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VARIABLE STRUCTURE CONTROL OF PWM RECTIFIER WITH SLIDING-MODE-BASED MODULATION AND MINIMIZED NUMBER OF SENSORS

The paper presents the novel approach to the sliding-mode-based modulation technique in application to the PWM rectifier with the minimized number of the necessary voltage and current transducers. The crucial and technically most inconvenient part of this sensorless technique is the preliminary reconstruction of the line three-phase currents based on the proper sampling of the DC-link current within the specified PWM periods. The reconstructed grid currents are then used to estimate their smooth counterparts as well as the grid three-phase voltages. This can be realized through the application of the sliding-mode current observer for the grid voltage mixed with the conventional nonlinear asymptotic observer for the grid voltage. The novel approach to the design of the sliding-mode control for the PWM rectifier is based on the application of the nonlinear current controllers in the d-q coordinate frame and the direct look-up table-based modulation. This solution allows eliminating the back transformation into the three-phase coordinate system. The selected computational results based on the multi-rate simulations have been presented and discussed.

1. INTRODUCTION

The three-phase two-level PWM rectifiers are greedily applied in the modern frequency converters since they provide the four-quadrant operation with the

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possibility of the regenerative motor braking. Since the PWM rectifiers are mostly subjected to the varying line and load conditions and disturbances it is convenient to apply the nonlinear control strategies that provide robustness and better effectiveness of the rectifier's control system [1,2]. The PWM rectifiers represent a class of the variable structure systems since the three-phase current is conducted along with the six different electrical routes determined by the six active states of the power switches under control.

The sliding-mode control stems from the nonlinear control strategies designed for the variable structure systems. During the closed-loop operation this kind of the nonlinear control method provides the control system insensitiveness to the particular extent of the uncertainties and disturbances. Moreover the sliding-mode control turns out to be a most effective tool in approach to the design of the state-variable observers for sensorless control [7,8].

2. MODEL OF THE BOOST-TYPE PWM RECTIFIER

The boost-type PWM rectifier called also the AC/DC line-side converter has the topology of the three-phase PWM voltage inverter commonly used to feed the squirrel-cage induction motors. The converter topology presented in Fig.1 provides the synchronous rectification and the power regeneration.

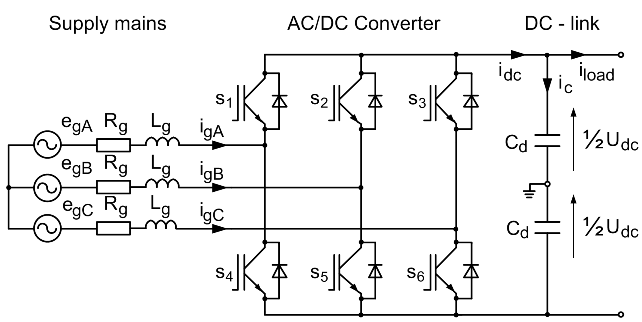

Fig.1. Boost-type PWM rectifier

The three-phase two-level PWM rectifier consists of the six fully-controlled IGBT transistors connected to the grid through the three symmetrical line inductors. The voltage-source output of the rectifier requires the DC-link capacitor to provide the step-up operation. In order to make the implementation of the sliding-mode control more intelligible the DC-link has been divided into two equal parts with the ground connection between them. The equations (1) describe the dynamic model of the PWM rectifier in the three-phase coordinate frame:

$$\begin{aligned}
\frac{d}{dt}i_{gA} &= \frac{1}{L_g}\left[e_{gA} - R_g i_{gA} - \frac{u_{dc}}{6}\left(2K_a - K_b - K_c\right)\right] \\
\frac{d}{dt}i_{gB} &= \frac{1}{L_g}\left[e_{gB} - R_g i_{gB} - \frac{u_{dc}}{6}\left(-K_a + 2K_b - K_c\right)\right] \\
\frac{d}{dt}i_{gC} &= \frac{1}{L_g}\left[e_{gC} - R_g i_{gC} - \frac{u_{dc}}{6}\left(-K_a - K_b + 2K_c\right)\right] \\
\frac{d}{dt}u_{dc} &= -\frac{i_{load}}{C_d} + \frac{1}{2C_d}\left(K_a \cdot i_{gA} + K_b \cdot i_{gB} + K_c \cdot i_{gC}\right)
\end{aligned} \tag{1}$$

