Homework 5

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1 Problem 1

Given

$$Z_{at}(\lambda) = (1 + e^{\beta\mu}) \prod_{n=-\infty}^{\infty} (1 - \frac{\lambda}{i\omega_n + \mu}), \tag{1}$$

1.1 1(a)

clearly

$$\begin{aligned} -\frac{1}{Z_{at}} \frac{\partial Z_{at}}{\partial \lambda_n} &= -\frac{1}{Z_{at}} \frac{\partial}{\partial \lambda_n} (1 + e^{\beta\mu}) \prod_{n=-\infty}^{\infty} (1 - \frac{\lambda}{i\omega_n + \mu}) \\ &= -\frac{1}{Z_{at}} (1 + e^{\beta\mu}) \prod (-\frac{1}{i\omega_n + \mu}) \\ &= \frac{(1 + e^{\beta\mu}) \prod (-\frac{1}{i\omega_n+\mu})}{(1 + e^{\beta\mu}) \prod (1 - \frac{\lambda}{i\omega_n+\mu})} \\ &= \boxed{\frac{1}{i\omega_n + \mu - \lambda_n}}. \end{aligned} \tag{2}$$

1.2 1(b)

The mystery of such infinite product is still unsolved...

1.3 1(c)

$$\begin{aligned} \mathrm{Tr}\left\{ e^{\beta\mu c^+ c} \right\} &= \langle 0 | e^{\beta\mu c^+ c} | 0 \rangle + \langle 1 | e^{\beta\mu c^+ c} | 1 \rangle \\ &= \langle 0 | 0 \rangle e^{\beta\mu 0} + \langle 1 | 1 \rangle e^{\beta\mu 1} \\ &= \boxed{1 + e^{\beta\mu}}. \end{aligned} \tag{3}$$

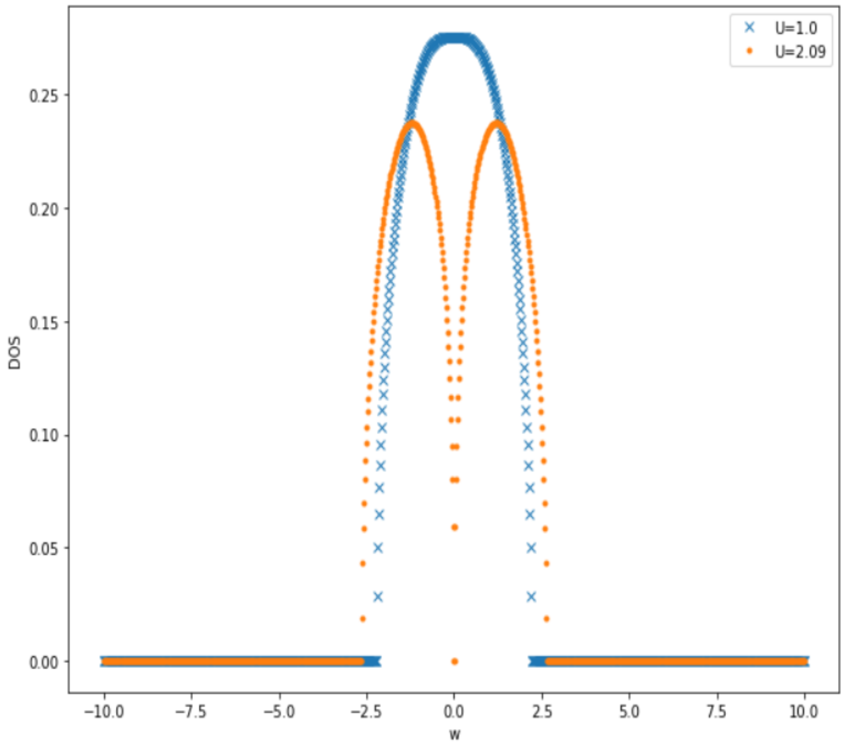

Figure 1: At $T = 2.09$ (which I think is supposed to be 2.0, but due to blunt-ness of the code, it can only split as early as 2.09), the DOS reaches 0, hence showing the split of the graph.

