A Simple Two-Quadrant DC Motor Controller

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***Abstract* - DC drives are still very important especially for low voltages and low power (e.g. in cars and robots). In this paper a converter is analyzed which enables to control the voltage across the machine (and therefore the speed) from zero to three times of the input voltage. The model of the drive, the design of the devices, and some experimental results are given.**

I. INTRODUCTION

The basic converter is shown in Fig.1 [1] and consists of an inductor L, an active switch S, a capacitor C, and a passive switch D. The positive pole of the output voltage is the cathode of the diode. The machine is modelled by a voltage source U_q and an inductor L_A. Due to the additional capacitor C and the additional inductor L, the classical one-quadrant drive which is only a step-down converter is transferred into a step-up-down converter. The mean value of the output voltage U_2 (the voltage across the diode, this is also the voltage across the armature terminals), with the duty ratio of the active switch d (the on-time of the active switch referred to the switching period) and neglecting the losses is

$$U_2 = \frac{d}{1-d} \cdot U_1 \ . \tag{1}$$

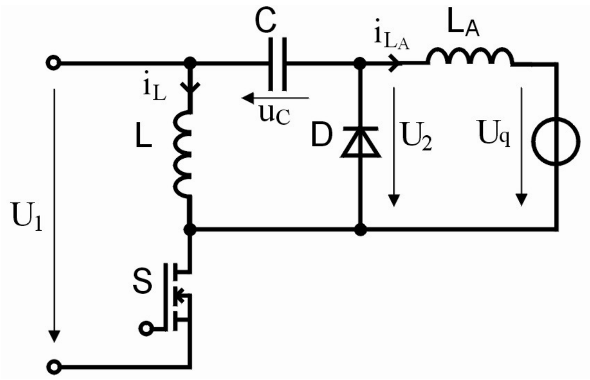

Fig. 1. One-quadrant step-up-down DC motor drive

The converter is a one-quadrant step-up step-down converter, which enables to drive a DC motor in one direction, controlled braking is not possible with this circuit. For other concepts, control applications, and detail about the electrical machine refer to the references [2 till 10].

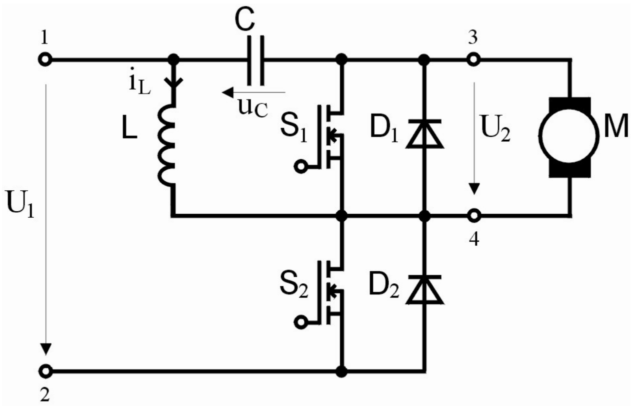

Fig. 2. Two-quadrant step-up-down DC motor drive

Neglecting all losses, the source voltage of the machine in steady case is equal to the mean value of the output voltage. Therefore, the speed of the motor (with C_E as voltage constant of the machine) is

$$n_0 = \frac{1}{C_E} \cdot \frac{d}{1-d} \cdot U_1 \ . \tag{2}$$

Figure 2 shows a two-quadrant converter derivated from Fig. 1. The converter consists now of a half-bridge with two active and two passive switches and again one inductor and one capacitor as storage elements. Now both current directions are possible and therefore controlled braking with feeding back energy into the input source is achieved.

The equations for the output voltage and the speed are the same as in the case of the one-quadrant drive.

II. BASIC ANALYSIS

The basic analysis has to be done with idealized components (that means no parasitic resistors, no switching losses) and for the continuous mode in steady (stationary) state. A good way to start is to consider the voltage across the inductors.

