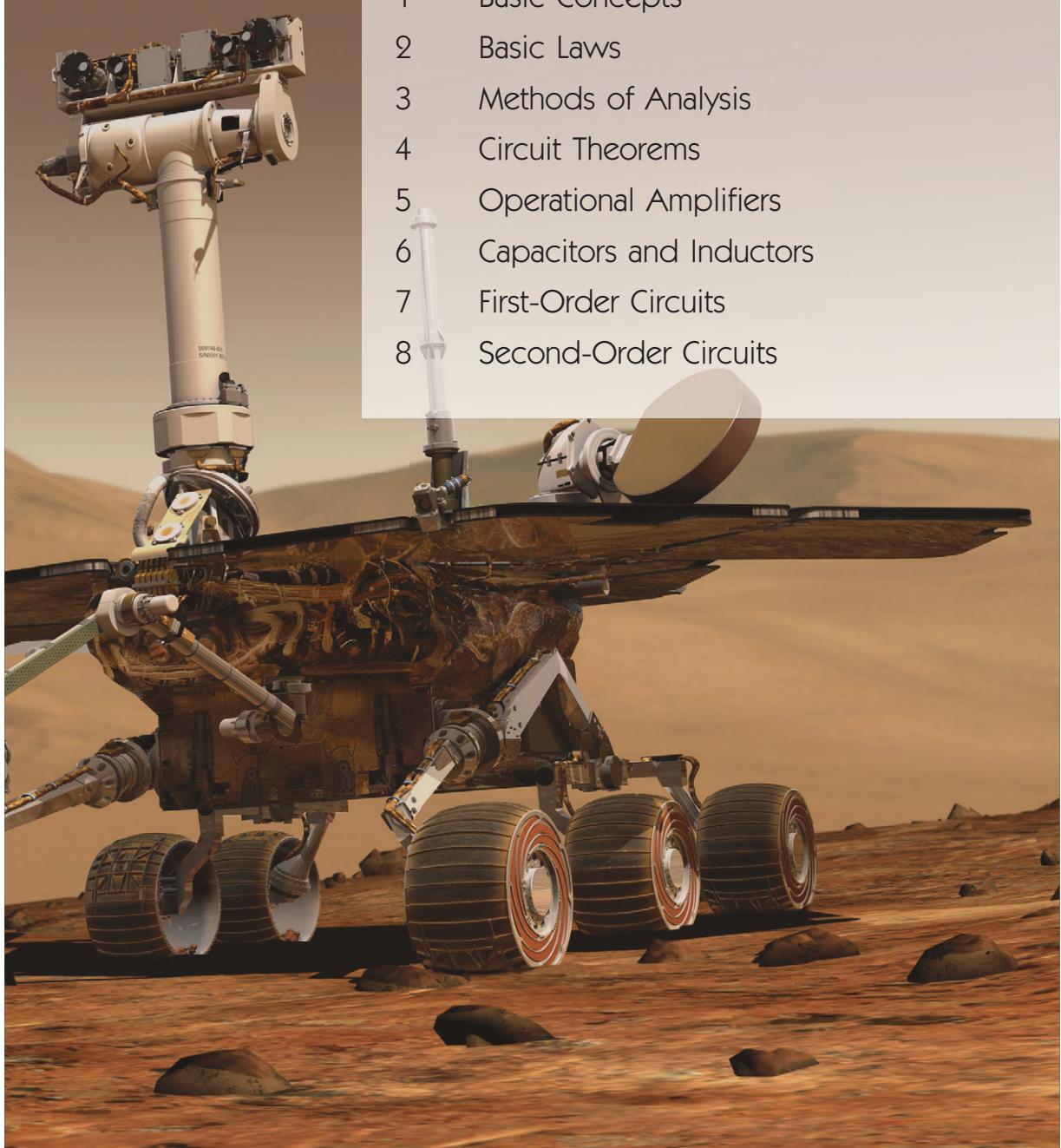


Fundamentals of Electric Circuits

DC Circuits

OUTLINE

- 1 Basic Concepts
- 2 Basic Laws
- 3 Methods of Analysis
- 4 Circuit Theorems
- 5 Operational Amplifiers
- 6 Capacitors and Inductors
- 7 First-Order Circuits
- 8 Second-Order Circuits



Basic Concepts

Some books are to be tasted, others to be swallowed, and some few to be chewed and digested.

—Francis Bacon

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.a), “an ability to apply knowledge of mathematics, science, and engineering.”

As students, you are required to study mathematics, science, and engineering with the purpose of being able to apply that knowledge to the solution of engineering problems. The skill here is the ability to apply the fundamentals of these areas in the solution of a problem. So how do you develop and enhance this skill?

The best approach is to work as many problems as possible in all of your courses. However, if you are really going to be successful with this, you must spend time analyzing where and when and why you have difficulty in easily arriving at successful solutions. You may be surprised to learn that most of your problem-solving problems are with mathematics rather than your understanding of theory. You may also learn that you start working the problem too soon. Taking time to think about the problem and how you should solve it will always save you time and frustration in the end.

What I have found that works best for me is to apply our six-step problem-solving technique. Then I carefully identify the areas where I have difficulty solving the problem. Many times, my actual deficiencies are in my understanding and ability to use correctly certain mathematical principles. I then return to my fundamental math texts and carefully review the appropriate sections, and in some cases, work some example problems in that text. This brings me to another important thing you should always do: Keep nearby all your basic mathematics, science, and engineering textbooks.

This process of continually looking up material you thought you had acquired in earlier courses may seem very tedious at first; however, as your skills develop and your knowledge increases, this process will become easier and easier. On a personal note, it is this very process that led me from being a much less than average student to someone who could earn a Ph.D. and become a successful researcher.



Photo by Charles Alexander

1.1 Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

An **electric circuit** is an interconnection of electrical elements.

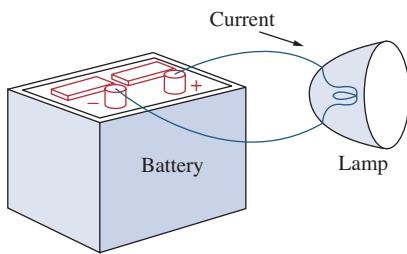


Figure 1.1
A simple electric circuit.

A simple electric circuit is shown in Fig. 1.1. It consists of three basic elements: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a flashlight, a search light, and so forth.

A complicated real circuit is displayed in Fig. 1.2, representing the schematic diagram for a radio receiver. Although it seems complicated, this circuit can be analyzed using the techniques we cover in this book. Our goal in this text is to learn various analytical techniques and computer software applications for describing the behavior of a circuit like this.

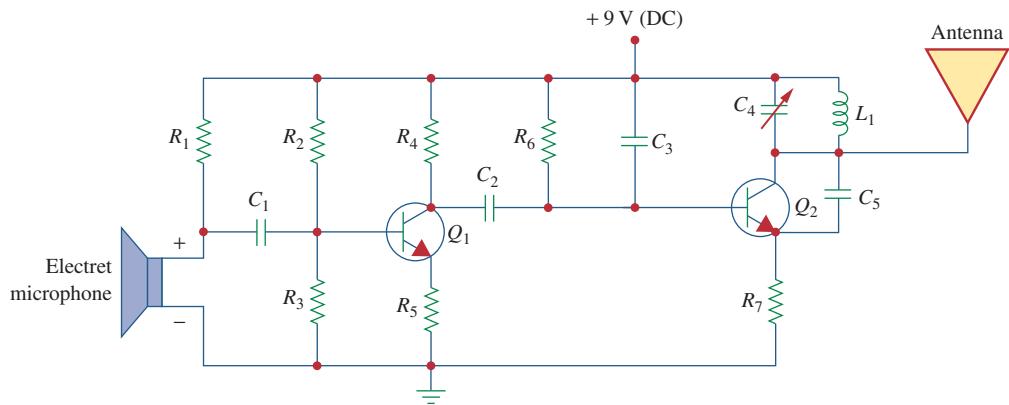


Figure 1.2
Electric circuit of a radio transmitter.

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather, our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

1.2 Systems of Units

As electrical engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system, there are seven principal units from which the units of all other physical quantities can be derived. Table 1.1 shows the six units and one derived unit that are relevant to this text. The SI units are used throughout this text.

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

600,000,000 mm 600,000 m 600 km

1.3 Charge and Current

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge*. We all experience the effect of electric

TABLE 1.1

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	K	
Luminous intensity	candela	cd
Charge	coulomb	C

TABLE 1.2

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC .¹
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

We now consider the flow of electric charges. A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges. That is, opposite to the flow of negative charges, as Fig. 1.3 illustrates. This convention was introduced by Benjamin Franklin (1706–1790), the American scientist and inventor. Although we now know that current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges. Thus,

Electric current is the time rate of change of charge, measured in amperes (A).

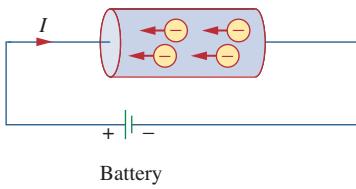


Figure 1.3

Electric current due to flow of electronic charge in a conductor.

A convention is a standard way of describing something so that others in the profession can understand what we mean. We will be using IEEE conventions throughout this book.

Mathematically, the relationship between current i , charge q , and time t is

$$i \triangleq \frac{dq}{dt} \quad (1.1)$$

¹ However, a large power supply capacitor can store up to 0.5 C of charge.

Historical

Andre-Marie Ampere (1775–1836), a French mathematician and physicist, laid the foundation of electrodynamics. He defined the electric current and developed a way to measure it in the 1820s.

Born in Lyons, France, Ampere at age 12 mastered Latin in a few weeks, as he was intensely interested in mathematics and many of the best mathematical works were in Latin. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and the ammeter. The unit of electric current, the ampere, was named after him.



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San Marino, California.

where current is measured in amperes (A), and

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$Q \triangleq \int_{t_0}^t i dt \quad (1.2)$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways.

If the current does not change with time, but remains constant, we call it a *direct current* (dc).

A **direct current** (dc) is a current that remains constant with time.

By convention the symbol I is used to represent such a constant current.

A time-varying current is represented by the symbol i . A common form of time-varying current is the sinusoidal current or *alternating current* (ac).

An **alternating current** (ac) is a current that varies sinusoidally with time.

Such current is used in your household to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4

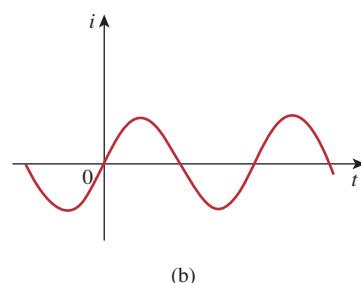
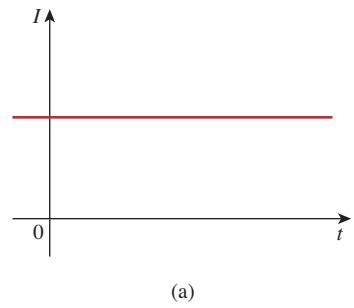
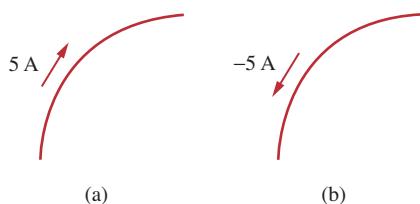


Figure 1.4

Two common types of current: (a) direct current (dc), (b) alternating current (ac).

**Figure 1.5**

Conventional current flow: (a) positive current flow, (b) negative current flow.

shows direct current and alternating current; these are the two most common types of current. We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of -5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of $+5\text{ A}$ flowing in the opposite direction.

Example 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has $-1.602 \times 10^{-19}\text{ C}$. Hence 4,600 electrons will have
 $-1.602 \times 10^{-19}\text{ C/electron} \times 4,600 \text{ electrons} = -7.369 \times 10^{-16}\text{ C}$

Practice Problem 1.1

Calculate the amount of charge represented by six million protons.

Answer: $+9.612 \times 10^{-13}\text{ C}$.

Example 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t \text{ mC}$. Calculate the current at $t = 0.5\text{ s}$.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem 1.2

If in Example 1.2, $q = (10 - 10e^{-2t}) \text{ mC}$, find the current at $t = 1.0\text{ s}$.

Answer: 2.707 mA.

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Example 1.3

Solution:

$$\begin{aligned} Q &= \int_{t=1}^2 i \, dt = \int_1^2 (3t^2 - t) \, dt \\ &= \left(t^3 - \frac{t^2}{2} \right) \Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2} \right) = 5.5 \text{ C} \end{aligned}$$

The current flowing through an element is

Practice Problem 1.3

$$i = \begin{cases} 4 \text{ A}, & 0 < t < 1 \\ 4t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 13.333 C.

1.4 Voltage

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b ; mathematically,

$$v_{ab} \triangleq \frac{dw}{dq} \quad (1.3)$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Thus,

Voltage (or **potential difference**) is the energy required to move a unit charge through an element, measured in volts (V).

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b . The plus (+) and minus (−) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) Point a is at a potential of v_{ab}

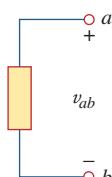


Figure 1.6
Polarity of voltage v_{ab} .

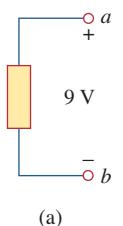
Historical



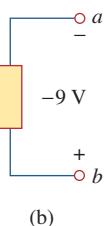
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Alessandro Antonio Volta (1745–1827), an Italian physicist, invented the electric battery—which provided the first continuous flow of electricity—and the capacitor.

Born into a noble family in Como, Italy, Volta was performing electrical experiments at age 18. His invention of the battery in 1796 revolutionized the use of electricity. The publication of his work in 1800 marked the beginning of electric circuit theory. Volta received many honors during his lifetime. The unit of voltage or potential difference, the volt, was named in his honor.



(a)



(b)

Figure 1.7

Two equivalent representations of the same voltage v_{ab} : (a) Point a is 9 V above point b ; (b) point b is -9 V above point a .

Keep in mind that electric current is always through an element and that electric voltage is always across the element or between two points.

volts higher than point b , or (2) the potential at point a with respect to point b is v_{ab} . It follows logically that in general

$$v_{ab} = -v_{ba} \quad (1.4)$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b ; in Fig. 1.7(b), point b is -9 V above point a . We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a . In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a .

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V , whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v . A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

1.5 Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus, power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$p \triangleq \frac{dw}{dt} \quad (1.5)$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \quad (1.6)$$

or

$$p = vi \quad (1.7)$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a + sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a - sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention*. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of +12 W because a positive current enters the negative terminal. Of course, an absorbing power of -12 W is equivalent to a supplying power of +12 W. In general,

$$+\text{Power absorbed} = -\text{Power supplied}$$

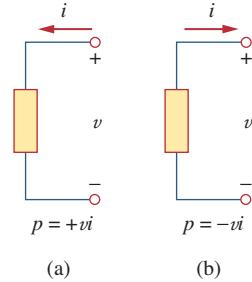


Figure 1.8

Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

When the voltage and current directions conform to Fig. 1.8(b), we have the *active sign convention* and $p = +vi$.

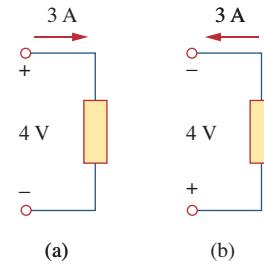


Figure 1.9

Two cases of an element with an absorbing power of 12 W: (a) $p = 4 \times 3 = 12$ W, (b) $p = 4 \times 3 = 12$ W.

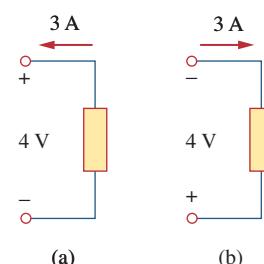


Figure 1.10

Two cases of an element with a supplying power of 12 W: (a) $p = -4 \times 3 = -12$ W, (b) $p = -4 \times 3 = -12$ W.

In fact, the *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0 \quad (1.8)$$

This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.

From Eq. (1.6), the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt \quad (1.9)$$

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3,600 \text{ J}$$

Example 1.4

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i \Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

Practice Problem 1.4

To move charge q from point a to point b requires -30 J . Find the voltage drop v_{ab} if: (a) $q = 6 \text{ C}$, (b) $q = -3 \text{ C}$.

Answer: (a) -5 V , (b) 10 V .

Example 1.5

Find the power delivered to an element at $t = 3 \text{ ms}$ if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3 di/dt$.

Solution:

(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3 \text{ ms}$,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3 \frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3 \text{ ms}$,

$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \end{aligned}$$

Find the power delivered to the element in Example 1.5 at $t = 5 \text{ ms}$ if the current remains the same but the voltage is: (a) $v = 2i \text{ V}$,

Practice Problem 1.5

(b) $v = \left(10 + 5 \int_0^t i \, dt \right) \text{ V}$.

Answer: (a) 17.27 W, (b) 29.7 W.

How much energy does a 100-W electric bulb consume in two hours?

Example 1.6**Solution:**

$$\begin{aligned} w &= pt = 100 \text{ (W)} \times 2 \text{ (h)} \times 60 \text{ (min/h)} \times 60 \text{ (s/min)} \\ &= 720,000 \text{ J} = 720 \text{ kJ} \end{aligned}$$

This is the same as

$$w = pt = 100 \text{ W} \times 2 \text{ h} = 200 \text{ Wh}$$

A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 180 kJ?

Practice Problem 1.6

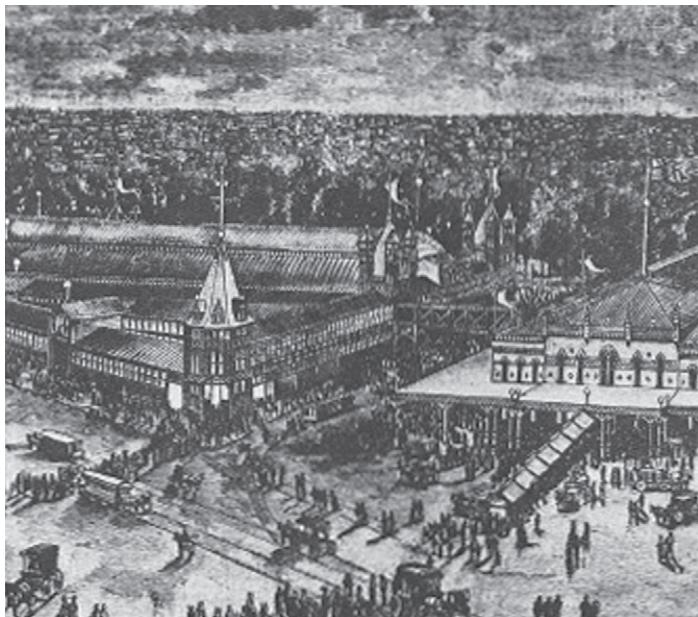
Answer: 50 s.

Historical

1884 Exhibition In the United States, nothing promoted the future of electricity like the 1884 International Electrical Exhibition. Just imagine a world without electricity, a world illuminated by candles and gaslights, a world where the most common transportation was by walking and riding on horseback or by horse-drawn carriage. Into this world an exhibition was created that highlighted Thomas Edison and reflected his highly developed ability to promote his inventions and products. His exhibit featured spectacular lighting displays powered by an impressive 100-kW “Jumbo” generator.

Edward Weston’s dynamos and lamps were featured in the United States Electric Lighting Company’s display. Weston’s well known collection of scientific instruments was also shown.

Other prominent exhibitors included Frank Sprague, Elihu Thompson, and the Brush Electric Company of Cleveland. The American Institute of Electrical Engineers (AIEE) held its first technical meeting on October 7–8 at the Franklin Institute during the exhibit. AIEE merged with the Institute of Radio Engineers (IRE) in 1964 to form the Institute of Electrical and Electronics Engineers (IEEE).



Smithsonian Institution.

1.6 Circuit Elements

As we discussed in Section 1.1, an element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers. Our aim in this section is to gain familiarity with some important active elements.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1.11 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.11(a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.11(a) can be used for a time-varying voltage source. Similarly, an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol for an independent current source is displayed in Fig. 1.12, where the arrow indicates the direction of current i .

An **ideal dependent (or controlled) source** is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCSV).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

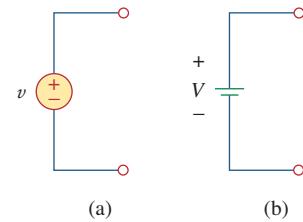


Figure 1.11
Symbols for independent voltage sources:
(a) used for constant or time-varying voltage,
(b) used for constant voltage (dc).

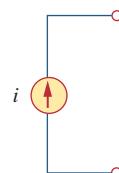


Figure 1.12
Symbol for independent current source.

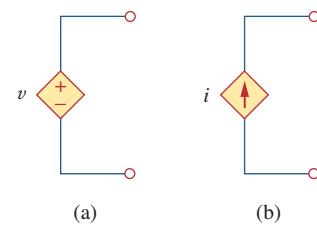
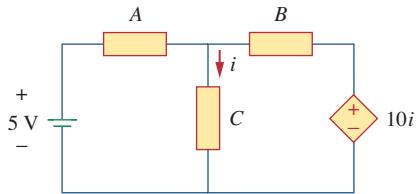


Figure 1.13
Symbols for: (a) dependent voltage source,
(b) dependent current source.

**Figure 1.14**

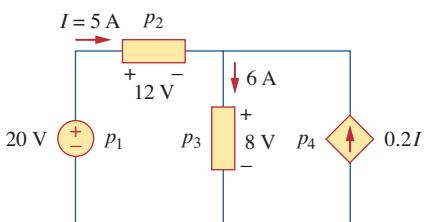
The source on the right-hand side is a current-controlled voltage source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.14, where the voltage $10i$ of the voltage source depends on the current i through element C . Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus, an ideal source could in theory supply an infinite amount of energy. It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

Example 1.7

Calculate the power supplied or absorbed by each element in Fig. 1.15.

**Figure 1.15**

For Example 1.7.

Solution:

We apply the sign convention for power shown in Figs. 1.8 and 1.9. For p_1 , the 5-A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100 \text{ W} \quad \text{Supplied power}$$

For p_2 and p_3 , the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60 \text{ W} \quad \text{Absorbed power}$$

$$p_3 = 8(6) = 48 \text{ W} \quad \text{Absorbed power}$$

For p_4 , we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3 , since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \quad \text{Supplied power}$$

We should observe that the 20-V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the two passive elements are absorbing power. Also,

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

In agreement with Eq. (1.8), the total power supplied equals the total power absorbed.

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

Answer: $p_1 = -45 \text{ W}$, $p_2 = 18 \text{ W}$, $p_3 = 12 \text{ W}$, $p_4 = 15 \text{ W}$.

1.7 [†]Applications²

In this section, we will consider two practical applications of the concepts developed in this chapter. The first one deals with the TV picture tube and the other with how electric utilities determine your electric bill.

1.7.1 TV Picture Tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver.³ The CRT is depicted in Fig. 1.17. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun, maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus, the beam can be made to “paint” a picture on the TV screen.

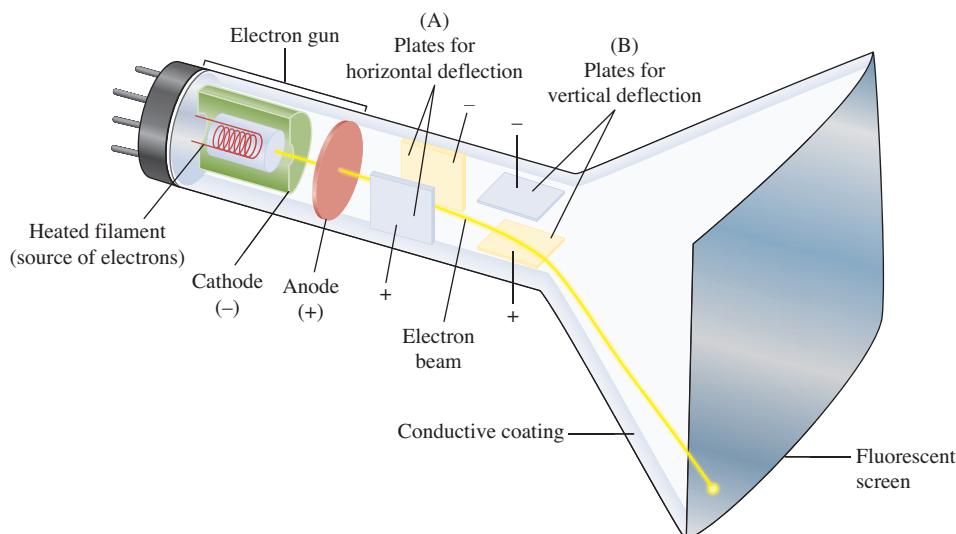


Figure 1.17
Cathode-ray tube.

Practice Problem 1.7

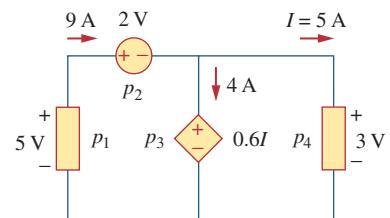


Figure 1.16
For Practice Prob. 1.7.

² The dagger sign preceding a section heading indicates the section that may be skipped, explained briefly, or assigned as homework.

³ Modern TV tubes use a different technology.

Historical

Karl Ferdinand Braun and Vladimir K. Zworykin

Karl Ferdinand Braun (1850–1918), of the University of Strasbourg, invented the Braun cathode-ray tube in 1879. This then became the basis for the picture tube used for so many years for televisions. It is still the most economical device today, although the price of flat-screen systems is rapidly becoming competitive. Before the Braun tube could be used in television, it took the inventiveness of **Vladimir K. Zworykin** (1889–1982) to develop the iconoscope so that the modern television would become a reality. The iconoscope developed into the orthicon and the image orthicon, which allowed images to be captured and converted into signals that could be sent to the television receiver. Thus, the television camera was born.

Example 1.8

The electron beam in a TV picture tube carries 10^{15} electrons per second. As a design engineer, determine the voltage V_o needed to accelerate the electron beam to achieve 4 W.

Solution:

The charge on an electron is

$$e = -1.6 \times 10^{-19} \text{ C}$$

If the number of electrons is n , then $q = ne$ and

$$i = \frac{dq}{dt} = e \frac{dn}{dt} = (-1.6 \times 10^{-19})(10^{15}) = -1.6 \times 10^{-4} \text{ A}$$

The negative sign indicates that the current flows in a direction opposite to electron flow as shown in Fig. 1.18, which is a simplified diagram of the CRT for the case when the vertical deflection plates carry no charge. The beam power is

$$p = V_o i \quad \text{or} \quad V_o = \frac{p}{i} = \frac{4}{1.6 \times 10^{-4}} = 25,000 \text{ V}$$

Thus, the required voltage is 25 kV.

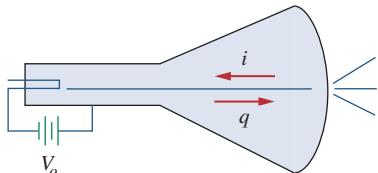


Figure 1.18

A simplified diagram of the cathode-ray tube; for Example 1.8.

Practice Problem 1.8

If an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 30 kV, calculate the power in the beam.

Answer: 48 mW.

TABLE 1.3

Typical average monthly consumption of household appliances.

Appliance	kWh consumed	Appliance	kWh consumed
Water heater	500	Washing machine	120
Freezer	100	Stove	100
Lighting	100	Dryer	80
Dishwasher	35	Microwave oven	25
Electric iron	15	Personal computer	12
TV	10	Radio	8
Toaster	4	Clock	2

1.7.2 Electricity Bills

The second application deals with how an electric utility company charges their customers. The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh). (Other factors that affect the cost include demand and power factors; we will ignore these for now.) However, even if a consumer uses no energy at all, there is a minimum service charge the customer must pay because it costs money to stay connected to the power line. As energy consumption increases, the cost per kWh drops. It is interesting to note the average monthly consumption of household appliances for a family of five, shown in Table 1.3.

A homeowner consumes 700 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

Example 1.9

Base monthly charge of \$12.00.

First 100 kWh per month at 16 cents/kWh.

Next 200 kWh per month at 10 cents/kWh.

Over 300 kWh per month at 6 cents/kWh.

Solution:

We calculate the electricity bill as follows.

$$\text{Base monthly charge} = \$12.00$$

$$\text{First } 100 \text{ kWh @ } \$0.16/\text{kWh} = \$16.00$$

$$\text{Next } 200 \text{ kWh @ } \$0.10/\text{kWh} = \$20.00$$

$$\text{Remaining } 400 \text{ kWh @ } \$0.06/\text{kWh} = \$24.00$$

$$\text{Total charge} = \$72.00$$

$$\text{Average cost} = \frac{\$72}{100 + 200 + 400} = 10.2 \text{ cents/kWh}$$

Referring to the residential rate schedule in Example 1.9, calculate the average cost per kWh if only 350 kWh are consumed in July when the family is on vacation most of the time.

Practice Problem 1.9

Answer: 14.571 cents/kWh.

1.8 †Problem Solving

Although the problems to be solved during one's career will vary in complexity and magnitude, the basic principles to be followed remain the same. The process outlined here is the one developed by the authors over many years of problem solving with students, for the solution of engineering problems in industry, and for problem solving in research.

We will list the steps simply and then elaborate on them.

1. Carefully **define** the problem.
2. **Present** everything you know about the problem.
3. Establish a set of **alternative** solutions and determine the one that promises the greatest likelihood of success.
4. **Attempt** a problem solution.
5. **Evaluate** the solution and check for accuracy.
6. Has the problem been solved **satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.

1. *Carefully define the problem.* This may be the most important part of the process, because it becomes the foundation for all the rest of the steps. In general, the presentation of engineering problems is somewhat incomplete. You must do all you can to make sure you understand the problem as thoroughly as the presenter of the problem understands it. Time spent at this point clearly identifying the problem will save you considerable time and frustration later. As a student, you can clarify a problem statement in a textbook by asking your professor. A problem presented to you in industry may require that you consult several individuals. At this step, it is important to develop questions that need to be addressed before continuing the solution process. If you have such questions, you need to consult with the appropriate individuals or resources to obtain the answers to those questions. With those answers, you can now refine the problem, and use that refinement as the problem statement for the rest of the solution process.

2. *Present everything you know about the problem.* You are now ready to write down everything you know about the problem and its possible solutions. This important step will save you time and frustration later.

3. *Establish a set of alternative solutions and determine the one that promises the greatest likelihood of success.* Almost every problem will have a number of possible paths that can lead to a solution. It is highly desirable to identify as many of those paths as possible. At this point, you also need to determine what tools are available to you, such as *PSpice* and *MATLAB* and other software packages that can greatly reduce effort and increase accuracy. Again, we want to stress that time spent carefully defining the problem and investigating alternative approaches to its solution will pay big dividends later. Evaluating the alternatives and determining which promises the greatest likelihood of success may be difficult but will be well worth the effort. Document this process well since you will want to come back to it if the first approach does not work.

4. *Attempt a problem solution.* Now is the time to actually begin solving the problem. The process you follow must be well documented

in order to present a detailed solution if successful, and to evaluate the process if you are not successful. This detailed evaluation may lead to corrections that can then lead to a successful solution. It can also lead to new alternatives to try. Many times, it is wise to fully set up a solution before putting numbers into equations. This will help in checking your results.

5. *Evaluate the solution and check for accuracy.* You now thoroughly evaluate what you have accomplished. Decide if you have an acceptable solution, one that you want to present to your team, boss, or professor.

6. *Has the problem been solved satisfactorily? If so, present the solution; if not, then return to step 3 and continue through the process again.* Now you need to present your solution or try another alternative. At this point, presenting your solution may bring closure to the process. Often, however, presentation of a solution leads to further refinement of the problem definition, and the process continues. Following this process will eventually lead to a satisfactory conclusion.

Now let us look at this process for a student taking an electrical and computer engineering foundations course. (The basic process also applies to almost every engineering course.) Keep in mind that although the steps have been simplified to apply to academic types of problems, the process as stated always needs to be followed. We consider a simple example.

Solve for the current flowing through the $8\text{-}\Omega$ resistor in Fig. 1.19.

Example 1.10

Solution:

1. *Carefully define the problem.* This is only a simple example, but we can already see that we do not know the polarity on the 3-V source. We have the following options. We can ask the professor what the polarity should be. If we cannot ask, then we need to make a decision on what to do next. If we have time to work the problem both ways, we can solve for the current when the 3-V source is plus on top and then plus on the bottom. If we do not have the time to work it both ways, assume a polarity and then carefully document your decision. Let us assume that the professor tells us that the source is plus on the bottom as shown in Fig. 1.20.

2. *Present everything you know about the problem.* Presenting all that we know about the problem involves labeling the circuit clearly so that we define what we seek.

Given the circuit shown in Fig. 1.20, solve for $i_{8\Omega}$.

We now check with the professor, if reasonable, to see if the problem is properly defined.

3. *Establish a set of alternative solutions and determine the one that promises the greatest likelihood of success.* There are essentially three techniques that can be used to solve this problem. Later in the text you will see that you can use circuit analysis (using Kirchhoff's laws and Ohm's law), nodal analysis, and mesh analysis.

To solve for $i_{8\Omega}$ using circuit analysis will eventually lead to a solution, but it will likely take more work than either nodal or mesh

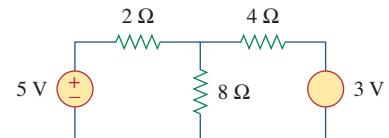


Figure 1.19
Illustrative example.

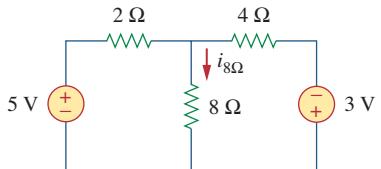


Figure 1.20
Problem definition.

analysis. To solve for $i_{8\Omega}$ using mesh analysis will require writing two simultaneous equations to find the two loop currents indicated in Fig. 1.21. Using nodal analysis requires solving for only one unknown. This is the easiest approach.

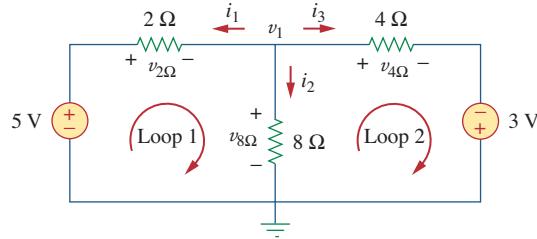


Figure 1.21
Using nodal analysis.

Therefore, we will solve for $i_{8\Omega}$ using nodal analysis.

4. **Attempt a problem solution.** We first write down all of the equations we will need in order to find $i_{8\Omega}$.

$$i_{8\Omega} = i_2, \quad i_2 = \frac{v_1}{8}, \quad i_{8\Omega} = \frac{v_1}{8}$$

$$\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} = 0$$

Now we can solve for v_1 .

$$8 \left[\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} \right] = 0$$

$$\text{leads to } (4v_1 - 20) + (v_1) + (2v_1 + 6) = 0$$

$$7v_1 = +14, \quad v_1 = +2 \text{ V}, \quad i_{8\Omega} = \frac{v_1}{8} = \frac{2}{8} = \mathbf{0.25 \text{ A}}$$

5. **Evaluate the solution and check for accuracy.** We can now use Kirchhoff's voltage law (KVL) to check the results.

$$i_1 = \frac{v_1 - 5}{2} = \frac{2 - 5}{2} = -\frac{3}{2} = -1.5 \text{ A}$$

$$i_2 = i_{8\Omega} = 0.25 \text{ A}$$

$$i_3 = \frac{v_1 + 3}{4} = \frac{2 + 3}{4} = \frac{5}{4} = 1.25 \text{ A}$$

$$i_1 + i_2 + i_3 = -1.5 + 0.25 + 1.25 = 0 \quad (\text{Checks.})$$

Applying KVL to loop 1,

$$\begin{aligned} -5 + v_{2\Omega} + v_{8\Omega} &= -5 + (-i_1 \times 2) + (i_2 \times 8) \\ &= -5 + [-(1.5)2] + (0.25 \times 8) \\ &= -5 + 3 + 2 = 0 \quad (\text{Checks.}) \end{aligned}$$

Applying KVL to loop 2,

$$\begin{aligned} -v_{8\Omega} + v_{4\Omega} - 3 &= -(i_2 \times 8) + (i_3 \times 4) - 3 \\ &= -(0.25 \times 8) + (1.25 \times 4) - 3 \\ &= -2 + 5 - 3 = 0 \quad (\text{Checks.}) \end{aligned}$$

So we now have a very high degree of confidence in the accuracy of our answer.

6. *Has the problem been solved satisfactorily? If so, present the solution; if not, then return to step 3 and continue through the process again.* This problem has been solved satisfactorily.

The current through the $8\text{-}\Omega$ resistor is 0.25 A flowing down through the $8\text{-}\Omega$ resistor.

Try applying this process to some of the more difficult problems at the end of the chapter.

Practice Problem 1.10

1.9 Summary

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the seven principal units, the units of other physical quantities can be derived.
3. Current is the rate of charge flow past a given point in a given direction.

$$i = \frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge through an element.

$$v = \frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p = \frac{dw}{dt} = vi$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
9. Two areas of application of the concepts covered in this chapter are the TV picture tube and electricity billing procedure.

Review Questions

- 1.1** One millivolt is one millionth of a volt.
 (a) True (b) False
- 1.2** The prefix *micro* stands for:
 (a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}
- 1.3** The voltage 2,000,000 V can be expressed in powers of 10 as:
 (a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV
- 1.4** A charge of 2 C flowing past a given point each second is a current of 2 A.
 (a) True (b) False
- 1.5** The unit of current is:
 (a) coulomb (b) ampere
 (c) volt (d) joule
- 1.6** Voltage is measured in:
 (a) watts (b) amperes
 (c) volts (d) joules per second
- 1.7** A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.
 (a) True (b) False
- 1.8** The voltage across a 1.1-kW toaster that produces a current of 10 A is:
 (a) 11 kV (b) 1100 V (c) 110 V (d) 11 V
- 1.9** Which of these is not an electrical quantity?
 (a) charge (b) time (c) voltage
 (d) current (e) power
- 1.10** The dependent source in Fig. 1.22 is:
 (a) voltage-controlled current source
 (b) voltage-controlled voltage source
 (c) current-controlled voltage source
 (d) current-controlled current source

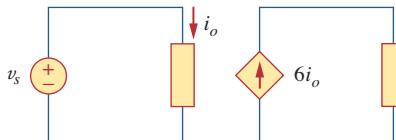


Figure 1.22

For Review Question 1.10.

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5b, 1.6c, 1.7a, 1.8c, 1.9b, 1.10d.

Problems

Section 1.3 Charge and Current

- 1.1** How many coulombs are represented by these amounts of electrons?
 (a) 6.482×10^{17} (b) 1.24×10^{18}
 (c) 2.46×10^{19} (d) 1.628×10^{20}
- 1.2** Determine the current flowing through an element if the charge flow is given by
 (a) $q(t) = (3t + 8)$ mC
 (b) $q(t) = (8t^2 + 4t - 2)$ C
 (c) $q(t) = (3e^{-t} - 5e^{-2t})$ nC
 (d) $q(t) = 10 \sin 120\pi t$ pC
 (e) $q(t) = 20e^{-4t} \cos 50t$ μ C
- 1.3** Find the charge $q(t)$ flowing through a device if the current is:
 (a) $i(t) = 3$ A, $q(0) = 1$ C
 (b) $i(t) = (2t + 5)$ mA, $q(0) = 0$
 (c) $i(t) = 20 \cos(10t + \pi/6)$ μ A, $q(0) = 2$ μ C
 (d) $i(t) = 10e^{-30t} \sin 40t$ A, $q(0) = 0$

- 1.4** A current of 7.4 A flows through a conductor. Calculate how much charge passes through any cross-section of the conductor in 20 s.
- 1.5** Determine the total charge transferred over the time interval of $0 \leq t \leq 10$ s when $i(t) = \frac{1}{2}t$ A.
- 1.6** The charge entering a certain element is shown in Fig. 1.23. Find the current at:
 (a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

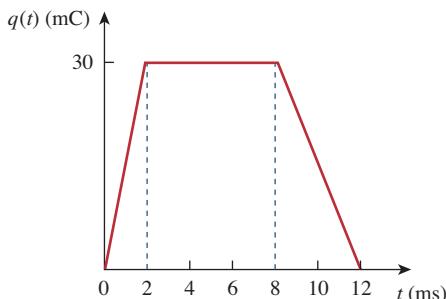
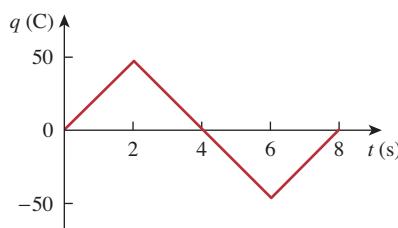


Figure 1.23

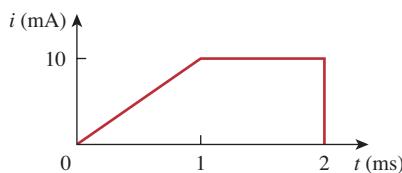
For Prob. 1.6.

- 1.7** The charge flowing in a wire is plotted in Fig. 1.24. Sketch the corresponding current.

**Figure 1.24**

For Prob. 1.7.

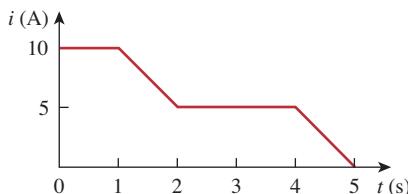
- 1.8** The current flowing past a point in a device is shown in Fig. 1.25. Calculate the total charge through the point.

**Figure 1.25**

For Prob. 1.8.

- 1.9** The current through an element is shown in Fig. 1.26. Determine the total charge that passed through the element at:

$$(a) t = 1 \text{ s} \quad (b) t = 3 \text{ s} \quad (c) t = 5 \text{ s}$$

**Figure 1.26**

For Prob. 1.9.

Sections 1.4 and 1.5 Voltage, Power, and Energy

- 1.10** A lightning bolt with 10 kA strikes an object for 15 μs . How much charge is deposited on the object?

- 1.11** A rechargeable flashlight battery is capable of delivering 90 mA for about 12 h. How much charge can it release at that rate? If its terminal voltage is 1.5 V, how much energy can the battery deliver?

- 1.12** If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t \text{ A}, & 0 \leq t < 6 \text{ s} \\ 18 \text{ A}, & 6 \leq t < 10 \text{ s} \\ -12 \text{ A}, & 10 \leq t < 15 \text{ s} \\ 0, & t \geq 15 \text{ s} \end{cases}$$

Plot the charge stored in the element over $0 < t < 20 \text{ s}$.

- 1.13** The charge entering the positive terminal of an element is

$$q = 5 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 3 \cos 4\pi t \text{ V}$$

- (a) Find the power delivered to the element at $t = 0.3 \text{ s}$.
 (b) Calculate the energy delivered to the element between 0 and 0.6 s.

- 1.14** The voltage v across a device and the current i through it are

$$v(t) = 10 \cos 2t \text{ V}, \quad i(t) = 20(1 - e^{-0.5t}) \text{ mA}$$

Calculate:

- (a) the total charge in the device at $t = 1 \text{ s}$
 (b) the power consumed by the device at $t = 1 \text{ s}$.

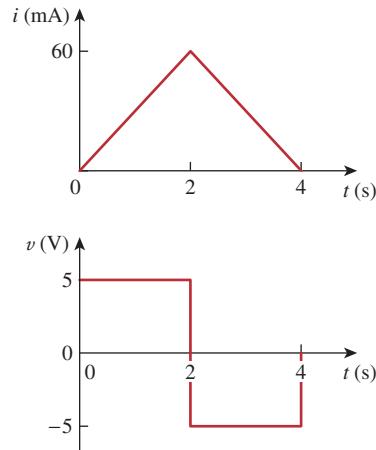
- 1.15** The current entering the positive terminal of a device is $i(t) = 6e^{-2t} \text{ mA}$ and the voltage across the device is $v(t) = 10di/dt \text{ V}$.

- (a) Find the charge delivered to the device between $t = 0$ and $t = 2 \text{ s}$.
 (b) Calculate the power absorbed.
 (c) Determine the energy absorbed in 3 s.

Section 1.6 Circuit Elements

- 1.16** Figure 1.27 shows the current through and the voltage across an element.

- (a) Sketch the power delivered to the element for $t > 0$.
 (b) Find the total energy absorbed by the element for the period of $0 < t < 4 \text{ s}$.

**Figure 1.27**

For Prob. 1.16.

- 1.17** Figure 1.28 shows a circuit with five elements. If $p_1 = -205 \text{ W}$, $p_2 = 60 \text{ W}$, $p_4 = 45 \text{ W}$, $p_5 = 30 \text{ W}$, calculate the power p_3 received or delivered by element 3.

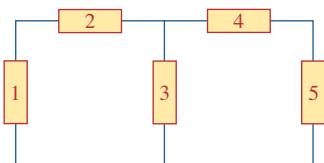


Figure 1.28

For Prob. 1.17.

- 1.18** Find the power absorbed by each of the elements in Fig. 1.29.

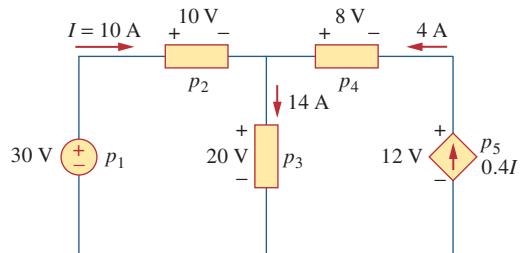


Figure 1.29

For Prob. 1.18.

- 1.19** Find I and the power absorbed by each element in the network of Fig. 1.30.

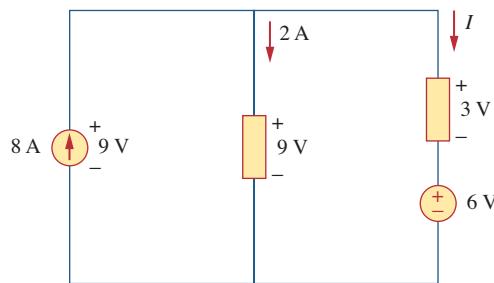


Figure 1.30

For Prob. 1.19.

- 1.20** Find V_o and the power absorbed by each element in the circuit of Fig. 1.31.

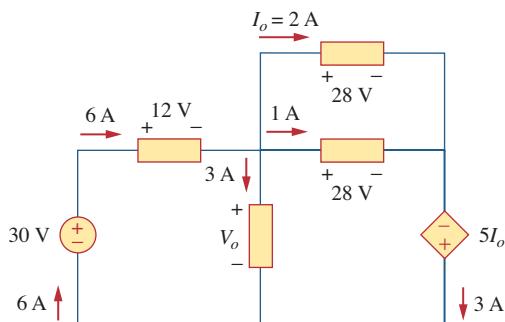


Figure 1.31

For Prob. 1.20.

Section 1.7 Applications

- 1.21** A 60-W incandescent bulb operates at 120 V. How many electrons and coulombs flow through the bulb in one day?

- 1.22** A lightning bolt strikes an airplane with 40 kA for 1.7 ms. How many coulombs of charge are deposited on the plane?

- 1.23** A 1.8-kW electric heater takes 15 min to boil a quantity of water. If this is done once a day and power costs 10 cents/kWh, what is the cost of its operation for 30 days?

- 1.24** A utility company charges 8.2 cents/kWh. If a consumer operates a 60-W light bulb continuously for one day, how much is the consumer charged?

- 1.25** A 1.5-kW toaster takes roughly 3.5 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 8.2 cents/kWh.

- 1.26** A flashlight battery has a rating of 0.8 ampere-hours (Ah) and a lifetime of 10 hours.

- (a) How much current can it deliver?
- (b) How much power can it give if its terminal voltage is 6 V?
- (c) How much energy is stored in the battery in Wh?

- 1.27** A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2 \text{ V}$, where t is in hours,

- (a) how much charge is transported as a result of the charging?
- (b) how much energy is expended?
- (c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

- 1.28** A 60-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:

- (a) the current through the lamp.
- (b) the cost of operating the light for one non-leap year if electricity costs 9.5 cents per kWh.

- 1.29** An electric stove with four burners and an oven is used in preparing a meal as follows.

Burner 1: 20 minutes Burner 2: 40 minutes

Burner 3: 15 minutes Burner 4: 45 minutes

Oven: 30 minutes

If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

- 1.30** Reliant Energy (the electric company in Houston, Texas) charges customers as follows:

Monthly charge \$6
First 250 kWh @ \$0.02/kWh
All additional kWh @ \$0.07/kWh

If a customer uses 2,436 kWh in one month, how much will Reliant Energy charge?

- 1.31** In a household, a 120-W personal computer (PC) is run for 4 h/day, while a 60-W bulb runs for 8 h/day. If the utility company charges \$0.12/kWh, calculate how much the household pays per year on the PC and the bulb.

Comprehensive Problems

- 1.32** A telephone wire has a current of $20 \mu\text{A}$ flowing through it. How long does it take for a charge of 15 C to pass through the wire?

- 1.33** A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

- 1.34** Figure 1.32 shows the power consumption of a certain household in 1 day. Calculate:
(a) the total energy consumed in kWh,
(b) the average power per hour over the total 24 hour period.

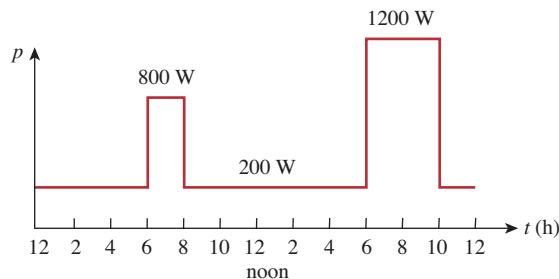


Figure 1.32

For Prob. 1.34.

- 1.35** The graph in Fig. 1.33 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

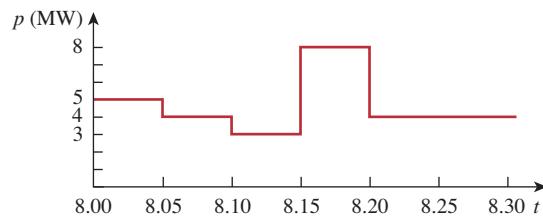


Figure 1.33

For Prob. 1.35.

- 1.36** A battery may be rated in ampere-hours (Ah). A lead-acid battery is rated at 160 Ah.

- (a) What is the maximum current it can supply for 40 h?
(b) How many days will it last if it is discharged at 1 mA?

- 1.37** A 12-V battery requires a total charge of 40 ampere-hours during recharging. How many joules are supplied to the battery?

- 1.38** How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

- 1.39** A 600-W TV receiver is turned on for 4 h with nobody watching it. If electricity costs 10 cents/kWh, how much money is wasted?

2

Basic Laws

There are too many people praying for mountains of difficulty to be removed, when what they really need is the courage to climb them!

—Unknown

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.b), “an ability to design and conduct experiments, as well as to analyze and interpret data.”

Engineers must be able to design and conduct experiments, as well as analyze and interpret data. Most students have spent many hours performing experiments in high school and in college. During this time, you have been asked to analyze the data and to interpret the data. Therefore, you should already be skilled in these two activities. My recommendation is that, in the process of performing experiments in the future, you spend more time in analyzing and interpreting the data in the context of the experiment. What does this mean?

If you are looking at a plot of voltage versus resistance or current versus resistance or power versus resistance, what do you actually see? Does the curve make sense? Does it agree with what the theory tells you? Does it differ from expectation, and, if so, why? Clearly, practice with analyzing and interpreting data will enhance this skill.

Since most, if not all, the experiments you are required to do as a student involve little or no practice in designing the experiment, how can you develop and enhance this skill?

Actually, developing this skill under this constraint is not as difficult as it seems. What you need to do is to take the experiment and analyze it. Just break it down into its simplest parts, reconstruct it trying to understand why each element is there, and finally, determine what the author of the experiment is trying to teach you. Even though it may not always seem so, every experiment you do was designed by someone who was sincerely motivated to teach you something.

2.1 Introduction

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built.

In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations. The application of these laws and techniques will be restricted to resistive circuits in this chapter. We will finally apply the laws and techniques to real-life problems of electrical lighting and the design of dc meters.

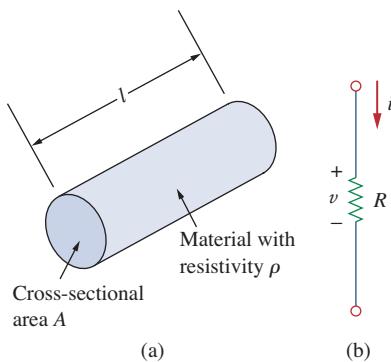


Figure 2.1
(a) Resistor, (b) Circuit symbol for resistance.

2.2 Ohm's Law

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R . The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ , as shown in Fig. 2.1(a). We can represent resistance (as measured in the laboratory), in mathematical form,

$$R = \rho \frac{\ell}{A} \quad (2.1)$$

where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

The circuit element used to model the current-resisting behavior of a material is the *resistor*. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit

TABLE 2.1
Resistivities of common materials.

Material	Resistivity ($\Omega \cdot \text{m}$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^2	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

symbol for the resistor is shown in Fig. 2.1(b), where R stands for the resistance of the resistor. The resistor is the simplest passive element.

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as *Ohm's law*.

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \quad (2.2)$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R . (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.2) becomes

$$v = iR \quad (2.3)$$

which is the mathematical form of Ohm's law. R in Eq. (2.3) is measured in the unit of ohms, designated Ω . Thus,

The *resistance R* of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

We may deduce from Eq. (2.3) that

$$R = \frac{v}{i} \quad (2.4)$$

so that

$$1 \Omega = 1 \text{ V/A}$$

To apply Ohm's law as stated in Eq. (2.3), we must pay careful attention to the current direction and voltage polarity. The direction of current i and the polarity of voltage v must conform with the passive

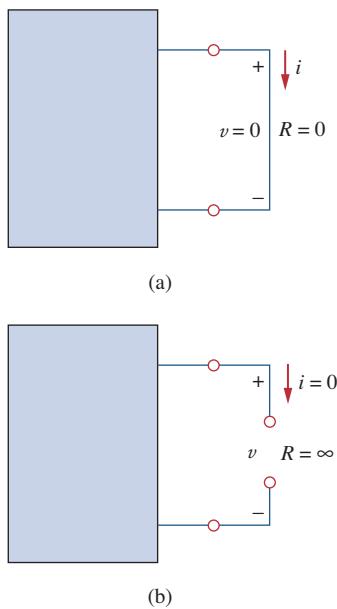
Historical

Georg Simon Ohm (1787–1854), a German physicist, in 1826 experimentally determined the most basic law relating voltage and current for a resistor. Ohm's work was initially denied by critics.

Born of humble beginnings in Erlangen, Bavaria, Ohm threw himself into electrical research. His efforts resulted in his famous law. He was awarded the Copley Medal in 1841 by the Royal Society of London. In 1849, he was given the Professor of Physics chair by the University of Munich. To honor him, the unit of resistance was named the ohm.



© SSPL via Getty Images

**Figure 2.2**

(a) Short circuit ($R = 0$), (b) Open circuit ($R = \infty$).

sign convention, as shown in Fig. 2.1(b). This implies that current flows from a higher potential to a lower potential in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R . An element with $R = 0$ is called a *short circuit*, as shown in Fig. 2.2(a). For a short circuit,

$$v = iR = 0 \quad (2.5)$$

showing that the voltage is zero but the current could be anything. In practice, a short circuit is usually a connecting wire assumed to be a perfect conductor. Thus,

A **short circuit** is a circuit element with resistance approaching zero.

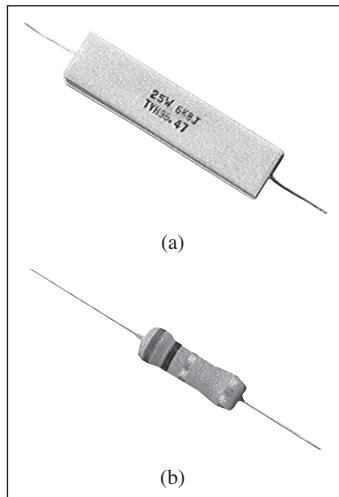
Similarly, an element with $R = \infty$ is known as an *open circuit*, as shown in Fig. 2.2(b). For an open circuit,

$$i = \lim_{R \rightarrow \infty} \frac{v}{R} = 0 \quad (2.6)$$

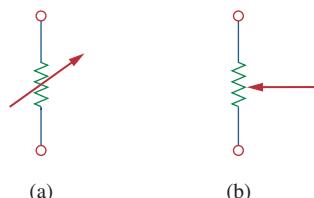
indicating that the current is zero though the voltage could be anything. Thus,

An **open circuit** is a circuit element with resistance approaching infinity.

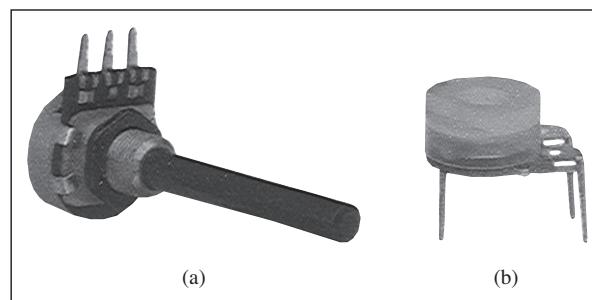
A resistor is either fixed or variable. Most resistors are of the fixed type, meaning their resistance remains constant. The two common types of fixed resistors (wirewound and composition) are shown in Fig. 2.3. The composition resistors are used when large resistance is needed. The circuit symbol in Fig. 2.1(b) is for a fixed resistor. Variable resistors have adjustable resistance. The symbol for a variable resistor is shown in Fig. 2.4(a). A common variable resistor is known as a *potentiometer* or *pot* for short, with the symbol shown in Fig. 2.4(b). The pot is a three-terminal element with a sliding contact or wiper. By sliding the wiper, the resistances between the wiper terminal and the fixed terminals vary. Like fixed resistors, variable resistors can be of either wirewound or composition type, as shown in Fig. 2.5. Although resistors like those in Figs. 2.3 and 2.5 are used in circuit designs, today most

**Figure 2.3**

Fixed resistors: (a) wirewound type, (b) carbon film type.
Courtesy of Tech America.

**Figure 2.4**

Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

**Figure 2.5**

Variable resistors: (a) composition type, (b) slider pot.
Courtesy of Tech America.

circuit components including resistors are either surface mounted or integrated, as typically shown in Fig. 2.6.

It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a *linear* resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in Fig. 2.7(a): Its i - v graph is a straight line passing through the origin. A *nonlinear* resistor does not obey Ohm's law. Its resistance varies with current and its i - v characteristic is typically shown in Fig. 2.7(b). Examples of devices with nonlinear resistance are the light bulb and the diode. Although all practical resistors may exhibit nonlinear behavior under certain conditions, we will assume in this book that all elements actually designated as resistors are linear.

A useful quantity in circuit analysis is the reciprocal of resistance R , known as *conductance* and denoted by G :

$$G = \frac{1}{R} = \frac{i}{v} \quad (2.7)$$

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the *mho* (ohm spelled backward) or reciprocal ohm, with symbol \Omega , the inverted omega. Although engineers often use the mho, in this book we prefer to use the siemens (S), the SI unit of conductance:

$$1 \text{ S} = 1 \text{ \Omega} = 1 \text{ A/V} \quad (2.8)$$

Thus,

Conductance is the ability of an element to conduct electric current; it is measured in mhos (\Omega) or siemens (S).

The same resistance can be expressed in ohms or siemens. For example, 10Ω is the same as 0.1 S . From Eq. (2.7), we may write

$$i = Gv \quad (2.9)$$

The power dissipated by a resistor can be expressed in terms of R . Using Eqs. (1.7) and (2.3),

$$p = vi = i^2R = \frac{v^2}{R} \quad (2.10)$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G} \quad (2.11)$$

We should note two things from Eqs. (2.10) and (2.11):

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

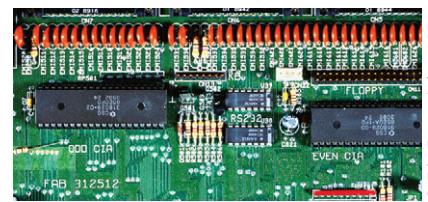
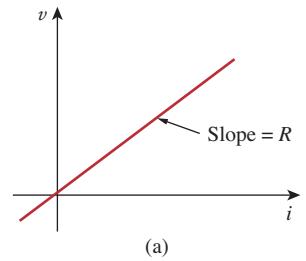
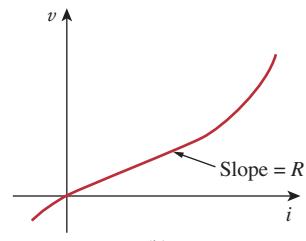


Figure 2.6

Resistors in an integrated circuit board.



(a)



(b)

Figure 2.7

The i - v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

Example 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60 \Omega$$

Practice Problem 2.1

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

Answer: 7.333 A.

Example 2.2

In the circuit shown in Fig. 2.8, calculate the current i , the conductance G , and the power p .

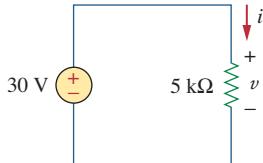


Figure 2.8

For Example 2.2.

Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^25 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^20.2 \times 10^{-3} = 180 \text{ mW}$$

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v , the conductance G , and the power p .

Answer: 30 V, 100 μS , 90 mW.

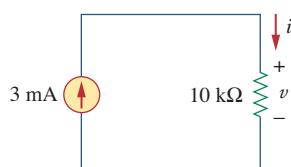


Figure 2.9

For Practice Prob. 2.2

A voltage source of $20 \sin \pi t$ V is connected across a $5\text{-k}\Omega$ resistor. Find the current through the resistor and the power dissipated.

Example 2.3

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R .

Practice Problem 2.3

Answer: $2 \cos t$ mA, $7.5 \text{ k}\Omega$.

2.3

Nodes, Branches, and Loops

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. To differentiate between a circuit and a network, we may regard a network as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths. The convention, when addressing network topology, is to use the word network rather than circuit. We do this even though the word network and circuit mean the same thing when used in this context. In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes, and loops.

A **branch** represents a single element such as a voltage source or a resistor.

In other words, a branch represents any two-terminal element. The circuit in Fig. 2.10 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

A **node** is the point of connection between two or more branches.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 2.10 has three nodes a , b , and c . Notice that the three points that form node b are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node c . We demonstrate that the circuit in Fig. 2.10 has only three nodes by redrawing the circuit in Fig. 2.11. The two circuits in Figs. 2.10 and 2.11 are identical. However, for the sake of clarity, nodes b and c are spread out with perfect conductors as in Fig. 2.10.

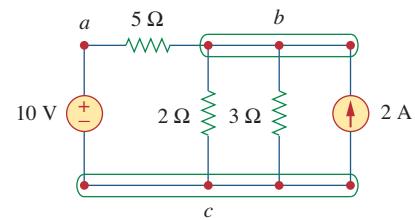


Figure 2.10
Nodes, branches, and loops.

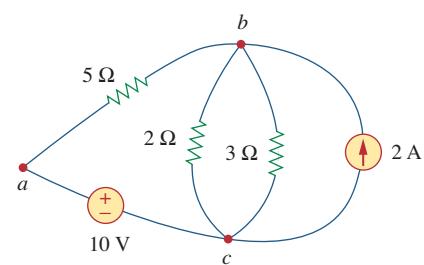


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

A **loop** is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be *independent* if it contains at least one branch which is not a part of any other independent loop. Independent loops or paths result in independent sets of equations.

It is possible to form an independent set of loops where one of the loops does not contain such a branch. In Fig. 2.11, *abca* with the 2Ω resistor is independent. A second loop with the 3Ω resistor and the current source is independent. The third loop could be the one with the 2Ω resistor in parallel with the 3Ω resistor. This does form an independent set of loops.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1 \quad (2.12)$$

As the next two definitions show, circuit topology is of great value to the study of voltages and currents in an electric circuit.

Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.

Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

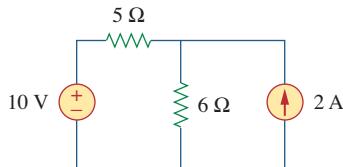
Elements are in series when they are chain-connected or connected sequentially, end to end. For example, two elements are in series if they share one common node and no other element is connected to that common node. Elements in parallel are connected to the same pair of terminals. Elements may be connected in a way that they are neither in series nor in parallel. In the circuit shown in Fig. 2.10, the voltage source and the $5\text{-}\Omega$ resistor are in series because the same current will flow through them. The $2\text{-}\Omega$ resistor, the $3\text{-}\Omega$ resistor, and the current source are in parallel because they are connected to the same two nodes *b* and *c* and consequently have the same voltage across them. The $5\text{-}\Omega$ and $2\text{-}\Omega$ resistors are neither in series nor in parallel with each other.

Example 2.4

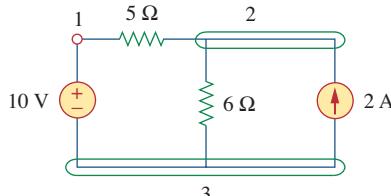
Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.

Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V , $5\text{ }\Omega$, $6\text{ }\Omega$, and 2 A . The circuit has three nodes as identified in Fig. 2.13. The $5\text{-}\Omega$ resistor is in series with the 10-V voltage source because the same current would flow in both. The $6\text{-}\Omega$ resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

**Figure 2.12**

For Example 2.4.

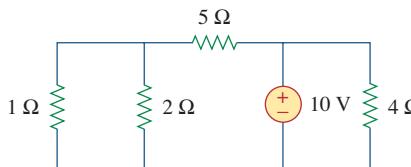
**Figure 2.13**

The three nodes in the circuit of Fig. 2.12.

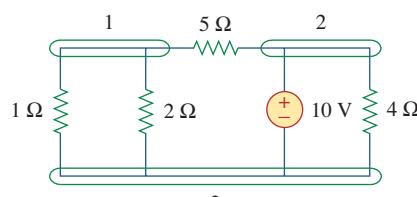
How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Practice Problem 2.4

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

**Figure 2.14**

For Practice Prob. 2.4.

**Figure 2.15**

Answer for Practice Prob. 2.4.

2.4 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

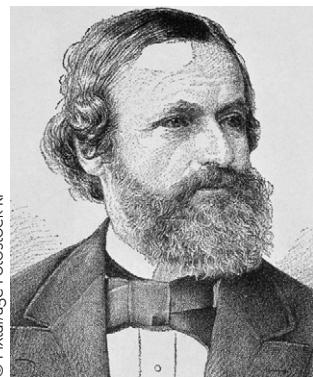
Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^N i_n = 0 \quad (2.13)$$

where N is the number of branches connected to the node and i_n is the n th current entering (or leaving) the node. By this law, currents

Historical



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Gustav Robert Kirchhoff (1824–1887), a German physicist, stated two basic laws in 1847 concerning the relationship between the currents and voltages in an electrical network. Kirchhoff's laws, along with Ohm's law, form the basis of circuit theory.

Born the son of a lawyer in Königsberg, East Prussia, Kirchhoff entered the University of Königsberg at age 18 and later became a lecturer in Berlin. His collaborative work in spectroscopy with German chemist Robert Bunsen led to the discovery of cesium in 1860 and rubidium in 1861. Kirchhoff was also credited with the Kirchhoff law of radiation. Thus Kirchhoff is famous among engineers, chemists, and physicists.

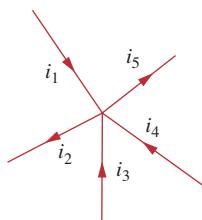


Figure 2.16

Currents at a node illustrating KCL.

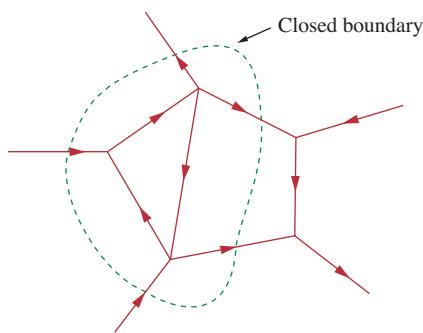


Figure 2.17

Applying KCL to a closed boundary.

Two sources (or circuits in general) are said to be equivalent if they have the same i - v relationship at a pair of terminals.

entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

To prove KCL, assume a set of currents $i_k(t)$, $k = 1, 2, \dots$, flow into a node. The algebraic sum of currents at the node is

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \dots \quad (2.14)$$

Integrating both sides of Eq. (2.14) gives

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \dots \quad (2.15)$$

where $q_k(t) = \int i_k(t) dt$ and $q_T(t) = \int i_T(t) dt$. But the law of conservation of electric charge requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus $q_T(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

Consider the node in Fig. 2.16. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \quad (2.16)$$

since currents i_1 , i_3 , and i_4 are entering the node, while currents i_2 and i_5 are leaving it. By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5 \quad (2.17)$$

Equation (2.17) is an alternative form of KCL:

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. As typically illustrated in the circuit of Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in

Fig. 2.18(a) can be combined as in Fig. 2.18(b). The combined or equivalent current source can be found by applying KCL to node a .

$$I_T + I_2 = I_1 + I_3$$

or

$$I_T = I_1 - I_2 + I_3 \quad (2.18)$$

A circuit cannot contain two different currents, I_1 and I_2 , in series, unless $I_1 = I_2$; otherwise KCL will be violated.

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^M v_m = 0 \quad (2.19)$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m th voltage.

To illustrate KVL, consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \quad (2.20)$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \quad (2.21)$$

which may be interpreted as

$$\text{Sum of voltage drops} = \text{Sum of voltage rises} \quad (2.22)$$

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1$, $-v_5$, $+v_4$, $-v_3$, and $-v_2$, which is the same as before except that the signs are reversed. Hence, Eqs. (2.20) and (2.21) remain the same.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 2.20(a), the combined or equivalent voltage source in Fig. 2.20(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

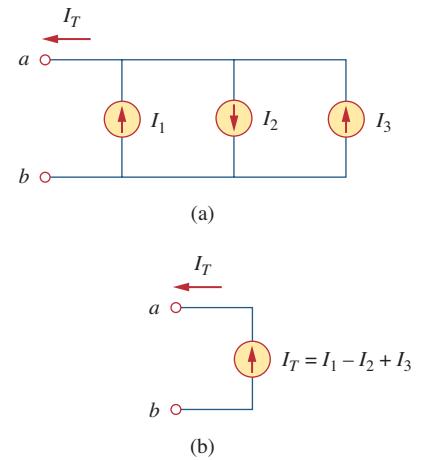


Figure 2.18

Current sources in parallel: (a) original circuit, (b) equivalent circuit.

KVL can be applied in two ways: by taking either a clockwise or a counterclockwise trip around the loop. Either way, the algebraic sum of voltages around the loop is zero.

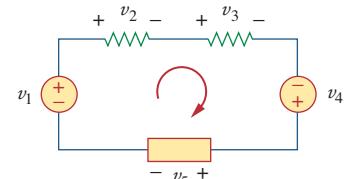


Figure 2.19

A single-loop circuit illustrating KVL.

or

$$V_{ab} = V_1 + V_2 - V_3 \quad (2.23)$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.

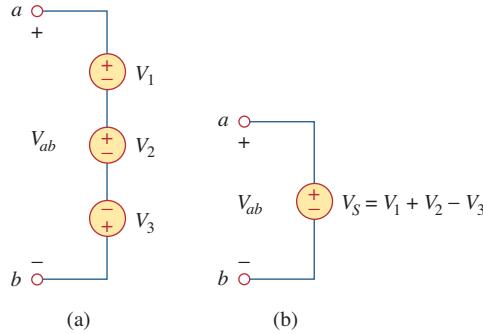


Figure 2.20

Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Example 2.5

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2 .

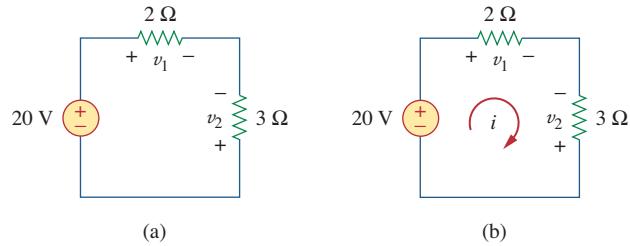


Figure 2.21

For Example 2.5.

Solution:

To find v_1 and v_2 , we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \quad v_2 = -3i \quad (2.5.1)$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \quad (2.5.2)$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Rightarrow \quad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \quad v_2 = -12 \text{ V}$$

Find v_1 and v_2 in the circuit of Fig. 2.22.

Answer: 16 V, -8 V.

Practice Problem 2.5

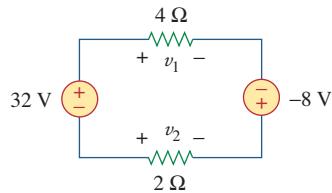


Figure 2.22

For Practice Prob. 2.5.

Determine v_o and i in the circuit shown in Fig. 2.23(a).

Example 2.6

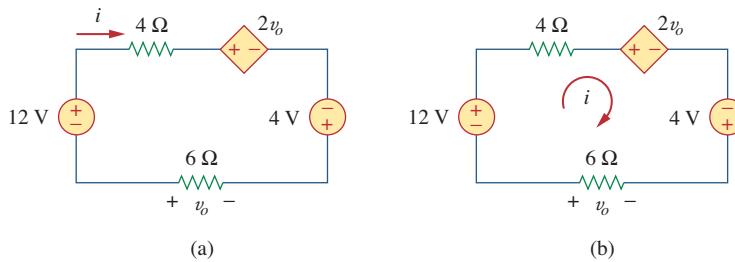


Figure 2.23

For Example 2.6.

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \quad (2.6.1)$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \quad (2.6.2)$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48 \text{ V}$.

Find v_x and v_o in the circuit of Fig. 2.24.

Practice Problem 2.6

Answer: 20 V, -10 V.

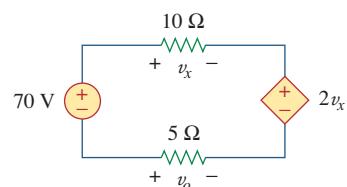
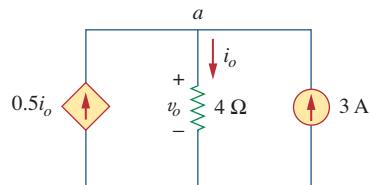


Figure 2.24

For Practice Prob. 2.6.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

**Solution:**

Applying KCL to node a , we obtain

$$3 + 0.5i_o = i_o \Rightarrow i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

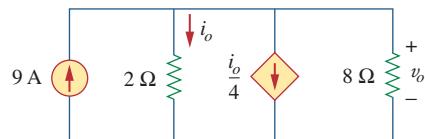
Figure 2.25

For Example 2.7.

Practice Problem 2.7

Find v_o and i_o in the circuit of Fig. 2.26.

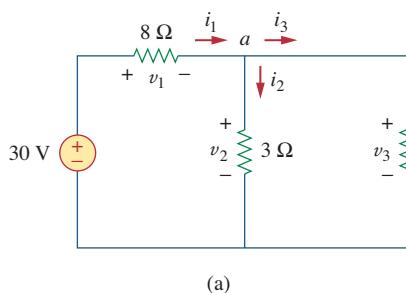
Answer: 12 V, 6 A.

**Figure 2.26**

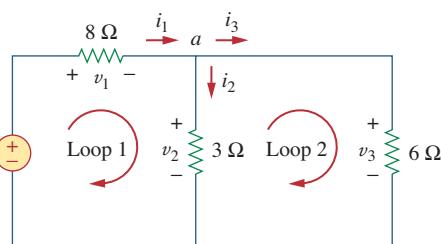
For Practice Prob. 2.7.

Example 2.8

Find currents and voltages in the circuit shown in Fig. 2.27(a).



(a)



(b)

Figure 2.27

For Example 2.8.

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \quad v_2 = 3i_2, \quad v_3 = 6i_3 \quad (2.8.1)$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3) . At node a , KCL gives

$$i_1 - i_2 - i_3 = 0 \quad (2.8.2)$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \quad (2.8.3)$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \Rightarrow v_3 = v_2 \quad (2.8.4)$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2} \quad (2.8.5)$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2 , we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

Find the currents and voltages in the circuit shown in Fig. 2.28.

Practice Problem 2.8

Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.

2.5 Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 2.29. The two resistors

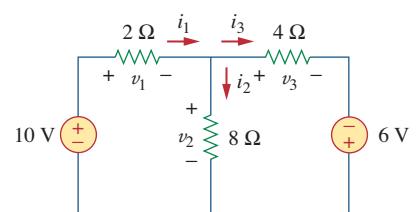
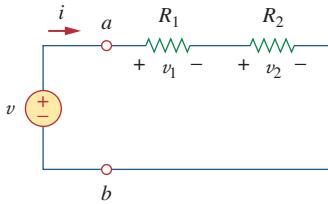
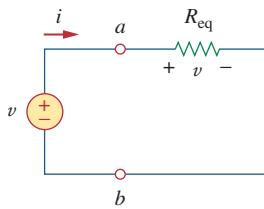


Figure 2.28
For Practice Prob. 2.8.

**Figure 2.29**

A single-loop circuit with two resistors in series.

**Figure 2.30**

Equivalent circuit of the Fig. 2.29 circuit.

Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \quad v_2 = iR_2 \quad (2.24)$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \quad (2.25)$$

Combining Eqs. (2.24) and (2.25), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \quad (2.26)$$

or

$$i = \frac{v}{R_1 + R_2} \quad (2.27)$$

Notice that Eq. (2.26) can be written as

$$v = iR_{eq} \quad (2.28)$$

implying that the two resistors can be replaced by an equivalent resistor R_{eq} ; that is,

$$R_{eq} = R_1 + R_2 \quad (2.29)$$

Thus, Fig. 2.29 can be replaced by the equivalent circuit in Fig. 2.30. The two circuits in Figs. 2.29 and 2.30 are equivalent because they exhibit the same voltage-current relationships at the terminals $a-b$. An equivalent circuit such as the one in Fig. 2.30 is useful in simplifying the analysis of a circuit. In general,

The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^N R_n \quad (2.30)$$

To determine the voltage across each resistor in Fig. 2.29, we substitute Eq. (2.26) into Eq. (2.24) and obtain

$$v_1 = \frac{R_1}{R_1 + R_2} v, \quad v_2 = \frac{R_2}{R_1 + R_2} v \quad (2.31)$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*. In general, if a voltage divider has N resistors (R_1, R_2, \dots, R_N) in series with the source voltage v , the n th resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v \quad (2.32)$$

2.6 Parallel Resistors and Current Division

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

or

$$i_1 = \frac{v}{R_1}, \quad i_2 = \frac{v}{R_2} \quad (2.33)$$

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2 \quad (2.34)$$

Substituting Eq. (2.33) into Eq. (2.34), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v}{R_{eq}} \quad (2.35)$$

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (2.36)$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad (2.37)$$

Thus,

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (2.37), if $R_1 = R_2$, then $R_{eq} = R_1/2$.

We can extend the result in Eq. (2.36) to the general case of a circuit with *N* resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \quad (2.38)$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \quad (2.39)$$

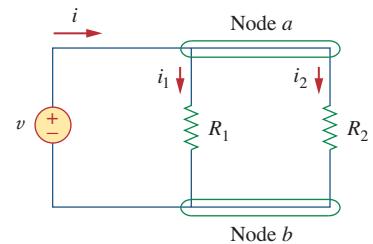


Figure 2.31

Two resistors in parallel.

Conductances in parallel behave as a single conductance whose value is equal to the sum of the individual conductances.

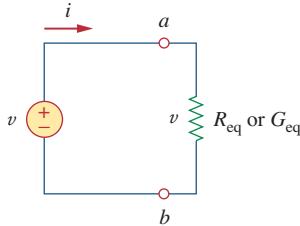


Figure 2.32

Equivalent circuit to Fig. 2.31.

For example, if four $100\text{-}\Omega$ resistors are connected in parallel, their equivalent resistance is $25\ \Omega$.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (2.38), the equivalent conductance for N resistors in parallel is

$$G_{\text{eq}} = G_1 + G_2 + G_3 + \cdots + G_N \quad (2.40)$$

where $G_{\text{eq}} = 1/R_{\text{eq}}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, ..., $G_N = 1/R_N$. Equation (2.40) states:

The **equivalent conductance** of resistors connected in parallel is the sum of their individual conductances.

This means that we may replace the circuit in Fig. 2.31 with that in Fig. 2.32. Notice the similarity between Eqs. (2.30) and (2.40). The equivalent conductance of parallel resistors is obtained the same way as the equivalent resistance of series resistors. In the same manner, the equivalent conductance of resistors in series is obtained just the same way as the resistance of resistors in parallel. Thus the equivalent conductance G_{eq} of N resistors in series (such as shown in Fig. 2.29) is

$$\frac{1}{G_{\text{eq}}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N} \quad (2.41)$$

Given the total current i entering node a in Fig. 2.31, how do we obtain current i_1 and i_2 ? We know that the equivalent resistor has the same voltage, or

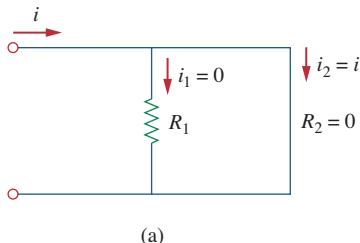
$$v = iR_{\text{eq}} = \frac{iR_1 R_2}{R_1 + R_2} \quad (2.42)$$

Combining Eqs. (2.33) and (2.42) results in

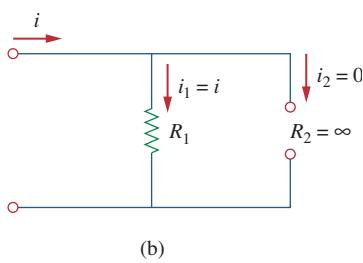
$$i_1 = \frac{R_2 i}{R_1 + R_2}, \quad i_2 = \frac{R_1 i}{R_1 + R_2} \quad (2.43)$$

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the *principle of current division*, and the circuit in Fig. 2.31 is known as a *current divider*. Notice that the larger current flows through the smaller resistance.

As an extreme case, suppose one of the resistors in Fig. 2.31 is zero, say $R_2 = 0$; that is, R_2 is a short circuit, as shown in Fig. 2.33(a). From Eq. (2.43), $R_2 = 0$ implies that $i_1 = 0$, $i_2 = i$. This means that the entire current i bypasses R_1 and flows through the short circuit $R_2 = 0$, the path of least resistance. Thus when a circuit



(a)



(b)

Figure 2.33

(a) A shorted circuit, (b) an open circuit.

is short circuited, as shown in Fig. 2.33(a), two things should be kept in mind:

1. The equivalent resistance $R_{\text{eq}} = 0$. [See what happens when $R_2 = 0$ in Eq. (2.37).]
2. The entire current flows through the short circuit.

As another extreme case, suppose $R_2 = \infty$, that is, R_2 is an open circuit, as shown in Fig. 2.33(b). The current still flows through the path of least resistance, R_1 . By taking the limit of Eq. (2.37) as $R_2 \rightarrow \infty$, we obtain $R_{\text{eq}} = R_1$ in this case.

If we divide both the numerator and denominator by $R_1 R_2$, Eq. (2.43) becomes

$$i_1 = \frac{G_1}{G_1 + G_2} i \quad (2.44\text{a})$$

$$i_2 = \frac{G_2}{G_1 + G_2} i \quad (2.44\text{b})$$

Thus, in general, if a current divider has N conductors (G_1, G_2, \dots, G_N) in parallel with the source current i , the n th conductor (G_n) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \dots + G_N} i \quad (2.45)$$

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single *equivalent resistance* R_{eq} . Such an equivalent resistance is the resistance between the designated terminals of the network and must exhibit the same i - v characteristics as the original network at the terminals.

Find R_{eq} for the circuit shown in Fig. 2.34.

Example 2.9

Solution:

To get R_{eq} , we combine resistors in series and in parallel. The 6Ω and 3Ω resistors are in parallel, so their equivalent resistance is

$$6\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\Omega$$

(The symbol \parallel is used to indicate a parallel combination.) Also, the 1Ω and 5Ω resistors are in series; hence their equivalent resistance is

$$1\Omega + 5\Omega = 6\Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2Ω resistors are in series, so the equivalent resistance is

$$2\Omega + 2\Omega = 4\Omega$$

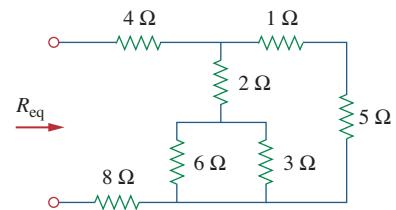
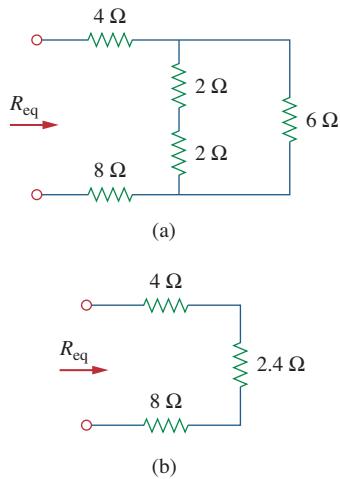


Figure 2.34
For Example 2.9.

**Figure 2.35**

Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4 \Omega \parallel 6 \Omega = \frac{4 \times 6}{4 + 6} = 2.4 \Omega$$

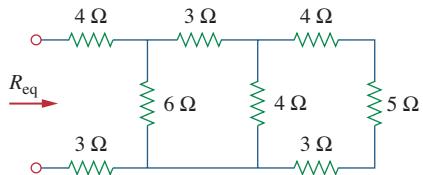
The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4 \Omega + 2.4 \Omega + 8 \Omega = 14.4 \Omega$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq} .

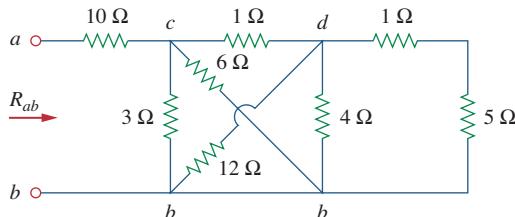
Answer: 10 Ω.

**Figure 2.36**

For Practice Prob. 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

**Figure 2.37**

For Example 2.10.

Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b . Their combined resistance is

$$3 \Omega \parallel 6 \Omega = \frac{3 \times 6}{3 + 6} = 2 \Omega \quad (2.10.1)$$

Similarly, the 12- Ω and 4- Ω resistors are in parallel since they are connected to the same two nodes d and b . Hence

$$12 \Omega \parallel 4 \Omega = \frac{12 \times 4}{12 + 4} = 3 \Omega \quad (2.10.2)$$

Also the 1- Ω and 5- Ω resistors are in series; hence, their equivalent resistance is

$$1 \Omega + 5 \Omega = 6 \Omega \quad (2.10.3)$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3- Ω in parallel with 6- Ω gives 2- Ω , as calculated in Eq. (2.10.1). This 2- Ω equivalent resistance is now in series with the 1- Ω resistance to give a combined resistance of $1 \Omega + 2 \Omega = 3 \Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2- Ω and 3- Ω resistors in parallel to get

$$2 \Omega \parallel 3 \Omega = \frac{2 \times 3}{2 + 3} = 1.2 \Omega$$

This 1.2- Ω resistor is in series with the 10- Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2 \Omega$$

Find R_{ab} for the circuit in Fig. 2.39.

Answer: 19 Ω .

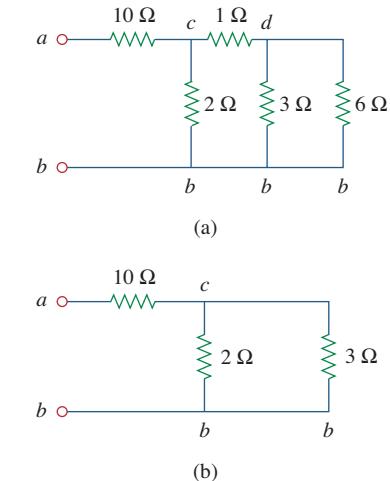


Figure 2.38
Equivalent circuits for Example 2.10.

Practice Problem 2.10

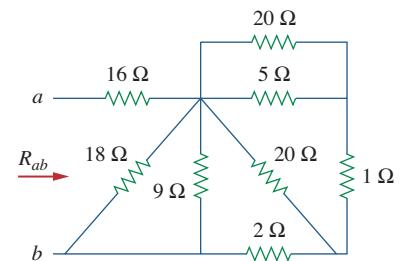


Figure 2.39
For Practice Prob. 2.10.

Find the equivalent conductance G_{eq} for the circuit in Fig. 2.40(a).

Example 2.11

Solution:

The 8-S and 12-S resistors are in parallel, so their conductance is

$$8 \text{ S} + 12 \text{ S} = 20 \text{ S}$$

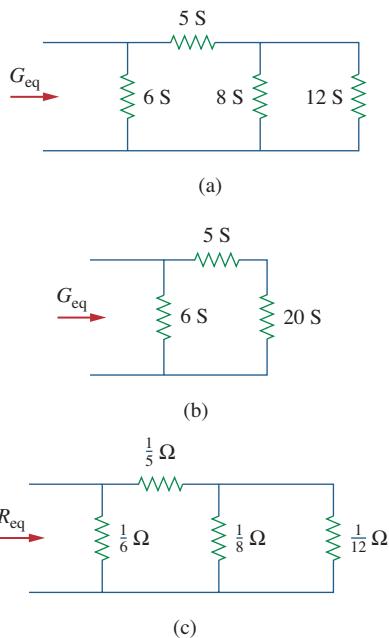
This 20-S resistor is now in series with 5 S as shown in Fig. 2.40(b) so that the combined conductance is

$$\frac{20 \times 5}{20 + 5} = 4 \text{ S}$$

This is in parallel with the 6-S resistor. Hence,

$$G_{eq} = 6 + 4 = 10 \text{ S}$$

We should note that the circuit in Fig. 2.40(a) is the same as that in Fig. 2.40(c). While the resistors in Fig. 2.40(a) are expressed in

**Figure 2.40**

For Example 2.11: (a) original circuit,
(b) its equivalent circuit, (c) same circuit as
in (a) but resistors are expressed in ohms.

siemens, those in Fig. 2.40(c) are expressed in ohms. To show that the circuits are the same, we find \$R_{eq}\$ for the circuit in Fig. 2.40(c).

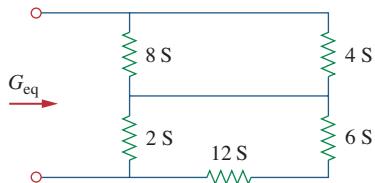
$$\begin{aligned}
 R_{eq} &= \frac{1}{6} \left(\frac{1}{5} + \frac{1}{8} \right) = \frac{1}{6} \left(\frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \left(\frac{1}{4} \right) \\
 &= \frac{\frac{1}{6} \times \frac{1}{4}}{\frac{1}{6} + \frac{1}{4}} = \frac{1}{10} \Omega \\
 G_{eq} &= \frac{1}{R_{eq}} = 10 \text{ S}
 \end{aligned}$$

This is the same as we obtained previously.

Practice Problem 2.11

Calculate \$G_{eq}\$ in the circuit of Fig. 2.41.

Answer: 4 S.

**Figure 2.41**

For Practice Prob. 2.11.

Example 2.12

Find \$i_o\$ and \$v_o\$ in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the \$3\Omega\$ resistor.

Solution:

The \$6\Omega\$ and \$3\Omega\$ resistors are in parallel, so their combined resistance is

$$6\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\Omega$$

Thus our circuit reduces to that shown in Fig. 2.42(b). Notice that \$v_o\$ is not affected by the combination of the resistors because the resistors are in parallel and therefore have the same voltage \$v_o\$. From Fig. 2.42(b), we can obtain \$v_o\$ in two ways. One way is to apply Ohm's law to get

$$i = \frac{12}{4 + 2} = 2 \text{ A}$$

and hence, $v_o = 2i = 2 \times 2 = 4$ V. Another way is to apply voltage division, since the 12 V in Fig. 2.42(b) is divided between the 4- Ω and 2- Ω resistors. Hence,

$$v_o = \frac{2}{2+4} (12 \text{ V}) = 4 \text{ V}$$

Similarly, i_o can be obtained in two ways. One approach is to apply Ohm's law to the 3- Ω resistor in Fig. 2.42(a) now that we know v_o , thus,

$$v_o = 3i_o = 4 \quad \Rightarrow \quad i_o = \frac{4}{3} \text{ A}$$

Another approach is to apply current division to the circuit in Fig. 2.42(a) now that we know i , by writing

$$i_o = \frac{6}{6+3}i = \frac{2}{3}(2 \text{ A}) = \frac{4}{3} \text{ A}$$

The power dissipated in the 3- Ω resistor is

$$p_o = v_o i_o = 4 \left(\frac{4}{3} \right) = 5.333 \text{ W}$$

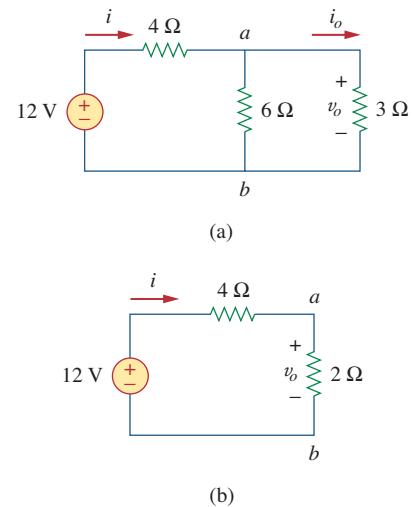


Figure 2.42

For Example 2.12: (a) original circuit, (b) its equivalent circuit.

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12- Ω and 40- Ω resistors.

Practice Problem 2.12

Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 = 500$ mA, $p_2 = 10$ W.

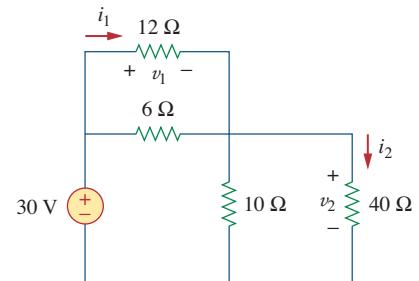


Figure 2.43

For Practice Prob. 2.12.

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o , (b) the power supplied by the current source, (c) the power absorbed by each resistor.

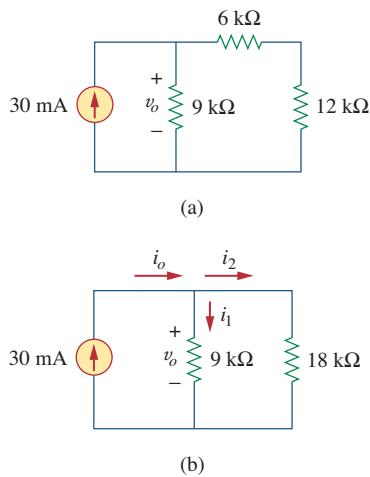
Example 2.13

Solution:

(a) The 6-k Ω and 12-k Ω resistors are in series so that their combined value is $6 + 12 = 18$ k Ω . Thus the circuit in Fig. 2.44(a) reduces to that shown in Fig. 2.44(b). We now apply the current division technique to find i_1 and i_2 .

$$i_1 = \frac{18,000}{9,000 + 18,000} (30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9,000}{9,000 + 18,000} (30 \text{ mA}) = 10 \text{ mA}$$

**Figure 2.44**

For Example 2.13: (a) original circuit, (b) its equivalent circuit.

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9,000i_1 = 18,000i_2 = 180$ V, as expected.

(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30) \text{ mW} = 5.4 \text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2 (12,000) = 1.2 \text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2 (6,000) = 0.6 \text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9,000} = 3.6 \text{ W}$$

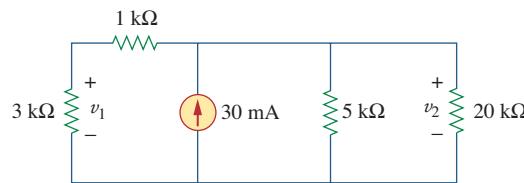
or

$$p = v_o i_1 = 180(20) \text{ mW} = 3.6 \text{ W}$$

Notice that the power supplied (5.4 W) equals the power absorbed ($1.2 + 0.6 + 3.6 = 5.4$ W). This is one way of checking results.

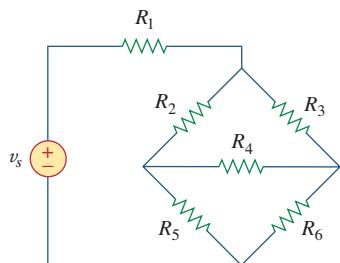
Practice Problem 2.13

For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2 , (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

**Figure 2.45**

For Practice Prob. 2.13.

Answer: (a) 45 V, 60 V, (b) 675 mW, 180 mW, (c) 1.8 W.

**Figure 2.46**

The bridge network.

2.7

†Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are

the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta (Δ) or pi (Π) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

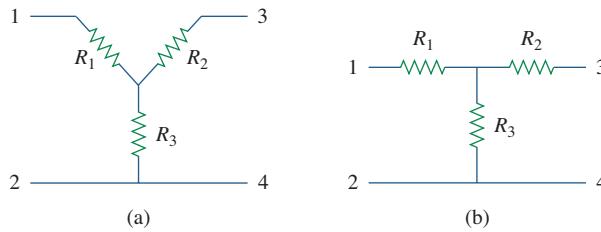


Figure 2.47

Two forms of the same network: (a) Y, (b) T.

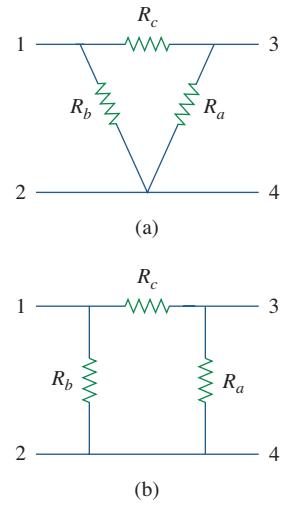


Figure 2.48

Two forms of the same network: (a) Δ , (b) Π .

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

$$R_{12}(Y) = R_1 + R_3 \quad (2.46)$$

$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(Y) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad (2.47a)$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad (2.47b)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad (2.47c)$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \quad (2.48)$$

Adding Eqs. (2.47b) and (2.48) gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad (2.49)$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad (2.50)$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad (2.51)$$

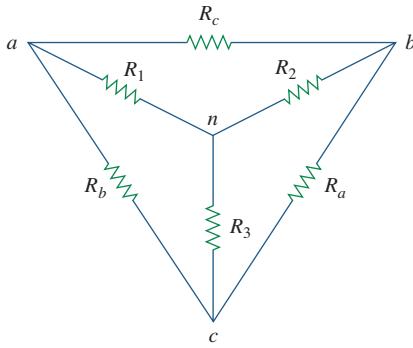


Figure 2.49

Superposition of Y and Δ networks as an aid in transforming one to the other.

We do not need to memorize Eqs. (2.49) to (2.51). To transform a Δ network to Y, we create an extra node n as shown in Fig. 2.49 and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

One can follow this rule and obtain Eqs. (2.49) to (2.51) from Fig. 2.49.

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$\begin{aligned} R_1 R_2 + R_2 R_3 + R_3 R_1 &= \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \\ &= \frac{R_a R_b R_c}{R_a + R_b + R_c} \end{aligned} \quad (2.52)$$

Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \quad (2.53)$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \quad (2.54)$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad (2.55)$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to Δ is as follows:

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \quad R_a = R_b = R_c = R_\Delta \quad (2.56)$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y \quad (2.57)$$

One may wonder why R_Y is less than R_Δ . Well, we notice that the Y-connection is like a “series” connection while the Δ -connection is like a “parallel” connection.

Note that in making the transformation, we do not take anything out of the circuit or put in anything new. We are merely substituting different but mathematically equivalent three-terminal network patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate R_{eq} if necessary.

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

Example 2.14

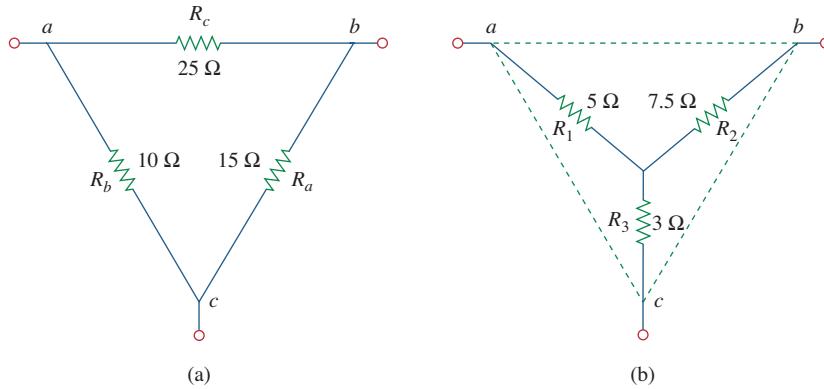


Figure 2.50

For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:

Using Eqs. (2.49) to (2.51), we obtain

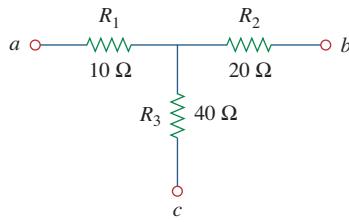
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5 \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

Practice Problem 2.14

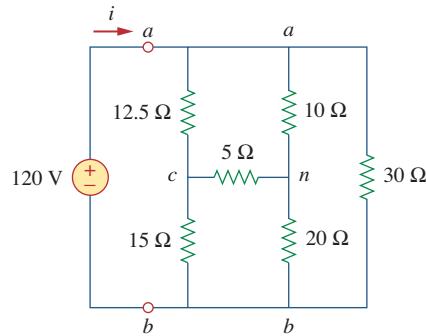
**Figure 2.51**

For Practice Prob. 2.14.

Transform the wye network in Fig. 2.51 to a delta network.

Answer: $R_a = 140 \Omega$, $R_b = 70 \Omega$, $R_c = 35 \Omega$.

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i .**Figure 2.52**

For Example 2.15.

Solution:

- Define.** The problem is clearly defined. Please note, this part normally will deservedly take much more time.
- Present.** Clearly, when we remove the voltage source, we end up with a purely resistive circuit. Since it is composed of deltas and wyes, we have a more complex process of combining the elements together. We can use wye-delta transformations as one approach to find a solution. It is useful to locate the wyes (there are two of them, one at n and the other at c) and the deltas (there are three: can , abn , cnb).
- Alternative.** There are different approaches that can be used to solve this problem. Since the focus of Sec. 2.7 is the wye-delta transformation, this should be the technique to use. Another approach would be to solve for the equivalent resistance by injecting one amp into the circuit and finding the voltage between a and b ; we will learn about this approach in Chap. 4.
- The approach we can apply here as a check would be to use a wye-delta transformation as the first solution to the problem. Later we can check the solution by starting with a delta-wye transformation.
- Attempt.** In this circuit, there are two Y networks and three Δ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

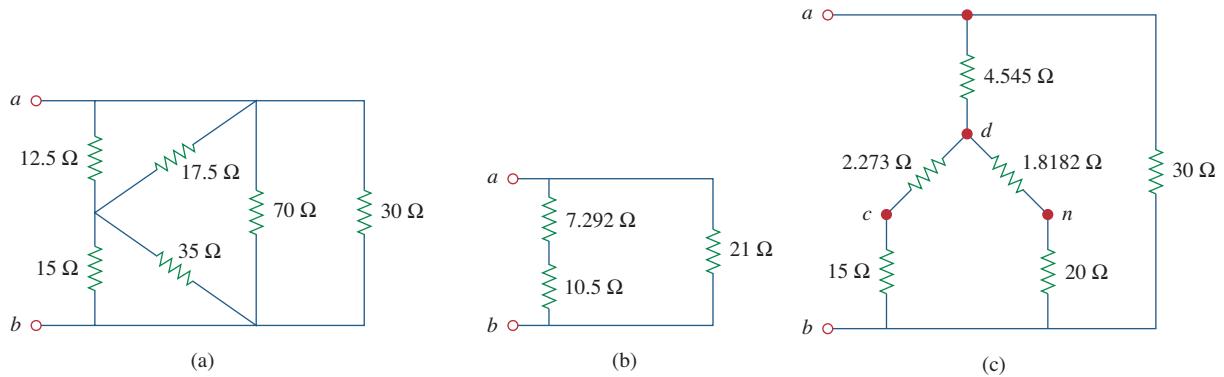
$$R_1 = 10 \Omega, \quad R_2 = 20 \Omega, \quad R_3 = 5 \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} \\ = \frac{350}{10} = 35 \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5 \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70 \Omega$$

**Figure 2.53**

Equivalent circuits to Fig. 2.52, with the voltage source removed.

With the Y converted to Δ , the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a).

Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21 \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292 \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5 \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632 \Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458 \text{ A}$$

We observe that we have successfully solved the problem.

Now we must evaluate the solution.

- 5. Evaluate.** Now we must determine if the answer is correct and then evaluate the final solution.

It is relatively easy to check the answer; we do this by solving the problem starting with a delta-wye transformation. Let us transform the delta, *can*, into a wye.

Let $R_c = 10 \Omega$, $R_a = 5 \Omega$, and $R_n = 12.5 \Omega$. This will lead to (let d represent the middle of the wye):

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545 \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273 \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182 \Omega$$

This now leads to the circuit shown in Figure 2.53(c). Looking at the resistance between *d* and *b*, we have two series combination in parallel, giving us

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642 \Omega$$

This is in series with the 4.545- Ω resistor, both of which are in parallel with the 30- Ω resistor. This then gives us the equivalent resistance of the circuit.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = 9.631 \Omega$$

This now leads to

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = 12.46 \text{ A}$$

We note that using two variations on the wye-delta transformation leads to the same results. This represents a very good check.

6. **Satisfactory?** Since we have found the desired answer by determining the equivalent resistance of the circuit first and the answer checks, then we clearly have a satisfactory solution. This represents what can be presented to the individual assigning the problem.

Practice Problem 2.15

For the bridge network in Fig. 2.54, find R_{ab} and i .

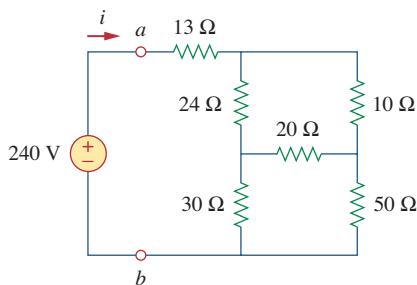


Figure 2.54

For Practice Prob. 2.15.

Answer: 40 Ω , 6 A.

2.8 Applications

Resistors are often used to model devices that convert electrical energy into heat or other forms of energy. Such devices include conducting wire, light bulbs, electric heaters, stoves, ovens, and loudspeakers. In this section, we will consider two real-life problems that apply the concepts developed in this chapter: electrical lighting systems and design of dc meters.

2.8.1 Lighting Systems

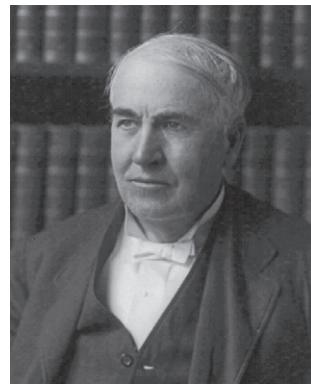
Lighting systems, such as in a house or on a Christmas tree, often consist of N lamps connected either in parallel or in series, as shown in Fig. 2.55. Each lamp is modeled as a resistor. Assuming that all the lamps are identical and V_o is the power-line voltage, the voltage across each lamp is V_o for the parallel connection and V_o/N for the series connection. The series connection is easy to manufacture but is seldom used in practice, for at least two reasons. First, it is less reliable; when a lamp fails, all the lamps go out. Second, it is harder to maintain; when a lamp is bad, one must test all the lamps one by one to detect the faulty one.

So far, we have assumed that connecting wires are perfect conductors (i.e., conductors of zero resistance). In real physical systems, however, the resistance of the connecting wire may be appreciably large, and the modeling of the system must include that resistance.

Historical

Thomas Alva Edison (1847–1931) was perhaps the greatest American inventor. He patented 1093 inventions, including such history-making inventions as the incandescent electric bulb, the phonograph, and the first commercial motion pictures.

Born in Milan, Ohio, the youngest of seven children, Edison received only three months of formal education because he hated school. He was home-schooled by his mother and quickly began to read on his own. In 1868, Edison read one of Faraday's books and found his calling. He moved to Menlo Park, New Jersey, in 1876, where he managed a well-staffed research laboratory. Most of his inventions came out of this laboratory. His laboratory served as a model for modern research organizations. Because of his diverse interests and the overwhelming number of his inventions and patents, Edison began to establish manufacturing companies for making the devices he invented. He designed the first electric power station to supply electric light. Formal electrical engineering education began in the mid-1880s with Edison as a role model and leader.



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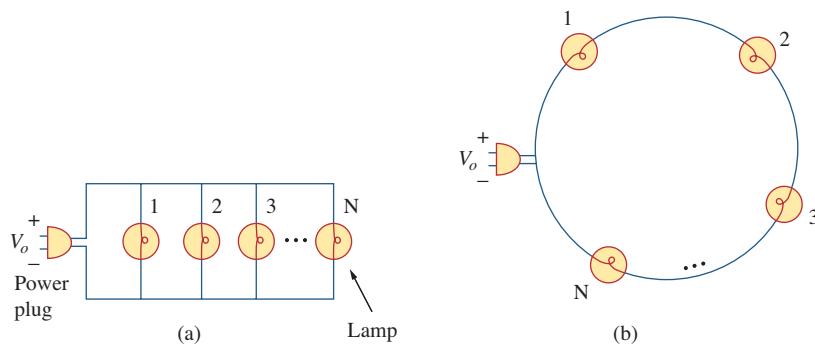


Figure 2.55

(a) Parallel connection of light bulbs, (b) series connection of light bulbs.

Three light bulbs are connected to a 9-V battery as shown in Fig. 2.56(a). Calculate: (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.

Example 2.16

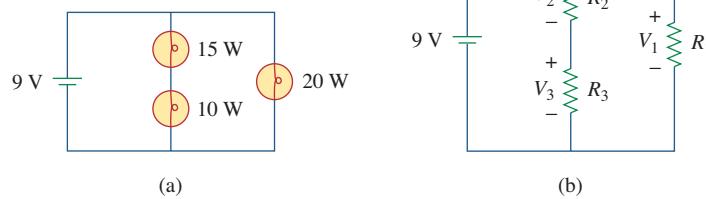


Figure 2.56

(a) Lighting system with three bulbs, (b) resistive circuit equivalent model.

Solution:

(a) The total power supplied by the battery is equal to the total power absorbed by the bulbs; that is,

$$p = 15 + 10 + 20 = 45 \text{ W}$$

Since $p = VI$, then the total current supplied by the battery is

$$I = \frac{p}{V} = \frac{45}{9} = 5 \text{ A}$$

(b) The bulbs can be modeled as resistors as shown in Fig. 2.56(b). Since R_1 (20-W bulb) is in parallel with the battery as well as the series combination of R_2 and R_3 ,

$$V_1 = V_2 + V_3 = 9 \text{ V}$$

The current through R_1 is

$$I_1 = \frac{p_1}{V_1} = \frac{20}{9} = 2.222 \text{ A}$$

By KCL, the current through the series combination of R_2 and R_3 is

$$I_2 = I - I_1 = 5 - 2.222 = 2.778 \text{ A}$$

(c) Since $p = I^2R$,

$$R_1 = \frac{p_1}{I_1^2} = \frac{20}{2.222^2} = 4.05 \Omega$$

$$R_2 = \frac{p_2}{I_2^2} = \frac{15}{2.777^2} = 1.945 \Omega$$

$$R_3 = \frac{p_3}{I_3^2} = \frac{10}{2.777^2} = 1.297 \Omega$$

Practice Problem 2.16

Refer to Fig. 2.55 and assume there are 10 light bulbs that can be connected in parallel and 10 light bulbs that can be connected in series, each with a power rating of 40 W. If the voltage at the plug is 110 V for the parallel and series connections, calculate the current through each bulb for both cases.

Answer: 364 mA (parallel), 3.64 A (series).

2.8.2 Design of DC Meters

By their nature, resistors are used to control the flow of current. We take advantage of this property in several applications, such as in a potentiometer (Fig. 2.57). The word *potentiometer*, derived from the words *potential* and *meter*, implies that potential can be metered out. The potentiometer (or pot for short) is a three-terminal device that operates on the principle of voltage division. It is essentially an adjustable voltage divider. As a voltage regulator, it is used as a volume or level control on radios, TVs, and other devices. In Fig. 2.57,

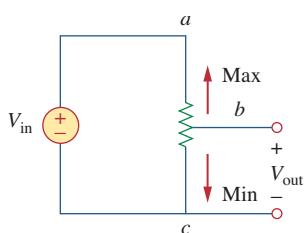


Figure 2.57

The potentiometer controlling potential levels.

$$V_{\text{out}} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{\text{in}} \quad (2.58)$$

where $R_{ac} = R_{ab} + R_{bc}$. Thus, V_{out} decreases or increases as the sliding contact of the pot moves toward c or a , respectively.

Another application where resistors are used to control current flow is in the analog dc meters—the ammeter, voltmeter, and ohmmeter, which measure current, voltage, and resistance, respectively. Each of these meters employs the d'Arsonval meter movement, shown in Fig. 2.58. The movement consists essentially of a movable iron-core coil mounted on a pivot between the poles of a permanent magnet. When current flows through the coil, it creates a torque which causes the pointer to deflect. The amount of current through the coil determines the deflection of the pointer, which is registered on a scale attached to the meter movement. For example, if the meter movement is rated 1 mA, 50 Ω, it would take 1 mA to cause a full-scale deflection of the meter movement. By introducing additional circuitry to the d'Arsonval meter movement, an ammeter, voltmeter, or ohmmeter can be constructed.

Consider Fig. 2.59, where an analog voltmeter and ammeter are connected to an element. The voltmeter measures the voltage across a *load* and is therefore connected in parallel with the element. As shown

An instrument capable of measuring voltage, current, and resistance is called a *multimeter* or a *volt-ohm meter* (VOM).

A load is a component that is receiving energy (an energy sink), as opposed to a generator supplying energy (an energy source). More about loading will be discussed in Section 4.9.1.

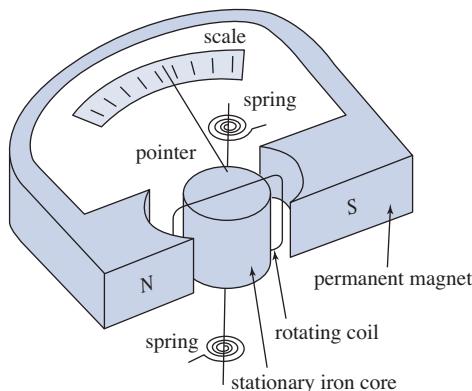


Figure 2.58
A d'Arsonval meter movement.

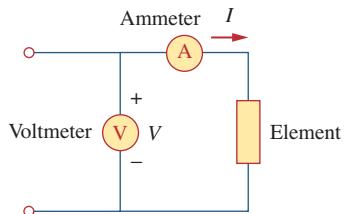
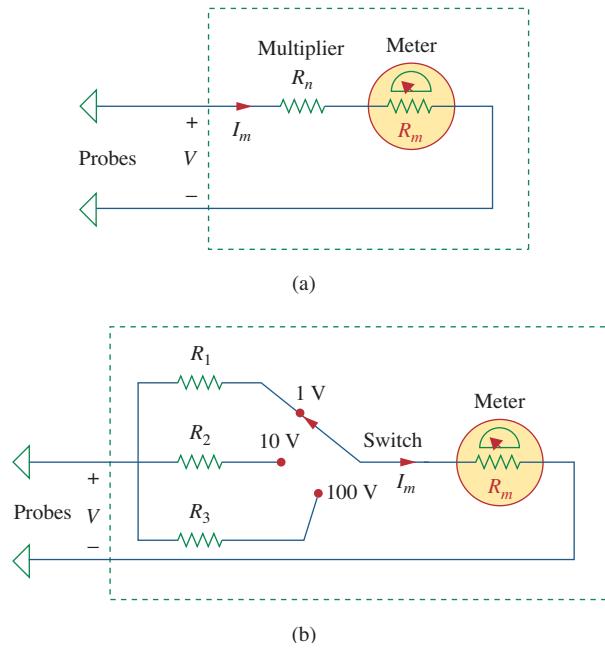


Figure 2.59
Connection of a voltmeter and an ammeter to an element.

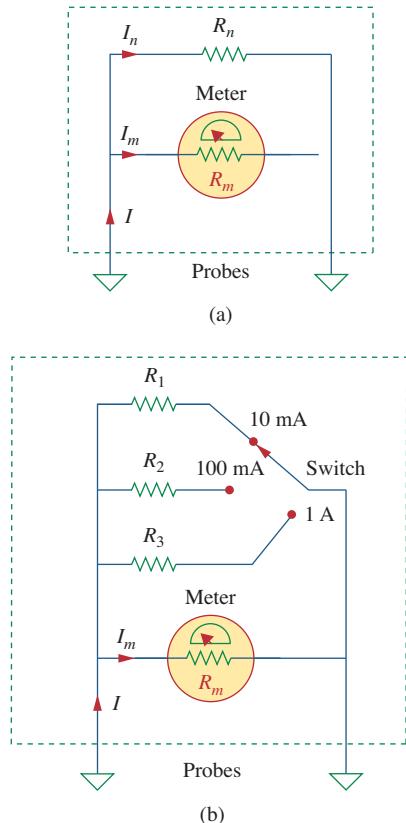
in Fig. 2.60(a), the voltmeter consists of a d'Arsonval movement in series with a resistor whose resistance R_m is deliberately made very large (theoretically, infinite), to minimize the current drawn from the circuit. To extend the range of voltage that the meter can measure, series multiplier resistors are often connected with the voltmeters, as shown in Fig. 2.60(b). The multiple-range voltmeter in Fig. 2.60(b) can measure voltage from 0 to 1 V, 0 to 10 V, or 0 to 100 V, depending on whether the switch is connected to R_1 , R_2 , or R_3 , respectively.

Let us calculate the multiplier resistor R_n for the single-range voltmeter in Fig. 2.60(a), or $R_n = R_1, R_2$, or R_3 for the multiple-range voltmeter in Fig. 2.60(b). We need to determine the value of R_n to be connected in series with the internal resistance R_m of the voltmeter. In any design, we consider the worst-case condition. In this case, the worst case occurs when the full-scale current $I_{fs} = I_m$ flows through the meter. This should also correspond to the maximum voltage reading or the full-scale voltage V_{fs} . Since the multiplier resistance R_n is in series with the internal resistance R_m ,

$$V_{fs} = I_{fs}(R_n + R_m) \quad (2.59)$$

**Figure 2.60**

Voltmeters: (a) single-range type, (b) multiple-range type.

**Figure 2.61**

Ammeters: (a) single-range type, (b) multiple-range type.

From this, we obtain

$$R_n = \frac{V_{fs}}{I_{fs}} - R_m \quad (2.60)$$

Similarly, the ammeter measures the current through the load and is connected in series with it. As shown in Fig. 2.61(a), the ammeter consists of a d'Arsonval movement in parallel with a resistor whose resistance R_m is deliberately made very small (theoretically, zero) to minimize the voltage drop across it. To allow multiple ranges, shunt resistors are often connected in parallel with R_m as shown in Fig. 2.61(b). The shunt resistors allow the meter to measure in the range 0–10 mA, 0–100 mA, or 0–1 A, depending on whether the switch is connected to R_1 , R_2 , or R_3 , respectively.

