Speed Control of DC Motor Using PI and SMC

R. K. Munje
Electrical Department
MET's Institute of Engineering,
BKC, Nashik, Maharashtra, India
E-mail: ravimunje@yahoo.co.in

M. R. Roda
Electrical Department
K.K.W.I.E.E. & R., Panchavati,
Nashik, Maharashtra, India
E-mail: mrroda@rediffmail.com

B. E. Kushare
Electrical Department
K.K.W.I.E.E. & R., Panchavati,
Nashik, Maharashtra, India
E-mail: be_kushare@rediffmail.com

***Abstract*–In this paper, sliding mode control (SMC) technique is used to control the speed of DC motor. The performance of the SMC is judged via MATLAB simulations using linear model of the DC motor and known disturbance. SMC is then compared with PI controller. The simulation result shows that the sliding mode controller (SMCr) is superior controller than PI for the speed control of DC motor. Since the SMC is robust in presence of disturbances, the desired speed is perfectly tracked. The problem of chattering, resulting from discontinuous controller, is handled by pseudo sliding with smooth control action.**

Keywords–Variable Structure Control; Sliding Mode Controller; DC Motor.

I INTRODUCTION

Variable structure control (VSC) with sliding mode made its first appearance in early 1950's in Soviet Union by Emelyanov and several co-researchers. In their Pioneer works, they considered a linear second order system modelled in phase variable form. Since then the significant interest on Variable Structure System (VSS) and Sliding Mode Control (SMC) has been generated in the control world community worldwide [1]. SMC theory first appeared outside Russia after 1970's when a book by Itkis in 1976 and first survey paper [2] by Utkin in 1977 were published in English. The VSS in sliding modes are known to be insensitive to variations in the process parameters and external disturbances. Various advantages of SMC include robustness, ability to deal with non-linear systems, time varying systems, multi-input/multi-output systems; it can be designed for the fast dynamic response and good stability over the wide range [3]. SMC results in better dynamic performance in terms of rise time, settling time, overshoot and steady state error [3, 4, 5].

PID controllers have been successfully used in control applications since 1940s and are the most often used industrial controller today. PID controllers have several important features. It provides feedback, it has the ability to eliminate steady state offset trough integral action and it can anticipate the changes through derivative action. In addition to this, PID controllers have very simple control structure and inexpensive cost. Following extensive industrial experience, various tuning methods have been developed. In spite of this, PID controllers are not perfectly able to stabilize known nonlinear system, particularly when the nonlinearity is very high [4, 5].

In many practical problems, almost perfect disturbance rejection and set point tracking are required. SMC may be applied to such systems to obtain these performances. VSC has non-linear feedback, which is discontinuous in the nature [1]. The control is called non-linear because the control input switches rapidly between two or more control limits. Using this control as a feedback the structure of the system can be altered or switched as its state crosses each discontinuity surface. This closed loop system is described as Variable Structure Control System or Variable Structure System (VSS) [2]. The state crosses and re-crosses the surface, called switching surface or sliding surface and then continuously lies on the switching surface, when error and rate of change of error become zero. This type of motion is called Sliding Motion. To emphasize the importance of the sliding motion, the control is often called as Sliding Mode Control (SMC) [3]. When the trajectory moves on the sliding surface, the system is internally controlled by the so called equivalent control [5].

DC motors are extensively used in robotics and electrical equipments. Therefore, the control of the speed of the DC motor is very important and has been studied since the early decades in the last century [3, 5]. Generally, the DC motor systems have uncertain and nonlinear characteristics which degrade performance of controllers. Based on these reasons, Sliding Mode Control (SMC) is one of the popular control strategies and powerful control technology to deal with the nonlinear uncertain system [2, 6]. It is often used to handle any worst-case control environment such as parametric perturbations with lower and upper bounds, external disturbances, stick-slip friction, and etc. Precise dynamic models are not required and its control algorithms can be easily implemented. However, the robustness of the sliding control strongly depends on specified parameters in designing of the sliding function [6, 8].

