Lecture 4: Kirchhoff's Laws

ELEC1111 Electrical and Telecommunications Engineering

UNSW AUSTRALIA

Never Stand Still

Faculty of Engineering

School of Electrical Engineering and Telecommunications

CONTENTS

1. Introduction

Why?

- ✓ To determine voltages and currents at any point in the circuit.

How?

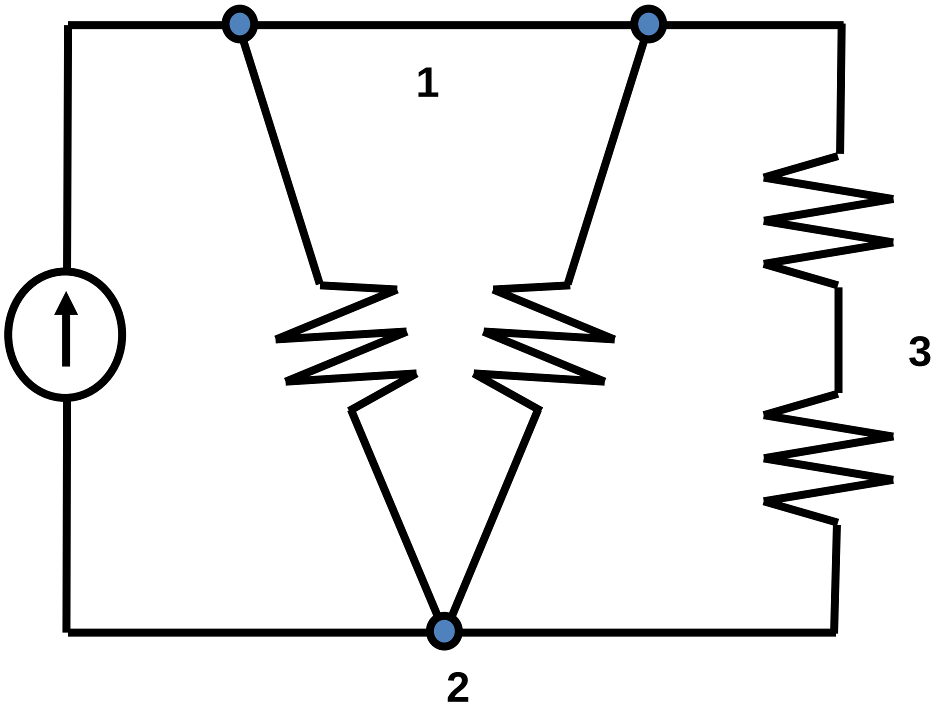
- ✓ To simplify the circuit as much as possible, e.g. series and parallel resistors, voltage division, sources in series & parallel, etc.

- ✓ Element laws, such as Ohm’s law, relate terminal voltages and currents of individual elements.

- ✓ Circuit laws (Kirchhoff’s laws) relate the voltages and currents shared at the interconnections of elements

2. Nodes, Branches, Loops, Meshes

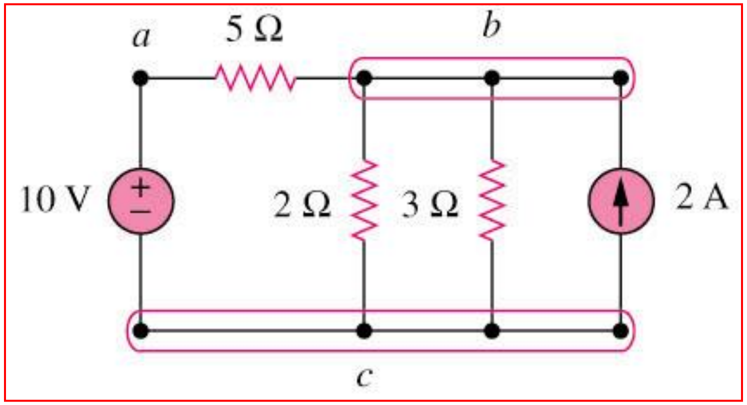
- ✓ A branch represents a single element such as a voltage source or a resistor. Wire connectors are assumed to have zero resistance.
- ✓ A node is the point of connection between two or more branches.
- ✓ All leads attached to a node are considered as part of that node.
- ✓ In moving from one node to another, if no node was encountered more than once, then the set of node and elements that we passed through is defined as a path.
- ✓ A loop is any closed path in a circuit.
- ✓ A mesh is a loop that contains no other loop.



