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# DC Motor Identification Using Speed Step Responses

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***Abstract*— Based on the DC motor speed response measurement under a constant voltage input, important motor parameters such as the electrical time constant, the mechanical time constant, and the friction can be estimated. A power series expansion of the motor speed response is presented, whose coefficients are used to calculate the motor parameters. Experimental results are presented to demonstrate the application of this approach and its effectiveness.**

## I. Introduction

DC motors have wide applications in control systems because they are easy to control and model. For analytical control system design and optimization, sometimes, a precise model of the DC motor used in a control system may be needed. In this case, the values for reference of the motor parameters given in the motor specifications provided by the motor manufacturer may not be considered adequate, especially for cheaper DC motors which tend to have relatively large tolerances in their electrical and mechanical parameters.

Without expensive testing apparatus and a long testing cycle, a quick and effective system identification approach based on the motor input and output is desirable and valuable, especially for the field applications and quick controller prototyping. In this paper, a DC motor parameter identification approach based on the Tyler series expansion of the motor constant voltage speed response is presented. The relationships between the motor parameters and the coefficients of the Tyler series are established. In the implementation, the motor speed response under a constant voltage is sampled, then fit the samples to obtain the coefficients of power terms in the Tyler series. The mechanical and electrical time constants, Back-emf and the friction of the DC motor can be estimated using these coefficients. With the knowledge of these parameters, a precise motor model is obtained for the subsequent controller design.

For application point if view, this approach requires only a speed/position sensor, such as an optical encoder, and a voltage power supply, and the motor is run in the open loop, thus it is practical and cost effective.

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## II. Algorithms

Consider the following DC motor governing equations

$$L\frac{di}{dt} + iR + k_b\omega = V, \tag{1}$$

$$J\frac{d\omega}{dt} = k_t i + T_d, \tag{2}$$

where $\omega$ is the motor speed, $V$ is the motor terminal voltage, $i$ is the winding current, $k_b$ is the back-EMF constant of the motor, $k_t$ is the torque constant, $R$ is the terminal resistance, $L$ is the terminal inductance, $J$ is the motor and load inertia, and $T_d$ is the disturbance torque. $T_d$ is a combination of the cogging torque, $T_{cog}$, the kinetic friction, $T_f$, and the viscous friction(viscous damping force)

$$T_d = T_{cog} + T_f + c\dot{\omega}, \tag{3}$$

where $c$ is the damping coefficient. According to Eqs. 1 and 2, the velocity response in the Laplace domain is

$$\omega(s) = \frac{\frac{1}{k_b}}{t_m t_e s^2 + t_m s + 1}V(s) + \frac{\frac{1}{J}t_m(t_e s + 1)}{t_m t_e s^2 + t_m s + 1}T_d(s), \tag{4}$$

where $t_e = \frac{L}{R}$ is the electrical time constant, $t_m = \frac{RJ}{k_t k_b}$ is the mechanical time constant, and $s$ is the Laplace variable.

Based on these equations, we would like to know $t_m$, $t_e$, $T_d$, $J$, etc, by measuring the velocity response under a known, controlled voltage input.

In this paper, We consider two application situations: The first situation is that the disturbance torque is negligible; while in the second one, the disturbance needs to be considered.

### A. Without the Disturbance Torque

When the voltage speed response dominates, e.g. the input voltage is large, we can ignore the disturbance torque in the speed response, see Eq. 4. In this case, we can consider the following DC motor model

$$\frac{\omega(s)}{V(s)} = \frac{\frac{1}{k_b}}{t_m t_e s^2 + t_m s + 1}. \tag{5}$$

The transfer function can be factorized into

$$\frac{\omega(s)}{V(s)} = \frac{\frac{1}{k_b}}{t_m t_e (s+a)(s+b)}, \tag{6}$$

where

$$a, b = \frac{1 \mp \sqrt{1 - 4t_e/t_m}}{2t_e}. \tag{7}$$

***Assumption:*** It is assumed here that there are two distinct real poles, that is $t_m > 4t_e$.

For a constant voltage input $V(s) = V_0/s$, the speed response is

$$\omega(s) = \frac{\frac{V_0}{k_b}}{t_m t_e s(s+a)(s+b)} = \frac{\alpha_1}{s} + \frac{\alpha_2}{s+a} + \frac{\alpha_3}{s+b}, \tag{8}$$

where

$$\alpha_1 = \frac{V_0}{k_b},\ \alpha_2 = \frac{V_0}{k_b}\frac{b}{a-b},\ \alpha_3 = \frac{V_0}{k_b}\frac{a}{b-a}.$$

