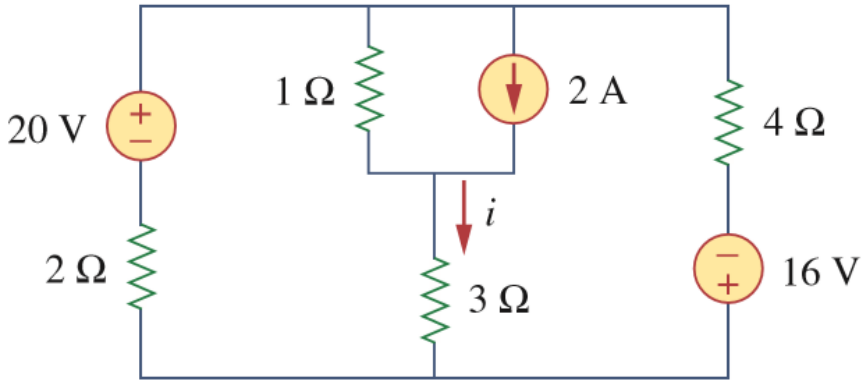
Weeks 1-4 Revision

Basis methods for circuit analysis

- Ohm’s Law
- Voltage/Current Division Laws
- Kirchhoff’s Current Law (KCL)
- Kirchhoff’s Voltage Law (KVL)
- Nodal Analysis
- Mesh Analysis
- Superposition
- Source Transformation
- Thévenin’s Theorem
- Norton’s Theorem
- Maximum Power Transfer



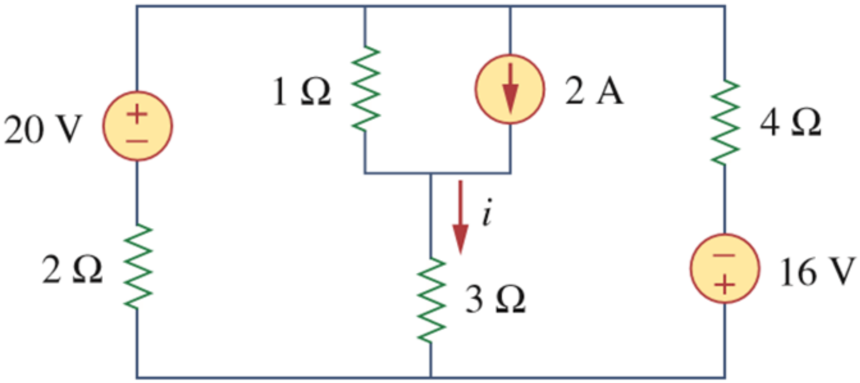
Indicative Course Structure

Wk. No	Summary of Lecture Program
1	Introduction, Circuit Basics Overview + Lab Safety.
2	Kirchhoff’s laws - Resistive circuits, Series & Parallel circuits, Power & Energy
3	Node Equations & Circuit analysis
4	Circuit theorems – Thévenin, Norton, Superposition Theorems, MPT
5	Energy storage elements - inductors and capacitors, energy storage
6	First order circuits – RL & RC circuits, transient responses
7	Introduction to AC/sinusoidal analysis, phasors & phasor diagrams
8	Sinusoidal steady-state analysis, AC circuit analysis, AC power analysis
9	Transformers and voltage shaping circuits
10	Operational amplifiers
11	Digital systems, number representation
12	Combination logic, digital circuit analysis

Weeks 5-12 Contents

Other electrical/electronic elements

- Capacitor
- Inductor
- OpAmp
- Transformers



AC signals (sinusoid signal)

- Phasor
- AC Power

Combination logic, digital circuit analysis

The basic laws & analysis methods in weeks 1-4 can be applied in all electrical circuits and will be used in the rest of semester

Lecture 7: Capacitors and RC Transients

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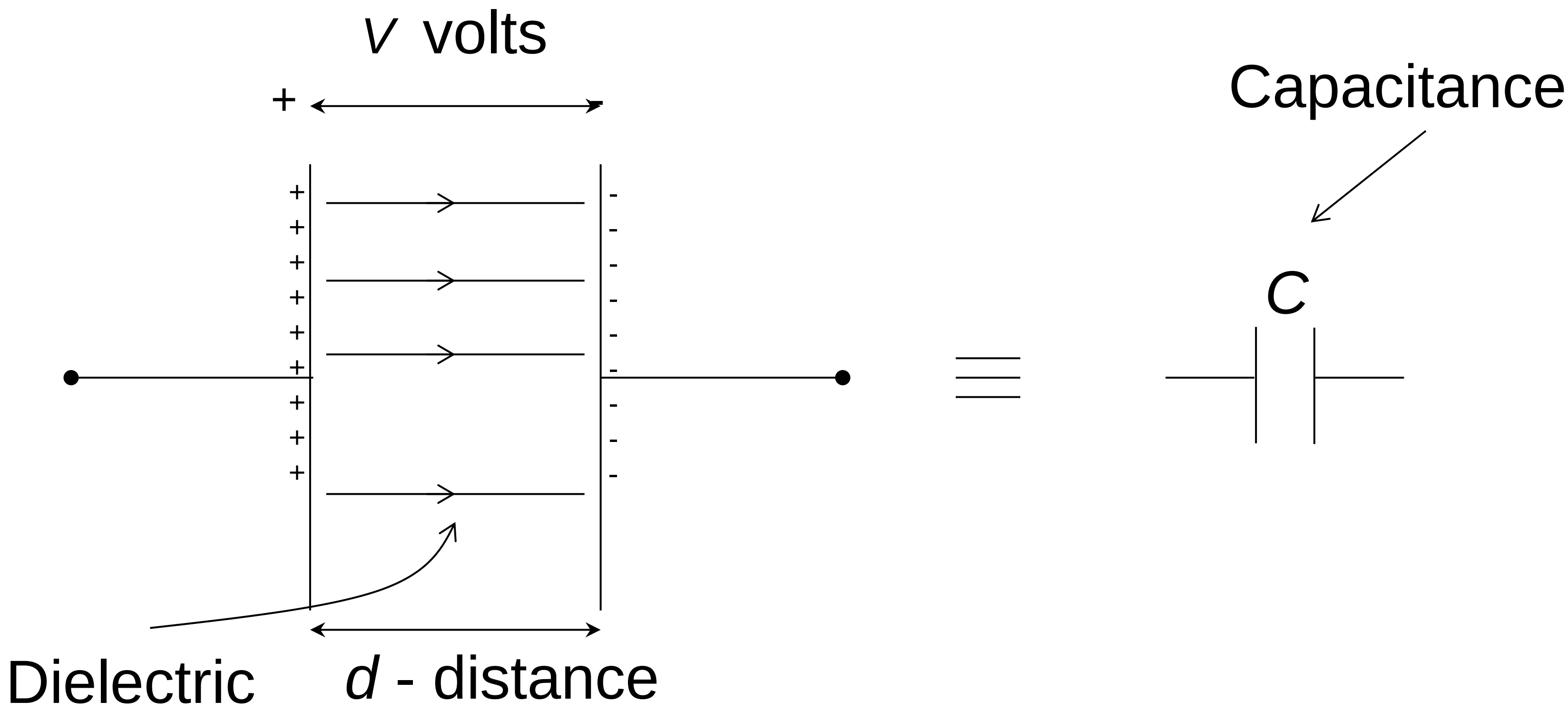
Capacitors

- **Capacitor:** Constructed using two parallel conducting plates separated by an insulating material (e.g. air)



- **<u>Capacitance</u>** is a measure of capacitor's ability to store charge on its plates. (i.e. its storage capacity)

- A capacitor has a **capacitance** of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt across the plates.
- Expressed as an equation, the capacitance is determined by

$$C = \frac{Q}{V}$$

C = farads (F)

Q = coulombs (C)

V = volts (V)

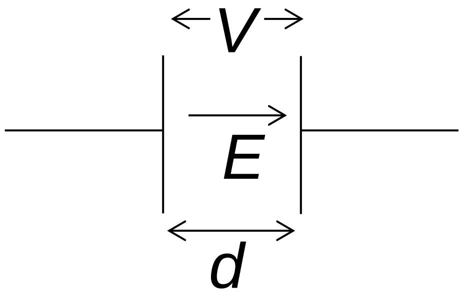
Note: C = 4 F

C = 6 μF (microfarad) = 6 x 10^{-6} F

C = 2 nF (nano farad) = 2 x 10^{-9} F

C = 3 pF (pico farad) = 3 x 10^{-12} F

$$\uparrow C = \frac{\uparrow Q}{V}$$



$$E = \frac{V}{d}$$

Potential difference

distance between the plates

Electric field strength

(volts/metres)

$$C = \varepsilon \frac{A}{d}$$

area of the plates

distance between the plates)

dielectric constant
or
relative permittivity

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

permittivity for vacuum
[8.85 x 10^{-12} F/m]

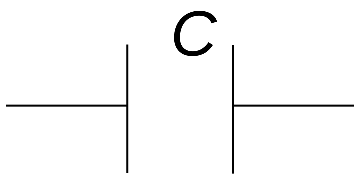
ε_r = 1 for vacuum

= 7 for glass

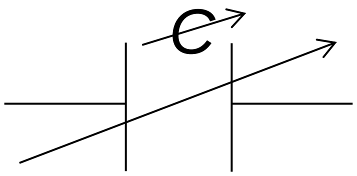
= 2 for nylon

- **Breakdown Voltage**: The maximum voltage that can be applied across a capacitor is known as the breakdown voltage.

Note:



Fixed capacitor

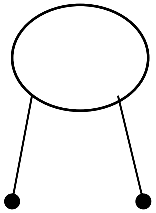


Variable capacitor

Standard values of capacitor:

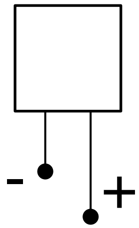
10pF, ..., 0.01μF, 0.1μF, 0.22μF, 0.33μF, ...> 1μF, 2.2μF, 3.3μF, 4.7μF, ...

No polarity



0.01 μF

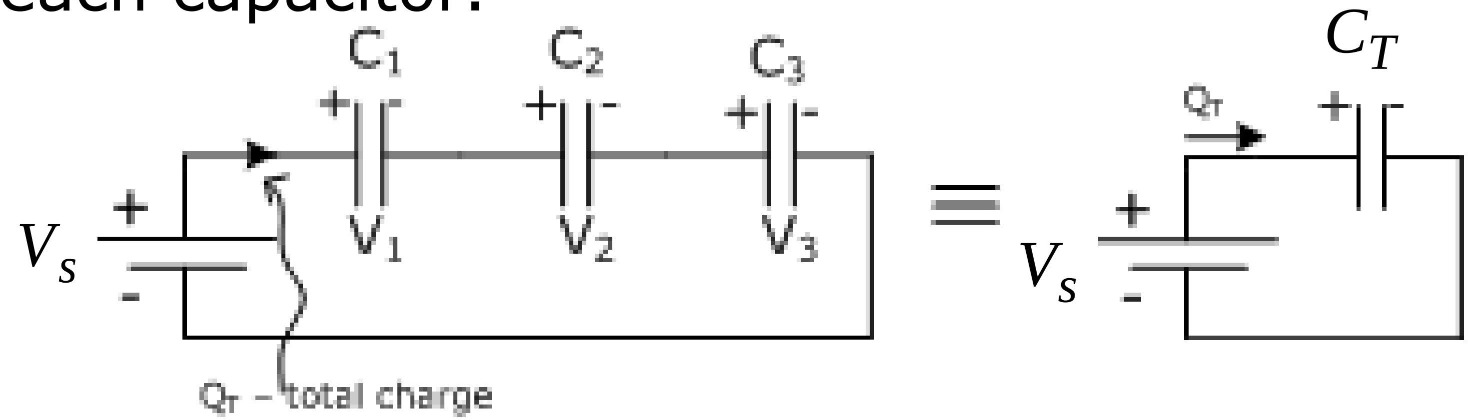
polarity is important



0.01 μF

Capacitors in Series and Parallel

- Capacitors, like resistors, can be placed in series and in parallel.
- **Capacitors in Series**: the charge is the same on each capacitor.