where R_g, L_g – grid resistance [Ω] and inductance [H]; C_d – DC-link capacitance [F]; i_{gA}, i_{gB}, i_{gC} – grid currents in three-phase coordinates frame [A]; e_{gA}, e_{gB}, e_{gC} – grid phase voltages in the three-phase coordinates frame [V]; u_{dc} – DC-link voltage [V]; i_{load} – load DC current [A]; K_a, K_b, K_c – converter's conduction states [-1,1].

The vector description of the PWM rectifier is presented by the equations (2):

$$\begin{aligned}
\frac{d}{dt}\mathbf{i_g} &= \frac{1}{L_g}\left[\mathbf{e_g} - R_g \mathbf{i_g} - \frac{u_{dc}}{6}\mathbf{A}\cdot\mathbf{K}\right] \\
\frac{d}{dt}u_{dc} &= -\frac{i_{load}}{C_d} + \frac{1}{2C_d}\mathbf{K^T}\cdot\mathbf{i_g}
\end{aligned} \tag{2}$$

where $\mathbf{A}$ is a gain matrix with the following elements that provide its singularity:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \qquad \det(\mathbf{A}) = 0 \tag{3}$$

and $\mathbf{K}$ is a vector of control states:

$$\mathbf{K} = \begin{bmatrix} K_a \\ K_b \\ K_c \end{bmatrix} \tag{4}$$

3. SLIDING-MODE CONTROL DESIGN FOR PWM RECTIFIER

In case of the design of any control system the deviations between the real plant and its simplified model are unavoidable due to the unknown variations of the parameters and the assumption of the meaningful constraints.

According to the cascade operation the inner sliding-mode loop controls the dynamics of the converter input currents, while the outer loop provides the desired value of the converter output voltage through a linear PI control.

The crucial step is the choice of the switching functions for the current control:

$$\begin{aligned} s_d &= i_{gdref} - i_{gd} = 0 \\ s_q &= i_{gqref} - i_{gq} = 0 \end{aligned} \tag{5}$$

The sliding-mode current controllers are applied directly in a (d–q) rotating frame and return the sign of the current errors as follows:

$$\begin{aligned} \Delta i_{gd} &= -sign(s_d) \\ \Delta i_{gq} &= -sign(s_q) \end{aligned} \tag{6}$$

Similarly to Direct Power Control for the PWM rectifiers, the control signals are selected out of a pre-defined look-up table according to the instantaneous converter's reference voltage rotating in the (α–β) coordinate frame [5,6]. Fig.2a presents the six active voltage vectors on the complex plane divided into twelve sectors.

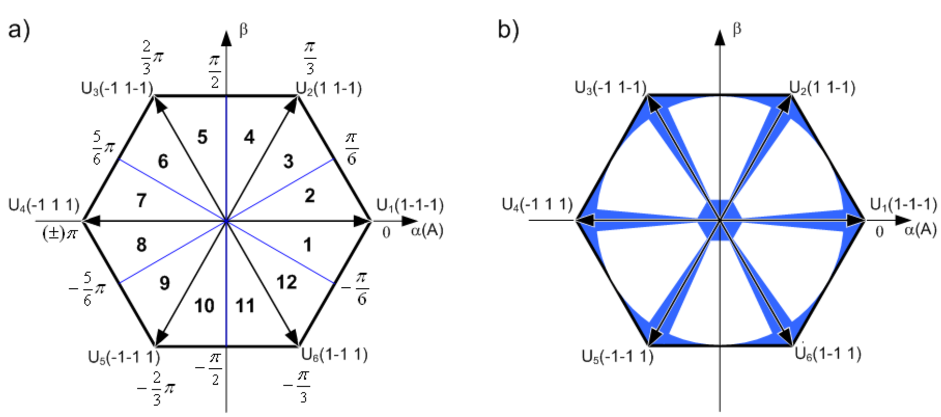

Fig.2. Six active positions of the converter's reference voltage: a) voltage plane divided into twelve equal sectors; b) constraints of the line current reconstruction due to dead-time of power transistors

