Statistics

MATH2089

Semester 1, 2018 – Lecture 5

This lecture

5. Special random variables

Additional reading:
Sections 1.6 + pp. 28-29, 32-34 in the textbook (2nd edition)
Sections 1.6 + pp. 31-33, 36-37 in the textbook (3rd edition)

5. Special random variables

Introduction

In practice, certain "types" of random variables come up over and over again from similar (random) experiments.

In this chapter, we will study a variety of those **special random variables**.

You can also go to

http://socr.stat.ucla.edu/htmls/SOCR_Distributions.html

and have a look at the numerous 'special' distributions there.

Example

Consider the following random experiments and random variables:

- Flip a coin 10 times. Let $X =$ number of heads obtained
- A worn machine tool produces defective parts 1% of the time . Let $X =$ number of defective parts in the next 25 parts produced
- Each sample of air has a 10% chance of containing a particular molecule. Let $X =$ the number of air samples that contain the molecule in the next 18 samples analysed
- Of all bits transmitted through a digital channel, 15% are received in error. Let $X =$ the number of bits in error in the next five bits transmitted
- A multiple-choice test contains 10 questions, each with 4 choices, and you guess at each question. Let $X =$ the number of questions answered correctly

$\rightarrow$ Similar experiments, similar random variables

$\rightarrow$ A general framework that include these experiments as particular cases would be very useful

The Binomial distribution

Assume:

- the outcome of a random experiment can be classified as either a "Success" or a "Failure" $\quad (\rightarrow S = \{\text{Success}, \text{Failure}\})$
- we observe a Success with probability π
- n independent repetitions of this experiment are performed

Define $X =$ number of Successes observed over the n repetitions. We say that X is a **binomial random variable** with parameters n and π:

$$\boxed{X \sim \text{Bin}(n, \pi)}$$

See that $S_X = \{0, 1, 2, \dots, n\}$ ($\rightarrow$ discrete r.v.) and the binomial probability mass function is given by

$$p(x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}, \qquad \text{for } x \in S_X$$

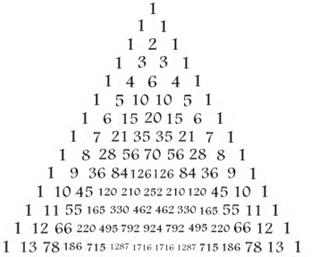
where $\binom{n}{x}$ is the number of different groups of x objects that can be chosen from a set of n objects.

The Binomial distribution

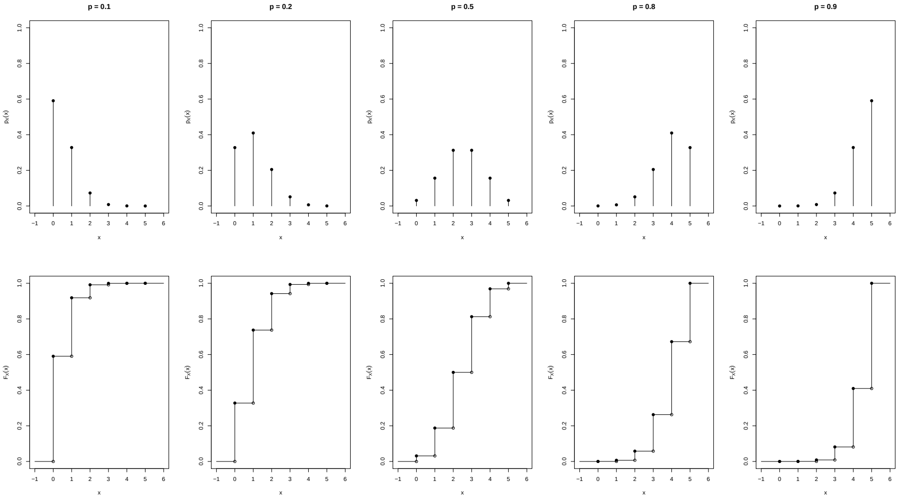
Note: the coefficients $\binom{n}{x} = n!/(x!(n-x)!)$ are called the binomial coefficients, they are the coefficients arising in Newton's famous binomial expansion.