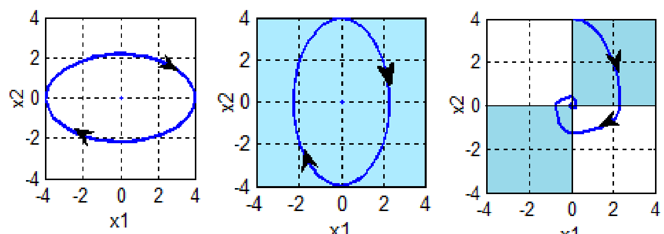
In this paper, sliding mode control has been adopted to control the speed of DC motor to achieve robustness against the system parameter variations and any external disturbances. After introducing the brief VSS theory and SMC design approach, model for the DC motor is developed. Finally paper is concluded by comparing results of the SMC with the PI controller.

II Variable Structure Systems With SMC

having two structures defined by $k = 0.3$ and $k = 3$ The phase plane plots consist of ellipses as illustrated in Fig. l(a), (b) and hence, neither structure is asymptotically stable. However, asymptotic stability is achieved if the structure of the system is changed as per the switching logic:

$$k = \begin{cases} 0.3 & x_1 x_2 < 0 \\ 3 & x_1 x_2 > 0 \end{cases} \tag{2}$$

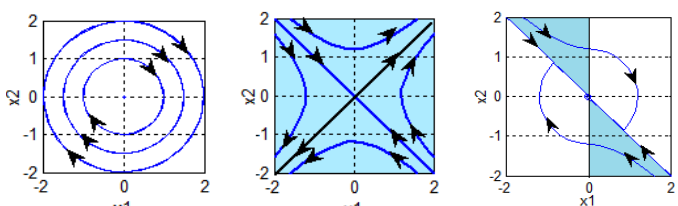
Resulting phase plane plot is shown in Fig. 1(c).



(a) *k=0.3* (b) *k=3* (c) Combined

Fig. 1 Asymptotically stable VSS consisting of two stable structures

There are other possibilities also such as, when $k = 1$ and $k = -1$, as shown in Fig. 2(a), (b) respectively. Both the structures are not stable. Fig. 2(a) is vortex or centre (structurally unstable) and Fig. 2(b) is saddle point.



(a) *k*=1 (b) *k*=-1 (c) combined

Fig. 2 Asymptotically stable VSS consisting of two unstable structures

If the switching occurs with the switching law

$$k = \begin{cases} -1 & \sigma x_1 < 0 \\ 1 & \sigma x_1 > 0 \end{cases} \quad \text{where } \sigma = x_1 + x_2 \tag{3}$$

Then, the resulting variable structure system is asymptotically stable, as shown in Fig. 2(c).

In the above examples, new system properties are obtained by composing a desired trajectory from the parts of trajectories of different structures. An even more fundamental

Consider second order system

$$\ddot{y}(t) = -ky(t) \tag{1}$$

aspect of VSS is the possibility to obtain trajectories not inherent in any of the structures. These trajectories describe a new type of motion-the so-called sliding mode [2].

To show how such motion occures, reconsider second example with $\sigma = 0.2x_1 + x_2$ as illustrated in the Fig. 3. The complete phase portrait of the system shows that there are no usual motion characteristics on the line $x_1 = 0$ other than possible discontinuities on motion direction. However, the line $\sigma = 0$ contains only the endpoints of those trajectories coming from both sides of the line. Locus of these points constitute a special trajectory along the $\sigma = 0$ line, representing motion called sliding mode. Thus, a phase trajectory of this system ganerally consists of two parts, representating the two modes of the system. The first part is the reaching mode, also called non-sliding mode, in which the trajectory starting from any where on the phase plane moves towards a switching line and reaches it in finite time. The second part is the sliding mode in which the trajectory asymptotically tends to the equilibrium point of the phase plane [1, 2].