A circuit containing 3 nodes and 5 branches

2. Nodes, Branches, Loops, Meshes

Example

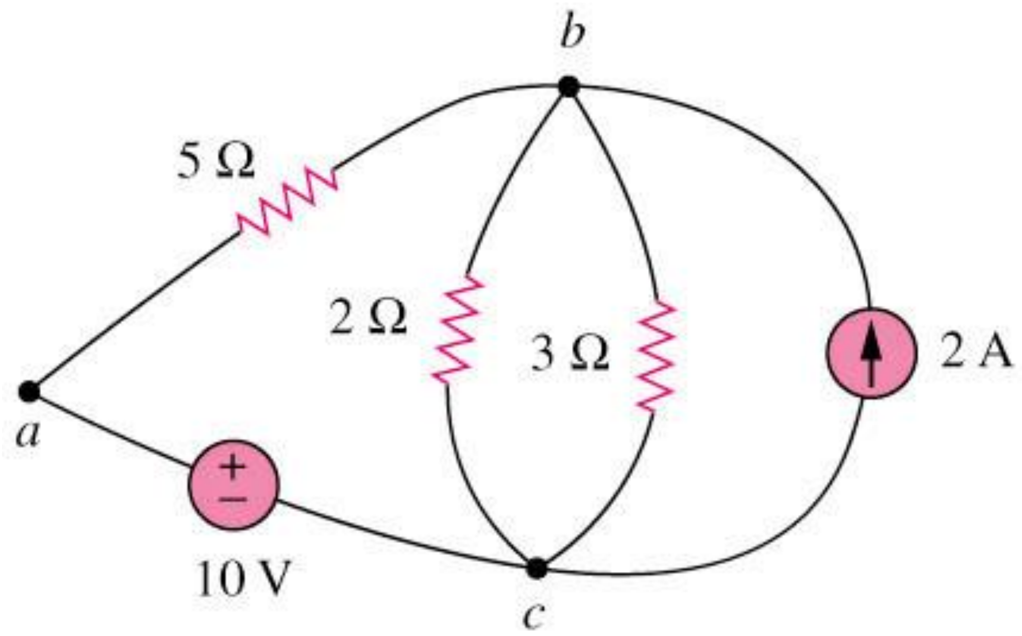


Original circuit

Nodes =

Branches =

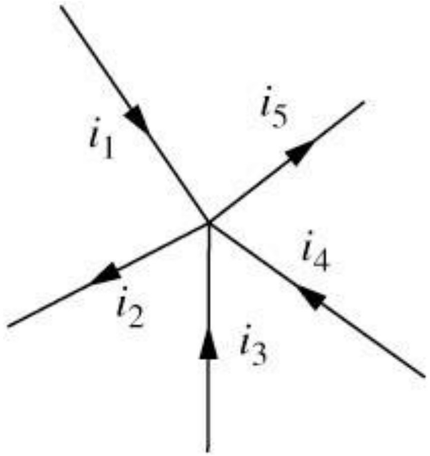
Meshes =

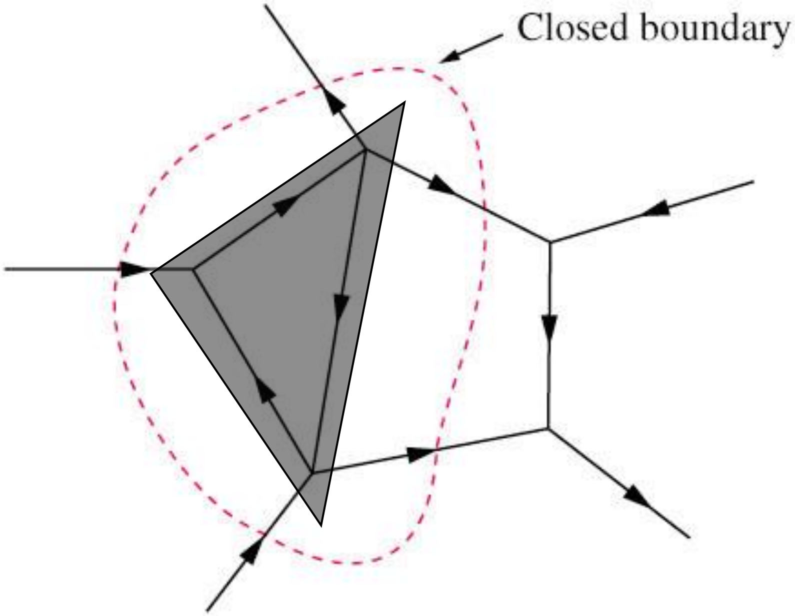


Equivalent circuit

3. Kirchhoff's Current Law (KCL)

✓ **KCL**: *The algebraic sum of currents into a node (or closed boundary) at any instant is zero.* (That is, charge cannot accumulate at a node)





$$i_1 - i_2 + i_3 + i_4 - i_5 = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

✓ **KCL** (restatement): *At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node or equivalently*

