

**National University of Singapore  
Department of Electrical and Computer Engineering  
MCH5002 Applications of Mechatronics**

**E1: MODELLING AND CONTROL OF A DC SERVO MOTOR  
WITH LABVIEW**

**1. OBJECTIVES**

This is a hands-on session on the application of computer-based control to a voltage-controllable electro-mechanical system – the DC motor. The session is mainly concerned with the modelling and control of a DC servo motor system, fully instrumented with position and velocity measurements. National Instrument's LabVIEW will be the control software for the experiment. At the end of the experiment, you should have some experience in

- Simple static and dynamic modelling of the DC motor system,
- Manual and feedback control of the system for velocity tracking

To benefit more fully from this session, students should read the manual and answer the pre-laboratory questions (Q1-Q3) before going to the laboratory.

**2. A MATHEMATICAL MODEL FOR DC MOTOR - BACKGROUND**

A simple mathematical relationship between the shaft angular velocity and voltage input to the DC motor may be derived from physical laws. The angular position may then be obtained from an integration of the angular velocity. With Newton's law of motion, we may obtain the following relationship,

$$J \frac{d\omega(t)}{dt} = \Gamma(t) \quad (1)$$

where  $\omega(t)$  is the velocity in rad/sec at time t, J is the inertia of motor and its load, and  $\Gamma(t)$  is the net torque given effectively by

$$\Gamma(t) = \tau_e(t) - \tau_L(t) - k_\omega \omega(t) \quad (2)$$

where  $k_\omega$  is a damping coefficient,  $\tau_L(t)$  is the load torque and  $\tau_e(t)$  is the generated torque. If we further assume that the generated torque is proportional to the applied voltage, v(t), we have the following differential equation describing the motor:

$$J \frac{d\omega(t)}{dt} = kv(t) - \tau_L(t) - k_\omega \omega(t) \quad (3)$$

The velocity may be measured with a tachogenerator, albeit in an electrical voltage form. Suppose we assume that the actual velocity is linearly related to the voltage output from the tachogenerator,  $y$ (volts).

$$\omega = my + c \quad (4)$$

Our differential equation in terms of tachogenerator output voltage and the applied voltage input is thus

$$Jm \frac{dy(t)}{dt} = kv(t) - \tau_L(t) - k_\omega my(t) - k_\omega c \quad (5)$$

or equivalently

$$T \frac{dy(t)}{dt} + y(t) = Kv(t) - k_L \tau_L(t) - k_c \quad (6)$$

where the time constant  $T = J / k_\omega$ , the steady state gain  $K = k / k_\omega m$  and the constants  $k_L = 1 / k_\omega m$ ,  $k_c = c / m$ . This is a simple first order differential equation.  $T$ ,  $K$ ,  $k_L \tau_L$ ,  $k_c$  are the model parameters.

We shall regard  $v(t)$ , the applied voltage as the input to the system,  $y(t)$  the measured output,  $\tau_L(t)$  the disturbance variable and  $c, m$  are constants.

### **Q1:**

If both the input voltage and the load disturbance are constant, what is the steady-state tachogenerator voltage, i.e.  $y(t)$  as  $t \rightarrow \infty$  ?

(HINT: For stable systems,  $\frac{d}{dt} \rightarrow 0, t \rightarrow \infty$ )

### **Q2:**

A proportional controller of the form  $v(t) = k_p(y_{SP} - y)$  is used on this plant. If  $y_{SP}$  is also constant, what is the steady-state tachogenerator voltage, i.e.  $y(t)$  as  $t \rightarrow \infty$  ? (HINT: For stable systems,  $\frac{d}{dt} \rightarrow 0, t \rightarrow \infty$ )

