PHYS327
Assignment #3:
Controlling the Speed of a DC motor

Thomas Bolstad, Øystein Bachmann Strand

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1 introduction

This assignment is meant to serve as an introduction of how to control a DC motor. and carrying out simple experiments. Because of this, the assignment is split into several smaller parts which in this text will be treated individually initially, and will be followed by a discussion about the sum of all these exercises. The sections and their respective lengths will vary depending on the nature of the different exercises.

2 Exercise 3-a: Process characteristics

2.1 Introduction

In this exercise, the student will

- Create a LabVIEW VI which communicates with the DC motor through the MIO board, measuring the output voltage fro it, while sending n input voltage, which is increased in steps from 0 V to 5 V and then back to 0 V.
- Plot the result graphically, i.e the number of rotations per minute (in %) should be plotted as a function of the drive signal u (in %.) The plotted data should demonstrate possible hysteresis.
- Comment on the results.

2.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS insrumentation, design and prototyping platform.

2.3 Results

To get an effective plot of the hysteresis, we would have to adjust the output to the DC-motor as fast as possible. The DC-motor does take some time to adjust it's rotation to the new output voltage, so balancing between those two conditions, we managed to get a good plot demonstrating clearly the hysteresis of the system.

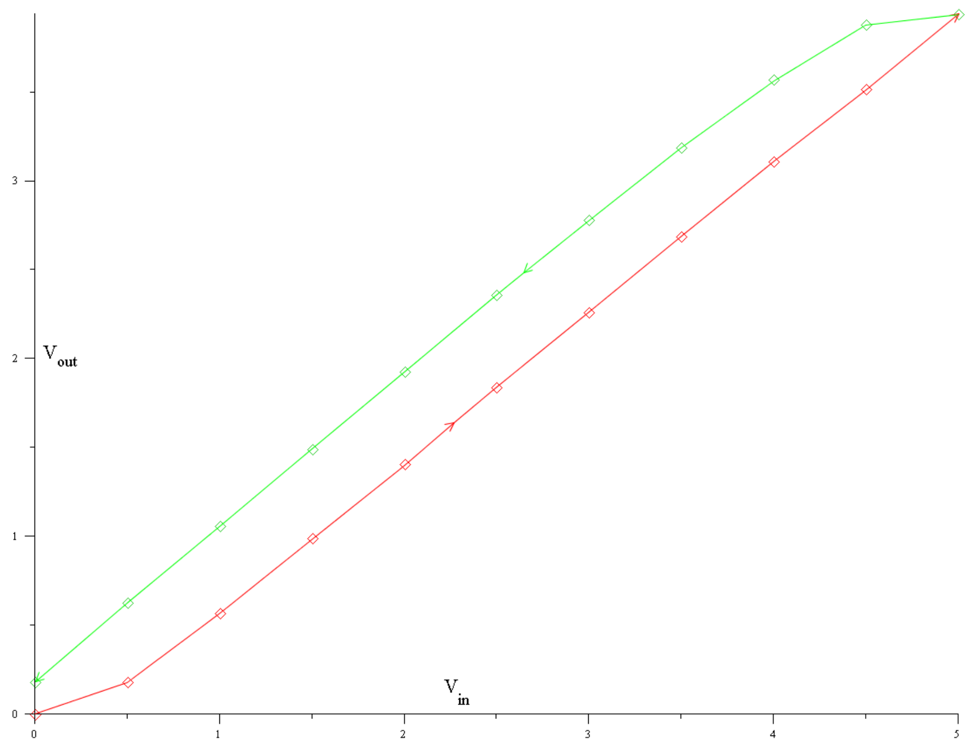

Figure 1: The output voltage (y-axis) vs the input voltage (x-axis), demonstrating the hysteresis of the DC-motor, when the output voltage to the DC motor is going up (red) versus going down(green.)