$$V_s = V_1 + V_2 + V_3$$

$$\frac{Q_T}{C_T} = \frac{Q_T}{C_1} + \frac{Q_T}{C_2} + \frac{Q_T}{C_3}$$

$$\boxed{\therefore \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

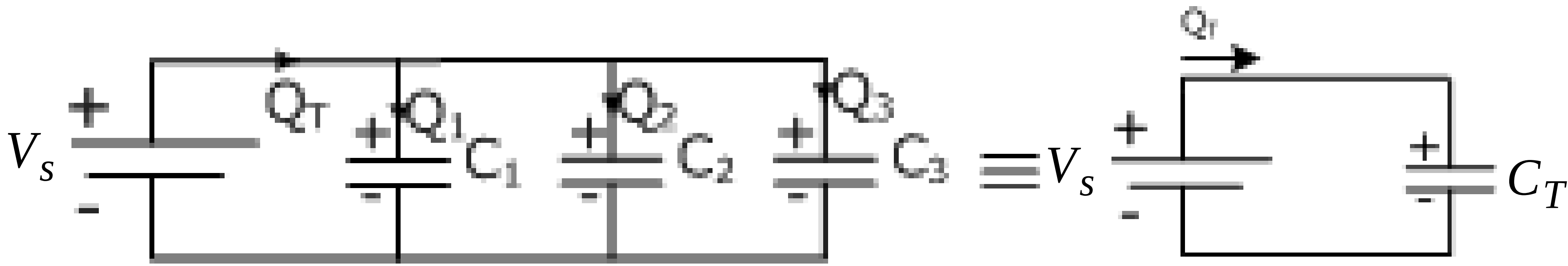
- The total capacitance of two capacitors in series is

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\therefore C_T = \frac{C_1 C_2}{C_1 + C_2}}$$

Capacitors in Parallel

- For capacitors in parallel, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor.



Q_T – total charge

$$Q_T = Q_1 + Q_2 + Q_3$$

$$C_T V_s = C_1 V_s + C_2 V_s + C_3 V_s$$

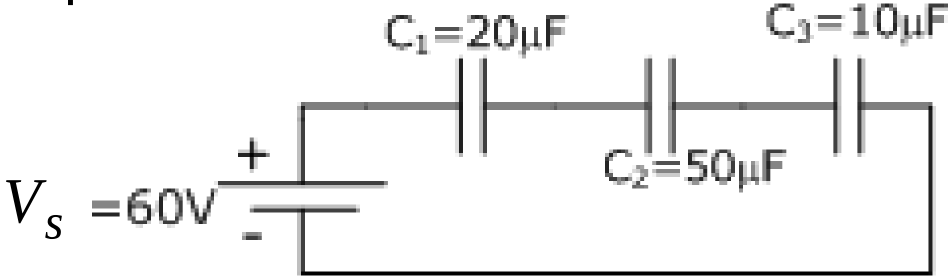
$$\boxed{\therefore C_T = C_1 + C_2 + C_3}$$

For the circuit shown below,

(a) Find the total capacitance

(b) Determine the charge on each plate

(c) Find the voltage across each capacitor



(a) $$\frac{1}{C_T}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$=\frac{1}{20\times10^{-6}}+\frac{1}{50\times10^{-6}}+\frac{1}{10\times10^{-6}}$$

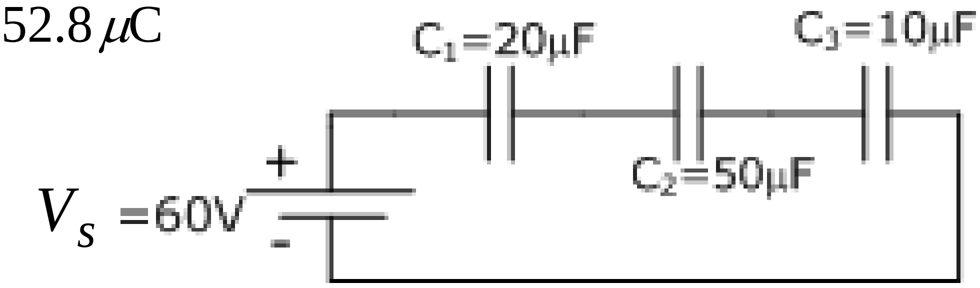
$$=0.05\times10^{6}+0.02\times10^{6}+0.1\times10^{6}$$

$$\frac{1}{C_T}=0.17\times10^{6}$$

$$\therefore\ C_T=\frac{1}{0.17\times10^{6}}$$

$$=5.88\mu\text{F}$$

(b) $Q_T = C_T V_s = 5.88\mu F \times 60 = 352.8\,\mu\text{C}$



(c) $$V_1 = \frac{Q_T}{C_1} = \frac{352.8\times10^{-6}\ C}{20\times10^{-6}\ F} = 17.64\ \text{V}$$

$$V_2 = \frac{Q_T}{C_2} = \frac{352.8\times10^{-6}\ C}{50\times10^{-6}\ F} = 7.056\ \text{V}$$

$$V_3 = \frac{Q_T}{C_3} = \frac{352.8\times10^{-6}\ C}{10\times10^{-6}\ F} = 35.28\ \text{V}$$

and $$V_s = V_1 + V_2 + V_3 = 60\text{V}$$

For the network shown below,

(a) Find the total capacitance

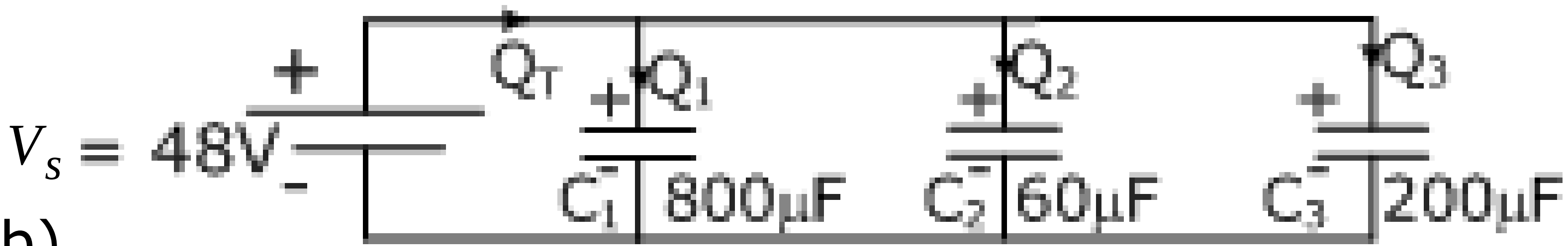
(b) Determine the charge on each plate

(c) Find the total charge

(a) $C_T = C_1 + C_2 + C_3$

$$= 800\mu\text{F} + 60\mu\text{F} + 200\mu\text{F}$$

$$= 1060\mu\text{F}$$



(b)

$$Q_1 = C_1 V_s = (800 \times 10^{-6}\ F)(48\text{V}) = 38.4\text{mC}$$

$$Q_2 = C_2 V_s = (60 \times 10^{-6}\ F)(48\text{V}) = 2.88\text{mC}$$

$$Q_3 = C_3 V_s = (200 \times 10^{-6}\ F)(48\text{V}) = 9.6\text{mC}$$

(c)

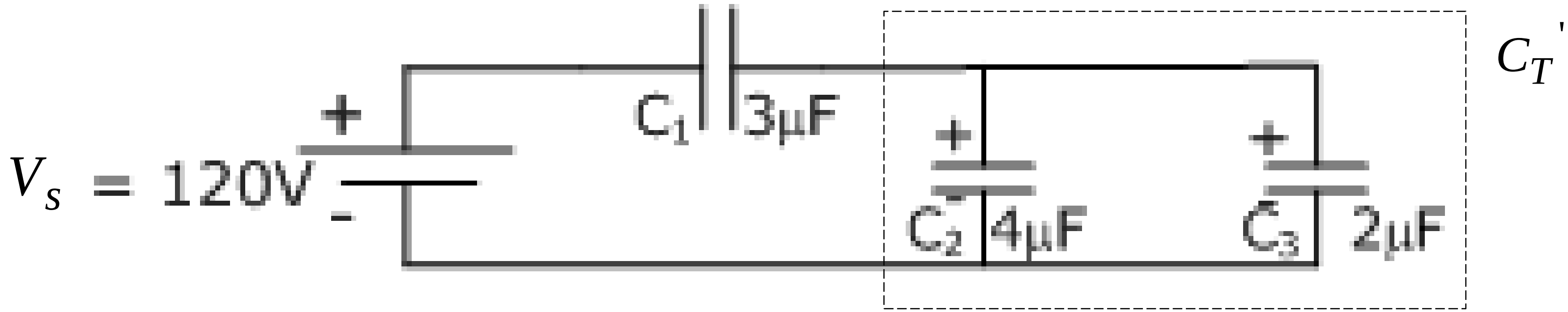
$$Q_T = Q_1 + Q_2 + Q_3$$

$$= 38.4\,\text{mC} + 2.88\,\text{mC} + 9.6\,\text{mC}$$

$$= 50.88\,\text{mC}$$

Example

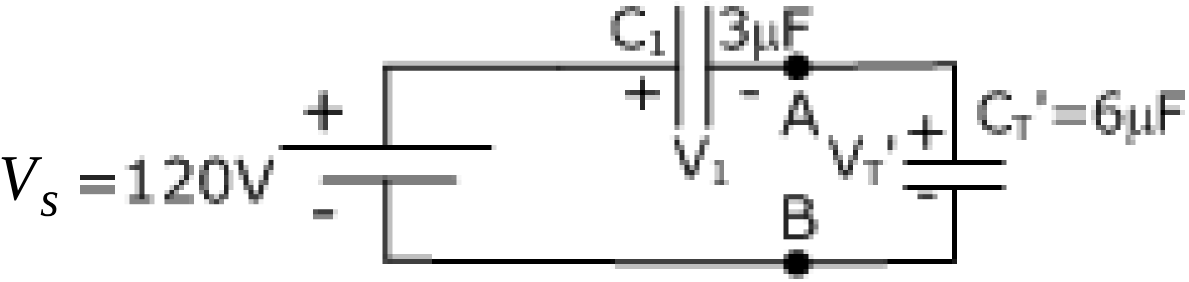
Find the voltage across and charge on each capacitor.