The values of the control signals as a function of the signs of the current errors and the grid voltage position in the (α–β) coordinates have been presented in Table 1. The sliding-mode occurs in the system when the system trajectory (current errors) converges the sliding line (5) and remains on it. This can be achieved only for the active converter states due to the singularity of the A matrix (3), otherwise the control system may lose controllability. Hence the look-up table does not contain the zero states.

Table 1. Twelve-sector look-up table for sliding-mode control of PWM rectifier

	N:	1	2	3	4	5	6
$\Delta i_d>0$	$\Delta i_q>0$	1 1−1	−1 1−1	−1 1−1	−1 1 1	−1 1 1	−1−1 1
$\Delta i_d>0$	$\Delta i_q<0$	1−1 1	1−1 1	1−1−1	1−1−1	1 1−1	1 1−1
$\Delta i_d<0$	$\Delta i_q>0$	1−1−1	1 1−1	1 1−1	−1 1−1	−1 1−1	−1 1 1
$\Delta i_d<0$	$\Delta i_q<0$	1−1 1	1−1−1	1−1−1	1 1−1	1 1−1	−1 1−1

	N:	7	8	9	10	11	12
$\Delta i_d>0$	$\Delta i_q>0$	−1−1 1	1−1 1	1−1 1	1−1−1	1−1−1	1 1−1
$\Delta i_d>0$	$\Delta i_q<0$	−1 1−1	−1 1−1	−1 1 1	−1 1 1	−1−1 1	−1−1 1
$\Delta i_d<0$	$\Delta i_q>0$	−1 1 1	−1−1 1	−1−1 1	1−1 1	1−1 1	1−1−1
$\Delta i_d<0$	$\Delta i_q<0$	−1 1−1	−1 1 1	−1 1 1	−1−1 1	−1−1 1	1−1 1

The fundamental requirement for the proper operation of the PWM rectifier is the power balance condition. The instantaneous values of the electrical power at the converter input must be equal to their corresponding quantities at the converter output assuming that the power conversion process is lossless and the line voltages and currents are of sinusoidal form (7). The coefficient 3/2 is a consequence of the applied orthogonal Clarke vector transformation.

$$\frac{3}{2}\left(e_{gd} i_{gdref} + e_{gq} i_{gqref}\right) = u_{dcref} i_{load} \tag{7}$$

The linear dynamics of the DC-link voltage control loop is presented in the differential equation (8) and its transient depends only on the time constant that results from the load resistance and the capacitance values:

$$\frac{d}{dt}u_{dc} = \frac{(u_{dcref} - u_{dc}) \cdot i_{load}}{u_{dc}C_d} = \frac{u_{dcref} - u_{dc}}{R_{load}C_d} \tag{8}$$

4. SLIDING-MODE-BASED OBSERVERS OF AC-SIDE VARIABLES

The strict knowledge of the AC voltage and AC current is necessary to determine the line voltage vector position and the line current errors, which is the crucial information for the design of all vector-oriented control methods for the PWM rectifiers. It has already been proved in [3,4] that the grid voltages and grid currents may be successfully estimated based only on the information from the DC-link voltage sensor, DC-link current sensor and the actual PWM pattern. The following section presents in detail the proposed methodology of the observer design.

4.1. LINE CURRENT PRELIMINARY RECONSTRUCTION

The mathematical relationship between the DC-link current and the line currents is based on the following equation involving the PWM pattern:

$$i_{dc} = K_a \cdot i_{gA} + K_b \cdot i_{gB} + K_c \cdot i_{gC} \tag{9}$$

According to the equation (9) the partial information about each line phase current can be achieved using the conditions presented in Table 2.