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

These coefficients are often represented in Pascal's triangle, named after the French mathematician Blaise Pascal (1623-1662)



The Binomial distribution: pmf and cdf



Binomial pmf and cdf, for $n = 5$ and $\pi = \{0.1, 0.2, 0.5, 0.8, 0.9\}$

The Bernoulli distribution

Particular case: if $n = 1 \rightarrow$ the Bernoulli distribution,

named after the Swiss scientist Jakob Bernoulli (1654-1705)

$$\boxed{X \sim \text{Bern}(\pi)}$$

pmf:

$$p(x) = \begin{cases} 1-\pi & \text{if } x = 0 \\ \pi & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Note: if $X \sim \text{Bin}(n, \pi)$, we can represent it as

$$X = \sum_{i=1}^{n} X_i$$

where X_i's are n independent Bernoulli r.v. with parameters π

$\rightarrow$ Each repetition of the experiment in the Binomial framework is called a Bernoulli trial

Binomial distribution: properties

First note that

$$\sum_{x \in S_X} p(x) = \sum_{x=0}^{n} \binom{n}{x} \pi^x (1-\pi)^{n-x} = (\pi + (1-\pi))^n = 1$$

using the binomial expansion

Second, it is easy to see that if $X_1 \sim \text{Bin}(n_1, \pi)$, $X_2 \sim \text{Bin}(n_2, \pi)$ and X_1 is independent of X_2, then

$$X_1 + X_2 \sim \text{Bin}(n_1 + n_2, \pi)$$

Binomial distribution: expectation and variance

Recall the representation $X = \sum_{i=1}^{n} X_i$, with $X_i \sim \text{Bern}(\pi)$

We know that

$$\mathbb{E}(X_i) = \pi \qquad \text{and} \qquad \mathbb{V}\text{ar}(X_i) = \pi(1-\pi)$$

It follows $\mathbb{E}(X) = \mathbb{E}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathbb{E}(X_i) = \sum_{i=1}^{n} \pi = n\pi$ and

$$\mathbb{V}\text{ar}(X) = \mathbb{V}\text{ar}\left(\sum_{i=1}^{n} X_i\right) \overset{\text{ind.}}{=} \sum_{i=1}^{n} \mathbb{V}\text{ar}(X_i) = \sum_{i=1}^{n} \pi(1-\pi) = n\pi(1-\pi)$$

Mean and variance of the binomial distribution

If $X \sim \text{Bin}(n, \pi)$,

$$\mu = \mathbb{E}(X) = n\pi \qquad \text{and} \qquad \sigma^2 = \mathbb{V}\text{ar}(X) = n\pi(1-\pi)$$

Binomial distribution: examples

Example

It is known that disks produced by a certain company will be defective with probability 0.01 independently of each other. The company sells the disk in packages of 10 and offers a money-back guarantee if more than 1 of the disks are defective. a) In the long-run, what proportion of packages is returned? b) If someone buys three packages, what is the probability that exactly one of them will be returned?

Binomial distribution: examples

Example

It is known that disks produced by a certain company will be defective with probability 0.01 independently of each other. The company sells the disk in packages of 10 and offers a money-back guarantee if more than 1 of the disks are defective. a) In the long-run, what proportion of packages is returned? b) If someone buys three packages, what is the probability that exactly one of them will be returned?

Binomial distribution: examples

Example (Ex. 54 p.54 in the textbook)

Suppose that 10% of all bits transmitted through a digital communication channel are erroneously received and that whether any is erroneously received is independent of whether any other bit is erroneously received. Consider sending a large number of messages, each consisting of 20 bits. a) What proportion of these messages will have exactly 2 erroneously received bits? b) What proportion of these messages will have at least 5 erroneously received bits? c) What proportion of these messages will more than half the bits be erroneously received?