As can be seen, the hysteresis is very linear, almost to the point of being a school book example of hysteresis. This illustrates very clearly that the DC motor does take a certain time to adjust to the new voltage, and measurements should only be taken a certain while after the output voltage to the DC-motor has been adjusted.

2.4 LabVIEW

This LabVIEW VI is an automated one. Before entering the while loop, the input and output are initalized. The output is then through a combination of formula node scripts and boolean case structures, automated to rise rapidly from 0 V to 5 V and then back to 0 V again. The input is all the while measured and averaged, and written into a file. Then, when the program is outside the while loop, the measurements are stopped, and the input reset to 0 V.

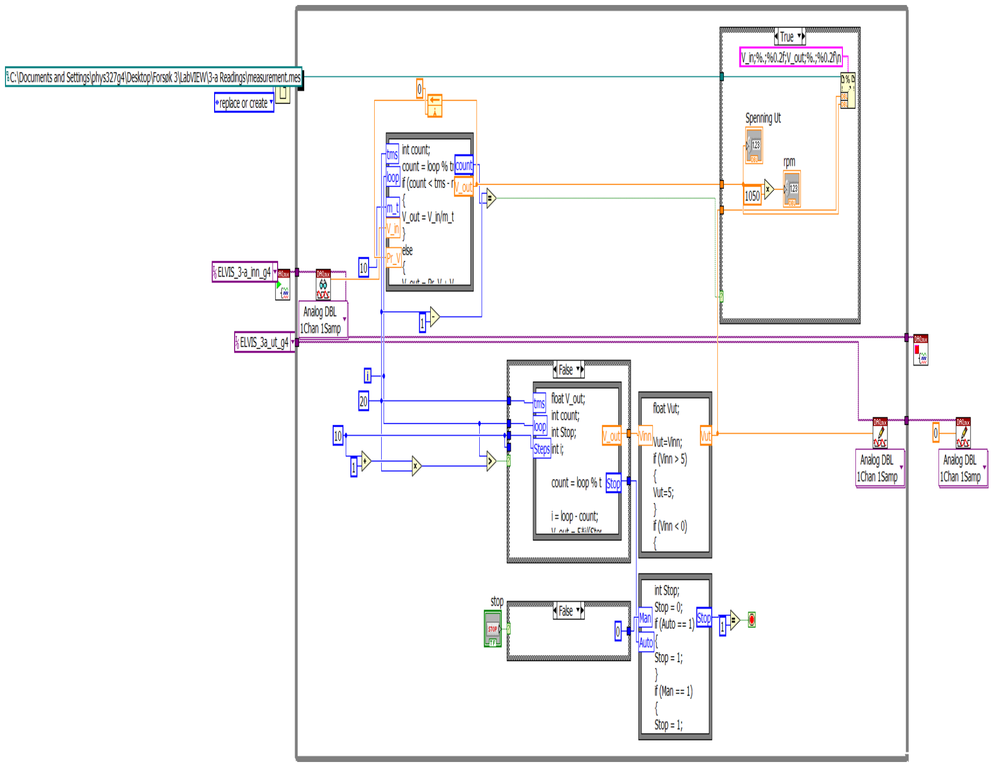

Figure 2: An image of the working VI's block diagram

3 Exercise 3-b : State space representation and transfer function of the DC-motor system

3.1 Introduction

In this exercise, the student will

- Determine the state space representation of the DC motor system using the armature current i_a and the rotational speed ω as state space variables.
- Based on the data sheet available for the DC-motor system, draw the block diagram and determine the transfer function $H_p(s)$ of the DC-motor system assuming the armature inductance, $L_a \approx 0$, the damping constant, B ≈ 0 and the external torque, $m_y \approx 0$

3.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS insrumentation, design and prototyping platform.

