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Design of Mixed H∞ and Optimal Controller for Three-Phase PWM Rectifiers

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***Abstract* –The mathematic model of three-phase pulse-width modulated (PWM) voltage rectifiers is nonlinear, strong coupling. And the mathematic model error, time-varying uncertainties and external disturbance will influence the performance of the system. This paper presents a linear quadratic regulator (LQR) for the inner current loop control without decoupling, and a robust H∞ control method for the outer voltage loop. For the inner current loop, a Riccati equation is solved and a constant value adjoint matrix is calculated to derive the designed controller. For the outer voltage loop, the state equations are established and the suitable weighting functions are selected. And then, the Riccati inequality is solved to derive the H∞ controller. The simulation results show that the designed controller can achieve unit power factor control, and has faster response and better external disturbance rejection capabilities compared with the conventional PI controller.**

Index Terms - H• control, PWM rectifiers, LQR.

I. INTRODUCTION

Three-phase PWM rectifiers have rapidly attracted the research interest over the past few years due to some of their significant advantages, such as power regeneration capability, controllable of dc-bus voltage, high power factor, and low harmonic distortion of input currents, etc [1] [2]. The mathematic model of three-phase PWM rectifiers is nonlinear, strong coupling. And the mathematic model error, time-varying uncertainties and external disturbance will influence the performance of the control system. Many studies have been conducted about performance of the three-phase PWM rectifiers. The widely used control method is the PI controller. A sliding-mode control algorithm on synchronous rotating reference frame for the out-voltage-loop is proposed in [3]. A fuzzy logic controller is presented in [4]. The feedback linearization technique for the three-phase AC/DC converters is proposed in [5] [6]. Three linearization-based and two passivity-based control concepts are introduced and compared in [7].

The robust H• control which is a comparative perfect theory now, is proposed by Zames in 1981. And the control method has taken account of all the uncertainties in designing of the control system [8]. This paper presents a robust H• control for the outer voltage loop and a LQR for the inner current loop. The simulation results show that the designed controller can achieve unit power factor control, and has faster response and better external disturbance rejection capabilities compared with the conventional PI controller.

II. MATHEMATIC MODEL OF PWM RECTIFIER

Three-phase PWM rectifier topology is shown as figure1.

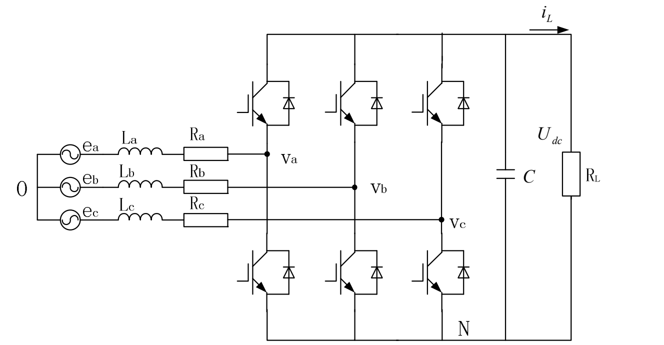

Fig. 1 Three-phase PWM rectifier topology

In figure 1, e_a , e_b and e_c are the input source voltages; L and R are equivalent inductance and resistance of reactor in supply network, respectively; C is capacitance of DC links, and R_L is load resistance.

A set of differential equations can describe the dynamic model of PWM rectifier. The three-phase PWM rectifier in stationary reference frame could be modelled as follow.

$$\begin{cases} L\dfrac{di_a}{dt} + Ri_a = e_a - \left(U_{dc}s_a + u_{NO}\right) \\ L\dfrac{di_b}{dt} + Ri_b = e_b - \left(U_{dc}s_b + u_{NO}\right) \\ L\dfrac{di_c}{dt} + Ri_c = e_c - \left(U_{dc}s_c + u_{NO}\right) \\ C\dfrac{dU_{dc}}{dt} = i_a s_a + i_b s_b + i_c s_c - \dfrac{U_{dc}}{R_L} \end{cases} \tag{1}$$