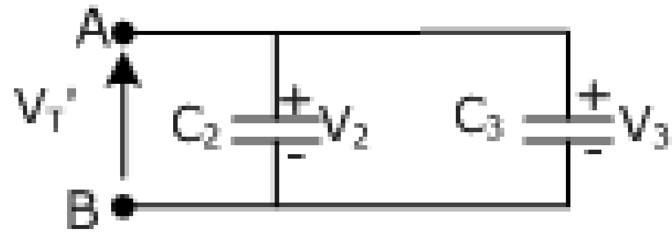


$$C_T' = C_2 + C_3 = 4\,\mu\text{F} + 2\,\mu\text{F} = 6\,\mu\text{F}$$

$$C_T = \frac{C_1 C_T'}{C_1 + C_T'} = \frac{(3\,\mu\text{F})(6\,\mu\text{F})}{3\,\mu\text{F} + 6\,\mu\text{F}} = 2\,\mu\text{F}$$

$$\begin{aligned} Q_T &= C_T V_S \\ &= (2\,\mu\text{F})(120\text{V}) \\ &= 240\,\mu\text{C} \end{aligned}$$





$$V_1 = \frac{Q_T}{C_1} = \frac{240\times10^{-6}\,\mathrm{C}}{3\times10^{-6}\,\mathrm{F}} = 80\,\mathrm{V}$$

$$V_T^{'} = \frac{Q_T}{C_T^{'}} = \frac{240\times10^{-6}\,\mathrm{C}}{6\times10^{-6}\,\mathrm{F}} = 40\,\mathrm{V}$$

$$\therefore Q_2 = C_2V_2 = C_2V_T^{'} = (4\times10^{-6}\,\mathrm{F})(40\,\mathrm{V}) = 160\,\mu\mathrm{C}$$

$$Q_3 = C_3V_3 = C_3V_T^{'} = (2\times10^{-6}\,\mathrm{F})(40\,\mathrm{V}) = 80\,\mu\mathrm{C}$$

Energy stored by a capacitor

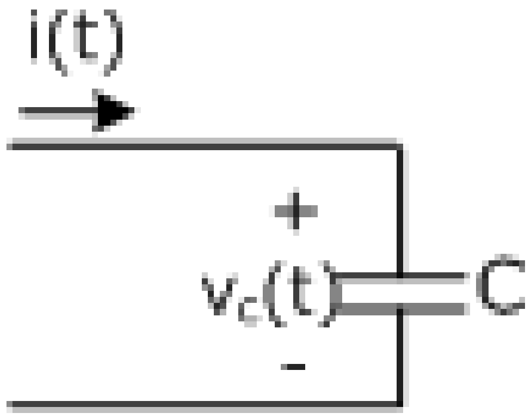
- An ideal capacitor stores the energy in the form of an electric field between the capacitor plates.
- Energy stored

$$W_c = \frac{1}{2} C V^2 \quad \text{Joules}$$

$$= \frac{Q^2}{2C} \quad (\mathrm{J})$$

- V is the **steady-state** voltage across the capacitor.

Instantaneous current, voltage, charge on a capacitor



$i(t)$ – instantaneous current

$v_c(t)$ – instantaneous voltage across the capacitor

$q(t)$ – instantaneous charge on the capacitor

$$q(t) = Cv_c(t)$$

$$\frac{dq(t)}{dt} = C\frac{dv_c(t)}{dt}$$

$$\parallel$$

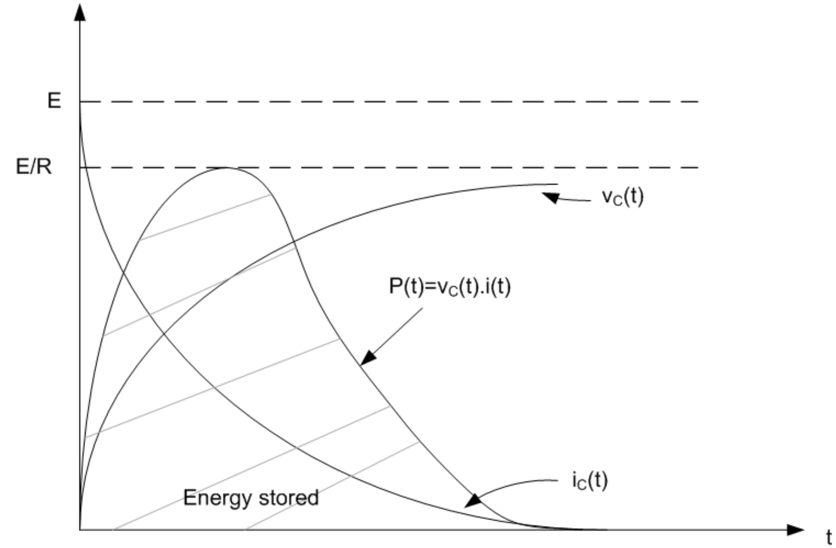
$$i(t) = C\frac{dv_c(t)}{dt} \quad \Rightarrow \quad v_c(t) = \frac{1}{C}\int_0^t i(t)dt$$

- **Instantaneous** power delivered to the capacitor

$$p(t) = v_c(t)i(t)$$

$$= v_c(t)C\frac{dv_c(t)}{dt}$$

$$= Cv_c(t)\frac{dv_c(t)}{dt}$$



- The energy stored in the capacitor

$$w_c(t) = \int_0^t p(t)dt$$

$$= \int_0^t v_c(t)C\frac{dv_c(t)}{dt}dt$$

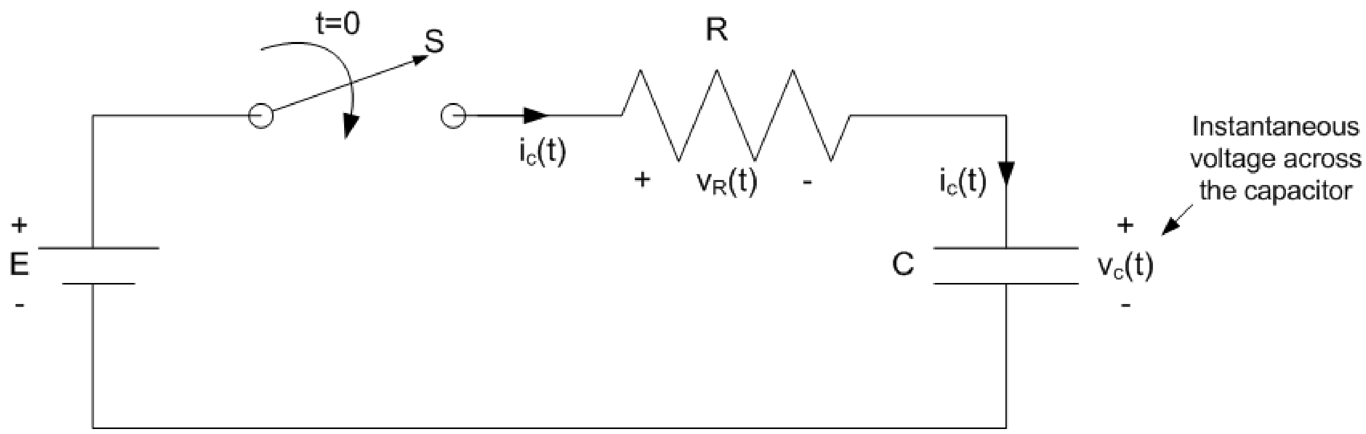
$$w_c(t) = C\int_0^t v_c(t)dv_c(t)$$

$$\boxed{w_c(t) = \frac{1}{2}Cv^2(t)}$$

Transients in Capacitive Networks

Reference(s): Introductory Circuit Analysis – Boylestad, Prentice Hall, 2000

Transients in Capacitive Networks
RC circuit: Charging Phase



- ✓ when the switch **s** is closed at $t = 0$, $i_c(t)$ is the changing current, t is time

$$E = R\, i_c(t) + v_C(t)$$

$$= RC\,\frac{dv_c(t)}{dt} + v_C(t)$$

$$i_c(t) = C\,\frac{dv_c(t)}{dt}$$

$$\therefore \boxed{\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{E}{RC}}$$

Transients in Capacitive Networks

RC circuit: Charging Phase

$$\frac{dv_c(t)}{dt} = \frac{1}{RC}[E - v_c(t)]$$

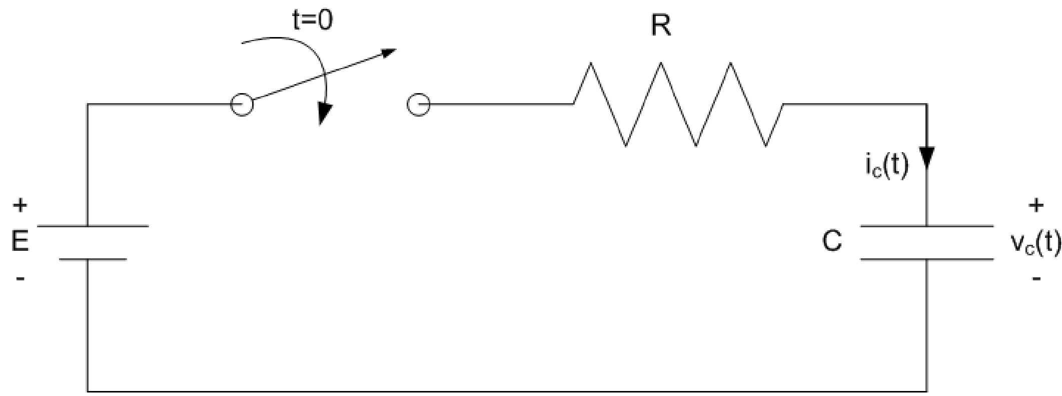
$$\therefore \int_0^t \frac{dv_c(t)}{E - v_c(t)} = \frac{1}{RC}\int_0^t dt$$

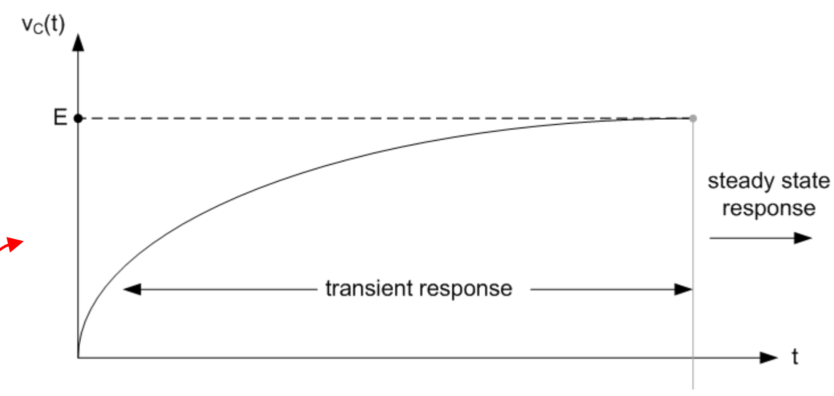
$$\therefore \frac{t}{RC} = -\left[\ln\left(E - v_C(t)\right)\right]_0^t$$

$$-\frac{t}{RC} = \ln\left[\frac{E - v_c(t)}{E}\right]$$

$$\frac{E - v_c(t)}{E} = e^{-\frac{t}{RC}}$$

$$\boxed{\therefore v_C(t) = E\left(1 - e^{-\frac{t}{RC}}\right)} \quad (1)$$





Transients in Capacitive Networks
RC circuit: Charging Phase

$$v_C(t) = E\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$= C \cdot E\left[0 - \left(-\frac{1}{RC}\right) \cdot e^{-\frac{t}{RC}}\right]$$

$$\boxed{i_c(t) = \frac{E}{R}e^{-\frac{t}{RC}}} \qquad (2)$$

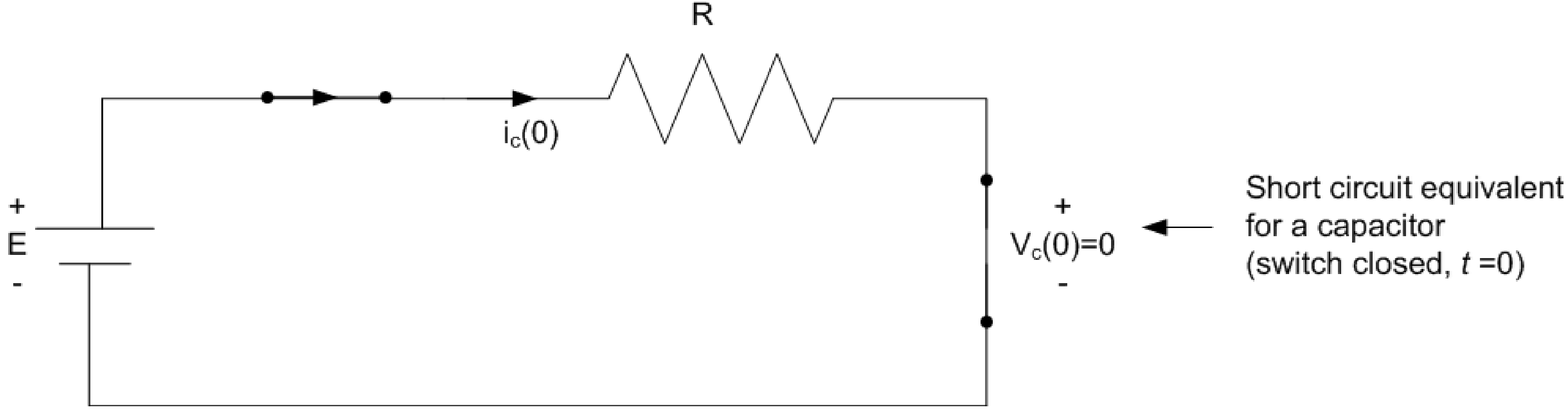
$$\boxed{\begin{aligned} v_R(t) &= i_c(t)R \\ &= Ee^{-\frac{t}{RC}} \end{aligned}} \qquad (3)$$

Transients in Capacitive Networks
RC circuit: Charging Phase

Current $i_c(t)$

✓ When the switch is closed at t = 0 sec, the current jumps to a value limited by the resistance of the RC circuit and then decays to zero as the capacitor is charged.

