The Universal Speed Controller for Time-Minimal and without Overshoot Speed Control of DC Motor

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***Abstract* —This paper concerns the universal speed controller for time-minimal and without overshoot speed control of DC motor. The nonlinear shape of speed controller characteristic was obtained with use dynamic programming approach on the basis of motor speed and motor current step responses. The motor current and speed responses could be periodical or non-periodical depending on the relationship between DC motor parameters. All the parameters relationships were considered. The speed controller is universal, because its is right not depending on the motor parameters relationships.**

***Keywords* — Control of drive, highly dynamic drive.**

I. Introduction

THE electric drives of robots and machine tools ought to feature with a high productivity. The productivity is usually increased by shortening the time of position or speed control process of a tool. Generally speaking, if speed control time of the tool is shorter, the productivity of the machine tool is higher. The bang-bang control methods of robots allow to obtain short control time [1] – [4]. Mentioned methods set reference motor torque from maximal to minimal value. The mentioned control methods are time - minimal, if the time of motor current (torque) control process is neglected. However, the current (the torque) of the electrical motor can not change immediately, because of motor inductance.

The current paper concerns the extremely fast and without overshoot speed control method for the electric drives with DC motor with taking into account the motor current (torque) control process.

The DC motors with mechanical armature are usually supplied by DC/DC transistor switching converters. It is assumed that the motor armature voltage changes in the shape of square wave (not current, not torque). The motor current and torque change in the shape of triangle. The motor voltage is constant, between converter switching processes, not depending on the shape of motor current and reference current responses. This information is important, because the motor voltage can be analyzed as a many steps. To continue, motor current and motor speed can be analyzed as step responses, between switching processes. Moreover, the whole speed control process can be analyzed as composition of many speed step responses, like it was shown in paper [5]. The shape of motor current and motor speed step responses depend on relationship between DC motor parameters. To continue, the motor speed and the motor current step responses can be periodical or non-periodical, depending on relationship between motor parameters. In paper [5], the time-minimal and without overshoot speed control system was worked out. The speed controller characteristic was found only for non-periodical motor speed and motor current responses [5]. In addition, it was assumed that the load torque was constant during speed control process.

In current paper, all the cases of motor parameters relationships are analyzed. In addition, the linear change of load torque change is assumed.

II. Speed Control System Structure

The speed control system consists of DC motor, DC/DC switching converter, current controller with delta-modulation, current limiter, nonlinear speed controller and load torque and speed observer, like it is shown in Fig. 1.

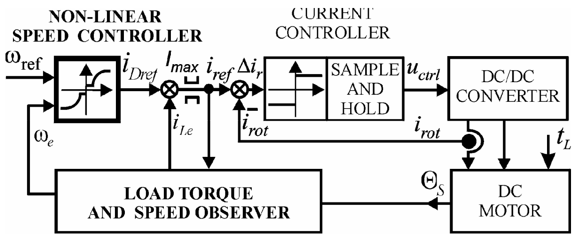

Fig. 1. The scheme of speed control system.

The DC motor is supplied by the four-quadrant DC/DC converter containing the four-transistors bridge. The structure of DC/DC converter is shown in Fig. 2. The transistors converter is controlled by binary output signal u_{ctrl} of the non-linear current controller. The sign of the converter voltage u_r is equal to the sign of the current control error Δi_r. The value of the motor voltage u_r, between switching converter processes, is equal to the capacitor voltage U_{DC}. The current control error Δi_r is the difference between the reference current i_{ref} and the motor

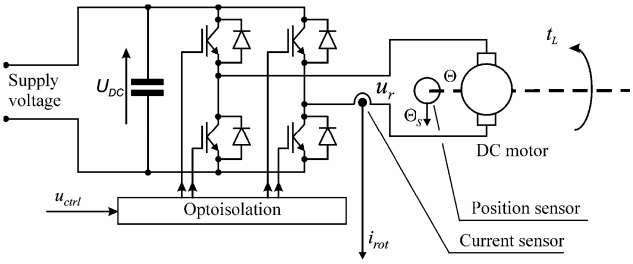

Fig. 2. The DC/DC converter supplying the DC motor.

current Δi_{rot}. The reference current is limited to the value I_{MAX} in the motor current limiter, see in Fig.1. Limitation is necessary in order not to damage the converter, the motor and the driven machine tool. It is possible to distinguish two states of sped control system depending on limiter work. If the reference current is limited to value I_{MAX}, there is the saturation state of speed control system. When the absolute value of reference current is lower than value I_{MAX}, there is the active state of speed control system. In the active state of speed control system, the reference current i_{ref} is calculated as a sum of output signal i_{Dref} of speed controller and output signal i_{Le} of load torque and speed observer.