### **Q3:**

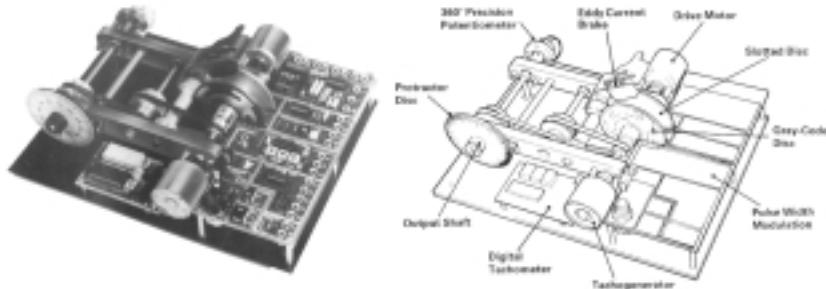
With the proportional controller, what is the time constant of the closed loop system? How is the magnitude compared to the open-loop time constant? (HINT: Put in the form below (where coefficient of  $y=1$ )

$$T_{cl} \frac{dy}{dt} + y = \dots$$

$T_{cl}$  is closed-loop time constant. )

### 3. APPARATUS

- LJ DC motor set (Fig. 1)



**Fig.1: DC Servomotor**

- PC with installed National Instruments VI data acquisition cards (2 channels of analog output and 8 channels of analog input), MATLAB and LabVIEW software
- Power supply for DC motor set
- Printer

### 4. INPUT-OUTPUT CONFIGURATION

The data acquisition box should be configured so that analog input channel 1 reads the motor output position voltage signal, analog input channel 2 reads the tacho generator output voltage signal and analog output channel 0 drives the motor.

### 5. INSPECTING THE SYSTEM

Identify the motor, the tachogenerator, the output shaft, the output position potentiometer, the load magnet arm, the RPM display indicator, and the angular position indicator. Ensure that the load arm is down so that the maximum load torque is applied.

## 6. STARTING LABVIEW

The experiment uses the software LabVIEW where various control configurations can be easily organised. LabVIEW, or Laboratory Virtual Instrument Engineering Workbench, is a graphical programming language that has been widely adopted throughout industry, academic, and research labs as the standard for data acquisition and instrument control software. It is a powerful and flexible instrumentation and analysis software system. Computers are much more flexible than standard instruments, and creating your own LabVIEW program, or virtual instrument (VI), is simple.



Double click **Labview** on the desktop of WINDOWS 95/98/2000 to start LabVIEW. Fig. 2 is a snapshot of the screen you should see.

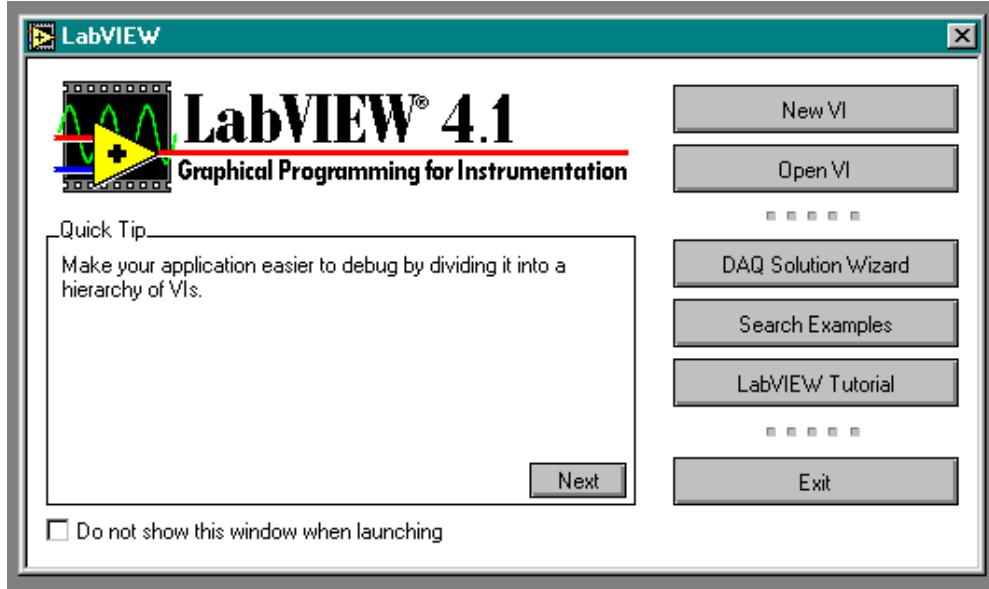


Fig. 2: Starting LabVIEW

## 7. PROCEDURES

### 7.1 Modelling of the DC Motor

The motor speed as measured by the tachogenerator output is the plant output and the applied voltage is the plant input. This part of the experiment will let you obtain the important model parameters ( $T$ ,  $K$ ,  $k_L\tau_L$ ,  $k_c$ ) in (6) of the system. The model of a system allows us to perform simulation, control and many other applications. We begin by finding out the relationship between motor speed measured in volts by the tachogenerator and the speed as displayed on the red digital display.