✓ At the instant the switch is closed, the capacitor behaves as a short circuit

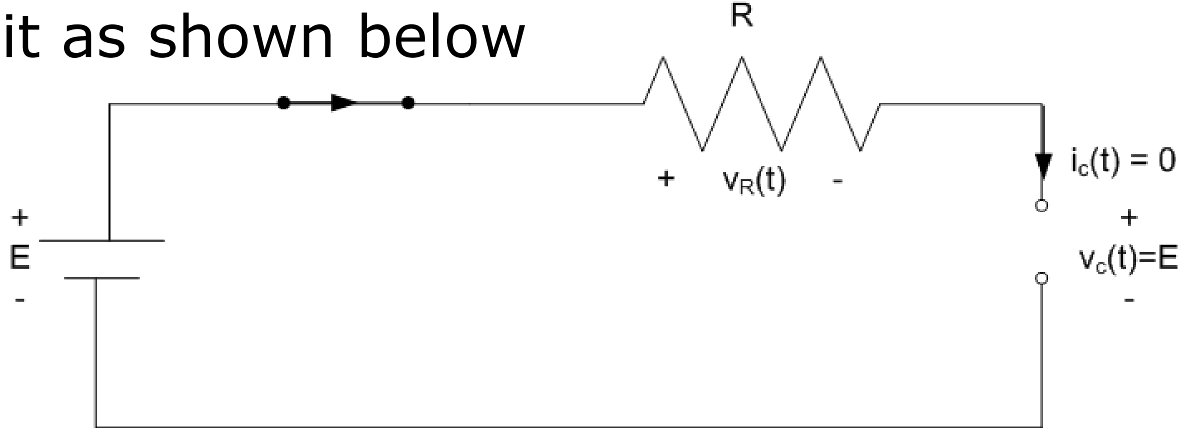


Transients in Capacitive Networks
RC circuit: Charging Phase

Voltage $v_c(t)$

- ✓ When the switch is closed at t = 0, the voltage across the capacitor is zero [assuming no initial charge on the plates of the capacitor]. As the current $i_c(t)$ decreases, the voltage $v_c(t)$ increases. Eventually the current $i_c(t)$ will be zero and the voltage $v_c(t)$ will be E.

- ✓ At this point, a capacitor can be replaced by an open circuit as shown below



[as $t \to \infty$]

Time Constant

✓ The factor *RC* in the following capacitor charging phase equations

$$\begin{bmatrix} V_C(t) = E\left[1 - e^{-\frac{t}{RC}}\right] \\ i_c(t) = \frac{E}{R} e^{-\frac{t}{RC}} \end{bmatrix}$$

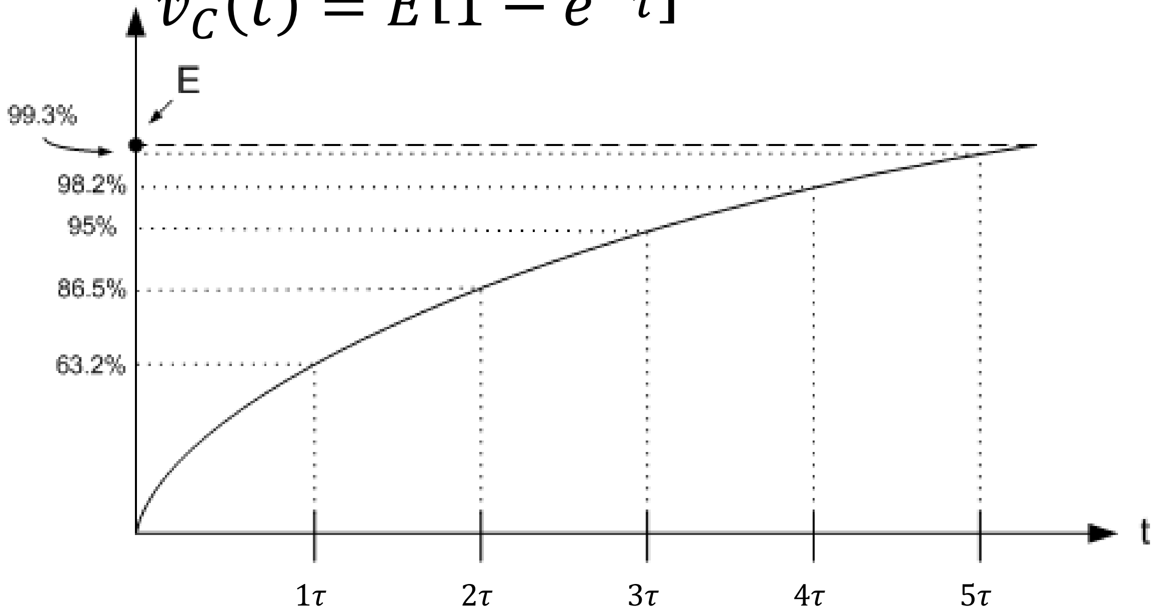
is called the **time constant**. Its symbol is τ (*tau*) and its unit of measure is seconds.

$$\boxed{\tau = RC} \text{ seconds}$$

$$\therefore \begin{bmatrix} V_C(t) = E\left[1 - e^{-\frac{t}{\tau}}\right] \\ i_c(t) = \frac{E}{R} e^{-\frac{t}{\tau}} \end{bmatrix}$$

Time Constant

$$v_C(t) = E\left[1 - e^{-\frac{t}{\tau}}\right]$$

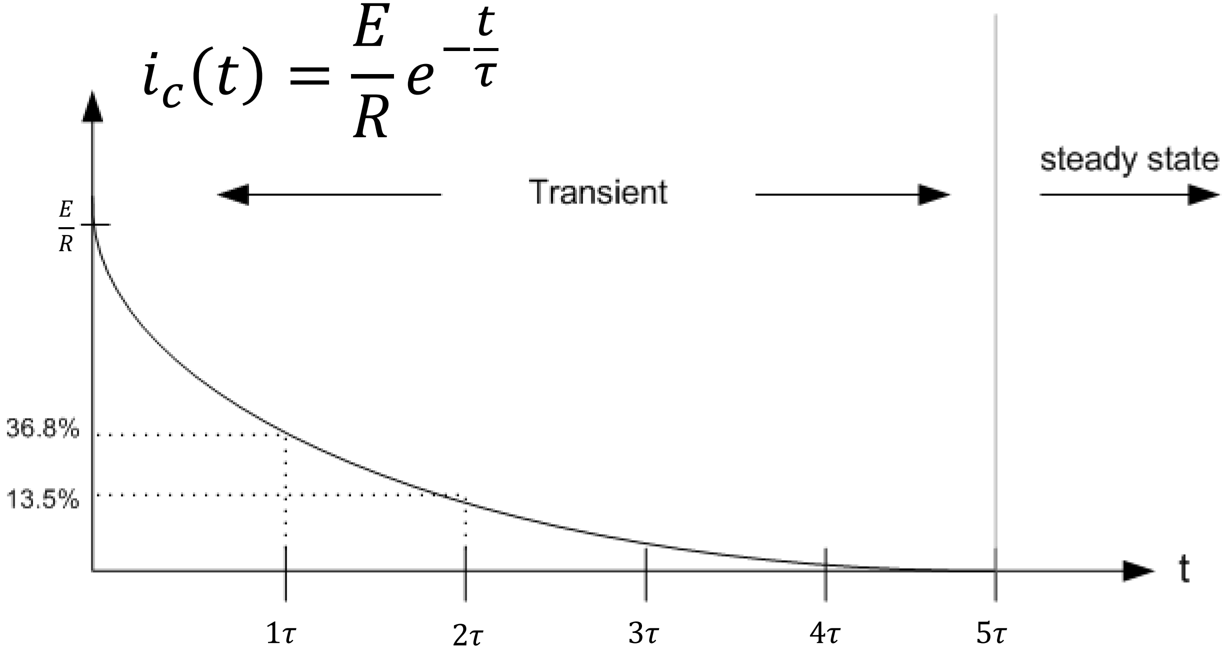


$$\boxed{\tau = RC}$$

The voltage across a capacitor cannot change instantaneously. The voltage across the capacitor is approximately equal to the supply voltage after five time constants, 5τ, of the charging phase have passed.

Time Constant

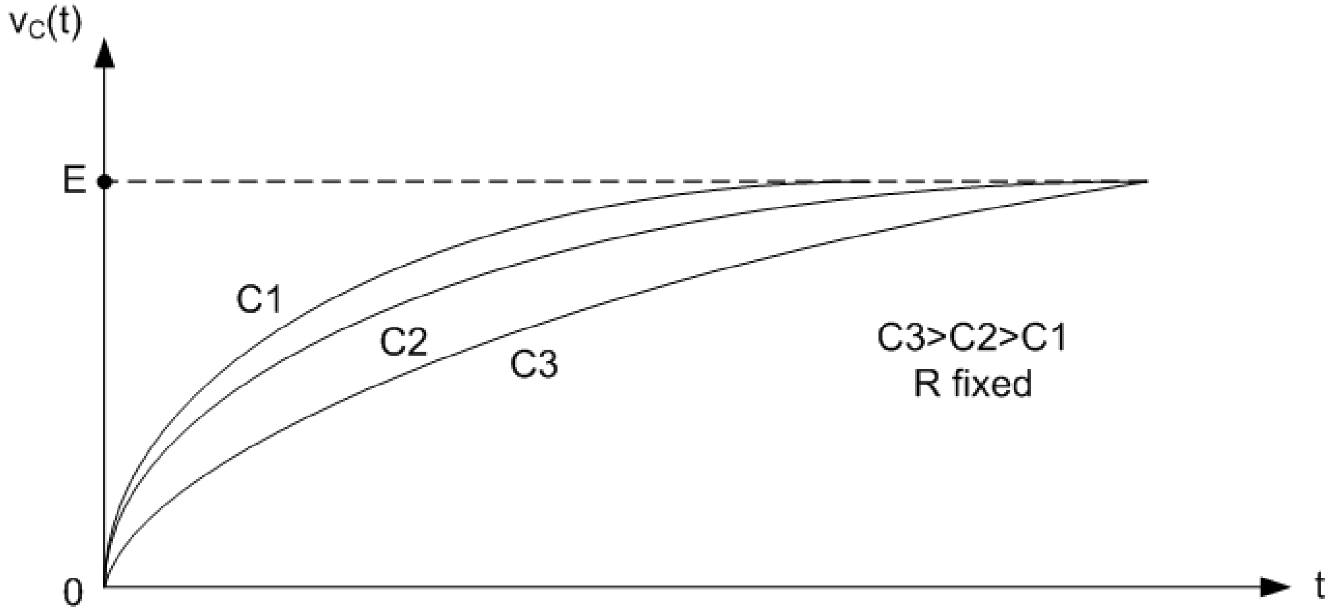
$$i_c(t) = \frac{E}{R} e^{-\frac{t}{\tau}}$$



The current $i_c(t)$ of a capacitive network is approximately zero after five time constants of the charging phase have passed in a DC circuit.

