Journal of Process Control 21 (2011) 1164–1171

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Performance assessment of PID control loops subject to setpoint changes☆

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ARTICLE INFO

Article history:
Received 19 November 2010
Received in revised form 16 June 2011
Accepted 20 June 2011
Available online 22 July 2011

Keywords:
PID control
IAE
IMC
Performance assessment

ABSTRACT

This paper aims at assessing the setpoint tracking performance of PID control loops, under certain constraints such as the processes under control are linear-time invariant and the instrumentations of control loops work properly. The lower bounds of integrated absolute errors (IAEs) are established, based on the widely used internal model control (IMC) principle, from closed-loop responses subject to setpoint changes in the form of step, ramp or other general types. Taking the lower bound as a benchmark, an IMC-IAE-based index is proposed to assess the setpoint tracking performance of PID control loops. Numerical and experimental examples, as well as an industrial case study, are provided to verify the lower bound as the performance benchmark and to illustrate the effectiveness of the proposed performance index.
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1. Introduction

Many industrial feedback control loops suffer from performance problems, possibly due to incorrect tuning, large unmeasured disturbances, inappropriate control structures [1,2]. These problems may cause inferior products, large waste and even safety issues. The objectives of control loop performance assessment are to develop tools that deliver information to users such as how well control loop performance meets with their control targets. To reach this objective, one of the most important tasks is to find a benchmark against which the current control-loop performance can be assessed. The celebrated minimum variance control (MVC) benchmark introduced by Harris [3] is such a benchmark to assess the control-loop performance.

Proportional-integral-derivative (PID) controllers are extensively used in industrial feedback control loops. Improper PID settings may result in sluggish, aggressive or oscillatory loop responses, poor disturbance rejection ability, low robustness and even safety problems. Hence, finding a benchmark to assess the performance of PID control loops is very important for industrial practice. The traditional performance indices include the overshooting, rising time, and settling time. Ko and Edgar [4,5], Jain and Lakshminarayanan [6], and Sendjaja and Kariwala [7] took the same idea of the MVC benchmark, with a specific consideration of the restricted PID controller structure, to calculate the best achievable minimum variance bound for PID/PI controllers. Grimble [8], Huang [9] and Horton et al. [10] studied the benchmark based on the LQG cost function with consideration on the restricted structure imposed by PID/PI controllers. Huagglund [11], Kuehl and Horch [12] and Visioli [13] used the area index and the idle index to tell whether a PID control loop is sluggish or oscillatory when the load disturbance presents. Recently, researchers have been focusing on the integrated absolute error (IAE) based indices. Swanda and Seborg [14] and Huang and Jeng [15] estimated the lower IAE bound for PI/PID control loops from step response by simulations. Veronesi and Visioli [16,17] proposed two indices for PID controllers based on IAE from closed-loop step responses by following the Skogestad internal model control tuning rule; they [18] later generalized the results based on both setpoint and load disturbance step responses for integral processes.

This paper studies the performance assessment of PID control loops subject to setpoint changes, under certain constraints, e.g., the processes under control are linear-time invariant (LTI) and the instrumentations of control loops work properly (to be clarified later in Section 4). While the existing studies in this category are limited to closed-loop step responses, we would also like to consider other types of setpoint changes that appear even more often than step changes in industrial practice, in order to avoid adverse effects caused by abrupt step changes. In addition, the current control loop performance is evaluated against the expected one using a PID controller following the internal model control (IMC) tuning rule. This tuning rule is preferred, because it usually leads to a

☆ This research was partially supported by the National Natural Science Foundation of China under grants No. 61074105 and No. 61061130559 and by Shandong Electric Power Research Institute.
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doi:10.1016/j.jprocont.2011.06.012

control loop with a good balance among setpoint tracking, control cost and robustness for closed-loop stability, and has been widely adopted in industrial practice for years. To be specific, we will establish in theory the lower bounds of IAEs based on the IMC principle from closed-loop responses subject to step, ramp and other general types of setpoint changes. By taking the lower bound of the IAE as a benchmark, we propose an IMC-IAE-based index to assess the setpoint tracking performance of PID control loops. If the index is close to one, then the performance of the current PID control loop is close to the ideal one using a PID controller following the IMC tuning rule.

This article is organized as follows. Section 2 reviews the IMC tuning rule briefly. Section 3 establishes the lower abound of the IAE based on the IMC principle. Section 4 proposes an IMC-IAE-based performance index. Sections 5–7 provide numerical examples, experimental results and an industrial case study, respectively, to validate the performance benchmark, and to illustrate the effectiveness of the proposed performance index. Concluding remarks are given in Section 8.

2. IMC tuning rule for PID controllers

In this section, the IMC tuning rule for PID controllers is briefly reviewed.

Consider a SISO feedback control loop depicted in Fig. 1. Here $P(s)$ and $C(s)$ are the process and the PID controller, respectively; $r(t)$, $u(t)$ and $y(t)$ are the setpoint, the control signal, and the process output, respectively. In this context, $P(s)$ is confined to be an LTI process that is stable, without integrals and negative zeros; hence, the process can be approximated by a first-order plus dead time (FOPDT) model,

$$P(s) = \frac{K}{T_1 s + 1} e^{-\theta s}, \tag{1}$$

or a second-order plus dead time (SOPDT) model,

$$P(s) = \frac{K}{T_1 s^2 + T_2 s + 1} e^{-\theta s}. \tag{2}$$

The approximation is applicable to many practical processes [19]. As θ is crucial to the subsequent performance index, it is worthy to mention that θ is the time delay of the low-order model (1) or (2), instead of the time delay of the actual process. For instance, the positive zero of a process can be removed by lumping it into the time delay part of a low-order approximated model [20].