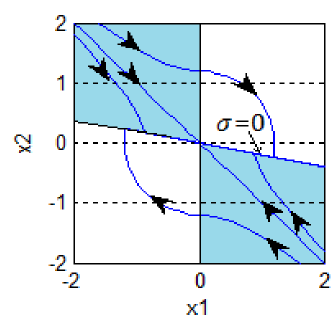


Fig. 3 Sliding Modes in second order system

III SMC Design

Consider the uncertain linear time invariant (LTI) system with m inputs given by

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t, x, u) \tag{4}$$

where $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$ are system state matrix and input matrix respectively, with $1 \le m < n$. $u \in \mathrm{R}^{m\times 1}$ is a control input. $f(t, x, u) \in \mathrm{R}^{n\times 1}$ is the disturbance input or unmodelled dynamics term, bounded by some known functions of the state.

Define the sliding function as

$$\sigma(x) = C_\sigma x(t) \tag{5}$$

where, the sliding matrix $C_\sigma \in \mathrm{R}^{m\times n}$ is full rank such that det $[C_\sigma B] \neq 0$. Then

$$\sigma(x) = \begin{bmatrix} C_{\sigma 1} \\ \vdots \\ C_{\sigma m} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{6}$$

where $C_{\sigma i} = [c_{i1}\ c_{i2}\ \cdots\ c_{in}]$. Without loss of generality, assume that $c_{in} = 1$.

In describing the method of equivalent control it will initially be assumed that the uncertain function in (4) is identically zero, i.e.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

Now suppose there exists a finite time t_s such that the solution to (7) represented by *x(t)* satisties

$$\sigma(t) = C_\sigma x(t) = 0 \qquad \text{for all } t \geq t_s \tag{8}$$

Then an ideal sliding mode is said to be taking place for all $t \geq t_s$. Mathematically this can be expressed as

$$\dot{\sigma}(t) = C_\sigma \dot{x}(t) = 0 \qquad \text{for all } t \geq t_s \tag{9}$$

Substituting $\dot{x}(t)$ from (7) in (9)

$$C_\sigma \dot{x}(t) = C_\sigma[Ax(t) + Bu(t)] = 0 \tag{10}$$

As already mentioned $C_\sigma B$ is non-singular, then the control input u associated with the nominal system (7) is denoted byu_{eq}, is given by solving (10)

$$u_{eq} = -[C_\sigma B]^{-1} C_\sigma A x(t) \tag{11}$$

This is defined to be the unique solution to the algebraic equation (7). The ideal sliding motion is then given by substituting the expression of equivalent control (equation (11)) into (7), which results in the free motion i.e. motion independent of the control action, given by

$$\dot{x}(t) = [I_n - B[C_\sigma B]^{-1} C_\sigma] A x(t) \text{ for all } t \geq t_s \tag{12}$$

The above system is sometimes referred to as the equivalent system. The system is governed by a reduced set of differential equations that are obtained by combining two sets of the equations represented by (7) and (12).

Define

$$P_s = [I_n - B[C_\sigma B]^{-1} C_\sigma] \tag{13}$$

then P_s is a projection operator and satisfies two simple yet important equations:

$$C_\sigma P_s = 0 \text{ and } P_s B = 0 \tag{14}$$

The m eigen values of P_s are zero and C_σ is selected such that $n - m$ remaining eigen values lie in the left half of s-plane.

Now consider the equation (4) where $f(t, x, u)$is unknown. This function can be thought of as representing uncertainty in the known matrices A and B. Suppose a controller exists which induces sliding motion on the surface σ despite the presence of uncertainity or disturbance. If at time t_s the states lie on σ and subsequently remain there, then the control action necessary to maintain such motion is given by

$$u_{eq} = -[C_\sigma B]^{-1}[C_\sigma A x(t) + K sign(\sigma)] \tag{15}$$

for all $t \geq t_s$. Where $K = diag[K_1, K_2, \dots, K_m]$ is controller gain matrix, with sufficiently large positive elements and

$$sign(\sigma) = [sign(\sigma_1)\ sign(\sigma_2)\ \ \dots\ \ sign(\sigma_m)]^T \tag{16}$$

enforces the movement of the system trajectories on to a sliding surface in the finite time and remain on it thereafter. The sliding mode reaching condition $\sigma\dot{\sigma} < 0$ is satisfied if the gain matrix *K* is selected with sufficiently large norm.