Let X be the number of erroneously received bits in a message of 20 bits. Clearly, we have $X \sim \text{Bin}(20, 0.1)$. Thus we have

a) $\mathbb{P}(X = 2) = \binom{20}{2} 0.1^2 0.9^{18} = 0.2852$

b) $\mathbb{P}(X \geq 5) = 1 - \mathbb{P}(X = 0) - \mathbb{P}(X = 1) - \mathbb{P}(X = 2) - \mathbb{P}(X = 3) - \mathbb{P}(X = 4) = \ldots$

c) $\mathbb{P}(X > 10) = \mathbb{P}(X = 11) + \mathbb{P}(X = 12) + \ldots + \mathbb{P}(X = 20) = \ldots$

$\rightarrow$ very tedious! $\rightarrow$ use **statistical software**

Binomial distribution: examples

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

```
>> binopdf(2,20,0.1)

ans =

    0.2852

>> 1-binocdf(4,20,0.1)

ans =

    0.0432

>> 1-binocdf(10,20,0.1)

ans =

  7.0886e-007

>>
```

The Poisson distribution

Assume you are interested in the number of occurrences of some random phenomenon in a fixed period of time.

Define $X =$ number of occurrences. We say that X is a **Poisson random variable** with parameter λ, i.e.

$$X \sim \mathcal{P}(\lambda),$$

if

$$p(x) = e^{-\lambda}\frac{\lambda^x}{x!} \qquad \text{for } x \in S_X = \{0, 1, 2, \ldots\}$$

Note: Simeon-Denis Poisson (1781-1840) was a French mathematician

Poisson distribution: how does it arise?

- Think of the time period of interest as being split up into a large number, say n, of sub-periods
- Assume that the phenomenon could occur at most one time in each of those subperiods, with some constant probability π
- If what happens within one interval is independent to others,

$$X \sim \text{Bin}(n, \pi)$$

- Now, as n increases, π should decrease (the shorter the period, the less likely the occurrence of the phenomenon) $\rightarrow$ let $\pi = \lambda/n$ for some $\lambda > 0$
- Then, for any $x \in \{0, 1, \ldots, n\}$,

$$\begin{aligned} \mathbb{P}(X = x) &= \frac{n!}{x!(n-x)!} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x} \quad \text{(binomial pmf)} \\ &= \frac{n!}{n^x (n-x)!(1-\lambda/n)^x} \frac{\lambda^x}{x!} \left(1 - \frac{\lambda}{n}\right)^n \end{aligned}$$

Poisson distribution: how does it arise?

- Finally, as $n \to \infty$

$$\frac{n!}{n^x(n-x)!(1-\lambda/n)^x} \to 1 \quad \text{and} \quad \left(1-\frac{\lambda}{n}\right)^n \to e^{-\lambda}$$

  Therefore,

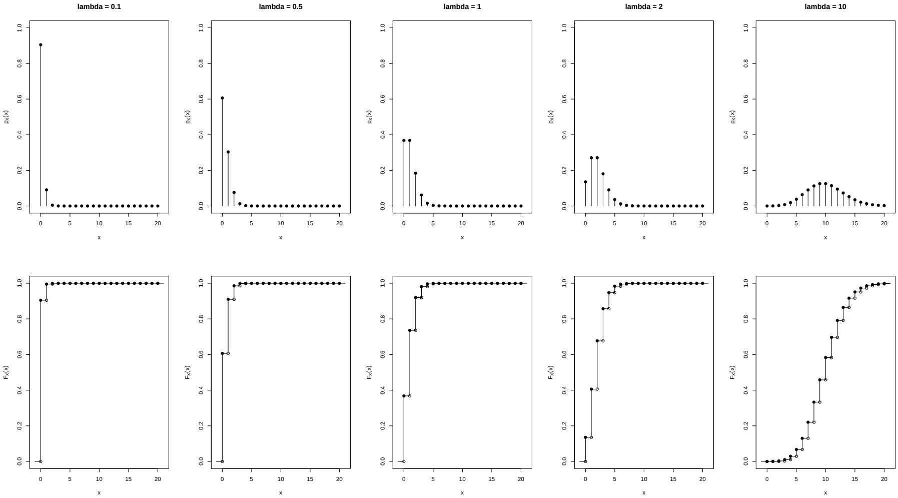
$$\mathbb{P}(X=x) = e^{-\lambda}\frac{\lambda^x}{x!} \qquad \text{for } x \in \{0,1,\dots\}$$

  which is the Poisson pmf as given on Slide 16

- The Poisson distribution is thus suitable for modelling the number of occurrences of a random phenomenon satisfying some assumptions of continuity, stationarity, independence and non-simultaneity
- λ is called the intensity of the phenomenon