3.3 Results

Using basic circuit relations, as well as some torque properties, we deduct the following:

$$
\begin{array}{ll}
u_a = k_v\omega + R_a i_a + L_a \cdot \frac{di_a}{dt}, & m_m = k_T i_a = J\frac{d\omega}{dt} + m_y + B\omega \\
\Downarrow & \Downarrow m_y = 0 \\
\frac{di}{dt} = \frac{1}{L_a}(u_a - k_v\omega - R_a i_a), & \frac{d\omega}{dt} = \frac{1}{J} \cdot (k_T i_a - B\omega)
\end{array}
\tag{1}
$$

Where

u_a is the input voltage	k_T is the torque constant for the motor
ω is the rotational speed of the DC motor	R_a is the armature resistance
i_a is the armature current	L_a is the armature inductance
m_m is the motor torque	J is the moment of inertia
m_y is the external torque	B is the damping constant
k_v is the motor back emf constant	

Applying those conditions on the general state space model, we obtain the following equations for the state space:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \frac{-R}{L_a} & \frac{-k_T}{L_a} \\ \\ \frac{k_T}{J} & \frac{-B}{J} \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ \\ 0 \end{bmatrix} \cdot u_a$$

$$\dot{\mathbf{y}}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix}$$

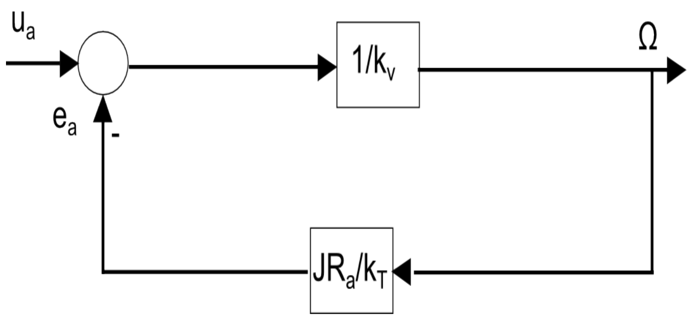

Figure 3: Block diagram of the DC-motor system

Using the relations we obtained in equation (1), and shifting to frequency space, the following relations hold true:

$$u_a(s) = k_v\Omega + R_a i_a(s) + s \cdot L_a i_a(s) \ , \quad k_T i_a(s) = sJ\Omega + m_y + B\Omega \tag{2}$$

Inserting this into the transfer function and assuming that the feedback of the mechanical part, B = 0, and $L_a = 0$, we obtain:

$$\begin{gathered} H_p(s) = \frac{\Omega(s)}{u_a(s)} = \frac{k_T i_a}{Js} \cdot \frac{1}{R_a i_a + sL_a i_a + K_v K_t i_a / Js} \\ \Downarrow \\ H_p(s) = \frac{k_T}{JsR_a + Js^2 L_a + K_v K_t} \\ \Downarrow \\ H_p(s) = \frac{1/k_v}{1 + (JR_a/k_v k_T) \cdot s + (JL_a/k_v k_T) \cdot s^2} \\ \Downarrow L_a = 0 \\ H_p(s) = \frac{1/k_v}{1 + (JR_a/k_v k_T) \cdot s} \end{gathered} \tag{3}$$

4 Exercise 3-c: Step response

4.1 Introduction

In this exercise, the student will

- Create a LabVIEW VI which records the step response of the DC-motor experimentally.
- Determine the time constant and gain of the motor system, assuming a 1. order system using the following steps of the drive signal:
  - From 0 to 5 V
  - From 1 to 3 V
- In the remainder of the laboratory assignement, use the experimental value of the time constant and gain found in this exercise.

4.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS insrumentation, design and prototyping platform.