The sliding mode control is insensitive to matched uncertainties. A drawback of SMC is the chattering resulting from discontinuous control. There are many methods to reduce chattering. One can consider pseudo sliding with smooth control action [6].

$$u_{nl} = K \frac{\sigma}{|\sigma| + \delta} \tag{17}$$

where δ is a small positive scalar also called as tuning parameter used to reduce chattering $(0 < \delta < 1)$. It can be visualized that as $\delta \to 0$, the function (17) tends pointwise to the signum function. The variable δ can be used to trade off the requirement of maintaining ideal performance with that of ensuring a smooth control action.

IV PID CONTROLLER

Proportional, integral and derivative are the basic modes of PID controller. Proportional mode provides a rapid adjustment of the manipulating variable, reduces error and speeds up dynamic response. Integral mode achieves zero offset. Derivative mode provides rapid correction based on the rate of change of controlled variable. The controller transfer function is given by

$$G_C = K_C\left[1+\frac{1}{\tau_i s}+\tau_d s\right] \tag{18}$$

where, K_C, τ_i and τ_d are the proportional, integral and derivative constants of PID controller respectively. PI controller tuning algorithm is based on Ziegler-Nichols open loop method. And the preference is given to the load disturbance rejection.

V MODEL OF DC MOTOR

DC motors are widely used for industrial and domestic applications. Examples are as robotic and actuator for automation process, mechanical motion, and others. Accurate speed control of the motor is the basic requirement in such applications. The electric circuit of the DC motor is shown in Fig. 4. Objective is to control the speed of the motor by armature voltage control. The reference signal determines the desired speed. For simplicity, a constant value as a reference signal is given to the system to obtain desired speed.

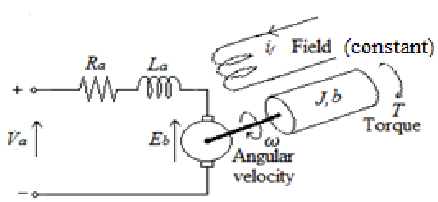


Fig.4 The structure of DC motor

The differential equations governing the dynamics of the system is given by

$$T(t) = J\frac{d\omega(t)}{dt} + b\omega(t) \tag{19}$$

Where ω represents angular velocity in rad/s, J represents the moment of inertia in Kg m^2/s^2 and b is the coefficient of viscous friction which opposes the direction of motion in Nms. The torque T generated by the armature current in Nm is given by

$$T(t) = K_t i_a(t) \tag{20}$$

Where $i_a(t)$ is the armature current in Amp and K_t is torque factor constant in Nm/Amp. This in turn is assumed to satisfy Kirchhoff's voltage law

$$v_a(t) - E_b(t) = R_a i_a(t) + L_a\frac{di_a(t)}{dt} \tag{21}$$

Where L_a and R_a are the armature inductance in H and resistance in ohm respectively and E_b represents electromotive force in V given by

$$E_b(t) = K_b\omega(t) \tag{22}$$

Where K_b is the back emf constant in Vs/rad. The input terminal voltage v_a is taken to be the controlling variable.

Using (19-22), one can write state model with the ω and i_a as state variables and v_a as manipulating variable, as given below

$$\begin{bmatrix}\frac{d\omega(t)}{dt}\\ \frac{di_a(t)}{dt}\end{bmatrix} = \begin{bmatrix}\frac{-b}{J} & \frac{K_t}{J}\\ \frac{-K_b}{L_a} & \frac{-R_a}{L_a}\end{bmatrix}\begin{bmatrix}\omega(t)\\ i_a(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L_a}\end{bmatrix} v_a(t) \tag{23}$$

TABLE 1
PARAMETERS OF DC MOTOR [8]