The observer and the nonlinear speed controller fulfill different tasks in the speed control system in Fig. 1. The tasks division follows from division of reference current i_{ref} into two components. The reference current i_{ref} is composed of the estimated load current i_{Le} and the reference dynamic current i_{Dref}. The estimated load current i_{Le} compensates the load torque and the current control error Δi_r, whereas the reference dynamic current i_{Dref} influences only the motor acceleration torque. Therefore, the observer is used for compensation of the load torque t_L. As the result of compensation, the steady–state speed control error $\Delta\omega$ is equal to zero not depending on the motor load. In contrast, the nonlinear speed controller influences only the motor acceleration torque. This fact makes possible to use the dynamic programming approach for finding the speed controller characteristic. This paper concerns the universal nonlinear characteristic of the speed controller.

III. The Load Torque and Speed Observer

The load torque observer functions as a precise load torque compensator. This issue will be explained with use equivalent observer scheme.

The electromechanical subsystem of DC motor is the observation object. The subsystem is electromechanical, because the input signal is the reference electrical current i_{ref}, whereas the subsystem output signal is the output signal of the mechanical position Θ_s sensor. The acceleration of the motor electromechanical subsystem is described with differential equation:

$$d^2\Theta_s / dt^2 = (k_M / J)\left(i_{ref} - \Delta i_r - (t_L / k_M)\right), \qquad (1)$$

in which, k_M is motor torque constant, J is moment of inertia.

The observer input signals are reference motor current i_{ref} and output signal Θ_s of position sensor.

The observer is divided into subsystems such as: the parallel model of the electromechanical subsystem, the filter of position error, the adjusting unit and the limiter of load torque derivative.

The observer equivalent scheme is derived on the basis of mathematical observer description. It is assumed that position sensor is very accurate so that the input signal Θ and the output signal Θ_s of position sensor are the same. It is also assumed the input signal Θ_e and the output signal Θ_{es} of position sensor model are the same. The parallel model of the electromechanical subsystem is described with differential equation:

$$\begin{aligned}\left(d^2\Theta_e / dt^2\right) &= (k_M / J)\left(i_{ref} - i_{Le}\right) + \\ &+ 15\cdot\Omega^4\left(d\Delta\Theta_{ef} / dt\right) + 6\cdot\Omega^5\cdot\Delta\Theta_{ef},\end{aligned} \qquad (2)$$

where Θ_e is the output signal of parallel model of the electromechanical subsystem, $\Delta\Theta_{ef}$ is the output signal of filter of position error, Ω is the inverse of observer time constant in units [1/s]. The position error filter is described with differential equation:

$$\begin{aligned}&\left(d^3\Delta\Theta_{ef} / dt^3\right) - 6\cdot\Omega\cdot\left(d^2\Delta\Theta_{ef} / dt^2\right) - \\ &- 15\cdot\Omega^2\cdot\left(d\Delta\Theta_{ef} / dt\right) - 20\cdot\Omega^3\cdot\Delta\Theta_{ef} = \Delta\Theta_e\end{aligned} \qquad (3)$$

