WEEK 6-2017

Synchronous Sequential Circuit II

ELEC2141: Digital Circuit Design

UNSW | ENGINEERING

THE UNIVERSITY OF NEW SOUTH WALES

Characteristic Tables

- A characteristic table defines the logical properties of a flip-flop by describing its operation in tabular form.
- It defines the next state as a function of the inputs and the presents state.
- The next state is the state of the flip-flop after the clock transition and denoted by Q(t+1).
- The present state is the state of the flip-flop immediately before the clock edge and denoted by Q(t).

Characteristic Tables

J	K	Q (t+1)	
0	0		
0	1		
1	0		
1	1		

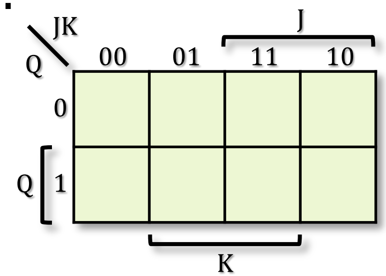
D	Q (t+1)	
0		
1		

T	Q (t+1)	
0		
1		

Characteristic Equations

Q	J	K	Q (t+1)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

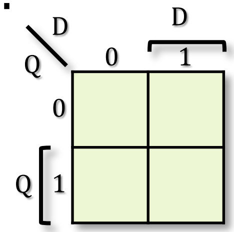
Q(t+1):



Characteristic Equations

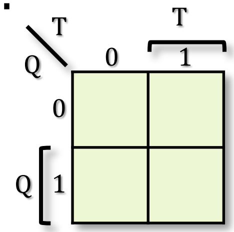
Q	D	Q (t+1)
0	0	
0	1	
1	0	
1	1	

Q(t+1):



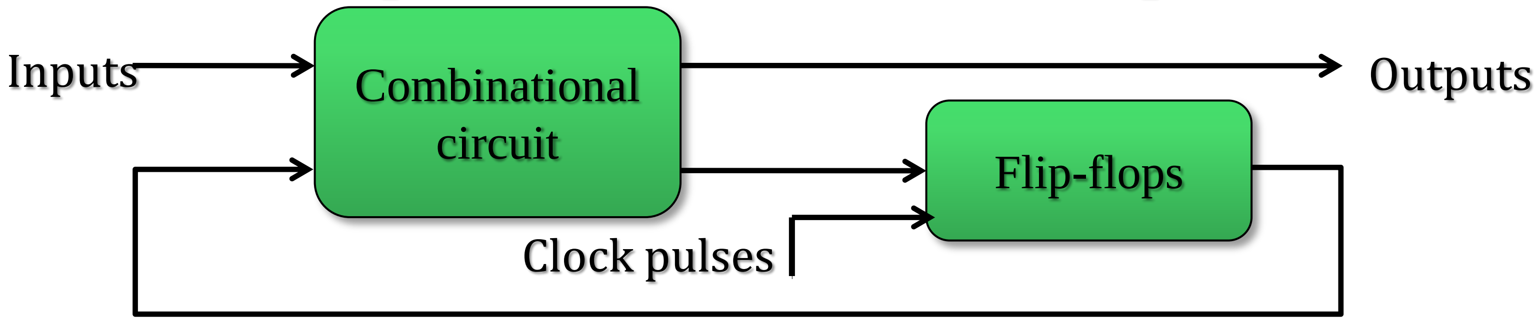
Q	T	Q (t+1)
0	0	
0	1	
1	0	
1	1	

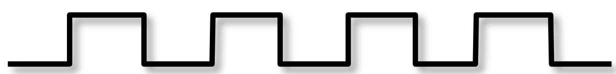
Q(t+1):



Analysis of clocked Sequential Circuit

- **The behavior of a clocked sequential circuit is determined from the inputs, internal states and outputs.**





