Lecture 5: Nodal and Mesh Analysis

ELEC1111 Electrical and Telecommunications Engineering

Never Stand Still | Faculty of Engineering | School of Electrical Engineering and Telecommunications

NETWORK ANALYSIS

Recall: Things we need to know in solving any resistive circuit with current and voltage sources only:

- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)
- Ohm's Law

How else should we apply these laws?

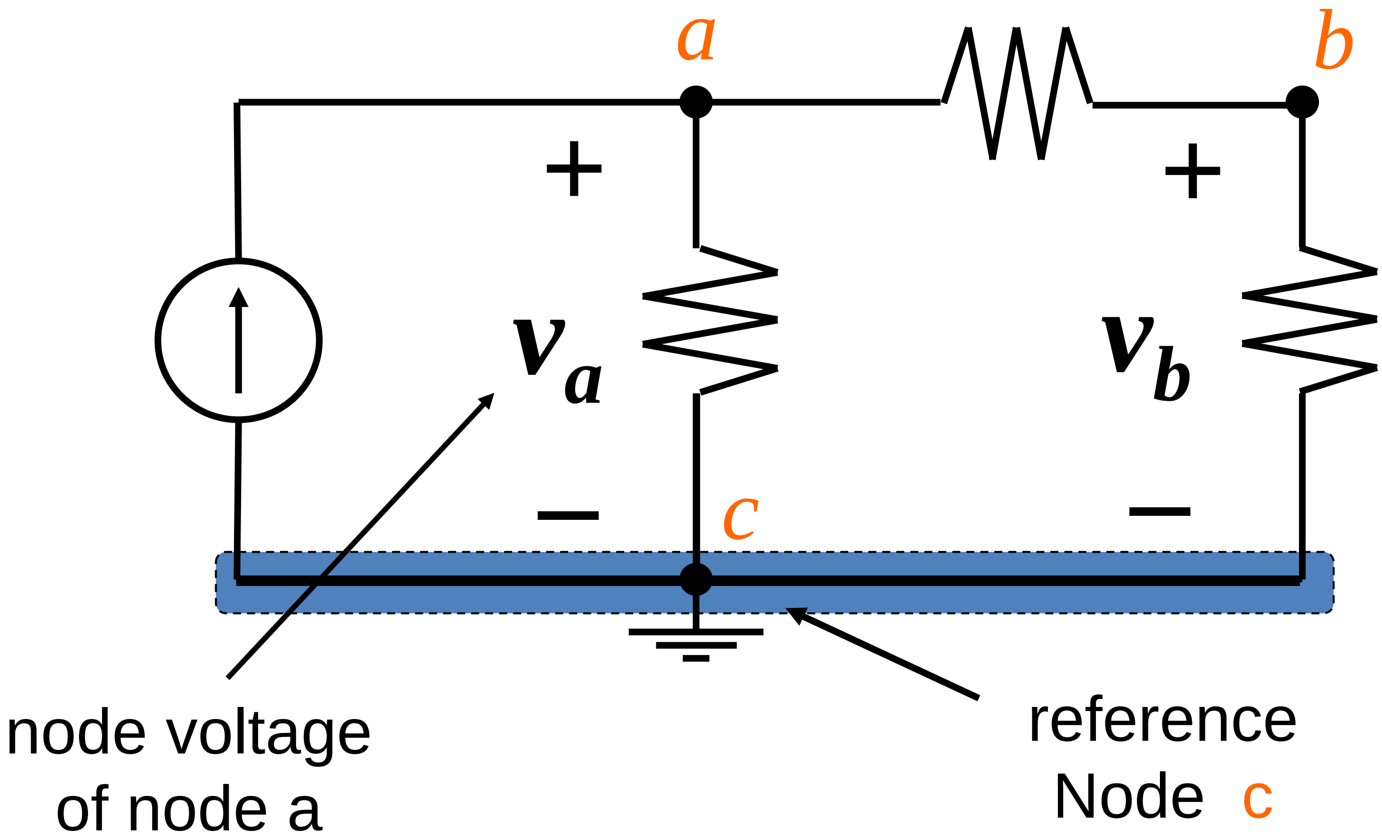
NODAL ANALYSIS

It provides a general procedure for analyzing circuits using <u>node voltages</u> as the circuit variables.

Objective: To solve for these node voltages

- ✓ In general, an *N-node* circuit will need *N*-1 voltages and *N*-1 equations.
- ✓ Apply KCL at each node except for one node – the *reference node*.
- ✓ Any node can be chosen as the reference node. Most common choices are: the ground node, top or bottom node, node connected to the highest number of branches

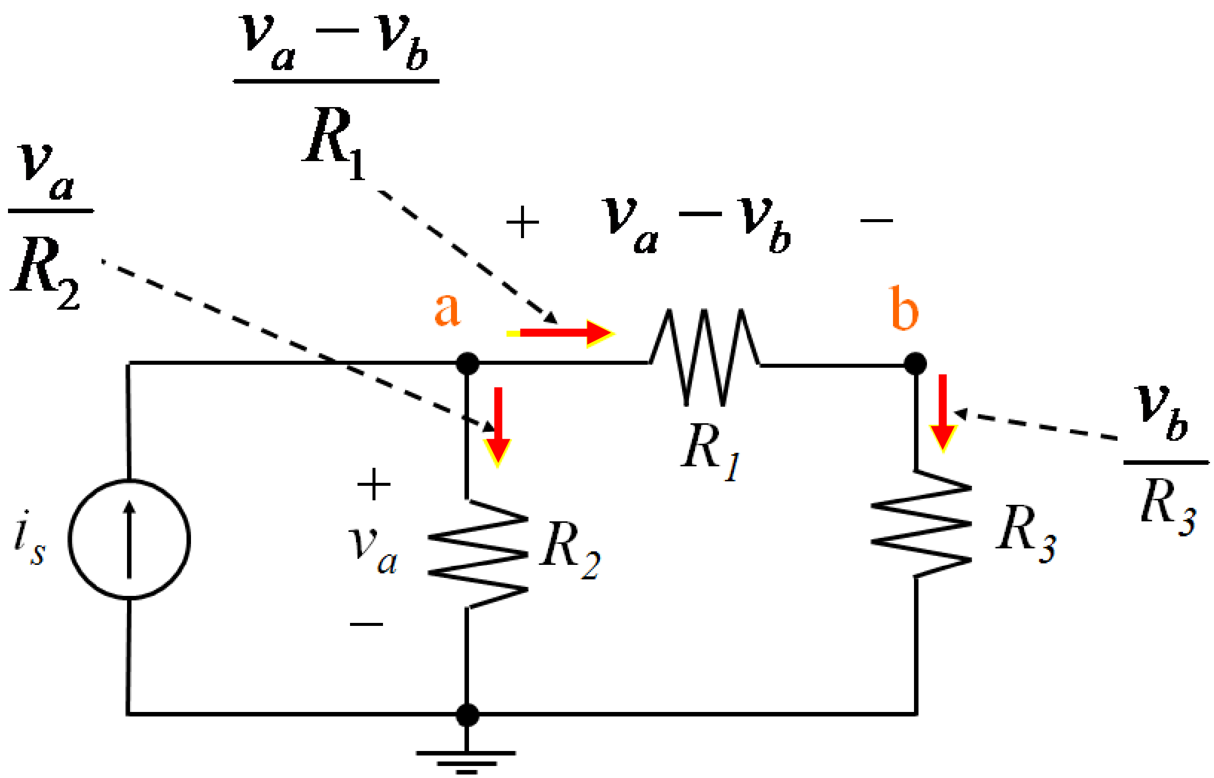
NODE VOLTAGE



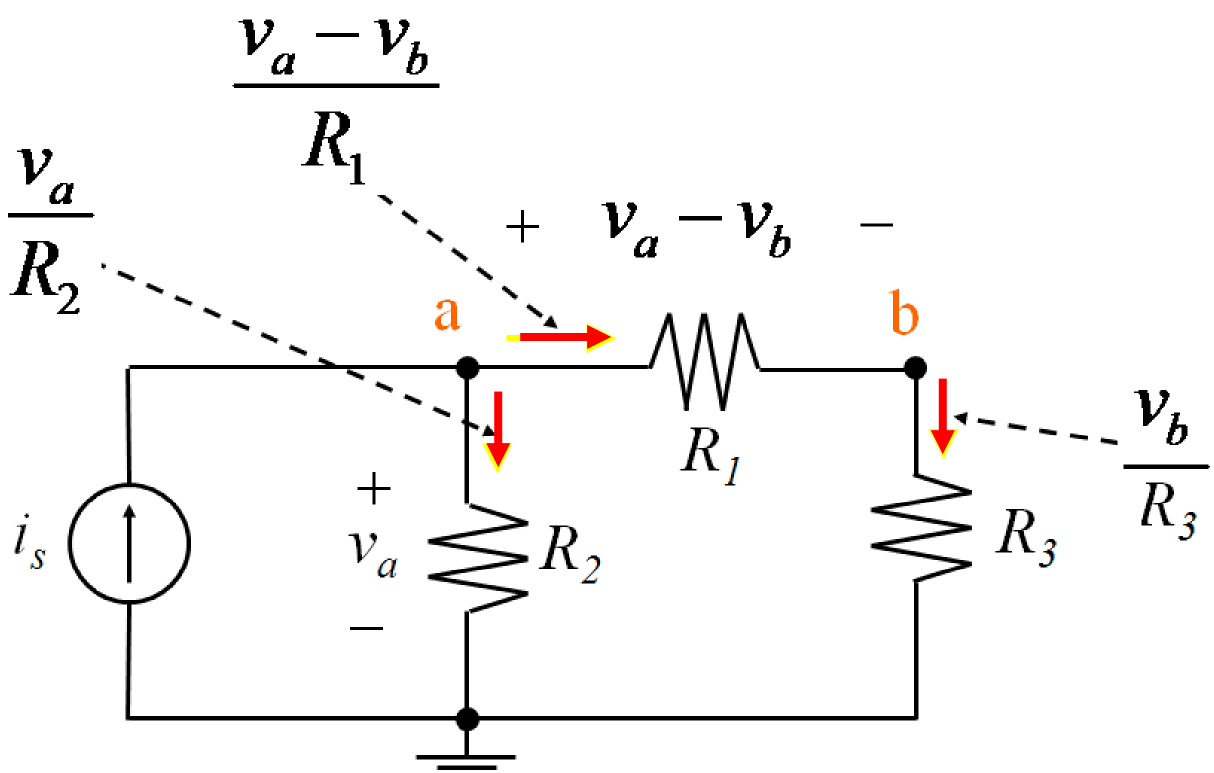
$$v_a \equiv v_{ac}$$

$$v_b \equiv v_{bc}$$

NODAL ANALYSIS
(INDEPENDENT CURRENT SOURCES ONLY)



NODAL ANALYSIS
(INDEPENDENT CURRENT SOURCES ONLY)



KCL for node a:

$$i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1}$$

KCL for node b:

$$\frac{v_a - v_b}{R_1} - \frac{v_b}{R_3} = 0$$

PROCEDURE FOR NODAL ANALYSIS

1. Find all essential nodes (more than 2 branches).

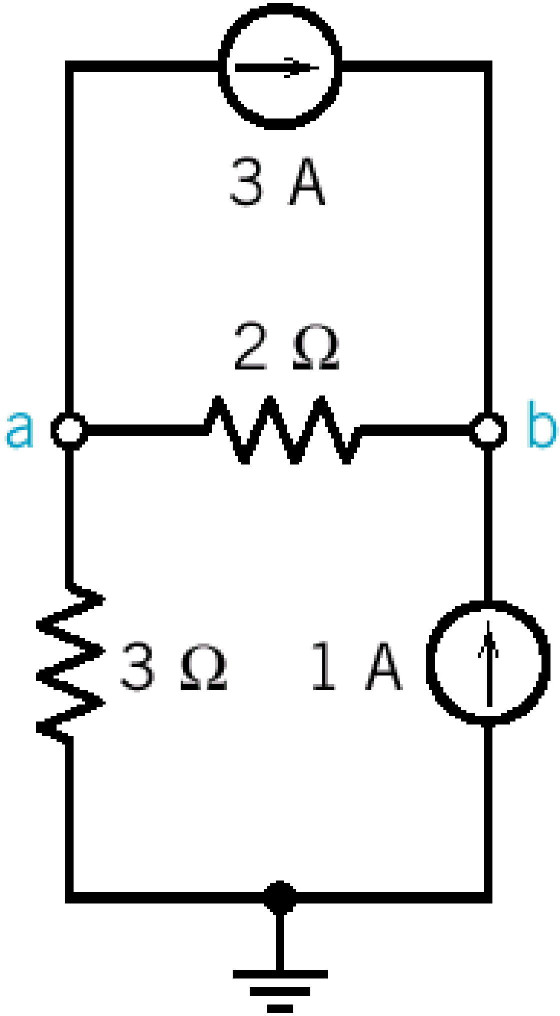
2. Select a node as the reference node.

3. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.

4. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

5. Solve the resulting simultaneous equations to obtain the unknown node voltages.

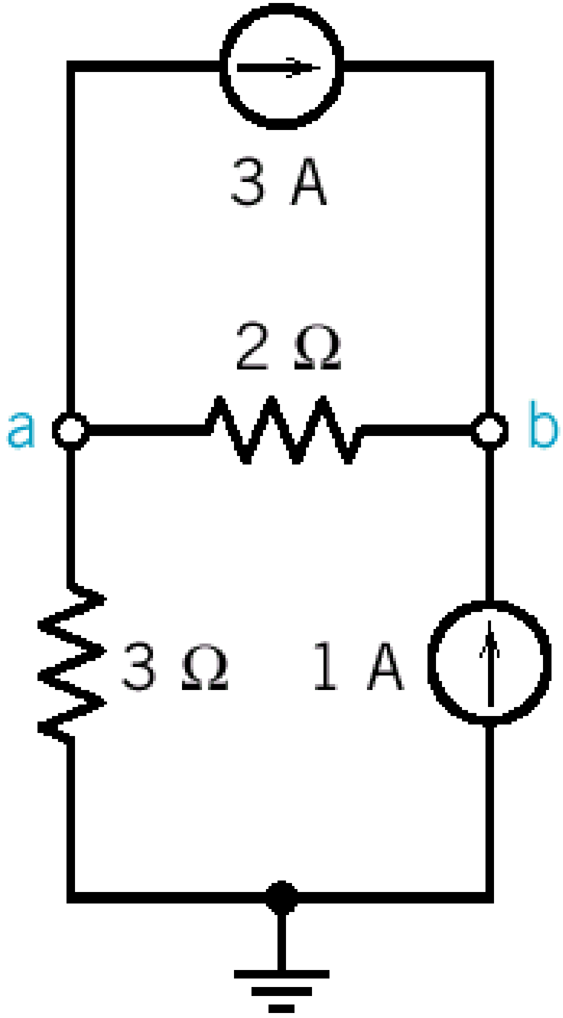
NODAL ANALYSIS – EXAMPLE 1



KCL for node a:

KCL for node b:

NODAL ANALYSIS – EXAMPLE 1



KCL for node b:

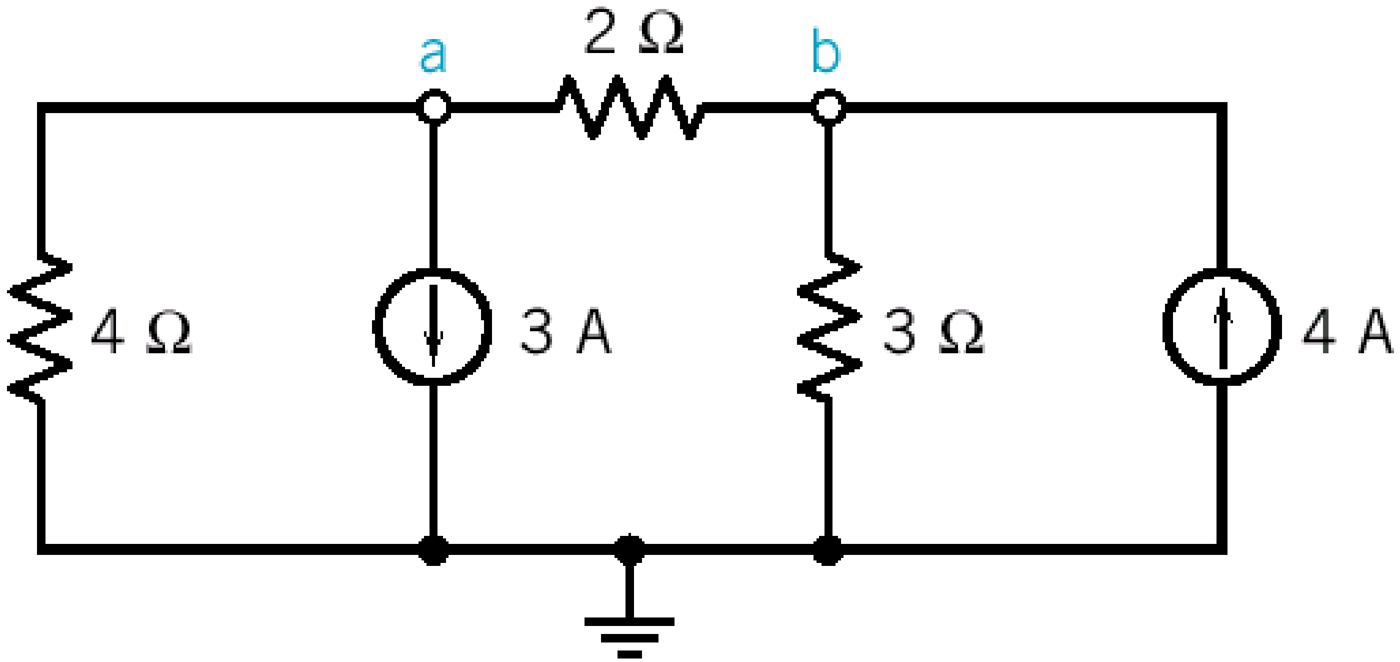
$$\frac{v_a}{3}+\frac{v_a-v_b}{2}+3=0$$

$$\frac{v_a-v_b}{2}+3+1=0$$

NODAL ANALYSIS – EXAMPLE 2

KCL for node a:

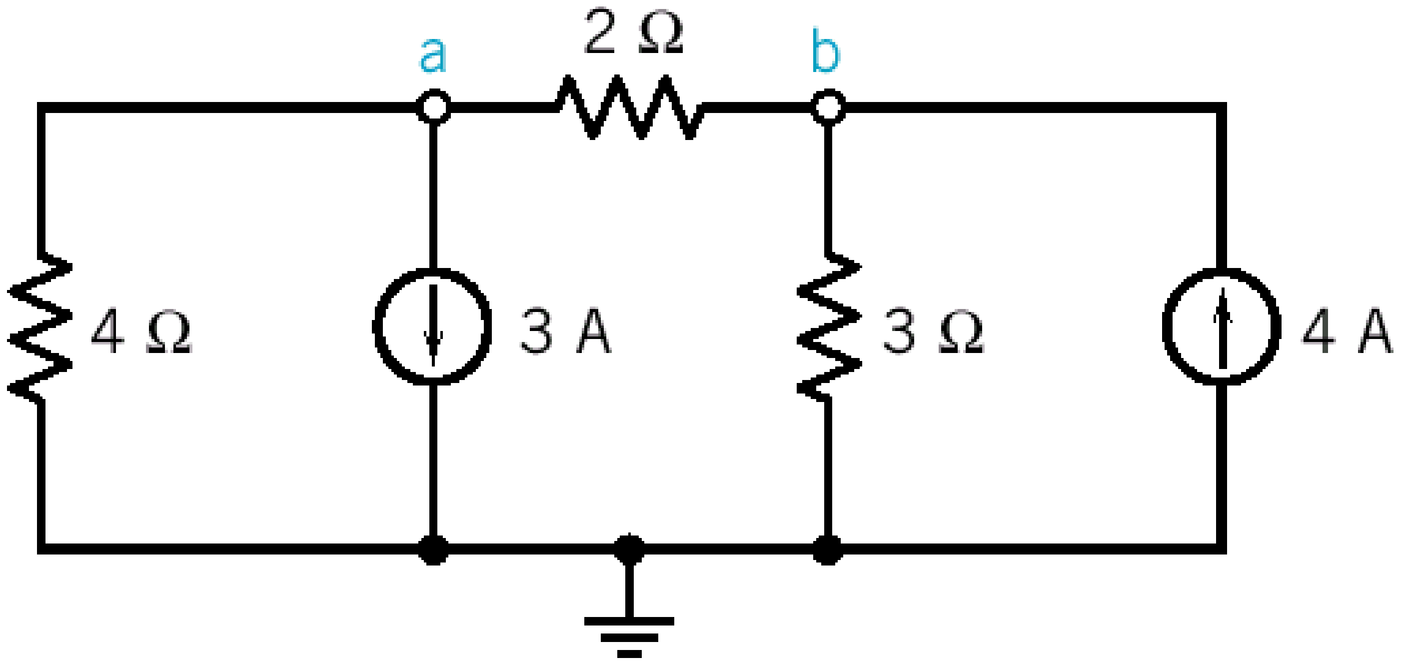
KCL for node b:



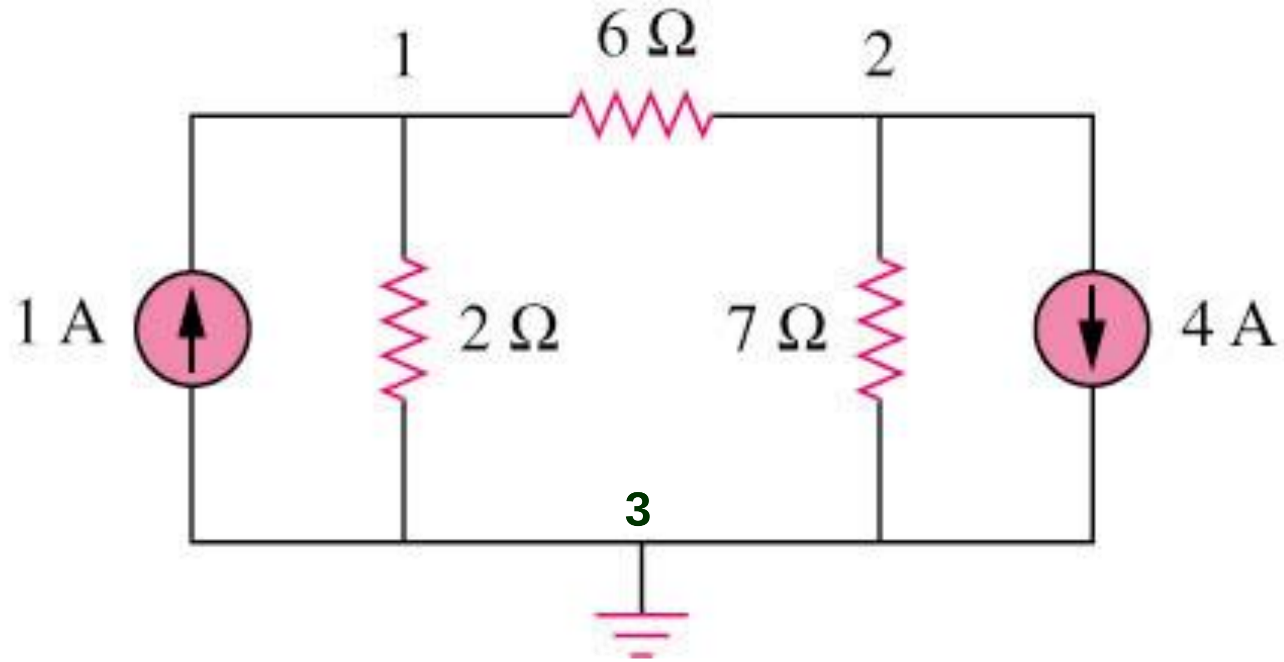
NODAL ANALYSIS – EXAMPLE 2

$$\frac{v_a}{4} + \frac{v_a - v_b}{2} + 3 = 0$$

$$\frac{v_a - v_b}{2} - \frac{v_b}{3} = -4$$

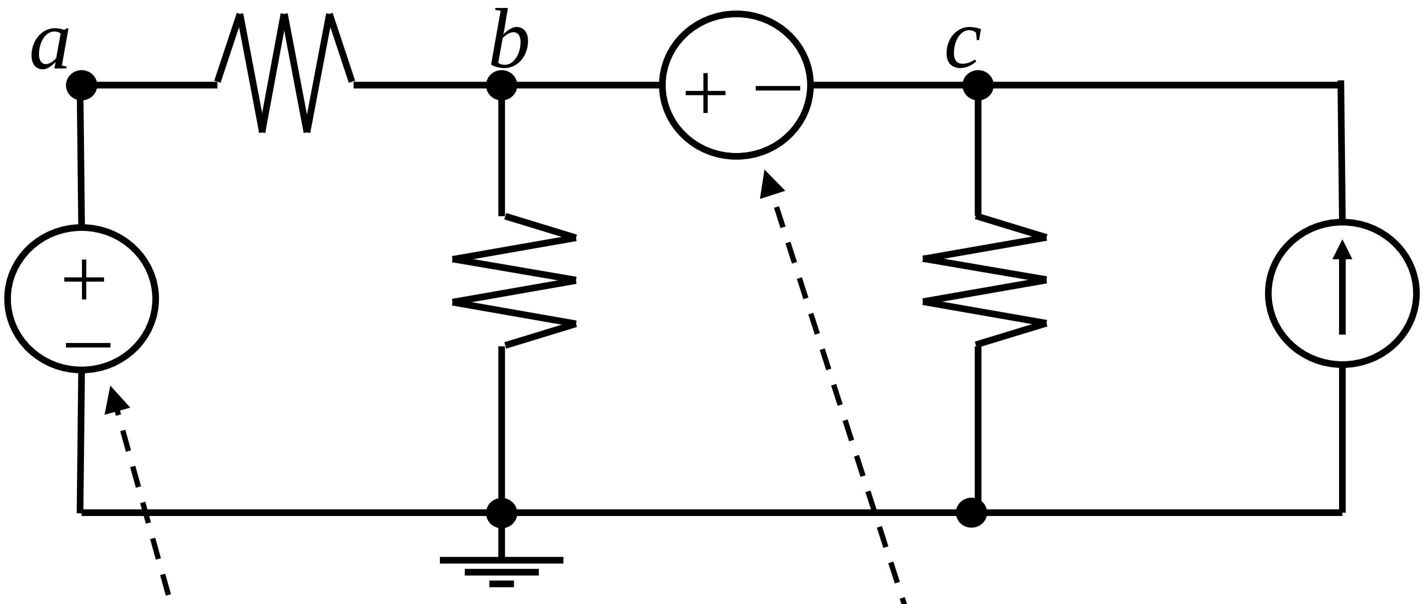


NODAL ANALYSIS – EXAMPLE 3



What are the voltages at node 1 and 2?