2 Problem 2

2.1 2(a)

2.2 2(b)

3 Problem 3

In the small U region, the DMFT algorithm and the Kramer-Kronig analysis agree with over 0.9999 accuracy, however, the accuracy decreases as the absolute value of U becomes larger.

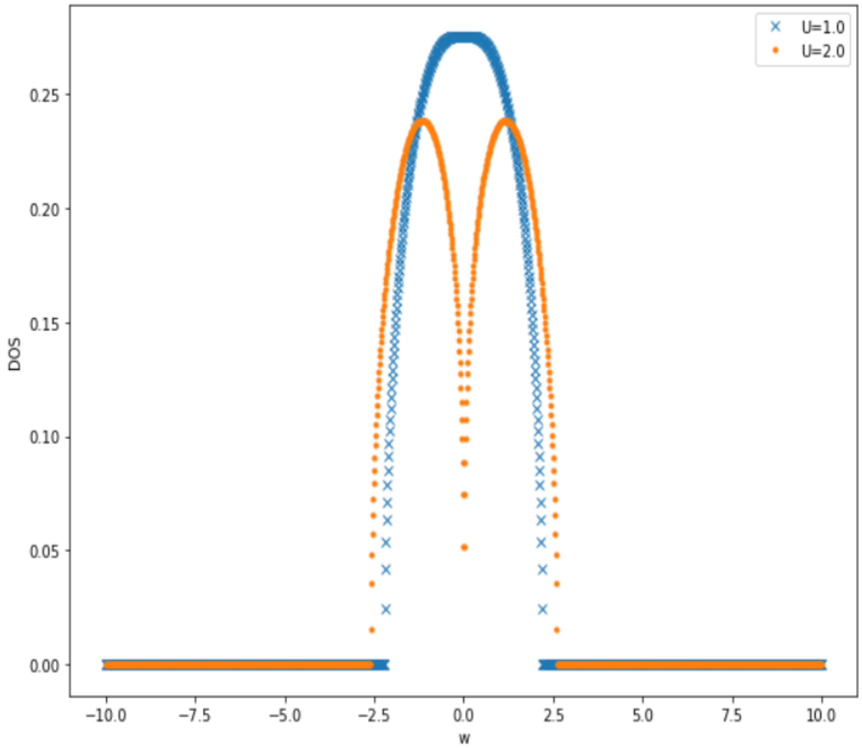

Figure 2: At $T = 2.0$, the DOS should have a split.

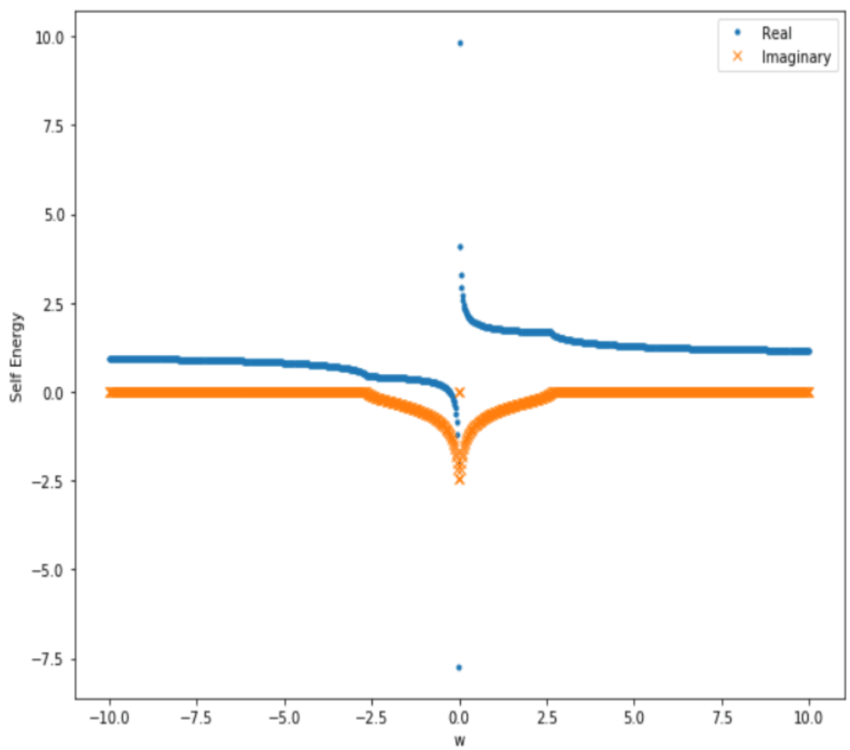

Figure 3: At 0, both real and imaginary part of the self energy develop a pole.