Time diagram of clock pulses

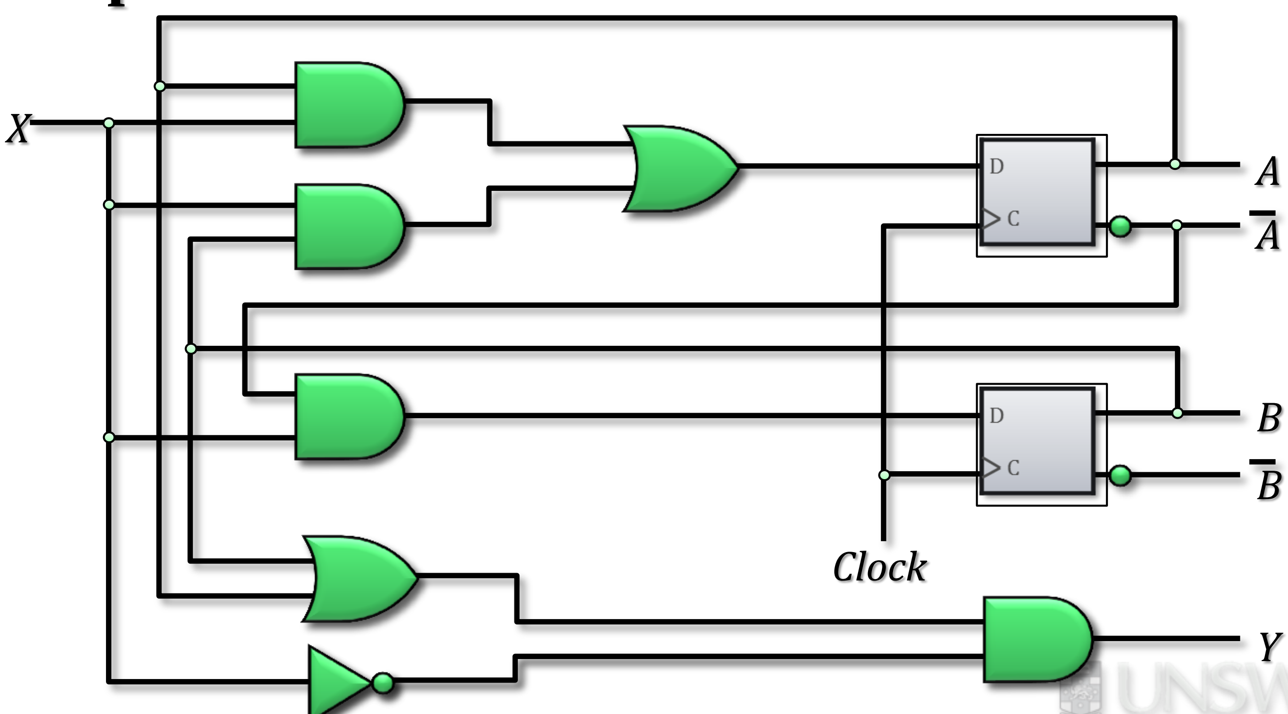
- **The outputs and the next state are both the function of the inputs and the present state.**
- **Analysis of sequential circuit consists of obtaining a table or a diagram for the time sequence of inputs, internal states and outputs.**
- **Boolean expressions can be used to describe the behavior of the sequential circuit. In this case, it should include the necessary time sequence**

Analysis of clocked Sequential Circuit

- **State equation, state table and state diagram will be used to describe the behavior of any clocked sequential circuit.**