4.3 Results

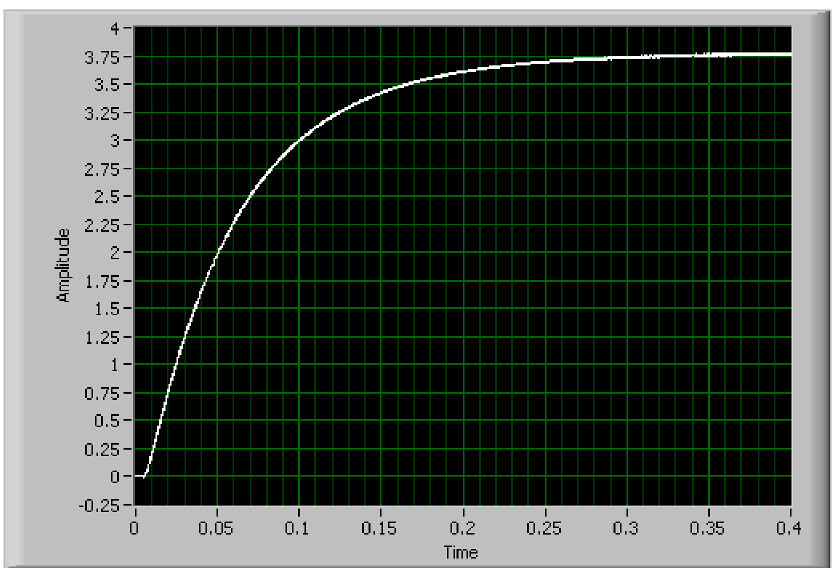

Figure 4: The step response of DC motor from 0 to 5 Volts.

From the step response from 0 to 5 volts, the time constant and gain, can be read out to be τ = 0.065s, K=0.756

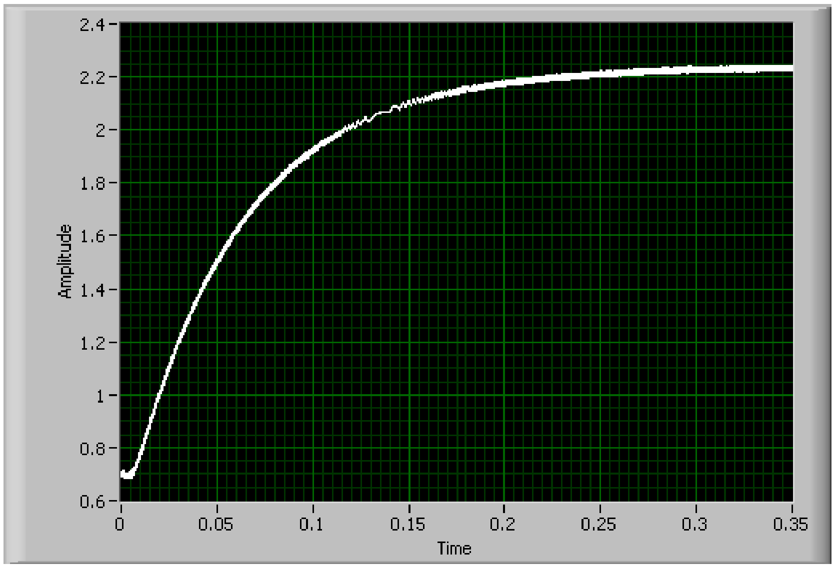

Figure 5: The step response of DC motor from 1 to 3 Volts.

From the step response from 1 to 3 volts, the time constant and gain, can be read out to be τ = 0.065s, K=0.770

Calculating the transfer function we found in 3b (3), and taking into account the Tachometer calibration from the data sheet (1050 rpm/V) and the PWM amplifier gain from the data sheet (2.3 V/V), we get:

$$K = \frac{2.3}{(1050 \cdot 0.0280)} = 0.0782$$
$$\tau = \frac{9.64 \cdot 10^{-6} \cdot 3.30}{0.0280 \cdot 0.0280} s = 0.0405 s$$

Compared with the experimental results, we see that the time constant isn't far from what we obtained, while the gain is suspiciously close to what we obtained, except that it's offset by a factor of 10. We were informed that he figures we were given, weren't all correct, so a certain deviation from the experimental results was expected.

4.4 LabVIEW

In LabVIEW, a sequence structure was used. First the in and out channels are initialized. Then the output voltage is set to be 0 V. After that, the input channel initiates measuring 10000 samples. While it is measuring, the output voltage is set to 5 V again. After the input channel is done measuring, it's stopped. And after it is done, it sends the data to the graph and sets the output voltage back to 0 V.

