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Optimization of IMC-PID Tuning Parameters for Adaptive Control: Part 1

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Abstract

This paper describes Part 1 of a two-part strategy for robust certainty equivalence adaptive PID control. In Part 1 we develop the strategy for simple PID controller tuning which maximizes the bandwidth subject to gain and phase margin constraints. An implementation of the non-adaptive strategy in a real time environment using model estimation based on non-convex optimization is described. The test shows the potential of the tuning method. In the next part, which due to space limitations could not be included here, we describe adaptive implementation.

Keywords: IMC-PID controller, robustness, optimization, adaptive control.

1. Introduction

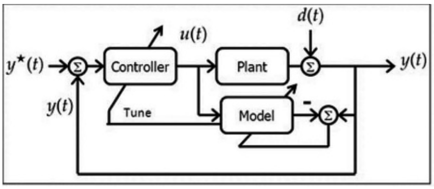
Surveys on PID control [1] show that the majority of PIDs are left on factory settings. This observation shows that the PID has inherent robustness properties when it is applied to typical chemical processes. However, one might suspect that significant gains could be achieved if the controllers were optimized since the accumulated effect of millions of poorly tuned PIDs may be large.

Many methods have been proposed for on and off line PID tuning. Most of these are not suitable for adaptive control since they do not tune performance subject to robust performance. For example, classical methods for PID tuning taught in undergraduate classes on process control (e.g. [2]) do not include any tuning knobs. In this respect the Internal Model Control (IMC) tuning procedure by Rivera et al. [3] is better suited since includes the filter parameter τ_c which tune closed loop performance [4,5] to achieve robust stability.

In this paper we develop a tuning procedure for IMC which minimizes the filter-parameter to maximize bandwidth subject to pre-specified gain and phase margin [6-9]. An analytical solution is developed for the first order dead time process. In the next section we show that the approach meshes with certainty equivalence adaptive control.

2. Robustness of Certainty Equivalence Adaptive Control

A compelling paradigm for adaptive control was developed in the 1970s under the banner of *Certainty Equivalence Adaptive Control*. In this approach the parameters of a transfer function model $G_p(s)$ is estimated in real time by matching the model to process data. The resulting model is used to update the controller as shown in Figure 1A. The figure does not highlight that it is critical update the controller tuning to achieve robust performance. This property is better illustrated in the equivalent diagram Figure 1B where the adaptive system is viewed as two composite systems. The first system shows the controller in feedback with the model. The second system shows how the model adapts to the plant. Robust performance is achieved when the nominal feedback system on the left does not generate high frequency inputs.



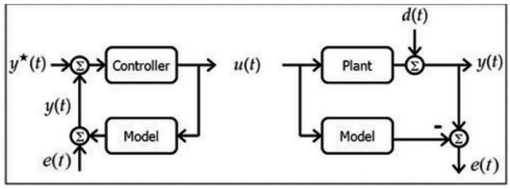


Figure1. Figure 1A on the left shows the classical representation of the certainty equivalence approach to adaptive control. Figure 1B on the right shows the structure used in stability analysis.

The controller tuning only needs to be robust with respect to unstructured (additive) uncertainty. Closed loop stability is ensured if $|G(s)_{cl}\Delta(s)| < 1$ where $\Delta(s)$ is the model uncertainty. It follows that the PID controller should be tuned so that it has pre-specified gain and phase margins to compensate for given unstructured uncertainty. In this sense adaptive control achieves better performance and robustness than robust control theory alone since we do not need to tune for parametric uncertainty.

3. PID Control with Pre-specified Gain and Phase Margins

The IMC design achieves optimal performance robustness by minimizing τ_c subject to gain margin and phase margin constraints. Thus we want to solve the problem

$$\begin{aligned} &min \quad \tau_c \\ &s.t. \quad A_m \geq A_m^* \\ &\qquad\; \Phi_m \geq \Phi_m^* \\ &\qquad\; \tau_c > 0 \end{aligned}$$

where A_m^* is the desired phase margin (typically $1/3\pi$) and Φ_m^* is the desired gain margin (typically 1.7). Denoting the process and controller transfer functions by $G_p(s)$ and $G_c(s)$ we get

$$A_m = \frac{1}{|G_c(j\omega_p)G_p(j\omega_p)|} \tag{1}$$

$$arg\left[G_c(j\omega_p)G_p(j\omega_p)\right] = -\pi \tag{2}$$

$$\Phi_m = arg\left[G_c(j\omega_g)G_p(j\omega_g)\right] + \pi \tag{3}$$

$$\left|G_c(j\omega_g)G_p(j\omega_g)\right| = 1 \tag{4}$$

where ω_p, ω_g are the phase and gain crossover frequency. Below we simplify this problem.

