Lecture 12: Op-Amps – Part II

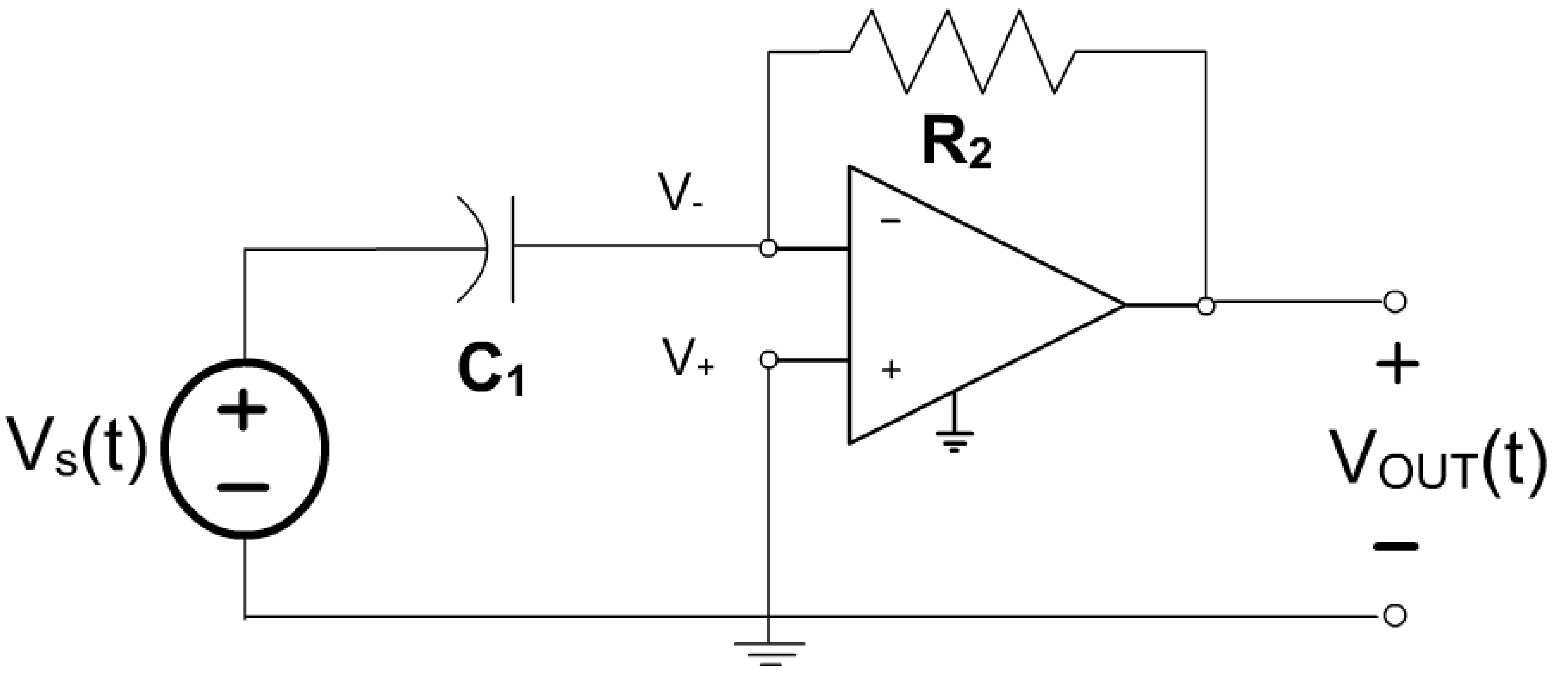
ELEC1111 Electrical and Telecommunications Engineering

UNSW AUSTRALIA

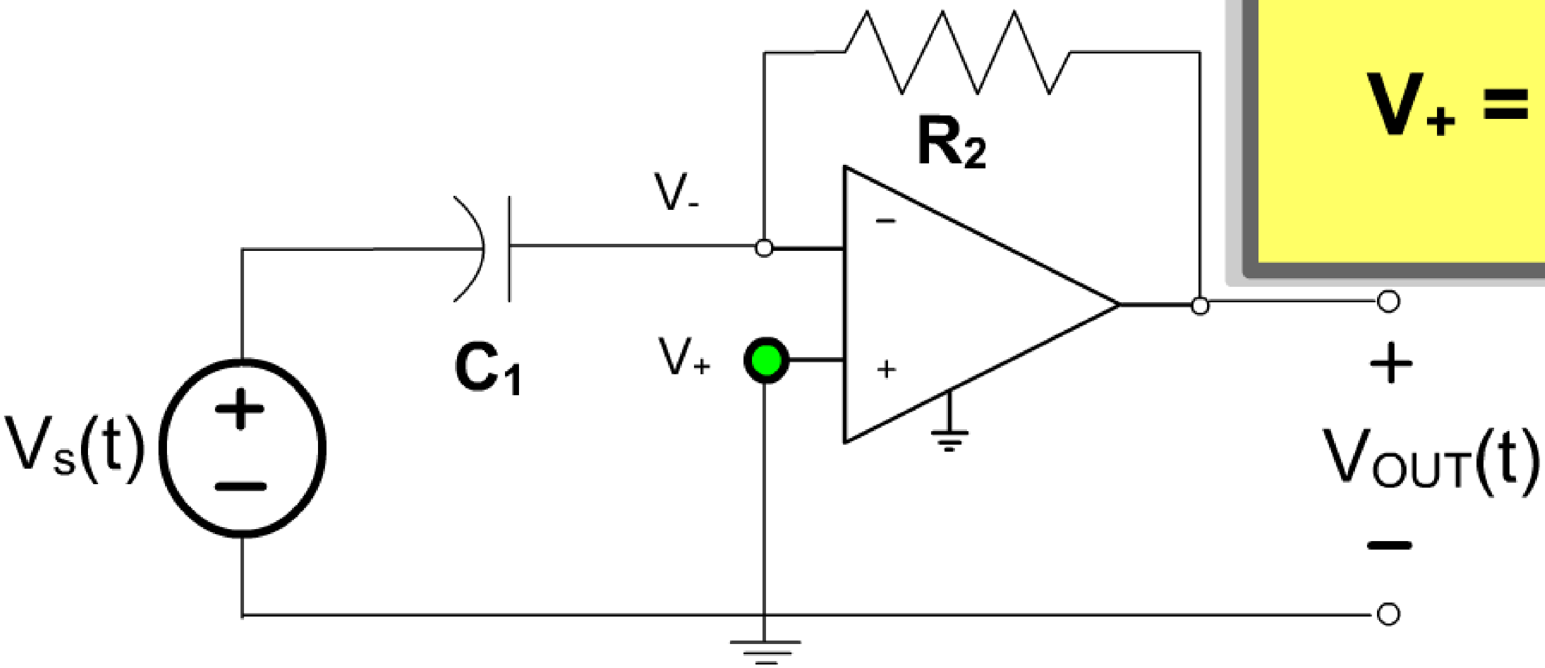
Never Stand Still | Faculty of Engineering | School of Electrical Engineering and Telecommunications

Op-Amp Differentiator

Operational amplifiers can be combined with reactive elements in such a manner that the output voltage is the time derivative of the input voltage.

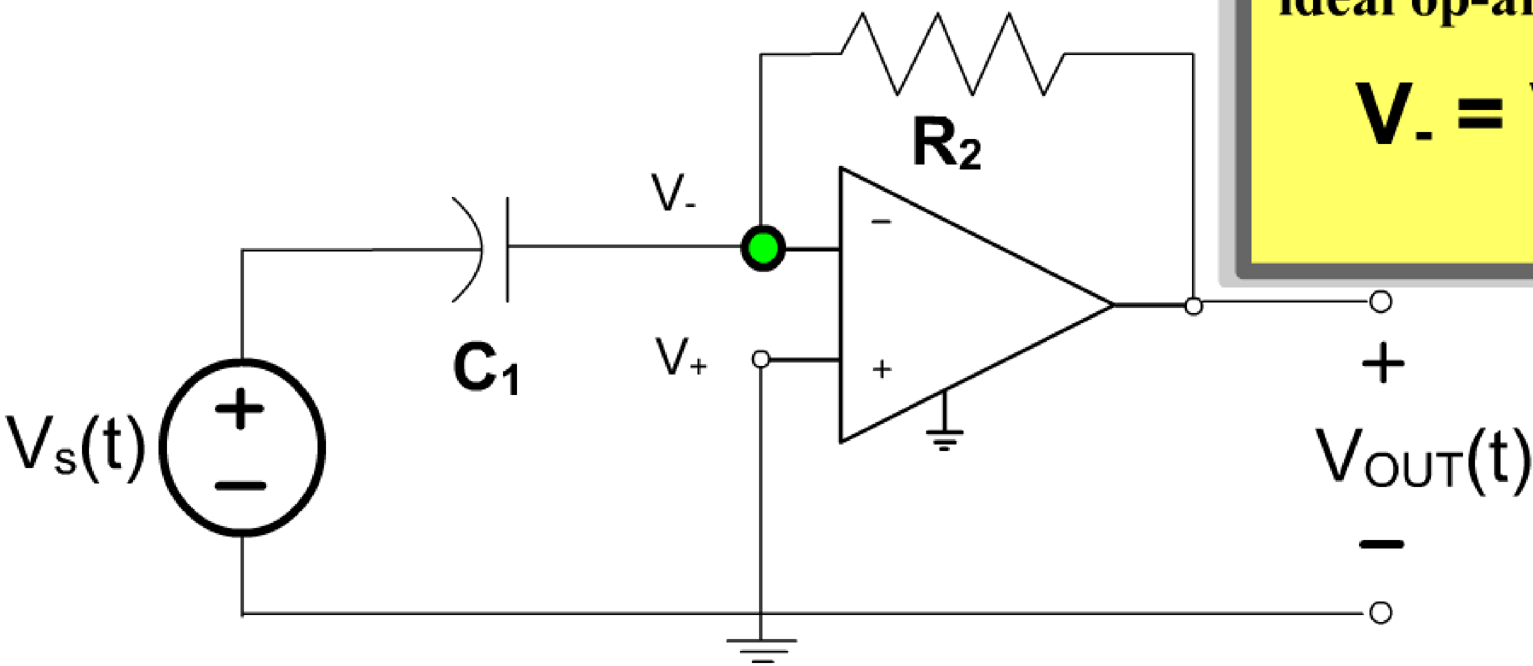


Since V_- is grounded, the node has a potential of zero volts.



$$V_+ = V_g = 0$$

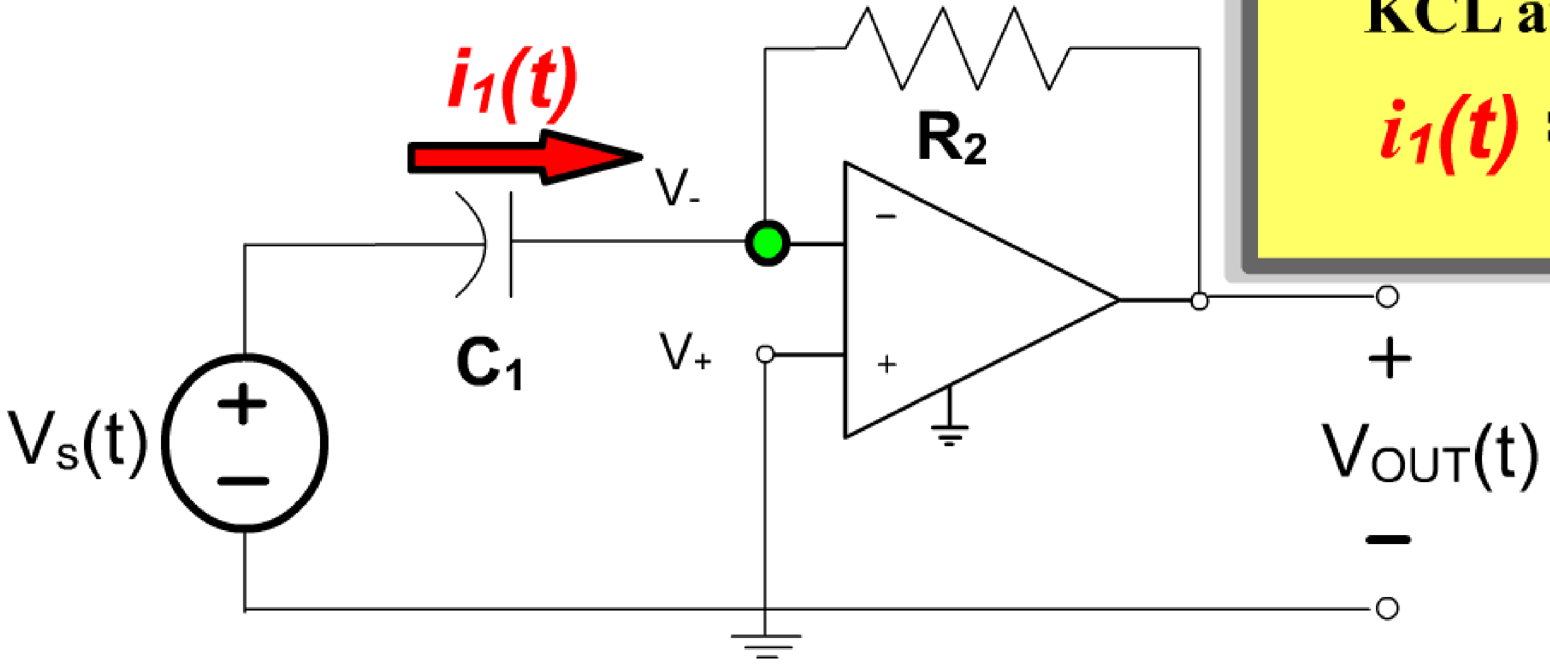
From the first ideal op-amp relation, $V_- = V_+$.



ideal op-amp equation

$$V_- = V_+ = 0$$

Now write KCL at node V_-. The current across C_1 will be denoted as $i_1(t)$.