Then the same process is repeated for the next element in the sequence structure encapsulating the inner sequence structure. This time with the voltage going from 1 V to 3 V and back to 0 V.

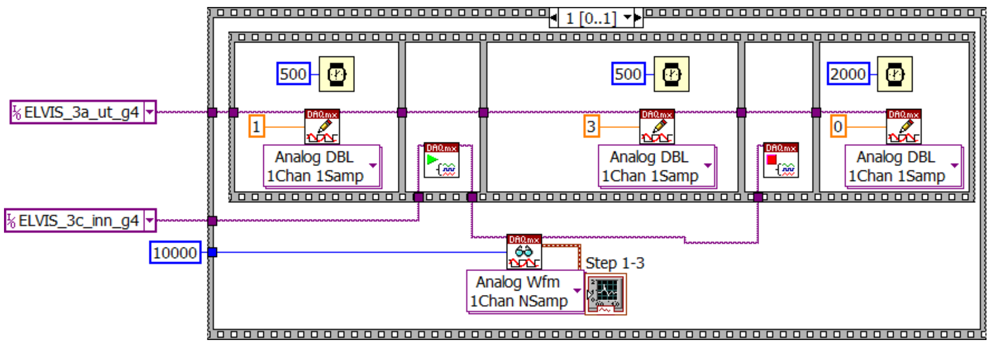

Figure 6: An image of the working VI's block diagram

5 Exercise 3-d: Process simulation using LabVIEW

5.1 Introduction

In this exercise, the student will

- Use the "Control Design & Simulation" toolkit in LabVIEW to simulate the dynamic behaviour of the DC-motor provess model as a voltage step appears on the unput.
- Present and comment on the simulation results.

5.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS insrumentation, design and prototyping platform.

5.3 Results

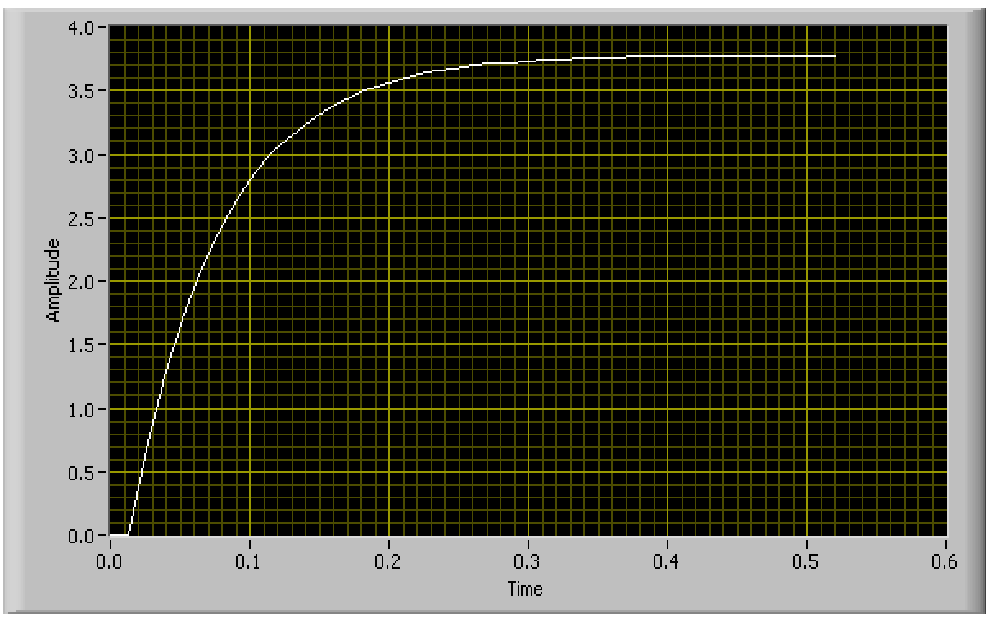

Figure 7: An image of the simulated step response from 0 to 5 V.