The PID controller $C(s)$ takes a non-interactive formulation,

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right). \tag{3}$$

This formulation can be transformed in a straightforward manner to the interactive PID controller, which is more widely used in industry for the convenience of controller design [21]. The difference between the two formulations is that non-interactive PID controllers can provide a pair of imaginary zeros to counteract imaginary poles of under-damped processes, while interactive PID controllers cannot. Hence, non-interactive PID controllers are used in this context to deal with under-damped processes. Note that both formulations of PID controllers lead to the same lower bound of the IAE to be derived in Section 3.

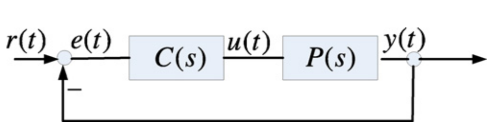

Fig. 1. A SISO feedback control loop.

The IMC tuning rule gives the controller setting for the FOPDT model as

$$K_p = \frac{T_1}{K(\tau_c + \theta)}, \quad T_i = T_1,\ T_d = 0,$$

and that for the SOPDT model as

$$K_p = \frac{T_2}{K(\tau_c + \theta)}, \quad T_i = T_2,\ T_d = \frac{T_1}{T_2}. \tag{4}$$

By taking the IMC tuning rule, the desired closed-loop response will be

$$\begin{aligned} G_{CL}(s) &= \frac{1}{(\theta + \tau_c)s + e^{-\theta s}} e^{-\theta s} \approx \frac{1}{(\theta + \tau_c)s + 1 - \theta s} e^{-\theta s} \\ &= \frac{1}{\tau_c s + 1} e^{-\theta s}. \end{aligned} \tag{5}$$

Here the first-order Taylor approximation is used for the term $e^{-\theta s} \approx 1 - \theta s$ in the denominator of $G_{CL}(s)$ [20]. The user-selected parameter τ_c stands for the desired closed-loop time constant. In general, τ_c should be larger than the time delay θ, since the loop response will usually be too aggressive or even oscillating if $\tau_c < \theta$. A good value of τ_c can be chosen by balancing the following mutually conflicting factors: fast closed-loop response, small variation of control efforts and high robustness for closed-loop stability. It is worthy to point out that if the controller structure is fixed to the PID form and the desired closed-loop response is the FOPDT one in (5), then the resulted parameters of the PID controller will follow the IMC tuning rule as shown in [20]. Based on (5), the lower bound of the IAE can be derived as shown in the next section.

3. IAE benchmark based on IMC tuning rule

This section derives the lower bound of the IAE for an IMC-based PID controller in the closed-loop response subject to step, ramp or other general types of setpoint changes. By taking the lower bound as a benchmark, an IMC-IAE-based index is proposed for performance assessment of PID control loops.

3.1. The lower bound of the IAE for step response

Without loss of generality, consider the setpoint $r(t)$ in Fig. 1 to be a step change from 0 to Δ_{SP}, where Δ_{SP} is a positive real number. From (5), we have

$$Y(s) = G_{CL}(s)R(s) = \frac{\Delta_{SP}}{s(\tau_c s + 1)} e^{-\theta s}.$$

The inverse Laplace transform gives the time-domain expression of $y(t)$,

$$y(t) = \begin{cases} 0, & 0 \le t < \theta, \\ \Delta_{SP}(1 - e^{-((t-\theta)/\tau_c)}), & \theta \le t < \infty. \end{cases}$$

Thus, the error signal $e(t) := r(t) - y(t)$ becomes

$$e(t) = \begin{cases} \Delta_{SP}, & 0 \le t < \theta, \\ \Delta_{SP} e^{-(t-\theta/\tau_c)}, & \theta \le t < \infty. \end{cases}$$

The resulted lower bound of the IAE for step response is

$$\begin{aligned} \mathrm{IAE}_0 &= \int_0^{\infty} |e(t)| dt \\ &= \int_0^{\theta} \Delta_{SP} dt + \int_{\theta}^{\infty} \Delta_{SP} e^{-((t-\theta)/\tau_c)} dt \\ &= \Delta_{SP}(\tau_c + \theta). \end{aligned} \tag{6}$$

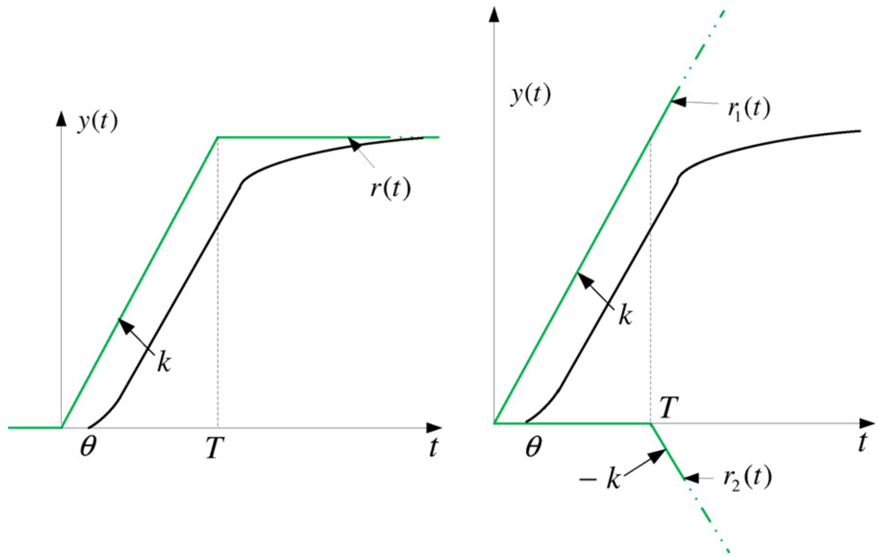

Fig. 2. The setpoint $r(t)$ as a ramp signal and its decomposition consisting of $r_1(t)$ and $r_2(t)$.

