Week 6-2017

Synchronous Sequential Circuit II

Characteristic Tables

JK
D
T

- **A characteristic table defines the logical properties of a flip-flop by describing its operation in tabular form.**

- **It defines the next state as a function of the inputs and the presents state.**

X J K Q(t+1) Y

- **The next state is the state of the flip-flop after the clock transition and denoted by Q(t+1).**

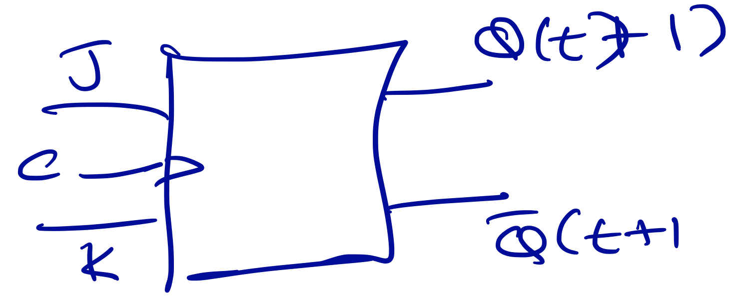
next

- **The present state is the state of the flip-flop immediately before the clock edge and denoted by Q(t).**

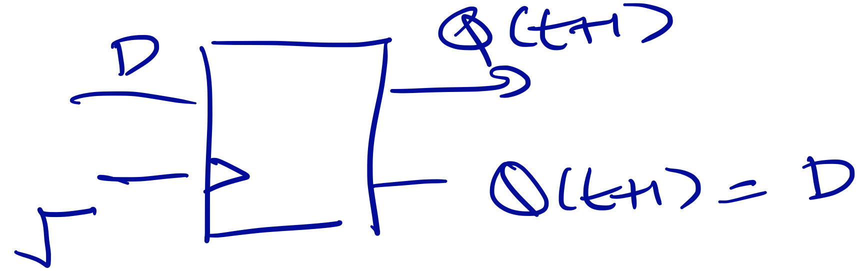
present

Characteristic Tables

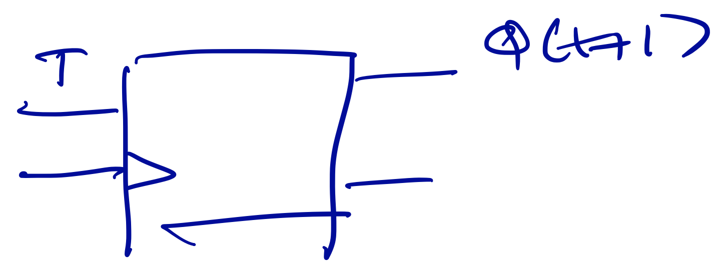
J	K	Q (t+1)	
0	0	$Q(t)$	unchanged
0	1	0	Reset
1	0	1	Set
1	1	$\overline{Q}(t)$	complement



D	Q (t+1)	
0	0	reset
1	1	set



T	Q (t+1)	
0	$Q(t)$	unchanged
1	$\overline{Q}(t)$	complemented



Characteristic Equations

Q	J	K	Q (t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Q(t+1):

Q \ JK	00	01	11	10
0	0	0	1	1
1	1	0	0	1

J spans columns 11 and 10; K spans columns 01 and 11; Q marks row 1.

$$Q(t+1) = J\bar{Q} + \bar{K}Q$$

Characteristic Equations

Q	D	Q (t+1)
0	0	0
0	1	1
1	0	0
1	1	1

Q(t+1):

Q \ D	0	1
0	0	1
1	0	1

$Q(t+1) = D$

Q	T	Q (t+1)
0	0	0
0	1	1
1	0	1
1	1	0

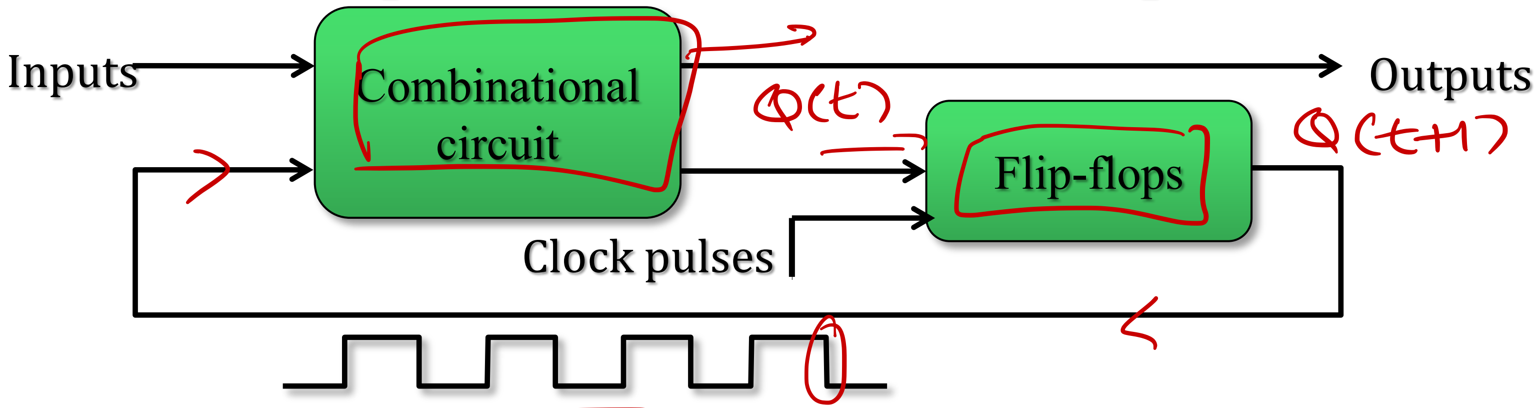
Q(t+1):

Q \ T	0	1
0	0	1
1	1	0

$Q(t+1) = \bar{Q}T$
$+ Q\bar{T}$
$= T \oplus Q$

Analysis of clocked Sequential Circuit

- The behavior of a clocked sequential circuit is determined from the inputs, internal states and outputs.



Time diagram of clock pulses

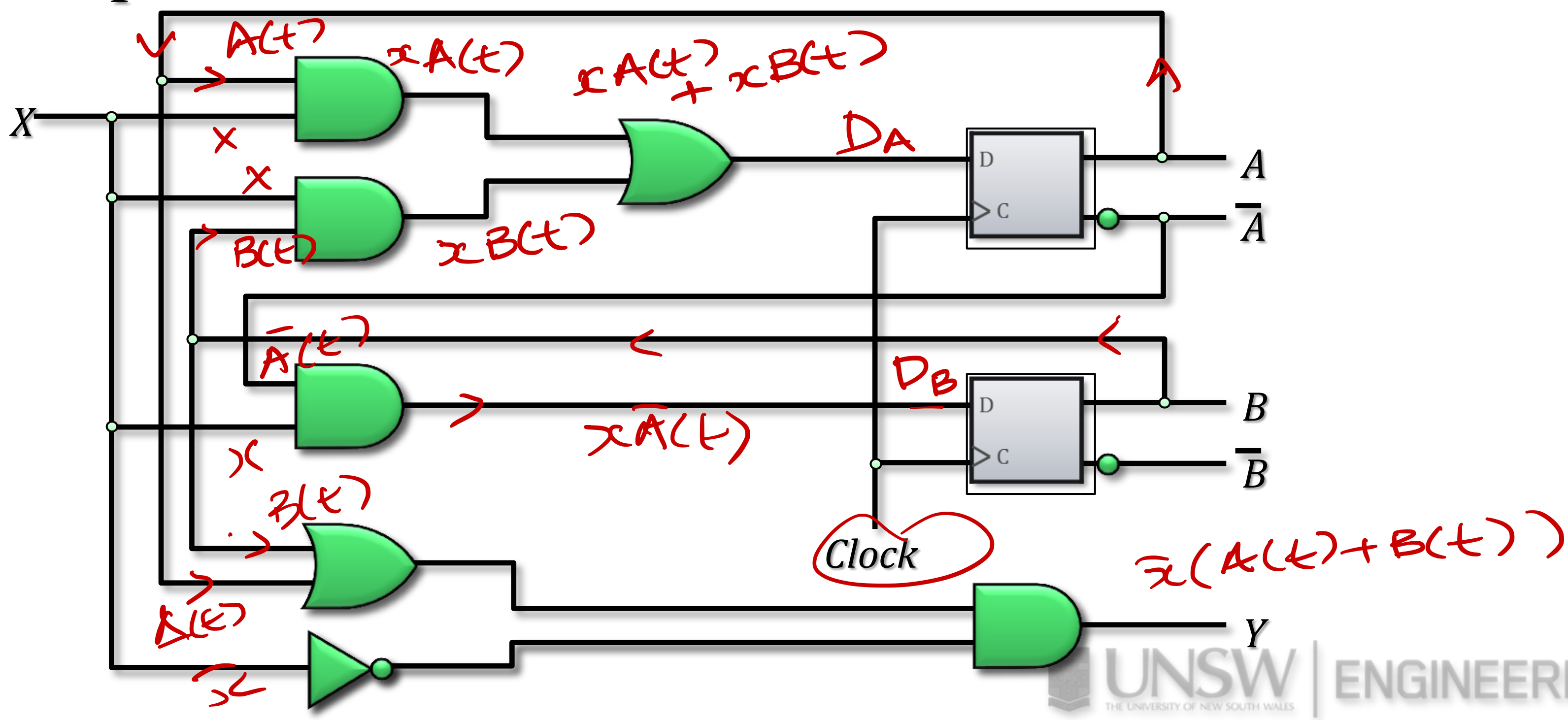
- The outputs and the next state are both the function of the inputs and the present state.
- Analysis of sequential circuit consists of obtaining a table or a diagram for the time sequence of inputs, internal states and outputs.
- Boolean expressions can be used to describe the behavior of the sequential circuit. In this case, it should include the necessary time sequence

Analysis of clocked Sequential Circuit

- State equation, state table and state diagram will be used to describe the behavior of any clocked sequential circuit.