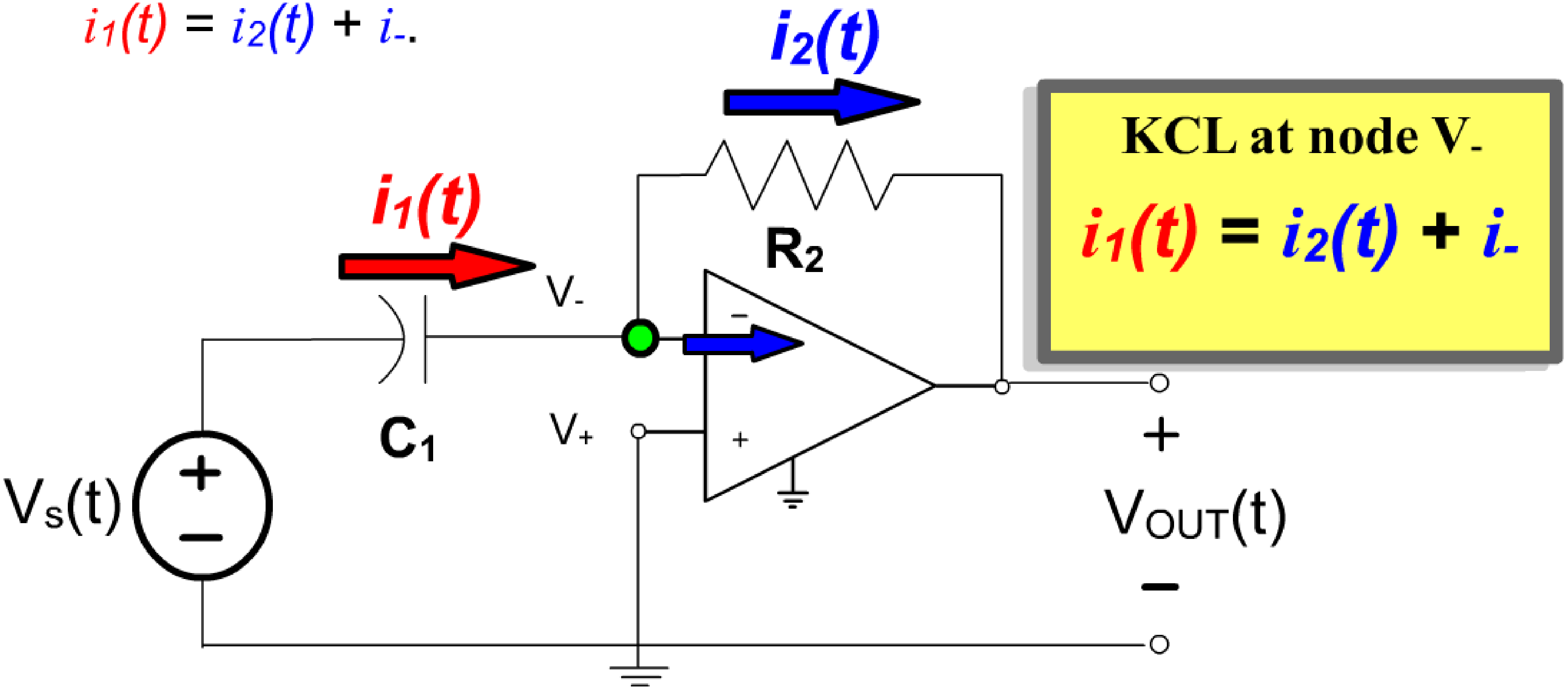


KCL at node V_-

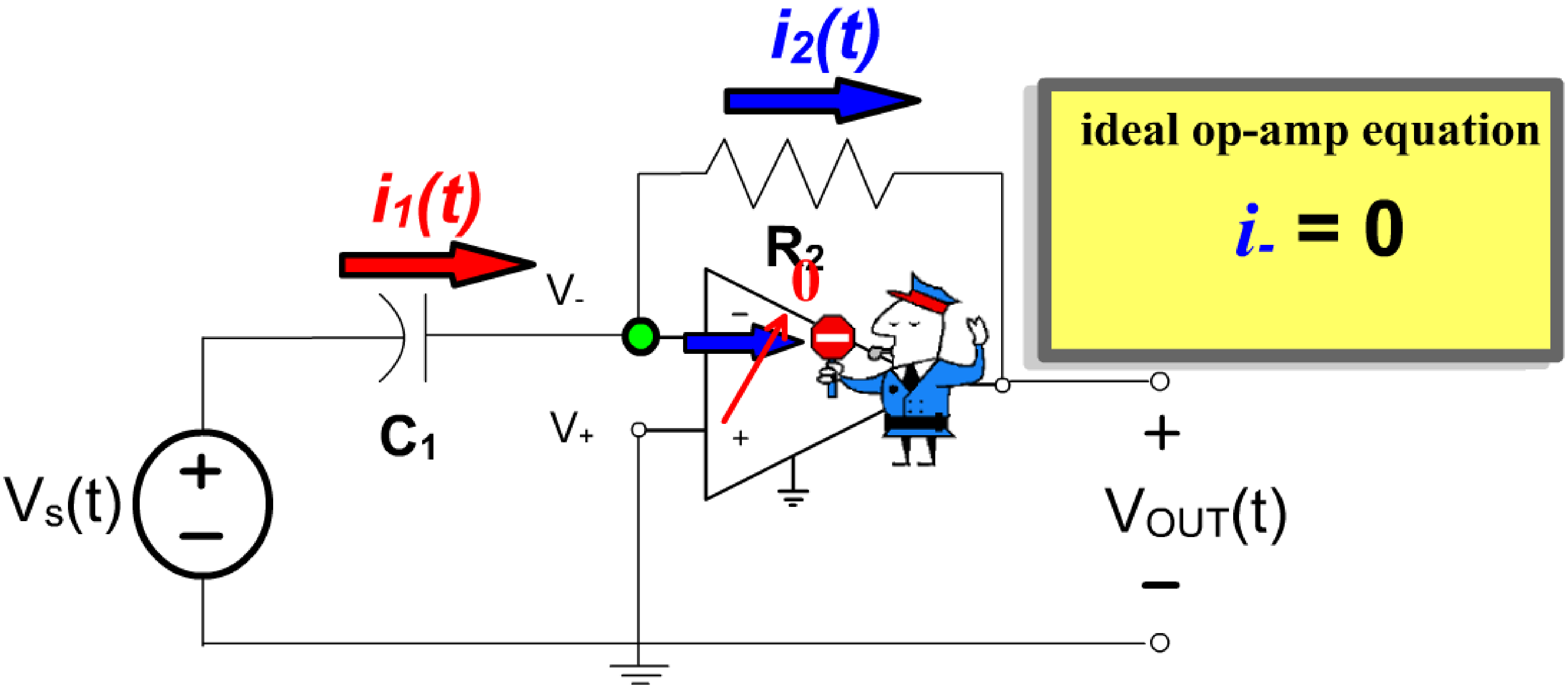
$i_1(t) = \ldots$

The other two currents will be labeled as $i_2(t)$ and i_-, which is denoted as leaving the node V-. The KCL equation is

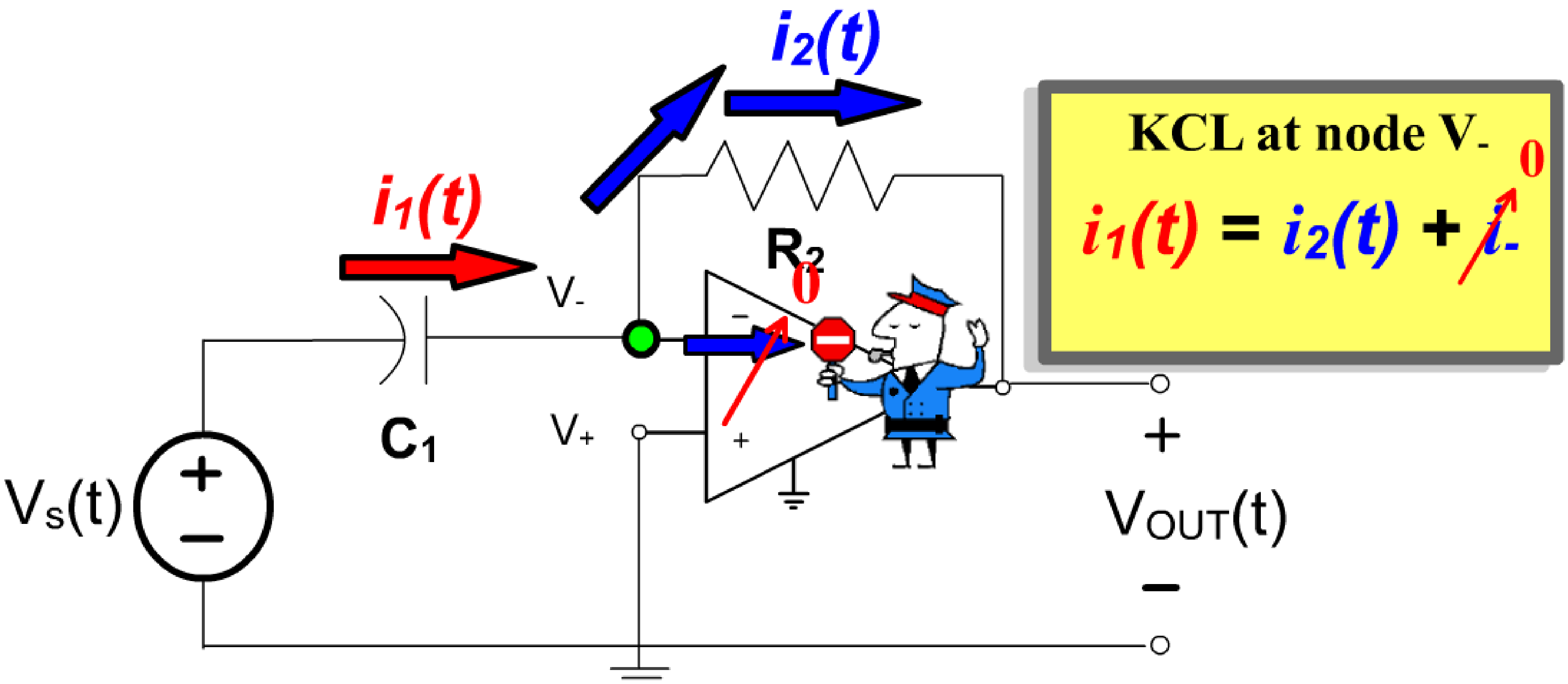
$i_1(t) = i_2(t) + i_-$.



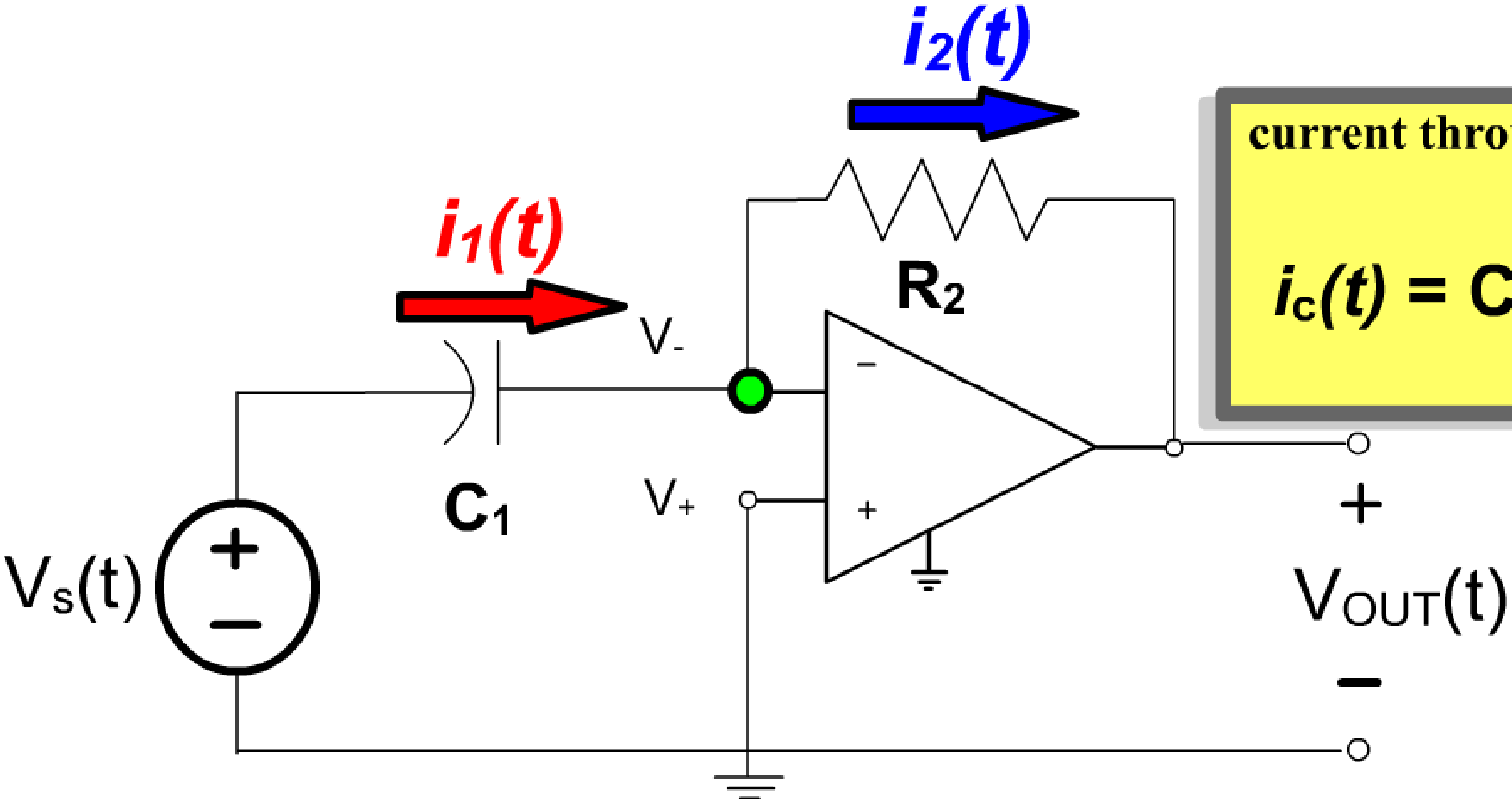
The second ideal op-amp equation requires that the i_- = 0. No current enters this terminal.



The KCL equation can be simplified to $i_1(t) = i_2(t)$.