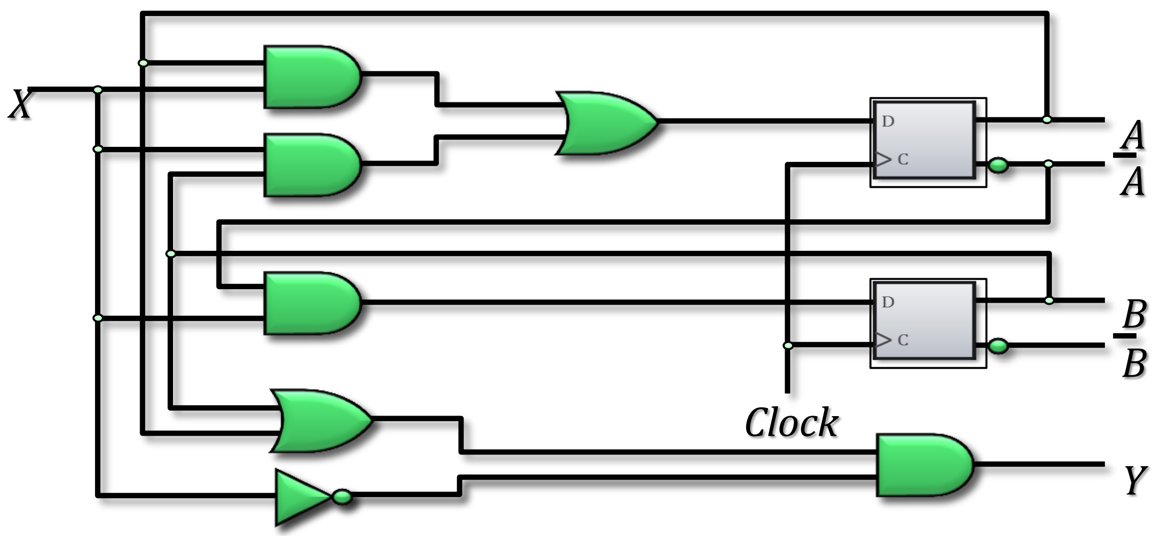
State Equations (Transition Equation):

- **A state equation (transition equation) specifies the next state as a Boolean function of the present state and inputs.**



State Equations

State Equations (Transition Equation):



Since the D input of a flip-flop determines the value of the next state (i.e, the state reached after the clock transition)

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

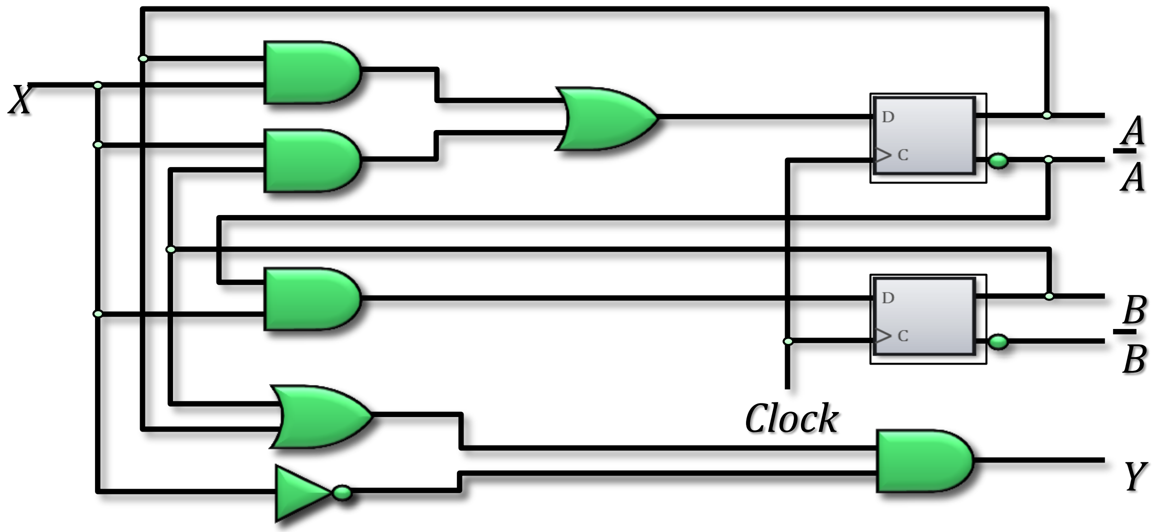
$$B(t+1) = \overline{A(t)}x(t)$$

The present-state value of the output can be expressed algebraically as

$$Y(t) = [A(t) + B(t)]\overline{x(t)}$$

State Equations

State Equations (Transition Equation):