Note: we defined the $\mathcal{P}(\lambda)$ distribution by partitioning a time period, however the same reasoning can be applied to any interval, area or volume

Poisson distribution: pmf and cdf



Poisson pmf and cdf, for $\lambda = \{0.1, 0.5, 1, 2, 10\}$

Poisson distribution: properties

First we have, as expected,

$$\sum_{x \in S_X} p(x) = \sum_{x=0}^{\infty} e^{-\lambda} \frac{\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} e^{\lambda} = 1$$

Similarly,

$$\mathbb{E}(X) = \sum_{x \in S_X} x p(x) = \sum_{x=0}^{\infty} x e^{-\lambda} \frac{\lambda^x}{x!} = \lambda \sum_{x=1}^{\infty} e^{-\lambda} \frac{\lambda^{x-1}}{(x-1)!} = \lambda$$

$$\mathbb{E}(X^2) = \sum_{x \in S_X} x^2 p(x) = \ldots = \lambda^2 + \lambda$$

$\to \mathbb{V}\mathrm{ar}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$

Mean and variance of the Poisson distribution

If $X \sim \mathcal{P}(\lambda)$,

$$\mathbb{E}(X) = \lambda \qquad \text{and} \qquad \mathbb{V}\mathrm{ar}(X) = \lambda$$

Poisson distribution: examples

Example

Over a 10-minute period, a counter records an average of 1.3 gamma particles per millisecond coming from a radioactive substance. To a good approximation, the distribution of the count, X, of gamma particles during the next millisecond is Poisson distributed. Determine a) λ, b) the probability of observing one or more gamma particles during the next millisecond and c) the variance of this number.

Poisson distribution: examples

Example (Ex. 56 p.55 in the textbook)

Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has Poisson distribution with parameter $\lambda = 20$. In the long-run, in what proportion of time periods will the number of drivers a) be at most 10? b) exceed 20? c) be between 10 and 20, inclusive? Strictly between 10 and 20?

Let X be the number of drivers. It is given that $X \sim \mathcal{P}(20)$

a)

$$\begin{aligned}
\mathbb{P}(X \leq 10) &= \mathbb{P}(X = 0) + \mathbb{P}(X = 1) + \ldots + \mathbb{P}(X = 10) \\
&= e^{-20} + e^{-20} \times 20 + e^{-20}\frac{20^2}{2} + \ldots + e^{-20}\frac{20^{10}}{10!} \\
&= \ldots
\end{aligned}$$

$\rightarrow$ tedious ! $\rightarrow$ use **statistical software**

Poisson distribution: examples

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

```
>> poisscdf(10,20)

ans =

    0.0108

>> 1-poisscdf(20,20)

ans =

    0.4409

>> poisscdf(20,20)-poisscdf(9,20)

ans =

    0.5541

>> poisscdf(19,20)-poisscdf(10,20)

ans =

    0.4594

>>
```

Poisson approximation to the Binomial distribution

Since it was derived as a limit case of the Binomial distribution when n is 'large' and π is 'small', one can expect the Poisson distribution to be a good approximation to Bin(n, π) in that case.

As it involves only one parameter, the Poisson pmf or the Poisson tables are usually easier to handle than the corresponding Binomial pmf and tables.

Example

It is known that 1% of the books at a certain bindery have defective bindings. Compare the probabilities that x ($x = 0, 1, 2, \ldots$) of 100 books will have defective bindings using the (exact) formula for the binomial distribution and its Poisson approximation.

The exact Binomial pmf is $p(x) = \binom{100}{x} \times 0.01^x \times 0.99^{100-x}$, while its Poisson approximation is

$$p^*(x) = e^{-\lambda} \frac{x^\lambda}{\lambda!}$$

with $\lambda = n \times p = 100 \times 0.01 = 1$

Poisson distribution: examples

Matlab computations give:

```
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.

>> x=[0:100];
Bino=binopdf(x,100,0.01);
Poiss=poisspdf(x,1);
A=[x;Bino;Poiss];
A(:,1:8)

ans =

        0    1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    7.0000
   0.3660    0.3697    0.1849    0.0610    0.0149    0.0029    0.0005    0.0001
   0.3679    0.3679    0.1839    0.0613    0.0153    0.0031    0.0005    0.0001

>>
```

We see that the error we would make by using the Poisson approximation instead of the true distribution is only of order 10^{-3}

$\rightarrow$ very good approximation

The Uniform distribution

There are also numerous **continuous** distributions which are of great interest. The simplest one is certainly the **uniform distribution**.