A special case is to adopt the Skogestad internal model control tuning rule, where τ_c is recommended to be equal to θ; in this case, the lower bound becomes

$$\mathrm{IAE}_0 = 2\theta\Delta_{SP}. \quad (7)$$

The lower bound (7) has already proposed in [16,17], and is also consistent with the counterpart in [14] obtained via simulation: $\mathrm{IAE}_0 \in [1.9\theta\Delta_{SP}, 2.1\theta\Delta_{SP}]$. However, to our best knowledge, the lower bounds of IAEs for ramp and other general types of setpoint changes have not been developed in literature, and will be given in the next two subsections.

3.2. The lower bound of the IAE for ramp response

Let the setpoint be a ramp signal,

$$r(t) = \begin{cases} kt, & 0 \le t < T, \\ kT, & T \le t < \infty. \end{cases}$$

Here k is the slope of the ramp signal, and takes a positive real value for the time being (the case of $k < 0$ will be discussed later); T is the time duration for the ramp signal to increase into its final constant value. As showed in Fig. 2, $r(t)$ can be decomposed into two parts, namely, $r_1(t)$ and $r_2(t)$, i.e.,

$$r(t) = r_1(t) + r_2(t), \quad (8)$$

where

$$\begin{aligned} r_1(t) &= kt, 0 \le t < \infty, \\ r_2(t) &= \begin{cases} 0, & 0 \le t < T, \\ -k(t-T), & T \le t < \infty. \end{cases} \end{aligned}$$

The Laplace transform of $r(t)$ is

$$R(s) = R_1(s) + R_2(s) = \frac{k}{s^2} + \frac{-k}{s^2}e^{-Ts},$$

from which, together with (5), we reach the Laplace transform of the desired closed-loop response,

$$\begin{aligned} Y(s) &= G_{CL}(s)R(s) \\ &= \frac{k}{\tau_c}\left(\frac{e^{-\theta s}}{s^2(s+(1/\tau_c))} - \frac{e^{-(\theta+T)s}}{s^2(s+(1/\tau_c))}\right). \end{aligned}$$

The inverse Laplace transform of $Y(s)$ is,

$$y(t) = \begin{cases} 0, & 0 \le t < \theta, \\ k(\tau_c e^{-((t-\theta)/\tau_c)} + t - \theta - \tau_c), & \theta \le t < \theta + T, \\ k(\tau_c e^{-((t-\theta)/\tau_c)} - \tau_c e^{-((t-\theta-T)/\tau_c)} + T), & \theta + T \le t < \infty. \end{cases} \quad (9)$$

From (8) and (9), the error signal $e(t) := r(t) - y(t)$ and the lower bound of the associated IAE are calculated as follows. If $\theta \le T$, then

$$e(t) = \begin{cases} kt, & 0 \le t < \theta, \\ k(-\tau_c e^{-((t-\theta)/\tau_c)} + \theta + \tau_c), & \theta \le t < T, \\ k(-\tau_c e^{-((t-\theta)/\tau_c)} + \theta + \tau_c + T - t), & T \le t < \theta + T, \\ k(-\tau_c e^{-((t-\theta)/\tau_c)} + \tau_c e^{-((t-\theta-T)/\tau_c)}), & \theta + T \le t < \infty. \end{cases}$$

Under the IMC tuning rule, the desired closed-loop response will have no overshooting, i.e., $e(t) \ge 0, \forall t \ge 0$; thus, the lower bound of the corresponding IAE is

$$\begin{aligned} \mathrm{IAE}_{0,\theta \le T} &= \int_0^\infty |e(t)| dt \\ &= k\left[\int_0^\theta t dt + \int_\theta^{\theta+T} (\tau_c + \theta) dt + \int_T^{\theta+T} (T-t) dt \right. \\ &\quad \left. - \int_\theta^\infty \tau_c e^{-((t-\theta)/\tau_c)} dt + \int_{\theta+T}^\infty \tau_c e^{-((t-\theta-T)/\tau_c)} dt\right] \quad (10) \\ &= k\left[\frac{\theta^2}{2} + T(\tau_c + \theta) + \frac{T^2}{2} + \theta T - \frac{(\theta+T)^2}{2}\right] \\ &= kT(\tau_c + \theta). \end{aligned}$$

If $\theta > T$, then,

$$e(t) = \begin{cases} kt, & 0 \le t < T, \\ kT, & T \le t < \theta, \\ kT - k\tau_c e^{-((t-\theta)/\tau_c)} + k(\theta + \tau_c - t), & \theta \le t < \theta + T, \\ -k\tau_c e^{-((t-\theta)/\tau_c)} + k\tau_c e^{-((t-\theta-T)/\tau_c)}, & \theta + T \le t < \infty. \end{cases}$$