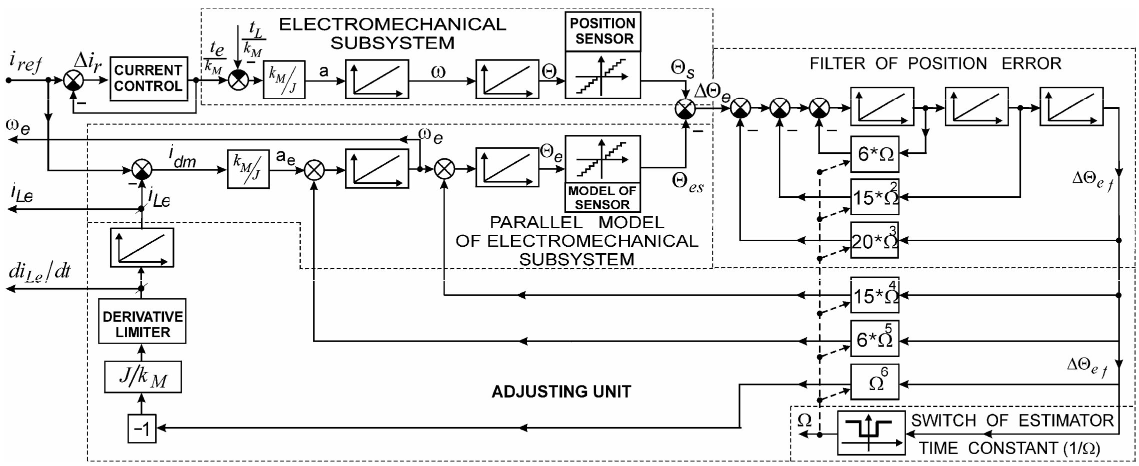

Fig. 3. The load torque and speed observer.

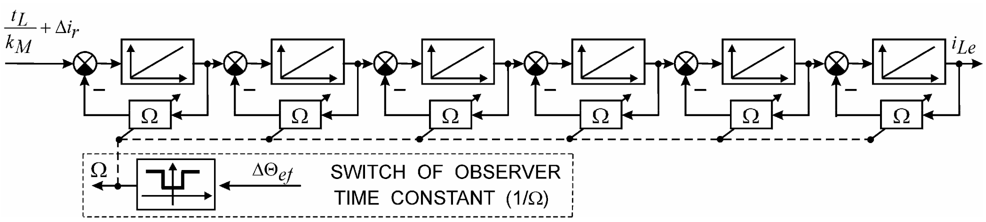

Fig. 4. The equivalent scheme of load torque estimation loop.

The position error $\Delta\Theta_e$ is the difference between the electromechanical subsystem position Θ_s and the output signal Θ_e of parallel model of the electromechanical subsystem. The adjusting unit is described with differential equation:

$$\frac{d}{dt} i_{Le} = -\Omega^6 \cdot (J/k_M) \cdot \Delta\Theta_{ef} \tag{4}$$

if we neglect the derivative limiter influence.

Next, the equations (1) - (4) are converted. The differential equation of position error $\Delta\Theta_e$:

$$\begin{aligned} \left(d^2\Delta\Theta_e / dt^2\right) &= -15 \cdot \Omega^4 \cdot \left(d\Delta\Theta_{ef} / dt\right) + \\ &- 6 \cdot \Omega^5 \cdot \Delta\Theta_{ef} - (k_M / J) \cdot \Delta i_L \end{aligned} \tag{5}$$

is obtained by subtracting (2) from (1). The symbol Δi_L means load current error:

$$\Delta i_L = (t_L / k_M) + \Delta i_r - i_{Le} \,. \tag{6}$$

The differential equation describing the influence of load current error Δi_L on filtered position error $\Delta\Theta_{ef}$:

$$\begin{aligned} &\left(d^5\Delta\Theta_{ef} / dt^5\right) + 6 \cdot \Omega \cdot \left(d^4\Delta\Theta_{ef} / dt^4\right) + \\ &+ 15 \cdot \Omega^2 \cdot \left(d^3\Delta\Theta_{ef} / dt^3\right) + 20 \cdot \Omega^3 \cdot \left(d^2\Delta\Theta_{ef} / dt^2\right) + \\ &+ 15 \cdot \Omega^4 \cdot \left(d\Delta\Theta_{ef} / dt\right) + 6 \cdot \Omega^5 \cdot \Delta\Theta_{ef} = \\ &= -(k_M / J) \cdot \Delta i_L \end{aligned} \tag{7}$$

is obtained by putting (5) to the result of two times differential calculus of (3). Next, the influence of load torque t_L and current control error Δi_r on estimated load current i_{Le}:

$$\begin{aligned} &\left(d^6 i_{Le} / dt^6\right) + 6 \cdot \Omega \cdot \left(d^5 i_{Le} / dt^5\right) + \\ &+ 15 \cdot \Omega^2 \cdot \left(d^4 i_{Le} / dt^4\right) + 20 \cdot \Omega^3 \cdot \left(d^3 i_{Le} / dt^3\right) + \\ &+ 15 \cdot \Omega^4 \cdot \left(d^2 i_{Le} / dt^2\right) + 6 \cdot \Omega^5 \cdot \left(d i_{Le} / dt\right) + \Omega^6 \cdot i_{Le} = \\ &= \Omega^6 \cdot \left((t_L / k_M) + \Delta i_r\right). \end{aligned} \tag{8}$$

is obtained by putting the result of differential calculus of (4) into (8). The equation (8) shows that the relationship between the load torque and the estimated load current. On the basis of equation (8) scheme in Fig. 4 was drawn. The Fig. 4 shows the equivalent scheme of load torque estimation loop. The input signal of this dynamic system is the sum of unknown load torque t_L and the current control error Δi_r. The output signal of this loop is the estimated load current i_{Le}.