Table 2. Relationship between DC-link current and phase currents due to PWM pattern

PWM pattern	$i_{dc}=$
1−1−1	i_{gA}
1 1−1	$-i_{gC}$
−1 1−1	i_{gB}
−1 1 1	$-i_{gA}$
−1−1 1	i_{gC}
1−1 1	$-i_{gB}$

Since no AC current transducers exist physically in the rectifier application there is no hardware-based protection in the control system. The most crucial problem in the proposed line current reconstruction is the high rate of the sampling of the DC-link current, which from a technical point of view is inconvenient. Fig.2b presents the ranges where the DC-link current can be properly sampled. The problem occurs when the converter's reference voltage vector crosses the boundary between each of the six sectors. These blank regions depend on the system delays and the IGBT dead-time.

4.2. SLIDING-MODE OBSERVER FOR LINE VOLTAGE AND LINE CURRENT

The starting point of designing a sliding-mode current observer for source voltage is the following discontinuous formula:

$$\dot{\hat{i}}_{gA} = \frac{1}{L_g}\left[m \cdot sign\left(i'_{gA} - \hat{i}_{gA}\right) - R_g \hat{i}_{gA} - \frac{u_{dc}}{6}\left(2K_a - K_b - K_c\right)\right] \tag{10}$$

where m is an observer gain, i'_{gA} is the observed line current described as:

$$i'_{gA} = \begin{cases} i_{gAreconstr} & \\ i_{gAest} \quad when & i_{gAreconstr} = 0 \end{cases} \tag{11}$$

The switching component $m \cdot sign\left(i'_{gA} - \hat{i}_{gA}\right)$ refers to the line voltage estimate and must be low-pass filtered (LPF) in order to obtain a useful line voltage signal:

$$\hat{e}_{gA} = LPF\left(m \cdot sign\left(i'_{gA} - \hat{i}_{gA}\right)\right)\Big|_{T_{filter}=1e-4} \tag{12}$$

The total line voltage signal can be obtain using the following formula:

$$e'_{gA} = -R_g\left(i'_{gA} - i_{gAest}\right) + \hat{e}_{gA} \tag{13}$$

The voltage signal e'_{gA} is next submitted to the nonlinear asymptotic observer that filters out the input signal giving the resulting smooth form of the line voltage e''_{gA}:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - L_1 W_1 \bar{x}_1 \\ \dot{\hat{x}}_2 &= -\left(\omega_g^2 + \frac{3\alpha^2}{\omega_g^2}\right)\hat{x}_1 + \frac{3\alpha}{\omega_g}\hat{x}_2 - L_2 W_1 \bar{x}_1 \end{aligned} \tag{14}$$

where $\bar{x}_1 = \hat{x}_1 - e'_{gA}$, ω_g is the grid pulsation [rad/s], L_1, L_2, α are observer gains.

W_1 is the "window signal" for the phase A that is defined by the following formula:

$$W_1 = \begin{cases} 1 & if \quad i_{gAreconstr} \neq 0 \\ 0 & otherwise \end{cases} \tag{15}$$

The estimated line voltage signal e''_{gA} is next the input signal of the line current observer that is described by the following equation:

$$\dot{i}_{gAest} = \frac{1}{L_g}\left[e''_{gA} - R_g \hat{i}_{gA} - \frac{u_{dc}}{6}\left(2K_a - K_b - K_c\right)\right] \tag{16}$$

The line voltage and line current for the phase *B* and *C* can be obtained in the similar way. The block diagram of the proposed sliding-mode control system for the PWM rectifier with the observers for AC-side variables is presented in Fig.3.