Consider the three terms in the step response one at a time. $\frac{\alpha_1}{s}$ is a step function in the time domain, both $\frac{\alpha_2}{s+a}$ and $\frac{\alpha_1}{s+b}$ are exponential functions in the time domain and can be expanded using Taylor series. Expand the term $\frac{\alpha_2}{s+a}$, we get

$$\frac{V_0}{k_b}\frac{b}{a-b}(1 - at + \frac{1}{2}a^2t^2 - \frac{1}{6}a^3t^3 + \ldots).$$

Expand the term $\frac{\alpha_3}{s+b}$, we get

$$\frac{V_0}{k_b}\frac{a}{b-a}(1 - bt + \frac{1}{2}b^2t^2 - \frac{1}{6}b^3t^3 + \ldots).$$

Combine the three terms together, we have the total speed response

$$\frac{V_0}{k_b}(\frac{1}{2}\beta_0 t^2 + \frac{1}{6}\beta_1 t^3 + \frac{1}{24}\beta_2 t^4 + \ldots) \tag{9}$$

where $\beta_0 = ab$, $\beta_1 = -ab(a+b)$, and $\beta_2 = ab(a^2 + ab + b^2)$.

According to Eq. 7,

$$ab = \frac{1}{t_m t_e},\ a + b = \frac{1}{t_e}.$$

Thus, we have

$$t_m = -\frac{\beta_1}{\beta_0^2},\ \ t_e = -\frac{\beta_0}{\beta_1}. \tag{10}$$

The above equation allows us to calculate the mechanical and electrical time constants $t_m$ and $t_e$ using the coefficients of the power series in Eq. 9. These coefficients can be obtained by curve fitting the motor speed step response data using power functions.

### B. With Disturbance Torque

Consider that the disturbance torque in the DC motor is not negligible. The disturbance transfer function is

$$\frac{\omega(s)}{T_d(s)} = \frac{\frac{1}{J}t_m(t_e s + 1)}{t_m t_e s^2 + t_m s + 1}. \tag{11}$$

Disturbance torque generally consists of the cogging torque and the friction torque. The cogging torque is quite complicated and is not addressed here. Both the kinetic and viscous frictions are considered and are assumed to be constant on average under a constant motor speed.

Given a constant motor terminal voltage $V(s) = V_0/s$ and the constant disturbance(ignore the cogging torque or consider the average cogging torque effect on speed over one revolution is zero) $T_d(s) = T_0/s$, the speed response is

$$\omega(s) = \frac{\frac{1}{k_b}}{t_m t_e s^2 + t_m s + 1}\frac{V_0}{s} + \frac{\frac{1}{J}t_m(t_e s + 1)}{t_m t_e s^2 + t_m s + 1}\frac{T_0}{s}. \tag{12}$$

As in the previous section, applying the partial fraction expansion of the step response in the Laplace domain, then expanding the exponential terms in the time domain using Taylor series, we obtain the total step response in the time domain

$$\omega(t) = \beta_0 t + \beta_1 t^2 - \beta_2 t^3 + \beta_3 t^4 - \ldots \tag{13}$$

Based on these coefficients, we have

$$ab = \frac{18\beta_2^2 - 24\beta_3\beta_1}{3\beta_0\beta_2 + 2\beta_1^2}, \quad (14)$$

$$a + b = \frac{6\beta_2 - \beta_0 ab}{2\beta 1}, \quad (15)$$

and another equation for $a+b$,

$$a + b = \frac{12\beta_3 + \beta_1 ab}{2\beta 2}. \quad (16)$$

Then, we can express the motor parameters as

$$t_m = \frac{a+b}{ab}, \quad (17)$$

$$t_e = \frac{1}{a+b}, \quad (18)$$

$$\frac{T_0}{J} = \beta_0, \quad (19)$$

$$k_b = \frac{ab}{2\beta_1} V_0. \quad (20)$$

In practice, fit the measured motor speed step response using power functions according to Eq. 13, then calculate the motor parameters using Eqs. 17-20.

***Remark***: Another relationship useful for checking the algorithm is based on the steady state response of Eq. 12, expressed by the following equation

$$\frac{V_0}{k_b} + \beta_0 t_m = \omega_{ss}, \quad (21)$$

where $\beta_0 = \frac{T_0}{J}$ and $\omega_{ss}$ is the motor steady state angular speed.