Since all variables in the Boolean expressions are a function of the present state, the designation of (t) after each variable can be omitted

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \overline{A}x$$

The present-state value of the output can be expressed algebraically as

$$Y(t) = [A + B]\overline{x}$$

State table

State table (Transition table):

- It is possible to enumerate the time sequences of inputs, outputs and states of the sequential circuit using state table (transition table)

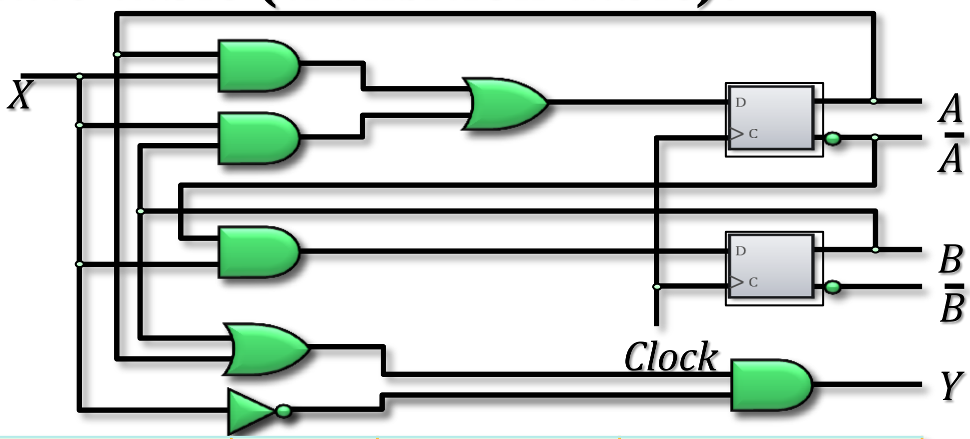
- The table consists of four sections labelled present state, input, next state and output.

- The derivation of a state table requires listing all possible binary combinations of present states and inputs.

- The next-states and the output values are then determined from the logic diagram or state equations for each possible binary combination.

State table

State table (transition table):



Present state		Input	Next state		Output
A	B	X	A	B	Y
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \overline{A}x$$
$$Y(t) = [A+B]\overline{x}$$

State table

State table (transition table):

- **The state transition may be conveniently expressed in a slightly different form having only three sections: present state, next state and output.**

Present state		Next state				Output	
		X=0		X=1		X=0	X=1
A	B	A	B	A	B		
0	0	0	0	0	1		
0	1						
1	0						
1	1						

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \bar{A}x$$
$$Y(t) = [A+B]\bar{x}$$

State diagram

- **The information available in state tables can be represented graphically in the form of a state diagram.**

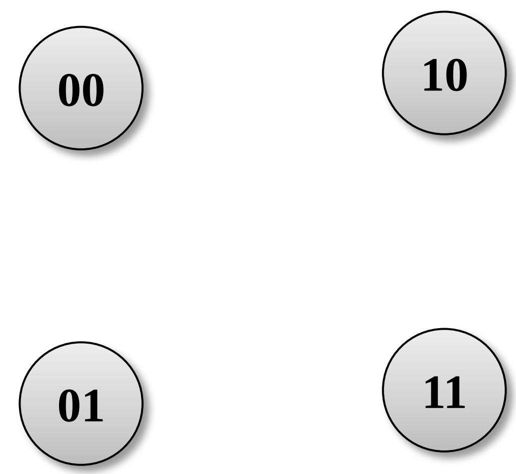
- **In a state diagram, a state is indicated by a circle and the transitions between the states are represented by directed lines connecting the circles.**

- **The directed lines are labeled with two binary numbers separated by a slash. The first binary number is for the input value and the second number shows the output value for the given input value during the present state which is represented by a circle from which the directed line emanates.**

State diagram

- The state diagram for the following state table can be drawn as:

Present state	Next state				Output	
	X=0		X=1		X=0	X=1
A B	A	B	A	B		
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0



- The same information is conveyed by both state diagram and state table.
- State diagrams provides pictorial representation that is easier for human representation.