Now our objective is to obtain the multiplier shunt R_n for the single-range ammeter in Fig. 2.61(a), or $R_n = R_1$, R_2 , or R_3 for the multiple-range ammeter in Fig. 2.61(b). We notice that R_m and R_n are in parallel and that at full-scale reading $I = I_{fs} = I_m + I_n$, where I_n is the current through the shunt resistor R_n . Applying the current division principle yields

$$I_m = \frac{R_n}{R_n + R_m} I_{fs}$$

or

$$R_n = \frac{I_m}{I_{fs} - I_m} R_m \quad (2.61)$$

The resistance R_x of a linear resistor can be measured in two ways. An indirect way is to measure the current I that flows through it by

connecting an ammeter in series with it and the voltage V across it by connecting a voltmeter in parallel with it, as shown in Fig. 2.62(a). Then

$$R_x = \frac{V}{I} \quad (2.62)$$

The direct method of measuring resistance is to use an ohmmeter. An ohmmeter consists basically of a d'Arsonval movement, a variable resistor or potentiometer, and a battery, as shown in Fig. 2.62(b). Applying KVL to the circuit in Fig. 2.62(b) gives

$$E = (R + R_m + R_x)I_m$$

or

$$R_x = \frac{E}{I_m} - (R + R_m) \quad (2.63)$$

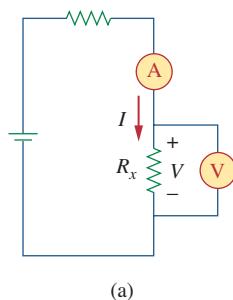
The resistor R is selected such that the meter gives a full-scale deflection; that is, $I_m = I_{fs}$ when $R_x = 0$. This implies that

$$E = (R + R_m)I_{fs} \quad (2.64)$$

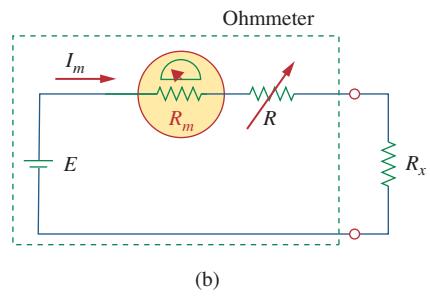
Substituting Eq. (2.64) into Eq. (2.63) leads to

$$R_x = \left(\frac{I_{fs}}{I_m} - 1 \right) (R + R_m) \quad (2.65)$$

As mentioned, the types of meters we have discussed are known as *analog* meters and are based on the d'Arsonval meter movement. Another type of meter, called a *digital meter*, is based on active circuit elements such as op amps. For example, a digital multimeter displays measurements of dc or ac voltage, current, and resistance as discrete numbers, instead of using a pointer deflection on a continuous scale as in an analog multimeter. Digital meters are what you would most likely use in a modern lab. However, the design of digital meters is beyond the scope of this book.



(a)



(b)

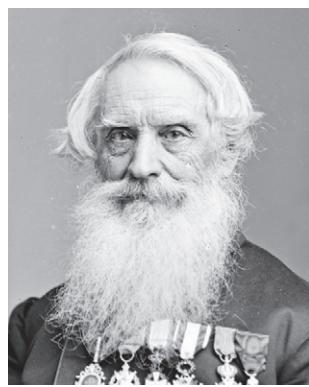
Figure 2.62

Two ways of measuring resistance:
(a) using an ammeter and a voltmeter,
(b) using an ohmmeter.

Historical

Samuel F. B. Morse (1791–1872), an American painter, invented the telegraph, the first practical, commercialized application of electricity.

Morse was born in Charlestown, Massachusetts and studied at Yale and the Royal Academy of Arts in London to become an artist. In the 1830s, he became intrigued with developing a telegraph. He had a working model by 1836 and applied for a patent in 1838. The U.S. Senate appropriated funds for Morse to construct a telegraph line between Baltimore and Washington, D.C. On May 24, 1844, he sent the famous first message: "What hath God wrought!" Morse also developed a code of dots and dashes for letters and numbers, for sending messages on the telegraph. The development of the telegraph led to the invention of the telephone.



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Example 2.17

Following the voltmeter setup of Fig. 2.60, design a voltmeter for the following multiple ranges:

- (a) 0–1 V (b) 0–5 V (c) 0–50 V (d) 0–100 V

Assume that the internal resistance $R_m = 2 \text{ k}\Omega$ and the full-scale current $I_{fs} = 100 \mu\text{A}$.

Solution:

We apply Eq. (2.60) and assume that R_1, R_2, R_3 , and R_4 correspond with ranges 0–1 V, 0–5 V, 0–50 V, and 0–100 V, respectively.

- (a) For range 0–1 V,

$$R_1 = \frac{1}{100 \times 10^{-6}} - 2000 = 10,000 - 2000 = 8 \text{ k}\Omega$$

- (b) For range 0–5 V,

$$R_2 = \frac{5}{100 \times 10^{-6}} - 2000 = 50,000 - 2000 = 48 \text{ k}\Omega$$

- (c) For range 0–50 V,

$$R_3 = \frac{50}{100 \times 10^{-6}} - 2000 = 500,000 - 2000 = 498 \text{ k}\Omega$$

- (d) For range 0–100 V,

$$R_4 = \frac{100}{100 \times 10^{-6}} - 2000 = 1,000,000 - 2000 = 998 \text{ k}\Omega$$

Note that the ratio of the total resistance ($R_n + R_m$) to the full-scale voltage V_{fs} is constant and equal to $1/I_{fs}$ for the four ranges. This ratio (given in ohms per volt, or Ω/V) is known as the *sensitivity* of the voltmeter. The larger the sensitivity, the better the voltmeter.

Practice Problem 2.17

Following the ammeter setup of Fig. 2.61, design an ammeter for the following multiple ranges:

- (a) 0–1 A (b) 0–100 mA (c) 0–10 mA

Take the full-scale meter current as $I_m = 1 \text{ mA}$ and the internal resistance of the ammeter as $R_m = 50 \Omega$.

Answer: Shunt resistors: 50 m Ω , 505 m Ω , 5.556 Ω .

2.9 Summary

1. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,

$$v = iR$$

where R is the resistance of the resistor.

2. A short circuit is a resistor (a perfectly, conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).
3. The conductance G of a resistor is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

4. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b , the number of nodes n , and the number of independent loops l in a network are related as

$$b = l + n - 1$$

5. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
6. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
7. Two elements are in series when they are connected sequentially, end to end. When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).
8. When two resistors $R_1 (=1/G_1)$ and $R_2 (=1/G_2)$ are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{\text{eq}} = R_1 + R_2, \quad G_{\text{eq}} = \frac{G_1 G_2}{G_1 + G_2}$$

9. When two resistors $R_1 (=1/G_1)$ and $R_2 (=1/G_2)$ are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}, \quad G_{\text{eq}} = G_1 + G_2$$

10. The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2} v, \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

11. The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2} i, \quad i_2 = \frac{R_1}{R_1 + R_2} i$$

12. The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

13. The formulas for a wye-to-delta transformation are

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}, \quad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

14. The basic laws covered in this chapter can be applied to the problems of electrical lighting and design of dc meters.

Review Questions

2.1 The reciprocal of resistance is:

- (a) voltage
- (b) current
- (c) conductance
- (d) coulombs

2.2 An electric heater draws 10 A from a 120-V line. The resistance of the heater is:

- (a) 1200 Ω
- (b) 120 Ω
- (c) 12 Ω
- (d) 1.2 Ω

2.3 The voltage drop across a 1.5-kW toaster that draws 12 A of current is:

- (a) 18 kV
- (b) 125 V
- (c) 120 V
- (d) 10.42 V

2.4 The maximum current that a 2W, 80 k Ω resistor can safely conduct is:

- (a) 160 kA
- (b) 40 kA
- (c) 5 mA
- (d) 25 μ A

2.5 A network has 12 branches and 8 independent loops. How many nodes are there in the network?

- (a) 19
- (b) 17
- (c) 5
- (d) 4

2.6 The current I in the circuit of Fig. 2.63 is:

- (a) -0.8 A
- (b) -0.2 A
- (c) 0.2 A
- (d) 0.8 A

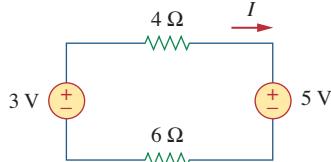


Figure 2.63

For Review Question 2.6.

2.7 The current I_o of Fig. 2.64 is:

- (a) -4 A
- (b) -2 A
- (c) 4 A
- (d) 16 A

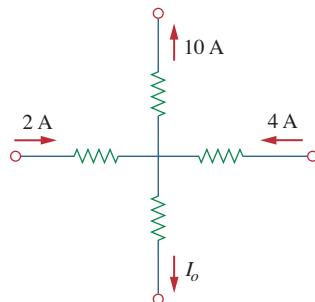


Figure 2.64

For Review Question 2.7.

2.8 In the circuit in Fig. 2.65, V is:

- (a) 30 V
- (b) 14 V
- (c) 10 V
- (d) 6 V

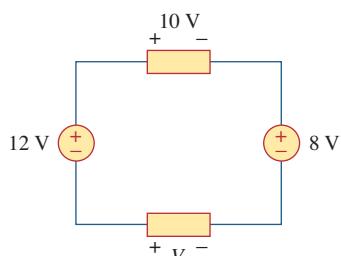
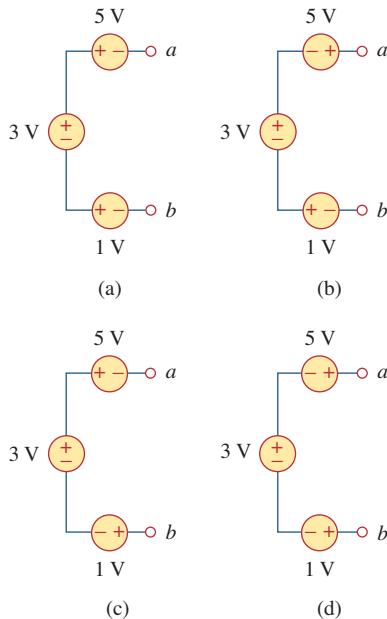


Figure 2.65

For Review Question 2.8.

- 2.9** Which of the circuits in Fig. 2.66 will give you $V_{ab} = 7 \text{ V}$?

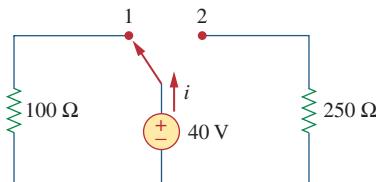
**Figure 2.66**

For Review Question 2.9.

Problems

Section 2.2 Ohm's Law

- 2.1** Design a problem, complete with a solution, to help students to better understand Ohm's Law. Use at least two resistors and one voltage source. Hint, you could use both resistors at once or one at a time, it is up to you. Be creative.
- 2.2** Find the hot resistance of a light bulb rated 60 W, 120 V.
- 2.3** A bar of silicon is 4 cm long with a circular cross section. If the resistance of the bar is 240Ω at room temperature, what is the cross-sectional radius of the bar?
- 2.4** (a) Calculate current i in Fig. 2.68 when the switch is in position 1.
 (b) Find the current when the switch is in position 2.

**Figure 2.68**

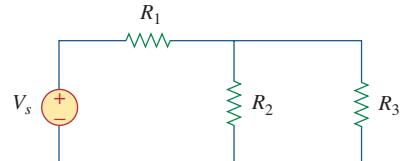
For Prob. 2.4.

Section 2.3 Nodes, Branches, and Loops

- 2.5** For the network graph in Fig. 2.69, find the number of nodes, branches, and loops.

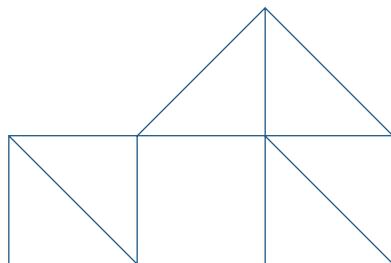
- 2.10** In the circuit of Fig. 2.67, a decrease in R_3 leads to a decrease of, select all that apply:

- (a) current through R_3
- (b) voltage across R_3
- (c) voltage across R_1
- (d) power dissipated in R_2
- (e) none of the above

**Figure 2.67**

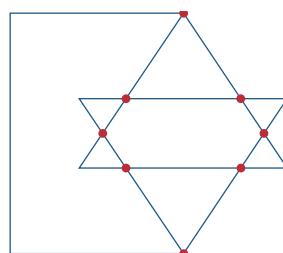
For Review Question 2.10.

Answers: 2.1c, 2.2c, 2.3b, 2.4c, 2.5c, 2.6b, 2.7a, 2.8d, 2.9d, 2.10b, d.

**Figure 2.69**

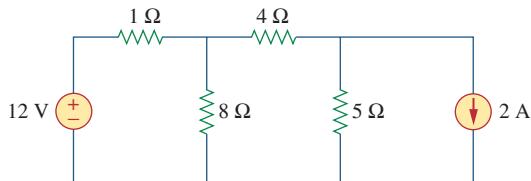
For Prob. 2.5.

- 2.6** In the network graph shown in Fig. 2.70, determine the number of branches and nodes.

**Figure 2.70**

For Prob. 2.6.

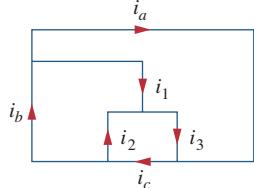
- 2.7** Determine the number of branches and nodes in the circuit of Fig. 2.71.

**Figure 2.71**

For Prob. 2.7.

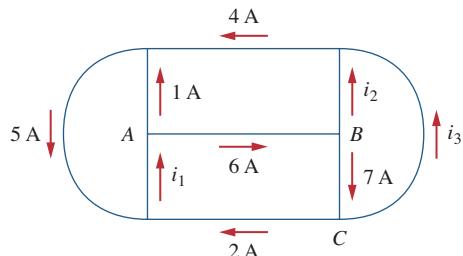
Section 2.4 Kirchhoff's Laws

- 2.8** Design a problem, complete with a solution, to help other students better understand Kirchhoff's Current Law. Design the problem by specifying values of i_a , i_b , and i_c , shown in Fig. 2.72, and asking them to solve for values of i_1 , i_2 , and i_3 . Be careful to specify realistic currents.

**Figure 2.72**

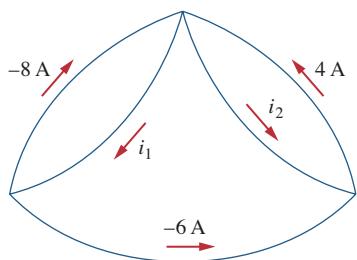
For Prob. 2.8.

- 2.9** Find i_1 , i_2 , and i_3 in Fig. 2.73.

**Figure 2.73**

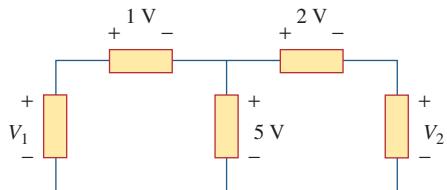
For Prob. 2.9.

- 2.10** Determine i_1 and i_2 in the circuit of Fig. 2.74.

**Figure 2.74**

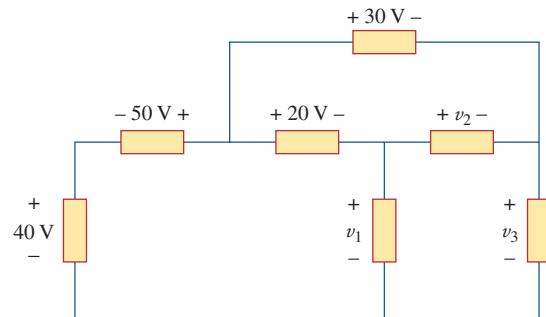
For Prob. 2.10.

- 2.11** In the circuit of Fig. 2.75, calculate V_1 and V_2 .

**Figure 2.75**

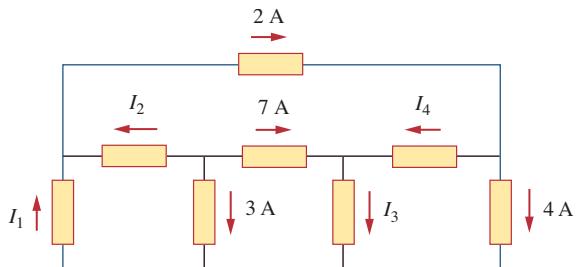
For Prob. 2.11.

- 2.12** In the circuit in Fig. 2.76, obtain v_1 , v_2 , and v_3 .

**Figure 2.76**

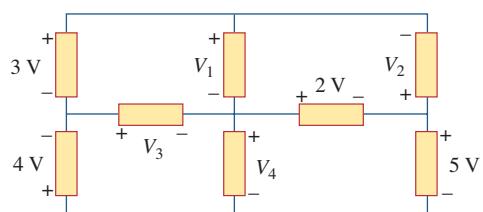
For Prob. 2.12.

- 2.13** For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4 .

**Figure 2.77**

For Prob. 2.13.

- 2.14** Given the circuit in Fig. 2.78, use KVL to find the branch voltages V_1 to V_4 .

**Figure 2.78**

For Prob. 2.14.

2.15 Calculate v and i_x in the circuit of Fig. 2.79.

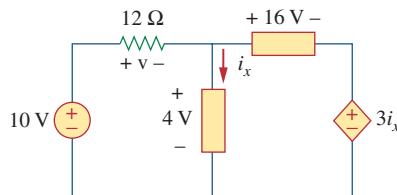


Figure 2.79

For Prob. 2.15.

2.16 Determine V_o in the circuit in Fig. 2.80.

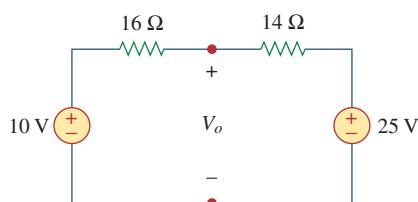


Figure 2.80

For Prob. 2.16.

2.17 Obtain v_1 through v_3 in the circuit of Fig. 2.81.

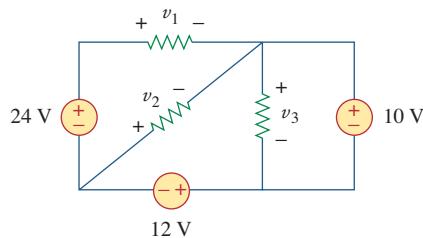


Figure 2.81

For Prob. 2.17.

2.18 Find I and V_{ab} in the circuit of Fig. 2.82.

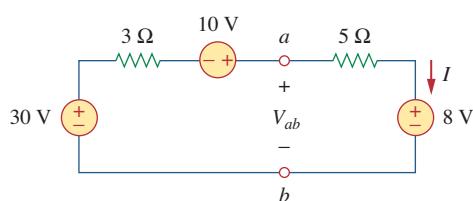


Figure 2.82

For Prob. 2.18.

2.19 From the circuit in Fig. 2.83, find I , the power dissipated by the resistor, and the power supplied by each source.

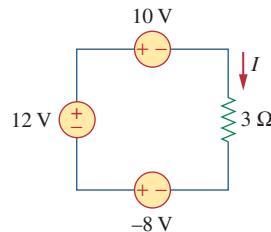


Figure 2.83

For Prob. 2.19.

2.20 Determine i_o in the circuit of Fig. 2.84.

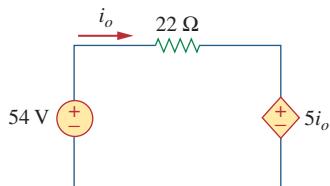


Figure 2.84

For Prob. 2.20.

2.21 Find V_x in the circuit of Fig. 2.85.

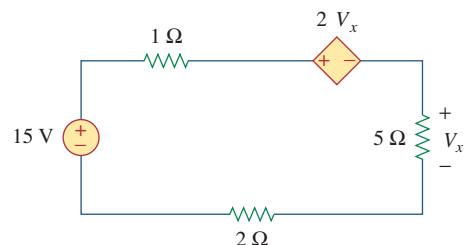


Figure 2.85

For Prob. 2.21.

2.22 Find V_o in the circuit in Fig. 2.86 and the power absorbed by the dependent source.

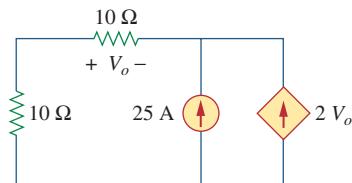


Figure 2.86

For Prob. 2.22.

- 2.23** In the circuit shown in Fig. 2.87, determine v_x and the power absorbed by the $12\text{-}\Omega$ resistor.

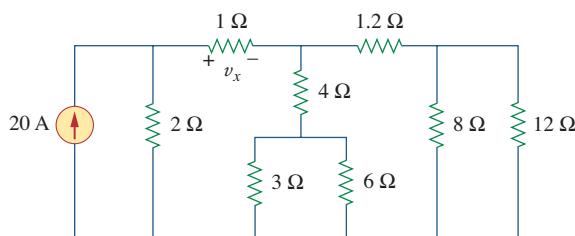


Figure 2.87

For Prob. 2.23.

- 2.24** For the circuit in Fig. 2.88, find V_o/V_s in terms of α , R_1 , R_2 , R_3 , and R_4 . If $R_1 = R_2 = R_3 = R_4$, what value of α will produce $|V_o/V_s| = 10$?

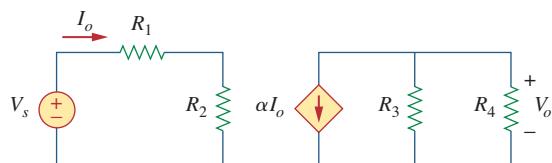


Figure 2.88

For Prob. 2.24.

- 2.25** For the network in Fig. 2.89, find the current, voltage, and power associated with the $20\text{-k}\Omega$ resistor.

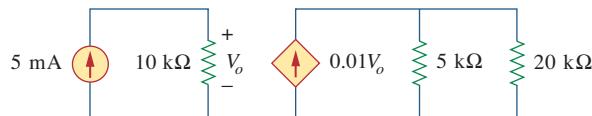


Figure 2.89

For Prob. 2.25.

Sections 2.5 and 2.6 Series and Parallel Resistors

- 2.26** For the circuit in Fig. 2.90, $i_o = 3 \text{ A}$. Calculate i_x and the total power absorbed by the entire circuit.

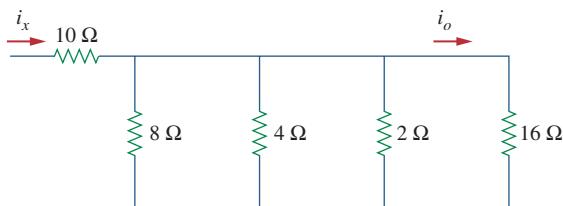


Figure 2.90

For Prob. 2.26.

- 2.27** Calculate I_o in the circuit of Fig. 2.91.

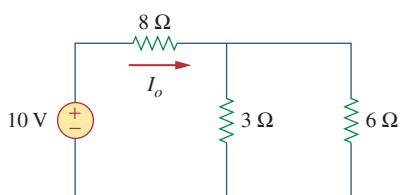


Figure 2.91

For Prob. 2.27.

- 2.28** Design a problem, using Fig. 2.92, to help other students better understand series and parallel circuits.

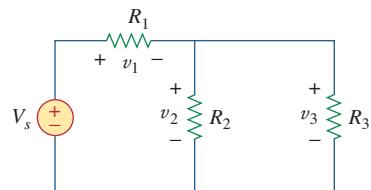


Figure 2.92

For Prob. 2.28.

- 2.29** All resistors in Fig. 2.93 are $5 \text{ }\Omega$ each. Find R_{eq} .

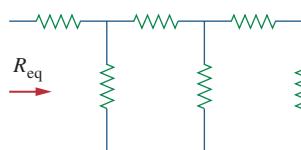


Figure 2.93

For Prob. 2.29.

- 2.30** Find R_{eq} for the circuit in Fig. 2.94.

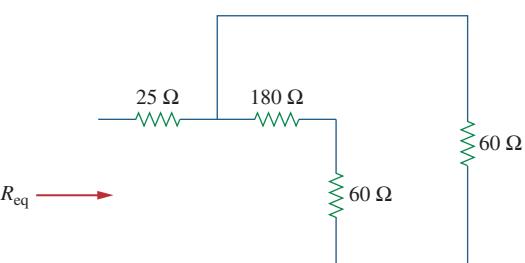


Figure 2.94

For Prob. 2.30.

2.31 For the circuit in Fig. 2.95, determine i_1 to i_5 .

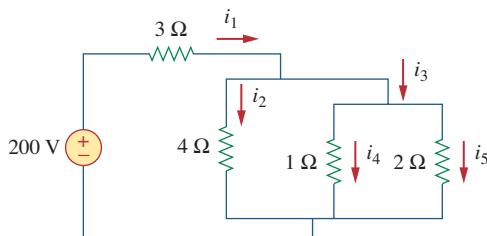


Figure 2.95

For Prob. 2.31.

2.32 Find i_1 through i_4 in the circuit in Fig. 2.96.

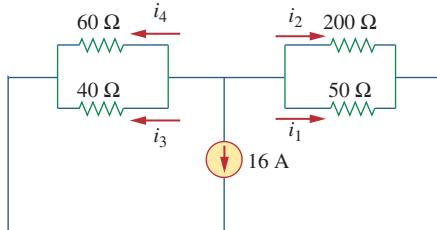


Figure 2.96

For Prob. 2.32.

2.33 Obtain v and i in the circuit of Fig. 2.97.

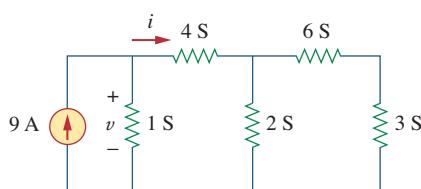


Figure 2.97

For Prob. 2.33.

2.34 Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall absorbed power by the resistor network.

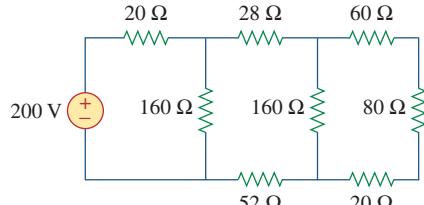


Figure 2.98

For Prob. 2.34.

2.35 Calculate V_o and I_o in the circuit of Fig. 2.99.

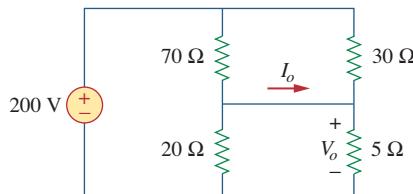


Figure 2.99

For Prob. 2.35.

2.36 Find i and V_o in the circuit of Fig. 2.100.

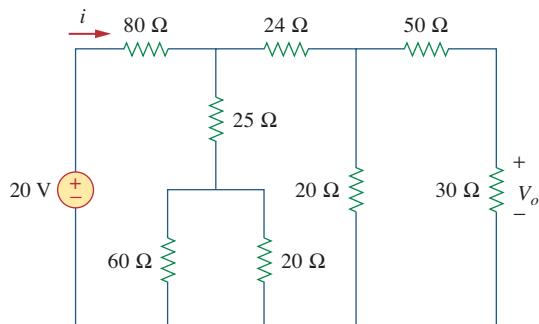


Figure 2.100

For Prob. 2.36.

2.37 Find R for the circuit in Fig. 2.101.

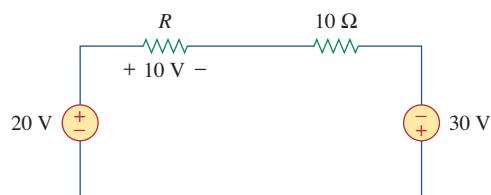


Figure 2.101

For Prob. 2.37.

2.38 Find R_{eq} and i_o in the circuit of Fig. 2.102.

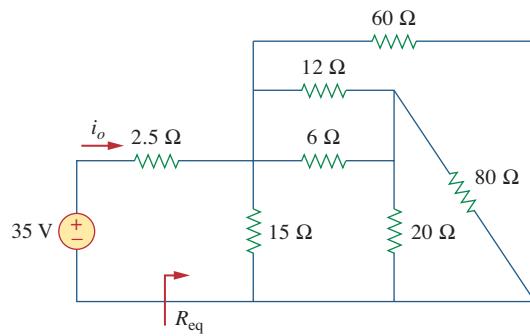


Figure 2.102

For Prob. 2.38.

- 2.39** Evaluate R_{eq} for each of the circuits shown in Fig. 2.103.

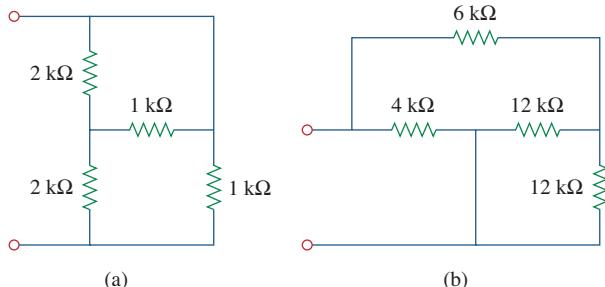


Figure 2.103

For Prob. 2.39.

- 2.40** For the ladder network in Fig. 2.104, find I and R_{eq} .

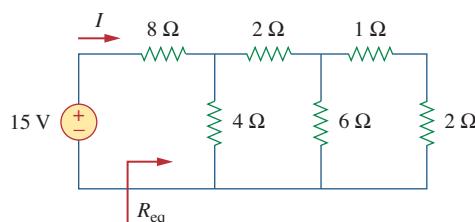


Figure 2.104

For Prob. 2.40.

- 2.41** If $R_{eq} = 50\text{ }\Omega$ in the circuit of Fig. 2.105, find R .

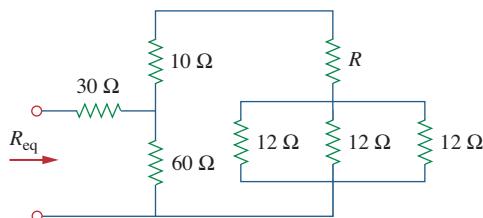


Figure 2.105

For Prob. 2.41.

- 2.42** Reduce each of the circuits in Fig. 2.106 to a single resistor at terminals $a-b$.

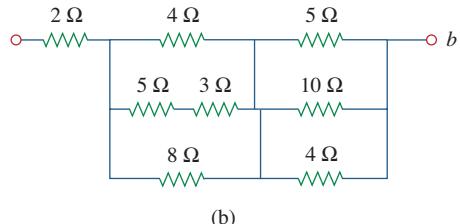
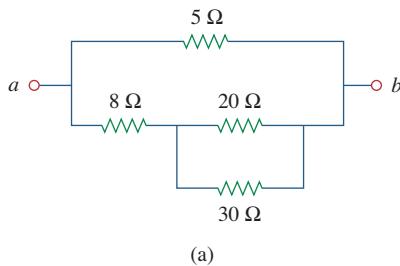


Figure 2.106

For Prob. 2.42.

- 2.43** Calculate the equivalent resistance R_{ab} at terminals $a-b$ for each of the circuits in Fig. 2.107.

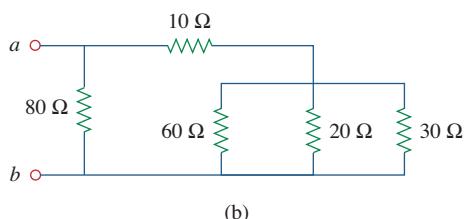
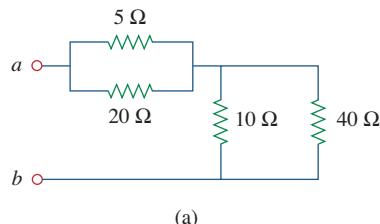


Figure 2.107

For Prob. 2.43.

- 2.44** For the circuits in Fig. 2.108, obtain the equivalent resistance at terminals $a-b$.

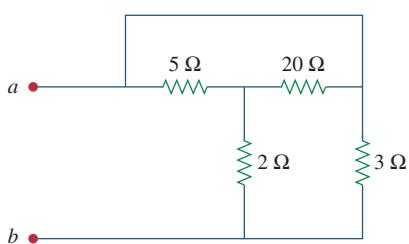
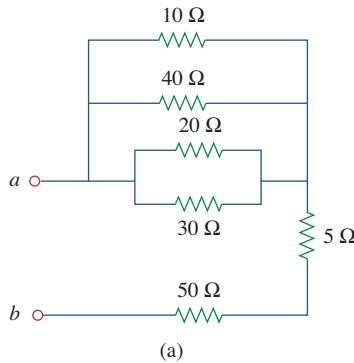


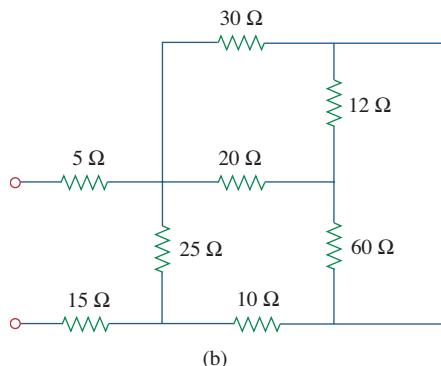
Figure 2.108

For Prob. 2.44.

- 2.45** Find the equivalent resistance at terminals *a*-*b* of each circuit in Fig. 2.109.



(a)

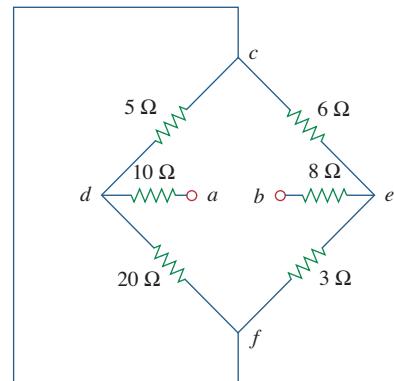


(b)

Figure 2.109

For Prob. 2.45.

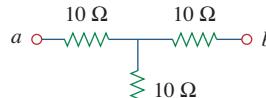
- 2.47** Find the equivalent resistance R_{ab} in the circuit of Fig. 2.111.

**Figure 2.111**

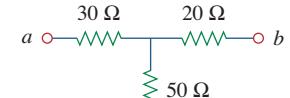
For Prob. 2.47.

Section 2.7 Wye-Delta Transformations

- 2.48** Convert the circuits in Fig. 2.112 from Y to Δ .



(a)



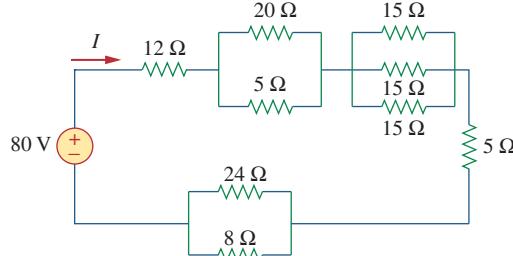
(b)

Figure 2.112

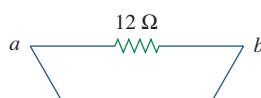
For Prob. 2.48.

- 2.49** Transform the circuits in Fig. 2.113 from Δ to Y.

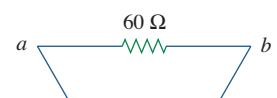
- 2.46** Find I in the circuit of Fig. 2.110.

**Figure 2.110**

For Prob. 2.46.



(a)



(b)

Figure 2.113

For Prob. 2.49.

- 2.50** Design a problem to help other students better understand wye-delta transformations using Fig. 2.114.

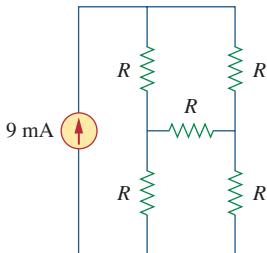
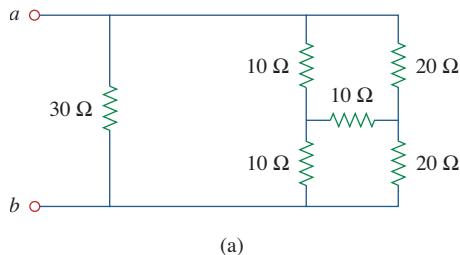


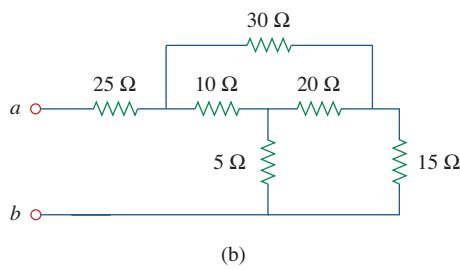
Figure 2.114

For Prob. 2.50.

- 2.51** Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits in Fig. 2.115.



(a)



(b)

Figure 2.115

For Prob. 2.51.

- ***2.52** For the circuit shown in Fig. 2.116, find the equivalent resistance. All resistors are 3 Ω.

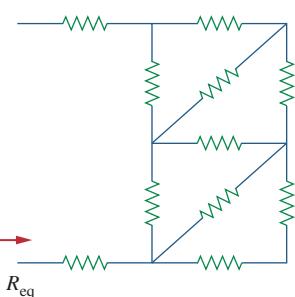
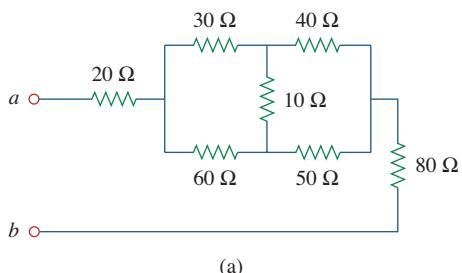


Figure 2.116

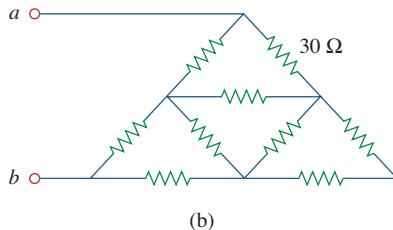
For Prob. 2.52.

* An asterisk indicates a challenging problem.

- *2.53** Obtain the equivalent resistance R_{ab} in each of the circuits of Fig. 2.117. In (b), all resistors have a value of 30 Ω.



(a)



(b)

Figure 2.117

For Prob. 2.53.

- 2.54** Consider the circuit in Fig. 2.118. Find the equivalent resistance at terminals: (a) *a*-*b*, (b) *c*-*d*.

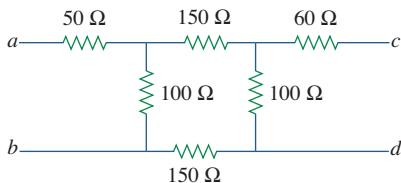


Figure 2.118

For Prob. 2.54.

- 2.55** Calculate I_o in the circuit of Fig. 2.119.

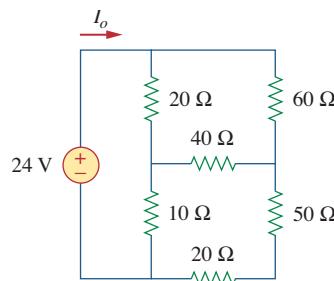


Figure 2.119

For Prob. 2.55.

- 2.56** Determine V in the circuit of Fig. 2.120.

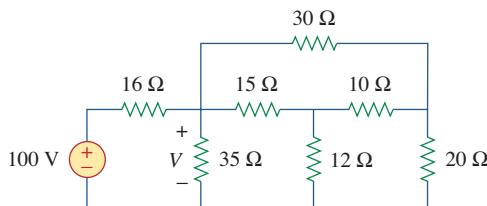


Figure 2.120

For Prob. 2.56.

- *2.57** Find R_{eq} and I in the circuit of Fig. 2.121.

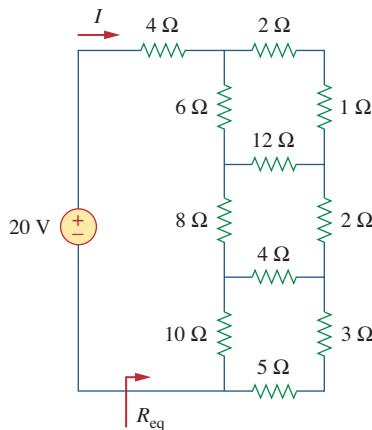


Figure 2.121

For Prob. 2.57.

Section 2.8 Applications

- 2.58** The 60 W light bulb in Fig. 2.122 is rated at 120 volts. Calculate V_s to make the light bulb operate at the rated conditions.

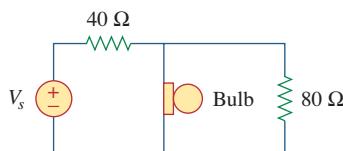


Figure 2.122

For Prob. 2.58.

- 2.59** Three light bulbs are connected in series to a 120-V source as shown in Fig. 2.123. Find the current I through the bulbs. Each bulb is rated at 120 volts. How much power is each bulb absorbing? Do they generate much light?

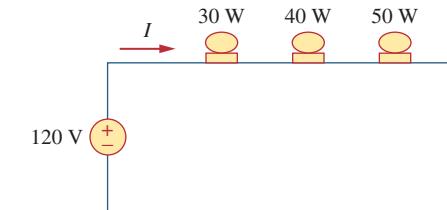


Figure 2.123

For Prob. 2.59.

- 2.60** If the three bulbs of Prob. 2.59 are connected in parallel to the 120-V source, calculate the current through each bulb.

- e2d** As a design engineer, you are asked to design a lighting system consisting of a 70-W power supply and two light bulbs as shown in Fig. 2.124. You must select the two bulbs from the following three available bulbs.

$$\begin{aligned} R_1 &= 80 \Omega, \text{ cost} = \$0.60 \text{ (standard size)} \\ R_2 &= 90 \Omega, \text{ cost} = \$0.90 \text{ (standard size)} \\ R_3 &= 100 \Omega, \text{ cost} = \$0.75 \text{ (nonstandard size)} \end{aligned}$$

The system should be designed for minimum cost such that lies within the range $I = 1.2 \text{ A} \pm 5 \text{ percent}$.

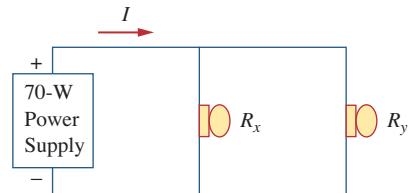


Figure 2.124

For Prob. 2.61.

- 2.62** A three-wire system supplies two loads A and B as shown in Fig. 2.125. Load A consists of a motor drawing a current of 8 A, while load B is a PC drawing 2 A. Assuming 10 h/day of use for 365 days and 6 cents/kWh, calculate the annual energy cost of the system.

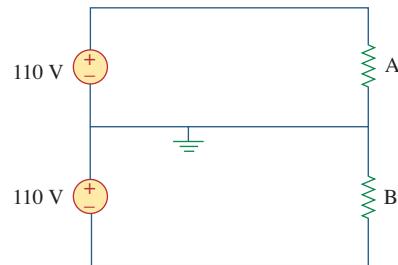


Figure 2.125

For Prob. 2.62.

- 2.63** If an ammeter with an internal resistance of 100 Ω and a current capacity of 2 mA is to measure 5 A, determine the value of the resistance needed.

Calculate the power dissipated in the shunt resistor.

- 2.64** The potentiometer (adjustable resistor) R_x in Fig. 2.126 is to be designed to adjust current i_x from 1 A to 10 A. Calculate the values of R and R_x to achieve this.

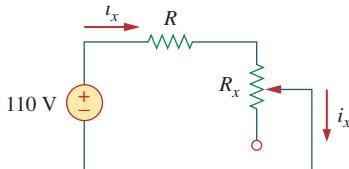


Figure 2.126

For Prob. 2.64.

- 2.65** A d'Arsonval meter with an internal resistance of $1\text{ k}\Omega$ requires 10 mA to produce full-scale deflection. Calculate the value of a series resistance needed to measure 50 V of full scale.

- 2.66** A 20-k Ω /V voltmeter reads 10 V full scale.

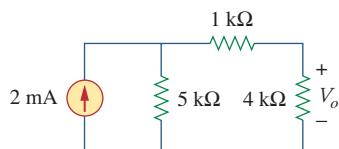
- (a) What series resistance is required to make the meter read 50 V full scale?
- (b) What power will the series resistor dissipate when the meter reads full scale?

- 2.67** (a) Obtain the voltage V_o in the circuit of Fig. 2.127(a).

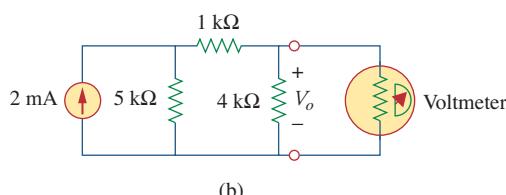
- (b) Determine the voltage V'_o measured when a voltmeter with 6-k Ω internal resistance is connected as shown in Fig. 2.127(b).
- (c) The finite resistance of the meter introduces an error into the measurement. Calculate the percent error as

$$\left| \frac{V_o - V'_o}{V_o} \right| \times 100\%$$

- (d) Find the percent error if the internal resistance were 36 k Ω .



(a)



(b)

Figure 2.127

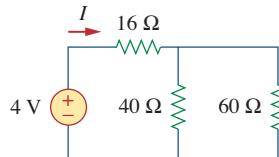
For Prob. 2.67.

- 2.68** (a) Find the current I in the circuit of Fig. 2.128(a).

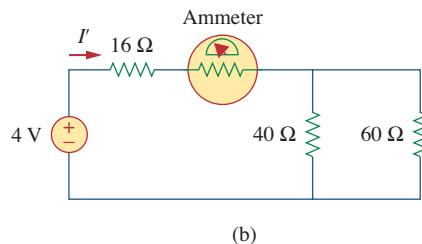
- (b) An ammeter with an internal resistance of $1\text{ }\Omega$ is inserted in the network to measure I' as shown in Fig. 2.128(b). What is I' ?

- (c) Calculate the percent error introduced by the meter as

$$\left| \frac{I - I'}{I} \right| \times 100\%$$



(a)



(b)

Figure 2.128

For Prob. 2.68.

- 2.69** A voltmeter is used to measure V_o in the circuit in Fig. 2.129. The voltmeter model consists of an ideal voltmeter in parallel with a $100\text{-k}\Omega$ resistor. Let $V_s = 40\text{ V}$, $R_s = 10\text{ k}\Omega$, and $R_1 = 20\text{ k}\Omega$. Calculate V_o with and without the voltmeter when

- (a) $R_2 = 1\text{ k}\Omega$
- (b) $R_2 = 10\text{ k}\Omega$
- (c) $R_2 = 100\text{ k}\Omega$

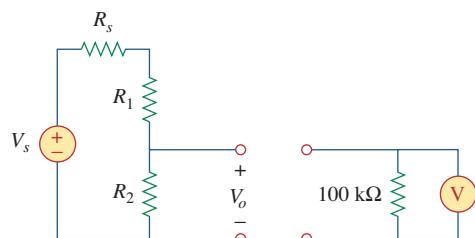
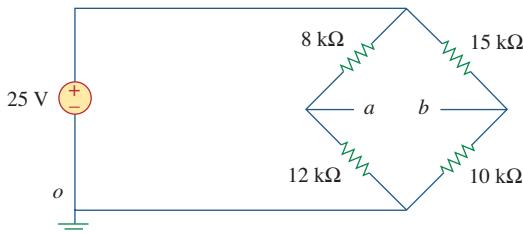


Figure 2.129

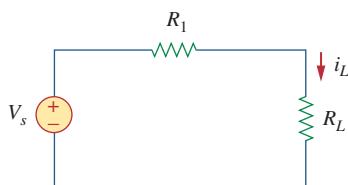
For Prob. 2.69.

- 2.70** (a) Consider the Wheatstone bridge shown in Fig. 2.130. Calculate v_a , v_b , and v_{ab} .
 (b) Rework part (a) if the ground is placed at a instead of o .

**Figure 2.130**

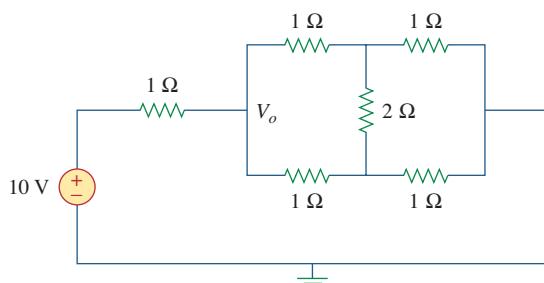
For Prob. 2.70.

- 2.71** Figure 2.131 represents a model of a solar photovoltaic panel. Given that $V_s = 30 \text{ V}$, $R_1 = 20 \Omega$, and $i_L = 1 \text{ A}$, find R_L .

**Figure 2.131**

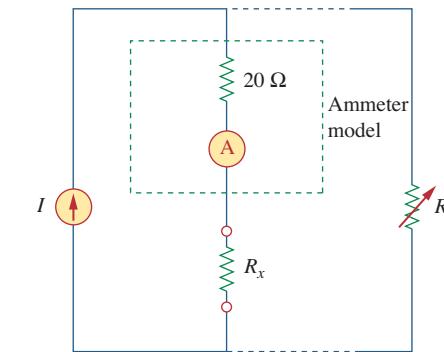
For Prob. 2.71.

- 2.72** Find V_o in the two-way power divider circuit in Fig. 2.132.

**Figure 2.132**

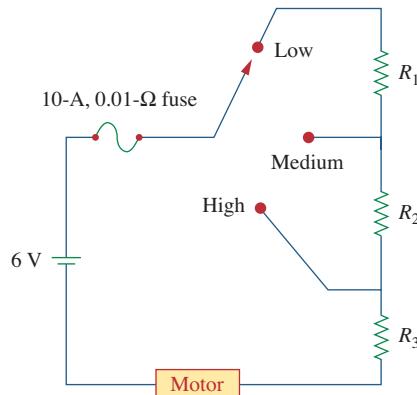
For Prob. 2.72.

- 2.73** An ammeter model consists of an ideal ammeter in series with a 20-Ω resistor. It is connected with a current source and an unknown resistor R_x as shown in Fig. 2.133. The ammeter reading is noted. When a potentiometer R is added and adjusted until the ammeter reading drops to one half its previous reading, then $R = 65 \Omega$. What is the value of R_x ?

**Figure 2.133**

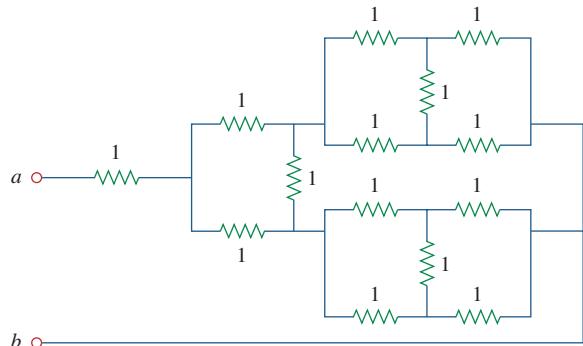
For Prob. 2.73.

- 2.74** The circuit in Fig. 2.134 is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 20 mΩ. Determine the series dropping resistances R_1 , R_2 , and R_3 .

**Figure 2.134**

For Prob. 2.74.

- 2.75** Find R_{ab} in the four-way power divider circuit in Fig. 2.135. Assume each element is 1 Ω.

**Figure 2.135**

For Prob. 2.75.

Comprehensive Problems

- 2.76** Repeat Prob. 2.75 for the eight-way divider shown in Fig. 2.136.

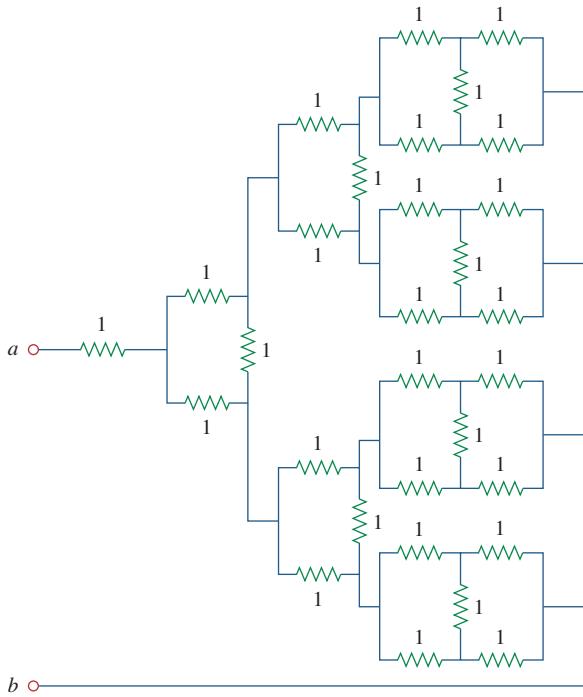


Figure 2.136

For Prob. 2.76.

- 2.77** Suppose your circuit laboratory has the following standard commercially available resistors in large quantities:

1.8 Ω 20 Ω 300 Ω 24 k Ω 56 k Ω

Using series and parallel combinations and a minimum number of available resistors, how would you obtain the following resistances for an electronic circuit design?

- (a) 5 Ω (b) 311.8 Ω
 (c) 40 k Ω (d) 52.32 k Ω

- 2.78** In the circuit in Fig. 2.137, the wiper divides the potentiometer resistance between αR and $(1 - \alpha)R$, $0 \leq \alpha \leq 1$. Find v_o/v_s .

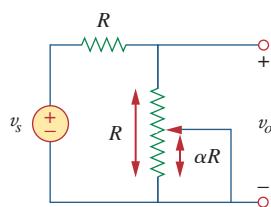


Figure 2.137

For Prob. 2.78.

- 2.79** An electric pencil sharpener rated 240 mW, 6 V is connected to a 9-V battery as shown in Fig. 2.138. Calculate the value of the series-dropping resistor R_x needed to power the sharpener.

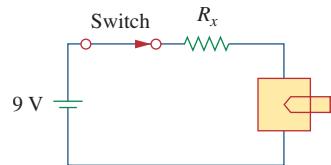


Figure 2.138

For Prob. 2.79.

- 2.80** A loudspeaker is connected to an amplifier as shown in Fig. 2.139. If a 10- Ω loudspeaker draws the maximum power of 12 W from the amplifier, determine the maximum power a 4- Ω loudspeaker will draw.



Figure 2.139

For Prob. 2.80.

- 2.81** In a certain application, the circuit in Fig. 2.140 must be designed to meet these two criteria:

$$(a) V_o/V_s = 0.05 \quad (b) R_{eq} = 40 \text{ k}\Omega$$

If the load resistor 5 k Ω is fixed, find R_1 and R_2 to meet the criteria.

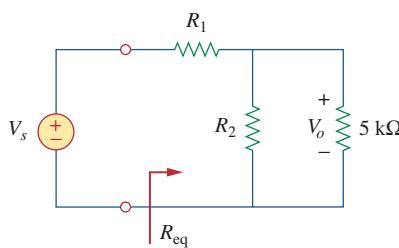


Figure 2.140

For Prob. 2.81.

- 2.82** The pin diagram of a resistance array is shown in Fig. 2.141. Find the equivalent resistance between the following:

- (a) 1 and 2
- (b) 1 and 3
- (c) 1 and 4

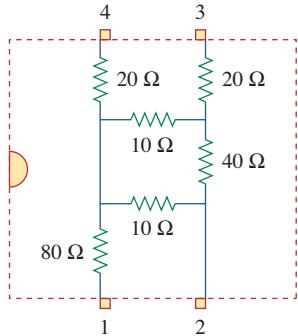


Figure 2.141

For Prob. 2.82.

- 2.83** Two delicate devices are rated as shown in Fig. 2.142. Find the values of the resistors R_1 and R_2 needed to power the devices using a 24-V battery.

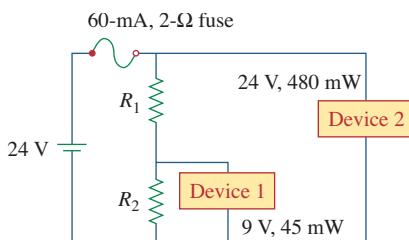


Figure 2.142

For Prob. 2.83.

3

Methods of Analysis

No great work is ever done in a hurry. To develop a great scientific discovery, to print a great picture, to write an immortal poem, to become a minister, or a famous general—to do anything great requires time, patience, and perseverance. These things are done by degrees, “little by little.”

—W. J. Wilmont Buxton

Enhancing Your Career

Career in Electronics

One area of application for electric circuit analysis is electronics. The term *electronics* was originally used to distinguish circuits of very low current levels. This distinction no longer holds, as power semiconductor devices operate at high levels of current. Today, electronics is regarded as the science of the motion of charges in a gas, vacuum, or semiconductor. Modern electronics involves transistors and transistor circuits. The earlier electronic circuits were assembled from components. Many electronic circuits are now produced as integrated circuits, fabricated in a semiconductor substrate or chip.

Electronic circuits find applications in many areas, such as automation, broadcasting, computers, and instrumentation. The range of devices that use electronic circuits is enormous and is limited only by our imagination. Radio, television, computers, and stereo systems are but a few.

An electrical engineer usually performs diverse functions and is likely to use, design, or construct systems that incorporate some form of electronic circuits. Therefore, an understanding of the operation and analysis of electronics is essential to the electrical engineer. Electronics has become a specialty distinct from other disciplines within electrical engineering. Because the field of electronics is ever advancing, an electronics engineer must update his/her knowledge from time to time. The best way to do this is by being a member of a professional organization such as the Institute of Electrical and Electronics Engineers (IEEE). With a membership of over 300,000, the IEEE is the largest professional organization in the world. Members benefit immensely from the numerous magazines, journals, transactions, and conference/symposium proceedings published yearly by IEEE. You should consider becoming an IEEE member.



Troubleshooting an electronic circuit board.

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3.1 Introduction

Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of Kirchhoff's current law (KCL), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (KVL). The two techniques are so important that this chapter should be regarded as the most important in the book. Students are therefore encouraged to pay careful attention.

With the two techniques to be developed in this chapter, we can analyze any linear circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage. One method of solving simultaneous equations involves Cramer's rule, which allows us to calculate circuit variables as a quotient of determinants. The examples in the chapter will illustrate this method; Appendix A also briefly summarizes the essentials the reader needs to know for applying Cramer's rule. Another method of solving simultaneous equations is to use *MATLAB*, a computer software discussed in Appendix E.

Also in this chapter, we introduce the use of *PSpice for Windows*, a circuit simulation computer software program that we will use throughout the text. Finally, we apply the techniques learned in this chapter to analyze transistor circuits.

3.2 Nodal Analysis

Nodal analysis is also known as the *node-voltage method*.

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

In *nodal analysis*, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, v_2, \dots, v_{n-1} to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps.

The first step in nodal analysis is selecting a node as the *reference* or *datum node*. The reference node is commonly called the *ground*

since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in Fig. 3.1. The type of ground in Fig. 3.1(c) is called a *chassis ground* and is used in devices where the case, enclosure, or chassis acts as a reference point for all circuits. When the potential of the earth is used as reference, we use the *earth ground* in Fig. 3.1(a) or (b). We shall always use the symbol in Fig. 3.1(b).

Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in Fig. 3.2(a). Node 0 is the reference node ($v = 0$), while nodes 1 and 2 are assigned voltages v_1 and v_2 , respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in Fig. 3.2(a), each node voltage is the voltage rise from the reference node to the corresponding nonreference node or simply the voltage of that node with respect to the reference node.

As the second step, we apply KCL to each nonreference node in the circuit. To avoid putting too much information on the same circuit, the circuit in Fig. 3.2(a) is redrawn in Fig. 3.2(b), where we now add i_1 , i_2 , and i_3 as the currents through resistors R_1 , R_2 , and R_3 , respectively. At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2 \quad (3.1)$$

At node 2,

$$I_2 + i_2 = i_3 \quad (3.2)$$

We now apply Ohm's law to express the unknown currents i_1 , i_2 , and i_3 in terms of node voltages. The key idea to bear in mind is that, since resistance is a passive element, by the passive sign convention, current must always flow from a higher potential to a lower potential.

Current flows from a **higher** potential to a **lower** potential in a resistor.

We can express this principle as

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R} \quad (3.3)$$

Note that this principle is in agreement with the way we defined resistance in Chapter 2 (see Fig. 2.1). With this in mind, we obtain from Fig. 3.2(b),

$$\begin{aligned} i_1 &= \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1 \\ i_2 &= \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2) \\ i_3 &= \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2 \end{aligned} \quad (3.4)$$

Substituting Eq. (3.4) in Eqs. (3.1) and (3.2) results, respectively, in

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \quad (3.5)$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \quad (3.6)$$

The number of nonreference nodes is equal to the number of independent equations that we will derive.

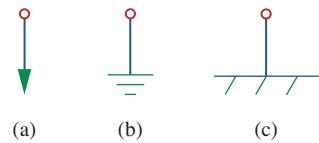
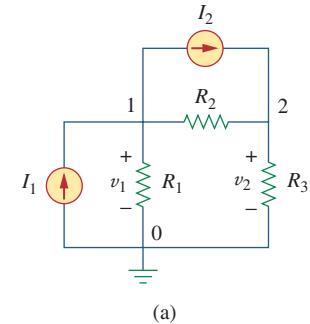
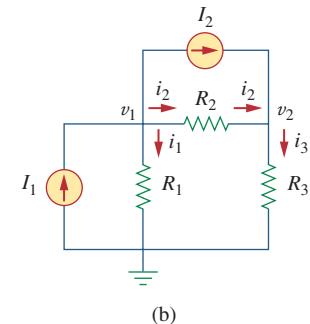


Figure 3.1

Common symbols for indicating reference node, (a) common ground, (b) ground, (c) chassis ground.



(a)



(b)

Figure 3.2

Typical circuit for nodal analysis.

In terms of the conductances, Eqs. (3.5) and (3.6) become

$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2) \quad (3.7)$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2 \quad (3.8)$$

The third step in nodal analysis is to solve for the node voltages. If we apply KCL to $n - 1$ nonreference nodes, we obtain $n - 1$ simultaneous equations such as Eqs. (3.5) and (3.6) or (3.7) and (3.8). For the circuit of Fig. 3.2, we solve Eqs. (3.5) and (3.6) or (3.7) and (3.8) to obtain the node voltages v_1 and v_2 using any standard method, such as the substitution method, the elimination method, Cramer's rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, Eqs. (3.7) and (3.8) can be cast in matrix form as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \quad (3.9)$$

which can be solved to get v_1 and v_2 . Equation 3.9 will be generalized in Section 3.6. The simultaneous equations may also be solved using calculators or with software packages such as *MATLAB*, *Mathcad*, *Maple*, and *Quattro Pro*.

Example 3.1

Calculate the node voltages in the circuit shown in Fig. 3.3(a).

Solution:

Consider Fig. 3.3(b), where the circuit in Fig. 3.3(a) has been prepared for nodal analysis. Notice how the currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that i_2 enters the 4- Ω resistor from the left-hand side, i_2 must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages v_1 and v_2 are now to be determined.

At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

or

$$3v_1 - v_2 = 20 \quad (3.1.1)$$

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

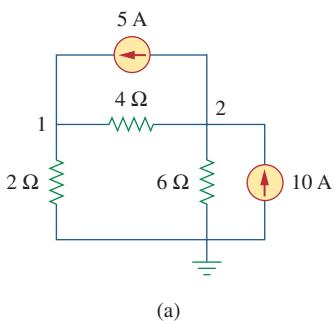
$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

or

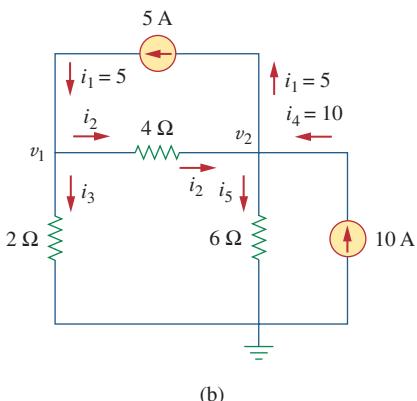
$$-3v_1 + 5v_2 = 60 \quad (3.1.2)$$

Figure 3.3

For Example 3.1: (a) original circuit, (b) circuit for analysis.



(a)



(b)

Now we have two simultaneous Eqs. (3.1.1) and (3.1.2). We can solve the equations using any method and obtain the values of v_1 and v_2 .

METHOD 1 Using the elimination technique, we add Eqs. (3.1.1) and (3.1.2).

$$4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V}$$

METHOD 2 To use Cramer's rule, we need to put Eqs. (3.1.1) and (3.1.2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \quad (3.1.3)$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain v_1 and v_2 as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}$$

giving us the same result as did the elimination method.

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5 \text{ A}, \quad i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}, \quad i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

$$i_4 = 10 \text{ A}, \quad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that i_2 is negative shows that the current flows in the direction opposite to the one assumed.

Obtain the node voltages in the circuit of Fig. 3.4.

Practice Problem 3.1

Answer: $v_1 = -6 \text{ V}$, $v_2 = -42 \text{ V}$.

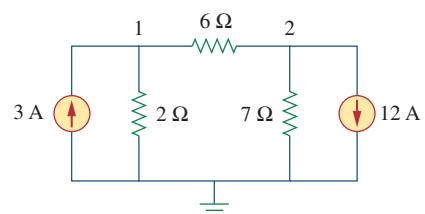


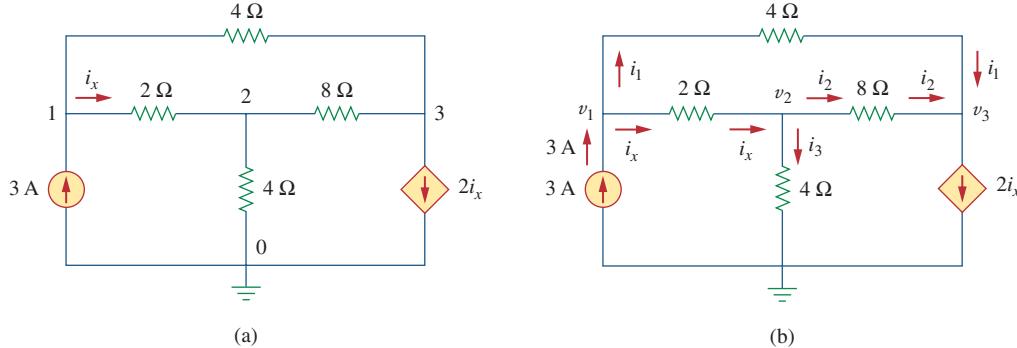
Figure 3.4
For Practice Prob. 3.1.

Example 3.2

Determine the voltages at the nodes in Fig. 3.5(a).

Solution:

The circuit in this example has three nonreference nodes, unlike the previous example which has two nonreference nodes. We assign voltages to the three nodes as shown in Fig. 3.5(b) and label the currents.

**Figure 3.5**

For Example 3.2: (a) original circuit, (b) circuit for analysis.

At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \quad (3.2.1)$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \quad (3.2.2)$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \quad (3.2.3)$$

We have three simultaneous equations to solve to get the node voltages v_1 , v_2 , and v_3 . We shall solve the equations in three ways.

METHOD 1 Using the elimination technique, we add Eqs. (3.2.1) and (3.2.3).

$$5v_1 - 5v_2 = 12$$

or

$$v_1 - v_2 = \frac{12}{5} = 2.4 \quad (3.2.4)$$

Adding Eqs. (3.2.2) and (3.2.3) gives

$$-2v_1 + 4v_2 = 0 \quad \Rightarrow \quad v_1 = 2v_2 \quad (3.2.5)$$

Substituting Eq. (3.2.5) into Eq. (3.2.4) yields

$$2v_2 - v_2 = 2.4 \Rightarrow v_2 = 2.4, \quad v_1 = 2v_2 = 4.8 \text{ V}$$

From Eq. (3.2.3), we get

$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4 \text{ V}$$

Thus,

$$v_1 = 4.8 \text{ V}, \quad v_2 = 2.4 \text{ V}, \quad v_3 = -2.4 \text{ V}$$

METHOD 2 To use Cramer's rule, we put Eqs. (3.2.1) to (3.2.3) in matrix form.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \quad (3.2.6)$$

From this, we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}$$

where Δ , Δ_1 , Δ_2 , and Δ_3 are the determinants to be calculated as follows. As explained in Appendix A, to calculate the determinant of a 3 by 3 matrix, we repeat the first two rows and cross multiply.

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{|ccc|} \hline & 3 & -2 & -1 \\ & -4 & 7 & -1 \\ & 2 & -3 & 1 \\ \hline & 3 & -2 & -1 \\ & -4 & 7 & -1 \\ & 2 & -3 & 1 \\ \hline & 3 & -2 & -1 \\ & -4 & 7 & -1 \\ & 2 & -3 & 1 \\ \hline \end{array} = 21 - 12 + 4 + 14 - 9 - 8 = 10$$

Similarly, we obtain

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$

as we obtained with Method 1.

METHOD 3 We now use *MATLAB* to solve the matrix. Equation (3.2.6) can be written as

$$\mathbf{AV} = \mathbf{B} \Rightarrow \mathbf{V} = \mathbf{A}^{-1}\mathbf{B}$$

where \mathbf{A} is the 3 by 3 square matrix, \mathbf{B} is the column vector, and \mathbf{V} is a column vector comprised of v_1 , v_2 , and v_3 that we want to determine. We use *MATLAB* to determine \mathbf{V} as follows:

```
>>A = [3 -2 -1; -4 7 -1; 2 -3 1];
>>B = [12 0 0]';
>>V = inv(A) * B
4.8000
V =
2.4000
-2.4000
```

Thus, $v_1 = 4.8 \text{ V}$, $v_2 = 2.4 \text{ V}$, and $v_3 = -2.4 \text{ V}$, as obtained previously.

Practice Problem 3.2

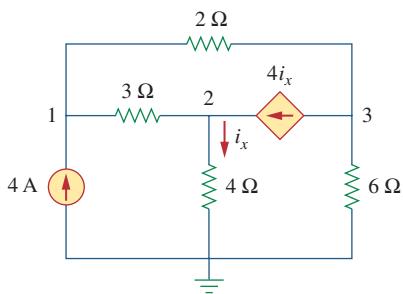


Figure 3.6
For Practice Prob. 3.2.

Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

Answer: $v_1 = 32 \text{ V}$, $v_2 = -25.6 \text{ V}$, $v_3 = 62.4 \text{ V}$.

3.3 Nodal Analysis with Voltage Sources

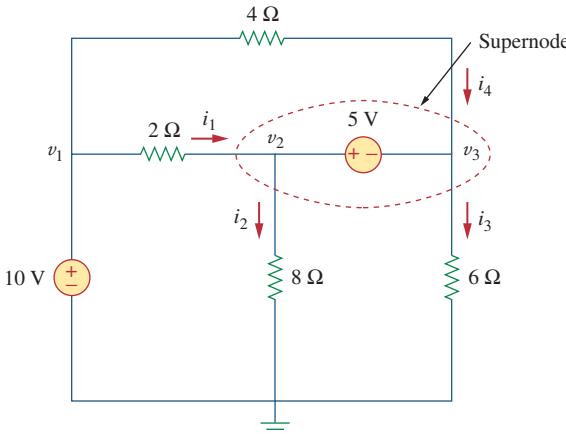
We now consider how voltage sources affect nodal analysis. We use the circuit in Fig. 3.7 for illustration. Consider the following two possibilities.

CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 3.7, for example,

$$v_1 = 10 \text{ V} \quad (3.10)$$

Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes

**Figure 3.7**

A circuit with a supernode.

form a *generalized node* or *supernode*; we apply both KCL and KVL to determine the node voltages.

A supernode may be regarded as a closed surface enclosing the voltage source and its two nodes.

A **supernode** is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

In Fig. 3.7, nodes 2 and 3 form a supernode. (We could have more than two nodes forming a single supernode. For example, see the circuit in Fig. 3.14.) We analyze a circuit with supernodes using the same three steps mentioned in the previous section except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, KCL must be satisfied at a supernode like any other node. Hence, at the supernode in Fig. 3.7,

$$i_1 + i_4 = i_2 + i_3 \quad (3.11a)$$

or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \quad (3.11b)$$

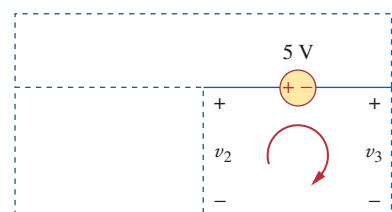
To apply Kirchhoff's voltage law to the supernode in Fig. 3.7, we redraw the circuit as shown in Fig. 3.8. Going around the loop in the clockwise direction gives

$$-v_2 + 5 + v_3 = 0 \Rightarrow v_2 - v_3 = 5 \quad (3.12)$$

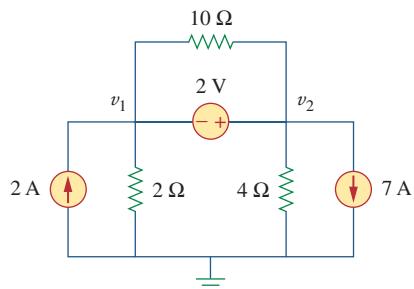
From Eqs. (3.10), (3.11b), and (3.12), we obtain the node voltages.

Note the following properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.

**Figure 3.8**

Applying KVL to a supernode.

Example 3.3**Figure 3.9**

For Example 3.3.

Solution:

The supernode contains the 2-V source, nodes 1 and 2, and the 10Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \Rightarrow 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \quad (3.3.1)$$

To get the relationship between v_1 and v_2 , we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \Rightarrow v_2 = v_1 + 2 \quad (3.3.2)$$

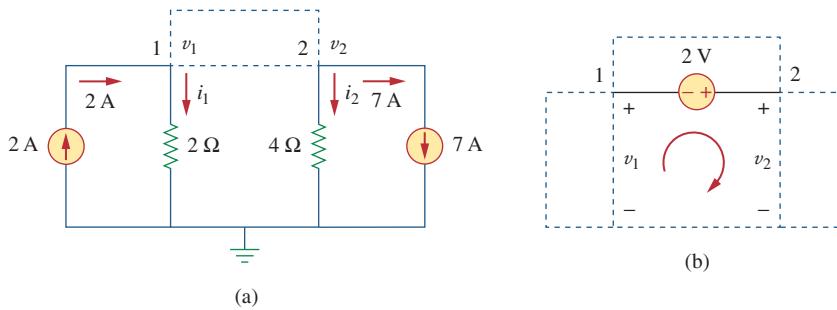
From Eqs. (3.3.1) and (3.3.2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

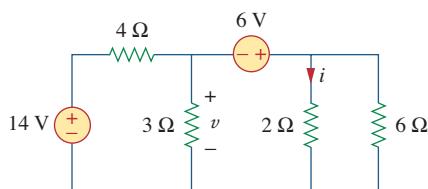
or

$$3v_1 = -22 \Rightarrow v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333 \text{ V}$. Note that the 10Ω resistor does not make any difference because it is connected across the supernode.

**Figure 3.10**

Applying: (a) KCL to the supernode, (b) KVL to the loop.

Practice Problem 3.3Find v and i in the circuit of Fig. 3.11.**Answer:** -400 mV , 2.8 A .**Figure 3.11**

For Practice Prob. 3.3.

Find the node voltages in the circuit of Fig. 3.12.

Example 3.4

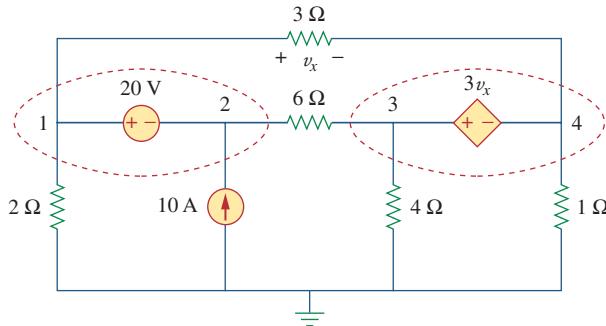


Figure 3.12

For Example 3.4.

Solution:

Nodes 1 and 2 form a supernode; so do nodes 3 and 4. We apply KCL to the two supernodes as in Fig. 3.13(a). At supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$

Expressing this in terms of the node voltages,

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

or

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \quad (3.4.1)$$

At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

or

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \quad (3.4.2)$$

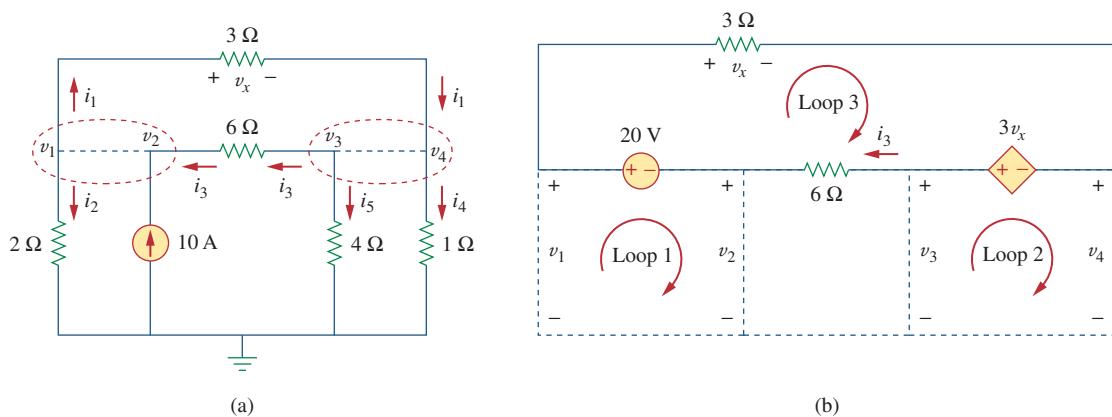


Figure 3.13

Applying: (a) KCL to the two supernodes, (b) KVL to the loops.

We now apply KVL to the branches involving the voltage sources as shown in Fig. 3.13(b). For loop 1,

$$-v_1 + 20 + v_2 = 0 \quad \Rightarrow \quad v_1 - v_2 = 20 \quad (3.4.3)$$

For loop 2,

$$-v_3 + 3v_x + v_4 = 0$$

But $v_x = v_1 - v_4$ so that

$$3v_1 - v_3 - 2v_4 = 0 \quad (3.4.4)$$

For loop 3,

$$v_x - 3v_x + 6i_3 - 20 = 0$$

But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence,

$$-2v_1 - v_2 + v_3 + 2v_4 = 20 \quad (3.4.5)$$

We need four node voltages, v_1 , v_2 , v_3 , and v_4 , and it requires only four out of the five Eqs. (3.4.1) to (3.4.5) to find them. Although the fifth equation is redundant, it can be used to check results. We can solve Eqs. (3.4.1) to (3.4.4) directly using *MATLAB*. We can eliminate one node voltage so that we solve three simultaneous equations instead of four. From Eq. (3.4.3), $v_2 = v_1 - 20$. Substituting this into Eqs. (3.4.1) and (3.4.2), respectively, gives

$$6v_1 - v_3 - 2v_4 = 80 \quad (3.4.6)$$

and

$$6v_1 - 5v_3 - 16v_4 = 40 \quad (3.4.7)$$

Equations (3.4.4), (3.4.6), and (3.4.7) can be cast in matrix form as

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Using Cramer's rule gives

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \quad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480,$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \quad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67 \text{ V}, \quad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33 \text{ V},$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67 \text{ V}$$

and $v_2 = v_1 - 20 = 6.667 \text{ V}$. We have not used Eq. (3.4.5); it can be used to cross check results.

Find v_1 , v_2 , and v_3 in the circuit of Fig. 3.14 using nodal analysis.

Practice Problem 3.4

Answer: $v_1 = 7.608 \text{ V}$, $v_2 = -17.39 \text{ V}$, $v_3 = 1.6305 \text{ V}$.

3.4 Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it.

Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is *nonplanar*. A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches. For example, the circuit in Fig. 3.15(a) has two crossing branches, but it can be redrawn as in Fig. 3.15(b). Hence, the circuit in Fig. 3.15(a) is planar. However, the circuit in Fig. 3.16 is nonplanar, because there is no way to redraw it and avoid the branches crossing. Nonplanar circuits can be handled using nodal analysis, but they will not be considered in this text.

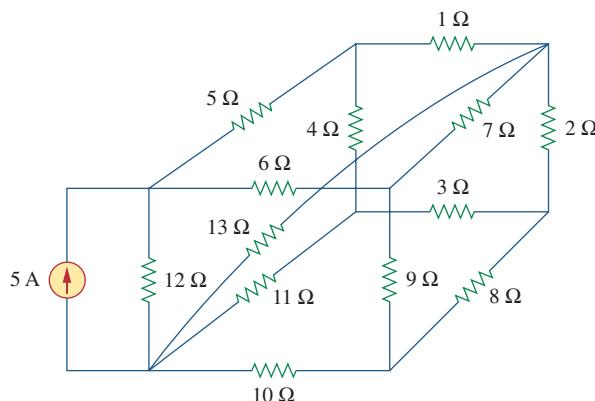


Figure 3.16
A nonplanar circuit.

To understand mesh analysis, we should first explain more about what we mean by a mesh.

A **mesh** is a loop which does not contain any other loops within it.

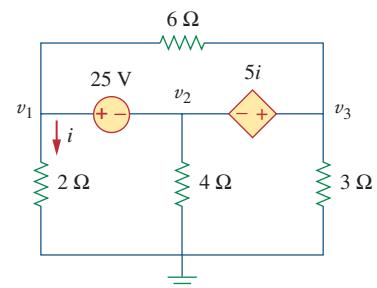
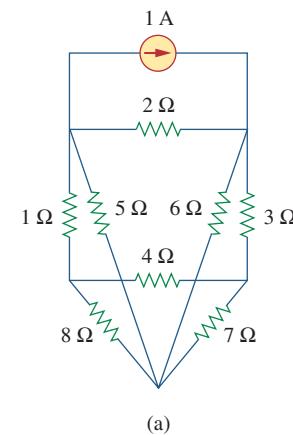
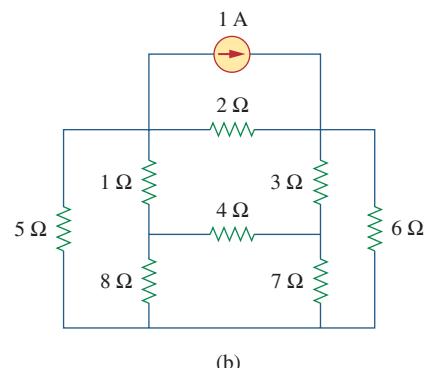


Figure 3.14
For Practice Prob. 3.4.

Mesh analysis is also known as *loop analysis* or the *mesh-current method*.

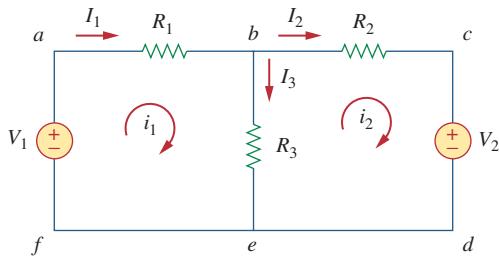


(a)



(b)

Figure 3.15
(a) A planar circuit with crossing branches,
(b) the same circuit redrawn with no crossing branches.

**Figure 3.17**

A circuit with two meshes.

Although path $abcdfa$ is a loop and not a mesh, KVL still holds. This is the reason for loosely using the terms *loop analysis* and *mesh analysis* to mean the same thing.

In Fig. 3.17, for example, paths $abefa$ and $bcdeb$ are meshes, but path $abcdefa$ is not a mesh. The current through a mesh is known as *mesh current*. In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit.

In this section, we will apply mesh analysis to planar circuits that do not contain current sources. In the next section, we will consider circuits with current sources. In the mesh analysis of a circuit with n meshes, we take the following three steps.

Steps to Determine Mesh Currents:

1. Assign mesh currents i_1, i_2, \dots, i_n to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

The direction of the mesh current is arbitrary—(clockwise or counterclockwise)—and does not affect the validity of the solution.

To illustrate the steps, consider the circuit in Fig. 3.17. The first step requires that mesh currents i_1 and i_2 are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

or

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1 \quad (3.13)$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \quad (3.14)$$

The shortcut way will not apply if one mesh current is assumed clockwise and the other assumed counterclockwise, although this is permissible.

Note in Eq. (3.13) that the coefficient of i_1 is the sum of the resistances in the first mesh, while the coefficient of i_2 is the negative of the resistance common to meshes 1 and 2. Now observe that the same is true in Eq. (3.14). This can serve as a shortcut way of writing the mesh equations. We will exploit this idea in Section 3.6.

The third step is to solve for the mesh currents. Putting Eqs. (3.13) and (3.14) in matrix form yields

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \quad (3.15)$$

which can be solved to obtain the mesh currents i_1 and i_2 . We are at liberty to use any technique for solving the simultaneous equations. According to Eq. (2.12), if a circuit has n nodes, b branches, and l independent loops or meshes, then $l = b - n + 1$. Hence, l independent simultaneous equations are required to solve the circuit using mesh analysis.

Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use i for a mesh current and I for a branch current. The current elements I_1 , I_2 , and I_3 are algebraic sums of the mesh currents. It is evident from Fig. 3.17 that

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2 \quad (3.16)$$

For the circuit in Fig. 3.18, find the branch currents I_1 , I_2 , and I_3 using mesh analysis.

Example 3.5

Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \quad (3.5.1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \quad (3.5.2)$$

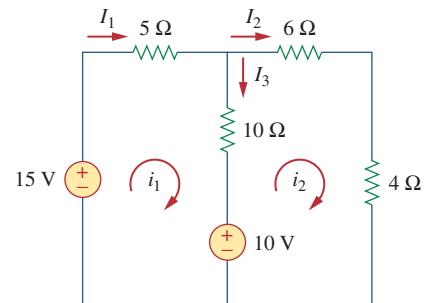


Figure 3.18
For Example 3.5.

METHOD 1 Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \Rightarrow i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1 \text{ A}$. Thus,

$$I_1 = i_1 = 1 \text{ A}, \quad I_2 = i_2 = 1 \text{ A}, \quad I_3 = i_1 - i_2 = 0$$

METHOD 2 To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \quad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \quad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.

Practice Problem 3.5

Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 3.19.

Answer: $i_1 = 2.5 \text{ A}$, $i_2 = 0 \text{ A}$.

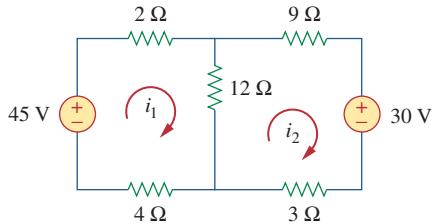


Figure 3.19

For Practice Prob. 3.5.

Example 3.6

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \quad (3.6.1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

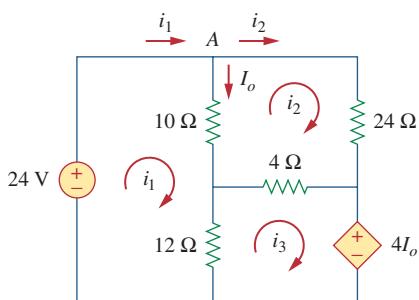
$$-5i_1 + 19i_2 - 2i_3 = 0 \quad (3.6.2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Figure 3.20

For Example 3.6.



But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \quad (3.6.3)$$

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\begin{aligned} \Delta &= \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{vmatrix} \\ &= 418 - 30 - 10 - 114 - 22 - 50 = 192 \\ \Delta_1 &= \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 12 & -5 & -6 \end{vmatrix} = 456 - 24 = 432 \\ \Delta_2 &= \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 24 + 120 = 144 \\ \Delta_3 &= \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 60 + 228 = 288 \end{aligned}$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \quad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5 \text{ A}$.

Practice Problem 3.6

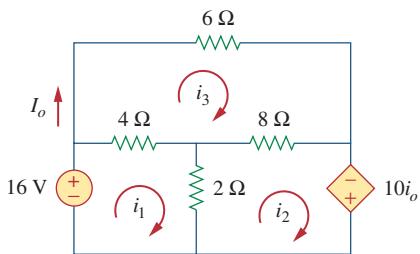


Figure 3.21

For Practice Prob. 3.6.

Using mesh analysis, find I_o in the circuit of Fig. 3.21.

Answer: -4 A .

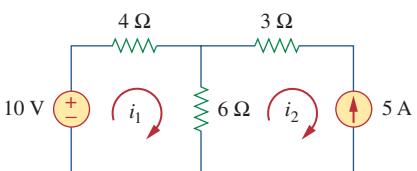


Figure 3.22

A circuit with a current source.

3.5 Mesh Analysis with Current Sources

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

CASE 1 When a current source exists only in one mesh: Consider the circuit in Fig. 3.22, for example. We set $i_2 = -5 \text{ A}$ and write a mesh equation for the other mesh in the usual way; that is,

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2 \text{ A} \quad (3.17)$$

CASE 2 When a current source exists between two meshes: Consider the circuit in Fig. 3.23(a), for example. We create a *supermesh* by excluding the current source and any elements connected in series with it, as shown in Fig. 3.23(b). Thus,

A **supermesh** results when two meshes have a (dependent or independent) current source in common.

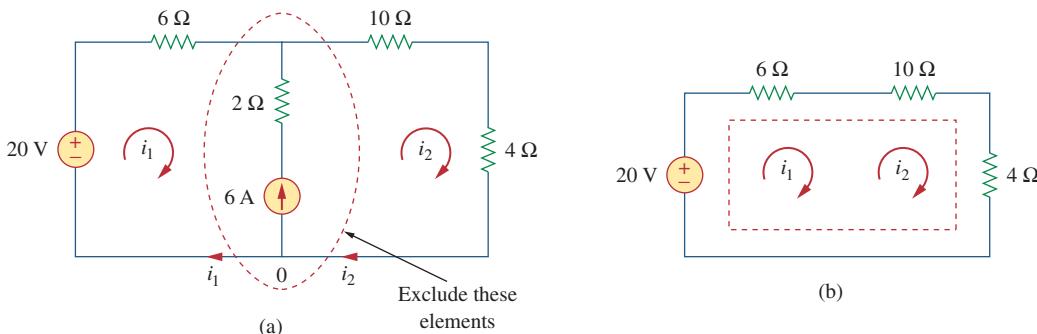


Figure 3.23

(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

As shown in Fig. 3.23(b), we create a supermesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.) Why treat the supermesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh in Fig. 3.23(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

or

$$6i_1 + 14i_2 = 20 \quad (3.18)$$

We apply KCL to a node in the branch where the two meshes intersect. Applying KCL to node 0 in Fig. 3.23(a) gives

$$i_2 = i_1 + 6 \quad (3.19)$$

Solving Eqs. (3.18) and (3.19), we get

$$i_1 = -3.2 \text{ A}, \quad i_2 = 2.8 \text{ A} \quad (3.20)$$

Note the following properties of a supermesh:

1. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.

Example 3.7

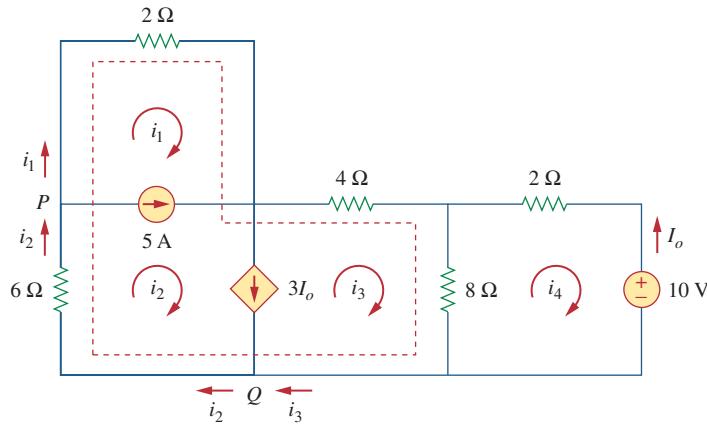


Figure 3.24

For Example 3.7.

Solution:

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad (3.7.1)$$

For the independent current source, we apply KCL to node P :

$$i_2 = i_1 + 5 \quad (3.7.2)$$

For the dependent current source, we apply KCL to node Q :

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \quad (3.7.3)$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \quad (3.7.4)$$

From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5 \text{ A}, \quad i_2 = -2.5 \text{ A}, \quad i_3 = 3.93 \text{ A}, \quad i_4 = 2.143 \text{ A}$$

Practice Problem 3.7

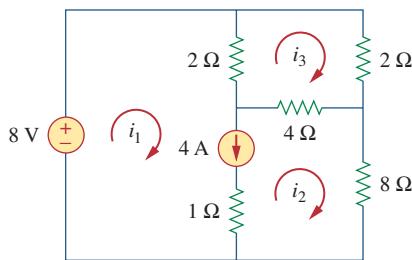


Figure 3.25

For Practice Prob. 3.7.

Use mesh analysis to determine i_1 , i_2 , and i_3 in Fig. 3.25.

Answer: $i_1 = 4.632 \text{ A}$, $i_2 = 631.6 \text{ mA}$, $i_3 = 1.4736 \text{ A}$.

3.6

† Nodal and Mesh Analyses by Inspection

This section presents a generalized procedure for nodal or mesh analysis. It is a shortcut approach based on mere inspection of a circuit.

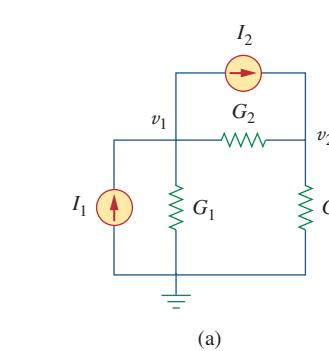
When all sources in a circuit are independent current sources, we do not need to apply KCL to each node to obtain the node-voltage equations as we did in Section 3.2. We can obtain the equations by mere inspection of the circuit. As an example, let us reexamine the circuit in Fig. 3.2, shown again in Fig. 3.26(a) for convenience. The circuit has two nonreference nodes and the node equations were derived in Section 3.2 as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \quad (3.21)$$

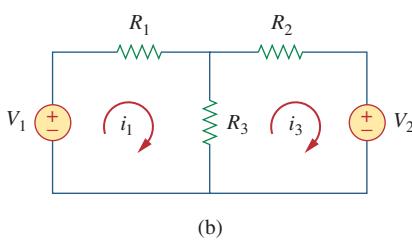
Observe that each of the diagonal terms is the sum of the conductances connected directly to node 1 or 2, while the off-diagonal terms are the negatives of the conductances connected between the nodes. Also, each term on the right-hand side of Eq. (3.21) is the algebraic sum of the currents entering the node.

In general, if a circuit with independent current sources has N nonreference nodes, the node-voltage equations can be written in terms of the conductances as

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \quad (3.22)$$



(a)



(b)

Figure 3.26

(a) The circuit in Fig. 3.2, (b) the circuit in Fig. 3.17.

or simply

$$\mathbf{Gv} = \mathbf{i} \quad (3.23)$$

where

- G_{kk} = Sum of the conductances connected to node k
- $G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j , $k \neq j$
- v_k = Unknown voltage at node k
- i_k = Sum of all independent current sources directly connected to node k , with currents entering the node treated as positive

\mathbf{G} is called the *conductance matrix*; \mathbf{v} is the output vector; and \mathbf{i} is the input vector. Equation (3.22) can be solved to obtain the unknown node voltages. Keep in mind that this is valid for circuits with only independent current sources and linear resistors.

Similarly, we can obtain mesh-current equations by inspection when a linear resistive circuit has only independent voltage sources. Consider the circuit in Fig. 3.17, shown again in Fig. 3.26(b) for convenience. The circuit has two nonreference nodes and the node equations were derived in Section 3.4 as

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \quad (3.24)$$

We notice that each of the diagonal terms is the sum of the resistances in the related mesh, while each of the off-diagonal terms is the negative of the resistance common to meshes 1 and 2. Each term on the right-hand side of Eq. (3.24) is the algebraic sum taken clockwise of all independent voltage sources in the related mesh.

In general, if the circuit has N meshes, the mesh-current equations can be expressed in terms of the resistances as

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \quad (3.25)$$

or simply

$$\mathbf{Ri} = \mathbf{v} \quad (3.26)$$

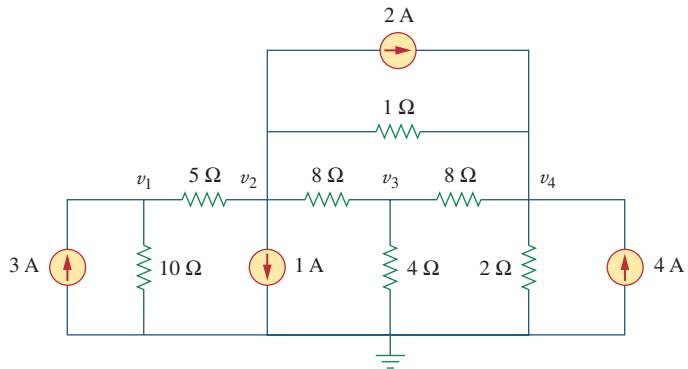
where

- R_{kk} = Sum of the resistances in mesh k
- $R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j , $k \neq j$
- i_k = Unknown mesh current for mesh k in the clockwise direction
- v_k = Sum taken clockwise of all independent voltage sources in mesh k , with voltage rise treated as positive

\mathbf{R} is called the *resistance matrix*; \mathbf{i} is the output vector; and \mathbf{v} is the input vector. We can solve Eq. (3.25) to obtain the unknown mesh currents.

Example 3.8

Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

**Figure 3.27**

For Example 3.8.

Solution:

The circuit in Fig. 3.27 has four nonreference nodes, so we need four node equations. This implies that the size of the conductance matrix \mathbf{G} , in siemens, is 4 by 4. The diagonal terms of \mathbf{G} , in siemens, are

$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \quad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \quad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \quad G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \quad G_{32} = -0.125, \quad G_{34} = -\frac{1}{8} = -0.125$$

$$G_{41} = 0, \quad G_{42} = -1, \quad G_{43} = -0.125$$

The input current vector \mathbf{i} has the following terms, in amperes:

$$i_1 = 3, \quad i_2 = -1 - 2 = -3, \quad i_3 = 0, \quad i_4 = 2 + 4 = 6$$

Thus the node-voltage equations are

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

which can be solved using *MATLAB* to obtain the node voltages v_1 , v_2 , v_3 , and v_4 .

By inspection, obtain the node-voltage equations for the circuit in Fig. 3.28.

Practice Problem 3.8

Answer:

$$\begin{bmatrix} 1.25 & -0.2 & -1 & 0 \\ -0.2 & 0.2 & 0 & 0 \\ -1 & 0 & 1.25 & -0.25 \\ 0 & 0 & -0.25 & 1.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -3 \\ 2 \end{bmatrix}$$

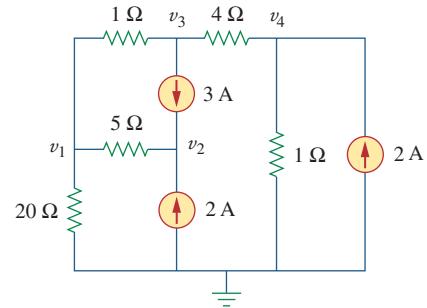


Figure 3.28
For Practice Prob. 3.8.

By inspection, write the mesh-current equations for the circuit in Fig. 3.29.

Example 3.9

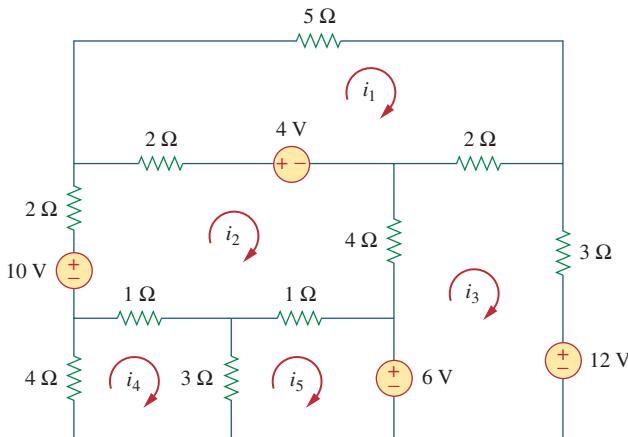


Figure 3.29

For Example 3.9.

Solution:

We have five meshes, so the resistance matrix is 5 by 5. The diagonal terms, in ohms, are:

$$R_{11} = 5 + 2 + 2 = 9, \quad R_{22} = 2 + 4 + 1 + 1 + 2 = 10, \\ R_{33} = 2 + 3 + 4 = 9, \quad R_{44} = 1 + 3 + 4 = 8, \quad R_{55} = 1 + 3 = 4$$

The off-diagonal terms are:

$$R_{12} = -2, \quad R_{13} = -2, \quad R_{14} = 0 = R_{15}, \\ R_{21} = -2, \quad R_{23} = -4, \quad R_{24} = -1, \quad R_{25} = -1, \\ R_{31} = -2, \quad R_{32} = -4, \quad R_{34} = 0 = R_{35}, \\ R_{41} = 0, \quad R_{42} = -1, \quad R_{43} = 0, \quad R_{45} = -3, \\ R_{51} = 0, \quad R_{52} = -1, \quad R_{53} = 0, \quad R_{54} = -3$$

The input voltage vector \mathbf{v} has the following terms in volts:

$$v_1 = 4, \quad v_2 = 10 - 4 = 6, \\ v_3 = -12 + 6 = -6, \quad v_4 = 0, \quad v_5 = -6$$

Thus, the mesh-current equations are:

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

From this, we can use *MATLAB* to obtain mesh currents i_1, i_2, i_3, i_4 , and i_5 .

Practice Problem 3.9

By inspection, obtain the mesh-current equations for the circuit in Fig. 3.30.

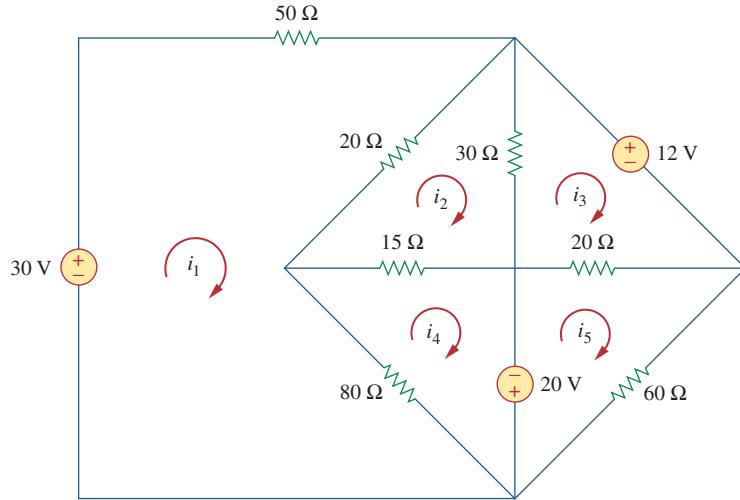


Figure 3.30

For Practice Prob. 3.9.

Answer:

$$\begin{bmatrix} 150 & -40 & 0 & -80 & 0 \\ -40 & 65 & -30 & -15 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -15 & 0 & 95 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix}$$

3.7

Nodal Versus Mesh Analysis

Both nodal and mesh analyses provide a systematic way of analyzing a complex network. Someone may ask: Given a network to be analyzed, how do we know which method is better or more efficient? The choice of the better method is dictated by two factors.

The first factor is the nature of the particular network. Networks that contain many series-connected elements, voltage sources, or supermeshes are more suitable for mesh analysis, whereas networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis. Also, a circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis. The key is to select the method that results in the smaller number of equations.

The second factor is the information required. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

It is helpful to be familiar with both methods of analysis, for at least two reasons. First, one method can be used to check the results from the other method, if possible. Second, since each method has its limitations, only one method may be suitable for a particular problem. For example, mesh analysis is the only method to use in analyzing transistor circuits, as we shall see in Section 3.9. But mesh analysis cannot easily be used to solve an op amp circuit, as we shall see in Chapter 5, because there is no direct way to obtain the voltage across the op amp itself. For nonplanar networks, nodal analysis is the only option, because mesh analysis only applies to planar networks. Also, nodal analysis is more amenable to solution by computer, as it is easy to program. This allows one to analyze complicated circuits that defy hand calculation. A computer software package based on nodal analysis is introduced next.

3.8 Circuit Analysis with *PSpice*

PSpice is a computer software circuit analysis program that we will gradually learn to use throughout the course of this text. This section illustrates how to use *PSpice for Windows* to analyze the dc circuits we have studied so far.

The reader is expected to review Sections D.1 through D.3 of Appendix D before proceeding in this section. It should be noted that *PSpice* is only helpful in determining branch voltages and currents when the numerical values of all the circuit components are known.

Appendix D provides a tutorial on using *PSpice for Windows*.

Use *PSpice* to find the node voltages in the circuit of Fig. 3.31.

Example 3.10

Solution:

The first step is to draw the given circuit using Schematics. If one follows the instructions given in Appendix sections D.2 and D.3, the schematic in Fig. 3.32 is produced. Since this is a dc analysis, we use voltage source VDC and current source IDC. The pseudocomponent VIEWPOINTS are added to display the required node voltages. Once the circuit is drawn and saved as *exam310.sch*, we run *PSpice* by selecting **Analysis/Simulate**. The circuit is simulated and the results

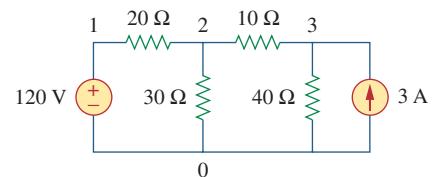
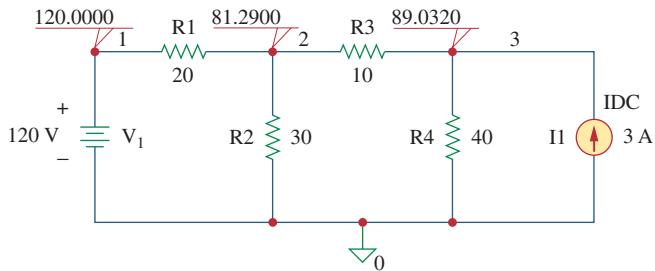


Figure 3.31
For Example 3.10.

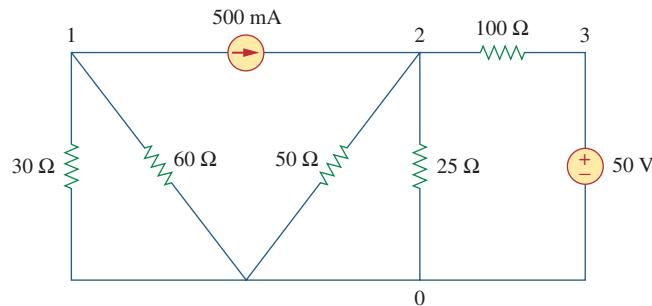
**Figure 3.32**

For Example 3.10; the schematic of the circuit in Fig. 3.31.

are displayed on VIEWPOINTS and also saved in output file *exam310.out*. The output file includes the following:

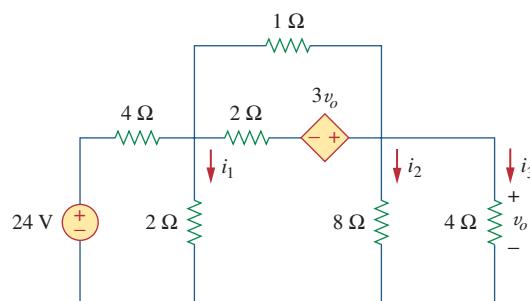
NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
(1)	120.0000	(2)	81.2900	(3)	89.0320

indicating that $V_1 = 120 \text{ V}$, $V_2 = 81.29 \text{ V}$, $V_3 = 89.032 \text{ V}$.

Practice Problem 3.10For the circuit in Fig. 3.33, use *PSpice* to find the node voltages.**Figure 3.33**

For Practice Prob. 3.10.

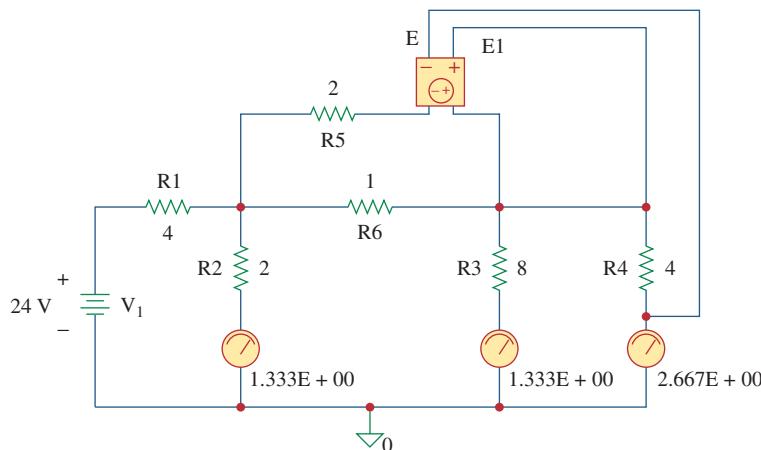
Answer: $V_1 = -10 \text{ V}$, $V_2 = 14.286 \text{ V}$, $V_3 = 50 \text{ V}$.

Example 3.11In the circuit of Fig. 3.34, determine the currents i_1 , i_2 , and i_3 .**Figure 3.34**

For Example 3.11.

Solution:

The schematic is shown in Fig. 3.35. (The schematic in Fig. 3.35 includes the output results, implying that it is the schematic displayed on the screen *after* the simulation.) Notice that the voltage-controlled voltage source E1 in Fig. 3.35 is connected so that its input is the voltage across the 4Ω resistor; its gain is set equal to 3. In order to display the required currents, we insert pseudocomponent IPROBES in the appropriate branches. The schematic is saved as *exam311.sch* and simulated by selecting **Analysis/Simulate**. The results are displayed on IPROBES as shown in Fig. 3.35 and saved in output file *exam311.out*. From the output file or the IPROBES, we obtain $i_1 = i_2 = 1.333 \text{ A}$ and $i_3 = 2.667 \text{ A}$.

**Figure 3.35**

The schematic of the circuit in Fig. 3.34.

Use *PSpice* to determine currents i_1 , i_2 , and i_3 in the circuit of Fig. 3.36.

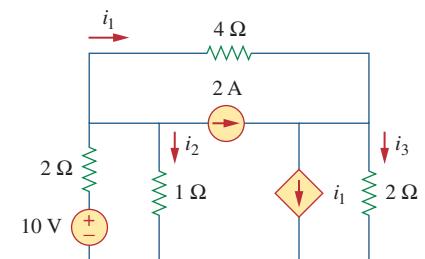
Practice Problem 3.11

Answer: $i_1 = -428.6 \text{ mA}$, $i_2 = 2.286 \text{ A}$, $i_3 = 2 \text{ A}$.

3.9 [†]**Applications: DC Transistor Circuits**

Most of us deal with electronic products on a routine basis and have some experience with personal computers. A basic component for the integrated circuits found in these electronics and computers is the active, three-terminal device known as the *transistor*. Understanding the transistor is essential before an engineer can start an electronic circuit design.

Figure 3.37 depicts various kinds of transistors commercially available. There are two basic types of transistors: *bipolar junction transistors* (BJTs) and *field-effect transistors* (FETs). Here, we consider only the BJTs, which were the first of the two and are still used today. Our objective is to present enough detail about the BJT to enable us to apply the techniques developed in this chapter to analyze dc transistor circuits.

**Figure 3.36**

For Practice Prob. 3.11.

Historical



Courtesy of Lucent Technologies/Bell Labs

William Shockley (1910–1989), **John Bardeen** (1908–1991), and **Walter Brattain** (1902–1987) co-invented the transistor.

Nothing has had a greater impact on the transition from the “Industrial Age” to the “Age of the Engineer” than the transistor. I am sure that Dr. Shockley, Dr. Bardeen, and Dr. Brattain had no idea they would have this incredible effect on our history. While working at Bell Laboratories, they successfully demonstrated the point-contact transistor, invented by Bardeen and Brattain in 1947, and the junction transistor, which Shockley conceived in 1948 and successfully produced in 1951.

It is interesting to note that the idea of the field-effect transistor, the most commonly used one today, was first conceived in 1925–1928 by J. E. Lilienfeld, a German immigrant to the United States. This is evident from his patents of what appears to be a field-effect transistor. Unfortunately, the technology to realize this device had to wait until 1954 when Shockley’s field-effect transistor became a reality. Just think what today would be like if we had this transistor 30 years earlier!

For their contributions to the creation of the transistor, Dr. Shockley, Dr. Bardeen, and Dr. Brattain received, in 1956, the Nobel Prize in physics. It should be noted that Dr. Bardeen is the only individual to win two Nobel prizes in physics; the second came later for work in superconductivity at the University of Illinois.

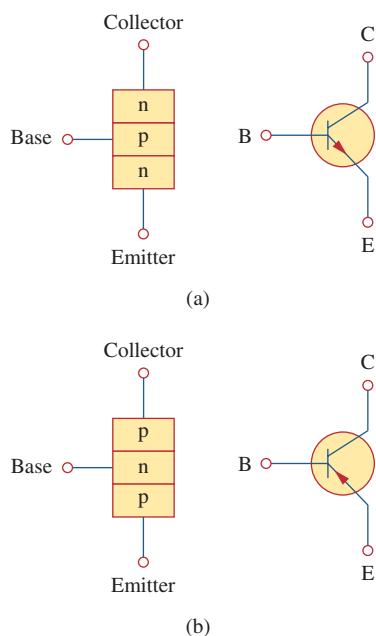


Figure 3.38

Two types of BJTs and their circuit symbols: (a) *npn*, (b) *pnp*.

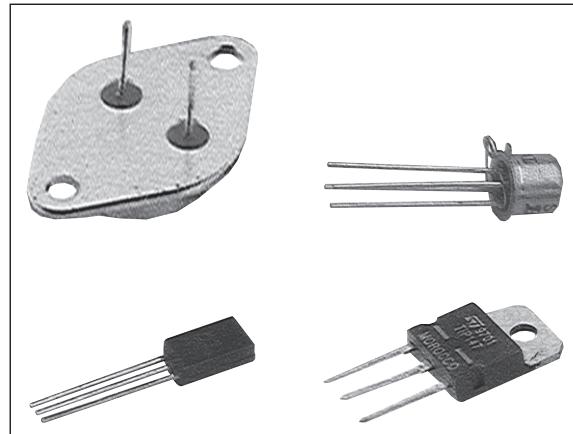


Figure 3.37
Various types of transistors.
(Courtesy of Tech America.)

There are two types of BJTs: *npn* and *pnp*, with their circuit symbols as shown in Fig. 3.38. Each type has three terminals, designated as emitter (E), base (B), and collector (C). For the *npn* transistor, the currents and voltages of the transistor are specified as in Fig. 3.39. Applying KCL to Fig. 3.39(a) gives

$$I_E = I_B + I_C \quad (3.27)$$

where I_E , I_C , and I_B are emitter, collector, and base currents, respectively. Similarly, applying KVL to Fig. 3.39(b) gives

$$V_{CE} + V_{EB} + V_{BC} = 0 \quad (3.28)$$

where V_{CE} , V_{EB} , and V_{BC} are collector-emitter, emitter-base, and base-collector voltages. The BJT can operate in one of three modes: active, cutoff, and saturation. When transistors operate in the active mode, typically $V_{BE} \approx 0.7$ V,

$$I_C = \alpha I_E \quad (3.29)$$

where α is called the *common-base current gain*. In Eq. (3.29), α denotes the fraction of electrons injected by the emitter that are collected by the collector. Also,

$$I_C = \beta I_B \quad (3.30)$$

where β is known as the *common-emitter current gain*. The α and β are characteristic properties of a given transistor and assume constant values for that transistor. Typically, α takes values in the range of 0.98 to 0.999, while β takes values in the range of 50 to 1000. From Eqs. (3.27) to (3.30), it is evident that

$$I_E = (1 + \beta)I_B \quad (3.31)$$

and

$$\beta = \frac{\alpha}{1 - \alpha} \quad (3.32)$$

These equations show that, in the active mode, the BJT can be modeled as a dependent current-controlled current source. Thus, in circuit analysis, the dc equivalent model in Fig. 3.40(b) may be used to replace the *npn* transistor in Fig. 3.40(a). Since β in Eq. (3.32) is large, a small base current controls large currents in the output circuit. Consequently, the bipolar transistor can serve as an amplifier, producing both current gain and voltage gain. Such amplifiers can be used to furnish a considerable amount of power to transducers such as loudspeakers or control motors.

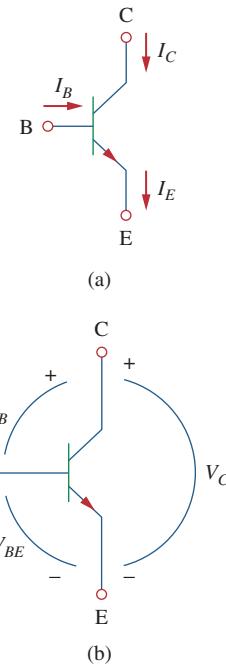


Figure 3.39

The terminal variables of an *npn* transistor:
(a) currents, (b) voltages.

In fact, transistor circuits provide motivation to study dependent sources.

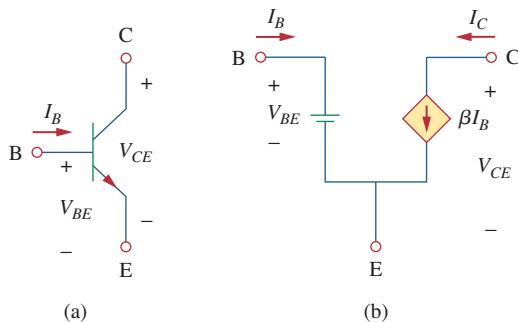


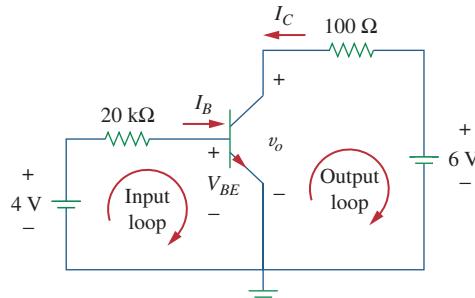
Figure 3.40

(a) An *npn* transistor, (b) its dc equivalent model.

It should be observed in the following examples that one cannot directly analyze transistor circuits using nodal analysis because of the potential difference between the terminals of the transistor. Only when the transistor is replaced by its equivalent model can we apply nodal analysis.

Example 3.12

Find I_B , I_C , and v_o in the transistor circuit of Fig. 3.41. Assume that the transistor operates in the active mode and that $\beta = 50$.

**Figure 3.41**

For Example 3.12.

Solution:

For the input loop, KVL gives

$$-4 + I_B(20 \times 10^3) + V_{BE} = 0$$

Since $V_{BE} = 0.7$ V in the active mode,

$$I_B = \frac{4 - 0.7}{20 \times 10^3} = 165 \mu\text{A}$$

But

$$I_C = \beta I_B = 50 \times 165 \mu\text{A} = 8.25 \text{ mA}$$

For the output loop, KVL gives

$$-v_o - 100I_C + 6 = 0$$

or

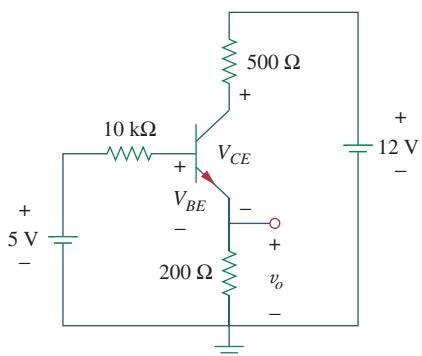
$$v_o = 6 - 100I_C = 6 - 0.825 = 5.175 \text{ V}$$

Note that $v_o = V_{CE}$ in this case.

Practice Problem 3.12

For the transistor circuit in Fig. 3.42, let $\beta = 100$ and $V_{BE} = 0.7$ V. Determine v_o and V_{CE} .

Answer: 2.876 V, 1.984 V.

**Figure 3.42**

For Practice Prob. 3.12.

For the BJT circuit in Fig. 3.43, $\beta = 150$ and $V_{BE} = 0.7$ V. Find v_o .