A random variable is said to be uniformly distributed over an interval $[\alpha, \beta]$, i.e.

$$\boxed{X \sim U_{[\alpha,\beta]}}$$

if its probability density function is given by

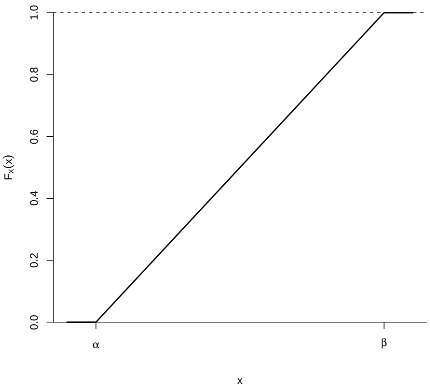
$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases} \qquad (\rightarrow S_X = [\alpha, \beta])$$

Constant density $\rightarrow$ X is just as likely to be "close" to any value in S_X.

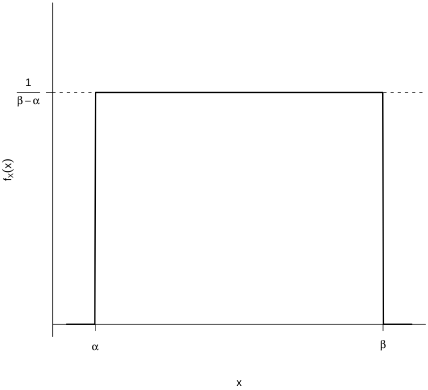
By integration, it is easy to show that

$$F(x) = \begin{cases} 0 & \text{if } x < \alpha \\ \frac{x-\alpha}{\beta-\alpha} & \text{if } \alpha \leq x \leq \beta \\ 1 & \text{if } x > \beta \end{cases}$$

The Uniform distribution



cdf $F(x)$



pdf $f(x) = F'(x)$

Uniform distribution: properties

Note that the Uniform density is constant at $1/(\beta-\alpha)$ on $[\alpha,\beta]$ so as to ensure that $\int_\alpha^\beta f(x)\,dx = 1$

Now,

$$\mathbb{E}(X) = \int_\alpha^\beta x\,\frac{1}{\beta-\alpha}dx = \frac{1}{\beta-\alpha}\left[\frac{x^2}{2}\right]_\alpha^\beta = \frac{\beta^2-\alpha^2}{2(\beta-\alpha)} = \frac{\alpha+\beta}{2}$$

Similarly,

$$\mathbb{E}(X^2) = \int_\alpha^\beta x^2\frac{1}{\beta-\alpha}\,dx = \frac{1}{\beta-\alpha}\left[\frac{x^3}{3}\right]_\alpha^\beta = \frac{\beta^3-\alpha^3}{3(\beta-\alpha)} = \frac{\beta^2+\alpha\beta+\alpha^2}{3}$$

which implies $\mathbb{V}\mathrm{ar}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \ldots = \frac{(\beta-\alpha)^2}{12}$

Mean and variance of the Uniform distribution

If $X \sim U_{[\alpha,\beta]}$,

$$\mathbb{E}(X) = \frac{\alpha+\beta}{2} \qquad \text{and} \qquad \mathbb{V}\mathrm{ar}(X) = \frac{(\beta-\alpha)^2}{12}$$