NODAL ANALYSIS (INDEPENDENT VOLTAGE SOURCES)



Case 1: between a node and reference.

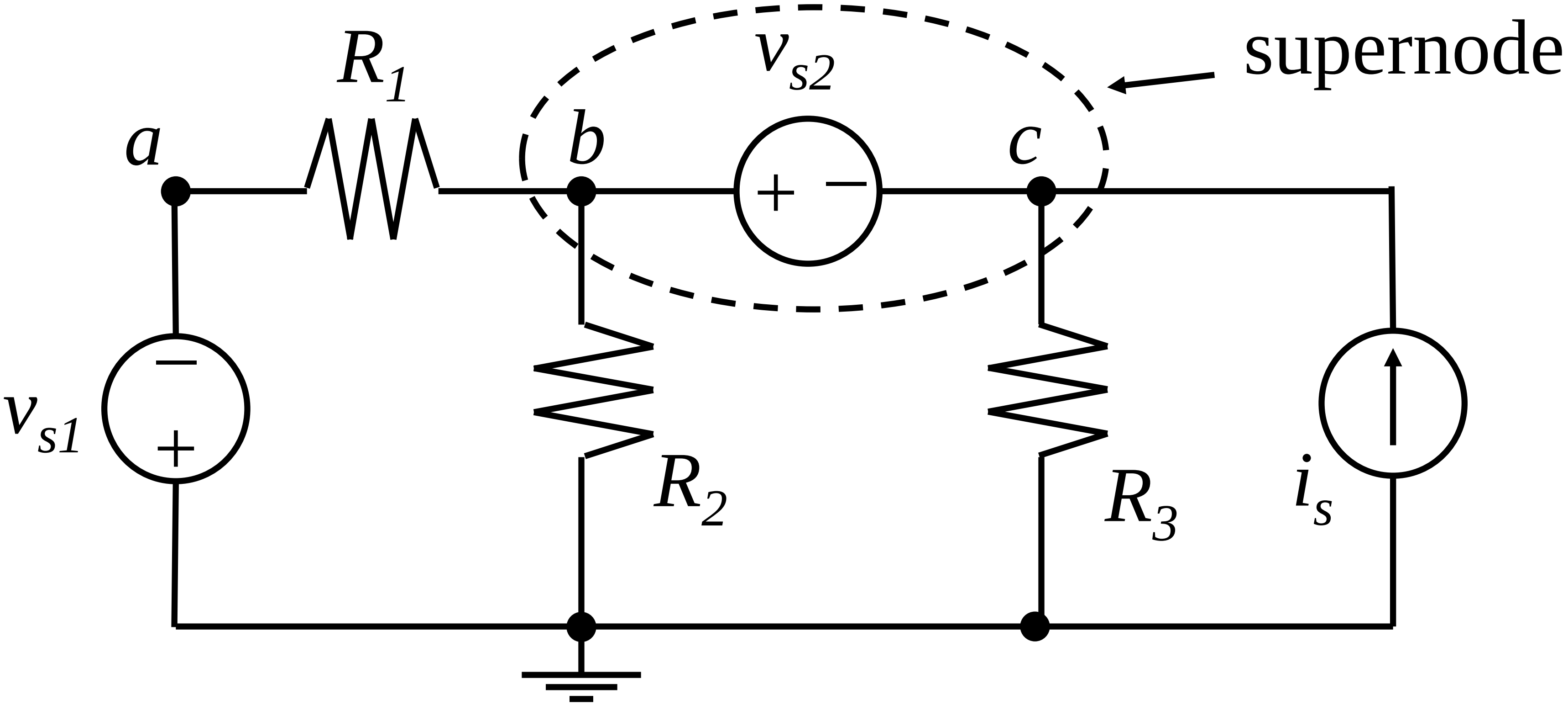
Case 2: between two non-reference nodes.

NODAL ANALYSIS – CONCEPT OF SUPERNODE

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

*Note: We analyse a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.

CONCEPT OF SUPERNODE



Algebraic sum of currents at a supernode = 0

$$\frac{v_b - v_a}{R_1} + \frac{v_b}{R_2} + \frac{v_c}{R_3} = i_s \qquad v_b - v_c = v_{s2}$$

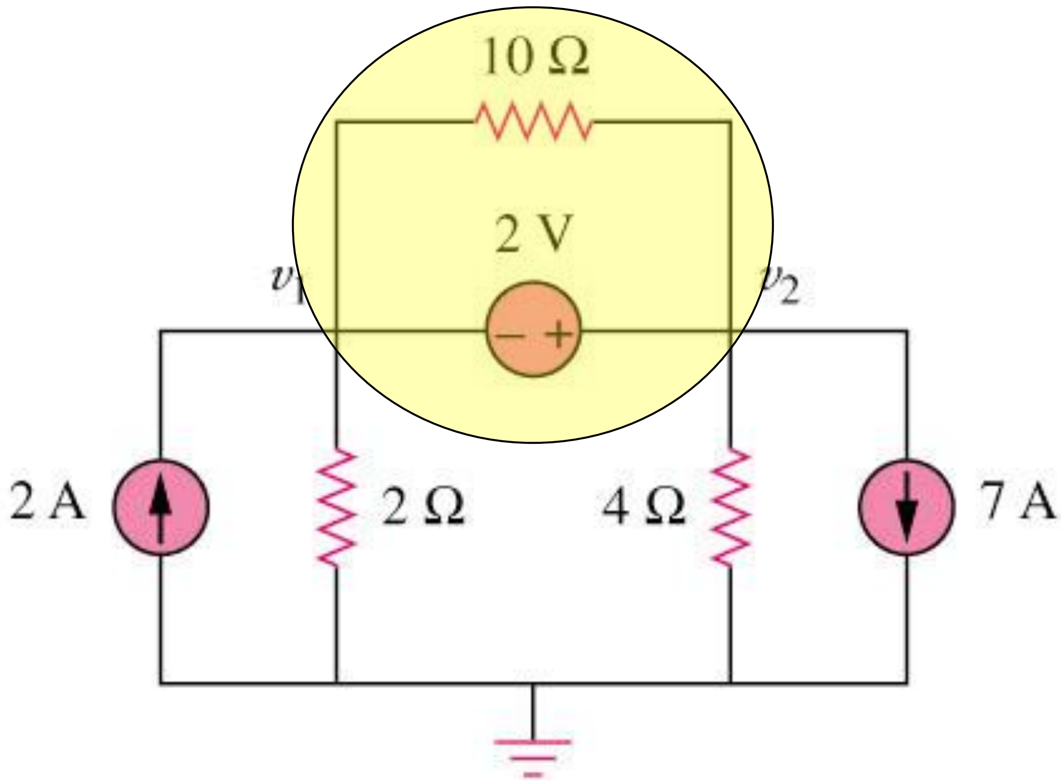
METHOD FOR A SUPERNODE

1. Create a supernode combining the two nodes (with voltage source lying between them)

2. Apply KCL to supernode(s) – sum of all currents into this node is zero.

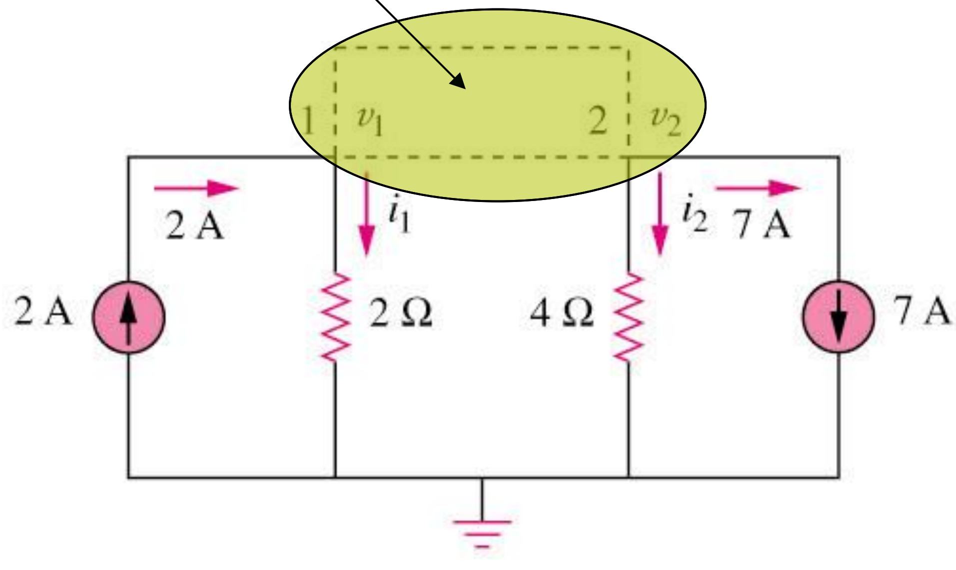
3. Apply voltage equation to supernode(s).

METHOD FOR A SUPERNODE

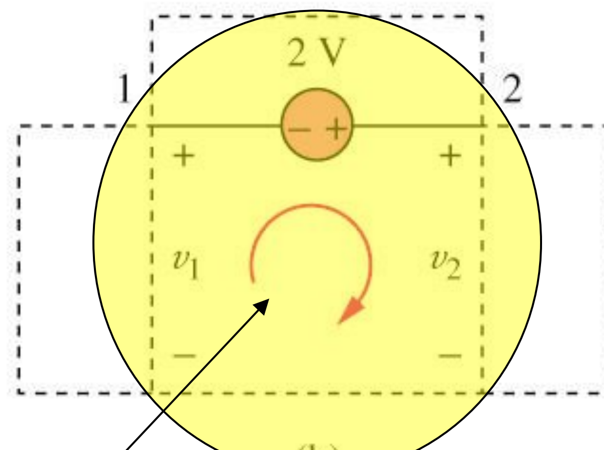


METHOD FOR A SUPERNODE

Supernode => $2 = i_1 + i_2 + 7$



(a)



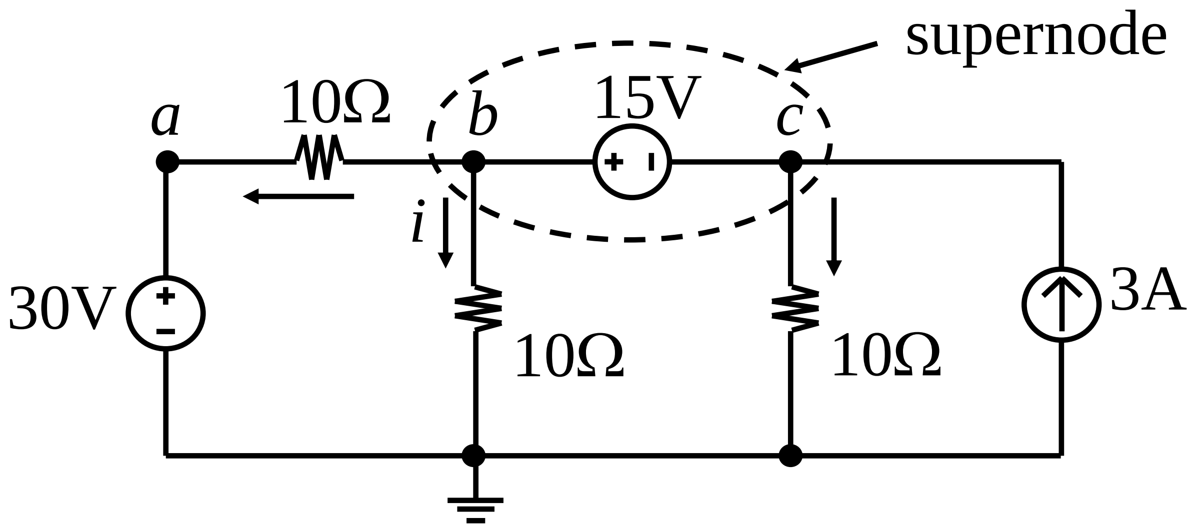
(b)

Apply KVL => $-v_1 - 2 + v_2 = 0$

(or) $v_2 - v_1 = 2$

SUPERNODE- EXAMPLE 4

$i = ?$



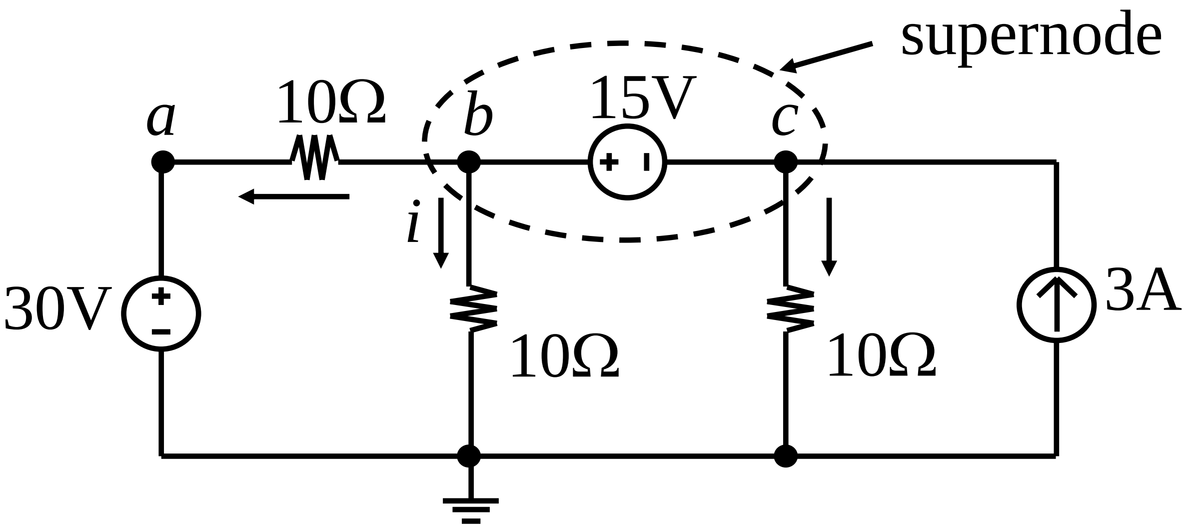
For node *a*:

Source between *b* and *c*:

KCL to supernode:

SUPERNODE- EXAMPLE 4

$i = ?$



For node *a*: $v_a = 30$

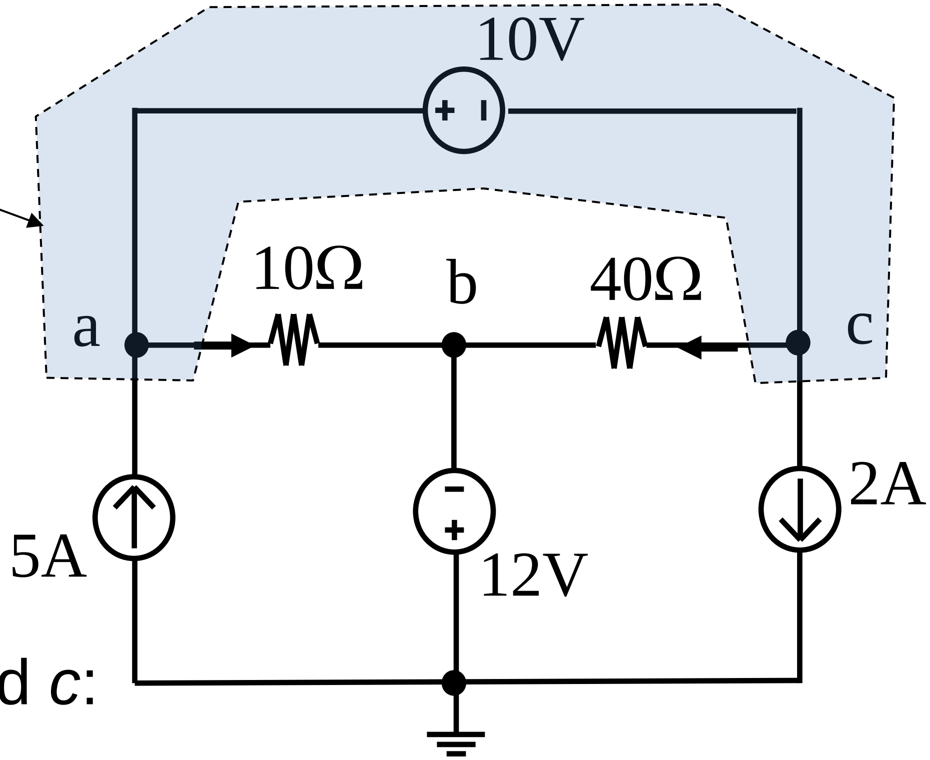
Source between *b* and *c*: $v_b - v_c = 15$

KCL to supernode: $3 = \frac{v_b - v_a}{10} + \frac{v_b}{10} + \frac{v_c}{10}$

Solve: $v_b = 25$ $\quad i = \frac{v_b}{10} = 2.5A$

SUPERNODE- EXAMPLE 5

supernode

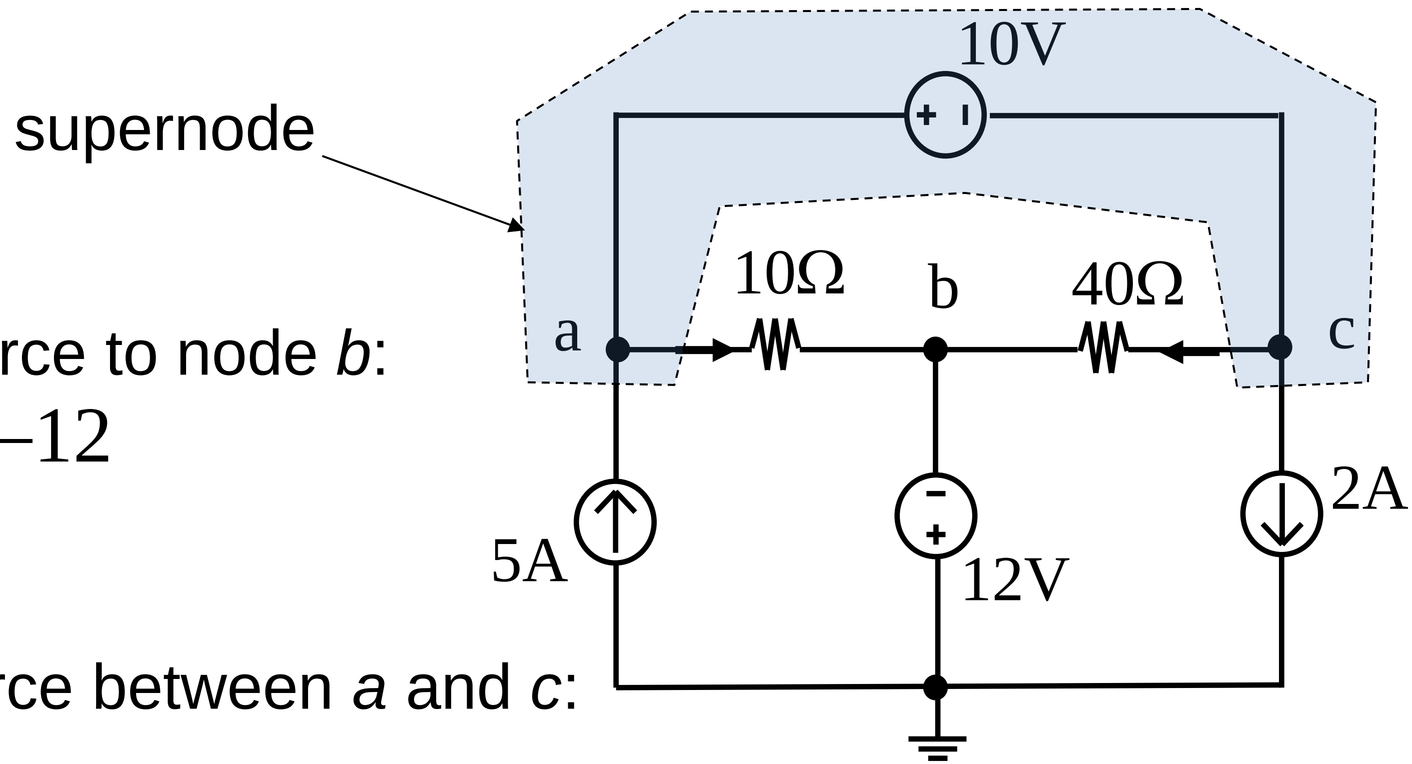


Voltage source to node *b*:

Voltage source between *a* and *c*:

KCL for supernode:

SUPERNODE- EXAMPLE 5



Voltage source to node *b*:

$$v_b = -12$$

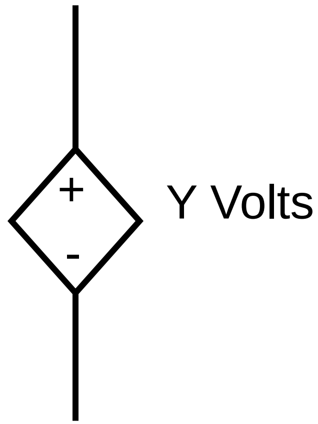
Voltage source between *a* and *c*:

$$v_a - v_c = 10$$

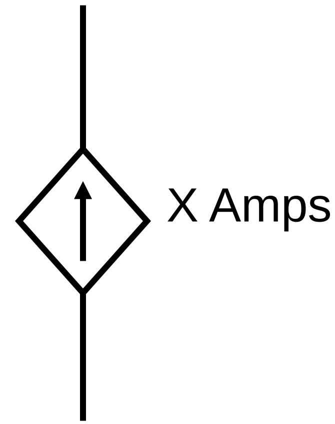
KCL for supernode: $\dfrac{v_a - v_b}{10} + \dfrac{v_c - v_b}{40} + 2 = 5$

CIRCUITS WITH DEPENDENT SOURCES

V source



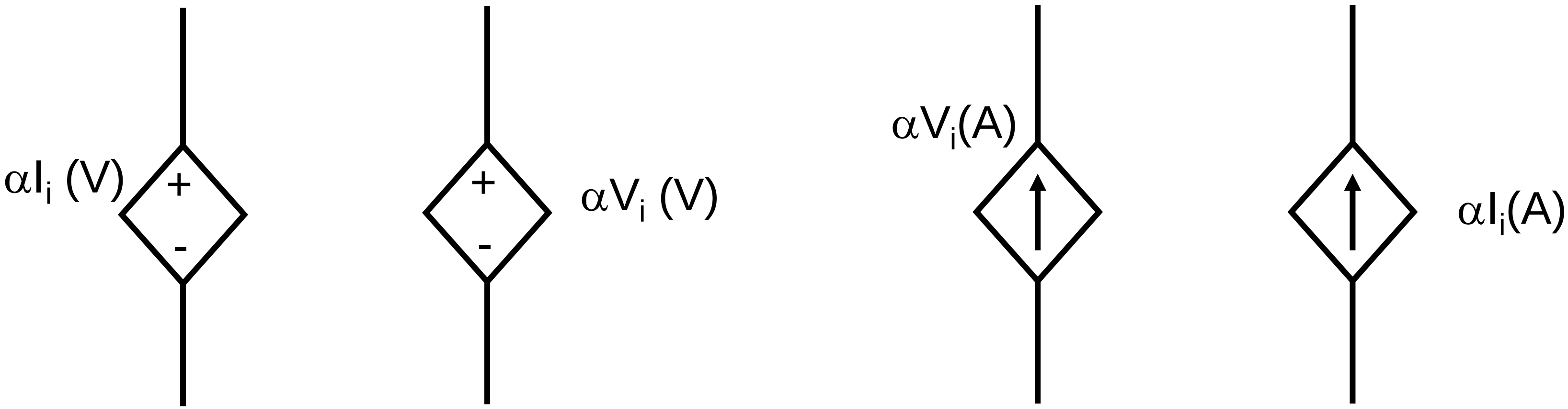
I source



- Dependent source **= a voltage or current generator whose value is controlled by another circuit variable**
- **e.g. transistors, amplifiers**

CIRCUITS WITH DEPENDENT SOURCES

V source　　　　I source



Amplifier types:

current controlled voltage source (CCVS)

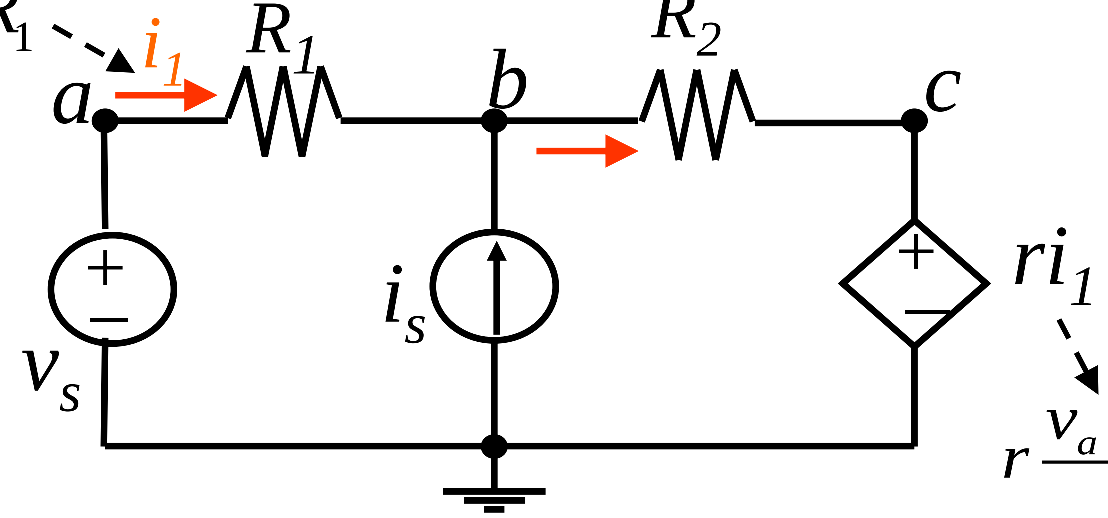
current controlled current source (CCCS)

voltage controlled voltage source (VCVS)

voltage controlled current source (VCCS)

CIRCUITS WITH DEPENDENT SOURCES

$$i_1 = \frac{v_a - v_b}{R_1}$$



$$r\frac{v_a - v_b}{R_1}$$

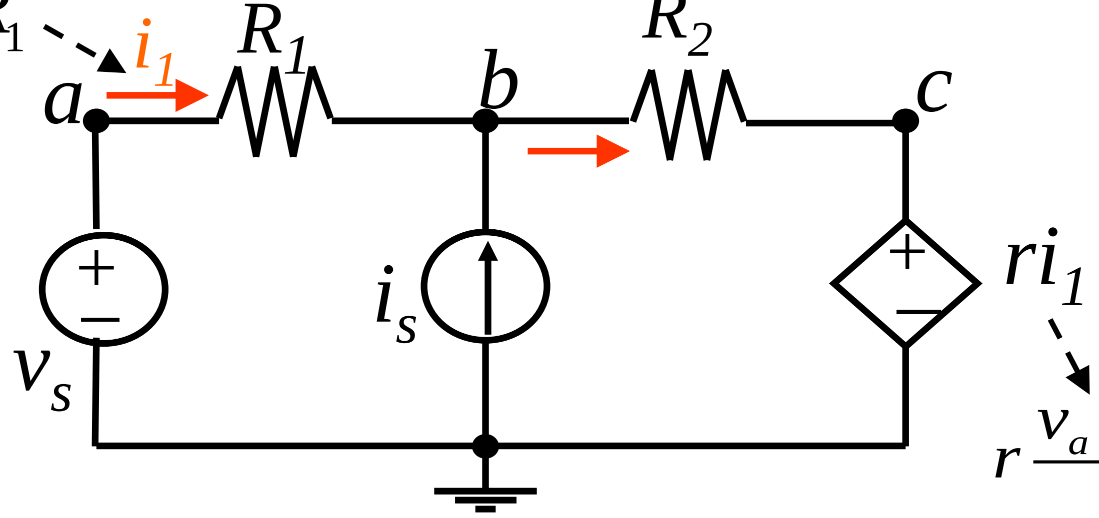
Node *a*:

Node *c*:

Node *b*:

CIRCUITS WITH DEPENDENT SOURCES

$$i_1 = \frac{v_a - v_b}{R_1}$$



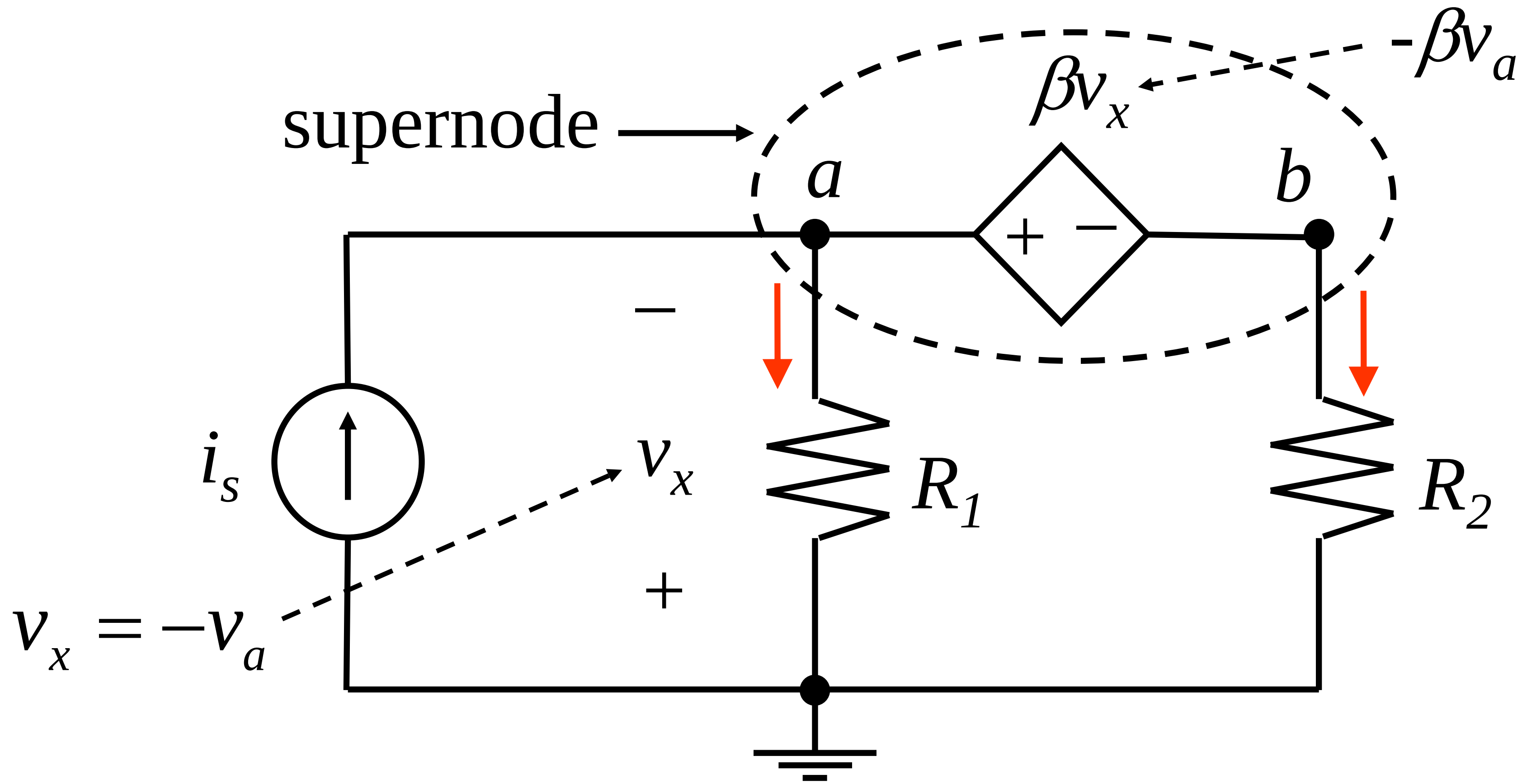
$$r\frac{v_a - v_b}{R_1}$$

Node a: $v_a = v_s$

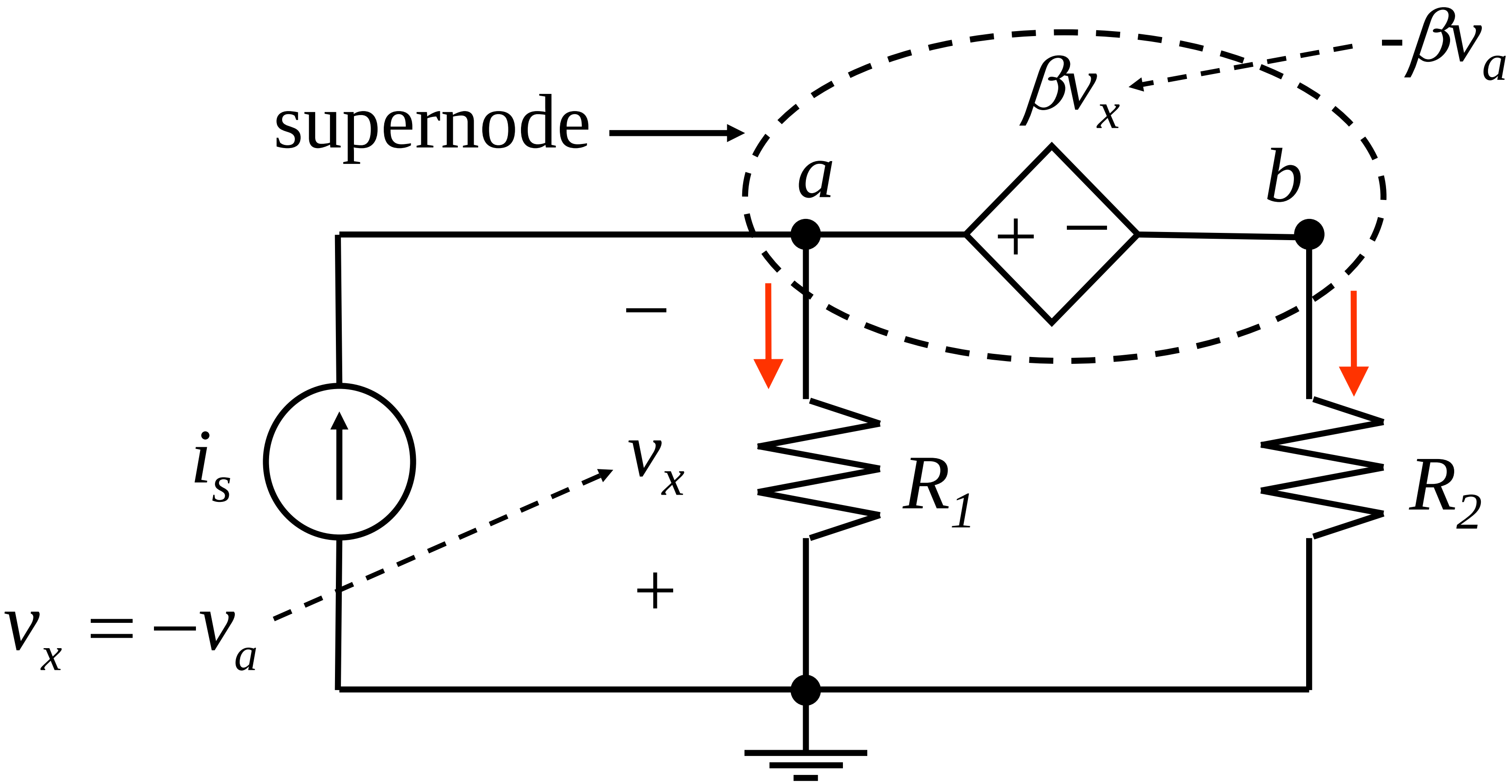
Node b: $\dfrac{v_a - v_b}{R_1} + i_s = \dfrac{v_b - v_c}{R_2}$