Example 3.13

Solution:

- Define.** The circuit is clearly defined and the problem is clearly stated. There appear to be no additional questions that need to be asked.
- Present.** We are to determine the output voltage of the circuit shown in Fig. 3.43. The circuit contains an ideal transistor with $\beta = 150$ and $V_{BE} = 0.7$ V.
- Alternative.** We can use mesh analysis to solve for v_o . We can replace the transistor with its equivalent circuit and use nodal analysis. We can try both approaches and use them to check each other. As a third check, we can use the equivalent circuit and solve it using PSpice.
- Attempt.**

■ METHOD 1

Working with Fig. 3.44(a), we start with the first loop.

$$-2 + 100kI_1 + 200k(I_1 - I_2) = 0 \quad \text{or} \quad 3I_1 - 2I_2 = 2 \times 10^{-5} \quad (3.13.1)$$

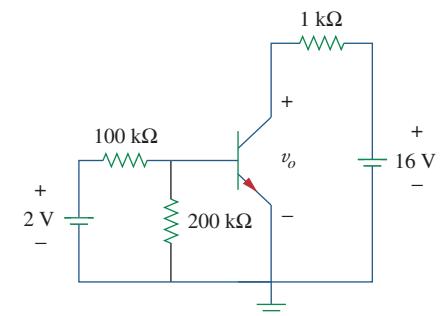
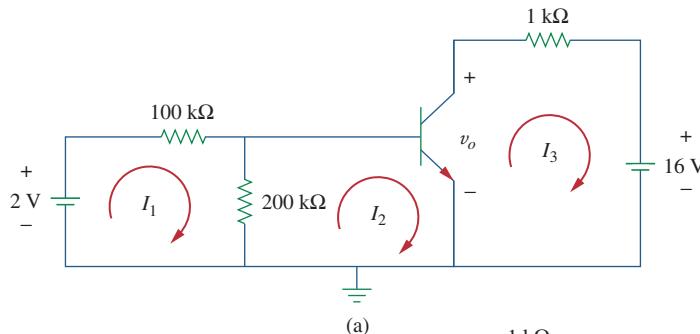
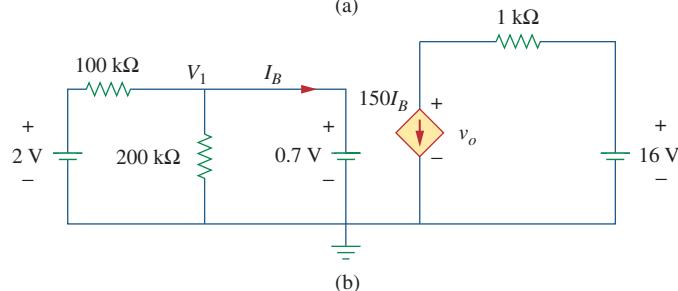


Figure 3.43

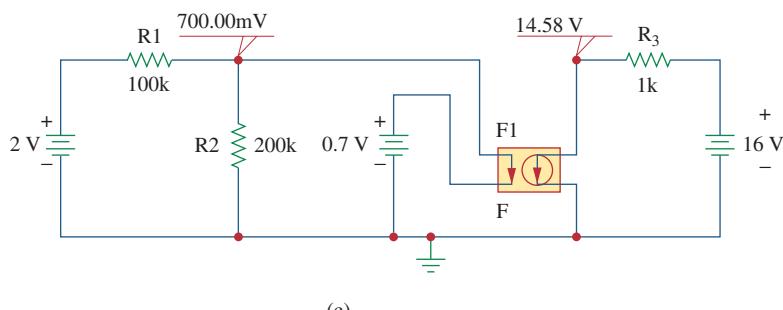
For Example 3.13.



(a)



(b)



(c)

Figure 3.44

Solution of the problem in Example 3.13: (a) Method 1, (b) Method 2, (c) Method 3.

Now for loop 2.

$$200k(I_2 - I_1) + V_{BE} = 0 \quad \text{or} \quad -2I_1 + 2I_2 = -0.7 \times 10^{-5} \quad (3.13.2)$$

Since we have two equations and two unknowns, we can solve for I_1 and I_2 . Adding Eq. (3.13.1) to (3.13.2) we get;

$$I_1 = 1.3 \times 10^{-5} \text{ A} \quad \text{and} \quad I_2 = (-0.7 + 2.6)10^{-5}/2 = 9.5 \mu\text{A}$$

Since $I_3 = -150I_2 = -1.425 \text{ mA}$, we can now solve for v_o using loop 3:

$$-v_o + 1\text{k}I_3 + 16 = 0 \quad \text{or} \quad v_o = -1.425 + 16 = \mathbf{14.575 \text{ V}}$$

■ METHOD 2 Replacing the transistor with its equivalent circuit produces the circuit shown in Fig. 3.44(b). We can now use nodal analysis to solve for v_o .

At node number 1: $V_1 = 0.7 \text{ V}$

$$(0.7 - 2)/100\text{k} + 0.7/200\text{k} + I_B = 0 \quad \text{or} \quad I_B = 9.5 \mu\text{A}$$

At node number 2 we have:

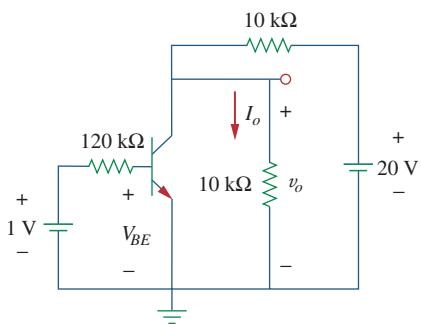
$$150I_B + (v_o - 16)/1\text{k} = 0 \quad \text{or} \quad v_o = 16 - 150 \times 10^3 \times 9.5 \times 10^{-6} = \mathbf{14.575 \text{ V}}$$

5. Evaluate. The answers check, but to further check we can use *PSpice* (Method 3), which gives us the solution shown in Fig. 3.44(c).

6. Satisfactory? Clearly, we have obtained the desired answer with a very high confidence level. We can now present our work as a solution to the problem.

Practice Problem 3.13

The transistor circuit in Fig. 3.45 has $\beta = 80$ and $V_{BE} = 0.7 \text{ V}$. Find v_o and I_o .



Answer: 12 V, $600 \mu\text{A}$.

3.10 Summary

1. Nodal analysis is the application of Kirchhoff's current law at the nonreference nodes. (It is applicable to both planar and nonplanar circuits.) We express the result in terms of the node voltages. Solving the simultaneous equations yields the node voltages.
2. A supernode consists of two nonreference nodes connected by a (dependent or independent) voltage source.
3. Mesh analysis is the application of Kirchhoff's voltage law around meshes in a planar circuit. We express the result in terms of mesh currents. Solving the simultaneous equations yields the mesh currents.

Figure 3.45

For Practice Prob. 3.13.

4. A supermesh consists of two meshes that have a (dependent or independent) current source in common.
5. Nodal analysis is normally used when a circuit has fewer node equations than mesh equations. Mesh analysis is normally used when a circuit has fewer mesh equations than node equations.
6. Circuit analysis can be carried out using *PSpice*.
7. DC transistor circuits can be analyzed using the techniques covered in this chapter.

Review Questions

- 3.1** At node 1 in the circuit of Fig. 3.46, applying KCL gives:

- (a) $2 + \frac{12 - v_1}{3} = \frac{v_1}{6} + \frac{v_1 - v_2}{4}$
- (b) $2 + \frac{v_1 - 12}{3} = \frac{v_1}{6} + \frac{v_2 - v_1}{4}$
- (c) $2 + \frac{12 - v_1}{3} = \frac{0 - v_1}{6} + \frac{v_1 - v_2}{4}$
- (d) $2 + \frac{v_1 - 12}{3} = \frac{0 - v_1}{6} + \frac{v_2 - v_1}{4}$

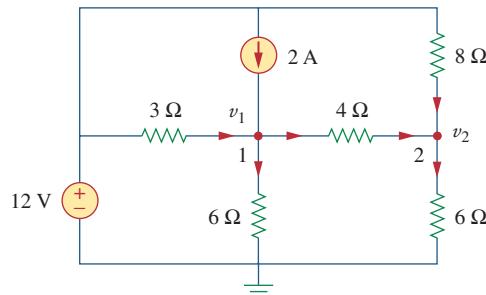


Figure 3.46

For Review Questions 3.1 and 3.2.

- 3.2** In the circuit of Fig. 3.46, applying KCL at node 2 gives:

- (a) $\frac{v_2 - v_1}{4} + \frac{v_2}{8} = \frac{v_2}{6}$
- (b) $\frac{v_1 - v_2}{4} + \frac{v_2}{8} = \frac{v_2}{6}$
- (c) $\frac{v_1 - v_2}{4} + \frac{12 - v_2}{8} = \frac{v_2}{6}$
- (d) $\frac{v_2 - v_1}{4} + \frac{v_2 - 12}{8} = \frac{v_2}{6}$

- 3.3** For the circuit in Fig. 3.47, v_1 and v_2 are related as:

- (a) $v_1 = 6i + 8 + v_2$ (b) $v_1 = 6i - 8 + v_2$
 (c) $v_1 = -6i + 8 + v_2$ (d) $v_1 = -6i - 8 + v_2$

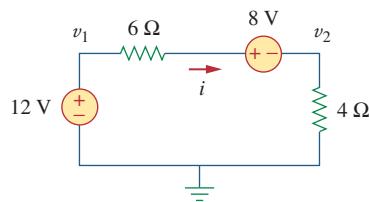


Figure 3.47

For Review Questions 3.3 and 3.4.

- 3.4** In the circuit of Fig. 3.47, the voltage v_2 is:

- (a) -8 V (b) -1.6 V
 (c) 1.6 V (d) 8 V

- 3.5** The current i in the circuit of Fig. 3.48 is:

- (a) -2.667 A (b) -0.667 A
 (c) 0.667 A (d) 2.667 A

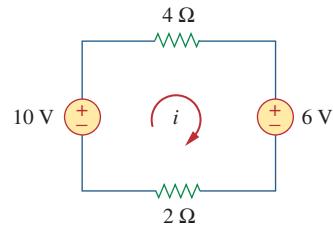


Figure 3.48

For Review Questions 3.5 and 3.6.

- 3.6** The loop equation for the circuit in Fig. 3.48 is:

- (a) $-10 + 4i + 6 + 2i = 0$
 (b) $10 + 4i + 6 + 2i = 0$
 (c) $10 + 4i - 6 + 2i = 0$
 (d) $-10 + 4i - 6 + 2i = 0$

- 3.7 In the circuit of Fig. 3.49, current i_1 is:

(a) 4 A (b) 3 A (c) 2 A (d) 1 A

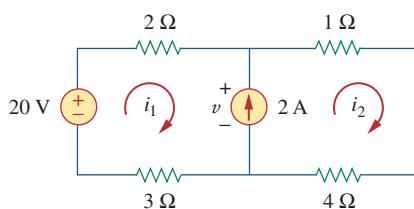


Figure 3.49

For Review Questions 3.7 and 3.8.

- 3.8 The voltage v across the current source in the circuit of Fig. 3.49 is:

(a) 20 V (b) 15 V (c) 10 V (d) 5 V

- 3.9 The PSpice part name for a current-controlled voltage source is:

(a) EX (b) FX (c) HX (d) GX

- 3.10 Which of the following statements are not true of the pseudocomponent IPROBE:

- (a) It must be connected in series.
- (b) It plots the branch current.
- (c) It displays the current through the branch in which it is connected.
- (d) It can be used to display voltage by connecting it in parallel.
- (e) It is used only for dc analysis.
- (f) It does not correspond to a particular circuit element.

Answers: 3.1a, 3.2c, 3.3a, 3.4c, 3.5c, 3.6a, 3.7d, 3.8b, 3.9c, 3.10b,d.

Problems

Sections 3.2 and 3.3 Nodal Analysis

- 3.1 Using Fig. 3.50, design a problem to help other students better understand nodal analysis.

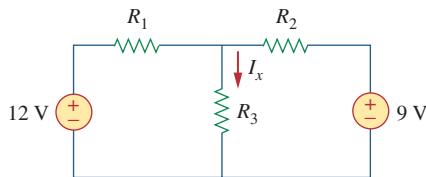


Figure 3.50

For Prob. 3.1 and Prob. 3.39.

- 3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2 .

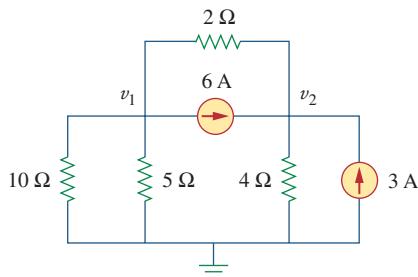


Figure 3.51

For Prob. 3.2.

- 3.3 Find the currents I_1 through I_4 and the voltage v_o in the circuit of Fig. 3.52.

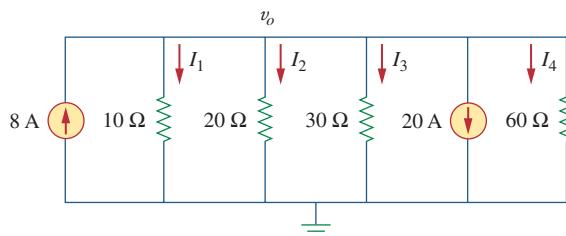


Figure 3.52

For Prob. 3.3.

- 3.4 Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4 .

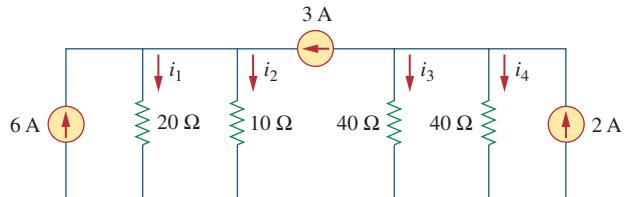


Figure 3.53

For Prob. 3.4.

- 3.5** Obtain v_o in the circuit of Fig. 3.54.

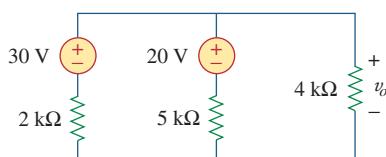


Figure 3.54

For Prob. 3.5.

- 3.6** Solve for V_1 in the circuit of Fig. 3.55 using nodal analysis.

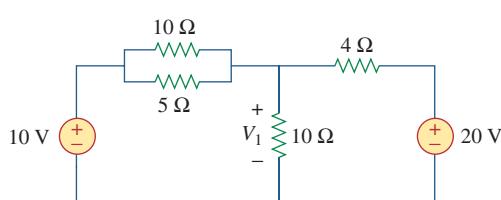


Figure 3.55

For Prob. 3.6.

- 3.7** Apply nodal analysis to solve for V_x in the circuit of Fig. 3.56.

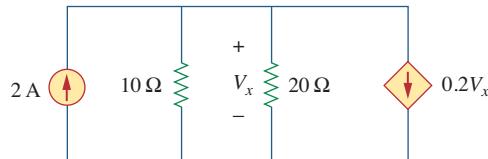


Figure 3.56

For Prob. 3.7.

- 3.8** Using nodal analysis, find v_o in the circuit of Fig. 3.57.

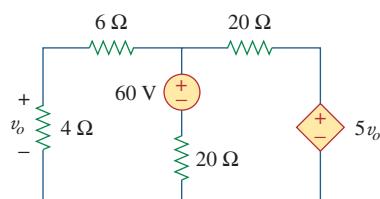


Figure 3.57

For Prob. 3.8 and Prob. 3.37.

- 3.9** Determine I_b in the circuit in Fig. 3.58 using nodal analysis.

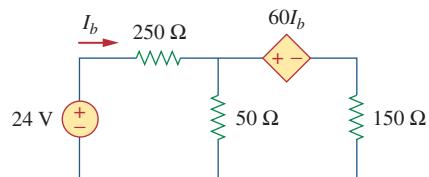


Figure 3.58

For Prob. 3.9.

- 3.10** Find I_o in the circuit of Fig. 3.59.

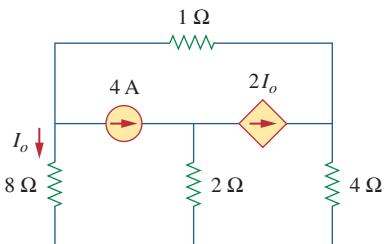


Figure 3.59

For Prob. 3.10.

- 3.11** Find V_o and the power dissipated in all the resistors in the circuit of Fig. 3.60.

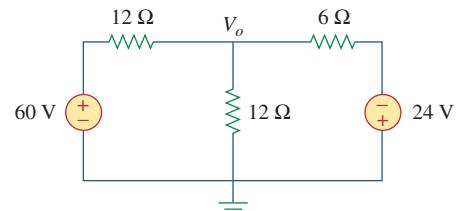


Figure 3.60

For Prob. 3.11.

- 3.12** Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

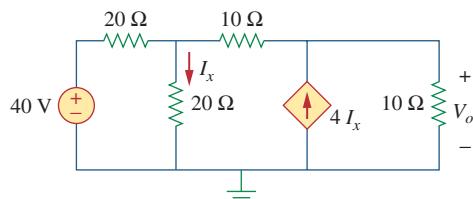
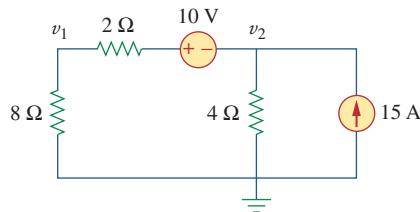


Figure 3.61

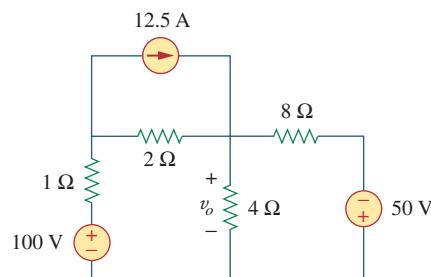
For Prob. 3.12.

- 3.13** Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

**Figure 3.62**

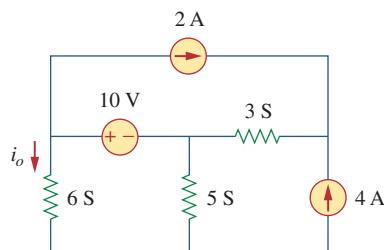
For Prob. 3.13.

- 3.14** Using nodal analysis, find v_o in the circuit of Fig. 3.63.

**Figure 3.63**

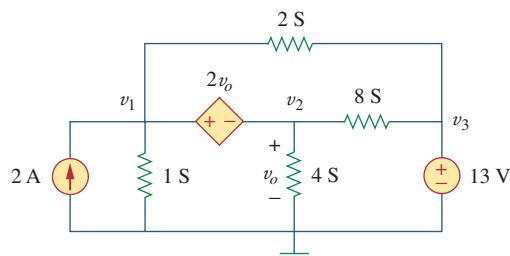
For Prob. 3.14.

- 3.15** Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

**Figure 3.64**

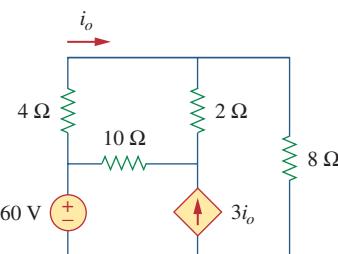
For Prob. 3.15.

- 3.16** Determine voltages v_1 through v_3 in the circuit of Fig. 3.65 using nodal analysis.

**Figure 3.65**

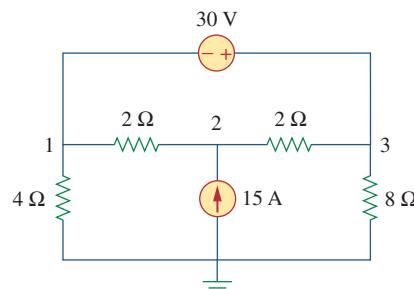
For Prob. 3.16.

- 3.17** Using nodal analysis, find current i_o in the circuit of Fig. 3.66.

**Figure 3.66**

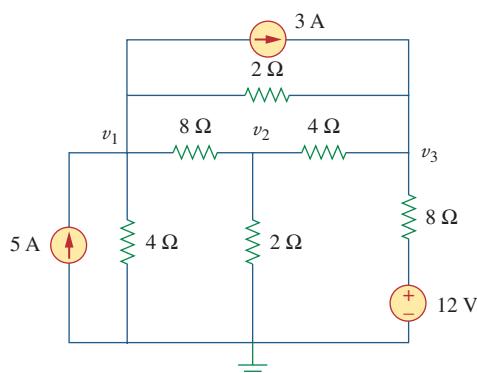
For Prob. 3.17.

- 3.18** Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

**Figure 3.67**

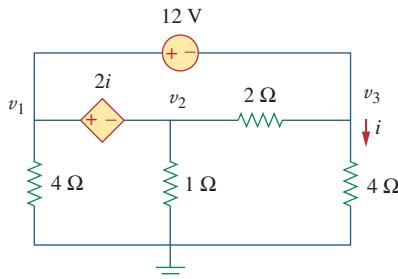
For Prob. 3.18.

- 3.19** Use nodal analysis to find v_1 , v_2 , and v_3 in the circuit of Fig. 3.68.

ML**Figure 3.68**

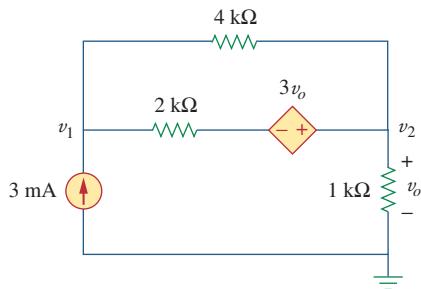
For Prob. 3.19.

- 3.20** For the circuit in Fig. 3.69, find v_1 , v_2 , and v_3 using nodal analysis.

**Figure 3.69**

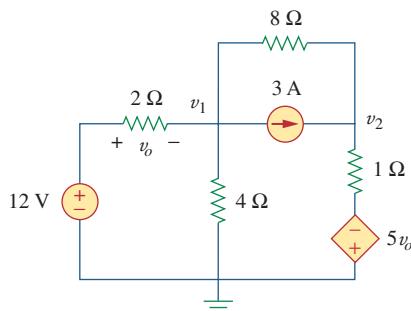
For Prob. 3.20.

- 3.21** For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

**Figure 3.70**

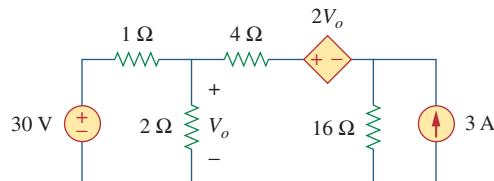
For Prob. 3.21.

- 3.22** Determine v_1 and v_2 in the circuit of Fig. 3.71.

**Figure 3.71**

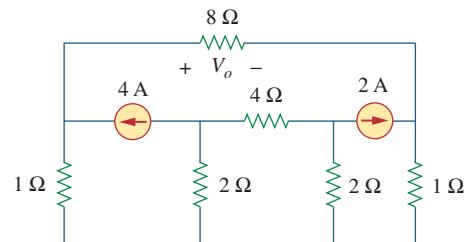
For Prob. 3.22.

- 3.23** Use nodal analysis to find V_o in the circuit of Fig. 3.72.

**Figure 3.72**

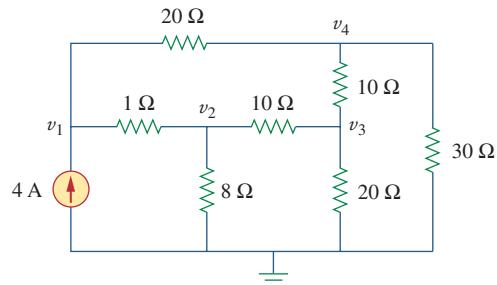
For Prob. 3.23.

- 3.24** Use nodal analysis and MATLAB to find V_o in the circuit of Fig. 3.73.

ML**Figure 3.73**

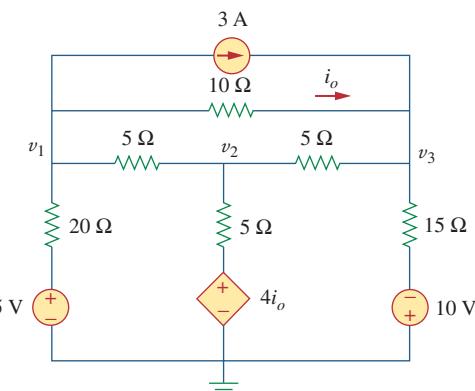
For Prob. 3.24.

- 3.25** Use nodal analysis along with MATLAB to determine the node voltages in Fig. 3.74.

ML**Figure 3.74**

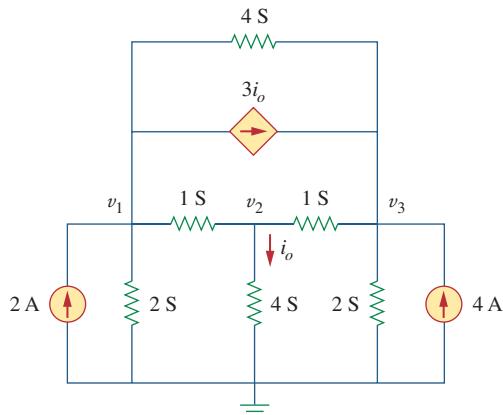
For Prob. 3.25.

- 3.26** Calculate the node voltages v_1 , v_2 , and v_3 in the circuit of Fig. 3.75.

ML**Figure 3.75**

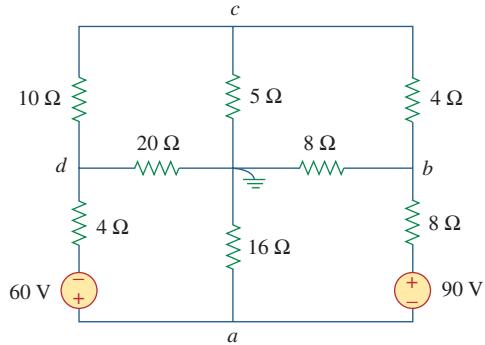
For Prob. 3.26.

- *3.27 Use nodal analysis to determine voltages v_1 , v_2 , and v_3 in the circuit of Fig. 3.76.

ML**Figure 3.76**

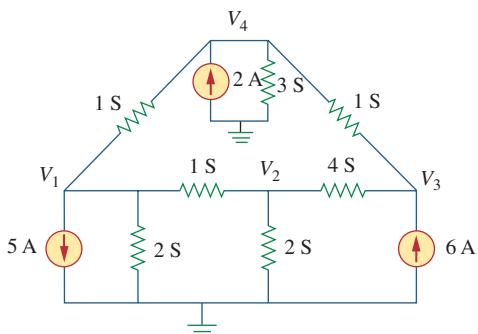
For Prob. 3.27.

- *3.28 Use MATLAB to find the voltages at nodes a , b , c , and d in the circuit of Fig. 3.77.

ML**Figure 3.77**

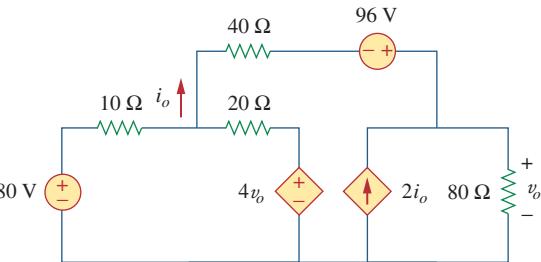
For Prob. 3.28.

- 3.29 Use MATLAB to solve for the node voltages in the circuit of Fig. 3.78.

ML**Figure 3.78**

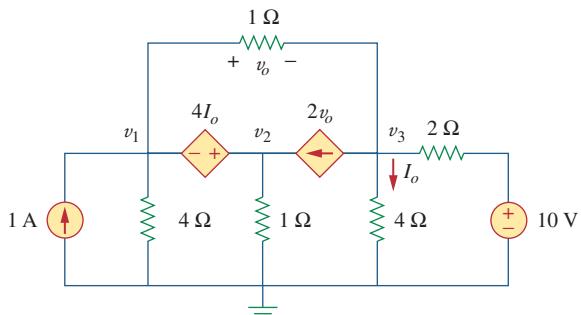
For Prob. 3.29.

- 3.30 Using nodal analysis, find v_o and i_o in the circuit of Fig. 3.79.

**Figure 3.79**

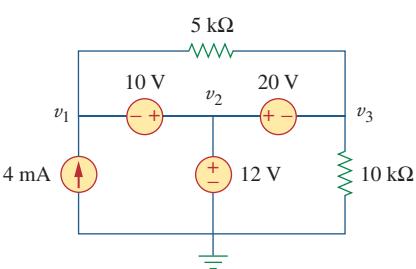
For Prob. 3.30.

- 3.31 Find the node voltages for the circuit in Fig. 3.80.

ML**Figure 3.80**

For Prob. 3.31.

- 3.32 Obtain the node voltages v_1 , v_2 , and v_3 in the circuit of Fig. 3.81.

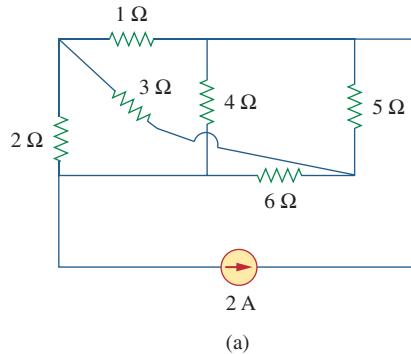
**Figure 3.81**

For Prob. 3.32.

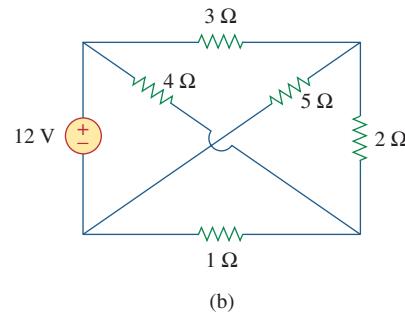
* An asterisk indicates a challenging problem.

Sections 3.4 and 3.5 Mesh Analysis

- 3.33** Which of the circuits in Fig. 3.82 is planar? For the planar circuit, redraw the circuit with no crossing branches.



(a)

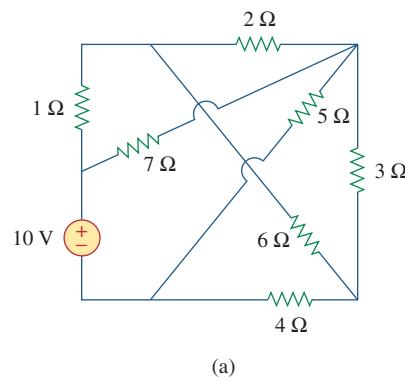


(b)

Figure 3.82

For Prob. 3.33.

- 3.34** Determine which of the circuits in Fig. 3.83 is planar and redraw it with no crossing branches.



(a)

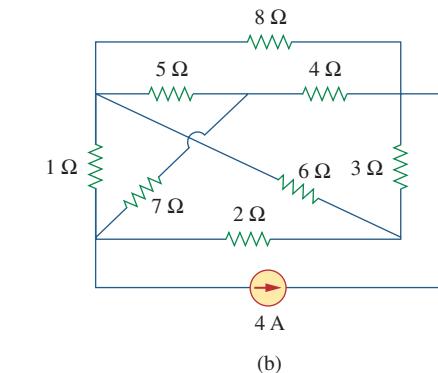


Figure 3.83

For Prob. 3.34.

- 3.35** Rework Prob. 3.5 using mesh analysis.

- 3.36** Use mesh analysis to obtain i_1 , i_2 , and i_3 in the circuit in Fig. 3.84.

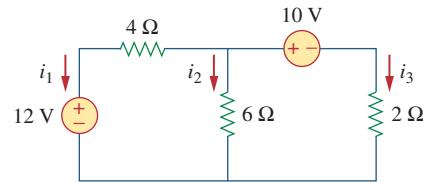


Figure 3.84

For Prob. 3.36.

- 3.37** Solve Prob. 3.8 using mesh analysis.

- 3.38** Apply mesh analysis to the circuit in Fig. 3.85 and obtain I_o .

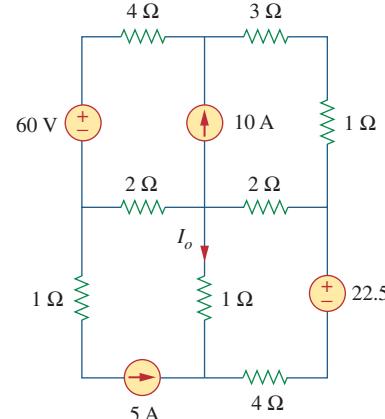
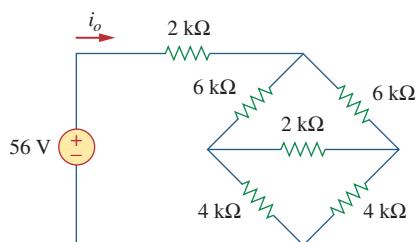


Figure 3.85

For Prob. 3.38.

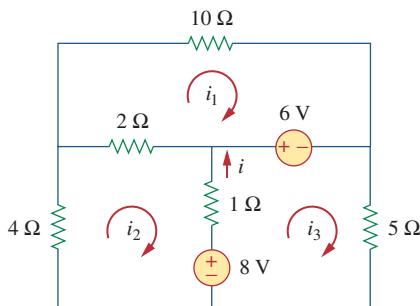
- 3.39** Using Fig. 3.50 from Prob. 3.1, design a problem to help other students better understand mesh analysis.

- 3.40** For the bridge network in Fig. 3.86, find i_o using mesh analysis.

ML**Figure 3.86**

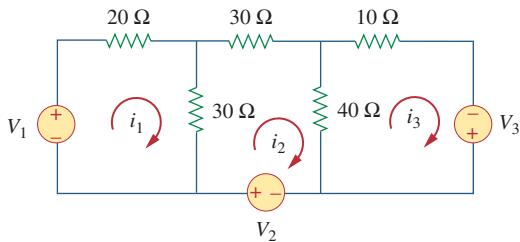
For Prob. 3.40.

- 3.41** Apply mesh analysis to find i in Fig. 3.87.

ML**Figure 3.87**

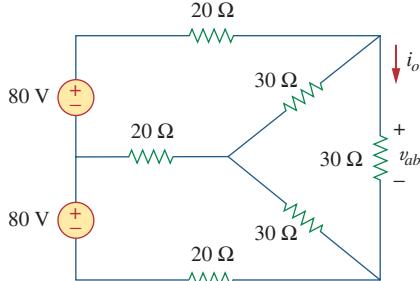
For Prob. 3.41.

- 3.42** Using Fig. 3.88, design a problem to help students better understand mesh analysis using matrices.

**Figure 3.88**

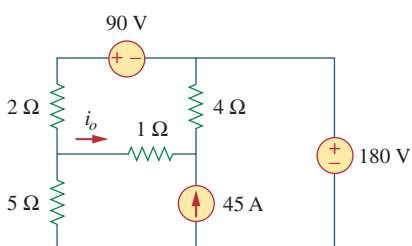
For Prob. 3.42.

- 3.43** Use mesh analysis to find v_{ab} and i_o in the circuit of Fig. 3.89.

ML**Figure 3.89**

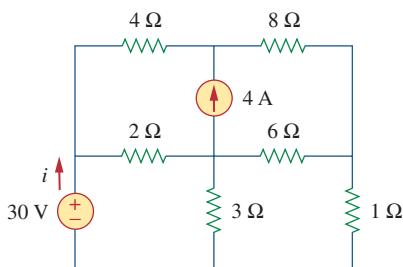
For Prob. 3.43.

- 3.44** Use mesh analysis to obtain i_o in the circuit of Fig. 3.90.

**Figure 3.90**

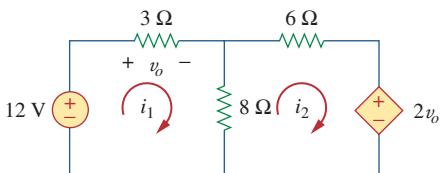
For Prob. 3.44.

- 3.45** Find current i in the circuit of Fig. 3.91.

ML**Figure 3.91**

For Prob. 3.45.

- 3.46** Calculate the mesh currents i_1 and i_2 in Fig. 3.92.

ML**Figure 3.92**

For Prob. 3.46.

- 3.47** Rework Prob. 3.19 using mesh analysis.

ML

- 3.48** Determine the current through the $10\text{-k}\Omega$ resistor in the circuit of Fig. 3.93 using mesh analysis.
ML

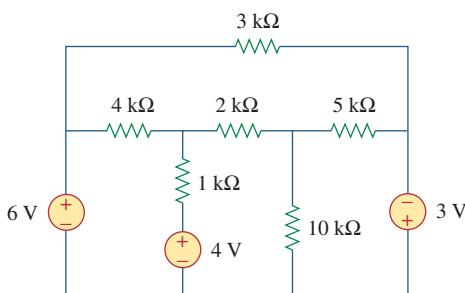


Figure 3.93
For Prob. 3.48.

- 3.49** Find v_o and i_o in the circuit of Fig. 3.94.

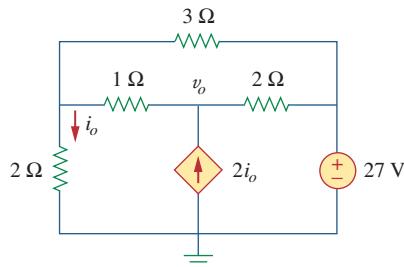


Figure 3.94
For Prob. 3.49.

- 3.50** Use mesh analysis to find the current i_o in the circuit of Fig. 3.95.
ML

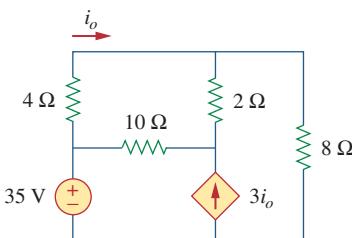


Figure 3.95
For Prob. 3.50.

- 3.51** Apply mesh analysis to find v_o in the circuit of Fig. 3.96.

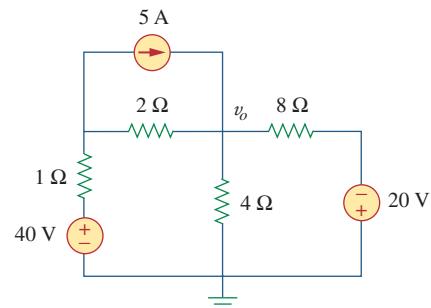


Figure 3.96
For Prob. 3.51.

- 3.52** Use mesh analysis to find i_1 , i_2 , and i_3 in the circuit of Fig. 3.97.
ML

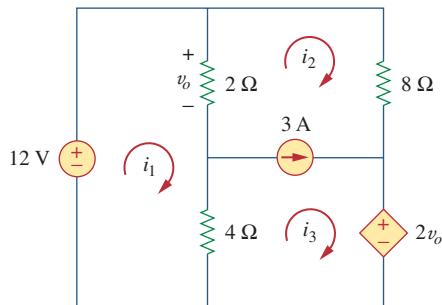


Figure 3.97
For Prob. 3.52.

- 3.53** Find the mesh currents in the circuit of Fig. 3.98 using MATLAB.
ML

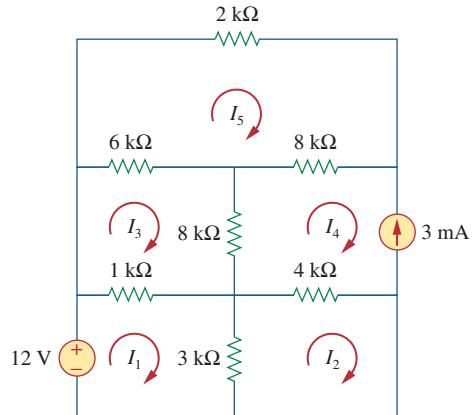
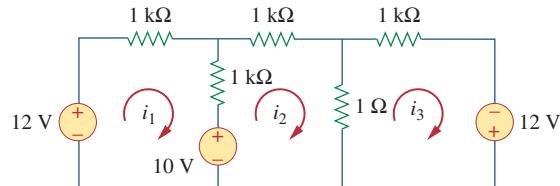


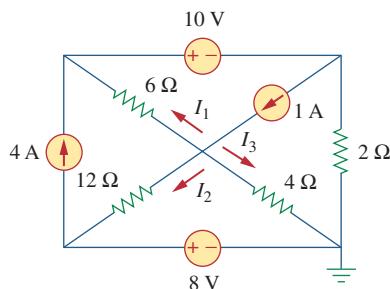
Figure 3.98
For Prob. 3.53.

- 3.54** Find the mesh currents i_1 , i_2 , and i_3 in the circuit in Fig. 3.99.

ML**Figure 3.99**

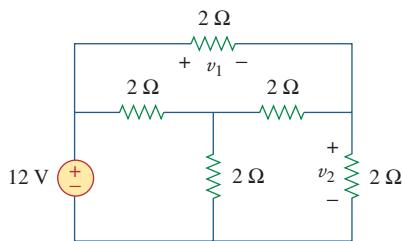
For Prob. 3.54.

- ***3.55** In the circuit of Fig. 3.100, solve for I_1 , I_2 , and I_3 .

ML**Figure 3.100**

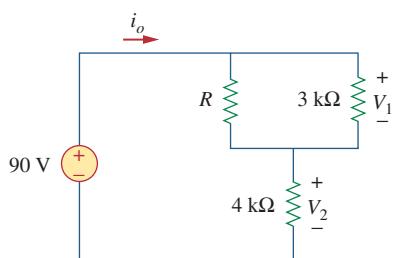
For Prob. 3.55.

- 3.56** Determine v_1 and v_2 in the circuit of Fig. 3.101.

**Figure 3.101**

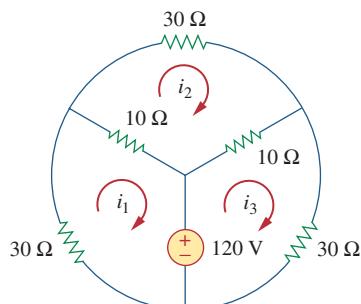
For Prob. 3.56.

- 3.57** In the circuit of Fig. 3.102, find the values of R , V_1 , and V_2 given that $i_o = 15 \text{ mA}$.

**Figure 3.102**

For Prob. 3.57.

- 3.58** Find i_1 , i_2 , and i_3 in the circuit of Fig. 3.103.

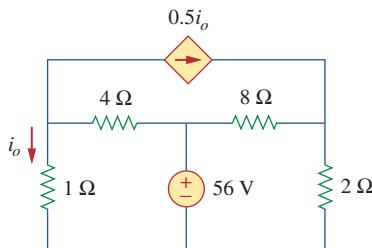
ML**Figure 3.103**

For Prob. 3.58.

- 3.59** Rework Prob. 3.30 using mesh analysis.

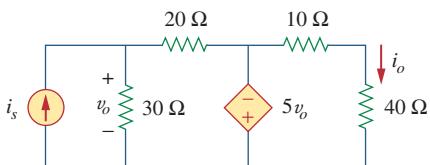
ML

- 3.60** Calculate the power dissipated in each resistor in the circuit of Fig. 3.104.

**Figure 3.104**

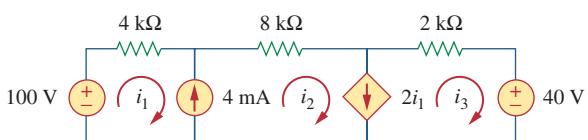
For Prob. 3.60.

- 3.61** Calculate the current gain i_o/i_s in the circuit of Fig. 3.105.

**Figure 3.105**

For Prob. 3.61.

- 3.62** Find the mesh currents i_1 , i_2 , and i_3 in the network of Fig. 3.106.

ML**Figure 3.106**

For Prob. 3.62.

3.63 Find v_x and i_x in the circuit shown in Fig. 3.107.

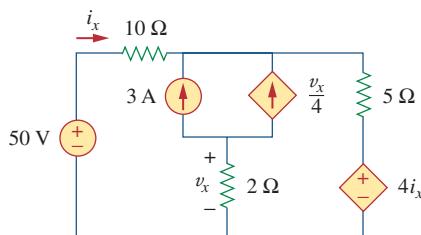


Figure 3.107

For Prob. 3.63.

3.64 Find v_o and i_o in the circuit of Fig. 3.108.

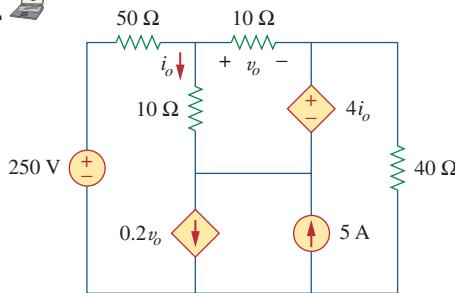


Figure 3.108

For Prob. 3.64.

3.65 Use MATLAB to solve for the mesh currents in the circuit of Fig. 3.109.

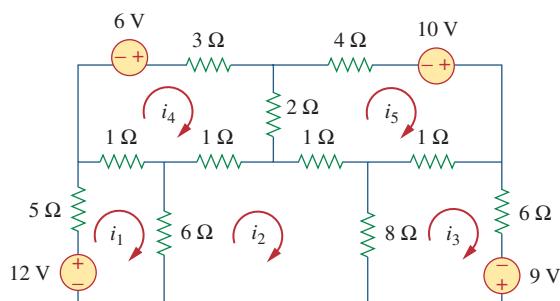


Figure 3.109

For Prob. 3.65.

3.66 Write a set of mesh equations for the circuit in Fig. 3.110. Use MATLAB to determine the mesh currents.

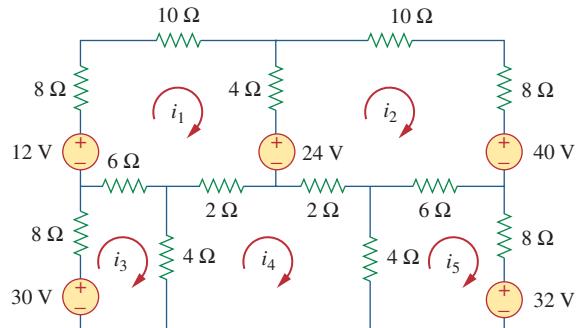


Figure 3.110

For Prob. 3.66.

Section 3.6 Nodal and Mesh Analyses by Inspection

3.67 Obtain the node-voltage equations for the circuit in Fig. 3.111 by inspection. Then solve for V_o .

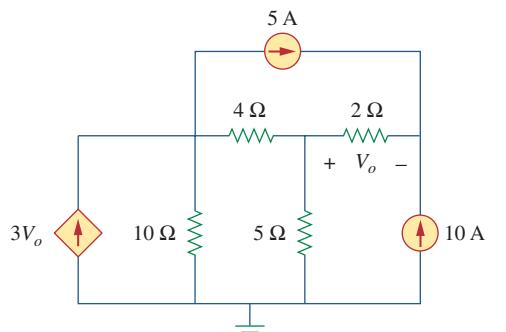


Figure 3.111

For Prob. 3.67.

3.68 Using Fig. 3.112, design a problem, to solve for V_o , to help other students better understand nodal analysis. Try your best to come up with values to make the calculations easier.

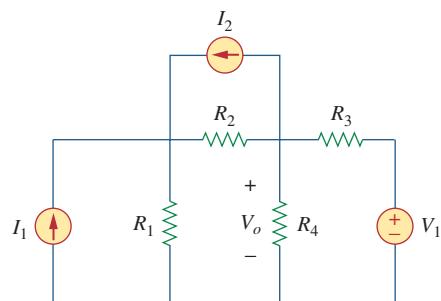
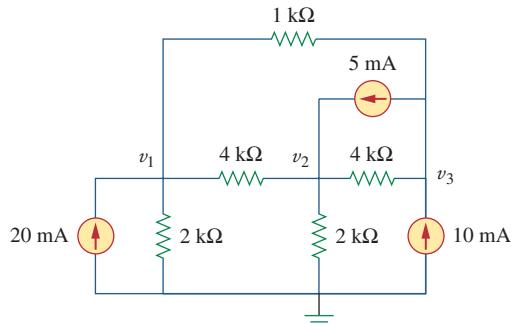


Figure 3.112

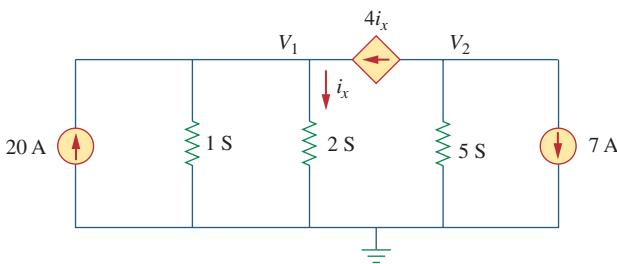
For Prob. 3.68.

- 3.69** For the circuit shown in Fig. 3.113, write the node-voltage equations by inspection.

**Figure 3.113**

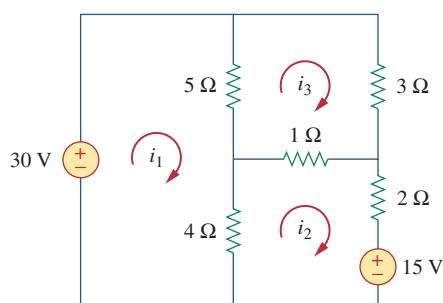
For Prob. 3.69.

- 3.70** Write the node-voltage equations by inspection and then determine values of V_1 and V_2 in the circuit of Fig. 3.114.

**Figure 3.114**

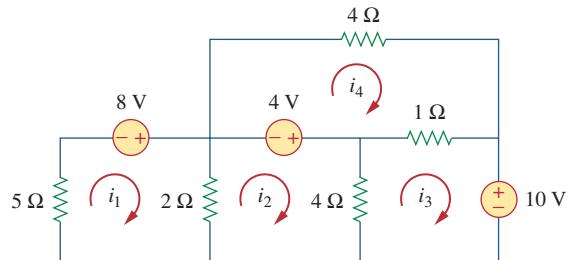
For Prob. 3.70.

- 3.71** Write the mesh-current equations for the circuit in Fig. 3.115. Next, determine the values of i_1 , i_2 , and i_3 .

**Figure 3.115**

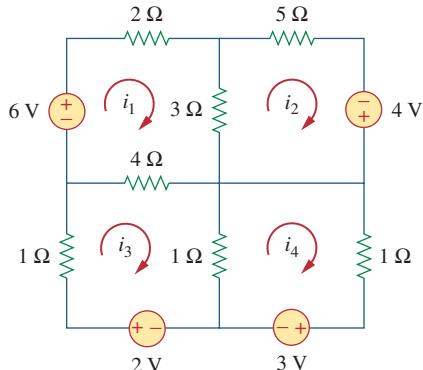
For Prob. 3.71.

- 3.72** By inspection, write the mesh-current equations for the circuit in Fig. 3.116.

**Figure 3.116**

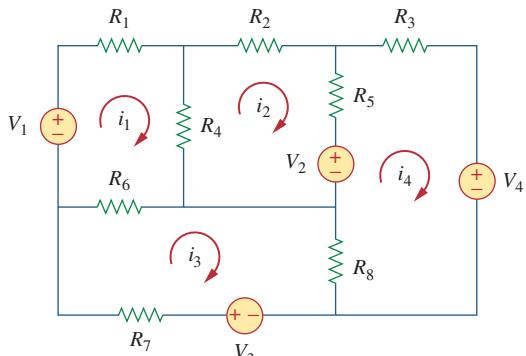
For Prob. 3.72.

- 3.73** Write the mesh-current equations for the circuit in Fig. 3.117.

**Figure 3.117**

For Prob. 3.73.

- 3.74** By inspection, obtain the mesh-current equations for the circuit in Fig. 3.118.

**Figure 3.118**

For Prob. 3.74.

Section 3.8 Circuit Analysis with PSpice or MultiSim

- 3.75** Use PSpice or MultiSim to solve Prob. 3.58.

- 3.76** Use PSpice or MultiSim to solve Prob. 3.27.

- 3.77** Solve for V_1 and V_2 in the circuit of Fig. 3.119 using *PSpice* or *MultiSim*.

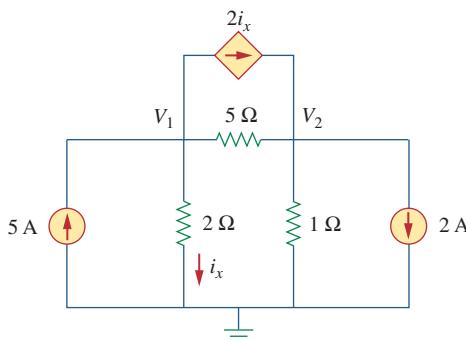


Figure 3.119

For Prob. 3.77.

- 3.78** Solve Prob. 3.20 using *PSpice* or *MultiSim*.

- 3.79** Rework Prob. 3.28 using *PSpice* or *MultiSim*.

- 3.80** Find the nodal voltages v_1 through v_4 in the circuit of Fig. 3.120 using *PSpice* or *MultiSim*.

ML

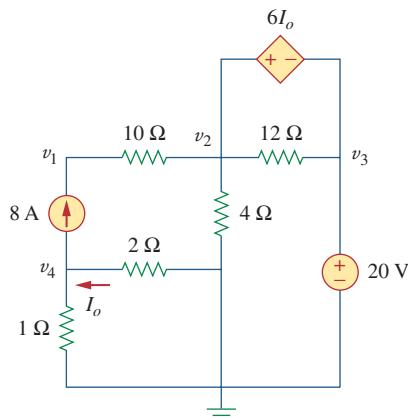


Figure 3.120

For Prob. 3.80.

- 3.81** Use *PSpice* or *MultiSim* to solve the problem in Example 3.4.

- 3.82** If the Schematics Netlist for a network is as follows, draw the network.

```
R_R1 1 2 2K
R_R2 2 0 4K
R_R3 3 0 8K
R_R4 3 4 6K
R_R5 1 3 3K
V_VS 4 0 DC 100
I_IS 0 1 DC 4
F_F1 1 3 VF_F1 2
VF_F1 5 0 0V
E_E1 3 2 1 3 3
```

- 3.83** The following program is the Schematics Netlist of a particular circuit. Draw the circuit and determine the voltage at node 2.

```
R_R1 1 2 20
R_R2 2 0 50
R_R3 2 3 70
R_R4 3 0 30
V_VS 1 0 20V
I_IS 2 0 DC 2A
```

Section 3.9 Applications

- 3.84** Calculate v_o and I_o in the circuit of Fig. 3.121.

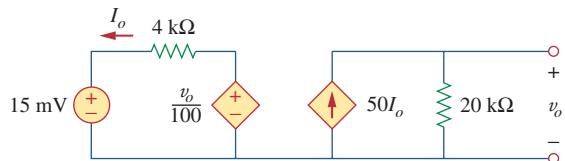


Figure 3.121

For Prob. 3.84.

- 3.85** An audio amplifier with a resistance of 9Ω supplies power to a speaker. What should be the resistance of the speaker for maximum power to be delivered?

- 3.86** For the simplified transistor circuit of Fig. 3.122, calculate the voltage v_o .

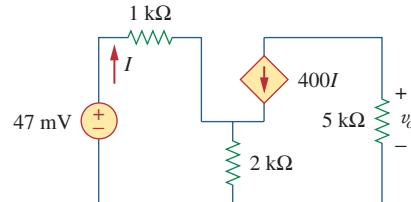


Figure 3.122

For Prob. 3.86.

- 3.87** For the circuit in Fig. 3.123, find the gain v_o/v_s .

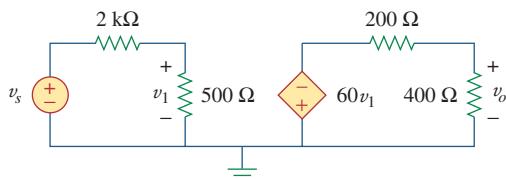


Figure 3.123

For Prob. 3.87.

- *3.88 Determine the gain v_o/v_s of the transistor amplifier circuit in Fig. 3.124.

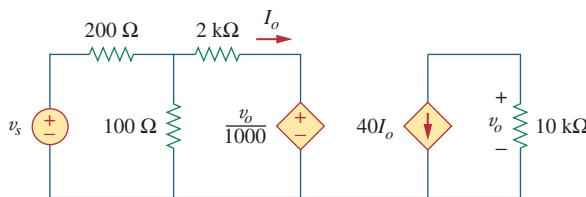


Figure 3.124

For Prob. 3.88.

- 3.89 For the transistor circuit shown in Fig. 3.125, find I_B and V_{CE} . Let $\beta = 100$, and $V_{BE} = 0.7$ V.

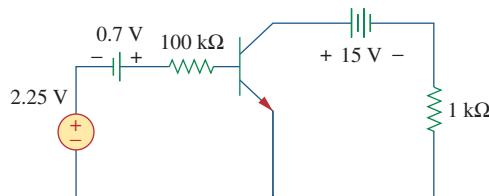


Figure 3.125

For Prob. 3.89.

- 3.90 Calculate v_s for the transistor in Fig. 3.126 given that $v_o = 4$ V, $\beta = 150$, $V_{BE} = 0.7$ V.

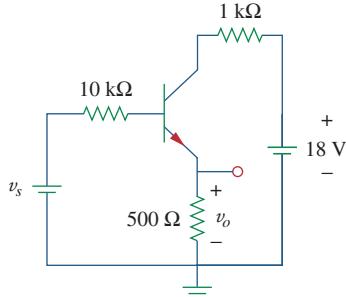


Figure 3.126

For Prob. 3.90.

- 3.91 For the transistor circuit of Fig. 3.127, find I_B , V_{CE} , and v_o . Take $\beta = 200$, $V_{BE} = 0.7$ V.

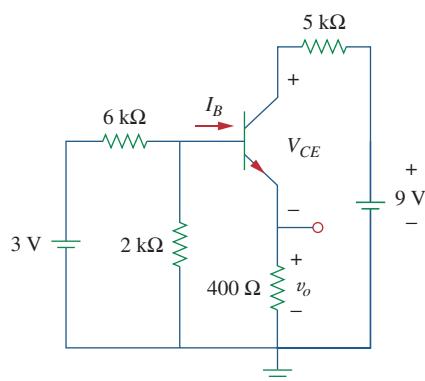


Figure 3.127

For Prob. 3.91.

- 3.92 Using Fig. 3.128, design a problem to help other students better understand transistors. Make sure you use reasonable numbers!

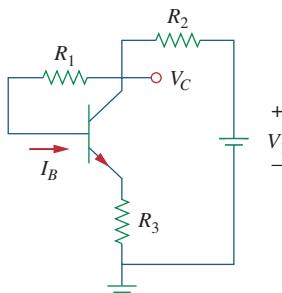


Figure 3.128

For Prob. 3.92.

Comprehensive Problem

- *3.93 Rework Example 3.11 with hand calculation.

4

Circuit Theorems

Your success as an engineer will be directly proportional to your ability to communicate!

—Charles K. Alexander

Enhancing Your Skills and Your Career

Enhancing Your Communication Skills

Taking a course in circuit analysis is one step in preparing yourself for a career in electrical engineering. Enhancing your communication skills while in school should also be part of that preparation, as a large part of your time will be spent communicating.

People in industry have complained again and again that graduating engineers are ill-prepared in written and oral communication. An engineer who communicates effectively becomes a valuable asset.

You can probably speak or write easily and quickly. But how *effectively* do you communicate? The art of effective communication is of the utmost importance to your success as an engineer.

For engineers in industry, communication is key to promotability. Consider the result of a survey of U.S. corporations that asked what factors influence managerial promotion. The survey includes a listing of 22 personal qualities and their importance in advancement. You may be surprised to note that “technical skill based on experience” placed fourth from the bottom. Attributes such as self-confidence, ambition, flexibility, maturity, ability to make sound decisions, getting things done with and through people, and capacity for hard work all ranked higher. At the top of the list was “ability to communicate.” The higher your professional career progresses, the more you will need to communicate. Therefore, you should regard effective communication as an important tool in your engineering tool chest.

Learning to communicate effectively is a lifelong task you should always work toward. The best time to begin is while still in school. Continually look for opportunities to develop and strengthen your reading, writing, listening, and speaking skills. You can do this through classroom presentations, team projects, active participation in student organizations, and enrollment in communication courses. The risks are less now than later in the workplace.



Ability to communicate effectively is regarded by many as the most important step to an executive promotion.

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4.1 Introduction

A major advantage of analyzing circuits using Kirchhoff's laws as we did in Chapter 3 is that we can analyze a circuit without tampering with its original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved.

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to *linear* circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer in this chapter. The concepts we develop are applied in the last section to source modeling and resistance measurement.

4.2 Linearity Property

Linearity is the property of an element describing a linear relationship between cause and effect. Although the property applies to many circuit elements, we shall limit its applicability to resistors in this chapter. The property is a combination of both the homogeneity (scaling) property and the additivity property.

The homogeneity property requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input i to the output v ,

$$v = iR \quad (4.1)$$

If the current is increased by a constant k , then the voltage increases correspondingly by k ; that is,

$$kiR = kv \quad (4.2)$$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$v_1 = i_1 R \quad (4.3a)$$

and

$$v_2 = i_2 R \quad (4.3b)$$

then applying $(i_1 + i_2)$ gives

$$v = (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2 \quad (4.4)$$

We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

In general, a circuit is linear if it is both additive and homogeneous. A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

A **linear circuit** is one whose output is linearly related (or directly proportional) to its input.

Throughout this book we consider only linear circuits. Note that since $p = i^2R = v^2/R$ (making it a quadratic function rather than a linear one), the relationship between power and voltage (or current) is nonlinear. Therefore, the theorems covered in this chapter are not applicable to power.

To illustrate the linearity principle, consider the linear circuit shown in Fig. 4.1. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s , which serves as the input. The circuit is terminated by a load R . We may take the current i through R as the output. Suppose $v_s = 10$ V gives $i = 2$ A. According to the linearity principle, $v_s = 1$ V will give $i = 0.2$ A. By the same token, $i = 1$ mA must be due to $v_s = 5$ mV.

For the circuit in Fig. 4.2, find I_o when $v_s = 12$ V and $v_s = 24$ V.

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0 \quad (4.1.1)$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \quad (4.1.2)$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \quad (4.1.3)$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Rightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$I_o = i_2 = \frac{12}{76} \text{ A}$$

When $v_s = 24$ V,

$$I_o = i_2 = \frac{24}{76} \text{ A}$$

showing that when the source value is doubled, I_o doubles.

For the circuit in Fig. 4.3, find v_o when $i_s = 30$ and $i_s = 45$ A.

Answer: 40 V, 60 V.

For example, when current i_1 flows through resistor R , the power is $p_1 = R i_1^2$, and when current i_2 flows through R , the power is $p_2 = R i_2^2$. If current $i_1 + i_2$ flows through R , the power absorbed is $p_3 = R(i_1 + i_2)^2 = R i_1^2 + R i_2^2 + 2R i_1 i_2 \neq p_1 + p_2$. Thus, the power relation is nonlinear.

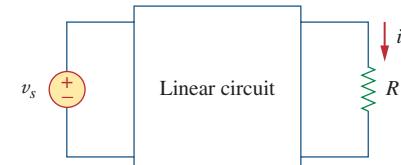


Figure 4.1

A linear circuit with input v_s and output i .

Example 4.1

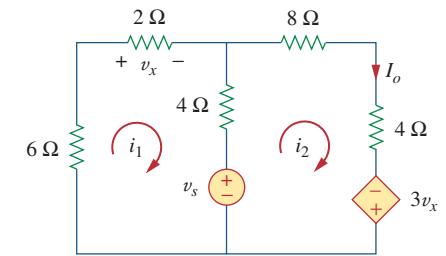


Figure 4.2

For Example 4.1.

Practice Problem 4.1

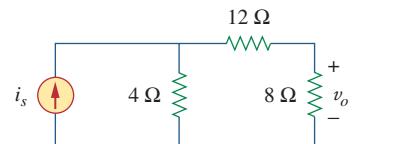


Figure 4.3

For Practice Prob. 4.1.

Example 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

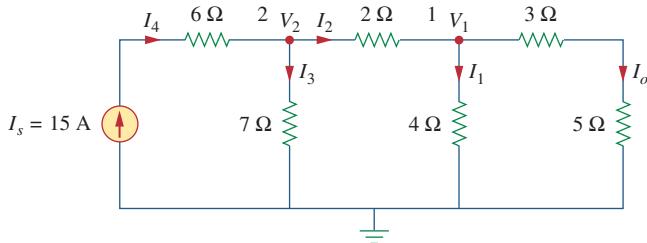


Figure 4.4
For Example 4.2.

Solution:

If $I_o = 1$ A, then $V_1 = (3 + 5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \quad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

Practice Problem 4.2

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

Answer: 16 V.

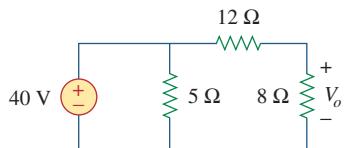


Figure 4.5

For Practice Prob. 4.2.

4.3 Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis as in Chapter 3. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition*.

The idea of superposition rests on the linearity property.

The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

With these in mind, we apply the superposition principle in three steps:

Other terms such as *killed*, *made inactive*, *deadened*, or *set equal to zero* are often used to convey the same idea.

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: It may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Use the superposition theorem to find v in the circuit of Fig. 4.6.

Example 4.3

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1 , we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

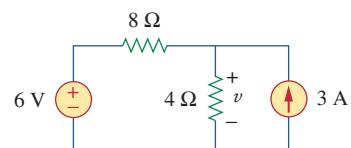
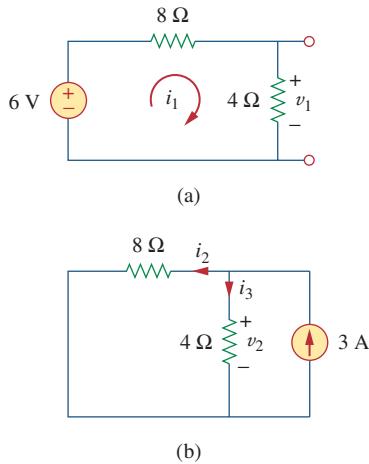


Figure 4.6
For Example 4.3.

**Figure 4.7**

For Example 4.3: (a) calculating v_1 ,
(b) calculating v_2 .

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4 + 8}(6) = 2 \text{ V}$$

To get v_2 , we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4 + 8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

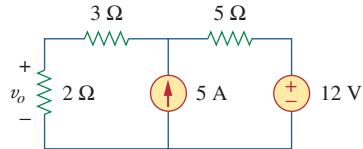
And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Practice Problem 4.3

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.

Answer: 7.4 V.

**Figure 4.8**

For Practice Prob. 4.3.

Example 4.4

Find i_o in the circuit of Fig. 4.9 using superposition.

Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i'_o + i''_o \quad (4.4.1)$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o , we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o . For loop 1,

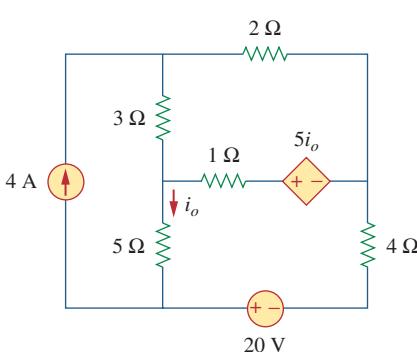
$$i_1 = 4 \text{ A} \quad (4.4.2)$$

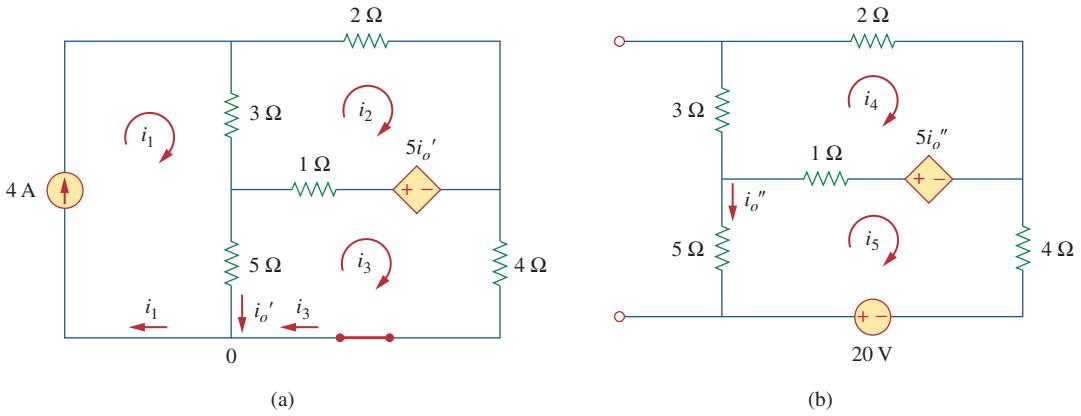
For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \quad (4.4.3)$$

Figure 4.9

For Example 4.4.



**Figure 4.10**

For Example 4.4: Applying superposition to (a) obtain i'_o , (b) obtain i''_o .

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \quad (4.4.4)$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \quad (4.4.5)$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \quad (4.4.6)$$

$$i_2 + 5i'_o = 20 \quad (4.4.7)$$

which can be solved to get

$$i'_o = \frac{52}{17} \text{ A} \quad (4.4.8)$$

To obtain i''_o , we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \quad (4.4.9)$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \quad (4.4.10)$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \quad (4.4.11)$$

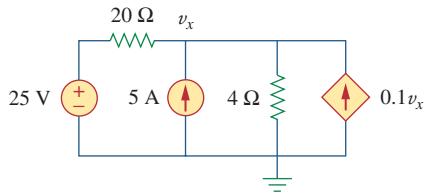
$$i_4 + 5i''_o = -20 \quad (4.4.12)$$

which we solve to get

$$i''_o = -\frac{60}{17} \text{ A} \quad (4.4.13)$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706 \text{ A}$$

Practice Problem 4.4

Use superposition to find v_x in the circuit of Fig. 4.11.

Answer: $v_x = 31.25$ V.

Figure 4.11

For Practice Prob. 4.4.

Example 4.5

For the circuit in Fig. 4.12, use the superposition theorem to find i .

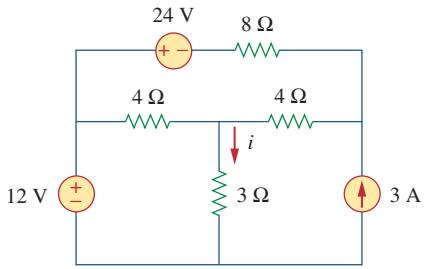


Figure 4.12

For Example 4.5.

Solution:

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1 , i_2 , and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1 , consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4 / 16 = 3$ Ω. Thus,

$$i_1 = \frac{12}{6} = 2 \text{ A}$$

To get i_2 , consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

$$16i_a - 4i_b + 24 = 0 \Rightarrow 4i_a - i_b = -6 \quad (4.5.1)$$

$$7i_b - 4i_a = 0 \Rightarrow i_a = \frac{7}{4}i_b \quad (4.5.2)$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3 , consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \Rightarrow 24 = 3v_2 - 2v_1 \quad (4.5.3)$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \Rightarrow v_2 = \frac{10}{3}v_1 \quad (4.5.4)$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

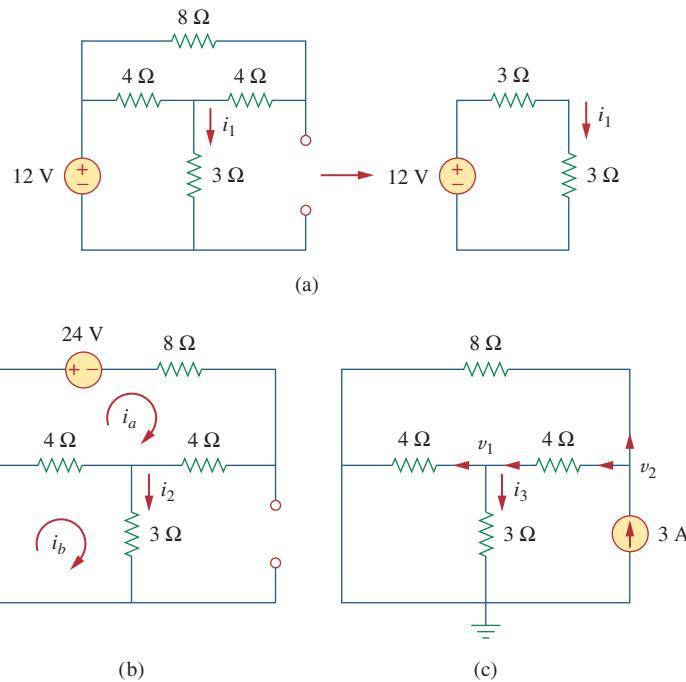


Figure 4.13
For Example 4.5.

Find I in the circuit of Fig. 4.14 using the superposition principle.

Practice Problem 4.5

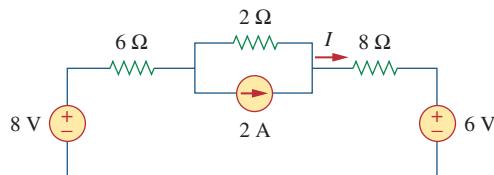


Figure 4.14
For Practice Prob. 4.5.

Answer: 375 mA.

4.4 Source Transformation

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equality*. We recall that an equivalent circuit is one whose $v-i$ characteristics are identical with the original circuit.

In Section 3.6, we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources. It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a

resistor, or vice versa, as shown in Fig. 4.15. Either substitution is known as a *source transformation*.

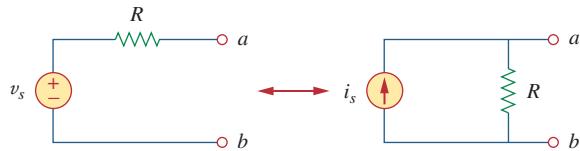


Figure 4.15

Transformation of independent sources.

A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R , or vice versa.

The two circuits in Fig. 4.15 are equivalent—provided they have the same voltage-current relation at terminals $a-b$. It is easy to show that they are indeed equivalent. If the sources are turned off, the equivalent resistance at terminals $a-b$ in both circuits is R . Also, when terminals $a-b$ are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R} \quad (4.5)$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 4.16, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa where we make sure that Eq. (4.5) is satisfied.

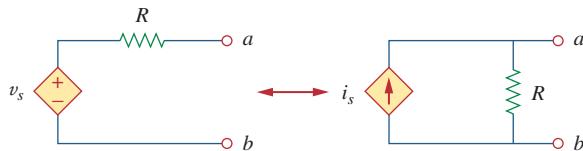


Figure 4.16

Transformation of dependent sources.

Like the wye-delta transformation we studied in Chapter 2, a source transformation does not affect the remaining part of the circuit. When applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis. However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 4.15 (or Fig. 4.16) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from Eq. (4.5) that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source. More will be said on ideal and nonideal sources in Section 4.10.1.

Use source transformation to find v_o in the circuit of Fig. 4.17.

Example 4.6

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4- Ω and 2- Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3- Ω and 6- Ω resistors in parallel to get 2- Ω . We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

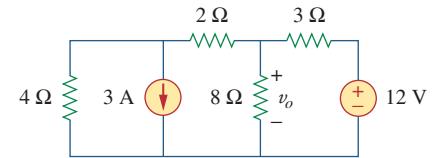


Figure 4.17

For Example 4.6.

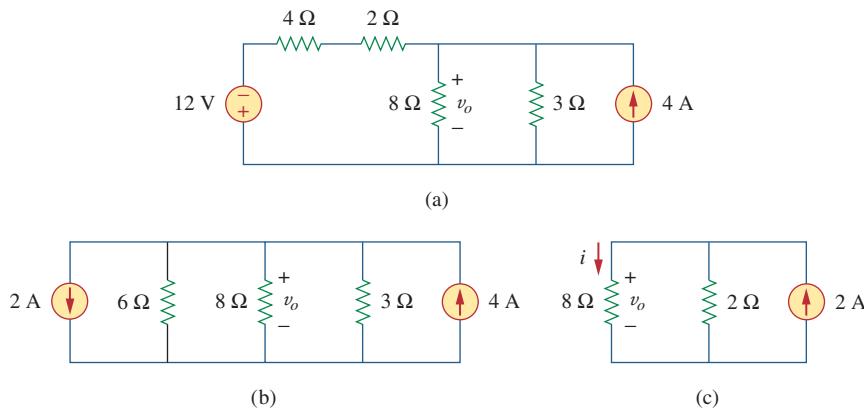


Figure 4.18

For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2 + 8}(2) = 0.4 \text{ A}$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8- Ω and 2- Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Find i_o in the circuit of Fig. 4.19 using source transformation.

Practice Problem 4.6

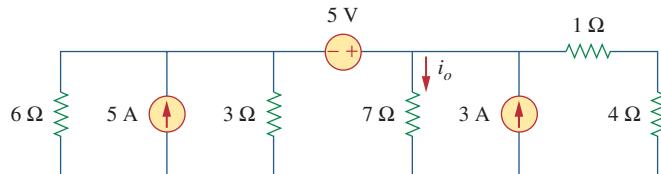


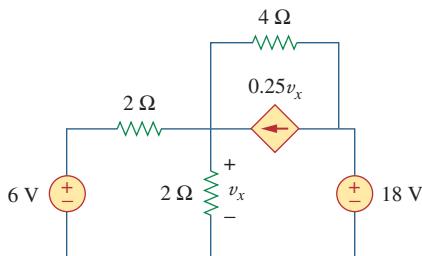
Figure 4.19

For Practice Prob. 4.6.

Answer: 1.78 A.

Example 4.7

Find v_x in Fig. 4.20 using source transformation.

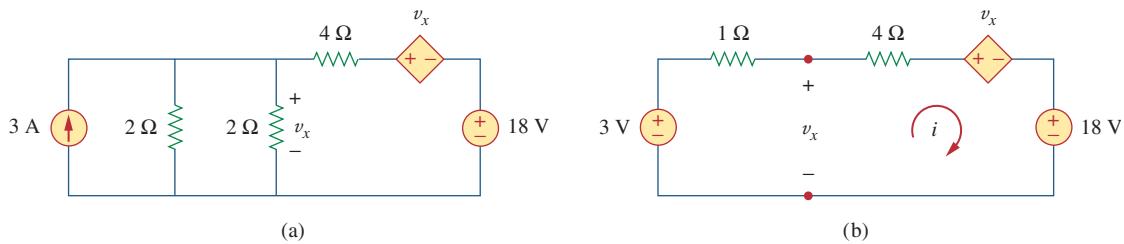
**Figure 4.20**

For Example 4.7.

Solution:

The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current source is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \quad (4.7.1)$$

**Figure 4.21**

For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and v_x yields

$$-3 + 1i + v_x = 0 \Rightarrow v_x = 3 - i \quad (4.7.2)$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \Rightarrow i = -4.5 \text{ A}$$

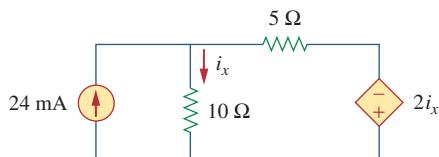
Alternatively, we may apply KVL to the loop containing v_x , the 4-Ω resistor, the voltage-controlled dependent voltage source, and the 18-V voltage source in Fig. 4.21(b). We obtain

$$-v_x + 4i + v_x + 18 = 0 \Rightarrow i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5 \text{ V}$.

Practice Problem 4.7

Use source transformation to find i_x in the circuit shown in Fig. 4.22.



Answer: 7.059 mA.

Figure 4.22

For Practice Prob. 4.7.

4.5 Thevenin's Theorem

It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in Fig. 4.23(a) can be replaced by that in Fig. 4.23(b). (The load in Fig. 4.23 may be a single resistor or another circuit.) The circuit to the left of the terminals *a-b* in Fig. 4.23(b) is known as the *Thevenin equivalent circuit*; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} , where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

The proof of the theorem will be given later, in Section 4.7. Our major concern right now is how to find the Thevenin equivalent voltage V_{Th} and resistance R_{Th} . To do so, suppose the two circuits in Fig. 4.23 are equivalent. Two circuits are said to be *equivalent* if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. 4.23 equivalent. If the terminals *a-b* are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals *a-b* in Fig. 4.23(a) must be equal to the voltage source V_{Th} in Fig. 4.23(b), since the two circuits are equivalent. Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 4.24(a); that is,

$$V_{Th} = v_{oc} \quad (4.6)$$

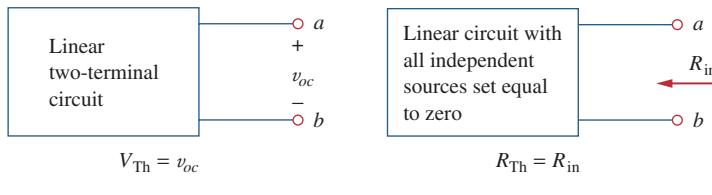


Figure 4.24
Finding V_{Th} and R_{Th} .

Again, with the load disconnected and terminals *a-b* open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals *a-b* in Fig. 4.23(a) must be equal to R_{Th} in Fig. 4.23(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. 4.24(b); that is,

$$R_{Th} = R_{in} \quad (4.7)$$

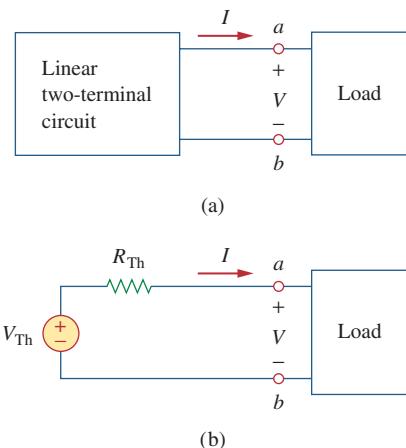
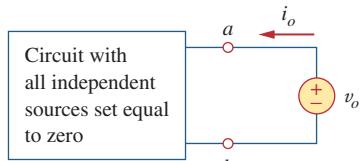
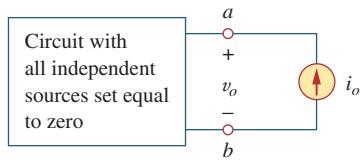


Figure 4.23

Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.



(a)



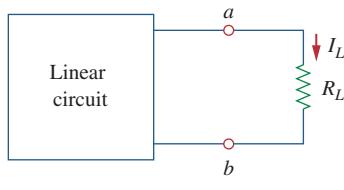
$$R_{\text{Th}} = \frac{v_o}{i_o}$$

(b)

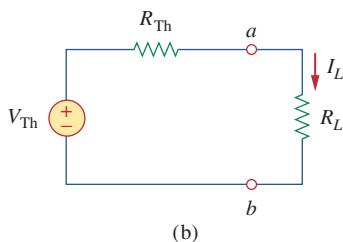
Figure 4.25

Finding R_{Th} when circuit has dependent sources.

Later we will see that an alternative way of finding R_{Th} is $R_{\text{Th}} = V_{\text{OC}}/i_{\text{SC}}$.



(a)



(b)

Figure 4.26

A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

To apply this idea in finding the Thevenin resistance R_{Th} , we need to consider two cases.

CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals *a* and *b*, as shown in Fig. 4.24(b).

CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals *a* and *b* and determine the resulting current i_o . Then $R_{\text{Th}} = v_o/i_o$, as shown in Fig. 4.25(a). Alternatively, we may insert a current source i_o at terminals *a*-*b* as shown in Fig. 4.25(b) and find the terminal voltage v_o . Again $R_{\text{Th}} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o . For example, we may use $v_o = 1 \text{ V}$ or $i_o = 1 \text{ A}$, or even use unspecified values of v_o or i_o .

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v = -iR$) implies that the circuit is supplying power. This is possible in a circuit with dependent sources; Example 4.10 will illustrate this.

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

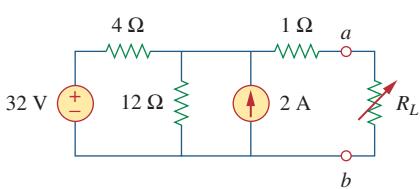
As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L , as shown in Fig. 4.26(a). The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. 4.26(b). From Fig. 4.26(b), we obtain

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} \quad (4.8a)$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}} \quad (4.8b)$$

Note from Fig. 4.26(b) that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

Example 4.8

**Figure 4.27**

For Example 4.8.

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16$, and 36Ω .

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an

open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4 \Omega$$

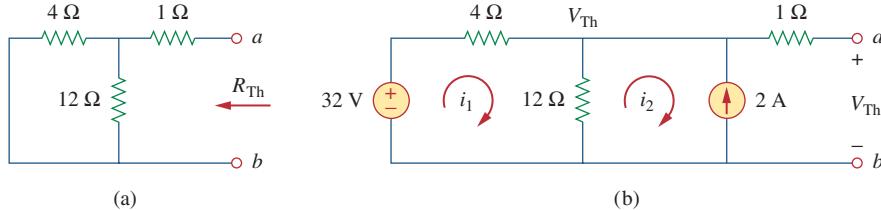


Figure 4.28

For Example 4.8: (a) finding R_{Th} , (b) finding V_{Th} .

To find V_{Th} , consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \quad i_2 = -2 \text{ A}$$

Solving for i_1 , we get $i_1 = 0.5 \text{ A}$. Thus,

$$V_{\text{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{\text{Th}}}{4} + 2 = \frac{V_{\text{Th}}}{12}$$

or

$$96 - 3V_{\text{Th}} + 24 = V_{\text{Th}} \Rightarrow V_{\text{Th}} = 30 \text{ V}$$

as obtained before. We could also use source transformation to find V_{Th} .

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

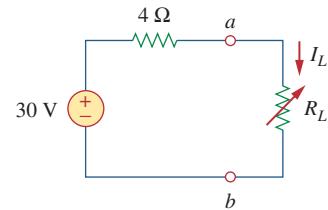
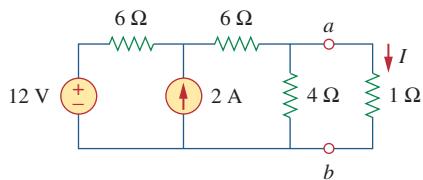


Figure 4.29

The Thevenin equivalent circuit for Example 4.8.

Practice Problem 4.8

**Figure 4.30**

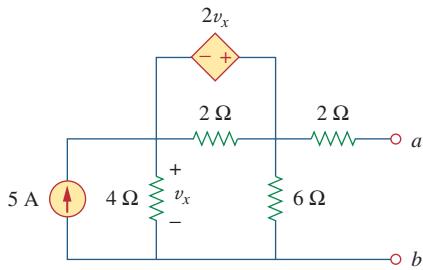
For Practice Prob. 4.8.

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Fig. 4.30. Then find I .

Answer: $V_{Th} = 6 \text{ V}$, $R_{Th} = 3 \Omega$, $I = 1.5 \text{ A}$.

Example 4.9

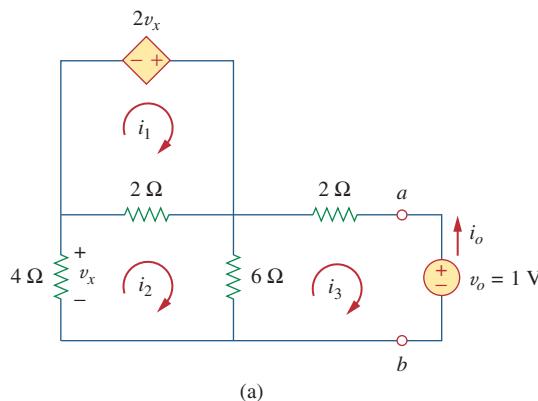
Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals $a-b$.

**Figure 4.31**

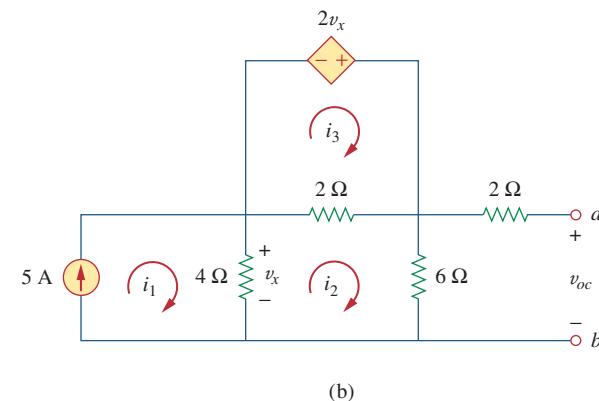
For Example 4.9.

Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th} , we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1 \text{ V}$ to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o , and obtain $R_{Th} = v_o/1$.)



(a)



(b)

Figure 4.32Finding R_{Th} and V_{Th} for Example 4.9.

Applying mesh analysis to loop 1 in the circuit of Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \quad (4.9.1)$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \quad (4.9.2)$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \quad (4.9.3)$$

Solving these equations gives

$$i_3 = -\frac{1}{6} \text{ A}$$

But $i_o = -i_3 = 1/6 \text{ A}$. Hence,

$$R_{\text{Th}} = \frac{1 \text{ V}}{i_o} = 6 \Omega$$

To get V_{Th} , we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \quad (4.9.4)$$

$$\begin{aligned} -2v_x + 2(i_3 - i_2) &= 0 \Rightarrow v_x = i_3 - i_2 \quad (4.9.5) \\ 4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 &= 0 \end{aligned}$$

or

$$12i_2 - 4i_1 - 2i_3 = 0 \quad (4.9.6)$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20 \text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

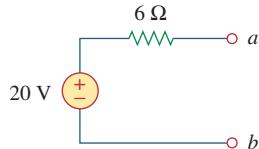


Figure 4.33

The Thevenin equivalent of the circuit in Fig. 4.31.

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Practice Problem 4.9

Answer: $V_{\text{Th}} = 5.333 \text{ V}$, $R_{\text{Th}} = 444.4 \text{ m}\Omega$.

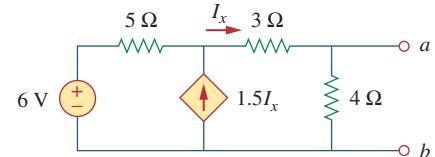


Figure 4.34

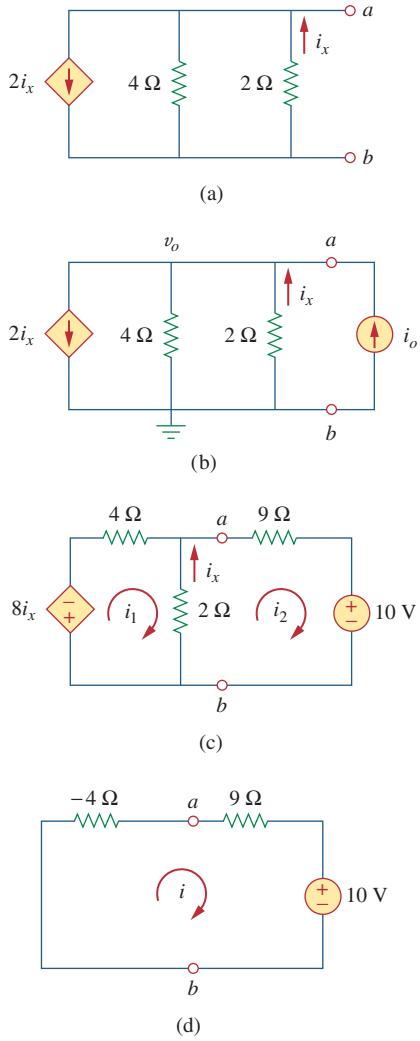
For Practice Prob. 4.9.

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a) at terminals $a-b$.

Example 4.10

Solution:

1. **Define.** The problem is clearly defined; we are to determine the Thevenin equivalent of the circuit shown in Fig. 4.35(a).
2. **Present.** The circuit contains a $2-\Omega$ resistor in parallel with a $4-\Omega$ resistor. These are, in turn, in parallel with a dependent current source. It is important to note that there are no independent sources.
3. **Alternative.** The first thing to consider is that, since we have no independent sources in this circuit, we must excite the circuit externally. In addition, when you have no independent sources you will not have a value for V_{Th} ; you will only have to find R_{Th} .

**Figure 4.35**

For Example 4.10.

The simplest approach is to excite the circuit with either a 1-V voltage source or a 1-A current source. Since we will end up with an equivalent resistance (either positive or negative), I prefer to use the current source and nodal analysis which will yield a voltage at the output terminals equal to the resistance (with 1 A flowing in, \$v_o\$ is equal to 1 times the equivalent resistance).

As an alternative, the circuit could also be excited by a 1-V voltage source and mesh analysis could be used to find the equivalent resistance.

4. **Attempt.** We start by writing the nodal equation at *a* in Fig. 4.35(b) assuming \$i_o = 1\$ A.

$$2i_x + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0 \quad (4.10.1)$$

Since we have two unknowns and only one equation, we will need a constraint equation.

$$i_x = (0 - v_o)/2 = -v_o/2 \quad (4.10.2)$$

Substituting Eq. (4.10.2) into Eq. (4.10.1) yields

$$\begin{aligned} 2(-v_o/2) + (v_o - 0)/4 + (v_o - 0)/2 + (-1) &= 0 \\ = (-1 + \frac{1}{4} + \frac{1}{2})v_o - 1 &= 0 \quad \text{or} \quad v_o = -4 \text{ V} \end{aligned}$$

Since \$v_o = 1 \times R_{Th}\$, then \$R_{Th} = v_o/1 = -4 \Omega\$.

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in Fig. 4.35(a) is supplying power. Of course, the resistors in Fig. 4.35(a) cannot supply power (they absorb power); it is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

5. **Evaluate.** First of all, we note that the answer has a negative value. We know this is not possible in a passive circuit, but in this circuit we do have an active device (the dependent current source). Thus, the equivalent circuit is essentially an active circuit that can supply power.

Now we must evaluate the solution. The best way to do this is to perform a check, using a different approach, and see if we obtain the same solution. Let us try connecting a \$9\Omega\$ resistor in series with a \$10V\$ voltage source across the output terminals of the original circuit and then the Thevenin equivalent. To make the circuit easier to solve, we can take and change the parallel current source and \$4\Omega\$ resistor to a series voltage source and \$4\Omega\$ resistor by using source transformation. This, with the new load, gives us the circuit shown in Fig. 4.35(c).

We can now write two mesh equations.

$$\begin{aligned} 8i_x + 4i_1 + 2(i_1 - i_2) &= 0 \\ 2(i_2 - i_1) + 9i_2 + 10 &= 0 \end{aligned}$$

Note, we only have two equations but have 3 unknowns, so we need a constraint equation. We can use

$$i_x = i_2 - i_1$$

This leads to a new equation for loop 1. Simplifying leads to

$$(4 + 2 - 8)i_1 + (-2 + 8)i_2 = 0$$

or

$$-2i_1 + 6i_2 = 0 \quad \text{or} \quad i_1 = 3i_2$$

$$-2i_1 + 11i_2 = -10$$

Substituting the first equation into the second gives

$$-6i_2 + 11i_2 = -10 \quad \text{or} \quad i_2 = -10/5 = -2 \text{ A}$$

Using the Thevenin equivalent is quite easy since we have only one loop, as shown in Fig. 4.35(d).

$$-4i + 9i + 10 = 0 \quad \text{or} \quad i = -10/5 = -2 \text{ A}$$

6. **Satisfactory?** Clearly we have found the value of the equivalent circuit as required by the problem statement. Checking does validate that solution (we compared the answer we obtained by using the equivalent circuit with one obtained by using the load with the original circuit). We can present all this as a solution to the problem.

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Practice Problem 4.10

Answer: $V_{\text{Th}} = 0 \text{ V}$, $R_{\text{Th}} = -7.5 \Omega$.

4.6 Norton's Theorem

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 4.37(a) can be replaced by the one in Fig. 4.37(b).

The proof of Norton's theorem will be given in the next section. For now, we are mainly concerned with how to get R_N and I_N . We find R_N in the same way we find R_{Th} . In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{\text{Th}} \quad (4.9)$$

To find the Norton current I_N , we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 4.37. It is evident

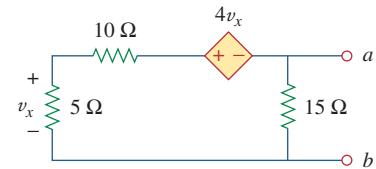


Figure 4.36
For Practice Prob. 4.10.

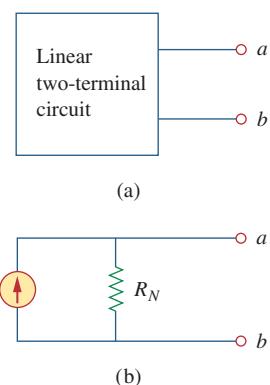
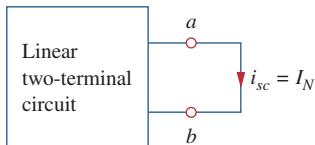


Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

**Figure 4.38**Finding Norton current I_N .

The Thevenin and Norton equivalent circuits are related by a source transformation.

that the short-circuit current in Fig. 4.37(b) is I_N . This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \quad (4.10)$$

shown in Fig. 4.38. Dependent and independent sources are treated the same way as in Thevenin's theorem.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in Eq. (4.9), and

$$I_N = \frac{V_{Th}}{R_{Th}} \quad (4.11)$$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th} , I_N , and R_{Th} are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a and b .
- The short-circuit current i_{sc} at terminals a and b .
- The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Example 4.11 will illustrate this. Also, since

$$V_{Th} = v_{oc} \quad (4.12a)$$

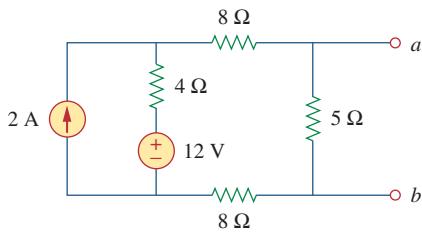
$$I_N = i_{sc} \quad (4.12b)$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N \quad (4.12c)$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent, of a circuit which contains at least one independent source.

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals $a-b$.

**Figure 4.39**

For Example 4.11.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N . Thus,

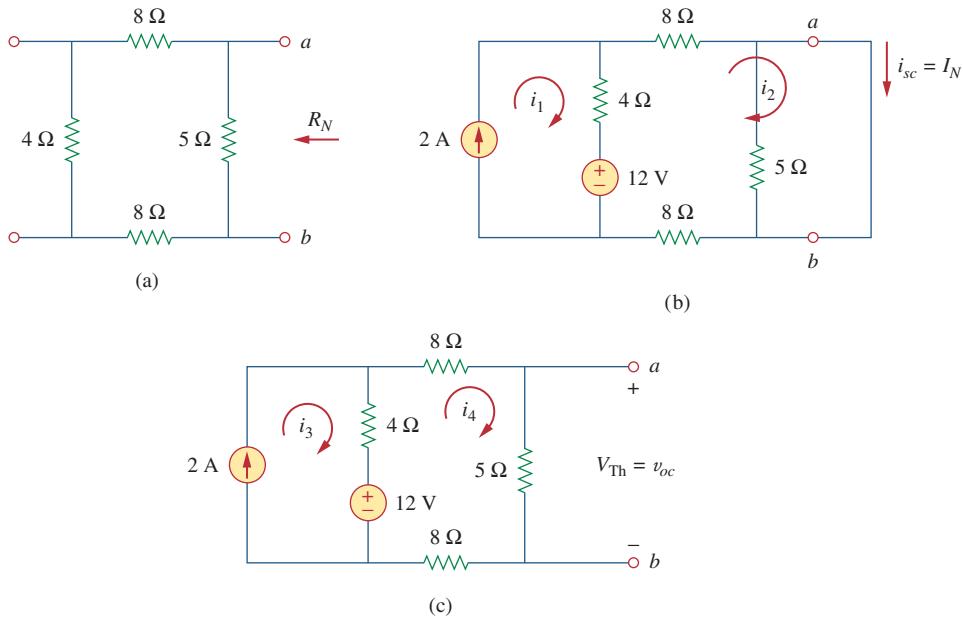
$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4 \Omega$$

To find I_N , we short-circuit terminals a and b , as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2 \text{ A}, \quad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1 \text{ A} = i_{sc} = I_N$$

**Figure 4.40**For Example 4.11; finding: (a) R_N , (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Alternatively, we may determine I_N from V_{Th}/R_{Th} . We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

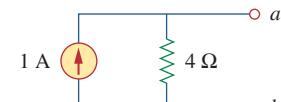
$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4 \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

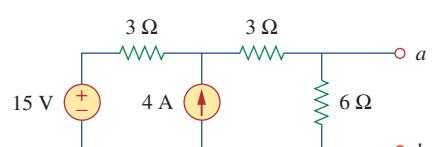
Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals $a-b$.

Practice Problem 4.11

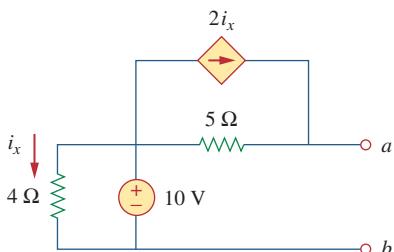
Answer: $R_N = 3 \Omega$, $I_N = 4.5 \text{ A}$.

**Figure 4.41**

Norton equivalent of the circuit in Fig. 4.39.

**Figure 4.42**

For Practice Prob. 4.11.

Example 4.12**Figure 4.43**

For Example 4.12.

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals $a-b$.

Solution:

To find R_N , we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = 0$. At node a , $i_o = \frac{v_o}{5\Omega} = 0.2$ A, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5 \Omega$$

To find I_N , we short-circuit terminals a and b and find the current i_{sc} , as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

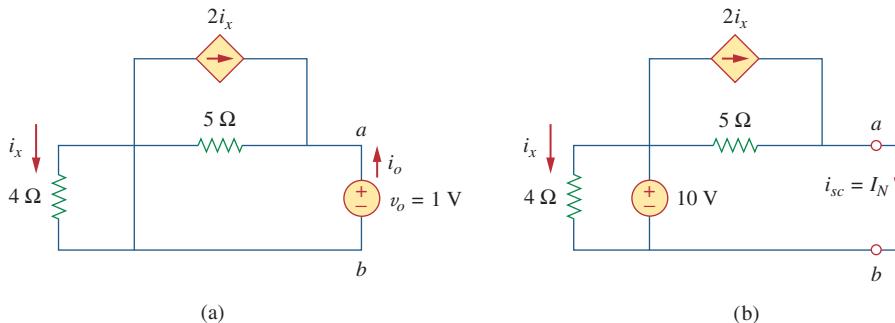
$$i_x = \frac{10}{4} = 2.5 \text{ A}$$

At node a , KCL gives

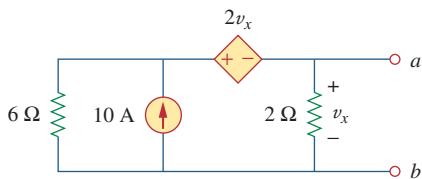
$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7 \text{ A}$$

Thus,

$$I_N = 7 \text{ A}$$

**Figure 4.44**For Example 4.12: (a) finding R_N , (b) finding I_N .**Practice Problem 4.12**

Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals $a-b$.



Answer: $R_N = 1 \Omega$, $I_N = 10 \text{ A}$.

Figure 4.45

For Practice Prob. 4.12.

4.7 [†]Derivations of Thevenin's and Norton's Theorems

In this section, we will prove Thevenin's and Norton's theorems using the superposition principle.

Consider the linear circuit in Fig. 4.46(a). It is assumed that the circuit contains resistors and dependent and independent sources. We have access to the circuit via terminals a and b , through which current from an external source is applied. Our objective is to ensure that the voltage-current relation at terminals a and b is identical to that of the Thevenin equivalent in Fig. 4.46(b). For the sake of simplicity, suppose the linear circuit in Fig. 4.46(a) contains two independent voltage sources v_{s1} and v_{s2} and two independent current sources i_{s1} and i_{s2} . We may obtain any circuit variable, such as the terminal voltage v , by applying superposition. That is, we consider the contribution due to each independent source including the external source i . By superposition, the terminal voltage v is

$$v = A_0i + A_1v_{s1} + A_2v_{s2} + A_3i_{s1} + A_4i_{s2} \quad (4.13)$$

where A_0, A_1, A_2, A_3 , and A_4 are constants. Each term on the right-hand side of Eq. (4.13) is the contribution of the related independent source; that is, A_0i is the contribution to v due to the external current source i , A_1v_{s1} is the contribution due to the voltage source v_{s1} , and so on. We may collect terms for the internal independent sources together as B_0 , so that Eq. (4.13) becomes

$$v = A_0i + B_0 \quad (4.14)$$

where $B_0 = A_1v_{s1} + A_2v_{s2} + A_3i_{s1} + A_4i_{s2}$. We now want to evaluate the values of constants A_0 and B_0 . When the terminals a and b are open-circuited, $i = 0$ and $v = B_0$. Thus, B_0 is the open-circuit voltage v_{oc} , which is the same as V_{Th} , so

$$B_0 = V_{Th} \quad (4.15)$$

When all the internal sources are turned off, $B_0 = 0$. The circuit can then be replaced by an equivalent resistance R_{eq} , which is the same as R_{Th} , and Eq. (4.14) becomes

$$v = A_0i = R_{Th}i \Rightarrow A_0 = R_{Th} \quad (4.16)$$

Substituting the values of A_0 and B_0 in Eq. (4.14) gives

$$v = R_{Th}i + V_{Th} \quad (4.17)$$

which expresses the voltage-current relation at terminals a and b of the circuit in Fig. 4.46(b). Thus, the two circuits in Fig. 4.46(a) and 4.46(b) are equivalent.

When the same linear circuit is driven by a voltage source v as shown in Fig. 4.47(a), the current flowing into the circuit can be obtained by superposition as

$$i = C_0v + D_0 \quad (4.18)$$

where C_0v is the contribution to i due to the external voltage source v and D_0 contains the contributions to i due to all internal independent sources. When the terminals $a-b$ are short-circuited, $v = 0$ so that

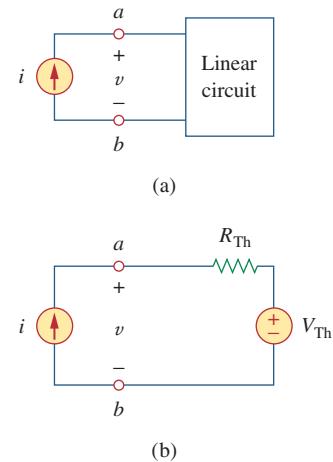


Figure 4.46
Derivation of Thevenin equivalent: (a) a current-driven circuit, (b) its Thevenin equivalent.

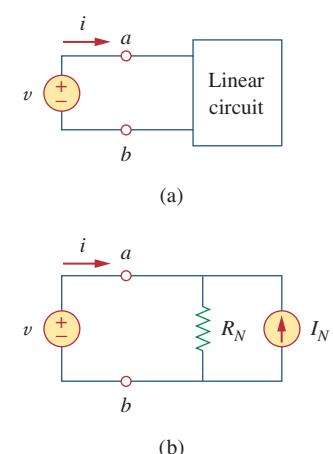


Figure 4.47
Derivation of Norton equivalent: (a) a voltage-driven circuit, (b) its Norton equivalent.

$i = D_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal a , which is the same as the Norton current I_N , i.e.,

$$D_0 = -I_N \quad (4.19)$$

When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance $G_{eq} = 1/R_{eq}$), which is the same as R_{Th} or R_N . Thus Eq. (4.19) becomes

$$i = \frac{v}{R_{Th}} - I_N \quad (4.20)$$

This expresses the voltage-current relation at terminals $a-b$ of the circuit in Fig. 4.47(b), confirming that the two circuits in Fig. 4.47(a) and 4.47(b) are equivalent.

4.8 Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L . If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 4.48, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L \quad (4.21)$$

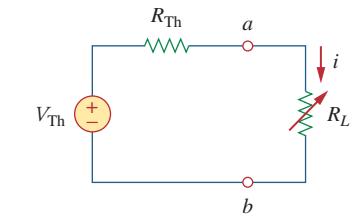


Figure 4.48

The circuit used for maximum power transfer.

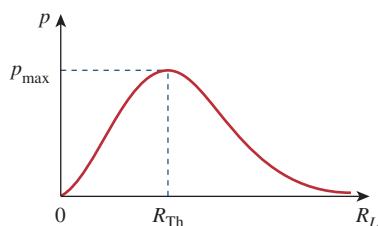


Figure 4.49

Power delivered to the load as a function of R_L .

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.21) with respect to R_L and set the result equal to zero. We obtain

$$\begin{aligned} \frac{dp}{dR_L} &= V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] \\ &= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} \right] = 0 \end{aligned}$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \quad (4.22)$$

which yields

$$R_L = R_{\text{Th}} \quad (4.23)$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th} . We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$p_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} \quad (4.24)$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

The source and load are said to be matched when $R_L = R_{\text{Th}}$.

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

Example 4.13

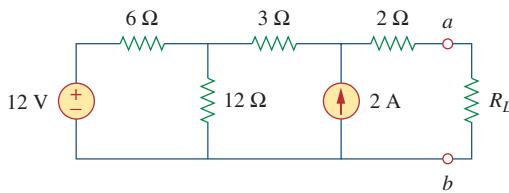


Figure 4.50

For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals $a-b$. To get R_{Th} , we use the circuit in Fig. 4.51(a) and obtain

$$R_{\text{Th}} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9 \Omega$$

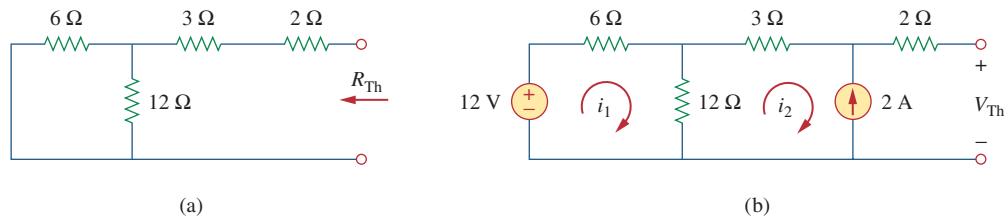


Figure 4.51

For Example 4.13: (a) finding R_{Th} , (b) finding V_{Th} .

To get V_{Th} , we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \quad i_2 = -2 \text{ A}$$

Solving for i_1 , we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals $a-b$, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \Rightarrow V_{\text{Th}} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{\text{Th}} = 9 \Omega$$

and the maximum power is

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Practice Problem 4.13

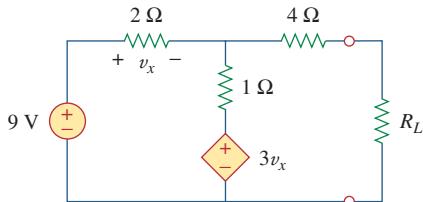


Figure 4.52

For Practice Prob. 4.13.

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.222Ω , 2.901 W .

4.9 Verifying Circuit Theorems with PSpice

In this section, we learn how to use *PSpice* to verify the theorems covered in this chapter. Specifically, we will consider using DC Sweep analysis to find the Thevenin or Norton equivalent at any pair of nodes in a circuit and the maximum power transfer to a load. The reader is advised to read Section D.3 of Appendix D in preparation for this section.

To find the Thevenin equivalent of a circuit at a pair of open terminals using *PSpice*, we use the schematic editor to draw the circuit and insert an independent probing current source, say, I_p , at the terminals. The probing current source must have a part name ISRC. We then perform a DC Sweep on I_p , as discussed in Section D.3. Typically, we may let the current through I_p vary from 0 to 1 A in 0.1-A increments. After saving and simulating the circuit, we use Probe to display a plot of the voltage across I_p versus the current through I_p . The zero intercept of the plot gives us the Thevenin equivalent voltage, while the slope of the plot is equal to the Thevenin resistance.

To find the Norton equivalent involves similar steps except that we insert a probing independent voltage source (with a part name VSRC), say, V_p , at the terminals. We perform a DC Sweep on V_p and let V_p vary from 0 to 1 V in 0.1-V increments. A plot of the current through V_p versus the voltage across V_p is obtained using the Probe menu after simulation. The zero intercept is equal to the Norton current, while the slope of the plot is equal to the Norton conductance.

To find the maximum power transfer to a load using *PSpice* involves performing a DC parametric Sweep on the component value of R_L in Fig. 4.48 and plotting the power delivered to the load as a function of R_L . According to Fig. 4.49, the maximum power occurs

when $R_L = R_{Th}$. This is best illustrated with an example, and Example 4.15 provides one.

We use VSRC and ISRC as part names for the independent voltage and current sources, respectively.

Consider the circuit in Fig. 4.31 (see Example 4.9). Use *PSpice* to find the Thevenin and Norton equivalent circuits.

Example 4.14

Solution:

(a) To find the Thevenin resistance R_{Th} and Thevenin voltage V_{Th} at the terminals $a-b$ in the circuit in Fig. 4.31, we first use Schematics to draw the circuit as shown in Fig. 4.53(a). Notice that a probing current source $I2$ is inserted at the terminals. Under **Analysis/Setup**, we select DC Sweep. In the DC Sweep dialog box, we select Linear for the *Sweep Type* and Current Source for the *Sweep Var. Type*. We enter $I2$ under the *Name* box, 0 as *Start Value*, 1 as *End Value*, and 0.1 as *Increment*. After simulation, we add trace $V(I2:-)$ from the *PSpice A/D* window and obtain the plot shown in Fig. 4.53(b). From the plot, we obtain

$$V_{Th} = \text{Zero intercept} = 20 \text{ V}, \quad R_{Th} = \text{Slope} = \frac{26 - 20}{1} = 6 \Omega$$

These agree with what we got analytically in Example 4.9.

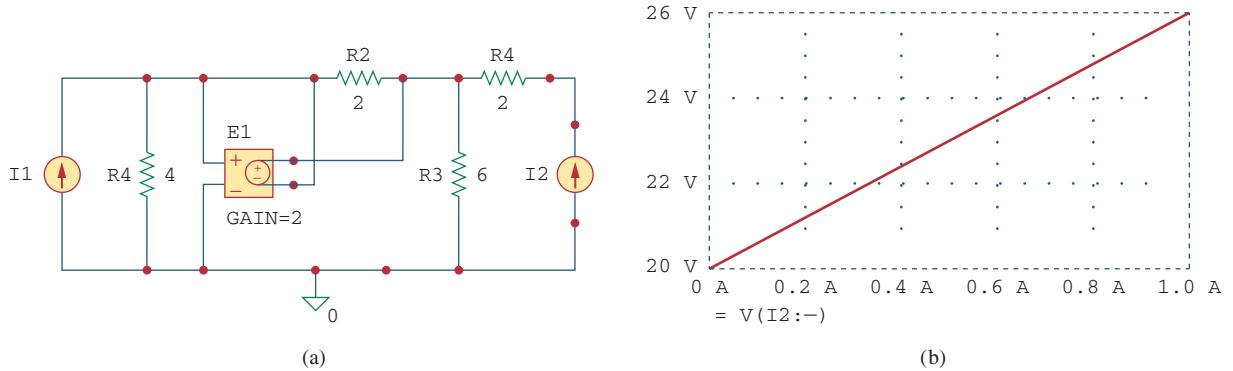


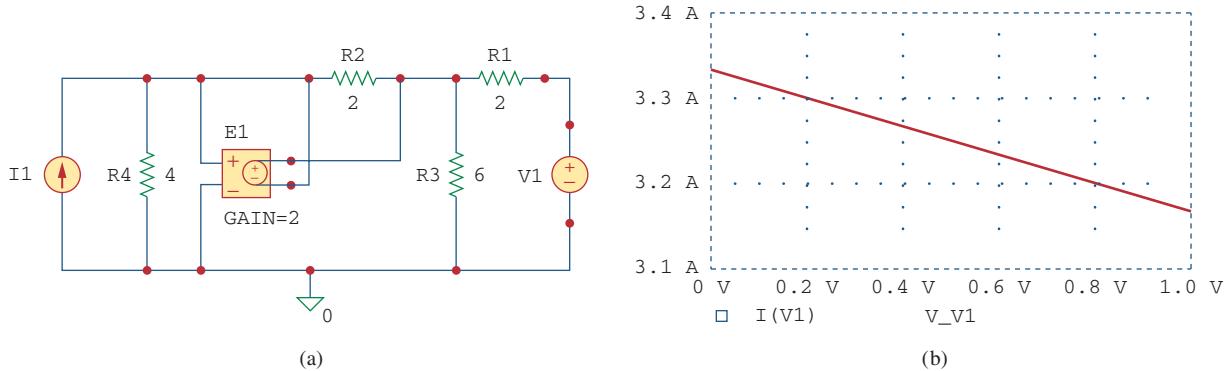
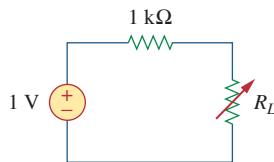
Figure 4.53

For Example 4.14: (a) schematic and (b) plot for finding R_{Th} and V_{Th} .

(b) To find the Norton equivalent, we modify the schematic in Fig. 4.53(a) by replacing the probing current source with a probing voltage source $V1$. The result is the schematic in Fig. 4.54(a). Again, in the DC Sweep dialog box, we select Linear for the *Sweep Type* and Voltage Source for the *Sweep Var. Type*. We enter $V1$ under *Name* box, 0 as *Start Value*, 1 as *End Value*, and 0.1 as *Increment*. Under the *PSpice A/D Window*, we add trace $I(V1)$ and obtain the plot in Fig. 4.54(b). From the plot, we obtain

$$I_N = \text{Zero intercept} = 3.335 \text{ A}$$

$$G_N = \text{Slope} = \frac{3.335 - 3.165}{1} = 0.17 \text{ S}$$

**Figure 4.54**For Example 4.14: (a) schematic and (b) plot for finding G_N and I_N .**Practice Problem 4.14**Rework Practice Prob. 4.9 using *PSpice*.**Answer:** $V_{Th} = 5.333 \text{ V}$, $R_{Th} = 444.4 \text{ m}\Omega$.**Example 4.15**Refer to the circuit in Fig. 4.55. Use *PSpice* to find the maximum power transfer to R_L .**Figure 4.55**

For Example 4.15.

Solution:

We need to perform a DC Sweep on R_L to determine when the power across it is maximum. We first draw the circuit using Schematics as shown in Fig. 4.56. Once the circuit is drawn, we take the following three steps to further prepare the circuit for a DC Sweep.

The first step involves defining the value of R_L as a parameter, since we want to vary it. To do this:

1. **DCLICKL** the value 1k of R2 (representing R_L) to open up the *Set Attribute Value* dialog box.
2. Replace 1k with {RL} and click **OK** to accept the change.

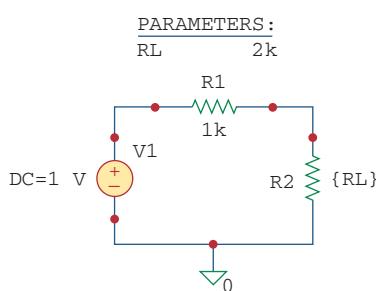
Note that the curly brackets are necessary.

The second step is to define parameter. To achieve this:

1. Select **Draw/Get New Part/Libraries** \cdots /special.slb.
2. Type PARAM in the *PartName* box and click **OK**.
3. **DRAG** the box to any position near the circuit.
4. **CLICKL** to end placement mode.
5. **DCLICKL** to open up the *PartName: PARAM* dialog box.
6. **CLICKL** on *NAME1* = and enter RL (with no curly brackets) in the *Value* box, and **CLICKL Save Attr** to accept change.
7. **CLICKL** on *VALUE1* = and enter 2k in the *Value* box, and **CLICKL Save Attr** to accept change.
8. Click **OK**.

Figure 4.56

Schematic for the circuit in Fig. 4.55.



The value 2k in item 7 is necessary for a bias point calculation; it cannot be left blank.

The third step is to set up the DC Sweep to sweep the parameter. To do this:

1. Select **Analysis/Setup** to bring up the DC Sweep dialog box.
2. For the *Sweep Type*, select Linear (or Octave for a wide range of R_L).
3. For the *Sweep Var. Type*, select Global Parameter.
4. Under the *Name* box, enter RL .
5. In the *Start Value* box, enter 100.
6. In the *End Value* box, enter 5k.
7. In the *Increment* box, enter 100.
8. Click **OK** and **Close** to accept the parameters.