As can be seen from figure 8, we based our simulation on the transfer function of the system. Since this was done experimentally, it was expected that the resuls in this exercise would correlate well to what we found in the previous exercise which figure 7 confirms.

5.4 LabVIEW

In this LabVIEW VI, the transfer function obtained experimentally for the DC motor was used. What it does is basically taking the numerator and denominator as an input to the transfer function, then it sends it to "CD Step Response" which simulates a step response on the function. This is then plotted in an XY Graph. The transfer function is multiplied with the final voltage to obtain the correct amplitude in the plot. It is also drawn in the block diagram in the "Equation" block.

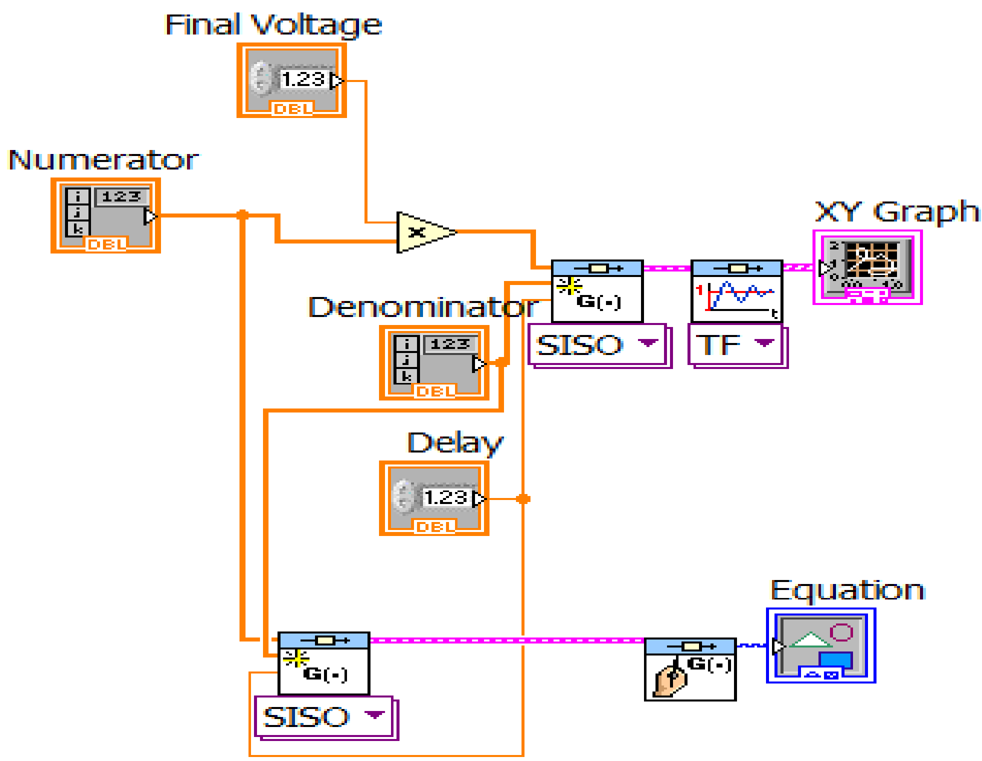

Figure 8: An image of the working VI's block diagram

6 Exercise 3-e: Design of a PID controller

6.1 Introduction

In this exercise, the student will

- Design a PI controller for the system as is shown in figure 9 using the transfer function given in equation 4.

$$M(s) = \frac{y(s)}{r(s)} = \frac{h_p(s)h_r(s)}{1 + h_p(s)h_r(s)h_m(s)} \tag{4}$$

Where

$$\begin{aligned} h_p(s) &= \tfrac{K}{1+T_m \cdot s} \\ h_r(s) &= \tfrac{u(s)}{e(s)} \approx K_P \tfrac{(T_i s+1)(T_d s+1)}{T_i s} \\ h_m(s) &= K_{tacho} \end{aligned} \tag{5}$$