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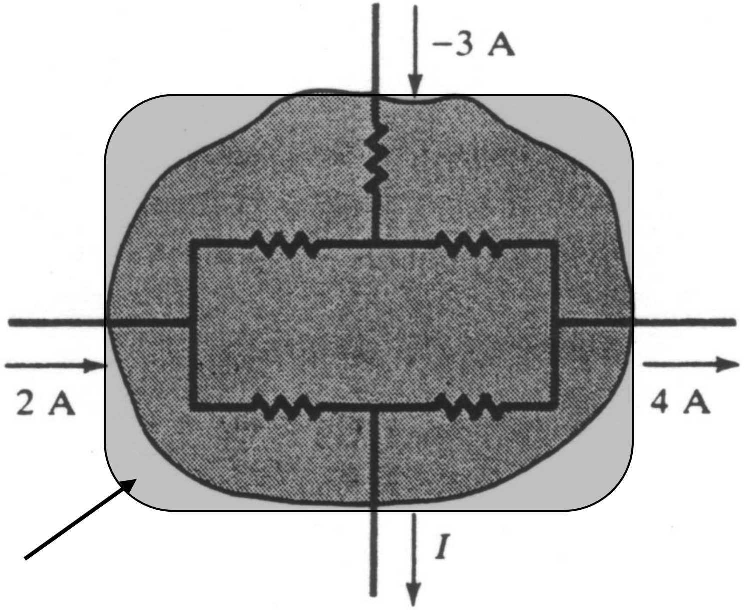
3. Kirchhoff’s Current Law (KCL)

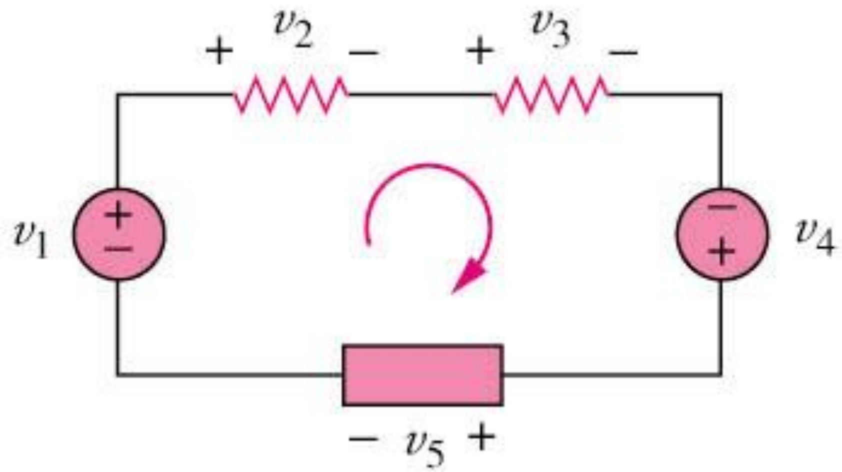
Example



We can consider the whole enclosed area as one “node”.

4. Kirchhoff's Voltage Law (KVL)

- ✓ **KVL**: *The algebraic sum of voltages around any closed path is zero.*
- ✓ **One method**: Move around the closed path in a clockwise direction and write down the voltage of each element using the sign on the first (or second) terminal encountered.



$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_1 + v_4 = v_2 + v_3 + v_5$$

- ✓ Total amount of energy gained per unit charge are equal to the amount of energy lost per unit charge (energy and charge are both conserved)
- ✓ Key – first label all the voltages and currents on the diagram