Time Constant

- ✓ The capacitance of a DC (assuming *R* is fixed) circuit is also a measure of how quickly the capacitor could be charged.
- ✓ The larger the capacitance, the larger the time constant, and longer it takes to charge up to its fixed value



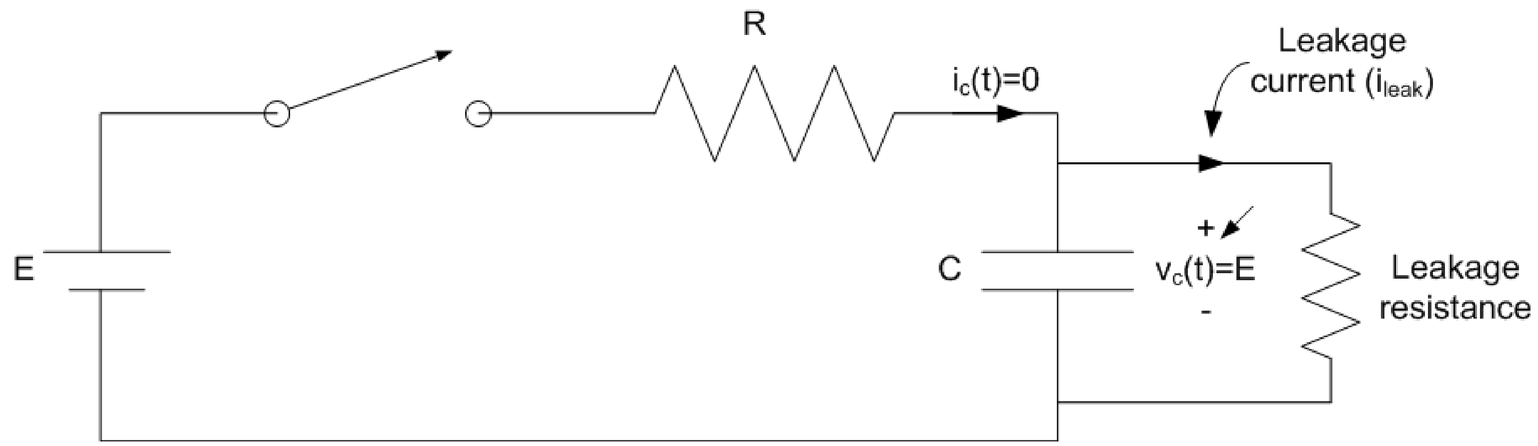
Time Constant

- ✓ A smaller capacitance would permit the voltage to build up more quickly since the time constant is smaller

- ✓ Once the voltage across the capacitor has reached the input voltage E, the capacitor is fully charged

RC circuit: Discharge Phase

- ✓ If the switch is opened (after fully charging the capacitor) see figure below



- ✓ The capacitance will retain its charge for a period of time determined by its leakage current.

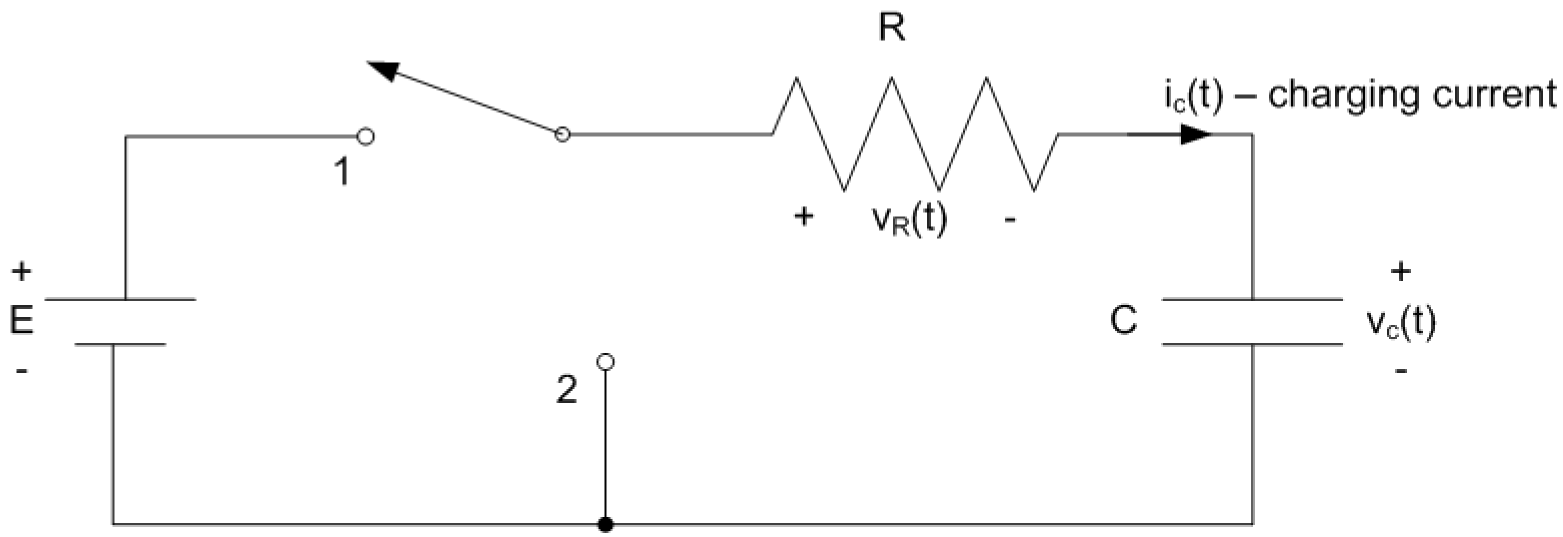
- ✓ For capacitors such as the mica and ceramic, the leakage current is very small, enabling the capacitor to retain its charge for a long time.

RC circuit: Discharge Phase

- ✓ For electrolytic capacitors, which have a very high leakage currents, the capacitor will discharge more rapidly
- ✓ Note: a charged capacitor should be completely discharged by a lead (connecting both terminals) before they are handled

RC circuit: Discharge Phase

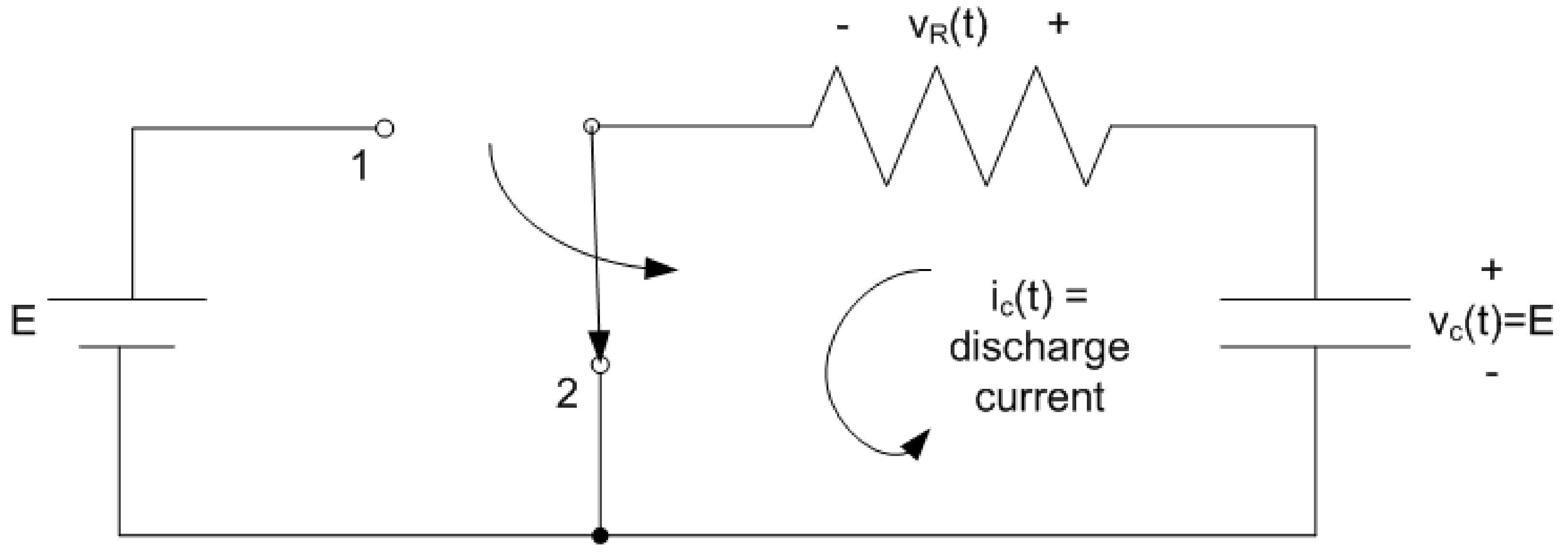
✓ The circuit below is designed to both charge and discharge the capacitor



When the switch is placed in position 1, the capacitor will charge toward the supply voltage.

RC circuit: Discharge Phase

If the capacitor is charged to the full battery voltage and then the switch is moved to position 2, the capacitor will begin to discharge at a rate sensitive to the same time constant, $\tau = RC$.



<u>Discharging behaviour of a capacitor circuit</u>

RC circuit: Discharge Phase

The voltage

$$v_c(t) = v_R(t)$$
$$v_c(t) = i_R(t) \cdot R$$
$$v_c(t) = R\left\{-C\frac{dv_c(t)}{dt}\right\}$$
$$\therefore \frac{-1}{RC}\int_0^t dt = \int_0^t \frac{dv_c(t)}{v_c(t)}$$
$$\frac{-t}{RC} = [ln\, v_c(t)]_0{}^t \leftarrow$$

$v_C(t) = E$
when $t = 0$

$$\frac{-t}{RC} = ln\frac{v_c(t)}{E}$$

$$\boxed{\therefore v_c(t) = \underbrace{E\, e^{\frac{-t}{RC}}}}$$

$$i_c(t) = -C\frac{dv_c(t)}{dt}$$
$$= -C \cdot \left(\frac{-E}{RC}\right) e^{\frac{-t}{RC}}$$

$$\boxed{i_c(t) = \frac{E}{R}e^{\frac{-t}{RC}}}$$

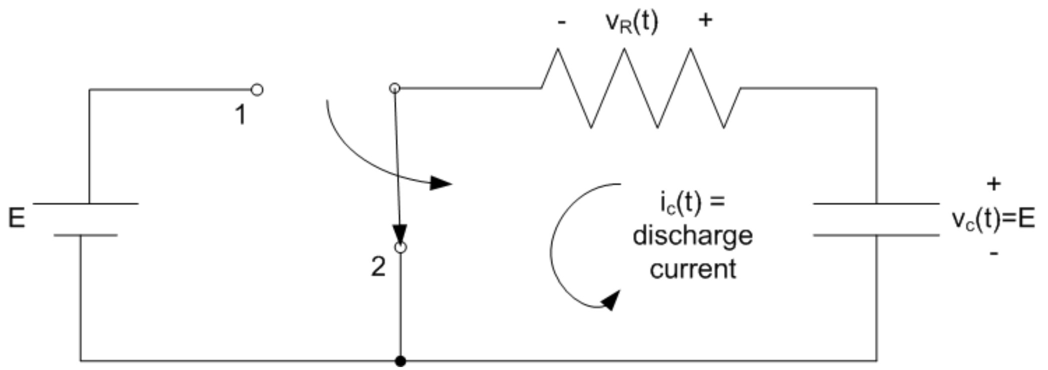
$$\boxed{v_R(t) = v_c(t) = E e^{\frac{-t}{RC}}}$$

Charging current

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$i_c(t) = -C\frac{dv_c(t)}{dt}$$