4. Tuning algorithm

A first-order-plus-time-delay (FOPTD) plant model models process control systems in this paper. The first order Pade approximation gives

$$G_p(s) = \frac{K_p}{\tau s+1}e^{-\theta s} \cong \frac{K_p}{\tau s+1}\frac{1-\frac{1}{2}\theta s}{1+\frac{1}{2}\theta s} \tag{5}$$

The IMC-PID formula for FOPTD is given as [9]:

$$G_c(s) = \frac{\left(1+\frac{\theta}{2}s\right)(\tau s+1)}{K_p\left(\tau_c+\frac{\theta}{2}\right)s} \tag{6}$$

$$K_c = \frac{1}{K_p}\frac{2\left(\frac{\tau}{\theta}\right)+1}{2\left(\frac{\tau_c}{\theta}\right)+1} \tag{7}$$

$$T_i = \frac{\theta}{2} + \tau \tag{8}$$

$$T_d = \frac{\tau}{2\left(\frac{\tau}{\theta}\right)+1} \tag{9}$$

The open-loop and closed-loop transfer function are given by

$$G_{ol}(s) = G_p(s)G_c(s) = \frac{\left(1+\frac{\theta}{2}s\right)}{\left(\tau_c+\frac{\theta}{2}\right)s}e^{-\theta s} \tag{10}$$

$$G_{cl}(s) = \frac{G_{ol}(s)}{1+G_{ol}(s)} \cong \frac{1-\frac{1}{2}\theta s}{\tau_c s+1} \tag{11}$$

Substituting Eq. (10) into (1) – (4) results in

$$A_m = \frac{\left(\tau_c+\frac{\theta}{2}\right)\omega_p}{\sqrt{\left(\frac{\omega_p\theta}{2}\right)^2+1}} \tag{12}$$

$$arctan\left(\frac{\omega_p\theta}{2}\right) - \omega_p\theta = -\frac{\pi}{2} \tag{13}$$

$$\Phi_m = arctan\left(\frac{\omega_g\theta}{2}\right) - \omega_g\theta + \frac{\pi}{2} \tag{14}$$

$$\omega_g = \frac{1}{\sqrt{\tau_c{}^2+\tau_c\theta}} \tag{15}$$

Thus the optimization problem becomes

$$\begin{aligned}
&min \quad \tau_c\\
&s.t. \quad \frac{\left(\tau_c+\frac{\theta}{2}\right)\omega_p}{\sqrt{\left(\frac{\omega_p\theta}{2}\right)^2+1}} \geq A_m^*\\
&\qquad arctan\left(\frac{\omega_p\theta}{2}\right) - \omega_p\theta = -\frac{\pi}{2}\\
&\qquad arctan\left(\frac{\omega_g\theta}{2}\right) - \omega_g\theta + \frac{\pi}{2} \geq \Phi_m^*\\
&\qquad \omega_g = \frac{1}{\sqrt{\tau_c{}^2+\tau_c\theta}} \qquad \tau_c,\ \omega_p,\ \omega_g > 0
\end{aligned}$$

This problem has 3 variables (τ_c, ω_p, ω_g) and 3 parameters (θ, A_m^*, Φ_m^*). However, the problem needs the time delay information, and an explicit analytical relation between the tuning parameter, τ_c, gain margin, A_m, and phase margin, Φ_m can be found from Eq. (12) – (15). Solving Eq. (12) gives a constant and for convenience denoted as α:

$$\omega_p\theta = \alpha = 2.458 \tag{16}$$

By substituting Eq. (12), (15) and (16) into (14) and using Eq. (12) and (16) we express Φ_m and τ_c as functions of A_m so that

$$\Phi_m = \frac{\pi}{2} - \frac{2}{\sqrt{A_m^2\left(1+\frac{4}{\alpha^2}\right)-1}} + arctan\frac{1}{\sqrt{A_m^2\left(1+\frac{4}{\alpha^2}\right)-1}} \tag{17}$$

$$\tau_c = \frac{\theta}{2}\left(A_m\sqrt{1+\frac{4}{\alpha^2}}-1\right) \tag{18}$$

The plot in Fig. 2 shows that for a given process that the gain and phase margins are coupled so that only one of the two constraints will be active.