Uniform distribution: example

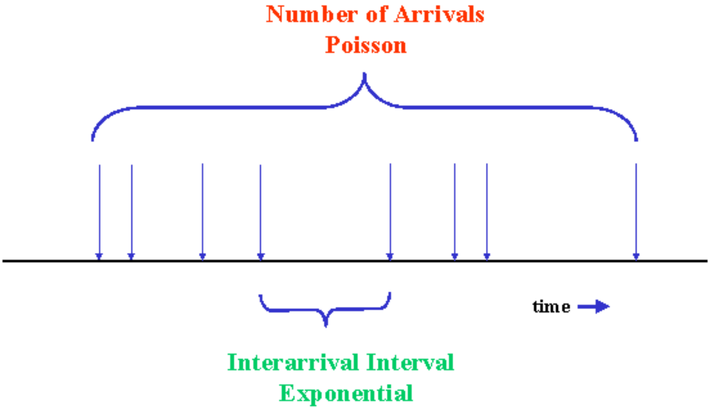
The probability that X lies in any subinterval $[a, b]$ of $[\alpha, \beta]$ is:

$$\boxed{\mathbb{P}(a < X < b) = \frac{b-a}{\beta-\alpha}} \qquad \text{(area of a rectangle)}$$

Example

Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, etc. If a passenger arrives at the stop at a time uniformly distributed between 7 and 7:30, find the probability that he waits less than 5 minutes for a bus

The Exponential distribution



Poisson and exponential distributions

The Exponential distribution

- Recall that a Poisson distributed r.v. counts the number of occurrences of a given phenomenon over a unit period of time
- The (random) amount of time before the first occurrence of that phenomenon is often of interest as well
- If $N \sim \mathcal{P}(\lambda)$ denote the number of occurrences over a unit period of time, then the number of occurrences of the phenomenon by a time x, say N_x, is $\sim \mathcal{P}(\lambda x)$ ("Poisson process")
- Denote X the amount of time before the first occurrence
- This time will exceed x ($x \geq 0$) if and only if there have been no occurrences of the phenomenon by time x, that is, $N_x = 0$

As $N_x \sim \mathcal{P}(\lambda x)$, it follows $\mathbb{P}(X > x) = \mathbb{P}(N_x = 0) = e^{-\lambda x} \frac{(\lambda x)^0}{0!} = e^{-\lambda x}$, which yields the cdf of X:

$$F(x) = \mathbb{P}(X \leq x) = 1 - e^{-\lambda x} \qquad \text{for } x \geq 0$$

This particular distribution is called the Exponential distribution.

The Exponential distribution

A random variable is said to be an **Exponential random variable** with parameter μ ($\mu > 0$), i.e.

$$X \sim \text{Exp}(\mu),$$

if its probability density function is given by

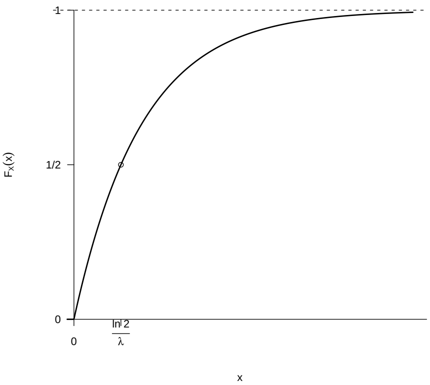
$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (\rightarrow S_X = \mathbb{R}^+)$$

By integration, we find

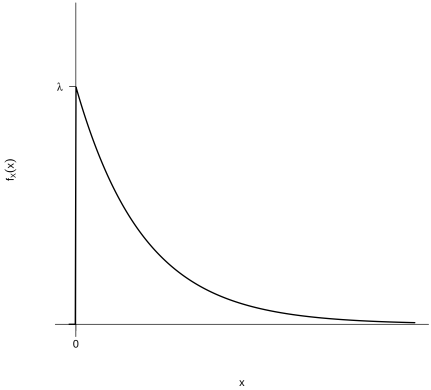
$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\frac{x}{\mu}} & \text{if } x \geq 0 \end{cases}$$

This distribution is often useful for representing random amounts of time, like the amount of time required to complete a task, the waiting time at a counter, the amount of time until you receive a phone call, etc.
Note: the parameter μ is related to λ by $\mu = 1/\lambda$.