#### 7.1.1 Calibrating the Tachogenerator

The motor velocity is measured with a tachogenerator. The voltage output from the tachogenerator is approximately linearly related to the velocity. In this part of the experiment, we will find this linear relationship.

Click <Open VI> to load c:\motor\step1.vi. You can change the voltage applied to the motor by clicking the up/down arrow of Applied Voltage.

Set applied voltage output to -5 volts. Allow a few seconds for the speed to settle down. In Table 1 below, record the tachogenerator output in volts as well as the displayed speed in rpm. Complete Table 1 below in your report.

Table 1 Calibration of Speed Sensor

Applied voltage	Tachogenerator (Volts) (y)	Speed in rpm	Speed in rad/sec ( $\omega$ )
-5 volts			
-3 volts			
-1 volts			
0 volts			
1 volts			
3 volts			
5 volts			

**Q4:**

Using the data in Table 1, plot a graph of the speed ( $\omega$ ) in rad/sec vs the tachogenerator output voltage (y). From the graph estimate the coefficients m and c in Equation 4 above. Thus,  $k_c = c/m$  can be determined.

**Q5:**

Using the data in Table 1, plot a graph of the speed ( $\omega$ ) in rad/sec vs the applied voltage (v).

From (3), at steady state and assuming load torque is constant, we have

$$\omega = \frac{k}{k_\omega} v - \frac{1}{k_\omega} \tau_L$$

From plot and knowing m, we can therefore obtain  $K = k/(k_\omega m)$  and  $k_L \tau_L$  where  $k_L = 1/(k_\omega m)$ .

Comment on the linearity of this relationship, giving possible reasons for nonlinearities, if any. The only parameter left to be estimated is the time constant T which we will obtain next.

### 7.1.2 Estimating K and T from Step Tests

- Load the program c:\motor\step2.vi. This program sets up LabVIEW to apply a square wave (a series of steps). The magnitude and bias of the square wave is set by corresponding number controls.
- Using the number controls, set the bias to 1 volt and the magnitude to 1.5 volts. The applied voltage is then a square wave with voltages 1 volt and 2.5 volts. Observe the step response on the chart. Obtain a printout of the chart by pressing CTRL P.

**Q6:**

From the chart, we can estimate the time constant T in Equation 6. From the same chart, we can also obtain K to compare with what is obtained earlier. Let's see how to obtain these parameters.

Assuming the system is in a steady state with  $v_0$  and  $y_0$ , related by:

$$y_0 = Kv_0 - k_L\tau_L - k_C$$

Then let v changes to  $v_0 + \Delta v$ , and y becomes  $y_0 + \Delta y(t)$ . Substituting into (6), we have

$$T \frac{dy_0 + \Delta y(t)}{dt} + y_0 + \Delta y(t) = Kv_0 + K\Delta v - k_L\tau_L - k_C$$

Since  $y_0$  is a constant,  $dy_0/dt=0$  and this equation simplifies to

$$T \frac{d\Delta y(t)}{dt} + \Delta y(t) = K\Delta v$$

The solution to this differential equation is:

$$\Delta y(t) = K(1 - e^{-t/T})\Delta v.$$

Put t=T,

$$\Delta y(t) = 0.63K\Delta v = 0.63\Delta y_{ss},$$

where  $\Delta y_{ss}$  is the steady state change in y. i.e.  $\Delta y_{ss} = \Delta y(t \rightarrow \infty) = y(t \rightarrow \infty) - y_0$ .