This is important to notice that the estimated load current i_{Le} does not depend on the speed control error $\Delta\omega$ and the motor acceleration torque. This feature takes only place under the condition that the parameters of the parallel model are the same as parameters of the motor electromechanical subsystem. In order to fulfill this condition, the load torque observer should be developed into the electromechanical subsystem estimator [5]. The electromechanical subsystem estimator contains additional loop for the moment inertia and the motor constant estimation. The moment inertia and the motor constant estimation loop is used for the adaptation the whole speed control system (observer-estimator and speed controller) to the actual moment of inertia. The adaptation to the actual moment of inertia and the motor constant is not the aim of this work.

The observer structure allows to change the time constant (1/Ω) with ease, like it is shown in Fig. 4. The observer time constant is changed by the switch of observer time constant, see in Fig. 3. When filtered position error is high, the observer time constant (1/Ω) is low, in order to obtain fast observer response. When filtered position error is low, the observer time constant (1/Ω) is high, for better reducing noise of the filtered position error $\Delta\Theta_{ef}$. This issue is explained in the text below.

If the noise of filtered position error is lower the noise of the estimated speed is lower. As the result of position error noise reducing, the speed control range is wider. Next, the relationship between the speed estimation error $\Delta\omega_e$ and the filtered position error $\Delta\Theta_{ef}$ is found. The speed estimation error $\Delta\omega_e$:

$$\Delta\omega_e = (d\Theta / dt) - \omega_e . \tag{9}$$

is the difference between the mechanical motor speed (dΘ/dt) and the estimated speed ω_e. According to the observer scheme in Fig. 3, the estimated speed error:

$$(d\Theta_e / dt) = \omega_e + 15 \cdot \Omega^4 \cdot \Delta\Theta_{ef} \,, \tag{10}$$

depends on the filtered position error $\Delta\Theta_{ef}$ and the output signal Θ_e of parallel model. The formula:

$$\Delta\omega_e = (d\Delta\Theta_e / dt) + 15 \cdot \Omega^4 \cdot \Delta\Theta_{ef} \tag{11}$$

is obtained by putting (10) into (9). Next by putting (3) into (11), the relationship between speed estimation error $\Delta\omega_e$ and filtered position error $\Delta\Theta_{ef}$ is found:

$$\begin{aligned} \Delta\omega_e &= \left(d^4\Delta\Theta_{ef} / dt^4\right) + 6 \cdot \Omega \cdot \left(d^3\Delta\Theta_{ef} / dt^3\right) + \\ &+ 15 \cdot \Omega^2 \cdot \left(d^2\Delta\Theta_{ef} / dt^2\right) + 20 \cdot \Omega^3 \cdot \left(d\Delta\Theta_{ef} / dt\right) + \\ &+ 15 \cdot \Omega^4 \cdot \Delta\Theta_{ef} . \end{aligned} \tag{12}$$

The equation (12) acknowledges the following rule. If the parameter Ω is lower, the influence the filtered position error derivatives on the speed estimation error is lower. Therefore, the position noise is better filtered if the observer time constant (1/Ω) is higher.

The differential equations (7) and (12) proves also that, the load torque and speed observer assure the load torque compensation. To continue, the compensation is necessary in order to decrease the steady–state speed control error $\Delta\omega$ to zero not depending on the load torque. If the load current error Δi_L is equal to zero, the speed estimation error $\Delta\omega_e$ is equal to zero. It means that, there are no difference between estimated speed ω_e and the motor speed ω in the case of the accurate compensation. It is important, because the estimated speed is used as the feedback signal for the speed controller. To continue, if we analyze (1) in case the load torque error Δi_L is equal to zero, we will come to conclusion that the speed controller output signal i_{Dref} influences only the motor acceleration:

$$d^2\Theta_s / dt^2 = (k_M / J) \cdot i_{Dref}, \qquad (13)$$

The motor acceleration does not depend on load torque t_L. It means that the load torque observer functions as a precise load torque compensator.