Where, i_a, i_b, i_c are the input source currents. s_a , s_b , s_c are the switching functions. U_{dc} is DC voltage.
And

$$u_{NO} = -\frac{U_{dc}}{3} \sum_{k=a,b,c} s_k \tag{2}$$

The mathematic model of three-phase PWM rectifier in dq reference frame is:

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$$\begin{cases} \dfrac{dU_{dc}^2}{dt} = -\dfrac{2}{R_L C}U_{dc}^2 + \dfrac{3}{C}e_d i_d + \dfrac{3}{C}e_q i_q \\ \dfrac{di_d}{dt} = -\dfrac{R}{L}i_d + \omega i_q + \dfrac{1}{L}u_d \\ \dfrac{di_q}{dt} = -\omega i_d - \dfrac{R}{L}i_q + \dfrac{1}{L}u_q \end{cases} \tag{3}$$

Where, i_d and i_q are the d-axis and q-axis current respectively.

III. THE CONTROLLER DESIGN

A. the inner current loop

The state equations for the LQR are built as equation (5).

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{5}$$

Where the state variables vector x is selected as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{6}$$

Then the parameters of matrixes in system state equation can be achieved as follows.

$$A = \begin{bmatrix} -\dfrac{R}{L} & w \\ -w & -\dfrac{R}{L} \end{bmatrix},\ B = \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & \dfrac{1}{L} \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A reasonable cost function to use is:

$$J(u(\cdot)) = \frac{1}{2}\int_0^{\infty} [(y-y_r)^T Q (y-y_r) + u^T R u]dt \tag{7}$$

The aim of designed optimal controller is to make the cost function J minimum.

Since the optimal controller is shown as equation (8).

$$u(t) = -R^{-1}B^T P x(t) + R^{-1}B^T g \tag{8}$$

Where, P can be derived from solving the Riccati equation as follow.

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{9}$$

And g is:

$$g = [PBR^{-1}B^T - A^T]^{-1}C^T Q y_r \tag{10}$$

B. the outer voltage loop

Consider the standard setup of the H∞ control problem described by the block diagram.

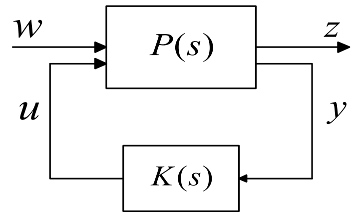

Fig. 2 Standard setup of the H∞ control problem

In the diagram, P(s) notes the plant model and K(s) is the feedback controller. The mathematical object of H∞ control is to make the closed-loop MIMO transfer function H_{zw} satisfy $\|H_{zw}\|_{\infty} < \gamma$.

The state equations which are taken in account of a variety of disturbances and combined with the mathematic model can be built as follows.

$$\begin{aligned} \dot{x} &= Ax + B_1\varepsilon + B_2 u \\ z &= C_1 x + D_{11}\varepsilon + D_{12}u \\ y &= C_2 x + D_{21}\varepsilon + D_{22}u \end{aligned} \tag{11}$$

Where x is the state variable vector. z is the weighted state variable vector. ε is the exogenous input of a matrix. A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22} are the constant matrix.

For the voltage loop controller, the state variable vector x_v is selected as follow.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} U_{dc} \\ \dfrac{e}{s+K_a} \\ \dfrac{U_{dc}}{s+K_b} \end{bmatrix} \tag{12}$$

Where, $e = U_{dc}^* - U_{dc}$, U_{dc}^* is the reference DC voltage input. K_a, K_b are the underdetermined parameters.

ε is selected as follow.

$$\varepsilon = \begin{bmatrix} {U_{dc}}^* \\ i_L \\ n \end{bmatrix} \tag{13}$$

Where n is measure noise of the DC voltage sensing.