Therefore, we can obtain T from the chart as the time for y to reach 63% of its final value. K can also be obtained from

$$K = \frac{\Delta y_{ss}}{\Delta v}.$$

## 7.2 Velocity Control

We will experiment with both the manual (open-loop) control and automatic (closed-loop) control of the motor velocity.

### 7.2.1 Open-Loop (Manual) Control

Now, we would like the motor speed to track the set point (desired speed). We would also like to achieve this tracking of the set point even if there are plant load changes or disturbances. An example of a load disturbance is when we change the load torque by moving the load arm. Alternatively, you can try to hold the output disk with your fingers. Naturally, we also need a stable response.

In this part of the experiment, we examine the performance of open loop control or manual control.

- Reload the program c:\motor\step1.vi.
- From Table 1 or your graph, estimate the voltage you need so that the motor velocity is 100 rpm. Using the control, set the applied voltage to your estimated value.
- Now raise (or lower) the load arm. Observe carefully the measured speed. You can obtain a plot of the chart by pressing CTRL P.
- Now try to adjust the applied voltage to get the speed back to 100 rpm.

#### Q7:

Comment on the performance of open loop control. Consider the steady state error and the response to load disturbance.

### 7.2.2 Feedback Control

(i) Proportional control (P) only

In this part of the experiment, we examine the performance of a simple proportional feedback controller. Here we compare the measured speed with a desired set point. The applied voltage is proportional to the resulting error signal. Thus  $v(t) = k_p[y_{SP} - y]$  where  $y_{SP}$  is the set point voltage (desired speed in terms of voltage).

- Load the program c:\motor\step3.vi. The set point  $y_{SP}$  is a square wave with a base voltage of 1 volt and an amplitude specified by the control magnitude. The proportional gain  $k_p$  is set by the control P. Begin with a small value of proportional gain .
- Record the transient and steady state response using the CTRL P print screen function. Note the steady state error.
- Now increase the proportional gain until the speed is unstable. Note the gain value which just causes instability. Obtain a chart of the unstable response at this point.
- Using a trial and error approach, find a value of proportional gain so that step response to a set point change has at most one overshoot and one undershoot. Set the controller for proportional control with this proportional gain determined. Print this chart.
- Now set the magnitude to 0 so that the set point is constant. Apply a load disturbance using the load arm. On one chart, obtain a record of the disturbance response.

**Q8:**

Use your experimental data to verify that the steady state error decreases as the proportional gain increases. From your experimental data, describe what happens to the transient response as the proportional gain becomes large? Without going into the mathematics, is this consistent with what you would expect from theory?

**Q9:**

Comment on the effect of the load disturbance.

**Q10:**

Summarise the conclusions of your experiment in terms of the effect of proportional feedback control on (a) the steady state error to set point change (b) the transient response to set point change (c) the steady state error to disturbance (d) the transient response to disturbance. (e) what you understand by an unstable feedback system.

(ii) Proportional plus integral (PI) control

With proportional control, it was not possible to remove steady state error. In proportional plus integral control, the applied voltage is proportional to the sum of the error and its integral with respect to time. This means that the error signal will increase or decrease until the error is zero

$$v(t) = k_p \left( e(t) + \frac{1}{T_i} \int e(t) dt \right) \quad e(t) = y_{SP} - y(t)$$

$K_p$  is the proportional gain and  $T_i$  is the integral (reset) time constant.

- Load the program c:\motor\step4.vi.
- Using the number controls, set the proportional gain based on the proportional control results. Using the control, set the parameter  $T_i$  to the largest value permitted.
- Record the transient and steady state response.
- Reduce the integral (reset) time. Record the transient and steady state response.
- Record the transient response to a load disturbance
- Using trial and error, select a set of proportional gain and integral (reset) time which gives a satisfactory response to step change in set point.

**Q11:**

Demonstrate with your experimental data that with any positive value of integral (reset) time, the steady state error to a set point change is zero.

**Q12:**

Comment on the transient response (both to set point change and to disturbance) when the integral (reset) time is reduced.

## 8. REPORT

The report should log the results from the experiment with your interpretations, observations and conclusions. You should try to answer all questions in the manual.