4. Kirchhoff’s Voltage Law (KVL)

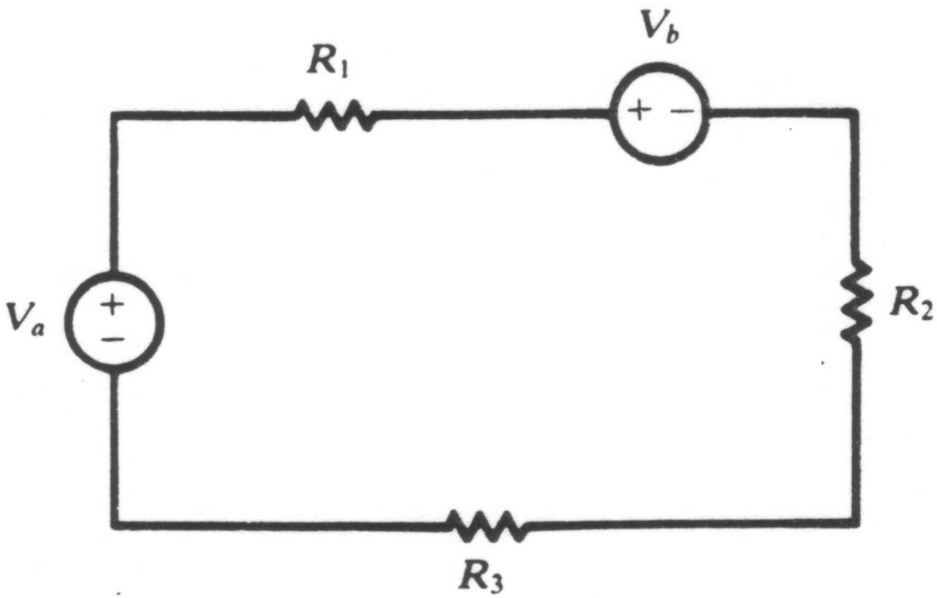
The Single Loop Circuit

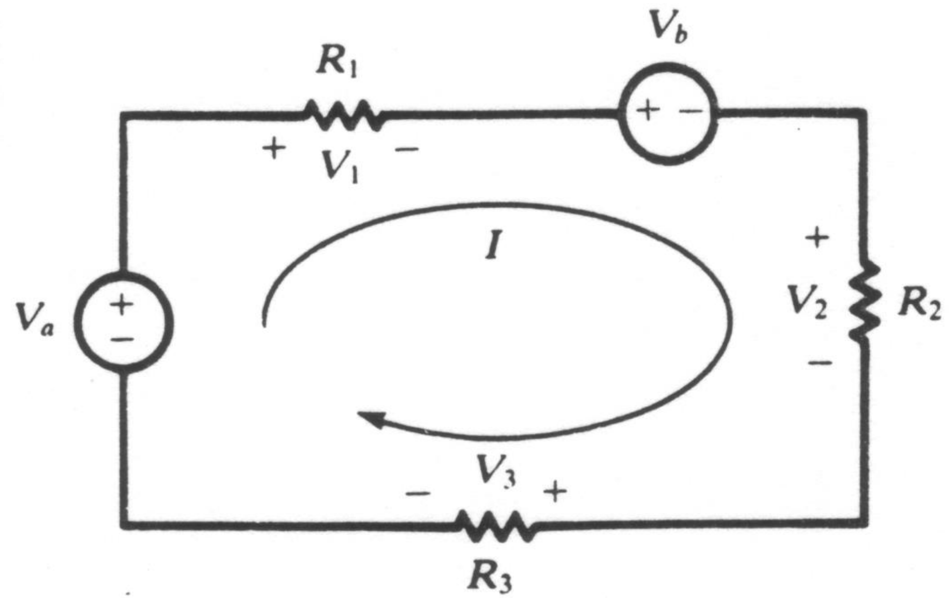
- ✓ **Circuit:** – circuits elements connected in series to form a single loop
- ✓ **What**:- determine the current, voltage (and power) associated with each element.
- ✓ **Steps**:-

1. Assign current direction (e.g. clockwise)
2. Label all voltages (prefer passive convention)
3. Apply KVL
4. Apply Ohm’s law to resistors
5. Solve for current
6. Solve for *v*, *p* for any element