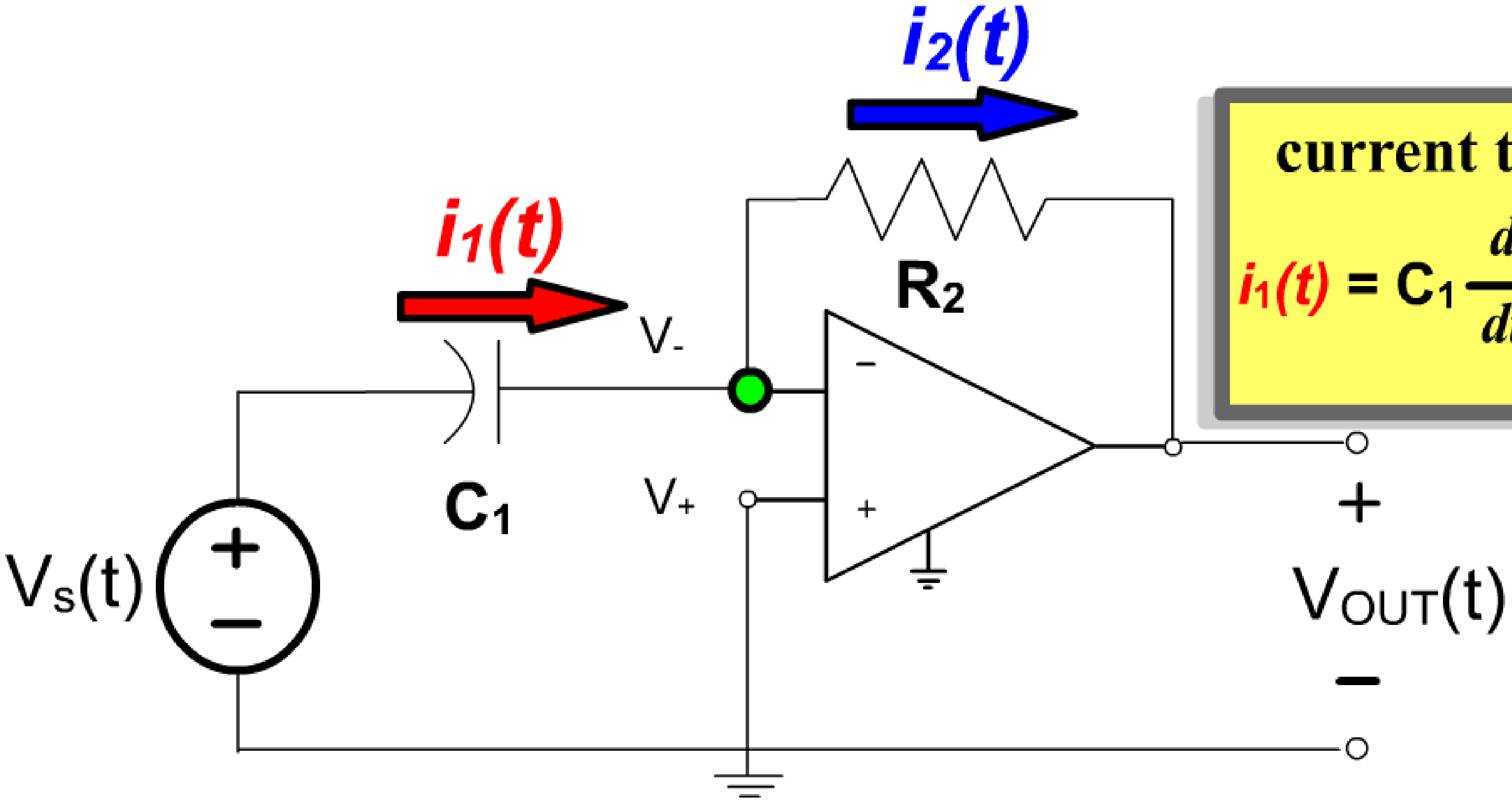
To solve for $i_1(t)$, recall that the current through a capacitor is proportional to the time derivative of the capacitor voltage.



current through capacitor

$$i_c(t) = C \frac{d}{dt} V_c(t)$$

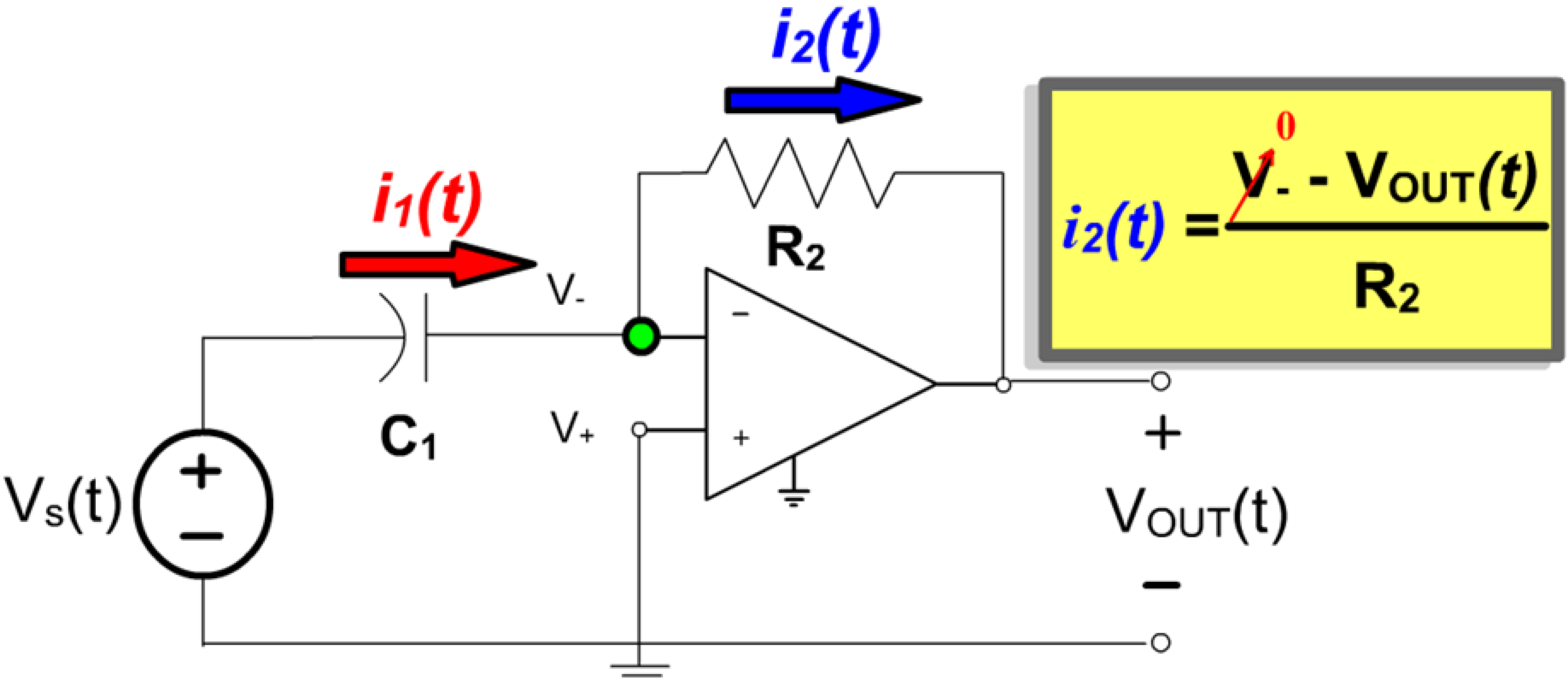
The current $i_1(t)$ through C_1 is the capacitance times the time derivative of the potential difference.



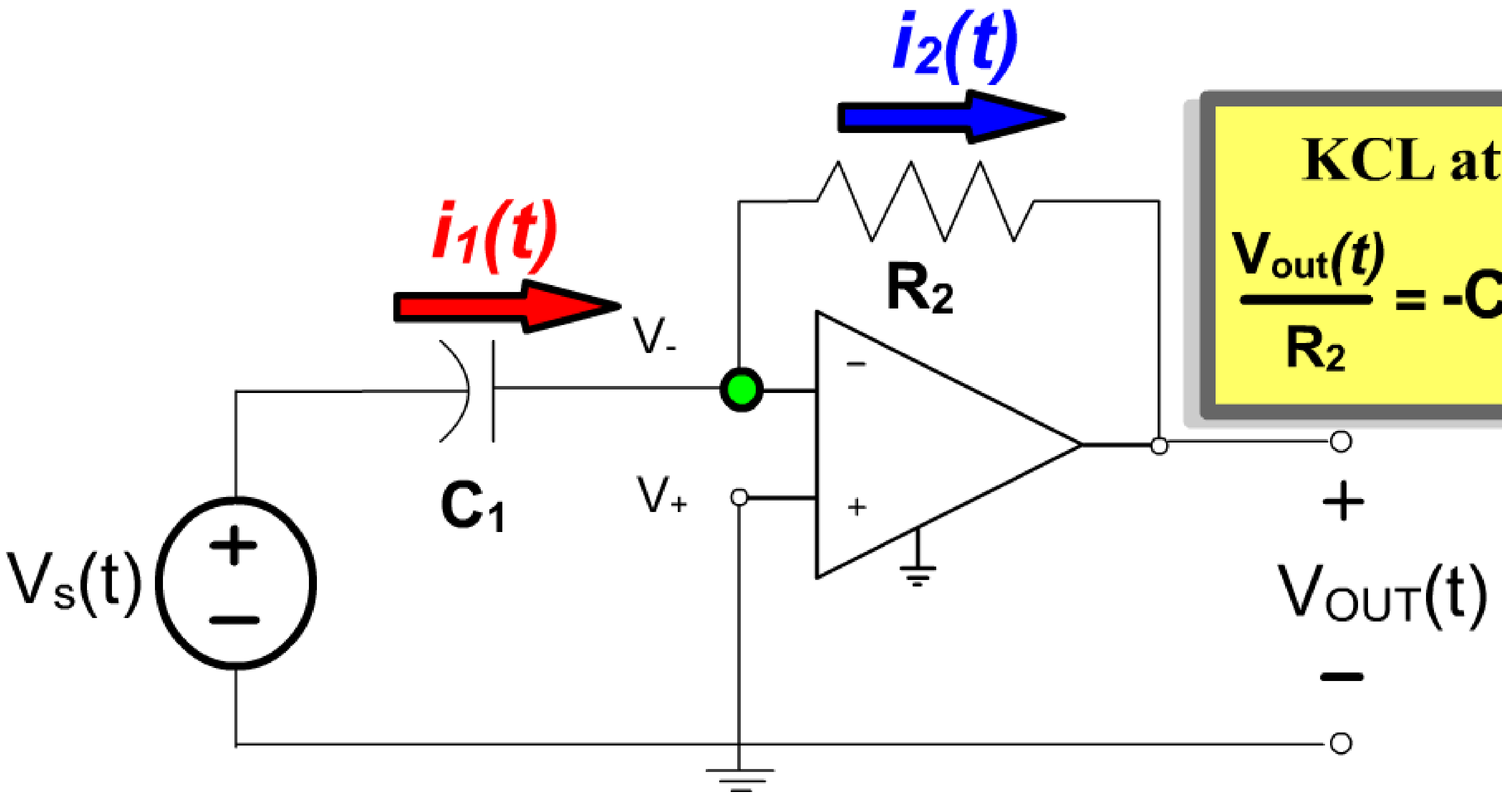
current throuth C_1

$$i_1(t) = C_1 \frac{d}{dt}[V_S(t) - V_-]$$

The current through $i_2(t)$ is found using Ohm's law.



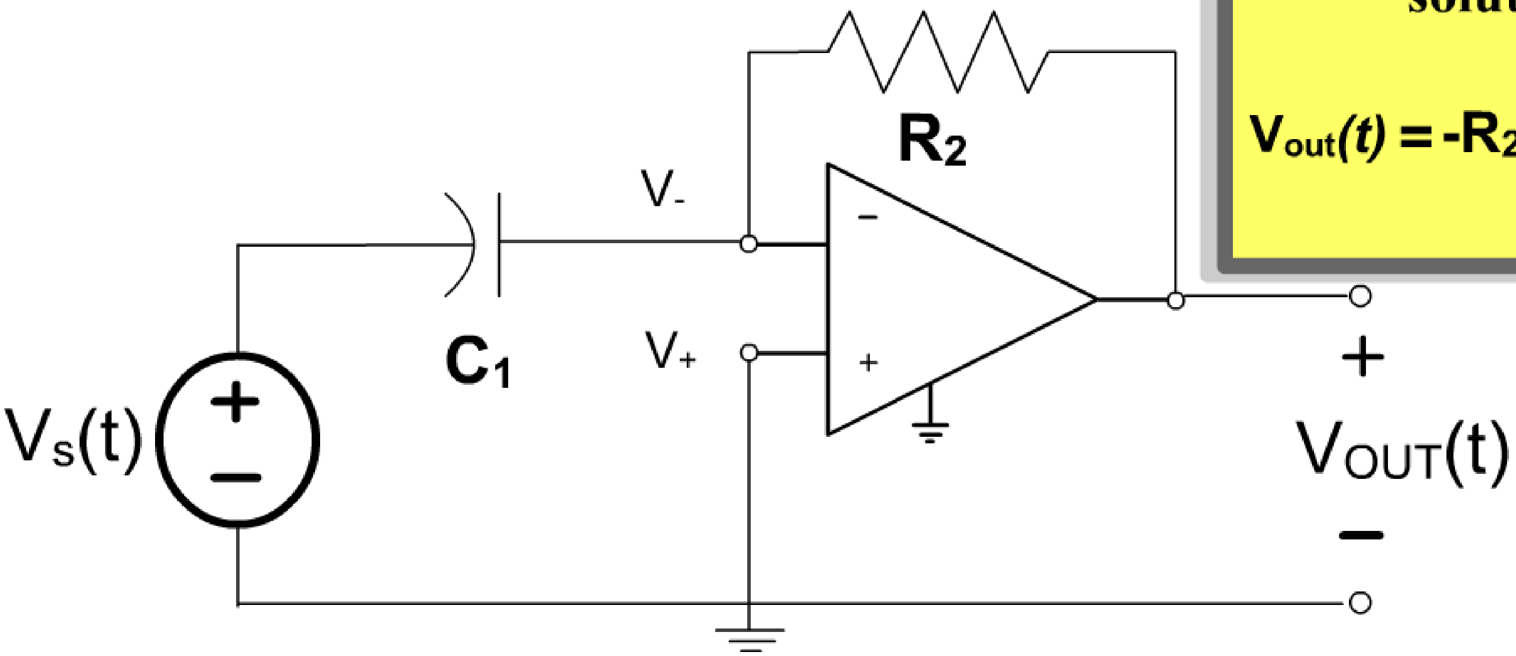
By writing KCL at node V_- we then have



KCL at node V_-

$$\frac{V_{out}(t)}{R_2} = -C_1 \frac{d}{dt} V_S(t)$$

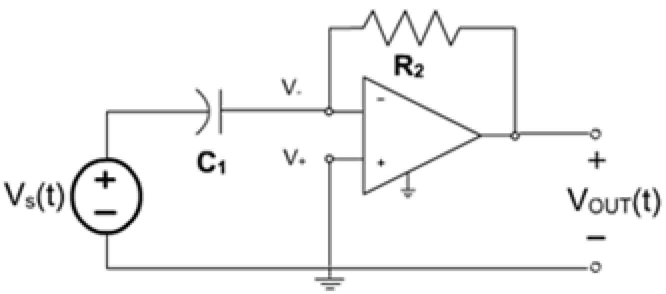
Rearranging the equation yields the solution