## III. Implementation and Results

The proposed approaches were applied to a Mabuchi RK370CA motor. To implement the algorithms, a LabVIEW program was created to interface a motor pulse width modulated (PWM) driver and an optical encoder with quadrature digital outputs mounted on the motor shaft. The determinism of the sample time was assured by the LabVIEW real-time module. And, a National Instrument (NI) LabVIEW FPGA (field programmable gate array) card was utilized to receive the quadrature encoder signals to obtain the motor speed and to control the motor PWM drive.

TABLE I

Given Motor Parameter Values

| Parameter | Value | Unit |
|---|---|---|
| Terminal resistance | 17±15% | $\Omega$ |
| Terminal inductance | N/A | $Henry$ |
| Torque constant | 18.3±18% | $mNm/A$ |
| Mass moment of inertia | 9.0 | $gcm^2$ |
| Counter-electromotive force | 0.0233 | $\frac{volt}{rad/sec}$ |

Values of the motor parameters given in the motor specifications for reference are presented in Table 1.

Note that the Back-emf and torque constant are not equal. Inductance value is not given and was measured as $20.25Henry$. The resistance was measured as $16.4\Omega$. Thus $t_e = \frac{L}{R} = 0.00122sec$.

First, apply the approach for no disturbance torque. In this case, the speed response part due to the voltage input is assumed to dominate. To meet this condition, e.g. the speed variation at the steady state is small compared to the steady state speed, we sent a large voltage to drive the motor, $V = 20volt$. Next, we apply the approach considering disturbance torque. We drove the motor at two different voltage levels to demonstrate that the viscous friction varies with the speed.

Usually $t_e$ is very small compared to $t_m$, a good estimate of both $t_e$ and $t_m$ at the same time is difficult. Many factors, such as the sampling rate, noise level in the system, the order of the power series, can affect it's estimation. Because $t_m$ is much larger than $t_e$, two separate procedures were used in estimating $t_m$ and $t_e$. $t_m$ and $t_e$ were estimated separately using different data collected in separate tests with different sample rates and different time durations. For estimating $t_m$, the motor speed in both the transient phase and the steady state was sampled at 1kHz for one second; for estimating $t_e$, the motor speed in the transient phase was sampled at 8kHz for 200 msec. In each test, the motor was driven multiple times and parameter estimates were avaraged.

Results are summarized in Table 2. Note $R = 17\Omega$, $J = 9gcm^2$, $k_b = 0.0233\frac{volt}{rad/sec}$ and $k_t = 0.0183\frac{Nm}{A}$ are used to calculate $t_m$ in the fourth column in the table. According to Table 2, the estimates of $k_t$, $t_m$, and $t_e$ are in good agreement with those given by the motor specifications.

TABLE II

TEST RESULTS

| Parameter | w/o Dist. | w/ Dist. 2v/10v | Spec.(Meas.) | Unit |
|---|---|---|---|---|
| $k_t$ | 0.0238 | 0.0207/0.0169 | 0.0183±18% | $\frac{Nm}{A}$ |
| $t_m$ | 0.0407 | 0.0211/0.0203 | 0.0359 | $sec$ |
| $t_e$ | 0.00554 | 0.00122/0.00134 | (0.00122) | $sec$ |
| $\frac{T_0}{J}$ | N/A | 10.551/115.758 | N/A | $\frac{Nm}{kgm^2}$ |

Time responses sampled at 1kHz for 1s are given in Fig. 1 to 3. Compare these figures, it is obvious that the approach with disturbance consideration approximates the measurements much better, because of the existence of the linear term, $\beta_0 t$, in the power series due to the presence of the constant disturbance in the motor.

***Remark*** $\frac{T_0}{J}$ may be used to calculate the friction (both kinetic and viscous) if $J$ is known. First, calculate the viscous friction $c = \frac{T_1 - T_0}{\omega_1 - \omega_0}$. Then, calculate the dynamic friction, $T_f = T_0 - c\omega_0$. For example, $\omega_0 = 1.21 ips$ under $2volt$, $\omega_1 = 6.274 ips$ under $10volt$, $J = 9.0 gcm^2$, it renders $c = 0.0187 mNm/ips$.

***Remark*** The number of terms in the power series included for fitting the data was determined through trail and error. When disturbance was not considered, twenty five terms were included; when disturbance was considered, including fourty terms gave the best results. Since the coefficients were calculated using the polynomial curve fitting function from the math libarary provided inside LabVIEW, it was not difficult and time consuming to try different number of terms. Including more terms does not necessarily improve the parameter estimation accuracy. Convergence of this algorithm may require further investigation.

