DCMS

DC MOTOR SYSTEM

User Manual

release 1.2

June 3, 2010

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1. INTRODUCTION

DC motors are widely used to convert electrical energy into mechanical energy. They are relatively small, fast, efficient and inexpensive. DC motors are important components of numerous control systems and several basic features of control systems can be investigated by studying DC motor control.

The DCMS has been developed to offer a low-cost experimental setup for students, educators and researchers. The developed system is modeled very accurately and fully compatible with Hilink (hardware-in-the-loop real-time control platform for Matlab/Simulink).

1.1. System Features

- Fascinating demonstrations and experiments
- Optical incremental encoder position sensing
- Compatible with Hilink (hardware-in-the-loop real-time control platform for Matlab/Simulink)
- Accurate model with precise model parameters
- Compact and low-cost
- Position and speed control experiments

1.2. Possible Experiments

- System modeling and identification
- Simulation and real-time systems
- P, PD, PI, PID, lead controller design
- Root locus, time-domain and frequency-domain analysis and design
- Reference tracking and disturbance rejection
- State-space design (LQR, LQG, observer, $\mathcal{H}_2$, $\mathcal{H}_\infty$, μ-synthesis, sliding-mode, adaptive)
- Intelligent control (neural networks, fuzzy logic, genetic algorithms)

1.3. Requirements

- PC with Windows XP or later
- Matlab R2007b or later with Simulink, Real-Time Workshop and Real-Time Windows Target
- Hilink (hardware-in-the-loop real-time control platform for Matlab/Simulink) 1.0 or later
- Power supply ($6 - 15$ V, 2 A)

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2. SYSTEM MODEL

The model of the permanent magnet DC motor is shown in Figure 1, where R is the armature resistance, L is the armature inductance, v is the voltage applied to the motor, i is the current through the motor, e is the back emf voltage, J is the moment of inertia of the load, B is the viscous friction coefficient, τ is the torque generated by the motor, θ is the angular position of the motor, ω is the angular velocity of the motor and m is the external torque. The angular position of the motor is measured by an incremental encoder and the motor voltage is applied through an H bridge.
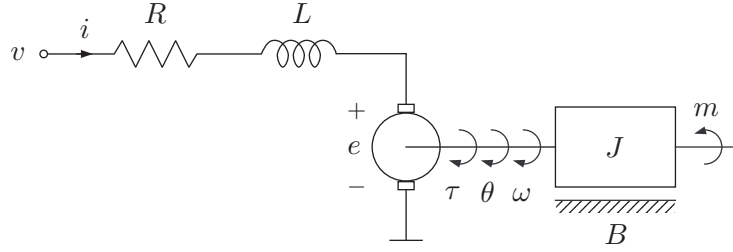

Figure 1. Permanent magnet DC motor system model.

The back emf voltage is proportional to the shaft angular velocity and can be expressed as

$$e = k_\omega \omega, \tag{2.1}$$

where k_ω is the back emf constant of the motor [1]. The motor torque is proportional to the armature current and can be expressed as

$$\tau = k_i i, \tag{2.2}$$

where k_i is the torque constant of the motor [1]. In SI units, the numerical values of k_ω and k_i are the same. It follows from Kirchhoff's voltage law that

$$v = L\dot{i} + Ri + e = L\dot{i} + Ri + k_\omega \omega. \tag{2.3}$$

Moreover, it follows from Newton's second law that

$$\tau = J\dot{\omega} + B\omega + m = k_i i. \tag{2.4}$$

A block diagram representation of the DC motor system obtained from (2.3) and (2.4) is as shown in Figure 2. Thus, the transfer function from v to ω (by assuming $m = 0$) is

$$\frac{\Omega(s)}{V(s)} = \frac{k_i}{(Ls + R)(Js + B) + k_\omega k_i}, \tag{2.5}$$

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where $V(s)$ and $\Omega(s)$ are the Laplace transforms of $v(t)$ and $\omega(t)$, respectively. Since $w = \dot{\theta}$, the transfer function from v to θ (by assuming again $m = 0$) is