Since for the stationary case the absolute values of the voltage-time-areas of the inductors have to be equal (the voltage across the inductor has to be zero in the average), we can easily draw the shapes according to Fig. 3 and Fig. 4. (Here the capacitor is assumed so large that the voltages can be regarded constant during a pulse period.) Figure 3 shows the current through and the voltage across L. The current

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rate of rise of course depends on the values of L and U_1. Figure 4 shows the current through and the voltage across armature inductor L_A. Based on the equality of the voltage-time-areas in the stationary case, it is easy to give the transformation relationship for the source voltage U_q dependent on the input voltage U_1 and the duty ratio d. From Fig. 4 we get

$$(U_C + U_1 - U_q) \cdot d = (1-d) \cdot U_q \tag{3}$$

and from Fig. 3

$$U_1 \cdot d = (1-d) \cdot U_C \quad . \tag{4}$$

After a few steps we get the voltage transformation law

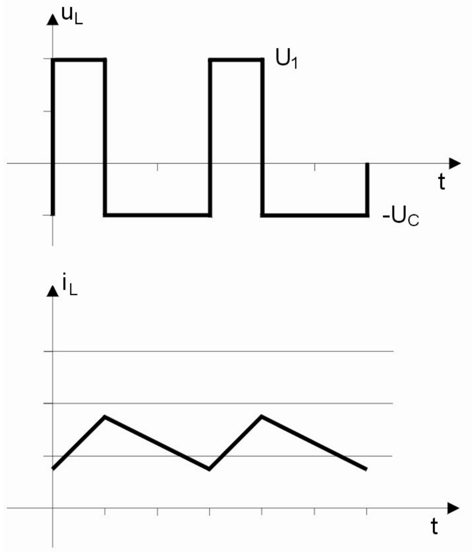

Fig. 3. Voltage across and current through the inductor L

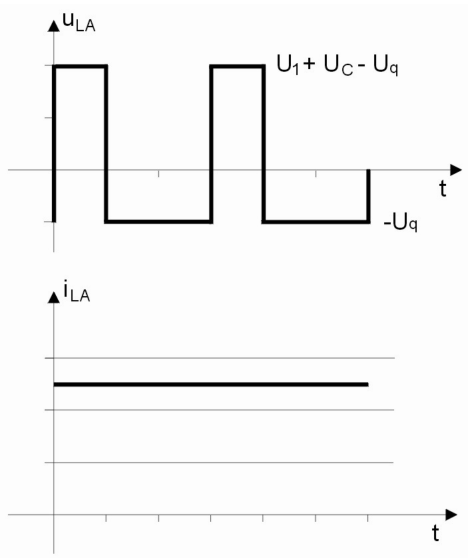

Fig. 4. Voltage across and current through the inductor L_A

$$M = \frac{U_q}{U_1} = \frac{d}{(1-d)} \quad . \tag{5}$$

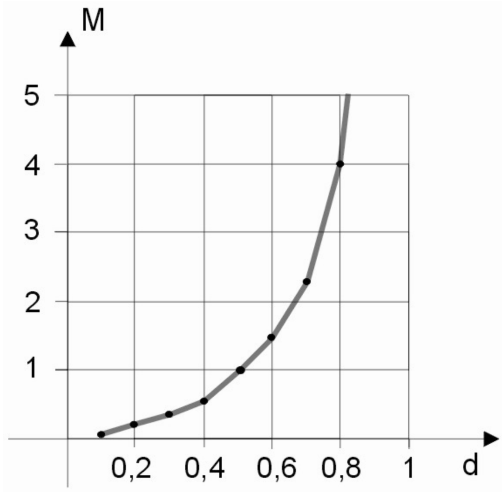

Fig. 5. Voltage transformation rate in dependence on the duty cycle

Figure 5 shows the voltage transformation factor $M = \frac{U_q}{U_1}$ [the ratio of mean value of the output voltage of the converter (or the source voltage of the machine) to input voltage] in dependence on the duty cycle d. The converter is a step-up-down converter.