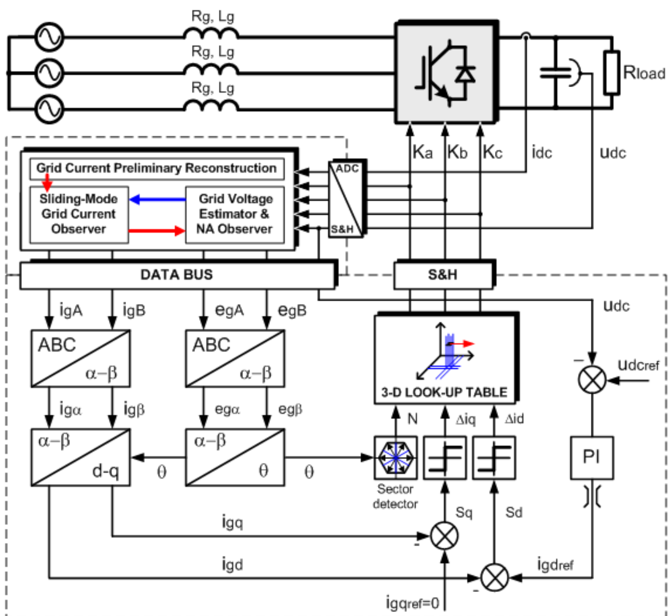

Fig.3. AC-sensorless sliding-mode control system for the PWM rectifier

5. INVESTIGATIONS OF PROPOSED TECHNIQUE AND SELECTED SIMULATION RESULTS

The computer multi-rate simulations have been carried out for the AC-sensorless sliding-mode control system of the PWM rectifier shown in Fig.3. The proposed control method has been tested using Matlab/Simulink. The simulation model has been divided into three subsystems which have been computed with the different rates. The parameters of the simulation model are presented in Tab.3.

Table 3. Simulation parameters

Line phase voltage e_g:	230 V
Line voltage frequency f_g:	50 Hz
Line resistance R_g:	100 mΩ
Line inductance L_g:	10 mH
DC-link capacitance C_d:	1000 μF
DC-link nominal voltage U_{dc}:	600 V
Load resistance R_{load}:	100 Ω
Observer gain m	500
Observer gain L_1	4000
Observer gain L_2	400000
Observer gain α	10
PWM rectifier's sample time T_{p1}:	1 μs
Line current reconstruction rate T_{p2}:	10 μs
Control system's sample time T_{p3}:	30 μs

Fig.4a shows the transients of the estimated phase voltage e''_{gA} and the line currents by applying the full load resistance R_{load}. Fig.4b presents the transients of the line current in the (d-q) coordinate frame under the step change of the converter load. The converter fulfills the unity power factor condition, since i_{gq}=0.

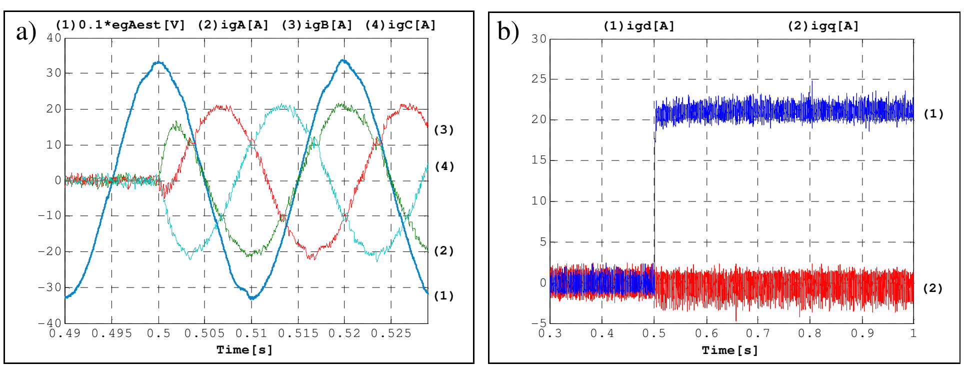

Fig.4. a) estimated line phase voltage and three-phase line currents;
b) (d–q) components of line current

Fig.5a presents the transient of the DC-link voltage under the step change of the rectifier load. In sliding-mode due to the equation (8) the DC-link voltage tends to its reference value u_{dcref} with the time constant $\tau = R_{load} \cdot C_d$ irrespective of the values of the parameters of the control system and the AC grid. The DC-link current is presented in Fig.5b and corresponds to the active component i_{gd} of the line current.

