# Linear Control of DC Motor Drive with Field Weakening

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*Abstract*—**This paper considers a linear control design of separately excited DC motor system (SEDCM) operated in the field weakening high speed regime. Nonlinear multiple input – multiple output (MIMO) feedback linearization is applied and a linear controller schema is proposed. It is shown that very good control performances can be obtained with the classical PI cascaded linear controllers in conjunction with the linearized SEDCM model. Simulation results presented validate the proposed control strategy.**

***Index Terms*—D.C. drive, feedback linearization, field weakening, nonlinear control.**

## I. INTRODUCTION

A D.C. motor drive system can operate in two different regions: constant field with armature control and variable field. In the first case linear control techniques are easily applied because the equations of system are linear. In the second case two product nonlinearities appear: the electromagnetic torque (ET) and back electromotive force (EMF) calculus equations. The circumventing of these nonlinearities is usually realized by a well known method, the system equations linearization around an operating point and linear controller design [3]. Recent advances in nonlinear control systems and development of the intensive real time control using DSP have allowed new approaches regarding to DC weakening field drive [1], [2], [4], [5], [6], [9]. Such a control is required for high dynamic performances in the closed-loop design of DC motor drives applicable to motion control.

In this paper, applying MIMO feedback linearization technique, a linear model of DC weakening field drive system was built. The new model allows a linear behaviour of ET and EMF. Using new state variable, ET and back EMF, the traditional closed loop control, speed and field current, was developed. Also the new model allows the controllers design using well known methods for electric drive control, modulus and symmetrical optimum criterions [3]. The proposed control is feasible for the both regions, constant and weakening field.

The solution, assessed using numerical procedures, is applied to a drive system powered by an A.C. – D.C. four quadrants converter. The numerical results confirm the utility as well as the properties of the solution.

## II. DYNAMICAL SYSTEM MODELLING

A separately excited dc motor is described by the differential equations:

$$\begin{aligned}
\frac{di_A(t)}{dt} &= -\frac{R_A}{L_A} i_A(t) - \frac{k}{L_A} i_E(t).\omega(t) + \frac{1}{L_A} u_A(t) \\
\frac{di_E(t)}{dt} &= -\frac{R_E}{L_E} i_E(t) + \frac{1}{L_E} u_E(t) \\
\frac{d\omega(t)}{dt} &= \frac{k}{J} i_E(t) i_A - \frac{F_V}{J}\omega(t) - \frac{1}{J} T_L(t)
\end{aligned} \tag{1}$$

where $u_A$, $i_A$, $L_A$, $R_A$ are the armature parameters, $u_E$, $i_E$, $L_E$, $R_E$ are the stator parameters, $F_v$ viscous friction, $T_L$ load torque, $J$ inertia,. The back EMF and the developed torque are given by:

$$\begin{aligned}
e(t) &= k i_E(t)\omega(t) \\
T_m(t) &= k i_A(t) i_E(t)
\end{aligned} \tag{2}$$

The system (1) can be rewritten in the form:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^{i=3} g_i(x) u_i \tag{3}$$

with the components:
- state vector

$$\boldsymbol{x}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T = [i_A(t) \quad i_E(t) \quad \omega(t)]^T, \tag{4}$$

- input vector

$$\boldsymbol{u}(t) = [u_1(t) \quad u_2(t) \quad u_3(t)]^T = [u_A(t) \quad u_E(t) \quad T_L(t)]^T, \tag{5}$$

- function of $i_A$, $i_E$, $\omega$

$$f(x) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} -x_1 R_A / L_A - x_2 x_3 k / L_A \\ -x_2 R_E / L_E \\ x_1 x_2 k / J - x_3 F_V / J \end{bmatrix}, \tag{6}$$

and coefficient vectors:

$$\boldsymbol{g}_1 = [1/L_A \quad 0 \quad 0]^T, \boldsymbol{g}_2 = [0 \quad 1/L_E \quad 0]^T, \boldsymbol{g}_3 = [0 \quad 0 \quad -1/J]^T \tag{7}$$

In the aim of system linearization new outputs are chosen:

$$\boldsymbol{h}(t) = [e(t) \quad T_m(t) \quad o(t)]^T = [kx_2x_3 \quad kx_1x_2 \quad kx_1x_3]^T \tag{8}$$

where $o(t)$ has not a physical signification, being introduced from the symmetrical problem formulation

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reasons.