In the same manner a relationship for the current through the inductors can be derived based on the equality of the absolute values of the current-time-areas of the capacitor during on- and off-times of the active switch. From Fig. 6 we get

$$\bar{I}_L (1-d) = I_{LA} \cdot d \, , \tag{6}$$

$$\bar{I}_L = I_{LA} \cdot \frac{d}{(1-d)} \quad . \tag{7}$$

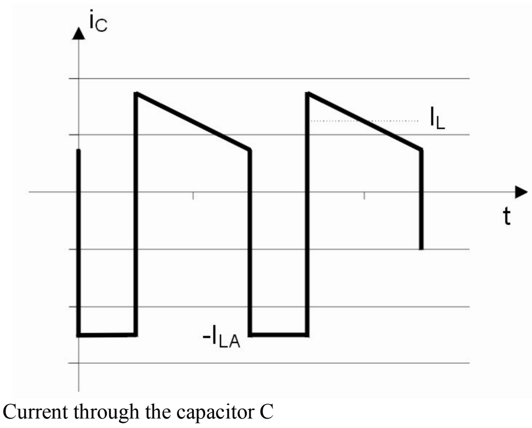

Fig. 6. Current through the capacitor C

From the schematic Fig. 1 one can immediately see that the current through the semiconductor devices is the sum of the inductor currents i_L and i_{LA} (through the switch S during T on, through the diode D during T off). Therefore, the current maximum values for the semiconductor devices are

$$I_{S,\max} = I_{D,\max} = \bar{I}_L + I_{LA} \tag{8}$$

For the calculation of the on-state losses the *rms* values are important. For the switch S we get approximately