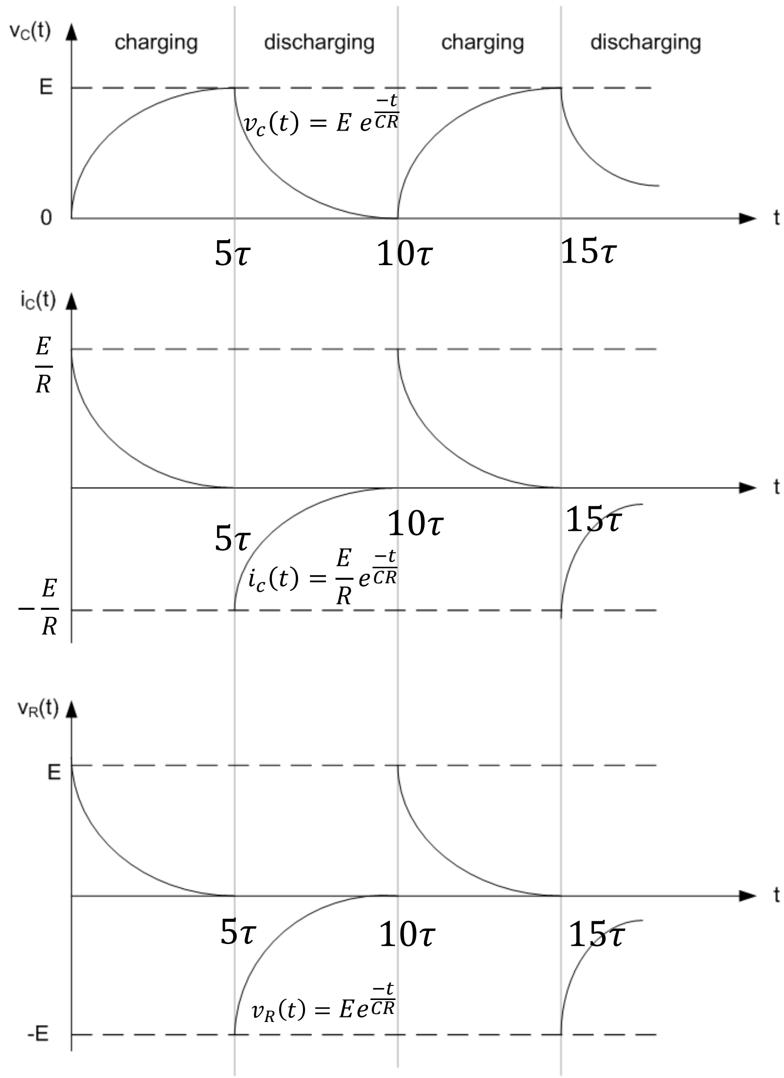
discharging current

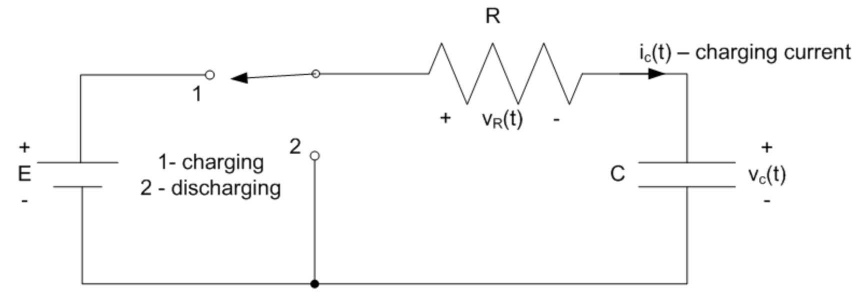


The complete discharge will occur for all practical purposes, in about five time constants.

RC circuit: Discharge Phase

If the switch is moved to between terminals 1 and 2 every five time constants, sketch $v_c(t), i_c(t), v_R(t)$



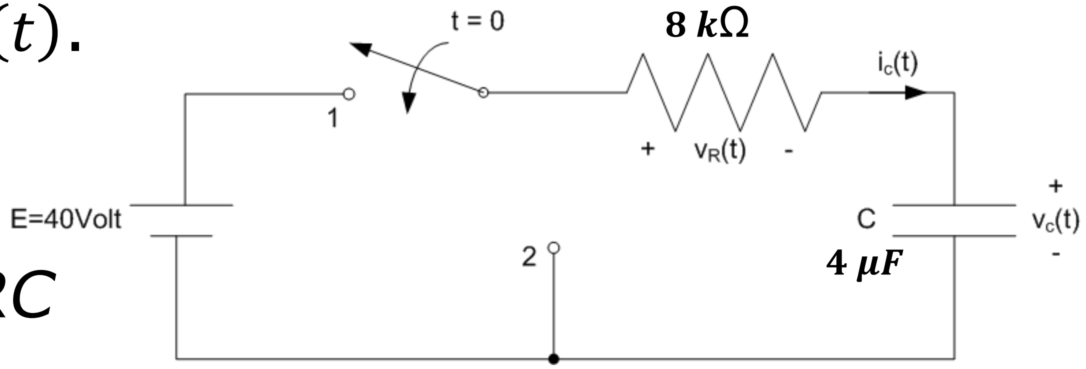


- ✓ Since the polarity of $v_c(t)$ is the same for both the charging and discharging phases $v_c(t)$ is always above the t-axis.

- ✓ The current $i_c(t)$ reverse direction during the charging and discharging phase, producing a negative pulse for both the current and the voltage $v_R(t)$

Example

(a) Find the mathematical expressions for the transient behaviour of $v_c(t), i_c(t), v_R(t)$ for the circuit below when the switch is moved to position 1. Plot the curves $v_c(t), i_c(t), v_R(t)$.



(a)Time constant (τ)= RC

$$\tau = 8 \times 10^3 \times 4 \times 10^{-6} = 32\text{ms}$$

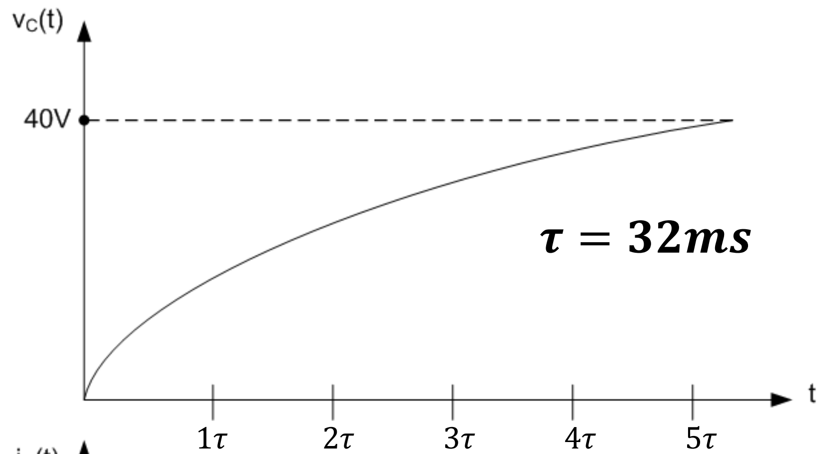
$$v_c(t) = E\left[1 - e^{\frac{-t}{\tau}}\right] = 40\left[1 - e^{\frac{-t}{(32\times10^{-3})}}\right]$$

$$v_R(t) = Ee^{-\frac{t}{\tau}} = 40e^{-\frac{t}{(32\times10^{-3})}}$$

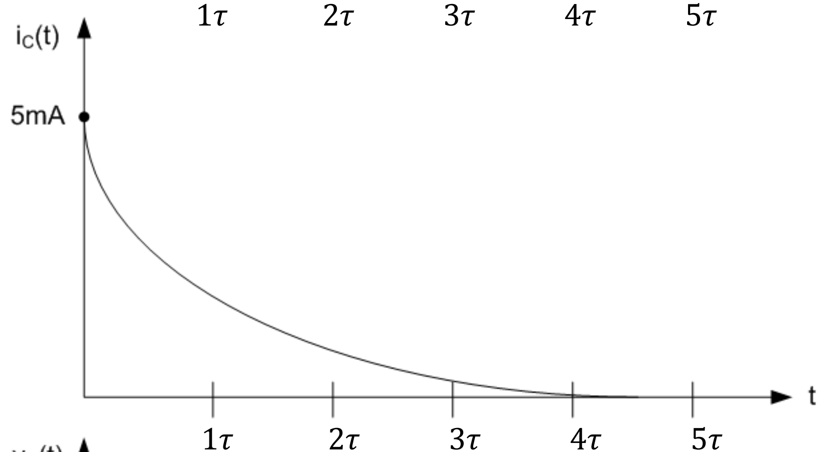
$$i_c(t) = \frac{E}{R}e^{\frac{-t}{\tau}} = \frac{40}{8 \times 10^{-3}}\mathrm{e}^{-\frac{t}{(32\times10^{-3})}}$$

Example

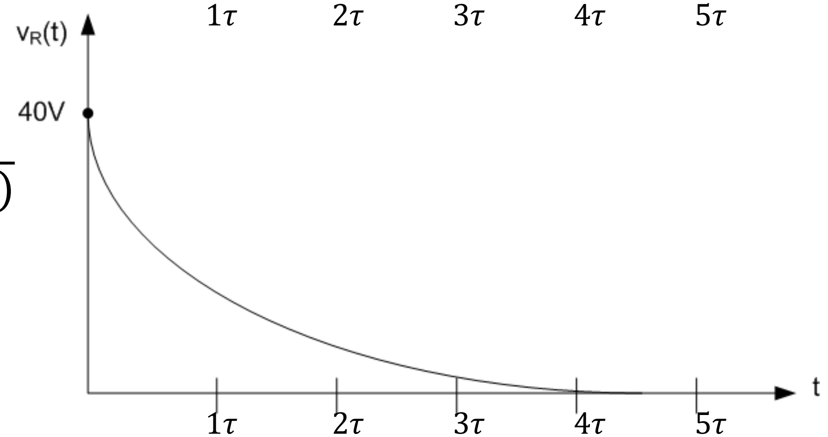
$$v_c(t) = E\left[1 - e^{\frac{-t}{\tau}}\right]$$
$$= 40\left[1 - e^{\frac{-t}{(32\times10^{-3})}}\right]$$



$$i_c(t) = \frac{E}{R} e^{\frac{-t}{\tau}}$$
$$= \frac{40}{8\text{x}10^{-3}}\,\mathrm{e}^{-\frac{t}{(32\times10^{-3})}}$$

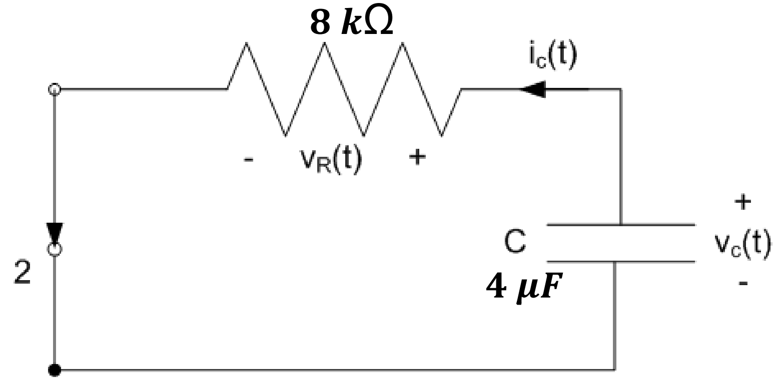


$$v_R(t) = Ee^{-\frac{t}{\tau}} = 40e^{-\frac{t}{(32\times10^{-3})}}$$



Example

(b) After $v_c(t)$ has reached its final value of 40V, the switch is thrown into position 2 (see diagram below) assume that when $t = 0$, the switch is moved to position 2. Plot the curves $v_c(t), i_c(t), v_R(t)$.