4. Kirchhoff's Voltage Law (KVL)

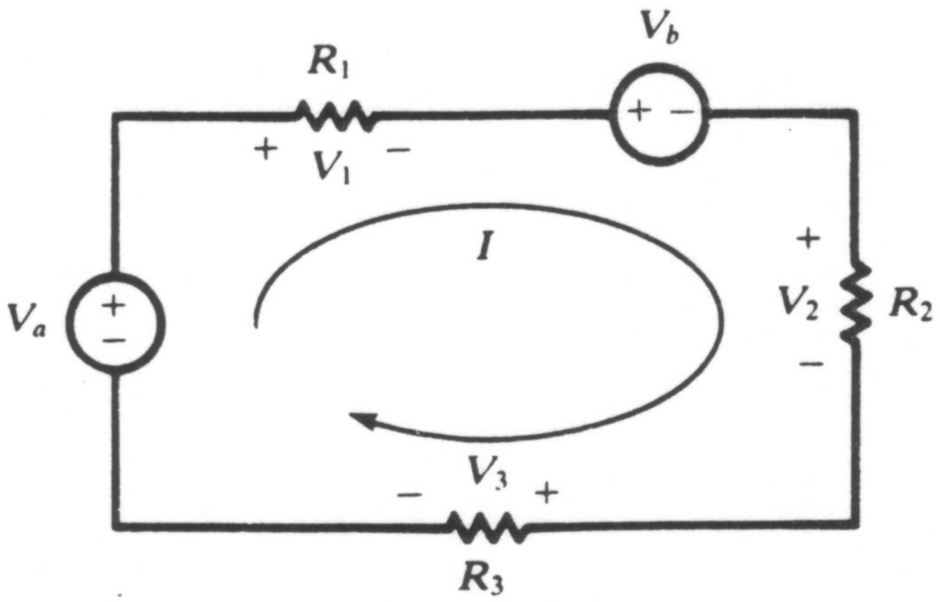
Example





4. Kirchhoff's Voltage Law (KVL)

Example



$$-V_a + V_1 + V_b + V_2 + V_3 = 0$$

$$V_1 = IR_1\,;\ V_2 = IR_2\,;\ V_3 = IR_3$$

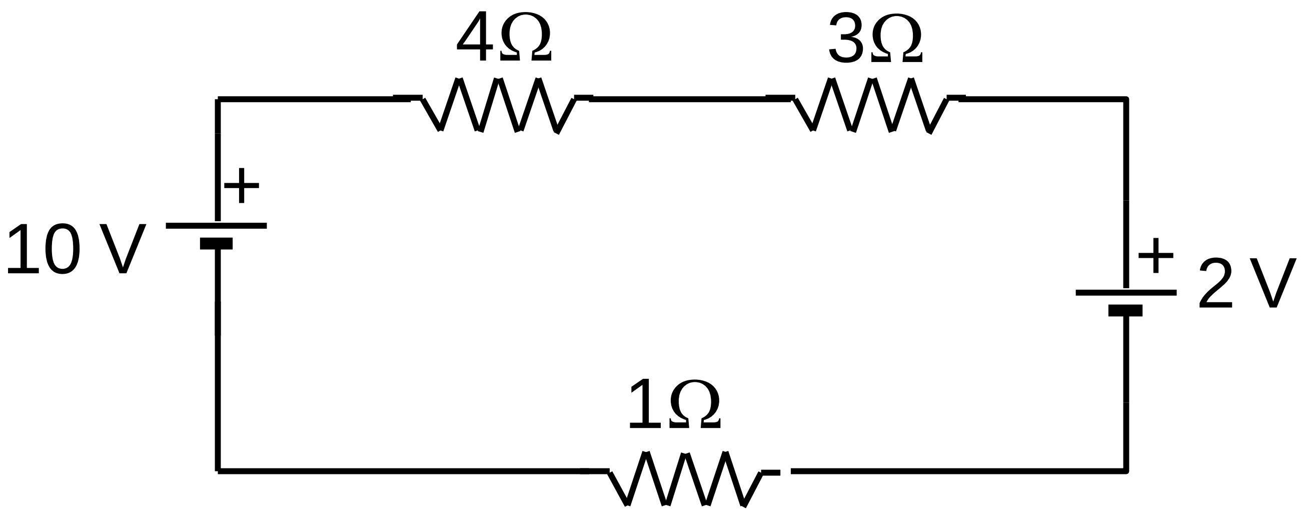
$$V_a - V_b = I(R_1 + R_2 + R_3)$$

$$I = \frac{V_a - V_b}{R_1 + R_2 + R_3}$$

Voltage & Power?

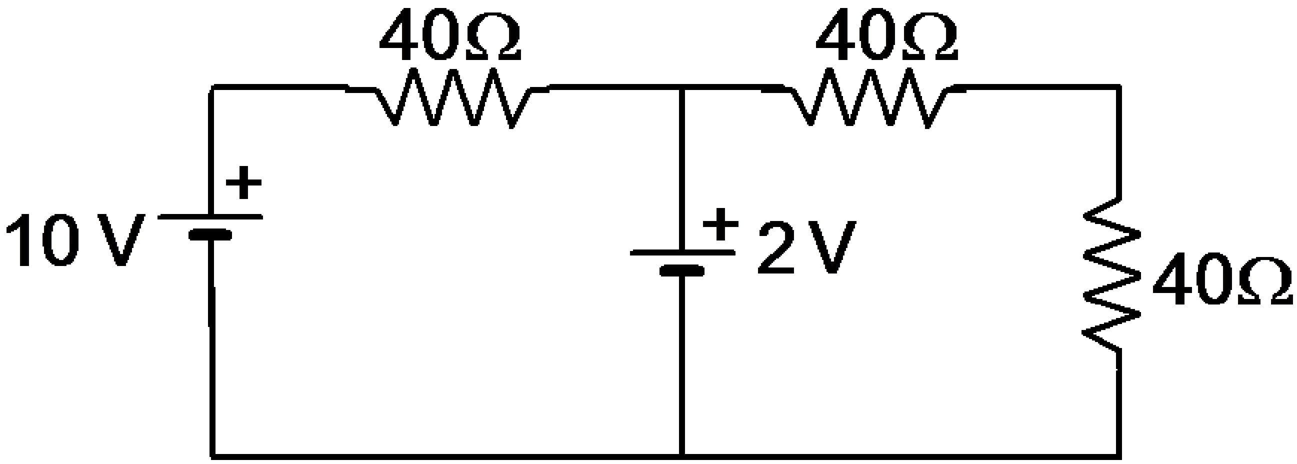
5. KVL & KCL Examples

Example 1



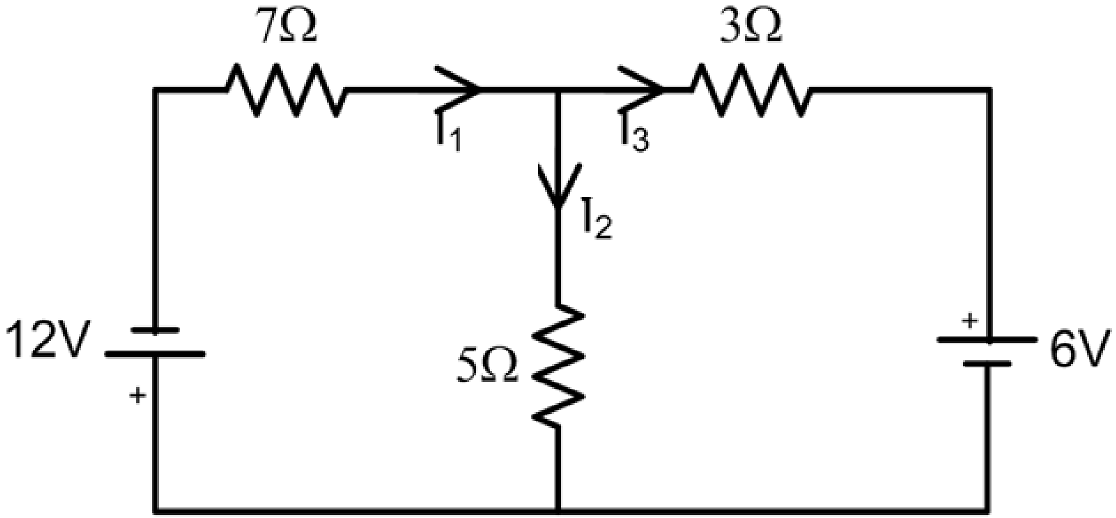
5. KVL & KCL Examples

Example 2



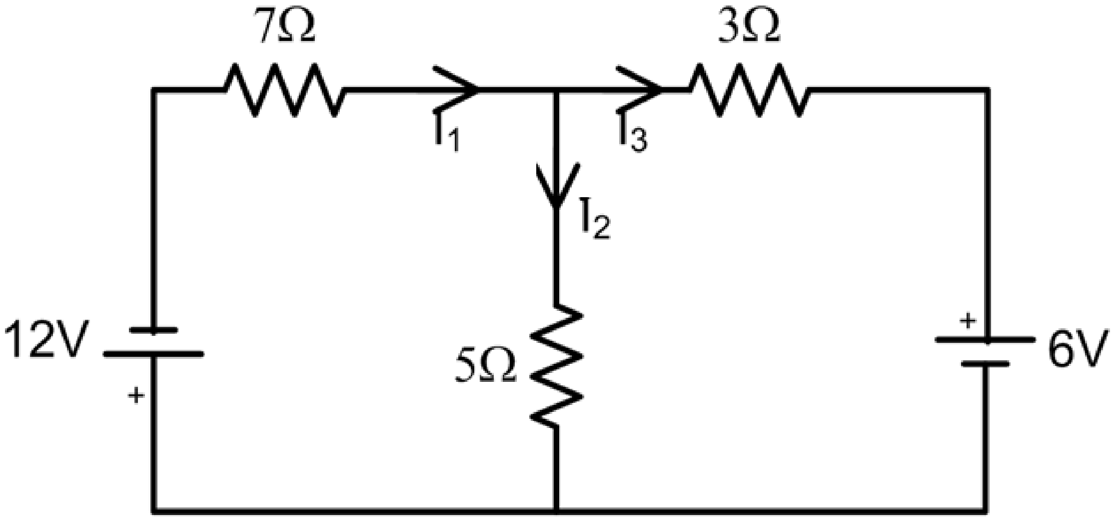
5. KVL & KCL Examples

Example 3



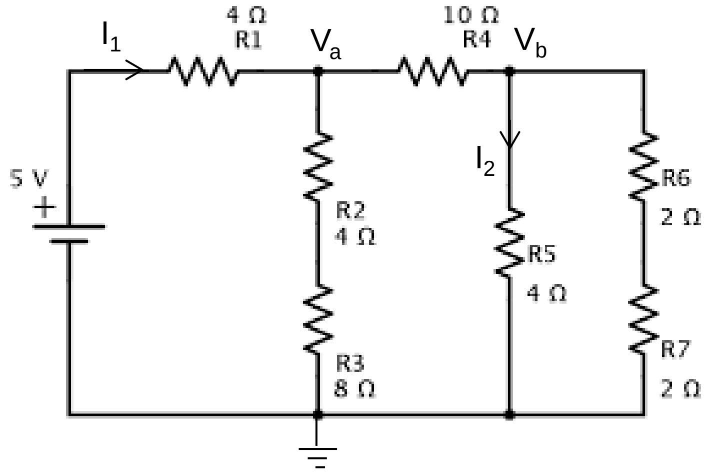
5. KVL & KCL Examples

Example 3



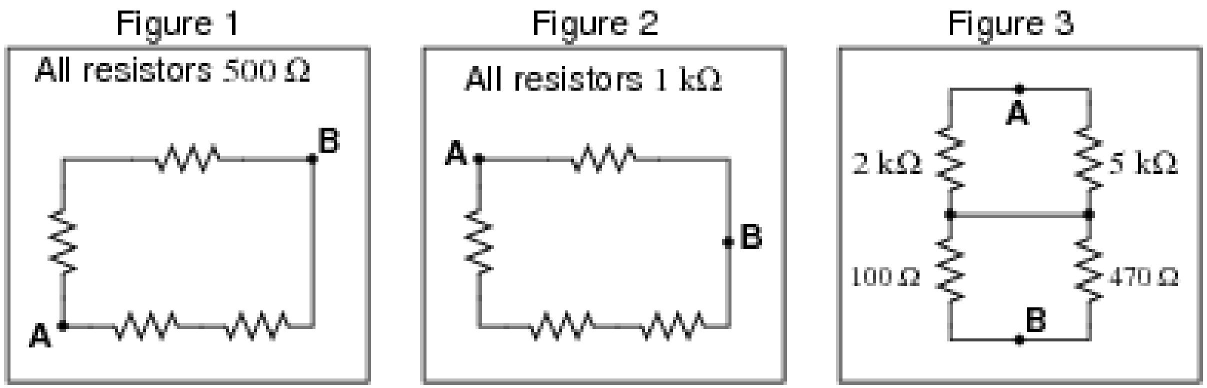
5. KVL & KCL Examples

Example 4: In the following circuit, find the value of I_1, I_2, V_a, V_b and validate the power conservation condition of the circuit.