$$I_{S,\max} = (\bar{I}_L + I_{LA})\sqrt{d} \tag{9}$$

and for the diode D

$$I_{D,\max} = (\bar{I}_L + I_{LA})\sqrt{(1-d)} \ . \tag{10}$$

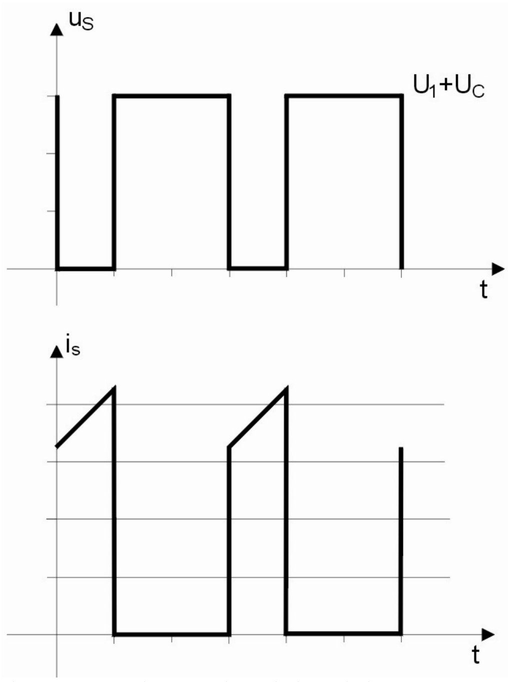

Fig. 7. Voltage across and current through the switch S

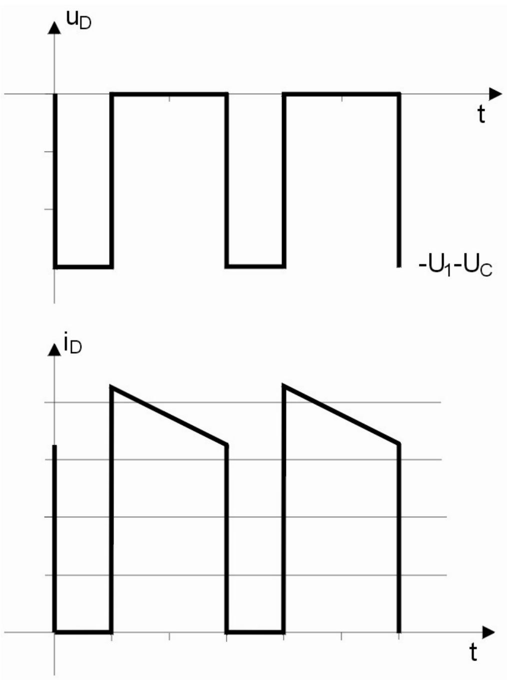

Fig. 8. Voltage across and current through the diode D

Figure 7 shows the current through the active switch and Fig. 8 through the passive switch (diode). For the dimensioning of the circuit the voltage rates for the semiconductors are important. According to the on-state (S on, D off), the voltage across the switch S is

$$\begin{aligned} U_{S,\max} &= U_1 + U_C = U_1 + U_1 \frac{d}{(1-d)} = \\ &= U_1(1 + \frac{d}{(1-d)}) = U_1 \frac{1-d+d}{(1-d)} = U_1 \frac{1}{(1-d)} \end{aligned} \ . \tag{11}$$

The same value occurs in the other state (S off, D on) across the diode

$$\begin{aligned} U_{D,\max} &= U_1 + U_C = U_1 + U_1 \frac{d}{(1-d)} = \\ &= U_1(1 + \frac{d}{(1-d)}) = U_1 \frac{1-d+d}{(1-d)} = U_1 \frac{1}{(1-d)} \end{aligned} \ . \tag{12}$$

For continuous operation mode it is easy to calculate a rough approximation of the efficiency of the converter. With R_D as on-resistance of the semiconductors we can calculate the conduction losses (without switching losses) according to

$$P_v = (I_L + I_{LA})^2 \cdot [R_{DS} \cdot d + R_{DD} \cdot (1-d)] \tag{13}$$

and with synchronous rectification $R_D = R_{DS} = R_{DD}$

$$\eta = \frac{P_n}{P_g} = \frac{(U_q \cdot I_{LA})}{(U_q \cdot I_{LA}) + P_v} = \frac{1}{1 + \frac{(I_L + I_{LA})^2 \cdot R_D}{U_q \cdot I_{LA}}} \ . \tag{14}$$

III. Converter Model

For the DC motor the standard model based on [2] with constant field flux which can be assumed if a permanent magnet is employed or a constant extinction, is used. R_A is the armature resistance, L_A the armature inductance, C_T and C_E the motor constants for torque and source voltage, J the inertia, and B the damping. The state variables are the inductor current i_L, the armature current i_A, the capacitor voltage u_C, and the speed n. The input variables are the input voltage u_1, and the work load (load torque) t_{WL}. The fixed forward voltage of the diode (the diode is modeled as a fixed forward voltage V_{FD} and an additional voltage drop depending on the differential resistor of the diode R_D) is included as an additional vector. The parasitic resistances are the on-resistance of the active switch R_S, the series resistance of the coil R_L, the series resistor of the capacitor R_C, and the differential resistor of the diode R_D.

In continuous inductor current mode there are two states. In state one the active switch is turned on (or the first active switch is turned on in case of the two-quadrant drive) and the passive switch is turned off (or the second active switch is turned off in case of the two-quadrant drive). Figure 9 shows this switching state one.

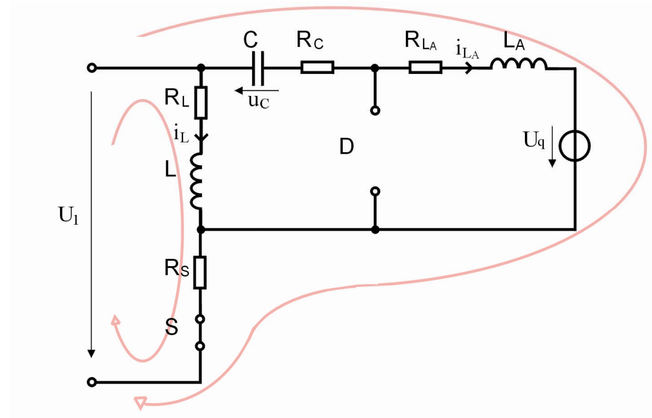

Fig. 9. Equivalent circuit for state one for the one-quadrant drive

The state space equations are now

$$\frac{di_L}{dt} = \frac{-i_L(R_L + R_S) - i_{LA} \cdot R_S + u_1}{L} \tag{15}$$