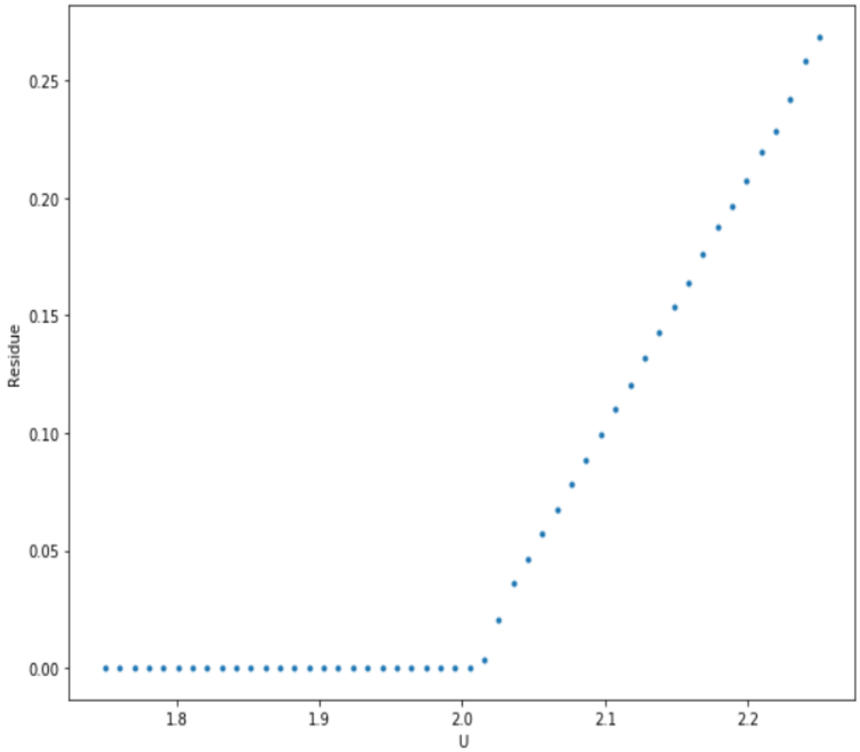

Figure 4: At 2.09, the residue starts to deviate from 0 and grow linearly.