$$\frac{\Theta(s)}{V(s)} = \frac{k_i}{s[(Ls + R)(Js + B) + k_\omega k_i]}, \tag{2.6}$$
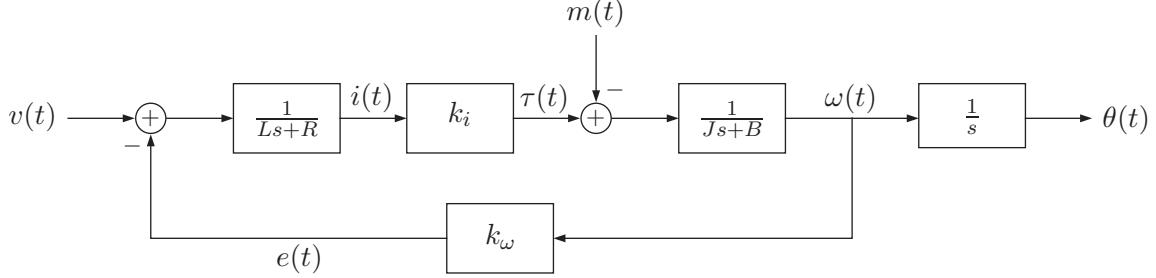
where $\Theta(s)$ is the Laplace transform of $\theta(t)$.

Figure 2. Permanent magnet DC motor system block diagram.

Letting $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} \theta & \omega & i \end{bmatrix}$ be the state, $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix} = \begin{bmatrix} \theta & \omega \end{bmatrix}$ be the output and $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix} = \begin{bmatrix} v & m \end{bmatrix}$ be the input of the system, the standard state equation description of the system can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\dfrac{B}{J} & \dfrac{k_i}{J} \\ 0 & -\dfrac{k_\omega}{L} & -\dfrac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{J} \\ \dfrac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{2.7}$$

The transfer matrix of the system from $u(t)$ to $y(t)$ is

$$P(s) = \begin{bmatrix} \dfrac{k_i}{s[(Ls + R)(Js + B) + k_\omega k_i]} & \dfrac{-(Ls + R)}{s[(Ls + R)(Js + B) + k_\omega k_i]} \\ \dfrac{k_i}{(Ls + R)(Js + B) + k_\omega k_i} & \dfrac{-(Ls + R)}{(Ls + R)(Js + B) + k_\omega k_i} \end{bmatrix} \tag{2.8}$$

and

$$Y(s) = P(s)U(s), \tag{2.9}$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of $y(t)$ and $u(t)$, respectively.

In this derivation, the nonlinear effects in the DC motor (hysteresis, saturation, static friction, etc.) are ignored. Moreover, the quantization due to the encoder and the distortion due to the H bridge are assumed to be negligible.

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3. MODEL PARAMETERS

Using measurement obtained by the Hilink platform, the parameters of the DC motor system are determined as

$$R = 12.04 \ \Omega, \qquad L = 9.61 \times 10^{-3} \ \text{H}, \qquad B = 3.43 \times 10^{-5} \ \text{N m s},$$
$$J = 1.85 \times 10^{-6} \ \text{N m s}^2, \quad k_\omega = 4.21 \times 10^{-2} \ \text{V s}, \quad k_i = 4.21 \times 10^{-2} \ \text{N m/A}. \tag{3.1}$$

With these parameter values, the matrices in the state equation are

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -18.5405 & 2.2757 \times 10^4 \\ 0 & -4.3809 & -1.2529 \times 10^3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & -5.4054 \times 10^5 \\ 104.0583 & 0 \end{bmatrix},$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad\qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{3.2}$$

and

$$P(s) = \begin{bmatrix} \dfrac{2.3680 \times 10^6}{s(s^2 + 1.2714 \times 10^3 s + 1.2292 \times 10^5)} & \dfrac{-5.4054 \times 10^5 s - 6.7722 \times 10^8}{s(s^2 + 1.2714 \times 10^3 s + 1.2292 \times 10^5)} \\ \dfrac{2.3680 \times 10^6}{s^2 + 1.2714 \times 10^3 s + 1.2292 \times 10^5} & \dfrac{-5.4054 \times 10^5 s - 6.7722 \times 10^8}{s^2 + 1.2714 \times 10^3 s + 1.2292 \times 10^5} \end{bmatrix}. \tag{3.3}$$

The model becomes more accurate if the static friction is also taken into account. For this, equation (2.4) should be modified as

$$\tau = J\dot{\omega} + B\omega + C \,\text{sgn}(\omega) + m = k_i i. \tag{3.4}$$

The values of B and C are determined as

$$B = 7.33 \times 10^{-6} \ \text{N m s}, \quad C = 1.21 \times 10^{-3} \ \text{N m}. \tag{3.5}$$

In addition, it should be noted that the output of the H bridge saturates at ± 12 V.

4. EXPERIMENTS

A few possible experiments with the DCMS are outlined below. It is assumed that the supply voltage is 12 V and the encoder resolution is 500 ppr throughout these experiments.

4.1. System Identification

- Measure the resistance between the terminals of the motor to determine the armature resistance R of the motor.