$$\frac{di_{LA}}{dt} = \frac{-i_L \cdot R_S - i_{LA}(R_{LA} + R_C + R_S) + u_C + u_1 - c_E \cdot n}{L_A} \tag{16}$$

$$\frac{du_C}{dt} = \frac{-i_{LA}}{C} \tag{17}$$

$$\frac{dn}{dt} = \frac{c_T \cdot i_{LA} - t_{WL}}{2\pi J} \tag{18}$$

leading to the state space description

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} = \begin{bmatrix} \frac{-(R_L+R_S)}{L} & \frac{-R_S}{L} & 0 & 0 \\ \frac{-R_S}{L_A} & \frac{-(R_{LA}+R_C+R_S)}{L_A} & \frac{1}{L_A} & \frac{-c_E}{L_A} \\ 0 & \frac{-1}{C} & 0 & 0 \\ 0 & \frac{C_T}{2\pi J} & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ \frac{1}{L_A} & 0 \\ 0 & 0 \\ 0 & \frac{-1}{2\pi J} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix} \tag{19}$$

In state two the active switch is turned off (or the first active switch is turned off and the second switch is turned on in case of the two-quadrant drive) and the passive switch is turned on. Figure 10 shows this switching state two.

The describing equations are

$$\frac{di_L}{dt} = \frac{-i_L(R_L + R_C + R_D) - i_{LA} \cdot R_D - V_{FD} - u_C}{L} \tag{20}$$

$$\frac{di_{LA}}{dt} = \frac{-i_L \cdot R_D - i_{LA}(R_{LA} + R_D) - V_{FD} - c_E \cdot n}{L_A} \tag{21}$$

$$\frac{du_C}{dt} = \frac{i_L}{C} \tag{22}$$

$$\frac{dn}{dt} = \frac{C_T \cdot i_{LA} - t_{WL}}{2\pi J} \tag{23}$$

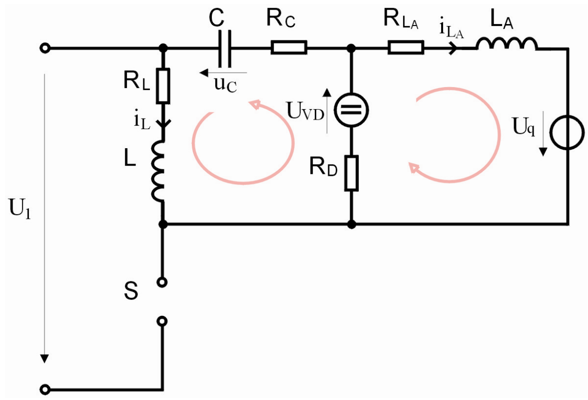

Fig. 10. Equivalent circuit for state two for the one-quadrant drive

leading to the state space description

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} = \begin{bmatrix} \frac{-(R_L+R_C+R_D)}{L} & \frac{-R_D}{L} & -\frac{1}{L} & 0 \\ \frac{-R_D}{L_A} & \frac{-(R_{LA}+R_D)}{L_A} & 0 & \frac{-c_E}{L_A} \\ \frac{1}{C} & 0 & 0 & 0 \\ 0 & \frac{C_T}{2\pi J} & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{-1}{2\pi J} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix} + \begin{pmatrix} -\frac{1}{L} \\ -\frac{1}{L_A} \\ 0 \\ 0 \end{pmatrix} \cdot V_{FD} \tag{24}$$

When using two active switches in push-pull mode, the diode is shunted and V_{FD} can be set to zero. R_D is then the on-resistance of the second switch. Combining the two systems by the state-space averaging method leads to a model, which describes the drive in the mean. On condition that the system time constants are large compared to the switching period, we can combine these two sets of equations.