After taking these steps and saving the circuit, we are ready to simulate. Select **Analysis/Simulate**. If there are no errors, we select **Add Trace** in the PSpice A/D window and type $-V(R2:2)*I(R2)$ in the *Trace Command* box. [The negative sign is needed since $I(R2)$ is negative.] This gives the plot of the power delivered to R_L as R_L varies from 100Ω to $5 \text{ k}\Omega$. We can also obtain the power absorbed by R_L by typing $V(R2:2)*V(R2:2)/RL$ in the *Trace Command* box. Either way, we obtain the plot in Fig. 4.57. It is evident from the plot that the maximum power is $250 \mu\text{W}$. Notice that the maximum occurs when $R_L = 1 \text{ k}\Omega$, as expected analytically.

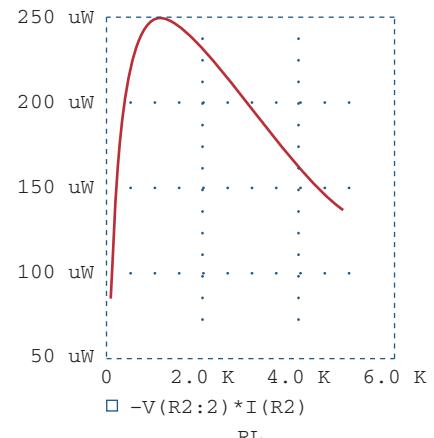


Figure 4.57

For Example 4.15: the plot of power across R_L .

Find the maximum power transferred to R_L if the $1\text{-k}\Omega$ resistor in Fig. 4.55 is replaced by a $2\text{-k}\Omega$ resistor.

Practice Problem 4.15

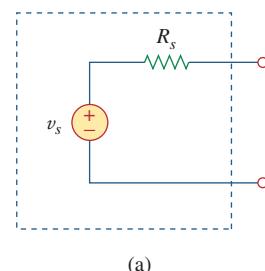
Answer: $125 \mu\text{W}$.

4.10 [†]Applications

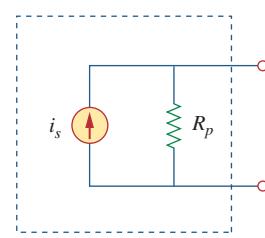
In this section we will discuss two important practical applications of the concepts covered in this chapter: source modeling and resistance measurement.

4.10.1 Source Modeling

Source modeling provides an example of the usefulness of the Thevenin or the Norton equivalent. An active source such as a battery is often characterized by its Thevenin or Norton equivalent circuit. An ideal voltage source provides a constant voltage irrespective of the current drawn by the load, while an ideal current source supplies a constant current regardless of the load voltage. As Fig. 4.58 shows, practical voltage and current sources are not ideal, due to their *internal resistances* or *source resistances* R_s and R_p . They become ideal as $R_s \rightarrow 0$ and $R_p \rightarrow \infty$. To show that this is the case, consider the effect



(a)



(b)

Figure 4.58

(a) Practical voltage source, (b) practical current source.

of the load on voltage sources, as shown in Fig. 4.59(a). By the voltage division principle, the load voltage is

$$v_L = \frac{R_L}{R_s + R_L} v_s \quad (4.25)$$

As R_L increases, the load voltage approaches a source voltage v_s , as illustrated in Fig. 4.59(b). From Eq. (4.25), we should note that:

1. The load voltage will be constant if the internal resistance R_s of the source is zero or, at least, $R_s \ll R_L$. In other words, the smaller R_s is compared with R_L , the closer the voltage source is to being ideal.

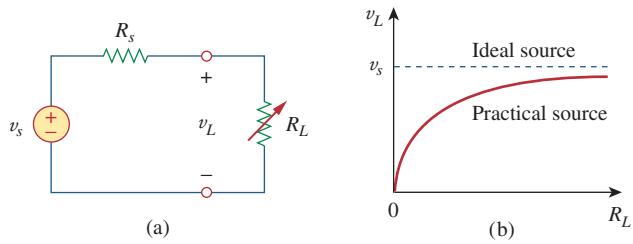


Figure 4.59

(a) Practical voltage source connected to a load R_L , (b) load voltage decreases as R_L decreases.

2. When the load is disconnected (i.e., the source is open-circuited so that $R_L \rightarrow \infty$), $v_{oc} = v_s$. Thus, v_s may be regarded as the *unloaded source* voltage. The connection of the load causes the terminal voltage to drop in magnitude; this is known as the *loading effect*.

The same argument can be made for a practical current source when connected to a load as shown in Fig. 4.60(a). By the current division principle,

$$i_L = \frac{R_p}{R_p + R_L} i_s \quad (4.26)$$

Figure 4.60(b) shows the variation in the load current as the load resistance increases. Again, we notice a drop in current due to the load (loading effect), and load current is constant (ideal current source) when the internal resistance is very large (i.e., $R_p \rightarrow \infty$ or, at least, $R_p \gg R_L$).

Sometimes, we need to know the unloaded source voltage v_s and the internal resistance R_s of a voltage source. To find v_s and R_s , we follow the procedure illustrated in Fig. 4.61. First, we measure the open-circuit voltage v_{oc} as in Fig. 4.61(a) and set

$$v_s = v_{oc} \quad (4.27)$$

Then, we connect a variable load R_L across the terminals as in Fig. 4.61(b). We adjust the resistance R_L until we measure a load voltage of exactly one-half of the open-circuit voltage, $v_L = v_{oc}/2$, because now $R_L = R_{Th} = R_s$. At that point, we disconnect R_L and measure it. We set

$$R_s = R_L \quad (4.28)$$

For example, a car battery may have $v_s = 12 \text{ V}$ and $R_s = 0.05 \Omega$.

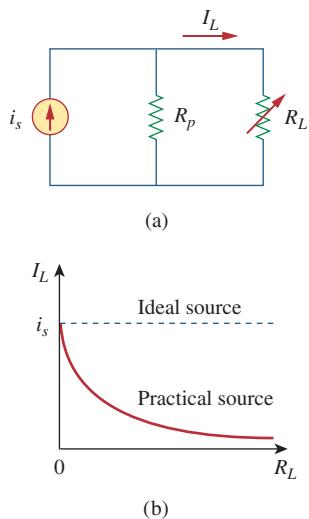


Figure 4.60

(a) Practical current source connected to a load R_L , (b) load current decreases as R_L increases.

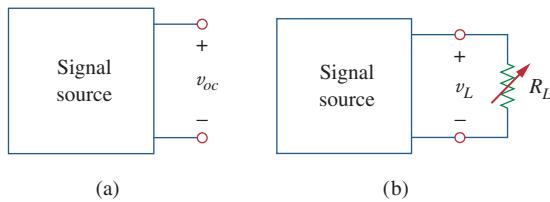


Figure 4.61

(a) Measuring v_{oc} , (b) measuring v_L .

The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V. (a) Calculate the source voltage v_s and internal resistance R_s . (b) Determine the voltage when an 8- Ω load is connected to the source.

Example 4.16

Solution:

- (a) We replace the source by its Thevenin equivalent. The terminal voltage when the load is disconnected is the open-circuit voltage,

$$v_s = v_{oc} = 12.4 \text{ V}$$

When the load is connected, as shown in Fig. 4.62(a), $v_L = 12 \text{ V}$ and $p_L = 2 \text{ W}$. Hence,

$$p_L = \frac{v_L^2}{R_L} \quad \Rightarrow \quad R_L = \frac{v_L^2}{p_L} = \frac{12^2}{2} = 72 \Omega$$

The load current is

$$i_L = \frac{v_L}{R_L} = \frac{12}{72} = \frac{1}{6} \text{ A}$$

The voltage across R_s is the difference between the source voltage v_s and the load voltage v_L , or

$$12.4 - 12 = 0.4 = R_s i_L, \quad R_s = \frac{0.4}{I_L} = 2.4 \Omega$$

(b) Now that we have the Thevenin equivalent of the source, we connect the $8\text{-}\Omega$ load across the Thevenin equivalent as shown in Fig. 4.62(b). Using voltage division, we obtain

$$v = \frac{8}{8 + 24}(12.4) = 9.538 \text{ V}$$

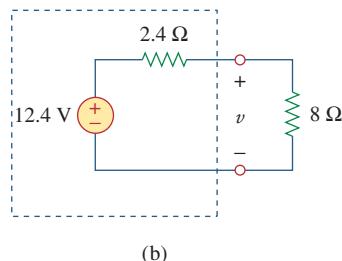
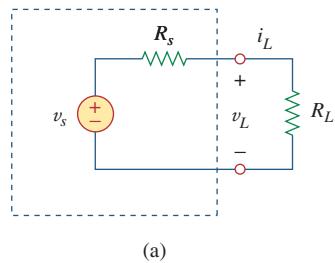


Figure 4.62

For Example 4.16.

The measured open-circuit voltage across a certain amplifier is 9 V. The voltage drops to 8 V when a $20\text{-}\Omega$ loudspeaker is connected to the amplifier. Calculate the voltage when a $10\text{-}\Omega$ loudspeaker is used instead.

Practice Problem 4.16

Answer: 7.2 V.

4.10.2 Resistance Measurement

Although the ohmmeter method provides the simplest way to measure resistance, more accurate measurement may be obtained using the Wheatstone bridge. While ohmmeters are designed to measure resistance in low, mid, or high range, a Wheatstone bridge is used to measure resistance in the mid range, say, between $1\ \Omega$ and $1\ M\Omega$. Very low values of resistances are measured with a *milliohmmeter*, while very high values are measured with a *Megger tester*.

Historical note: The bridge was invented by Charles Wheatstone (1802–1875), a British professor who also invented the telegraph, as Samuel Morse did independently in the United States.

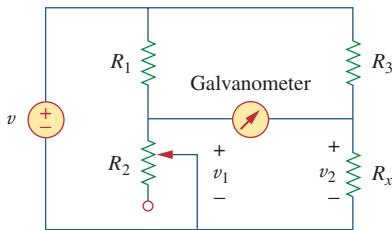


Figure 4.63

The Wheatstone bridge; R_x is the resistance to be measured.

The Wheatstone bridge (or resistance bridge) circuit is used in a number of applications. Here we will use it to measure an unknown resistance. The unknown resistance R_x is connected to the bridge as shown in Fig. 4.63. The variable resistance is adjusted until no current flows through the galvanometer, which is essentially a d'Arsonval movement operating as a sensitive current-indicating device like an ammeter in the microamp range. Under this condition $v_1 = v_2$, and the bridge is said to be *balanced*. Since no current flows through the galvanometer, R_1 and R_2 behave as though they were in series; so do R_3 and R_x . The fact that no current flows through the galvanometer also implies that $v_1 = v_2$. Applying the voltage division principle,

$$v_1 = \frac{R_2}{R_1 + R_2}v = v_2 = \frac{R_x}{R_3 + R_x}v \quad (4.29)$$

Hence, no current flows through the galvanometer when

$$\frac{R_2}{R_1 + R_2} = \frac{R_x}{R_3 + R_x} \Rightarrow R_2R_3 = R_1R_x$$

or

$$R_x = \frac{R_3}{R_1}R_2 \quad (4.30)$$

If $R_1 = R_3$, and R_2 is adjusted until no current flows through the galvanometer, then $R_x = R_2$.

How do we find the current through the galvanometer when the Wheatstone bridge is *unbalanced*? We find the Thevenin equivalent (V_{Th} and R_{Th}) with respect to the galvanometer terminals. If R_m is the resistance of the galvanometer, the current through it under the unbalanced condition is

$$I = \frac{V_{Th}}{R_{Th} + R_m} \quad (4.31)$$

Example 4.18 will illustrate this.

Example 4.17

In Fig. 4.63, $R_1 = 500\ \Omega$ and $R_3 = 200\ \Omega$. The bridge is balanced when R_2 is adjusted to be $125\ \Omega$. Determine the unknown resistance R_x .

Solution:

Using Eq. (4.30) gives

$$R_x = \frac{R_3}{R_1}R_2 = \frac{200}{500}125 = 50\ \Omega$$

A Wheatstone bridge has $R_1 = R_3 = 1\text{ k}\Omega$. R_2 is adjusted until no current flows through the galvanometer. At that point, $R_2 = 3.2\text{ k}\Omega$. What is the value of the unknown resistance?

Practice Problem 4.17

Answer: $3.2\text{ k}\Omega$.

The circuit in Fig. 4.64 represents an unbalanced bridge. If the galvanometer has a resistance of $40\ \Omega$, find the current through the galvanometer.

Example 4.18

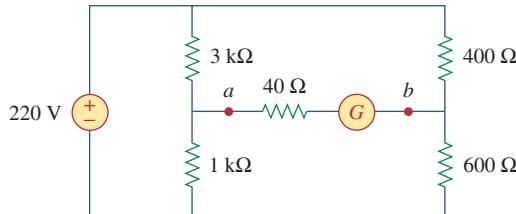


Figure 4.64

Unbalanced bridge of Example 4.18.

Solution:

We first need to replace the circuit by its Thevenin equivalent at terminals a and b . The Thevenin resistance is found using the circuit in Fig. 4.65(a). Notice that the $3\text{-k}\Omega$ and $1\text{-k}\Omega$ resistors are in parallel; so are the $400\text{-}\Omega$ and $600\text{-}\Omega$ resistors. The two parallel combinations form a series combination with respect to terminals a and b . Hence,

$$\begin{aligned} R_{\text{Th}} &= 3000 \parallel 1000 + 400 \parallel 600 \\ &= \frac{3000 \times 1000}{3000 + 1000} + \frac{400 \times 600}{400 + 600} = 750 + 240 = 990\ \Omega \end{aligned}$$

To find the Thevenin voltage, we consider the circuit in Fig. 4.65(b). Using the voltage division principle gives

$$v_1 = \frac{1000}{1000 + 3000}(220) = 55\ \text{V}, \quad v_2 = \frac{600}{600 + 400}(220) = 132\ \text{V}$$

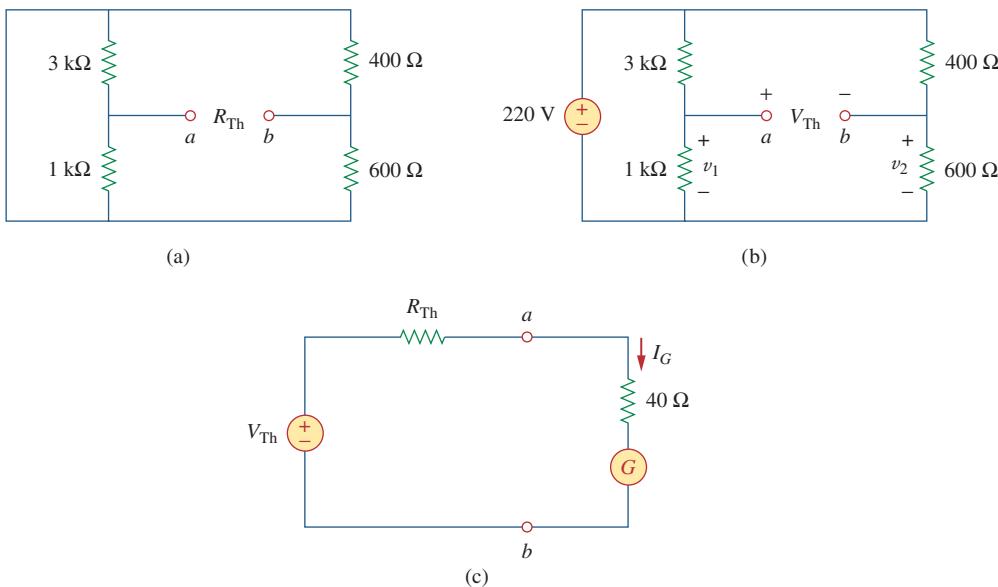
Applying KVL around loop ab gives

$$-v_1 + V_{\text{Th}} + v_2 = 0 \quad \text{or} \quad V_{\text{Th}} = v_1 - v_2 = 55 - 132 = -77\ \text{V}$$

Having determined the Thevenin equivalent, we find the current through the galvanometer using Fig. 4.65(c).

$$I_G = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_m} = \frac{-77}{990 + 40} = -74.76\ \text{mA}$$

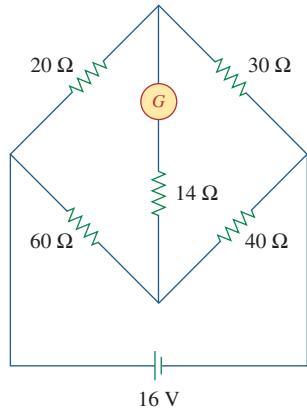
The negative sign indicates that the current flows in the direction opposite to the one assumed, that is, from terminal b to terminal a .

**Figure 4.65**

For Example 4.18: (a) Finding R_{Th} , (b) finding V_{Th} , (c) determining the current through the galvanometer.

Practice Problem 4.18

Obtain the current through the galvanometer, having a resistance of 14Ω , in the Wheatstone bridge shown in Fig. 4.66.

**Figure 4.66**

For Practice Prob. 4.18.

Answer: 64 mA.

4.11 Summary

1. A linear network consists of linear elements, linear dependent sources, and linear independent sources.
2. Network theorems are used to reduce a complex circuit to a simpler one, thereby making circuit analysis much simpler.
3. The superposition principle states that for a circuit having multiple independent sources, the voltage across (or current through) an element is equal to the algebraic sum of all the individual voltages (or currents) due to each independent source acting one at a time.
4. Source transformation is a procedure for transforming a voltage source in series with a resistor to a current source in parallel with a resistor, or vice versa.
5. Thevenin's and Norton's theorems allow us to isolate a portion of a network while the remaining portion of the network is replaced by an equivalent network. The Thevenin equivalent consists of a voltage source V_{Th} in series with a resistor R_{Th} , while the Norton equivalent consists of a current source I_N in parallel with a resistor R_N . The two theorems are related by source transformation.

$$R_N = R_{\text{Th}}, \quad I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$

- For a given Thevenin equivalent circuit, maximum power transfer occurs when $R_L = R_{Th}$; that is, when the load resistance is equal to the Thevenin resistance.
 - The maximum power transfer theorem states that the maximum power is delivered by a source to the load R_L when R_L is equal to R_{Th} , the Thevenin resistance at the terminals of the load.
 - PSpice* can be used to verify the circuit theorems covered in this chapter.
 - Source modeling and resistance measurement using the Wheatstone bridge provide applications for Thevenin's theorem.

Review Questions

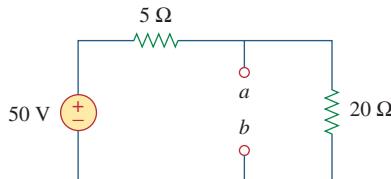


Figure 4.67

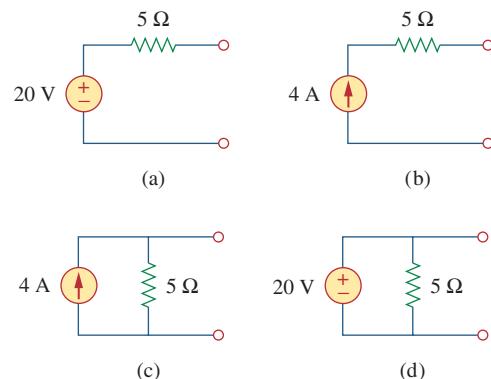


Figure 4.68

Answers: 4.1b, 4.2a, 4.3b, 4.4d, 4.5b, 4.6a, 4.7a, 4.8c, 4.9c, 4.10a.

Problems

Section 4.2 Linearity Property

- 4.1** Calculate the current i_o in the circuit of Fig. 4.69. What value of input voltage is necessary to make i_o equal to 5 amps?

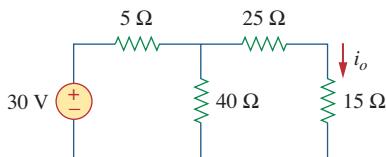


Figure 4.69

For Prob. 4.1.

- 4.2** Using Fig. 4.70, design a problem to help other **end** students better understand linearity.

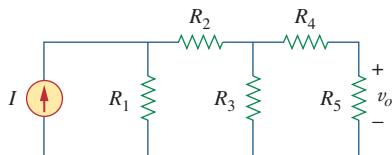


Figure 4.70

For Prob. 4.2.

- 4.3** (a) In the circuit of Fig. 4.71, calculate v_o and i_o when $v_s = 1$ V.
 (b) Find v_o and i_o when $v_s = 10$ V.
 (c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

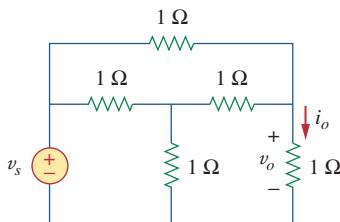


Figure 4.71

For Prob. 4.3.

- 4.4** Use linearity to determine i_o in the circuit of Fig. 4.72.

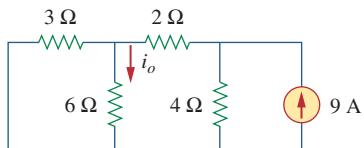


Figure 4.72

For Prob. 4.4.

- 4.5** For the circuit in Fig. 4.73, assume $v_o = 1$ V, and use linearity to find the actual value of v_o .

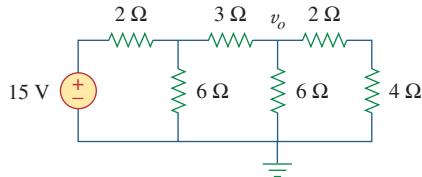


Figure 4.73

For Prob. 4.5.

- 4.6** For the linear circuit shown in Fig. 4.74, use linearity to complete the following table.

Experiment	V_s	V_o
1		12 V
2		16 V
3		1 V
4		-2 V

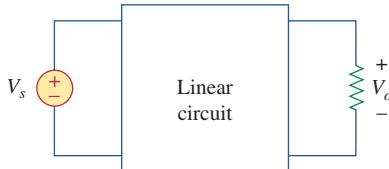


Figure 4.74

For Prob. 4.6.

- 4.7** Use linearity and the assumption that $V_o = 1$ V to find the actual value of V_o in Fig. 4.75.

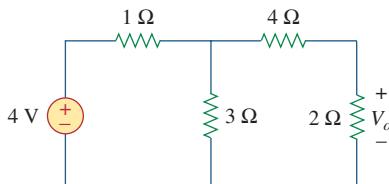


Figure 4.75

For Prob. 4.7.

Section 4.3 Superposition

- 4.8** Using superposition, find V_o in the circuit of Fig. 4.76. Check with *PSpice or MultiSim*.

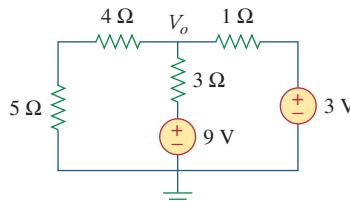
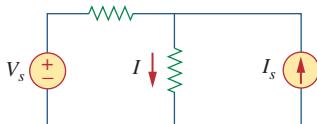


Figure 4.76

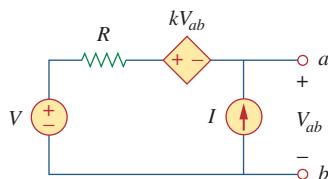
For Prob. 4.8.

- 4.9** Given that $I = 4$ amps when $V_s = 40$ volts and $I_s = 4$ amps and $I = 1$ amp when $V_s = 20$ volts and $I_s = 0$, use superposition and linearity to determine the value of I when $V_s = 60$ volts and $I_s = -2$ amps.

**Figure 4.77**

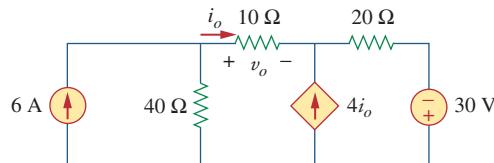
For Prob. 4.9.

- 4.10** Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter k is a gain you can specify to make the problem easier to solve but must not be zero.

**Figure 4.78**

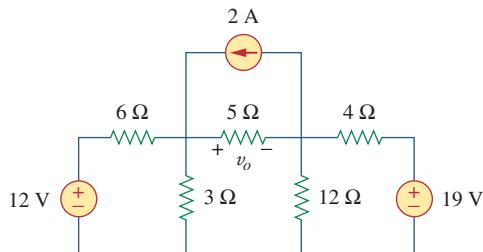
For Prob. 4.10.

- 4.11** Use the superposition principle to find i_o and v_o in the circuit of Fig. 4.79.

**Figure 4.79**

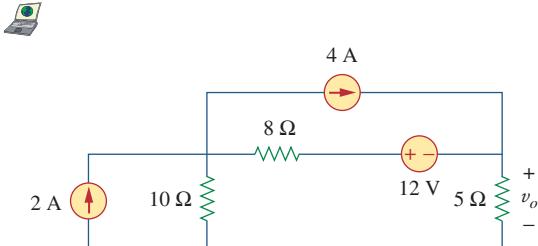
For Prob. 4.11.

- 4.12** Determine v_o in the circuit of Fig. 4.80 using the superposition principle.

**Figure 4.80**

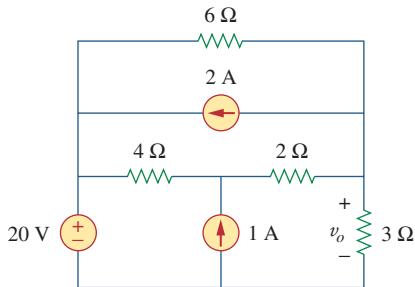
For Prob. 4.12.

- 4.13** Use superposition to find v_o in the circuit of Fig. 4.81.

**Figure 4.81**

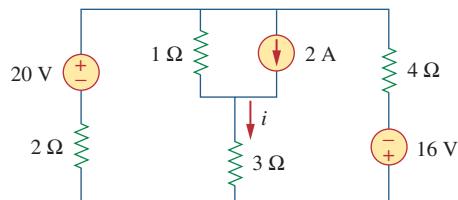
For Prob. 4.13.

- 4.14** Apply the superposition principle to find v_o in the circuit of Fig. 4.82.

**Figure 4.82**

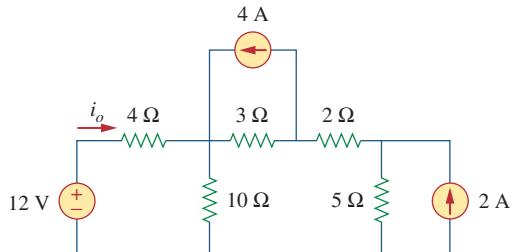
For Prob. 4.14.

- 4.15** For the circuit in Fig. 4.83, use superposition to find i . Calculate the power delivered to the 3Ω resistor.

**Figure 4.83**

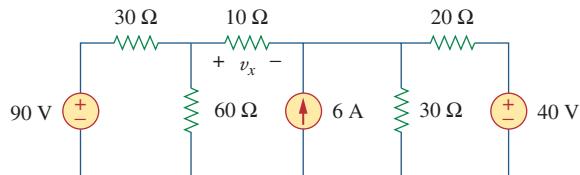
For Probs. 4.15 and 4.56.

- 4.16** Given the circuit in Fig. 4.84, use superposition to obtain i_o .

**Figure 4.84**

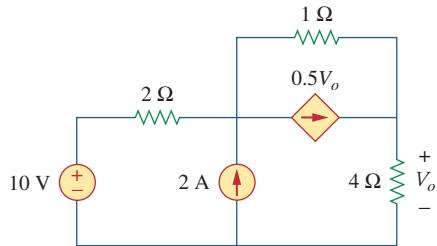
For Prob. 4.16.

- 4.17** Use superposition to obtain v_x in the circuit of Fig. 4.85. Check your result using PSpice or **ML MultiSim**.

**Figure 4.85**

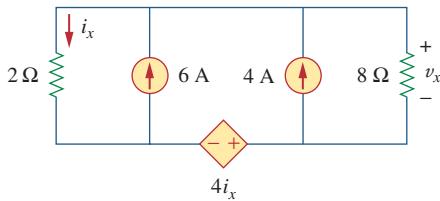
For Prob. 4.17.

- 4.18** Use superposition to find V_o in the circuit of Fig. 4.86.

**Figure 4.86**

For Prob. 4.18.

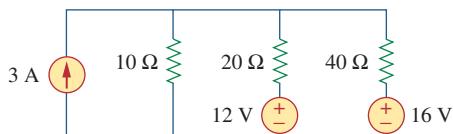
- 4.19** Use superposition to solve for v_x in the circuit of Fig. 4.87.

**Figure 4.87**

For Prob. 4.19.

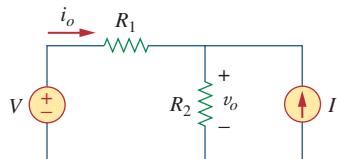
Section 4.4 Source Transformation

- 4.20** Use source transformation to reduce the circuit in Fig. 4.88 to a single voltage source in series with a single resistor.

**Figure 4.88**

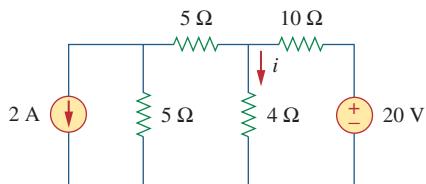
For Prob. 4.20.

- 4.21** Using Fig. 4.89, design a problem to help other **e2d** students better understand source transformation.

**Figure 4.89**

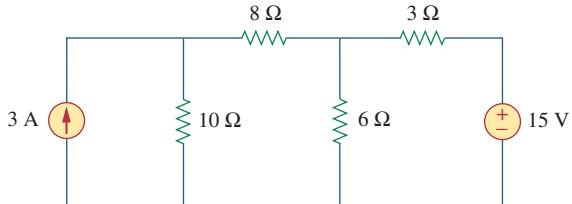
For Prob. 4.21.

- 4.22** For the circuit in Fig. 4.90, use source transformation to find i .

**Figure 4.90**

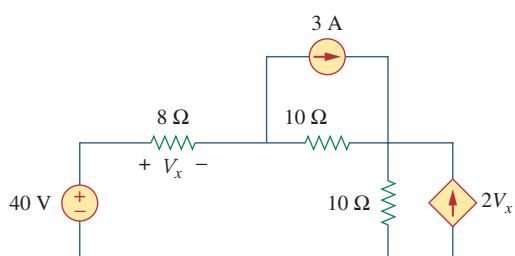
For Prob. 4.22.

- 4.23** Referring to Fig. 4.91, use source transformation to determine the current and power absorbed by the 8-Ω resistor.

**Figure 4.91**

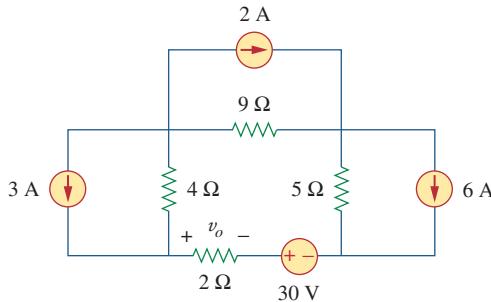
For Prob. 4.23.

- 4.24** Use source transformation to find the voltage V_x in the circuit of Fig. 4.92.

**Figure 4.92**

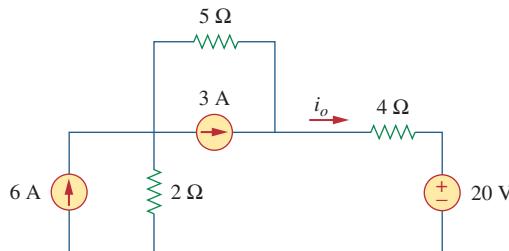
For Prob. 4.24.

- 4.25** Obtain v_o in the circuit of Fig. 4.93 using source transformation. Check your result using PSpice or MultiSim.

**Figure 4.93**

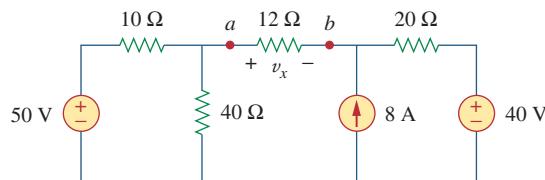
For Prob. 4.25.

- 4.26** Use source transformation to find i_o in the circuit of Fig. 4.94.

**Figure 4.94**

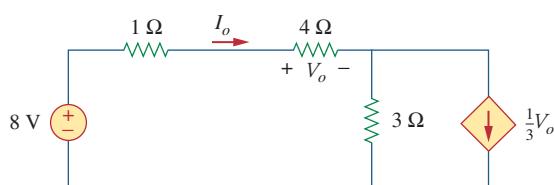
For Prob. 4.26.

- 4.27** Apply source transformation to find v_x in the circuit of Fig. 4.95.

**Figure 4.95**

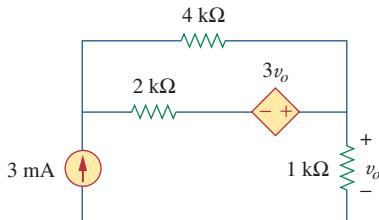
For Probs. 4.27 and 4.40.

- 4.28** Use source transformation to find I_o in Fig. 4.96.

**Figure 4.96**

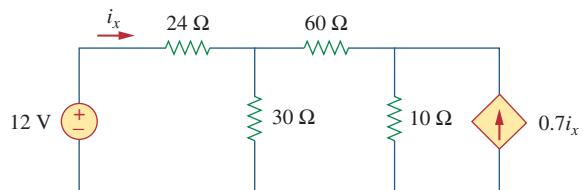
For Prob. 4.28.

- 4.29** Use source transformation to find v_o in the circuit of Fig. 4.97.

**Figure 4.97**

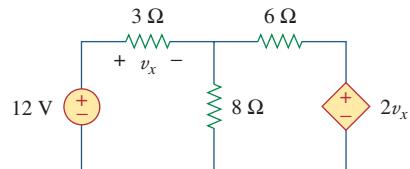
For Prob. 4.29.

- 4.30** Use source transformation on the circuit shown in Fig. 4.98 to find i_x .

**Figure 4.98**

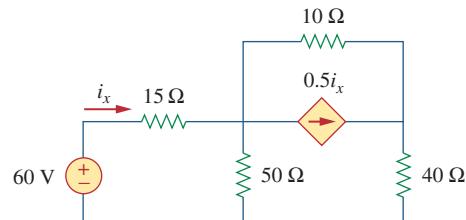
For Prob. 4.30.

- 4.31** Determine v_x in the circuit of Fig. 4.99 using source transformation.

**Figure 4.99**

For Prob. 4.31.

- 4.32** Use source transformation to find i_x in the circuit of Fig. 4.100.

**Figure 4.100**

For Prob. 4.32.

Sections 4.5 and 4.6 Thevenin's and Norton's Theorems

- 4.33** Determine the Thevenin equivalent circuit, shown in Fig. 4.101, as seen by the 5- Ω resistor.
Then calculate the current flowing through the 5- Ω resistor.

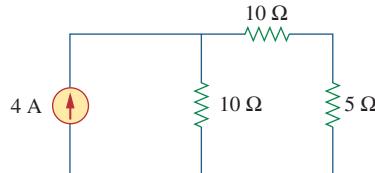


Figure 4.101
For Prob. 4.33.

- 4.34** Using Fig. 4.102, design a problem that will help other students better understand Thevenin equivalent circuits.

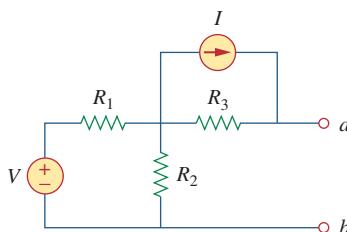


Figure 4.102
For Probs. 4.34 and 4.49.

- 4.35** Use Thevenin's theorem to find v_o in Prob. 4.12.

- 4.36** Solve for the current i in the circuit of Fig. 4.103 using Thevenin's theorem. (Hint: Find the Thevenin equivalent seen by the 12- Ω resistor.)

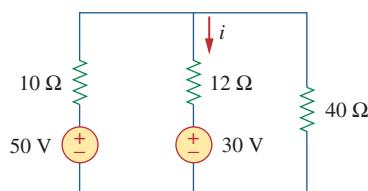


Figure 4.103
For Prob. 4.36.

- 4.37** Find the Norton equivalent with respect to terminals a - b in the circuit shown in Fig. 4.104.

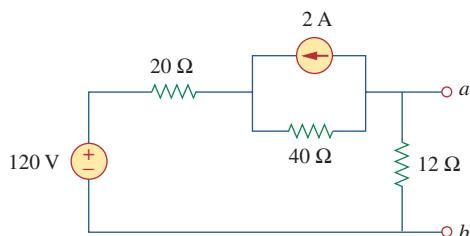


Figure 4.104
For Prob. 4.37.

- 4.38** Apply Thevenin's theorem to find V_o in the circuit of Fig. 4.105.

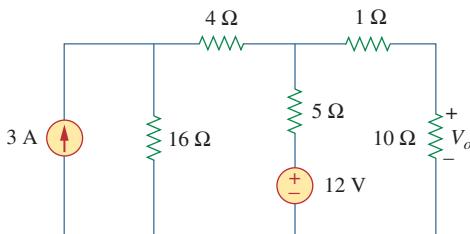


Figure 4.105
For Prob. 4.38.

- 4.39** Obtain the Thevenin equivalent at terminals a - b of the circuit shown in Fig. 4.106.

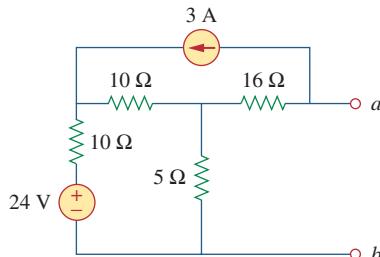


Figure 4.106
For Prob. 4.39.

- 4.40** Find the Thevenin equivalent at terminals a - b of the circuit in Fig. 4.107.

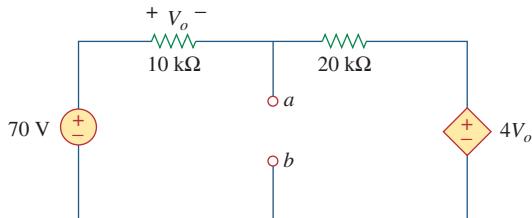
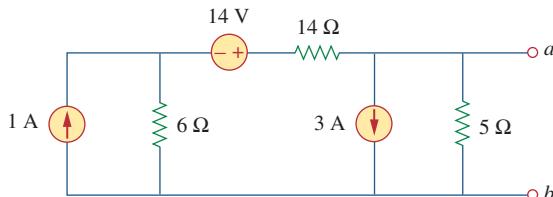


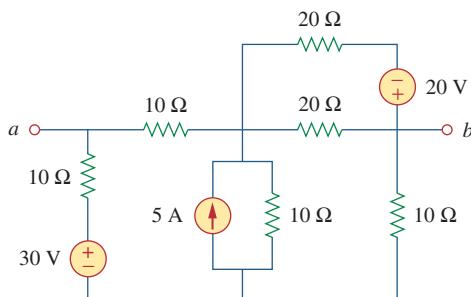
Figure 4.107
For Prob. 4.40.

- 4.41** Find the Thevenin and Norton equivalents at terminals *a*-*b* of the circuit shown in Fig. 4.108.

**Figure 4.108**

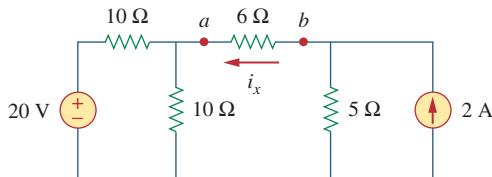
For Prob. 4.41.

- *4.42** For the circuit in Fig. 4.109, find the Thevenin equivalent between terminals *a* and *b*.

**Figure 4.109**

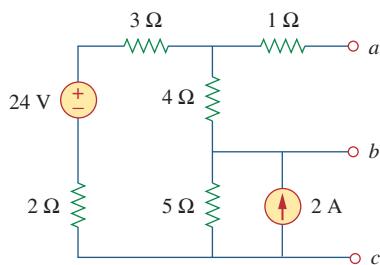
For Prob. 4.42.

- 4.43** Find the Thevenin equivalent looking into terminals *a*-*b* of the circuit in Fig. 4.110 and solve for i_x .

**Figure 4.110**

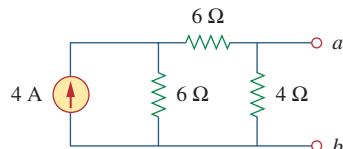
For Prob. 4.43.

- 4.44** For the circuit in Fig. 4.111, obtain the Thevenin equivalent as seen from terminals:

(a) *a*-*b*(b) *b*-*c***Figure 4.111**

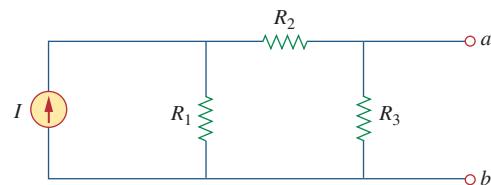
For Prob. 4.44.

- 4.45** Find the Thevenin equivalent of the circuit in Fig. 4.112 as seen by looking into terminals *a* and *b*.

**Figure 4.112**

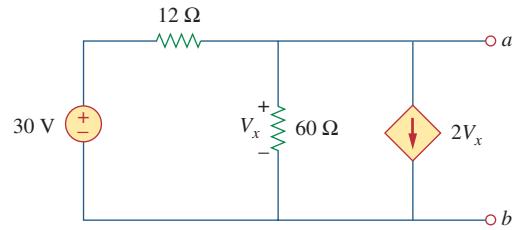
For Prob. 4.45.

- 4.46** Using Fig. 4.113, design a problem to help other **e2d** students better understand Norton equivalent circuits.

**Figure 4.113**

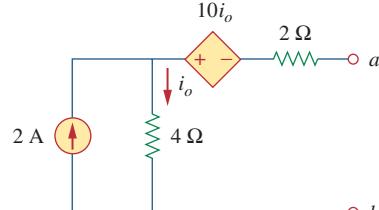
For Prob. 4.46.

- 4.47** Obtain the Thevenin and Norton equivalent circuits of the circuit in Fig. 4.114 with respect to terminals *a* and *b*.

**Figure 4.114**

For Prob. 4.47.

- 4.48** Determine the Norton equivalent at terminals *a*-*b* for the circuit in Fig. 4.115.

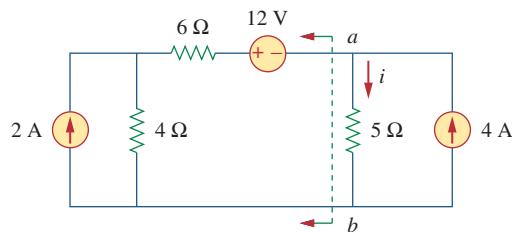
**Figure 4.115**

For Prob. 4.48.

- 4.49** Find the Norton equivalent looking into terminals *a*-*b* of the circuit in Fig. 4.102. Let $V = 40 \text{ V}$, $I = 3 \text{ A}$, $R_1 = 10 \Omega$, $R_2 = 40 \Omega$, and $R_3 = 20 \Omega$.

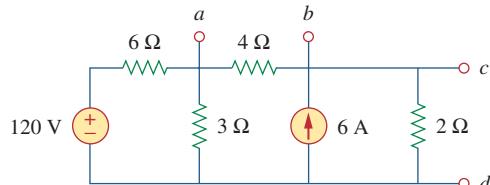
* An asterisk indicates a challenging problem.

- 4.50** Obtain the Norton equivalent of the circuit in Fig. 4.116 to the left of terminals *a*-*b*. Use the result to find current *i*.

**Figure 4.116**

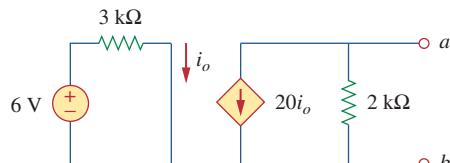
For Prob. 4.50.

- 4.51** Given the circuit in Fig. 4.117, obtain the Norton equivalent as viewed from terminals:

(a) *a*-*b*(b) *c*-*d***Figure 4.117**

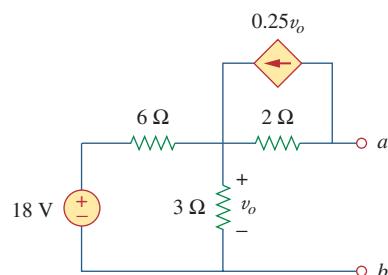
For Prob. 4.51.

- 4.52** For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals *a*-*b*.

**Figure 4.118**

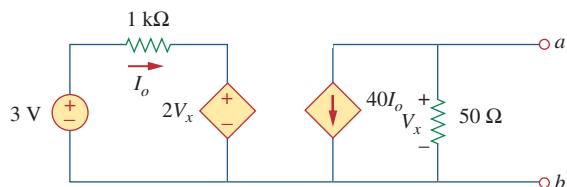
For Prob. 4.52.

- 4.53** Find the Norton equivalent at terminals *a*-*b* of the circuit in Fig. 4.119.

**Figure 4.119**

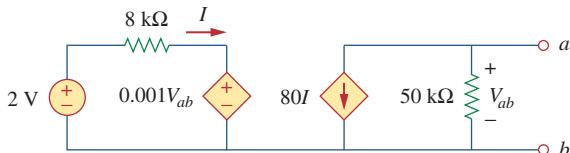
For Prob. 4.53.

- 4.54** Find the Thevenin equivalent between terminals *a*-*b* of the circuit in Fig. 4.120.

**Figure 4.120**

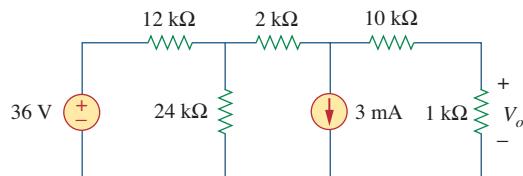
For Prob. 4.54.

- *4.55** Obtain the Norton equivalent at terminals *a*-*b* of the circuit in Fig. 4.121.

**Figure 4.121**

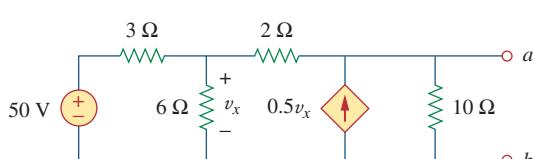
For Prob. 4.55.

- 4.56** Use Norton's theorem to find V_o in the circuit of Fig. 4.122.

**Figure 4.122**

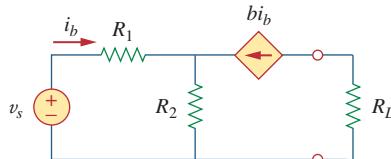
For Prob. 4.56.

- 4.57** Obtain the Thevenin and Norton equivalent circuits at terminals *a*-*b* for the circuit in Fig. 4.123.

**Figure 4.123**

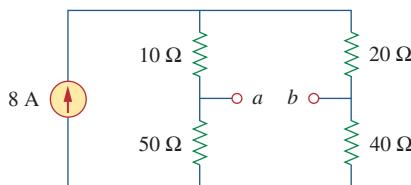
For Probs. 4.57 and 4.79.

- 4.58** The network in Fig. 4.124 models a bipolar transistor common-emitter amplifier connected to a load. Find the Thevenin resistance seen by the load.

**Figure 4.124**

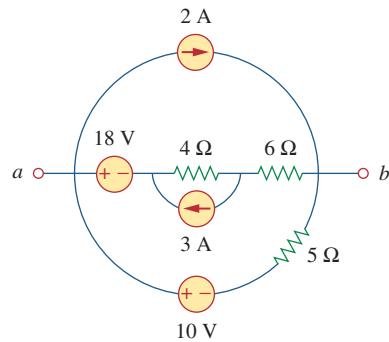
For Prob. 4.58.

- 4.59** Determine the Thevenin and Norton equivalents at terminals *a-b* of the circuit in Fig. 4.125.

**Figure 4.125**

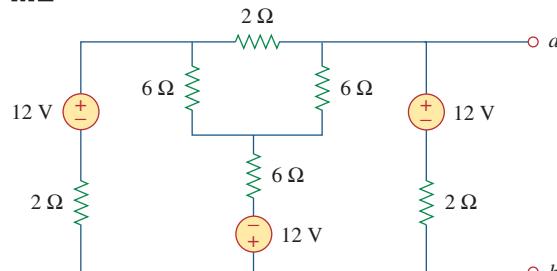
For Probs. 4.59 and 4.80.

- *4.60** For the circuit in Fig. 4.126, find the Thevenin and Norton equivalent circuits at terminals *a-b*.

**Figure 4.126**

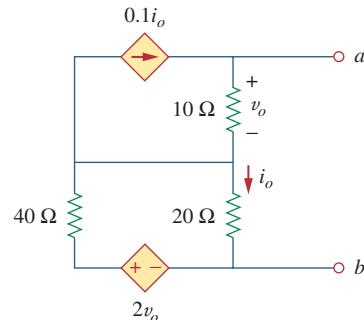
For Probs. 4.60 and 4.81.

- *4.61** Obtain the Thevenin and Norton equivalent circuits at terminals *a-b* of the circuit in Fig. 4.127.

ML**Figure 4.127**

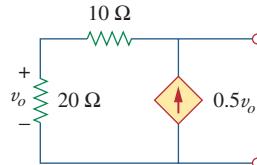
For Prob. 4.61.

- *4.62** Find the Thevenin equivalent of the circuit in Fig. 4.128.

ML**Figure 4.128**

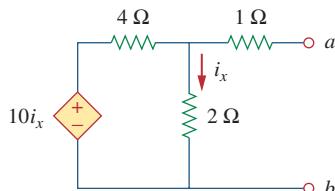
For Prob. 4.62.

- 4.63** Find the Norton equivalent for the circuit in Fig. 4.129.

**Figure 4.129**

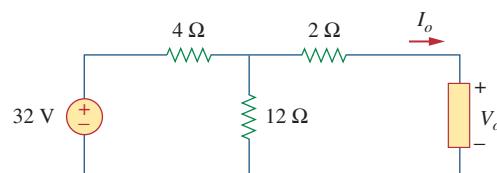
For Prob. 4.63.

- 4.64** Obtain the Thevenin equivalent seen at terminals *a-b* of the circuit in Fig. 4.130.

**Figure 4.130**

For Prob. 4.64.

- 4.65** For the circuit shown in Fig. 4.131, determine the relationship between V_o and I_o .

**Figure 4.131**

For Prob. 4.65.

Section 4.8 Maximum Power Transfer

- 4.66** Find the maximum power that can be delivered to the resistor R in the circuit of Fig. 4.132.

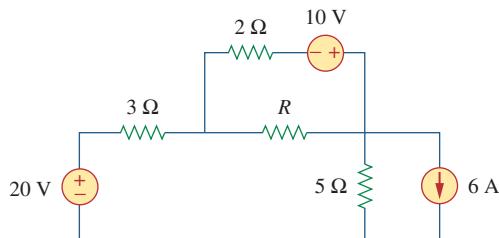


Figure 4.132

For Prob. 4.66.

- 4.67** The variable resistor R in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit.
 (a) Calculate the value of R for maximum power.
 (b) Determine the maximum power absorbed by R .

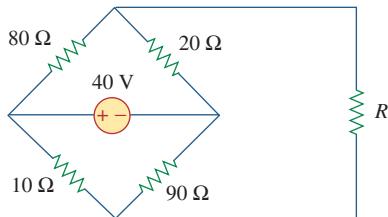


Figure 4.133

For Prob. 4.67.

- *4.68** Compute the value of R that results in maximum power transfer to the $10\text{-}\Omega$ resistor in Fig. 4.134. Find the maximum power.

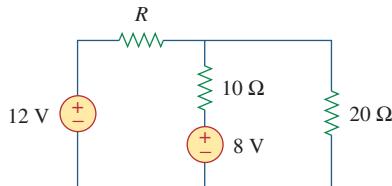


Figure 4.134

For Prob. 4.68.

- 4.69** Find the maximum power transferred to resistor R in the circuit of Fig. 4.135.

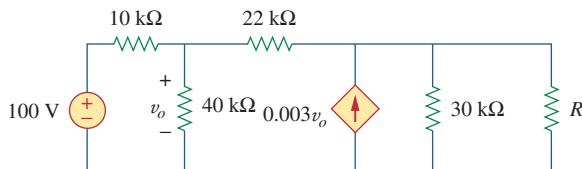


Figure 4.135

For Prob. 4.69.

- 4.70** Determine the maximum power delivered to the variable resistor R shown in the circuit of Fig. 4.136.

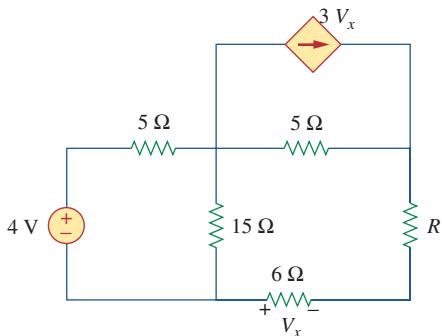


Figure 4.136

For Prob. 4.70.

- 4.71** For the circuit in Fig. 4.137, what resistor connected across terminals a - b will absorb maximum power from the circuit? What is that power?

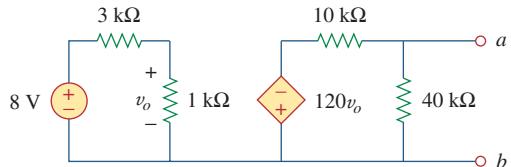


Figure 4.137

For Prob. 4.71.

- 4.72** (a) For the circuit in Fig. 4.138, obtain the Thevenin equivalent at terminals a - b .
 (b) Calculate the current in $R_L = 8\text{ }\Omega$.
 (c) Find R_L for maximum power deliverable to R_L .
 (d) Determine that maximum power.

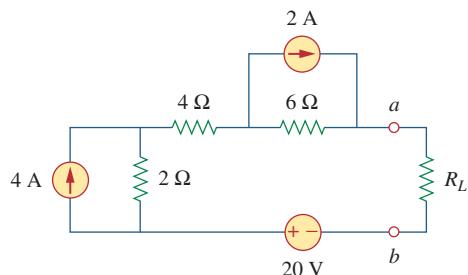


Figure 4.138

For Prob. 4.72.

- 4.73** Determine the maximum power that can be delivered to the variable resistor R in the circuit of Fig. 4.139.

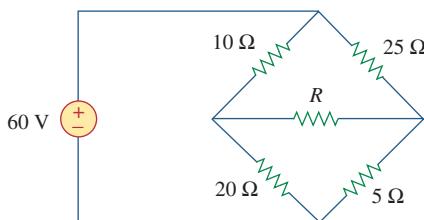


Figure 4.139

For Prob. 4.73.

- 4.74** For the bridge circuit shown in Fig. 4.140, find the load R_L for maximum power transfer and the maximum power absorbed by the load.

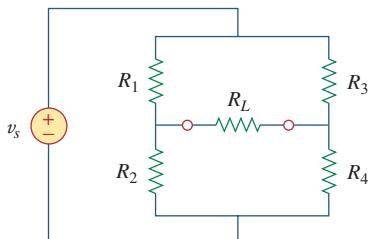


Figure 4.140

For Prob. 4.74.

- *4.75** For the circuit in Fig. 4.141, determine the value of R such that the maximum power delivered to the load is 3 mW.

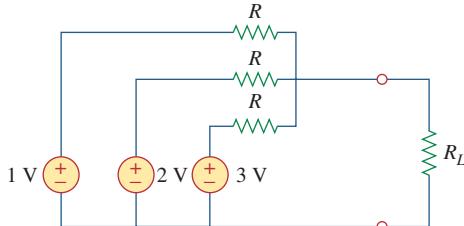


Figure 4.141

For Prob. 4.75.

Section 4.9 Verifying Circuit Theorems with PSpice

- 4.76** Solve Prob. 4.34 using PSpice or MultiSim. Let $V = 40 \text{ V}$, $I = 3 \text{ A}$, $R_1 = 10 \Omega$, $R_2 = 40 \Omega$, and $R_3 = 20 \Omega$.

- 4.77** Use PSpice or MultiSim to solve Prob. 4.44.

- 4.78** Use PSpice or MultiSim to solve Prob. 4.52.

- 4.79** Obtain the Thevenin equivalent of the circuit in Fig. 4.123 using PSpice or MultiSim.

- 4.80** Use PSpice or MultiSim to find the Thevenin equivalent circuit at terminals $a-b$ of the circuit in Fig. 4.125.

- 4.81** For the circuit in Fig. 4.126, use PSpice or MultiSim to find the Thevenin equivalent at terminals $a-b$.

Section 4.10 Applications

- 4.82** A battery has a short-circuit current of 20 A and an open-circuit voltage of 12 V. If the battery is connected to an electric bulb of resistance 2 Ω, calculate the power dissipated by the bulb.

- 4.83** The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	12 V	0 V
Terminal Current	0 A	1.5 A

Find the Thevenin equivalent of the network.

- 4.84** When connected to a 4-Ω resistor, a battery has a terminal voltage of 10.8 V but produces 12 V on an open circuit. Determine the Thevenin equivalent circuit for the battery.

- 4.85** The Thevenin equivalent at terminals $a-b$ of the linear network shown in Fig. 4.142 is to be determined by measurement. When a 10-kΩ resistor is connected to terminals $a-b$, the voltage V_{ab} is measured as 6 V. When a 30-kΩ resistor is connected to the terminals, V_{ab} is measured as 12 V. Determine: (a) the Thevenin equivalent at terminals $a-b$, (b) V_{ab} when a 20-kΩ resistor is connected to terminals $a-b$.

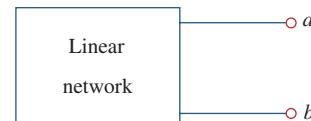


Figure 4.142

For Prob. 4.85.

- 4.86** A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.143. The results are shown in the table on the next page.

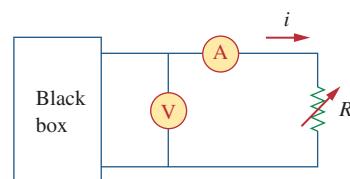


Figure 4.143

For Prob. 4.86.

- (a) Find i when $R = 4 \Omega$.
 (b) Determine the maximum power from the box.

$R(\Omega)$	$V(V)$	$i(A)$
2	3	1.5
8	8	1.0
14	10.5	0.75

- 4.87** A transducer is modeled with a current source I_s and a parallel resistance R_s . The current at the terminals of the source is measured to be 9.975 mA when an ammeter with an internal resistance of 20Ω is used.

- (a) If adding a $2\text{-k}\Omega$ resistor across the source terminals causes the ammeter reading to fall to 9.876 mA, calculate I_s and R_s .
 (b) What will the ammeter reading be if the resistance between the source terminals is changed to $4\text{ k}\Omega$?

- 4.88** Consider the circuit in Fig. 4.144. An ammeter with internal resistance R_i is inserted between A and B to measure I_o . Determine the reading of the ammeter if:
 (a) $R_i = 500 \Omega$, (b) $R_i = 0 \Omega$. (*Hint:* Find the Thevenin equivalent circuit at terminals $a-b$.)

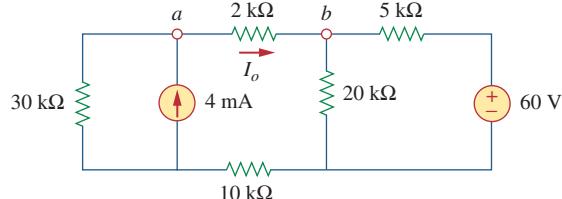


Figure 4.144
For Prob. 4.88.

- 4.89** Consider the circuit in Fig. 4.145. (a) Replace the resistor R_L by a zero resistance ammeter and determine the ammeter reading. (b) To verify the reciprocity theorem, interchange the ammeter and the 12-V source and determine the ammeter reading again.

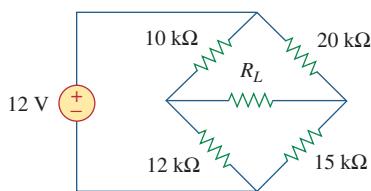


Figure 4.145
For Prob. 4.89.

- 4.90** The Wheatstone bridge circuit shown in Fig. 4.146 is used to measure the resistance of a strain gauge. The adjustable resistor has a linear taper with a maximum value of 100Ω . If the resistance of the strain gauge is found to be 42.6Ω , what fraction of the full slider travel is the slider when the bridge is balanced?

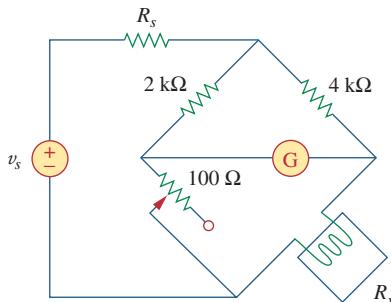


Figure 4.146
For Prob. 4.90.

- 4.91** (a) In the Wheatstone bridge circuit of Fig. 4.147, select the values of R_1 and R_3 such that the bridge can measure R_x in the range of $0-10 \Omega$.

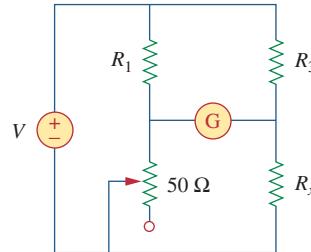


Figure 4.147
For Prob. 4.91.

- (b) Repeat for the range of $0-100 \Omega$.

- *4.92** Consider the bridge circuit of Fig. 4.148. Is the bridge balanced? If the $10\text{-k}\Omega$ resistor is replaced by an $18\text{-k}\Omega$ resistor, what resistor connected between terminals $a-b$ absorbs the maximum power? What is this power?

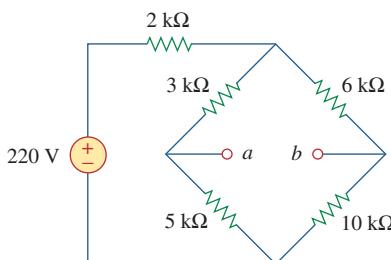


Figure 4.148
For Prob. 4.92.

Comprehensive Problems

- 4.93** The circuit in Fig. 4.149 models a common-emitter transistor amplifier. Find i_x using source transformation.

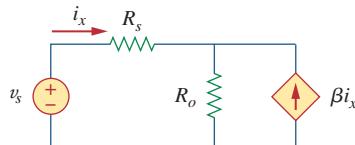


Figure 4.149

For Prob. 4.93.

- 4.94** An attenuator is an interface circuit that reduces the **e2d** voltage level without changing the output resistance.

- (a) By specifying R_s and R_p of the interface circuit in Fig. 4.150, design an attenuator that will meet the following requirements:

$$\frac{V_o}{V_g} = 0.125, \quad R_{eq} = R_{Th} = R_g = 100 \Omega$$

- (b) Using the interface designed in part (a), calculate the current through a load of $R_L = 50 \Omega$ when $V_g = 12 \text{ V}$.

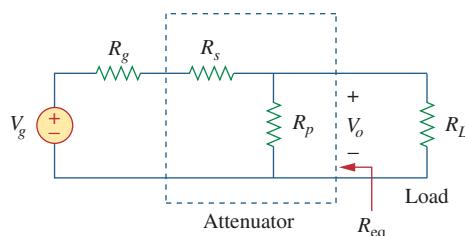


Figure 4.150

For Prob. 4.94.

- *4.95** A dc voltmeter with a sensitivity of $20 \text{ k}\Omega/\text{V}$ is used **e2d** to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:

- (a) 0–10 V scale: 4 V (b) 0–50 V scale: 5 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

- *4.96** A resistance array is connected to a load resistor R and a 9-V battery as shown in Fig. 4.151.

- (a) Find the value of R such that $V_o = 1.8 \text{ V}$.
 (b) Calculate the value of R that will draw the maximum current. What is the maximum current?

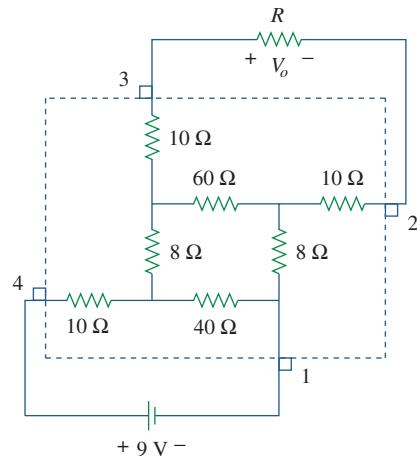


Figure 4.151

For Prob. 4.96.

- 4.97** A common-emitter amplifier circuit is shown in Fig. 4.152. Obtain the Thevenin equivalent to the left of points B and E .

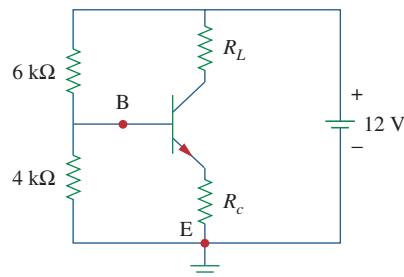


Figure 4.152

For Prob. 4.97.

- *4.98** For Practice Prob. 4.18, determine the current through the 40- Ω resistor and the power dissipated by the resistor.

5

Operational Amplifiers

He who will not reason is a bigot; he who cannot is a fool; and he who dares not is a slave.

—Lord Byron

Enhancing Your Career

Career in Electronic Instrumentation

Engineering involves applying physical principles to design devices for the benefit of humanity. But physical principles cannot be understood without measurement. In fact, physicists often say that physics is the science that measures reality. Just as measurements are a tool for understanding the physical world, instruments are tools for measurement. The operational amplifier introduced in this chapter is a building block of modern electronic instrumentation. Therefore, mastery of operational amplifier fundamentals is paramount to any practical application of electronic circuits.

Electronic instruments are used in all fields of science and engineering. They have proliferated in science and technology to the extent that it would be ridiculous to have a scientific or technical education without exposure to electronic instruments. For example, physicists, physiologists, chemists, and biologists must learn to use electronic instruments. For electrical engineering students in particular, the skill in operating digital and analog electronic instruments is crucial. Such instruments include ammeters, voltmeters, ohmmeters, oscilloscopes, spectrum analyzers, and signal generators.

Beyond developing the skill for operating the instruments, some electrical engineers specialize in designing and constructing electronic instruments. These engineers derive pleasure in building their own instruments. Most of them invent and patent their inventions. Specialists in electronic instruments find employment in medical schools, hospitals, research laboratories, aircraft industries, and thousands of other industries where electronic instruments are routinely used.



Electronic Instrumentation used in medical research.

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5.1 Introduction

The term operational amplifier was introduced in 1947 by John Ragazzini and his colleagues, in their work on analog computers for the National Defense Research Council after World War II. The first op amps used vacuum tubes rather than transistors.

An op amp may also be regarded as a voltage amplifier with very high gain.

Having learned the basic laws and theorems for circuit analysis, we are now ready to study an active circuit element of paramount importance: the *operational amplifier*, or *op amp* for short. The op amp is a versatile circuit building block.

The **op amp** is an electronic unit that behaves like a voltage-controlled voltage source.

It can also be used in making a voltage- or current-controlled current source. An op amp can sum signals, amplify a signal, integrate it, or differentiate it. The ability of the op amp to perform these mathematical operations is the reason it is called an *operational amplifier*. It is also the reason for the widespread use of op amps in analog design. Op amps are popular in practical circuit designs because they are versatile, inexpensive, easy to use, and fun to work with.

We begin by discussing the ideal op amp and later consider the nonideal op amp. Using nodal analysis as a tool, we consider ideal op amp circuits such as the inverter, voltage follower, summer, and difference amplifier. We will also analyze op amp circuits with *PSpice*. Finally, we learn how an op amp is used in digital-to-analog converters and instrumentation amplifiers.

5.2 Operational Amplifiers

An operational amplifier is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals. Thus,

The **op amp** is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes. A full discussion of what is inside the op amp is beyond the scope of this book. It will suffice to treat the op amp as a circuit building block and simply study what takes place at its terminals.

Op amps are commercially available in integrated circuit packages in several forms. Figure 5.1 shows a typical op amp package. A typical one is the eight-pin dual in-line package (or DIP), shown in Fig. 5.2(a). Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:

1. The inverting input, pin 2.
2. The noninverting input, pin 3.
3. The output, pin 6.
4. The positive power supply V^+ , pin 7.
5. The negative power supply V^- , pin 4.

The circuit symbol for the op amp is the triangle in Fig. 5.2(b); as shown, the op amp has two inputs and one output. The inputs are

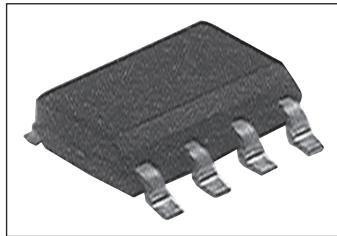
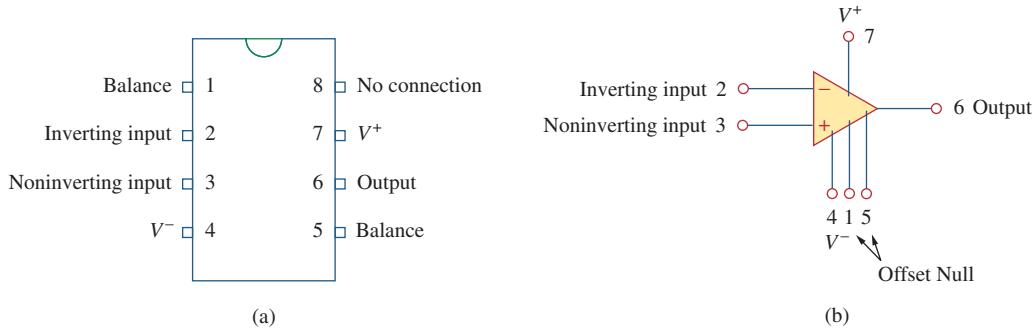


Figure 5.1

A typical operational amplifier.
Courtesy of Tech America.

The pin diagram in Fig. 5.2(a) corresponds to the 741 general-purpose op amp made by Fairchild Semiconductor.

**Figure 5.2**

A typical op amp: (a) pin configuration, (b) circuit symbol.

marked with minus (−) and plus (+) to specify *inverting* and *noninverting* inputs, respectively. An input applied to the noninverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.

As an active element, the op amp must be powered by a voltage supply as typically shown in Fig. 5.3. Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL,

$$i_o = i_1 + i_2 + i_+ + i_- \quad (5.1)$$

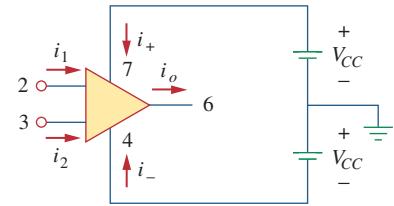
The equivalent circuit model of an op amp is shown in Fig. 5.4. The output section consists of a voltage-controlled source in series with the output resistance R_o . It is evident from Fig. 5.4 that the input resistance R_i is the Thevenin equivalent resistance seen at the input terminals, while the output resistance R_o is the Thevenin equivalent resistance seen at the output. The differential input voltage v_d is given by

$$v_d = v_2 - v_1 \quad (5.2)$$

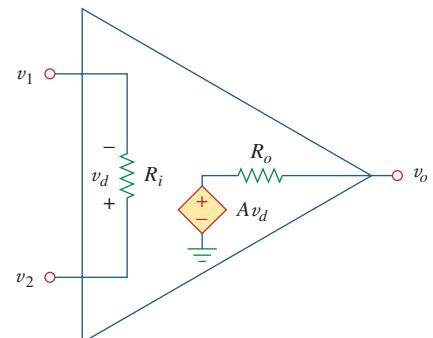
where v_1 is the voltage between the inverting terminal and ground and v_2 is the voltage between the noninverting terminal and ground. The op amp senses the difference between the two inputs, multiplies it by the gain A , and causes the resulting voltage to appear at the output. Thus, the output v_o is given by

$$v_o = Av_d = A(v_2 - v_1) \quad (5.3)$$

A is called the *open-loop voltage gain* because it is the gain of the op amp without any external feedback from output to input. Table 5.1

**Figure 5.3**

Powering the op amp.

**Figure 5.4**

The equivalent circuit of the nonideal op amp.

Sometimes, voltage gain is expressed in decibels (dB), as discussed in Chapter 14.

$$A \text{ dB} = 20 \log_{10} A$$

TABLE 5.1

Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

shows typical values of voltage gain A , input resistance R_i , output resistance R_o , and supply voltage V_{CC} .

The concept of feedback is crucial to our understanding of op amp circuits. A negative feedback is achieved when the output is fed back to the inverting terminal of the op amp. As Example 5.1 shows, when there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the *closed-loop gain*. As a result of the negative feedback, it can be shown that the closed-loop gain is almost insensitive to the open-loop gain A of the op amp. For this reason, op amps are used in circuits with feedback paths.

A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$. In other words, the output voltage is dependent on and is limited by the power supply voltage. Figure 5.5 illustrates that the op amp can operate in three modes, depending on the differential input voltage v_d :

1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.

If we attempt to increase v_d beyond the linear range, the op amp becomes saturated and yields $v_o = V_{CC}$ or $v_o = -V_{CC}$. Throughout this book, we will assume that our op amps operate in the linear mode. This means that the output voltage is restricted by

$$-V_{CC} \leq v_o \leq V_{CC} \quad (5.4)$$

Throughout this book, we assume that an op amp operates in the linear range. Keep in mind the voltage constraint on the op amp in this mode.

Although we shall always operate the op amp in the linear region, the possibility of saturation must be borne in mind when one designs with op amps, to avoid designing op amp circuits that will not work in the laboratory.

Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5 , input resistance of $2 \text{ M}\Omega$, and output resistance of 50Ω . The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s . Determine current i when $v_s = 2 \text{ V}$.

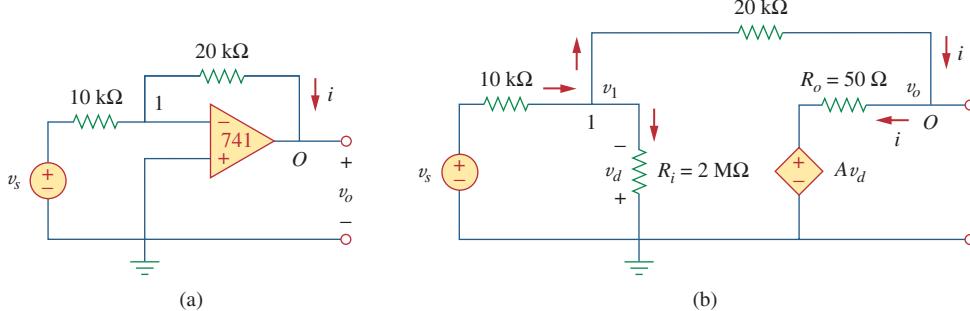


Figure 5.6

For Example 5.1: (a) original circuit, (b) the equivalent circuit.

Solution:

Using the op amp model in Fig. 5.4, we obtain the equivalent circuit of Fig. 5.6(a) as shown in Fig. 5.6(b). We now solve the circuit in Fig. 5.6(b) by using nodal analysis. At node 1, KCL gives

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

Multiplying through by 2000×10^3 , we obtain

$$200v_s = 301v_1 - 100v_o$$

or

$$2v_s = 3v_1 - v_o \quad \Rightarrow \quad v_1 = \frac{2v_s + v_o}{3} \quad (5.1.1)$$

At node O ,

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50}$$

But $v_d = -v_1$ and $A = 200,000$. Then

$$v_1 - v_o = 400(v_o + 200,000v_1) \quad (5.1.2)$$

Substituting v_1 from Eq. (5.1.1) into Eq. (5.1.2) gives

$$0 \approx 26,667,067v_o + 53,333,333v_s \quad \Rightarrow \quad \frac{v_o}{v_s} = -1.9999699$$

This is closed-loop gain, because the $20\text{-k}\Omega$ feedback resistor closes the loop between the output and input terminals. When $v_s = 2\text{ V}$, $v_o = -3.9999398\text{ V}$. From Eq. (5.1.1), we obtain $v_1 = 20.066667\text{ }\mu\text{V}$. Thus,

$$i = \frac{v_1 - v_o}{20 \times 10^3} = 0.19999\text{ mA}$$

It is evident that working with a nonideal op amp is tedious, as we are dealing with very large numbers.

If the same 741 op amp in Example 5.1 is used in the circuit of Fig. 5.7, calculate the closed-loop gain v_o/v_s . Find i_o when $v_s = 1\text{ V}$.

Practice Problem 5.1

Answer: 9.00041, 657 μA .

5.3 Ideal Op Amp

To facilitate the understanding of op amp circuits, we will assume ideal op amps. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, $A \approx \infty$.
2. Infinite input resistance, $R_i \approx \infty$.
3. Zero output resistance, $R_o \approx 0$.

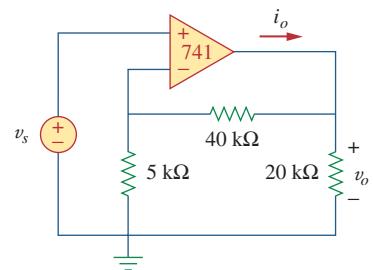


Figure 5.7
For Practice Prob. 5.1.

An **ideal op amp** is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.

Although assuming an ideal op amp provides only an approximate analysis, most modern amplifiers have such large gains and input impedances that the approximate analysis is a good one. Unless stated otherwise, we will assume from now on that every op amp is ideal.

For circuit analysis, the ideal op amp is illustrated in Fig. 5.8, which is derived from the nonideal model in Fig. 5.4. Two important characteristics of the ideal op amp are:

1. The currents into both input terminals are zero:

$$i_1 = 0, \quad i_2 = 0 \quad (5.5)$$

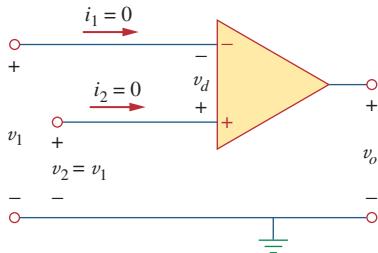


Figure 5.8

Ideal op amp model.

The two characteristics can be exploited by noting that for voltage calculations the input port behaves as a short circuit, while for current calculations the input port behaves as an open circuit.

This is due to infinite input resistance. An infinite resistance between the input terminals implies that an open circuit exists there and current cannot enter the op amp. But the output current is not necessarily zero according to Eq. (5.1).

2. The voltage across the input terminals is equal to zero; i.e.,

$$v_d = v_2 - v_1 = 0 \quad (5.6)$$

or

$$v_1 = v_2 \quad (5.7)$$

Thus, an ideal op amp has zero current into its two input terminals and the voltage between the two input terminals is equal to zero. Equations (5.5) and (5.7) are extremely important and should be regarded as the key handles to analyzing op amp circuits.

Example 5.2

Rework Practice Prob. 5.1 using the ideal op amp model.

Solution:

We may replace the op amp in Fig. 5.7 by its equivalent model in Fig. 5.9 as we did in Example 5.1. But we do not really need to do this. We just need to keep Eqs. (5.5) and (5.7) in mind as we analyze the circuit in Fig. 5.7. Thus, the Fig. 5.7 circuit is presented as in Fig. 5.9. Notice that

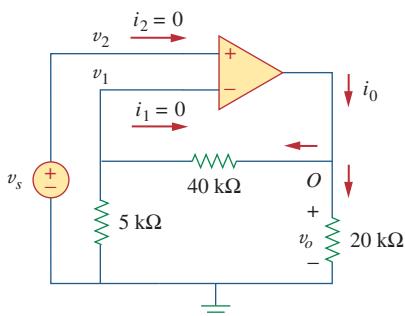
$$v_2 = v_s \quad (5.2.1)$$

Since $i_1 = 0$, the $40\text{-k}\Omega$ and $5\text{-k}\Omega$ resistors are in series; the same current flows through them. v_1 is the voltage across the $5\text{-k}\Omega$ resistor. Hence, using the voltage division principle,

$$v_1 = \frac{5}{5 + 40} v_o = \frac{v_o}{9} \quad (5.2.2)$$

Figure 5.9

For Example 5.2.



According to Eq. (5.7),

$$v_2 = v_1 \quad (5.2.3)$$

Substituting Eqs. (5.2.1) and (5.2.2) into Eq. (5.2.3) yields the closed-loop gain,

$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9 \quad (5.2.4)$$

which is very close to the value of 9.00041 obtained with the nonideal model in Practice Prob. 5.1. This shows that negligibly small error results from assuming ideal op amp characteristics.

At node O ,

$$i_o = \frac{v_o}{40 + 5} + \frac{v_o}{20} \text{ mA} \quad (5.2.5)$$

From Eq. (5.2.4), when $v_s = 1 \text{ V}$, $v_o = 9 \text{ V}$. Substituting for $v_o = 9 \text{ V}$ in Eq. (5.2.5) produces

$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

This, again, is close to the value of 0.657 mA obtained in Practice Prob. 5.1 with the nonideal model.

Repeat Example 5.1 using the ideal op amp model.

Practice Problem 5.2

Answer: $-2, 200 \mu\text{A}$.

5.4 Inverting Amplifier

In this and the following sections, we consider some useful op amp circuits that often serve as modules for designing more complex circuits. The first of such op amp circuits is the inverting amplifier shown in Fig. 5.10. In this circuit, the noninverting input is grounded, v_i is connected to the inverting input through R_1 , and the feedback resistor R_f is connected between the inverting input and output. Our goal is to obtain the relationship between the input voltage v_i and the output voltage v_o . Applying KCL at node 1,

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \quad (5.8)$$

But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

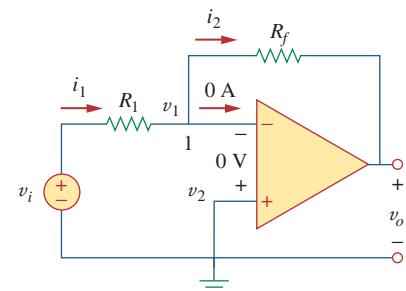


Figure 5.10

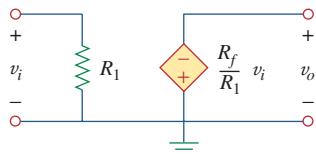
The inverting amplifier.

A key feature of the inverting amplifier is that both the input signal and the feedback are applied at the inverting terminal of the op amp.

or

$$v_o = -\frac{R_f}{R_1} v_i \quad (5.9)$$

Note there are two types of gains: The one here is the closed-loop voltage gain A_v , while the op amp itself has an open-loop voltage gain A .

**Figure 5.11**

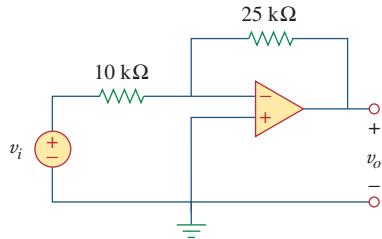
An equivalent circuit for the inverter in Fig. 5.10.

The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit in Fig. 5.10 as an *inverter* arises from the negative sign. Thus,

An inverting amplifier reverses the polarity of the input signal while amplifying it.

Notice that the gain is the feedback resistance divided by the input resistance which means that the gain depends only on the external elements connected to the op amp. In view of Eq. (5.9), an equivalent circuit for the inverting amplifier is shown in Fig. 5.11. The inverting amplifier is used, for example, in a current-to-voltage converter.

Example 5.3

**Figure 5.12**

For Example 5.3.

Refer to the op amp in Fig. 5.12. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o , and (b) the current in the 10-kΩ resistor.

Solution:

(a) Using Eq. (5.9),

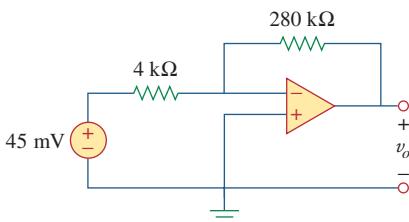
$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50 \mu\text{A}$$

Practice Problem 5.3

**Figure 5.13**

For Practice Prob. 5.3.

Find the output of the op amp circuit shown in Fig. 5.13. Calculate the current through the feedback resistor.

Answer: -3.15 V, $26.25 \mu\text{A}$.

Determine v_o in the op amp circuit shown in Fig. 5.14.

Example 5.4

Solution:

Applying KCL at node a ,

$$\frac{v_a - v_o}{40 \text{ k}\Omega} = \frac{6 - v_a}{20 \text{ k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \Rightarrow v_o = 3v_a - 12$$

But $v_a = v_b = 2 \text{ V}$ for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6 \text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$, as expected from Eq. (5.9).

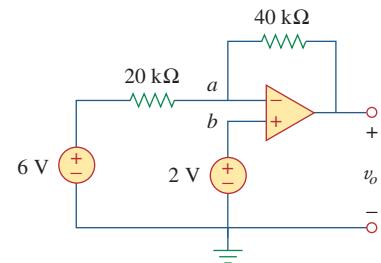


Figure 5.14
For Example 5.4.

Two kinds of current-to-voltage converters (also known as *transresistance amplifiers*) are shown in Fig. 5.15.

Practice Problem 5.4

(a) Show that for the converter in Fig. 5.15(a),

$$\frac{v_o}{i_s} = -R$$

(b) Show that for the converter in Fig. 5.15(b),

$$\frac{v_o}{i_s} = -R_1 \left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2} \right)$$

Answer: Proof.

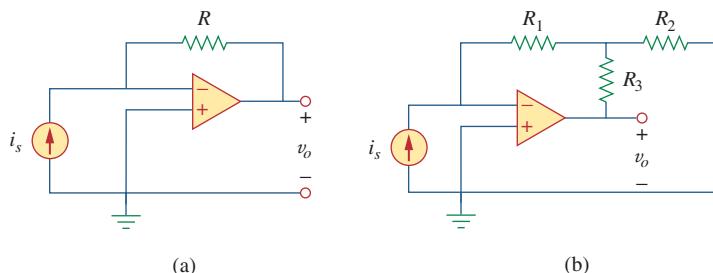


Figure 5.15
For Practice Prob. 5.4.

5.5 Noninverting Amplifier

Another important application of the op amp is the noninverting amplifier shown in Fig. 5.16. In this case, the input voltage v_i is applied directly at the noninverting input terminal, and resistor R_1 is connected

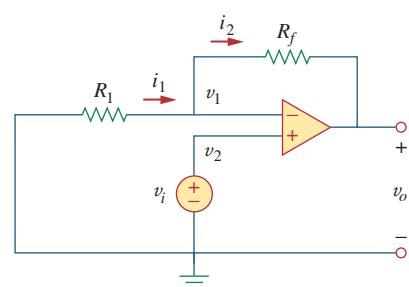


Figure 5.16
The noninverting amplifier.

between the ground and the inverting terminal. We are interested in the output voltage and the voltage gain. Application of KCL at the inverting terminal gives

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \quad (5.10)$$

But $v_1 = v_2 = v_i$. Equation (5.10) becomes

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

or

$$v_o = \left(1 + \frac{R_f}{R_1}\right)v_i \quad (5.11)$$

The voltage gain is $A_v = v_o/v_i = 1 + R_f/R_1$, which does not have a negative sign. Thus, the output has the same polarity as the input.

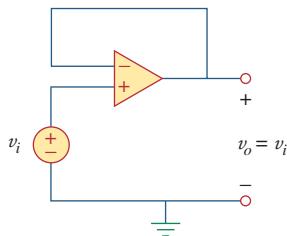


Figure 5.17

The voltage follower.

A noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.

Again we notice that the gain depends only on the external resistors.

Notice that if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. Under these conditions ($R_f = 0$ and $R_1 = \infty$), the circuit in Fig. 5.16 becomes that shown in Fig. 5.17, which is called a *voltage follower* (or *unity gain amplifier*) because the output follows the input. Thus, for a voltage follower

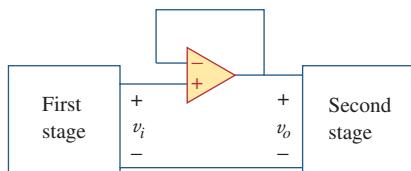


Figure 5.18

A voltage follower used to isolate two cascaded stages of a circuit.

$$v_o = v_i \quad (5.12)$$

Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another, as portrayed in Fig. 5.18. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.

Example 5.5

For the op amp circuit in Fig. 5.19, calculate the output voltage v_o .

Solution:

We may solve this in two ways: using superposition and using nodal analysis.

■ **METHOD 1** Using superposition, we let

$$v_o = v_{o1} + v_{o2}$$

where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input. To get v_{o1} , we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter. Hence Eq. (5.9) gives

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2} , we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier so that Eq. (5.11) applies.

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

Thus,

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

METHOD 2 Applying KCL at node a ,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \Rightarrow 5 = 4 - v_o$$

or $v_o = -1 \text{ V}$, as before.

Calculate v_o in the circuit of Fig. 5.20.

Practice Problem 5.5

Answer: 7 V.

5.6 Summing Amplifier

Besides amplification, the op amp can perform addition and subtraction. The addition is performed by the summing amplifier covered in this section; the subtraction is performed by the difference amplifier covered in the next section.

A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

The summing amplifier, shown in Fig. 5.21, is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time. We keep in mind

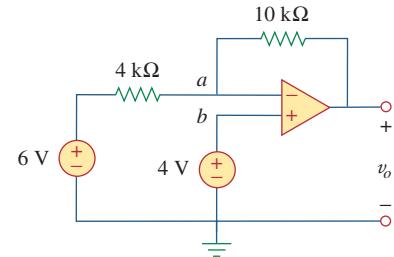


Figure 5.19

For Example 5.5.

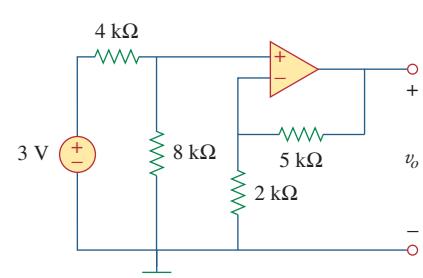


Figure 5.20

For Practice Prob. 5.5.

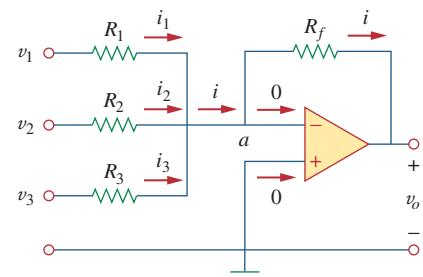


Figure 5.21

The summing amplifier.

that the current entering each op amp input is zero. Applying KCL at node a gives

$$i = i_1 + i_2 + i_3 \quad (5.13)$$

But

$$\begin{aligned} i_1 &= \frac{v_1 - v_a}{R_1}, & i_2 &= \frac{v_2 - v_a}{R_2} \\ i_3 &= \frac{v_3 - v_a}{R_3}, & i &= \frac{v_a - v_o}{R_f} \end{aligned} \quad (5.14)$$

We note that $v_a = 0$ and substitute Eq. (5.14) into Eq. (5.13). We get

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \quad (5.15)$$

indicating that the output voltage is a weighted sum of the inputs. For this reason, the circuit in Fig. 5.21 is called a *summer*. Needless to say, the summer can have more than three inputs.

Example 5.6

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.

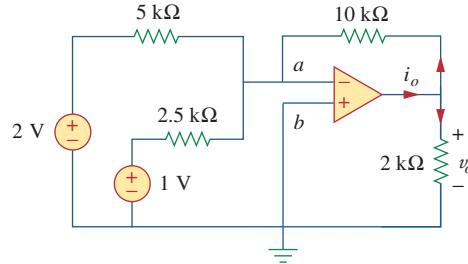


Figure 5.22

For Example 5.6.

Solution:

This is a summer with two inputs. Using Eq. (5.15) gives

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8 \text{ V}$ across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 4 = -4.8 \text{ mA}$$

Find v_o and i_o in the op amp circuit shown in Fig. 5.23.

Practice Problem 5.6

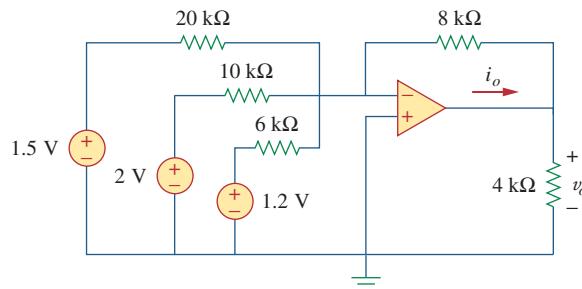


Figure 5.23
For Practice Prob. 5.6.

Answer: -3.8 V , -1.425 mA .

5.7 Difference Amplifier

Difference (or differential) amplifiers are used in various applications where there is a need to amplify the difference between two input signals. They are first cousins of the *instrumentation amplifier*, the most useful and popular amplifier, which we will discuss in Section 5.10.

The difference amplifier is also known as the *subtractor*, for reasons to be shown later.

A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Consider the op amp circuit shown in Fig. 5.24. Keep in mind that zero currents enter the op amp terminals. Applying KCL to node a ,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

or

$$v_o = \left(\frac{R_2}{R_1} + 1 \right) v_a - \frac{R_2}{R_1} v_1 \quad (5.16)$$

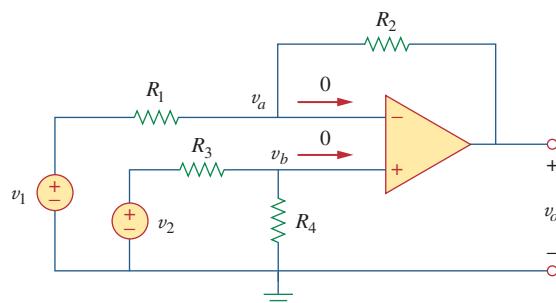


Figure 5.24
Difference amplifier.

Applying KCL to node b ,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

or

$$v_b = \frac{R_4}{R_3 + R_4} v_2 \quad (5.17)$$

But $v_a = v_b$. Substituting Eq. (5.17) into Eq. (5.16) yields

$$v_o = \left(\frac{R_2}{R_1} + 1 \right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

or

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1 \quad (5.18)$$

Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This property exists when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (5.19)$$

Thus, when the op amp circuit is a difference amplifier, Eq. (5.18) becomes

$$v_o = \frac{R_2}{R_1} (v_2 - v_1) \quad (5.20)$$

If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1 \quad (5.21)$$

Example 5.7

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Solution:

The circuit requires that

$$v_o = 3v_2 - 5v_1 \quad (5.7.1)$$

This circuit can be realized in two ways.

Design 1 If we desire to use only one op amp, we can use the op amp circuit of Fig. 5.24. Comparing Eq. (5.7.1) with Eq. (5.18), we see

$$\frac{R_2}{R_1} = 5 \quad \Rightarrow \quad R_2 = 5R_1 \quad (5.7.2)$$

Also,

$$5 \frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \Rightarrow \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

or

$$2 = 1 + \frac{R_3}{R_4} \Rightarrow R_3 = R_4 \quad (5.7.3)$$

If we choose $R_1 = 10 \text{ k}\Omega$ and $R_3 = 20 \text{ k}\Omega$, then $R_2 = 50 \text{ k}\Omega$ and $R_4 = 20 \text{ k}\Omega$.

Design 2 If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig. 5.25. For the summer,

$$v_o = -v_a - 5v_1 \quad (5.7.4)$$

and for the inverter,

$$v_a = -3v_2 \quad (5.7.5)$$

Combining Eqs. (5.7.4) and (5.7.5) gives

$$v_o = 3v_2 - 5v_1$$

which is the desired result. In Fig. 5.25, we may select $R_1 = 10 \text{ k}\Omega$ and $R_3 = 20 \text{ k}\Omega$ or $R_1 = R_3 = 10 \text{ k}\Omega$.

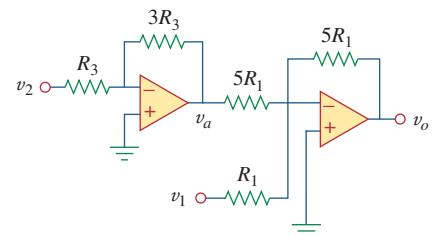


Figure 5.25
For Example 5.7.

Design a difference amplifier with gain 7.5.

Practice Problem 5.7

Answer: Typical: $R_1 = R_3 = 20 \text{ k}\Omega$, $R_2 = R_4 = 150 \text{ k}\Omega$.

An *instrumentation amplifier* shown in Fig. 5.26 is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units. Show that

Example 5.8

$$v_o = \frac{R_2}{R_1} \left(1 + \frac{2R_3}{R_4} \right) (v_2 - v_1)$$

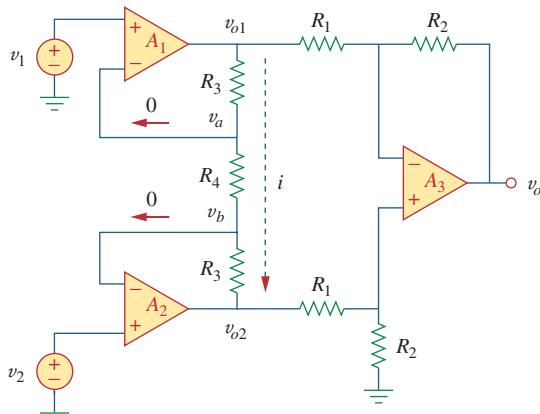
Solution:

We recognize that the amplifier A_3 in Fig. 5.26 is a difference amplifier. Thus, from Eq. (5.20),

$$v_o = \frac{R_2}{R_1} (v_{o2} - v_{o1}) \quad (5.8.1)$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series. Hence,

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4) \quad (5.8.2)$$

**Figure 5.26**

Instrumentation amplifier; for Example 5.8.

But

$$i = \frac{v_a - v_b}{R_4}$$

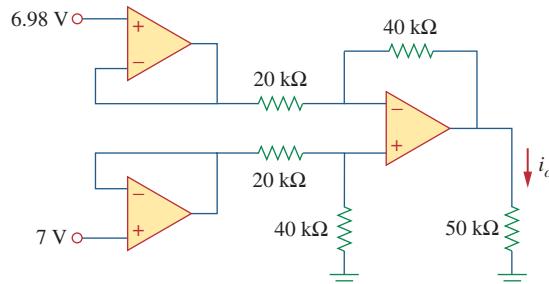
and $v_a = v_1$, $v_b = v_2$. Therefore,

$$i = \frac{v_1 - v_2}{R_4} \quad (5.8.3)$$

Inserting Eqs. (5.8.2) and (5.8.3) into Eq. (5.8.1) gives

$$v_o = \frac{R_2}{R_1} \left(1 + \frac{2R_3}{R_4} \right) (v_2 - v_1)$$

as required. We will discuss the instrumentation amplifier in detail in Section 5.10.

Practice Problem 5.8Obtain i_o in the instrumentation amplifier circuit of Fig. 5.27.**Figure 5.27**

Instrumentation amplifier; for Practice Prob. 5.8.

Answer: $-800 \mu\text{A}$.

5.8 Cascaded Op Amp Circuits

As we know, op amp circuits are modules or building blocks for designing complex circuits. It is often necessary in practical applications to connect op amp circuits in cascade (i.e., head to tail) to achieve a large overall gain. In general, two circuits are cascaded when they are connected in tandem, one behind another in a single file.

A cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next.

When op amp circuits are cascaded, each circuit in the string is called a *stage*; the original input signal is increased by the gain of the individual stage. Op amp circuits have the advantage that they can be cascaded without changing their input-output relationships. This is due to the fact that each (ideal) op amp circuit has infinite input resistance and zero output resistance. Figure 5.28 displays a block diagram representation of three op amp circuits in cascade. Since the output of one stage is the input to the next stage, the overall gain of the cascade connection is the product of the gains of the individual op amp circuits, or

$$A = A_1 A_2 A_3 \quad (5.22)$$

Although the cascade connection does not affect the op amp input-output relationships, care must be exercised in the design of an actual op amp circuit to ensure that the load due to the next stage in the cascade does not saturate the op amp.

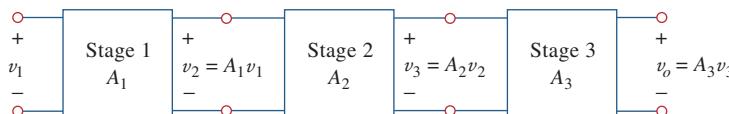


Figure 5.28

A three-stage cascaded connection.

Find v_o and i_o in the circuit in Fig. 5.29.

Example 5.9

Solution:

This circuit consists of two noninverting amplifiers cascaded. At the output of the first op amp,

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

At the output of the second op amp,

$$v_o = \left(1 + \frac{10}{4}\right)v_a = (1 + 2.5)100 = 350 \text{ mV}$$

The required current i_o is the current through the $10\text{-k}\Omega$ resistor.

$$i_o = \frac{v_o - v_b}{10} \text{ mA}$$

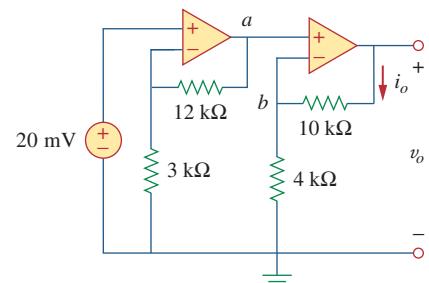


Figure 5.29

For Example 5.9.

But $v_b = v_a = 100 \text{ mV}$. Hence,

$$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25 \mu\text{A}$$

Practice Problem 5.9

Determine v_o and i_o in the op amp circuit in Fig. 5.30.

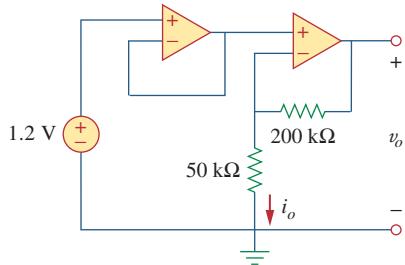


Figure 5.30

For Practice Prob. 5.9.

Answer: 6 V, 24 μA .

Example 5.10

If $v_1 = 1 \text{ V}$ and $v_2 = 2 \text{ V}$, find v_o in the op amp circuit of Fig. 5.31.

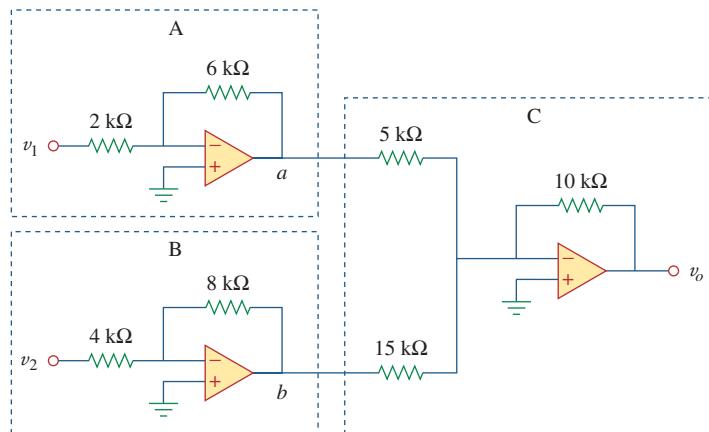


Figure 5.31

For Example 5.10.

Solution:

- Define.** The problem is clearly defined.
- Present.** With an input of v_1 of 1 V and of v_2 of 2 V, determine the output voltage of the circuit shown in Figure 5.31. The op amp circuit is actually composed of three circuits. The first circuit acts as an amplifier of gain $-3(-6 \text{ k}\Omega / 2 \text{ k}\Omega)$ for v_1 and the second functions as an amplifier of gain $-2(-8 \text{ k}\Omega / 4 \text{ k}\Omega)$ for v_2 . The last circuit serves as a summer of two different gains for the output of the other two circuits.
- Alternative.** There are different ways of working with this circuit. Since it involves ideal op amps, then a purely mathematical

approach will work quite easily. A second approach would be to use *PSpice* as a confirmation of the math.

4. **Attempt.** Let the output of the first op amp circuit be designated as v_{11} and the output of the second op amp circuit be designated as v_{22} . Then we get

$$v_{11} = -3v_1 = -3 \times 1 = -3 \text{ V},$$

$$v_{22} = -2v_2 = -2 \times 2 = -4 \text{ V}$$

In the third circuit we have

$$\begin{aligned} v_o &= -(10 \text{ k}\Omega / 5 \text{ k}\Omega) v_{11} + [-(10 \text{ k}\Omega / 15 \text{ k}\Omega) v_{22}] \\ &= -2(-3) - (2/3)(-4) \\ &= 6 + 2.667 = \mathbf{8.667 \text{ V}} \end{aligned}$$

5. **Evaluate.** In order to properly evaluate our solution, we need to identify a reasonable check. Here we can easily use *PSpice* to provide that check.

Now we can simulate this in *PSpice*. We see the results are shown in Fig. 5.32.

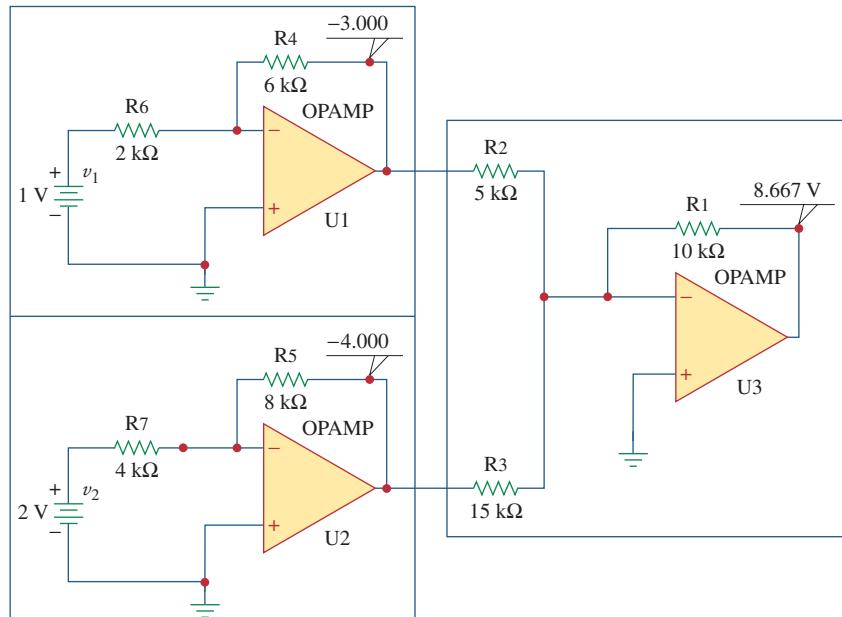


Figure 5.32

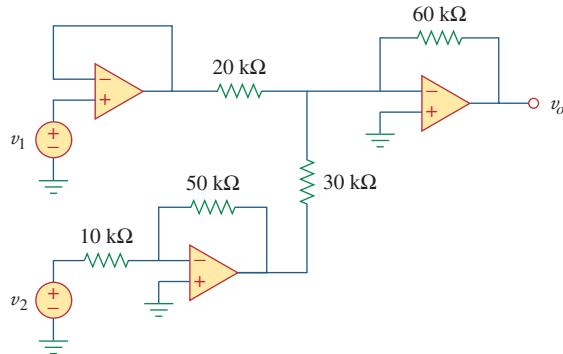
For Example 5.10.

We note that we obtain the same results using two entirely different techniques (the first is to treat the op amp circuits as just gains and a summer and the second is to use circuit analysis with *PSpice*). This is a very good method of assuring that we have the correct answer.

6. **Satisfactory?** We are satisfied we have obtained the asked for results. We can now present our work as a solution to the problem.

Practice Problem 5.10

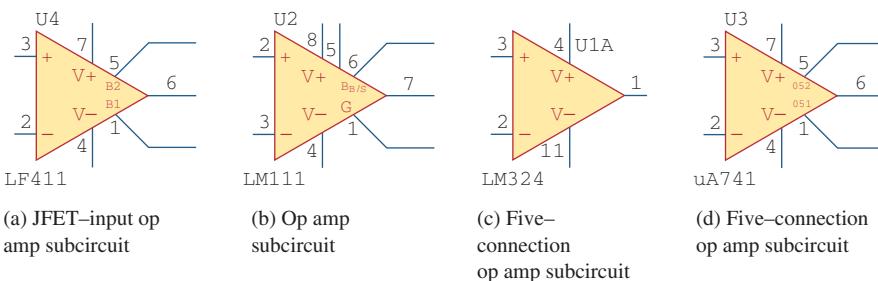
If $v_1 = 7$ V and $v_2 = 3.1$ V, find v_o in the op amp circuit of Fig. 5.33.

**Figure 5.33**

For Practice Prob. 5.10.

Answer: 10 V.**5.9 Op Amp Circuit Analysis with PSpice**

PSpice for Windows does not have a model for an ideal op amp, although one may create one as a subcircuit using the *Create Subcircuit* line in the *Tools* menu. Rather than creating an ideal op amp, we will use one of the four nonideal, commercially available op amps supplied in the *PSpice* library *eval.slb*. The op amp models have the part names LF411, LM111, LM324, and uA741, as shown in Fig. 5.34. Each of them can be obtained from **Draw/Get New Part/libraries ··· /eval.lib** or by simply selecting **Draw/Get New Part** and typing the part name in the *PartName* dialog box, as usual. Note that each of them requires dc supplies, without which the op amp will not work. The dc supplies should be connected as shown in Fig. 5.3.

**Figure 5.34**Nonideal op amp model available in *PSpice*.

Use PSpice to solve the op amp circuit for Example 5.1.

Example 5.11

Solution:

Using Schematics, we draw the circuit in Fig. 5.6(a) as shown in Fig. 5.35. Notice that the positive terminal of the voltage source v_s is connected to the inverting terminal (pin 2) via the 10-k Ω resistor, while the noninverting terminal (pin 3) is grounded as required in Fig. 5.6(a). Also, notice how the op amp is powered; the positive power supply terminal V+ (pin 7) is connected to a 15-V dc voltage source, while the negative power supply terminal V- (pin 4) is connected to -15 V. Pins 1 and 5 are left floating because they are used for offset null adjustment, which does not concern us in this chapter. Besides adding the dc power supplies to the original circuit in Fig. 5.6(a), we have also added pseudocomponents VIEWPOINT and IPROBE to respectively measure the output voltage v_o at pin 6 and the required current i through the 20-k Ω resistor.

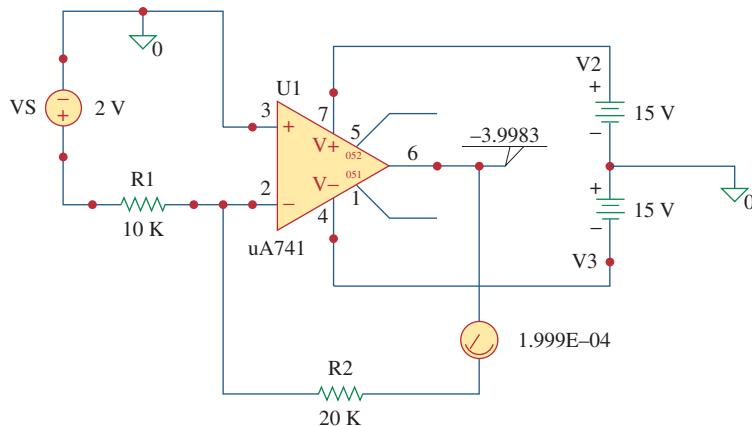


Figure 5.35

Schematic for Example 5.11.

After saving the schematic, we simulate the circuit by selecting **Analysis/Simulate** and have the results displayed on VIEWPOINT and IPROBE. From the results, the closed-loop gain is

$$\frac{v_o}{v_s} = \frac{-3.9983}{2} = -1.99915$$

and $i = 0.1999$ mA, in agreement with the results obtained analytically in Example 5.1.

Rework Practice Prob. 5.1 using PSpice.

Practice Problem 5.11

Answer: 9.0027, 650.2 μ A.

5.10 \dagger Applications

The op amp is a fundamental building block in modern electronic instrumentation. It is used extensively in many devices, along with resistors and other passive elements. Its numerous practical applications include instrumentation amplifiers, digital-to-analog converters, analog computers, level shifters, filters, calibration circuits, inverters, summers, integrators, differentiators, subtractors, logarithmic amplifiers, comparators, gyrators, oscillators, rectifiers, regulators, voltage-to-current converters, current-to-voltage converters, and clippers. Some of these we have already considered. We will consider two more applications here: the digital-to-analog converter and the instrumentation amplifier.

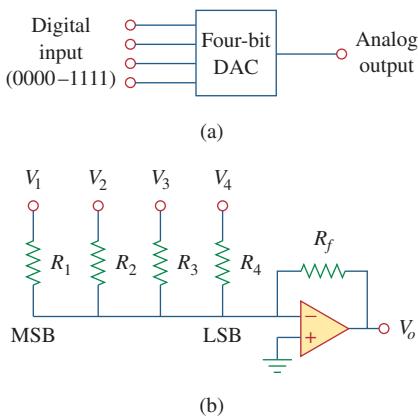


Figure 5.36

Four-bit DAC: (a) block diagram, (b) binary weighted ladder type.

In practice, the voltage levels may be typically 0 and ± 5 V.

5.10.1 Digital-to-Analog Converter

The digital-to-analog converter (DAC) transforms digital signals into analog form. A typical example of a four-bit DAC is illustrated in Fig. 5.36(a). The four-bit DAC can be realized in many ways. A simple realization is the *binary weighted ladder*, shown in Fig. 5.36(b). The bits are weights according to the magnitude of their place value, by descending value of R_f/R_n so that each lesser bit has half the weight of the next higher. This is obviously an inverting summing amplifier. The output is related to the inputs as shown in Eq. (5.15). Thus,

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 \quad (5.23)$$

Input V_1 is called the *most significant bit* (MSB), while input V_4 is the *least significant bit* (LSB). Each of the four binary inputs V_1, \dots, V_4 can assume only two voltage levels: 0 or 1 V. By using the proper input and feedback resistor values, the DAC provides a single output that is proportional to the inputs.

Example 5.12

In the op amp circuit of Fig. 5.36(b), let $R_f = 10 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $R_3 = 40 \text{ k}\Omega$, and $R_4 = 80 \text{ k}\Omega$. Obtain the analog output for binary inputs [0000], [0001], [0010], ..., [1111].

Solution:

Substituting the given values of the input and feedback resistors in Eq. (5.23) gives

$$\begin{aligned} -V_o &= \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 \\ &= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4 \end{aligned}$$

Using this equation, a digital input $[V_1 V_2 V_3 V_4] = [0000]$ produces an analog output of $-V_o = 0 \text{ V}$; $[V_1 V_2 V_3 V_4] = [0001]$ gives $-V_o = 0.125 \text{ V}$.

Similarly,

$$\begin{aligned}
 [V_1 V_2 V_3 V_4] = [0010] &\Rightarrow -V_o = 0.25 \text{ V} \\
 [V_1 V_2 V_3 V_4] = [0011] &\Rightarrow -V_o = 0.25 + 0.125 = 0.375 \text{ V} \\
 [V_1 V_2 V_3 V_4] = [0100] &\Rightarrow -V_o = 0.5 \text{ V} \\
 &\vdots \\
 [V_1 V_2 V_3 V_4] = [1111] &\Rightarrow -V_o = 1 + 0.5 + 0.25 + 0.125 \\
 &= 1.875 \text{ V}
 \end{aligned}$$

Table 5.2 summarizes the result of the digital-to-analog conversion. Note that we have assumed that each bit has a value of 0.125 V. Thus, in this system, we cannot represent a voltage between 1.000 and 1.125, for example. This lack of resolution is a major limitation of digital-to-analog conversions. For greater accuracy, a word representation with a greater number of bits is required. Even then a digital representation of an analog voltage is never exact. In spite of this inexact representation, digital representation has been used to accomplish remarkable things such as audio CDs and digital photography.

TABLE 5.2

Input and output values of the four-bit DAC.

Binary input [$V_1 V_2 V_3 V_4$]	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875

A three-bit DAC is shown in Fig. 5.37.

Practice Problem 5.12

- Determine $|V_o|$ for $[V_1 V_2 V_3] = [010]$.
- Find $|V_o|$ if $[V_1 V_2 V_3] = [110]$.
- If $|V_o| = 1.25 \text{ V}$ is desired, what should be $[V_1 V_2 V_3]$?
- To get $|V_o| = 1.75 \text{ V}$, what should be $[V_1 V_2 V_3]$?

Answer: 0.5 V, 1.5 V, [101], [111].

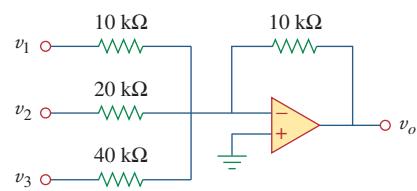


Figure 5.37

Three-bit DAC; for Practice Prob. 5.12.

5.10.2 Instrumentation Amplifiers

One of the most useful and versatile op amp circuits for precision measurement and process control is the *instrumentation amplifier* (IA), so called because of its widespread use in measurement systems. Typical applications of IAs include isolation amplifiers, thermocouple amplifiers, and data acquisition systems.

The instrumentation amplifier is an extension of the difference amplifier in that it amplifies the difference between its input signals. As shown in Fig. 5.26 (see Example 5.8), an instrumentation amplifier typically consists of three op amps and seven resistors. For convenience, the amplifier is shown again in Fig. 5.38(a), where the resistors are made equal except for the external gain-setting resistor R_G , connected between the gain set terminals. Figure 5.38(b) shows its schematic symbol. Example 5.8 showed that

$$v_o = A_v(v_2 - v_1) \quad (5.24)$$

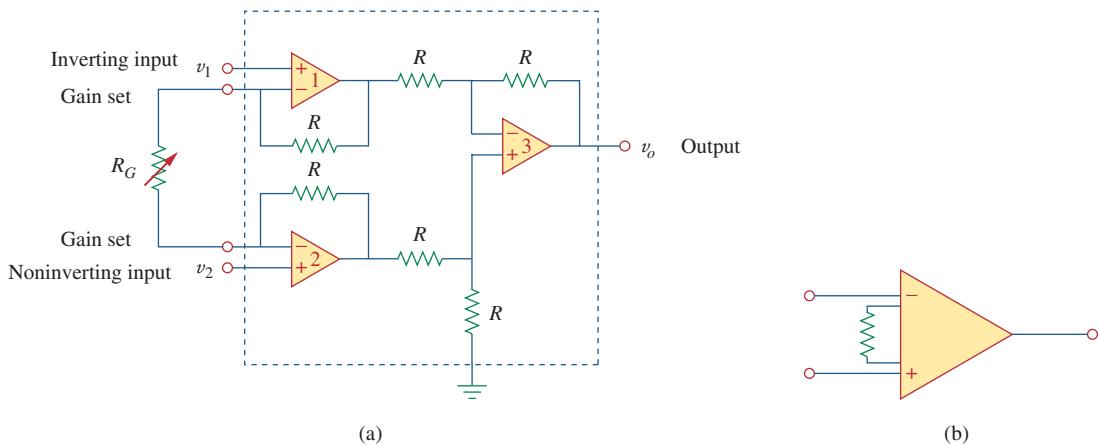


Figure 5.38

(a) The instrumentation amplifier with an external resistance to adjust the gain, (b) schematic diagram.

where the voltage gain is

$$A_v = 1 + \frac{2R}{R_G} \quad (5.25)$$

As shown in Fig. 5.39, the instrumentation amplifier amplifies small differential signal voltages superimposed on larger common-mode

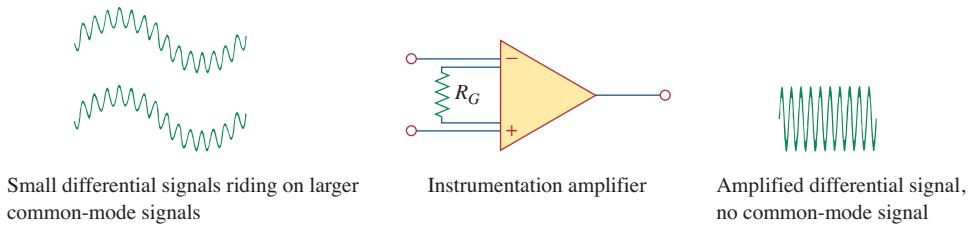


Figure 5.39

The IA rejects common voltages but amplifies small signal voltages.

Floyd, Thomas, L., *Electronic Devices*, 4th edition, © 1995, p. 795. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.

voltages. Since the common-mode voltages are equal, they cancel each other.

The IA has three major characteristics:

1. The voltage gain is adjusted by *one* external resistor R_G .
2. The input impedance of both inputs is very high and does not vary as the gain is adjusted.
3. The output v_o depends on the difference between the inputs v_1 and v_2 , not on the voltage common to them (common-mode voltage).