5. KVL & KCL Examples

Example 4: In the following circuits, find the resistance R_{AB} between A and B.



5. KVL & KCL Examples

Example 4 (cont.): In the following circuits, find the resistance R_{AB} between A and B.

Figure 4

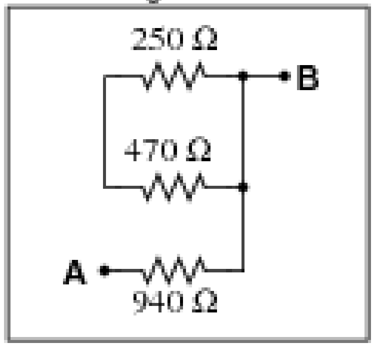


Figure 5

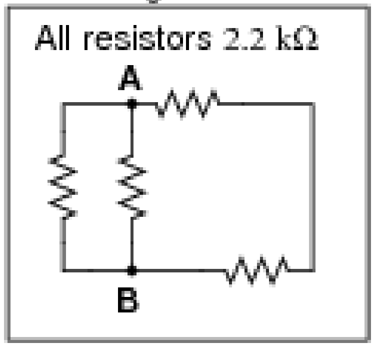
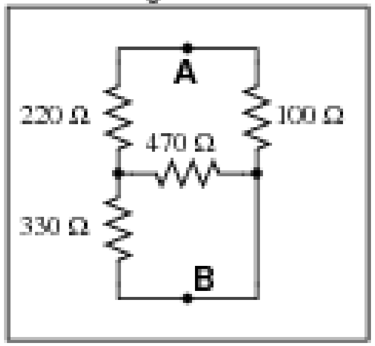
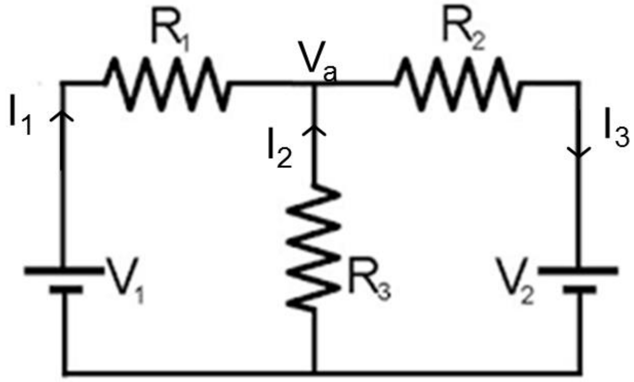


Figure 6



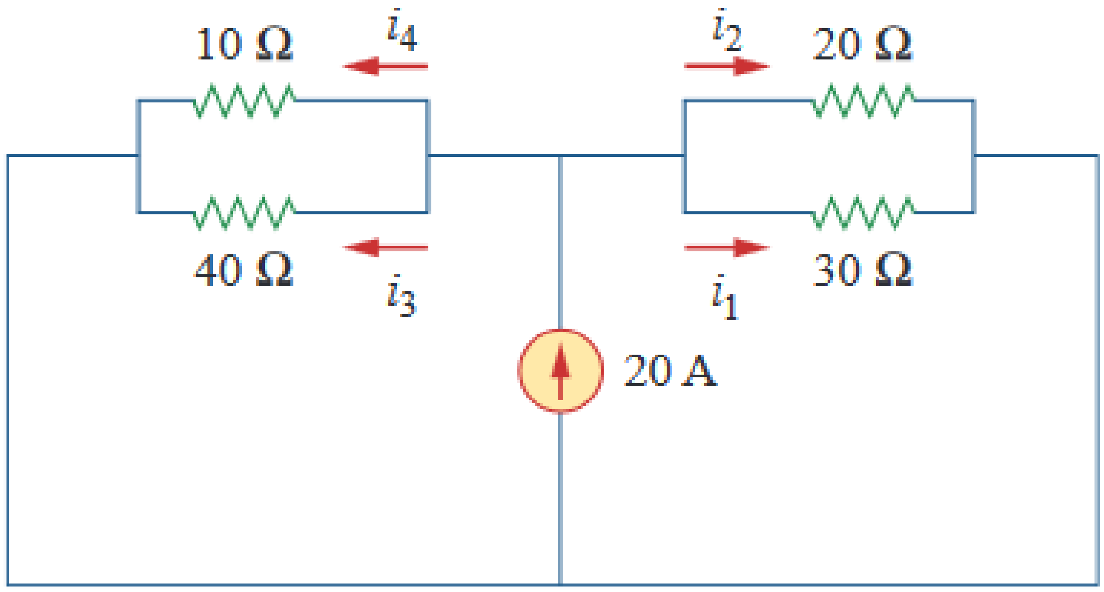
5. KVL & KCL Examples

Example 5: In the following circuit, V_1=10V, V_2=5V, R_1=2Ω, R_2=R_3=5Ω, find currents I_1, I_2, I_3 and voltage V_a. Validate the power conservation condition.