Weighed by the duty ratio, the combination of the two sets yields to

$$\frac{d}{dt}\begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} =$$

$$= \begin{bmatrix} \frac{-[d \cdot R_S + (1-d)\cdot(R_C + R_D) + R_L\ ]}{L} & \frac{-[d \cdot R_S + (1-d)\cdot R_D\ ]}{L} & \frac{(d-1)}{L} & 0 \\ \frac{-[d \cdot R_S + (1-d)\cdot R_D\ ]}{L_A} & \frac{-[d\cdot(R_C + R_S) + R_{LA} + (1-d)\cdot R_D]}{L_A} & \frac{d}{L_A} & \frac{-C_E}{L_A} \\ \frac{(1-d)}{C} & \frac{-d}{C} & 0 & 0 \\ 0 & \frac{C_T}{2\pi J} & 0 & 0 \end{bmatrix} \cdot$$

$$\cdot \begin{pmatrix} i_L \\ i_{LA} \\ u_C \\ n \end{pmatrix} + \begin{bmatrix} \frac{d}{L} & 0 \\ \frac{d}{L_A} & 0 \\ 0 & 0 \\ 0 & \frac{-1}{2\pi J} \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ t_{WL} \end{pmatrix} + \begin{pmatrix} \frac{d-1}{L} \\ \frac{d-1}{L_A} \\ 0 \\ 0 \end{pmatrix} \cdot V_{FD} \tag{25}$$

By the given system of equations the dynamic behavior of the idealized converter is described correctly in the average, thus quickly giving us a general view of the dynamic behavior of the converter. The superimposed ripple (which appears very pronounced in the coils) is of no importance for qualifying the dynamic behavior. This model is also appropriate as large-signal model, because no limitations with respect to the signal values have been made.

Linearizing this system around the operating point enables us to calculate transfer functions for constructing Bode plots (cf. e.g. [3], which gives a detailed analysis of a different converter including signal flow graphs).

The weighed matrix differential equation representing the dynamic behavior of the converter is a nonlinear one. To use the possibilities of the linear control theory a linearization is necessary. With capital letters for the operating point values and small letters for the disturbance around the operating point

$$\begin{aligned} i_L &= I_{L0} + \hat{i}_L & u_1 &= U_{10} + \hat{u}_1 \\ i_{LA} &= I_{LA0} + \hat{i}_{LA} & t_{WL} &= T_{WL0} + \hat{t}_{WL} \\ u_C &= U_{C0} + \hat{u}_C & d &= D_0 + \hat{d} \\ n &= N_0 + \hat{n}\ . \end{aligned} \tag{26}$$

For the stationary (working point) values the following equation can be specified

$$\begin{aligned} 0 = {} & I_{L0} \cdot [-D_0 \cdot R_S + (R_C + R_D)\cdot(D_0 - 1) - R_L] + \\ & + I_{LA0} \cdot [-D_0 \cdot R_S + R_D \cdot (D_0 - 1)] + \\ & + U_{C0} \cdot (D_0 - 1) + D_0 \cdot U_{10} + V_{FD} \cdot (D_0 - 1) \end{aligned} \tag{27}$$

$$\begin{aligned} 0 = {} & I_{L0} \cdot [-D_0 \cdot R_S + (D_0 - 1)\cdot R_D] + \\ & + I_{LA0} \cdot [-D_0 \cdot (R_C + R_S) - R_{LA} + R_D \cdot (D_0 - 1)] + \\ & + U_{C0} \cdot D_0 + N_0 \cdot (-C_E) + D_0 \cdot U_{10} + V_{FD} \cdot (D_0 - 1) \end{aligned} \tag{28}$$