- Connect the positive terminal of the motor to P0_A, and connect the negative terminal of the motor to GND of the Hilink board through a 10 Ω (5 W) resistor. Note that the motor will run even when nothing is done on the Matlab side. Connect also the negative terminal of the motor to A0 of the Hilink board. Finally, connect the encoder to VDD, E0_A, E0_B and GND of the Hilink board.

- Construct a Simulink model with a constant of value $2/12$ connected to a H0 block in normal conversion mode (to apply 2 V to the motor and the series resistor), an E0 block connected to a Scope through either a derivative block or a transfer function block with $s/(0.01s+1)$ (to measure the angular speed of the motor), and an A0 block in normal conversion mode connected to a Scope (to observe the current through the motor).

- Run the model and measure the steady state current i through the motor and its steady state angular speed ω. Determine the back emf constant k_ω, and thus the torque constant k_i, from $v = Ri + k_\omega\omega$. Moreover, determine the viscous friction coefficient B from $B\omega = k_i i$.

- Run the model briefly and then disconnect the motor while observing the speed waveform. Measure the time constant τ_m of the observed waveform and determine the moment of inertia J of the motor from $\tau_m = J/B$.

- Lock the shaft of the motor. In the Simulink model, Replace the constant with a square wave block of amplitude $2/12$ and frequency 10 Hz. Run the model and observe the voltage across the resistor. Measure the time constant τ_e of the voltage waveform and determine the armature inductance L of the motor from $\tau_e = L/R$.

Questions

- What is a mathematical model and why is it important in engineering?

- Describe a method for determining the time constant of an exponential rise or decay.

- Using the DC motor equations, describe a method to measure the parameters of the motor.

- Show that the back emf constant is equal to the torque constant for a DC motor. Hint: Use the conservation of energy principle.

- Propose a method to determine both viscous friction coefficient and static friction coefficient of a DC motor.

- Observe the waveform applied to the motor by an oscilloscope and derive an equation between the voltage that the motor "feels" and the applied voltage.

4.2. Position Control

- Using the model of the DC motor obtained, design a controller (using root-locus, time-domain, frequency domain or state-space techniques) to control the position of the motor using only the position measurement. The specifications are: settling time < 2 s and overshoot $< 10\%$ in its unit step (step height of 1 rad) response.

- Connect the positive terminal of the motor to P0_A and the negative terminal to P0_B of the Hilink board, and connect the encoder to VDD, E0_A, E0_B and GND of the Hilink board.

- Implement the controller in a Simulink model and run it.

- Show that the controller satisfies the design specifications.

- Change some of the parameters of the controller while the system is running and observe the corresponding change in the response.

Questions

- Describe a method for determining the step response of a system using the Hilink platform.

- Why is the step response important in control engineering?

- Determine the type of the system with the designed controller.

- Apply a square waveform with amplitude ± 2 V and frequency 0.5 Hz directly to the motor terminals (open-loop) and observe its position using the Hilink platform. Is the response of the motor repeatable?

- Determine if the designed controller ever saturates the H bridge driver. If it does, propose a practical method to avoid saturation. Hint: What happens if the unit step input is smoothed by a command preshaping filter?

- What happens if the unit step input is replaced by a sinusoidal input?

- Describe a method to measure the frequency response of the system.

4.3. Speed Control

- Using the model of the DC motor obtained, design a controller (using root-locus, time-domain, frequency domain or state-space techniques) to control the speed of the motor using only the speed measurement. The specifications are: settling time < 2 s and overshoot $< 10\%$ in its step (step height of 10 rad/s) response.

- Connect the positive terminal of the motor to P0_A and the negative terminal to P0_B of the Hilink board. Connect the encoder to VDD, E0_A, E0_B and GND of the Hilink board, and also connect E0_A (phase A of the encoder) to C0.

- Implement the controller in a Simulink model and run it.

- Show that the controller satisfies the design specifications.

- Change some of the parameters of the controller while the system is running and observe the corresponding change in the response.

Questions

- Describe two methods for measuring the speed of the DC motor using the Hilink platform. Which one is better?

- Relate the step response of the DC motor in this part to the step response obtained in the previous part.

- Determine the type of the system with the designed controller.

- Apply a square waveform with amplitude ± 2 V and frequency 0.5 Hz directly to the motor (open-loop) and observe its speed. Is the response of the motor repeatable?

- Determine if the designed controller ever saturates the H bridge driver. If it does not, describe how the H bridge can be forced into saturation?

- What happens if the unit step input is replaced by a sinusoidal input?

- Describe a method to measure the frequency response of the system.

REFERENCES

[1] B. C. Kuo, *Automatic Control Systems.* NJ: Prentice-Hall, 1995.