Due to the widespread use of IAs, manufacturers have developed these amplifiers on single-package units. A typical example is the LH0036, developed by National Semiconductor. The gain can be varied from 1 to 1,000 by an external resistor whose value may vary from $100\ \Omega$ to $10\ k\Omega$.

Example 5.13

In Fig. 5.38, let $R = 10\ k\Omega$, $v_1 = 2.011\ V$, and $v_2 = 2.017\ V$. If R_G is adjusted to $500\ \Omega$, determine: (a) the voltage gain, (b) the output voltage v_o .

Solution:

(a) The voltage gain is

$$A_v = 1 + \frac{2R}{R_G} = 1 + \frac{2 \times 10,000}{500} = 41$$

(b) The output voltage is

$$v_o = A_v(v_2 - v_1) = 41(2.017 - 2.011) = 41(6)\ mV = 246\ mV$$

Practice Problem 5.13

Determine the value of the external gain-setting resistor R_G required for the IA in Fig. 5.38 to produce a gain of 142 when $R = 25\ k\Omega$.

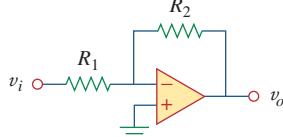
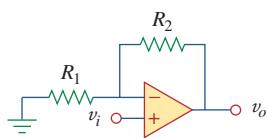
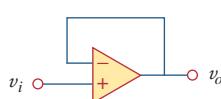
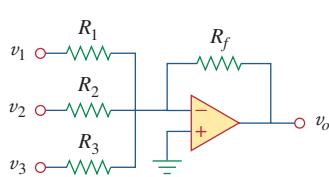
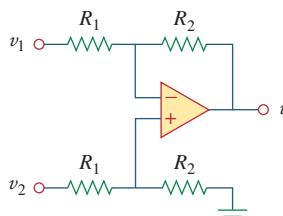
Answer: $354.6\ \Omega$.

5.11 Summary

1. The op amp is a high-gain amplifier that has high input resistance and low output resistance.
2. Table 5.3 summarizes the op amp circuits considered in this chapter. The expression for the gain of each amplifier circuit holds whether the inputs are dc, ac, or time-varying in general.

TABLE 5.3

Summary of basic op amp circuits.

Op amp circuit	Name/output-input relationship
	Inverting amplifier $v_o = -\frac{R_2}{R_1}v_i$
	Noninverting amplifier $v_o = \left(1 + \frac{R_2}{R_1}\right)v_i$
	Voltage follower $v_o = v_i$
	Summer $v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$
	Difference amplifier $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

3. An ideal op amp has an infinite input resistance, a zero output resistance, and an infinite gain.
4. For an ideal op amp, the current into each of its two input terminals is zero, and the voltage across its input terminals is negligibly small.
5. In an inverting amplifier, the output voltage is a negative multiple of the input.
6. In a noninverting amplifier, the output is a positive multiple of the input.
7. In a voltage follower, the output follows the input.
8. In a summing amplifier, the output is the weighted sum of the inputs.
9. In a difference amplifier, the output is proportional to the difference of the two inputs.
10. Op amp circuits may be cascaded without changing their input-output relationships.
11. *PSpice* can be used to analyze an op amp circuit.
12. Typical applications of the op amp considered in this chapter include the digital-to-analog converter and the instrumentation amplifier.

Review Questions

- 5.1** The two input terminals of an op amp are labeled as:
 (a) high and low.
 (b) positive and negative.
 (c) inverting and noninverting.
 (d) differential and nondifferential.
- 5.2** For an ideal op amp, which of the following statements are not true?
 (a) The differential voltage across the input terminals is zero.
 (b) The current into the input terminals is zero.
 (c) The current from the output terminal is zero.
 (d) The input resistance is zero.
 (e) The output resistance is zero.
- 5.3** For the circuit in Fig. 5.40, voltage v_o is:
 (a) -6 V (b) -5 V
 (c) -1.2 V (d) -0.2 V

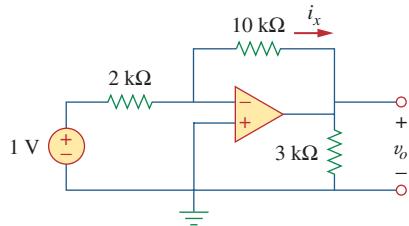


Figure 5.40
For Review Questions 5.3 and 5.4.

- 5.4** For the circuit in Fig. 5.40, current i_x is:
 (a) 0.6 mA (b) 0.5 mA
 (c) 0.2 mA (d) $1/12 \text{ mA}$
- 5.5** If $v_s = 0$ in the circuit of Fig. 5.41, current i_o is:
 (a) -10 mA (b) -2.5 mA
 (c) $10/12 \text{ mA}$ (d) $10/14 \text{ mA}$

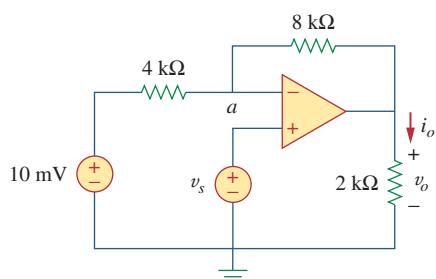


Figure 5.41
For Review Questions 5.5, 5.6, and 5.7.

- 5.6** If $v_s = 8 \text{ mV}$ in the circuit of Fig. 5.41, the output voltage is:
 (a) -44 mV (b) -8 mV
 (c) 4 mV (d) 7 mV
- 5.7** Refer to Fig. 5.41. If $v_s = 8 \text{ mV}$, voltage v_a is:
 (a) -8 mV (b) 0 mV
 (c) $10/3 \text{ mV}$ (d) 8 mV
- 5.8** The power absorbed by the $4\text{-k}\Omega$ resistor in Fig. 5.42 is:
 (a) 9 mW (b) 4 mW
 (c) 2 mW (d) 1 mW

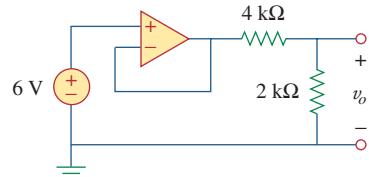


Figure 5.42
For Review Questions 5.8.

- 5.9** Which of these amplifiers is used in a digital-to-analog converter?
 (a) noninverter
 (b) voltage follower
 (c) summer
 (d) difference amplifier
- 5.10** Difference amplifiers are used in (please check all that apply):
 (a) instrumentation amplifiers
 (b) voltage followers
 (c) voltage regulators
 (d) buffers
 (e) summing amplifiers
 (f) subtracting amplifiers

Answers: 5.1c, 5.2c,d, 5.3b, 5.4b, 5.5a, 5.6c, 5.7d, 5.8b, 5.9c, 5.10a,f.

Problems

Section 5.2 Operational Amplifiers

- 5.1** The equivalent model of a certain op amp is shown in Fig. 5.43. Determine:

- the input resistance
- the output resistance
- the voltage gain in dB

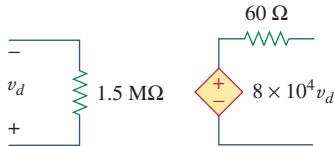


Figure 5.43

For Prob. 5.1.

- 5.2** The open-loop gain of an op amp is 100,000. Calculate the output voltage when there are inputs of $+10 \mu\text{V}$ on the inverting terminal and $+20 \mu\text{V}$ on the noninverting terminal.
- 5.3** Determine the output voltage when $-20 \mu\text{V}$ is applied to the inverting terminal of an op amp and $+30 \mu\text{V}$ to its noninverting terminal. Assume that the op amp has an open-loop gain of 200,000.
- 5.4** The output voltage of an op amp is -4 V when the noninverting input is 1 mV . If the open-loop gain of the op amp is 2×10^6 , what is the inverting input?
- 5.5** For the op amp circuit of Fig. 5.44, the op amp has an open-loop gain of 100,000, an input resistance of $10 \text{ k}\Omega$, and an output resistance of 100Ω . Find the voltage gain v_o/v_i using the nonideal model of the op amp.

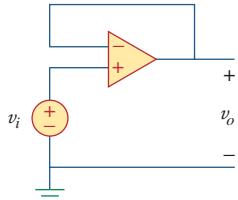


Figure 5.44

For Prob. 5.5.

- 5.6** Using the same parameters for the 741 op amp in Example 5.1, find v_o in the op amp circuit of Fig. 5.45.

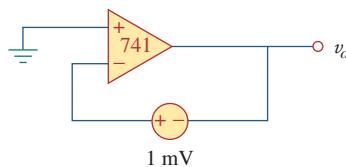


Figure 5.45

For Prob. 5.6.

- 5.7** The op amp in Fig. 5.46 has $R_i = 100 \text{ k}\Omega$, $R_o = 100 \Omega$, $A = 100,000$. Find the differential voltage v_d and the output voltage v_o .

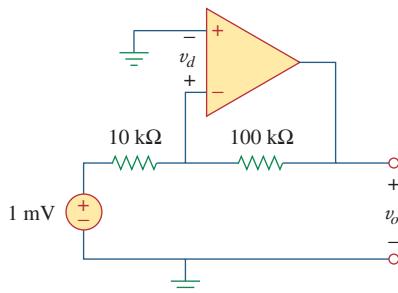


Figure 5.46

For Prob. 5.7.

Section 5.3 Ideal Op Amp

- 5.8** Obtain v_o for each of the op amp circuits in Fig. 5.47.

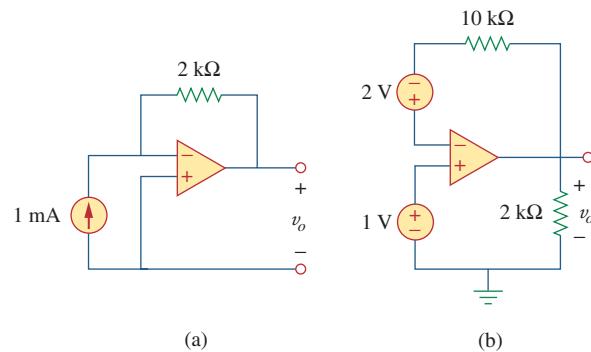
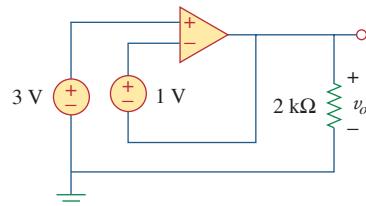
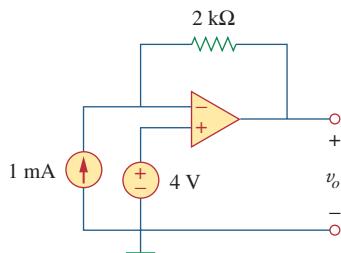


Figure 5.47

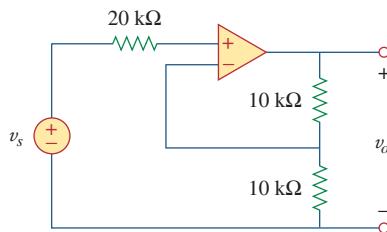
For Prob. 5.8.

- 5.9** Determine v_o for each of the op amp circuits in Fig. 5.48.

**Figure 5.48**

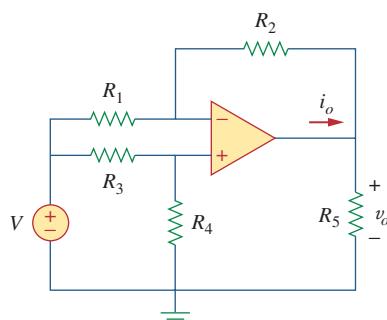
For Prob. 5.9.

- 5.10** Find the gain v_o/v_s of the circuit in Fig. 5.49.

**Figure 5.49**

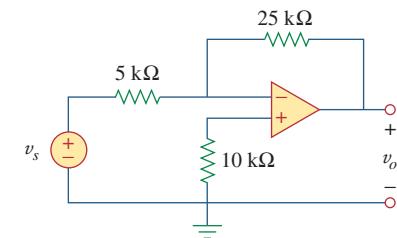
For Prob. 5.10.

- 5.11** Using Fig. 5.50, design a problem to help other **e2d** students better understand how ideal op amps work.

**Figure 5.50**

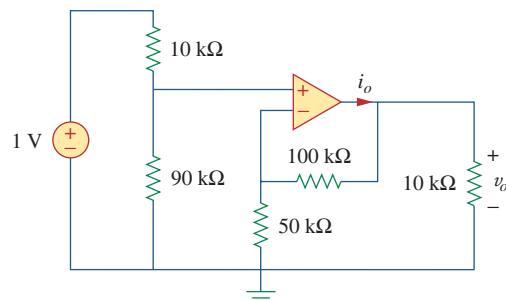
For Prob. 5.11.

- 5.12** Calculate the voltage ratio v_o/v_s for the op amp circuit of Fig. 5.51. Assume that the op amp is ideal.

**Figure 5.51**

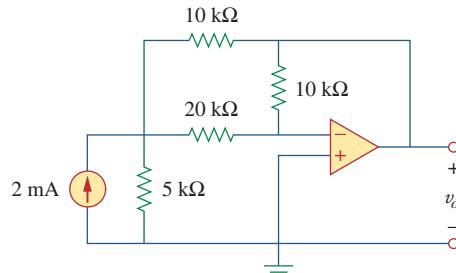
For Prob. 5.12.

- 5.13** Find v_o and i_o in the circuit of Fig. 5.52.

**Figure 5.52**

For Prob. 5.13.

- 5.14** Determine the output voltage v_o in the circuit of Fig. 5.53.

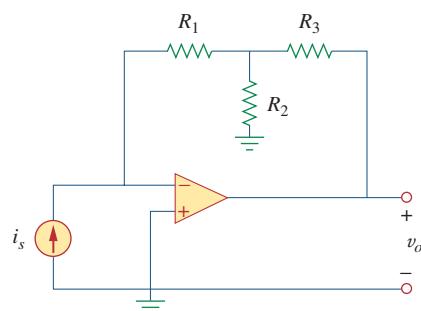
**Figure 5.53**

For Prob. 5.14.

Section 5.4 Inverting Amplifier

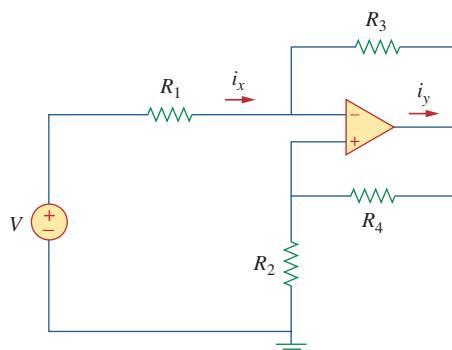
- 5.15** (a) Determine the ratio v_o/i_s in the op amp circuit of Fig. 5.54.

- (b) Evaluate the ratio for $R_1 = 20 \text{ k}\Omega$, $R_2 = 25 \text{ k}\Omega$, $R_3 = 40 \text{ k}\Omega$.

**Figure 5.54**

For Prob. 5.15.

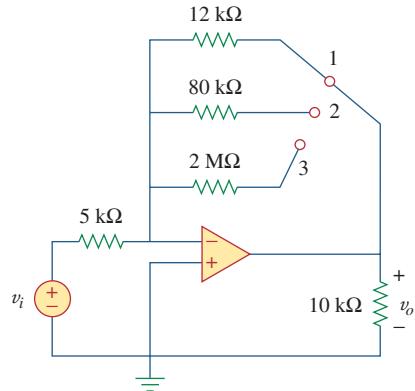
- 5.16** Using Fig. 5.55, design a problem to help students better understand inverting op amps.

**Figure 5.55**

For Prob. 5.16.

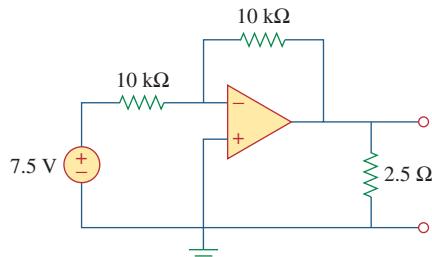
- 5.17** Calculate the gain v_o/v_i when the switch in Fig. 5.56 is in:

- (a) position 1 (b) position 2 (c) position 3.

**Figure 5.56**

For Prob. 5.17.

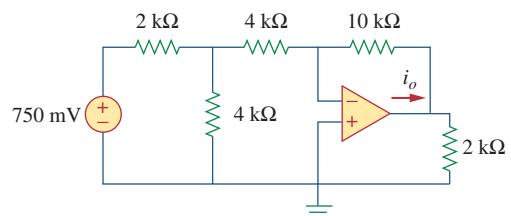
- *5.18** For the circuit shown in Figure 5.57, solve for the Thevenin equivalent circuit looking into terminals A and B.

**Figure 5.57**

For Prob. 5.18.

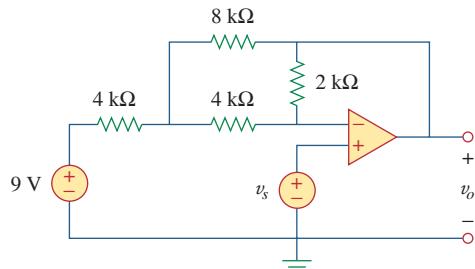
* An asterisk indicates a challenging problem.

- 5.19** Determine i_o in the circuit of Fig. 5.58.

**Figure 5.58**

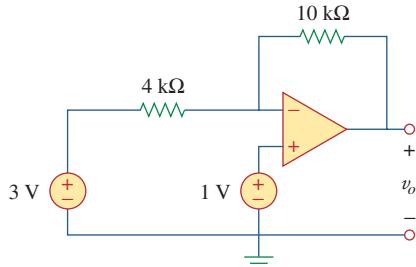
For Prob. 5.19.

- 5.20** In the circuit of Fig. 5.59, calculate v_o of $v_s = 2$ V.

**Figure 5.59**

For Prob. 5.20.

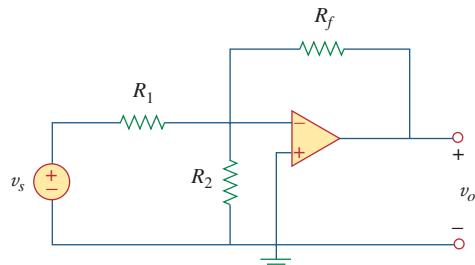
- 5.21** Calculate v_o in the op amp circuit of Fig. 5.60.

**Figure 5.60**

For Prob. 5.21.

- 5.22** Design an inverting amplifier with a gain of -15 .

- 5.23** For the op amp circuit in Fig. 5.61, find the voltage gain v_o/v_s .

**Figure 5.61**

For Prob. 5.23.

- 5.24** In the circuit shown in Fig. 5.62, find k in the voltage transfer function $v_o = kv_s$.

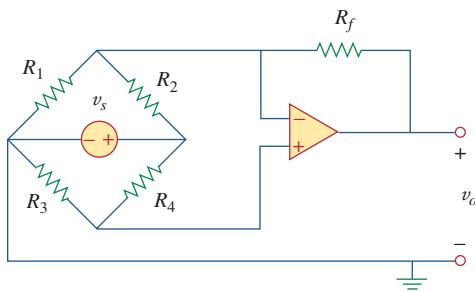


Figure 5.62

For Prob. 5.24.

Section 5.5 Noninverting Amplifier

- 5.25** Calculate v_o in the op amp circuit of Fig. 5.63.

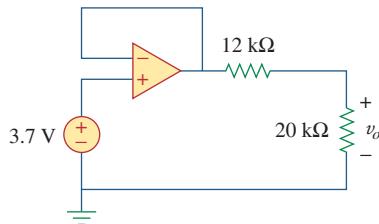


Figure 5.63

For Prob. 5.25.

- 5.26** Using Fig. 5.64, design a problem to help other **e2d** students better understand noninverting op amps.

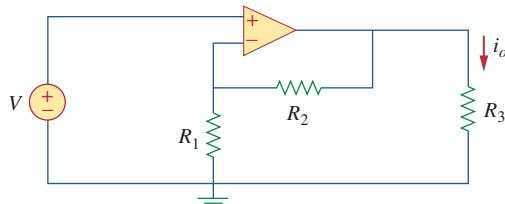


Figure 5.64

For Prob. 5.26.

- 5.27** Find v_o in the op amp circuit of Fig. 5.65.

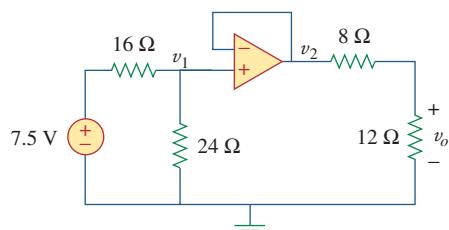


Figure 5.65

For Prob. 5.27.

- 5.28** Find i_o in the op amp circuit of Fig. 5.66.

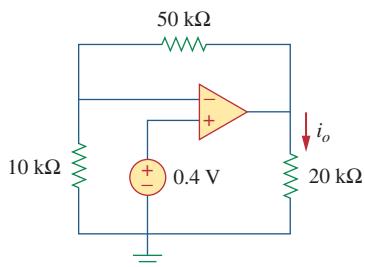


Figure 5.66

For Prob. 5.28.

- 5.29** Determine the voltage gain v_o/v_i of the op amp circuit in Fig. 5.67.

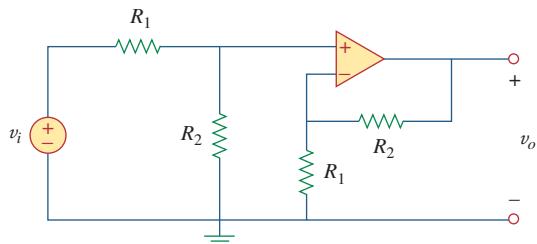


Figure 5.67

For Prob. 5.29.

- 5.30** In the circuit shown in Fig. 5.68, find i_x and the power absorbed by the 20-kΩ resistor.

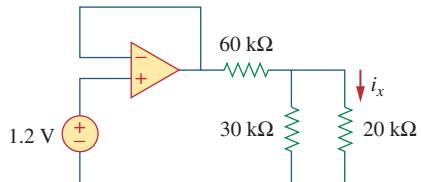


Figure 5.68

For Prob. 5.30.

- 5.31** For the circuit in Fig. 5.69, find i_x .

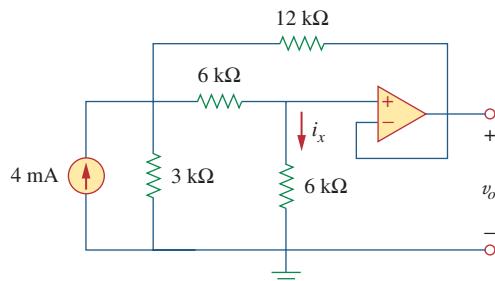
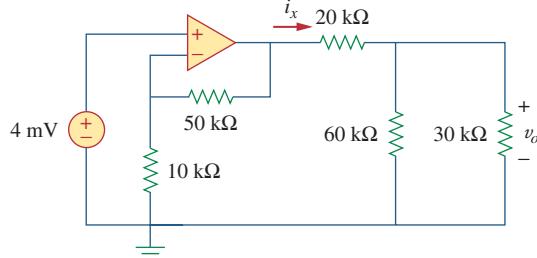


Figure 5.69

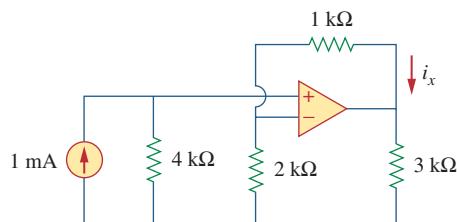
For Prob. 5.31.

- 5.32** Calculate i_x and v_o in the circuit of Fig. 5.70. Find the power dissipated by the $60\text{-k}\Omega$ resistor.

**Figure 5.70**

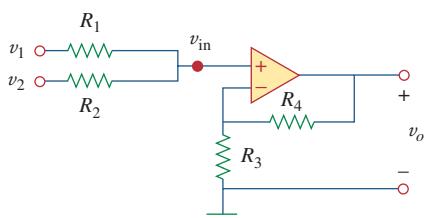
For Prob. 5.32.

- 5.33** Refer to the op amp circuit in Fig. 5.71. Calculate i_x and the power absorbed by the $3\text{-k}\Omega$ resistor.

**Figure 5.71**

For Prob. 5.33.

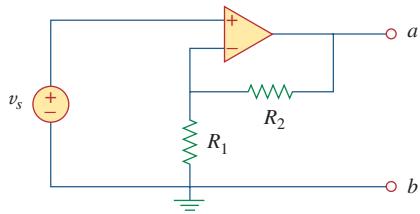
- 5.34** Given the op amp circuit shown in Fig. 5.72, express v_o in terms of v_1 and v_2 .

**Figure 5.72**

For Prob. 5.34.

- 5.35** Design a noninverting amplifier with a gain of 7.5. **e2d**

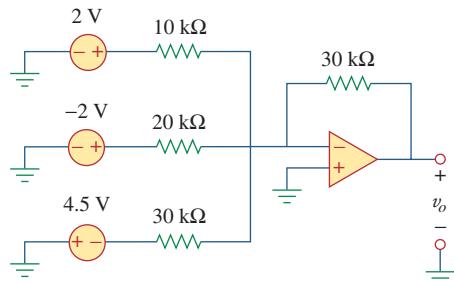
- 5.36** For the circuit shown in Fig. 5.73, find the Thevenin equivalent at terminals $a-b$. (*Hint:* To find R_{Th} , apply a current source i_o and calculate v_o .)

**Figure 5.73**

For Prob. 5.36.

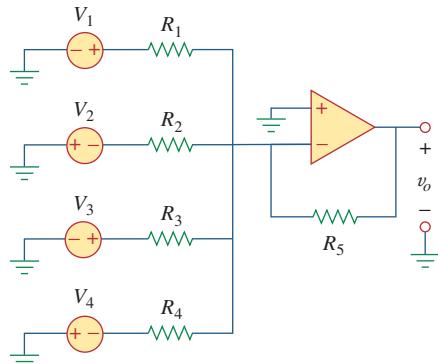
Section 5.6 Summing Amplifier

- 5.37** Determine the output of the summing amplifier in Fig. 5.74.

**Figure 5.74**

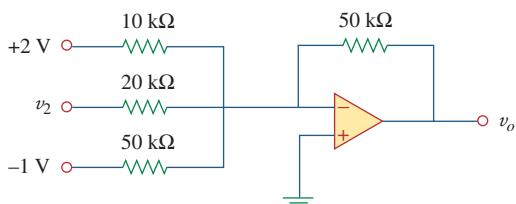
For Prob. 5.37.

- 5.38** Using Fig. 5.75, design a problem to help other students better understand summing amplifiers. **e2d**

**Figure 5.75**

For Prob. 5.38.

- 5.39** For the op amp circuit in Fig. 5.76, determine the value of v_2 in order to make $v_o = -16.5 \text{ V}$.

**Figure 5.76**

For Prob. 5.39.

- 5.40** Referring to the circuit shown in Fig. 5.77, determine V_o in terms of V_1 and V_2 .

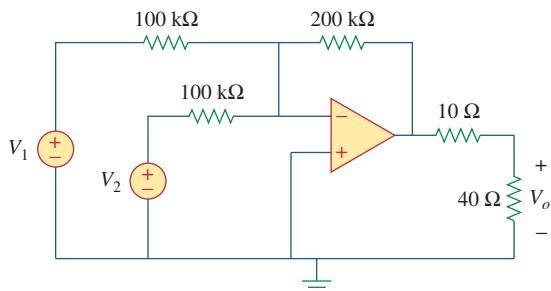


Figure 5.77

For Prob. 5.40.

- 5.41** An *averaging amplifier* is a summer that provides an output equal to the average of the inputs. By using proper input and feedback resistor values, one can get

$$-v_{\text{out}} = \frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

Using a feedback resistor of $10\text{ k}\Omega$, design an averaging amplifier with four inputs.

- 5.42** A three-input summing amplifier has input resistors with $R_1 = R_2 = R_3 = 75\text{ k}\Omega$. To produce an averaging amplifier, what value of feedback resistor is needed?

- 5.43** A four-input summing amplifier has $R_1 = R_2 = R_3 = R_4 = 80\text{ k}\Omega$. What value of feedback resistor is needed to make it an averaging amplifier?

- 5.44** Show that the output voltage v_o of the circuit in Fig. 5.78 is

$$v_o = \frac{(R_3 + R_4)}{R_3(R_1 + R_2)}(R_2v_1 + R_1v_2)$$

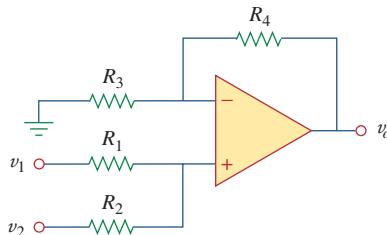


Figure 5.78

For Prob. 5.44.

- 5.45** Design an op amp circuit to perform the following operation:

$$v_o = 3v_1 - 2v_2$$

All resistances must be $\leq 100\text{ k}\Omega$.

- 5.46** Using only two op amps, design a circuit to solve

$$-v_{\text{out}} = \frac{v_1 - v_2}{3} + \frac{v_3}{2}$$

Section 5.7 Difference Amplifier

- 5.47** The circuit in Fig. 5.79 is for a difference amplifier. Find v_o given that $v_1 = 1\text{ V}$ and $v_2 = 2\text{ V}$.

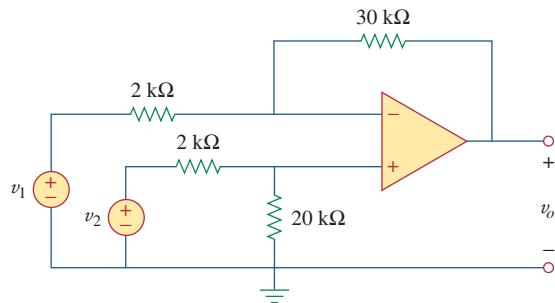


Figure 5.79

For Prob. 5.47.

- 5.48** The circuit in Fig. 5.80 is a differential amplifier driven by a bridge. Find v_o .

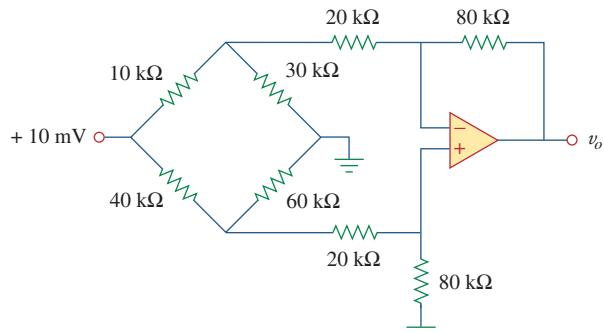


Figure 5.80

For Prob. 5.48.

- 5.49** Design a difference amplifier to have a gain of 4 and a common-mode input resistance of $20\text{ k}\Omega$ at each input.

- 5.50** Design a circuit to amplify the difference between two inputs by 2.5.

(a) Use only one op amp.

(b) Use two op amps.

5.51 Using two op amps, design a subtractor.
end

***5.52** Design an op amp circuit such that
end

$$v_o = 4v_1 + 6v_2 - 3v_3 - 5v_4$$

Let all the resistors be in the range of 20 to 200 kΩ.

***5.53** The ordinary difference amplifier for fixed-gain operation is shown in Fig. 5.81(a). It is simple and reliable unless gain is made variable. One way of providing gain adjustment without losing simplicity and accuracy is to use the circuit in Fig. 5.81(b). Another way is to use the circuit in Fig. 5.81(c). Show that:

(a) for the circuit in Fig. 5.81(a),

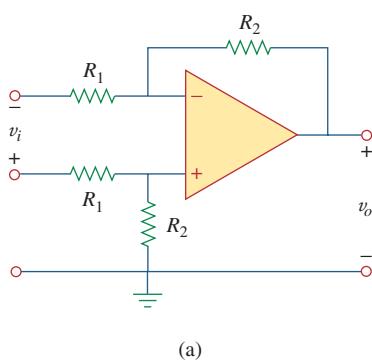
$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

(b) for the circuit in Fig. 5.81(b),

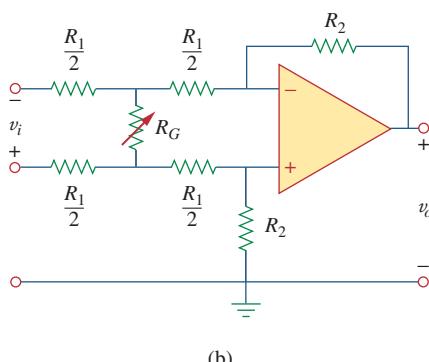
$$\frac{v_o}{v_i} = \frac{R_2 - 1}{R_1 + \frac{R_1}{2R_G}}$$

(c) for the circuit in Fig. 5.81(c),

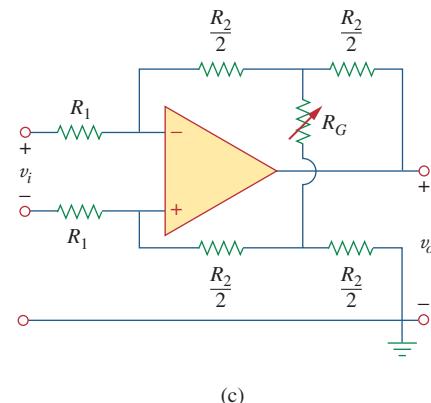
$$\frac{v_o}{v_i} = \frac{R_2}{R_1} \left(1 + \frac{R_2}{2R_G} \right)$$



(a)



(b)



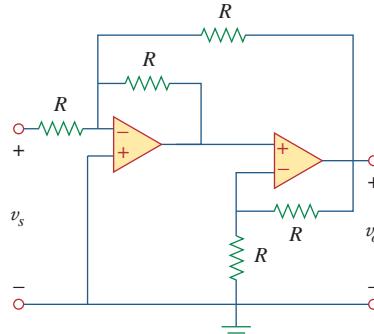
(c)

Figure 5.81

For Prob. 5.53.

Section 5.8 Cascaded Op Amp Circuits

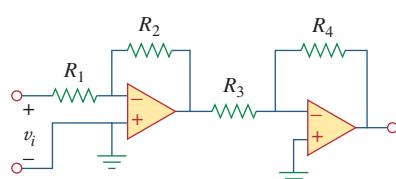
5.54 Determine the voltage transfer ratio v_o/v_s in the op amp circuit of Fig. 5.82, where $R = 10 \text{ k}\Omega$.

**Figure 5.82**

For Prob. 5.54.

5.55 In a certain electronic device, a three-stage amplifier is desired, whose overall voltage gain is 42 dB. The individual voltage gains of the first two stages are to be equal, while the gain of the third is to be one-fourth of each of the first two. Calculate the voltage gain of each.

5.56 Using Fig. 5.83, design a problem to help other students better understand cascaded op amps.

**Figure 5.83**

For Prob. 5.56.

5.57 Find v_o in the op amp circuit of Fig. 5.84.

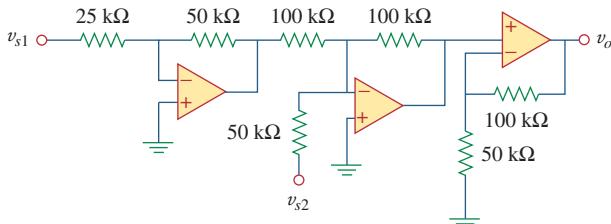


Figure 5.84

For Prob. 5.57.

5.58 Calculate i_o in the op amp circuit of Fig. 5.85.

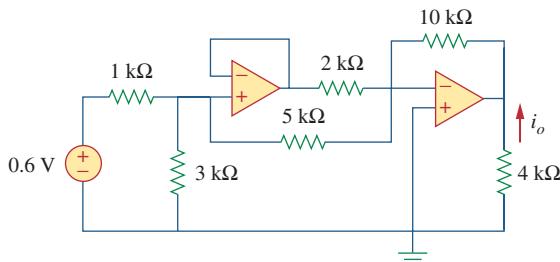


Figure 5.85

For Prob. 5.58.

5.59 In the op amp circuit of Fig. 5.86, determine the voltage gain v_o/v_s . Take $R = 10 \text{ k}\Omega$.

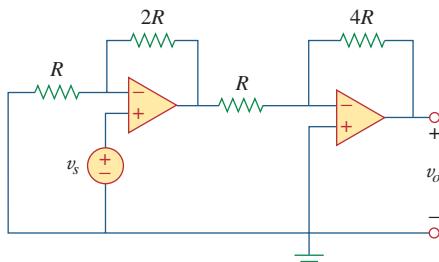


Figure 5.86

For Prob. 5.59.

5.60 Calculate v_o/v_i in the op amp circuit of Fig. 5.87.

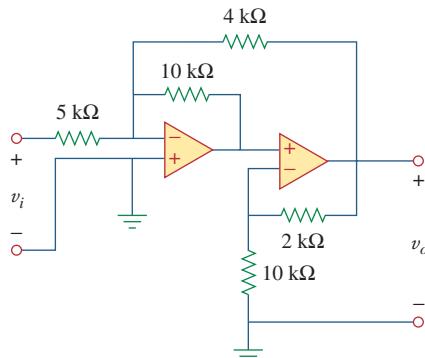


Figure 5.87

For Prob. 5.60.

5.61 Determine v_o in the circuit of Fig. 5.88.

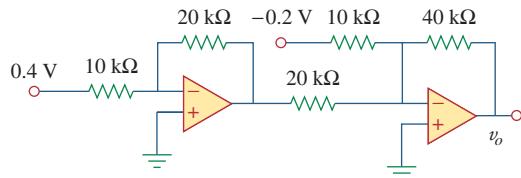


Figure 5.88

For Prob. 5.61.

5.62 Obtain the closed-loop voltage gain v_o/v_i of the circuit in Fig. 5.89.

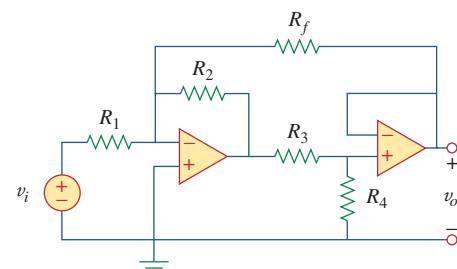


Figure 5.89

For Prob. 5.62.

5.63 Determine the gain v_o/v_i of the circuit in Fig. 5.90.

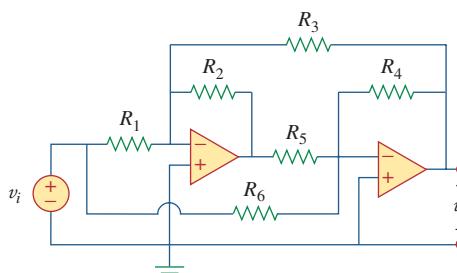


Figure 5.90

For Prob. 5.63.

5.64 For the op amp circuit shown in Fig. 5.91, find v_o/v_s .

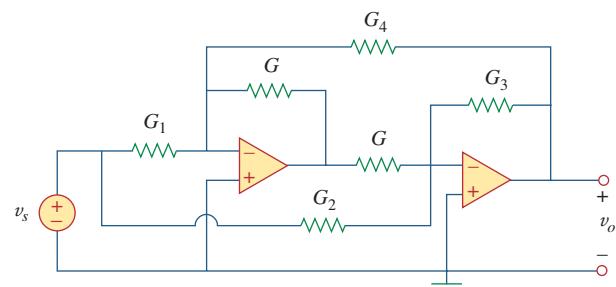


Figure 5.91

For Prob. 5.64.

5.65 Find v_o in the op amp circuit of Fig. 5.92.

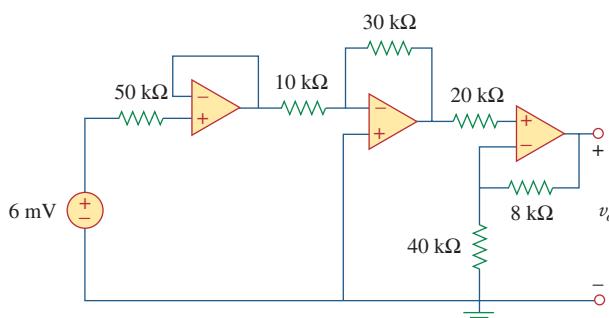


Figure 5.92

For Prob. 5.65.

5.66 For the circuit in Fig. 5.93, find v_o .

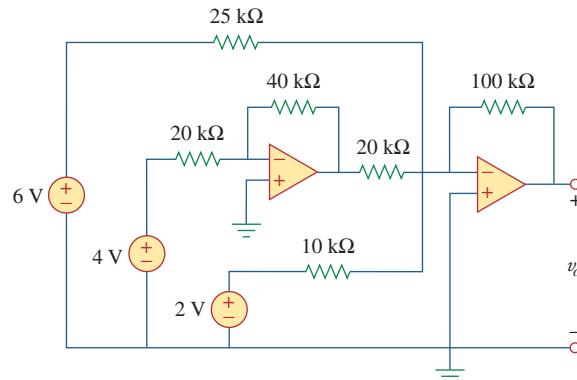


Figure 5.93

For Prob. 5.66.

5.67 Obtain the output v_o in the circuit of Fig. 5.94.

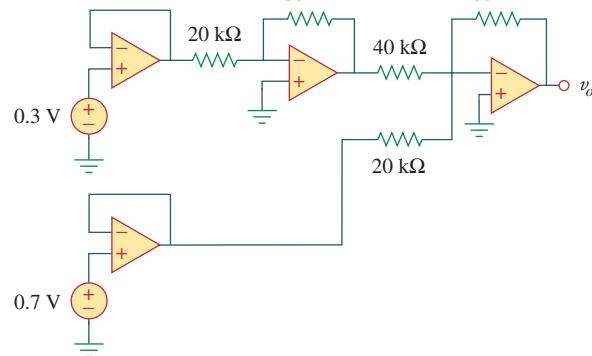


Figure 5.94

For Prob. 5.67.

5.68 Find v_o in the circuit of Fig. 5.95, assuming that $R_f = \infty$ (open circuit).

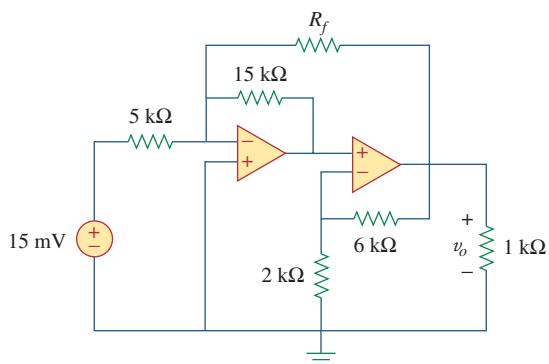


Figure 5.95

For Probs. 5.68 and 5.69.

5.69 Repeat the previous problem if $R_f = 10 \text{ k}\Omega$.

5.70 Determine v_o in the op amp circuit of Fig. 5.96.

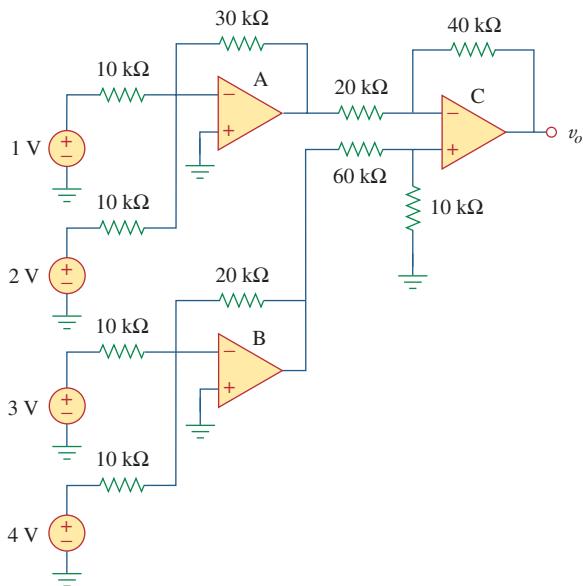


Figure 5.96

For Prob. 5.70.

5.71 Determine v_o in the op amp circuit of Fig. 5.97.

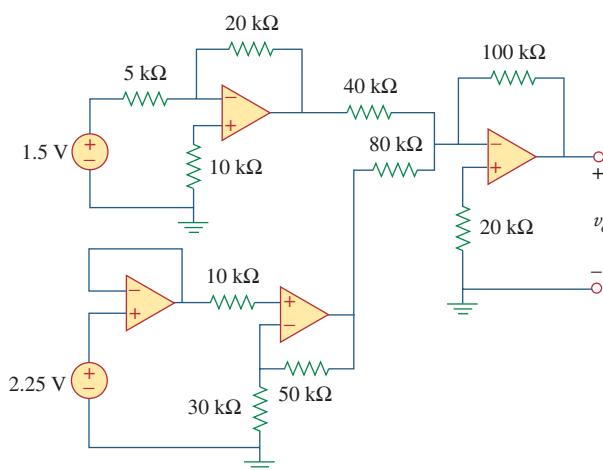


Figure 5.97
For Prob. 5.71.

5.72 Find the load voltage v_L in the circuit of Fig. 5.98.

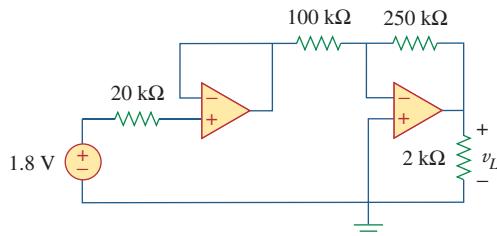


Figure 5.98
For Prob. 5.72.

5.73 Determine the load voltage v_L in the circuit of Fig. 5.99.

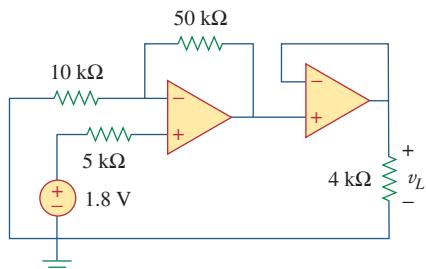


Figure 5.99
For Prob. 5.73.

5.74 Find i_o in the op amp circuit of Fig. 5.100.

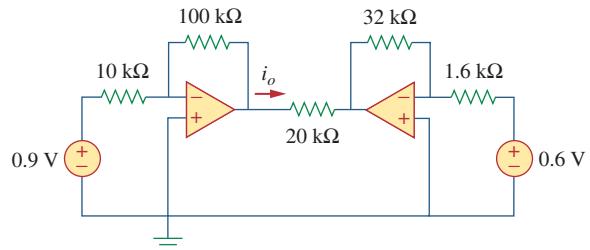


Figure 5.100

For Prob. 5.74.

Section 5.9 Op Amp Circuit Analysis with PSpice

5.75 Rework Example 5.11 using the nonideal op amp LM324 instead of uA741.

5.76 Solve Prob. 5.19 using PSpice or MultiSim and op amp uA741.

5.77 Solve Prob. 5.48 using PSpice or MultiSim and op amp LM324.

5.78 Use PSpice or MultiSim to obtain v_o in the circuit of Fig. 5.101.

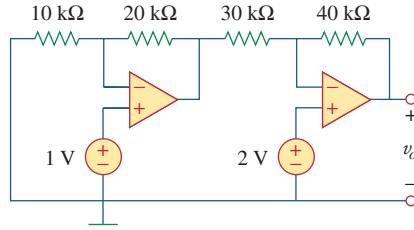


Figure 5.101

For Prob. 5.78.

5.79 Determine v_o in the op amp circuit of Fig. 5.102, using PSpice or MultiSim.

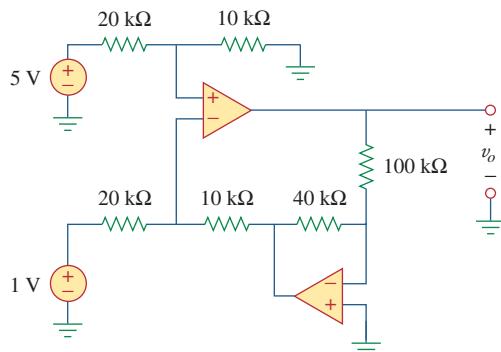


Figure 5.102

For Prob. 5.79.

5.80 Use *PSpice* or *MultiSim* to solve Prob. 5.70.

5.81 Use *PSpice* or *MultiSim* to verify the results in Example 5.9. Assume nonideal op amps LM324.

Section 5.10 Applications

5.82 A five-bit DAC covers a voltage range of 0 to 7.75 V.

e2d Calculate how much voltage each bit is worth.

5.83 Design a six-bit digital-to-analog converter.

e2d

- (a) If $|V_o| = 1.1875$ V is desired, what should $[V_1 V_2 V_3 V_4 V_5 V_6]$ be?
- (b) Calculate $|V_o|$ if $[V_1 V_2 V_3 V_4 V_5 V_6] = [011011]$.
- (c) What is the maximum value $|V_o|$ can assume?

***5.84** A four-bit *R-2R ladder* DAC is presented in Fig. 5.103.

(a) Show that the output voltage is given by

$$-V_o = R_f \left(\frac{V_1}{2R} + \frac{V_2}{4R} + \frac{V_3}{8R} + \frac{V_4}{16R} \right)$$

(b) If $R_f = 12 \text{ k}\Omega$ and $R = 10 \text{ k}\Omega$, find $|V_o|$ for $[V_1 V_2 V_3 V_4] = [1011]$ and $[V_1 V_2 V_3 V_4] = [0101]$.

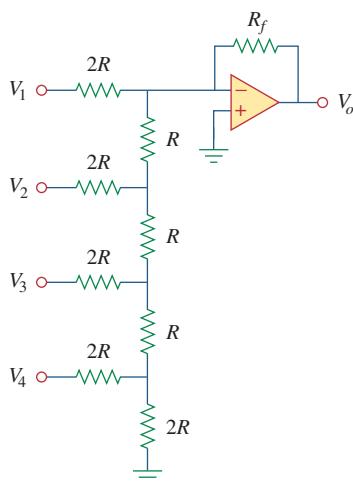


Figure 5.103

For Prob. 5.84.

5.85 In the op amp circuit of Fig. 5.104, find the value of R so that the power absorbed by the $10\text{-k}\Omega$ resistor is 10 mW . Take $v_s = 2 \text{ V}$.

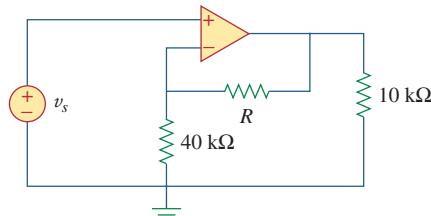


Figure 5.104

For Prob. 5.85.

5.86 Design a voltage controlled ideal current source

e2d (within the operating limits of the op amp) where the output current is equal to $200 v_s(t) \mu\text{A}$.

5.87 Figure 5.105 displays a two-op-amp instrumentation amplifier. Derive an expression for v_o in terms of v_1 and v_2 . How can this amplifier be used as a subtractor?

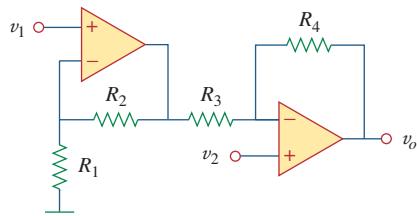


Figure 5.105

For Prob. 5.87.

***5.88** Figure 5.106 shows an instrumentation amplifier driven by a bridge. Obtain the gain v_o/v_i of the amplifier.

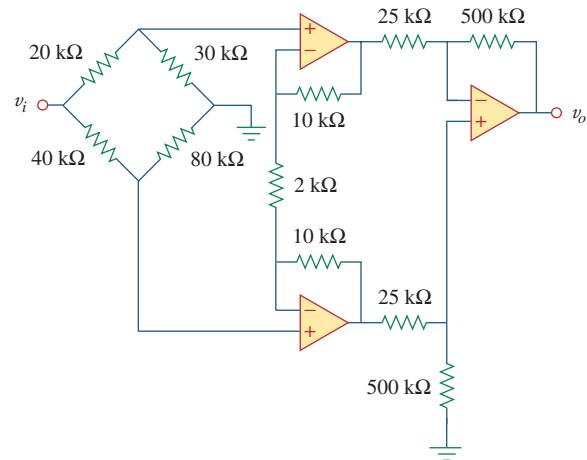


Figure 5.106

For Prob. 5.88.

Comprehensive Problems

- 5.89** Design a circuit that provides a relationship between output voltage v_o and input voltage v_s such that $v_o = 12v_s - 10$. Two op amps, a 6-V battery, and several resistors are available.

- 5.90** The op amp circuit in Fig. 5.107 is a *current amplifier*. Find the current gain i_o/i_s of the amplifier.

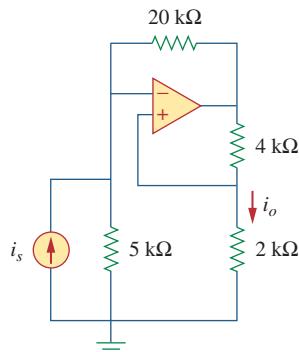


Figure 5.107

For Prob. 5.90.

- 5.91** A noninverting current amplifier is portrayed in Fig. 5.108. Calculate the gain i_o/i_s . Take $R_1 = 8 \text{ k}\Omega$ and $R_2 = 1 \text{ k}\Omega$.

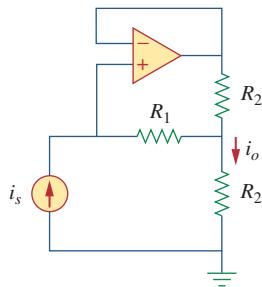


Figure 5.108

For Prob. 5.91.

- 5.92** Refer to the *bridge amplifier* shown in Fig. 5.109. Determine the voltage gain v_o/v_i .

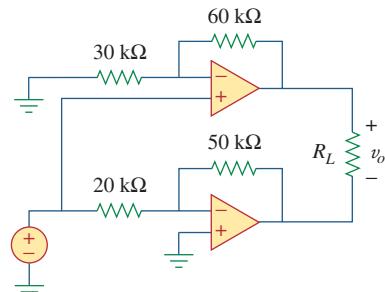


Figure 5.109

For Prob. 5.92.

- *5.93** A voltage-to-current converter is shown in Fig. 5.110, which means that $i_L = Av_i$ if $R_1R_2 = R_3R_4$. Find the constant term A .

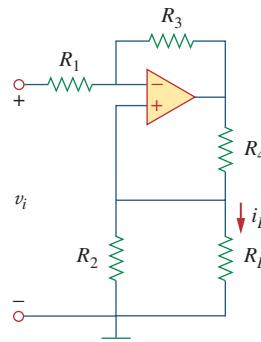


Figure 5.110

For Prob. 5.93.

6

Capacitors and Inductors

But in science the credit goes to the man who convinces the world, not to the man to whom the idea first occurs.

—Francis Darwin

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.c), “an ability to design a system, component, or process to meet desired needs.”

The “ability to design a system, component, or process to meet desired needs” is why engineers are hired. That is why this is the most important *technical* skill that an engineer has. Interestingly, your success as an engineer is directly proportional to your ability to communicate but your being able to design is why you will be hired in the first place.

Design takes place when you have what is termed an open-ended problem that eventually is defined by the solution. Within the context of this course or textbook, we can only explore some of the elements of design. Pursuing all of the steps of our problem-solving technique teaches you several of the most important elements of the design process.

Probably the most important part of design is clearly defining what the system, component, process, or, in our case, problem is. Rarely is an engineer given a perfectly clear assignment. Therefore, as a student, you can develop and enhance this skill by asking yourself, your colleagues, or your professors questions designed to clarify the problem statement.

Exploring alternative solutions is another important part of the design process. Again, as a student, you can practice this part of the design process on almost every problem you work.

Evaluating your solutions is critical to any engineering assignment. Again, this is a skill that you as a student can practice on every problem you work.



Photo by Charles Alexander

6.1 Introduction

So far we have limited our study to resistive circuits. In this chapter, we shall introduce two new and important passive linear circuit elements: the capacitor and the inductor. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.

The application of resistive circuits is quite limited. With the introduction of capacitors and inductors in this chapter, we will be able to analyze more important and practical circuits. Be assured that the circuit analysis techniques covered in Chapters 3 and 4 are equally applicable to circuits with capacitors and inductors.

We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors. As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.

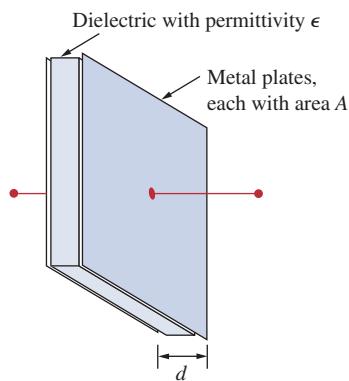


Figure 6.1
A typical capacitor.

6.2 Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

A capacitor is typically constructed as depicted in Fig. 6.1.

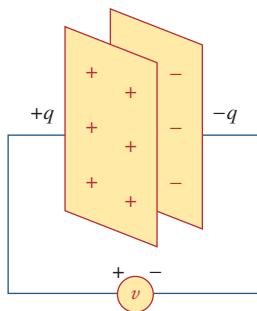


Figure 6.2
A capacitor with applied voltage v .

Alternatively, capacitance is the amount of charge stored per plate for a unit voltage difference in a capacitor.

A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

When a voltage source v is connected to the capacitor, as in Fig. 6.2, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q , is directly proportional to the applied voltage v so that

$$q = Cv \quad (6.1)$$

where C , the constant of proportionality, is known as the *capacitance* of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael Faraday (1791–1867). From Eq. (6.1), we may derive the following definition.

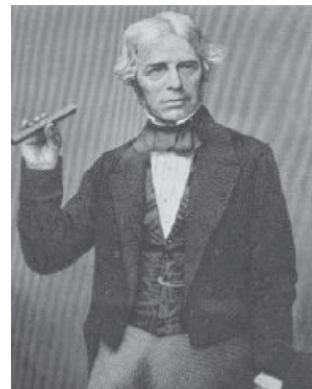
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Note from Eq. (6.1) that 1 farad = 1 coulomb/volt.

Historical

Michael Faraday (1791–1867), an English chemist and physicist, was probably the greatest experimentalist who ever lived.

Born near London, Faraday realized his boyhood dream by working with the great chemist Sir Humphry Davy at the Royal Institution, where he worked for 54 years. He made several contributions in all areas of physical science and coined such words as electrolysis, anode, and cathode. His discovery of electromagnetic induction in 1831 was a major breakthrough in engineering because it provided a way of generating electricity. The electric motor and generator operate on this principle. The unit of capacitance, the farad, was named in his honor.



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at The Huntington Library,
San Marino, California.

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v , it does not depend on q or v . It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. 6.1, the capacitance is given by

$$C = \frac{\epsilon A}{d} \quad (6.2)$$

where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates. Although Eq. (6.2) applies to only parallel-plate capacitors, we may infer from it that, in general, three factors determine the value of the capacitance:

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range. They are described by the dielectric material they are made of and by whether they are of fixed or variable type. Figure 6.3 shows the circuit symbols for fixed and variable capacitors. Note that according to the passive sign convention, if $v > 0$ and $i > 0$ or if $v < 0$ and $i < 0$, the capacitor is being charged, and if $v \cdot i < 0$, the capacitor is discharging.

Figure 6.4 shows common types of fixed-value capacitors. Polyester capacitors are light in weight, stable, and their change with temperature is predictable. Instead of polyester, other dielectric materials such as mica and polystyrene may be used. Film capacitors are rolled and housed in metal or plastic films. Electrolytic capacitors produce very high capacitance. Figure 6.5 shows the most common types of variable capacitors. The capacitance of a trimmer (or padder) capacitor

Capacitor voltage rating and capacitance are typically inversely rated due to the relationships in Eqs. (6.1) and (6.2). Arcing occurs if d is small and V is high.

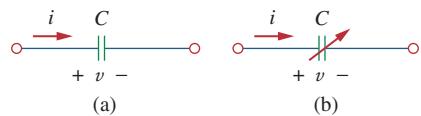
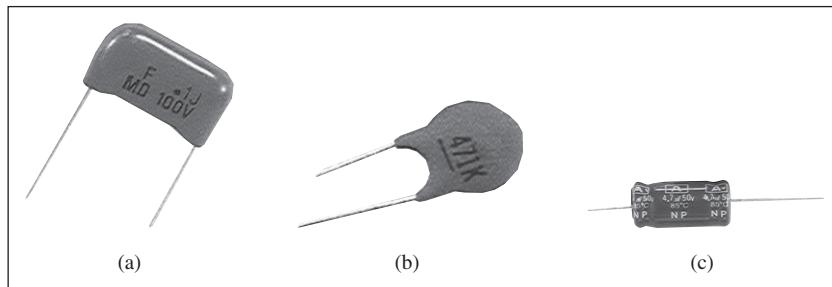
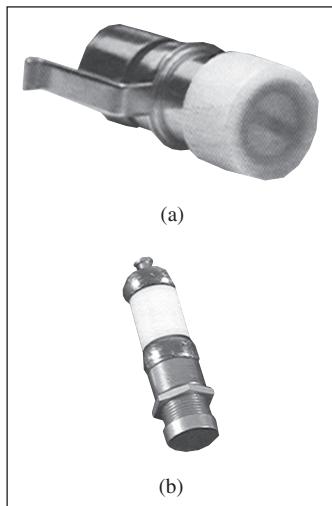


Figure 6.3

Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

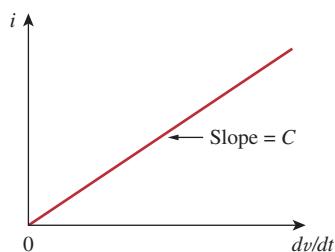
**Figure 6.4**

Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
Courtesy of Tech America.

**Figure 6.5**

Variable capacitors: (a) trimmer capacitor,
(b) filmtrim capacitor.
Courtesy of Johanson.

According to Eq. (6.4), for a capacitor to carry current, its voltage must vary with time. Hence, for constant voltage, $i = 0$.

**Figure 6.6**
Current-voltage relationship of a capacitor.

is often placed in parallel with another capacitor so that the equivalent capacitance can be varied slightly. The capacitance of the variable air capacitor (meshed plates) is varied by turning the shaft. Variable capacitors are used in radio receivers allowing one to tune to various stations. In addition, capacitors are used to block dc, pass ac, shift phase, store energy, start motors, and suppress noise.

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (6.1). Since

$$i = \frac{dq}{dt} \quad (6.3)$$

differentiating both sides of Eq. (6.1) gives

$$i = C \frac{dv}{dt} \quad (6.4)$$

This is the current-voltage relationship for a capacitor, assuming the passive sign convention. The relationship is illustrated in Fig. 6.6 for a capacitor whose capacitance is independent of voltage. Capacitors that satisfy Eq. (6.4) are said to be *linear*. For a *nonlinear capacitor*, the plot of the current-voltage relationship is not a straight line. Although some capacitors are nonlinear, most are linear. We will assume linear capacitors in this book.

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (6.4). We get

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau \quad (6.5)$$

or

$$v(t) = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0) \quad (6.6)$$

where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0 . Equation (6.6) shows that capacitor voltage depends on the past history

of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.

The instantaneous power delivered to the capacitor is

$$p = vi = Cv \frac{dv}{dt} \quad (6.7)$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^t p(\tau) d\tau = C \int_{-\infty}^t v \frac{dv}{d\tau} d\tau = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} Cv^2 \Big|_{v(-\infty)}^{v(t)} \quad (6.8)$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$w = \frac{1}{2} Cv^2 \quad (6.9)$$

Using Eq. (6.1), we may rewrite Eq. (6.9) as

$$w = \frac{q^2}{2C} \quad (6.10)$$

Equation (6.9) or (6.10) represents the energy stored in the electric field that exists between the plates of the capacitor. This energy can be retrieved, since an ideal capacitor cannot dissipate energy. In fact, the word *capacitor* is derived from this element's capacity to store energy in an electric field.

We should note the following important properties of a capacitor:

1. Note from Eq. (6.4) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

A capacitor is an open circuit to dc.

However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges.

2. The voltage on the capacitor must be continuous.

The voltage on a capacitor cannot change abruptly.

The capacitor resists an abrupt change in the voltage across it. According to Eq. (6.4), a discontinuous change in voltage requires an infinite current, which is physically impossible. For example, the voltage across a capacitor may take the form shown in Fig. 6.7(a), whereas it is not physically possible for the capacitor voltage to take the form shown in Fig. 6.7(b) because of the abrupt changes. Conversely, the current through a capacitor can change instantaneously.

3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.
4. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in Fig. 6.8. The leakage resistance may be as high as

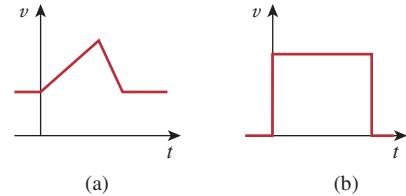


Figure 6.7

Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

An alternative way of looking at this is using Eq. (6.9), which indicates that energy is proportional to voltage squared. Since injecting or extracting energy can only be done over some finite time, voltage cannot change instantaneously across a capacitor.

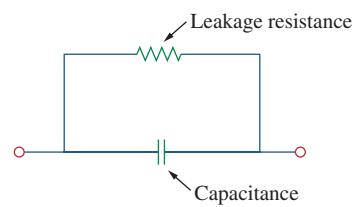


Figure 6.8

Circuit model of a nonideal capacitor.

$100 \text{ M}\Omega$ and can be neglected for most practical applications. For this reason, we will assume ideal capacitors in this book.

Example 6.1

- (a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
 (b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Practice Problem 6.1

What is the voltage across a $4.5\text{-}\mu\text{F}$ capacitor if the charge on one plate is 0.12 mC ? How much energy is stored?

Answer: 26.67 A, 1.6 mJ.

Example 6.2

The voltage across a $5\text{-}\mu\text{F}$ capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$\begin{aligned} i(t) &= C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt}(10 \cos 6000t) \\ &= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A} \end{aligned}$$

Practice Problem 6.2

If a $10\text{-}\mu\text{F}$ capacitor is connected to a voltage source with

$$v(t) = 75 \sin 2000t \text{ V}$$

determine the current through the capacitor.

Answer: $1.5 \cos 2000t \text{ A}$.

Example 6.3

Determine the voltage across a $2\text{-}\mu\text{F}$ capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \frac{1}{C} \int_0^t i dt + v(0)$ and $v(0) = 0$,

$$\begin{aligned} v &= \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t} dt \cdot 10^{-3} \\ &= \frac{3 \times 10^3}{-3000} e^{-3000t} \Big|_0^t = (1 - e^{-3000t}) \text{ V} \end{aligned}$$

The current through a $100\text{-}\mu\text{F}$ capacitor is $i(t) = 50 \sin 120\pi t \text{ mA}$. Calculate the voltage across it at $t = 1 \text{ ms}$ and $t = 5 \text{ ms}$. Take $v(0) = 0$.

Practice Problem 6.3

Answer: 93.14 mV, 1.736 V.

Determine the current through a $200\text{-}\mu\text{F}$ capacitor whose voltage is shown in Fig. 6.9.

Example 6.4**Solution:**

The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C dv/dt$ and $C = 200 \mu\text{F}$, we take the derivative of v to obtain

$$\begin{aligned} i(t) &= 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Thus the current waveform is as shown in Fig. 6.10.

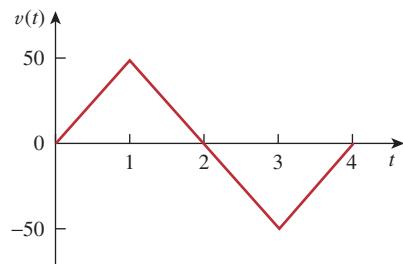


Figure 6.9
For Example 6.4.

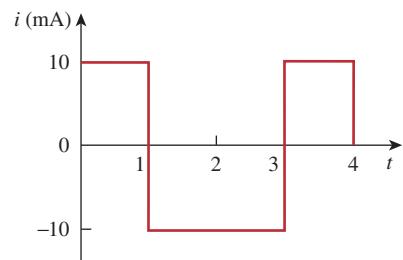


Figure 6.10
For Example 6.4.

An initially uncharged 1-mF capacitor has the current shown in Fig. 6.11 across it. Calculate the voltage across it at $t = 2 \text{ ms}$ and $t = 5 \text{ ms}$.

Practice Problem 6.4

Answer: 100 mV, 400 mV.

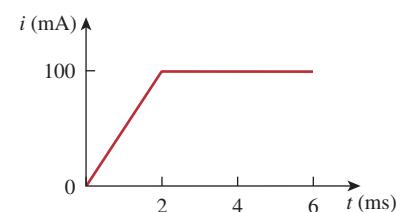
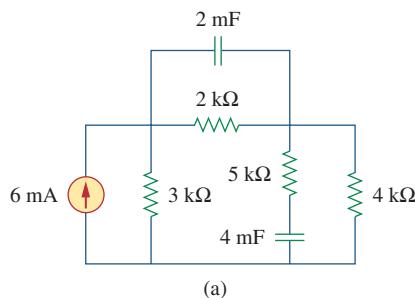


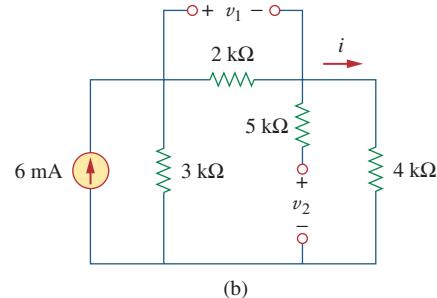
Figure 6.11
For Practice Prob. 6.4.

Example 6.5

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.



(a)



(b)

Figure 6.12

For Example 6.5.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \quad v_2 = 4000i = 8 \text{ V}$$

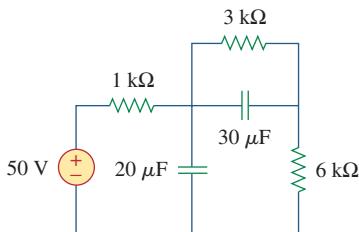
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

Practice Problem 6.5

Under dc conditions, find the energy stored in the capacitors in Fig. 6.13.

**Figure 6.13**

For Practice Prob. 6.5.

Answer: 20.25 mJ, 3.375 mJ.

6.3 Series and Parallel Capacitors

We know from resistive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor C_{eq} .

In order to obtain the equivalent capacitor C_{eq} of N capacitors in parallel, consider the circuit in Fig. 6.14(a). The equivalent circuit is

in Fig. 6.14(b). Note that the capacitors have the same voltage v across them. Applying KCL to Fig. 6.14(a),

$$i = i_1 + i_2 + i_3 + \cdots + i_N \quad (6.11)$$

But $i_k = C_k dv/dt$. Hence,

$$\begin{aligned} i &= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt} \\ &= \left(\sum_{k=1}^N C_k \right) \frac{dv}{dt} = C_{\text{eq}} \frac{dv}{dt} \end{aligned} \quad (6.12)$$

where

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots + C_N \quad (6.13)$$

The **equivalent capacitance** of N parallel-connected capacitors is the sum of the individual capacitances.

We observe that capacitors in parallel combine in the same manner as resistors in series.

We now obtain C_{eq} of N capacitors connected in series by comparing the circuit in Fig. 6.15(a) with the equivalent circuit in Fig. 6.15(b). Note that the same current i flows (and consequently the same charge) through the capacitors. Applying KVL to the loop in Fig. 6.15(a),

$$v = v_1 + v_2 + v_3 + \cdots + v_N \quad (6.14)$$

But $v_k = \frac{1}{C_k} \int_{t_0}^t i(\tau) d\tau + v_k(t_0)$. Therefore,

$$\begin{aligned} v &= \frac{1}{C_1} \int_{t_0}^t i(\tau) d\tau + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^t i(\tau) d\tau + v_2(t_0) \\ &\quad + \cdots + \frac{1}{C_N} \int_{t_0}^t i(\tau) d\tau + v_N(t_0) \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \right) \int_{t_0}^t i(\tau) d\tau + v_1(t_0) + v_2(t_0) \\ &\quad + \cdots + v_N(t_0) \\ &= \frac{1}{C_{\text{eq}}} \int_{t_0}^t i(\tau) d\tau + v(t_0) \end{aligned} \quad (6.15)$$

where

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N} \quad (6.16)$$

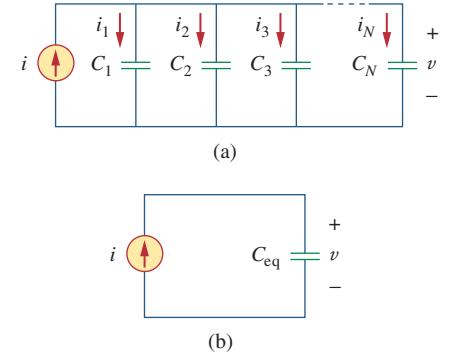


Figure 6.14
(a) Parallel-connected N capacitors,
(b) equivalent circuit for the parallel capacitors.

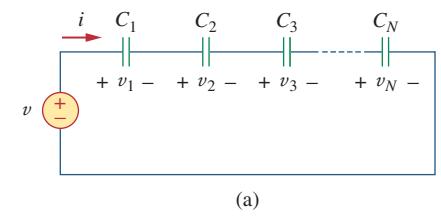


Figure 6.15
(a) Series-connected N capacitors,
(b) equivalent circuit for the series capacitor.

The initial voltage $v(t_0)$ across C_{eq} is required by KVL to be the sum of the capacitor voltages at t_0 . Or according to Eq. (6.15),

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

Thus, according to Eq. (6.16),

The **equivalent capacitance** of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Note that capacitors in series combine in the same manner as resistors in parallel. For $N = 2$ (i.e., two capacitors in series), Eq. (6.16) becomes

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \quad (6.17)$$

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.

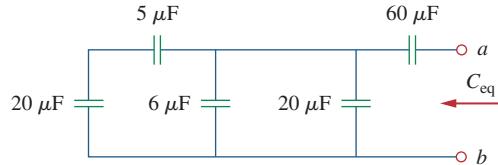


Figure 6.16

For Example 6.6.

Solution:

The $20\text{-}\mu\text{F}$ and $5\text{-}\mu\text{F}$ capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4 \mu\text{F}$$

This $4\text{-}\mu\text{F}$ capacitor is in parallel with the $6\text{-}\mu\text{F}$ and $20\text{-}\mu\text{F}$ capacitors; their combined capacitance is

$$4 + 6 + 20 = 30 \mu\text{F}$$

This $30\text{-}\mu\text{F}$ capacitor is in series with the $60\text{-}\mu\text{F}$ capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20 \mu\text{F}$$

Find the equivalent capacitance seen at the terminals of the circuit in Fig. 6.17.

Practice Problem 6.6

Answer: $40 \mu\text{F}$.

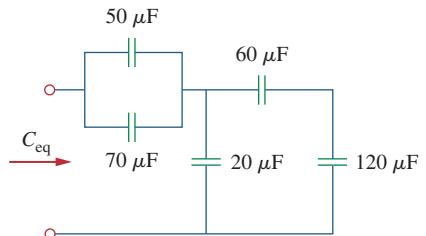


Figure 6.17

For Practice Prob. 6.6.

For the circuit in Fig. 6.18, find the voltage across each capacitor.

Example 6.7

Solution:

We first find the equivalent capacitance C_{eq} , shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get $40 + 20 = 60 \text{ mF}$. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \quad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2 , we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60 \text{ mF}$. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

Find the voltage across each of the capacitors in Fig. 6.20.

Practice Problem 6.7

Answer: $v_1 = 45 \text{ V}$, $v_2 = 45 \text{ V}$, $v_3 = 15 \text{ V}$, $v_4 = 30 \text{ V}$.

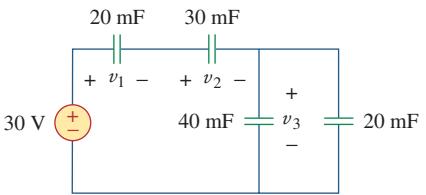


Figure 6.18

For Example 6.7.

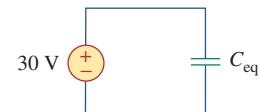


Figure 6.19

Equivalent circuit for Fig. 6.18.

Practice Problem 6.7

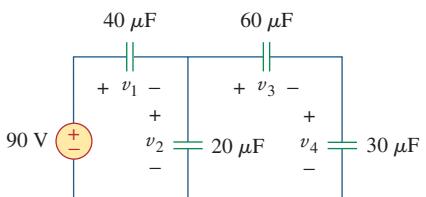
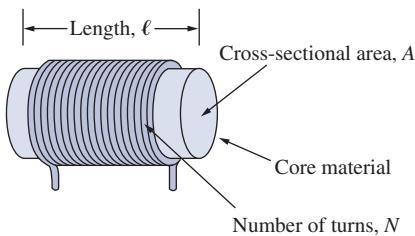


Figure 6.20

For Practice Prob. 6.7.

**Figure 6.21**

Typical form of an inductor.

6.4 Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig. 6.21.

An **inductor** consists of a coil of conducting wire.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$v = L \frac{di}{dt} \quad (6.18)$$

where L is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H), named in honor of the American inventor Joseph Henry (1797–1878). It is clear from Eq. (6.18) that 1 henry equals 1 volt-second per ampere.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

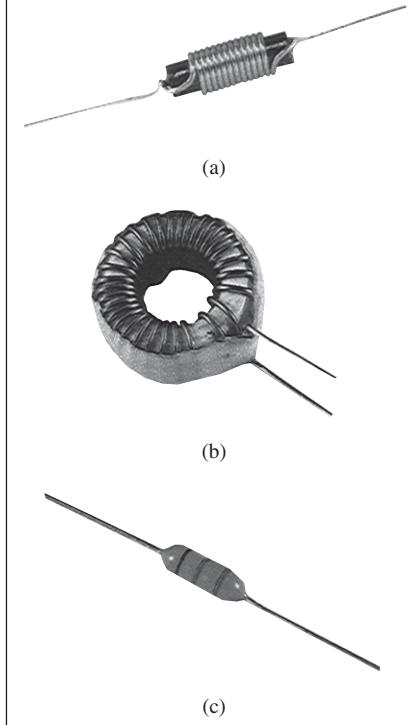
The inductance of an inductor depends on its physical dimension and construction. Formulas for calculating the inductance of inductors of different shapes are derived from electromagnetic theory and can be found in standard electrical engineering handbooks. For example, for the inductor, (solenoid) shown in Fig. 6.21,

$$L = \frac{N^2 \mu A}{\ell} \quad (6.19)$$

where N is the number of turns, ℓ is the length, A is the cross-sectional area, and μ is the permeability of the core. We can see from Eq. (6.19) that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μ H), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms *coil* and *choke* are also used for inductors. Common inductors are shown in Fig. 6.22. The circuit symbols for inductors are shown in Fig. 6.23, following the passive sign convention.

Equation (6.18) is the voltage-current relationship for an inductor. Figure 6.24 shows this relationship graphically for an inductor whose

**Figure 6.22**

Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.

Courtesy of Tech America.

Historical

Joseph Henry (1797–1878), an American physicist, discovered inductance and constructed an electric motor.

Born in Albany, New York, Henry graduated from Albany Academy and taught philosophy at Princeton University from 1832 to 1846. He was the first secretary of the Smithsonian Institution. He conducted several experiments on electromagnetism and developed powerful electromagnets that could lift objects weighing thousands of pounds. Interestingly, Joseph Henry discovered electromagnetic induction before Faraday but failed to publish his findings. The unit of inductance, the henry, was named after him.



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inductance is independent of current. Such an inductor is known as a *linear inductor*. For a *nonlinear inductor*, the plot of Eq. (6.18) will not be a straight line because its inductance varies with current. We will assume linear inductors in this textbook unless stated otherwise.

The current-voltage relationship is obtained from Eq. (6.18) as

$$di = \frac{1}{L}v dt$$

Integrating gives

$$i = \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau \quad (6.20)$$

or

$$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0) \quad (6.21)$$

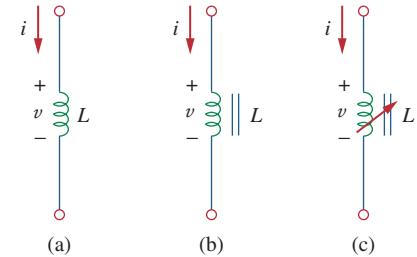
where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty) = 0$. The idea of making $i(-\infty) = 0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eq. (6.18). The power delivered to the inductor is

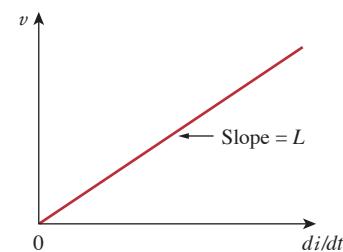
$$p = vi = \left(L \frac{di}{dt} \right) i \quad (6.22)$$

The energy stored is

$$\begin{aligned} w &= \int_{-\infty}^t p(\tau) d\tau = L \int_{-\infty}^t \frac{di}{d\tau} id\tau \\ &= L \int_{-\infty}^t i di = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty) \end{aligned} \quad (6.23)$$

**Figure 6.23**

Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

**Figure 6.24**

Voltage-current relationship of an inductor.

Since $i(-\infty) = 0$,

$$w = \frac{1}{2} L i^2 \quad (6.24)$$

We should note the following important properties of an inductor.

1. Note from Eq. (6.18) that the voltage across an inductor is zero when the current is constant. Thus,

An inductor acts like a short circuit to dc.

2. An important property of the inductor is its opposition to the change in current flowing through it.

The current through an inductor cannot change instantaneously.

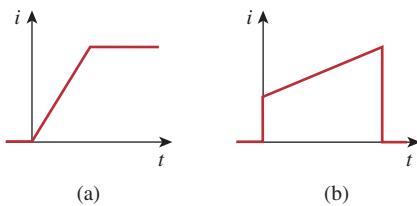


Figure 6.25

Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

Since an inductor is often made of a highly conducting wire, it has a very small resistance.

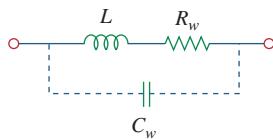


Figure 6.26

Circuit model for a practical inductor.

According to Eq. (6.18), a discontinuous change in the current through an inductor requires an infinite voltage, which is not physically possible. Thus, an inductor opposes an abrupt change in the current through it. For example, the current through an inductor may take the form shown in Fig. 6.25(a), whereas the inductor current cannot take the form shown in Fig. 6.25(b) in real-life situations due to the discontinuities. However, the voltage across an inductor can change abruptly.

3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.
4. A practical, nonideal inductor has a significant resistive component, as shown in Fig. 6.26. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance* R_w , and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The nonideal inductor also has a *winding capacitance* C_w due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies. We will assume ideal inductors in this book.

Example 6.8

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L di/dt$ and $L = 0.1$ H,

$$v = 0.1 \frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

If the current through a 1-mH inductor is $i(t) = 60 \cos 100t$ mA, find the terminal voltage and the energy stored.

Practice Problem 6.8

Answer: $-6 \sin 100t$ mV, $1.8 \cos^2(100t)$ μJ .

Find the current through a 5-H inductor if the voltage across it is

Example 6.9

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

Since $i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5} \int_0^t 30t^2 dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p dt = \int_0^5 60t^5 dt = 60 \frac{t^6}{6} \Big|_0^5 = 156.25 \text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.24), by writing

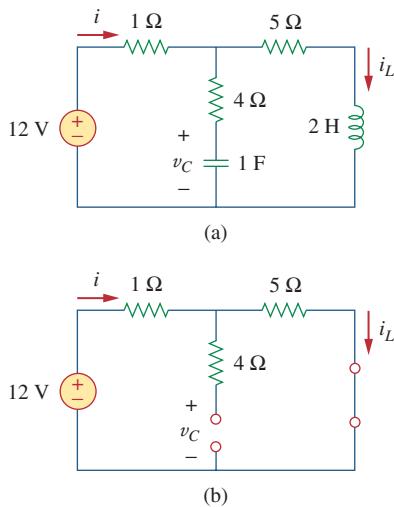
$$w|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

The terminal voltage of a 2-H inductor is $v = 10(1 - t)$ V. Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s. Assume $i(0) = 2$ A.

Practice Problem 6.9

Answer: -18 A, 320 J.

Example 6.10**Figure 6.27**

For Example 6.10.

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i , v_C , and i_L , (b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

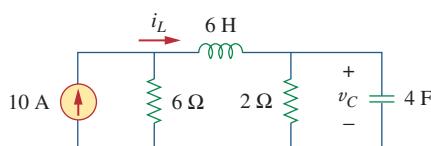
$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

Practice Problem 6.10**Figure 6.28**

For Practice Prob. 6.10.

Determine v_C , i_L , and the energy stored in the capacitor and inductor in the circuit of Fig. 6.28 under dc conditions.

Answer: 15 V, 7.5 A, 450 J, 168.75 J.

6.5 Series and Parallel Inductors

Now that the inductor has been added to our list of passive elements, it is necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

Consider a series connection of N inductors, as shown in Fig. 6.29(a), with the equivalent circuit shown in Fig. 6.29(b). The inductors have the same current through them. Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \cdots + v_N \quad (6.25)$$

Substituting $v_k = L_k di/dt$ results in

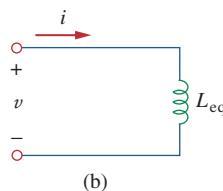
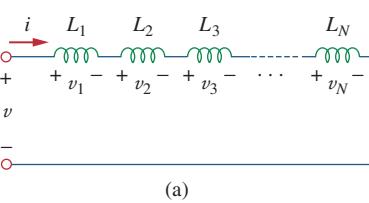
$$\begin{aligned} v &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt} \\ &= (L_1 + L_2 + L_3 + \cdots + L_N) \frac{di}{dt} \\ &= \left(\sum_{k=1}^N L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt} \end{aligned} \quad (6.26)$$

where

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N \quad (6.27)$$

Figure 6.29

(a) A series connection of N inductors,
(b) equivalent circuit for the series
inductors.



Thus,

The **equivalent inductance** of series-connected inductors is the sum of the individual inductances.

Inductors in series are combined in exactly the same way as resistors in series.

We now consider a parallel connection of N inductors, as shown in Fig. 6.30(a), with the equivalent circuit in Fig. 6.30(b). The inductors have the same voltage across them. Using KCL,

$$i = i_1 + i_2 + i_3 + \cdots + i_N \quad (6.28)$$

But $i_k = \frac{1}{L_k} \int_{t_0}^t v dt + i_k(t_0)$; hence,

$$\begin{aligned} i &= \frac{1}{L_1} \int_{t_0}^t v dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^t v dt + i_2(t_0) \\ &\quad + \cdots + \frac{1}{L_N} \int_{t_0}^t v dt + i_N(t_0) \\ &= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} \right) \int_{t_0}^t v dt + i_1(t_0) + i_2(t_0) \\ &\quad + \cdots + i_N(t_0) \\ &= \left(\sum_{k=1}^N \frac{1}{L_k} \right) \int_{t_0}^t v dt + \sum_{k=1}^N i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^t v dt + i(t_0) \quad (6.29) \end{aligned}$$

where

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \quad (6.30)$$

The initial current $i(t_0)$ through L_{eq} at $t = t_0$ is expected by KCL to be the sum of the inductor currents at t_0 . Thus, according to Eq. (6.29),

$$i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

According to Eq. (6.30),

The **equivalent inductance** of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

Note that the inductors in parallel are combined in the same way as resistors in parallel.

For two inductors in parallel ($N = 2$), Eq. (6.30) becomes

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \quad (6.31)$$

As long as all the elements are of the same type, the Δ -Y transformations for resistors discussed in Section 2.7 can be extended to capacitors and inductors.

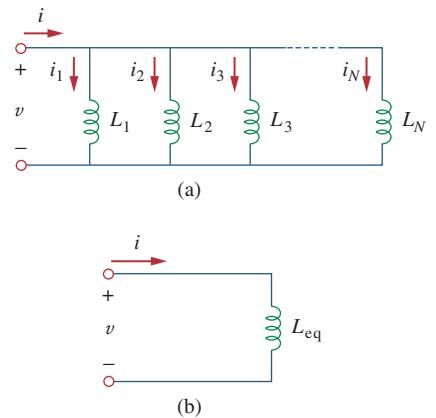


Figure 6.30

(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

TABLE 6.1

Important characteristics of the basic elements.[†]

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
$v-i$:	$v = iR$	$v = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$	$v = L \frac{di}{dt}$
$i-v$:	$i = v/R$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$
p or w :	$p = i^2 R = \frac{v^2}{R}$	$w = \frac{1}{2} Cv^2$	$w = \frac{1}{2} Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
	Circuit variable that cannot change abruptly:	Not applicable	v
			i

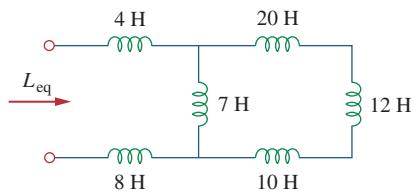
[†] Passive sign convention is assumed.

It is appropriate at this point to summarize the most important characteristics of the three basic circuit elements we have studied. The summary is given in Table 6.1.

The wye-delta transformation discussed in Section 2.7 for resistors can be extended to capacitors and inductors.

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.

**Figure 6.31**

For Example 6.11.

Solution:

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductor. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

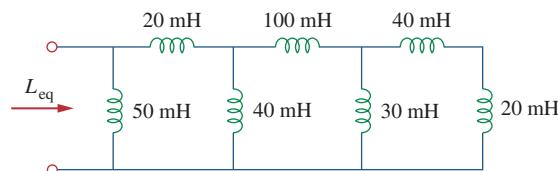
$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

Practice Problem 6.11

Calculate the equivalent inductance for the inductive ladder network in Fig. 6.32.

**Figure 6.32**

For Practice Prob. 6.11.

Answer: 25 mH.

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.

Example 6.12

Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{\text{eq}} = 2 + 4 \parallel 12 = 2 + 3 = 5 \text{ H}$$

Thus,

$$v(t) = L_{\text{eq}} \frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

and

$$v_1(t) = 2 \frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$\begin{aligned} i_1(t) &= \frac{1}{4} \int_0^t v_2 dt + i_1(0) = \frac{120}{4} \int_0^t e^{-10t} dt + 5 \text{ mA} \\ &= -3e^{-10t}|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA} \end{aligned}$$

Similarly,

$$\begin{aligned} i_2(t) &= \frac{1}{12} \int_0^t v_2 dt + i_2(0) = \frac{120}{12} \int_0^t e^{-10t} dt - 1 \text{ mA} \\ &= -e^{-10t}|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA} \end{aligned}$$

Note that $i_1(t) + i_2(t) = i(t)$.

In the circuit of Fig. 6.34, $i_1(t) = 0.6e^{-2t}$ A. If $i(0) = 1.4$ A, find: (a) $i_2(0)$; (b) $i_2(t)$ and $i(t)$; (c) $v_1(t)$, $v_2(t)$, and $v(t)$.

Answer: (a) 0.8 A, (b) $(-0.4 + 1.2e^{-2t})$ A, $(-0.4 + 1.8e^{-2t})$ A, (c) $-36e^{-2t}$ V, $-7.2e^{-2t}$ V, $-28.8e^{-2t}$ V.

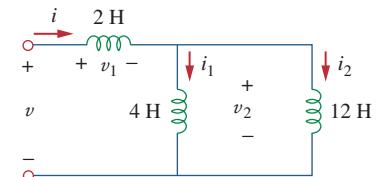


Figure 6.33

For Example 6.12.

Practice Problem 6.12

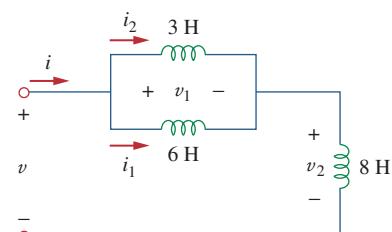


Figure 6.34

For Practice Prob. 6.12.

6.6 [†] Applications

Circuit elements such as resistors and capacitors are commercially available in either discrete form or integrated-circuit (IC) form. Unlike capacitors and resistors, inductors with appreciable inductance are difficult to produce on IC substrates. Therefore, inductors (coils) usually

come in discrete form and tend to be more bulky and expensive. For this reason, inductors are not as versatile as capacitors and resistors, and they are more limited in applications. However, there are several applications in which inductors have no practical substitute. They are routinely used in relays, delays, sensing devices, pick-up heads, telephone circuits, radio and TV receivers, power supplies, electric motors, microphones, and loudspeakers, to mention a few.

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a short period of time.
2. Capacitors oppose any abrupt change in voltage, while inductors oppose any abrupt change in current. This property makes inductors useful for spark or arc suppression and for converting pulsating dc voltage into relatively smooth dc voltage.
3. Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.

The first two properties are put to use in dc circuits, while the third one is taken advantage of in ac circuits. We will see how useful these properties are in later chapters. For now, consider three applications involving capacitors and op amps: integrator, differentiator, and analog computer.

6.6.1 Integrator

Important op amp circuits that use energy-storage elements include integrators and differentiators. These op amp circuits often involve resistors and capacitors; inductors (coils) tend to be more bulky and expensive.

The op amp integrator is used in numerous applications, especially in analog computers, to be discussed in Section 6.6.3.

An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

If the feedback resistor R_f in the familiar inverting amplifier of Fig. 6.35(a) is replaced by a capacitor, we obtain an ideal integrator, as shown in Fig. 6.35(b). It is interesting that we can obtain a mathematical representation of integration this way. At node a in Fig. 6.35(b),

$$i_R = i_C \quad (6.32)$$

But

$$i_R = \frac{v_i}{R}, \quad i_C = -C \frac{dv_o}{dt}$$

Substituting these in Eq. (6.32), we obtain

$$\frac{v_i}{R} = -C \frac{dv_o}{dt} \quad (6.33a)$$

$$dv_o = -\frac{1}{RC} v_i dt \quad (6.33b)$$

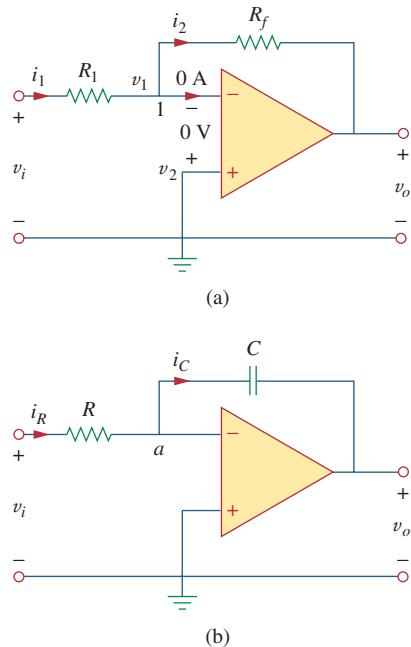


Figure 6.35

Replacing the feedback resistor in the inverting amplifier in (a) produces an integrator in (b).

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC} \int_0^t v_i(\tau) d\tau \quad (6.34)$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC} \int_0^t v_i(\tau) d\tau \quad (6.35)$$

which shows that the circuit in Fig. 6.35(b) provides an output voltage proportional to the integral of the input. In practice, the op amp integrator requires a feedback resistor to reduce dc gain and prevent saturation. Care must be taken that the op amp operates within the linear range so that it does not saturate.

If $v_1 = 10 \cos 2t$ mV and $v_2 = 0.5t$ mV, find v_o in the op amp circuit in Fig. 6.36. Assume that the voltage across the capacitor is initially zero.

Example 6.13

Solution:

This is a summing integrator, and

$$\begin{aligned} v_o &= -\frac{1}{R_1 C} \int v_1 dt - \frac{1}{R_2 C} \int v_2 dt \\ &= -\frac{1}{3 \times 10^6 \times 2 \times 10^{-6}} \int_0^t 10 \cos(2\tau) d\tau \\ &\quad - \frac{1}{100 \times 10^3 \times 2 \times 10^{-6}} \int_0^t 0.5\tau d\tau \\ &= -\frac{1}{6} \frac{10}{2} \sin 2t - \frac{1}{0.2} \frac{0.5t^2}{2} = -0.833 \sin 2t - 1.25t^2 \text{ mV} \end{aligned}$$

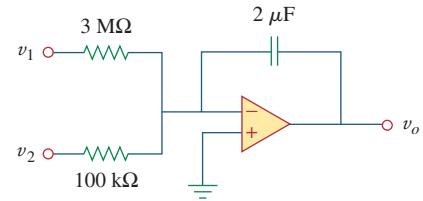


Figure 6.36
For Example 6.13.

The integrator in Fig. 6.35(b) has $R = 100$ k Ω , $C = 20$ μ F. Determine the output voltage when a dc voltage of 2.5 mV is applied at $t = 0$. Assume that the op amp is initially nulled.

Practice Problem 6.13

Answer: $-1.25t$ mV.

6.6.2 Differentiator

A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

In Fig. 6.35(a), if the input resistor is replaced by a capacitor, the resulting circuit is a differentiator, shown in Fig. 6.37. Applying KCL at node a ,

$$i_R = i_C \quad (6.36)$$

But

$$i_R = -\frac{v_o}{R}, \quad i_C = C \frac{dv_i}{dt}$$

Substituting these in Eq. (6.36) yields

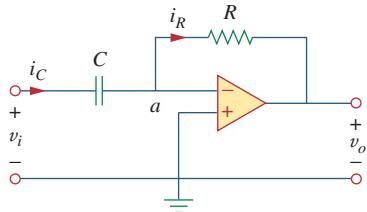


Figure 6.37
An op amp differentiator.

$$v_o = -RC \frac{dv_i}{dt} \quad (6.37)$$

showing that the output is the derivative of the input. Differentiator circuits are electronically unstable because any electrical noise within the circuit is exaggerated by the differentiator. For this reason, the differentiator circuit in Fig. 6.37 is not as useful and popular as the integrator. It is seldom used in practice.

Example 6.14

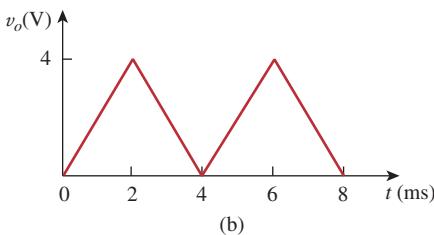
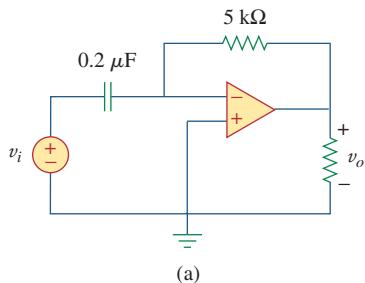


Figure 6.38
For Example 6.14.

Sketch the output voltage for the circuit in Fig. 6.38(a), given the input voltage in Fig. 6.38(b). Take $v_o = 0$ at $t = 0$.

Solution:

This is a differentiator with

$$RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 10^{-3} \text{ s}$$

For $0 < t < 4 \text{ ms}$, we can express the input voltage in Fig. 6.38(b) as

$$v_i = \begin{cases} 2000t & 0 < t < 2 \text{ ms} \\ 8 - 2000t & 2 < t < 4 \text{ ms} \end{cases}$$

This is repeated for $4 < t < 8 \text{ ms}$. Using Eq. (6.37), the output is obtained as

$$v_o = -RC \frac{dv_i}{dt} = \begin{cases} -2 \text{ V} & 0 < t < 2 \text{ ms} \\ 2 \text{ V} & 2 < t < 4 \text{ ms} \end{cases}$$

Thus, the output is as sketched in Fig. 6.39.

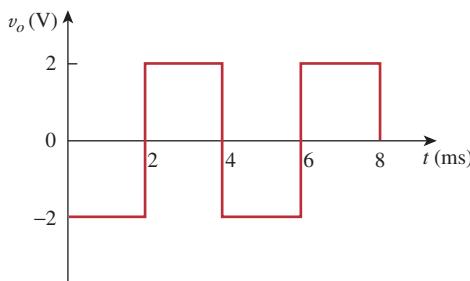


Figure 6.39
Output of the circuit in Fig. 6.38(a).

Practice Problem 6.14

The differentiator in Fig. 6.37 has $R = 100 \text{ k}\Omega$ and $C = 0.1 \mu\text{F}$. Given that $v_i = 1.25t \text{ V}$, determine the output v_o .

Answer: -12.5 mV .

6.6.3 Analog Computer

Op amps were initially developed for electronic analog computers. Analog computers can be programmed to solve mathematical models of mechanical or electrical systems. These models are usually expressed in terms of differential equations.

To solve simple differential equations using the analog computer requires cascading three types of op amp circuits: integrator circuits, summing amplifiers, and inverting/noninverting amplifiers for negative/positive scaling. The best way to illustrate how an analog computer solves a differential equation is with an example.

Suppose we desire the solution $x(t)$ of the equation

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = f(t), \quad t > 0 \quad (6.38)$$

where a , b , and c are constants, and $f(t)$ is an arbitrary forcing function. The solution is obtained by first solving the highest-order derivative term. Solving for d^2x/dt^2 yields

$$\frac{d^2x}{dt^2} = \frac{f(t)}{a} - \frac{b}{a} \frac{dx}{dt} - \frac{c}{a} x \quad (6.39)$$

To obtain dx/dt , the d^2x/dt^2 term is integrated and inverted. Finally, to obtain x , the dx/dt term is integrated and inverted. The forcing function is injected at the proper point. Thus, the analog computer for solving Eq. (6.38) is implemented by connecting the necessary summers, inverters, and integrators. A plotter or oscilloscope may be used to view the output x , or dx/dt , or d^2x/dt^2 , depending on where it is connected in the system.

Although the above example is on a second-order differential equation, any differential equation can be simulated by an analog computer comprising integrators, inverters, and inverting summers. But care must be exercised in selecting the values of the resistors and capacitors, to ensure that the op amps do not saturate during the solution time interval.

The analog computers with vacuum tubes were built in the 1950s and 1960s. Recently their use has declined. They have been superseded by modern digital computers. However, we still study analog computers for two reasons. First, the availability of integrated op amps has made it possible to build analog computers easily and cheaply. Second, understanding analog computers helps with the appreciation of the digital computers.

Design an analog computer circuit to solve the differential equation:

Example 6.15

$$\frac{d^2v_o}{dt^2} + 2 \frac{dv_o}{dt} + v_o = 10 \sin 4t, \quad t > 0$$

subject to $v_o(0) = -4$, $v'_o(0) = 1$, where the prime refers to the time derivative.

Solution:

- Define.** We have a clearly defined problem and expected solution. I might remind the student that many times the problem is not so well defined and this portion of the problem-solving process could

require much more effort. If this is so, then you should always keep in mind that time spent here will result in much less effort later and most likely save you a lot of frustration in the process.

2. **Present.** Clearly, using the devices developed in Section 6.6.3 will allow us to create the desired analog computer circuit. We will need the integrator circuits (possibly combined with a summing capability) and one or more inverter circuits.
3. **Alternative.** The approach for solving this problem is straightforward. We will need to pick the correct values of resistances and capacitors to allow us to realize the equation we are representing. The final output of the circuit will give the desired result.
4. **Attempt.** There are an infinite number of possibilities for picking the resistors and capacitors, many of which will result in correct solutions. Extreme values of resistors and capacitors will result in incorrect outputs. For example, low values of resistors will overload the electronics. Picking values of resistors that are too large will cause the op amps to stop functioning as ideal devices. The limits can be determined from the characteristics of the real op amp.

We first solve for the second derivative as

$$\frac{d^2v_o}{dt^2} = 10 \sin 4t - 2 \frac{dv_o}{dt} - v_o \quad (6.15.1)$$

Solving this requires some mathematical operations, including summing, scaling, and integration. Integrating both sides of Eq. (6.15.1) gives

$$\frac{dv_o}{dt} = - \int_0^t \left(-10 \sin(4\tau) + 2 \frac{dv_o(\tau)}{d\tau} + v_o(\tau) \right) d\tau + v'_o(0) \quad (6.15.2)$$

where $v'_o(0) = 1$. We implement Eq. (6.15.2) using the summing integrator shown in Fig. 6.40(a). The values of the resistors and capacitors have been chosen so that $RC = 1$ for the term

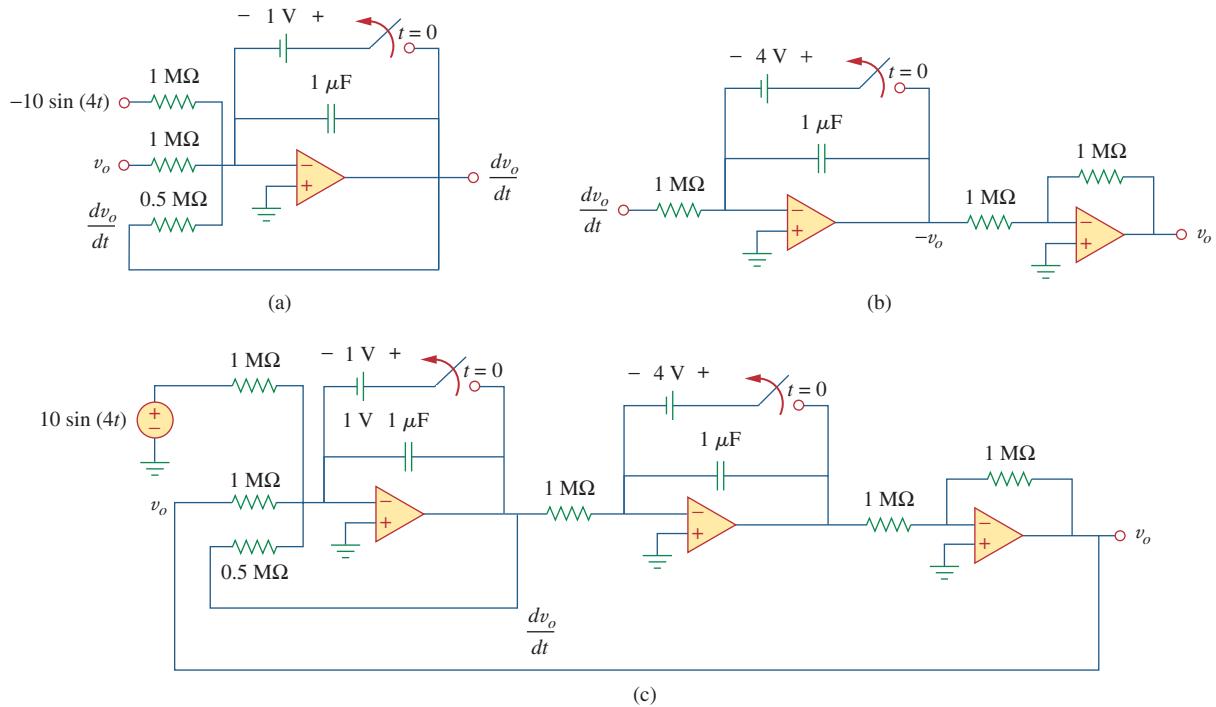
$$-\frac{1}{RC} \int_0^t v_o(\tau) d\tau$$

Other terms in the summing integrator of Eq. (6.15.2) are implemented accordingly. The initial condition $dv_o(0)/dt = 1$ is implemented by connecting a 1-V battery with a switch across the capacitor as shown in Fig. 6.40(a).

The next step is to obtain v_o by integrating dv_o/dt and inverting the result,

$$v_o = - \int_0^t \left(-\frac{dv_o(\tau)}{d\tau} \right) d\tau + v(0) \quad (6.15.3)$$

This is implemented with the circuit in Fig. 6.40(b) with the battery giving the initial condition of -4 V. We now combine the two circuits in Fig. 6.40(a) and (b) to obtain the complete circuit shown in Fig. 6.40(c). When the input signal $10 \sin 4t$ is applied, we open the switches at $t = 0$ to obtain the output waveform v_o , which may be viewed on an oscilloscope.

**Figure 6.40**

For Example 6.15.

5. Evaluate. The answer looks correct, but is it? If an actual solution for v_o is desired, then a good check would be to first find the solution by realizing the circuit in *PSpice*. This result could then be compared with a solution using the differential solution capability of *MATLAB*.

Since all we need to do is check the circuit and confirm that it represents the equation, we have an easier technique to use. We just go through the circuit and see if it generates the desired equation.

However, we still have choices to make. We could go through the circuit from left to right but that would involve differentiating the result to obtain the original equation. An easier approach would be to go from right to left. This is the approach we will use to check the answer.

Starting with the output, v_o , we see that the right-hand op amp is nothing more than an inverter with a gain of one. This means that the output of the middle circuit is $-v_o$. The following represents the action of the middle circuit.

$$\begin{aligned} -v_o &= -\left(\int_0^t \frac{dv_o}{dt} dt + v_o(0) \right) = -\left(v_o \Big|_0^t + v_o(0) \right) \\ &= -(v_o(t) - v_o(0) + v_o(0)) \end{aligned}$$

where $v_o(0) = -4 V$ is the initial voltage across the capacitor.

We check the circuit on the left the same way.

$$\frac{dv_o}{dt} = -\left(\int_0^t -\frac{d^2v_o}{dt^2} dt - v'_o(0) \right) = -\left(-\frac{dv_o}{dt} + v'_o(0) - v'_o(0) \right)$$

Now all we need to verify is that the input to the first op amp is $-d^2v_o/dt^2$.

Looking at the input we see that it is equal to

$$-10 \sin(4t) + v_o + \frac{1/10^{-6}}{0.5 \text{ M}\Omega} \frac{dv_o}{dt} = -10 \sin(4t) + v_o + 2 \frac{dv_o}{dt}$$

which does produce $-d^2v_o/dt^2$ from the original equation.

6. **Satisfactory?** The solution we have obtained is satisfactory. We can now present this work as a solution to the problem.

Practice Problem 6.15

Design an analog computer circuit to solve the differential equation:

$$\frac{d^2v_o}{dt^2} + 3 \frac{dv_o}{dt} + 2v_o = 4 \cos 10t, \quad t > 0$$

subject to $v_o(0) = 2$, $v'_o(0) = 0$.

Answer: See Fig. 6.41, where $RC = 1$ s.

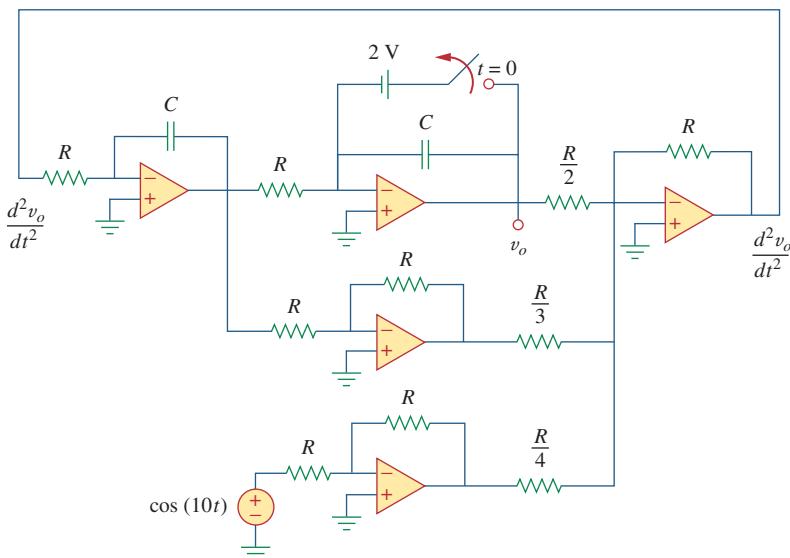


Figure 6.41
For Practice Prob. 6.15.

6.7 Summary

1. The current through a capacitor is directly proportional to the time rate of change of the voltage across it.

$$i = C \frac{dv}{dt}$$

The current through a capacitor is zero unless the voltage is changing. Thus, a capacitor acts like an open circuit to a dc source.

2. The voltage across a capacitor is directly proportional to the time integral of the current through it.

$$v = \frac{1}{C} \int_{-\infty}^t i \, dt = \frac{1}{C} \int_{t_0}^t i \, dt + v(t_0)$$

The voltage across a capacitor cannot change instantly.

3. Capacitors in series and in parallel are combined in the same way as conductances.
4. The voltage across an inductor is directly proportional to the time rate of change of the current through it.

$$v = L \frac{di}{dt}$$

The voltage across the inductor is zero unless the current is changing. Thus, an inductor acts like a short circuit to a dc source.

5. The current through an inductor is directly proportional to the time integral of the voltage across it.

$$i = \frac{1}{L} \int_{-\infty}^t v \, dt = \frac{1}{L} \int_{t_0}^t v \, dt + i(t_0)$$

The current through an inductor cannot change instantly.

6. Inductors in series and in parallel are combined in the same way resistors in series and in parallel are combined.
7. At any given time t , the energy stored in a capacitor is $\frac{1}{2}Cv^2$, while the energy stored in an inductor is $\frac{1}{2}Li^2$.
8. Three application circuits, the integrator, the differentiator, and the analog computer, can be realized using resistors, capacitors, and op amps.

Review Questions

- 6.1** What charge is on a 5-F capacitor when it is connected across a 120-V source?
- (a) 600 C (b) 300 C
 (c) 24 C (d) 12 C
- 6.2** Capacitance is measured in:
- (a) coulombs (b) joules
 (c) henrys (d) farads
- 6.3** When the total charge in a capacitor is doubled, the energy stored:
- (a) remains the same (b) is halved
 (c) is doubled (d) is quadrupled
- 6.4** Can the voltage waveform in Fig. 6.42 be associated with a real capacitor?
- (a) Yes (b) No

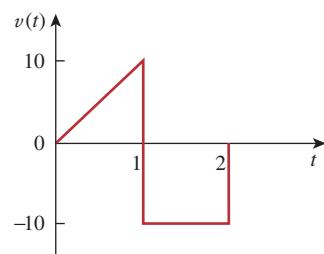
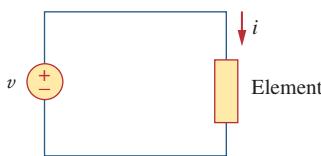


Figure 6.42

For Review Question 6.4.

- 6.5** The total capacitance of two 40-mF series-connected capacitors in parallel with a 4-mF capacitor is:
- (a) 3.8 mF (b) 5 mF (c) 24 mF
 (d) 44 mF (e) 84 mF

- 6.6** In Fig. 6.43, if $i = \cos 4t$ and $v = \sin 4t$, the element is:
- a resistor
 - a capacitor
 - an inductor

**Figure 6.43**

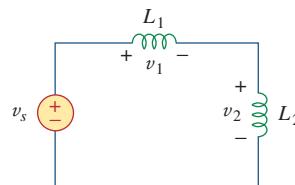
For Review Question 6.6.

- 6.7** A 5-H inductor changes its current by 3 A in 0.2 s. The voltage produced at the terminals of the inductor is:
- 75 V
 - 8.888 V
 - 3 V
 - 1.2 V
- 6.8** If the current through a 10-mH inductor increases from zero to 2 A, how much energy is stored in the inductor?
- 40 mJ
 - 20 mJ
 - 10 mJ
 - 5 mJ

- 6.9** Inductors in parallel can be combined just like resistors in parallel.
- True
 - False

- 6.10** For the circuit in Fig. 6.44, the voltage divider formula is:

$$\begin{array}{ll} \text{(a)} & v_1 = \frac{L_1 + L_2}{L_1} v_s \\ & \text{(b)} v_1 = \frac{L_1 + L_2}{L_2} v_s \\ \text{(c)} & v_1 = \frac{L_2}{L_1 + L_2} v_s \\ & \text{(d)} v_1 = \frac{L_1}{L_1 + L_2} v_s \end{array}$$

**Figure 6.44**

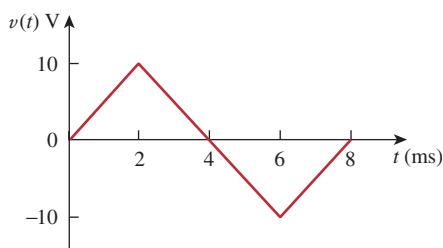
For Review Question 6.10.

Answers: 6.1a, 6.2d, 6.3d, 6.4b, 6.5c, 6.6b, 6.7a, 6.8b, 6.9a, 6.10d.

Problems

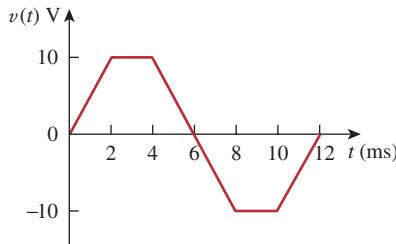
Section 6.2 Capacitors

- 6.1** If the voltage across a 7.5-F capacitor is $2te^{-3t}$ V, find the current and the power.
- 6.2** A 50- μ F capacitor has energy $w(t) = 10 \cos^2 377t$ J. Determine the current through the capacitor.
- 6.3** Design a problem to help other students better understand how capacitors work.
- 6.4** A current of $4 \sin 4t$ A flows through a 5-F capacitor. Find the voltage $v(t)$ across the capacitor given that $v(0) = 1$ V.
- 6.5** The voltage across a 4- μ F capacitor is shown in Fig. 6.45. Find the current waveform.

**Figure 6.45**

For Prob. 6.5.

- 6.6** The voltage waveform in Fig. 6.46 is applied across a 55- μ F capacitor. Draw the current waveform through it.

**Figure 6.46**

For Prob. 6.6.

- 6.7** At $t = 0$, the voltage across a 25-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current 5 mA flows through it.
- 6.8** A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50 \text{ V}, & t \leq 0 \\ Ae^{-100t} + Be^{-600t} \text{ V}, & t \geq 0 \end{cases}$$

If the capacitor has an initial current of 2 A, find:

- the constants A and B ,
- the energy stored in the capacitor at $t = 0$,
- the capacitor current for $t > 0$.

- 6.9** The current through a 0.5-F capacitor is $6(1 - e^{-t})$ A. Determine the voltage and power at $t = 2$ s. Assume $v(0) = 0$.

6.10 The voltage across a 5-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor.

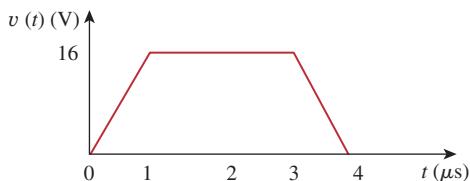


Figure 6.47

For Prob. 6.10.

- 6.11** A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that $v(0) = 10$ V, sketch the voltage waveform $v(t)$.

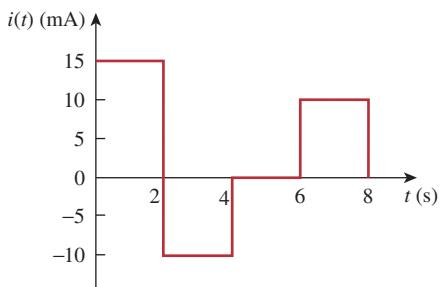


Figure 6.48

For Prob. 6.11.

- 6.12** A voltage of $30e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a $12\text{-}\Omega$ resistor. Calculate the power absorbed by the parallel combination.

- 6.13** Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.

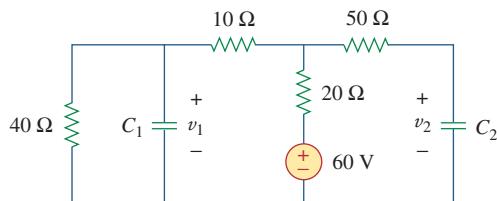


Figure 6.49

For Prob. 6.13.

Section 6.3 Series and Parallel Capacitors

- 6.14** Series-connected 20-pF and 60-pF capacitors are placed in parallel with series-connected 30-pF and 70-pF capacitors. Determine the equivalent capacitance.

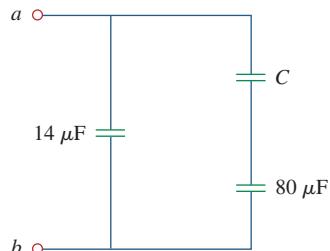


Figure 6.50

For Prob. 6.16.

- 6.17** Determine the equivalent capacitance for each of the circuits of Fig. 6.51.