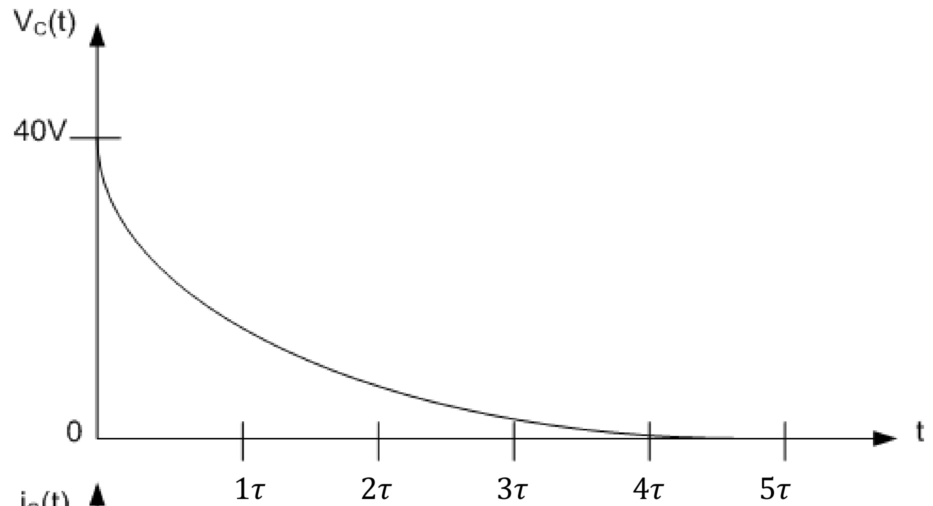
$\tau = 32\text{ms}$

$$v_c(t) = Ee^{\frac{-t}{\tau}} = 40e^{\frac{-t}{(32\times10^{-3})}} \qquad v_R(t) = -Ee^{-\frac{t}{\tau}} = -40e^{-\frac{t}{(32\times10^{-3})}}$$

$$i_c(t) = -\frac{E}{R}e^{\frac{-t}{\tau}} = -(5\times10^{-3})\mathrm{e}^{-\frac{t}{(32\times10^{-3})}}$$

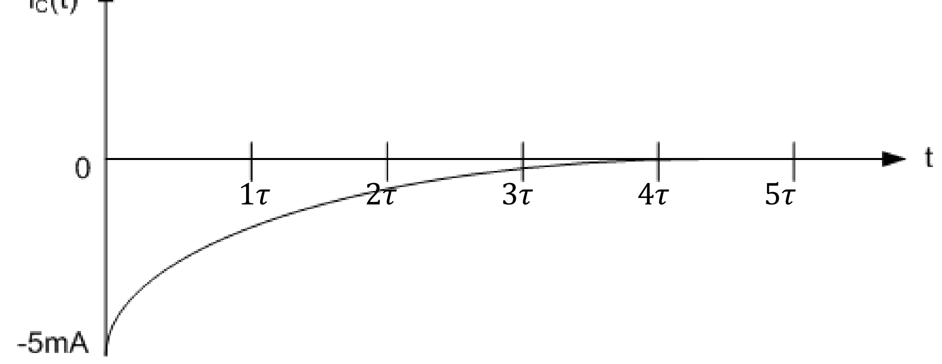
Example

$$v_c(t) = Ee^{\frac{-t}{\tau}} = 40e^{\frac{-t}{(32\times10^{-3})}}$$

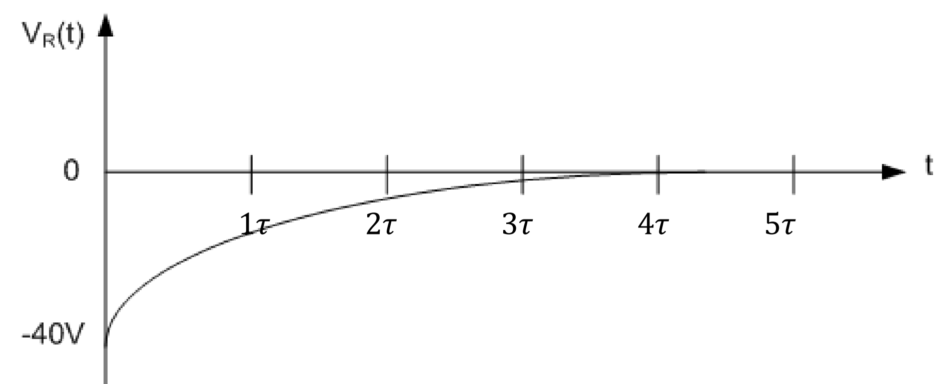


$$i_c(t) = -\frac{E}{R}e^{\frac{-t}{\tau}}$$

$$= -(5\times10^{-3})\mathrm{e}^{-\frac{t}{(32\times10^{-3})}}$$

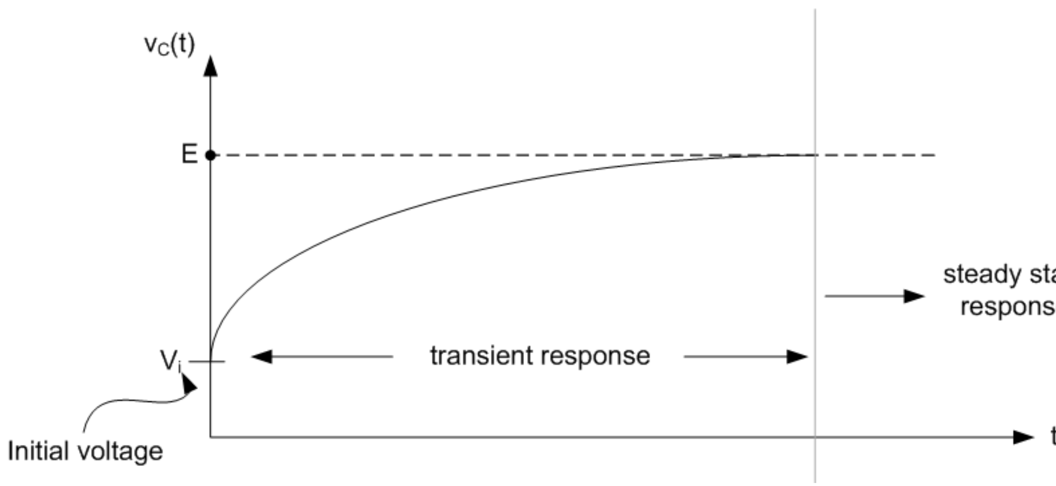


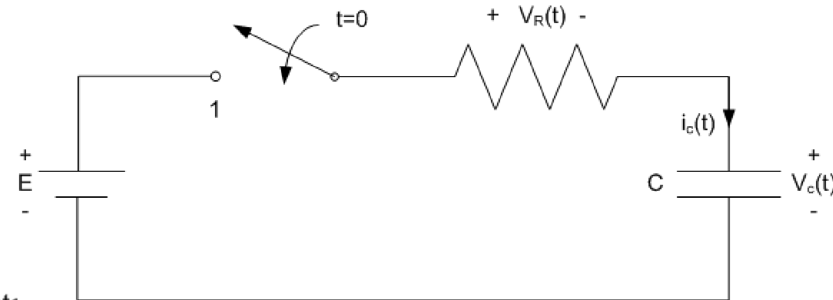
$$v_R(t) = -Ee^{-\frac{t}{\tau}} = -40e^{-\frac{t}{(32\times10^{-3})}}$$



Initial Value

In the previous sections, it was assumed that the capacitor was uncharged before the switch was thrown (i.e. the initial charge in the capacitor = 0).
We will now examine the effect of a charge (i.e. a voltage $V = \frac{Q}{C}$) on the plates at the instant the switch action takes place.





$$v_c(t) = V_i + (E - V_i)\left(1 - e^{-\frac{t}{\tau}}\right)$$

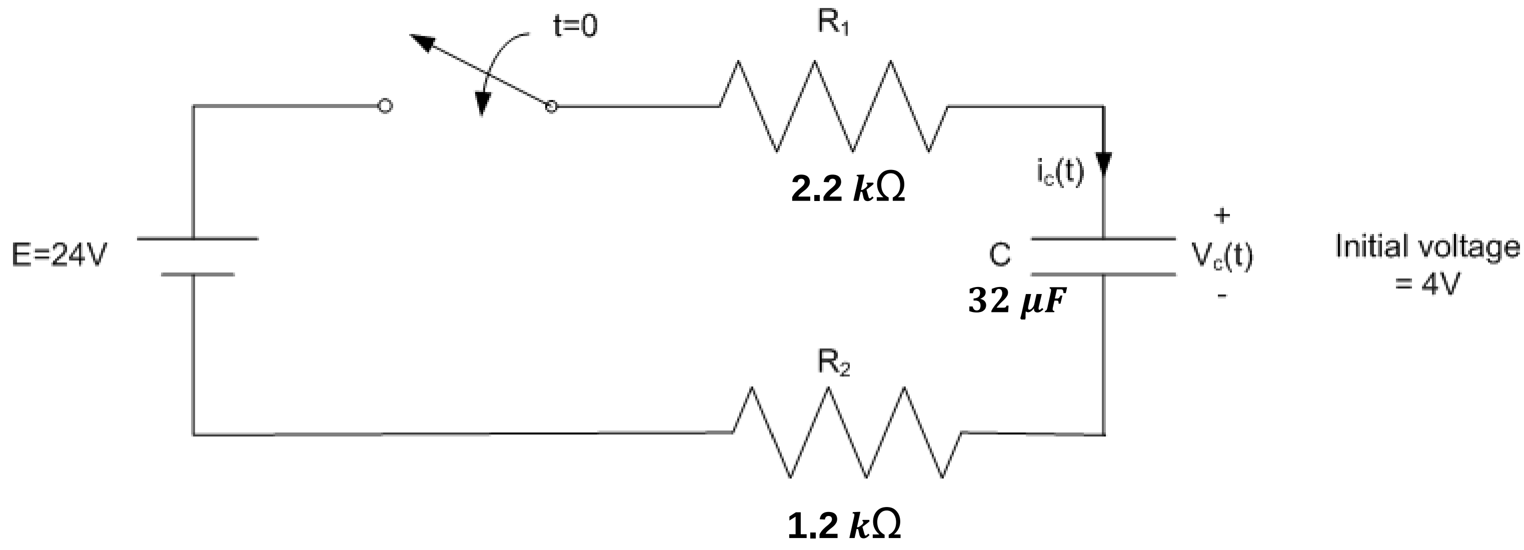
$$\boxed{\therefore v_c(t) = E + (V_i - E)\, e^{-\frac{t}{\tau}}}$$

If the capacitor is uncharged initially $V_i = 0$ then we get

$$v_c(t) = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

Example

Suppose a capacitor has an initial voltage of 4V



(a) Find the voltage $v_c(t)$ across the capacitor once the switch is closed

(b) Find the current $i_c(t)$ during the transient period

(c) Sketch $v_c(t)$ and $i_c(t)$

Example

(a) The time constant

$$\tau = (R_1 + R_2)C$$

$$= (2.2 + 1.2)\text{k}\Omega \times 3.3\mu\text{F}$$

$$= 11.22\text{ms and } 5\tau = 56.1\text{ms}$$

$$v_c(t) = E + (V_i - E)\, e^{-\frac{t}{\tau}}$$

$$= 24 + (4 - 24)e^{-\frac{t}{11.22ms}}$$

$$= 24 - 20e^{-\frac{t}{11.22\times 10^{-3}}}$$

Example

(b) At the instant the switch is closed, the voltage across the resistive elements is (24V - 4V) = 20V.