The lower bound of the associated IAE is

$$\begin{aligned}\mathrm{IAE}_{0,\theta>T} &= \int_0^\infty |e(t)|dt \\ &= k\left[\int_0^T tdt + \int_T^{\theta+T} Tdt + \int_\theta^{\theta+T}(\theta+\tau_c-t)dt \right.\\ &\quad \left. - \int_\theta^\infty \tau_c e^{-((t-\theta)/\tau_c)}dt + \int_{\theta+T}^\infty \tau_c e^{-((t-\theta-T)/\tau_c)}dt\right] \\ &= k\left[\frac{T^2}{2} + \theta T + T(\tau_c+\theta) + \frac{\theta^2}{2} - \frac{(\theta+T)^2}{2}\right] \\ &= kT(\tau_c+\theta).\end{aligned} \quad (11)$$

Eqs. (10) and (11) are based on the assumption that the slope k is positive; if $k<0$, by following a similar derivation, the counterpart of (10) and (11) is $\mathrm{IAE}_0 = -kT(\theta+\tau_c)$. Hence, in general, the lower bound of the IAE is

$$\mathrm{IAE}_0 = \int_0^\infty |e(t)|dt = |kT|(\theta+\tau_c). \quad (12)$$

This is somehow a surprising result, since a first thought may conjecture that the IAE in a closed-loop ramp response is dependent on the ramp slope k. That is, if the slope is larger, then the process output deviates more from the setpoint, and the IAE is expected to be increased. However, the lower bound of the IAE for ramp closed-loop response is the same as that for step response. This is explained by noticing that the ramp response having a smaller slope takes a longer settling time than the step response to make the same magnitude setpoint change, even though for each time instant, $e(t)$ is small in the ramp response; owing to this kind of balance, the lower bound of the IAE is independent to the slope variation.

3.3. The lower bound of the IAE for a general-type response

Suppose that the setpoint $r(t)$ changes from one steady value to another, by following a general-type path such as that shown in Fig. 3. The setpoint can be approximated by a series of ramp signals

$$r(t) \approx r_1(t) + r_2(t) + \cdots + r_N(t), \quad (13)$$

where

$$r_i(t) = \begin{cases} 0, & 0 \le t < t_{i-1}, \\ k_i(t-t_{i-1}), & t_{i-1} \le t < t_i, \\ k_i(t_i - t_{i-1}), & t_i \le t < \infty. \end{cases}$$

Here k_i is the slope of $r_i(t)$;$r(t_0)$ and $r(t_N)$ are the initial and final steady-state values of $r(t)$, respectively, i.e., $r(t)=r(t_0)$ for $t \le t_0$ and $r(t)=r(t_N)$ for $t \ge t_N$. From (12) and the superposition principle (the closed-loop system is LTI), it is ready to obtain the lower bound of the IAE as

$$\begin{aligned}\mathrm{IAE}_0[r(t)] &\approx \mathrm{IAE}_0[r_1(t) + \cdots + r_N(t)] \\ &\le \mathrm{IAE}_0[r_1(t)] + \cdots + IAE_0[r_N(t)] \\ &= (|k_1(t_1-t_0)| + \cdots + |k_N(t_N - t_{N-1})|)(\theta+\tau_c).\end{aligned} \quad (14)$$

Here $\mathrm{IAE}_0[r_i(t)]$ is the lower bound of the IAE for the ith ramp signal $r_i(t)$. The equality in (14) holds under two scenarios: (i) all slopes of $r_i(t)'s(i=1,\ldots,N)$ have the same sign; in other words, the setpoint is mononically increasing or decreasing and (ii) if there is a slope sign change between $r_i(t)$ and $r_{i+1}(t)$, that is, the setpoint changes into the opposite direction, then $r_i(t)$ has to keep invariant at the value $r(t_i)$ for a sufficient period of time for the control loop to arrive at a steady state. Under these scenarios, if the approximation error of $r(t)$ in (13) is ignored, the lower bound of the IAE is

$$\mathrm{IAE}_0 = \sum_{i=1}^\infty |r(t_{i+1}) - r(t_i)|(\theta+\tau_c). \quad (15)$$

Obviously, (6) for step response and (12) for ramp response are two special cases of (15).

4. IMC-IAE-based index

This section proposes an index for assessing the setpoint tracking performance of PID control loops, and discusses several relevant issues for its usage in practice.

Based on the lower bound IAE_0 in (15), a dimensionless index is defined as the ratio between IAE_0 and the actual IAE from closed-loop data under step, ramp or other types of setpoint changes,

$$\eta_{\mathrm{IAE}} = \frac{\min(\mathrm{IAE}_0, \mathrm{IAE}_{\mathrm{Actual}})}{\max(\mathrm{IAE}_0, \mathrm{IAE}_{\mathrm{Actual}})}, \quad (16)$$

where $\mathrm{IAE}_{\mathrm{Actual}} = \sum_{i=1}^\infty |e(t_{i+1}) - e(t_i)|T_s$ for T_s being the sampling period. Henceforth, it is referred to the IMC-IAE-based index. The index is able to evaluate the current control-loop setpoint tracking performance, no matter what the tuning rule the current PID controller follows. The index is in the range of [0, 1] with the ideal value equal to one: $\eta_{\mathrm{IAE}} \to 0$ means that the current control loop performance is far away from the one achieved using a PID controller following the IMC tuning rule, while $\eta_{\mathrm{IAE}} \to 1$ implies a satisfactory performance. Note that $\mathrm{IAE}_{\mathrm{Actual}}$ maybe smaller than IAE_0. For instance, the PID controller taking Ziegler–Nichols tuning rule may result in a smaller IAE than the IMC-based PID controller in a closed-loop ramp response. However, this does not say that an $\mathrm{IAE}_{\mathrm{Actual}}$ much lesser than IAE_0 is preferred, since the factors other than the IAE such as the overshoot or the robustness of closed-loop stability may become unacceptable in this case, e.g., Ziegler–Nichols tuning rule tends to produce oscillatory responses and large overshoots for setpoint changes [19].