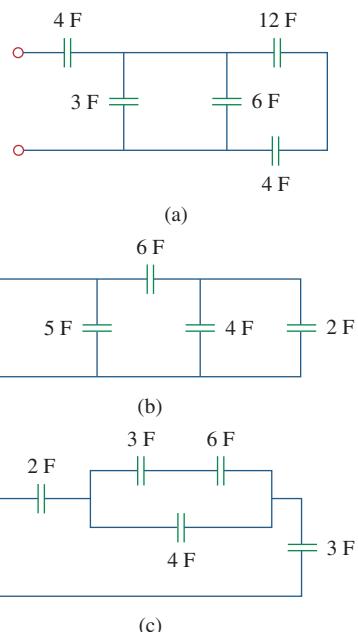


Figure 6.51

For Prob. 6.17.

- 6.18** Find C_{eq} in the circuit of Fig. 6.52 if all capacitors are $4 \mu F$.

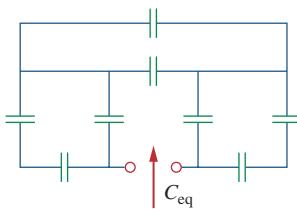
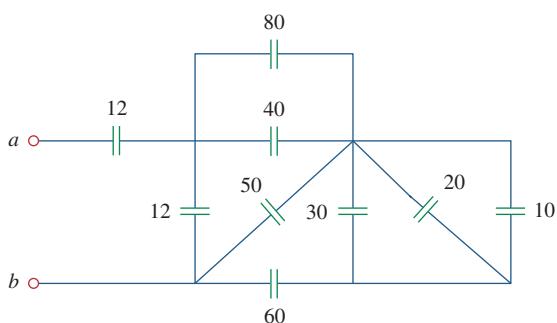


Figure 6.52

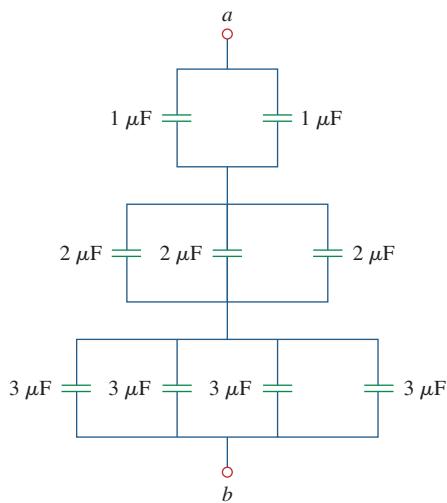
For Prob. 6.18.

- 6.19** Find the equivalent capacitance between terminals *a* and *b* in the circuit of Fig. 6.53. All capacitances are in μF .

**Figure 6.53**

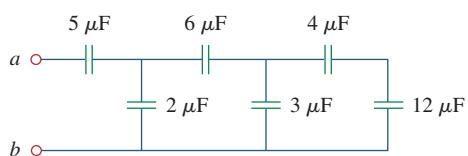
For Prob. 6.19.

- 6.20** Find the equivalent capacitance at terminals *a-b* of the circuit in Fig. 6.54.

**Figure 6.54**

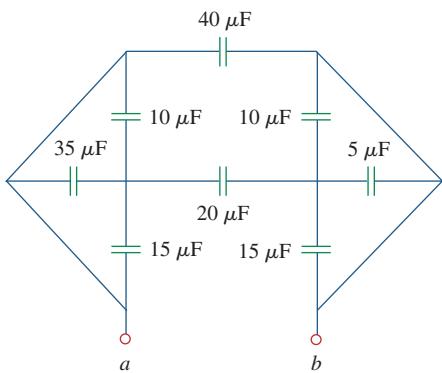
For Prob. 6.20.

- 6.21** Determine the equivalent capacitance at terminals *a-b* of the circuit in Fig. 6.55.

**Figure 6.55**

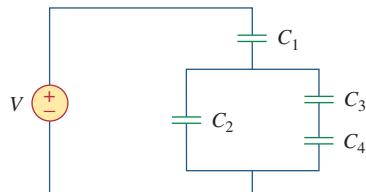
For Prob. 6.21.

- 6.22** Obtain the equivalent capacitance of the circuit in Fig. 6.56.

**Figure 6.56**

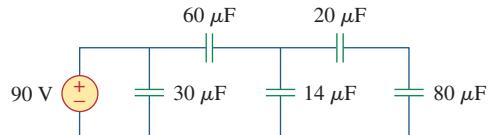
For Prob. 6.22.

- 6.23** Using Fig. 6.57, design a problem that will help other students better understand how capacitors work together when connected in series and in parallel.

**Figure 6.57**

For Prob. 6.23.

- 6.24** For the circuit in Figure 6.58, determine (a) the voltage across each capacitor and (b) the energy stored in each capacitor.

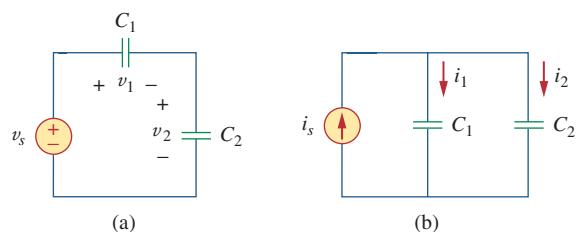
**Figure 6.58**

For Prob. 6.24.

- 6.25** (a) Show that the voltage-division rule for two capacitors in series as in Fig. 6.59(a) is

$$v_1 = \frac{C_2}{C_1 + C_2} v_s, \quad v_2 = \frac{C_1}{C_1 + C_2} v_s$$

assuming that the initial conditions are zero.

**Figure 6.59**

For Prob. 6.25.

- (b) For two capacitors in parallel as in Fig. 6.59(b), show that the current-division rule is

$$i_1 = \frac{C_1}{C_1 + C_2} i_s, \quad i_2 = \frac{C_2}{C_1 + C_2} i_s$$

assuming that the initial conditions are zero.

- 6.26** Three capacitors, $C_1 = 5 \mu\text{F}$, $C_2 = 10 \mu\text{F}$, and $C_3 = 20 \mu\text{F}$, are connected in parallel across a 150-V source. Determine:

- (a) the total capacitance,
- (b) the charge on each capacitor,
- (c) the total energy stored in the parallel combination.

- 6.27** Given that four $4-\mu\text{F}$ capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

- *6.28** Obtain the equivalent capacitance of the network shown in Fig. 6.60.

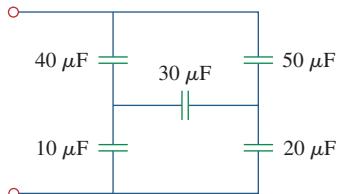


Figure 6.60
For Prob. 6.28.

- 6.29** Determine C_{eq} for each circuit in Fig. 6.61.

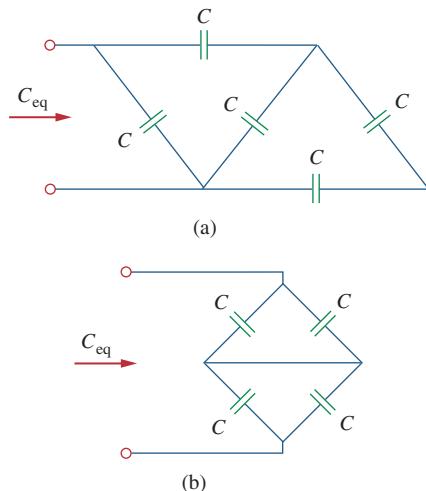


Figure 6.61
For Prob. 6.29.

* An asterisk indicates a challenging problem.

- 6.30** Assuming that the capacitors are initially uncharged, find $v_o(t)$ in the circuit of Fig. 6.62.

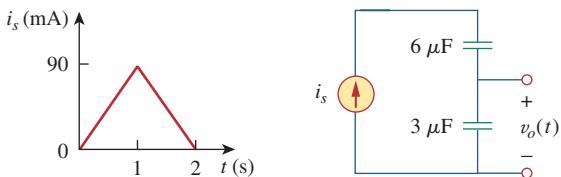


Figure 6.62

For Prob. 6.30.

- 6.31** If $v(0) = 0$, find $v(t)$, $i_1(t)$, and $i_2(t)$ in the circuit of Fig. 6.63.

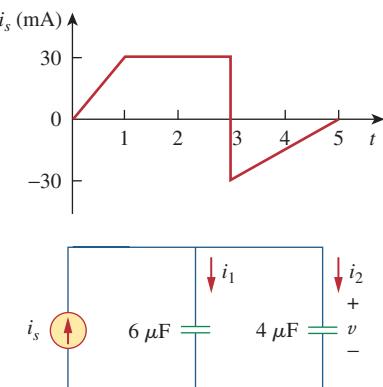


Figure 6.63

For Prob. 6.31.

- 6.32** In the circuit of Fig. 6.64, let $i_s = 50e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

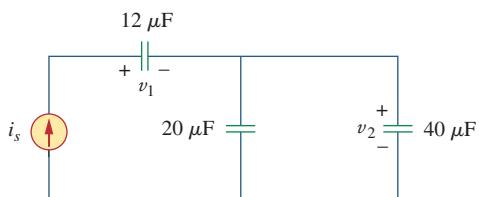
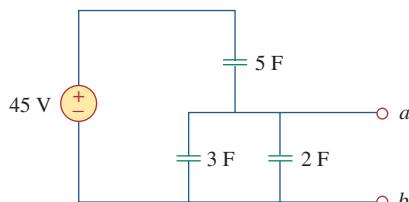


Figure 6.64

For Prob. 6.32.

- 6.33** Obtain the Thevenin equivalent at the terminals, *a*-*b*, of the circuit shown in Fig. 6.65. Please note that Thevenin equivalent circuits do not generally exist for circuits involving capacitors and resistors. This is a special case where the Thevenin equivalent circuit does exist.

**Figure 6.65**

For Prob. 6.33.

Section 6.4 Inductors

- 6.34** The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

- 6.35** An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

- 6.36** Design a problem to help other students better understand how inductors work.

- 6.37** The current through a 12-mH inductor is $4 \sin 100t$ A. Find the voltage, across the inductor for $0 < t < \pi/200$ s, and the energy stored at $t = \frac{\pi}{200}$ s.

- 6.38** The current through a 40-mH inductor is

$$i(t) = \begin{cases} 0, & t < 0 \\ te^{-2t} \text{ A}, & t > 0 \end{cases}$$

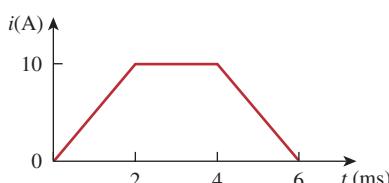
Find the voltage $v(t)$.

- 6.39** The voltage across a 200-mH inductor is given by

$$v(t) = 3t^2 + 2t + 4 \text{ V} \quad \text{for } t > 0.$$

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 1$ A.

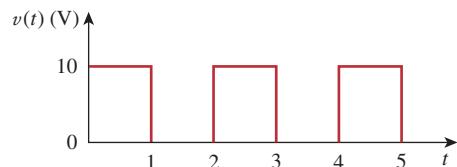
- 6.40** The current through a 5-mH inductor is shown in Fig. 6.66. Determine the voltage across the inductor at $t = 1$, 3, and 5 ms.

**Figure 6.66**

For Prob. 6.40.

- 6.41** The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

- 6.42** If the voltage waveform in Fig. 6.67 is applied across the terminals of a 5-H inductor, calculate the current through the inductor. Assume $i(0) = -1$ A.

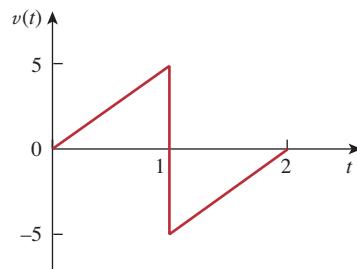
**Figure 6.67**

For Prob. 6.42.

- 6.43** The current in an 80-mH inductor increases from 0 to 60 mA. How much energy is stored in the inductor?

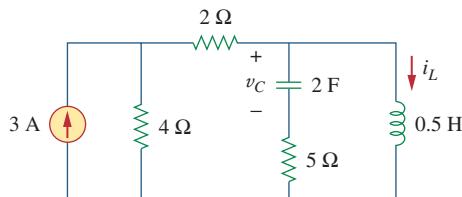
- *6.44** A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage v_L across the inductor. (b) Find the voltage v_R across the resistor. (c) Does $v_R(t) + v_L(t) = 0$? (d) Calculate the energy in the inductor at $t = 0$.

- 6.45** If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current $i(t)$. Assume $i(0) = 0$.

**Figure 6.68**

For Prob. 6.45.

- 6.46** Find v_C , i_L , and the energy stored in the capacitor and inductor in the circuit of Fig. 6.69 under dc conditions.

**Figure 6.69**

For Prob. 6.46.

- 6.47** For the circuit in Fig. 6.70, calculate the value of R that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

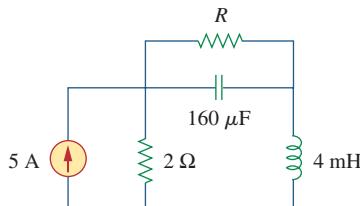


Figure 6.70
For Prob. 6.47.

- 6.48** Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

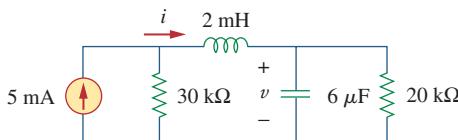


Figure 6.71
For Prob. 6.48.

Section 6.5 Series and Parallel Inductors

- 6.49** Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 10 mH.

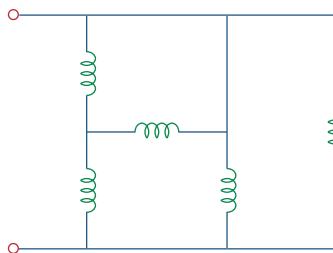


Figure 6.72
For Prob. 6.49.

- 6.50** An energy-storage network consists of series-connected 16-mH and 14-mH inductors in parallel with series-connected 24-mH and 36-mH inductors. Calculate the equivalent inductance.

- 6.51** Determine L_{eq} at terminals $a-b$ of the circuit in Fig. 6.73.

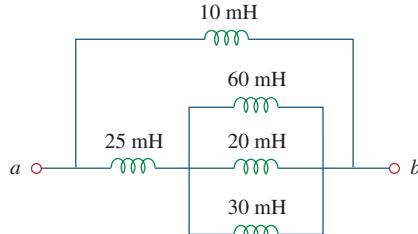


Figure 6.73
For Prob. 6.51.

- 6.52** Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.

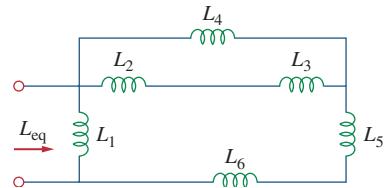


Figure 6.74
For Prob. 6.52.

- 6.53** Find L_{eq} at the terminals of the circuit in Fig. 6.75.

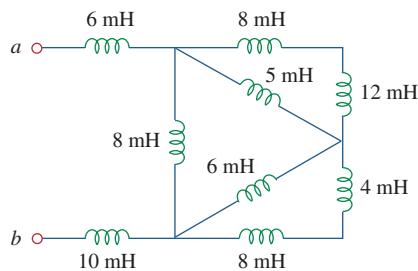


Figure 6.75
For Prob. 6.53.

- 6.54** Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

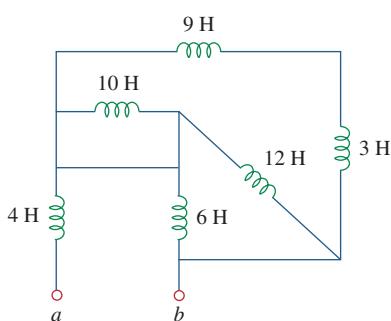
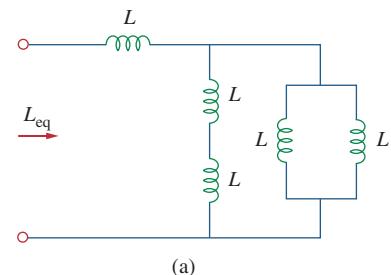
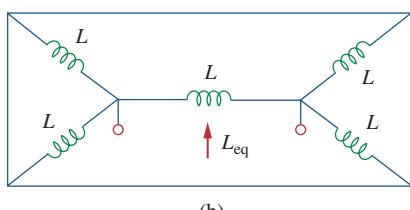


Figure 6.76
For Prob. 6.54.

6.55 Find L_{eq} in each of the circuits in Fig. 6.77.



(a)

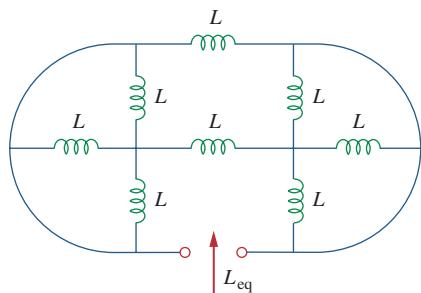


(b)

Figure 6.77

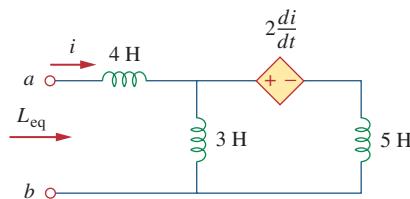
For Prob. 6.55.

6.56 Find L_{eq} in the circuit of Fig. 6.78.

**Figure 6.78**

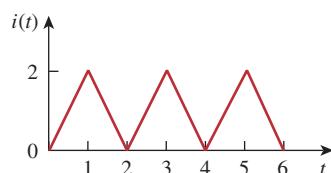
For Prob. 6.56.

***6.57** Determine L_{eq} that may be used to represent the inductive network of Fig. 6.79 at the terminals.

**Figure 6.79**

For Prob. 6.57.

6.58 The current waveform in Fig. 6.80 flows through a 3-H inductor. Sketch the voltage across the inductor over the interval $0 < t < 6$ s.

**Figure 6.80**

For Prob. 6.58.

6.59 (a) For two inductors in series as in Fig. 6.81(a), show that the voltage division principle is

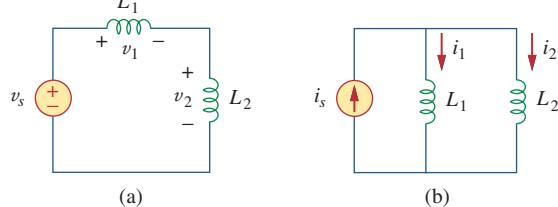
$$v_1 = \frac{L_1}{L_1 + L_2} v_s, \quad v_2 = \frac{L_2}{L_1 + L_2} v_s$$

assuming that the initial conditions are zero.

(b) For two inductors in parallel as in Fig. 6.81(b), show that the current-division principle is

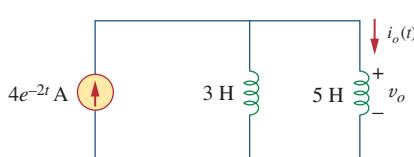
$$i_1 = \frac{L_2}{L_1 + L_2} i_s, \quad i_2 = \frac{L_1}{L_1 + L_2} i_s$$

assuming that the initial conditions are zero.

**Figure 6.81**

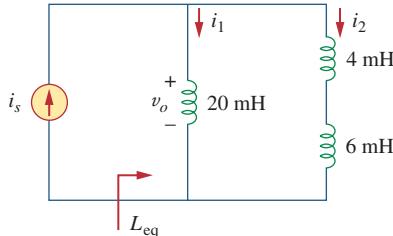
For Prob. 6.59.

6.60 In the circuit of Fig. 6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.

**Figure 6.82**

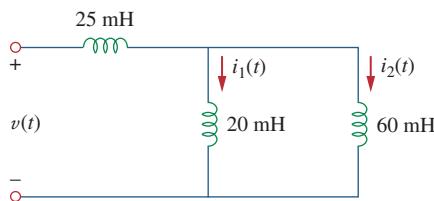
For Prob. 6.60.

- 6.61** Consider the circuit in Fig. 6.83. Find: (a) L_{eq} , $i_1(t)$, and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at $t = 1$ s.

**Figure 6.83**

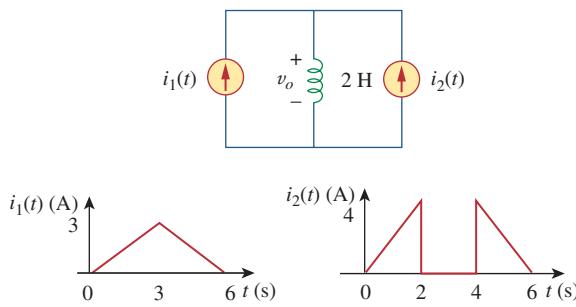
For Prob. 6.61.

- 6.62** Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

**Figure 6.84**

For Prob. 6.62.

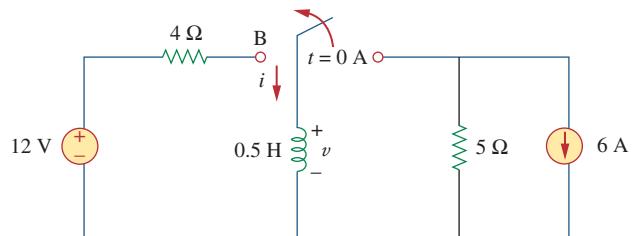
- 6.63** In the circuit of Fig. 6.85, sketch v_o .

**Figure 6.85**

For Prob. 6.63.

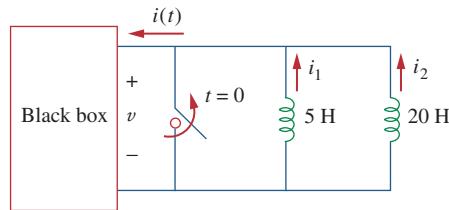
- 6.64** The switch in Fig. 6.86 has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

- $i(t)$ for $t > 0$,
- v just after the switch has been moved to position B,
- $v(t)$ long after the switch is in position B.

**Figure 6.86**

For Prob. 6.64.

- 6.65** The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$, find:
- the energy initially stored in each inductor,
 - the total energy delivered to the black box from $t = 0$ to $t = \infty$,
 - $i_1(t)$ and $i_2(t)$, $t \geq 0$,
 - $i(t)$, $t \geq 0$.

**Figure 6.87**

For Prob. 6.65.

- 6.66** The current $i(t)$ through a 20-mH inductor is equal, in magnitude, to the voltage across it for all values of time. If $i(0) = 2$ A, find $i(t)$.

Section 6.6 Applications

- 6.67** An op amp integrator has $R = 50$ kΩ and $C = 0.04$ μF. If the input voltage is $v_i = 10 \sin 50t$ mV, obtain the output voltage.

- 6.68** A 10-V dc voltage is applied to an integrator with $R = 50 \text{ k}\Omega$, $C = 100 \mu\text{F}$ at $t = 0$. How long will it take for the op amp to saturate if the saturation voltages are +12 V and -12 V? Assume that the initial capacitor voltage was zero.

- 6.69** An op amp integrator with $R = 4 \text{ M}\Omega$ and $C = 1 \mu\text{F}$ has the input waveform shown in Fig. 6.88. Plot the output waveform.

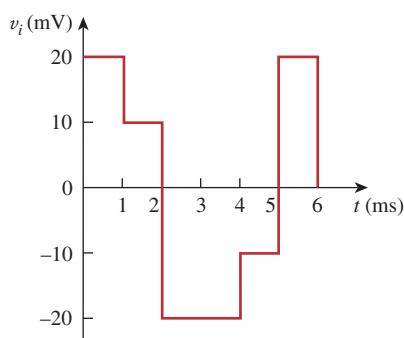


Figure 6.88

For Prob. 6.69.

- 6.70** Using a single op amp, a capacitor, and resistors of **e2d** 100 k Ω or less, design a circuit to implement

$$v_o = -50 \int_0^t v_i(t) dt$$

Assume $v_o = 0$ at $t = 0$.

- 6.71** Show how you would use a single op amp to generate

$$v_o = - \int_0^t (v_1 + 4v_2 + 10v_3) dt$$

If the integrating capacitor is $C = 2 \mu\text{F}$, obtain the other component values.

- 6.72** At $t = 1.5 \text{ ms}$, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at $t = 0$.

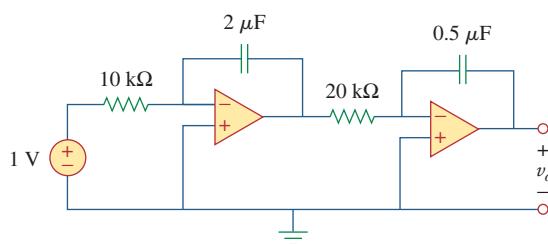


Figure 6.89

For Prob. 6.72.

- 6.73** Show that the circuit in Fig. 6.90 is a noninverting integrator.

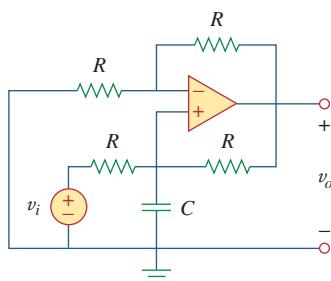
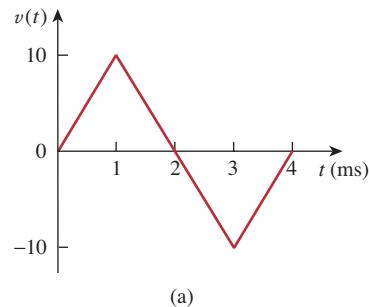


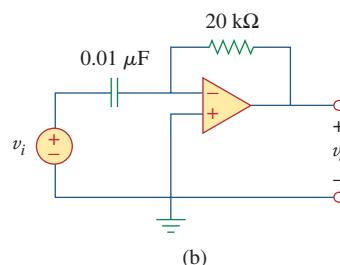
Figure 6.90

For Prob. 6.73.

- 6.74** The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output.



(a)



(b)

Figure 6.91

For Prob. 6.74.

- 6.75** An op amp differentiator has $R = 250 \text{ k}\Omega$ and $C = 10 \mu\text{F}$. The input voltage is a ramp $r(t) = 12t \text{ mV}$. Find the output voltage.

- 6.76** A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of 10 V/s for 10 ms. If the waveform is applied to a differentiator with $R = 50 \text{ k}\Omega$, $C = 10 \mu\text{F}$, sketch the output voltage waveform.

- *6.77 The output v_o of the op amp circuit in Fig. 6.92(a) is shown in Fig. 6.92(b). Let $R_i = R_f = 1 \text{ M}\Omega$ and $C = 1 \mu\text{F}$. Determine the input voltage waveform and sketch it.

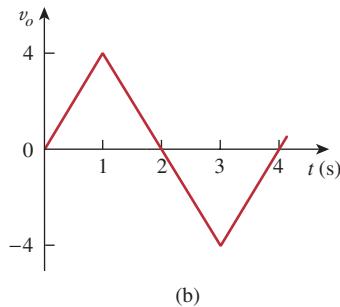
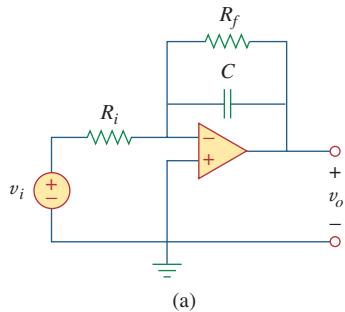


Figure 6.92

For Prob. 6.77.

- 6.78** Design an analog computer to simulate
e7d
$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10 \sin 2t$$
 where $v_o(0) = 2$ and $v'_o(0) = 0$.

Comprehensive Problems

- e2d** 6.83 Your laboratory has available a large number of $10\text{-}\mu\text{F}$ capacitors rated at 300 V. To design a capacitor bank of $40\ \mu\text{F}$ rated at 600 V, how many $10\text{-}\mu\text{F}$ capacitors are needed and how would you connect them?

- 6.79** Design an analog computer circuit to solve the following ordinary differential equation.

$$\frac{dy(t)}{dt} + 4y(t) = f(t)$$

where $y(0) = 1$ V.

- 6.80** Figure 6.93 presents an analog computer designed to solve a differential equation. Assuming $f(t)$ is known, set up the equation for $f(t)$.

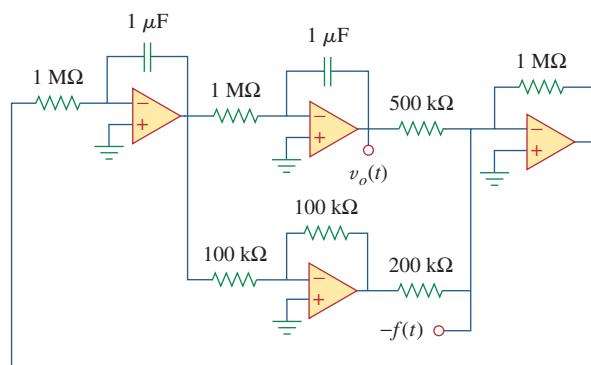


Figure 6.93

For Prob. 6.80.

- 6.81** Design an analog computer to simulate the following equation:

$$\frac{d^2v}{dt^2} + 5v = -2f(t)$$

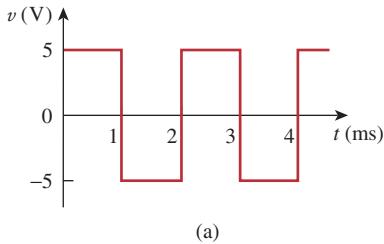
- 6.82 Design an op amp circuit such that

$$v_o = 10v_s + 2 \int v_s dt$$

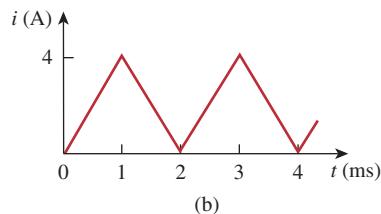
where v_s and v_o are the input voltage and output voltage, respectively.

- 6.84** An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 5 \sin^2 \pi t$ mA, $t > 0$, find the power being delivered to the inductor and the energy stored in it at $t = 0.5$ s.

- 6.85** A square-wave generator produces the voltage waveform shown in Fig. 6.94(a). What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. 6.94(b)? Calculate the value of the component, assuming that it is initially uncharged.



(a)

**Figure 6.94**

For Prob. 6.85.

- 6.86** An electric motor can be modeled as a series combination of a $12\text{-}\Omega$ resistor and 200-mH inductor. If a current $i(t) = 2te^{-10t}$ A flows through the series combination, find the voltage across the combination.

First-Order Circuits

We live in deeds, not years; in thoughts, not breaths; in feelings, not in figures on a dial. We should count time in heart-throbs. He most lives who thinks most, feels the noblest, acts the best.

—P. J. Bailey

Enhancing Your Career

Careers in Computer Engineering

Electrical engineering education has gone through drastic changes in recent decades. Most departments have come to be known as Department of Electrical and Computer Engineering, emphasizing the rapid changes due to computers. Computers occupy a prominent place in modern society and education. They have become commonplace and are helping to change the face of research, development, production, business, and entertainment. The scientist, engineer, doctor, attorney, teacher, airline pilot, businessperson—almost anyone benefits from a computer's abilities to store large amounts of information and to process that information in very short periods of time. The internet, a computer communication network, is essential in business, education, and library science. Computer usage continues to grow by leaps and bounds.

An education in computer engineering should provide breadth in software, hardware design, and basic modeling techniques. It should include courses in data structures, digital systems, computer architecture, microprocessors, interfacing, software engineering, and operating systems.

Electrical engineers who specialize in computer engineering find jobs in computer industries and in numerous fields where computers are being used. Companies that produce software are growing rapidly in number and size and providing employment for those who are skilled in programming. An excellent way to advance one's knowledge of computers is to join the IEEE Computer Society, which sponsors diverse magazines, journals, and conferences.



Computer design of very large scale integrated (VLSI) circuits.

Courtesy Brian Fast, Cleveland State University

7.1 Introduction

Now that we have considered the three passive elements (resistors, capacitors, and inductors) and one active element (the op amp) individually, we are prepared to consider circuits that contain various combinations of two or three of the passive elements. In this chapter, we shall examine two types of simple circuits: a circuit comprising a resistor and capacitor and a circuit comprising a resistor and an inductor. These are called *RC* and *RL* circuits, respectively. As simple as these circuits are, they find continual applications in electronics, communications, and control systems, as we shall see.

We carry out the analysis of *RC* and *RL* circuits by applying Kirchhoff's laws, as we did for resistive circuits. The only difference is that applying Kirchhoff's laws to purely resistive circuits results in algebraic equations, while applying the laws to *RC* and *RL* circuits produces differential equations, which are more difficult to solve than algebraic equations. The differential equations resulting from analyzing *RC* and *RL* circuits are of the first order. Hence, the circuits are collectively known as *first-order* circuits.

A **first-order** circuit is characterized by a first-order differential equation.

In addition to there being two types of first-order circuits (*RC* and *RL*), there are two ways to excite the circuits. The first way is by initial conditions of the storage elements in the circuits. In these so-called *source-free circuits*, we assume that energy is initially stored in the capacitive or inductive element. The energy causes current to flow in the circuit and is gradually dissipated in the resistors. Although source-free circuits are by definition free of independent sources, they may have dependent sources. The second way of exciting first-order circuits is by independent sources. In this chapter, the independent sources we will consider are dc sources. (In later chapters, we shall consider sinusoidal and exponential sources.) The two types of first-order circuits and the two ways of exciting them add up to the four possible situations we will study in this chapter.

Finally, we consider four typical applications of *RC* and *RL* circuits: delay and relay circuits, a photoflash unit, and an automobile ignition circuit.

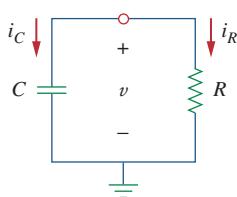


Figure 7.1

A source-free *RC* circuit.

A circuit response is the manner in which the circuit reacts to an excitation.

7.2 The Source-Free *RC* Circuit

A source-free *RC* circuit occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

Consider a series combination of a resistor and an initially charged capacitor, as shown in Fig. 7.1. (The resistor and capacitor may be the equivalent resistance and equivalent capacitance of combinations of resistors and capacitors.) Our objective is to determine the circuit response, which, for pedagogic reasons, we assume to be the voltage

$v(t)$ across the capacitor. Since the capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is

$$v(0) = V_0 \quad (7.1)$$

with the corresponding value of the energy stored as

$$w(0) = \frac{1}{2} CV_0^2 \quad (7.2)$$

Applying KCL at the top node of the circuit in Fig. 7.1 yields

$$i_C + i_R = 0 \quad (7.3)$$

By definition, $i_C = C dv/dt$ and $i_R = v/R$. Thus,

$$C \frac{dv}{dt} + \frac{v}{R} = 0 \quad (7.4a)$$

or

$$\frac{dv}{dt} + \frac{v}{RC} = 0 \quad (7.4b)$$

This is a *first-order differential equation*, since only the first derivative of v is involved. To solve it, we rearrange the terms as

$$\frac{dv}{v} = -\frac{1}{RC} dt \quad (7.5)$$

Integrating both sides, we get

$$\ln v = -\frac{t}{RC} + \ln A$$

where $\ln A$ is the integration constant. Thus,

$$\ln \frac{v}{A} = -\frac{t}{RC} \quad (7.6)$$

Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

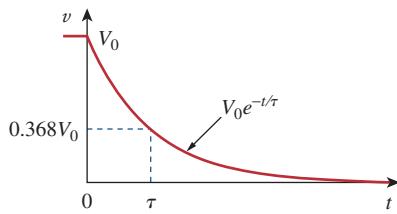
$$v(t) = V_0 e^{-t/RC} \quad (7.7)$$

This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage. Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the *natural response* of the circuit.

The **natural response** of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.

The natural response is illustrated graphically in Fig. 7.2. Note that at $t = 0$, we have the correct initial condition as in Eq. (7.1). As t increases, the voltage decreases toward zero. The rapidity with which

The natural response depends on the nature of the circuit alone, with no external sources. In fact, the circuit has a response only because of the energy initially stored in the capacitor.

**Figure 7.2**

The voltage response of the RC circuit.

the voltage decreases is expressed in terms of the *time constant*, denoted by τ , the lowercase Greek letter tau.

The **time constant** of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8 percent of its initial value.¹

This implies that at $t = \tau$, Eq. (7.7) becomes

$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368V_0$$

or

$$\tau = RC \quad (7.8)$$

In terms of the time constant, Eq. (7.7) can be written as

$$v(t) = V_0 e^{-t/\tau} \quad (7.9)$$

TABLE 7.1

Values of $v(t)/V_0 = e^{-t/\tau}$.

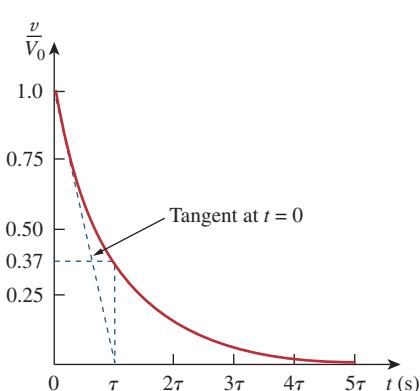
t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

With a calculator it is easy to show that the value of $v(t)/V_0$ is as shown in Table 7.1. It is evident from Table 7.1 that the voltage $v(t)$ is less than 1 percent of V_0 after 5τ (five time constants). Thus, it is customary to assume that the capacitor is fully discharged (or charged) after five time constants. In other words, it takes 5τ for the circuit to reach its final state or steady state when no changes take place with time. Notice that for every time interval of τ , the voltage is reduced by 36.8 percent of its previous value, $v(t + \tau) = v(t)/e = 0.368v(t)$, regardless of the value of t .

Observe from Eq. (7.8) that the smaller the time constant, the more rapidly the voltage decreases, that is, the faster the response. This is illustrated in Fig. 7.4. A circuit with a small time constant gives a fast response in that it reaches the steady state (or final state) quickly due to quick dissipation of energy stored, whereas a circuit with a large time constant gives a slow response because it takes longer to reach steady state. At any rate, whether the time constant is small or large, the circuit reaches steady state in five time constants.

With the voltage $v(t)$ in Eq. (7.9), we can find the current $i_R(t)$,

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau} \quad (7.10)$$

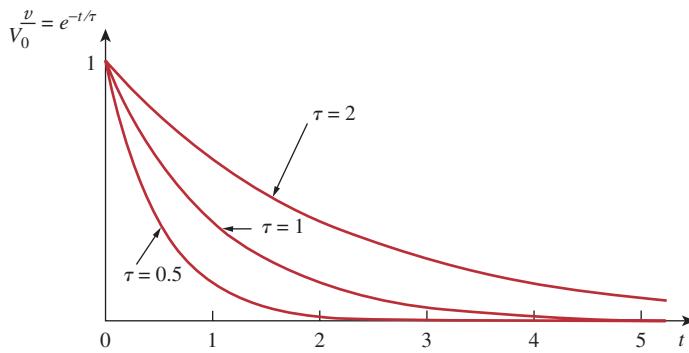
**Figure 7.3**

Graphical determination of the time constant τ from the response curve.

¹ The time constant may be viewed from another perspective. Evaluating the derivative of $v(t)$ in Eq. (7.7) at $t = 0$, we obtain

$$\left. \frac{d}{dt} \left(\frac{v}{V_0} \right) \right|_{t=0} = -\frac{1}{\tau} e^{-t/\tau} \Big|_{t=0} = -\frac{1}{\tau}$$

Thus, the time constant is the initial rate of decay, or the time taken for v/V_0 to decay from unity to zero, assuming a constant rate of decay. This initial slope interpretation of the time constant is often used in the laboratory to find τ graphically from the response curve displayed on an oscilloscope. To find τ from the response curve, draw the tangent to the curve at $t = 0$, as shown in Fig. 7.3. The tangent intersects with the time axis at $t = \tau$.

**Figure 7.4**Plot of $v/V_0 = e^{-t/\tau}$ for various values of the time constant.

The power dissipated in the resistor is

$$p(t) = vi_R = \frac{V_0^2}{R}e^{-2t/\tau} \quad (7.11)$$

The energy absorbed by the resistor up to time t is

$$\begin{aligned} w_R(t) &= \int_0^t p(\lambda)d\lambda = \int_0^t \frac{V_0^2}{R}e^{-2\lambda/\tau}d\lambda \\ &= -\frac{\tau V_0^2}{2R}e^{-2\lambda/\tau} \Big|_0^t = \frac{1}{2}CV_0^2(1 - e^{-2t/\tau}), \quad \tau = RC \end{aligned} \quad (7.12)$$

Notice that as $t \rightarrow \infty$, $w_R(\infty) \rightarrow \frac{1}{2}CV_0^2$, which is the same as $w_C(0)$, the energy initially stored in the capacitor. The energy that was initially stored in the capacitor is eventually dissipated in the resistor.

In summary:

The Key to Working with a Source-Free RC Circuit Is Finding:

1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant τ .

With these two items, we obtain the response as the capacitor voltage $v_C(t) = v(t) = v(0)e^{-t/\tau}$. Once the capacitor voltage is first obtained, other variables (capacitor current i_C , resistor voltage v_R , and resistor current i_R) can be determined. In finding the time constant $\tau = RC$, R is often the Thevenin equivalent resistance at the terminals of the capacitor; that is, we take out the capacitor C and find $R = R_{Th}$ at its terminals.

The time constant is the same regardless of what the output is defined to be.

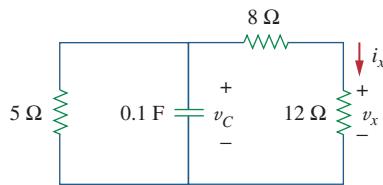
When a circuit contains a single capacitor and several resistors and dependent sources, the Thevenin equivalent can be found at the terminals of the capacitor to form a simple RC circuit. Also, one can use Thevenin's theorem when several capacitors can be combined to form a single equivalent capacitor.

In Fig. 7.5, let $v_C(0) = 15$ V. Find v_C , v_x , and i_x for $t > 0$.

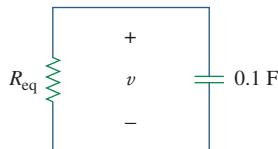
Example 7.1

Solution:

We first need to make the circuit in Fig. 7.5 conform with the standard RC circuit in Fig. 7.1. We find the equivalent resistance or the Thevenin

**Figure 7.5**

For Example 7.1.

**Figure 7.6**

Equivalent circuit for the circuit in Fig. 7.5.

resistance at the capacitor terminals. Our objective is always to first obtain capacitor voltage v_C . From this, we can determine v_x and i_x .

The 8-Ω and 12-Ω resistors in series can be combined to give a 20-Ω resistor. This 20-Ω resistor in parallel with the 5-Ω resistor can be combined so that the equivalent resistance is

$$R_{\text{eq}} = \frac{20 \times 5}{20 + 5} = 4 \Omega$$

Hence, the equivalent circuit is as shown in Fig. 7.6, which is analogous to Fig. 7.1. The time constant is

$$\tau = R_{\text{eq}}C = 4(0.1) = 0.4 \text{ s}$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4} \text{ V}, \quad v_C = v = 15e^{-2.5t} \text{ V}$$

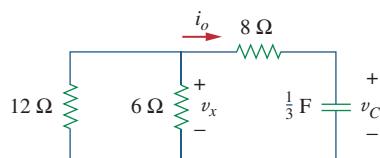
From Fig. 7.5, we can use voltage division to get v_x ; so

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t} \text{ V}$$

Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t} \text{ A}$$

Practice Problem 7.1

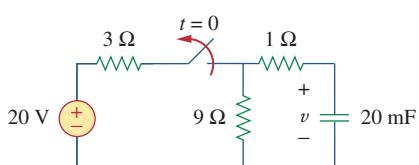
**Figure 7.7**

For Practice Prob. 7.1.

Refer to the circuit in Fig. 7.7. Let $v_C(0) = 60 \text{ V}$. Determine v_C , v_x , and i_o for $t \geq 0$.

Answer: $60e^{-0.25t} \text{ V}$, $20e^{-0.25t} \text{ V}$, $-5e^{-0.25t} \text{ A}$.

Example 7.2

**Figure 7.8**

For Example 7.2.

The switch in the circuit in Fig. 7.8 has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

Solution:

For $t < 0$, the switch is closed; the capacitor is an open circuit to dc, as represented in Fig. 7.9(a). Using voltage division

$$v_C(t) = \frac{9}{9 + 3}(20) = 15 \text{ V}, \quad t < 0$$

Since the voltage across a capacitor cannot change instantaneously, the voltage across the capacitor at $t = 0^-$ is the same as at $t = 0$, or

$$v_C(0) = V_0 = 15 \text{ V}$$

For $t > 0$, the switch is opened, and we have the RC circuit shown in Fig. 7.9(b). [Notice that the RC circuit in Fig. 7.9(b) is source free; the independent source in Fig. 7.8 is needed to provide V_0 or the initial energy in the capacitor.] The $1\text{-}\Omega$ and $9\text{-}\Omega$ resistors in series give

$$R_{\text{eq}} = 1 + 9 = 10 \text{ }\Omega$$

The time constant is

$$\tau = R_{\text{eq}}C = 10 \times 20 \times 10^{-3} = 0.2 \text{ s}$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2} \text{ V}$$

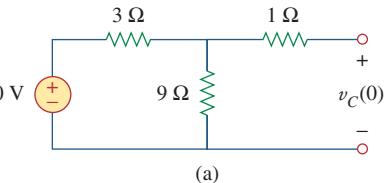
or

$$v(t) = 15e^{-5t} \text{ V}$$

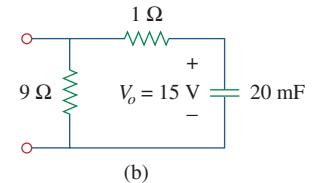
The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25 \text{ J}$$

If the switch in Fig. 7.10 opens at $t = 0$, find $v(t)$ for $t \geq 0$ and $w_C(0)$.



(a)



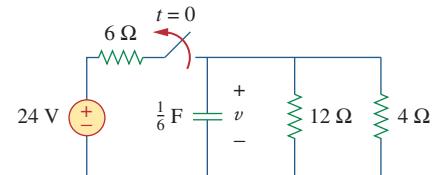
(b)

Figure 7.9

For Example 7.2: (a) $t < 0$, (b) $t > 0$.

Practice Problem 7.2

Answer: $8e^{-2t}$ V, 5.333 J.

**Figure 7.10**

For Practice Prob. 7.2.

7.3 The Source-Free RL Circuit

Consider the series connection of a resistor and an inductor, as shown in Fig. 7.11. Our goal is to determine the circuit response, which we will assume to be the current $i(t)$ through the inductor. We select the inductor current as the response in order to take advantage of the idea that the inductor current cannot change instantaneously. At $t = 0$, we assume that the inductor has an initial current I_0 , or

$$i(0) = I_0 \quad (7.13)$$

with the corresponding energy stored in the inductor as

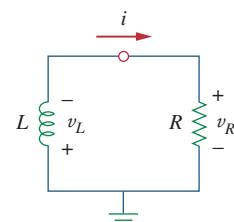
$$w(0) = \frac{1}{2}LI_0^2 \quad (7.14)$$

Applying KVL around the loop in Fig. 7.11,

$$v_L + v_R = 0 \quad (7.15)$$

But $v_L = L di/dt$ and $v_R = iR$. Thus,

$$L \frac{di}{dt} + Ri = 0$$

**Figure 7.11**

A source-free RL circuit.

or

$$\frac{di}{dt} + \frac{R}{L}i = 0 \quad (7.16)$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = - \int_0^t \frac{R}{L} dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L} \Big|_0^t \Rightarrow \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L} \quad (7.17)$$

Taking the powers of e , we have

$$i(t) = I_0 e^{-Rt/L} \quad (7.18)$$

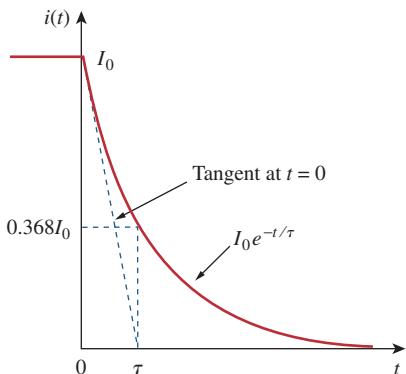


Figure 7.12

The current response of the RL circuit.

This shows that the natural response of the RL circuit is an exponential decay of the initial current. The current response is shown in Fig. 7.12. It is evident from Eq. (7.18) that the time constant for the RL circuit is

$$\tau = \frac{L}{R} \quad (7.19)$$

with τ again having the unit of seconds. Thus, Eq. (7.18) may be written as

$$i(t) = I_0 e^{-t/\tau} \quad (7.20)$$

With the current in Eq. (7.20), we can find the voltage across the resistor as

$$v_R(t) = iR = I_0 R e^{-t/\tau} \quad (7.21)$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau} \quad (7.22)$$

The energy absorbed by the resistor is

$$w_R(t) = \int_0^t p(\lambda) d\lambda = \int_0^t I_0^2 R e^{-2\lambda/\tau} d\lambda = -\frac{\tau}{2} I_0^2 R e^{-2\lambda/\tau} \Big|_0^t, \quad \tau = \frac{L}{R}$$

or

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}) \quad (7.23)$$

Note that as $t \rightarrow \infty$, $w_R(\infty) \rightarrow \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor as in Eq. (7.14). Again, the energy initially stored in the inductor is eventually dissipated in the resistor.

Figure 7.12 shows an initial slope interpretation may be given to τ .

In summary:

The Key to Working with a Source-Free RL Circuit Is to Find:

1. The initial current $i(0) = I_0$ through the inductor.
2. The time constant τ of the circuit.

With the two items, we obtain the response as the inductor current $i_L(t) = i(t) = i(0)e^{-t/\tau}$. Once we determine the inductor current i_L , other variables (inductor voltage v_L , resistor voltage v_R , and resistor current i_R) can be obtained. Note that in general, R in Eq. (7.19) is the Thevenin resistance at the terminals of the inductor.

Assuming that $i(0) = 10 \text{ A}$, calculate $i(t)$ and $i_x(t)$ in the circuit of Fig. 7.13.

Solution:

There are two ways we can solve this problem. One way is to obtain the equivalent resistance at the inductor terminals and then use Eq. (7.20). The other way is to start from scratch by using Kirchhoff's voltage law. Whichever approach is taken, it is always better to first obtain the inductor current.

METHOD 1 The equivalent resistance is the same as the Thevenin resistance at the inductor terminals. Because of the dependent source, we insert a voltage source with $v_o = 1 \text{ V}$ at the inductor terminals $a-b$, as in Fig. 7.14(a). (We could also insert a 1-A current source at the terminals.) Applying KVL to the two loops results in

$$2(i_1 - i_2) + 1 = 0 \quad \Rightarrow \quad i_1 - i_2 = -\frac{1}{2} \quad (7.3.1)$$

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \Rightarrow \quad i_2 = \frac{5}{6}i_1 \quad (7.3.2)$$

Substituting Eq. (7.3.2) into Eq. (7.3.1) gives

$$i_1 = -3 \text{ A}, \quad i_o = -i_1 = 3 \text{ A}$$

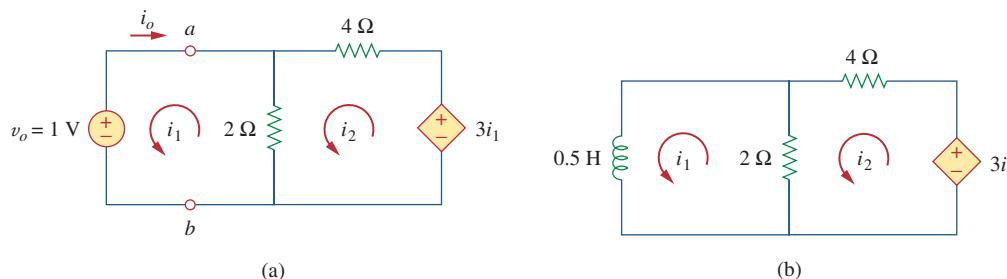


Figure 7.14
Solving the circuit in Fig. 7.13.

When a circuit has a single inductor and several resistors and dependent sources, the Thevenin equivalent can be found at the terminals of the inductor to form a simple RL circuit. Also, one can use Thevenin's theorem when several inductors can be combined to form a single equivalent inductor.

Example 7.3

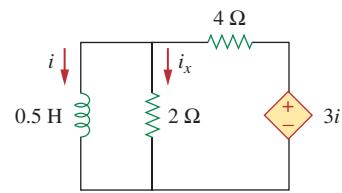


Figure 7.13
For Example 7.3.

Hence,

$$R_{\text{eq}} = R_{\text{Th}} = \frac{v_o}{i_o} = \frac{1}{3} \Omega$$

The time constant is

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2} \text{ s}$$

Thus, the current through the inductor is

$$i(t) = i(0)e^{-t/\tau} = 10e^{-(2/3)t} \text{ A}, \quad t > 0$$

METHOD 2 We may directly apply KVL to the circuit as in Fig. 7.14(b). For loop 1,

$$\frac{1}{2} \frac{di_1}{dt} + 2(i_1 - i_2) = 0$$

or

$$\frac{di_1}{dt} + 4i_1 - 4i_2 = 0 \quad (7.3.3)$$

For loop 2,

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \Rightarrow \quad i_2 = \frac{5}{6}i_1 \quad (7.3.4)$$

Substituting Eq. (7.3.4) into Eq. (7.3.3) gives

$$\frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

Rearranging terms,

$$\frac{di_1}{i_1} = -\frac{2}{3}dt$$

Since $i_1 = i$, we may replace i_1 with i and integrate:

$$\ln i \Big|_{i(0)}^{i(t)} = -\frac{2}{3}t \Big|_0^t$$

or

$$\ln \frac{i(t)}{i(0)} = -\frac{2}{3}t$$

Taking the powers of e , we finally obtain

$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t} \text{ A}, \quad t > 0$$

which is the same as by Method 1.

The voltage across the inductor is

$$v = L \frac{di}{dt} = 0.5(10) \left(-\frac{2}{3}\right) e^{-(2/3)t} = -\frac{10}{3} e^{-(2/3)t} \text{ V}$$

Since the inductor and the 2Ω resistor are in parallel,

$$i_x(t) = \frac{v}{2} = -1.6667e^{-(2/3)t} \text{ A}, \quad t > 0$$

Find i and v_x in the circuit of Fig. 7.15. Let $i(0) = 12 \text{ A}$.

Practice Problem 7.3

Answer: $12e^{-2t} \text{ A}$, $-12e^{-2t} \text{ V}$, $t > 0$.

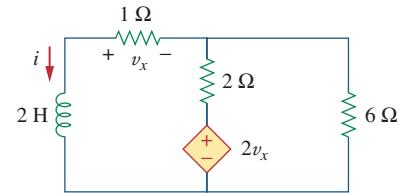


Figure 7.15
For Practice Prob. 7.3.

The switch in the circuit of Fig. 7.16 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

Example 7.4

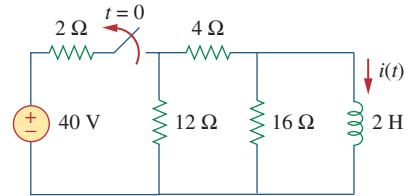


Figure 7.16
For Example 7.4.

Hence,

$$i_1 = \frac{40}{2 + 3} = 8 \text{ A}$$

We obtain $i(t)$ from i_1 in Fig. 7.17(a) using current division, by writing

$$i(t) = \frac{12}{12 + 4} i_1 = 6 \text{ A}, \quad t < 0$$

Since the current through an inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6 \text{ A}$$

When $t > 0$, the switch is open and the voltage source is disconnected. We now have the source-free RL circuit in Fig. 7.17(b). Combining the resistors, we have

$$R_{\text{eq}} = (12 + 4) \parallel 16 = 8 \Omega$$

The time constant is

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{2}{8} = \frac{1}{4} \text{ s}$$

Thus,

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t} \text{ A}$$

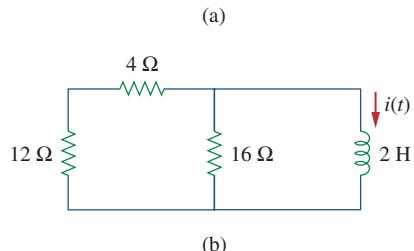
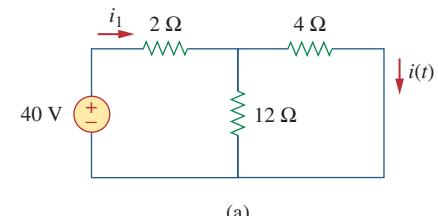
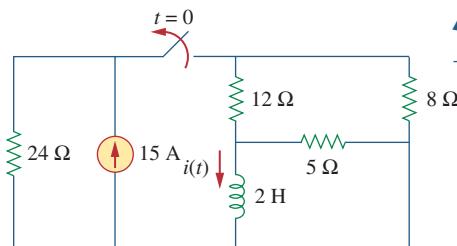


Figure 7.17
Solving the circuit of Fig. 7.16: (a) for $t < 0$, (b) for $t > 0$.

Practice Problem 7.4

For the circuit in Fig. 7.18, find $i(t)$ for $t > 0$.



Answer: $5e^{-2t}$ A, $t > 0$.

Figure 7.18

For Practice Prob. 7.4.

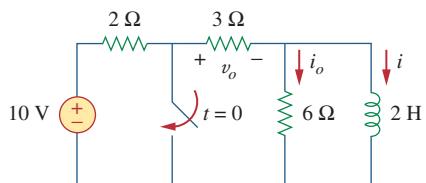
Example 7.5

Figure 7.19

For Example 7.5.

In the circuit shown in Fig. 7.19, find i_o , v_o , and i for all time, assuming that the switch was open for a long time.

Solution:

It is better to first find the inductor current i and then obtain other quantities from it.

For $t < 0$, the switch is open. Since the inductor acts like a short circuit to dc, the 6-Ω resistor is short-circuited, so that we have the circuit shown in Fig. 7.20(a). Hence, $i_o = 0$, and

$$i(t) = \frac{10}{2+3} = 2 \text{ A}, \quad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \quad t < 0$$

Thus, $i(0) = 2$.

For $t > 0$, the switch is closed, so that the voltage source is short-circuited. We now have a source-free RL circuit as shown in Fig. 7.20(b). At the inductor terminals,

$$R_{\text{Th}} = 3 \parallel 6 = 2 \Omega$$

so that the time constant is

$$\tau = \frac{L}{R_{\text{Th}}} = 1 \text{ s}$$

Hence,

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t} \text{ A}, \quad t > 0$$

Since the inductor is in parallel with the 6-Ω and 3-Ω resistors,

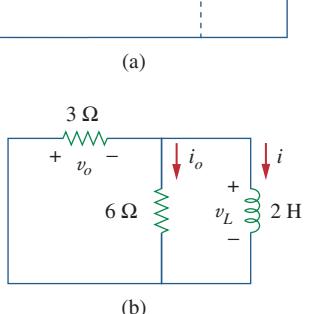
$$v_o(t) = -v_L = -L \frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t} \text{ V}, \quad t > 0$$

and

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t} \text{ A}, \quad t > 0$$

Figure 7.20

The circuit in Fig. 7.19 for: (a) $t < 0$, (b) $t > 0$.



Thus, for all time,

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \quad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

We notice that the inductor current is continuous at $t = 0$, while the current through the $6\text{-}\Omega$ resistor drops from 0 to $-2/3$ at $t = 0$, and the voltage across the $3\text{-}\Omega$ resistor drops from 6 to 4 at $t = 0$. We also notice that the time constant is the same regardless of what the output is defined to be. Figure 7.21 plots i and i_o .

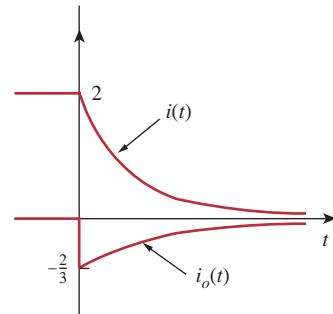


Figure 7.21

A plot of i and i_o .

Determine i , i_o , and v_o for all t in the circuit shown in Fig. 7.22. Assume that the switch was closed for a long time. It should be noted that opening a switch in series with an ideal current source creates an infinite voltage at the current source terminals. Clearly this is impossible. For the purposes of problem solving, we can place a shunt resistor in parallel with the source (which now makes it a voltage source in series with a resistor). In more practical circuits, devices that act like current sources are, for the most part, electronic circuits. These circuits will allow the source to act like an ideal current source over its operating range but voltage-limit it when the load resistor becomes too large (as in an open circuit).

Practice Problem 7.5

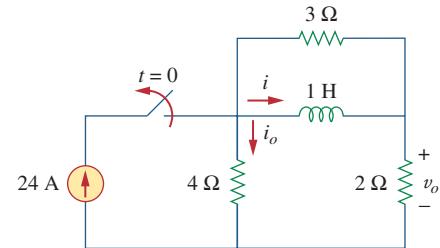


Figure 7.22

For Practice Prob. 7.5.

Answer:

$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \geq 0 \end{cases}, \quad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

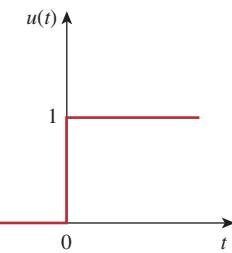
$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

7.4 Singularity Functions

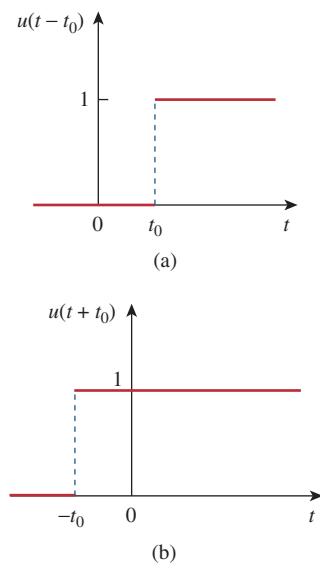
Before going on with the second half of this chapter, we need to digress and consider some mathematical concepts that will aid our understanding of transient analysis. A basic understanding of singularity functions will help us make sense of the response of first-order circuits to a sudden application of an independent dc voltage or current source.

Singularity functions (also called *switching functions*) are very useful in circuit analysis. They serve as good approximations to the switching signals that arise in circuits with switching operations. They are helpful in the neat, compact description of some circuit phenomena, especially the step response of RC or RL circuits to be discussed in the next sections. By definition,

Singularity functions are functions that either are discontinuous or have discontinuous derivatives.

**Figure 7.23**

The unit step function.

**Figure 7.24**

(a) The unit step function delayed by t_0 .
 (b) the unit step advanced by t_0 .

The three most widely used singularity functions in circuit analysis are the *unit step*, the *unit impulse*, and the *unit ramp* functions.

The **unit step function** $u(t)$ is 0 for negative values of t and 1 for positive values of t .

In mathematical terms,

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \quad (7.24)$$

The unit step function is undefined at $t = 0$, where it changes abruptly from 0 to 1. It is dimensionless, like other mathematical functions such as sine and cosine. Figure 7.23 depicts the unit step function. If the abrupt change occurs at $t = t_0$ (where $t_0 > 0$) instead of $t = 0$, the unit step function becomes

$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases} \quad (7.25)$$

which is the same as saying that $u(t)$ is delayed by t_0 seconds, as shown in Fig. 7.24(a). To get Eq. (7.25) from Eq. (7.24), we simply replace every t by $t - t_0$. If the change is at $t = -t_0$, the unit step function becomes

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases} \quad (7.26)$$

meaning that $u(t)$ is advanced by t_0 seconds, as shown in Fig. 7.24(b).

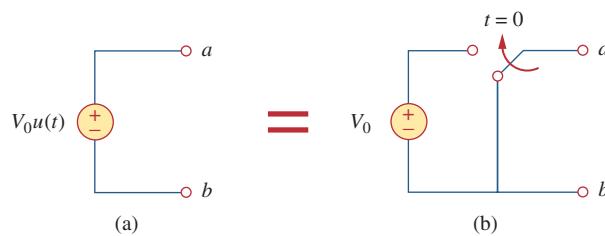
We use the step function to represent an abrupt change in voltage or current, like the changes that occur in the circuits of control systems and digital computers. For example, the voltage

$$v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases} \quad (7.27)$$

may be expressed in terms of the unit step function as

$$v(t) = V_0 u(t - t_0) \quad (7.28)$$

If we let $t_0 = 0$, then $v(t)$ is simply the step voltage $V_0 u(t)$. A voltage source of $V_0 u(t)$ is shown in Fig. 7.25(a); its equivalent circuit is shown in Fig. 7.25(b). It is evident in Fig. 7.25(b) that terminals $a-b$ are short-circuited ($v = 0$) for $t < 0$ and that $v = V_0$ appears at the terminals

**Figure 7.25**

(a) Voltage source of $V_0 u(t)$, (b) its equivalent circuit.

for $t > 0$. Similarly, a current source of $I_0 u(t)$ is shown in Fig. 7.26(a), while its equivalent circuit is in Fig. 7.26(b). Notice that for $t < 0$, there is an open circuit ($i = 0$), and that $i = I_0$ flows for $t > 0$.

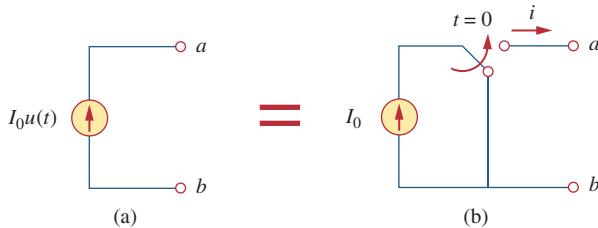


Figure 7.26

(a) Current source of $I_0 u(t)$, (b) its equivalent circuit.

The derivative of the unit step function $u(t)$ is the *unit impulse function* $\delta(t)$, which we write as

$$\delta(t) = \frac{d}{dt} u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases} \quad (7.29)$$

The unit impulse function—also known as the *delta* function—is shown in Fig. 7.27.

The **unit impulse function** $\delta(t)$ is zero everywhere except at $t = 0$, where it is undefined.

Impulsive currents and voltages occur in electric circuits as a result of switching operations or impulsive sources. Although the unit impulse function is not physically realizable (just like ideal sources, ideal resistors, etc.), it is a very useful mathematical tool.

The unit impulse may be regarded as an applied or resulting shock. It may be visualized as a very short duration pulse of unit area. This may be expressed mathematically as

$$\int_{0^-}^{0^+} \delta(t) dt = 1 \quad (7.30)$$

where $t = 0^-$ denotes the time just before $t = 0$ and $t = 0^+$ is the time just after $t = 0$. For this reason, it is customary to write 1 (denoting unit area) beside the arrow that is used to symbolize the unit impulse function, as in Fig. 7.27. The unit area is known as the *strength* of the impulse function. When an impulse function has a strength other than unity, the area of the impulse is equal to its strength. For example, an impulse function $10\delta(t)$ has an area of 10. Figure 7.28 shows the impulse functions $5\delta(t + 2)$, $10\delta(t)$, and $-4\delta(t - 3)$.

To illustrate how the impulse function affects other functions, let us evaluate the integral

$$\int_a^b f(t)\delta(t - t_0) dt \quad (7.31)$$

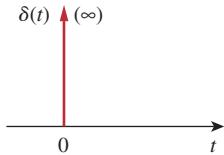


Figure 7.27

The unit impulse function.

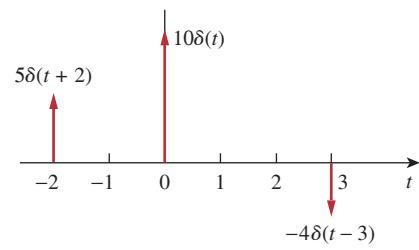


Figure 7.28

Three impulse functions.

where $a < t_0 < b$. Since $\delta(t - t_0) = 0$ except at $t = t_0$, the integrand is zero except at t_0 . Thus,

$$\begin{aligned}\int_a^b f(t)\delta(t - t_0)dt &= \int_a^b f(t_0)\delta(t - t_0)dt \\ &= f(t_0) \int_a^b \delta(t - t_0)dt = f(t_0)\end{aligned}$$

or

$$\int_a^b f(t)\delta(t - t_0)dt = f(t_0) \quad (7.32)$$

This shows that when a function is integrated with the impulse function, we obtain the value of the function at the point where the impulse occurs. This is a highly useful property of the impulse function known as the *sampling* or *sifting* property. The special case of Eq. (7.31) is for $t_0 = 0$. Then Eq. (7.32) becomes

$$\int_{0^-}^{0^+} f(t)\delta(t)dt = f(0) \quad (7.33)$$

Integrating the unit step function $u(t)$ results in the *unit ramp function* $r(t)$; we write

$$r(t) = \int_{-\infty}^t u(\lambda)d\lambda = tu(t) \quad (7.34)$$

or

$$r(t) = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases} \quad (7.35)$$

The **unit ramp function** is zero for negative values of t and has a unit slope for positive values of t .

Figure 7.29 shows the unit ramp function. In general, a ramp is a function that changes at a constant rate.

The unit ramp function may be delayed or advanced as shown in Fig. 7.30. For the delayed unit ramp function,

$$r(t - t_0) = \begin{cases} 0, & t \leq t_0 \\ t - t_0, & t \geq t_0 \end{cases} \quad (7.36)$$

and for the advanced unit ramp function,

$$r(t + t_0) = \begin{cases} 0, & t \leq -t_0 \\ t + t_0, & t \geq -t_0 \end{cases} \quad (7.37)$$

We should keep in mind that the three singularity functions (impulse, step, and ramp) are related by differentiation as

$$\delta(t) = \frac{du(t)}{dt}, \quad u(t) = \frac{dr(t)}{dt} \quad (7.38)$$

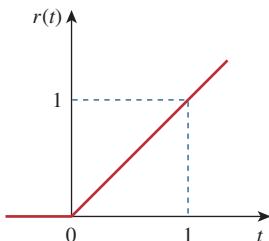
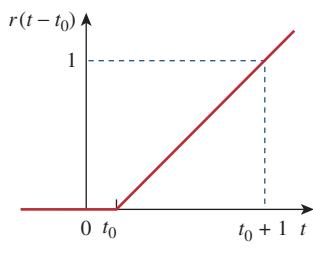
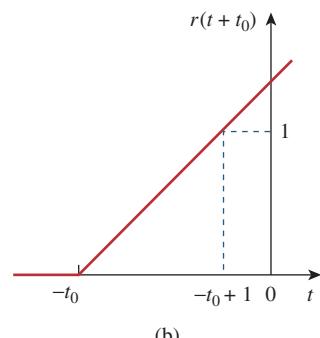


Figure 7.29

The unit ramp function.



(a)



(b)

Figure 7.30

The unit ramp function: (a) delayed by t_0 , (b) advanced by t_0 .

or by integration as

$$u(t) = \int_{-\infty}^t \delta(\lambda) d\lambda, \quad r(t) = \int_{-\infty}^t u(\lambda) d\lambda \quad (7.39)$$

Although there are many more singularity functions, we are only interested in these three (the impulse function, the unit step function, and the ramp function) at this point.

Express the voltage pulse in Fig. 7.31 in terms of the unit step. Calculate its derivative and sketch it.

Example 7.6

Gate functions are used along with switches to pass or block another signal.

Solution:

The type of pulse in Fig. 7.31 is called the *gate function*. It may be regarded as a step function that switches on at one value of t and switches off at another value of t . The gate function shown in Fig. 7.31 switches on at $t = 2$ s and switches off at $t = 5$ s. It consists of the sum of two unit step functions as shown in Fig. 7.32(a). From the figure, it is evident that

$$v(t) = 10u(t - 2) - 10u(t - 5) = 10[u(t - 2) - u(t - 5)]$$

Taking the derivative of this gives

$$\frac{dv}{dt} = 10[\delta(t - 2) - \delta(t - 5)]$$

which is shown in Fig. 7.32(b). We can obtain Fig. 7.32(b) directly from Fig. 7.31 by simply observing that there is a sudden increase by 10 V at $t = 2$ s leading to $10\delta(t - 2)$. At $t = 5$ s, there is a sudden decrease by 10 V leading to $-10\delta(t - 5)$.

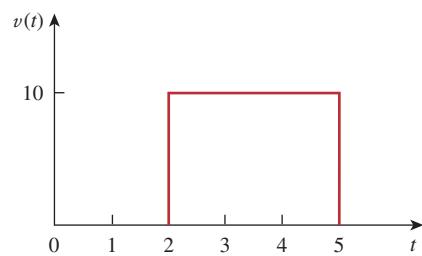
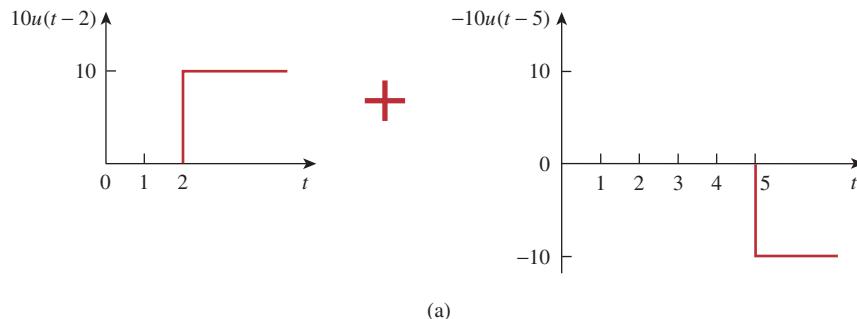
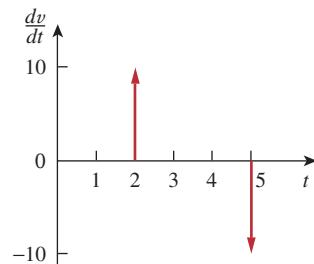


Figure 7.31
For Example 7.6.



(a)



(b)

Figure 7.32

(a) Decomposition of the pulse in Fig. 7.31, (b) derivative of the pulse in Fig. 7.31.

Practice Problem 7.6

Express the current pulse in Fig. 7.33 in terms of the unit step. Find its integral and sketch it.

Answer: $10[u(t) - 2u(t - 2) + u(t - 4)]$, $10[r(t) - 2r(t - 2) + r(t - 4)]$. See Fig. 7.34.

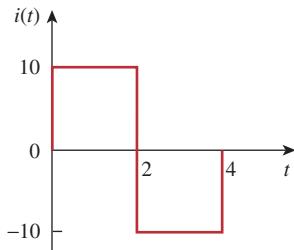


Figure 7.33
For Practice Prob. 7.6.

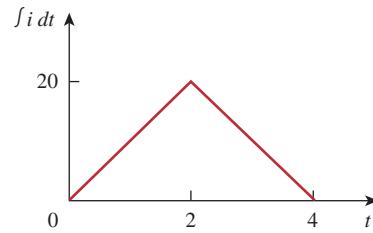


Figure 7.34
Integral of $i(t)$ in Fig. 7.33.

Example 7.7

Express the *sawtooth* function shown in Fig. 7.35 in terms of singularity functions.

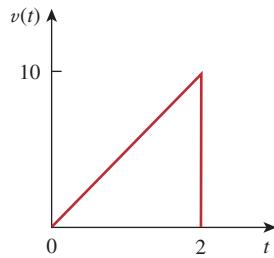


Figure 7.35
For Example 7.7.

Solution:

There are three ways of solving this problem. The first method is by mere observation of the given function, while the other methods involve some graphical manipulations of the function.

■ METHOD 1 By looking at the sketch of $v(t)$ in Fig. 7.35, it is not hard to notice that the given function $v(t)$ is a combination of singularity functions. So we let

$$v(t) = v_1(t) + v_2(t) + \dots \quad (7.7.1)$$

The function $v_1(t)$ is the ramp function of slope 5, shown in Fig. 7.36(a); that is,

$$v_1(t) = 5r(t) \quad (7.7.2)$$

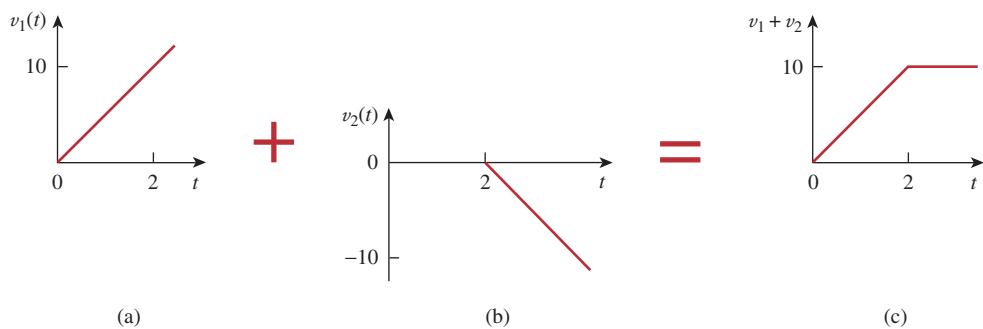


Figure 7.36
Partial decomposition of $v(t)$ in Fig. 7.35.

Since $v_1(t)$ goes to infinity, we need another function at $t = 2$ s in order to get $v(t)$. We let this function be v_2 , which is a ramp function of slope -5 , as shown in Fig. 7.36(b); that is,

$$v_2(t) = -5r(t - 2) \quad (7.7.3)$$

Adding v_1 and v_2 gives us the signal in Fig. 7.36(c). Obviously, this is not the same as $v(t)$ in Fig. 7.35. But the difference is simply a constant 10 units for $t > 2$ s. By adding a third signal v_3 , where

$$v_3 = -10u(t - 2) \quad (7.7.4)$$

we get $v(t)$, as shown in Fig. 7.37. Substituting Eqs. (7.7.2) through (7.7.4) into Eq. (7.7.1) gives

$$v(t) = 5r(t) - 5r(t - 2) - 10u(t - 2)$$

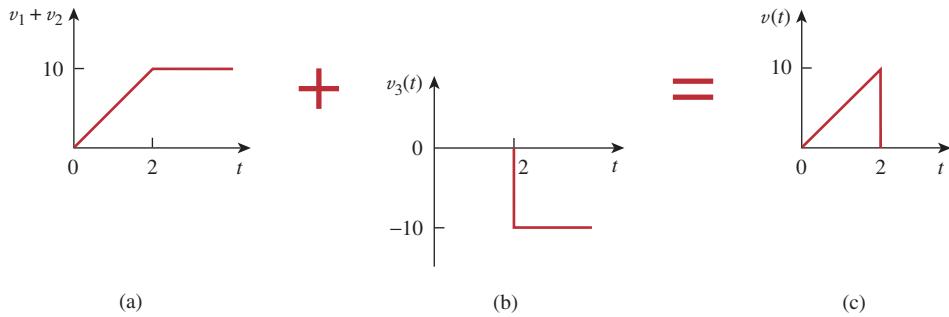


Figure 7.37

Complete decomposition of $v(t)$ in Fig. 7.35.

METHOD 2 A close observation of Fig. 7.35 reveals that $v(t)$ is a multiplication of two functions: a ramp function and a gate function. Thus,

$$\begin{aligned} v(t) &= 5t[u(t) - u(t - 2)] \\ &= 5tu(t) - 5tu(t - 2) \\ &= 5r(t) - 5(t - 2 + 2)u(t - 2) \\ &= 5r(t) - 5(t - 2)u(t - 2) - 10u(t - 2) \\ &= 5r(t) - 5r(t - 2) - 10u(t - 2) \end{aligned}$$

the same as before.

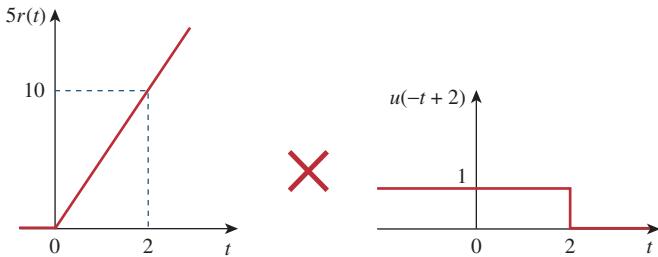
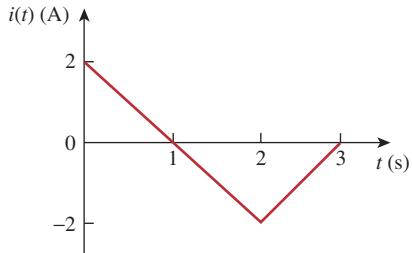
METHOD 3 This method is similar to Method 2. We observe from Fig. 7.35 that $v(t)$ is a multiplication of a ramp function and a unit step function, as shown in Fig. 7.38. Thus,

$$v(t) = 5r(t)u(-t + 2)$$

If we replace $u(-t)$ by $1 - u(t)$, then we can replace $u(-t + 2)$ by $1 - u(t - 2)$. Hence,

$$v(t) = 5r(t)[1 - u(t - 2)]$$

which can be simplified as in Method 2 to get the same result.

**Figure 7.38**Decomposition of $v(t)$ in Fig. 7.35.**Practice Problem 7.7**Refer to Fig. 7.39. Express $i(t)$ in terms of singularity functions.**Answer:** $2u(t) - 2r(t) + 4r(t - 2) - 2r(t - 3)$ A.**Figure 7.39**

For Practice Prob. 7.7.

Example 7.8

Given the signal

$$g(t) = \begin{cases} 3, & t < 0 \\ -2, & 0 < t < 1 \\ 2t - 4, & t > 1 \end{cases}$$

express $g(t)$ in terms of step and ramp functions.**Solution:**The signal $g(t)$ may be regarded as the sum of three functions specified within the three intervals $t < 0$, $0 < t < 1$, and $t > 1$.For $t < 0$, $g(t)$ may be regarded as 3 multiplied by $u(-t)$, where $u(-t) = 1$ for $t < 0$ and 0 for $t > 0$. Within the time interval $0 < t < 1$, the function may be considered as -2 multiplied by a gated function $[u(t) - u(t - 1)]$. For $t > 1$, the function may be regarded as $2t - 4$ multiplied by the unit step function $u(t - 1)$. Thus,

$$\begin{aligned} g(t) &= 3u(-t) - 2[u(t) - u(t - 1)] + (2t - 4)u(t - 1) \\ &= 3u(-t) - 2u(t) + (2t - 4 + 2)u(t - 1) \\ &= 3u(-t) - 2u(t) + 2(t - 1)u(t - 1) \\ &= 3u(-t) - 2u(t) + 2r(t - 1) \end{aligned}$$

One may avoid the trouble of using $u(-t)$ by replacing it with $1 - u(t)$. Then

$$g(t) = 3[1 - u(t)] - 2u(t) + 2r(t - 1) = 3 - 5u(t) + 2r(t - 1)$$

Alternatively, we may plot $g(t)$ and apply Method 1 from Example 7.7.

If

Practice Problem 7.8

$$h(t) = \begin{cases} 0, & t < 0 \\ -4, & 0 < t < 2 \\ 3t - 8, & 2 < t < 6 \\ 0, & t > 6 \end{cases}$$

express $h(t)$ in terms of the singularity functions.

Answer: $-4u(t) + 2u(t-2) + 3r(t-2) - 10u(t-6) - 3r(t-6)$.

Evaluate the following integrals involving the impulse function:

Example 7.9

$$\int_0^{10} (t^2 + 4t - 2)\delta(t-2)dt$$

$$\int_{-\infty}^{\infty} [\delta(t-1)e^{-t} \cos t + \delta(t+1)e^{-t} \sin t]dt$$

Solution:

For the first integral, we apply the sifting property in Eq. (7.32).

$$\int_0^{10} (t^2 + 4t - 2)\delta(t-2)dt = (t^2 + 4t - 2)|_{t=2} = 4 + 8 - 2 = 10$$

Similarly, for the second integral,

$$\int_{-\infty}^{\infty} [\delta(t-1)e^{-t} \cos t + \delta(t+1)e^{-t} \sin t]dt$$

$$= e^{-t} \cos t|_{t=1} + e^{-t} \sin t|_{t=-1}$$

$$= e^{-1} \cos 1 + e^1 \sin(-1) = 0.1988 - 2.2873 = -2.0885$$

Evaluate the following integrals:

Practice Problem 7.9

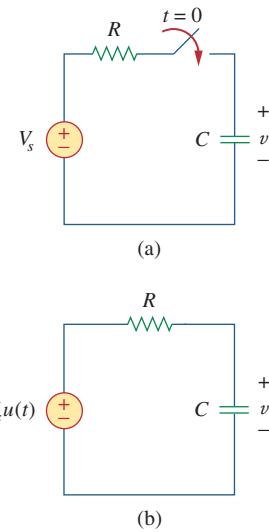
$$\int_{-\infty}^{\infty} (t^3 + 5t^2 + 10)\delta(t+3)dt, \quad \int_0^{10} \delta(t-\pi) \cos 3t dt$$

Answer: 28, -1.

7.5**Step Response of an *RC* Circuit**

When the dc source of an *RC* circuit is suddenly applied, the voltage or current source can be modeled as a step function, and the response is known as a *step response*.

The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.

**Figure 7.40**

An RC circuit with voltage step input.

The step response is the response of the circuit due to a sudden application of a dc voltage or current source.

Consider the RC circuit in Fig. 7.40(a) which can be replaced by the circuit in Fig. 7.40(b), where V_s is a constant dc voltage source. Again, we select the capacitor voltage as the circuit response to be determined. We assume an initial voltage V_0 on the capacitor, although this is not necessary for the step response. Since the voltage of a capacitor cannot change instantaneously,

$$v(0^-) = v(0^+) = V_0 \quad (7.40)$$

where $v(0^-)$ is the voltage across the capacitor just before switching and $v(0^+)$ is its voltage immediately after switching. Applying KCL, we have

$$C \frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0$$

or

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} u(t) \quad (7.41)$$

where v is the voltage across the capacitor. For $t > 0$, Eq. (7.41) becomes

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} \quad (7.42)$$

Rearranging terms gives

$$\frac{dv}{dt} = -\frac{v - V_s}{RC}$$

or

$$\frac{dv}{v - V_s} = -\frac{dt}{RC} \quad (7.43)$$

Integrating both sides and introducing the initial conditions,

$$\ln(v - V_s) \Big|_{V_0}^{v(t)} = -\frac{t}{RC} \Big|_0^t$$

$$\ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

or

$$\ln \frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC} \quad (7.44)$$

Taking the exponential of both sides

$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}, \quad \tau = RC$$

$$v - V_s = (V_0 - V_s)e^{-t/\tau}$$

or

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \quad t > 0 \quad (7.45)$$

Thus,

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases} \quad (7.46)$$

This is known as the *complete response* (or total response) of the *RC* circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged. The reason for the term “complete” will become evident a little later. Assuming that $V_s > V_0$, a plot of $v(t)$ is shown in Fig. 7.41.

If we assume that the capacitor is uncharged initially, we set $V_0 = 0$ in Eq. (7.46) so that

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases} \quad (7.47)$$

which can be written alternatively as

$$v(t) = V_s(1 - e^{-t/\tau})u(t) \quad (7.48)$$

This is the complete step response of the *RC* circuit when the capacitor is initially uncharged. The current through the capacitor is obtained from Eq. (7.47) using $i(t) = Cdv/dt$. We get

$$i(t) = C \frac{dv}{dt} = \frac{C}{\tau} V_s e^{-t/\tau}, \quad \tau = RC, \quad t > 0$$

or

$$i(t) = \frac{V_s}{R} e^{-t/\tau} u(t) \quad (7.49)$$

Figure 7.42 shows the plots of capacitor voltage $v(t)$ and capacitor current $i(t)$.

Rather than going through the derivations above, there is a systematic approach—or rather, a short-cut method—for finding the step response of an *RC* or *RL* circuit. Let us reexamine Eq. (7.45), which is more general than Eq. (7.48). It is evident that $v(t)$ has two components. Classically there are two ways of decomposing this into two components. The first is to break it into a “natural response and a forced response” and the second is to break it into a “transient response and a steady-state response.” Starting with the natural response and forced response, we write the total or complete response as

$$\text{Complete response} = \text{natural response}_{\text{stored energy}} + \text{forced response}_{\text{independent source}}$$

or

$$v = v_n + v_f \quad (7.50)$$

where

$$v_n = V_0 e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

We are familiar with the natural response v_n of the circuit, as discussed in Section 7.2. v_f is known as the *forced* response because it is produced by the circuit when an external “force” (a voltage source in this case) is applied. It represents what the circuit is forced to do by the input excitation. The natural response eventually dies out along with the transient component of the forced response, leaving only the steady-state component of the forced response.

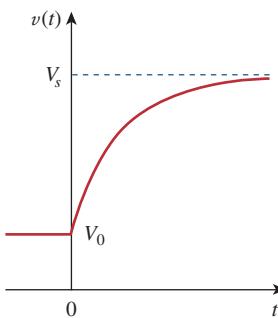
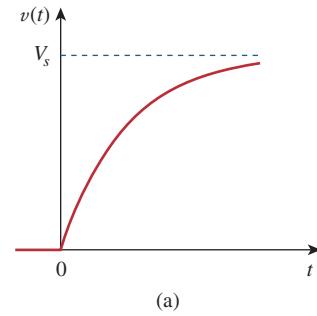
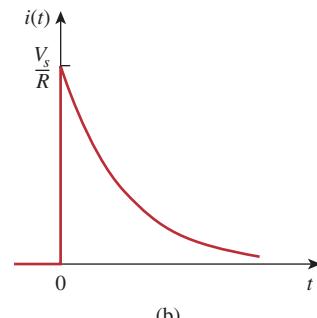


Figure 7.41
Response of an *RC* circuit with initially charged capacitor.



(a)



(b)

Figure 7.42
Step response of an *RC* circuit with initially uncharged capacitor: (a) voltage response, (b) current response.

Another way of looking at the complete response is to break into two components—one temporary and the other permanent, i.e.,

$$\text{Complete response} = \underset{\text{temporary part}}{\text{transient response}} + \underset{\text{permanent part}}{\text{steady-state response}}$$

or

$$v = v_t + v_{ss} \quad (7.51)$$

where

$$v_t = (V_o - V_s)e^{-t/\tau} \quad (7.52a)$$

and

$$v_{ss} = V_s \quad (7.52b)$$

The *transient response* v_t is temporary; it is the portion of the complete response that decays to zero as time approaches infinity. Thus,

The **transient response** is the circuit's temporary response that will die out with time.

The *steady-state response* v_{ss} is the portion of the complete response that remains after the transient response has died out. Thus,

The **steady-state response** is the behavior of the circuit a long time after an external excitation is applied.

The first decomposition of the complete response is in terms of the source of the responses, while the second decomposition is in terms of the permanency of the responses. Under certain conditions, the natural response and transient response are the same. The same can be said about the forced response and steady-state response.

Whichever way we look at it, the complete response in Eq. (7.45) may be written as

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \quad (7.53)$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an *RC* circuit requires three things:

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ .

This is the same as saying that the complete response is the sum of the transient response and the steady-state response.

Once we know $x(0)$, $x(\infty)$, and τ , almost all the circuit problems in this chapter can be solved using the formula

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}$$

We obtain item 1 from the given circuit for $t < 0$ and items 2 and 3 from the circuit for $t > 0$. Once these items are determined, we obtain

the response using Eq. (7.53). This technique equally applies to RL circuits, as we shall see in the next section.

Note that if the switch changes position at time $t = t_0$ instead of at $t = 0$, there is a time delay in the response so that Eq. (7.53) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau} \quad (7.54)$$

where $v(t_0)$ is the initial value at $t = t_0^+$. Keep in mind that Eq. (7.53) or (7.54) applies only to step responses, that is, when the input excitation is constant.

Example 7.10

The switch in Fig. 7.43 has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ s and 4 s.

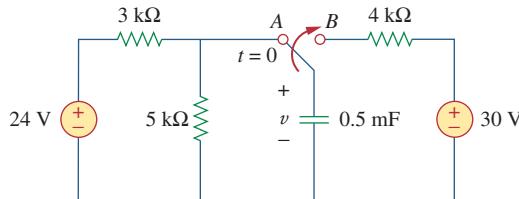


Figure 7.43
For Example 7.10.

Solution:

For $t < 0$, the switch is at position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across the $5\text{-k}\Omega$ resistor. Hence, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$, the switch is in position B. The Thevenin resistance connected to the capacitor is $R_{\text{Th}} = 4\text{k}\Omega$, and the time constant is

$$\tau = R_{\text{Th}}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30 \text{ V}$. Thus,

$$\begin{aligned} v(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\ &= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V} \end{aligned}$$

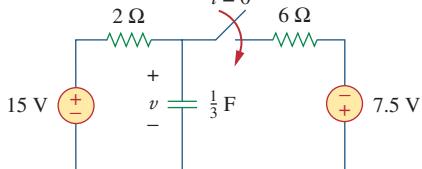
At $t = 1$,

$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

At $t = 4$,

$$v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

Practice Problem 7.10



Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.44. Assume the switch has been open for a long time and is closed at $t = 0$. Calculate $v(t)$ at $t = 0.5$.

Answer: $(9.375 + 5.625e^{-2t})$ V for all $t > 0$, 7.63 V.

Figure 7.44

For Practice Prob. 7.10.

Example 7.11

In Fig. 7.45, the switch has been closed for a long time and is opened at $t = 0$. Find i and v for all time.

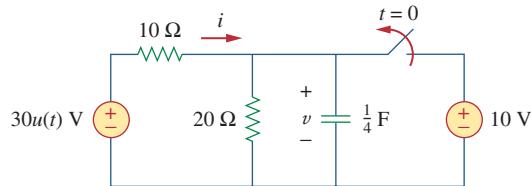


Figure 7.45

For Example 7.11.

Solution:

The resistor current i can be discontinuous at $t = 0$, while the capacitor voltage v cannot. Hence, it is always better to find v and then obtain i from v .

By definition of the unit step function,

$$30u(t) = \begin{cases} 0, & t < 0 \\ 30, & t > 0 \end{cases}$$

For $t < 0$, the switch is closed and $30u(t) = 0$, so that the $30u(t)$ voltage source is replaced by a short circuit and should be regarded as contributing nothing to v . Since the switch has been closed for a long time, the capacitor voltage has reached steady state and the capacitor acts like an open circuit. Hence, the circuit becomes that shown in Fig. 7.46(a) for $t < 0$. From this circuit we obtain

$$v = 10 \text{ V}, \quad i = -\frac{v}{10} = -1 \text{ A}$$

Since the capacitor voltage cannot change instantaneously,

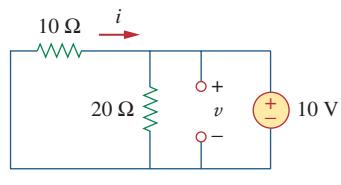
$$v(0) = v(0^-) = 10 \text{ V}$$

For $t > 0$, the switch is opened and the 10-V voltage source is disconnected from the circuit. The $30u(t)$ voltage source is now operative, so the circuit becomes that shown in Fig. 7.46(b). After a long time, the circuit reaches steady state and the capacitor acts like an open circuit again. We obtain $v(\infty)$ by using voltage division, writing

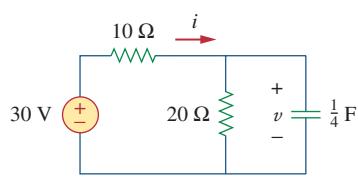
$$v(\infty) = \frac{20}{20 + 10}(30) = 20 \text{ V}$$

Figure 7.46

Solution of Example 7.11: (a) for $t < 0$, (b) for $t > 0$.



(a)



(b)

The Thevenin resistance at the capacitor terminals is

$$R_{Th} = 10 \parallel 20 = \frac{10 \times 20}{30} = \frac{20}{3} \Omega$$

and the time constant is

$$\tau = R_{Th}C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3} \text{ s}$$

Thus,

$$\begin{aligned} v(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\ &= 20 + (10 - 20)e^{-(3/5)t} = (20 - 10e^{-0.6t}) \text{ V} \end{aligned}$$

To obtain i , we notice from Fig. 7.46(b) that i is the sum of the currents through the $20\text{-}\Omega$ resistor and the capacitor; that is,

$$\begin{aligned} i &= \frac{v}{20} + C \frac{dv}{dt} \\ &= 1 - 0.5e^{-0.6t} + 0.25(-0.6)(-10)e^{-0.6t} = (1 + e^{-0.6t}) \text{ A} \end{aligned}$$

Notice from Fig. 7.46(b) that $v + 10i = 30$ is satisfied, as expected. Hence,

$$\begin{aligned} v &= \begin{cases} 10 \text{ V}, & t < 0 \\ (20 - 10e^{-0.6t}) \text{ V}, & t \geq 0 \end{cases} \\ i &= \begin{cases} -1 \text{ A}, & t < 0 \\ (1 + e^{-0.6t}) \text{ A}, & t > 0 \end{cases} \end{aligned}$$

Notice that the capacitor voltage is continuous while the resistor current is not.

The switch in Fig. 7.47 is closed at $t = 0$. Find $i(t)$ and $v(t)$ for all time. Note that $u(-t) = 1$ for $t < 0$ and 0 for $t > 0$. Also, $u(-t) = 1 - u(t)$.

Practice Problem 7.11

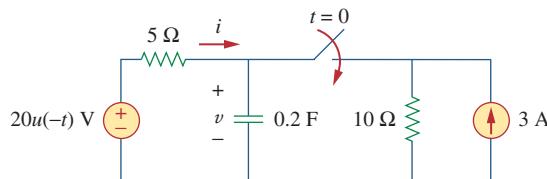
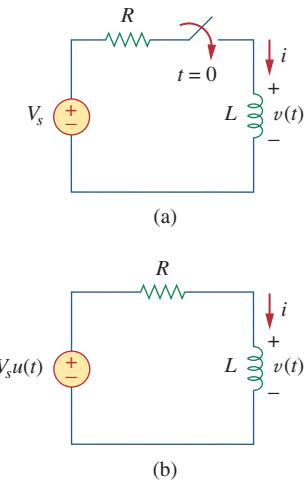


Figure 7.47
For Practice Prob. 7.11.

Answer: $i(t) = \begin{cases} 0, & t < 0 \\ -2(1 + e^{-1.5t}) \text{ A}, & t > 0, \end{cases}$

$$v = \begin{cases} 20 \text{ V}, & t < 0 \\ 10(1 + e^{-1.5t}) \text{ V}, & t > 0 \end{cases}$$

**Figure 7.48**

An *RL* circuit with a step input voltage.

7.6 Step Response of an *RL* Circuit

Consider the *RL* circuit in Fig. 7.48(a), which may be replaced by the circuit in Fig. 7.48(b). Again, our goal is to find the inductor current i as the circuit response. Rather than apply Kirchhoff's laws, we will use the simple technique in Eqs. (7.50) through (7.53). Let the response be the sum of the transient response and the steady-state response,

$$i = i_t + i_{ss} \quad (7.55)$$

We know that the transient response is always a decaying exponential, that is,

$$i_t = Ae^{-t/\tau}, \quad \tau = \frac{L}{R} \quad (7.56)$$

where A is a constant to be determined.

The steady-state response is the value of the current a long time after the switch in Fig. 7.48(a) is closed. We know that the transient response essentially dies out after five time constants. At that time, the inductor becomes a short circuit, and the voltage across it is zero. The entire source voltage V_s appears across R . Thus, the steady-state response is

$$i_{ss} = \frac{V_s}{R} \quad (7.57)$$

Substituting Eqs. (7.56) and (7.57) into Eq. (7.55) gives

$$i = Ae^{-t/\tau} + \frac{V_s}{R} \quad (7.58)$$

We now determine the constant A from the initial value of i . Let I_0 be the initial current through the inductor, which may come from a source other than V_s . Since the current through the inductor cannot change instantaneously,

$$i(0^+) = i(0^-) = I_0 \quad (7.59)$$

Thus, at $t = 0$, Eq. (7.58) becomes

$$I_0 = A + \frac{V_s}{R}$$

From this, we obtain A as

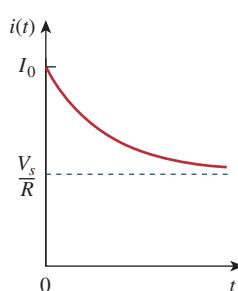
$$A = I_0 - \frac{V_s}{R}$$

Substituting for A in Eq. (7.58), we get

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-t/\tau} \quad (7.60)$$

This is the complete response of the *RL* circuit. It is illustrated in Fig. 7.49. The response in Eq. (7.60) may be written as

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \quad (7.61)$$

**Figure 7.49**

Total response of the *RL* circuit with initial inductor current I_0 .

where $i(0)$ and $i(\infty)$ are the initial and final values of i , respectively. Thus, to find the step response of an *RL* circuit requires three things:

1. The initial inductor current $i(0)$ at $t = 0$.
2. The final inductor current $i(\infty)$.
3. The time constant τ .

We obtain item 1 from the given circuit for $t < 0$ and items 2 and 3 from the circuit for $t > 0$. Once these items are determined, we obtain the response using Eq. (7.61). Keep in mind that this technique applies only for step responses.

Again, if the switching takes place at time $t = t_0$ instead of $t = 0$, Eq. (7.61) becomes

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau} \quad (7.62)$$

If $I_0 = 0$, then

$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{V_s}{R}(1 - e^{-t/\tau}), & t > 0 \end{cases} \quad (7.63a)$$

or

$$i(t) = \frac{V_s}{R}(1 - e^{-t/\tau})u(t) \quad (7.63b)$$

This is the step response of the *RL* circuit with no initial inductor current. The voltage across the inductor is obtained from Eq. (7.63) using $v = L di/dt$. We get

$$v(t) = L \frac{di}{dt} = V_s \frac{L}{\tau R} e^{-t/\tau}, \quad \tau = \frac{L}{R}, \quad t > 0$$

or

$$v(t) = V_s e^{-t/\tau} u(t) \quad (7.64)$$

Figure 7.50 shows the step responses in Eqs. (7.63) and (7.64).

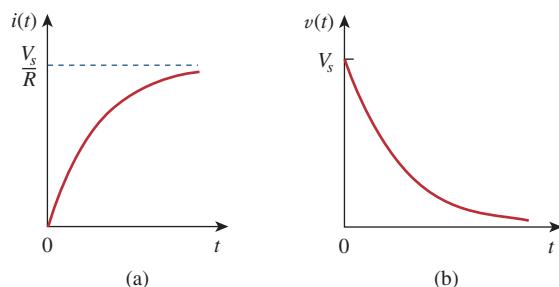
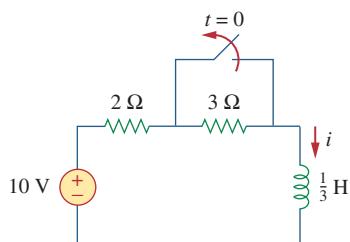


Figure 7.50

Step responses of an *RL* circuit with no initial inductor current: (a) current response, (b) voltage response.

Example 7.12**Figure 7.51**

For Example 7.12.

Find $i(t)$ in the circuit of Fig. 7.51 for $t > 0$. Assume that the switch has been closed for a long time.

Solution:

When $t < 0$, the 3-Ω resistor is short-circuited, and the inductor acts like a short circuit. The current through the inductor at $t = 0^-$ (i.e., just before $t = 0$) is

$$i(0^-) = \frac{10}{2} = 5 \text{ A}$$

Since the inductor current cannot change instantaneously,

$$i(0) = i(0^+) = i(0^-) = 5 \text{ A}$$

When $t > 0$, the switch is open. The 2-Ω and 3-Ω resistors are in series, so that

$$i(\infty) = \frac{10}{2 + 3} = 2 \text{ A}$$

The Thevenin resistance across the inductor terminals is

$$R_{\text{Th}} = 2 + 3 = 5 \Omega$$

For the time constant,

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{\frac{1}{3}}{5} = \frac{1}{15} \text{ s}$$

Thus,

$$\begin{aligned} i(t) &= i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \\ &= 2 + (5 - 2)e^{-15t} = 2 + 3e^{-15t} \text{ A}, \quad t > 0 \end{aligned}$$

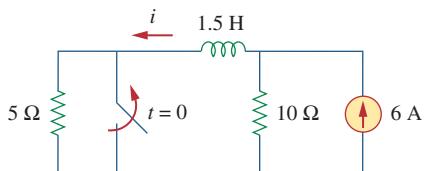
Check: In Fig. 7.51, for $t > 0$, KVL must be satisfied; that is,

$$\begin{aligned} 10 &= 5i + L \frac{di}{dt} \\ 5i + L \frac{di}{dt} &= [10 + 15e^{-15t}] + \left[\frac{1}{3}(3)(-15)e^{-15t} \right] = 10 \end{aligned}$$

This confirms the result.

Practice Problem 7.12

The switch in Fig. 7.52 has been closed for a long time. It opens at $t = 0$. Find $i(t)$ for $t > 0$.



Answer: $(4 + 2e^{-10t}) \text{ A}$ for all $t > 0$.

Figure 7.52

For Practice Prob. 7.12.

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.

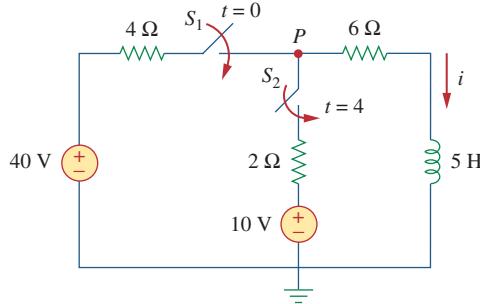
Example 7.13


Figure 7.53

For Example 7.13.

Solution:

We need to consider the three time intervals $t \leq 0$, $0 \leq t \leq 4$, and $t \geq 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

For $0 \leq t \leq 4$, S_1 is closed so that the $4\text{-}\Omega$ and $6\text{-}\Omega$ resistors are in series. (Remember, at this time, S_2 is still open.) Hence, assuming for now that S_1 is closed forever,

$$i(\infty) = \frac{40}{4 + 6} = 4 \text{ A}, \quad R_{\text{Th}} = 4 + 6 = 10 \Omega$$

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{5}{10} = \frac{1}{2} \text{ s}$$

Thus,

$$\begin{aligned} i(t) &= i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \\ &= 4 + (0 - 4)e^{-2t} = 4(1 - e^{-2t}) \text{ A}, \quad 0 \leq t \leq 4 \end{aligned}$$

For $t \geq 4$, S_2 is closed; the 10-V voltage source is connected, and the circuit changes. This sudden change does not affect the inductor current because the current cannot change abruptly. Thus, the initial current is

$$i(4) = i(4^-) = 4(1 - e^{-8}) \approx 4 \text{ A}$$

To find $i(\infty)$, let v be the voltage at node P in Fig. 7.53. Using KCL,

$$\frac{40 - v}{4} + \frac{10 - v}{2} = \frac{v}{6} \Rightarrow v = \frac{180}{11} \text{ V}$$

$$i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727 \text{ A}$$

The Thevenin resistance at the inductor terminals is

$$R_{\text{Th}} = 4 \parallel 2 + 6 = \frac{4 \times 2}{6} + 6 = \frac{22}{3} \Omega$$

and

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{5}{\frac{22}{3}} = \frac{15}{22} \text{ s}$$

Hence,

$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau}, \quad t \geq 4$$

We need $(t - 4)$ in the exponential because of the time delay. Thus,

$$\begin{aligned} i(t) &= 2.727 + (4 - 2.727)e^{-(t-4)/\tau}, \quad \tau = \frac{15}{22} \\ &= 2.727 + 1.273e^{-1.4667(t-4)}, \quad t \geq 4 \end{aligned}$$

Putting all this together,

$$i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273e^{-1.4667(t-4)}, & t \geq 4 \end{cases}$$

At $t = 2$,

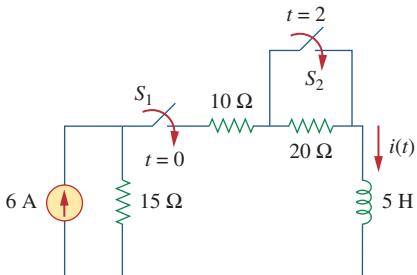
$$i(2) = 4(1 - e^{-4}) = 3.93 \text{ A}$$

At $t = 5$,

$$i(5) = 2.727 + 1.273e^{-1.4667} = 3.02 \text{ A}$$

Practice Problem 7.13

Switch S_1 in Fig. 7.54 is closed at $t = 0$, and switch S_2 is closed at $t = 2$ s. Calculate $i(t)$ for all t . Find $i(1)$ and $i(3)$.



Answer:

$$i(t) = \begin{cases} 0, & t < 0 \\ 2(1 - e^{-9t}), & 0 < t < 2 \\ 3.6 - 1.6e^{-5(t-2)}, & t > 2 \end{cases}$$

$$i(1) = 1.9997 \text{ A}, \quad i(3) = 3.589 \text{ A.}$$

Figure 7.54

For Practice Prob. 7.13.

7.7

First-Order Op Amp Circuits

An op amp circuit containing a storage element will exhibit first-order behavior. Differentiators and integrators treated in Section 6.6 are examples of first-order op amp circuits. Again, for practical reasons, inductors are hardly ever used in op amp circuits; therefore, the op amp circuits we consider here are of the *RC* type.

As usual, we analyze op amp circuits using nodal analysis. Sometimes, the Thevenin equivalent circuit is used to reduce the op amp circuit to one that we can easily handle. The following three examples illustrate the concepts. The first one deals with a source-free op amp circuit, while the other two involve step responses. The three examples have been carefully selected to cover all possible *RC* types of op amp circuits, depending on the location of the capacitor with respect to the op amp; that is, the capacitor can be located in the input, the output, or the feedback loop.

For the op amp circuit in Fig. 7.55(a), find v_o for $t > 0$, given that $v(0) = 3$ V. Let $R_f = 80 \text{ k}\Omega$, $R_1 = 20 \text{ k}\Omega$, and $C = 5 \mu\text{F}$.

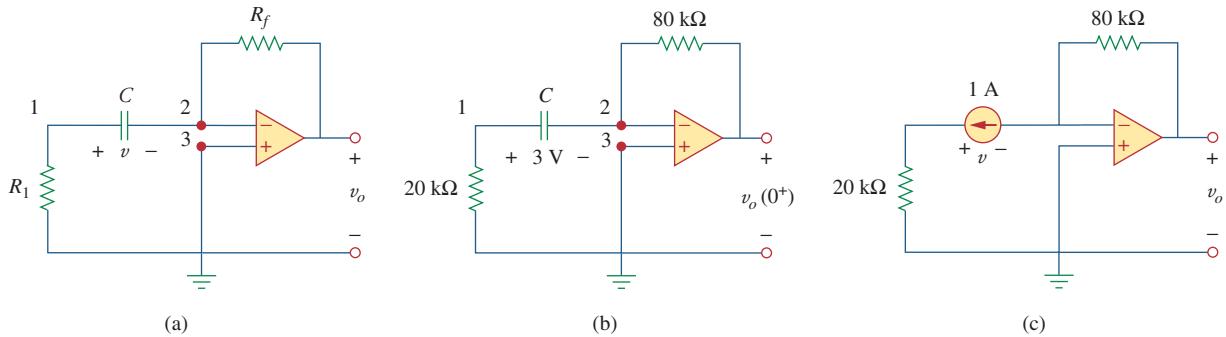
Example 7.14


Figure 7.55
For Example 7.14.

Solution:

This problem can be solved in two ways:

METHOD 1 Consider the circuit in Fig. 7.55(a). Let us derive the appropriate differential equation using nodal analysis. If v_1 is the voltage at node 1, at that node, KCL gives

$$\frac{0 - v_1}{R_1} = C \frac{dv}{dt} \quad (7.14.1)$$

Since nodes 2 and 3 must be at the same potential, the potential at node 2 is zero. Thus, $v_1 - 0 = v$ or $v_1 = v$ and Eq. (7.14.1) becomes

$$\frac{dv}{dt} + \frac{v}{CR_1} = 0 \quad (7.14.2)$$

This is similar to Eq. (7.4b) so that the solution is obtained the same way as in Section 7.2, i.e.,

$$v(t) = V_0 e^{-t/\tau}, \quad \tau = R_1 C \quad (7.14.3)$$

where V_0 is the initial voltage across the capacitor. But $v(0) = 3 = V_0$ and $\tau = 20 \times 10^3 \times 5 \times 10^{-6} = 0.1$. Hence,

$$v(t) = 3e^{-10t} \quad (7.14.4)$$

Applying KCL at node 2 gives

$$C \frac{dv}{dt} = \frac{0 - v_o}{R_f}$$

or

$$v_o = -R_f C \frac{dv}{dt} \quad (7.14.5)$$

Now we can find v_0 as

$$v_o = -80 \times 10^3 \times 5 \times 10^{-6}(-30e^{-10t}) = 12e^{-10t} \text{ V}, \quad t > 0$$

METHOD 2 Let us apply the short-cut method from Eq. (7.53). We need to find $v_o(0^+)$, $v_o(\infty)$, and τ . Since $v(0^+) = v(0^-) = 3 \text{ V}$, we apply KCL at node 2 in the circuit of Fig. 7.55(b) to obtain

$$\frac{3}{20,000} + \frac{0 - v_o(0^+)}{80,000} = 0$$

or $v_o(0^+) = 12 \text{ V}$. Since the circuit is source free, $v(\infty) = 0 \text{ V}$. To find τ , we need the equivalent resistance R_{eq} across the capacitor terminals. If we remove the capacitor and replace it by a 1-A current source, we have the circuit shown in Fig. 7.55(c). Applying KVL to the input loop yields

$$20,000(1) - v = 0 \quad \Rightarrow \quad v = 20 \text{ kV}$$

Then

$$R_{\text{eq}} = \frac{v}{1} = 20 \text{ k}\Omega$$

and $\tau = R_{\text{eq}}C = 0.1$. Thus,

$$\begin{aligned} v_o(t) &= v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} \\ &= 0 + (12 - 0)e^{-10t} = 12e^{-10t} \text{ V}, \quad t > 0 \end{aligned}$$

as before.

Practice Problem 7.14

For the op amp circuit in Fig. 7.56, find v_o for $t > 0$ if $v(0) = 4 \text{ V}$. Assume that $R_f = 50 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, and $C = 10 \mu\text{F}$.

Answer: $-4e^{-2t} \text{ V}$, $t > 0$.

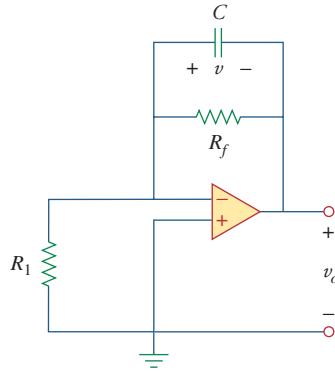


Figure 7.56

For Practice Prob. 7.14.

Example 7.15

Determine $v(t)$ and $v_o(t)$ in the circuit of Fig. 7.57.

Solution:

This problem can be solved in two ways, just like the previous example. However, we will apply only the second method. Since what we are looking for is the step response, we can apply Eq. (7.53) and write

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}, \quad t > 0 \quad (7.15.1)$$

where we need only find the time constant τ , the initial value $v(0)$, and the final value $v(\infty)$. Notice that this applies strictly to the capacitor voltage due a step input. Since no current enters the input terminals of the op amp, the elements on the feedback loop of the op amp constitute an RC circuit, with

$$\tau = RC = 50 \times 10^3 \times 10^{-6} = 0.05 \quad (7.15.2)$$

For $t < 0$, the switch is open and there is no voltage across the capacitor. Hence, $v(0) = 0$. For $t > 0$, we obtain the voltage at node 1 by voltage division as

$$v_1 = \frac{20}{20 + 10} 3 = 2 \text{ V} \quad (7.15.3)$$

Since there is no storage element in the input loop, v_1 remains constant for all t . At steady state, the capacitor acts like an open circuit so that the op amp circuit is a noninverting amplifier. Thus,

$$v_o(\infty) = \left(1 + \frac{50}{20}\right)v_1 = 3.5 \times 2 = 7 \text{ V} \quad (7.15.4)$$

But

$$v_1 - v_o = v \quad (7.15.5)$$

so that

$$v(\infty) = 2 - 7 = -5 \text{ V}$$

Substituting τ , $v(0)$, and $v(\infty)$ into Eq. (7.15.1) gives

$$v(t) = -5 + [0 - (-5)]e^{-20t} = 5(e^{-20t} - 1) \text{ V}, \quad t > 0 \quad (7.15.6)$$

From Eqs. (7.15.3), (7.15.5), and (7.15.6), we obtain

$$v_o(t) = v_1(t) - v(t) = 7 - 5e^{-20t} \text{ V}, \quad t > 0 \quad (7.15.7)$$

Find $v(t)$ and $v_o(t)$ in the op amp circuit of Fig. 7.58.

Practice Problem 7.15

Answer: (Note, the voltage across the capacitor and the output voltage must be both equal to zero, for $t < 0$, since the input was zero for all $t < 0$.) $40(1 - e^{-10t}) u(t)$ mV, $40(e^{-10t} - 1) u(t)$ mV.

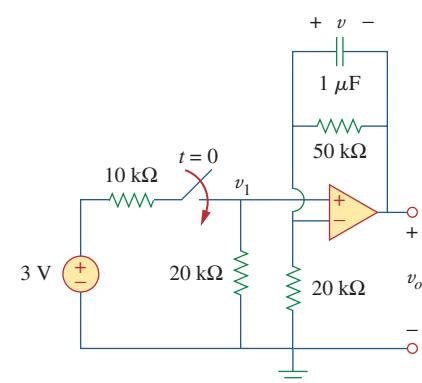


Figure 7.57

For Example 7.15.

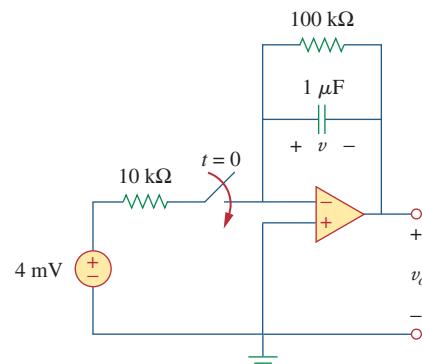
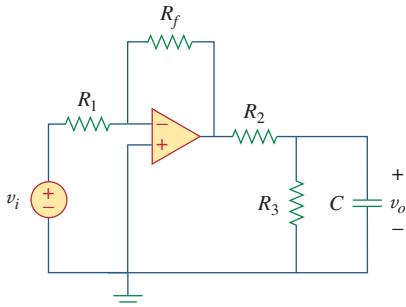


Figure 7.58

For Practice Prob. 7.15.

Example 7.16**Figure 7.59**

For Example 7.16.

Find the step response $v_o(t)$ for $t > 0$ in the op amp circuit of Fig. 7.59. Let $v_i = 2u(t)$ V, $R_1 = 20\text{ k}\Omega$, $R_f = 50\text{ k}\Omega$, $R_2 = R_3 = 10\text{ k}\Omega$, $C = 2\mu\text{F}$.

Solution:

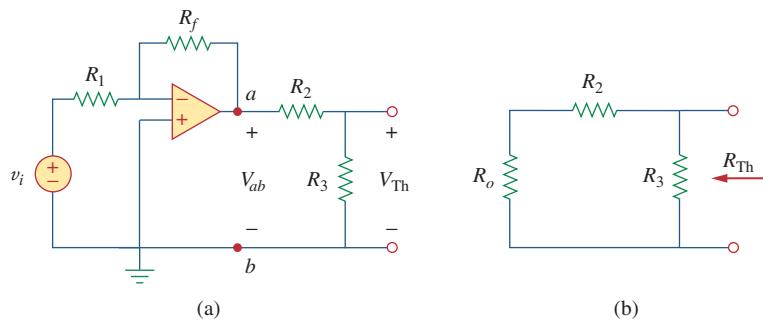
Notice that the capacitor in Example 7.14 is located in the input loop, while the capacitor in Example 7.15 is located in the feedback loop. In this example, the capacitor is located in the output of the op amp. Again, we can solve this problem directly using nodal analysis. However, using the Thevenin equivalent circuit may simplify the problem.