The Exponential distribution



cdf $F(x)$



pdf $f(x) = F'(x)$

Exponential distribution: properties

We can check that (as expected)

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_0^{+\infty} \frac{1}{\mu} e^{-\frac{x}{\mu}}\,dx = \frac{1}{\mu}\left[\frac{e^{-\frac{x}{\mu}}}{(-\frac{1}{\mu})}\right]_0^{+\infty} = 1$$

Moreover,

$$\mathbb{E}(X) = \int_0^{+\infty} x\,\frac{1}{\mu} e^{-\frac{x}{\mu}}\,dx = \left[-x\,e^{-\frac{x}{\mu}}\right]_0^{+\infty} + \int_0^{+\infty} e^{-\frac{x}{\mu}}\,dx \qquad \text{(by parts)}$$

$$= 0 + \left[-\frac{e^{-\frac{x}{\mu}}}{\frac{1}{\mu}}\right]_0^{+\infty} = \mu$$

Similarly, $\mathbb{E}(X^2) = \int_0^{+\infty} x^2 e^{-\frac{x}{\mu}}\,dx = \ldots = 2\mu^2$, so that $\mathbb{V}\mathrm{ar}(X) = \mu^2$

Mean and variance of the Exponential distribution

If $X \sim \mathrm{Exp}(\mu)$,

$$\mathbb{E}(X) = \mu \qquad \text{and} \qquad \mathbb{V}\mathrm{ar}(X) = \mu^2$$

Exponential distribution: example

Example

Suppose that, on average, 3 trucks arrive per hour to be unloaded at a warehouse. What is the probability that the time between the arrivals of two successive trucks will be a) less than 5 minutes? b) at least 45 minutes?

Other useful distributions

In the remainder of this course we will also encounter some other continuous distributions, among these are

- the Student-t (or just t) distribution, $X \sim t_\nu$;
- the χ^2 distribution, $X \sim \chi^2_\nu$;
- the Fisher-F (or just F) distribution, $X \sim \mathbf{F}_{d_1,d_2}$

We will return to them later when we will need them.

The several distributions that we have introduced so far are very useful in the application of statistics to problems of engineering and physical science.

The Normal distribution: introduction

However, the most widely used, and therefore the most important, statistical distribution is undoubtedly the

Normal distribution

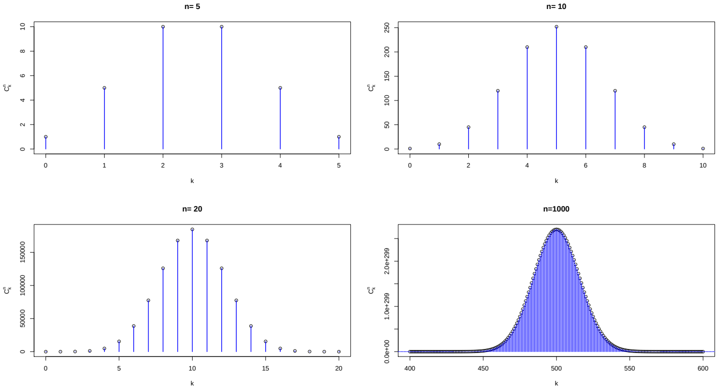
Its prevalence was first highlighted when it was observed that in many natural processes, random variation among individuals systematically conforms to a particular pattern:

- most of the observations concentrate around one single value (which is the mean)
- the number of observations smoothly decreases, symmetrically on either side, with the deviation from the mean
- it is very unlikely, yet not impossible, to find very extreme values

$\rightarrow$ this yields the famous **bell-shaped** curve

The Normal distribution: introduction

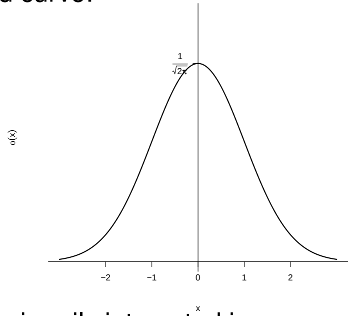
The bell-shaped curve was first spotted by the French mathematician Abraham de Moivre (1667-1754) who in his 1738 book "The Doctrine of Chances" showed that the coefficients $C_k^n = \binom{n}{k}$ in the binomial expansion of $(a+b)^n$ (see Slide 7) precisely follow the bell shape pattern when n is large



The Normal distribution: introduction

Later, Carl-Friedrich Gauss (1777-1855), a German mathematician (sometimes referred to as the *Princeps mathematicorum*, Latin for "the Prince of Mathematicians" or "the foremost of mathematicians") , was the first to write an explicit equation for the bell-shaped curve:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