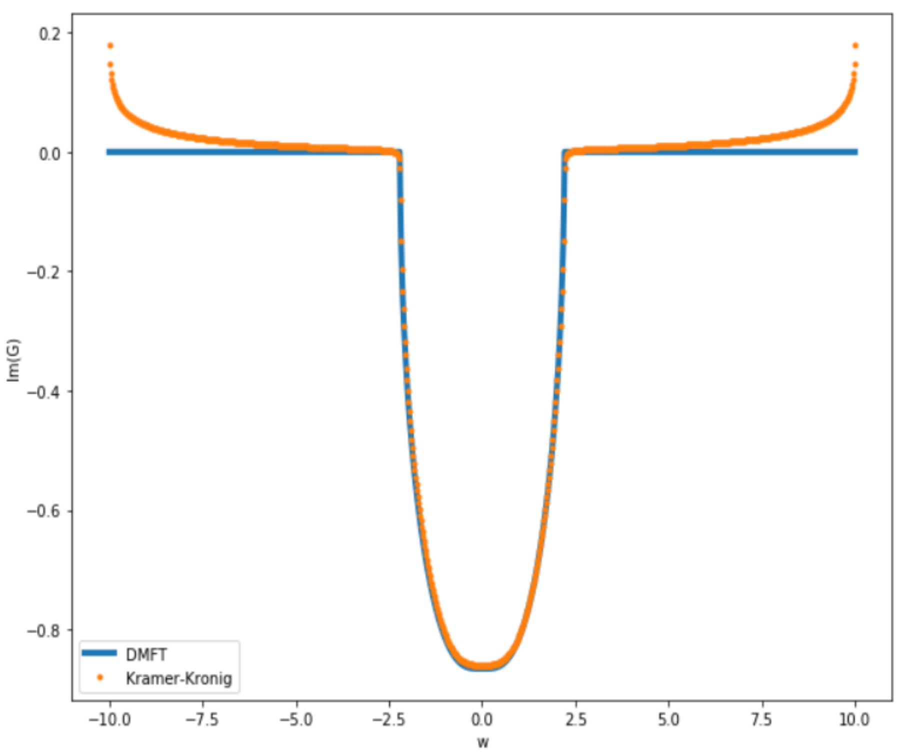

Figure 5: $U = 1$

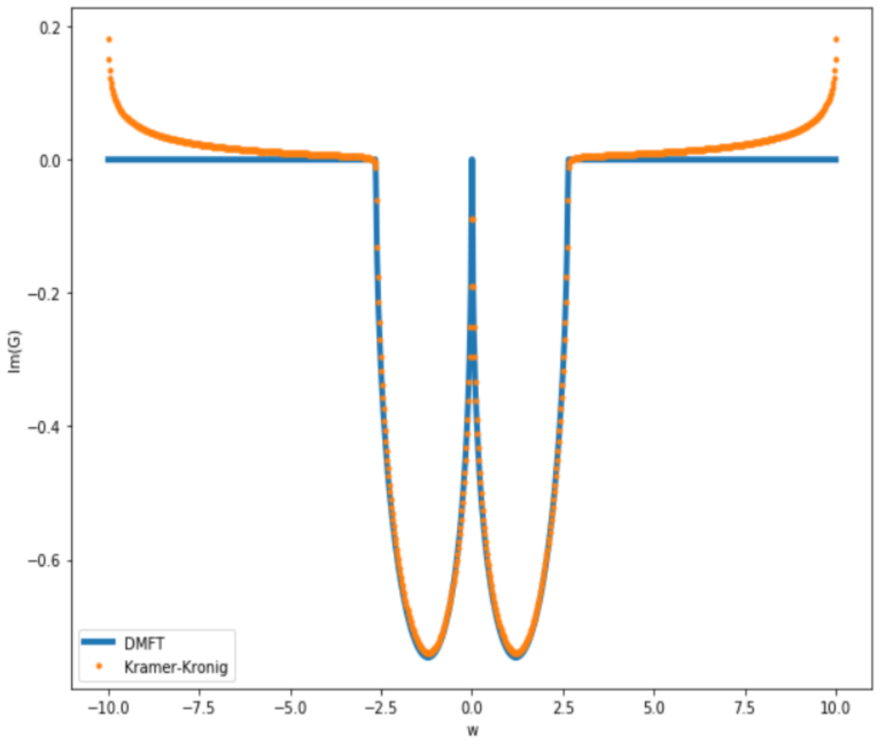

Figure 6: $U = 2$

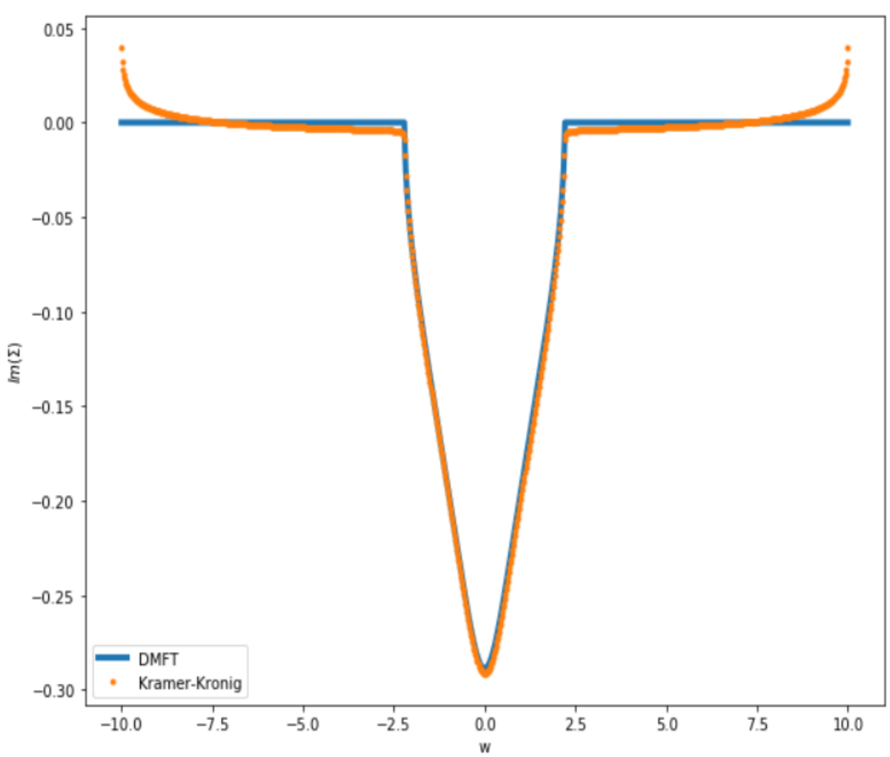

Figure 7: $U = 1$

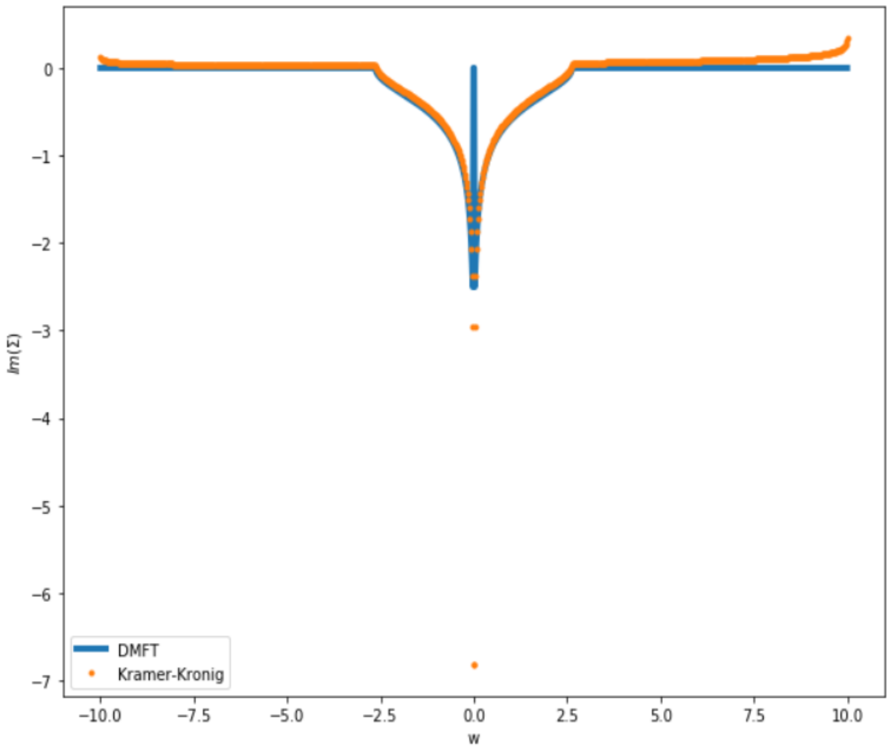

Figure 8: $U = 2$