We temporarily remove the capacitor and find the Thevenin equivalent at its terminals. To obtain V_{Th} , consider the circuit in Fig. 7.60(a). Since the circuit is an inverting amplifier,

$$V_{ab} = -\frac{R_f}{R_1}v_i$$

By voltage division,

$$V_{Th} = \frac{R_3}{R_2 + R_3}V_{ab} = -\frac{R_3}{R_2 + R_3}\frac{R_f}{R_1}v_i$$

**Figure 7.60**Obtaining V_{Th} and R_{Th} across the capacitor in Fig. 7.59.

To obtain R_{Th} , consider the circuit in Fig. 7.60(b), where R_o is the output resistance of the op amp. Since we are assuming an ideal op amp, $R_o = 0$, and

$$R_{Th} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

Substituting the given numerical values,

$$V_{Th} = -\frac{R_3}{R_2 + R_3}\frac{R_f}{R_1}v_i = -\frac{10}{20}\frac{50}{20}2u(t) = -2.5u(t)$$

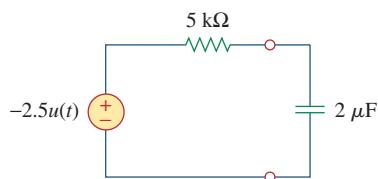
$$R_{Th} = \frac{R_2 R_3}{R_2 + R_3} = 5\text{ k}\Omega$$

The Thevenin equivalent circuit is shown in Fig. 7.61, which is similar to Fig. 7.40. Hence, the solution is similar to that in Eq. (7.48); that is,

$$v_o(t) = -2.5(1 - e^{-t/\tau})u(t)$$

where $\tau = R_{Th}C = 5 \times 10^3 \times 2 \times 10^{-6} = 0.01$. Thus, the step response for $t > 0$ is

$$v_o(t) = 2.5(e^{-100t} - 1)u(t) \text{ V}$$

**Figure 7.61**

Thevenin equivalent circuit of the circuit in Fig. 7.59.

Obtain the step response $v_o(t)$ for the circuit in Fig. 7.62. Let $v_i = 4.5u(t)$ V, $R_1 = 20 \text{ k}\Omega$, $R_f = 40 \text{ k}\Omega$, $R_2 = R_3 = 10 \text{ k}\Omega$, $C = 2 \mu\text{F}$.

Answer: $13.5(1 - e^{-50t})u(t)$ V.

Practice Problem 7.16

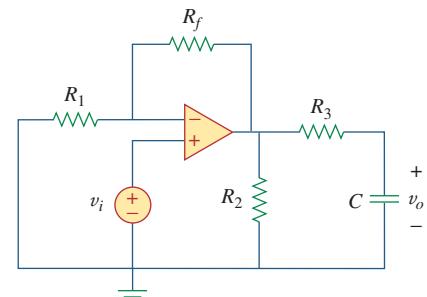


Figure 7.62

For Practice Prob. 7.16.

7.8 Transient Analysis with PSpice

As we discussed in Section 7.5, the transient response is the temporary response of the circuit that soon disappears. *PSpice* can be used to obtain the transient response of a circuit with storage elements. Section D.4 in Appendix D provides a review of transient analysis using *PSpice for Windows*. It is recommended that you read Section D.4 before continuing with this section.

If necessary, dc *PSpice* analysis is first carried out to determine the initial conditions. Then the initial conditions are used in the transient *PSpice* analysis to obtain the transient responses. It is recommended but not necessary that during this dc analysis, all capacitors should be open-circuited while all inductors should be short-circuited.

Use *PSpice* to find the response $i(t)$ for $t > 0$ in the circuit of Fig. 7.63.

Example 7.17

Solution:

Solving this problem by hand gives $i(0) = 0$, $i(\infty) = 2 \text{ A}$, $R_{\text{Th}} = 6$, $\tau = 3/6 = 0.5 \text{ s}$, so that

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 2(1 - e^{-2t}), \quad t > 0$$

To use *PSpice*, we first draw the schematic as shown in Fig. 7.64. We recall from Appendix D that the part name for a closed switch is `Sw_tclos`. We do not need to specify the initial condition of the inductor because *PSpice* will determine that from the circuit. By selecting **Analysis/Setup/Transient**, we set **Print Step** to 25 ms and **Final Step** to $5\tau = 2.5 \text{ s}$. After saving the circuit, we simulate by selecting **Analysis/Simulate**. In the *PSpice A/D* window, we select **Trace/Add** and display $-I(L1)$ as the current through the inductor. Figure 7.65 shows the plot of $i(t)$, which agrees with that obtained by hand calculation.

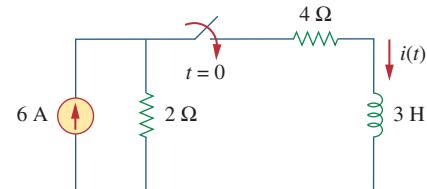


Figure 7.63

For Example 7.17.

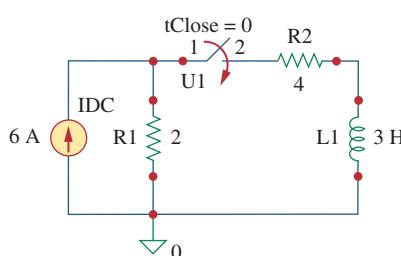


Figure 7.64

The schematic of the circuit in Fig. 7.63.

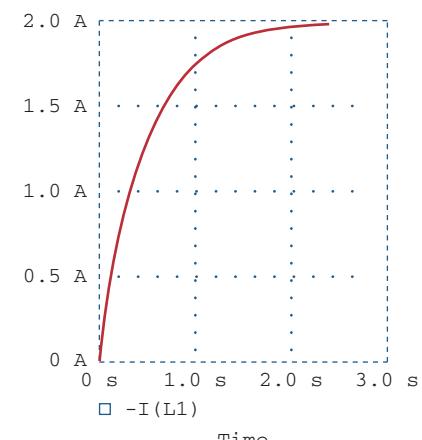


Figure 7.65

For Example 7.17; the response of the circuit in Fig. 7.63.

Note that the negative sign on $I(L1)$ is needed because the current enters through the upper terminal of the inductor, which happens to be the negative terminal after one counterclockwise rotation. A way to avoid the negative sign is to ensure that current enters pin 1 of the inductor. To obtain this desired direction of positive current flow, the initially horizontal inductor symbol should be rotated counterclockwise 270° and placed in the desired location.

Practice Problem 7.17

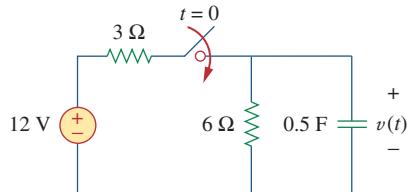


Figure 7.66

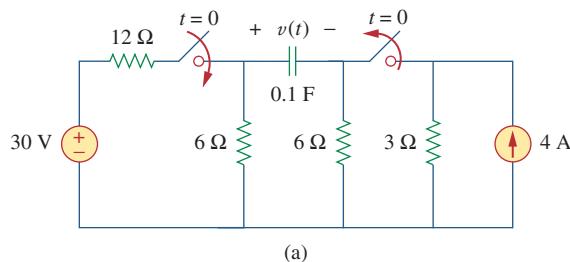
For Practice Prob. 7.17.

For the circuit in Fig. 7.66, use *Pspice* to find $v(t)$ for $t > 0$.

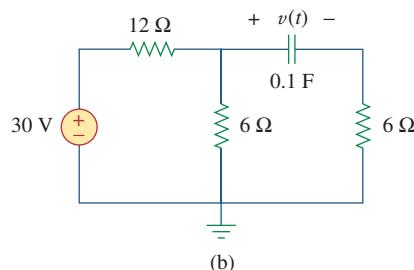
Answer: $v(t) = 8(1 - e^{-t})$ V, $t > 0$. The response is similar in shape to that in Fig. 7.65.

Example 7.18

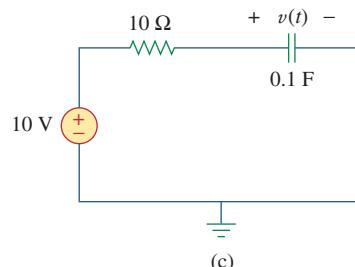
In the circuit of Fig. 7.67(a), determine the response $v(t)$.



(a)



(b)



(c)

Figure 7.67

For Example 7.18. Original circuit (a), circuit for $t > 0$ (b), and reduced circuit for $t > 0$ (c).

Solution:

1. **Define.** The problem is clearly stated and the circuit is clearly labeled.
2. **Present.** Given the circuit shown in Fig. 7.67(a), determine the response $v(t)$.
3. **Alternative.** We can solve this circuit using circuit analysis techniques, nodal analysis, mesh analysis, or *PSpice*. Let us solve the problem using circuit analysis techniques (this time Thevenin equivalent circuits) and then check the answer using two methods of *PSpice*.
4. **Attempt.** For time $t < 0$, the switch on the left is open and the switch on the right is closed. Assume that the switch on the right has been closed long enough for the circuit to reach steady state; then the capacitor acts like an open circuit and the current from the 4-A source flows through the parallel combination of the 6Ω and 3Ω resistors ($6 \parallel 3 = 18/9 = 2$), producing a voltage equal to $2 \times 4 = 8 \text{ V} = -v(0)$.

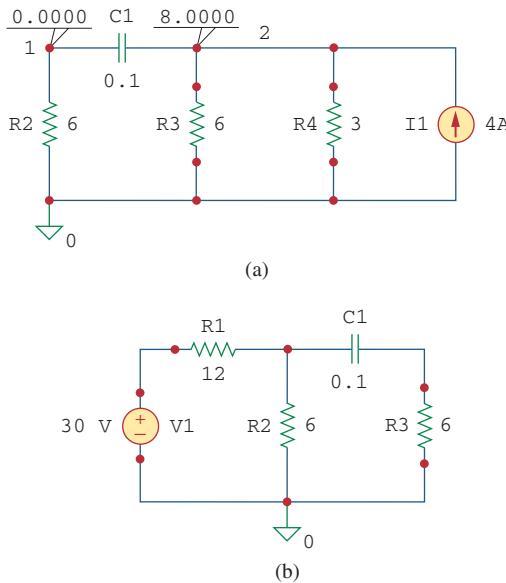
At $t = 0$, the switch on the left closes and the switch on the right opens, producing the circuit shown in Fig. 7.67(b).

The easiest way to complete the solution is to find the Thevenin equivalent circuit as seen by the capacitor. The open-circuit voltage (with the capacitor removed) is equal to the voltage drop across the 6Ω resistor on the left, or 10 V (the voltage drops uniformly across the 12Ω resistor, 20 V, and across the 6Ω resistor, 10 V). This is V_{Th} . The resistance looking in where the capacitor was is equal to $12 \parallel 6 + 6 = 72/18 + 6 = 10 \Omega$, which is R_{eq} . This produces the Thevenin equivalent circuit shown in Fig. 7.67(c). Matching up the boundary conditions ($v(0) = -8 \text{ V}$ and $v(\infty) = 10 \text{ V}$) and $\tau = RC = 1$, we get

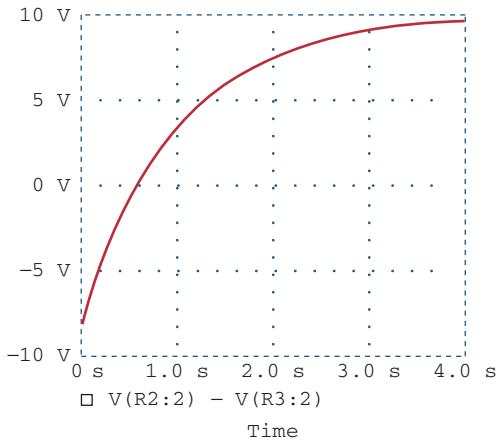
$$v(t) = 10 - 18e^{-t} \text{ V}$$

5. **Evaluate.** There are two ways of solving the problem using *PSpice*.

METHOD 1 One way is to first do the dc *PSpice* analysis to determine the initial capacitor voltage. The schematic of the relevant circuit is in Fig. 7.68(a). Two pseudocomponent VIEWPOINTS are inserted to measure the voltages at nodes 1 and 2. When the circuit is simulated, we obtain the displayed values in Fig. 7.68(a) as $V_1 = 0 \text{ V}$ and $V_2 = 8 \text{ V}$. Thus, the initial capacitor voltage is $v(0) = V_1 - V_2 = -8 \text{ V}$. The *PSpice* transient analysis uses this value along with the schematic in Fig. 7.68(b). Once the circuit in Fig. 7.68(b) is drawn, we insert the capacitor initial voltage as $IC = -8$. We select **Analysis/Setup/Transient** and set *Print Step* to 0.1 s and *Final Step* to $4\tau = 4 \text{ s}$. After saving the circuit, we select **Analysis/Simulate** to simulate the circuit. In the *PSpice* A/D window, we select **Trace/Add** and display $V(R2:2) - V(R3:2)$ or $V(C1:1) - V(C1:2)$ as the capacitor voltage $v(t)$. The plot of $v(t)$ is shown in Fig. 7.69. This agrees with the result obtained by hand calculation, $v(t) = 10 - 18 e^{-t} \text{ V}$.

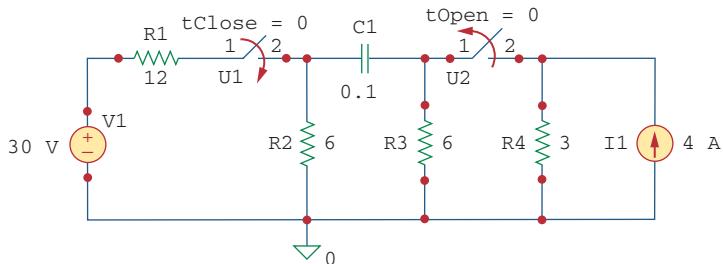
**Figure 7.68**

(a) Schematic for dc analysis to get $v(0)$, (b) schematic for transient analysis used in getting the response $v(t)$.

**Figure 7.69**

Response $v(t)$ for the circuit in Fig. 7.67.

METHOD 2 We can simulate the circuit in Fig. 7.67 directly, since PSpice can handle the open and closed switches and determine the initial conditions automatically. Using this approach, the schematic is drawn as shown in Fig. 7.70. After drawing the circuit, we select **Analysis/Setup/Transient** and set **Print Step** to 0.1 s and **Final Step** to $4\tau = 4$ s. We save the circuit, then select **Analysis/Simulate** to simulate the circuit. In the PSpice A/D window, we select **Trace/Add** and display $V(R2:2) - V(R3:2)$ as the capacitor voltage $v(t)$. The plot of $v(t)$ is the same as that shown in Fig. 7.69.

**Figure 7.70**

For Example 7.18.

6. **Satisfactory?** Clearly, we have found the value of the output response $v(t)$, as required by the problem statement. Checking does validate that solution. We can present all this as a complete solution to the problem.

The switch in Fig. 7.71 was open for a long time but closed at $t = 0$. If $i(0) = 10 \text{ A}$, find $i(t)$ for $t > 0$ by hand and also by PSpice analysis.

Practice Problem 7.18

Answer: $i(t) = 6 + 4e^{-5t} \text{ A}$. The plot of $i(t)$ obtained by PSpice analysis is shown in Fig. 7.72.

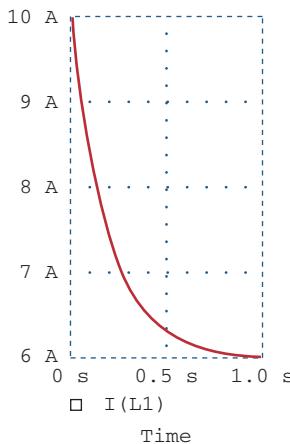


Figure 7.72
For Practice Prob. 7.18.

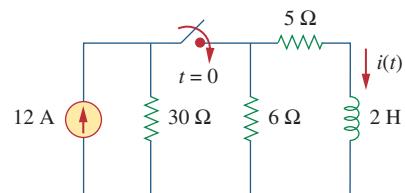


Figure 7.71
For Practice Prob. 7.18.

7.9 [†]Applications

The various devices in which RC and RL circuits find applications include filtering in dc power supplies, smoothing circuits in digital communications, differentiators, integrators, delay circuits, and relay circuits. Some of these applications take advantage of the short or long time constants of the RC or RL circuits. We will consider four simple applications here. The first two are RC circuits, the last two are RL circuits.

7.9.1 Delay Circuits

An RC circuit can be used to provide various time delays. Figure 7.73 shows such a circuit. It basically consists of an RC circuit with the capacitor connected in parallel with a neon lamp. The voltage source can provide enough voltage to fire the lamp. When the switch is closed, the capacitor voltage increases gradually toward 110 V at a rate determined by the circuit's time constant, $(R_1 + R_2)C$. The lamp will act as an open

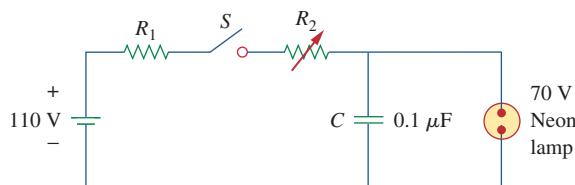


Figure 7.73
An RC delay circuit.

circuit and not emit light until the voltage across it exceeds a particular level, say 70 V. When the voltage level is reached, the lamp fires (goes on), and the capacitor discharges through it. Due to the low resistance of the lamp when on, the capacitor voltage drops fast and the lamp turns off. The lamp acts again as an open circuit and the capacitor recharges. By adjusting R_2 , we can introduce either short or long time delays into the circuit and make the lamp fire, recharge, and fire repeatedly every time constant $\tau = (R_1 + R_2)C$, because it takes a time period τ to get the capacitor voltage high enough to fire or low enough to turn off.

The warning blinkers commonly found on road construction sites are one example of the usefulness of such an RC delay circuit.

Example 7.19

Consider the circuit in Fig. 7.73, and assume that $R_1 = 1.5 \text{ M}\Omega$, $0 < R_2 < 2.5 \text{ M}\Omega$. (a) Calculate the extreme limits of the time constant of the circuit. (b) How long does it take for the lamp to glow for the first time after the switch is closed? Let R_2 assume its largest value.

Solution:

(a) The smallest value for R_2 is 0Ω , and the corresponding time constant for the circuit is

$$\tau = (R_1 + R_2)C = (1.5 \times 10^6 + 0) \times 0.1 \times 10^{-6} = 0.15 \text{ s}$$

The largest value for R_2 is $2.5 \text{ M}\Omega$, and the corresponding time constant for the circuit is

$$\tau = (R_1 + R_2)C = (1.5 + 2.5) \times 10^6 \times 0.1 \times 10^{-6} = 0.4 \text{ s}$$

Thus, by proper circuit design, the time constant can be adjusted to introduce a proper time delay in the circuit.

(b) Assuming that the capacitor is initially uncharged, $v_C(0) = 0$, while $v_C(\infty) = 110$. But

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau} = 110[1 - e^{-t/\tau}]$$

where $\tau = 0.4 \text{ s}$, as calculated in part (a). The lamp glows when $v_C = 70 \text{ V}$. If $v_C(t) = 70 \text{ V}$ at $t = t_0$, then

$$70 = 110[1 - e^{-t_0/\tau}] \quad \Rightarrow \quad \frac{7}{11} = 1 - e^{-t_0/\tau}$$

or

$$e^{-t_0/\tau} = \frac{4}{11} \quad \Rightarrow \quad e^{t_0/\tau} = \frac{11}{4}$$

Taking the natural logarithm of both sides gives

$$t_0 = \tau \ln \frac{11}{4} = 0.4 \ln 2.75 = 0.4046 \text{ s}$$

A more general formula for finding t_0 is

$$t_0 = \tau \ln \frac{-v(\infty)}{v(t_0) - v(\infty)}$$

The lamp will fire repeatedly every t_0 seconds if and only if $v(t_0) < v(\infty)$.

The RC circuit in Fig. 7.74 is designed to operate an alarm which activates when the current through it exceeds $120 \mu\text{A}$. If $0 \leq R \leq 6 \text{k}\Omega$, find the range of the time delay that the variable resistor can create.

Answer: Between 47.23 ms and 124 ms.

Practice Problem 7.19

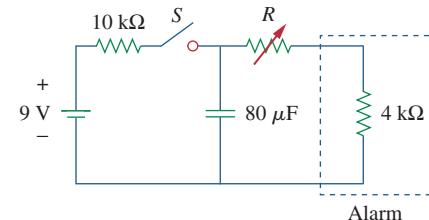


Figure 7.74
For Practice Prob. 7.19.

7.9.2 Photoflash Unit

An electronic flash unit provides a common example of an RC circuit. This application exploits the ability of the capacitor to oppose any abrupt change in voltage. Figure 7.75 shows a simplified circuit. It consists essentially of a high-voltage dc supply, a current-limiting large resistor R_1 , and a capacitor C in parallel with the flashlamp of low resistance R_2 . When the switch is in position 1, the capacitor charges slowly due to the large time constant ($\tau_1 = R_1 C$). As shown in Fig. 7.76(a), the capacitor voltage rises gradually from zero to V_s , while its current decreases gradually from $I_1 = V_s/R_1$ to zero. The charging time is approximately five times the time constant,

$$t_{\text{charge}} = 5R_1C \quad (7.65)$$

With the switch in position 2, the capacitor voltage is discharged. The low resistance R_2 of the photolamp permits a high discharge current with peak $I_2 = V_s/R_2$ in a short duration, as depicted in Fig. 7.76(b). Discharging takes place in approximately five times the time constant,

$$t_{\text{discharge}} = 5R_2C \quad (7.66)$$

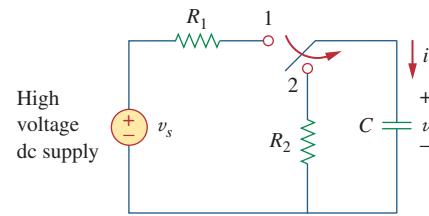
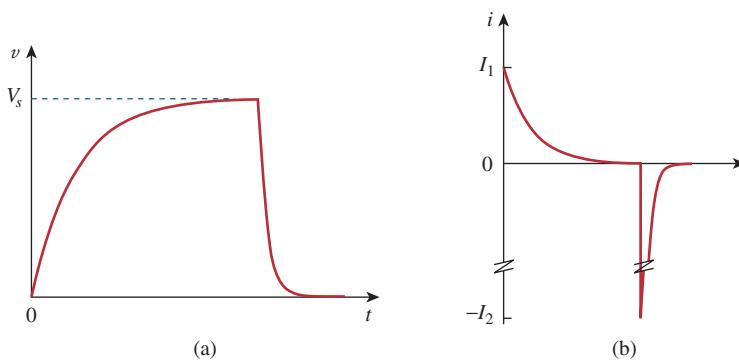


Figure 7.75
Circuit for a flash unit providing slow charge in position 1 and fast discharge in position 2.



Figures 7.76

(a) Capacitor voltage showing slow charge and fast discharge, (b) capacitor current showing low charging current $I_1 = V_s/R_1$ and high discharge current $I_2 = V_s/R_2$.

Thus, the simple RC circuit of Fig. 7.75 provides a short-duration, high-current pulse. Such a circuit also finds applications in electric spot welding and the radar transmitter tube.

Example 7.20

An electronic flashgun has a current-limiting $6\text{-k}\Omega$ resistor and $2000\text{-}\mu\text{F}$ electrolytic capacitor charged to 240 V. If the lamp resistance is $12\ \Omega$, find: (a) the peak charging current, (b) the time required for the capacitor to fully charge, (c) the peak discharging current, (d) the total energy stored in the capacitor, and (e) the average power dissipated by the lamp.

Solution:

(a) The peak charging current is

$$I_1 = \frac{V_s}{R_1} = \frac{240}{6 \times 10^3} = 40 \text{ mA}$$

(b) From Eq. (7.65),

$$t_{\text{charge}} = 5R_1C = 5 \times 6 \times 10^3 \times 2000 \times 10^{-6} = 60 \text{ s} = 1 \text{ minute}$$

(c) The peak discharging current is

$$I_2 = \frac{V_s}{R_2} = \frac{240}{12} = 20 \text{ A}$$

(d) The energy stored is

$$W = \frac{1}{2}CV_s^2 = \frac{1}{2} \times 2000 \times 10^{-6} \times 240^2 = 57.6 \text{ J}$$

(e) The energy stored in the capacitor is dissipated across the lamp during the discharging period. From Eq. (7.66),

$$t_{\text{discharge}} = 5R_2C = 5 \times 12 \times 2000 \times 10^{-6} = 0.12 \text{ s}$$

Thus, the average power dissipated is

$$P = \frac{W}{t_{\text{discharge}}} = \frac{57.6}{0.12} = 480 \text{ watts}$$

Practice Problem 7.20

The flash unit of a camera has a 2-mF capacitor charged to 80 V.

- (a) How much charge is on the capacitor?
- (b) What is the energy stored in the capacitor?
- (c) If the flash fires in 0.8 ms, what is the average current through the flashtube?
- (d) How much power is delivered to the flashtube?
- (e) After a picture has been taken, the capacitor needs to be recharged by a power unit that supplies a maximum of 5 mA. How much time does it take to charge the capacitor?

Answer: (a) 0.16 C, (b) 6.4 J, (c) 200 A, (d) 8 kW, (e) 32 s.

7.9.3 Relay Circuits

A magnetically controlled switch is called a *relay*. A relay is essentially an electromagnetic device used to open or close a switch that controls another circuit. Figure 7.77(a) shows a typical relay circuit.

The coil circuit is an RL circuit like that in Fig. 7.77(b), where R and L are the resistance and inductance of the coil. When switch S_1 in Fig. 7.77(a) is closed, the coil circuit is energized. The coil current gradually increases and produces a magnetic field. Eventually the magnetic field is sufficiently strong to pull the movable contact in the other circuit and close switch S_2 . At this point, the relay is said to be *pulled in*. The time interval t_d between the closure of switches S_1 and S_2 is called the *relay delay time*.

Relays were used in the earliest digital circuits and are still used for switching high-power circuits.

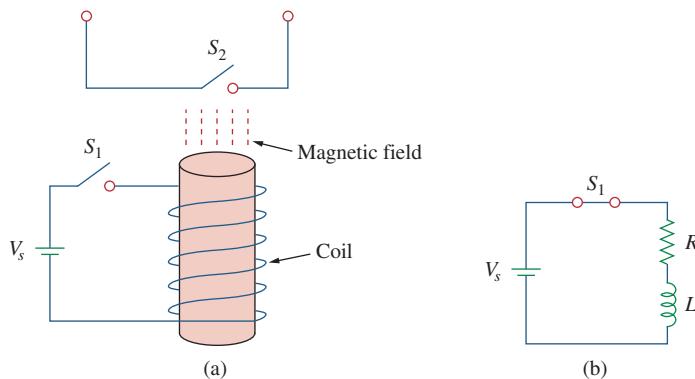


Figure 7.77

A relay circuit.

The coil of a certain relay is operated by a 12-V battery. If the coil has a resistance of $150\ \Omega$ and an inductance of $30\ \text{mH}$ and the current needed to pull in is $50\ \text{mA}$, calculate the relay delay time.

Example 7.21

Solution:

The current through the coil is given by

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

where

$$i(0) = 0, \quad i(\infty) = \frac{12}{150} = 80\ \text{mA}$$

$$\tau = \frac{L}{R} = \frac{30 \times 10^{-3}}{150} = 0.2\ \text{ms}$$

Thus,

$$i(t) = 80[1 - e^{-t/\tau}]\ \text{mA}$$

If $i(t_d) = 50\ \text{mA}$, then

$$50 = 80[1 - e^{-t_d/\tau}] \quad \Rightarrow \quad \frac{5}{8} = 1 - e^{-t_d/\tau}$$

or

$$e^{-t_d/\tau} = \frac{3}{8} \quad \Rightarrow \quad e^{t_d/\tau} = \frac{8}{3}$$

By taking the natural logarithm of both sides, we get

$$t_d = \tau \ln \frac{8}{3} = 0.2 \ln \frac{8}{3} \text{ ms} = 0.1962 \text{ ms}$$

Alternatively, we may find t_d using

$$t_d = \tau \ln \frac{i(0) - i(\infty)}{i(t_d) - i(\infty)}$$

Practice Problem 7.21

A relay has a resistance of 200Ω and an inductance of 500 mH . The relay contacts close when the current through the coil reaches 350 mA . What time elapses between the application of 110 V to the coil and contact closure?

Answer: 2.529 ms .

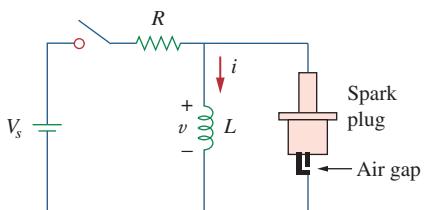


Figure 7.78

Circuit for an automobile ignition system.

7.9.4 Automobile Ignition Circuit

The ability of inductors to oppose rapid change in current makes them useful for arc or spark generation. An automobile ignition system takes advantage of this feature.

The gasoline engine of an automobile requires that the fuel-air mixture in each cylinder be ignited at proper times. This is achieved by means of a spark plug (Fig. 7.78), which essentially consists of a pair of electrodes separated by an air gap. By creating a large voltage (thousands of volts) between the electrodes, a spark is formed across the air gap, thereby igniting the fuel. But how can such a large voltage be obtained from the car battery, which supplies only 12 V ? This is achieved by means of an inductor (the spark coil) L . Since the voltage across the inductor is $v = L di/dt$, we can make di/dt large by creating a large change in current in a very short time. When the ignition switch in Fig. 7.78 is closed, the current through the inductor increases gradually and reaches the final value of $i = V_s/R$, where $V_s = 12 \text{ V}$. Again, the time taken for the inductor to charge is five times the *time constant* of the circuit ($\tau = L/R$),

$$t_{\text{charge}} = 5 \frac{L}{R} \quad (7.67)$$

Since at steady state, i is constant, $di/dt = 0$ and the inductor voltage $v = 0$. When the switch suddenly opens, a large voltage is developed across the inductor (due to the rapidly collapsing field) causing a spark or arc in the air gap. The spark continues until the energy stored in the inductor is dissipated in the spark discharge. In laboratories, when one is working with inductive circuits, this same effect causes a very nasty shock, and one must exercise caution.

A solenoid with resistance 4Ω and inductance 6 mH is used in an automobile ignition circuit similar to that in Fig. 7.78. If the battery supplies 12 V , determine: the final current through the solenoid when the switch is closed, the energy stored in the coil, and the voltage across the air gap, assuming that the switch takes $1 \mu\text{s}$ to open.

Example 7.22

Solution:

The final current through the coil is

$$I = \frac{V_s}{R} = \frac{12}{4} = 3 \text{ A}$$

The energy stored in the coil is

$$W = \frac{1}{2}LI^2 = \frac{1}{2} \times 6 \times 10^{-3} \times 3^2 = 27 \text{ mJ}$$

The voltage across the gap is

$$V = L \frac{\Delta I}{\Delta t} = 6 \times 10^{-3} \times \frac{3}{1 \times 10^{-6}} = 18 \text{ kV}$$

The spark coil of an automobile ignition system has a 20-mH inductance and a $5\text{-}\Omega$ resistance. With a supply voltage of 12 V , calculate: the time needed for the coil to fully charge, the energy stored in the coil, and the voltage developed at the spark gap if the switch opens in $2 \mu\text{s}$.

Practice Problem 7.22

Answer: 20 ms , 57.6 mJ , and 24 kV .

7.10 Summary

1. The analysis in this chapter is applicable to any circuit that can be reduced to an equivalent circuit comprising a resistor and a single energy-storage element (inductor or capacitor). Such a circuit is first-order because its behavior is described by a first-order differential equation. When analyzing RC and RL circuits, one must always keep in mind that the capacitor is an open circuit to steady-state dc conditions while the inductor is a short circuit to steady-state dc conditions.
2. The natural response is obtained when no independent source is present. It has the general form

$$x(t) = x(0)e^{-t/\tau}$$

where x represents current through (or voltage across) a resistor, a capacitor, or an inductor, and $x(0)$ is the initial value of x . Because most practical resistors, capacitors, and inductors always have losses, the natural response is a transient response, i.e. it dies out with time.

3. The time constant τ is the time required for a response to decay to $1/e$ of its initial value. For RC circuits, $\tau = RC$ and for RL circuits, $\tau = L/R$.

4. The singularity functions include the unit step, the unit ramp function, and the unit impulse functions. The unit step function $u(t)$ is

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

The unit impulse function is

$$\delta(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$$

The unit ramp function is

$$r(t) = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$$

5. The steady-state response is the behavior of the circuit after an independent source has been applied for a long time. The transient response is the component of the complete response that dies out with time.

6. The total or complete response consists of the steady-state response and the transient response.

7. The step response is the response of the circuit to a sudden application of a dc current or voltage. Finding the step response of a first-order circuit requires the initial value $x(0^+)$, the final value $x(\infty)$, and the time constant τ . With these three items, we obtain the step response as

$$x(t) = x(\infty) + [x(0^+) - x(\infty)]e^{-t/\tau}$$

A more general form of this equation is

$$x(t) = x(\infty) + [x(t_0^+) - x(\infty)]e^{-(t-t_0)/\tau}$$

Or we may write it as

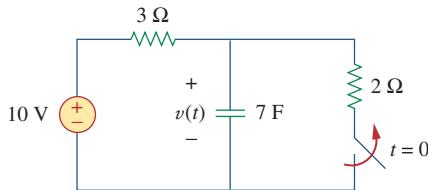
$$\text{Instantaneous value} = \text{Final} + [\text{Initial} - \text{Final}]e^{-(t-t_0)/\tau}$$

8. *PSpice* is very useful for obtaining the transient response of a circuit.
 9. Four practical applications of *RC* and *RL* circuits are: a delay circuit, a photoflash unit, a relay circuit, and an automobile ignition circuit.

Review Questions

- 7.1** An *RC* circuit has $R = 2 \Omega$ and $C = 4 \text{ F}$. The time constant is:
- (a) 0.5 s (b) 2 s (c) 4 s
 (d) 8 s (e) 15 s
- 7.2** The time constant for an *RL* circuit with $R = 2 \Omega$ and $L = 4 \text{ H}$ is:
- (a) 0.5 s (b) 2 s (c) 4 s
 (d) 8 s (e) 15 s
- 7.3** A capacitor in an *RC* circuit with $R = 2 \Omega$ and $C = 4 \text{ F}$ is being charged. The time required for the capacitor voltage to reach 63.2 percent of its steady-state value is:
- (a) 2 s (b) 4 s (c) 8 s
 (d) 16 s (e) none of the above
- 7.4** An *RL* circuit has $R = 2 \Omega$ and $L = 4 \text{ H}$. The time needed for the inductor current to reach 40 percent of its steady-state value is:
- (a) 0.5 s (b) 1 s (c) 2 s
 (d) 4 s (e) none of the above

- 7.5** In the circuit of Fig. 7.79, the capacitor voltage just before $t = 0$ is:
- 10 V
 - 7 V
 - 6 V
 - 4 V
 - 0 V

**Figure 7.79**

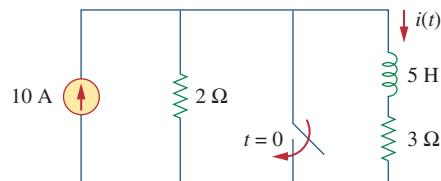
For Review Questions 7.5 and 7.6.

- 7.6** In the circuit in Fig. 7.79, $v(\infty)$ is:

- 10 V
- 7 V
- 6 V
- 4 V
- 0 V

- 7.7** For the circuit in Fig. 7.80, the inductor current just before $t = 0$ is:

- 8 A
- 6 A
- 4 A
- 2 A
- 0 A

**Figure 7.80**

For Review Questions 7.7 and 7.8.

- 7.8** In the circuit of Fig. 7.80, $i(\infty)$ is:

- 10 A
- 6 A
- 4 A
- 2 A
- 0 A

- 7.9** If v_s changes from 2 V to 4 V at $t = 0$, we may express v_s as:

- $\delta(t) \text{ V}$
- $2u(t) \text{ V}$
- $2u(-t) + 4u(t) \text{ V}$
- $2 + 2u(t) \text{ V}$
- $4u(t) - 2 \text{ V}$

- 7.10** The pulse in Fig. 7.116(a) can be expressed in terms of singularity functions as:

- $2u(t) + 2u(t - 1) \text{ V}$
- $2u(t) - 2u(t - 1) \text{ V}$
- $2u(t) - 4u(t - 1) \text{ V}$
- $2u(t) + 4u(t - 1) \text{ V}$

Answers: 7.1d, 7.2b, 7.3c, 7.4b, 7.5d, 7.6a, 7.7c, 7.8e, 7.9c,d, 7.10b.

Problems

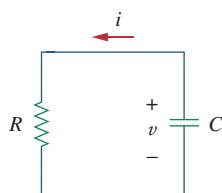
Section 7.2 The Source-Free RC Circuit

- 7.1** In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t} \text{ V}, \quad t > 0$$

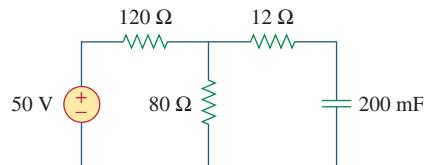
$$i(t) = 8e^{-200t} \text{ mA}, \quad t > 0$$

- Find the values of R and C .
- Calculate the time constant τ .
- Determine the time required for the voltage to decay half its initial value at $t = 0$.

**Figure 7.81**

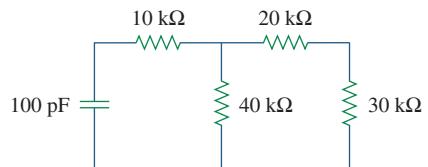
For Prob. 7.1.

- 7.2** Find the time constant for the RC circuit in Fig. 7.82.

**Figure 7.82**

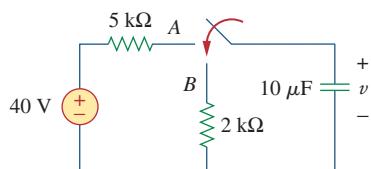
For Prob. 7.2.

- 7.3** Determine the time constant for the circuit in Fig. 7.83.

**Figure 7.83**

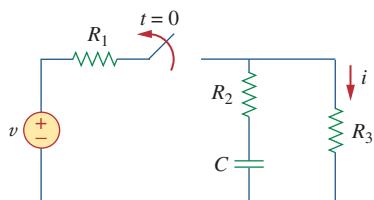
For Prob. 7.3.

- 7.4** The switch in Fig. 7.84 has been in position A for a long time. Assume the switch moves instantaneously from A to B at $t = 0$. Find v for $t > 0$.

**Figure 7.84**

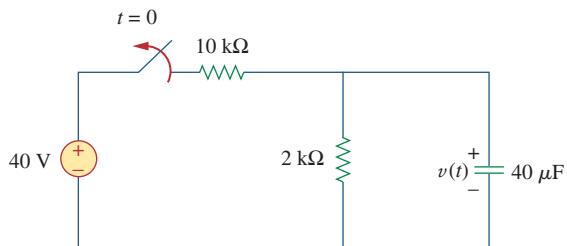
For Prob. 7.4.

- 7.5** Using Fig. 7.85, design a problem to help other students better understand source-free RC circuits.

**Figure 7.85**

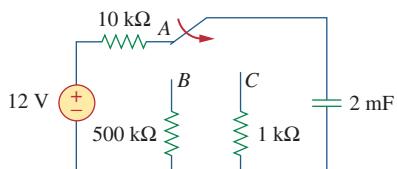
For Prob. 7.5.

- 7.6** The switch in Fig. 7.86 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

**Figure 7.86**

For Prob. 7.6.

- 7.7** Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at $t = 0$. Then at $t = 1$ second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.

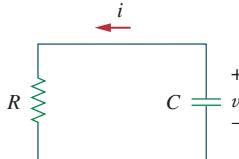
**Figure 7.87**

For Prob. 7.7.

- 7.8** For the circuit in Fig. 7.88, if

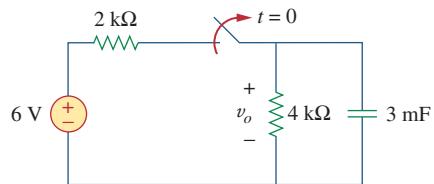
$$v = 10e^{-4t} \text{ V} \quad \text{and} \quad i = 0.2 e^{-4t} \text{ A}, \quad t > 0$$

- (a) Find R and C .
- (b) Determine the time constant.
- (c) Calculate the initial energy in the capacitor.
- (d) Obtain the time it takes to dissipate 50 percent of the initial energy.

**Figure 7.88**

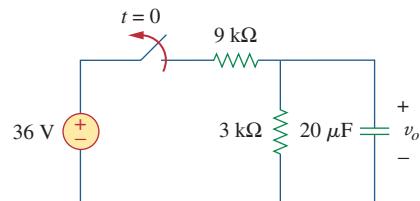
For Prob. 7.8.

- 7.9** The switch in Fig. 7.89 opens at $t = 0$. Find v_o for $t > 0$.

**Figure 7.89**

For Prob. 7.9.

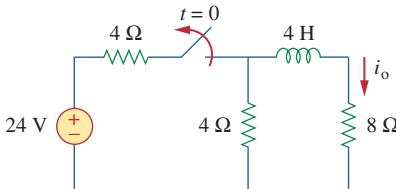
- 7.10** For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

**Figure 7.90**

For Prob. 7.10.

Section 7.3 The Source-Free RL Circuit

- 7.11** For the circuit in Fig. 7.91, find i_o for $t > 0$.

**Figure 7.91**

For Prob. 7.11.

- 7.12** Using Fig. 7.92, design a problem to help other **e2d** students better understand source-free *RL* circuits.

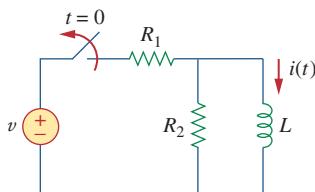


Figure 7.92

For Prob. 7.12.

- 7.13** In the circuit of Fig. 7.93,

$$v(t) = 80e^{-10^3t} \text{ V}, \quad t > 0$$

$$i(t) = 5e^{-10^3t} \text{ mA}, \quad t > 0$$

- (a) Find R , L , and τ .
 (b) Calculate the energy dissipated in the resistance for $0 < t < 0.5 \text{ ms}$.

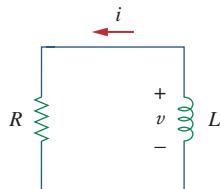


Figure 7.93

For Prob. 7.13.

- 7.14** Calculate the time constant of the circuit in Fig. 7.94.

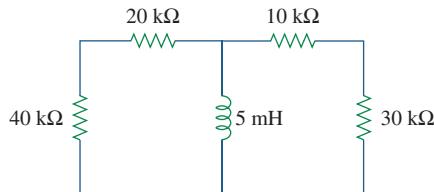


Figure 7.94

For Prob. 7.14.

- 7.15** Find the time constant for each of the circuits in Fig. 7.95.

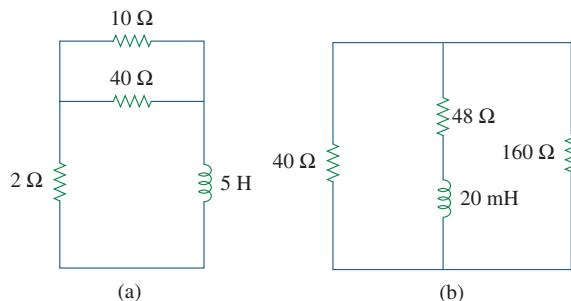


Figure 7.95

For Prob. 7.15.

- 7.16** Determine the time constant for each of the circuits in Fig. 7.96.

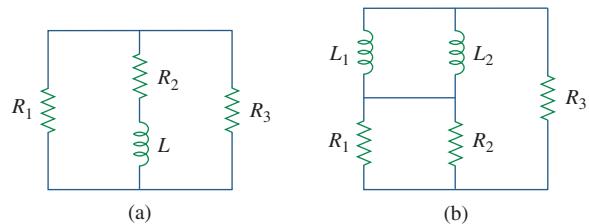


Figure 7.96

For Prob. 7.16.

- 7.17** Consider the circuit of Fig. 7.97. Find $v_o(t)$ if $i(0) = 6 \text{ A}$ and $v(t) = 0$.

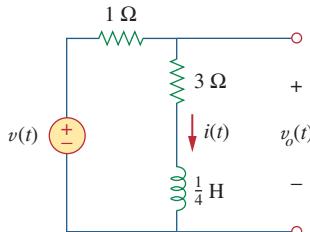


Figure 7.97

For Prob. 7.17.

- 7.18** For the circuit in Fig. 7.98, determine $v_o(t)$ when $i(0) = 5 \text{ A}$ and $v(t) = 0$.

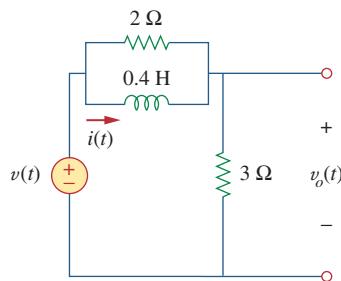


Figure 7.98

For Prob. 7.18.

- 7.19** In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 6 \text{ A}$.

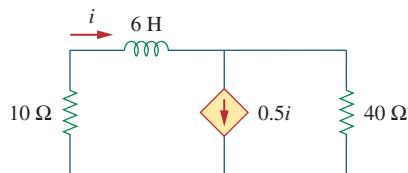


Figure 7.99

For Prob. 7.19.

- 7.20** For the circuit in Fig. 7.100,

$$v = 90e^{-50t} \text{ V}$$

and

$$i = 30e^{-50t} \text{ A}, \quad t > 0$$

- (a) Find L and R .
- (b) Determine the time constant.
- (c) Calculate the initial energy in the inductor.
- (d) What fraction of the initial energy is dissipated in 10 ms?

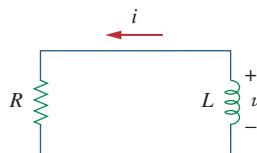


Figure 7.100

For Prob. 7.20.

- 7.21** In the circuit of Fig. 7.101, find the value of R for which the steady-state energy stored in the inductor will be 1 J.

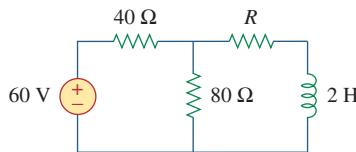


Figure 7.101

For Prob. 7.21.

- 7.22** Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 7.102 if $i(0) = 10 \text{ A}$.

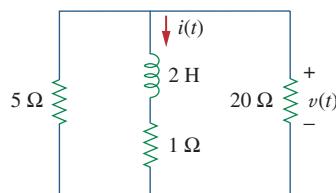


Figure 7.102

For Prob. 7.22.

- 7.23** Consider the circuit in Fig. 7.103. Given that $v_o(0) = 10 \text{ V}$, find v_o and v_x for $t > 0$.

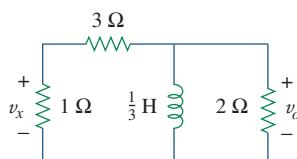


Figure 7.103

For Prob. 7.23.

Section 7.4 Singularity Functions

- 7.24** Express the following signals in terms of singularity functions.

$$(a) v(t) = \begin{cases} 0, & t < 0 \\ -5, & t > 0 \end{cases}$$

$$(b) i(t) = \begin{cases} 0, & t < 1 \\ -10, & 1 < t < 3 \\ 10, & 3 < t < 5 \\ 0, & t > 5 \end{cases}$$

$$(c) x(t) = \begin{cases} t - 1, & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 4 - t, & 3 < t < 4 \\ 0, & \text{Otherwise} \end{cases}$$

$$(d) y(t) = \begin{cases} 2, & t < 0 \\ -5, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$$

- 7.25** Design a problem to help other students better understand singularity functions.

- 7.26** Express the signals in Fig. 7.104 in terms of singularity functions.

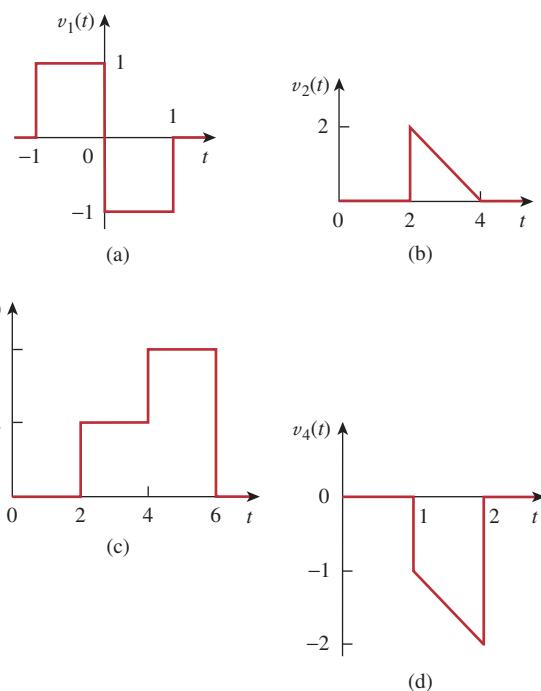
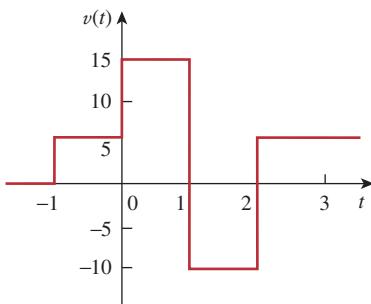


Figure 7.104

For Prob. 7.26.

- 7.27** Express $v(t)$ in Fig. 7.105 in terms of step functions.

**Figure 7.105**

For Prob. 7.27.

7.28 Sketch the waveform represented by

$$\begin{aligned} i(t) = & r(t) - r(t-1) - u(t-2) - r(t-2) \\ & + r(t-3) + u(t-4) \end{aligned}$$

7.29 Sketch the following functions:

- (a) $x(t) = 10e^{-t}u(t-1)$,
 (b) $y(t) = 10e^{-(t-1)}u(t)$,
 (c) $z(t) = \cos 4t\delta(t-1)$

7.30 Evaluate the following integrals involving the impulse functions:

- (a) $\int_{-\infty}^{\infty} 4t^2\delta(t-1)dt$
 (b) $\int_{-\infty}^{\infty} 4t^2 \cos 2\pi t\delta(t-0.5)dt$

7.31 Evaluate the following integrals:

- (a) $\int_{-\infty}^{\infty} e^{-4t^2}\delta(t-2)dt$
 (b) $\int_{-\infty}^{\infty} [5\delta(t) + e^{-t}\delta(t) + \cos 2\pi t\delta(t)]dt$

7.32 Evaluate the following integrals:

- (a) $\int_1^t u(\lambda)d\lambda$
 (b) $\int_0^4 r(t-1)dt$
 (c) $\int_1^5 (t-6)^2\delta(t-2)dt$

7.33 The voltage across a 10-mH inductor is $15\delta(t-2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.**7.34** Evaluate the following derivatives:

- (a) $\frac{d}{dt}[u(t-1)u(t+1)]$
 (b) $\frac{d}{dt}[r(t-6)u(t-2)]$
 (c) $\frac{d}{dt}[\sin 4tu(t-3)]$

7.35 Find the solution to the following differential equations:

$$(a) \frac{dv}{dt} + 2v = 0, \quad v(0) = -1 \text{ V}$$

$$(b) 2\frac{di}{dt} - 3i = 0, \quad i(0) = 2$$

7.36 Solve for v in the following differential equations, subject to the stated initial condition.

- (a) $dv/dt + v = u(t), \quad v(0) = 0$
 (b) $2dv/dt - v = 3u(t), \quad v(0) = -6$

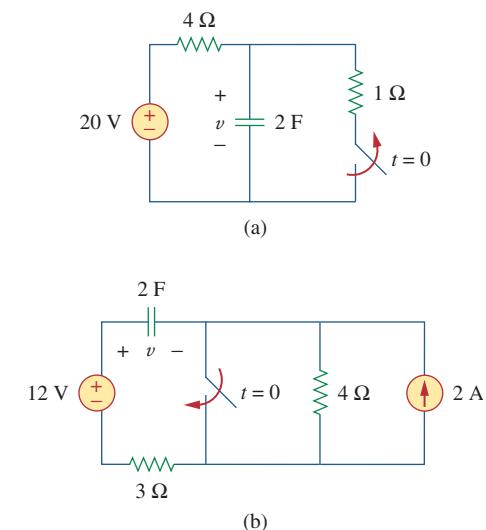
7.37 A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

- (a) What is the time constant of the circuit?
 (b) What is $v(\infty)$, the final value of v ?
 (c) If $v(0) = 2$, find $v(t)$ for $t \geq 0$.

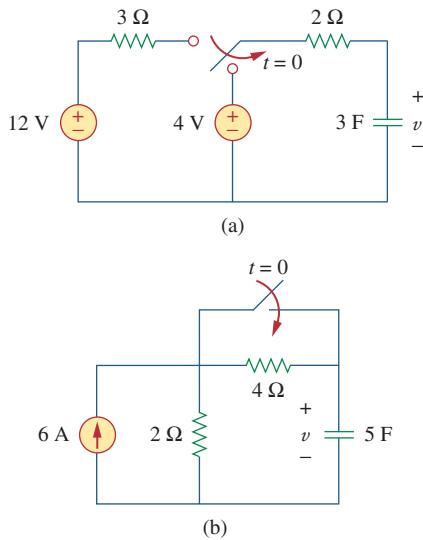
7.38 A circuit is described by

$$\frac{di}{dt} + 3i = 2u(t)$$

Find $i(t)$ for $t > 0$ given that $i(0) = 0$.**Section 7.5 Step Response of an RC Circuit****7.39** Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.**Figure 7.106**

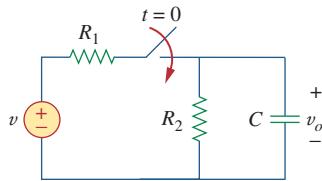
For Prob. 7.39.

7.40 Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.

**Figure 7.107**

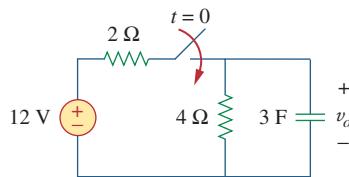
For Prob. 7.40.

- 7.41** Using Fig. 7.108, design a problem to help other students better understand the step response of an RC circuit.

**Figure 7.108**

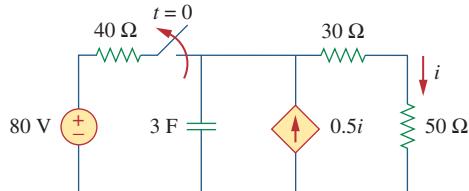
For Prob. 7.41.

- 7.42** (a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.
 (b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

**Figure 7.109**

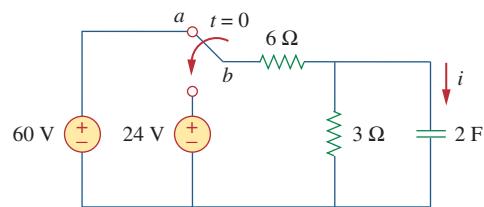
For Prob. 7.42.

- 7.43** Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.

**Figure 7.110**

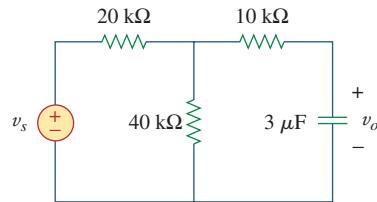
For Prob. 7.43.

- 7.44** The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$, it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

**Figure 7.111**

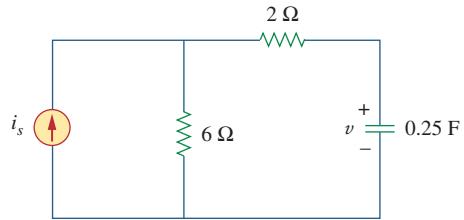
For Prob. 7.44.

- 7.45** Find v_o in the circuit of Fig. 7.112 when $v_s = 30u(t)$ V. Assume that $v_o(0) = 5$ V.

**Figure 7.112**

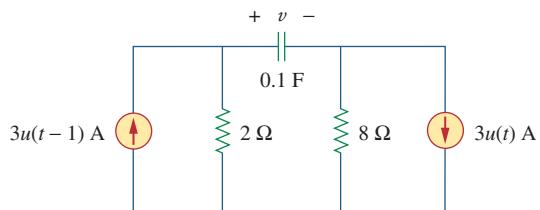
For Prob. 7.45.

- 7.46** For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$. Find $v(t)$.

**Figure 7.113**

For Prob. 7.46.

- 7.47** Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

**Figure 7.114**

For Prob. 7.47.

- 7.48** Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

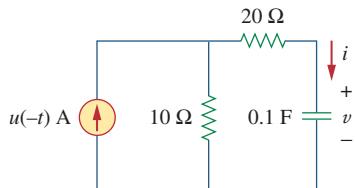
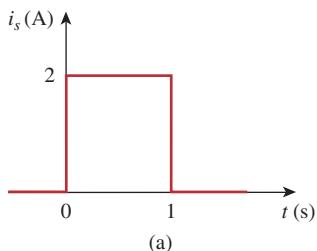


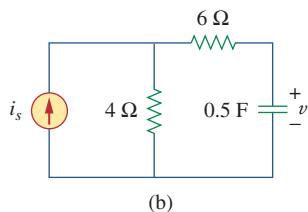
Figure 7.115

For Prob. 7.48.

- 7.49** If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.



(a)



(b)

Figure 7.116

For Prob. 7.49 and Review Question 7.10.

- *7.50** In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1 \text{ k}\Omega$, $R_3 = 2 \text{ k}\Omega$, and $C = 0.25 \text{ mF}$.

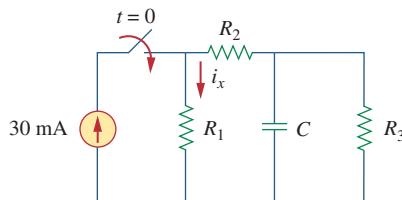


Figure 7.117

For Prob. 7.50.

Section 7.6 Step Response of an RL Circuit

- 7.51** Rather than applying the short-cut technique used in Section 7.6, use KVL to obtain Eq. (7.60).

- 7.52** Using Fig. 7.118, design a problem to help other **e2d** students better understand the step response of an RL circuit.

* An asterisk indicates a challenging problem.

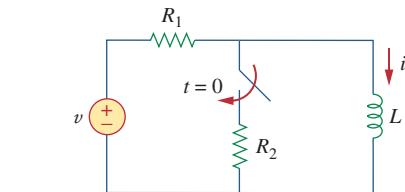


Figure 7.118

For Prob. 7.52.

- 7.53** Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.

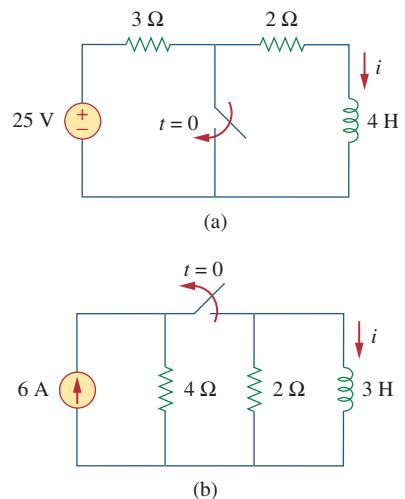


Figure 7.119

For Prob. 7.53.

- 7.54** Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.

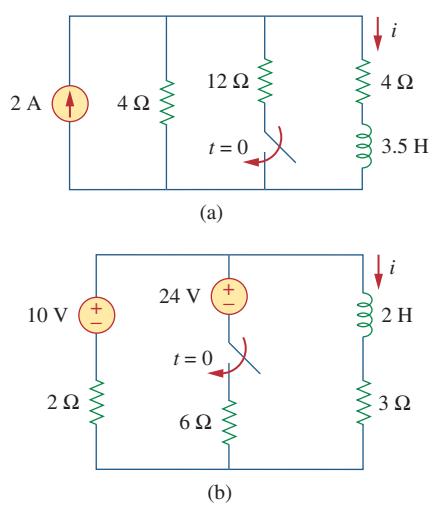
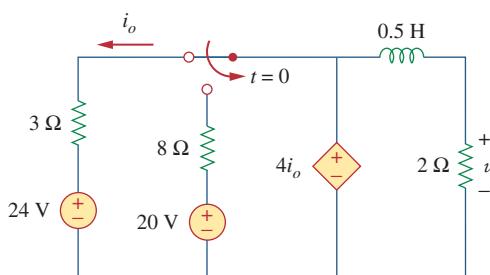


Figure 7.120

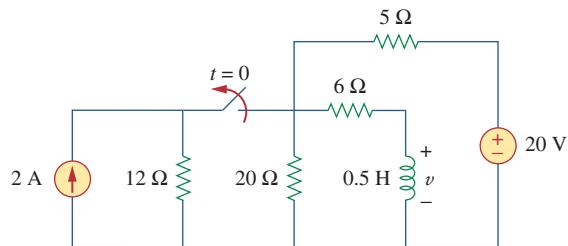
For Prob. 7.54.

- 7.55** Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

**Figure 7.121**

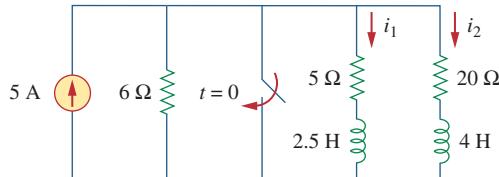
For Prob. 7.55.

- 7.56** For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

**Figure 7.122**

For Prob. 7.56.

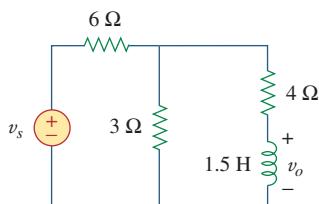
- *7.57** Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

**Figure 7.123**

For Prob. 7.57.

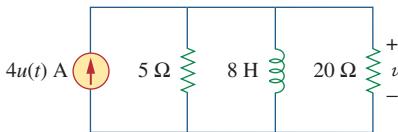
- 7.58** Rework Prob. 7.17 if $i(0) = 10$ A and $v(t) = 20u(t)$ V.

- 7.59** Determine the step response $v_o(t)$ to $v_s = 18u(t)$ in the circuit of Fig. 7.124.

**Figure 7.124**

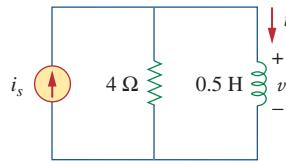
For Prob. 7.59.

- 7.60** Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.125 if the initial current in the inductor is zero.

**Figure 7.125**

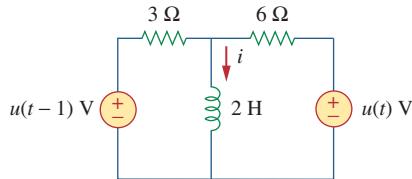
For Prob. 7.60.

- 7.61** In the circuit in Fig. 7.126, i_s changes from 5 A to 10 A at $t = 0$; that is, $i_s = 5u(-t) + 10u(t)$. Find v and i .

**Figure 7.126**

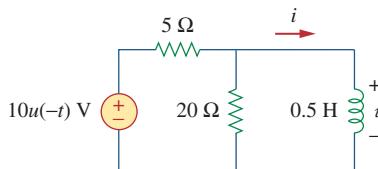
For Prob. 7.61.

- 7.62** For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

**Figure 7.127**

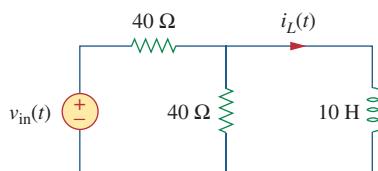
For Prob. 7.62.

- 7.63** Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

**Figure 7.128**

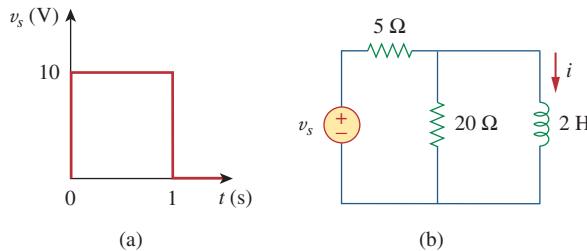
For Prob. 7.63.

- 7.64** Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from $t = 0$ sec to $t = \infty$ sec. The value of $v_{in}(t)$ is equal to $[40 - 40u(t)]$ volts.

**Figure 7.129**

For Prob. 7.64.

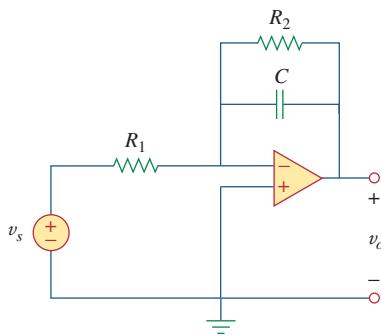
- 7.65** If the input pulse in Fig. 7.130(a) is applied to the circuit in Fig. 7.130(b), determine the response $i(t)$.

**Figure 7.130**

For Prob. 7.65.

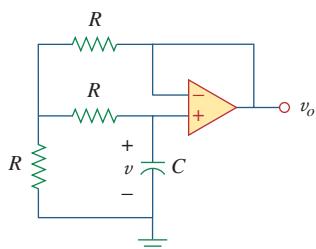
Section 7.7 First-order Op Amp Circuits

- 7.66** Using Fig. 7.131, design a problem to help other **e2d** students better understand first-order op amp circuits.

**Figure 7.131**

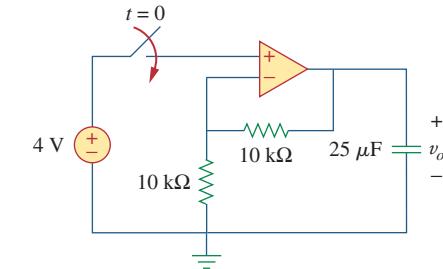
For Prob. 7.66.

- 7.67** If $v(0) = 5$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit in Fig. 7.132. Let $R = 10 \text{ k}\Omega$ and $C = 1 \mu\text{F}$.

**Figure 7.132**

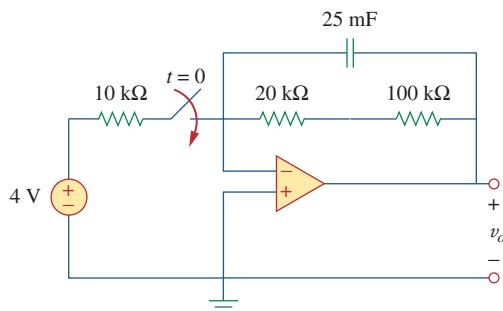
For Prob. 7.67.

- 7.68** Obtain v_o for $t > 0$ in the circuit of Fig. 7.133.

**Figure 7.133**

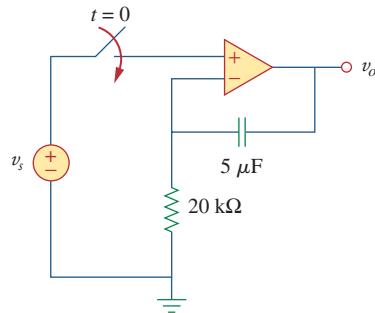
For Prob. 7.68.

- 7.69** For the op amp circuit in Fig. 7.134, find $v_o(t)$ for $t > 0$.

**Figure 7.134**

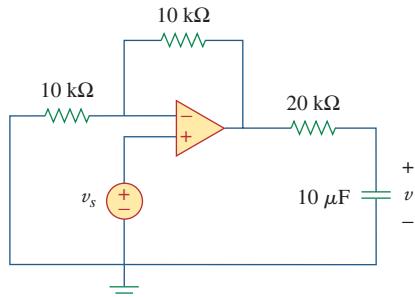
For Prob. 7.69.

- 7.70** Determine v_o for $t > 0$ when $v_s = 20 \text{ mV}$ in the op amp circuit of Fig. 7.135.

**Figure 7.135**

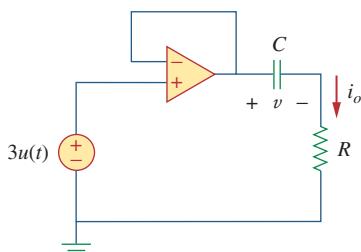
For Prob. 7.70.

- 7.71** For the op amp circuit in Fig. 7.136, suppose $v_o = 0$ and $v_s = 3 \text{ V}$. Find $v(t)$ for $t > 0$.

**Figure 7.136**

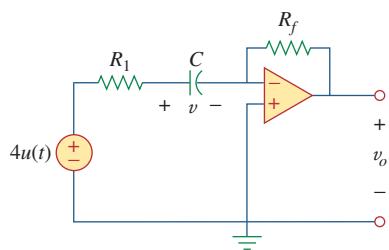
For Prob. 7.71.

- 7.72 Find i_o in the op amp circuit in Fig. 7.137. Assume that $v(0) = -2 \text{ V}$, $R = 10 \text{ k}\Omega$, and $C = 10 \mu\text{F}$.

**Figure 7.137**

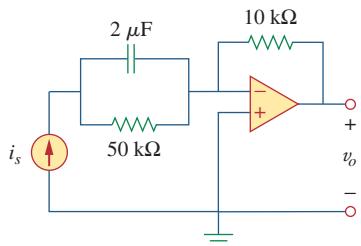
For Prob. 7.72.

- 7.73 For the op amp circuit of Fig. 7.138, let $R_1 = 10 \text{ k}\Omega$, $R_f = 20 \text{ k}\Omega$, $C = 20 \mu\text{F}$, and $v(0) = 1 \text{ V}$. Find v_o .

**Figure 7.138**

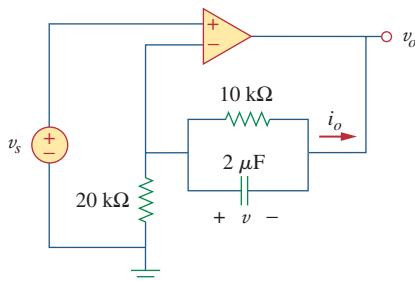
For Prob. 7.73.

- 7.74 Determine $v_o(t)$ for $t > 0$ in the circuit of Fig. 7.139. Let $i_s = 10u(t) \mu\text{A}$ and assume that the capacitor is initially uncharged.

**Figure 7.139**

For Prob. 7.74.

- 7.75 In the circuit of Fig. 7.140, find v_o and i_o , given that $v_s = 4u(t) \text{ V}$ and $v(0) = 1 \text{ V}$.

**Figure 7.140**

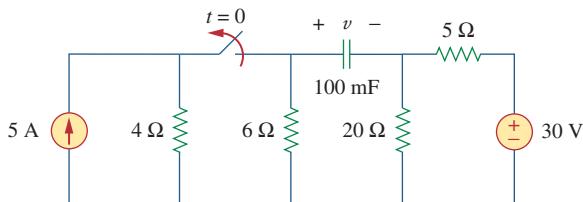
For Prob. 7.75.

Section 7.8 Transient Analysis with PSpice



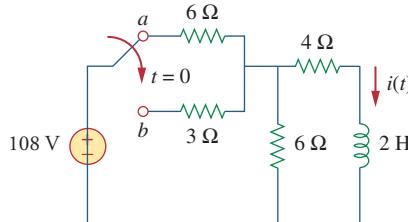
- 7.76 Repeat Prob. 7.49 using PSpice or MultiSim.

- 7.77 The switch in Fig. 7.141 opens at $t = 0$. Use PSpice or MultiSim to determine $v(t)$ for $t > 0$.

**Figure 7.141**

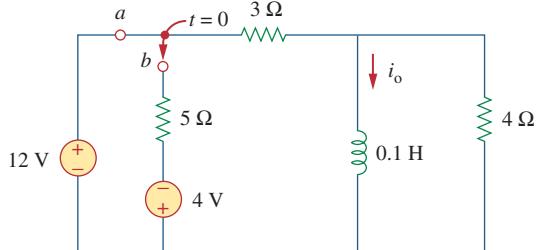
For Prob. 7.77.

- 7.78 The switch in Fig. 7.142 moves from position *a* to *b* at $t = 0$. Use PSpice or MultiSim to find $i(t)$ for $t > 0$.

**Figure 7.142**

For Prob. 7.78.

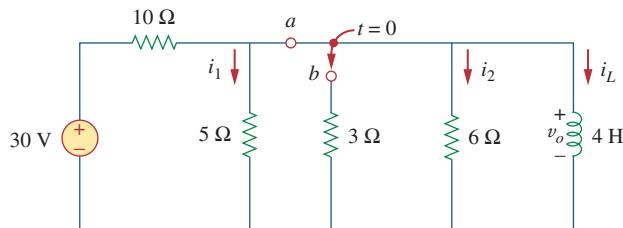
- 7.79 In the circuit of Fig. 7.143, the switch has been in position *a* for a long time but moves instantaneously to position *b* at $t = 0$. Determine $i_o(t)$.

**Figure 7.143**

For Prob. 7.79.

- 7.80 In the circuit of Fig. 7.144, assume that the switch has been in position *a* for a long time, find:

- $i_1(0)$, $i_2(0)$, and $v_o(0)$
- $i_L(t)$
- $i_1(\infty)$, $i_2(\infty)$, and $v_o(\infty)$.

**Figure 7.144**

For Prob. 7.80.

7.81 Repeat Prob. 7.65 using *PSpice* or *MultiSim*.

Section 7.9 Applications

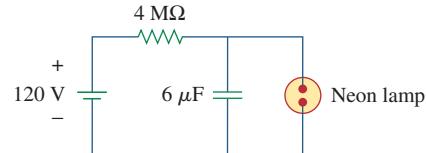
7.82 In designing a signal-switching circuit, it was found that a $100\text{-}\mu\text{F}$ capacitor was needed for a time constant of 3 ms. What value resistor is necessary for the circuit?

7.83 An *RC* circuit consists of a series connection of a **end** 120-V source, a switch, a $34\text{-M}\Omega$ resistor, and a $15\text{-}\mu\text{F}$ capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

7.84 The resistance of a 160-mH coil is $8\ \Omega$. Find the time required for the current to build up to 60 percent of its final value when voltage is applied to the coil.

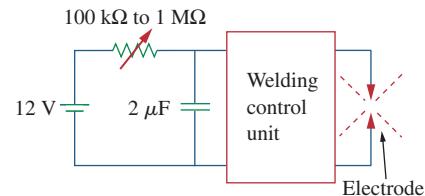
7.85 A simple relaxation oscillator circuit is shown in **end** Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is $120\ \Omega$ when on and infinitely high when off.

- For how long is the lamp on each time the capacitor discharges?
- What is the time interval between light flashes?

**Figure 7.145**

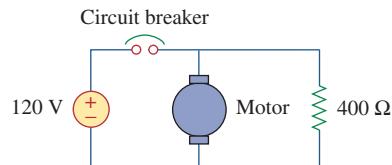
For Prob. 7.85.

7.86 Figure 7.146 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

**Figure 7.146**

For Prob. 7.86.

7.87 A 120-V dc generator energizes a motor whose coil has an inductance of $50\ \text{H}$ and a resistance of $100\ \Omega$. A field discharge resistor of $400\ \Omega$ is connected in parallel with the motor to avoid damage to the motor, as shown in Fig. 7.147. The system is at steady state. Find the current through the discharge resistor 100 ms after the breaker is tripped.

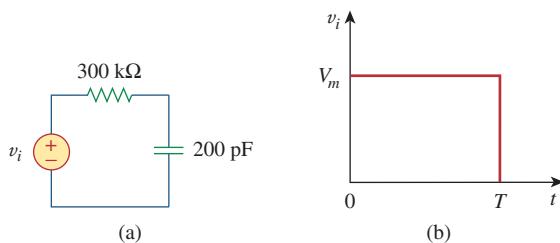
**Figure 7.147**

For Prob. 7.87.

Comprehensive Problems

7.88 The circuit in Fig. 7.148(a) can be designed as **end** an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant $\tau = RC$ of the circuit and the width T of the input pulse in Fig. 7.148(b). The circuit is a differentiator if $\tau \ll T$, say $\tau < 0.1T$, or an integrator if $\tau \gg T$, say $\tau > 10T$.

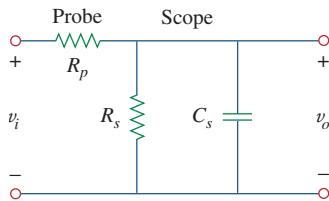
- What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?
- If the output is to be an integrated form of the input, what is the maximum value the pulse width can assume?

**Figure 7.148**

For Prob. 7.88.

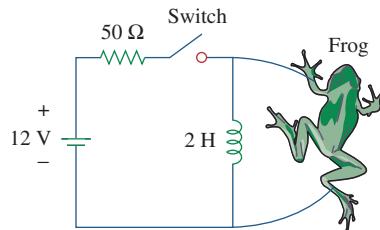
- 7.89** An RL circuit may be used as a differentiator if the output is taken across the inductor and $\tau \ll T$ (say $\tau < 0.1T$), where T is the width of the input pulse. If R is fixed at $200 \text{ k}\Omega$, determine the maximum value of L required to differentiate a pulse with $T = 10 \mu\text{s}$.

- 7.90** An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s , while the probe has an internal resistance R_p . If R_p is fixed at $6 \text{ M}\Omega$, find R_s and C_s for the circuit to have a time constant of $15 \mu\text{s}$.

**Figure 7.149**

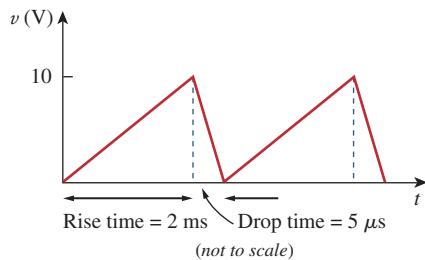
For Prob. 7.90.

- 7.91** The circuit in Fig. 7.150 is used by a biology student to study “frog kick.” She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.

**Figure 7.150**

For Prob. 7.91.

- 7.92** To move a spot of a cathode-ray tube across the screen requires a linear increase in the voltage across the deflection plates, as shown in Fig. 7.151. Given that the capacitance of the plates is 4 nF , sketch the current flowing through the plates.

**Figure 7.151**

For Prob. 7.92.

8

Second-Order Circuits

Everyone who can earn a masters degree in engineering must earn a masters degree in engineering in order to maximize the success of their career! If you want to do research, state-of-the-art engineering, teach in a university, or start your own business, you really need to earn a doctoral degree!

—Charles K. Alexander

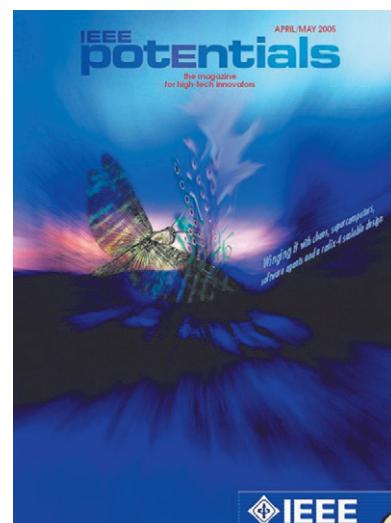
Enhancing Your Career

To increase your engineering career opportunities after graduation, develop a strong fundamental understanding in a broad set of engineering areas. When possible, this might best be accomplished by working toward a graduate degree immediately upon receiving your undergraduate degree.

Each degree in engineering represents certain skills the student acquires. At the Bachelor degree level, you learn the language of engineering and the fundamentals of engineering and design. At the Master's level, you acquire the ability to do advanced engineering projects and to communicate your work effectively both orally and in writing. The Ph.D. represents a thorough understanding of the fundamentals of electrical engineering and a mastery of the skills necessary both for working at the frontiers of an engineering area and for communicating one's effort to others.

If you have no idea what career you should pursue after graduation, a graduate degree program will enhance your ability to explore career options. Since your undergraduate degree will only provide you with the fundamentals of engineering, a Master's degree in engineering supplemented by business courses benefits more engineering students than does getting a Master's of Business Administration (MBA). The best time to get your MBA is after you have been a practicing engineer for some years and decide your career path would be enhanced by strengthening your business skills.

Engineers should constantly educate themselves, formally and informally, taking advantage of all means of education. Perhaps there is no better way to enhance your career than to join a professional society such as IEEE and be an active member.



Enhancing your career involves understanding your goals, adapting to changes, anticipating opportunities, and planning your own niche.

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8.1 Introduction

In the previous chapter we considered circuits with a single storage element (a capacitor or an inductor). Such circuits are first-order because the differential equations describing them are first-order. In this chapter we will consider circuits containing two storage elements. These are known as *second-order* circuits because their responses are described by differential equations that contain second derivatives.

Typical examples of second-order circuits are *RLC* circuits, in which the three kinds of passive elements are present. Examples of such circuits are shown in Fig. 8.1(a) and (b). Other examples are *RL* and *RC* circuits, as shown in Fig. 8.1(c) and (d). It is apparent from Fig. 8.1 that a second-order circuit may have two storage elements of different type or the same type (provided elements of the same type cannot be represented by an equivalent single element). An op amp circuit with two storage elements may also be a second-order circuit. As with first-order circuits, a second-order circuit may contain several resistors and dependent and independent sources.

A **second-order circuit** is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.

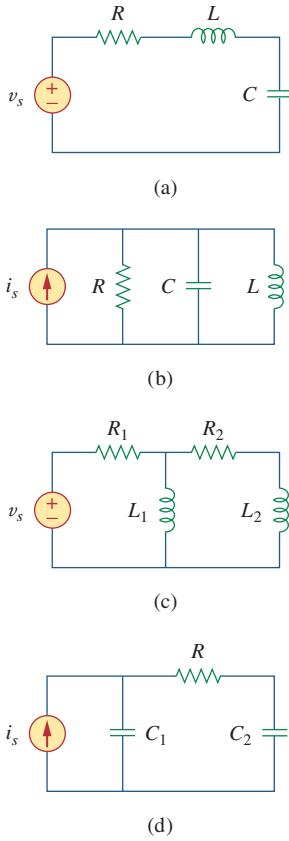


Figure 8.1

Typical examples of second-order circuits:
(a) series *RLC* circuit, (b) parallel *RLC* circuit, (c) *RL* circuit, (d) *RC* circuit.

Our analysis of second-order circuits will be similar to that used for first-order. We will first consider circuits that are excited by the initial conditions of the storage elements. Although these circuits may contain dependent sources, they are free of independent sources. These source-free circuits will give natural responses as expected. Later we will consider circuits that are excited by independent sources. These circuits will give both the transient response and the steady-state response. We consider only dc independent sources in this chapter. The case of sinusoidal and exponential sources is deferred to later chapters.

We begin by learning how to obtain the initial conditions for the circuit variables and their derivatives, as this is crucial to analyzing second-order circuits. Then we consider series and parallel *RLC* circuits such as shown in Fig. 8.1 for the two cases of excitation: by initial conditions of the energy storage elements and by step inputs. Later we examine other types of second-order circuits, including op amp circuits. We will consider *PSpice* analysis of second-order circuits. Finally, we will consider the automobile ignition system and smoothing circuits as typical applications of the circuits treated in this chapter. Other applications such as resonant circuits and filters will be covered in Chapter 14.

8.2 Finding Initial and Final Values

Perhaps the major problem students face in handling second-order circuits is finding the initial and final conditions on circuit variables. Students are usually comfortable getting the initial and final values of v and i but often have difficulty finding the initial values of their

derivatives: dv/dt and di/dt . For this reason, this section is explicitly devoted to the subtleties of getting $v(0)$, $i(0)$, $dv(0)/dt$, $di(0)/dt$, $i(\infty)$, and $v(\infty)$. Unless otherwise stated in this chapter, v denotes capacitor voltage, while i is the inductor current.

There are two key points to keep in mind in determining the initial conditions.

First—as always in circuit analysis—we must carefully handle the polarity of voltage $v(t)$ across the capacitor and the direction of the current $i(t)$ through the inductor. Keep in mind that v and i are defined strictly according to the passive sign convention (see Figs. 6.3 and 6.23). One should carefully observe how these are defined and apply them accordingly.

Second, keep in mind that the capacitor voltage is always continuous so that

$$v(0^+) = v(0^-) \quad (8.1a)$$

and the inductor current is always continuous so that

$$i(0^+) = i(0^-) \quad (8.1b)$$

where $t = 0^-$ denotes the time just before a switching event and $t = 0^+$ is the time just after the switching event, assuming that the switching event takes place at $t = 0$.

Thus, in finding initial conditions, we first focus on those variables that cannot change abruptly, capacitor voltage and inductor current, by applying Eq. (8.1). The following examples illustrate these ideas.

The switch in Fig. 8.2 has been closed for a long time. It is open at $t = 0$. Find: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)/dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.

Example 8.1

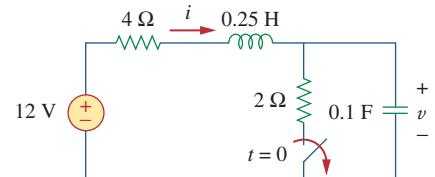


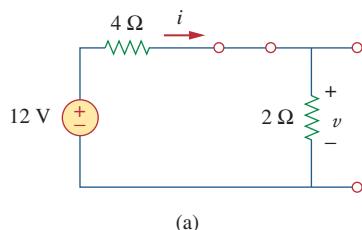
Figure 8.2

For Example 8.1.

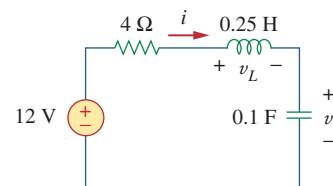
Solution:

(a) If the switch is closed a long time before $t = 0$, it means that the circuit has reached dc steady state at $t = 0$. At dc steady state, the inductor acts like a short circuit, while the capacitor acts like an open circuit, so we have the circuit in Fig. 8.3(a) at $t = 0^-$. Thus,

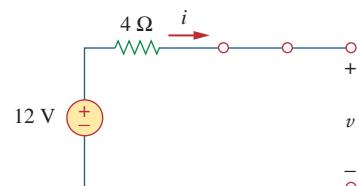
$$i(0^-) = \frac{12}{4 + 2} = 2 \text{ A}, \quad v(0^-) = 2i(0^-) = 4 \text{ V}$$



(a)



(b)



(c)

Figure 8.3

Equivalent circuit of that in Fig. 8.2 for: (a) $t = 0^-$, (b) $t = 0^+$, (c) $t \rightarrow \infty$.

As the inductor current and the capacitor voltage cannot change abruptly,

$$i(0^+) = i(0^-) = 2 \text{ A}, \quad v(0^+) = v(0^-) = 4 \text{ V}$$

(b) At $t = 0^+$, the switch is open; the equivalent circuit is as shown in Fig. 8.3(b). The same current flows through both the inductor and capacitor. Hence,

$$i_C(0^+) = i(0^+) = 2 \text{ A}$$

Since $C dv/dt = i_C$, $dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20 \text{ V/s}$$

Similarly, since $L di/dt = v_L$, $di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 8.3(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

or

$$v_L(0^+) = 12 - 8 - 4 = 0$$

Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0 \text{ A/s}$$

(c) For $t > 0$, the circuit undergoes transience. But as $t \rightarrow \infty$, the circuit reaches steady state again. The inductor acts like a short circuit and the capacitor like an open circuit, so that the circuit in Fig. 8.3(b) becomes that shown in Fig. 8.3(c), from which we have

$$i(\infty) = 0 \text{ A}, \quad v(\infty) = 12 \text{ V}$$

Practice Problem 8.1

The switch in Fig. 8.4 was open for a long time but closed at $t = 0$. Determine: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)/dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.

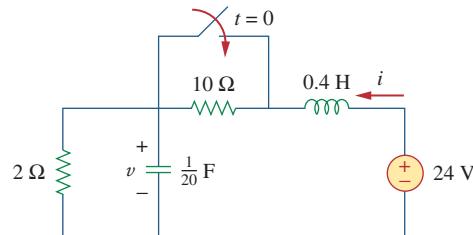


Figure 8.4

For Practice Prob. 8.1.

Answer: (a) 2 A, 4 V, (b) 50 A/s, 0 V/s, (c) 12 A, 24 V.

In the circuit of Fig. 8.5, calculate: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$, (c) $i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$.

Example 8.2

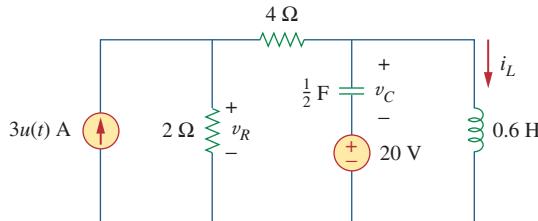


Figure 8.5
For Example 8.2.

Solution:

(a) For $t < 0$, $3u(t) = 0$. At $t = 0^-$, since the circuit has reached steady state, the inductor can be replaced by a short circuit, while the capacitor is replaced by an open circuit as shown in Fig. 8.6(a). From this figure we obtain

$$i_L(0^-) = 0, \quad v_R(0^-) = 0, \quad v_C(0^-) = -20 \text{ V} \quad (8.2.1)$$

Although the derivatives of these quantities at $t = 0^-$ are not required, it is evident that they are all zero, since the circuit has reached steady state and nothing changes.

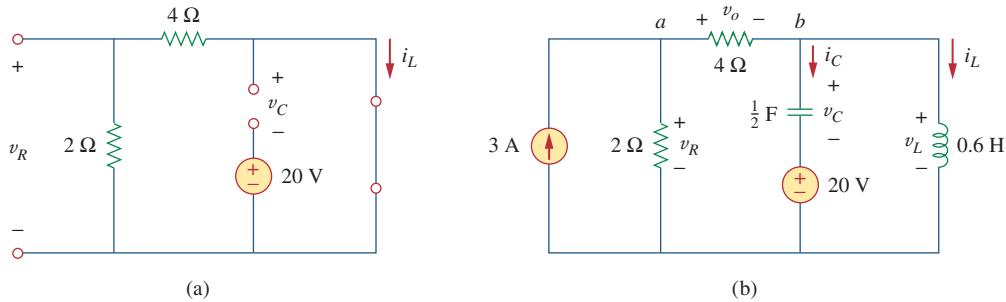


Figure 8.6
The circuit in Fig. 8.5 for: (a) $t = 0^-$, (b) $t = 0^+$.

For $t > 0$, $3u(t) = 3$, so that the circuit is now equivalent to that in Fig. 8.6(b). Since the inductor current and capacitor voltage cannot change abruptly,

$$i_L(0^+) = i_L(0^-) = 0, \quad v_C(0^+) = v_C(0^-) = -20 \text{ V} \quad (8.2.2)$$

Although the voltage across the 4-Ω resistor is not required, we will use it to apply KVL and KCL; let it be called v_o . Applying KCL at node a in Fig. 8.6(b) gives

$$3 = \frac{v_R(0^+)}{2} + \frac{v_o(0^+)}{4} \quad (8.2.3)$$

Applying KVL to the middle mesh in Fig. 8.6(b) yields

$$-v_R(0^+) + v_o(0^+) + v_C(0^+) + 20 = 0 \quad (8.2.4)$$

Since $v_C(0^+) = -20$ V from Eq. (8.2.2), Eq. (8.2.4) implies that

$$v_R(0^+) = v_o(0^+) \quad (8.2.5)$$

From Eqs. (8.2.3) and (8.2.5), we obtain

$$v_R(0^+) = v_o(0^+) = 4 \text{ V} \quad (8.2.6)$$

(b) Since $L \frac{di_L}{dt} = v_L$,

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

But applying KVL to the right mesh in Fig. 8.6(b) gives

$$v_L(0^+) = v_C(0^+) + 20 = 0$$

Hence,

$$\frac{di_L(0^+)}{dt} = 0 \quad (8.2.7)$$

Similarly, since $C \frac{dv_C}{dt} = i_C$, then $\frac{dv_C}{dt} = i_C/C$. We apply KCL at node b in Fig. 8.6(b) to get i_C :

$$\frac{v_o(0^+)}{4} = i_C(0^+) + i_L(0^+) \quad (8.2.8)$$

Since $v_o(0^+) = 4$ and $i_L(0^+) = 0$, $i_C(0^+) = 4/4 = 1$ A. Then

$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{1}{0.5} = 2 \text{ V/s} \quad (8.2.9)$$

To get $\frac{dv_R}{dt}(0^+)$, we apply KCL to node a and obtain

$$3 = \frac{v_R}{2} + \frac{v_o}{4}$$

Taking the derivative of each term and setting $t = 0^+$ gives

$$0 = 2 \frac{dv_R(0^+)}{dt} + \frac{dv_o(0^+)}{dt} \quad (8.2.10)$$

We also apply KVL to the middle mesh in Fig. 8.6(b) and obtain

$$-v_R + v_C + 20 + v_o = 0$$

Again, taking the derivative of each term and setting $t = 0^+$ yields

$$-\frac{dv_R(0^+)}{dt} + \frac{dv_C(0^+)}{dt} + \frac{dv_o(0^+)}{dt} = 0$$

Substituting for $\frac{dv_C}{dt}(0^+) = 2$ gives

$$\frac{dv_R(0^+)}{dt} = 2 + \frac{dv_o(0^+)}{dt} \quad (8.2.11)$$

From Eqs. (8.2.10) and (8.2.11), we get

$$\frac{dv_R(0^+)}{dt} = \frac{2}{3} \text{ V/s}$$

We can find $di_R(0^+)/dt$ although it is not required. Since $v_R = 5i_R$,

$$\frac{di_R(0^+)}{dt} = \frac{1}{5} \frac{dv_R(0^+)}{dt} = \frac{1}{5} \frac{2}{3} = \frac{2}{15} \text{ A/s}$$

(c) As $t \rightarrow \infty$, the circuit reaches steady state. We have the equivalent circuit in Fig. 8.6(a) except that the 3-A current source is now operative. By current division principle,

$$i_L(\infty) = \frac{2}{2+4} 3 \text{ A} = 1 \text{ A} \quad (8.2.12)$$

$$v_R(\infty) = \frac{4}{2+4} 3 \text{ A} \times 2 = 4 \text{ V}, \quad v_C(\infty) = -20 \text{ V}$$

For the circuit in Fig. 8.7, find: (a) $i_L(0^+), v_C(0^+), v_R(0^+)$, (b) $di_L(0^+)/dt, dv_C(0^+)/dt, dv_R(0^+)/dt$, (c) $i_L(\infty), v_C(\infty), v_R(\infty)$.

Practice Problem 8.2

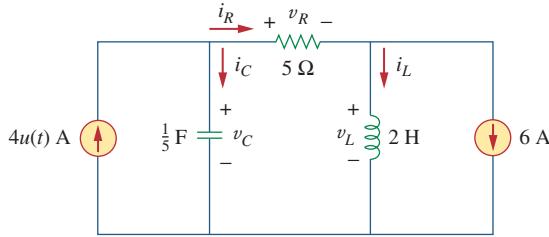


Figure 8.7
For Practice Prob. 8.2.

Answer: (a) $-6 \text{ A}, 0, 0$, (b) $0, 20 \text{ V/s}, 0$, (c) $-2 \text{ A}, 20 \text{ V}, 20 \text{ V}$.

8.3 The Source-Free Series RLC Circuit

An understanding of the natural response of the series RLC circuit is a necessary background for future studies in filter design and communications networks.

Consider the series RLC circuit shown in Fig. 8.8. The circuit is being excited by the energy initially stored in the capacitor and inductor. The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0 . Thus, at $t = 0$,

$$v(0) = \frac{1}{C} \int_{-\infty}^0 i \, dt = V_0 \quad (8.2a)$$

$$i(0) = I_0 \quad (8.2b)$$

Applying KVL around the loop in Fig. 8.8,

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = 0 \quad (8.3)$$

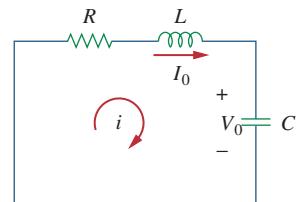


Figure 8.8
A source-free series RLC circuit.

To eliminate the integral, we differentiate with respect to t and rearrange terms. We get

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0 \quad (8.4)$$

This is a *second-order differential equation* and is the reason for calling the *RLC* circuits in this chapter second-order circuits. Our goal is to solve Eq. (8.4). To solve such a second-order differential equation requires that we have two initial conditions, such as the initial value of i and its first derivative or initial values of some i and v . The initial value of i is given in Eq. (8.2b). We get the initial value of the derivative of i from Eqs. (8.2a) and (8.3); that is,

$$Ri(0) + L \frac{di(0)}{dt} + V_0 = 0$$

or

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) \quad (8.5)$$

With the two initial conditions in Eqs. (8.2b) and (8.5), we can now solve Eq. (8.4). Our experience in the preceding chapter on first-order circuits suggests that the solution is of exponential form. So we let

$$i = Ae^{st} \quad (8.6)$$

where A and s are constants to be determined. Substituting Eq. (8.6) into Eq. (8.4) and carrying out the necessary differentiations, we obtain

$$As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0$$

or

$$Ae^{st} \left(s^2 + \frac{R}{L}s + \frac{1}{LC} \right) = 0 \quad (8.7)$$

Since $i = Ae^{st}$ is the assumed solution we are trying to find, only the expression in parentheses can be zero:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \quad (8.8)$$

This quadratic equation is known as the *characteristic equation* of the differential Eq. (8.4), since the roots of the equation dictate the character of i . The two roots of Eq. (8.8) are

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad (8.9a)$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad (8.9b)$$

A more compact way of expressing the roots is

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \quad (8.10)$$

See Appendix C.1 for the formula to find the roots of a quadratic equation.

where

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad (8.11)$$

The roots s_1 and s_2 are called *natural frequencies*, measured in nepers per second (Np/s), because they are associated with the natural response of the circuit; ω_0 is known as the *resonant frequency* or strictly as the *undamped natural frequency*, expressed in radians per second (rad/s); and α is the *neper frequency* or the *damping factor*, expressed in nepers per second. In terms of α and ω_0 , Eq. (8.8) can be written as

$$s^2 + 2\alpha s + \omega_0^2 = 0 \quad (8.8a)$$

The variables s and ω_0 are important quantities we will be discussing throughout the rest of the text.

The two values of s in Eq. (8.10) indicate that there are two possible solutions for i , each of which is of the form of the assumed solution in Eq. (8.6); that is,

$$i_1 = A_1 e^{s_1 t}, \quad i_2 = A_2 e^{s_2 t} \quad (8.12)$$

Since Eq. (8.4) is a linear equation, any linear combination of the two distinct solutions i_1 and i_2 is also a solution of Eq. (8.4). A complete or total solution of Eq. (8.4) would therefore require a linear combination of i_1 and i_2 . Thus, the natural response of the series RLC circuit is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (8.13)$$

where the constants A_1 and A_2 are determined from the initial values $i(0)$ and $di(0)/dt$ in Eqs. (8.2b) and (8.5).

From Eq. (8.10), we can infer that there are three types of solutions:

1. If $\alpha > \omega_0$, we have the *overdamped* case.
2. If $\alpha = \omega_0$, we have the *critically damped* case.
3. If $\alpha < \omega_0$, we have the *underdamped* case.

We will consider each of these cases separately.

Overdamped Case ($\alpha > \omega_0$)

From Eqs. (8.9) and (8.10), $\alpha > \omega_0$ implies $C > 4L/R^2$. When this happens, both roots s_1 and s_2 are negative and real. The response is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (8.14)$$

which decays and approaches zero as t increases. Figure 8.9(a) illustrates a typical overdamped response.

Critically Damped Case ($\alpha = \omega_0$)

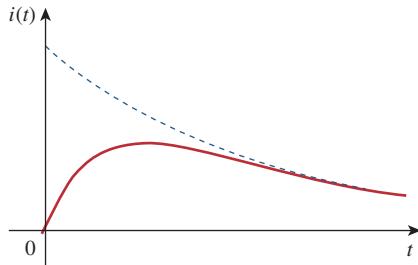
When $\alpha = \omega_0$, $C = 4L/R^2$ and

$$s_1 = s_2 = -\alpha = -\frac{R}{2L} \quad (8.15)$$

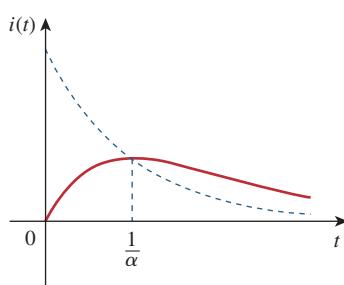
The *neper* (Np) is a dimensionless unit named after John Napier (1550–1617), a Scottish mathematician.

The ratio α/ω_0 is known as the *damping ratio* ζ .

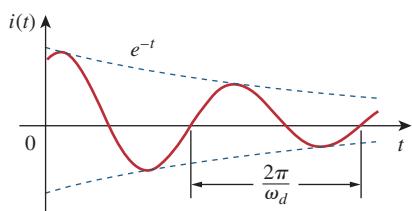
The response is *overdamped* when the roots of the circuit's characteristic equation are unequal and real, *critically damped* when the roots are equal and real, and *underdamped* when the roots are complex.



(a)



(b)



(c)

Figure 8.9

(a) Overdamped response, (b) critically damped response, (c) underdamped response.

For this case, Eq. (8.13) yields

$$i(t) = A_1 e^{-\alpha t} + A_2 e^{-\alpha t} = A_3 e^{-\alpha t}$$

where $A_3 = A_1 + A_2$. This cannot be the solution, because the two initial conditions cannot be satisfied with the single constant A_3 . What then could be wrong? Our assumption of an exponential solution is incorrect for the special case of critical damping. Let us go back to Eq. (8.4). When $\alpha = \omega_0 = R/2L$, Eq. (8.4) becomes

$$\frac{d^2i}{dt^2} + 2\alpha \frac{di}{dt} + \alpha^2 i = 0$$

or

$$\frac{d}{dt} \left(\frac{di}{dt} + \alpha i \right) + \alpha \left(\frac{di}{dt} + \alpha i \right) = 0 \quad (8.16)$$

If we let

$$f = \frac{di}{dt} + \alpha i \quad (8.17)$$

then Eq. (8.16) becomes

$$\frac{df}{dt} + \alpha f = 0$$

which is a first-order differential equation with solution $f = A_1 e^{-\alpha t}$, where A_1 is a constant. Equation (8.17) then becomes

$$\frac{di}{dt} + \alpha i = A_1 e^{-\alpha t}$$

or

$$e^{\alpha t} \frac{di}{dt} + e^{\alpha t} \alpha i = A_1 \quad (8.18)$$

This can be written as

$$\frac{d}{dt}(e^{\alpha t} i) = A_1 \quad (8.19)$$

Integrating both sides yields

$$e^{\alpha t} i = A_1 t + A_2$$

or

$$i = (A_1 t + A_2) e^{-\alpha t} \quad (8.20)$$

where A_2 is another constant. Hence, the natural response of the critically damped circuit is a sum of two terms: a negative exponential and a negative exponential multiplied by a linear term, or

$$i(t) = (A_2 + A_1 t) e^{-\alpha t} \quad (8.21)$$

A typical critically damped response is shown in Fig. 8.9(b). In fact, Fig. 8.9(b) is a sketch of $i(t) = te^{-\alpha t}$, which reaches a maximum value of e^{-1}/α at $t = 1/\alpha$, one time constant, and then decays all the way to zero.

Underdamped Case ($\alpha < \omega_0$)

For $\alpha < \omega_0$, $C < 4L/R^2$. The roots may be written as

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d \quad (8.22a)$$

$$s_2 = -\alpha - \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha - j\omega_d \quad (8.22b)$$

where $j = \sqrt{-1}$ and $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, which is called the *damping frequency*. Both ω_0 and ω_d are natural frequencies because they help determine the natural response; while ω_0 is often called the *undamped natural frequency*, ω_d is called the *damped natural frequency*. The natural response is

$$\begin{aligned} i(t) &= A_1 e^{-(\alpha - j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t} \\ &= e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) \end{aligned} \quad (8.23)$$

Using Euler's identities,

$$e^{j\theta} = \cos \theta + j \sin \theta, \quad e^{-j\theta} = \cos \theta - j \sin \theta \quad (8.24)$$

we get

$$\begin{aligned} i(t) &= e^{-\alpha t}[A_1(\cos \omega_d t + j \sin \omega_d t) + A_2(\cos \omega_d t - j \sin \omega_d t)] \\ &= e^{-\alpha t}[(A_1 + A_2) \cos \omega_d t + j(A_1 - A_2) \sin \omega_d t] \end{aligned} \quad (8.25)$$

Replacing constants $(A_1 + A_2)$ and $j(A_1 - A_2)$ with constants B_1 and B_2 , we write

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad (8.26)$$

With the presence of sine and cosine functions, it is clear that the natural response for this case is exponentially damped and oscillatory in nature. The response has a time constant of $1/\alpha$ and a period of $T = 2\pi/\omega_d$. Figure 8.9(c) depicts a typical underdamped response. [Figure 8.9 assumes for each case that $i(0) = 0$.]

Once the inductor current $i(t)$ is found for the *RLC* series circuit as shown above, other circuit quantities such as individual element voltages can easily be found. For example, the resistor voltage is $v_R = Ri$, and the inductor voltage is $v_L = L di/dt$. The inductor current $i(t)$ is selected as the key variable to be determined first in order to take advantage of Eq. (8.1b).

We conclude this section by noting the following interesting, peculiar properties of an *RLC* network:

1. The behavior of such a network is captured by the idea of *damping*, which is the gradual loss of the initial stored energy, as evidenced by the continuous decrease in the amplitude of the response. The damping effect is due to the presence of resistance R . The damping factor α determines the rate at which the response is damped. If $R = 0$, then $\alpha = 0$, and we have an *LC* circuit with $1/\sqrt{LC}$ as the undamped natural frequency. Since $\alpha < \omega_0$ in this case, the response is not only undamped but also oscillatory. The circuit is said to be *loss-less*, because the dissipating or damping element (R) is absent. By adjusting the value of R , the response may be made undamped, over-damped, critically damped, or under-damped.
2. Oscillatory response is possible due to the presence of the two types of storage elements. Having both L and C allows the flow of

$R = 0$ produces a perfectly sinusoidal response. This response cannot be practically accomplished with L and C because of the inherent losses in them. See Figs. 6.8 and 6.26. An electronic device called an *oscillator* can produce a perfectly sinusoidal response.

Examples 8.5 and 8.7 demonstrate the effect of varying R .

The response of a second-order circuit with two storage elements of the same type, as in Fig. 8.1(c) and (d), cannot be oscillatory.

What this means in most practical circuits is that we seek an overdamped circuit that is as close as possible to a critically damped circuit.

energy back and forth between the two. The damped oscillation exhibited by the underdamped response is known as *ringing*. It stems from the ability of the storage elements L and C to transfer energy back and forth between them.

3. Observe from Fig. 8.9 that the waveforms of the responses differ.

In general, it is difficult to tell from the waveforms the difference between the overdamped and critically damped responses. The critically damped case is the borderline between the underdamped and overdamped cases and it decays the fastest. With the same initial conditions, the overdamped case has the longest settling time, because it takes the longest time to dissipate the initial stored energy. If we desire the response that approaches the final value most rapidly without oscillation or ringing, the critically damped circuit is the right choice.

Example 8.3

In Fig. 8.8, $R = 40 \Omega$, $L = 4 \text{ H}$, and $C = 1/4 \text{ F}$. Calculate the characteristic roots of the circuit. Is the natural response overdamped, underdamped, or critically damped?

Solution:

We first calculate

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times \frac{1}{4}}} = 1$$

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

or

$$s_1 = -0.101, \quad s_2 = -9.899$$

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

Practice Problem 8.3

If $R = 10 \Omega$, $L = 5 \text{ H}$, and $C = 2 \text{ mF}$ in Fig. 8.8, find α , ω_0 , s_1 , and s_2 . What type of natural response will the circuit have?

Answer: $1, 10, -1 \pm j9.95$, underdamped.

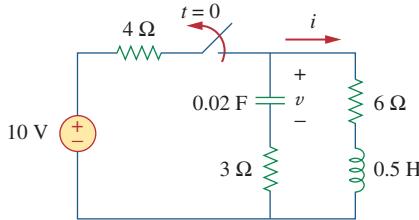
Example 8.4

Find $i(t)$ in the circuit of Fig. 8.10. Assume that the circuit has reached steady state at $t = 0^-$.

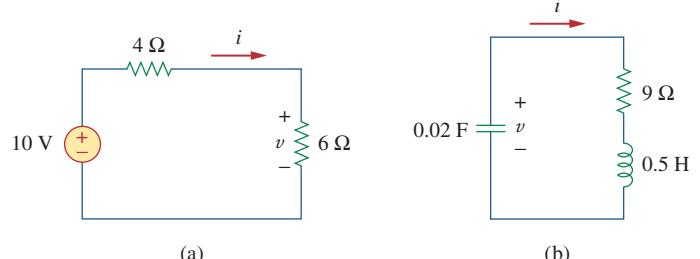
Solution:

For $t < 0$, the switch is closed. The capacitor acts like an open circuit while the inductor acts like a shunted circuit. The equivalent circuit is shown in Fig. 8.11(a). Thus, at $t = 0$,

$$i(0) = \frac{10}{4 + 6} = 1 \text{ A}, \quad v(0) = 6i(0) = 6 \text{ V}$$

**Figure 8.10**

For Example 8.4.

**Figure 8.11**The circuit in Fig. 8.10: (a) for $t < 0$, (b) for $t > 0$.

where $i(0)$ is the initial current through the inductor and $v(0)$ is the initial voltage across the capacitor.

For $t > 0$, the switch is opened and the voltage source is disconnected. The equivalent circuit is shown in Fig. 8.11(b), which is a source-free series RLC circuit. Notice that the 3- Ω and 6- Ω resistors, which are in series in Fig. 8.10 when the switch is opened, have been combined to give $R = 9 \Omega$ in Fig. 8.11(b). The roots are calculated as follows:

$$\alpha = \frac{R}{2L} = \frac{9}{2(\frac{1}{2})} = 9, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times \frac{1}{50}}} = 10$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100}$$

or

$$s_{1,2} = -9 \pm j4.359$$

Hence, the response is underdamped ($\alpha < \omega$); that is,

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) \quad (8.4.1)$$

We now obtain A_1 and A_2 using the initial conditions. At $t = 0$,

$$i(0) = 1 = A_1 \quad (8.4.2)$$

From Eq. (8.5),

$$\left. \frac{di}{dt} \right|_{t=0} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9(1) - 6] = -6 \text{ A/s} \quad (8.4.3)$$

Note that $v(0) = V_0 = -6 \text{ V}$ is used, because the polarity of v in Fig. 8.11(b) is opposite that in Fig. 8.8. Taking the derivative of $i(t)$ in Eq. (8.4.1),

$$\begin{aligned} \frac{di}{dt} &= -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) \\ &\quad + e^{-9t}(4.359)(-A_1 \sin 4.359t + A_2 \cos 4.359t) \end{aligned}$$

Imposing the condition in Eq. (8.4.3) at $t = 0$ gives

$$-6 = -9(A_1 + 0) + 4.359(-0 + A_2)$$

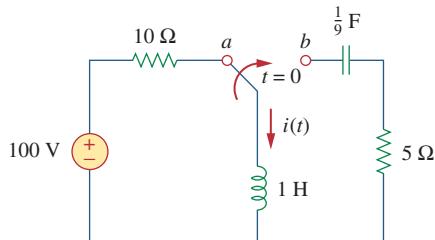
But $A_1 = 1$ from Eq. (8.4.2). Then

$$-6 = -9 + 4.359A_2 \quad \Rightarrow \quad A_2 = 0.6882$$

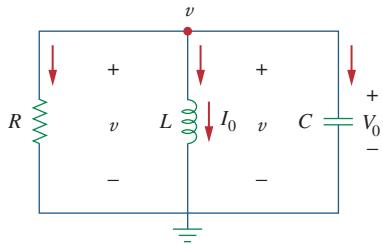
Substituting the values of A_1 and A_2 in Eq. (8.4.1) yields the complete solution as

$$i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t) \text{ A}$$

Practice Problem 8.4

**Figure 8.12**

For Practice Prob. 8.4.

**Figure 8.13**

A source-free parallel RLC circuit.

The circuit in Fig. 8.12 has reached steady state at $t = 0^-$. If the make-before-break switch moves to position *b* at $t = 0$, calculate $i(t)$ for $t > 0$.

Answer: $e^{-2.5t}(10 \cos 1.6583t - 15.076 \sin 1.6583t)$ A.

8.4 The Source-Free Parallel RLC Circuit

Parallel RLC circuits find many practical applications, notably in communications networks and filter designs.

Consider the parallel RLC circuit shown in Fig. 8.13. Assume initial inductor current I_0 and initial capacitor voltage V_0 ,

$$i(0) = I_0 = \frac{1}{L} \int_{-\infty}^0 v(t) dt \quad (8.27a)$$

$$v(0) = V_0 \quad (8.27b)$$

Since the three elements are in parallel, they have the same voltage v across them. According to passive sign convention, the current is entering each element; that is, the current through each element is leaving the top node. Thus, applying KCL at the top node gives

$$\frac{v}{R} + \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau + C \frac{dv}{dt} = 0 \quad (8.28)$$

Taking the derivative with respect to t and dividing by C results in

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0 \quad (8.29)$$

We obtain the characteristic equation by replacing the first derivative by s and the second derivative by s^2 . By following the same reasoning used in establishing Eqs. (8.4) through (8.8), the characteristic equation is obtained as

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad (8.30)$$

The roots of the characteristic equation are

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

or

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad (8.31)$$

where

$$\alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad (8.32)$$

The names of these terms remain the same as in the preceding section, as they play the same role in the solution. Again, there are three possible solutions, depending on whether $\alpha > \omega_0$, $\alpha = \omega_0$, or $\alpha < \omega_0$. Let us consider these cases separately.

Overdamped Case ($\alpha > \omega_0$)

From Eq. (8.32), $\alpha > \omega_0$ when $L > 4R^2C$. The roots of the characteristic equation are real and negative. The response is

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (8.33)$$

Critically Damped Case ($\alpha = \omega_0$)

For $\alpha = \omega_0$, $L = 4R^2C$. The roots are real and equal so that the response is

$$v(t) = (A_1 + A_2 t)e^{-\alpha t} \quad (8.34)$$

Underdamped Case ($\alpha < \omega_0$)

When $\alpha < \omega_0$, $L < 4R^2C$. In this case the roots are complex and may be expressed as

$$s_{1,2} = -\alpha \pm j\omega_d \quad (8.35)$$

where

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \quad (8.36)$$

The response is

$$v(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad (8.37)$$

The constants A_1 and A_2 in each case can be determined from the initial conditions. We need $v(0)$ and $dv(0)/dt$. The first term is known from Eq. (8.27b). We find the second term by combining Eqs. (8.27) and (8.28), as

$$\frac{V_0}{R} + I_0 + C \frac{dv(0)}{dt} = 0$$

or

$$\frac{dv(0)}{dt} = -\frac{(V_0 + RI_0)}{RC} \quad (8.38)$$

The voltage waveforms are similar to those shown in Fig. 8.9 and will depend on whether the circuit is overdamped, underdamped, or critically damped.

Having found the capacitor voltage $v(t)$ for the parallel RLC circuit as shown above, we can readily obtain other circuit quantities such as individual element currents. For example, the resistor current is $i_R = v/R$ and the capacitor voltage is $v_C = C dv/dt$. We have selected the capacitor voltage $v(t)$ as the key variable to be determined first in order to take advantage of Eq. (8.1a). Notice that we first found the inductor current $i(t)$ for the RLC series circuit, whereas we first found the capacitor voltage $v(t)$ for the parallel RLC circuit.

Example 8.5

In the parallel circuit of Fig. 8.13, find $v(t)$ for $t > 0$, assuming $v(0) = 5$ V, $i(0) = 0$, $L = 1$ H, and $C = 10$ mF. Consider these cases: $R = 1.923 \Omega$, $R = 5 \Omega$, and $R = 6.25 \Omega$.

Solution:**■ CASE 1** If $R = 1.923 \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 1.923 \times 10 \times 10^{-3}} = 26$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10 \times 10^{-3}}} = 10$$

Since $\alpha > \omega_0$ in this case, the response is overdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2, -50$$

and the corresponding response is

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t} \quad (8.5.1)$$

We now apply the initial conditions to get A_1 and A_2 .

$$v(0) = 5 = A_1 + A_2 \quad (8.5.2)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{1.923 \times 10 \times 10^{-3}} = -260$$

But differentiating Eq. (8.5.1),

$$\frac{dv}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

At $t = 0$,

$$-260 = -2A_1 - 50A_2 \quad (8.5.3)$$

From Eqs. (8.5.2) and (8.5.3), we obtain $A_1 = -0.2083$ and $A_2 = 5.208$. Substituting A_1 and A_2 in Eq. (8.5.1) yields

$$v(t) = -0.2083e^{-2t} + 5.208e^{-50t} \quad (8.5.4)$$

■ CASE 2 When $R = 5 \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 5 \times 10 \times 10^{-3}} = 10$$

while $\omega_0 = 10$ remains the same. Since $\alpha = \omega_0 = 10$, the response is critically damped. Hence, $s_1 = s_2 = -10$, and

$$v(t) = (A_1 + A_2 t)e^{-10t} \quad (8.5.5)$$

To get A_1 and A_2 , we apply the initial conditions

$$v(0) = 5 = A_1 \quad (8.5.6)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{5 \times 10 \times 10^{-3}} = -100$$

But differentiating Eq. (8.5.5),

$$\frac{dv}{dt} = (-10A_1 - 10A_2 t + A_2)e^{-10t}$$

At $t = 0$,

$$-100 = -10A_1 + A_2 \quad (8.5.7)$$

From Eqs. (8.5.6) and (8.5.7), $A_1 = 5$ and $A_2 = -50$. Thus,

$$v(t) = (5 - 50t)e^{-10t} \text{ V} \quad (8.5.8)$$

CASE 3 When $R = 6.25 \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 6.25 \times 10 \times 10^{-3}} = 8$$

while $\omega_0 = 10$ remains the same. As $\alpha < \omega_0$ in this case, the response is underdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8 \pm j6$$

Hence,

$$v(t) = (A_1 \cos 6t + A_2 \sin 6t)e^{-8t} \quad (8.5.9)$$

We now obtain A_1 and A_2 , as

$$v(0) = 5 = A_1 \quad (8.5.10)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{6.25 \times 10 \times 10^{-3}} = -80$$

But differentiating Eq. (8.5.9),

$$\frac{dv}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$$

At $t = 0$,

$$-80 = -8A_1 + 6A_2 \quad (8.5.11)$$

From Eqs. (8.5.10) and (8.5.11), $A_1 = 5$ and $A_2 = -6.667$. Thus,

$$v(t) = (5 \cos 6t - 6.667 \sin 6t)e^{-8t} \quad (8.5.12)$$

Notice that by increasing the value of R , the degree of damping decreases and the responses differ. Figure 8.14 plots the three cases.

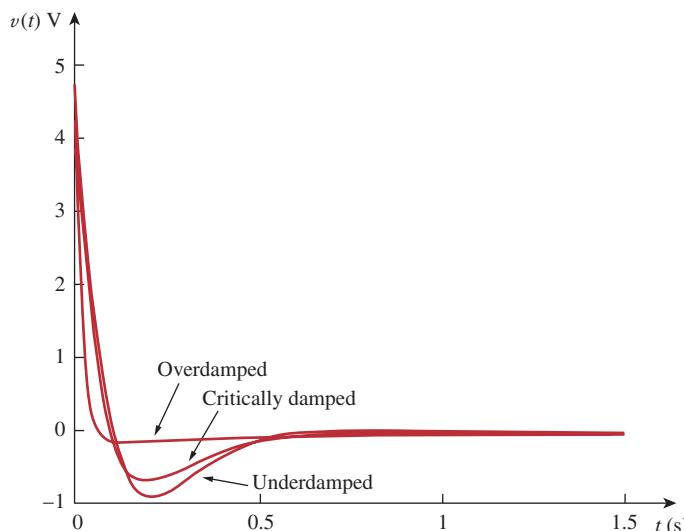


Figure 8.14

For Example 8.5: responses for three degrees of damping.