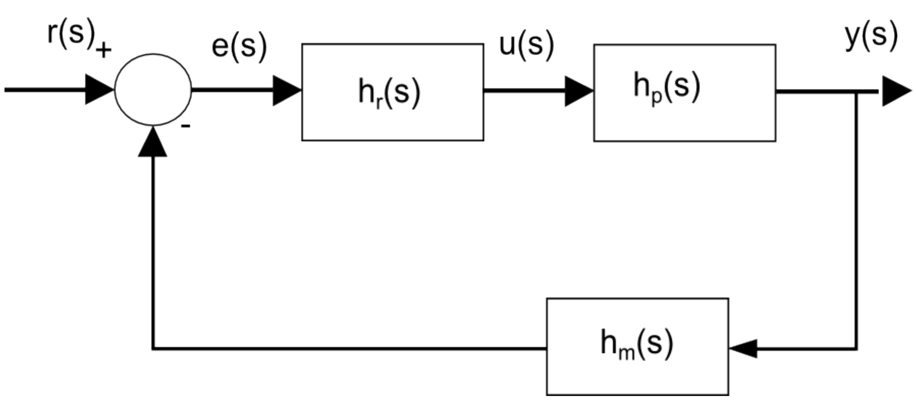

Figure 9: A block diagram of a PI controller for a DC motor

Use the following conditions:

- The integral time constant, T_i, of the controller should be equal to the mechanical time constant, T_M, of the process as in equation.
- The closed loop transfer function's time constant, T_r, should be equal to the mechanical time constant, T_M, of the process.

6.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS instrumentation, design and prototyping platform.

6.3 Results

Putting equation (5) into equation (4):

$$h_p(s)h_r(s) = \frac{1/k_v}{1+(JR_a/k_vk_T)\cdot s}K_P\frac{(T_is+1)(T_ds+1)}{T_is}$$
$$\Downarrow T_d = 0, T_i = T_M$$
$$h_p(s)h_r(s) = \frac{K_p}{K_vT_is}$$

Which gives us:

$$M(s) = \frac{K_p/(K_vT_is)}{1+K_pK_{tacho}/(K_vT_is)}$$
$$= \frac{1/K_{tacho}}{1+K_vT_i\cdot s/(K_pK_{tacho})}$$
$$\Downarrow T_r = \frac{K_vT_i}{K_pK_{tacho}}$$
$$M(s) = \frac{1/K_{tacho}}{1+T_rs}$$

Inserting for $T_r = T_m = 0.065s$ and $K_{tacho} = 952 \cdot 10^{10-6}\frac{V}{rpm}$ yields

$$M(s) = \frac{1050.4}{1+0.065[sec]s} \quad K_P = 0.124$$

So our PI controller design yields the following conditions:

$$T_i = T_r = 0.065s \ , \quad K_p = 0.124$$

7 Exercise 3-f: Development of a PID controller in LabVIEW

7.1 Introduction

In this exercise, the student will

- Create a LabVIEW VI for control of the DC motor using the PID algorithm included in the LabVIEW toolset.
- Show both reference and output in a waveform chart
- Experiment with the system by changing the control parameters and process disturbance, e.g. by holding a finger on the motor shaft and using an insufficient sampling time.
- Document the results in trend diagrams where the reference signal is shown.

7.2 Equipment

- QNET DC Motor Control Board suited for the NI ELVIS insrumentation, design and prototyping platform.