5. KVL & KCL Examples

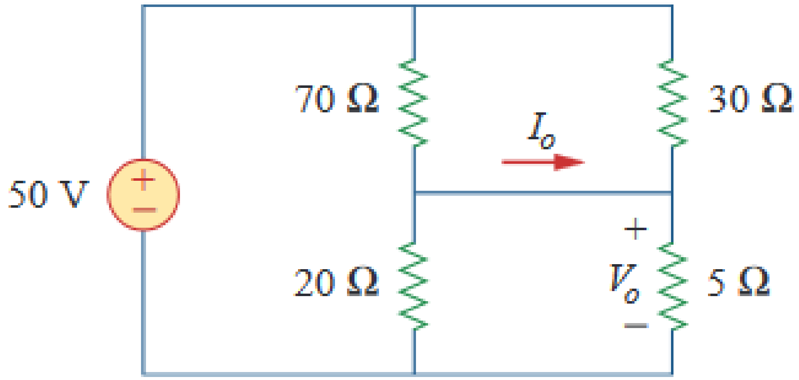
Example 6: Find the current i_1, i_2, i_3, i_4 in the following circuit



From "*Fundamentals of Electric Circuits*", Alexander and Sadiku, McGraw-Hill

5. KVL & KCL Examples

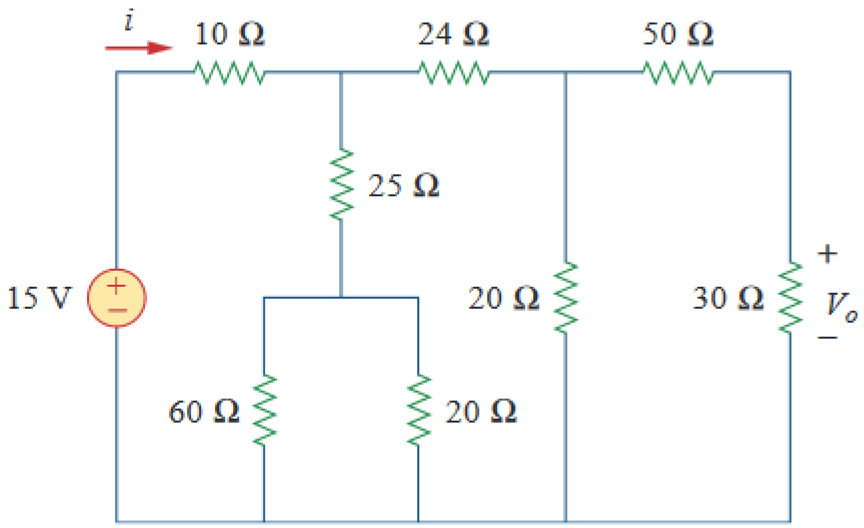
Example 7: Calculate V_o and I_o in the below circuit



From “*Fundamentals of Electric Circuits*”, Alexander and Sadiku, McGraw-Hill

5. KVL & KCL Examples

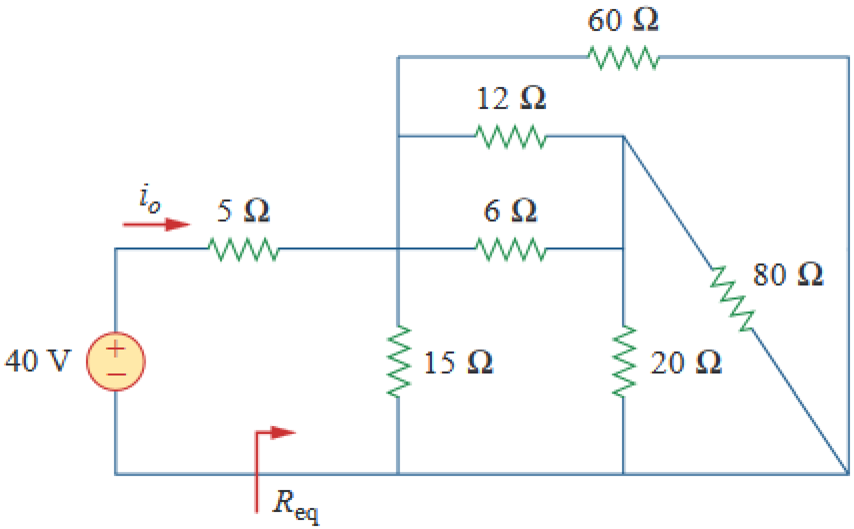
Example 8: Calculate i and V_o in the below circuit



From "*Fundamentals of Electric Circuits*", Alexander and Sadiku, McGraw-Hill

5. KVL & KCL Examples

Example 8: Find R_{eq} and i_o in the following circuit



From "*Fundamentals of Electric Circuits*", Alexander and Sadiku, McGraw-Hill