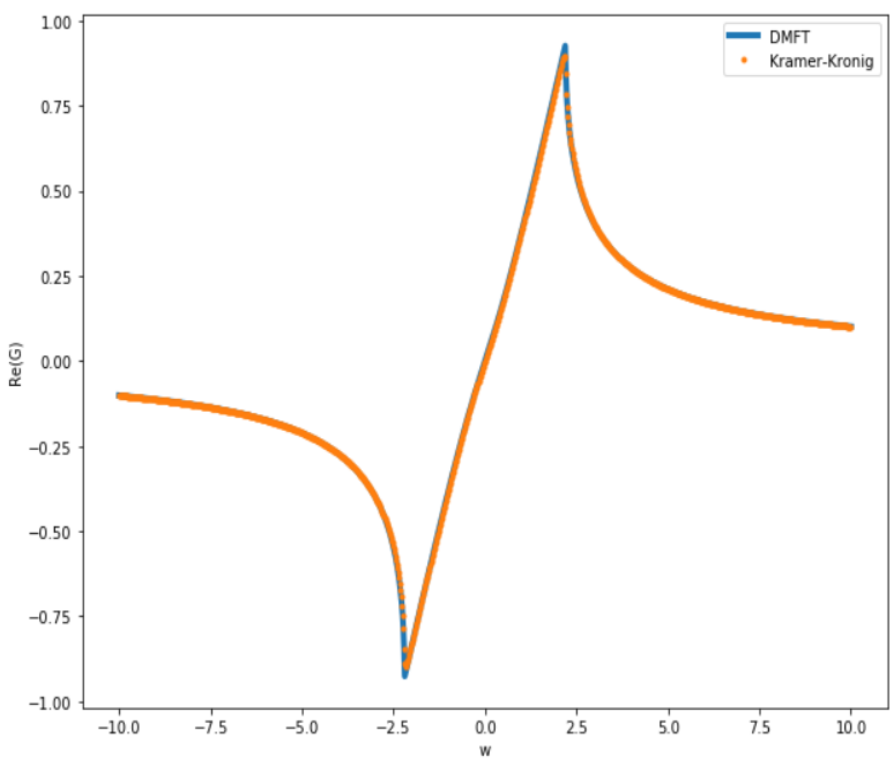

Figure 9: $U = 1$

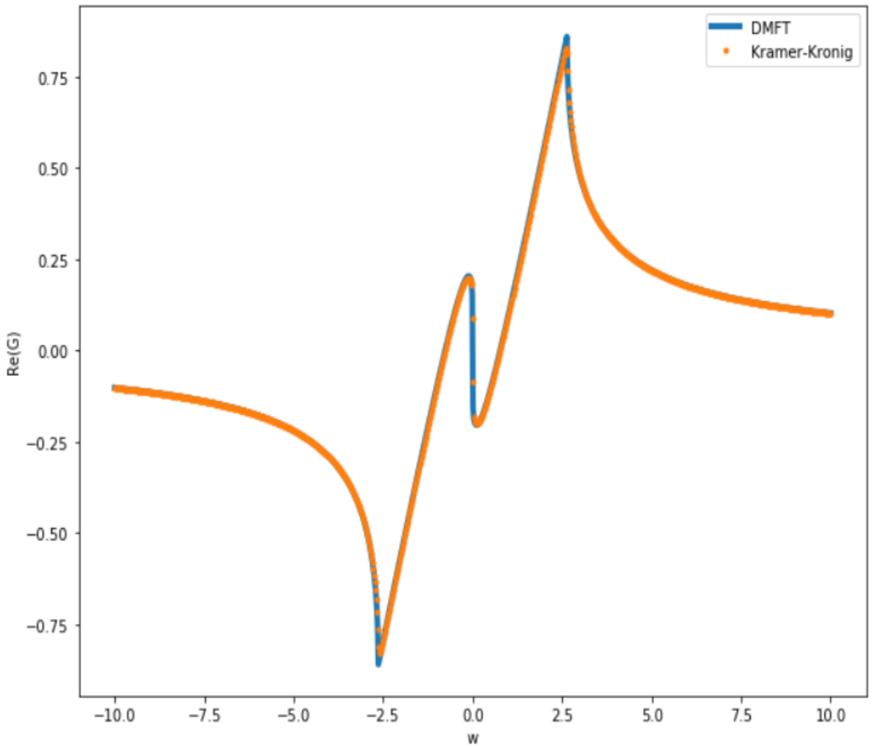

Figure 10: $U = 2$