$$I_{LA0} = I_{L0} \frac{(1 - D_0)}{D_0} \tag{29}$$

$$T_{WL0} = C_T \cdot I_{LA0}\ . \tag{30}$$

In the ideal case (assuming ideal devices with no losses) the same results as derived from the section basic analyzes can be achieved

$$0 = U_{C0} \cdot (D_0 - 1) + D_0 \cdot U_{10} \tag{31}$$

$$0 = U_{C0} \cdot D_0 + N_0 \cdot (-C_E) + D_0 \cdot U_{10} \tag{32}$$

leading again to

$$N_0 = \frac{1}{C_E} \cdot \frac{D_0}{1 - D_0} \cdot U_{10}\ . \tag{33}$$

One can calculate the linearized small signal model of the converter according to

$$\frac{d}{dt}\begin{pmatrix} \hat{i}_L \\ \hat{i}_{LA} \\ \hat{u}_C \\ \hat{n} \end{pmatrix} =$$

$$= \begin{bmatrix} \frac{-D_0 \cdot R_S + (D_0 - 1)\cdot(R_C + R_D) - R_L}{L} & \frac{-D_0 \cdot R_S + (D_0 - 1)\cdot R_D}{L} & \frac{(D_0 - 1)}{L} & 0 \\ \frac{-D_0 \cdot R_S + (D_0 - 1)\cdot R_D}{L_A} & \frac{-D_0 \cdot (R_C + R_S) - R_{LA} + (D_0 - 1)\cdot R_D}{L_A} & \frac{D_0}{L_A} & \frac{-C_E}{L_A} \\ \frac{(1 - D_0)}{C} & \frac{-D_0}{C} & 0 & 0 \\ 0 & \frac{C_T}{2\pi J} & 0 & 0 \end{bmatrix} \cdot$$

$$\cdot \begin{pmatrix} \hat{i}_L \\ \hat{i}_{LA} \\ \hat{u}_C \\ \hat{n} \end{pmatrix} + \begin{bmatrix} \frac{D_0}{L} & \frac{I_{L0}\cdot(-R_S + R_C + R_D) + I_{LA0}\cdot(-R_S + R_D) + U_{C0} + U_{10} + V_{FD}}{L} & 0 \\ \frac{D_0}{L_A} & \frac{I_{L0}(-R_S + R_D) + I_{LA0}(-R_C - R_S + R_D) + U_{C0} + U_{10} + V_{FD}}{L_A} & 0 \\ 0 & \frac{-I_{L0} - I_{LA0}}{C} & 0 \\ 0 & 0 & \frac{-1}{2\pi J} \end{bmatrix} \cdot$$

$$\cdot \begin{pmatrix} \hat{u}_1 \\ \hat{d} \\ \hat{t}_{WL} \end{pmatrix} + \begin{pmatrix} \frac{D_0 - 1}{L} \\ \frac{D_0 - 1}{L_A} \\ 0 \\ 0 \end{pmatrix} \cdot V_{FD}$$

$$. \tag{34}$$

IV. PRELIMINARY RESULTS

The DC motor is an MY1016 model from Unite Motor. The parameters of the motor have been obtained by measurements and are

$R_A = 0{,}6\ \Omega$ (measured DC resistance)

$L_A = 16\ mH$ (measured at 50 Hz)

$i_{L0} = 1{,}22\ A$ (measured with no load)

$C_T = \frac{1{,}22\ Nm}{12{,}8\ A} = 0{,}095 \frac{Nm}{A}$ (specification and measured).

$$C_E = \frac{36 - 1{,}22\ A \cdot 0{,}6\ \Omega}{\frac{3350 \frac{Rev}{Min}}{60} \cdot 2 \cdot \pi} =$$