Given collected data of $y(t)$ and $r(t)$ (see Fig. 1), the calculation of η_{IAE} requires the determination of the time delay θ and the desired closed-loop time constant τ_c.

- The time delay estimation has been an interesting research topic for years in various areas; see a recent survey in [22]. However, most of the existing methods have their own limitations; in particular, extra experiments are usually required to introduce special signals to excite the unknown process. By contrast, it is desired for industrial practice to estimate θ based on the closed-loop step, ramp or some other simple response, without introducing extra experiments. When the time delay θ is estimated with large uncertainties, we may follow the same idea as the extended MVC indices [23] by assuming a range of time

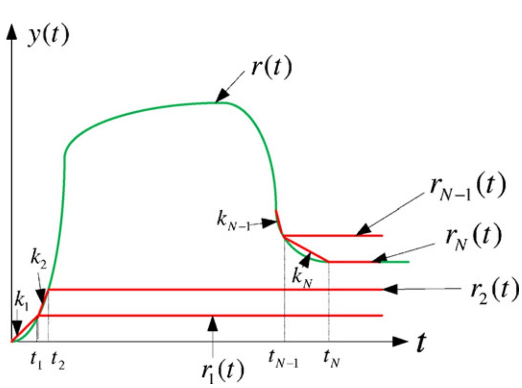

Fig. 3. General type of setpoint change $r(t)$ and its decomposition by $r_i(t)(i=1,\ldots,N)$.

delays and calculating the performance indices over this range. These indices are referred to as extended horizon IMC-IAE-based indices, and can also be exploited to assess the control loop performance.

- The appearance of τ_c makes IAE_0 a user specified benchmark, which is indeed an advantage comparing to the MVC benchmark. In other words, users can determine τ_c as the desired closed-loop time constant, and evaluate the performance of the current control loop against the desired one. However, an improper selection of τ_c could make η_{IAE} too large or too small, leading to erroneous conclusion on the current control loop performance. A fair selection of τ_c is to take the current closed-loop time constant (to be estimated based on the collected data) as τ_c. If $\eta_{\mathrm{IAE}} \to 1$, the current control-loop performance is close to the ideal one using the IMC tuning rule.

A practical issue is about the noise effect on η_{IAE}. First, the noise affects the estimates of τ_c and θ, whose accuracy is up to the corresponding estimation techniques. Thus, the quality control of the two estimates are out of context here. Second, the noise affects also the calculation of the actual IAE, and may result in an incorrect estimate of η_{IAE}, despite a fact that the summation in (16) may enable η_{IAE} somehow robust to noise. To resolve this issue, if $C(s)$ and $\hat{P}(s)$ (the estimate of the process $P(s)$) are available, the noise-free closed-loop response $\hat{y}(t)$ is obtained, i.e.,

$$\hat{Y}(s) = \frac{\hat{P}(s)C(s)}{1+\hat{P}(s)C(s)}R(s). \tag{17}$$

Based on $\hat{y}(t)$ and $r(t)$, a noise-free estimate of η_{IAE} can be calculated.

The time delay θ, the current closed-loop time constant as a choice of τ_c, and the estimate process model $\hat{P}(s)$ can be estimated by open- and closed-loop identification techniques. There have been some studies on identification of continuous-time systems based on open- and closed-loop step responses [24–26]; however, few existing methods could be applicable to the ramp or other general types of setpoint changes. We have proposed an effective novel identification approach to reach these required estimates based on the closed-loop step, ramp or some other simple response, which is beyond the scope of this paper and will be reported elsewhere.

The proposed index is applicable to industrial control loops under certain constraints. As stated in Section 2, the process $P(s)$ has to be an LTI one that can be approximated by the FOPDT model in (1) or the SOPDT one in (2). In addition, it is assumed that the instrumentations of the control loop including sensors, transmitters, converters, actuators work properly, and there is no saturation of control valves to induce any nonlinearities into the control loop. Due to these constraints, when the proposed index is used in practice, $\hat{y}(t)$ in (17) needs to be obtained first and is compared with $y(t)$. If $\hat{y}(t)$ can capture the main characteristics of $y(t)$, then we are confident that the above constraints are not violated and the proposed index can be applied to provide meaningful information. Otherwise, if $\hat{y}(t)$ severely deviates from $y(t)$, we would be aware of the fact that some of the above-mentioned constraints may be violated, and the proposed index does not provide reliable results.

5. Simulation examples

In this section, two simulation examples are presented to validate the lower bound of the IAE. The process in Fig. 1 is

$$P(s) = \frac{2.5}{5s^2+6.25s+1}e^{-2s},$$

Table 1
The PID controller parameters, the theoretical lower bounds of the IAEs, and the actual IAEs under $r(t)$ in (18) for different values of $\tau_c = \lambda\theta$.

