learning gain and repeated the experiment. The control result of E_i versus the number of iteration is now shown in Figure 11. In general, the convergent speed of E_i in this case is much slower. This proves the effectiveness of using fuzzy learning gain for this iterative learning control problem.

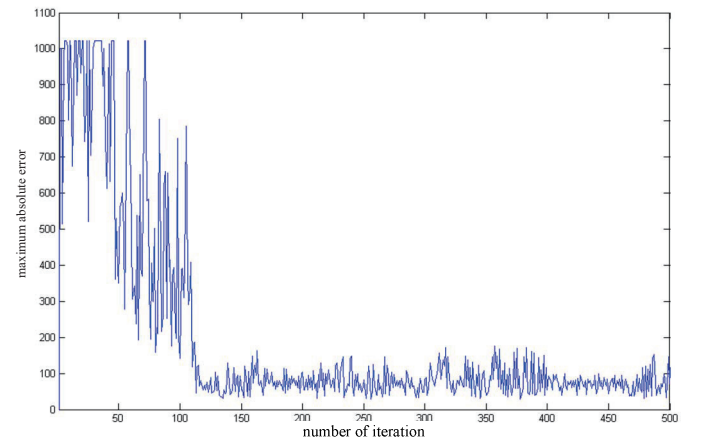


Figure 9: maximum absolute error E_i versus the number of iteration

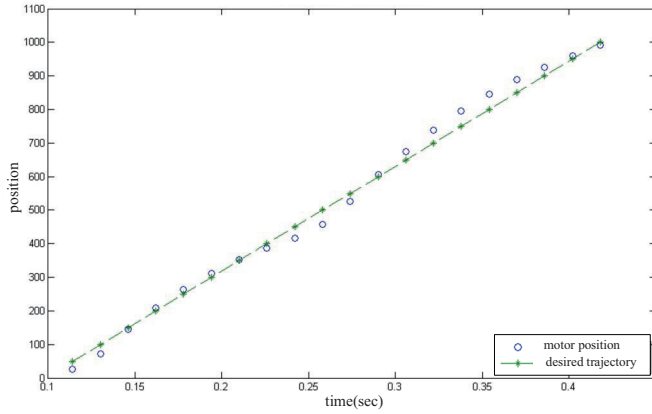


Figure 10: $y_{500}(n\Delta)$ and $y_d(n\Delta)$ versus time t

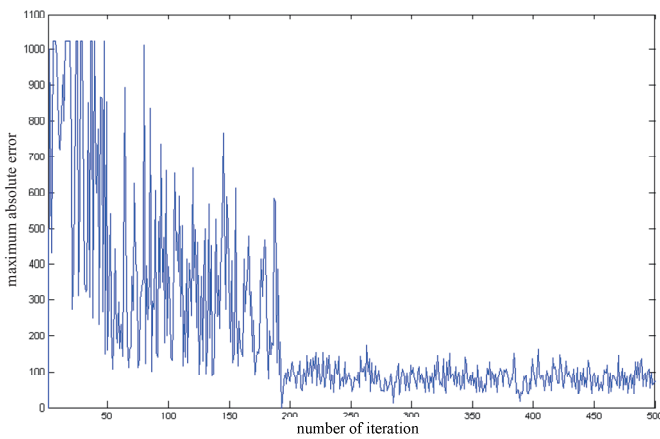


Figure 11: maximum absolute error E_i versus the number of iteration (constant learning gain)

V. CONCLUSION

In this paper, we propose a fuzzy iterative learning controller for repetitive output tracking control of linear time-invariant systems. In the first part of this paper, we study the convergence of the iterative learning control system. It is shown that under a sufficient condition for the learning gain, the asymptotic convergence the tracking error will be guaranteed even there exists uncertainties of input disturbance, output measurement noise and initial resetting error. We also presented a fuzzy mechanism to design the learning gain in order to increase the convergent speed. Based on the derived results, we investigated the digital circuit design for the proposed fuzzy iterative learning controller in the second part of this paper. The digital circuit consists of three main blocks which are named Hardware Control Circuit, Iterative Learning Control Circuit, and Fuzzy Learning Gain Tuning Circuit respectively. The designed circuit is realized by using the

VHDL code and implemented in an FPAG chip. An experiment of repetitive position tracking control of Dc servo motor is executed to demonstrate the effectiveness of the proposed controller.

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