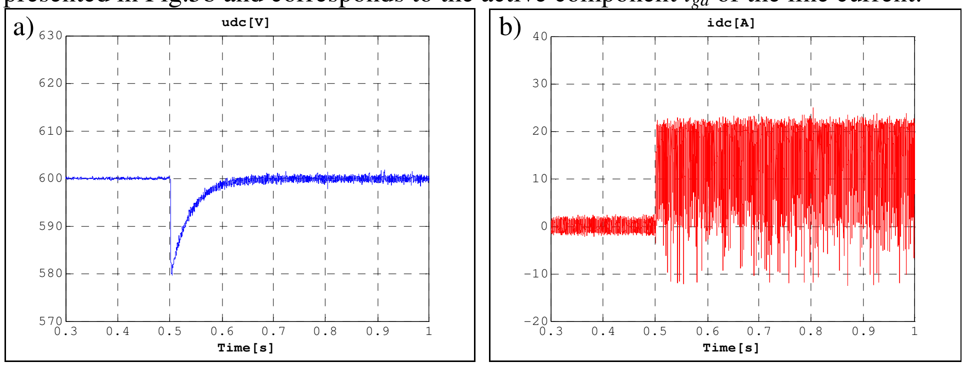

Fig.5. a) transient of DC-link voltage; b) DC-link current at step change of load

Fig.6a demonstrates the pre-estimated low-pass-filtered signal of the line voltage e'_{gA}. A compromise between the signal distortion of the line voltage e'_{gA} and the associated phase lag has been made by adjusting an appropriate value of the low-pass filter time constant T_f=1e-4. Fig.6b presents the estimated line voltage e''_{gA} smoothed by the nonlinear asymptotic observer with the "window" signal (14).

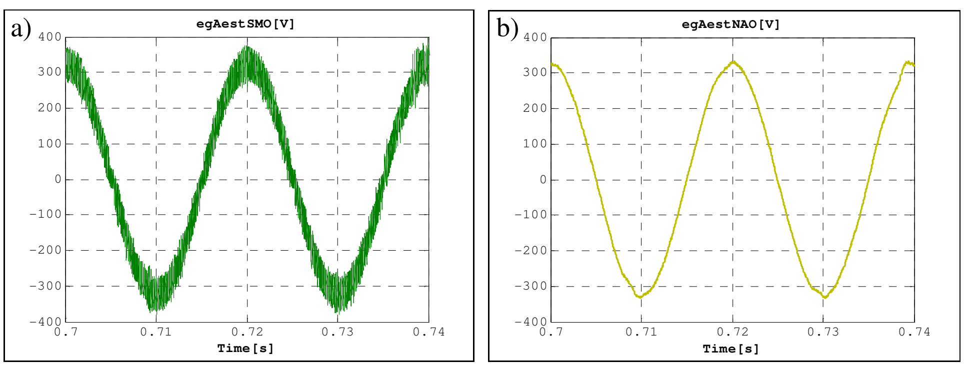

Fig.6. a) estimated line voltage e'_{gA} ; b) smoothed line voltage e''_{gA}

The information about the line currents after their reconstruction according to Table 2 is partial and insufficient for the direct application into the control system of the PWM rectifier. Hence the line current estimation has to be involved simultaneously with the line current reconstruction. According to (11) the resulting estimate of the line current i'_{gA} is made of the reconstructed line current and the estimated line current in points where the reconstructed line current equals zero. This procedure is presented in Fig.7a and allows using the resulting signal for the control purposes. Fig.7b presents the switching line during the sliding-mode. The state trajectory remains on the switching line and moves on it depending on the load conditions.