When deriving his distribution, Gauss was primarily interested in errors of measurement, whose distribution typically follows the bell-shaped curve as well. He called his curve the “normal curve of errors”, which was to become the **Normal distribution**. In honour of Gauss, the Normal distribution is also often referred to as the **Gaussian distribution**

The Normal distribution: introduction

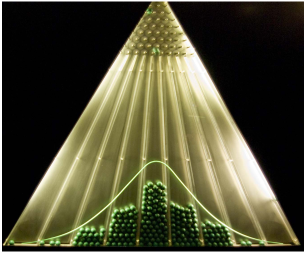
It is important to note that the Normal distribution is not just a convenient mathematical tool, but also occurs in natural phenomena.

For instance, in 1866 Maxwell, a Scottish physicist, determined the distribution of molecular velocity in a gas at equilibrium. As a result of unpredictable collisions with other molecules, molecular velocity in a given direction is randomly distributed, and from basic assumptions, that distribution can be shown to be the Normal distribution

James Maxwell (1831-1879)

The Normal distribution: introduction

Another famous example is the “bean machine”, invented by Sir Francis Galton (English scientist, 1822-1911) to demonstrate the Normal distribution. The machine consists of a vertical board with interleaved rows of pins. Balls are dropped from the top, and bounce left and right as they hit the pins. Eventually, they are collected into bins at the bottom. The height of ball columns in the bins approximately follows the bell-shaped curve.



The Normal distribution

A random variable is said to be normally distributed with parameters μ and σ ($\sigma > 0$), i.e.

$$X \sim \mathcal{N}(\mu, \sigma),$$

if its probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (\rightarrow S_X = \mathbb{R})$$

Unfortunately, no closed form exists for

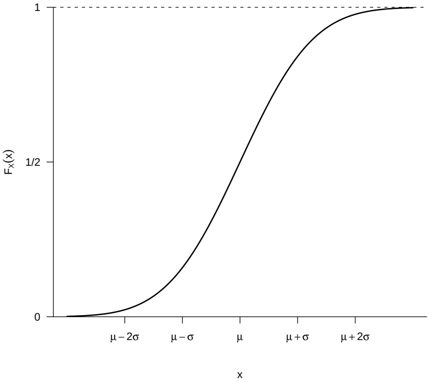
$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy$$

Important remark: Be careful! Many sources use the alternative notation

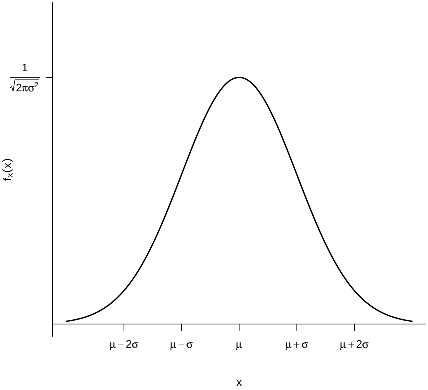
$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$\rightarrow$ in the textbook and in Matlab, the notation $\mathcal{N}(\mu, \sigma)$ is used, so we adopt it in these slides as well

The Normal distribution



cdf $F(x)$



pdf $f(x) = F'(x)$

Objectives

Now you should be able to:

- Understand the assumptions for some common discrete probability distributions
- Select an appropriate discrete probability distribution to calculate probabilities in specific applications
- Calculate probabilities, determine means and variances for some common discrete probability distributions
- Understand the assumptions for some common continuous probability distributions
- Select an appropriate continuous probability distribution to calculate probabilities in specific applications
- Calculate probabilities, determine means and variances for some common continuous probability distributions

Recommended exercises

$\rightarrow$ Q53 p.54, Q55 p.55, Q57 p.55, Q33 p.221, Q37 p.222, Q23 p.78, Q19 p.31, Q23 p.31, Q62 p.56 (2nd edition)

$\rightarrow$ Q54 p.56, Q56 p.57, Q58 p.57, Q35 p.226, Q39 p.227, Q24 p.79, Q19 p.34, Q23 p.35, Q64 p.58 (3rd edition)