Then the parameters of matrixes in system state equation can be got as follows.

$$A=\begin{bmatrix} -\frac{1}{Cr} & 0 & 0 \\ -1 & -K_a & 0 \\ 1 & 0 & -K_d \end{bmatrix}, \quad B_1=\begin{bmatrix} 0 & -\frac{1}{C} & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_2=\begin{bmatrix} \frac{1}{C} \\ 0 \\ 0 \end{bmatrix},$$

$$C_1=\begin{bmatrix} -K_b & K_c-K_aK_b & 0 \\ 0 & 0 & 0 \\ K_e & 0 & K_f-K_eK_d \end{bmatrix}$$

$$C_2=\begin{bmatrix} -1 & 0 & 0 \end{bmatrix}, \; D_{12}=\begin{bmatrix} 0 \\ K_g \\ 0 \end{bmatrix}, \; D_{21}=\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Where K_d, K_e, K_f and K_g are the underdetermined parameters.

In order to meet the demands of the system and simplify the designed controller, the designed weighting functions are selected as follows.

For voltage loop controller:

$$w_1=\frac{K_cs+K_d}{s+K_a} \tag{14}$$

$$w_2=K_g \tag{15}$$

$$w_3=\frac{K_es+K_f}{s+K_b} \tag{16}$$

Where, K_c, K_d, K_e, K_f, K_g are the underdetermined parameters.

In order to get a suitable controller K(s), the 0db cross-over frequency of weighting function w_1 should be less than that of weighting function w_3 in the bode diagram of the weighting functions [9] [10] [11].

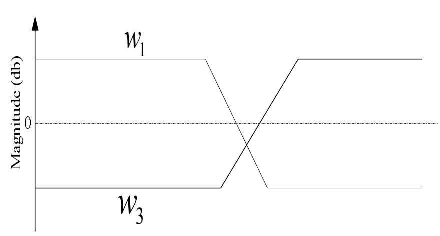

Fig. 3 The magnetic of w_1 and w_3

After the weighting functions w_1, w_2 and w_3 are determined, the controller K(s) can be derived from the Riccati inequality. The Riccati inequality is described as

$$A^TP+PA+\frac{1}{\gamma^2}PB_1B_1^TP+C_1^TC_1- \left(PB_2+C_1^TD_{12}\right)\left(D_{12}^TD_{12}\right)^{-1}\left(B_2^TP+D_{12}^TC_1\right)<0 \tag{17}$$

where γ is a selected constant. T is transpose of a matrix. And the matrix P is a constant. And the controller K(s) can be derived from equation (16) as follow.

$$K(x)=\left(D_{12}^TD_{12}\right)^{-1}\left(B_2^TP+D_{12}^TC_1\right) \tag{18}$$

The proposed PWM rectifier system based on the robust H ∞ control is shown as figure 4.

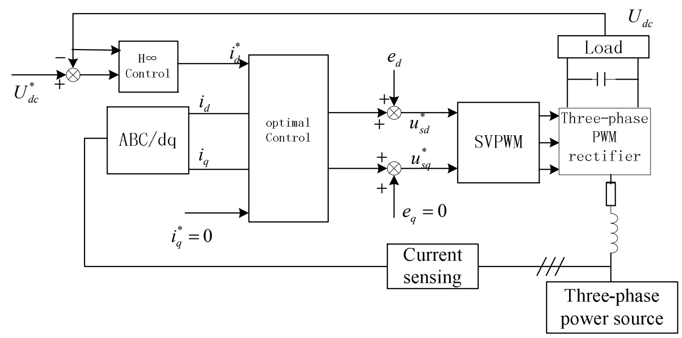

Fig. 4 The block diagram of proposed control system

IV. SIMULATION

The control system is simulated based on Matlab/Simulink. And the parameters of the PWM rectifier in this paper are given as table 1.

TABLE I. THE PARAMETERS OF PWM RECTIFIER

Three-phase power source	110V
Three-phase inductance	2 mH
DC capacitor	2000 uF

The conventional PI controller is designed to compare with the robust controller. The K_P and K_i are selected according to the root locus techniques.

Figure 5 shows DC voltage response with reference input change from 500V into 300V (t=0.3s) and 450V (t=0.6s). Figure 6 shows DC Voltage response with grid voltage disturbance. Figure 7 shows DC Voltage response with load disturbance. Figure 8 voltage and current of A phase power source.

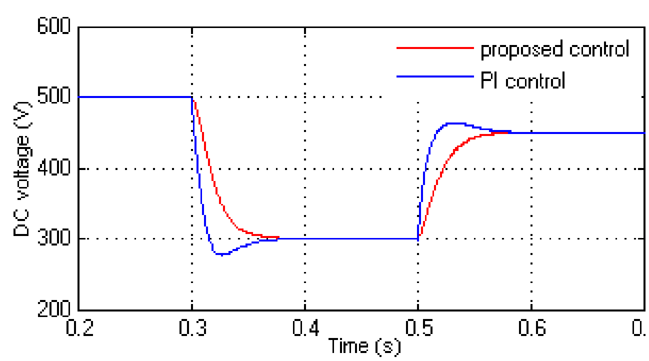

Fig. 5 DC Voltage response with reference input

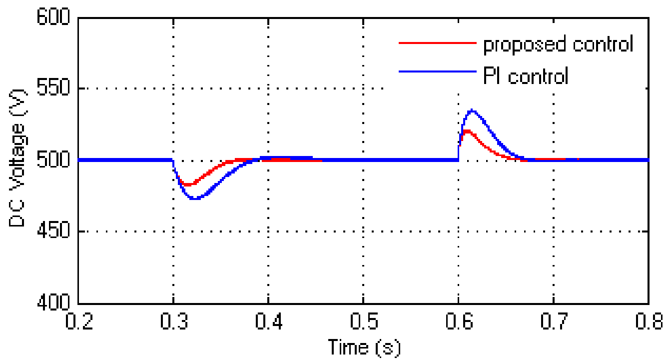

Fig. 6 DC Voltage response with grid voltage disturbance

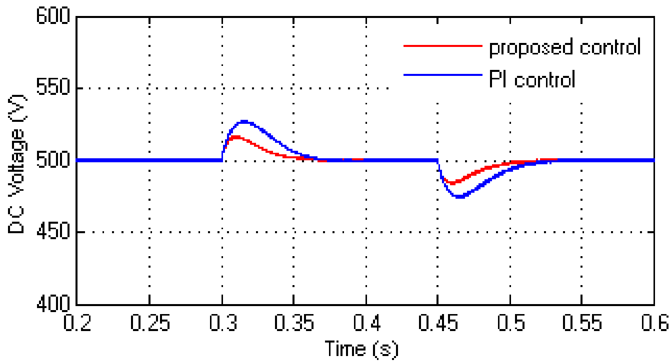

Fig. 7 DC Voltage response with load disturbance

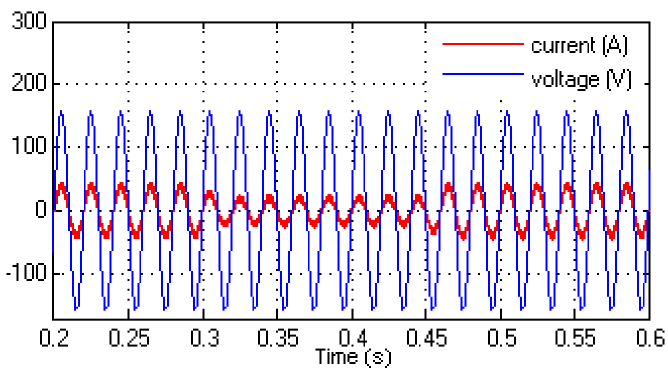

Fig. 8 voltage and current of A phase power source

V. CONCLUTIONS

In order to improve the performance of the three-phase PWM rectifiers, the paper presents a robust H• control for the outer voltage loop and a LQR for the inner current loop. The state equations are built. For the outer voltage loop, By building state equations, selecting the suitable weighting functions and solving the Riccati inequality, the H ∞ controller can be derived. For the inner current loop, a Riccati equation is solved and a constant value adjoint matrix is calculated to derive the designed controller. The simulation results show that the designed controller can achieve unit power factor control, and has faster response and better external disturbance rejection capabilities compared with the conventional PI controller.

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