There is a problem of load torque observation. According to (6), the observer is a strict load torque compensator when dynamic component of current control error is equal to zero. The problem follows from that the estimated load current i_{Le} is the component of reference motor current i_{ref}. There is the possibility, that the estimated load current i_{Le} could change so fast that could cause the high current control error Δi_r. In order to prevent high dynamic component of the current control error Δi_r, the limiter of the load current derivative is put into the observer structure. The scheme of the limiter of the load torque derivative di_{Le}/dt is show in Fig. 5.

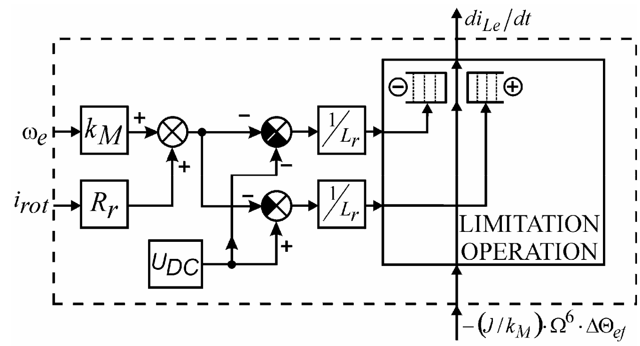

Fig. 5. The limiter of the load current derivative.

The current control error Δi_r is low, when absolute value of the motor current derivative is lower than the absolute value of the reference current derivative and the estimated load current derivative di_{Le}/dt. Therefore, the estimated load current derivative di_{Le}/dt is limited so that it can not be higher than the positive motor current derivative:

$$di_{r\_p} / dt = (1/L_r) \cdot (U_{DC} - \omega_e \cdot k_M - R_r \cdot i_{rot}), \qquad (14)$$

and can not be lower than negative motor current derivative:

$$di_{r\_n} / dt = (1/L_r) \cdot (-U_{DC} - \omega_e \cdot k_M - R_r \cdot i_{rot}). \qquad (15)$$

IV. Nonlinear Speed Controller

The speed controller makes possible to obtain the time - minimal and without overshoot speed control process of DC motor. The speed controller influences only the motor acceleration. This speed controller feature is important, because the nonlinear shape of the speed controller characteristic can be obtained with use Bellman dynamic programming method. The nonlinear shape of speed controller characteristic is obtained on the basis of the speed control error $\Delta\omega$ and the reference dynamic current i_{Dref}, see in Fig. 1, in active state of speed control system.

Firstly, the mathematical description the reference dynamic current is searched, as a function of time. As it was mentioned before, there is the condition of proper speed control system operation. The current control error Δi_r must be equal to zero, in order to ensure the precise work of the load torque observer. This condition contains information that the reference current i_{ref} and the motor current i_{rot} shapes have to be the same. The reference current i_{ref} is the sum of the reference dynamic current i_{Dref} and the estimated load current i_{Le}, see in Fig. 1. On the basis of above information the reference dynamic current:

$$i_{Dref} = i_{rot} - i_{Le} \qquad (16)$$

can be found as a subtract the estimated load current i_{Le} response from motor current i_{rot} step response.

To continue, the linear load torque change is assumed. The load torque is described as a linear function of time:

$$t_L(t) = T_L + t \cdot \alpha \qquad (17)$$

where T_L is constant component of load torque, α is the first derivative of load torque. Next, it is assumed that the estimated load current i_{Le} response time is short in relation to linear load torque rising (or falling) time. The fast estimator response is obtained by setting the properly short observer time constant (1/Ω), see in Fig. 4. As the result, the estimated load current is described with linear function of time:

$$i_{Le}(t) = I_{Le} + t \cdot (di_{Le}/dt) \qquad (18)$$

in which, I_{Le} is constant component of estimated load current response, (di_{Le}/dt) is the estimated load current derivative. The constant component of estimated load current I_{Le} is equal to ratio between the constant component of the load torque T_L and the motor torque constant k_M, compare (17) with (18).

The motor current step response is found on the basis of linear model of DC motor. DC motor was modeled as second order inertial system:

$$d\omega/dt = (k_M / J)(i_{rot} - (t_L / k_M)) \qquad (19)$$

$$di_{rot}/dt = (1/L_r)(u_r - R_r \cdot i_{rot} - \omega \cdot k_M) \qquad (20)$$

It is assumed that capacity of converter capacitor is such a high, that output converter voltage u_r can be considered as a constant between switching processes.