R_a =1.2Ω	K_b=0.6 V s/rad
L_a =0.05 H	J=0.1352 Kg m^2/s^2
Kt=0.6 Nm/Amp	b=0 Nms

Using the parameters given in Table 1, transfer function of the DC motor with angular velocity as controlled variable and input terminal voltage as manipulating variable is determined as given below

$$\frac{\omega(s)}{v_a(s)} = \frac{88.76}{s^2+24s+53.25} \tag{24}$$

In time domain the above equation can be written as

$$\ddot{\omega}(t) + 24\dot{\omega}(t) + 53.25\omega(t) = 88.76 v_a(t) \tag{25}$$

Now consider

$$x_1 = \omega(t) \text{ and } u = v_a(t) \tag{26}$$

Then the system can be converted in the following canonical form

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -53.25x_1 - 24x_2 + 88.76u \tag{27}$$

$$y = x_1$$

Now select the sliding surface

$$\sigma = c(r - x_1) + x_2 = 0 \tag{28}$$

where c is the constant of sliding matrix $C_\sigma \in R^{m\times n}$ such that $c < 0$. The total control law is then given by

$$u = u_l + u_{nl}$$

$$u = \frac{-1}{88.76}\left\{[-53.25x_1 + (c-24)x_2] + K\frac{\sigma}{|\sigma|+\delta}\right\} \qquad (29)$$

where $K > 0$ is selected sufficiently large. Larger the value of K the faster the trajectory converges to the sliding surface.

VI Simulation Results

Simulation results of the DC motor with control (29) are shown in Fig. 5 and 6. Results of the SMC are compared with the PI controller. Firstly the response of DC motor is observed under normal condition (Fig. 5) and secondly under disturbance (Fig. 6). Fig 5(a) and 5(b) give the control action of the PI and SMC respectively. Fig 5(c) and 5(d) show the sliding function and phase plane plot respectively. From the Fig 5(c) it is clear that the reaching time is near about 0.1 sec. Once the trajectory reaches to the sliding surface it remains there thereafter.

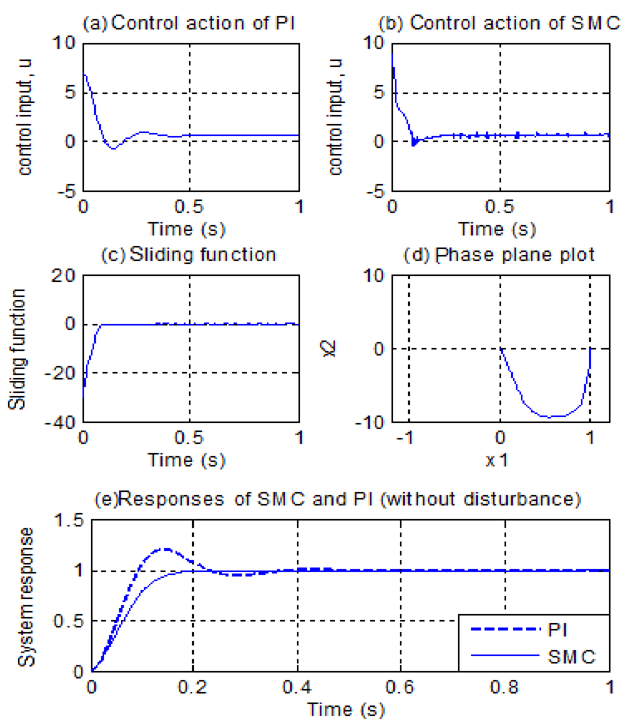


Fig. 5 Responses of the DC motor using SMC and PI controller without disturbance

Fig 6(b) shows the sliding surface under disturbance and it is nearly same to that of without disturbance. Step responses of PI and SMC under normal condition are illustrated in Fig. 5(e). Response of PI controller is underdamped, and that of SMCr is near about critically damped. Settling time of SMCr is less than the PI controller, however, rise time is more. Under disturbance (Fig. 6(a)) response of PI controller is degraded and is not stabilized as shown in Fig. 6(d). However, response of the SMC is smooth and almost critically damped. This shows that the disturbance do not affect the system in the sliding mode.

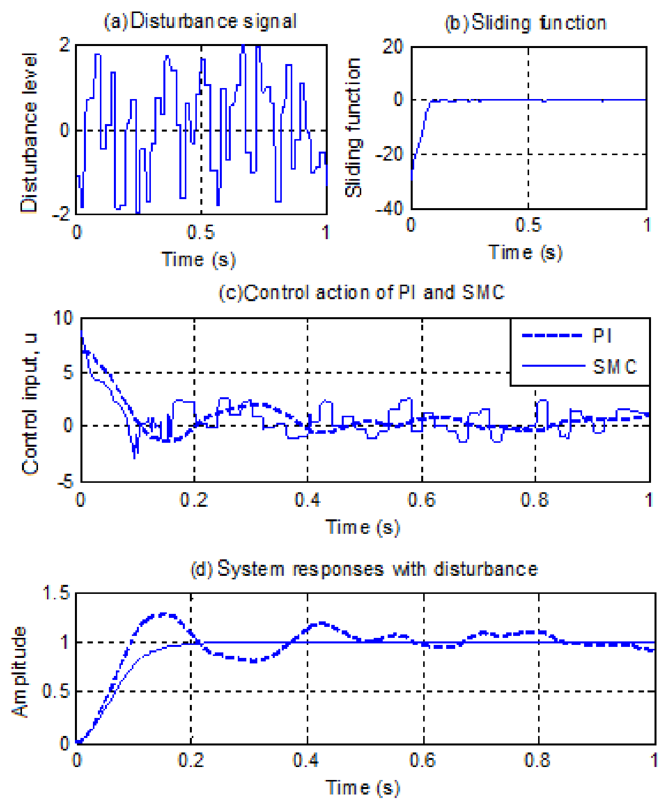


Fig. 6 Responses of the DC motor using SMC and PI controller with disturbance

VII Conclusion

In this paper PI controller and SMCr are applied for the speed control of DC motor. It is observed that SMCr is better than PI controller, both with and without disturbance. Specially PI response is observed to be degraded while SMCr response remains unchanged, when disturbance is applied. As a result SMCr is more effective and versatile as compared to PI controller. The sliding mode controller exhibits insensitivity to such disturbances. However, the cost of SMCr is high because of high switching, powerful computing device & advanced software compared to PI controller.

Refernces

[1] Hung J. Y., Gao W., and Hung J. C., "Variable Structure Control: A Survey", *IEEE Trans., on Industrial Electronics*, Vol. 40, No. 1, February 1993, pp. 2-22

[2] V. I Utkin., "Variable Structure System with Sliding Mode" survey paper *IEEE Trans., on Automatic Control,* Vol. 22, No. 2, April 1977, pp. 212-222

[3] Young K. D., Utkin V. I. and Ozguner U., "A Control Engineer's Guide to Sliding Mode Control", *IEEE Trans., on Control System Technology*, Vol. 7, No. 1, May 1993, pp. 328-342.

[4] Oscar Camacho, Carlos A. Smith.: "Sliding Mode Control: an Approach to Regulate Nonlinear Chemical Processes", *ISA Trans.,* Vol. 39, 2000, pp. 205-18.

[5] Ali J. Koshkouei, Keith J. Burnham, "Control of DC Motor Using Proportional Integral Sliding Mode", Control Theory and Application Centre, Coventry University, Conventry.

[6] Tossaporn Chamsai, Piyoros Jirawattana and Thana Radpukdee, "Sliding Mode Control with PID Tuning Technique: An Application to a DC Servo Motor Position Tracking Control", *Energy Research Journal* 1 (2): 2010, pp. 55-61.

[7] Igor Karaskakovski1, Goce Shutinoski2, "Comparison of Sliding Mode and Proportional Integral Control for Brushless DC Motor", ELECTRONICS' Sozopol, BULGARIA, September, 2005, pp. 21-23.

[8] C. Edwards, S. Spurgeon, "Sliding Mode Control: Theory and Applications", *Taylor and Francis*, Bristol, PA, 1998.