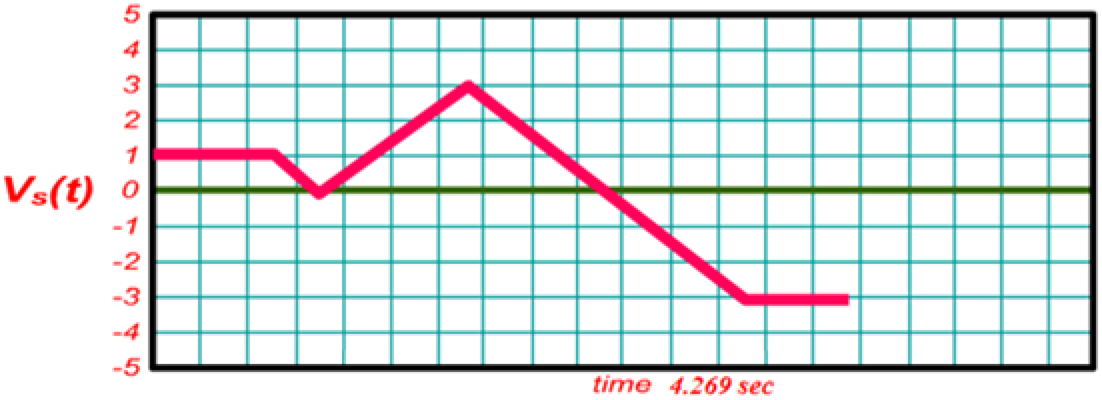


solution:

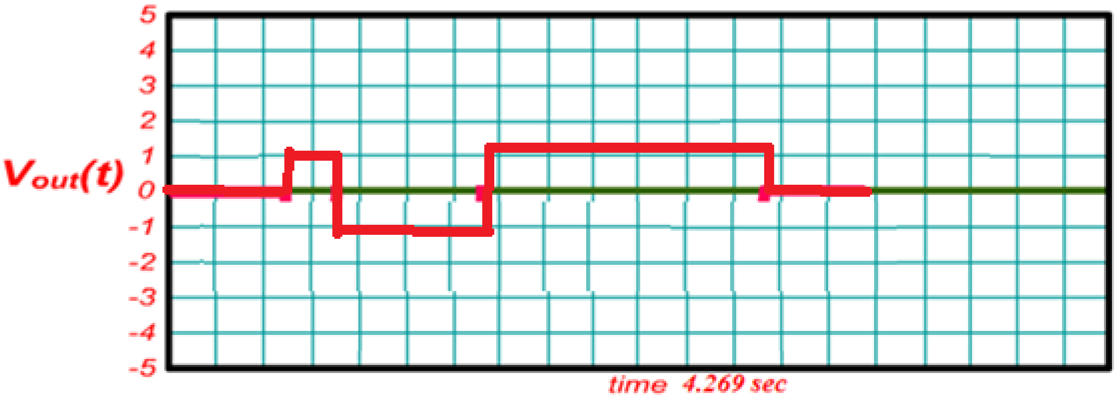
$$V_{out}(t) = -R_2C_1\frac{d}{dt}v_S(t)$$

The graphs below show the differentiation performed by the op-amp circuit for an arbitrary input voltage signal.



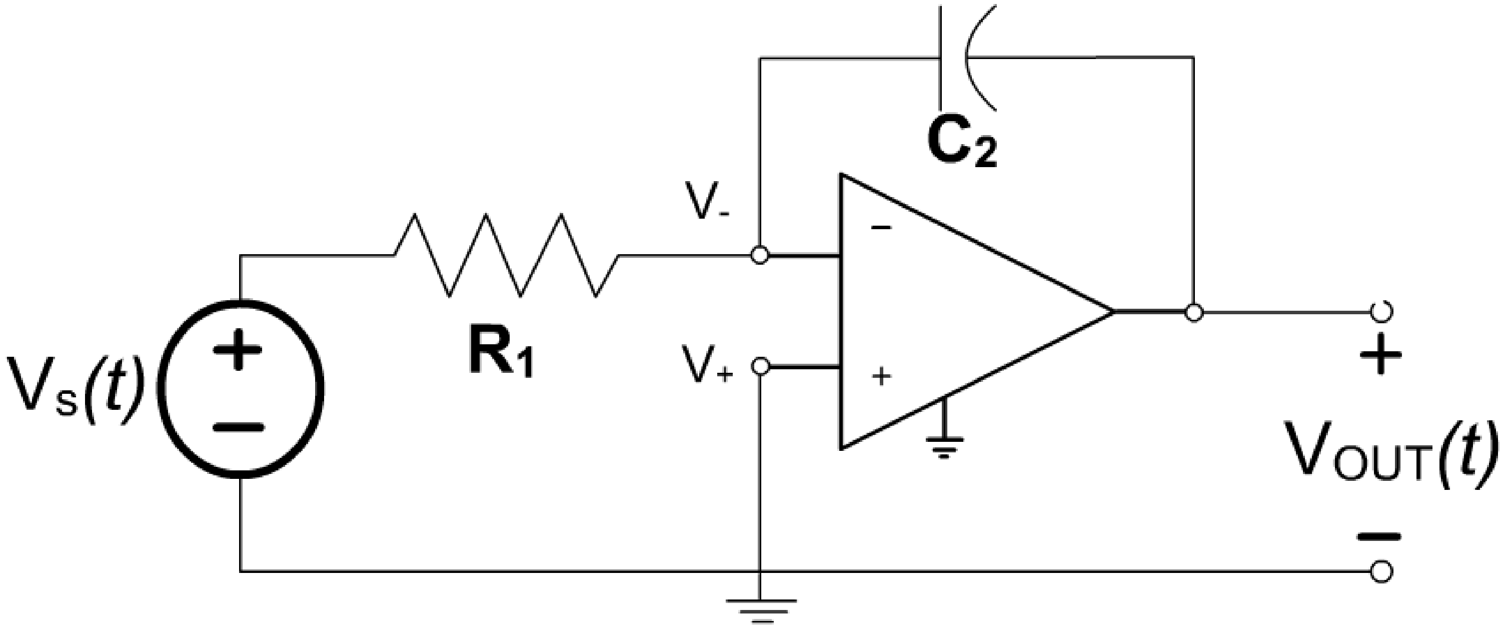


$$V_{out}(t) = -R_2C_1\frac{d}{dt}v_S(t)$$



Op-Amp Integrator

Operational Amplifiers can be combined with reactive elements in such a manner that the output voltage is the time integral of the input voltage.



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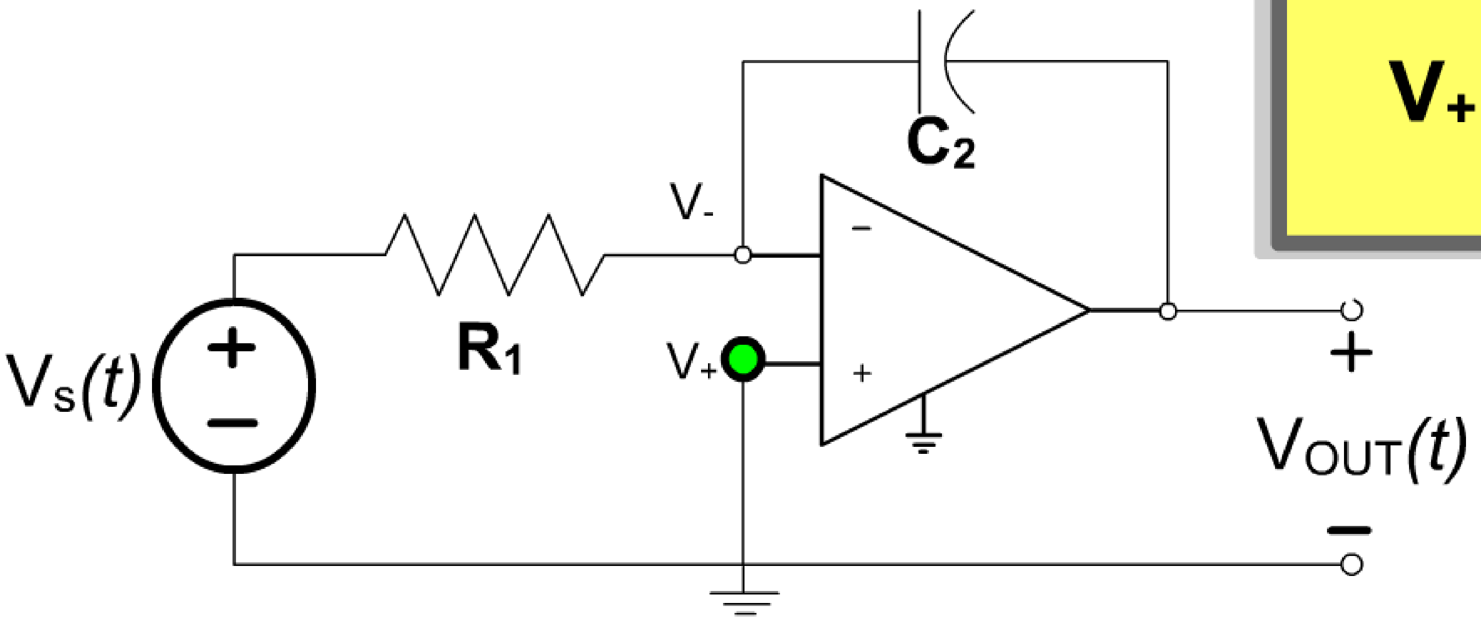
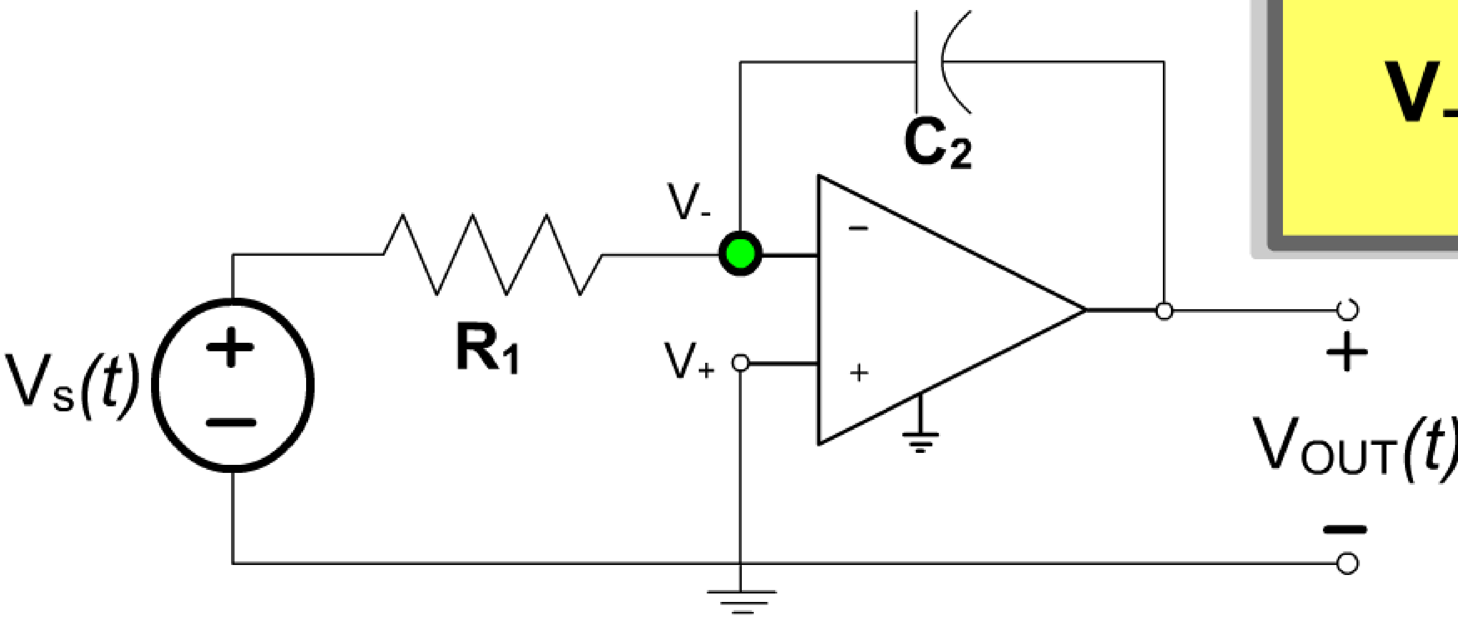
Notice that V_+ is connected to ground, and has a potential of zero volts.



$$V_+ = V_g = 0$$

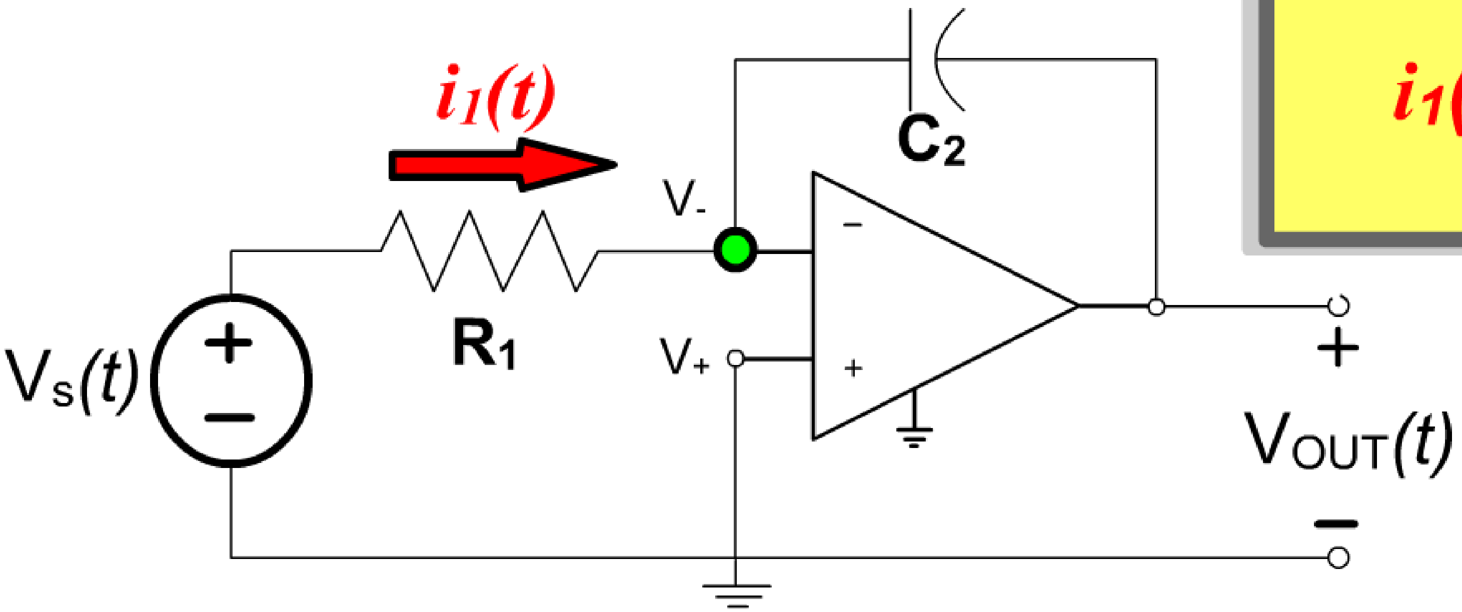
From the first ideal op-amp equation, $V_- = V_+$.