Let we consider the case of the positive speed control error. At the last stage of speed control process (in the active state of speed control system), the current control error should be negative [1] in order to decrease the motor current i_{rot} to value of the estimated load current i_{Le}, see in Fig. 6. In the active state of the speed control system, the converter output voltage:

$$U_r(s) = -U_{DC}/s \tag{21}$$

can be described as negative step. On the basis of linear DC motor model (19), (20) and its input voltage (21) and the load torque (17), the motor current step response will be obtained from formula:

$$I_{rot}(s) = \left((-U_{DC} - k_M \Omega(0)/L_r) + I_{rot}(0) \cdot s + \right. \\ \left. + \frac{k_M}{J} \cdot \frac{T_L}{L_r} \cdot \frac{1}{s} + \frac{k_M}{J} \cdot \frac{\alpha}{L_r} \cdot \frac{1}{s^2} \right) \Big/ ((s+a)(s+b)) \tag{22}$$

where $I_{rot}(0)$ is the initial value of the motor current, $\Omega(0)$ is the initial value of the motor speed at the beginning of the speed control process in the active state of speed control system. Constants a and b depend on motor parameters:

$$a = 0{,}5 \cdot (R_r/L_r) \left(1 - \sqrt{1 - 4 \cdot (k_M^2 \cdot L_r) / (R_r^2 \cdot J)} \right) \tag{23}$$

$$b = 0{,}5 \cdot (R_r/L_r) \left(1 + \sqrt{1 - 4 \cdot (k_M^2 \cdot L_r) / (R_r^2 \cdot J)} \right) \tag{24}$$

Secondly, the mathematical description the speed control error is searched, as a function of time. The speed control error is the subtract the estimated speed ω_e from reference speed ω_{ref}. If the load torque is compensated, the estimated speed ω_e is equal to the motor mechanical speed. On the basis of the linear motor model (19), (20) and its input voltage (21) and load torque (17), the motor speed step response will be obtained from formula:

$$\Omega(s) = \left(-(U_{DC} k_M + T_L \cdot R_r + L_r \cdot \alpha)/L_r \cdot J \cdot s + \right. \\ + (k_M I_{rot}(0) - T_L)/J + (R_r \cdot \Omega(0)/L_r) + \\ \left. + \Omega(0) \cdot s + (\alpha \cdot R_r / L_r \cdot J) \cdot s^{-2} \right) \Big/ ((s+a)(s+b)) \tag{25}$$

The motor speed and motor current step responses depend on relationship motor parameters. If the constants a and b are real and different (23), (24) the motor current and the motor speed step responses are following:

$$i_{rot}(t) = A_1 \cdot e^{-a \cdot t} + A_2 \cdot e^{-b \cdot t} + A_3 \cdot t + A_4, \tag{26}$$

$$\omega(t) = B_1 \cdot e^{-a \cdot t} + B_2 \cdot e^{-b \cdot t} + B_3 \cdot t + B_4, \tag{27}$$

where A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 are constants. Constants depend on the motor parameters. If the constants a and b are real and the same:

$$a = b = 0{,}5 \cdot R_r / L_r, \tag{28}$$

the motor current and the motor speed step responses are following:

$$i_{rot}(t) = C_1 \cdot t \cdot e^{-a \cdot t} + C_2 \cdot e^{-a \cdot t} + C_3 \cdot t + C_4, \tag{29}$$

$$\omega(t) = D_1 \cdot e^{-a \cdot t} + D_2 \cdot e^{-b \cdot t} + D_3 \cdot t + D_4, \tag{30}$$

where C_1, C_2, C_3, C_4, D_1, D_2, D_3, D_4 are constants. If the constants a and b are complex:

$$a = \sigma - j\omega_0, \tag{31}$$

$$b = \sigma + j\omega_0 \tag{32}$$

the motor current and the motor speed step responses are following:

$$i_{rot}(t) = E_1 \cdot e^{-\sigma \cdot t} \cdot \sin(\omega_0 \cdot t) + \\ + E_2 \cdot e^{-\sigma \cdot t} \cdot \cos(\omega_0 \cdot t) + E_3 \cdot t + E_4, \tag{33}$$

$$\omega(t) = F_1 \cdot e^{-\sigma \cdot t} \cdot \sin(\omega_0 \cdot t) + \\ + F_2 \cdot e^{-\sigma \cdot t} \cdot \cos(\omega_0 \cdot t) + F_3 \cdot t + F_4. \tag{34}$$