Practice Problem 8.5

In Fig. 8.13, let $R = 2 \Omega$, $L = 0.4 \text{ H}$, $C = 25 \text{ mF}$, $v(0) = 0$, $i(0) = 50 \text{ mA}$. Find $v(t)$ for $t > 0$.

Answer: $-2te^{-10t} u(t) \text{ V}$.

Example 8.6

Find $v(t)$ for $t > 0$ in the RLC circuit of Fig. 8.15.

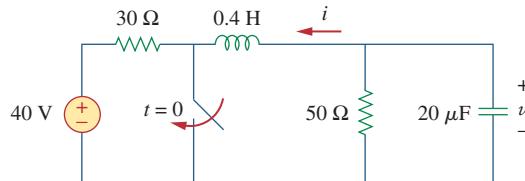


Figure 8.15

For Example 8.6.

Solution:

When $t < 0$, the switch is open; the inductor acts like a short circuit while the capacitor behaves like an open circuit. The initial voltage across the capacitor is the same as the voltage across the $50\text{-}\Omega$ resistor; that is,

$$v(0) = \frac{50}{30 + 50}(40) = \frac{5}{8} \times 40 = 25 \text{ V} \quad (8.6.1)$$

The initial current through the inductor is

$$i(0) = -\frac{40}{30 + 50} = -0.5 \text{ A}$$

The direction of i is as indicated in Fig. 8.15 to conform with the direction of I_0 in Fig. 8.13, which is in agreement with the convention that current flows into the positive terminal of an inductor (see Fig. 6.23). We need to express this in terms of dv/dt , since we are looking for v .

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{25 - 50 \times 0.5}{50 \times 20 \times 10^{-6}} = 0 \quad (8.6.2)$$

When $t > 0$, the switch is closed. The voltage source along with the $30\text{-}\Omega$ resistor is separated from the rest of the circuit. The parallel RLC circuit acts independently of the voltage source, as illustrated in Fig. 8.16. Next, we determine that the roots of the characteristic equation are

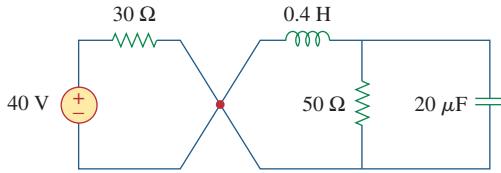
$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 50 \times 20 \times 10^{-6}} = 500$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 20 \times 10^{-6}}} = 354$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \\ = -500 \pm \sqrt{250,000 - 124,997.6} = -500 \pm 354$$

or

$$s_1 = -854, \quad s_2 = -146$$

**Figure 8.16**

The circuit in Fig. 8.15 when $t > 0$. The parallel RLC circuit on the right-hand side acts independently of the circuit on the left-hand side of the junction.

Since $\alpha > \omega_0$, we have the overdamped response

$$v(t) = A_1 e^{-854t} + A_2 e^{-146t} \quad (8.6.3)$$

At $t = 0$, we impose the condition in Eq. (8.6.1),

$$v(0) = 25 = A_1 + A_2 \Rightarrow A_2 = 25 - A_1 \quad (8.6.4)$$

Taking the derivative of $v(t)$ in Eq. (8.6.3),

$$\frac{dv}{dt} = -854A_1 e^{-854t} - 146A_2 e^{-146t}$$

Imposing the condition in Eq. (8.6.2),

$$\frac{dv(0)}{dt} = 0 = -854A_1 - 146A_2$$

or

$$0 = 854A_1 + 146A_2 \quad (8.6.5)$$

Solving Eqs. (8.6.4) and (8.6.5) gives

$$A_1 = -5.156, \quad A_2 = 30.16$$

Thus, the complete solution in Eq. (8.6.3) becomes

$$v(t) = -5.156e^{-854t} + 30.16e^{-146t} \text{ V}$$

Refer to the circuit in Fig. 8.17. Find $v(t)$ for $t > 0$.

Practice Problem 8.6

Answer: $150(e^{-10t} - e^{-2.5t}) \text{ V}$.

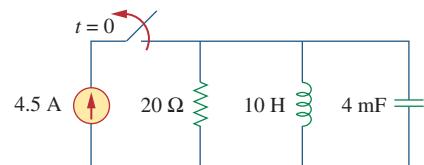
8.5 Step Response of a Series RLC Circuit

As we learned in the preceding chapter, the step response is obtained by the sudden application of a dc source. Consider the series RLC circuit shown in Fig. 8.18. Applying KVL around the loop for $t > 0$,

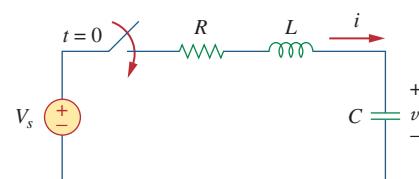
$$L \frac{di}{dt} + Ri + v = V_s \quad (8.39)$$

But

$$i = C \frac{dv}{dt}$$

**Figure 8.17**

For Practice Prob. 8.6.

**Figure 8.18**

Step voltage applied to a series RLC circuit.

Substituting for i in Eq. (8.39) and rearranging terms,

$$\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC} \quad (8.40)$$

which has the same form as Eq. (8.4). More specifically, the coefficients are the same (and that is important in determining the frequency parameters) but the variable is different. (Likewise, see Eq. (8.47).) Hence, the characteristic equation for the series RLC circuit is not affected by the presence of the dc source.

The solution to Eq. (8.40) has two components: the transient response $v_t(t)$ and the steady-state response $v_{ss}(t)$; that is,

$$v(t) = v_t(t) + v_{ss}(t) \quad (8.41)$$

The transient response $v_t(t)$ is the component of the total response that dies out with time. The form of the transient response is the same as the form of the solution obtained in Section 8.3 for the source-free circuit, given by Eqs. (8.14), (8.21), and (8.26). Therefore, the transient response $v_t(t)$ for the overdamped, underdamped, and critically damped cases are:

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (\text{Overdamped}) \quad (8.42a)$$

$$v_t(t) = (A_1 + A_2 t)e^{-\alpha t} \quad (\text{Critically damped}) \quad (8.42b)$$

$$v_t(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \quad (\text{Underdamped}) \quad (8.42c)$$

The steady-state response is the final value of $v(t)$. In the circuit in Fig. 8.18, the final value of the capacitor voltage is the same as the source voltage V_s . Hence,

$$v_{ss}(t) = v(\infty) = V_s \quad (8.43)$$

Thus, the complete solutions for the overdamped, underdamped, and critically damped cases are:

$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (\text{Overdamped}) \quad (8.44a)$$

$$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \quad (\text{Critically damped}) \quad (8.44b)$$

$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \quad (\text{Underdamped}) \quad (8.44c)$$

The values of the constants A_1 and A_2 are obtained from the initial conditions: $v(0)$ and $dv(0)/dt$. Keep in mind that v and i are, respectively, the voltage across the capacitor and the current through the inductor. Therefore, Eq. (8.44) only applies for finding v . But once the capacitor voltage $v_C = v$ is known, we can determine $i = C dv/dt$, which is the same current through the capacitor, inductor, and resistor. Hence, the voltage across the resistor is $v_R = iR$, while the inductor voltage is $v_L = L di/dt$.

Alternatively, the complete response for any variable $x(t)$ can be found directly, because it has the general form

$$x(t) = x_{ss}(t) + x_t(t) \quad (8.45)$$

where the $x_{ss} = x(\infty)$ is the final value and $x_t(t)$ is the transient response. The final value is found as in Section 8.2. The transient response has the same form as in Eq. (8.42), and the associated constants are determined from Eq. (8.44) based on the values of $x(0)$ and $dx(0)/dt$.

For the circuit in Fig. 8.19, find $v(t)$ and $i(t)$ for $t > 0$. Consider these cases: $R = 5 \Omega$, $R = 4 \Omega$, and $R = 1 \Omega$.

Example 8.7

Solution:

CASE 1 When $R = 5 \Omega$. For $t < 0$, the switch is closed for a long time. The capacitor behaves like an open circuit while the inductor acts like a short circuit. The initial current through the inductor is

$$i(0) = \frac{24}{5 + 1} = 4 \text{ A}$$

and the initial voltage across the capacitor is the same as the voltage across the 1Ω resistor; that is,

$$v(0) = 1i(0) = 4 \text{ V}$$

For $t > 0$, the switch is opened, so that we have the 1Ω resistor disconnected. What remains is the series RLC circuit with the voltage source. The characteristic roots are determined as follows:

$$\alpha = \frac{R}{2L} = \frac{5}{2 \times 1} = 2.5, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.25}} = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1, -4$$

Since $\alpha > \omega_0$, we have the overdamped natural response. The total response is therefore

$$v(t) = v_{ss} + (A_1 e^{-t} + A_2 e^{-4t})$$

where v_{ss} is the steady-state response. It is the final value of the capacitor voltage. In Fig. 8.19, $v_f = 24 \text{ V}$. Thus,

$$v(t) = 24 + (A_1 e^{-t} + A_2 e^{-4t}) \quad (8.7.1)$$

We now need to find A_1 and A_2 using the initial conditions.

$$v(0) = 4 = 24 + A_1 + A_2$$

or

$$-20 = A_1 + A_2 \quad (8.7.2)$$

The current through the inductor cannot change abruptly and is the same current through the capacitor at $t = 0^+$ because the inductor and capacitor are now in series. Hence,

$$i(0) = C \frac{dv(0)}{dt} = 4 \quad \Rightarrow \quad \frac{dv(0)}{dt} = \frac{4}{C} = \frac{4}{0.25} = 16$$

Before we use this condition, we need to take the derivative of v in Eq. (8.7.1).

$$\frac{dv}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t} \quad (8.7.3)$$

At $t = 0$,

$$\frac{dv(0)}{dt} = 16 = -A_1 - 4A_2 \quad (8.7.4)$$

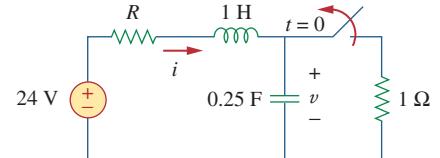


Figure 8.19
For Example 8.7.

From Eqs. (8.7.2) and (8.7.4), $A_1 = -64/3$ and $A_2 = 4/3$. Substituting A_1 and A_2 in Eq. (8.7.1), we get

$$v(t) = 24 + \frac{4}{3}(-16e^{-t} + e^{-4t}) \text{ V} \quad (8.7.5)$$

Since the inductor and capacitor are in series for $t > 0$, the inductor current is the same as the capacitor current. Hence,

$$i(t) = C \frac{dv}{dt}$$

Multiplying Eq. (8.7.3) by $C = 0.25$ and substituting the values of A_1 and A_2 gives

$$i(t) = \frac{4}{3}(4e^{-t} - e^{-4t}) \text{ A} \quad (8.7.6)$$

Note that $i(0) = 4 \text{ A}$, as expected.

CASE 2 When $R = 4 \Omega$. Again, the initial current through the inductor is

$$i(0) = \frac{24}{4 + 1} = 4.8 \text{ A}$$

and the initial capacitor voltage is

$$v(0) = 1i(0) = 4.8 \text{ V}$$

For the characteristic roots,

$$\alpha = \frac{R}{2L} = \frac{4}{2 \times 1} = 2$$

while $\omega_0 = 2$ remains the same. In this case, $s_1 = s_2 = -\alpha = -2$, and we have the critically damped natural response. The total response is therefore

$$v(t) = v_{ss} + (A_1 + A_2 t)e^{-2t}$$

and, as before $v_{ss} = 24 \text{ V}$,

$$v(t) = 24 + (A_1 + A_2 t)e^{-2t} \quad (8.7.7)$$

To find A_1 and A_2 , we use the initial conditions. We write

$$v(0) = 4.8 = 24 + A_1 \Rightarrow A_1 = -19.2 \quad (8.7.8)$$

Since $i(0) = C dv(0)/dt = 4.8$ or

$$\frac{dv(0)}{dt} = \frac{4.8}{C} = 19.2$$

From Eq. (8.7.7),

$$\frac{dv}{dt} = (-2A_1 - 2tA_2 + A_2)e^{-2t} \quad (8.7.9)$$

At $t = 0$,

$$\frac{dv(0)}{dt} = 19.2 = -2A_1 + A_2 \quad (8.7.10)$$

From Eqs. (8.7.8) and (8.7.10), $A_1 = -19.2$ and $A_2 = -19.2$. Thus, Eq. (8.7.7) becomes

$$v(t) = 24 - 19.2(1 + t)e^{-2t} \text{ V} \quad (8.7.11)$$

The inductor current is the same as the capacitor current; that is,

$$i(t) = C \frac{dv}{dt}$$

Multiplying Eq. (8.7.9) by $C = 0.25$ and substituting the values of A_1 and A_2 gives

$$i(t) = (4.8 + 9.6t)e^{-2t} \text{ A} \quad (8.7.12)$$

Note that $i(0) = 4.8 \text{ A}$, as expected.

■ CASE 3 When $R = 1 \Omega$

$$i(0) = \frac{24}{1 + 1} = 12 \text{ A}$$

and the initial voltage across the capacitor is the same as the voltage across the 1Ω resistor,

$$\begin{aligned} v(0) &= 1i(0) = 12 \text{ V} \\ \alpha &= \frac{R}{2L} = \frac{1}{2 \times 1} = 0.5 \end{aligned}$$

Since $\alpha = 0.5 < \omega_0 = 2$, we have the underdamped response

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -0.5 \pm j1.936$$

The total response is therefore

$$v(t) = 24 + (A_1 \cos 1.936t + A_2 \sin 1.936t)e^{-0.5t} \quad (8.7.13)$$

We now determine A_1 and A_2 . We write

$$v(0) = 12 = 24 + A_1 \Rightarrow A_1 = -12 \quad (8.7.14)$$

Since $i(0) = C dv(0)/dt = 12$,

$$\frac{dv(0)}{dt} = \frac{12}{C} = 48 \quad (8.7.15)$$

But

$$\begin{aligned} \frac{dv}{dt} &= e^{-0.5t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t) \\ &\quad - 0.5e^{-0.5t}(A_1 \cos 1.936t + A_2 \sin 1.936t) \end{aligned} \quad (8.7.16)$$

At $t = 0$,

$$\frac{dv(0)}{dt} = 48 = (-0 + 1.936A_2) - 0.5(A_1 + 0)$$

Substituting $A_1 = -12$ gives $A_2 = 21.694$, and Eq. (8.7.13) becomes

$$v(t) = 24 + (21.694 \sin 1.936t - 12 \cos 1.936t)e^{-0.5t} \text{ V} \quad (8.7.17)$$

The inductor current is

$$i(t) = C \frac{dv}{dt}$$

Multiplying Eq. (8.7.16) by $C = 0.25$ and substituting the values of A_1 and A_2 gives

$$i(t) = (3.1 \sin 1.936t + 12 \cos 1.936t)e^{-0.5t} \text{ A} \quad (8.7.18)$$

Note that $i(0) = 12 \text{ A}$, as expected.

Figure 8.20 plots the responses for the three cases. From this figure, we observe that the critically damped response approaches the step input of 24 V the fastest.

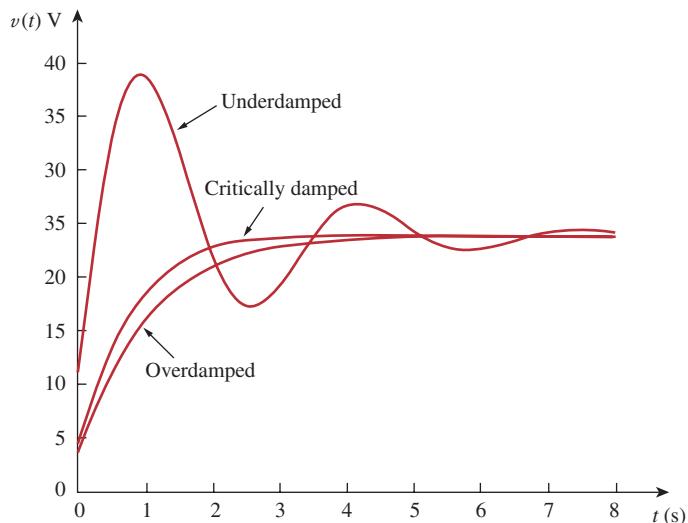


Figure 8.20

For Example 8.7: response for three degrees of damping.

Practice Problem 8.7

Having been in position *a* for a long time, the switch in Fig. 8.21 is moved to position *b* at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

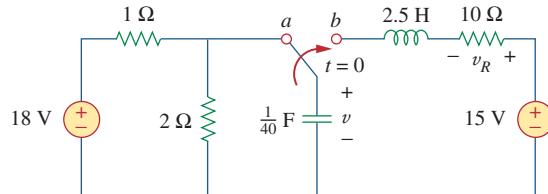


Figure 8.21

For Practice Prob. 8.7.

Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t} \text{ V}$, $3.464e^{-2t} \sin 3.464t \text{ V}$.

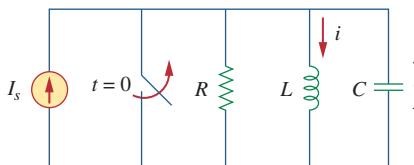


Figure 8.22

Parallel RLC circuit with an applied current.

8.6 Step Response of a Parallel RLC Circuit

Consider the parallel RLC circuit shown in Fig. 8.22. We want to find i due to a sudden application of a dc current. Applying KCL at the top node for $t > 0$,

$$\frac{v}{R} + i + C \frac{dv}{dt} = I_s \quad (8.46)$$

But

$$v = L \frac{di}{dt}$$

Substituting for v in Eq. (8.46) and dividing by LC , we get

$$\frac{d^2i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC} \quad (8.47)$$

which has the same characteristic equation as Eq. (8.29).

The complete solution to Eq. (8.47) consists of the transient response $i_t(t)$ and the steady-state response i_{ss} ; that is,

$$i(t) = i_t(t) + i_{ss}(t) \quad (8.48)$$

The transient response is the same as what we had in Section 8.4. The steady-state response is the final value of i . In the circuit in Fig. 8.22, the final value of the current through the inductor is the same as the source current I_s . Thus,

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (\text{Overdamped})$$

$$i(t) = I_s + (A_1 + A_2 t) e^{-\alpha t} \quad (\text{Critically damped})$$

$$i(t) = I_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t) e^{-\alpha t} \quad (\text{Underdamped})$$

(8.49)

The constants A_1 and A_2 in each case can be determined from the initial conditions for i and di/dt . Again, we should keep in mind that Eq. (8.49) only applies for finding the inductor current i . But once the inductor current $i_L = i$ is known, we can find $v = L di/dt$, which is the same voltage across inductor, capacitor, and resistor. Hence, the current through the resistor is $i_R = v/R$, while the capacitor current is $i_C = C dv/dt$. Alternatively, the complete response for any variable $x(t)$ may be found directly, using

$$x(t) = x_{ss}(t) + x_t(t) \quad (8.50)$$

where x_{ss} and x_t are its final value and transient response, respectively.

In the circuit of Fig. 8.23, find $i(t)$ and $i_R(t)$ for $t > 0$.

Example 8.8

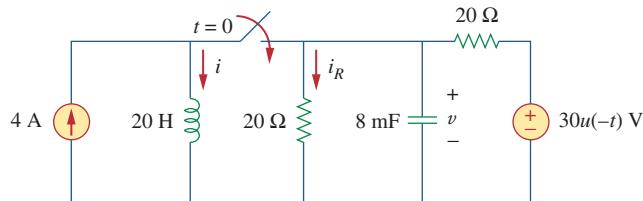


Figure 8.23

For Example 8.8.

Solution:

For $t < 0$, the switch is open, and the circuit is partitioned into two independent subcircuits. The 4-A current flows through the inductor, so that

$$i(0) = 4 \text{ A}$$

Since $30u(-t) = 30$ when $t < 0$ and 0 when $t > 0$, the voltage source is operative for $t < 0$. The capacitor acts like an open circuit and the voltage across it is the same as the voltage across the $20\text{-}\Omega$ resistor connected in parallel with it. By voltage division, the initial capacitor voltage is

$$v(0) = \frac{20}{20 + 20}(30) = 15 \text{ V}$$

For $t > 0$, the switch is closed, and we have a parallel RLC circuit with a current source. The voltage source is zero which means it acts like a short-circuit. The two $20\text{-}\Omega$ resistors are now in parallel. They are combined to give $R = 20 \parallel 20 = 10 \Omega$. The characteristic roots are determined as follows:

$$\begin{aligned} \alpha &= \frac{1}{2RC} = \frac{1}{2 \times 10 \times 8 \times 10^{-3}} = 6.25 \\ \omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 8 \times 10^{-3}}} = 2.5 \\ s_{1,2} &= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm \sqrt{39.0625 - 6.25} \\ &= -6.25 \pm 5.7282 \end{aligned}$$

or

$$s_1 = -11.978, \quad s_2 = -0.5218$$

Since $\alpha > \omega_0$, we have the overdamped case. Hence,

$$i(t) = I_s + A_1 e^{-11.978t} + A_2 e^{-0.5218t} \quad (8.8.1)$$

where $I_s = 4$ is the final value of $i(t)$. We now use the initial conditions to determine A_1 and A_2 . At $t = 0$,

$$i(0) = 4 = 4 + A_1 + A_2 \quad \Rightarrow \quad A_2 = -A_1 \quad (8.8.2)$$

Taking the derivative of $i(t)$ in Eq. (8.8.1),

$$\frac{di}{dt} = -11.978A_1 e^{-11.978t} - 0.5218A_2 e^{-0.5218t}$$

so that at $t = 0$,

$$\frac{di(0)}{dt} = -11.978A_1 - 0.5218A_2 \quad (8.8.3)$$

But

$$L \frac{di(0)}{dt} = v(0) = 15 \quad \Rightarrow \quad \frac{di(0)}{dt} = \frac{15}{L} = \frac{15}{20} = 0.75$$

Substituting this into Eq. (8.8.3) and incorporating Eq. (8.8.2), we get

$$0.75 = (11.978 - 0.5218)A_2 \quad \Rightarrow \quad A_2 = 0.0655$$

Thus, $A_1 = -0.0655$ and $A_2 = 0.0655$. Inserting A_1 and A_2 in Eq. (8.8.1) gives the complete solution as

$$i(t) = 4 + 0.0655(e^{-0.5218t} - e^{-11.978t}) \text{ A}$$

From $i(t)$, we obtain $v(t) = L di/dt$ and

$$i_R(t) = \frac{v(t)}{20} = \frac{L}{20} \frac{di}{dt} = 0.785e^{-11.978t} - 0.0342e^{-0.5218t} \text{ A}$$

Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 8.24.

Answer: $10(1 - \cos(0.25t))$ A, $50 \sin(0.25t)$ V.

Practice Problem 8.8

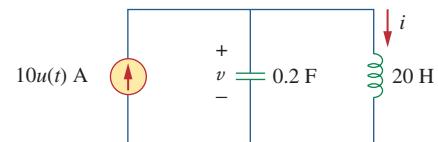


Figure 8.24

For Practice Prob. 8.8.

8.7 General Second-Order Circuits

Now that we have mastered series and parallel *RLC* circuits, we are prepared to apply the ideas to any second-order circuit having one or more independent sources with constant values. Although the series and parallel *RLC* circuits are the second-order circuits of greatest interest, other second-order circuits including op amps are also useful. Given a second-order circuit, we determine its step response $x(t)$ (which may be voltage or current) by taking the following four steps:

1. We first determine the initial conditions $x(0)$ and $dx(0)/dt$ and the final value $x(\infty)$, as discussed in Section 8.2.
2. We turn off the independent sources and find the form of the transient response $x_t(t)$ by applying KCL and KVL. Once a second-order differential equation is obtained, we determine its characteristic roots. Depending on whether the response is overdamped, critically damped, or underdamped, we obtain $x_t(t)$ with two unknown constants as we did in the previous sections.
3. We obtain the steady-state response as

$$x_{ss}(t) = x(\infty) \quad (8.51)$$

where $x(\infty)$ is the final value of x , obtained in step 1.

4. The total response is now found as the sum of the transient response and steady-state response

$$x(t) = x_t(t) + x_{ss}(t) \quad (8.52)$$

We finally determine the constants associated with the transient response by imposing the initial conditions $x(0)$ and $dx(0)/dt$, determined in step 1.

We can apply this general procedure to find the step response of any second-order circuit, including those with op amps. The following examples illustrate the four steps.

A circuit may look complicated at first. But once the sources are turned off in an attempt to find the form of the transient response, it may be reducible to a first-order circuit, when the storage elements can be combined, or to a parallel/series *RLC* circuit. If it is reducible to a first-order circuit, the solution becomes simply what we had in Chapter 7. If it is reducible to a parallel or series *RLC* circuit, we apply the techniques of previous sections in this chapter.

Find the complete response v and then i for $t > 0$ in the circuit of Fig. 8.25.

Example 8.9

Problems in this chapter can also be solved by using Laplace transforms, which are covered in Chapters 15 and 16.

Solution:

We first find the initial and final values. At $t = 0^-$, the circuit is at steady state. The switch is open; the equivalent circuit is shown in Fig. 8.26(a). It is evident from the figure that

$$v(0^-) = 12 \text{ V}, \quad i(0^-) = 0$$

At $t = 0^+$, the switch is closed; the equivalent circuit is in Fig. 8.26(b). By the continuity of capacitor voltage and inductor current, we know that

$$v(0^+) = v(0^-) = 12 \text{ V}, \quad i(0^+) = i(0^-) = 0 \quad (8.9.1)$$

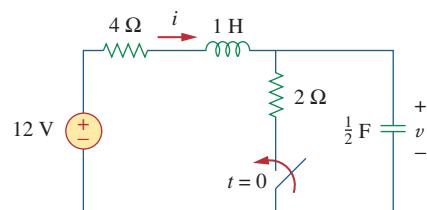
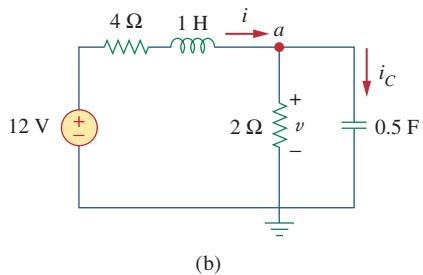
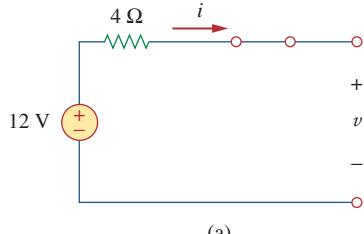
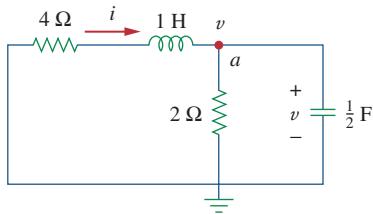


Figure 8.25

For Example 8.9.

**Figure 8.26**

Equivalent circuit of the circuit in Fig. 8.25 for: (a) $t < 0$, (b) $t > 0$.

**Figure 8.27**

Obtaining the form of the transient response for Example 8.9.

To get $dv(0^+)/dt$, we use $C dv/dt = i_C$ or $dv/dt = i_C/C$. Applying KCL at node a in Fig. 8.26(b),

$$i(0^+) = i_C(0^+) + \frac{v(0^+)}{2}$$

$$0 = i_C(0^+) + \frac{12}{2} \Rightarrow i_C(0^+) = -6 \text{ A}$$

Hence,

$$\frac{dv(0^+)}{dt} = \frac{-6}{0.5} = -12 \text{ V/s} \quad (8.9.2)$$

The final values are obtained when the inductor is replaced by a short circuit and the capacitor by an open circuit in Fig. 8.26(b), giving

$$i(\infty) = \frac{12}{4+2} = 2 \text{ A}, \quad v(\infty) = 2i(\infty) = 4 \text{ V} \quad (8.9.3)$$

Next, we obtain the form of the transient response for $t > 0$. By turning off the 12-V voltage source, we have the circuit in Fig. 8.27. Applying KCL at node a in Fig. 8.27 gives

$$i = \frac{v}{2} + \frac{1}{2} \frac{dv}{dt} \quad (8.9.4)$$

Applying KVL to the left mesh results in

$$4i + 1 \frac{di}{dt} + v = 0 \quad (8.9.5)$$

Since we are interested in v for the moment, we substitute i from Eq. (8.9.4) into Eq. (8.9.5). We obtain

$$2v + 2 \frac{dv}{dt} + \frac{1}{2} \frac{dv}{dt} + \frac{1}{2} \frac{d^2v}{dt^2} + v = 0$$

or

$$\frac{d^2v}{dt^2} + 5 \frac{dv}{dt} + 6v = 0$$

From this, we obtain the characteristic equation as

$$s^2 + 5s + 6 = 0$$

with roots $s = -2$ and $s = -3$. Thus, the natural response is

$$v_n(t) = Ae^{-2t} + Be^{-3t} \quad (8.9.6)$$

where A and B are unknown constants to be determined later. The steady-state response is

$$v_{ss}(t) = v(\infty) = 4 \quad (8.9.7)$$

The complete response is

$$v(t) = v_t + v_{ss} = 4 + Ae^{-2t} + Be^{-3t} \quad (8.9.8)$$

We now determine A and B using the initial values. From Eq. (8.9.1), $v(0) = 12$. Substituting this into Eq. (8.9.8) at $t = 0$ gives

$$12 = 4 + A + B \Rightarrow A + B = 8 \quad (8.9.9)$$

Taking the derivative of v in Eq. (8.9.8),

$$\frac{dv}{dt} = -2Ae^{-2t} - 3Be^{-3t} \quad (8.9.10)$$

Substituting Eq. (8.9.2) into Eq. (8.9.10) at $t = 0$ gives

$$-12 = -2A - 3B \Rightarrow 2A + 3B = 12 \quad (8.9.11)$$

From Eqs. (8.9.9) and (8.9.11), we obtain

$$A = 12, \quad B = -4$$

so that Eq. (8.9.8) becomes

$$v(t) = 4 + 12e^{-2t} - 4e^{-3t} \text{ V}, \quad t > 0 \quad (8.9.12)$$

From v , we can obtain other quantities of interest by referring to Fig. 8.26(b). To obtain i , for example,

$$\begin{aligned} i &= \frac{v}{2} + \frac{1}{2} \frac{dv}{dt} = 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t} \\ &= 2 - 6e^{-2t} + 4e^{-3t} \text{ A}, \quad t > 0 \end{aligned} \quad (8.9.13)$$

Notice that $i(0) = 0$, in agreement with Eq. (8.9.1).

Determine v and i for $t > 0$ in the circuit of Fig. 8.28. (See comments about current sources in Practice Prob. 7.5.)

Answer: $12(1 - e^{-5t}) \text{ V}$, $3(1 - e^{-5t}) \text{ A}$.

Practice Problem 8.9

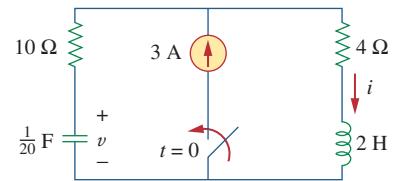


Figure 8.28
For Practice Prob. 8.9.

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 8.29.

Example 8.10

Solution:

This is an example of a second-order circuit with two inductors. We first obtain the mesh currents i_1 and i_2 , which happen to be the currents through the inductors. We need to obtain the initial and final values of these currents.

For $t < 0$, $7u(t) = 0$, so that $i_1(0^-) = 0 = i_2(0^-)$. For $t > 0$, $7u(t) = 7$, so that the equivalent circuit is as shown in Fig. 8.30(a). Due to the continuity of inductor current,

$$i_1(0^+) = i_1(0^-) = 0, \quad i_2(0^+) = i_2(0^-) = 0 \quad (8.10.1)$$

$$v_{L_2}(0^+) = v_o(0^+) = 1[(i_1(0^+) - i_2(0^+))] = 0 \quad (8.10.2)$$

Applying KVL to the left loop in Fig. 8.30(a) at $t = 0^+$,

$$7 = 3i_1(0^+) + v_{L_1}(0^+) + v_o(0^+)$$

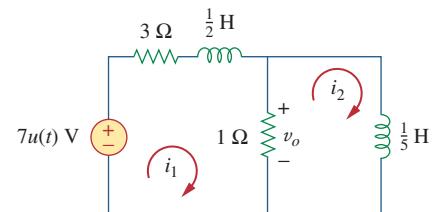
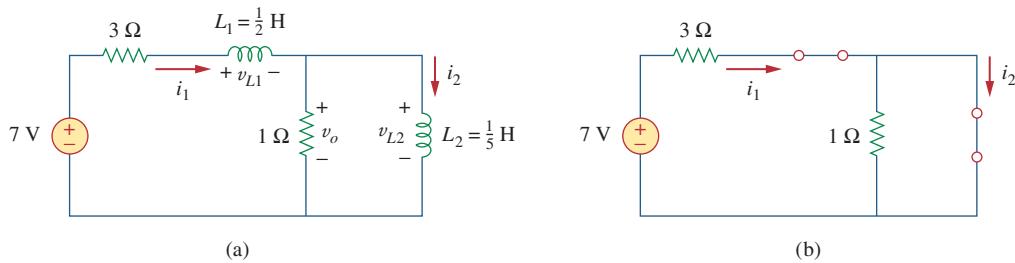


Figure 8.29
For Example 8.10.

**Figure 8.30**

Equivalent circuit of that in Fig. 8.29 for: (a) $t > 0$, (b) $t \rightarrow \infty$.

or

$$v_{L_1}(0^+) = 7 \text{ V}$$

Since $L_1 \frac{di_1}{dt} = v_{L_1}$,

$$\frac{di_1(0^+)}{dt} = \frac{v_{L_1}}{L_1} = \frac{7}{\frac{1}{2}} = 14 \text{ V/s} \quad (8.10.3)$$

Similarly, since $L_2 \frac{di_2}{dt} = v_{L_2}$,

$$\frac{di_2(0^+)}{dt} = \frac{v_{L_2}}{L_2} = 0 \quad (8.10.4)$$

As $t \rightarrow \infty$, the circuit reaches steady state, and the inductors can be replaced by short circuits, as shown in Fig. 8.30(b). From this figure,

$$i_1(\infty) = i_2(\infty) = \frac{7}{3} \text{ A} \quad (8.10.5)$$

Next, we obtain the form of the transient responses by removing the voltage source, as shown in Fig. 8.31. Applying KVL to the two meshes yields

$$4i_1 - i_2 + \frac{1}{2} \frac{di_1}{dt} = 0 \quad (8.10.6)$$

and

$$i_2 + \frac{1}{5} \frac{di_2}{dt} - i_1 = 0 \quad (8.10.7)$$

From Eq. (8.10.6),

$$i_2 = 4i_1 + \frac{1}{2} \frac{di_1}{dt} \quad (8.10.8)$$

Substituting Eq. (8.10.8) into Eq. (8.10.7) gives

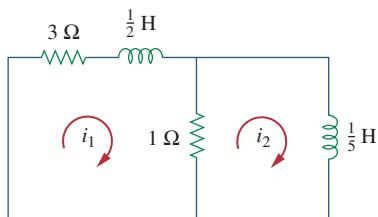
$$\begin{aligned} 4i_1 + \frac{1}{2} \frac{di_1}{dt} + \frac{4}{5} \frac{di_1}{dt} + \frac{1}{10} \frac{d^2i_1}{dt^2} - i_1 &= 0 \\ \frac{d^2i_1}{dt^2} + 13 \frac{di_1}{dt} + 30i_1 &= 0 \end{aligned}$$

From this we obtain the characteristic equation as

$$s^2 + 13s + 30 = 0$$

which has roots $s = -3$ and $s = -10$. Hence, the form of the transient response is

$$i_{1n} = Ae^{-3t} + Be^{-10t} \quad (8.10.9)$$

**Figure 8.31**

Obtaining the form of the transient response for Example 8.10.

where A and B are constants. The steady-state response is

$$i_{1ss} = i_1(\infty) = \frac{7}{3} \text{ A} \quad (8.10.10)$$

From Eqs. (8.10.9) and (8.10.10), we obtain the complete response as

$$i_1(t) = \frac{7}{3} + Ae^{-3t} + Be^{-10t} \quad (8.10.11)$$

We finally obtain A and B from the initial values. From Eqs. (8.10.1) and (8.10.11),

$$0 = \frac{7}{3} + A + B \quad (8.10.12)$$

Taking the derivative of Eq. (8.10.11), setting $t = 0$ in the derivative, and enforcing Eq. (8.10.3), we obtain

$$14 = -3A - 10B \quad (8.10.13)$$

From Eqs. (8.10.12) and (8.10.13), $A = -4/3$ and $B = -1$. Thus,

$$i_1(t) = \frac{7}{3} - \frac{4}{3}e^{-3t} - e^{-10t} \quad (8.10.14)$$

We now obtain i_2 from i_1 . Applying KVL to the left loop in Fig. 8.30(a) gives

$$7 = 4i_1 - i_2 + \frac{1}{2} \frac{di_1}{dt} \Rightarrow i_2 = -7 + 4i_1 + \frac{1}{2} \frac{di_1}{dt}$$

Substituting for i_1 in Eq. (8.10.14) gives

$$\begin{aligned} i_2(t) &= -7 + \frac{28}{3} - \frac{16}{3}e^{-3t} - 4e^{-10t} + 2e^{-3t} + 5e^{-10t} \\ &= \frac{7}{3} - \frac{10}{3}e^{-3t} + e^{-10t} \end{aligned} \quad (8.10.15)$$

From Fig. 8.29,

$$v_o(t) = 1[i_1(t) - i_2(t)] \quad (8.10.16)$$

Substituting Eqs. (8.10.14) and (8.10.15) into Eq. (8.10.16) yields

$$v_o(t) = 2(e^{-3t} - e^{-10t}) \quad (8.10.17)$$

Note that $v_o(0) = 0$, as expected from Eq. (8.10.2).

For $t > 0$, obtain $v_o(t)$ in the circuit of Fig. 8.32. (Hint: First find v_1 and v_2 .)

Answer: $8(e^{-t} - e^{-6t})$ V, $t > 0$.

Practice Problem 8.10

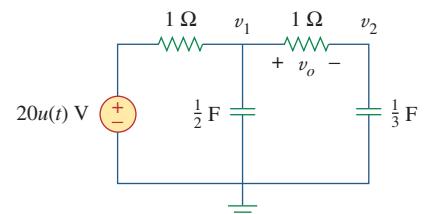


Figure 8.32
For Practice Prob. 8.10.

8.8 Second-Order Op Amp Circuits

The use of op amps in second-order circuits avoids the use of inductors, which are undesirable in some applications.

An op amp circuit with two storage elements that cannot be combined into a single equivalent element is second-order. Because inductors are bulky and heavy, they are rarely used in practical op amp circuits. For this reason, we will only consider RC second-order op amp circuits here. Such circuits find a wide range of applications in devices such as filters and oscillators.

The analysis of a second-order op amp circuit follows the same four steps given and demonstrated in the previous section.

Example 8.11

In the op amp circuit of Fig. 8.33, find $v_o(t)$ for $t > 0$ when $v_s = 10u(t)$ mV. Let $R_1 = R_2 = 10 \text{ k}\Omega$, $C_1 = 20 \mu\text{F}$, and $C_2 = 100 \mu\text{F}$.

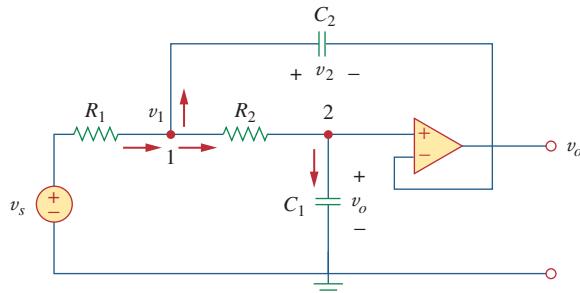


Figure 8.33
For Example 8.11.

Solution:

Although we could follow the same four steps given in the previous section to solve this problem, we will solve it a little differently. Due to the voltage follower configuration, the voltage across C_1 is v_o . Applying KCL at node 1,

$$\frac{v_s - v_1}{R_1} = C_2 \frac{dv_2}{dt} + \frac{v_1 - v_o}{R_2} \quad (8.11.1)$$

At node 2, KCL gives

$$\frac{v_1 - v_o}{R_2} = C_1 \frac{dv_o}{dt} \quad (8.11.2)$$

But

$$v_2 = v_1 - v_o \quad (8.11.3)$$

We now try to eliminate v_1 and v_2 in Eqs. (8.11.1) to (8.11.3). Substituting Eqs. (8.11.2) and (8.11.3) into Eq. (8.11.1) yields

$$\frac{v_s - v_1}{R_1} = C_2 \frac{dv_1}{dt} - C_2 \frac{dv_o}{dt} + C_1 \frac{dv_o}{dt} \quad (8.11.4)$$

From Eq. (8.11.2),

$$v_1 = v_o + R_2 C_1 \frac{dv_o}{dt} \quad (8.11.5)$$

Substituting Eq. (8.11.5) into Eq. (8.11.4), we obtain

$$\frac{v_s}{R_1} = \frac{v_o}{R_1} + \frac{R_2 C_1}{R_1} \frac{dv_o}{dt} + C_2 \frac{dv_o}{dt} + R_2 C_1 C_2 \frac{d^2 v_o}{dt^2} - C_2 \frac{dv_o}{dt} + C_1 \frac{dv_o}{dt}$$

or

$$\frac{d^2 v_o}{dt^2} + \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \right) \frac{dv_o}{dt} + \frac{v_o}{R_1 R_2 C_1 C_2} = \frac{v_s}{R_1 R_2 C_1 C_2} \quad (8.11.6)$$

With the given values of R_1 , R_2 , C_1 , and C_2 , Eq. (8.11.6) becomes

$$\frac{d^2 v_o}{dt^2} + 2 \frac{dv_o}{dt} + 5 v_o = 5 v_s \quad (8.11.7)$$

To obtain the form of the transient response, set $v_s = 0$ in Eq. (8.11.7), which is the same as turning off the source. The characteristic equation is

$$s^2 + 2s + 5 = 0$$

which has complex roots $s_{1,2} = -1 \pm j2$. Hence, the form of the transient response is

$$v_{ot} = e^{-t}(A \cos 2t + B \sin 2t) \quad (8.11.8)$$

where A and B are unknown constants to be determined.

As $t \rightarrow \infty$, the circuit reaches the steady-state condition, and the capacitors can be replaced by open circuits. Since no current flows through C_1 and C_2 under steady-state conditions and no current can enter the input terminals of the ideal op amp, current does not flow through R_1 and R_2 .

Thus,

$$v_o(\infty) = v_1(\infty) = v_s$$

The steady-state response is then

$$v_{oss} = v_o(\infty) = v_s = 10 \text{ mV}, \quad t > 0 \quad (8.11.9)$$

The complete response is

$$v_o(t) = v_{ot} + v_{oss} = 10 + e^{-t}(A \cos 2t + B \sin 2t) \text{ mV} \quad (8.11.10)$$

To determine A and B , we need the initial conditions. For $t < 0$, $v_s = 0$, so that

$$v_o(0^-) = v_2(0^-) = 0$$

For $t > 0$, the source is operative. However, due to capacitor voltage continuity,

$$v_o(0^+) = v_2(0^+) = 0 \quad (8.11.11)$$

From Eq. (8.11.3),

$$v_1(0^+) = v_2(0^+) + v_o(0^+) = 0$$

and, hence, from Eq. (8.11.2),

$$\frac{dv_o(0^+)}{dt} = \frac{v_1 - v_o}{R_2 C_1} = 0 \quad (8.11.12)$$

We now impose Eq. (8.11.11) on the complete response in Eq. (8.11.10) at $t = 0$, for

$$0 = 10 + A \quad \Rightarrow \quad A = -10 \quad (8.11.13)$$

Taking the derivative of Eq. (8.11.10),

$$\frac{dv_o}{dt} = e^{-t}(-A \cos 2t - B \sin 2t - 2A \sin 2t + 2B \cos 2t)$$

Setting $t = 0$ and incorporating Eq. (8.11.12), we obtain

$$0 = -A + 2B \quad (8.11.14)$$

From Eqs. (8.11.13) and (8.11.14), $A = -10$ and $B = -5$. Thus, the step response becomes

$$v_o(t) = 10 - e^{-t}(10 \cos 2t + 5 \sin 2t) \text{ mV}, \quad t > 0$$

Practice Problem 8.11

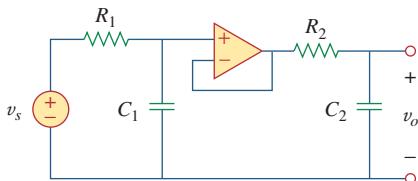


Figure 8.34
For Practice Prob. 8.11.

In the op amp circuit shown in Fig. 8.34, $v_s = 10u(t)$ V, find $v_o(t)$ for $t > 0$. Assume that $R_1 = R_2 = 10 \text{ k}\Omega$, $C_1 = 20 \mu\text{F}$, and $C_2 = 100 \mu\text{F}$.

Answer: $(10 - 12.5e^{-t} + 2.5e^{-5t})$ V, $t > 0$.

8.9 PSpice Analysis of RLC Circuits

RLC circuits can be analyzed with great ease using *PSpice*, just like the *RC* or *RL* circuits of Chapter 7. The following two examples will illustrate this. The reader may review Section D.4 in Appendix D on *PSpice* for transient analysis.

Example 8.12

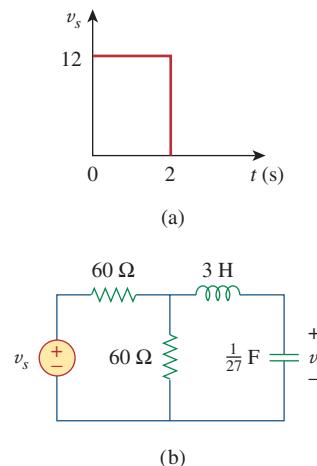


Figure 8.35
For Example 8.12.

The input voltage in Fig. 8.35(a) is applied to the circuit in Fig. 8.35(b). Use *PSpice* to plot $v(t)$ for $0 < t < 4$ s.

Solution:

- Define.** As true with most textbook problems, the problem is clearly defined.
- Present.** The input is equal to a single square wave of amplitude 12 V with a period of 2 s. We are asked to plot the output, using *PSpice*.
- Alternative.** Since we are required to use *PSpice*, that is the only alternative for a solution. However, we can check it using the technique illustrated in Section 8.5 (a step response for a series RLC circuit).
- Attempt.** The given circuit is drawn using Schematics as in Fig. 8.36. The pulse is specified using VPWL voltage source, but VPULSE could be used instead. Using the piecewise linear function, we set the attributes of VPWL as T1 = 0, V1 = 0, T2 = 0.001, V2 = 12, and so forth, as shown in Fig. 8.36. Two voltage markers are inserted to plot the input and output voltages. Once the circuit is drawn and the attributes are set, we select **Analysis/Setup/Transient** to open up the *Transient Analysis* dialog box. As a parallel RLC circuit, the roots of the characteristic equation are -1 and -9 . Thus, we may set *Final Time* as 4 s (four times the magnitude of the lower root). When

the schematic is saved, we select **Analysis/Simulate** and obtain the plots for the input and output voltages under the *PSpice A/D* window as shown in Fig. 8.37.

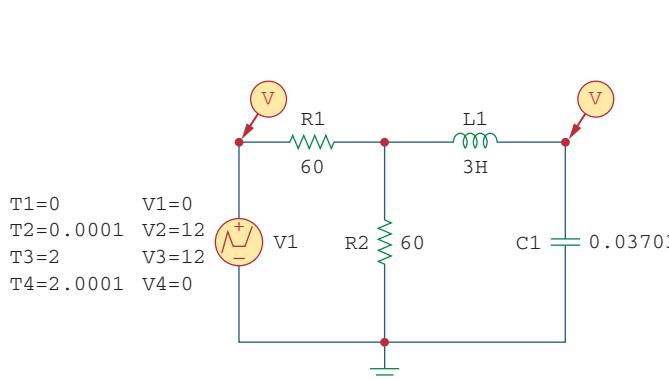


Figure 8.36

Schematic for the circuit in Fig. 8.35(b).

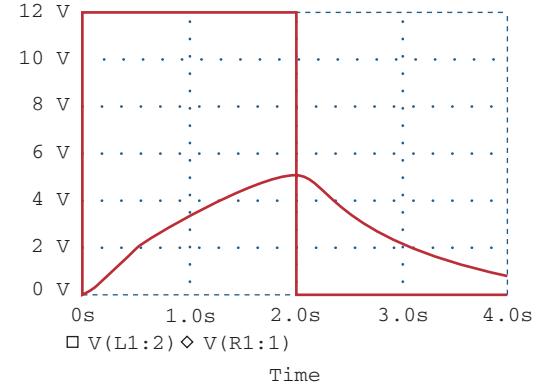


Figure 8.37

For Example 8.12: input and output.

Now we check using the technique from Section 8.5. We can start by realizing the Thevenin equivalent for the resistor-source combination is $V_{Th} = 12/2$ (the open circuit voltage divides equally across both resistors) = 6 V. The equivalent resistance is 30Ω ($60 \parallel 60$). Thus, we can now solve for the response using $R = 30 \Omega$, $L = 3 \text{ H}$, and $C = (1/27) \text{ F}$.

We first need to solve for α and ω_0 :

$$\alpha = R/(2L) = 30/6 = 5 \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{3 \frac{1}{27}}} = 3$$

Since 5 is greater than 3, we have the overdamped case

$$s_{1,2} = -5 \pm \sqrt{5^2 - 9} = -1, -9, \quad v(0) = 0, \\ v(\infty) = 6 \text{ V}, \quad i(0) = 0$$

$$i(t) = C \frac{dv(t)}{dt},$$

where

$$v(t) = A_1 e^{-t} + A_2 e^{-9t} + 6 \\ v(0) = 0 = A_1 + A_2 + 6 \\ i(0) = 0 = C(-A_1 - 9A_2)$$

which yields $A_1 = -9A_2$. Substituting this into the above, we get $0 = 9A_2 - A_2 + 6$, or $A_2 = 0.75$ and $A_1 = -6.75$.

$$v(t) = (-6.75e^{-t} + 0.75e^{-9t} + 6)u(t) \text{ V for all } 0 < t < 2 \text{ s.}$$

$$\text{At } t = 1 \text{ s}, v(1) = -6.75e^{-1} + 0.75e^{-9} = -2.483 + 0.0001 \\ + 6 = -3.552 \text{ V. At } t = 2 \text{ s}, v(2) = -6.75e^{-2} + 0 + 6 = 5.086 \text{ V.}$$

Note that from $2 < t < 4$ s, $V_{Th} = 0$, which implies that $v(\infty) = 0$. Therefore, $v(t) = (A_3 e^{-(t-2)} + A_4 e^{-9(t-2)})u(t-2)$ V.

At $t = 2$ s, $A_3 + A_4 = 5.086$.

$$i(t) = \frac{(-A_3 e^{-(t-2)} - 9A_4 e^{-9(t-2)})}{27}$$

and

$$i(2) = \frac{(6.75e^{-2} - 6.75e^{-18})}{27} = 33.83 \text{ mA}$$

Therefore, $-A_3 - 9A_4 = 0.9135$.

Combining the two equations, we get $-A_3 - 9(5.086 - A_3) = 0.9135$, which leads to $A_3 = 5.835$ and $A_4 = -0.749$.

$$v(t) = (5.835e^{-(t-2)} - 0.749e^{-9(t-2)}) u(t-2) \text{ V}$$

At $t = 3 \text{ s}$, $v(3) = (2.147 - 0) = 2.147 \text{ V}$. At $t = 4 \text{ s}$, $v(4) = 0.7897 \text{ V}$.

5. **Evaluate.** A check between the values calculated above and the plot shown in Figure 8.37 shows good agreement within the obvious level of accuracy.
6. **Satisfactory?** Yes, we have agreement and the results can be presented as a solution to the problem.

Practice Problem 8.12

Find $i(t)$ using PSpice for $0 < t < 4 \text{ s}$ if the pulse voltage in Fig. 8.35(a) is applied to the circuit in Fig. 8.38.

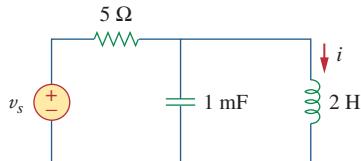


Figure 8.38

For Practice Prob. 8.12.

Answer: See Fig. 8.39.

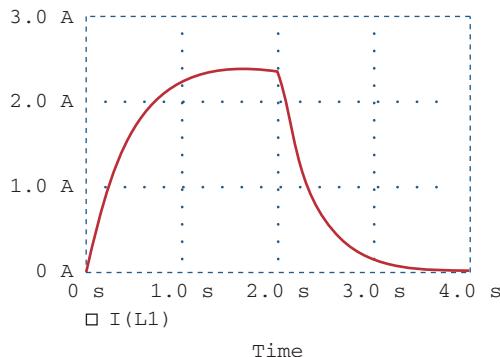


Figure 8.39

Plot of $i(t)$ for Practice Prob. 8.12.

Example 8.13

For the circuit in Fig. 8.40, use PSpice to obtain $i(t)$ for $0 < t < 3 \text{ s}$.

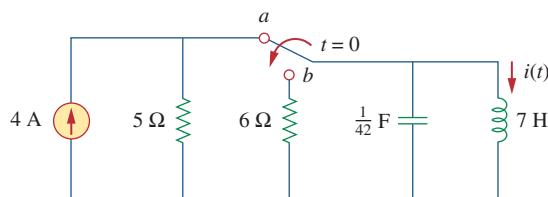
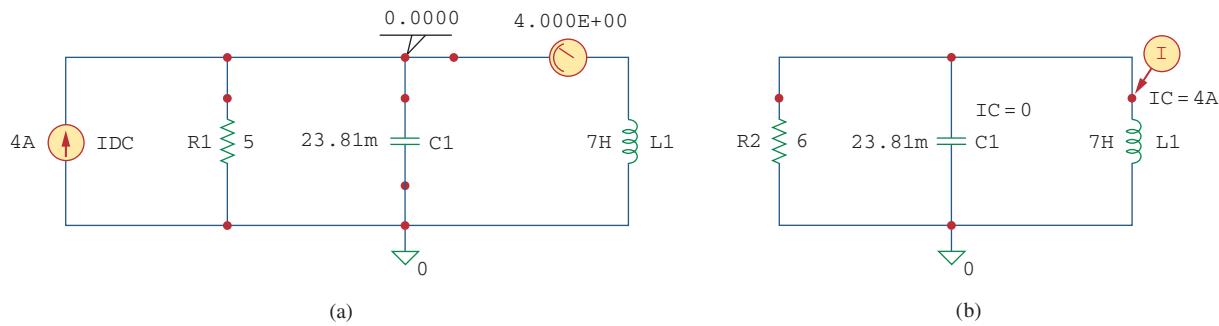


Figure 8.40

For Example 8.13.

Solution:

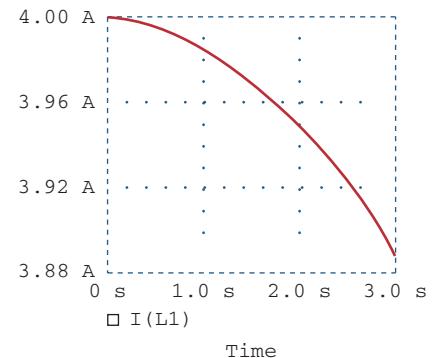
When the switch is in position *a*, the 6Ω resistor is redundant. The schematic for this case is shown in Fig. 8.41(a). To ensure that current

**Figure 8.41**

For Example 8.13: (a) for dc analysis, (b) for transient analysis.

$i(t)$ enters pin 1, the inductor is rotated three times before it is placed in the circuit. The same applies for the capacitor. We insert pseudo-components VIEWPOINT and IPROBE to determine the initial capacitor voltage and initial inductor current. We carry out a dc *PSpice* analysis by selecting **Analysis/Simulate**. As shown in Fig. 8.41(a), we obtain the initial capacitor voltage as 0 V and the initial inductor current $i(0)$ as 4 A from the dc analysis. These initial values will be used in the transient analysis.

When the switch is moved to position *b*, the circuit becomes a source-free parallel *RLC* circuit with the schematic in Fig. 8.41(b). We set the initial condition $IC = 0$ for the capacitor and $IC = 4$ A for the inductor. A current marker is inserted at pin 1 of the inductor. We select **Analysis/Setup/Transient** to open up the *Transient Analysis* dialog box and set *Final Time* to 3 s. After saving the schematic, we select **Analysis/Transient**. Figure 8.42 shows the plot of $i(t)$. The plot agrees with $i(t) = 4.8e^{-t} - 0.8e^{-6t}$ A, which is the solution by hand calculation.

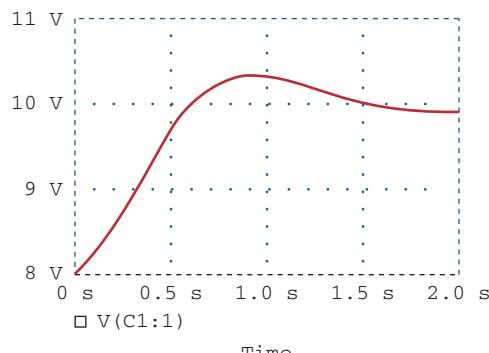
**Figure 8.42**

Plot of $i(t)$ for Example 8.13.

Refer to the circuit in Fig. 8.21 (see Practice Prob. 8.7). Use *PSpice* to obtain $v(t)$ for $0 < t < 2$.

Practice Problem 8.13

Answer: See Fig. 8.43.

**Figure 8.43**

Plot of $v(t)$ for Practice Prob. 8.13.

8.10 [†]Duality

The concept of duality is a time-saving, effort-effective measure of solving circuit problems. Consider the similarity between Eq. (8.4) and Eq. (8.29). The two equations are the same, except that we must interchange the following quantities: (1) voltage and current, (2) resistance and conductance, (3) capacitance and inductance. Thus, it sometimes occurs in circuit analysis that two different circuits have the same equations and solutions, except that the roles of certain complementary elements are interchanged. This interchangeability is known as the principle of *duality*.

The **duality principle** asserts a parallelism between pairs of characterizing equations and theorems of electric circuits.

TABLE 8.1

Dual pairs.

Resistance R	Conductance G
Inductance L	Capacitance C
Voltage v	Current i
Voltage source	Current source
Node	Mesh
Series path	Parallel path
Open circuit	Short circuit
KVL	KCL
Thevenin	Norton

Even when the principle of linearity applies, a circuit element or variable may not have a dual. For example, mutual inductance (to be covered in Chapter 13) has no dual.

Dual pairs are shown in Table 8.1. Note that power does not appear in Table 8.1, because power has no dual. The reason for this is the principle of linearity; since power is not linear, duality does not apply. Also notice from Table 8.1 that the principle of duality extends to circuit elements, configurations, and theorems.

Two circuits that are described by equations of the same form, but in which the variables are interchanged, are said to be dual to each other.

Two circuits are said to be **duals** of one another if they are described by the same characterizing equations with dual quantities interchanged.

The usefulness of the duality principle is self-evident. Once we know the solution to one circuit, we automatically have the solution for the dual circuit. It is obvious that the circuits in Figs. 8.8 and 8.13 are dual. Consequently, the result in Eq. (8.32) is the dual of that in Eq. (8.11). We must keep in mind that the method described here for finding a dual is limited to planar circuits. Finding a dual for a nonplanar circuit is beyond the scope of this textbook because nonplanar circuits cannot be described by a system of mesh equations.

To find the dual of a given circuit, we do not need to write down the mesh or node equations. We can use a graphical technique. Given a planar circuit, we construct the dual circuit by taking the following three steps:

1. Place a node at the center of each mesh of the given circuit. Place the reference node (the ground) of the dual circuit outside the given circuit.
2. Draw lines between the nodes such that each line crosses an element. Replace that element by its dual (see Table 8.1).
3. To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage source that produces a positive (clockwise) mesh current has as its dual a current source whose reference direction is from the ground to the nonreference node.

In case of doubt, one may verify the dual circuit by writing the nodal or mesh equations. The mesh (or nodal) equations of the original circuit are similar to the nodal (or mesh) equations of the dual circuit. The duality principle is illustrated with the following two examples.

Construct the dual of the circuit in Fig. 8.44.

Example 8.14

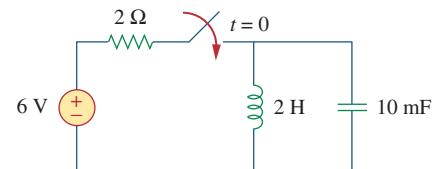


Figure 8.44

For Example 8.14.

Solution:

As shown in Fig. 8.45(a), we first locate nodes 1 and 2 in the two meshes and also the ground node 0 for the dual circuit. We draw a line between one node and another crossing an element. We replace the line joining the nodes by the duals of the elements which it crosses. For example, a line between nodes 1 and 2 crosses a 2-H inductor, and we place a 2-F capacitor (an inductor's dual) on the line. A line between nodes 1 and 0 crossing the 6-V voltage source will contain a 6-A current source. By drawing lines crossing all the elements, we construct the dual circuit on the given circuit as in Fig. 8.45(a). The dual circuit is redrawn in Fig. 8.45(b) for clarity.

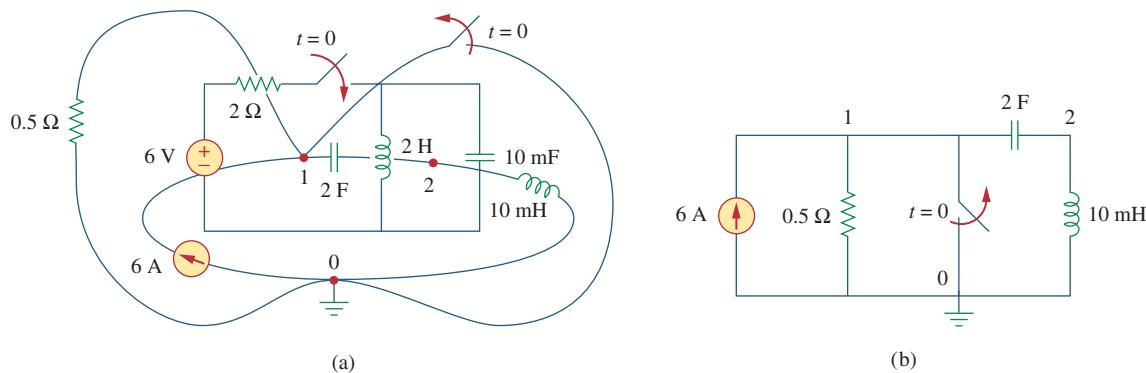


Figure 8.45

(a) Construction of the dual circuit of Fig. 8.44, (b) dual circuit redrawn.

Draw the dual circuit of the one in Fig. 8.46.

Practice Problem 8.14

Answer: See Fig. 8.47.

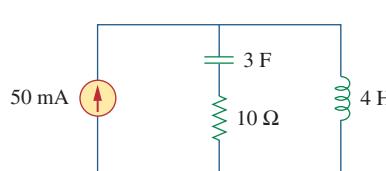


Figure 8.46

For Practice Prob. 8.14.

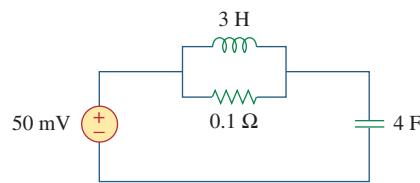


Figure 8.47

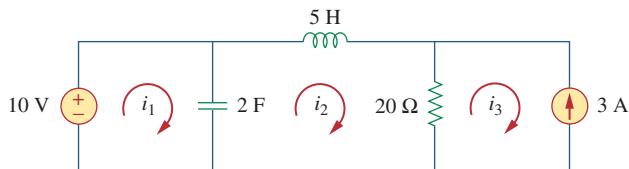
Dual of the circuit in Fig. 8.46.

Obtain the dual of the circuit in Fig. 8.48.

Example 8.15

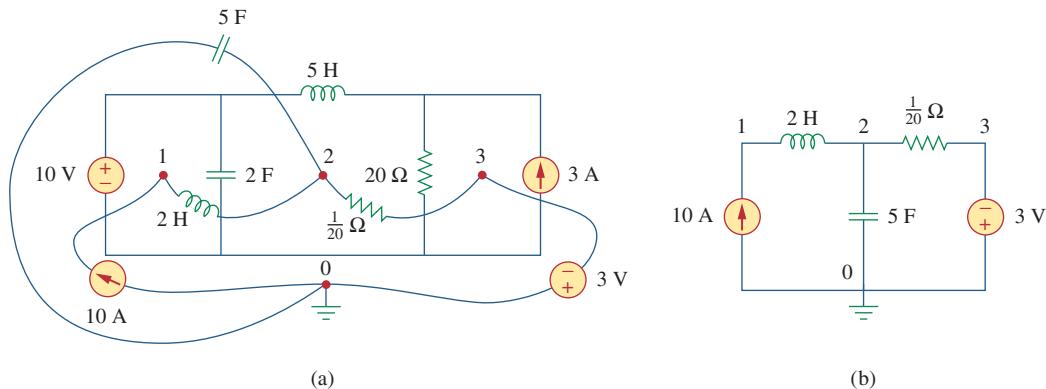
Solution:

The dual circuit is constructed on the original circuit as in Fig. 8.49(a). We first locate nodes 1 to 3 and the reference node 0. Joining nodes 1 and 2, we cross the 2-F capacitor, which is replaced by a 2-H inductor.

**Figure 8.48**

For Example 8.15.

Joining nodes 2 and 3, we cross the $20\text{-}\Omega$ resistor, which is replaced by a $\frac{1}{20}\text{-}\Omega$ resistor. We keep doing this until all the elements are crossed. The result is in Fig. 8.49(a). The dual circuit is redrawn in Fig. 8.49(b).

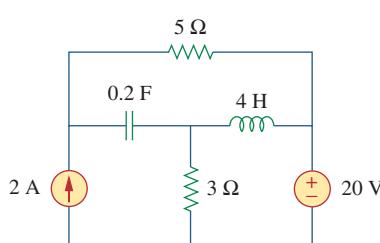
**Figure 8.49**

For Example 8.15: (a) construction of the dual circuit of Fig. 8.48, (b) dual circuit redrawn.

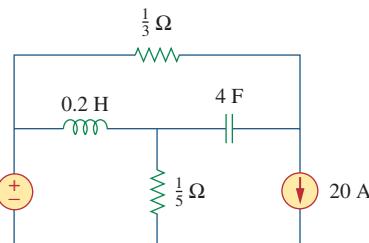
To verify the polarity of the voltage source and the direction of the current source, we may apply mesh currents i_1 , i_2 , and i_3 (all in the clockwise direction) in the original circuit in Fig. 8.48. The 10-V voltage source produces positive mesh current i_1 , so that its dual is a 10-A current source directed from 0 to 1. Also, $i_3 = -3 \text{ A}$ in Fig. 8.48 has as its dual $v_3 = -3 \text{ V}$ in Fig. 8.49(b).

Practice Problem 8.15

For the circuit in Fig. 8.50, obtain the dual circuit.

Answer: See Fig. 8.51.**Figure 8.50**

For Practice Prob. 8.15.

**Figure 8.51**

Dual of the circuit in Fig. 8.50.

8.11 [†]Applications

Practical applications of *RLC* circuits are found in control and communications circuits such as ringing circuits, peaking circuits, resonant circuits, smoothing circuits, and filters. Most of these circuits cannot be covered until we treat ac sources. For now, we will limit ourselves to two simple applications: automobile ignition and smoothing circuits.

8.11.1 Automobile Ignition System

In Section 7.9.4, we considered the automobile ignition system as a charging system. That was only a part of the system. Here, we consider another part—the voltage generating system. The system is modeled by the circuit shown in Fig. 8.52. The 12-V source is due to the battery and alternator. The 4- Ω resistor represents the resistance of the wiring. The ignition coil is modeled by the 8-mH inductor. The 1- μF capacitor (known as the *condenser* to automechanics) is in parallel with the switch (known as the *breaking points* or *electronic ignition*). In the following example, we determine how the *RLC* circuit in Fig. 8.52 is used in generating high voltage.

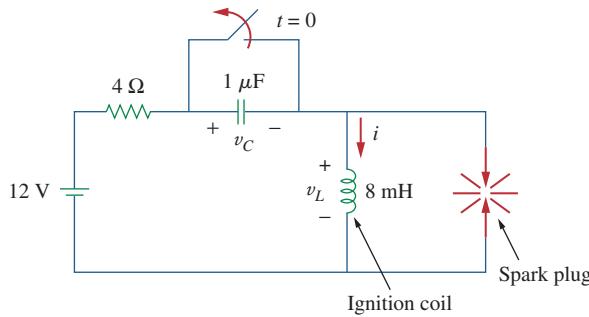


Figure 8.52
Automobile ignition circuit.

Assuming that the switch in Fig. 8.52 is closed prior to $t = 0^-$, find the inductor voltage v_L for $t > 0$.

Example 8.16

Solution:

If the switch is closed prior to $t = 0^-$ and the circuit is in steady state, then

$$i(0^-) = \frac{12}{4} = 3 \text{ A}, \quad v_C(0^-) = 0$$

At $t = 0^+$, the switch is opened. The continuity conditions require that

$$i(0^+) = 3 \text{ A}, \quad v_C(0^+) = 0 \quad (8.16.1)$$

We obtain $di(0^+)/dt$ from $v_L(0^+)$. Applying KVL to the mesh at $t = 0^+$ yields

$$\begin{aligned} -12 + 4i(0^+) + v_L(0^+) + v_C(0^+) &= 0 \\ -12 + 4 \times 3 + v_L(0^+) + 0 &= 0 \quad \Rightarrow \quad v_L(0^+) = 0 \end{aligned}$$

Hence,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = 0 \quad (8.16.2)$$

As $t \rightarrow \infty$, the system reaches steady state, so that the capacitor acts like an open circuit. Then

$$i(\infty) = 0 \quad (8.16.3)$$

If we apply KVL to the mesh for $t > 0$, we obtain

$$12 = Ri + L \frac{di}{dt} + \frac{1}{C} \int_0^t i \, dt + v_C(0)$$

Taking the derivative of each term yields

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0 \quad (8.16.4)$$

We obtain the form of the transient response by following the procedure in Section 8.3. Substituting $R = 4 \Omega$, $L = 8 \text{ mH}$, and $C = 1 \mu\text{F}$, we get

$$\alpha = \frac{R}{2L} = 250, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 1.118 \times 10^4$$

Since $\alpha < \omega_0$, the response is underdamped. The damped natural frequency is

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \approx \omega_0 = 1.118 \times 10^4$$

The form of the transient response is

$$i_t(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t) \quad (8.16.5)$$

where A and B are constants. The steady-state response is

$$i_{ss}(t) = i(\infty) = 0 \quad (8.16.6)$$

so that the complete response is

$$i(t) = i_t(t) + i_{ss}(t) = e^{-250t}(A \cos 11,180t + B \sin 11,180t) \quad (8.16.7)$$

We now determine A and B .

$$i(0) = 3 = A + 0 \quad \Rightarrow \quad A = 3$$

Taking the derivative of Eq. (8.16.7),

$$\begin{aligned} \frac{di}{dt} &= -250e^{-250t}(A \cos 11,180t + B \sin 11,180t) \\ &\quad + e^{-250t}(-11,180A \sin 11,180t + 11,180B \cos 11,180t) \end{aligned}$$

Setting $t = 0$ and incorporating Eq. (8.16.2),

$$0 = -250A + 11,180B \quad \Rightarrow \quad B = 0.0671$$

Thus,

$$i(t) = e^{-250t}(3 \cos 11,180t + 0.0671 \sin 11,180t) \quad (8.16.8)$$

The voltage across the inductor is then

$$v_L(t) = L \frac{di}{dt} = -268e^{-250t} \sin 11,180t \quad (8.16.9)$$

This has a maximum value when sine is unity, that is, at $11,180t_0 = \pi/2$ or $t_0 = 140.5 \mu\text{s}$. At time $= t_0$, the inductor voltage reaches its peak, which is

$$v_L(t_0) = -268e^{-250t_0} = -259 \text{ V} \quad (8.16.10)$$

Although this is far less than the voltage range of 6000 to 10,000 V required to fire the spark plug in a typical automobile, a device known as a *transformer* (to be discussed in Chapter 13) is used to step up the inductor voltage to the required level.

In Fig. 8.52, find the capacitor voltage v_C for $t > 0$.

Practice Problem 8.16

Answer: $12 - 12e^{-250t} \cos 11,180t + 267.7e^{-250t} \sin 11,180t \text{ V}$.

8.11.2 Smoothing Circuits

In a typical digital communication system, the signal to be transmitted is first sampled. Sampling refers to the procedure of selecting samples of a signal for processing, as opposed to processing the entire signal. Each sample is converted into a binary number represented by a series of pulses. The pulses are transmitted by a transmission line such as a coaxial cable, twisted pair, or optical fiber. At the receiving end, the signal is applied to a digital-to-analog (D/A) converter whose output is a “staircase” function, that is, constant at each time interval. In order to recover the transmitted analog signal, the output is smoothed by letting it pass through a “smoothing” circuit, as illustrated in Fig. 8.53. An *RLC* circuit may be used as the smoothing circuit.

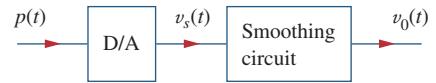


Figure 8.53

A series of pulses is applied to the digital-to-analog (D/A) converter, whose output is applied to the smoothing circuit.

The output of a D/A converter is shown in Fig. 8.54(a). If the *RLC* circuit in Fig. 8.54(b) is used as the smoothing circuit, determine the output voltage $v_o(t)$.

Example 8.17

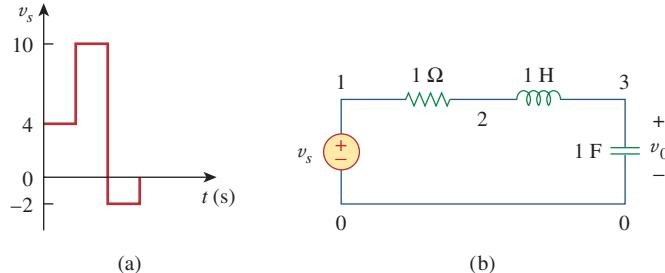
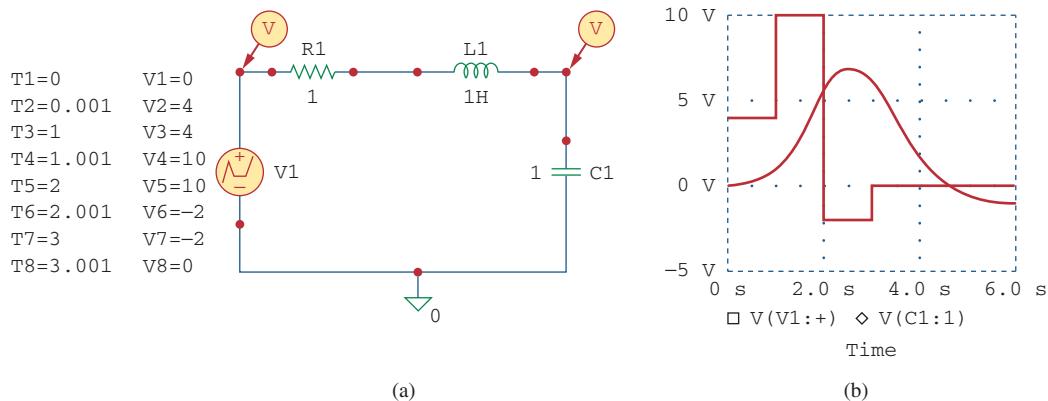


Figure 8.54

For Example 8.17: (a) output of a D/A converter, (b) an *RLC* smoothing circuit.

Solution:

This problem is best solved using *PSpice*. The schematic is shown in Fig. 8.55(a). The pulse in Fig. 8.54(a) is specified using the piecewise

**Figure 8.55**

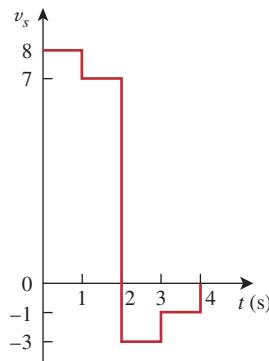
For Example 8.17: (a) schematic, (b) input and output voltages.

linear function. The attributes of V1 are set as $T1 = 0$, $V1 = 0$, $T2 = 0.001$, $V2 = 4$, $T3 = 1$, $V3 = 4$, and so on. To be able to plot both input and output voltages, we insert two voltage markers as shown. We select **Analysis/Setup/Transient** to open up the *Transient Analysis* dialog box and set *Final Time* as 6 s. Once the schematic is saved, we select **Analysis/Simulate** to run and obtain the plots shown in Fig. 8.55(b).

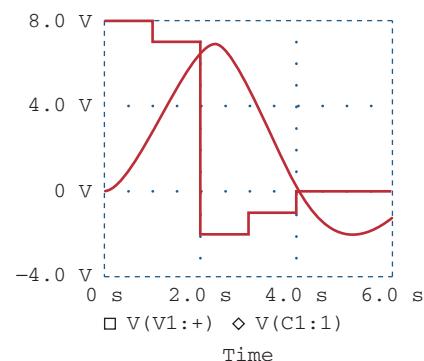
Practice Problem 8.17

Rework Example 8.17 if the output of the D/A converter is as shown in Fig. 8.56.

Answer: See Fig. 8.57.

**Figure 8.56**

For Practice Prob. 8.17.

**Figure 8.57**

Result of Practice Prob. 8.17.

8.12 Summary

1. The determination of the initial values $x(0)$ and $dx(0)/dt$ and final value $x(\infty)$ is crucial to analyzing second-order circuits.
2. The *RLC* circuit is second-order because it is described by a second-order differential equation. Its characteristic equation is

$s^2 + 2\alpha s + \omega_0^2 = 0$, where α is the damping factor and ω_0 is the undamped natural frequency. For a series circuit, $\alpha = R/2L$, for a parallel circuit $\alpha = 1/2RC$, and for both cases $\omega_0 = 1/0\sqrt{LC}$.

3. If there are no independent sources in the circuit after switching (or sudden change), we regard the circuit as source-free. The complete solution is the natural response.
4. The natural response of an *RLC* circuit is overdamped, underdamped, or critically damped, depending on the roots of the characteristic equation. The response is critically damped when the roots are equal ($s_1 = s_2$ or $\alpha = \omega_0$), overdamped when the roots are real and unequal ($s_1 \neq s_2$ or $\alpha > \omega_0$), or underdamped when the roots are complex conjugate ($s_1 = s_2^*$ or $\alpha < \omega_0$).
5. If independent sources are present in the circuit after switching, the complete response is the sum of the transient response and the steady-state response.
6. *PSpice* is used to analyze *RLC* circuits in the same way as for *RC* or *RL* circuits.
7. Two circuits are dual if the mesh equations that describe one circuit have the same form as the nodal equations that describe the other. The analysis of one circuit gives the analysis of its dual circuit.
8. The automobile ignition circuit and the smoothing circuit are typical applications of the material covered in this chapter.

Review Questions

- 8.1** For the circuit in Fig. 8.58, the capacitor voltage at $t = 0^-$ (just before the switch is closed) is:

(a) 0 V (b) 4 V (c) 8 V (d) 12 V

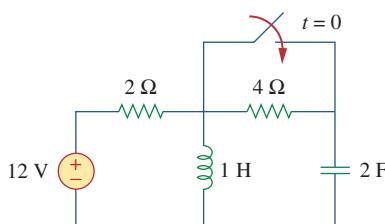


Figure 8.58

For Review Questions 8.1 and 8.2.

- 8.2** For the circuit in Fig. 8.58, the initial inductor current (at $t = 0$) is:
- (a) 0 A (b) 2 A (c) 6 A (d) 12 A
- 8.3** When a step input is applied to a second-order circuit, the final values of the circuit variables are found by:
- (a) Replacing capacitors with closed circuits and inductors with open circuits.
 (b) Replacing capacitors with open circuits and inductors with closed circuits.
 (c) Doing neither of the above.

- 8.4** If the roots of the characteristic equation of an *RLC* circuit are -2 and -3 , the response is:

(a) $(A \cos 2t + B \sin 2t)e^{-3t}$
 (b) $(A + 2Bt)e^{-3t}$
 (c) $Ae^{-2t} + Bte^{-3t}$
 (d) $Ae^{-2t} + Be^{-3t}$

where A and B are constants.

- 8.5** In a series *RLC* circuit, setting $R = 0$ will produce:

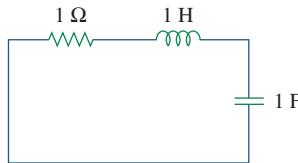
(a) an overdamped response
 (b) a critically damped response
 (c) an underdamped response
 (d) an undamped response
 (e) none of the above

- 8.6** A parallel *RLC* circuit has $L = 2$ H and $C = 0.25$ F. The value of R that will produce unity damping factor is:

(a) 0.5Ω (b) 1Ω (c) 2Ω (d) 4Ω

- 8.7** Refer to the series *RLC* circuit in Fig. 8.59. What kind of response will it produce?

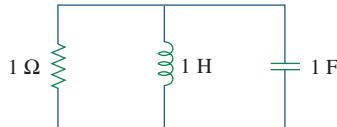
(a) overdamped
 (b) underdamped
 (c) critically damped
 (d) none of the above

**Figure 8.59**

For Review Question 8.7.

- 8.8** Consider the parallel RLC circuit in Fig. 8.60. What type of response will it produce?

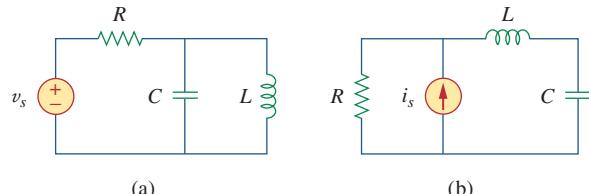
- (a) overdamped
- (b) underdamped
- (c) critically damped
- (d) none of the above

**Figure 8.60**

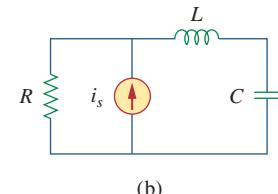
For Review Question 8.8.

- 8.9** Match the circuits in Fig. 8.61 with the following items:

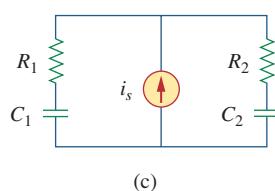
- (i) first-order circuit
- (ii) second-order series circuit
- (iii) second-order parallel circuit
- (iv) none of the above



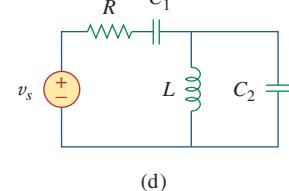
(a)



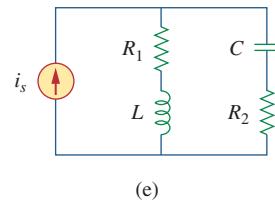
(b)



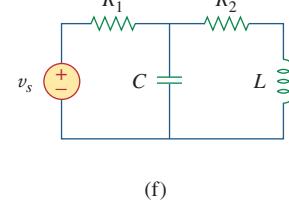
(c)



(d)



(e)



(f)

Figure 8.61

For Review Question 8.9.

- 8.10** In an electric circuit, the dual of resistance is:

- (a) conductance
- (b) inductance
- (c) capacitance
- (d) open circuit
- (e) short circuit

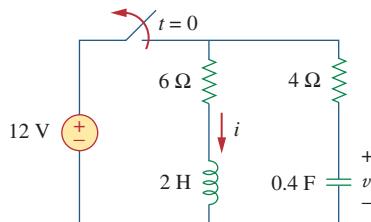
Answers: 8.1a, 8.2c, 8.3b, 8.4d, 8.5d, 8.6c, 8.7b, 8.8b, 8.9 (i)-c, (ii)-b, e, (iii)-a, (iv)-d, f, 8.10a.

Problems

Section 8.2 Finding Initial and Final Values

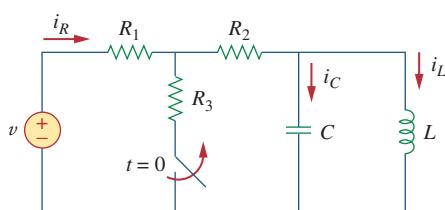
- 8.1** For the circuit in Fig. 8.62, find:

- (a) $i(0^+)$ and $v(0^+)$,
- (b) $di(0^+)/dt$ and $dv(0^+)/dt$,
- (c) $i(\infty)$ and $v(\infty)$.

**Figure 8.62**

For Prob. 8.1.

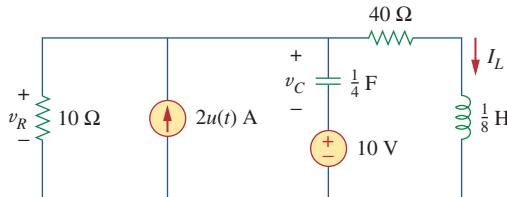
- 8.2** Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

**Figure 8.63**

For Prob. 8.2.

- 8.3** Refer to the circuit shown in Fig. 8.64. Calculate:

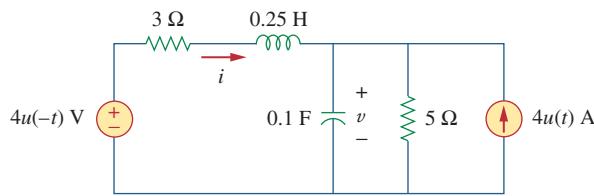
- $i_L(0^+)$, $v_C(0^+)$, and $v_R(0^+)$,
- $dv_L(0^+)/dt$, $dv_C(0^+)/dt$, and $dv_R(0^+)/dt$,
- $i_L(\infty)$, $v_C(\infty)$, and $v_R(\infty)$.

**Figure 8.64**

For Prob. 8.3.

- 8.4** In the circuit of Fig. 8.65, find:

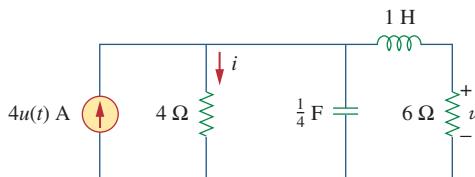
- $v(0^+)$ and $i(0^+)$,
- $dv(0^+)/dt$ and $di(0^+)/dt$,
- $v(\infty)$ and $i(\infty)$.

**Figure 8.65**

For Prob. 8.4.

- 8.5** Refer to the circuit in Fig. 8.66. Determine:

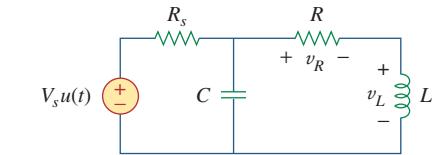
- $i(0^+)$ and $v(0^+)$,
- $di(0^+)/dt$ and $dv(0^+)/dt$,
- $i(\infty)$ and $v(\infty)$.

**Figure 8.66**

For Prob. 8.5.

- 8.6** In the circuit of Fig. 8.67, find:

- $v_R(0^+)$ and $v_L(0^+)$,
- $dv_R(0^+)/dt$ and $dv_L(0^+)/dt$,
- $v_R(\infty)$ and $v_L(\infty)$.

**Figure 8.67**

For Prob. 8.6.

Section 8.3 Source-Free Series RLC Circuit

- 8.7** A series RLC circuit has $R = 20 \text{ k}\Omega$, $L = 0.2 \text{ mH}$, and $C = 5 \mu\text{F}$. What type of damping is exhibited by the circuit?

- 8.8** Design a problem to help other students better understand source-free RLC circuits.

- 8.9** The current in an RLC circuit is described by

$$\frac{d^2i}{dt^2} + 10 \frac{di}{dt} + 25i = 0$$

If $i(0) = 10 \text{ A}$ and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.

- 8.10** The differential equation that describes the voltage in an RLC network is

$$\frac{d^2v}{dt^2} + 5 \frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 10 \text{ V/s}$, obtain $v(t)$.

- 8.11** The natural response of an RLC circuit is described by the differential equation

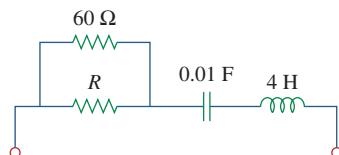
$$\frac{d^2v}{dt^2} + 2 \frac{dv}{dt} + v = 0$$

for which the initial conditions are $v(0) = 10 \text{ V}$ and $dv(0)/dt = 0$. Solve for $v(t)$.

- 8.12** If $R = 50 \Omega$, $L = 1.5 \text{ H}$, what value of C will make an RLC series circuit:

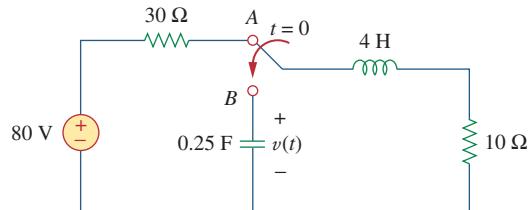
- overdamped,
- critically damped,
- underdamped?

- 8.13** For the circuit in Fig. 8.68, calculate the value of R needed to have a critically damped response.

**Figure 8.68**

For Prob. 8.13.

- 8.14** The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.

**Figure 8.69**

For Prob. 8.14.

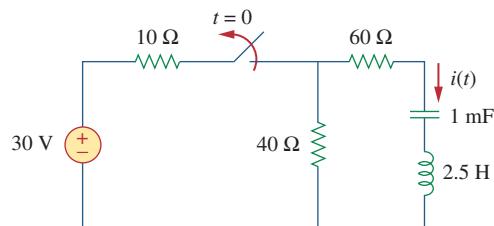
- 8.15** The responses of a series RLC circuit are

$$v_C(t) = 30 - 10e^{-20t} + 30e^{-10t} \text{ V}$$

$$i_L(t) = 40e^{-20t} - 60e^{-10t} \text{ mA}$$

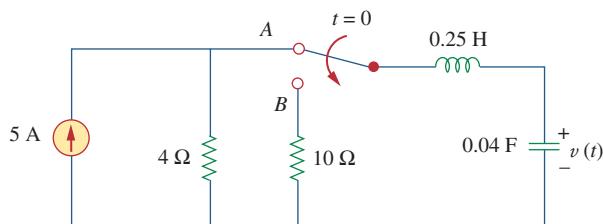
where v_C and i_L are the capacitor voltage and inductor current, respectively. Determine the values of R , L , and C .

- 8.16** Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.70.

**Figure 8.70**

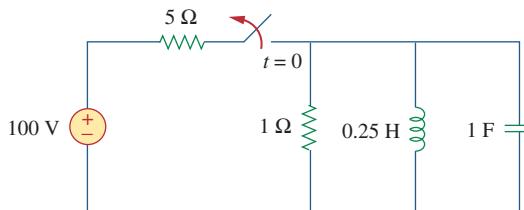
For Prob. 8.16.

- 8.17** In the circuit of Fig. 8.71, the switch instantaneously moves from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.

**Figure 8.71**

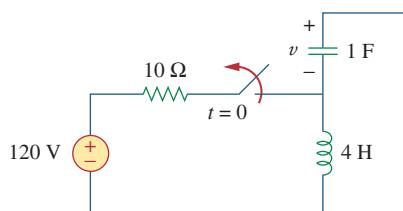
For Prob. 8.17.

- 8.18** Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 8.72. Assume steady-state conditions exist at $t = 0^-$.

**Figure 8.72**

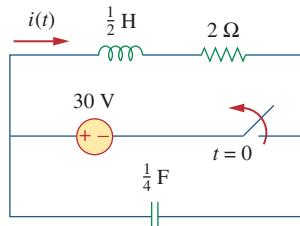
For Prob. 8.18.

- 8.19** Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 8.73.

**Figure 8.73**

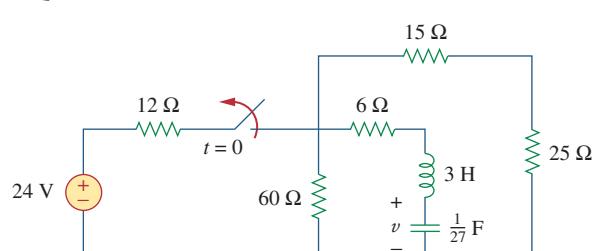
For Prob. 8.19.

- 8.20** The switch in the circuit of Fig. 8.74 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

**Figure 8.74**

For Prob. 8.20.

- *8.21** Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.

**Figure 8.75**

For Prob. 8.21.

* An asterisk indicates a challenging problem.

Section 8.4 Source-Free Parallel RLC Circuit

- 8.22** Assuming $R = 2 \text{ k}\Omega$, design a parallel RLC circuit that has the characteristic equation

$$s^2 + 100s + 10^6 = 0.$$

- 8.23** For the network in Fig. 8.76, what value of C is needed to make the response underdamped with unity damping factor ($\alpha = 1$)?

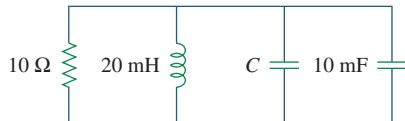


Figure 8.76

For Prob. 8.23.

- 8.24** The switch in Fig. 8.77 moves from position *A* to position *B* at $t = 0$ (please note that the switch must connect to point *B* before it breaks the connection at *A*, a make-before-break switch). Determine $i(t)$ for $t > 0$.

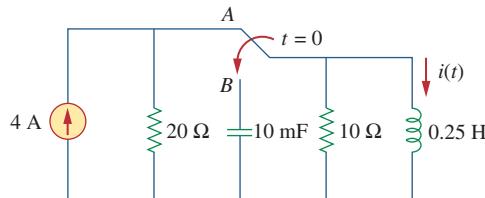


Figure 8.77

For Prob. 8.24.

- 8.25** Using Fig. 8.78, design a problem to help other **e2d** students better understand source-free RLC circuits.

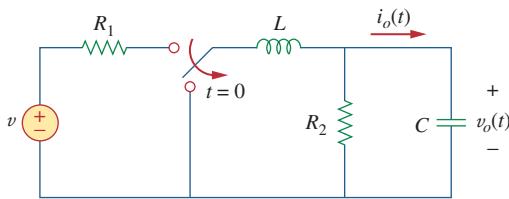


Figure 8.78

For Prob. 8.25.

Section 8.5 Step Response of a Series RLC Circuit

- 8.26** The step response of an RLC circuit is given by

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 2$ and $di(0)/dt = 4$, solve for $i(t)$.

- 8.27** A branch voltage in an RLC circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

- 8.28** A series RLC circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5 \text{ H}$, $R = 4 \Omega$, and $C = 0.2 \text{ F}$. Let $i(0) = 1$, $di(0)/dt = 0$.

- 8.29** Solve the following differential equations subject to the specified initial conditions

- (a) $d^2v/dt^2 + 4v = 12$, $v(0) = 0$, $dv(0)/dt = 2$
- (b) $d^2i/dt^2 + 5\frac{di}{dt} + 4i = 8$, $i(0) = -1$, $di(0)/dt = 0$
- (c) $d^2v/dt^2 + 2\frac{dv}{dt} + v = 3$, $v(0) = 5$, $dv(0)/dt = 1$
- (d) $d^2i/dt^2 + 2\frac{di}{dt} + 5i = 10$, $i(0) = 4$, $di(0)/dt = -2$

- 8.30** The step responses of a series RLC circuit are

$$v_C = 40 - 10e^{-2000t} - 10e^{-4000t} \text{ V}, \quad t > 0$$

$$i_L(t) = 3e^{-2000t} + 6e^{-4000t} \text{ mA}, \quad t > 0$$

- (a) Find C . (b) Determine what type of damping is exhibited by the circuit.

- 8.31** Consider the circuit in Fig. 8.79. Find $v_L(0^+)$ and $v_C(0^+)$.

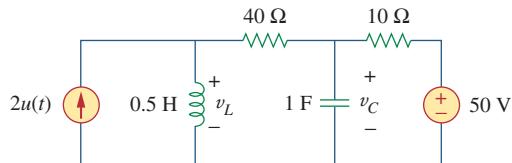


Figure 8.79

For Prob. 8.31.

- 8.32** For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

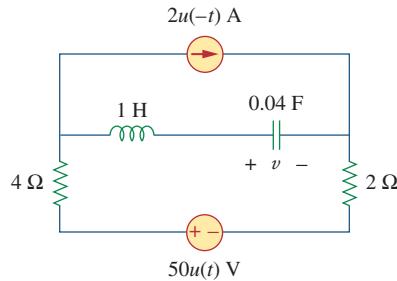


Figure 8.80

For Prob. 8.32.

8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

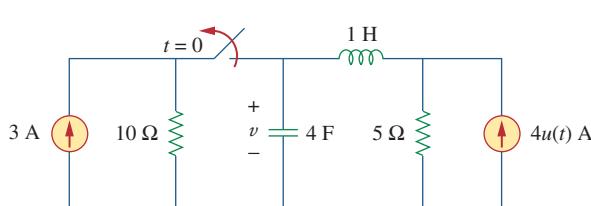


Figure 8.81

For Prob. 8.33.

8.34 Calculate $i(t)$ for $t > 0$ in the circuit of Fig. 8.82.

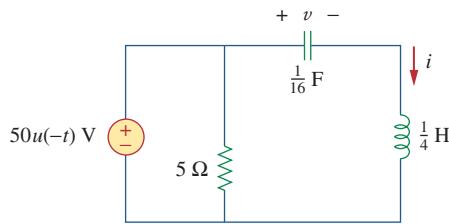


Figure 8.82

For Prob. 8.34.

8.35 Using Fig. 8.83, design a problem to help other students better understand the step response of series RLC circuits.

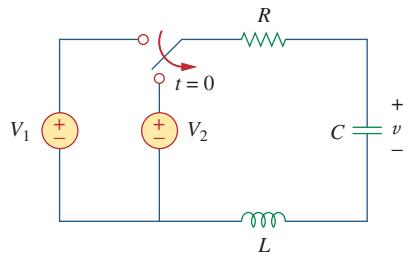


Figure 8.83

For Prob. 8.35.

8.36 Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 8.84.

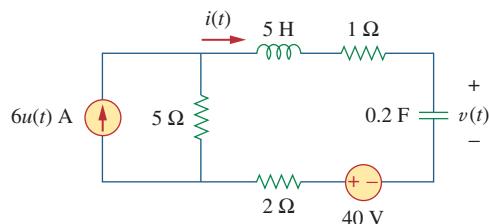


Figure 8.84

For Prob. 8.36.

***8.37** For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

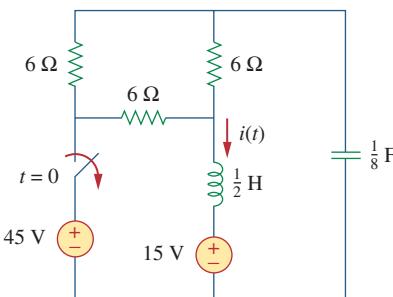


Figure 8.85

For Prob. 8.37.

8.38 Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$.

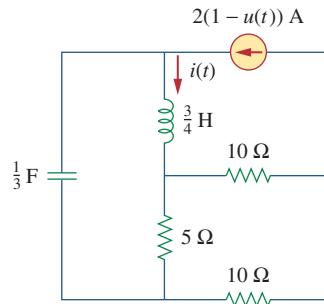


Figure 8.86

For Prob. 8.38.

8.39 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 8.87.

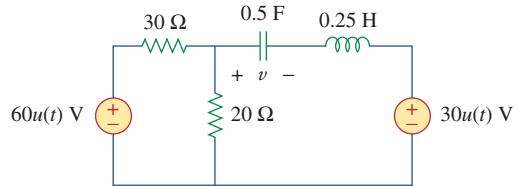


Figure 8.87

For Prob. 8.39.

8.40 The switch in the circuit of Fig. 8.88 is moved from position *a* to *b* at $t = 0$. Determine $i(t)$ for $t > 0$.

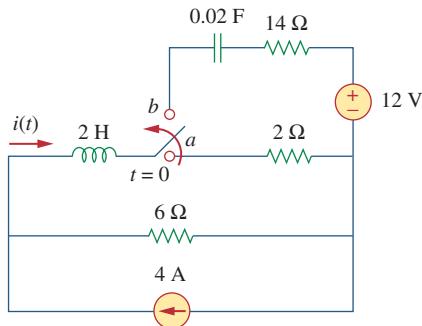


Figure 8.88

For Prob. 8.40.

- *8.41 For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

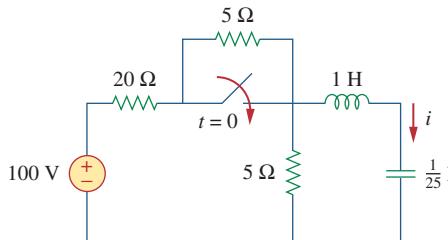


Figure 8.89

For Prob. 8.41.

- *8.42 Given the network in Fig. 8.90, find $v(t)$ for $t > 0$.

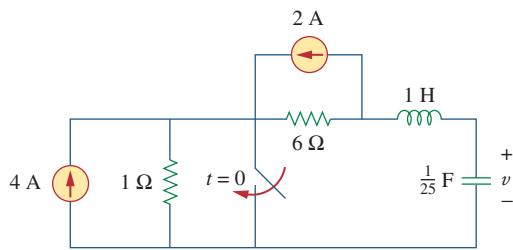


Figure 8.90

For Prob. 8.42.

- 8.43 The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8 \text{ Np/s}$ and $\omega_d = 30 \text{ rad/s}$.

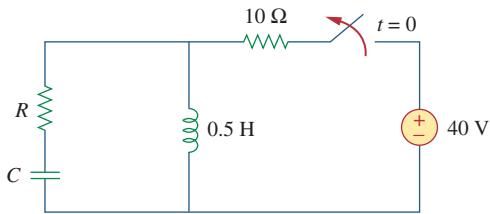


Figure 8.91

For Prob. 8.43.

- 8.44 A series RLC circuit has the following parameters: $R = 1 \text{ k}\Omega$, $L = 1 \text{ H}$, and $C = 10 \text{ nF}$. What type of damping does this circuit exhibit?

Section 8.6 Step Response of a Parallel RLC Circuit

- 8.45 In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0 \text{ V}$ and $i(0) = 1 \text{ A}$.

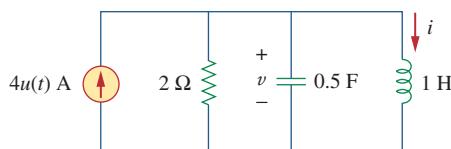


Figure 8.92

For Prob. 8.45.

- 8.46 Using Fig. 8.93, design a problem to help other students better understand the step response of a parallel RLC circuit.

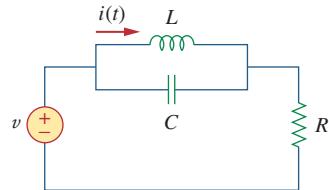


Figure 8.93

For Prob. 8.46.

- 8.47 Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

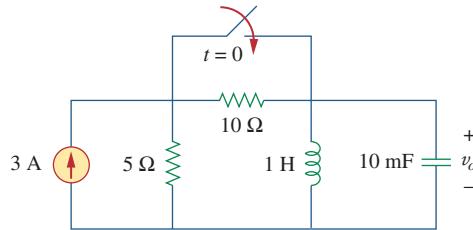


Figure 8.94

For Prob. 8.47.

- 8.48 Given the circuit in Fig. 8.95, find $i(t)$ and $v(t)$ for $t > 0$.

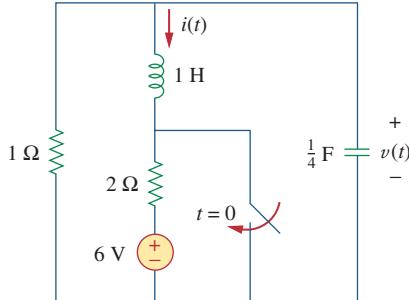


Figure 8.95

For Prob. 8.48.

- 8.49 Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

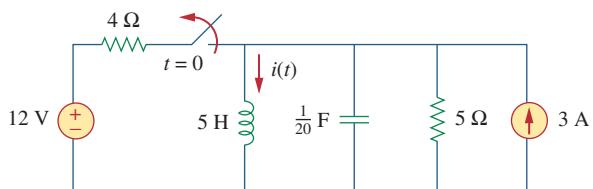


Figure 8.96

For Prob. 8.49.

- 8.50** For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

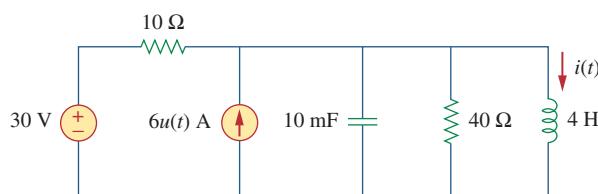


Figure 8.97

For Prob. 8.50.

- 8.51** Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.98.

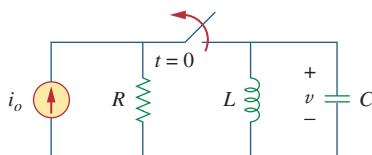


Figure 8.98

For Prob. 8.51.

- 8.52** The step response of a parallel RLC circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2 \sin 400t) \text{ V}, \quad t \geq 0$$

when the inductor is 50 mH. Find R and C .

Section 8.7 General Second-Order Circuits

- 8.53** After being open for a day, the switch in the circuit of Fig. 8.99 is closed at $t = 0$. Find the differential equation describing $i(t)$, $t > 0$.

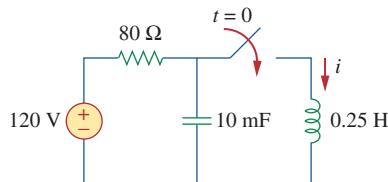


Figure 8.99

For Prob. 8.53.

- 8.54** Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

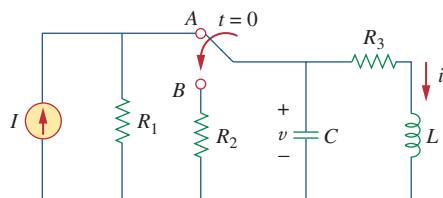


Figure 8.100

For Prob. 8.54.

- 8.55** For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4 \text{ V}$ and $i(0^+) = 2 \text{ A}$.

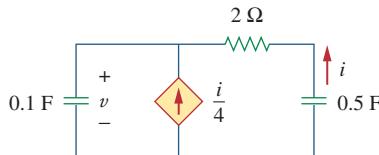


Figure 8.101

For Prob. 8.55.

- 8.56** In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

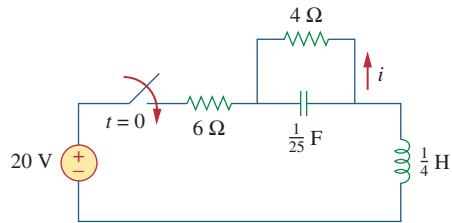


Figure 8.102

For Prob. 8.56.

- 8.57** If the switch in Fig. 8.103 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine:
- the characteristic equation of the circuit,
 - i_x and v_R for $t > 0$.

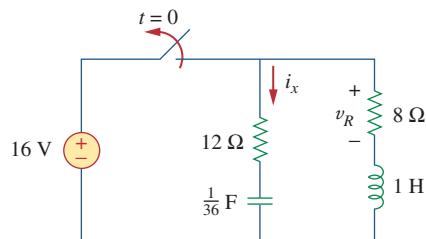


Figure 8.103

For Prob. 8.57.

- 8.58** In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

- $v(0^+)$, $dv(0^+)/dt$,
- $v(t)$ for $t \geq 0$.

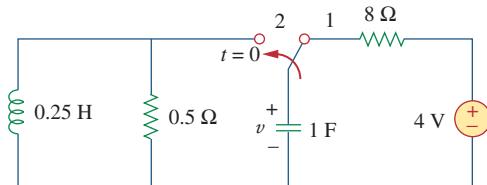
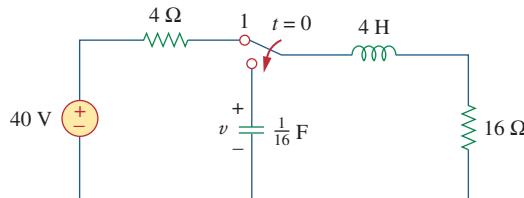


Figure 8.104

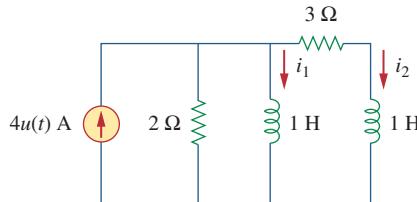
For Prob. 8.58.

- 8.59** The switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at $t = 0$. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

**Figure 8.105**

For Prob. 8.59.

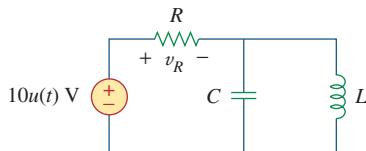
- 8.60** Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

**Figure 8.106**

For Prob. 8.60.

- 8.61** For the circuit in Prob. 8.5, find i and v for $t > 0$.

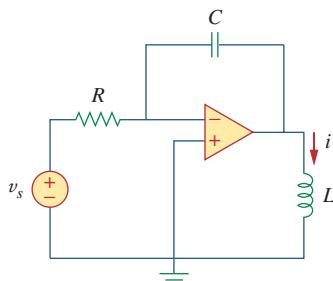
- 8.62** Find the response $v_R(t)$ for $t > 0$ in the circuit of Fig. 8.107. Let $R = 3 \Omega$, $L = 2 \text{ H}$, and $C = 1/18 \text{ F}$.

**Figure 8.107**

For Prob. 8.62.

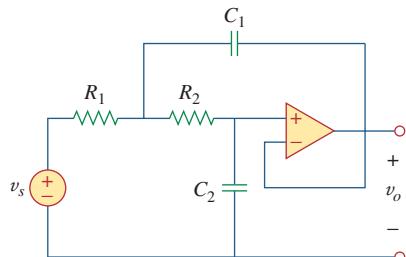
Section 8.8 Second-Order Op Amp Circuits

- 8.63** For the op amp circuit in Fig. 8.108, find the differential equation for $i(t)$.

**Figure 8.108**

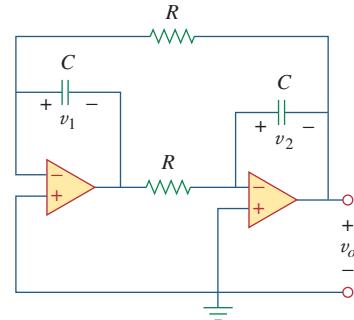
For Prob. 8.63.

- 8.64** Using Fig. 8.109, design a problem to help other students better understand second-order op amp circuits.

**Figure 8.109**

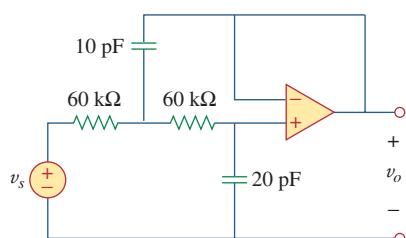
For Prob. 8.64.

- 8.65** Determine the differential equation for the op amp circuit in Fig. 8.110. If $v_1(0^+) = 2 \text{ V}$ and $v_2(0^+) = 0 \text{ V}$, find v_o for $t > 0$. Let $R = 100 \text{ k}\Omega$ and $C = 1 \mu\text{F}$.

**Figure 8.110**

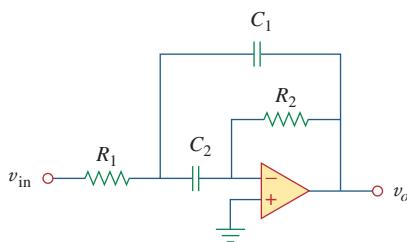
For Prob. 8.65.

- 8.66** Obtain the differential equations for $v_o(t)$ in the op amp circuit of Fig. 8.111.

**Figure 8.111**

For Prob. 8.66.

- *8.67 In the op amp circuit of Fig. 8.112, determine $v_o(t)$ for $t > 0$. Let $v_{in} = u(t)$ V, $R_1 = R_2 = 10 \text{ k}\Omega$, $C_1 = C_2 = 100 \mu\text{F}$.

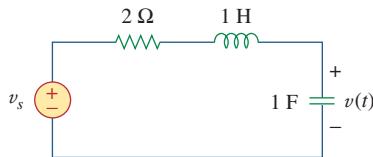
**Figure 8.112**

For Prob. 8.67.

Section 8.9 PSpice Analysis of RLC Circuit

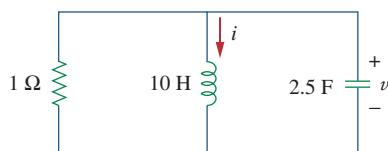


- 8.68 For the step function $v_s = u(t)$, use PSpice or MultiSim to find the response $v(t)$ for $0 < t < 6$ s in the circuit of Fig. 8.113.

**Figure 8.113**

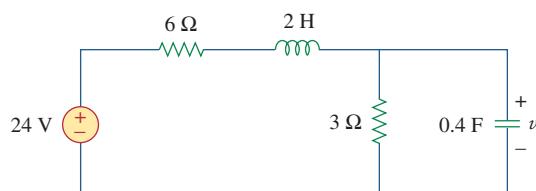
For Prob. 8.68.

- 8.69 Given the source-free circuit in Fig. 8.114, use PSpice or MultiSim to get $i(t)$ for $0 < t < 20$ s. Take $v(0) = 30$ V and $i(0) = 2$ A.

**Figure 8.114**

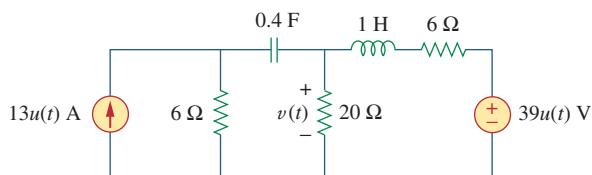
For Prob. 8.69.

- 8.70 For the circuit in Fig. 8.115, use PSpice or MultiSim to obtain $v(t)$ for $0 < t < 4$ s. Assume that the capacitor voltage and inductor current at $t = 0$ are both zero.

**Figure 8.115**

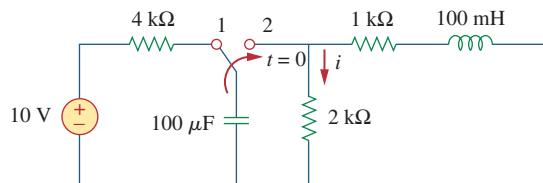
For Prob. 8.70.

- 8.71 Obtain $v(t)$ for $0 < t < 4$ s in the circuit of Fig. 8.116 using PSpice or MultiSim.

**Figure 8.116**

For Prob. 8.71.

- 8.72 The switch in Fig. 8.117 has been in position 1 for a long time. At $t = 0$, it is switched to position 2. Use PSpice or MultiSim to find $i(t)$ for $0 < t < 0.2$ s.

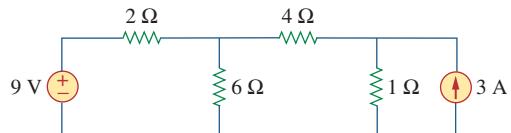
**Figure 8.117**

For Prob. 8.72.

- 8.73 Design a problem, to be solved using PSpice or **e2d** MultiSim, to help other students better understand source-free RLC circuits.

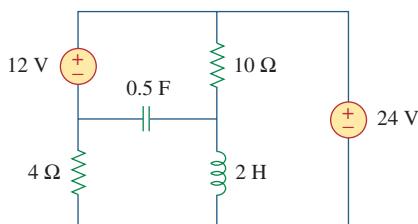
Section 8.10 Duality

- 8.74 Draw the dual of the circuit shown in Fig. 8.118.

**Figure 8.118**

For Prob. 8.74.

- 8.75 Obtain the dual of the circuit in Fig. 8.119.

**Figure 8.119**

For Prob. 8.75.

- 8.76** Find the dual of the circuit in Fig. 8.120.

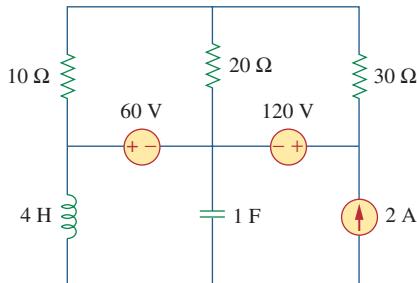


Figure 8.120

For Prob. 8.76.

- 8.77** Draw the dual of the circuit in Fig. 8.121.

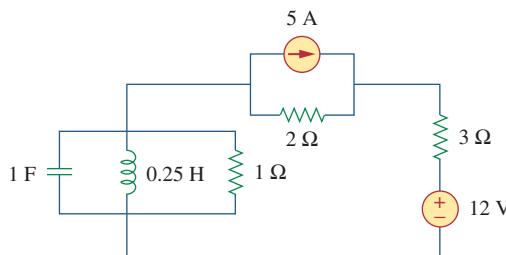


Figure 8.121

For Prob. 8.77.

Section 8.11 Applications

- 8.78** An automobile airbag igniter is modeled by the circuit in Fig. 8.122. Determine the time it takes the voltage across the igniter to reach its first peak after switching from A to B . Let $R = 3\ \Omega$, $C = 1/30\text{ F}$, and $L = 60\text{ mH}$.

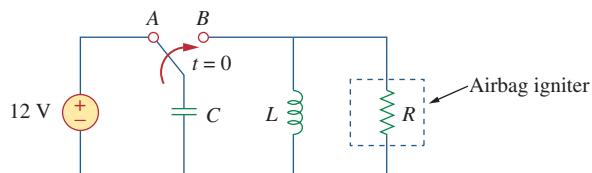


Figure 8.122

For Prob. 8.78.

- 8.79** A load is modeled as a 250-mH inductor in parallel with a $12\text{-}\Omega$ resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz . Calculate the size of the capacitor.

Comprehensive Problems

- 8.80** A mechanical system is modeled by a series RLC circuit. It is desired to produce an overdamped response with time constants 0.1 ms and 0.5 ms . If a series $50\text{-k}\Omega$ resistor is used, find the values of L and C .

- 8.81** An oscilloscope can be adequately modeled by a second-order system in the form of a parallel RLC circuit. It is desired to give an underdamped voltage across a $200\text{-}\Omega$ resistor. If the damping frequency is 4 kHz and the time constant of the envelope is 0.25 s , find the necessary values of L and C .

- 8.82** The circuit in Fig. 8.123 is the electrical analog of body functions used in medical schools to study convulsions. The analog is as follows:

C_1 = Volume of fluid in a drug

C_2 = Volume of blood stream in a specified region

R_1 = Resistance in the passage of the drug from the input to the blood stream

R_2 = Resistance of the excretion mechanism, such as kidney, etc.

v_0 = Initial concentration of the drug dosage

$v(t)$ = Percentage of the drug in the blood stream

Find $v(t)$ for $t > 0$ given that $C_1 = 0.5\ \mu\text{F}$, $C_2 = 5\ \mu\text{F}$, $R_1 = 5\text{ M}\Omega$, $R_2 = 2.5\text{ M}\Omega$, and $v_0 = 60u(t)\text{ V}$.

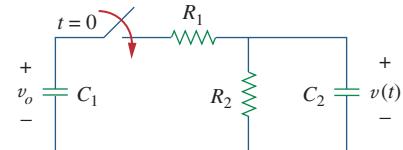


Figure 8.123

For Prob. 8.82.

- 8.83** Figure 8.124 shows a typical tunnel-diode oscillator circuit. The diode is modeled as a nonlinear resistor with $i_D = f(v_D)$, i.e., the diode current is a nonlinear function of the voltage across the diode. Derive the differential equation for the circuit in terms of v and i_D .

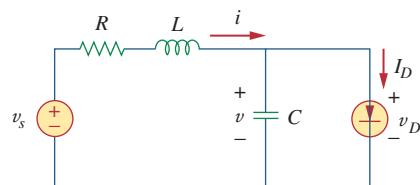


Figure 8.124

For Prob. 8.83.