State and output time sequence

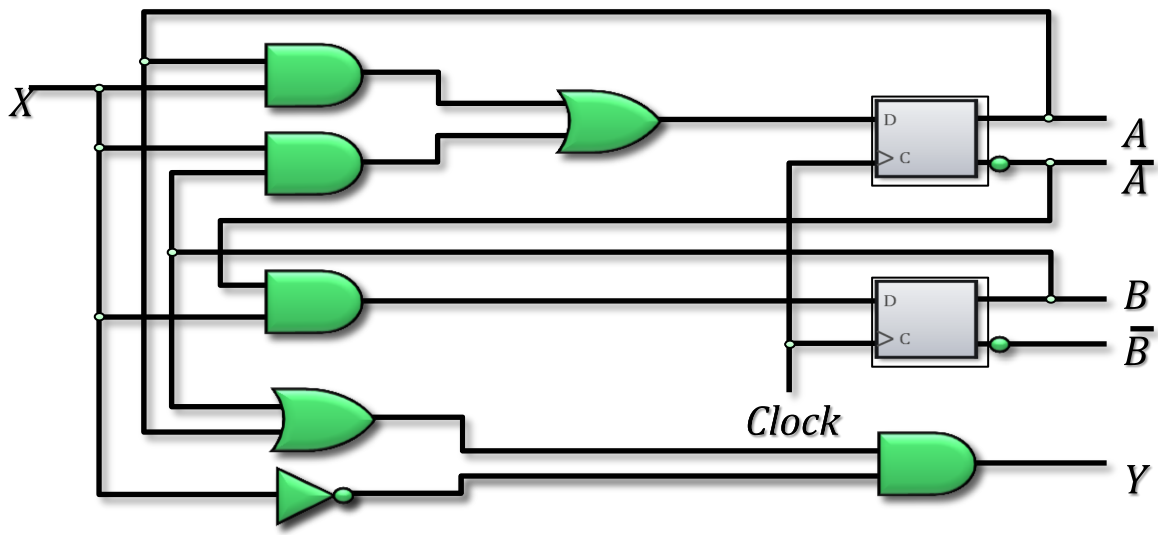
- **Find state and output time sequence for input sequence of 010100110 assuming the sequential is initially in state 00**

Clk										
X	0	1	0	1	0	0	1	1	0	0
A	0	0	0							
B	0	0	1							
Y	0	0								

Present state	Next state				Output	
	X=0		X=1		X=0	X=1
A B	A	B	A	B		
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0

Flip-flop input equations

- The set of Boolean functions that describes the part of the sequential circuit that generates the inputs to flip-flops.