Symbols σ, ω_0, mean constants:

$$\sigma = 0{,}5 R_r / (L_r), \tag{35}$$

$$\omega_0 = 0{,}5 (R_r / L_r) \sqrt{4 \cdot (k_M^2 \cdot L_r) / (R_r^2 \cdot J) - 1}. \tag{36}$$

The formulas describing the influence motor parameters on constants: A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4, D_1, D_2, D_3, D_4, E_1, E_2, E_3, E_4, F_1, F_2, F_3, F_4 are not in paper, because they are very long. However, they can be obtained on the basis of (22) and (25).

Formulas (26), (27), (29), (30), (33), (34) describing the motor current and the motor speed step responses are completely different. However, the aim of this work is to find the universal speed controller, which would be right for three sets of constants a and b. Therefore, the universal and simplified description of formulas (26), (27), (29), (30), (33), (34) is searched. The aim of simplification of mathematical description of the motor current and the motor speed step responses, is explained in [5]. The formulas are simplified by development into Maclauren's series. As the result of formulas (26), (29), (33) development into series, the same linear function of time is obtained:

$$i_{rot}(t) = I_{rot}(0) - ((U_{DC} + k_M \Omega(0) + R_r I_{rot}(0))/L_r) \cdot t \tag{37}$$

describing motor current step response. As the result of formulas (27), (30), (34) development into series, the same squarer function of time is obtained:

$$\omega(t) = \Omega(0) + (k_M/J)(I_{rot}(0) - (T_L/k_M)) \cdot t - \\ - 0{,}5 \cdot (k_M/(J \cdot L_r)) \cdot \\ \cdot (U_{DC} + k_M \Omega(0) + R_r I_{rot}(0) + (L_r \alpha / k_M)) \cdot t^2. \tag{38}$$

which describes the motor speed step response. The output signal of speed controller:

$$i_{Dref}(t) = I_{rot}(0) - I_{Le} - (U_{DC} + k_M \cdot \Omega(0) + \\ + R_r \cdot I_{rot}(0) + L_r \cdot (di_{Le}/dt)/L_r) \cdot t \tag{39}$$

is obtained by putting (18) and (37) into (16). The initial value of motor current $I_{rot}(0)$ is obtained from boundary condition, that motor current is equal to I_{MAX} at beginning of speed control process in the active state of the speed control system [5]. The input signal of speed controller:

$$\Delta\omega(t) = \omega_{ref} - \Omega(0) - (k_M/J)(I_{MAX} - I_{Le}) \cdot t + \\ + 0{,}5 \cdot (k_M/(J \cdot L_r)) \cdot \\ \cdot (U_{DC} + k_M \Omega(0) + R_r I_{MAX} + (L_r (di_{Le}/dt)/k_M)) \cdot t^2. \tag{40}$$

is obtained under the assumption, that the reference speed is changed in steps and it is constant between the converter switching processes. This condition is easy to fulfill e.g. by use of the sample and hold device triggered by the converter control signal. The unknown initial value $\Omega(0)$ can be obtained from boundary condition, that the speed control error is equal to zero, at the end of the speed control process [5]. The speed control time can be calculated from boundary condition, that the motor current is equal to the estimated load current at the end of the speed control process. The all boundary conditions are explained in [5]. Next, the formula:

$$i_{Dref} = \sqrt{\left|\omega_{ref} - \omega_e\right|} \cdot \sqrt{\left|u_1\right| \cdot J/\left(k_M \cdot L_r\right)} \cdot$$
$$\cdot \sqrt{1 + \sqrt{1 - \left(2k_M{}^2 \cdot L_r/J\right) \cdot \left(\left(I_{MAX} - I_{Le}\right)/u_1\right)^2}} \quad , \quad (41)$$

describing speed controller characteristic for positive speed control error is obtained, where:

$$u_1 = U_{DC} + I_{MAX} \cdot Rr + k_M \cdot \omega_{ref} + L_r \cdot \left(di_{Le}/dt\right). \quad (42)$$