λ	K_p	T_i	T_d	IAE_0	$\mathrm{IAE}_{\mathrm{Actual}}$
1	0.625	5	0.8	360	365.75
2	0.417	5	0.8	540	544.50
3	0.312	5	0.8	720	724.50
4	0.25	5	0.8	900	904.50
5	0.208	5	0.8	1080	1084.50

Table 2
The PID controller parameters, the theoretical lower bounds of the IAEs, and the actual IAEs under $r(t)$ in (19) for different values of $\tau_c = \lambda\theta$.

λ	K_p	T_i	T_d	IAE_0	$\mathrm{IAE}_{\mathrm{Actual}}$
1	0.625	5	0.8	128.56	131.71
2	0.417	5	0.8	192.84	194.45
3	0.312	5	0.8	257.12	258.73
4	0.25	5	0.8	321.41	323.01
5	0.208	5	0.8	385.69	387.30

and the PID controller $C(s)$ follows the IMC tuning rule in (4). The setpoint is a ramp signal,

$$r(t) = \begin{cases} 0, & 0 \le t < 10, \\ t-10, & 10 \le t < 100, \\ 90, & 100 \le t < \infty. \end{cases} \tag{18}$$

To validate the lower bound of the IAE, IAE_0 in (12), we vary the value of the desired closed-loop time constant as an integer multiple of the time delay θ, i.e., $\tau_c = \lambda\theta$. Table 1 compares the theoretical lower bound of the IAE in (12) and the actual IAE. Here the actual IAE is calculated as that in (16) with the sampling period $T_s = 0.1$ s. The two values are very close to each other for different values of λ. The minor difference between the actual and theoretical values is due to the first-order approximation for the time delay in (5).

To validate (15), we let the setpoint experience several continuous changes,

$$r(t) = \begin{cases} 0, & 0 \le t < 10, \\ (t-10)/10, & 10 \le t < 60, \\ 5+5\sin\left(\dfrac{(t-60)\pi}{80}\right), & 60 \le t < 100, \\ 9+e^{(t-100)\pi/50}, & 101 \le t < 150, \\ 32.14, & 151 \le t < \infty. \end{cases} \tag{19}$$

Table 2 is the counterpart of Table 1 for this more general setpoint signal. The theoretical lower bounds of the IAE in (15) fit the actual IAEs very well for different values of λ.

6. Experimental examples

This section provides experimental examples to illustrate the procedure of assessing the setpoint tracking performance of a PID control loop using the proposed IMC-IAE-based index.

In the experiments, the process in Fig. 1 is a water tank system, whose cross-sectional area is about 320 cm^2. The water level of the tank is selected as the process variable (PV), with the range [0, 100]. The opening of the outlet valve is fixed, while the input valve is driven by a frequency convertor to control the inlet flow, i.e., the frequency of the converter is the manipulated variable. The PID controller is in the non-interactive form as that in (3). The sampling period is 0.5 s. Two experiments are performed for two different sets of PID controller parameters. In both experiments, the setpoint has initially stayed at the value 20 for a sufficient long time for the

Table 3
Performance assessment results for the experimental examples.

	1st experiment	2nd experiment
τ_c	8.8196	16.6068
θ	1.2143	1.1122
IAE_0	200.6780	354.3800
$IAE_{y(t)}$	742.3969	446.9704
$IAE_{\hat{y}(t)}$	681.1547	364.5743
$\eta_{IAE,y(t)}$	0.2703	0.7928
$\eta_{IAE,\hat{y}(t)}$	0.2946	0.9720

PV to initiate at the steady state. The setpoint experiences a ramp change as

$$r(t) = \begin{cases} 20 + 0.8t, & 0 \le t < 25, \\ 40, & 25 \le t < 250. \end{cases} \tag{20}$$

In the first experiment, the PID controller parameters are $K_p = 1.2$, $T_i = 10$ and $T_d = 0$, and the corresponding ramp response is shown in Fig. 4 (solid line). Based on this closed-loop ramp response, an FOPDT model for the open-loop process is estimated,

$$\hat{P}(s) = \frac{5.7256}{211.28s + 1} e^{-1.2143s}. \tag{21}$$

The current closed-loop time constant is estimated by fitting another FOPDT model for the closed-loop system, $\tau_c = 8.8196$. Thus, the lower bound of the IAE is $IAE_0 = \Delta_{SP}(\tau_c + \theta) = 200.6780$. As the actual IAE of the response is 742.3969, the IMC-IAE-based index is $\eta_{IAE,y(t)} = 0.2703$. To avoid an incorrect estimate of η_{IAE} due to the noise, a noise-free closed-loop response $\hat{y}(t)$ is obtained as that in (17), based on $\hat{P}(s)$ in (21) subject to the same setpoint $r(t)$ in (20); the resulted IAE (based on $\hat{y}(t)$ and $r(t)$) is 681.1547 and the IMC-IAE-based index is $\eta_{IAE,\hat{y}(t)} = 0.2946$. The two indices are close to each other, saying that the current control loop performance has a quite large space for improvement. All these results are summarized in Table 3 for clarity.

In the second experiment, a different set of PID controller parameters is adopted, $K_p = 2.0249$, $T_i = 233.94$ and $T_d = 0$. The resulted ramp response is shown in Fig. 4 (dashed-dotted line). Based on this closed-loop ramp response, an FOPDT model for the open-loop process is estimated,

$$\hat{P}(s) = \frac{6.6469}{241.3700s + 1} e^{-1.1122s}. \tag{22}$$

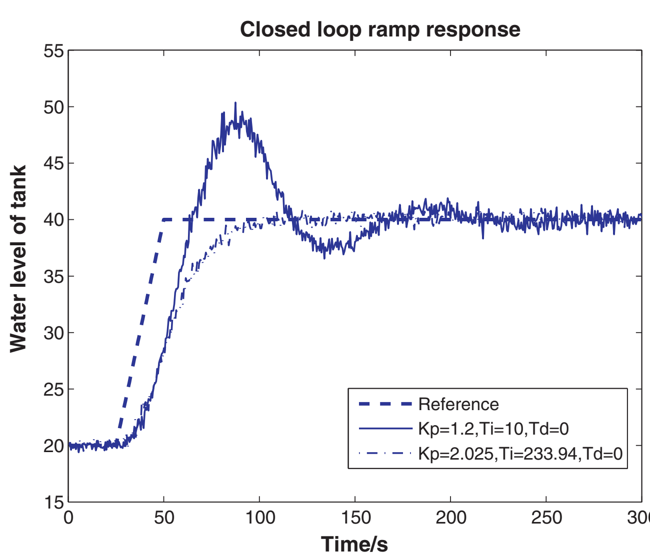

Fig. 4. The ramp setpoint (dashed), the measured PV (solid) for $K_p = 1.2$, $T_i = 10$ and $T_d = 0$, and the measured PV (dashed-dotted) for $K_p = 2.0249$, $T_i = 233.94$ and $T_d = 0$.

Analogously to the first experiment, the performance assessment results are obtained, as given in the last column of Table 3. Both $\eta_{IAE,y(t)}$ and $\eta_{IAE,\hat{y}(t)}$ imply that the control loop performance is very close to the ideal one using the IMC tuning rule. In fact, if the IMC tuning rule is used based on the model in (22), the resulted controller parameters are $K_p = 2.0494$, $T_i = 241.37$ and $T_d = 0$, which are almost the same as the PID controller parameters used in the second experiment.

7. Industrial case study

This section uses the IMC-IAE index to assess the setpoint tracking performance of an industrial PID control loop, which is the forced-draft control system for a thermal power plant at Shandong Province, China. That is, $r(t)$, $u(t)$ and $y(t)$ in Fig. 1 represent the air flow demand, the control command for two forced-draft fans, and the measured air flow, respectively. From the DCS database, 20,000 data points are collected with the sampling period 2 s, standing for the normal operation of the forced-draft control system in 11.1 h on May 21, 2010 ($r(t)$ and $y(t)$ are presented in Fig. 5 (top)). The PID controller takes the form as $C(s) = K_c + (1/T_i s)$ with $K_c = 0.021$ and $T_i = 1500$.

The performance assessment could be conducted for a closed-loop response subject to a step, ramp or other general type of setpoint change. We assess the performance of the forced-draft control system based on the whole 20,000 collected data set and two local data sets extracted from it. The two local data sets experience similar setpoint changes, in the sample-index range of [2500, 3500] with the increment of $r(t)$ from 1200 to 1450 for one set, and in the range of [17,500, 19,500] with the increment of $r(t)$ from 1130 to 1460 for the other.

Analogously to the experimental example in Section 5, the closed-loop time constant is estimated for each data set, by fitting an FOPDT model for the closed-loop system. Another FOPDT model is identified for each data set to capture the open-loop process dynamics. Taking the estimated closed-loop time constant as τ_c and the time delay of the identified FOPDT model for open-loop process dynamics as θ, the lower bound of IAE in (15) and the IMC-IAE-based index in (16) are calculated. Table 4 lists the estimated closed-loop time constant τ_c, the time delay θ, the lower bound of IAE (IAE_0), the actual IAE based on $y(t)$ ($IAE_{y(t)}$), and the IMC-IAE-based index $\eta_{IAE,y(t)}$ for each data set. To avoid an incorrect estimate of η_{IAE} due to the noise, the noise-free estimate $\hat{y}(t)$ of $y(t)$ is obtained as that in (17) based on the identified open-loop process model, also plotted together with $y(t)$ in Fig. 5. Table 4 gives the resulted IAE based on $\hat{y}(t)(IAE_{\hat{y}(t)})$ and the associated IMC-IAE-based index $\eta_{IAE,\hat{y}(t)}$. In Table 4, $\eta_{IAE,y(t)}$ varies quite a lot, from 0.4188 for the whole data set to 0.7308 for the data set in the range [2500, 3500], separating the control-loop performance into different levels. However, all values of the index $\eta_{IAE,\hat{y}(t)}$ based on $\hat{y}(t)$ are close to 1, implying that the control loop performance is very close to the ideal one using the IMC tuning rule.

For industrial applications of the proposed performance assessment technique, a related important issue is to validate the conclusion of the performance assessment based on the collected

Table 4
Performance assessment results for the industrial case study.