∴ The peak current is $I_{max} = \frac{20}{R_1+R_2} = 5.88mA$

∴ $i_c(t) = 5.88 \times 10^{-3} \mathrm{e}^{-\frac{t}{11.22ms}}$

Or use

$i_C(t) = C\frac{dv_C}{dt}$

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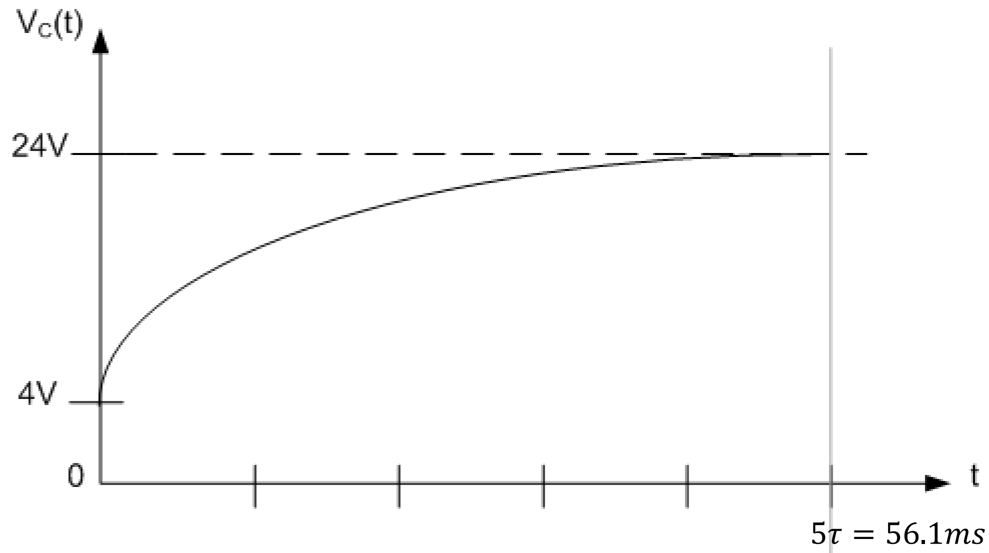
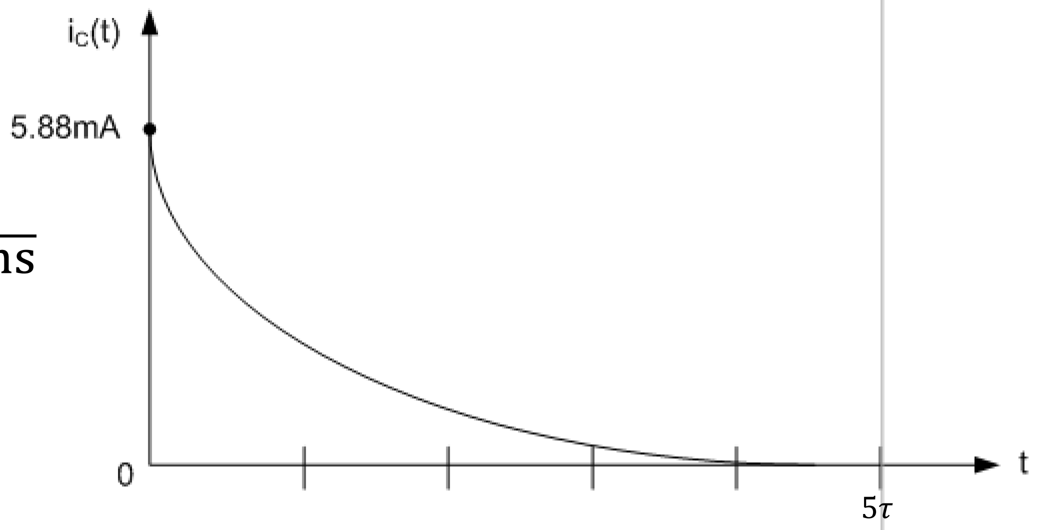
Example

$$v_c(t) = 24 - 20e^{-\frac{t}{11.22\times10^{-3}}}$$

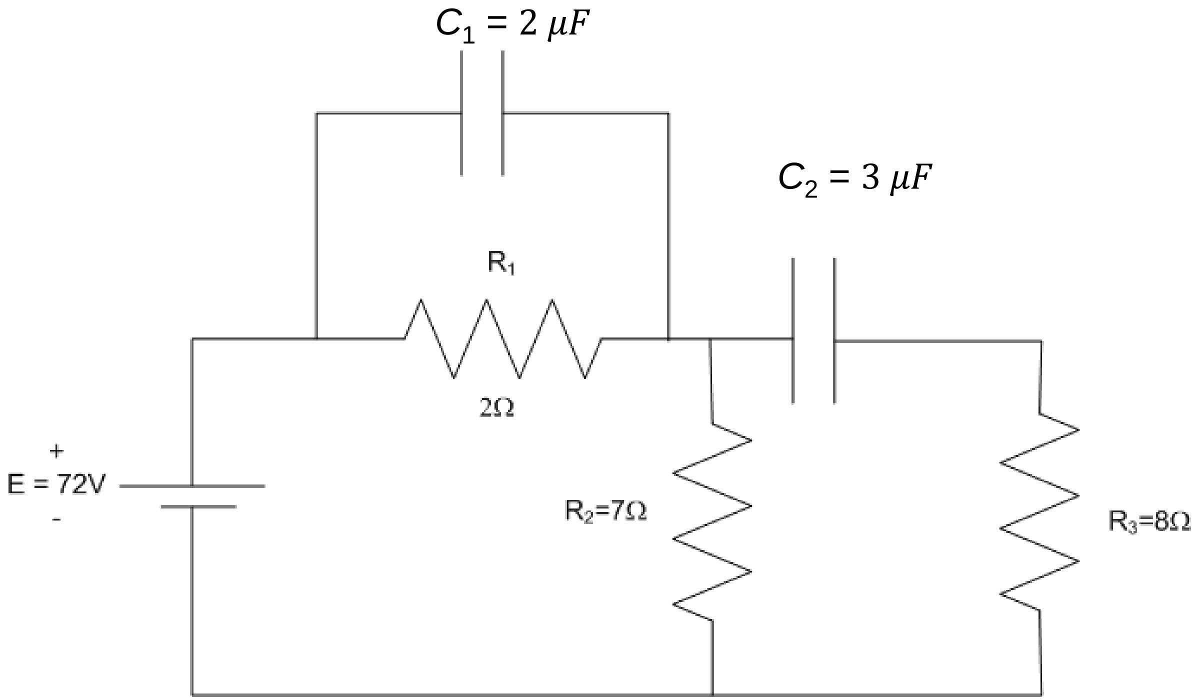


$$i_c(t) = 5.88 \times 10^{-3}\mathrm{e}^{-\frac{t}{11.22\mathrm{ms}}}$$



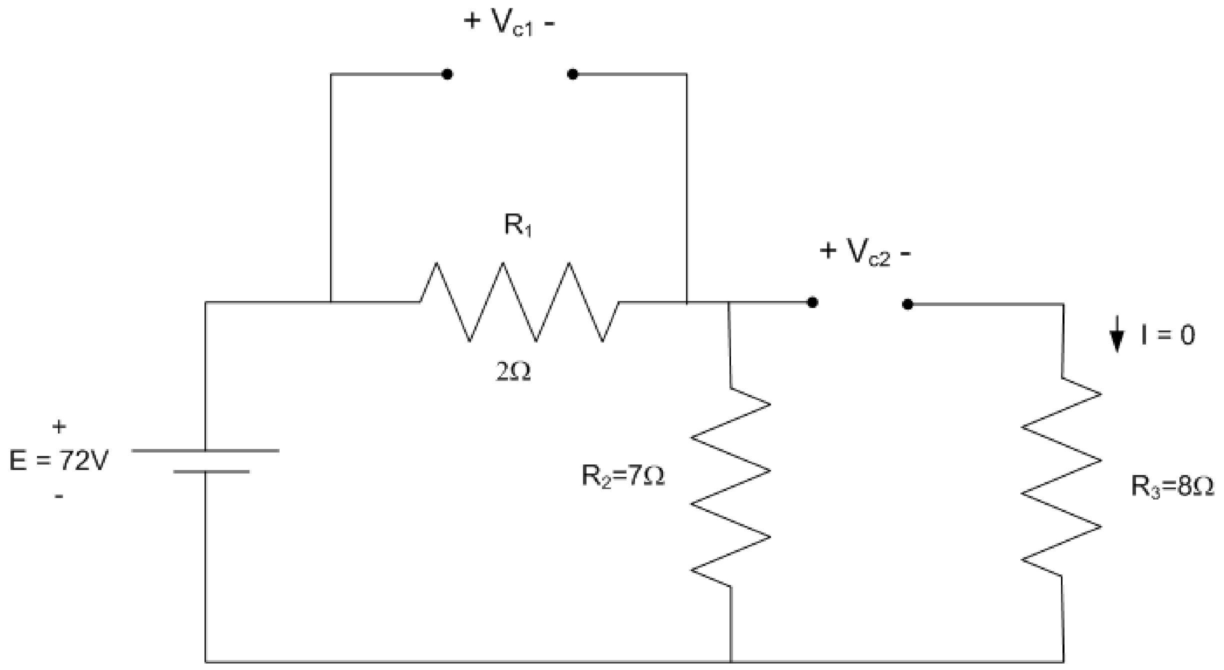
Example (Steady State)

Find the voltage across and charge on each capacitor of the circuit given below after each has charged up to its steady state value



Note: a capacitor can be replaced by an open-circuit equivalent once it has been charged up to its full value

Example (Steady State)



$$V_{c2} = \frac{7}{7+2} \times 72 = 56V$$

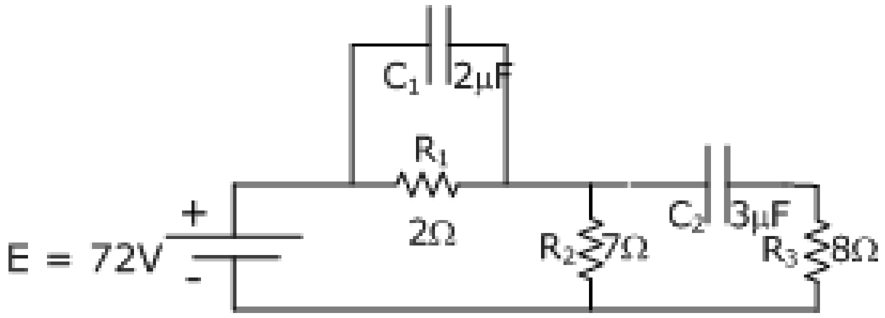
$$V_{c1} = \frac{2}{7+2} \times 72 = 16V$$

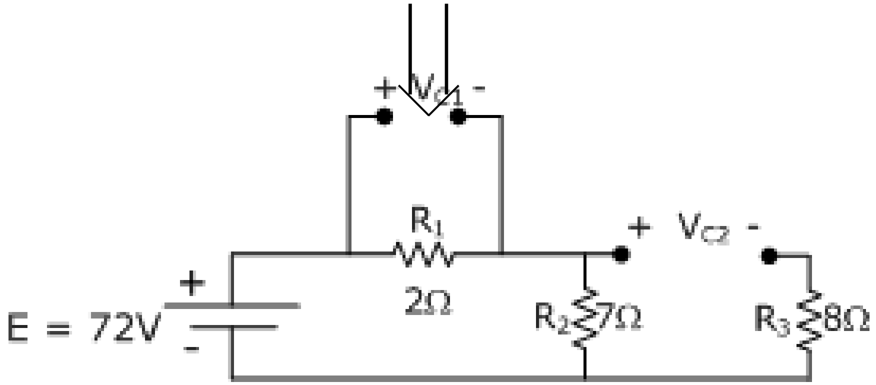
$$Q_1 = C_1 V_{c1} = (2 \times 10^{-6}\mathrm{F})(16\mathrm{V}) = 32\mu\mathrm{C}$$

$$Q_2 = C_2 V_{c2} = (3 \times 10^{-6}\mathrm{F})(56\mathrm{V}) = 168\mu\mathrm{C}$$

Example

Also determine the energy stored by each capacitor.





$$V_{C_2} = 7 \times \frac{72}{7+2} = 56\text{V}$$

$$V_{C_1} = 2 \times \frac{72}{7+2} = 16\text{V}$$

For C_1,

$$w_C = \frac{1}{2} C_1 V_{C_1}^2 = \frac{1}{2}(2 \times 10^{-6}(16\text{V})^2)$$

$$= 256\mu\text{J}$$

For C_2,

$$w_C = \frac{1}{2} C_2 V_{C_2}^2 = \frac{1}{2}(3 \times 10^{-6}(56\text{V})^2)$$

$$= 4704\mu\text{J}$$

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Reference: Introductory Circuit Analysis – Boylestad, Prentice Hall, 2000