It is possible to obtain the speed controller characteristic for the negative speed control error with the analogous way. The difference is in the assumed boundary condition and in the supply motor voltage sign. The initial value of motor current $I_{rot}(0)$ is equal to $-I_{MAX}$. If the speed control error is negative, the motor voltage u_r is equal $+U_{DC}$, in the active state of speed control system. The speed controller characteristic for the negative speed control error is described with formula:

$$i_{Dref} = -\sqrt{\left|\omega_{ref} - \omega_e\right|} \cdot \sqrt{\left|u_2\right| \cdot J/\left(k_M \cdot L_r\right)} \cdot$$
$$\cdot \sqrt{1 + \sqrt{1 - \left(2k_M{}^2 \cdot L_r/J\right) \cdot \left(\left(-I_{MAX} - I_{Le}\right)/u_2\right)^2}} \quad , \quad (43)$$

in which:

$$u_2 = -U_{DC} - I_{MAX} \cdot Rr + k_M \cdot \omega_{ref} + L_r \cdot \left(di_{Le}/dt\right). \quad (44)$$

Signals: I_{Le} and (di_{Le}/dt) are taken from the load torque observer, because estimated load current i_{Le} is very close to value I_{Le}, when the observer time constant $(1/\Omega)$ is short and the load current derivative (di_{Le}/dt) is relatively low.

V. Experimental Result

The experimental investigation were done on the test stand consisting on the DC/DC converter controlled by the microprocessor system with DSP 96002 (40MIPS) and the two DC motors PZOb44a. One motor was driving, the other motor was loading. The rated parameters of DC motors PZOb44a from KOMEL company are P_N= 1,2kW, U_N=230V, I_N=5,2A, n_n=1450rpm, and excitation winding parameters are U_{exc}=230V, I_{exc}=0,37A. The rated parameters of the DC/DC converter are I_{MAX}=7A, U_{MAX}=330V. The angular position is measured with use the position encoder PFI60A2048c22-DAF10 from INTRON company. The encoder resolution is 4*2048marks per rotation. The DC/DC converter and the microprocessor system was built at Bialystok Technical University.

The time - minimal speed response was tested. The laboratory investigation results are shown in Fig. 6. The results acknowledges, that described speed control system ensures the time – minimal and without overshoot speed control of loaded DC motor. At the beginning speed control process, the motor torque is controller with use of one switching process of DC/DC converter. This is time minimal control process, because number of switching converter processes can not be lower than one. Next, the maximal motor current is forced during next stage of the speed control process,. This is the time minimal speed response, because it is not allowed to force higher current value than maximal in order not to damage the motor. The value of maximal current depends on particular type of motor. However, the motor current is maximal. Next, the speed control system is active. During the active state of the speed control process, the speed control error is decreased to zero with use of one switching process of the DC/DC converter. This is the time-minimal speed control process, because it is not possible to obtain lower number of converter switching processes than one. After the time-minimal speed control process, the DC/DC converter is constantly switched in order to keep the motor current on value i_{Le}. The average motor current is equal to i_{Le} in order to compensate the load torque, after speed control process. During this test, the load torque was not constant. The load torque changed with the speed. The load torque change was obtained by switching on the resistor to the loading motor armature. In this way, the breaking was obtained.

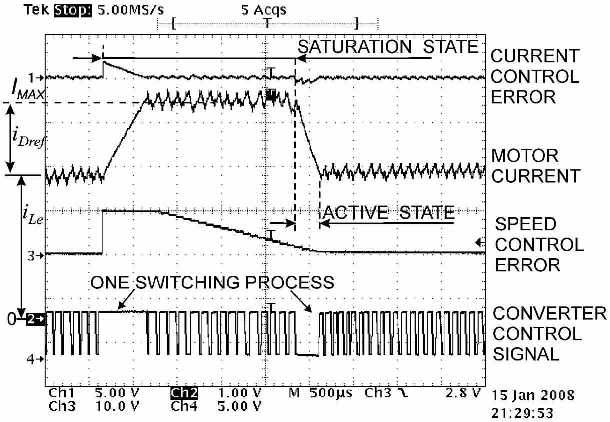

Fig. 6. The time – minimal speed control process.

VI. Conclusion

To conclude, this paper concerns the time-minimal speed control of DC motor. DC motor was modeled by second order inertial system. The time – minimal control was found for all parameters cases of second order inertial system, for periodical and for non-periodical step response. In addition, this paper concerns the first approach to the time - minimal speed control of DC motor with the linear load torque change. The new nonlinear speed controller characteristic was found. However, the speed controller characteristic was found under the assumption that load torque change is slow and the observer responses relatively faster.

The transient motor current response was taken into account in the whole time-minimal speed control process.

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