Data set	[2500, 3500]	[17,500, 19,500]	[1, 20,000]
τ_c	35.4389	39.3052	46.1849
θ	10.2040	6.3628	4.5918
IAE_0	13,324	19,739	129,527
$IAE_{y(t)}$	18,233	33,921	309,314
$IAE_{\hat{y}(t)}$	13,379	17,980	134,386
$\eta_{IAE,y(t)}$	0.7308	0.5819	0.4188
$\eta_{IAE,\hat{y}(t)}$	0.9959	0.9109	0.9638

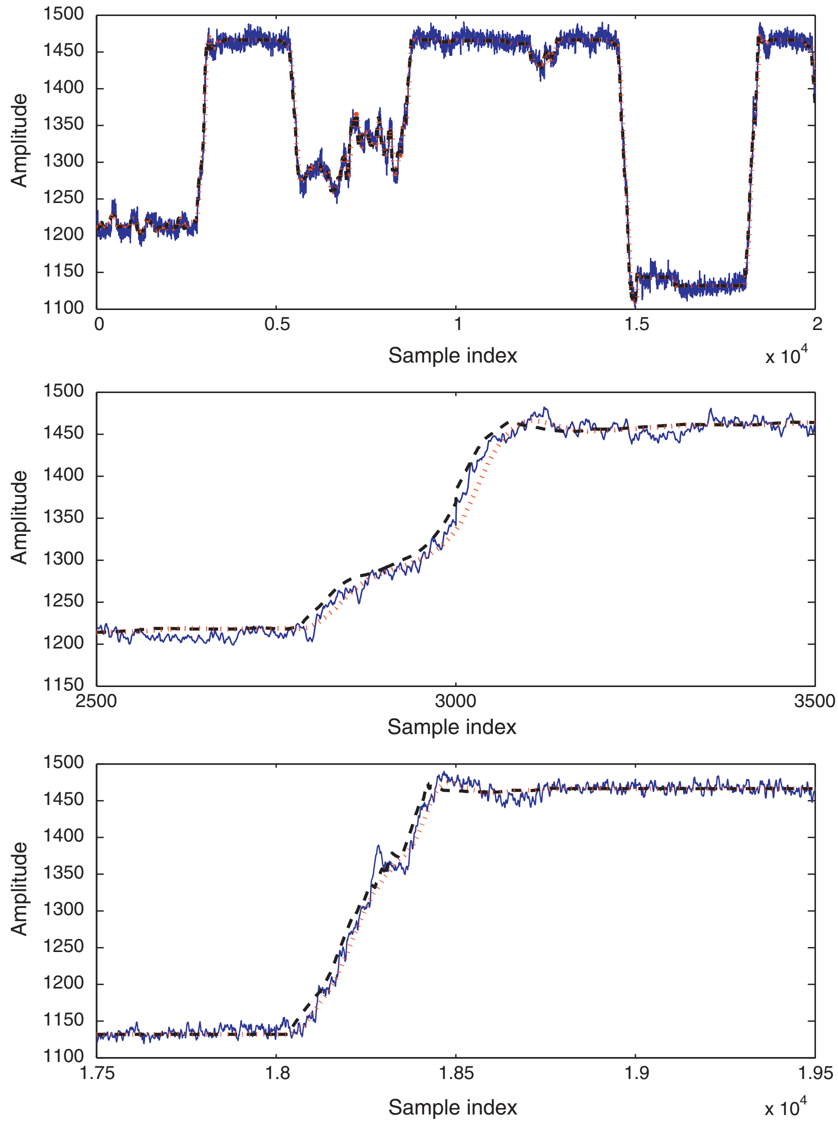

Fig. 5. The measured output $y(t)$ (solid), the setpoint $r(t)$ (dash) and the noise-free output estimate $\hat{y}(t)$ (dotted) for the whole data set in the range of [1, 20,000] (top), the data set in [2500, 3500] (middle), and the data set in [17,500, 19,500] (bottom).

data. To resolve this issue, we have proposed a series of specially designed model validation tests. That is, if the identified closed-loop and open-loop models have successfully passed the model validation tests, then the conclusion of the performance assessment would be trustworthy. One of the validation tests is to see the fitness[1] between $\hat{y}(t)$ and $y(t)$,

$$\text{Fitness} = 100\left(1 - \frac{\|\hat{y}(t) - y(t)\|_2}{\|y(t) - E\{y(t)\}\|_2}\right).$$

For instance, $\hat{y}(t)$ captures the main characteristic of $y(t)$ very well in Fig. 5.

8. Conclusions

This paper established the lower bound of the IAE for PID controllers by following the IMC tuning principle, from closed-loop responses subject to step, ramp or other types of setpoint changes. Based on the lower bound, an IMC-IAE-based index was proposed in (16) to assess the setpoint tracking performance of PID control loops. Numerical examples validated the obtained lower bound as the performance benchmark. Experimental examples and an industrial case study illustrated the effectiveness of the IMC-IAE-based index.

The proposed IMC-IAE-based index needs some further studies to extend its applicability. First, the proposed index is based on the assumption that the actual processes can be approximated by the low-order model in (1) or (2). If this assumption is not satisfied, e.g., for the process containing limiting factors including non-invertible elements and large model uncertainties, then the desired closed-loop response has to be more involved than the simple one in (5)[9,27–29]. Second, the proposed index aims at assessing the setpoint tracking performance. If the control loop contains significant input load disturbance, it is well-known that the IMC tuning rule results in a long settling time for input disturbance for lag dominant processes. A remedy is to use an SOPDT model as the desired closed-loop, instead of the FOPDT model in (5). For example, the SIMC tuning rule proposed in [20] used $T_i = 8\theta$. This modification greatly improves disturbance rejection ability of a process, with a slight sacrifice of setpoint tracking performance. Hence, one of the

[1] The fitness index is often adopted in the literature of system identification; see 'compare' command in Matlab System Identification Toolbox.

future works is to extend the proposed index for the control loops where the control objective is to achieve a good balance between setpoint tracking and disturbance attenuation.

Acknowledgments

The authors would like to thank the anonymous reviewers for their constructive comments and helpful suggestions.

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