7.3 Results

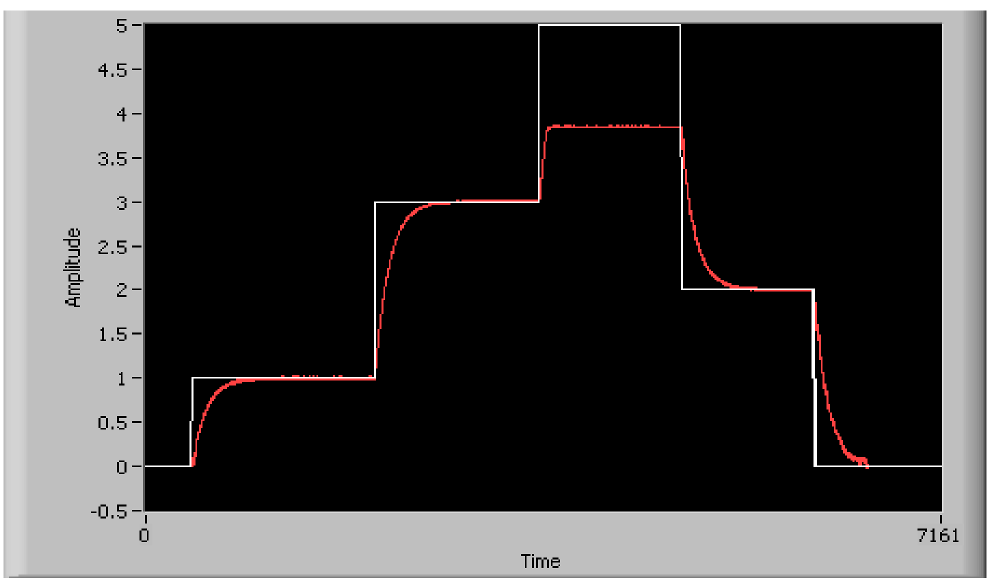

Figure 10: Measurements taken (the red line) using the PID controller, setting different drive voltages (the white line.)

7.4 LabVIEW

In this VI, the input and output are first initialized. Then it enters the while loop, where a PID controlled built into LabVIEW is used to control the DC motor. Apart from the constants the PID controller needs to operate well, the only inputs that are sent to it, are the measured value of the input voltage, and the desired voltage (named "setpoint".) Beyond that, it just makes a check if the input voltage is within the 0 to 5 voltage range, and the measured value along with the setpoint value is plotted in a chart. Exiting the while loop, the input value is reset to 0.

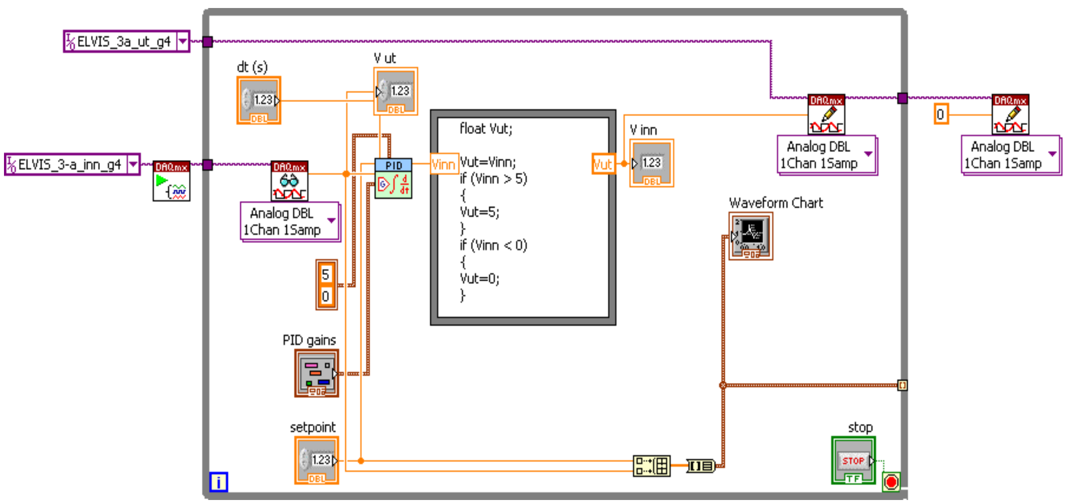

Figure 11: An image of the working VI's block diagram