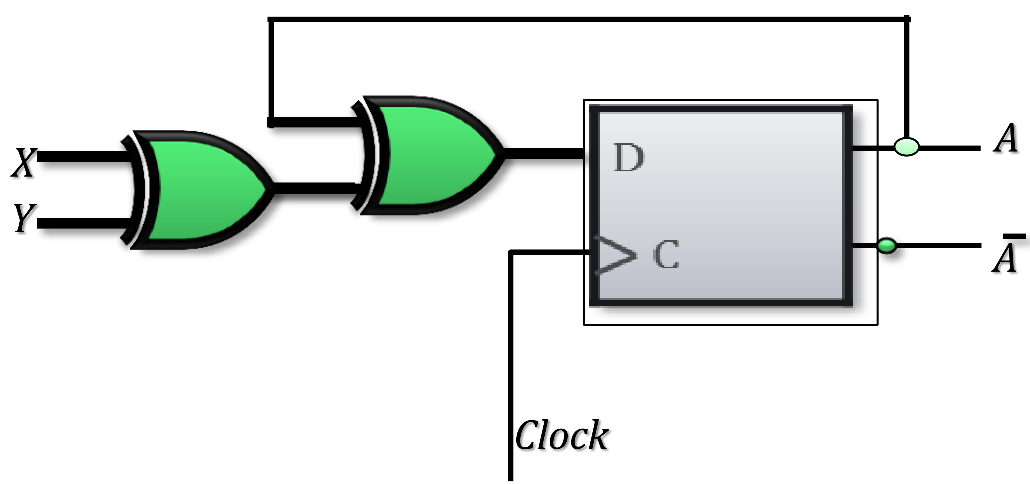
$$D_A = Ax + Bx$$
$$D_B = \overline{A}x$$
$$Y = [A + B]\overline{x}$$

- The three equations provide the necessary information for drawing the logic diagram of the sequential circuit.
- The symbol D_A specifies a D flip-flop labeled A and D_B is for another D flip-flop labeled B.
- The flip-flop equations specify the combination circuit that drives the flip-flops and the type of flip-flops used.

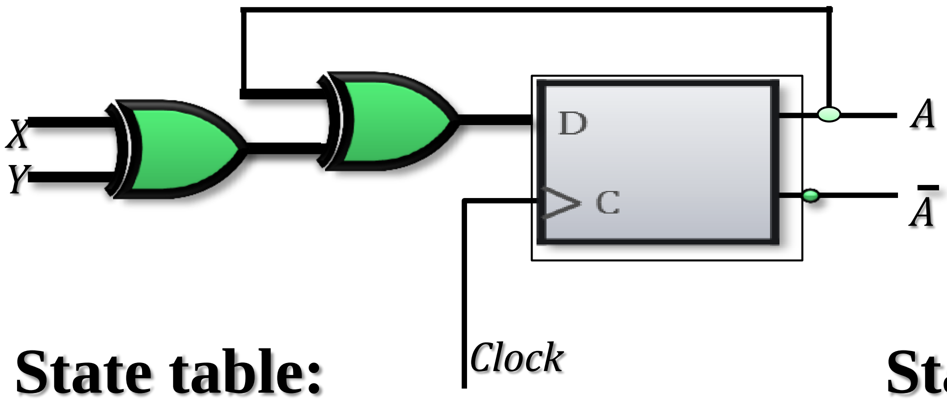
Analysis with D flip-flops

- **Steps in analyzing clocked sequential circuits with D flip-flops:**
  - **Find the flip-flop input equations.**
  - **The state equations are the same as the flip-flop input equations for D flip-flops.**
  - **Derive the state table from the state equations.**
  - **Draw the state diagram from the state table**

Example:



Analysis with D flip-flops



- **State table:**

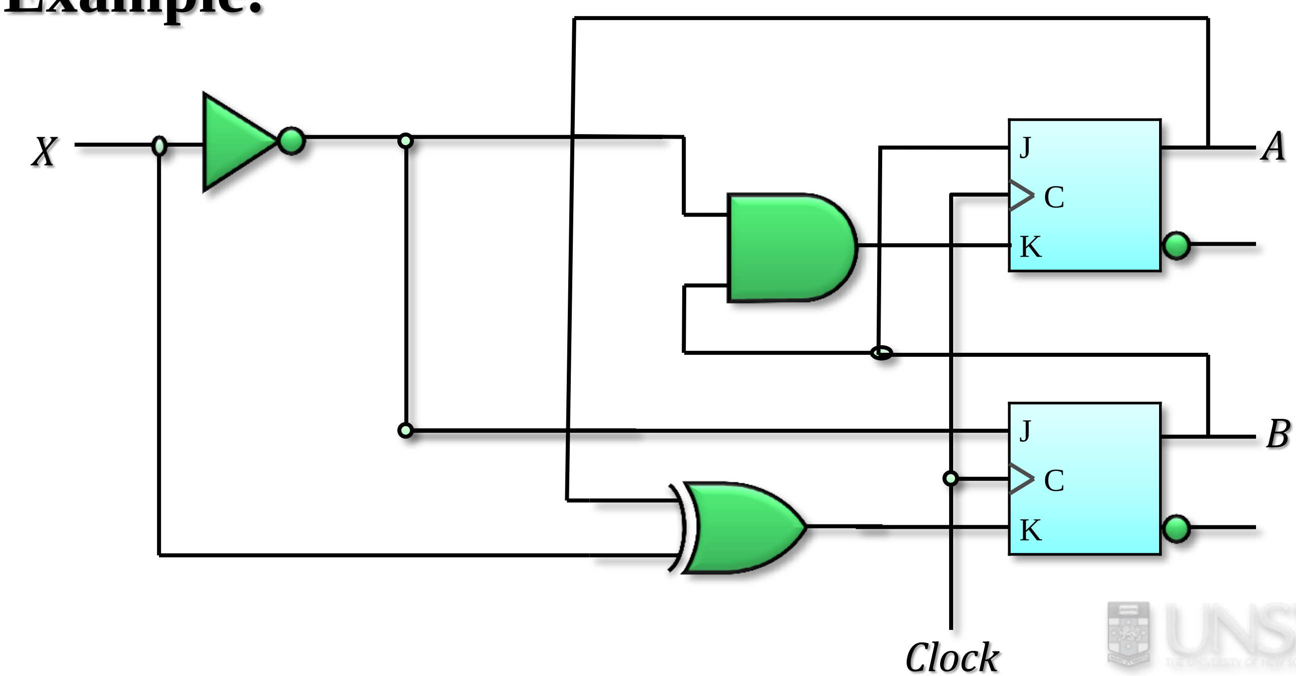
Present state	Input		Next state
A	**X**	**Y**	**A**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

State Diagram:

Analysis with JK flip-flops

- **Steps in analyzing clocked sequential circuits with JK flip-flops:**
  - **Find the flip-flop input equations in terms of present state and inputs.**
  - **List the binary values of each input equation.**
  - **Use the JK flip-flop characteristic table to determine the next states in the state table.**
  - **Use K-map to obtain minimized state equations**

Example:



Analysis with JK flip-flops

- **Flip-flop input equations:**

J_A=　　　　K_A=

J_B=　　　　K_B=

Present state		Input	Next state		Flip-Flop inputs			
A	B	X	A	B	J_A	K_B	J_B	K_B
0	0	0						
0	0	0						
0	1	1						
0	1	1						
1	0	0						
1	0	0						
1	1	1						
1	1	1						

Analysis with JK flip-flops

- Steps in analyzing clocked sequential circuits with JK flip-flops:
  - Find the flip-flop input equations in terms of present state and inputs.
  - Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.
  - Use the corresponding state equations to determine the next-values in the state table and state diagrams

For previous example:

$J_A = B$ $\qquad K_A = B\overline{X}$

$J_B = \overline{X}$ $\qquad K_B = \overline{A \oplus x}$

Characteristic equation for JK flip-flop:

$$A(t+1) = J_A\overline{A} + \overline{K_A}A$$

$$B(t+1) = J_B\overline{B} + \overline{K_B}B$$

Analysis with JK flip-flops

State Table:

Present state			Next State				
			X=0		X=1		
A	B		A	B	A	B	
0	0						
0	1						
1	0						
1	1						

State Diagram:

Analysis with T flip-flops

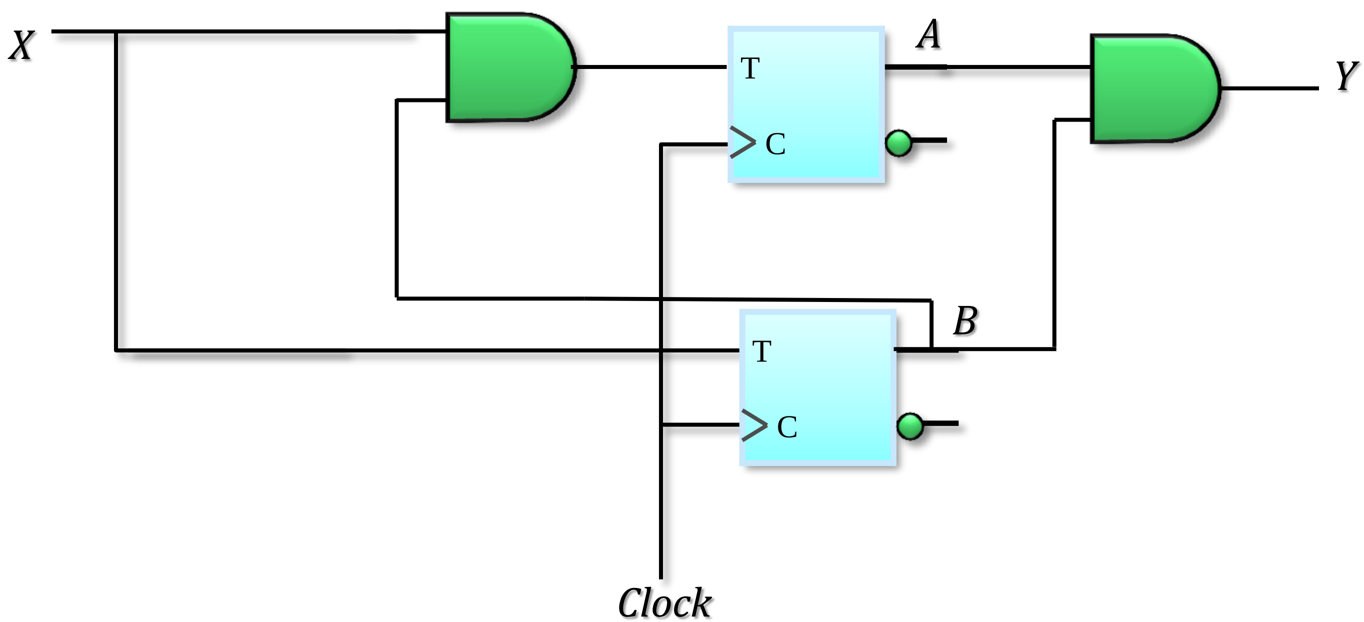
- **The same procedure as JK flip-flop is to be followed:**
  - Find the flip-flop input equations in terms of present state and inputs.
  - List the binary values of each input equation.
  - Use the T flip-flop characteristic table to determine the next states in the state table.
  - Use K-map to obtain minimized state equations

OR

  - Find the flip-flop input equations in terms of present state and inputs.
  - Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.
  - Use the corresponding state equations to determine the next-values in the state table and state diagrams

Analysis with T flip-flops

Example:



Flip-flop input equations:

$$T_A =$$

$$T_B =$$

Output equation:

$$Y = AB$$

Analysis with T flip-flops

Present state		Input	Next state		Flip-Flop inputs	
A	B	X	A	B	T_A	T_B
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Analysis with T flip-flops

For previous example:

T_A= K_A=$B\overline{X}$

T_B=

Characteristic equation for T flip-flop:

$$A(t+1) = \overline{T_A \oplus A}$$

$$B(t+1) = \overline{T_B \oplus B}$$

Analysis with T flip-flops

State Table:

Present state			Next State				Output state	
			X=0		X=1		X=0	X=1
A	B		A	B	A	B	Y	Y
0	0							
0	1							
1	0							
1	1							

State Diagram: