Backpropagation

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Backpropagation

- We've got the forward propagation, but how to train the neural network?
- many approaches, but Backward-Propagation is enabling the whole Deep-Learning Hype

for BP to work, **everything** needs to be differentiable!!!!

Intuition: Learning as an Optimization-Problem

in theory, it's easy!

1. assign every instance of neural network a score through a differentiable function (error function), e.g. derivation from target
2. optimize the score

Intuition: Learning as an Optimization-Problem

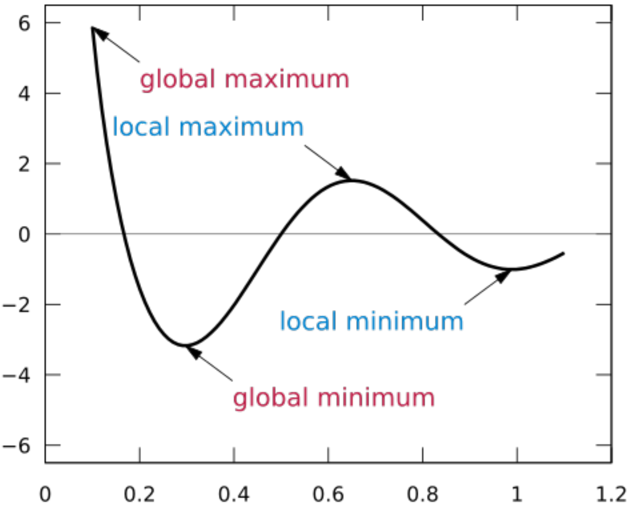


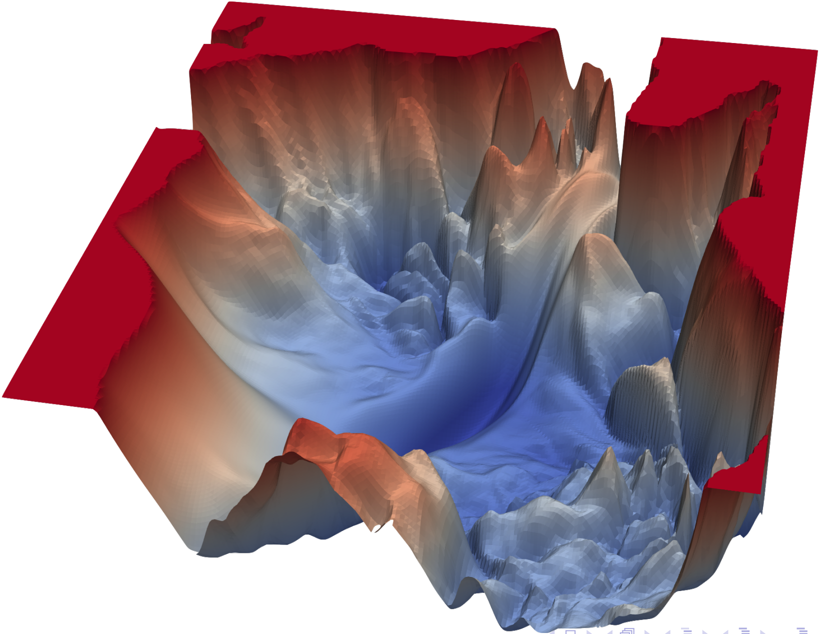
Figure: [Com07]

Intuition: Learning as an Optimization-Problem

common vocabulary:

1. a **Loss-Function** L is something I want to minimize
2. an **Error-Function** $E : Output_{NN} \times Target \to \mathbb{R}$
   $\Rightarrow$ in our context: Loss-Function = Error-Function

Intuition: Learning as an Optimization-Problem



The BP-Algorithm

essentially: nothing more than the chain rule!

A naive formulation

w_{ijz} is the weight z of neuron j in layer z
Gradients of our Error Function: $\nabla E = \{\frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, ..., \frac{\partial E}{\partial w_l}\}$

A naive formulation

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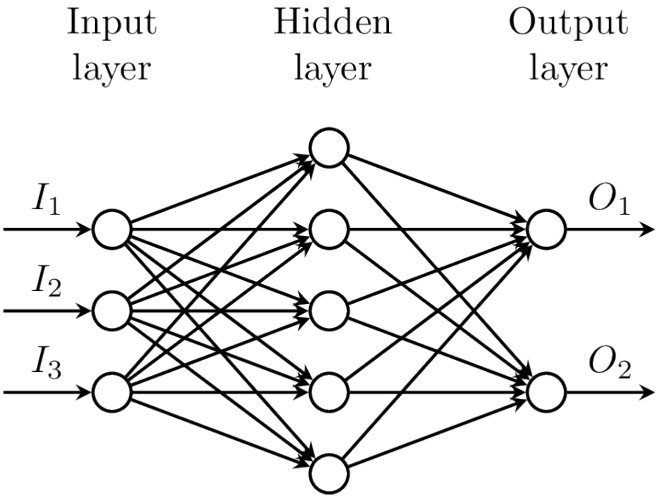
Definition

update weight: $\Delta w_{ij} = -\lambda \cdot \frac{\partial E}{\partial w_{ij}}$*, where* λ *is the learning rate ("step size")*

This is expensive!



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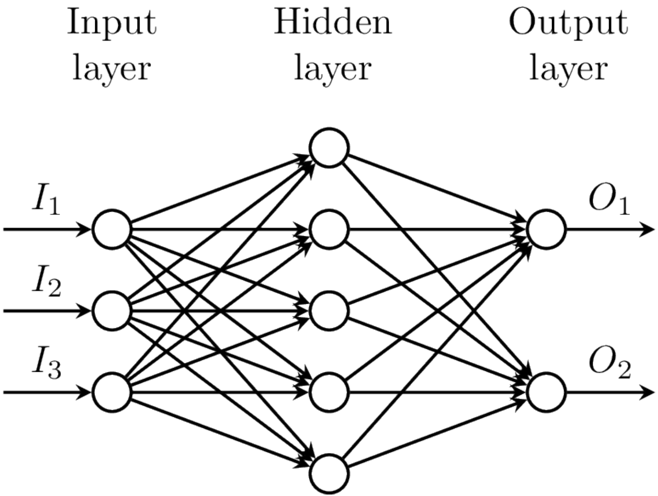

- $5 \cdot 3 + 5 * 2 = 25$ weights!

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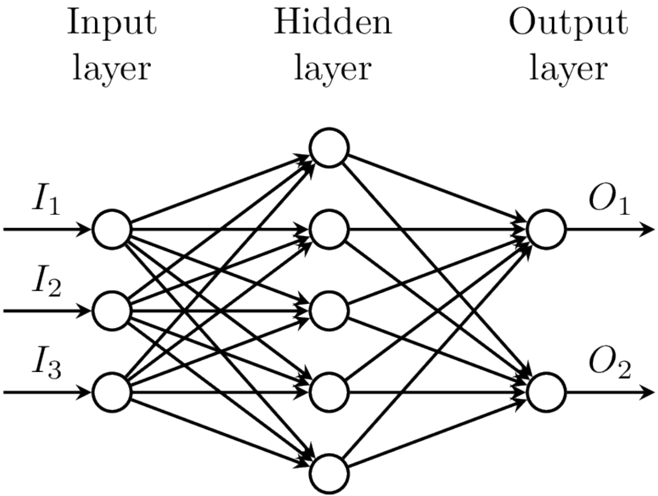

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- often hundreds of thousands of weights in real scenarios!

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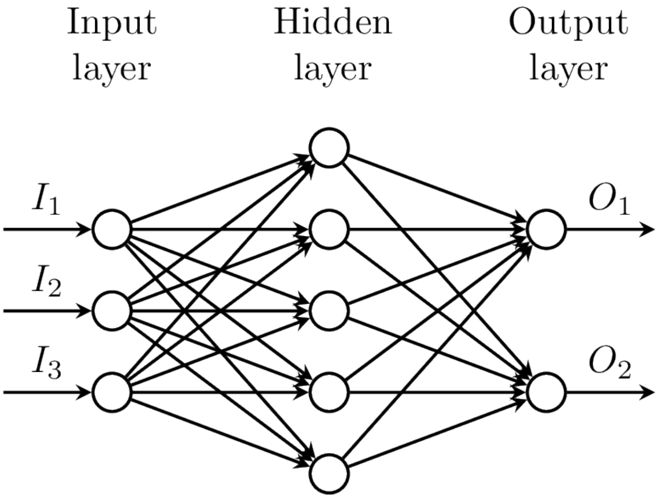

- $5 \cdot 3 + 5 * 2 = 25$ weights!
- often hundreds of thousands of weights in real scenarios!
- and this is **one** update

Recap: Generalization of the Chain-Rule

single variable: $[f \circ g(x)]' = (f' \circ g)(x) \cdot g'(x)$
different notation: for $y = f(t)$, $t = g(x)$: $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$

generalized to multivar.:
For $y = f(m_1, m_2)$,$m_1 = g(x_1, x_2)$ and $m_2 = h(x_1, x_2)$:
$\frac{\partial y}{\partial x_1} = \frac{\partial y}{\partial m_1}\frac{\partial m_1}{\partial x_1} + \frac{\partial y}{\partial m_2}\frac{\partial m_2}{\partial x_1}$

Chain rule to the rescue!

$y_{iz} = f(n_{iz})$, $n_{iz} = w_{iz}^T \cdot x_z + b_{iz}$ and trained with $H(\vec{target}, \vec{y_2})$

$$\frac{\partial H}{\partial w_{piz}} = \frac{\partial H}{\partial y_{iz}} \frac{\partial y_{iz}}{\partial w_{piz}} \tag{1}$$

$$\tag{3}$$

Chain rule to the rescue!

$$y_{iz} = f(n_{iz}),\; n_{iz} = w_{iz}^T \cdot x_z + b_{iz} \text{ and trained with } H(\vec{target}, \vec{y_2})$$

$$\frac{\partial H}{\partial w_{piz}} = \frac{\partial H}{\partial y_{iz}} \frac{\partial y_{iz}}{\partial w_{piz}} \tag{1}$$

$$= \frac{\partial H}{\partial y_{iz}} \frac{\partial y_{iz}}{\partial n_{iz}} \frac{\partial n_{iz}}{\partial w_{piz}} \tag{2}$$

$$\tag{3}$$

Chain rule to the rescue!

$y_{iz} = f(n_{iz})$, $n_{iz} = w_{iz}^T \cdot x_z + b_{iz}$ and trained with $H(\vec{target}, \vec{y_2})$

$$
\begin{align}
\frac{\partial H}{\partial w_{piz}} &= \frac{\partial H}{\partial y_{iz}} \frac{\partial y_{iz}}{\partial w_{piz}} \tag{1} \\
&= \frac{\partial H}{\partial y_{iz}} \frac{\partial y_{iz}}{\partial n_{iz}} \frac{\partial n_{iz}}{\partial w_{piz}} \tag{2} \\
&= \Big(\sum_{j_{(z+1)}=0}^{I_{(z+1)}} \frac{\partial H}{\partial y_{j_{(z+1)}(z+1)}} \frac{\partial y_{j_{(z+1)}(z+1)}}{\partial y_{iz}} \Big) \frac{\partial y_{iz}}{\partial n_{iz}} x_{piz} \tag{3}
\end{align}
$$

Chain rule to the rescue!

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$$\quad (5)$$

Chain rule to the rescue!

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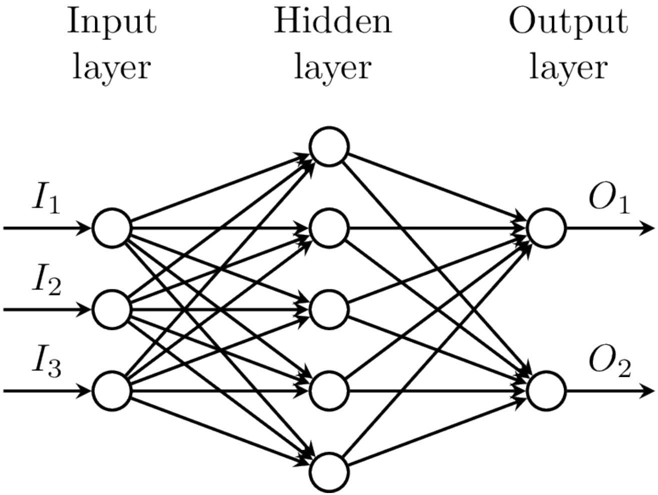
$$= \left(\sum_{j_{(z+1)}=0}^{I_{(z+1)}} \frac{\partial H}{\partial y_{j_{(z+1)}(z+1)}} \frac{\partial y_{j_{(z+1)}(z+1)}}{\partial n_{j_{(z+1)}(z+1)}} w_{ij_{(z+1)}(z+1)}\right) \frac{\partial y_{iz}}{\partial n_{iz}} x_{piz} \tag{5}$$

Chain rule to the rescue!



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Hao Li et al. "Visualizing the Loss Landscape of Neural Nets". In: *CoRR* abs/1712.09913 (2017). arXiv: 1712.09913. URL: http://arxiv.org/abs/1712.09913.