$= 0{,}1 \frac{V \cdot s}{rad}$ (specification and measured), $c_E = 0{,}63 Vs$

$$P_0 = 36\,V \cdot 1{,}22\,A = 43{,}92\,W$$

$$P_{Mech} = P_0 - (1{,}22\,A)^2 \cdot 0{,}6\Omega = 43\,W$$

$$B = \frac{P_{Mech}}{\omega^2} = 0{,}00035 \frac{Nm \cdot s}{rad} \ \text{(conversation of energy)}$$

$$, R = 2{,}31.10^{-5}\,Nms^2$$

$$J \approx \frac{B \cdot t_{Off}}{3} = 0{,}00073 \frac{Nm \cdot s^2}{rad} \ \text{(measured full speed to stop)}.$$

The DC motor operates within the following operating limits:

$$U_1 = 24\,V$$

$$f_S = 50\,kHz\ (switching\ \ frequency)$$

$$n = 0 - n_{MAX} = 0 - 3350 \frac{\text{Rev}}{\text{min}}$$

$$T_{WL} \leq 1\,Nm\ \text{(Below motor rating)}.$$

The coil L has about $60\mu H$ and a resistor of about 0.5 Ohm, the capacitor C (implemented with an electrolyte capacitor and a film capacitor) has about $100\,\mu F$ and a resistor of about $21\,m\Omega$. For the high frequency component of the capacitor current, a pulse capacitor is shunted parallel to the electrolyte capacitor. The diode has a current rating of $60A$, a differential on-resistance of $5\,m\Omega$, and a forward voltage of $550\,mV$. The transistor can carry a current of $55A$ at a maximum drain source voltage of $100V$. The on-resistance is $26\,m\Omega$.

Figure 11 shows (up to down) the control signal of the active switched (generated by the arbitrary pulse generator) (blue), the current through the machine (violet and nearly constant), the current through the inductor (turquoise and triangular shaped) and the voltage across the active switch (green) for a duty cycle of 34%, 50% and 66%.

V. CONCLUSION

DC drives are still very important especially, for low voltages and low power (e.g. in cars and robots). The converter is similar to the basic step-down motor controller. Due to a simple LC element which is added, the converter generates now a mean voltage across the motor terminals, which can be higher or lower than the input voltage. This is especially useful when only low input voltages are available. The system can be described as a fourth order system. From the control point of view the time constants are quite different: there is a large time constant caused by the mechanical inertia and due to the armature inductivity a smaller but still much higher one, compared to the time constant generated by the converter. Cascade control with a slow motion and a fast converter controller is therefore useful. The control and the influence of small parameter values of the converter will be studied in the future.

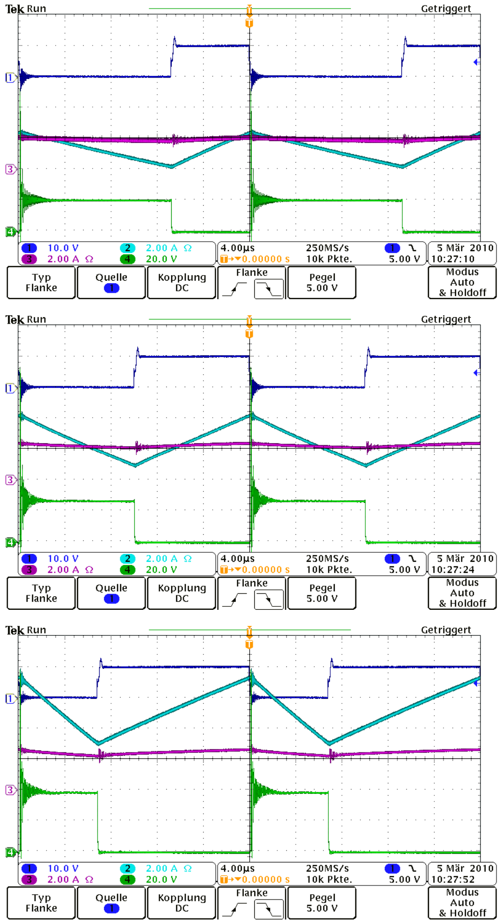

Fig. 11. control signal of the active switched, current through the machine, current through the inductor, voltage across the active switch for a duty cycle of 34%, 50% and 66%

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