ideal op-amp equation

$$V_- = V_+ = 0$$

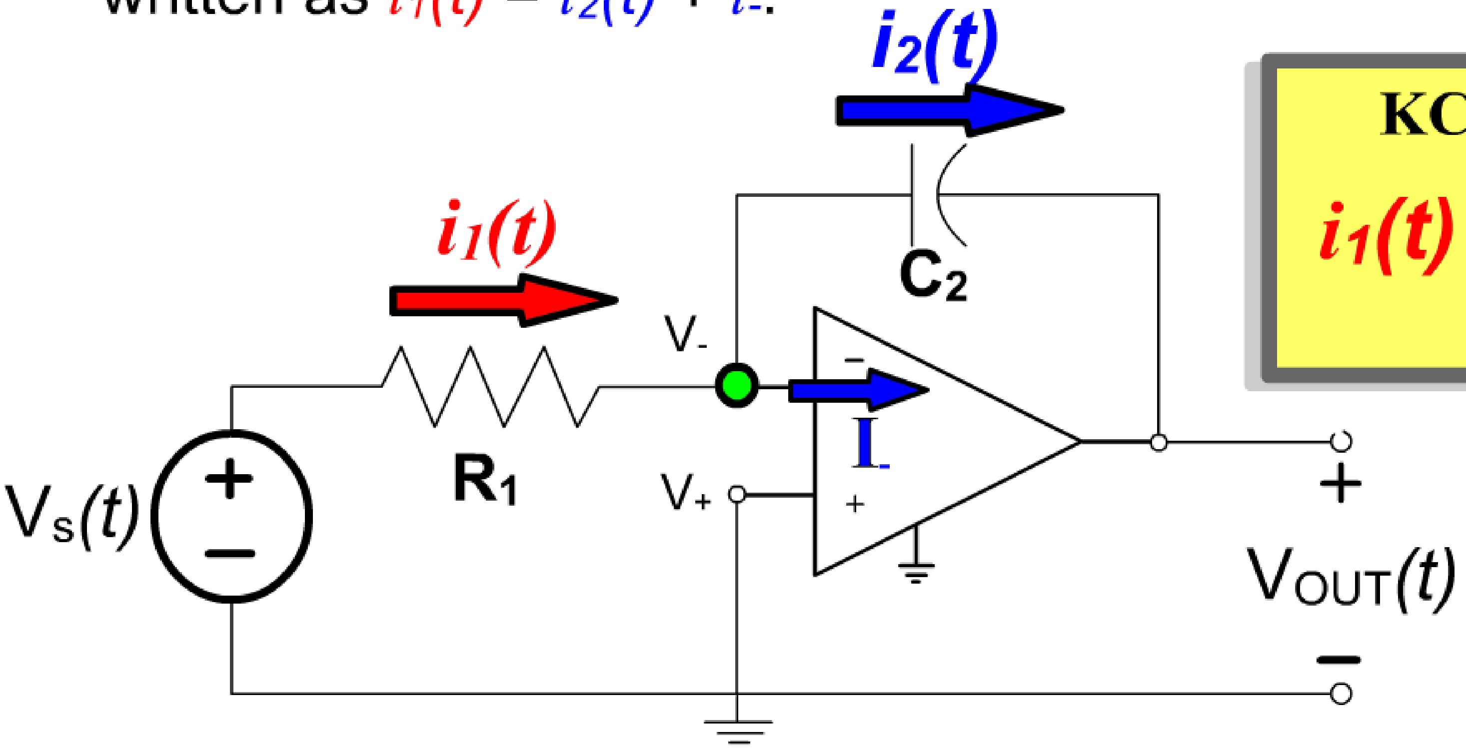
Now write KCL at node V_-. For convenience, the current across R_1 will be labeled as $i_1(t)$ and indicated as entering the node.



KCL at node V_-

$i_1(t)$ = …

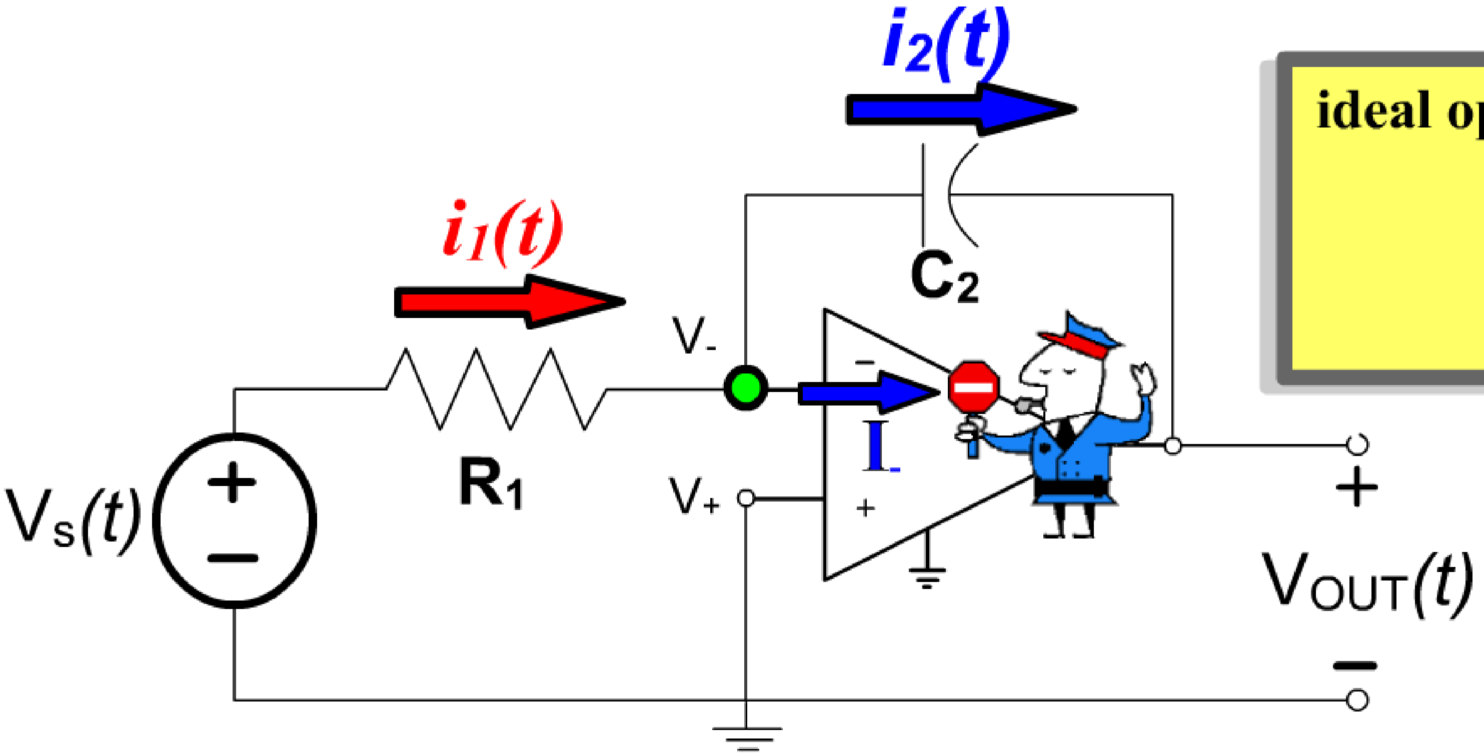
The other two currents are labeled as $i_2(t)$ and i_-, which is denoted as leaving the node V_-. The KCL equation can be written as $i_1(t) = i_2(t) + i_-$.



KCL at node V_-

$$i_1(t) = i_2(t) + i_-$$

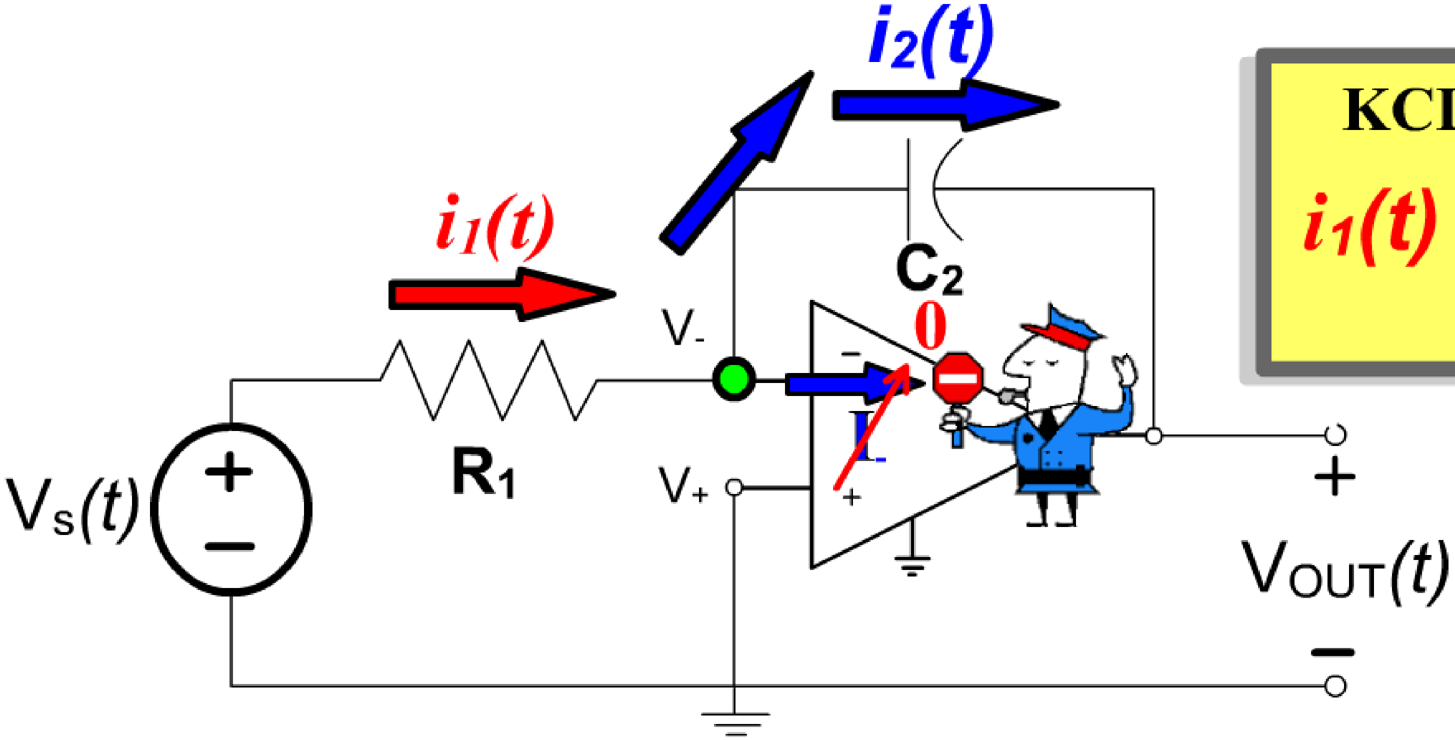
The second ideal op-amp equation requires that the i_- = 0. No current enters this terminal.



ideal op-amp equation

$$i_- = 0$$

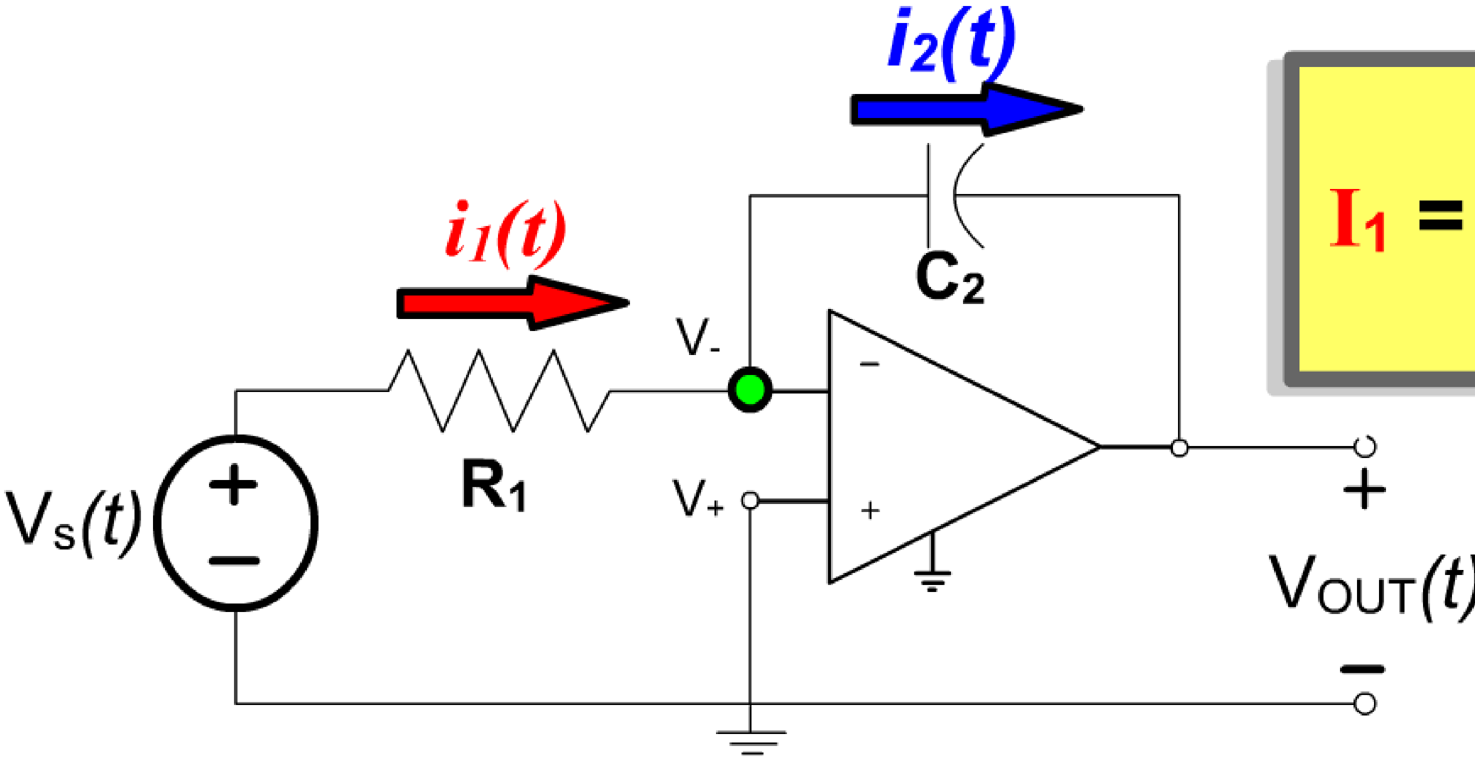
The KCL equation can be simplified to $i_1(t) = i_2(t)$.



KCL at node V-

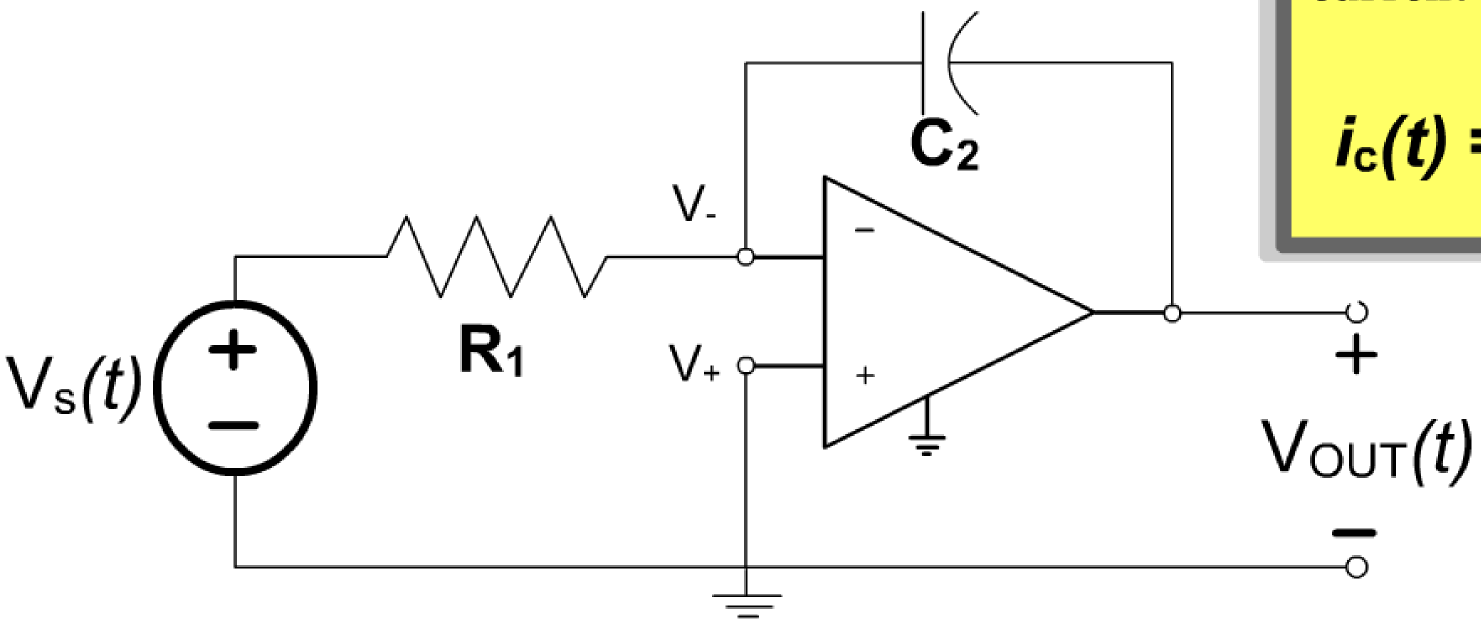
$$i_1(t) = i_2(t) + \cancel{i_-}^{\,0}$$

The current $i_1(t)$ across element R_1 is the difference in potential $V_S(t)$ - V_- = $V_S(t)$, divided by the resistance, R_1.



$$I_1 = \frac{V_S(t) - \cancel{V_-}^{\,0}}{R_1}$$

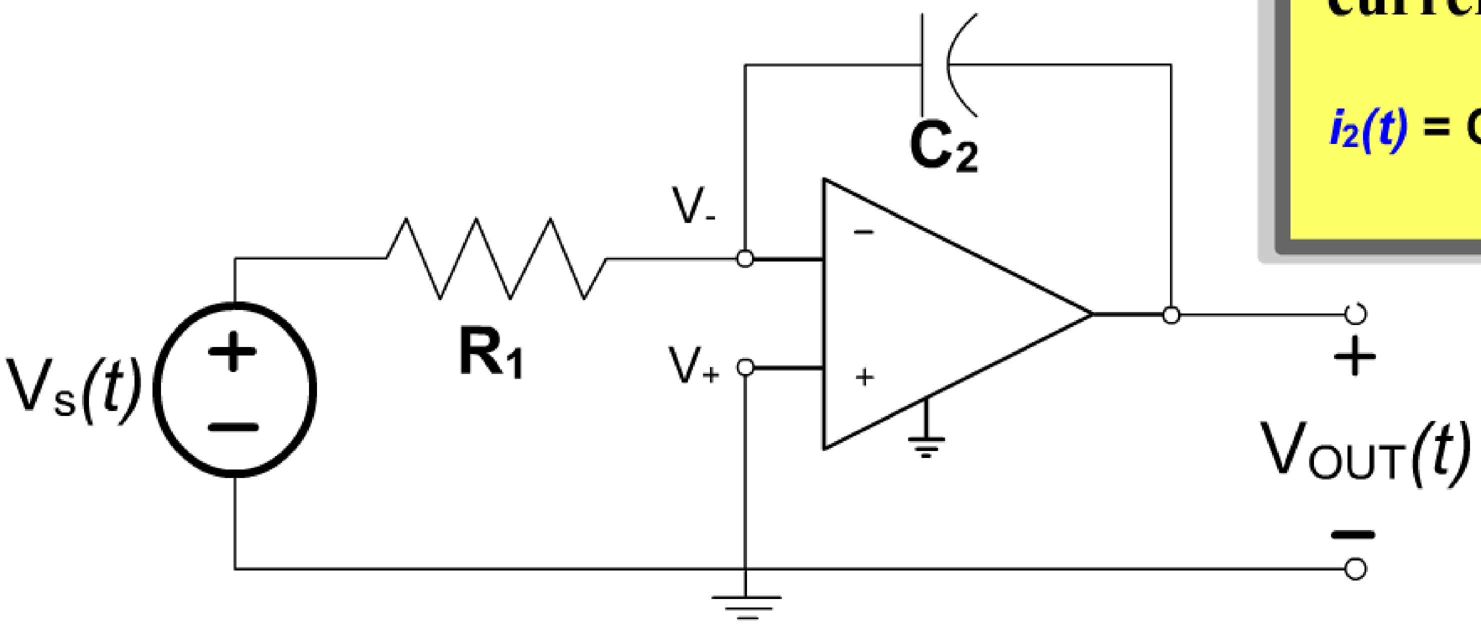
Recall that the current through a capacitor is proportional to the time derivative of the voltage across the capacitor.



current through capacitor

$$i_C(t) = C\,\frac{d}{dt}V_c(t)$$

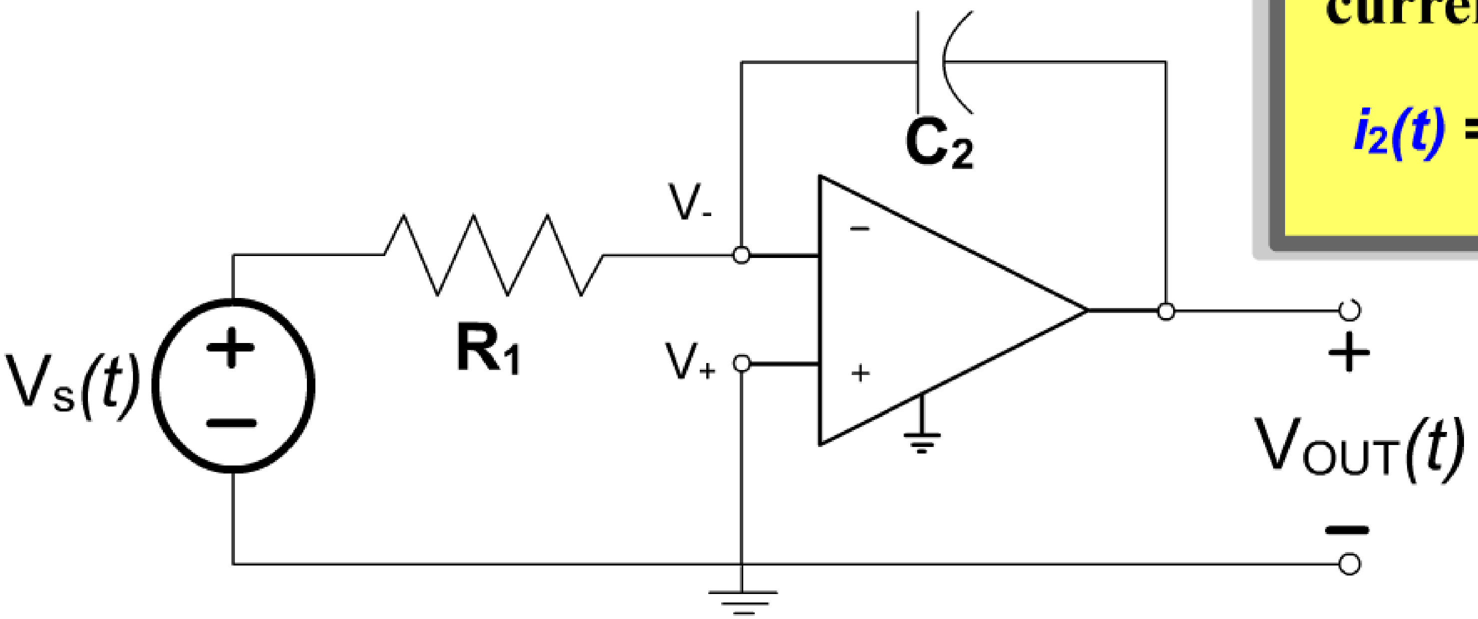
The current $i_2(t)$ through C_2 is the capacitance times the time derivative of the potential difference.



current through C_2

$$i_2(t) = C_2 \frac{d}{dt}[V_- - V_{out}(t)]$$

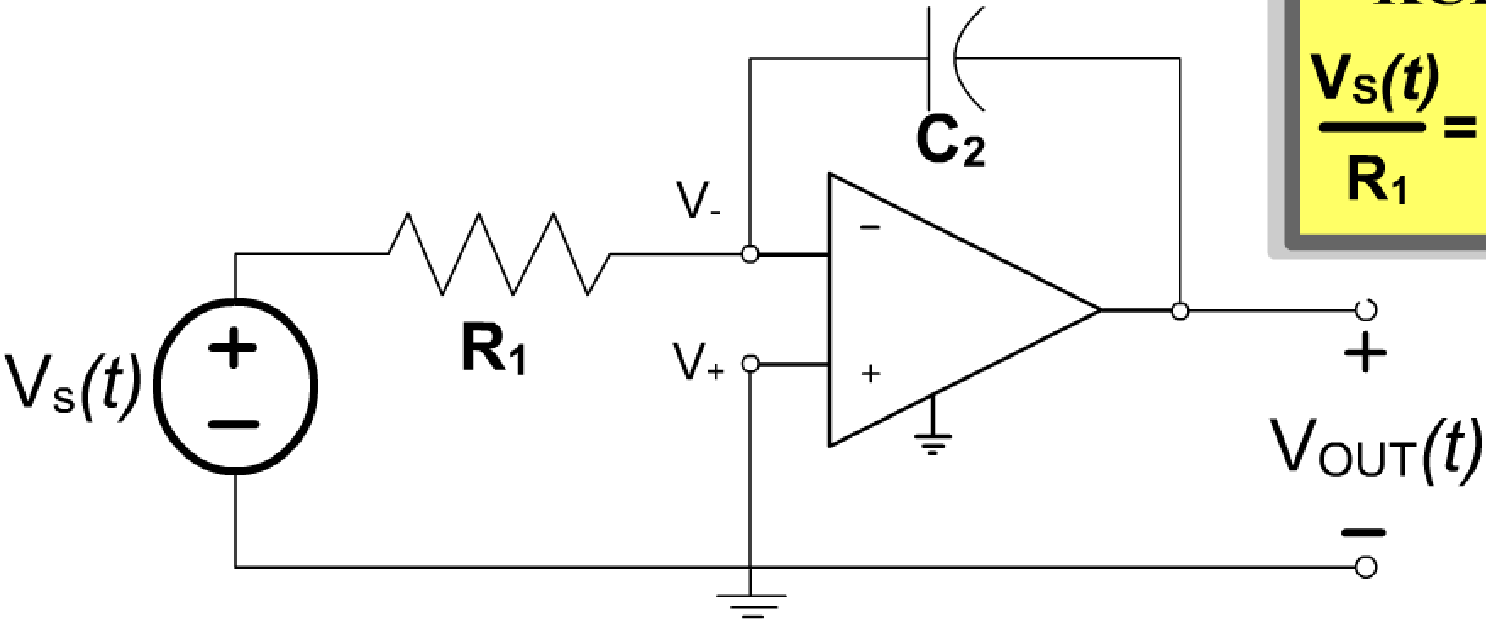
Since V- = 0, the equation for $i_2(t)$ is simply



current through C_2

$$i_2(t) = -C_2 \frac{d}{dt} V_{out}(t)$$

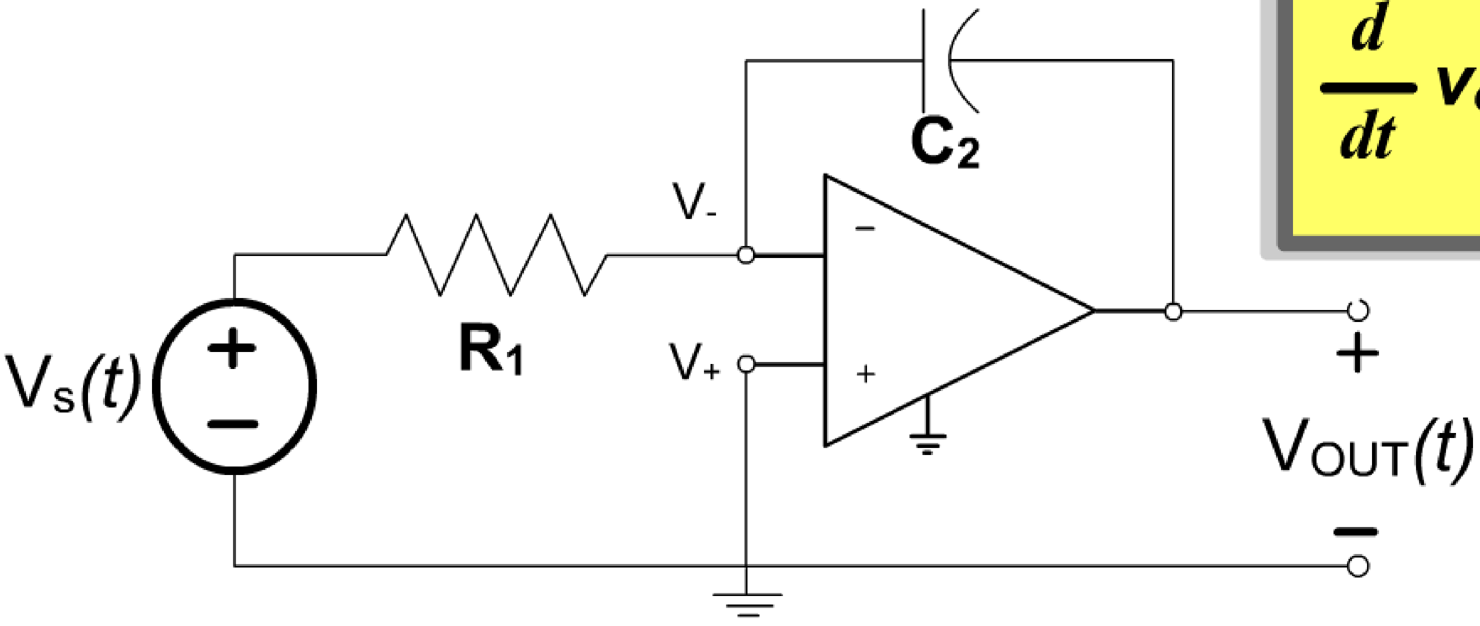
After substitution, KCL at node V- is written as



KCL at node V-

$$\frac{V_S(t)}{R_1} = -C_2 \frac{d}{dt} V_{out}(t)$$

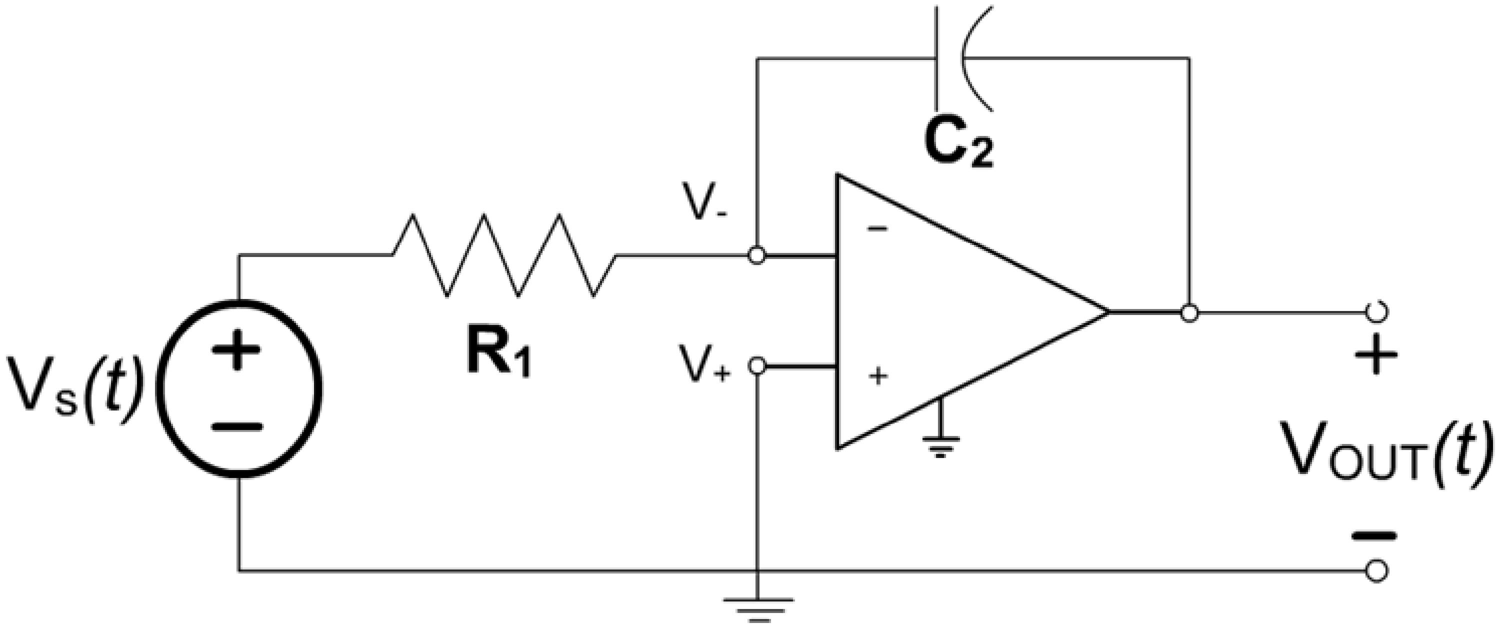
The equation is rearranged



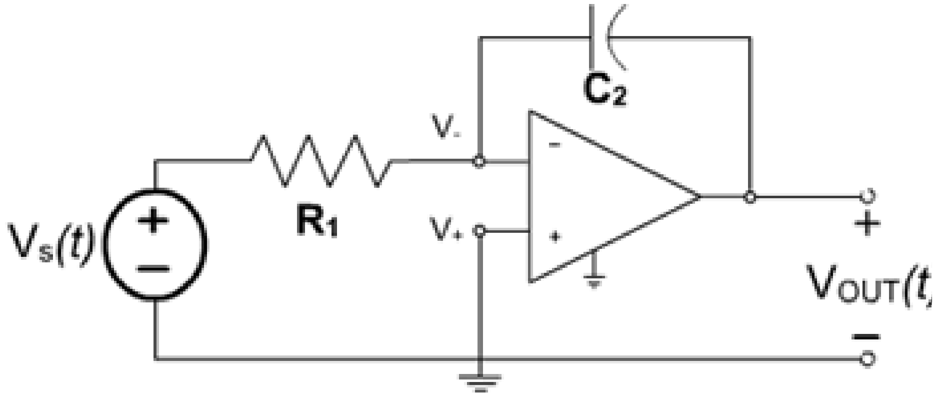
$$\frac{d}{dt}\,v_{out}(t) = \frac{-V_S(t)}{R_1 C_2}$$

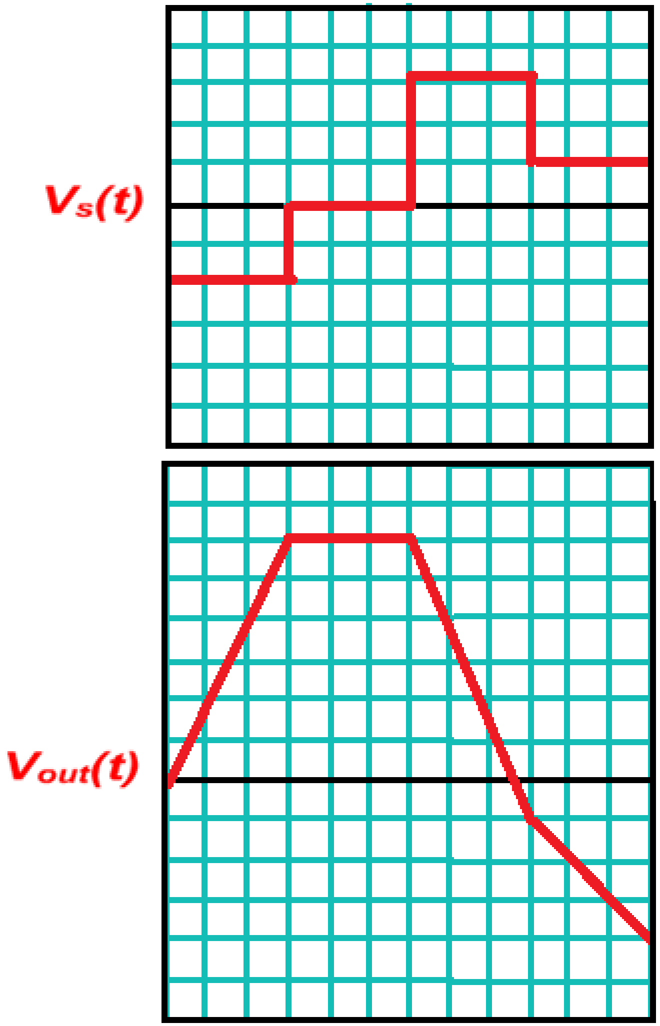
Integrate both sides of the equation up to the present time

$$V_{out}(t) = \int_{-\infty}^{t} \frac{-V_S(t)}{R_1 C_2}\, dt$$



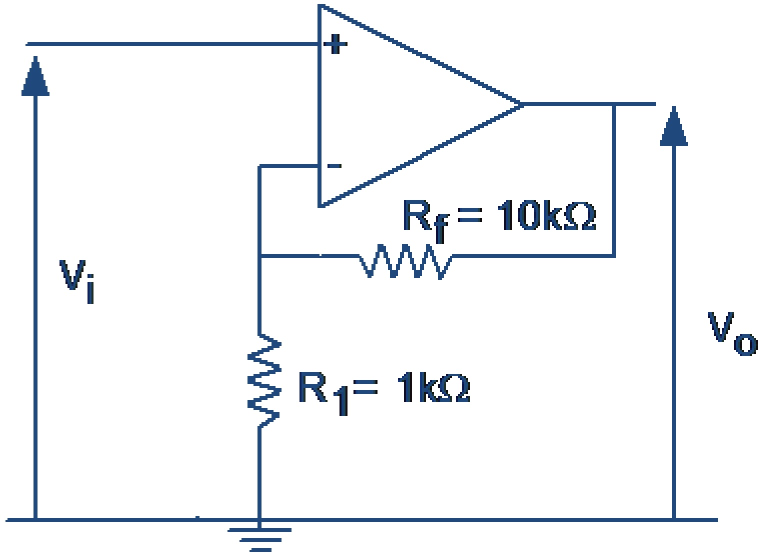
The graphs below show the integration performed by the op-amp circuit for an arbitrary input voltage signal.

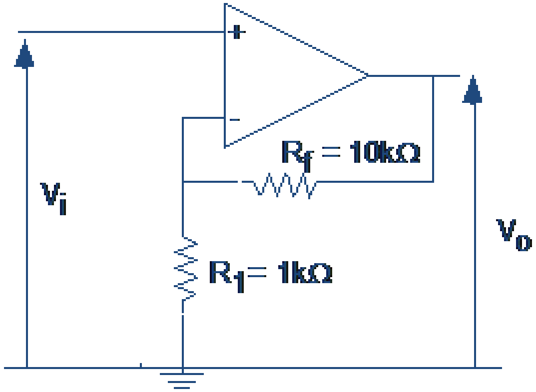




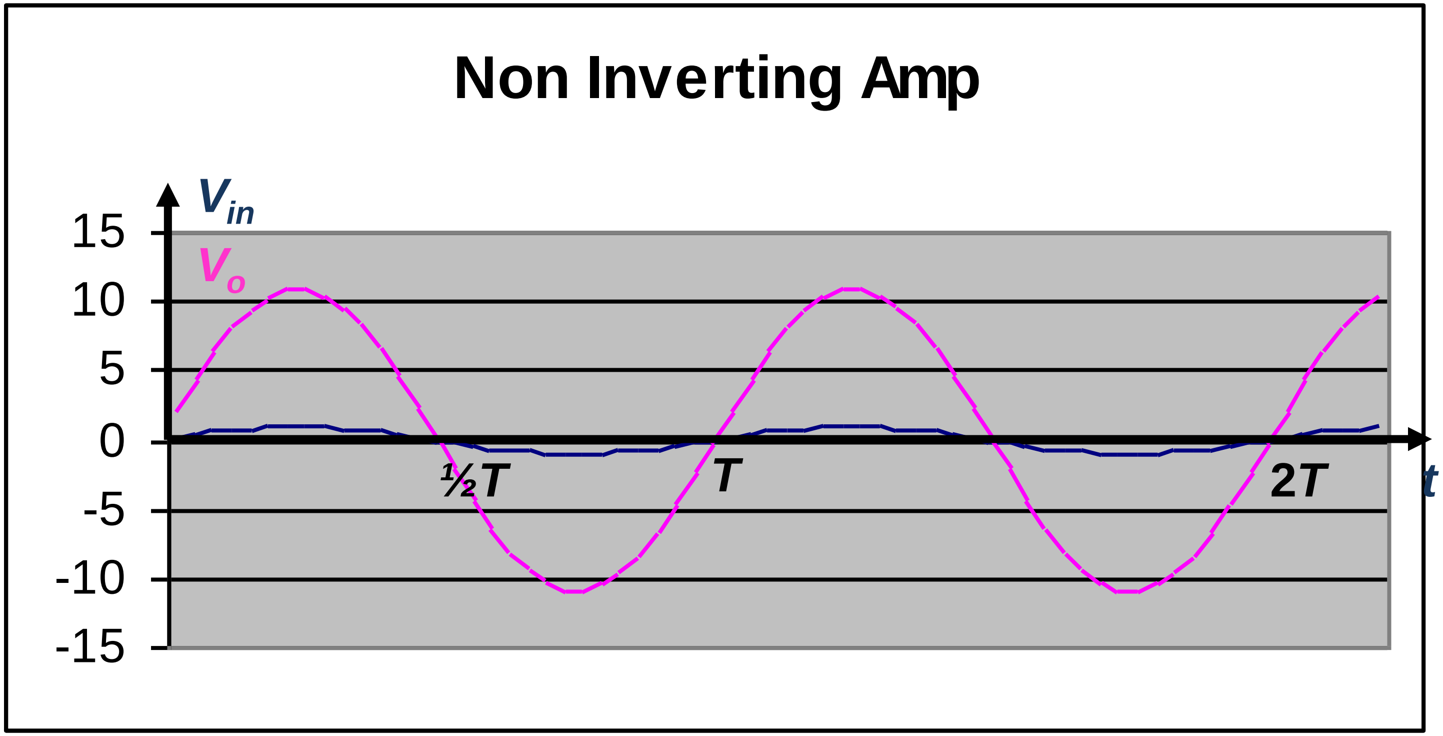
Example 1

Given $V_i = \sin \omega t$, sketch V_o.



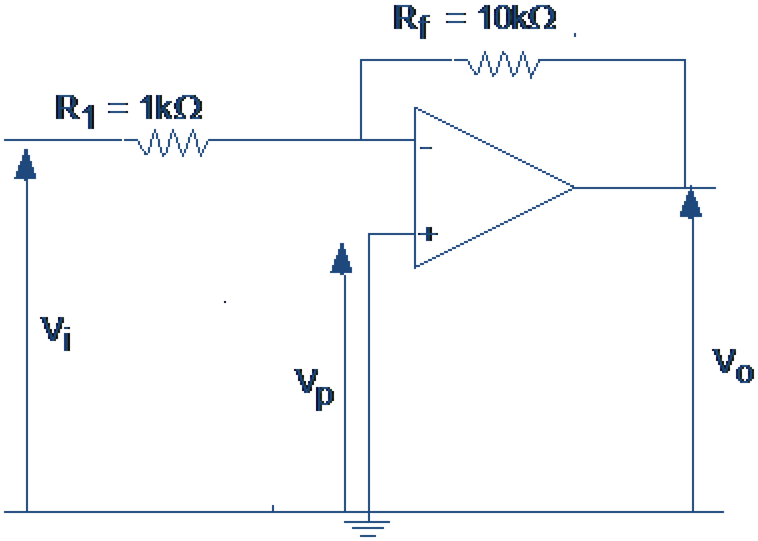


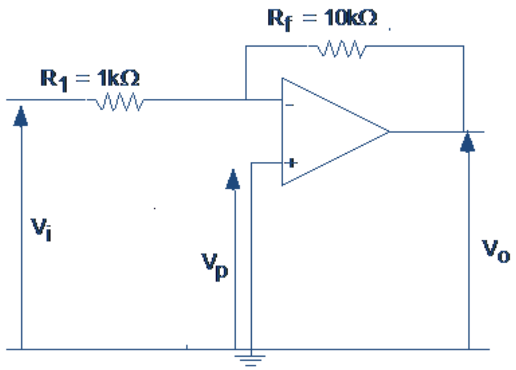
$$\frac{V_o}{V_{in}} = \frac{1+10}{1} = 11$$



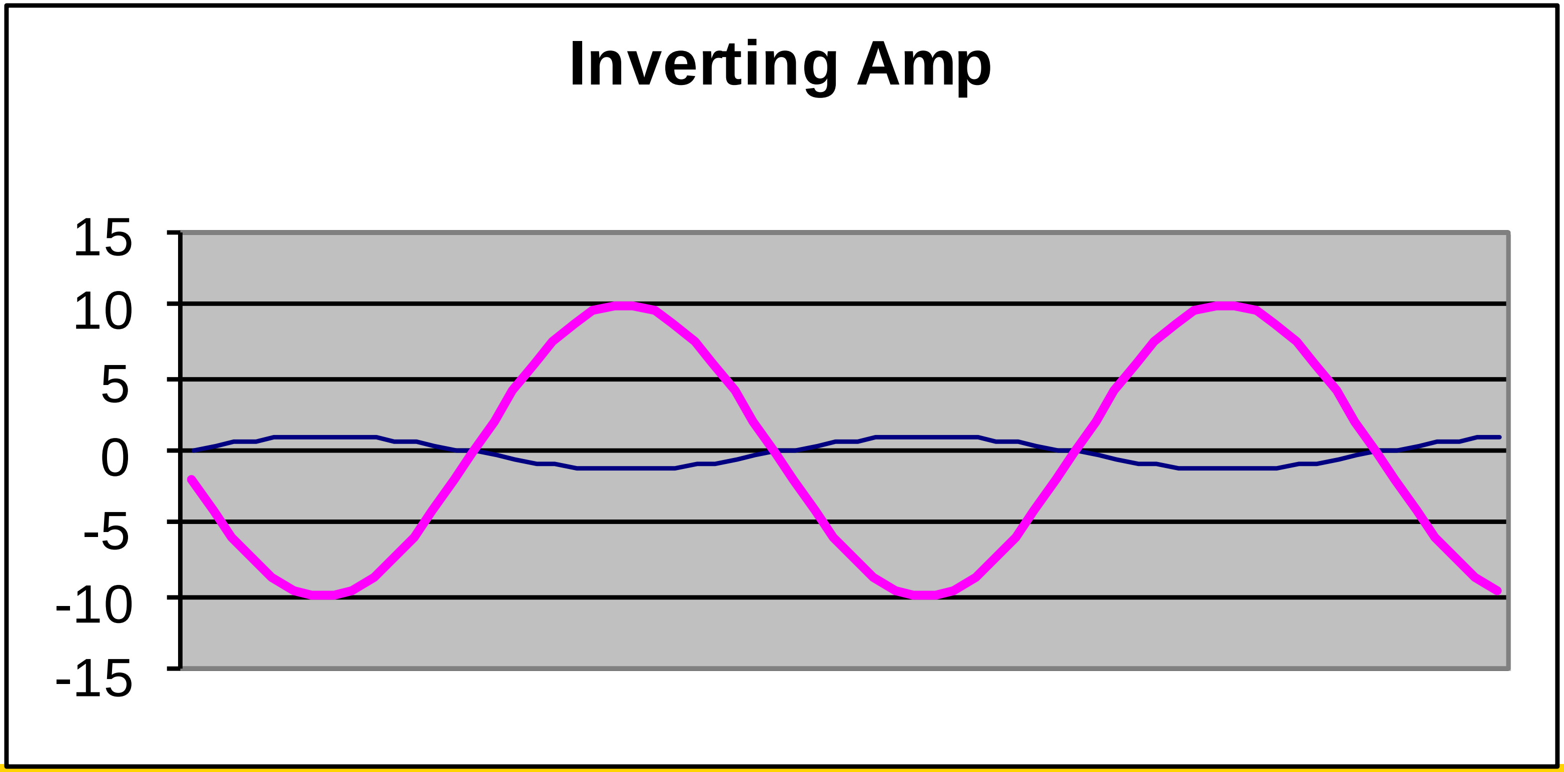
Example 2

Given $V_i = \sin \omega t$, sketch V_o.

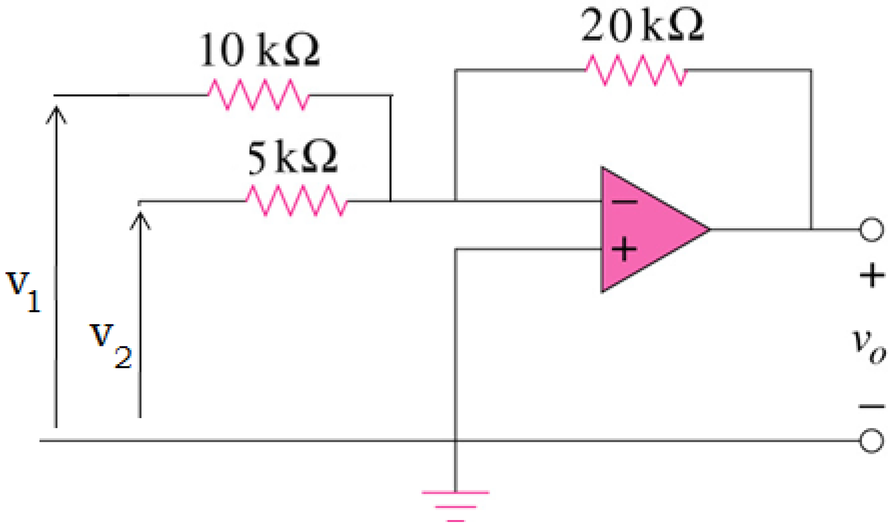




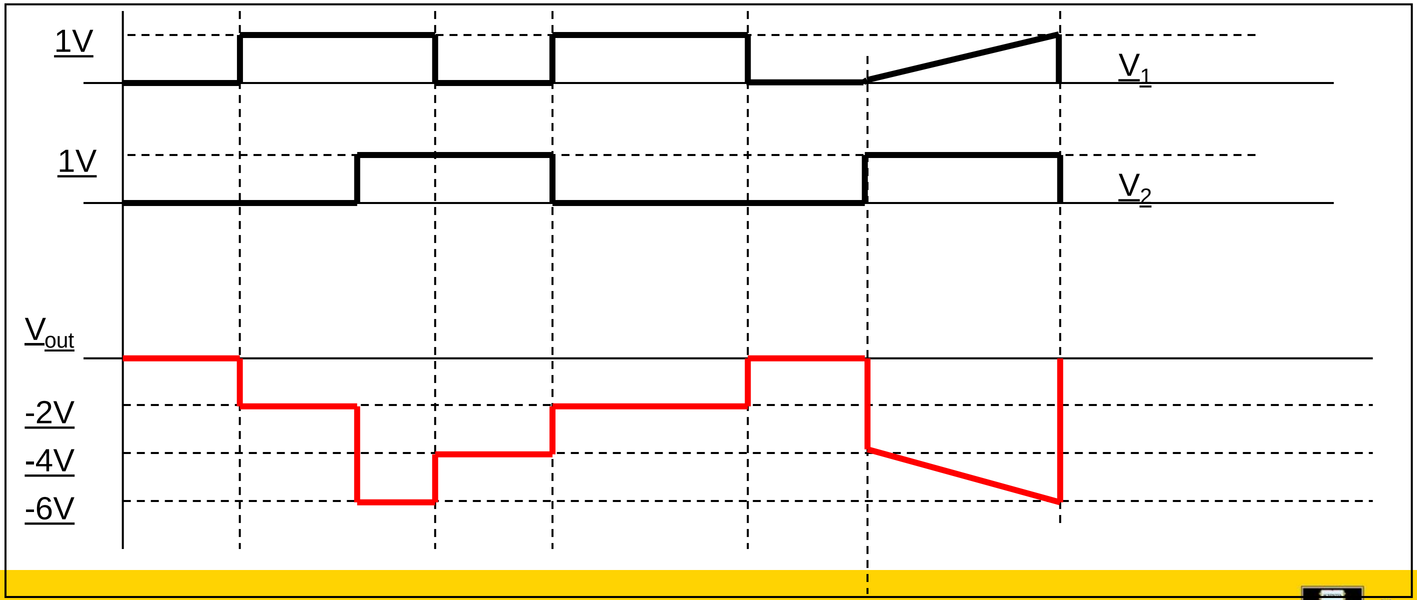
$$\frac{v_0}{v_i} = -\frac{R_f}{R_1} = -\frac{10}{1} = -10$$



Example 3

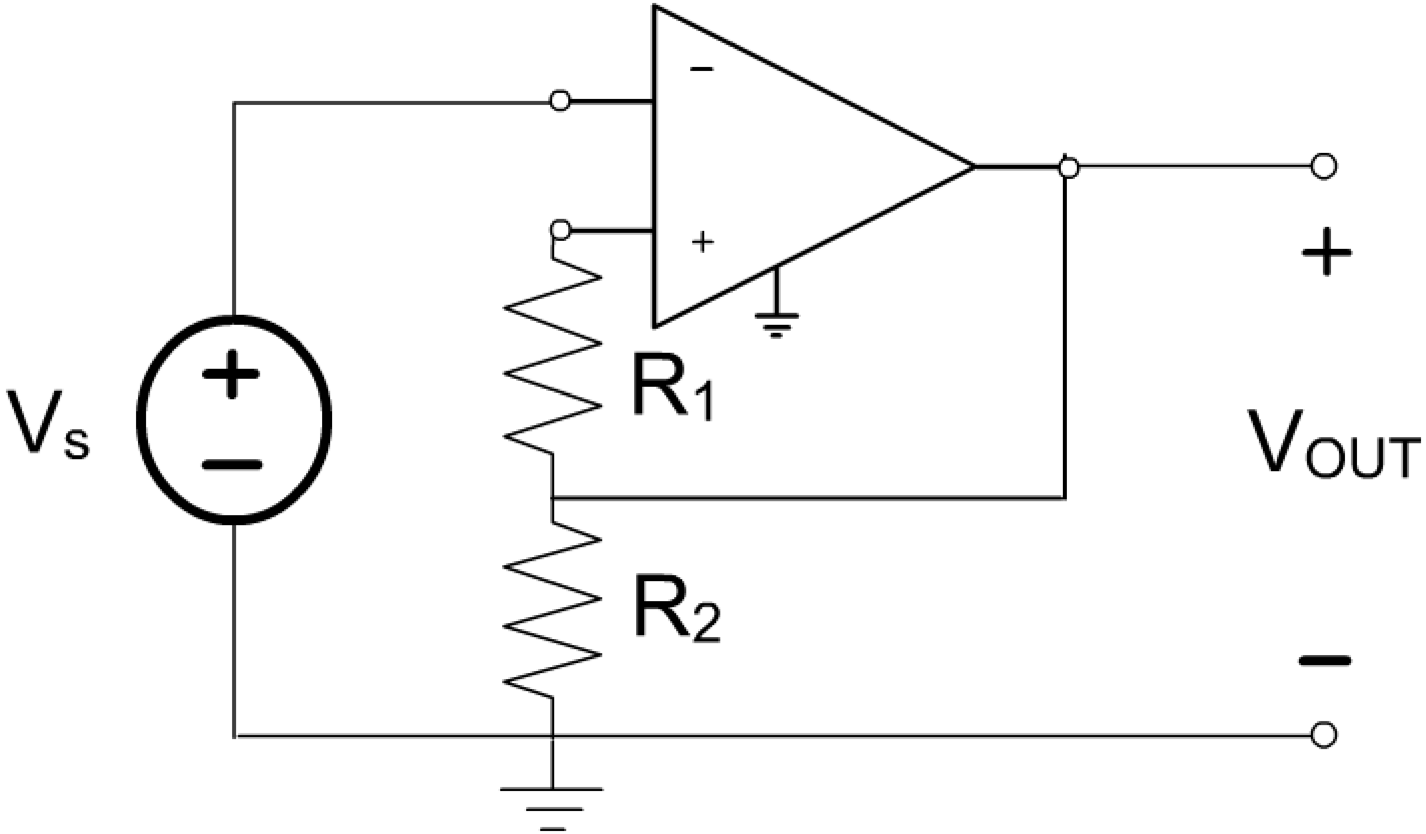


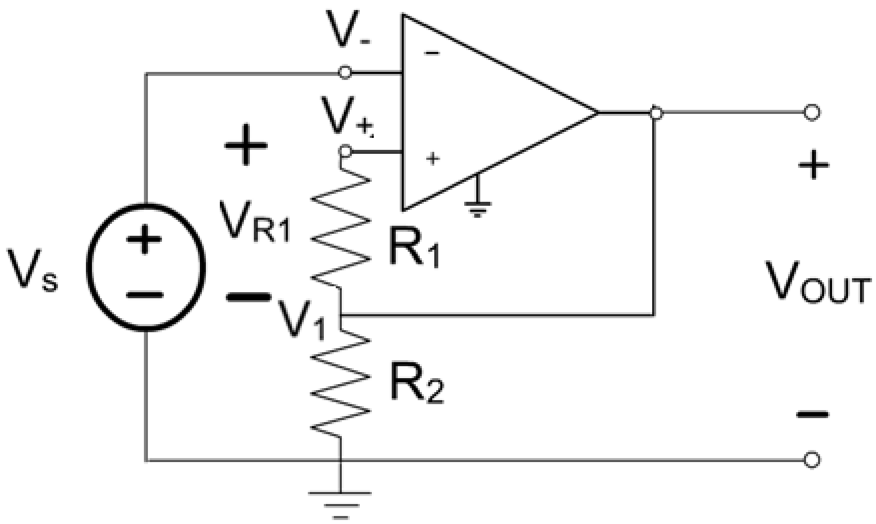
$$v_0 = -\frac{20}{10}v_1 - \frac{20}{5}v_2 = -2v_1 - 4v_2$$



Example 4

Find the Transfer Function





$V_- = V_S$

$V_+ = V_-$

$I_+ = 0$

$V_{R1} = I_+ R_1 = 0$

$V_1 = V_+ + V_{R1}$
$= V_+$

$V_{OUT} = V_1 = V_S$

So, Transfer Function = 1

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