Node c: $v_c = r\dfrac{v_a - v_b}{R_1}$

DEPENDENT SOURCES – EXAMPLE 6

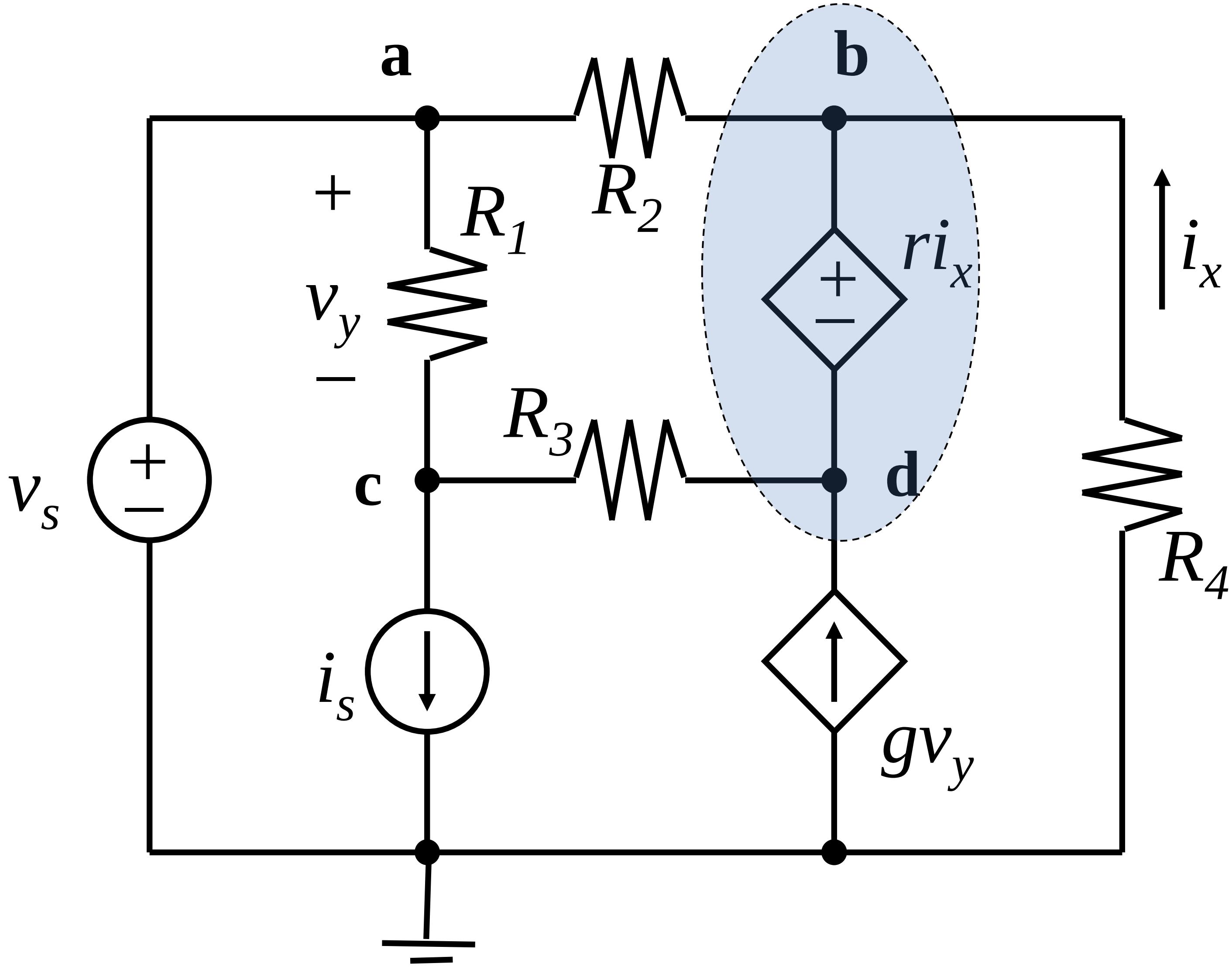


DEPENDENT SOURCES – EXAMPLE 6

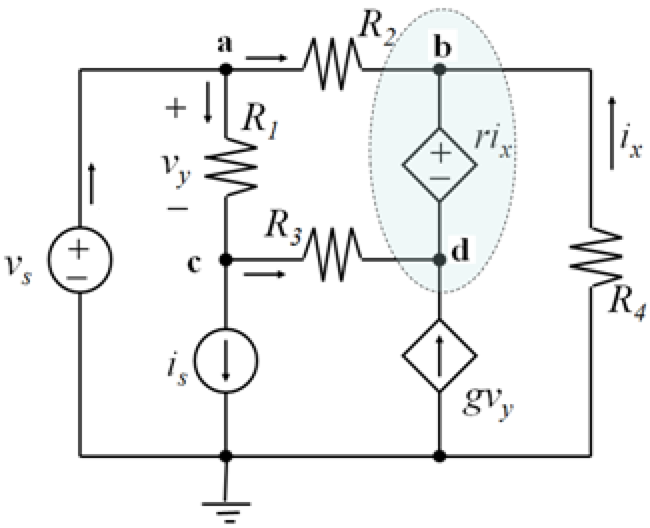


$$i_s = \frac{v_a}{R_1} + \frac{v_b}{R_2} \qquad v_a - v_b = -\beta v_a$$

DEPENDENT SOURCES – EXAMPLE 7



DEPENDENT SOURCES - EXAMPLE

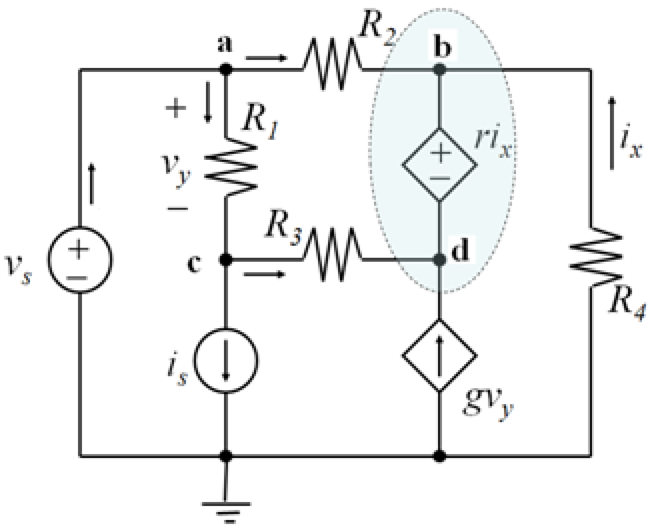


1 KCL equation for supernode

1 KCL equation for node *c*

2 equations for two voltage sources

DEPENDENT SOURCES - EXAMPLE



1 KCL equation for supernode
1 KCL equation for node *c*
2 equations for two voltage sources

$$\frac{v_a - v_b}{R_2} + i_x + \frac{v_c - v_d}{R_3} + gv_y = 0, \quad \text{where} \quad v_y = v_a - v_c \quad \text{and } i_x = -\frac{v_b}{R_4}$$

$$\frac{v_a - v_c}{R_1} = i_s + \frac{v_c - v_d}{R_3}$$

$$v_a = v_s \qquad v_b - v_d = ri_x = r\left(-\frac{v_b}{R_4}\right)$$

MESH CURRENT ANALYSIS

1. Mesh analysis provides another general procedure for analysing circuits using mesh currents as the circuit variables.

2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.

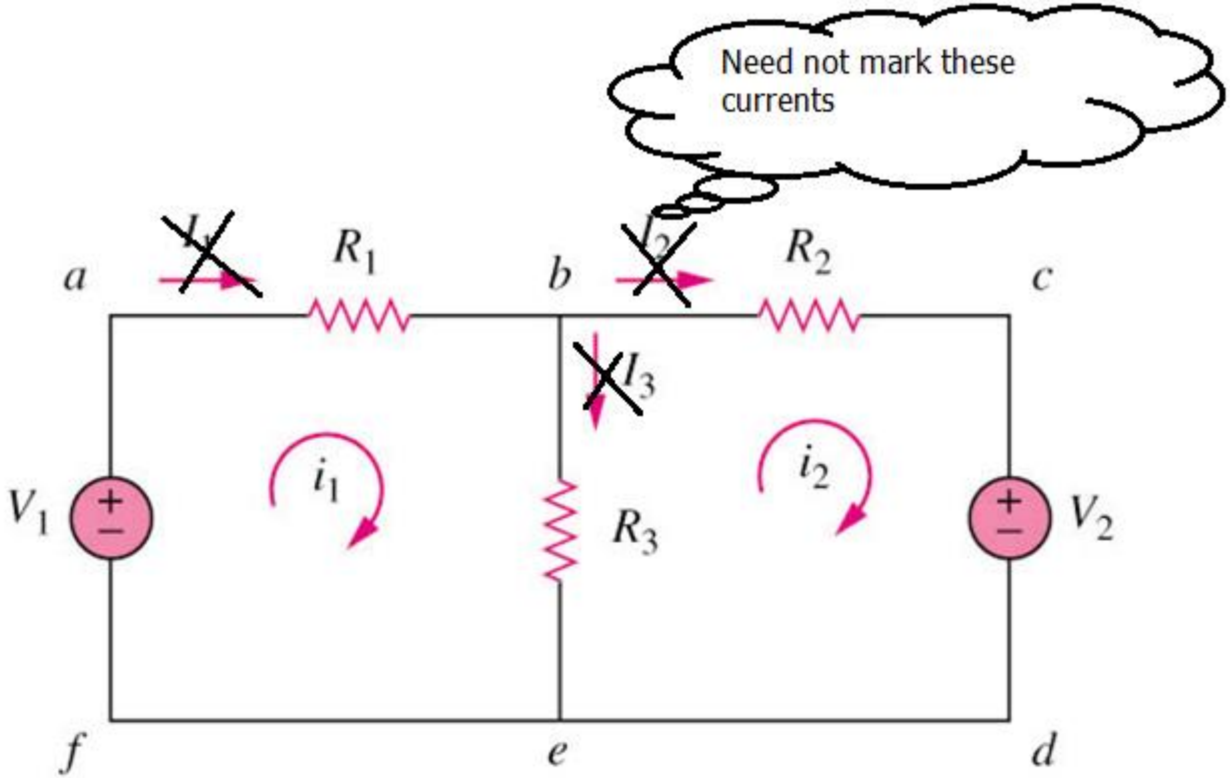
METHOD TO DETERMINE MESH CURRENT

1. Assign mesh currents i_1, i_2, …, i_n to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

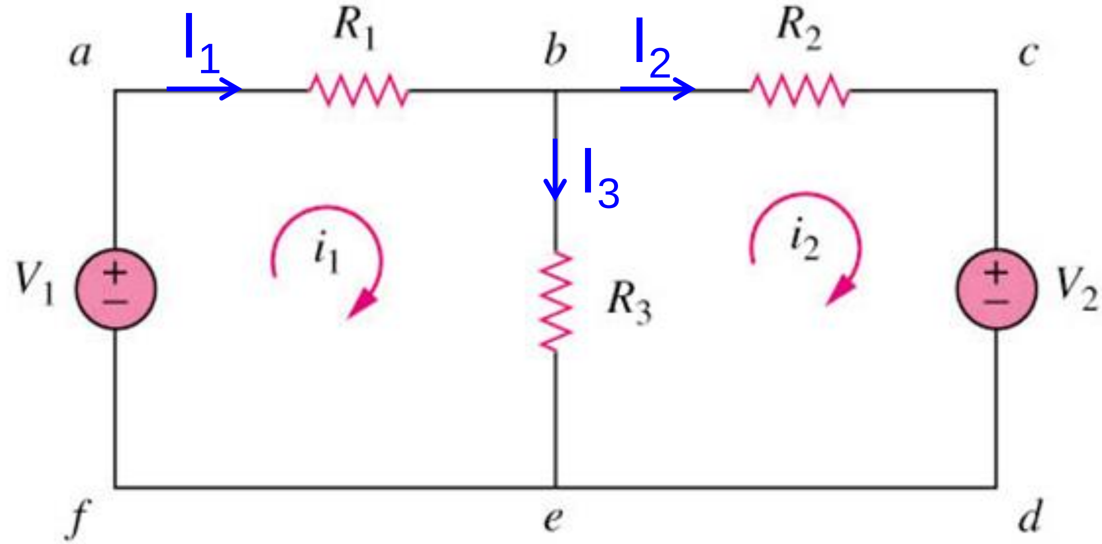
MESH ANALYSIS (INDEPENDENT VOLTAGE SOURCES ONLY)



Mesh equation: KVL around each mesh

Note: Remember the sign convention!

MESH ANALYSIS (INDEPENDENT VOLTAGE SOURCES ONLY)



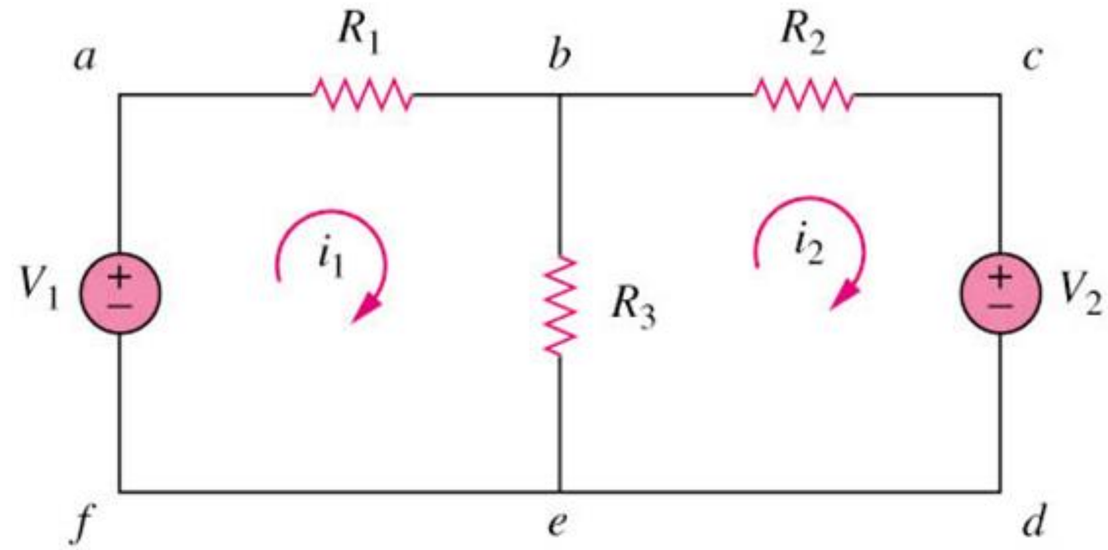
Note: I_1, I_2 and I_3 are branch currents

i_1 and i_2 are mesh currents

$I_1=i_1$; $I_2=i_2$

$I_3=i_1-i_2$ (two mesh currents i_1 & i_2 cover through branch I_3)

MESH ANALYSIS (INDEPENDENT VOLTAGE SOURCES ONLY)



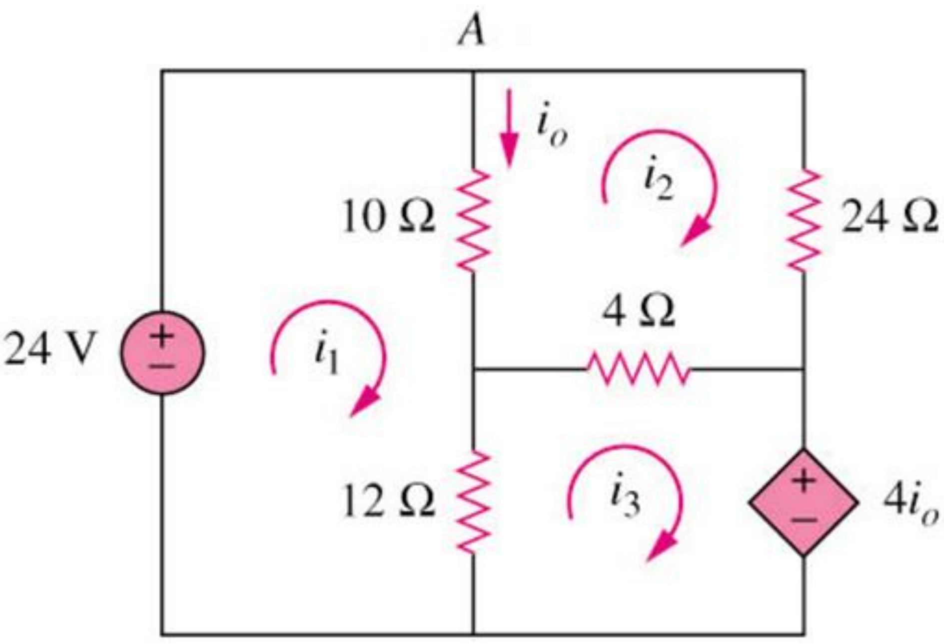
Mesh (Loop) 1

$$-v_1 + i_1R_1 + (i_1 - i_2)R_3 = 0$$

Mesh (Loop) 2

$$-(i_1 - i_2)R_3 + i_2R_2 + v_2 = 0$$

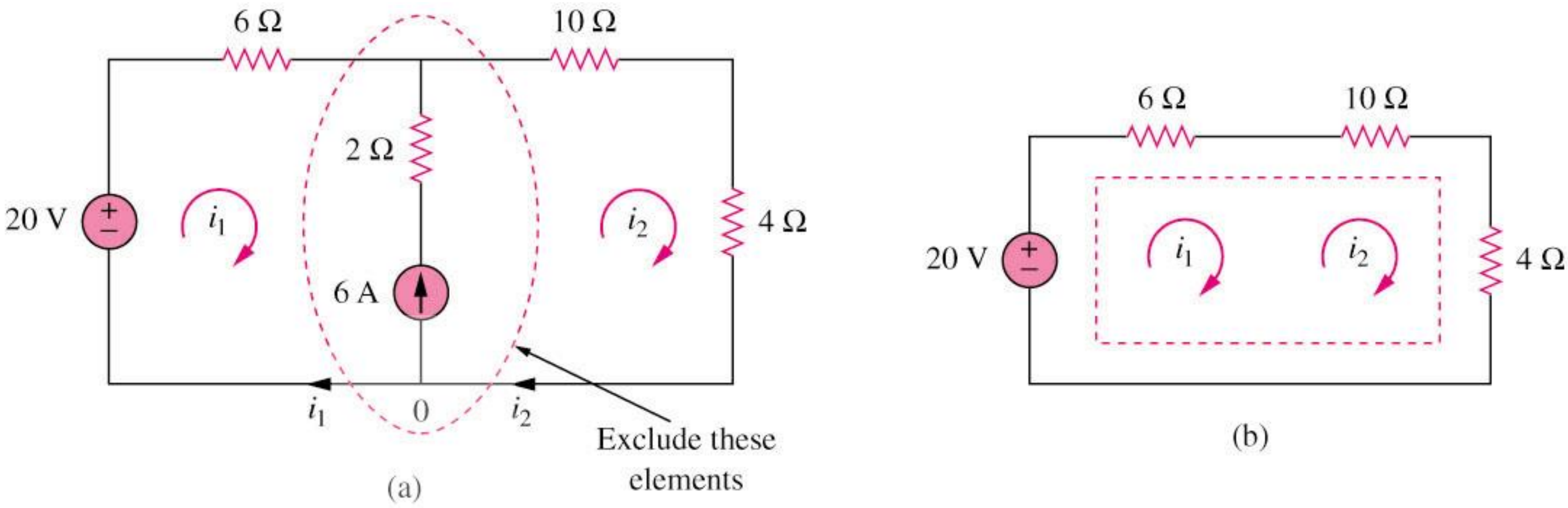
MESH ANALYSIS – EXAMPLE 8



Solve for i_0

(i_o = 1.5A)