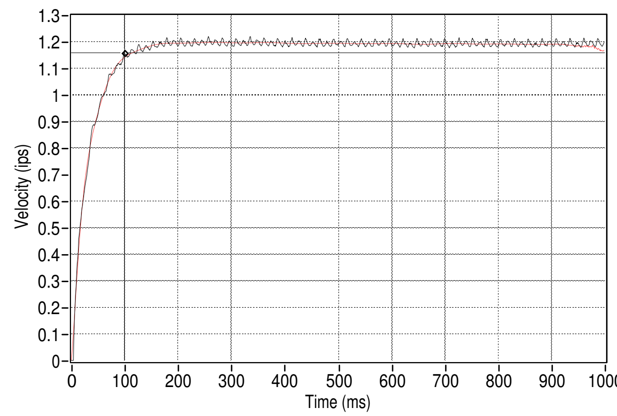


Fig. 1. Approach w/ consideration of disturbance under $2volt$ input: Black, measurement; Red, fitted.

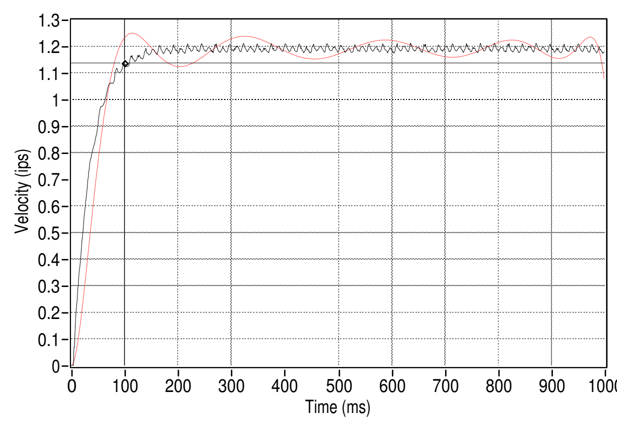


Fig. 2. Approach w/o consideration of disturbance under $2volt$ input:Black, measurement; Red, fitted.

## IV. CONCLUSIONS

A convenient, effective system identification approach is proposed to estimate the DC motor torque constant, mechanical time constant, electrical time constant, and friction. This approach was implemented on a Mabuchi motor, and the test results were presented. This open loop method requires little hardware, only a speed/position sensor and a voltage supply. The estimated motor parameters can be used to verify the DC motor performance, or be used to build a model of the motor for the subsequent controller design or system optimization. This approach is especially suited to quick field applications.

## REFERENCES


[1] Younkin, G.W., “Industrial Servo Control Systems: Fundamentals and Applications,” 2nd Edition, Marcel Dekker, 2003.

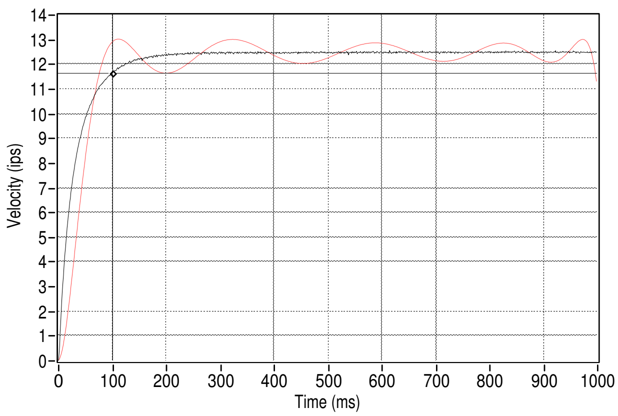


Fig. 3. Approach w/o consideration of disturbance under $20volt$ input:Black, measurement; Red, fitted.

## APPENDIX

### A. Coefficients for no disturbance case

$$\beta_0 = ab, \tag{22}$$

$$\beta_1 = -ab(a+b), \tag{23}$$

$$\beta_2 = ab(a^2 + ab + b^2). \tag{24}$$

### B. Coefficients for disturbance case

$$\beta_0 = \frac{T_0}{J}, \tag{25}$$

$$\beta_1 = \frac{1}{2}\frac{V_0}{k_b}ab, \tag{26}$$

$$\beta_2 = \frac{1}{6}[\frac{V_0}{k_b}ab(a+b) - \frac{T_0}{J}(a^2+ab+b^2) + \frac{T_0}{J}t_m ab(a+b)], \tag{27}$$

$$\beta_3 = \frac{1}{24}[\frac{V_0}{k_b}ab(a^2+ab+b^2) - \frac{T_0}{J}(a^3+a^2b+ab^2+b^3) + \frac{T_0}{J}t_m ab(a^2+ab+b^2)]. \tag{28}$$