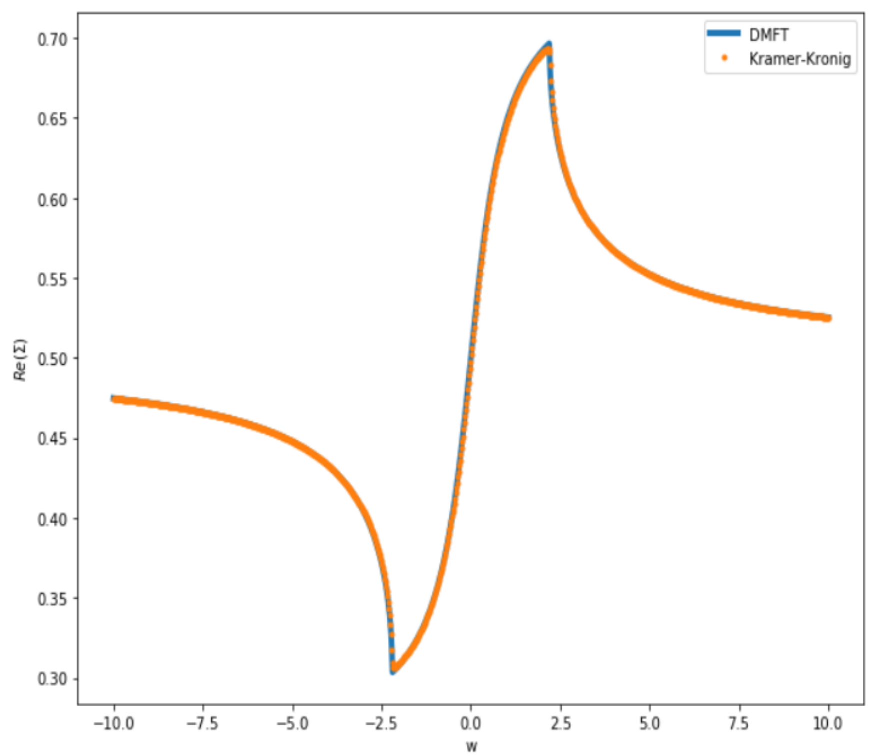

Figure 11: $U = 1$

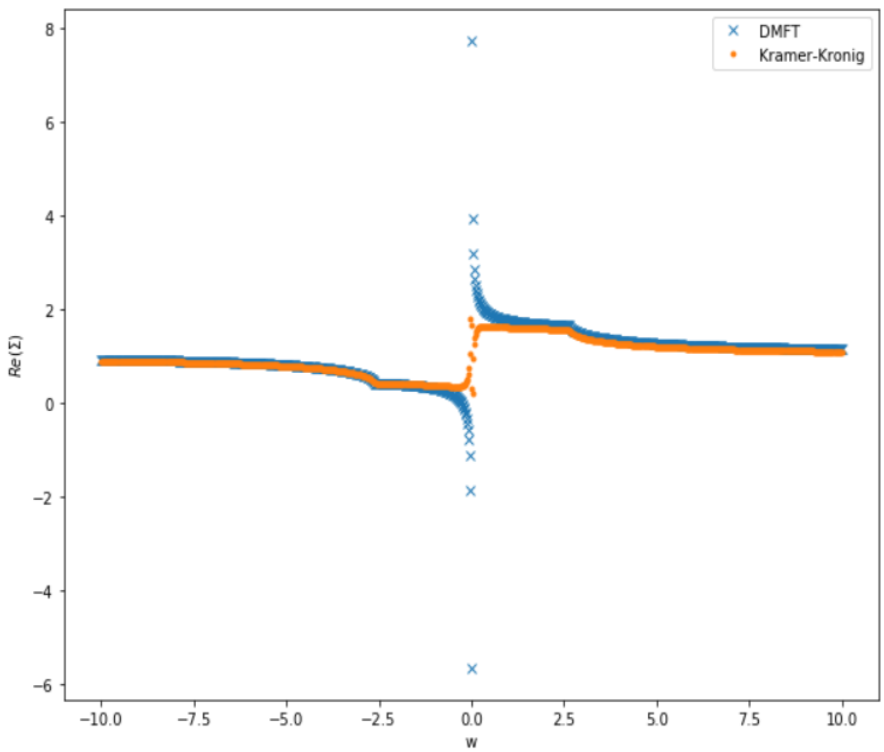

Figure 12: $U = 2$