MESH ANALYSIS WITH CURRENT SOURCE



A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

SUPERMESH PROPERTIES

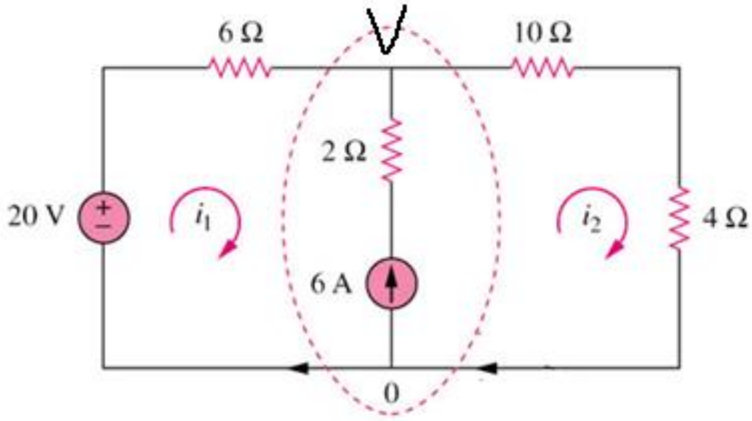
1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

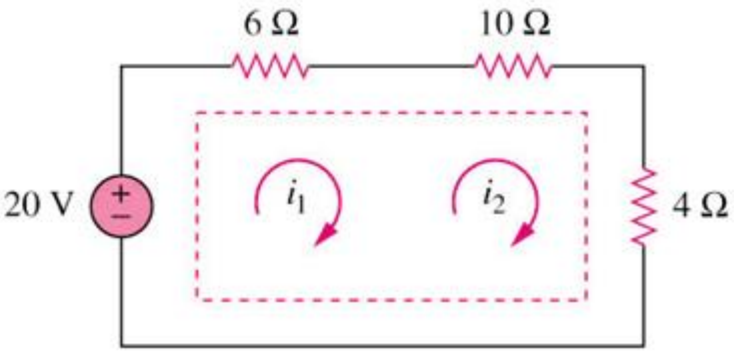
2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

SUPERMESH EXAMPLE 9

Solve for *V*





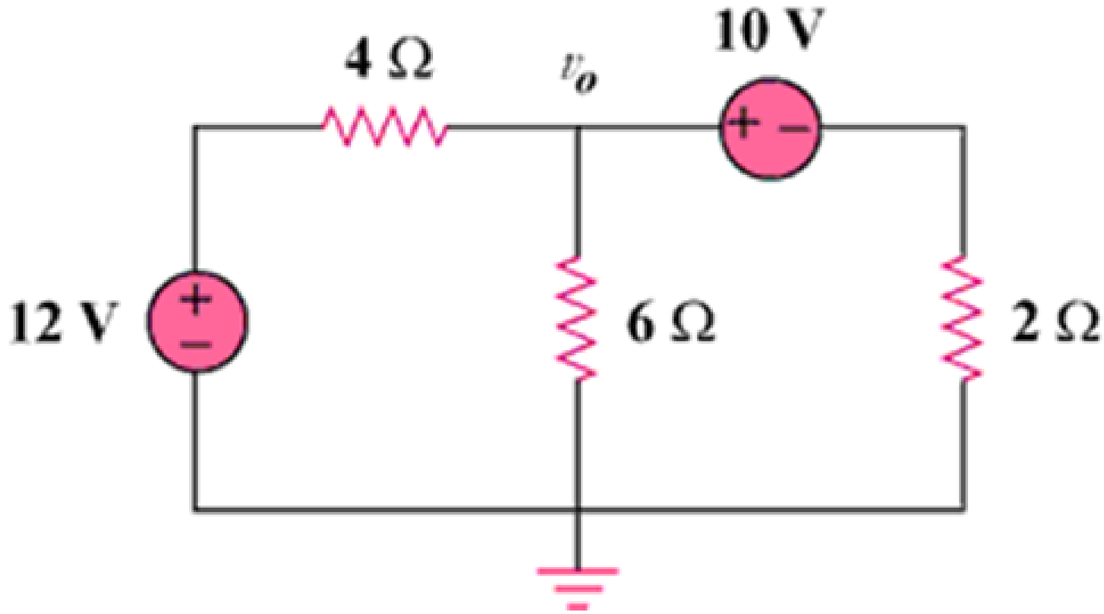
NODAL VERSUS MESH ANALYSIS

To select the method that results in the smaller number of equations.

- Choose nodal analysis for circuit with fewer nodes than meshes.
- Choose mesh analysis for circuit with fewer meshes than nodes.
- Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
- Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
- If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.
- Wherever possible, go for source conversion.

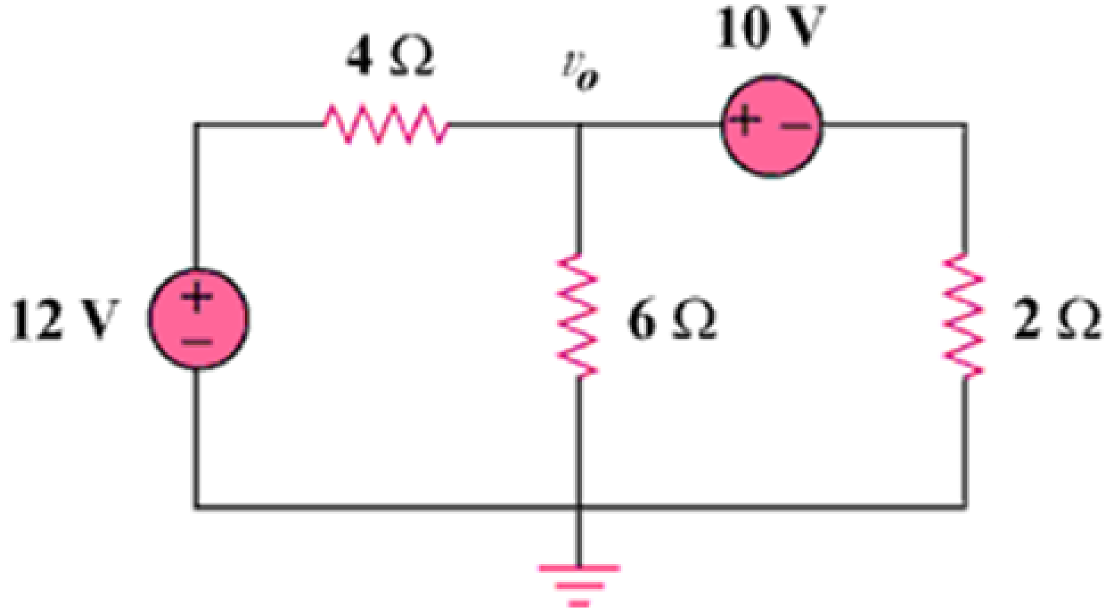
NODAL & MESH ANALYSIS EXAMPLE 10

Calculate v_0 using mesh analysis. Verify using nodal analysis.



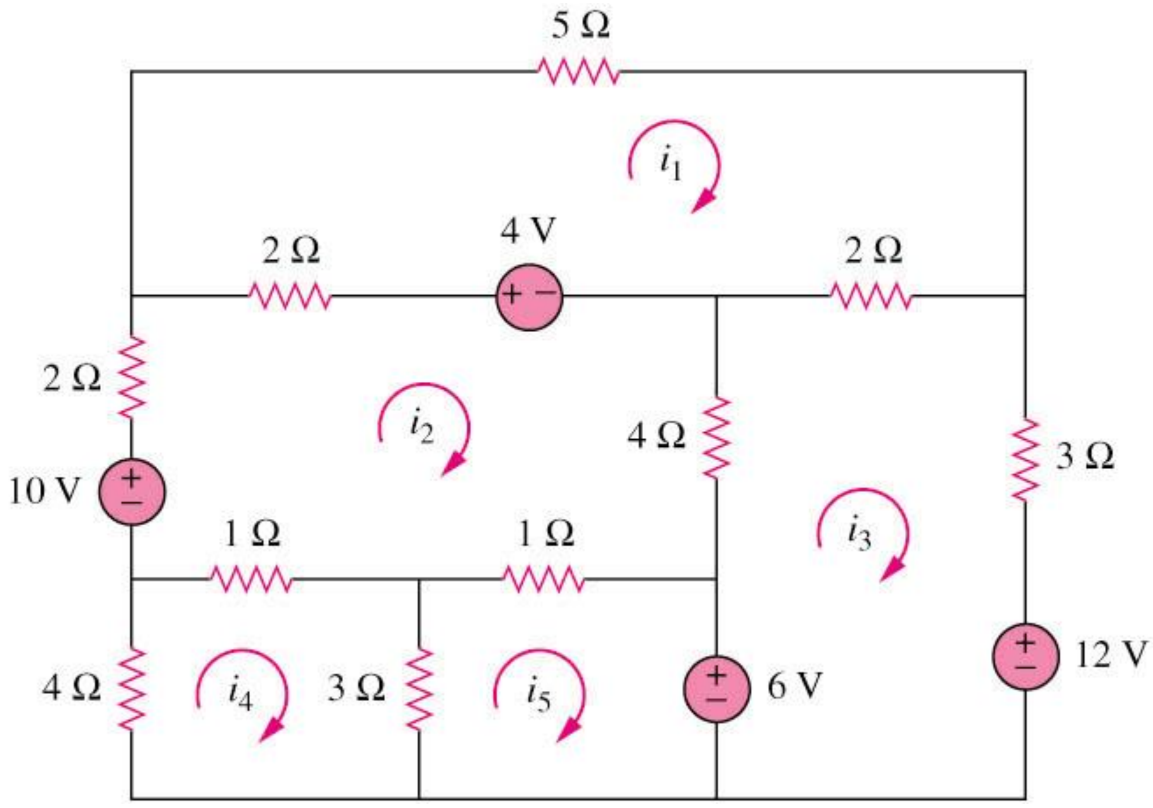
NODAL & MESH ANALYSIS EXAMPLE 10

Mesh analysis:



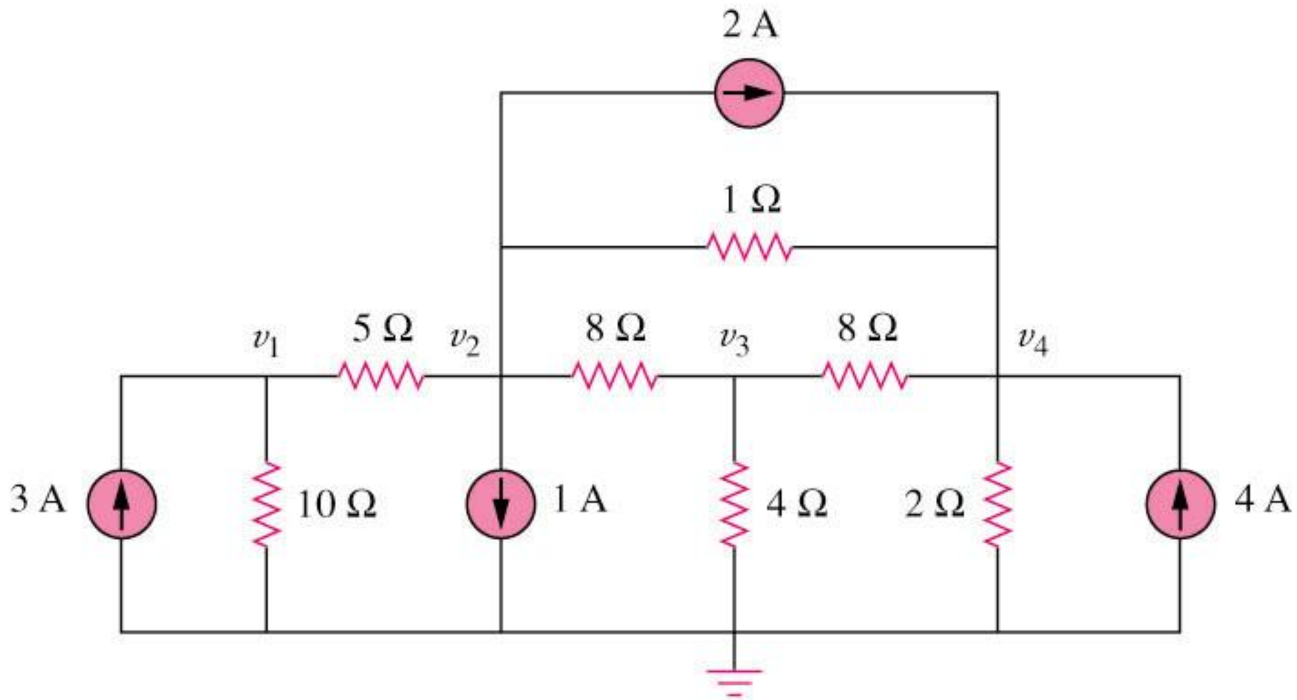
NODAL & MESH ANALYSIS EXAMPLE 11

Write the mesh-current equations for the circuit



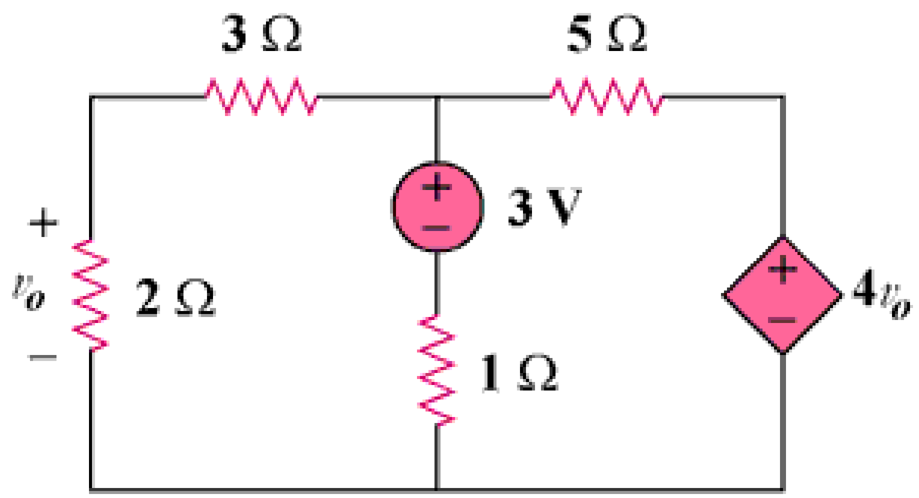
NODAL & MESH ANALYSIS EXAMPLE 12

Write the nodal voltage equations for the circuit



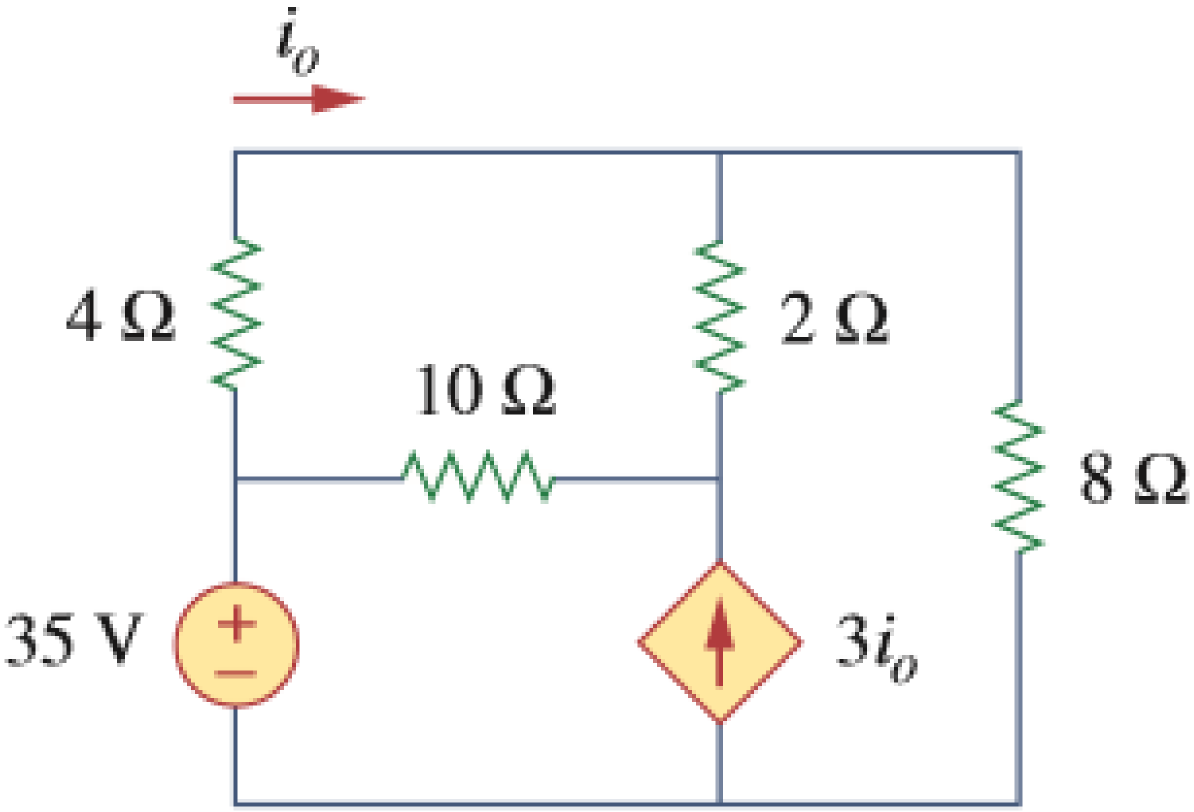
NODAL & MESH ANALYSIS EXAMPLE 13

Calculate v_0 using mesh analysis. Verify using nodal analysis.



NODAL & MESH ANALYSIS EXAMPLE 14

Use mesh analysis to find the current i_0 in the following circuit



CRAMER’S RULE

Cramer's rule is a theorem, which gives an expression for the solution of a system of linear equations with as many equations as unknowns

Consider the linear system of equations given by $\begin{cases} ax + by &= e \\ cx + dy &= f \end{cases}$

In matrix format it is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}.$

Then, *x* and *y* can be found with Cramer's rule as

$$x = \frac{\Delta x}{\Delta} = \begin{vmatrix} e & b \\ f & d \end{vmatrix} \Big/ \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{ed - bf}{ad - bc}$$

$$y = \frac{\Delta y}{\Delta} = \begin{vmatrix} a & e \\ c & f \end{vmatrix} \Big/ \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{af - ec}{ad - bc}$$

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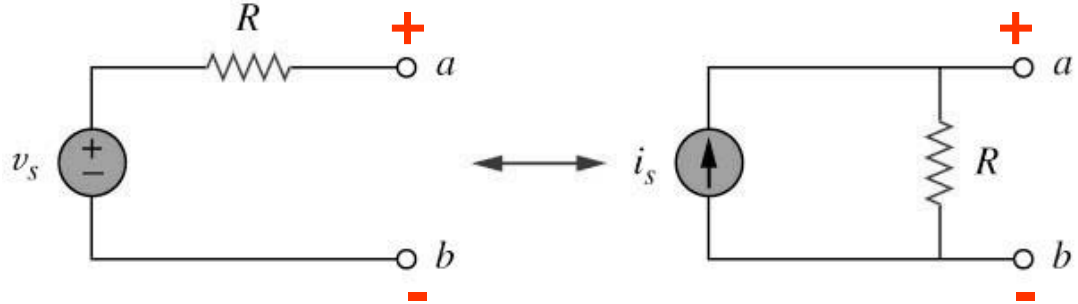
CRAMER'S RULE – 3x3 matrix

Given $\begin{cases} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{cases}$ In matrix form, $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$

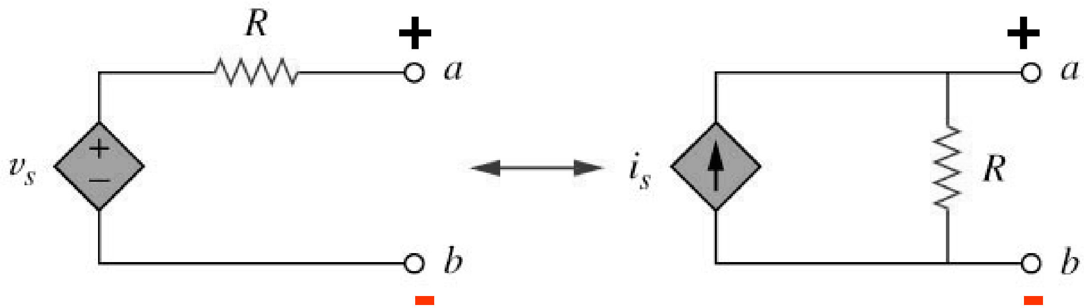
The values of *x*, *y* and *z* are,

$$x = \frac{\Delta x}{\Delta} = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \quad y = \frac{\Delta y}{\Delta} = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \text{ and } z = \frac{\Delta z}{\Delta} = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$$

SOURCE TRANSFORMATION



(a) Independent source transform

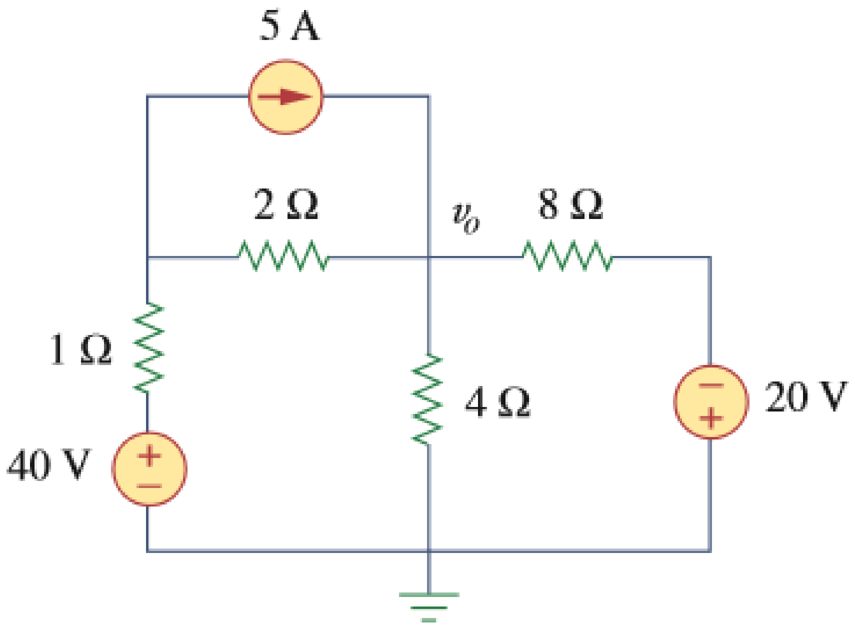


(b) Dependent source transform

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when $R = 0$ for voltage source and $R = \infty$ for current source.

NODAL & MESH ANALYSIS EXAMPLE 15

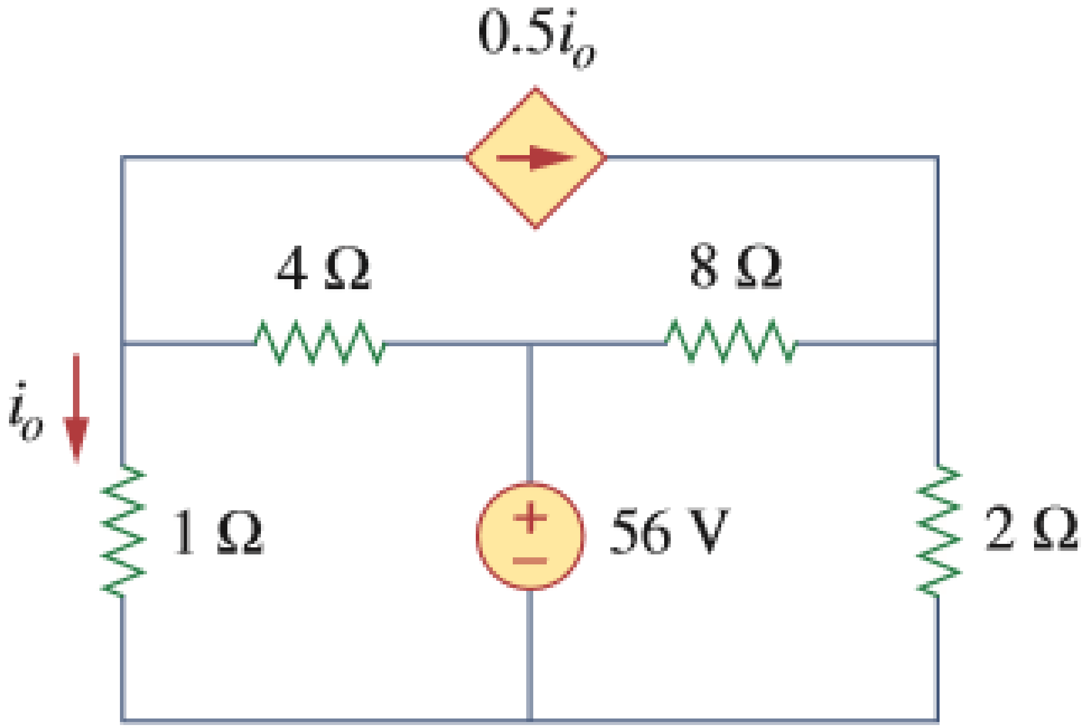
Find v_0 in the following circuit using source transformation



Ans: Vo = 20V

NODAL & MESH ANALYSIS EXAMPLE 16

Find i_0 in the following circuit



Ans: i_o=8A