The relative degree of problem is $r=3$ [1] [2], because the Lie derivatives of system have the values:

$$
\begin{aligned}
&L_{g1}h_1 = 0; L_{g2}h_1 \neq 0; L_{g3}h_1 \neq 0;\\
&L_{g1}h_2 \neq 0; L_{g2}h_2 \neq 0; L_{g3}h_2 = 0;\\
&L_{g1}h_3 \neq 0; L_{g2}h_3 = 0; L_{g3}h_3 \neq 0.
\end{aligned} \tag{9}
$$

The decupling matrix has the non-singular form:

$$
\boldsymbol{D}(\boldsymbol{x},t)=\begin{bmatrix} L_{g1}h_1 & L_{g2}h_1 & L_{g3}h_1 \\ L_{g1}h_2 & L_{g2}h_2 & L_{g3}h_2 \\ L_{g1}h_3 & L_{g2}h_3 & L_{g3}h_3 \end{bmatrix} = \begin{bmatrix} 0 & k/L_E x_3 & -k/Jx_2 \\ k/L_A x_2 & k/L_E x_1 & 0 \\ k/L_A x_3 & 0 & -k/Jx_1 \end{bmatrix} \tag{10}
$$

In order to find the new linear form of the system, the following change of coordinates is associated to the system (3)

$$
z(t)=\begin{bmatrix} z_1(t) & z_2(t) & z_3(t)\end{bmatrix}^T = \begin{bmatrix} kx_2x_3 & kx_1x_2 & kx_1x_3\end{bmatrix}^T \; . \tag{11}
$$

Using (10) the linear dynamic system gets the form:

$$
\overset{\circ}{z}(t) = \begin{bmatrix} \overset{\circ}{z}_1 \\ \overset{\circ}{z}_2 \\ \overset{\circ}{z}_3 \end{bmatrix} = \begin{bmatrix} L_f h_1 \\ L_f h_2 \\ L_f h_3 \end{bmatrix} + \boldsymbol{D}(\boldsymbol{x},t)\begin{bmatrix} u_A \\ u_E \\ T_L \end{bmatrix} \tag{12}
$$

or in another form

$$
\begin{bmatrix} \overset{\circ}{z}_1 \\ \overset{\circ}{z}_2 \\ \overset{\circ}{z}_3 \end{bmatrix} = \begin{bmatrix} -R_E/L_E - F_V/J & 0 & 0 \\ 0 & -R_A/L_A - R_E/L_E & 0 \\ 0 & 0 & -R_A/L_A - F_V/J \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} k^2x_1x_2^2/J \\ -k^2x_2^2x_3/L_A \\ -k^2x_2x_3^2/L_A + k^2x_1^2x_2/J \end{bmatrix} + \boldsymbol{D}(\boldsymbol{x},t)\begin{bmatrix} u_A \\ u_E \\ T_L \end{bmatrix} \tag{13}
$$

Taken in consideration the following notations:

- linearized system matrix

$$
\boldsymbol{A}=\begin{bmatrix} -R_E/L_E - F_V/J & 0 & 0 \\ 0 & -R_A/L_A - R_E/L_E & 0 \\ 0 & 0 & -R_A/L_A - F_V/J \end{bmatrix} \tag{14}
$$

- accumulated non-linear part vector

$$
\boldsymbol{F}(\boldsymbol{x},t)=\begin{bmatrix} k^2x_1x_2^2/J \\ -k^2x_2^2x_3/L_A \\ -k^2x_2x_3^2/L_A + k^2x_1^2x_2/J \end{bmatrix} \tag{15}
$$

the system (13) becomes:

$$
\overset{\circ}{z}(t) = \boldsymbol{A}z(t) + \boldsymbol{F}(\boldsymbol{x},t) + \boldsymbol{D}(\boldsymbol{x},t)\boldsymbol{u}(t) = \boldsymbol{A}z(t) + \boldsymbol{v}(\boldsymbol{x},t) \tag{16}
$$

where the new input is given by:

$$
\boldsymbol{v}(\boldsymbol{x},t) = \boldsymbol{F}(\boldsymbol{x},t) + \boldsymbol{D}(\boldsymbol{x},t)\boldsymbol{u}(t)\,, \tag{17}
$$

$\boldsymbol{u}(t)$ having the form in equation (13). From (16) results the real input:

$$
\boldsymbol{u}(t) = \boldsymbol{D}^{-1}(\boldsymbol{x},t)(\boldsymbol{v}(t) - \boldsymbol{F}(\boldsymbol{x},t))\,. \tag{18}
$$

Taken in consideration form of system matrix $\boldsymbol{A}$ (14) and linearized system (16), the new states $\boldsymbol{z}$ can be easily calculated because they are completely independent. Thus a state variable is the solution of an equation in the form:

$$
\overset{\circ}{z}_i(t) = a_{ii}z_i(t) + v_i(\boldsymbol{x},t)\,. \tag{19}
$$

A SEDCM with parameters in table 1 was adopted for linear modeling validation.

TABLE I
SEDCM PARAMETERS

| | |
|---|---|
| Rated power | $P_N$=2.2 [kW] |
| Rated speed | $n_N$=1840/4900[rpm] |
| Armature voltage | $U_{AN}$=420[V] |
| Field voltage | $U_{EN}$=220[V] |
| Rated motor torque | $T_m$=12.47[Nm] |
| Armature resistance | $R_A$=10.7 [Ω] |
| Field resistance | $R_E$=220[Ω] |
| Armature inductance | $L_A$=40.5[mH] |
| Field inductance | $L_E$=44[H] |
| Inertia | J=0.026[kgm$^2$] |
| Motor constant | k=1.795 |
| Damping coefficient | $F_v$=0.01 |

In Fig. 1 is presented the block operational diagram of model. The angular speed variations for a natural starting at rated armature and stator voltages, with a 50% field weakening at $t$=5 seconds, are plotted in Fig.2. It can be observed that speed variations are identically in the two cases, nonlinear and linearized. This is confirmed by Fig.3 in which the error

$$
\varepsilon(t) = \omega_L(t) - \omega_{NL}(t) \tag{20}
$$

is plotted. In (19) $\omega_L$ and $\omega_{NL}$ are the speeds for linearized and natural model. This behavior is valid for all electric and mechanic parameters, linearized model being a real one.

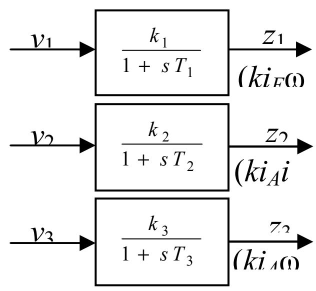


Fig.1. Operational block diagram

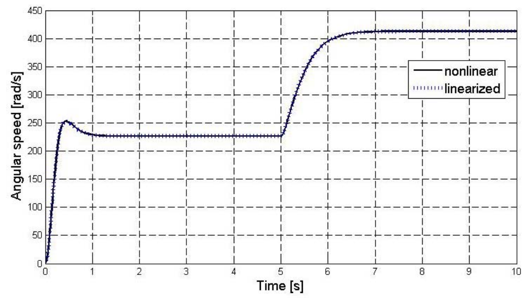


Fig.2. Angular speed variation

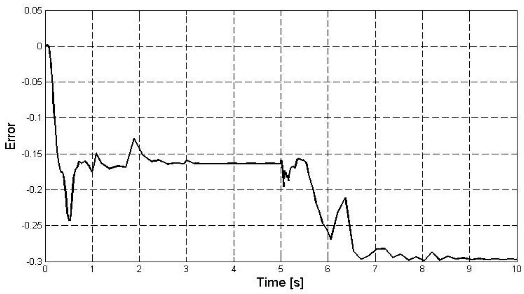


Fig.3. Error variation

The linearized model (16) has two major advantages: separate channels for state calculation and the removal of product type nonlinearities for torque and back EMF calculation. From the three new defined state variables only $z_1$ and $z_2$ will be used, the third one, introduced from problem formulation reasons, has no physical signification and is not used in the control design.

The block diagram in Fig.4 is proposed for the speed and torque control, with the following notations: $Y_1$ is SEDCM electromechanical part transfer function, $Y_2$ motor torque calculation block, $Y_3$ and $Y_4$ are the speed and torque sensors transfer functions. Linear controller design technique can be easily applied for speed ($R_\Omega$) and torque ($R_T$) PI controllers. The modulus and symmetrical optimum criterions were used for the controller tuning.

Field current control is achieved by Fig.5 structure, where $Y_1$ is the field loop transfer function, $Y_2$ back EMF calculation block, $Y_3$ and $Y_4$ the related sensors transfer functions.

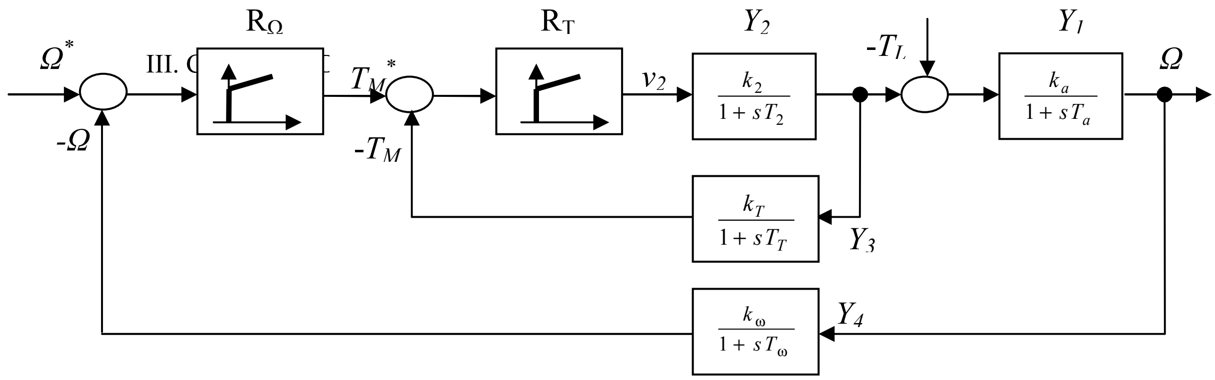


Fig.4. Speed and torque control

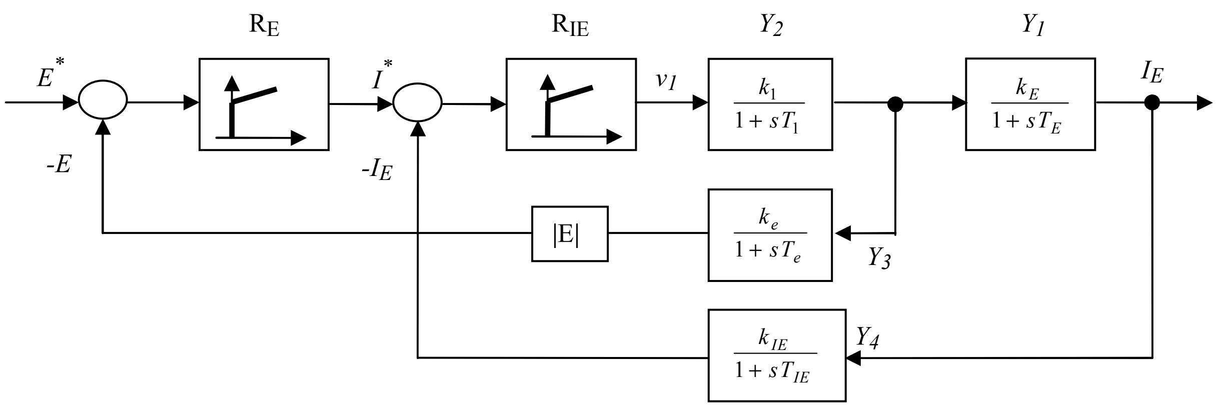


Fig.5. Field current and back EMF control

The objective of the control subsystem is to determine the two SEDCM input vectors $u_A(t)$ and $u_E(t)$. The calculation path is presented in Fig.6 and consist in finding of $v_1(t)$ and $v_2(t)$ based on the linearising model and feedback. BRA block contains the controllers.

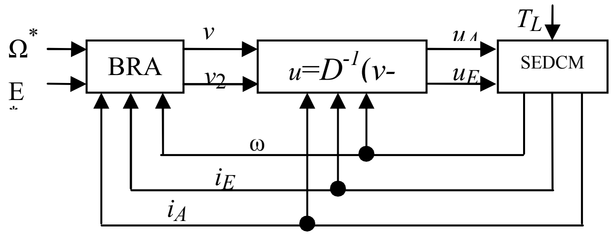


Fig.6. Block diagram of the control system

## IV. SIMULATION RESULTS

A 2.2 [kW] SEDCAM linearised model (16) and the proposed control subsystem are simulated for these studies using Matlab-SIMULINK, for several motor operating conditions.

The first simulation, Fig.7, is done for a start and a reversing, no-load, double rated speed, with a 1:2 field weakening ratio. At time $t$=0 the back EMF reference is applied to assure the rated field current, at $t$=1 and $t$=4 starting, respectively reversing speed reference are applied. The angular speed, motor torque, field current and back EMF variations are reported in fig.7. The results show a good dynamic, no steady state error and

The third test, Fig.9, comes also with a load torque step, but half the rated value (6.23 Nm) and with the double of the previous simulation angular speed due to a 1:2 field weakening ratio. The system response is similar to the previous test and demonstrates controller performance in the field weakening regime.

Fig.10 diagrams present a case of exceeded SEDCM drive rated regime. A step load torque 12.47 [Nm] is applied for 2 seconds, then the torque is stepped down to 6.23 [Nm], half the rate value, corresponding to the 1:2 field weakening ratio set for this test. First, the system electromagnetic torque in the prescribed limits (double of the rated torque). Also, it can be observed a good evolution of the field current, which tend to decrease in the reversing area, in order to maintain the back EMF within normal values. The back EMF and field proposed control is more stable than the classical one, in which case the control tend to oscillate along entire domain.

In the next simulation the SEDCM is tested to the rated load torque, at rated field. After no-load starting ($t$=1 s), at $t$=6 s the rated torque is applied. The results are reported in Fig.8. A very good system response can be observed, the torque step influence on speed variation is well damped by the controller, with short transient time and no steady-state error.

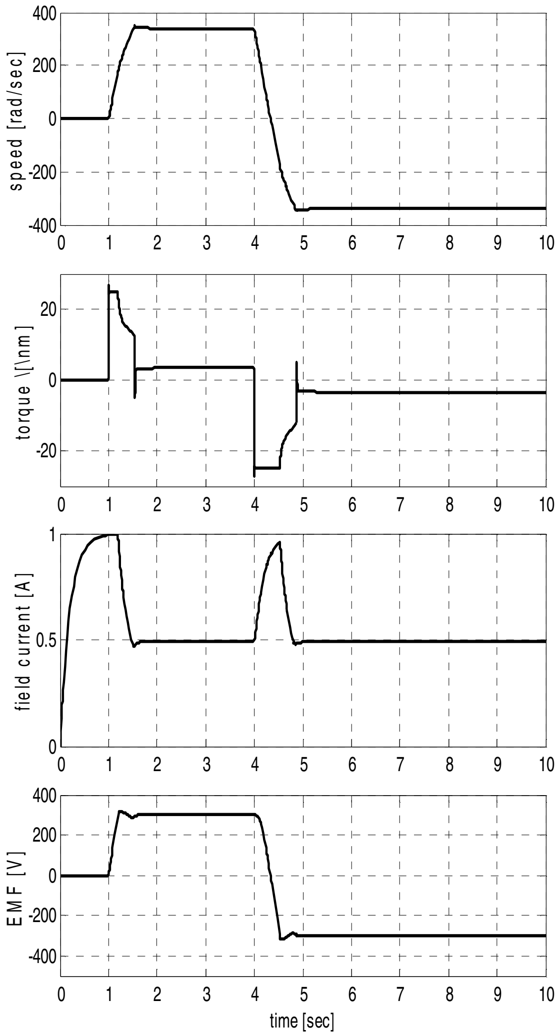


Fig.7. Starting and reversing

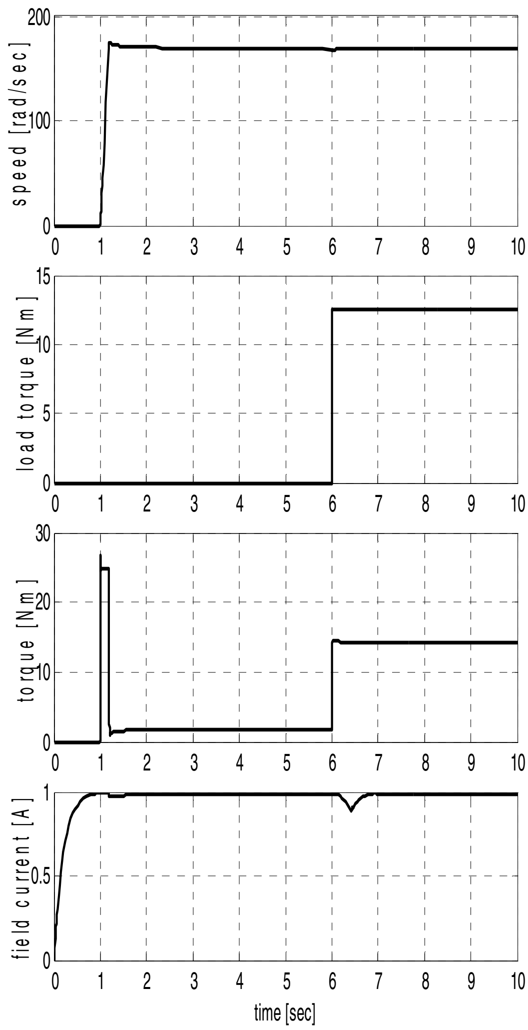


Fig.8. Load torque control – rated field

cannot track the speed reference because of the exceeded motor torque. After reducing the motor load within withstands values, the speed correction occurs. Secondly, the field current increase, tending to boost the electromagnetic torque, but in the same time decreasing the speed. This response is the result of the back EMF loop action. The system good dynamic stands and the response is normal for the given conditions.

Standard limitations regarding control angles for the two power converters feeding the SEDCM and maximum torque are used for the simulation. The armature current does not appear explicitly in the control circuits and thus cannot be directly limited. It is controlled by limiting the electromagnetic torque and by the back EMF loop action.

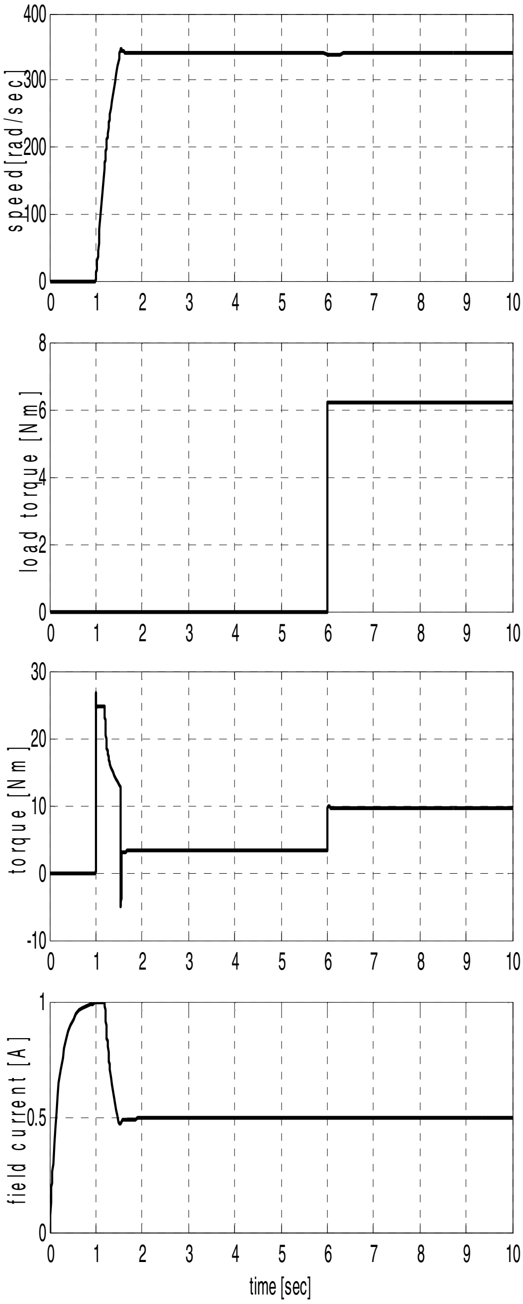

Fig.9. Load torque control – field weakening

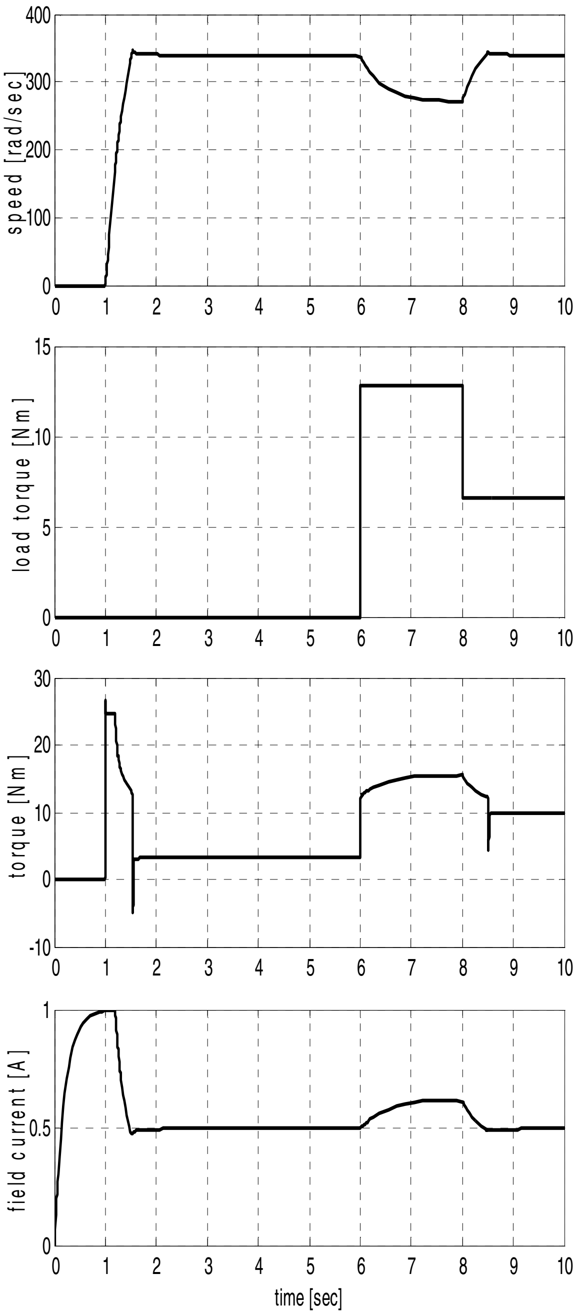

Fig.10. Load torque control - field weakening and rated load torque

## V. CONCLUSIONS

Using MIMO feedback linearization technique, a SEDCM linearized model with back EMF and ET as state vector was developed. Base on this model, a linear cascaded controller for a field weakening SEDCM was design. The linearization and the decoupling allow an independent control implementation for speed/torque and field/ back EMF loops. Given the linear structure, the controllers tuning is easily achieved with modulus and symmetrical optimum criteria. Along with the dynamic responses in the field weakening regime, similar to those from armature control, this could ease control configuration in the industrial electrical drive systems working in non linear regimes.

The simulation results validate the proposed control scheme, which show a high dynamic tracking performance and very good response at inputs step variations both in field weakening and constant field regimes.

Extending this approach, complex control structures can be developed in order to improve system performances in the nonlinear regimes of the electrical drives, similar to the classical linear ones.


## ACKNOWLEDGMENT

The results reported in this article have been obtained under IDEI grant no. 521, contract 715/2009.


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