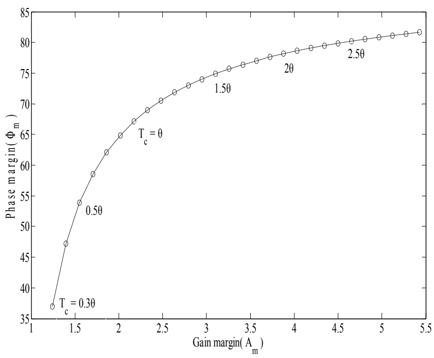


Figure 2. A_m vs. Φ_m respect to τ_c

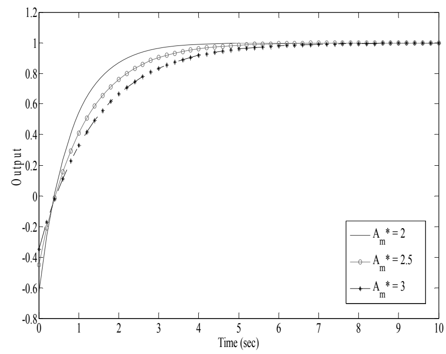


Figure 3. Closed-loop step responses.

The bandwidth ω_{BW} is is defined as the frequency at which

$$AR_{cl}(j\omega_{BW}) = |G_{cl}(j\omega_{BW})| = \frac{1}{\sqrt{2}} \tag{19}$$

Substituting Eq. (11) and (18) into (19) gives

$$\omega_{BW} = \frac{2}{\theta\sqrt{A_m{}^2\left(1+\frac{4}{\alpha^2}\right)-2A_m\sqrt{1+\frac{4}{\alpha^2}}-1}} \tag{20}$$

The relation between ω_{BW} and A_m provides an estimate for closed-loop performance. Now we can propose a tuning method based on gain margin specification. According to Eq. (12), A_m is proportional to τ_c. So for given A_m^*, the minimal τ_c can be located when A_m equals to its minimal value, A_m^*. Then the PID controller parameters, corresponding phase margin and bandwidth can be calculated from Eq. (7) – (9), (17) and (20). Fig. 3 shows the simulation result for the closed-loop step responses of the controller designed by different gain margin specifications. As A_m^* getting larger, the performance of the controller gets more conservative. Substituting into (20) gives

$$A_m^* > \frac{\sqrt{2}+1}{\sqrt{1+\frac{4}{\alpha^2}}} = 1.87 \tag{21}$$

5. Real-time experiments

The experimental set up comprises of a countercurrent shell and tube heat exchanger. Hot water flows through the shell side and the cold water flows through the tube side. Temperatures and flow rates are recorded at a sampling time of 0.1 seconds. The FOPTD model

$$G_{cc}(s) = \frac{-0.365}{1.63s+1} e^{-3s} \tag{22}$$

was identified using global optimization as shown [10]. Fig. 3 shows the response of the system output for a set-point change followed by a load disturbance on hot water flow rate change. The controller gives quick set-point response and well disturbance rejection.

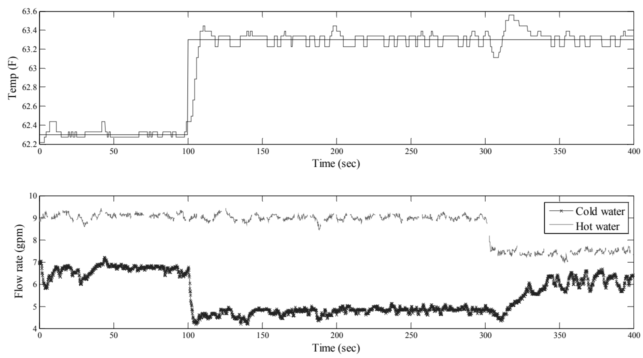


Figure 3. Real-time experimental result. The precision is limited by 8bit AtoD conversion

6. Conclusions

An optimization problem for the IMC-PID controller suitable for adaptive control is developed. The analytical solution for the optimization of bandwidth from gain and phase margin constraints is derived. We show that that gain and phase margins are coupled. The real time experiment result gives satisfied satisfactory set-point response and disturbance rejection. The proposed approach is ideally suited for application to adaptive control since the tuning criteria (gain margin and phase margin) are based on closed rather than open-loop performance.

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