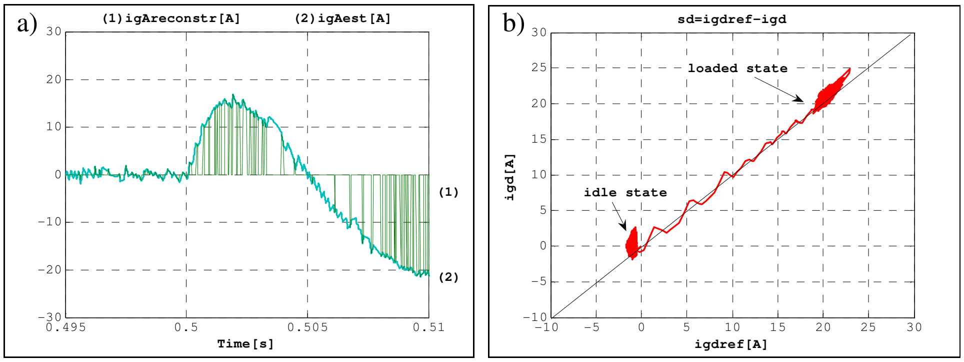

Fig.7. a) line current reconstruction; b) switching line in sliding-mode

5. CONCLUSIONS

The paper presents the design of the novel sliding-mode current control technique for the PWM rectifier. The special stress has been put on the designing of the sliding-mode line voltage and line current observer for the sensorless operation of the PWM rectifier without the AC-side transducers. The line currents have been reconstructed throughout the sampling of the DC-link current during the PWM period. The missing AC-side variables have been then estimated based on the reconstructed line currents. In the proposed control technique the sliding-mode current controllers have been located in the rotating (d–q) coordinate frame and the PWM pattern for the converter has been computed using the three-dimensional look-up table. This modification has reduced the necessary number of the transformations and has led to the significant simplification in the proposed control system.

The cascade control structure used in the multi-rate simulations has involved the line current vector transformation into the (d–q) components oriented with the line voltage vector. Hence the power factor of the PWM rectifier can be independently controlled by the reactive component of the line current. The simulation results demonstrate the excellent performance of the proposed sliding-mode current control system of the PWM rectifier with the sliding-mode observers for the AC-side sensorless operation.

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STEROWANIE ZE ZMIENNĄ STRUKTURĄ PROSTOWNIKIEM PWM Z MODULACJĄ ŚLIZGOWĄ I MINIMALNĄ LICZBĄ CZUJNIKÓW POMIAROWYCH

Artykuł prezentuje metodę modulacji ślizgowej w zastosowaniu do układów sterowania prostowników PWM bez czujników pomiarowych napięcia i prądu sieci zasilającej. Proces rekonstrukcji prądów przewodowych trójfazowej sieci zasilającej jest oparty na odpowiednim próbkowaniu sygnału prądu stałego w obwodzie pośredniczącym w czasie trwania impulsów sterujących kluczami przekształtnika według zdefiniowanego schematu. Następnie zrekonstruowane przebiegi prądów wejściowych prostownika PWM wraz z sygnałem napięcia stałego obwodu pośredniczącego przekształtnika są poddane dalszej obróbce w obserwatorach ślizgowych. Przebiegi napięcia sieci zasilającej uzyskano przy zastosowaniu połączenia obserwatora ślizgowego z klasycznym obserwatorem nieliniowym. Zaprezentowano metodę modulacji ślizgowej PWM opartą na zastosowaniu ślizgowych regulatorów prądu sieci w wirującym synchronicznie układzie odniesienia (d–q) i trójwymiarowej tablicy wyboru wektora napięcia przekształtnika. Przedstawiona metoda modulacji ślizgowej pozwala na wyeliminowanie transformacji prostych przełączających do stacjonarnego, trójfazowego układu odniesienia, co ma miejsce w klasycznym układzie sterowania ślizgowego prostownikiem PWM. W artykule przedstawiono i omówiono wybrane wyniki badań symulacyjnych.