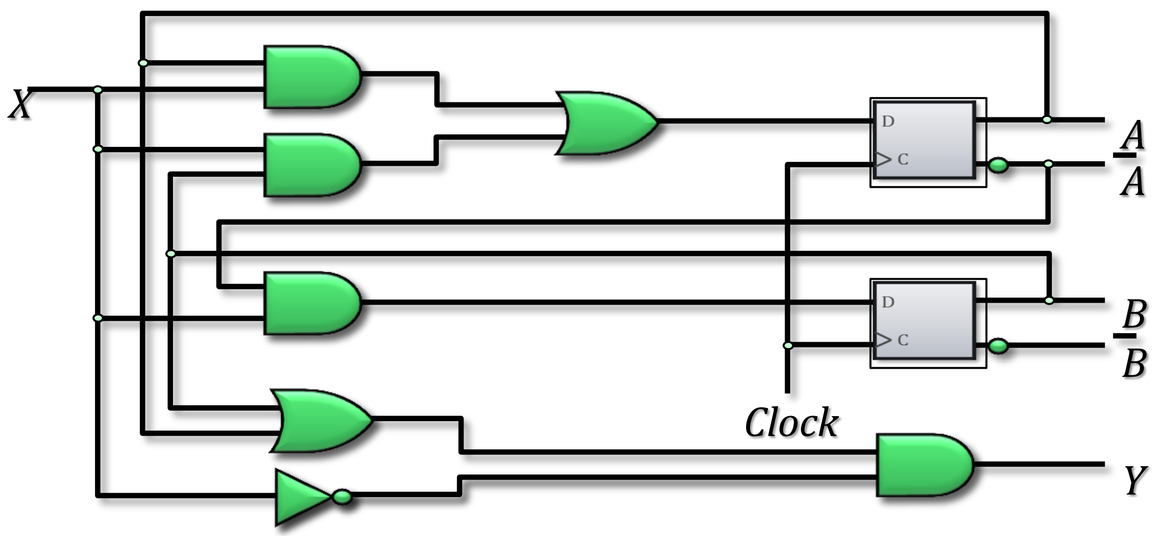
State Equations (Transition Equation):

- A state equation (transition equation) specifies the next state as a Boolean function of the present state and inputs.



State Equations

State Equations (Transition Equation):



Since the D input of a flip-flop determines the value of the next state (i.e, the state reached after the clock transition)

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

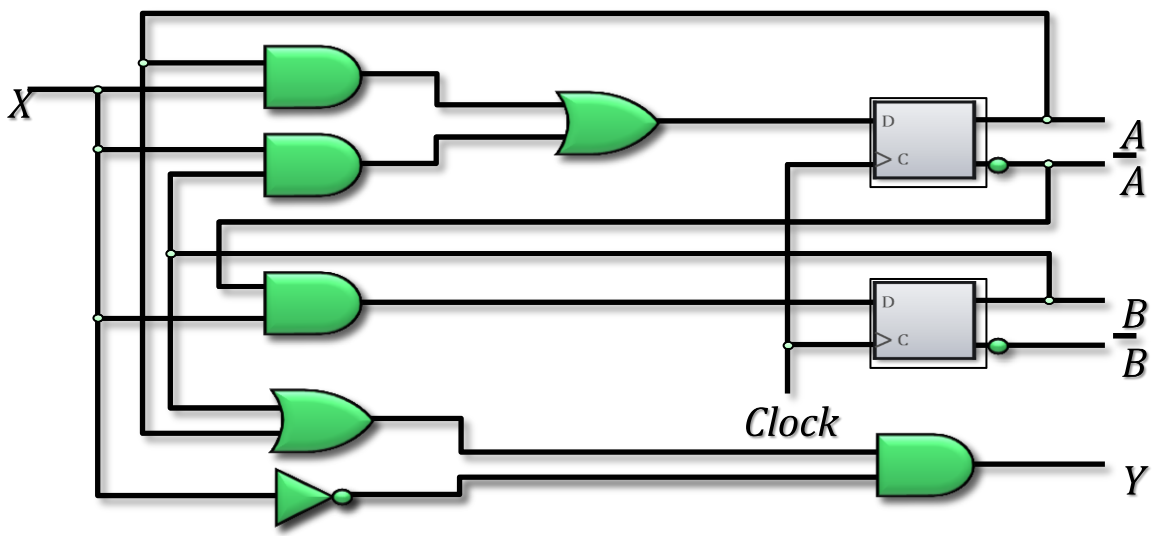
$$B(t+1) = \overline{A(t)}x(t)$$

The present-state value of the output can be expressed algebraically as

$$Y(t) = [A(t) + B(t)]\overline{x(t)}$$

State Equations

State Equations (Transition Equation):



x
A
B

$A(t+1)$
$B(t+1)$
Y

Since all variables in the Boolean expressions are a function of the present state, the designation of (t) after each variable can be omitted

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \overline{A}x$$

The present-state value of the output can be expressed algebraically as

$$Y(t) = [A+B]\overline{x}$$

State table

State table (Transition table):

- It is possible to enumerate the time sequences of inputs, outputs and states of the sequential circuit using state table (transition table)

- The table consists of four sections labelled present state, input, next state and output. → Y

  A(t) B(t)

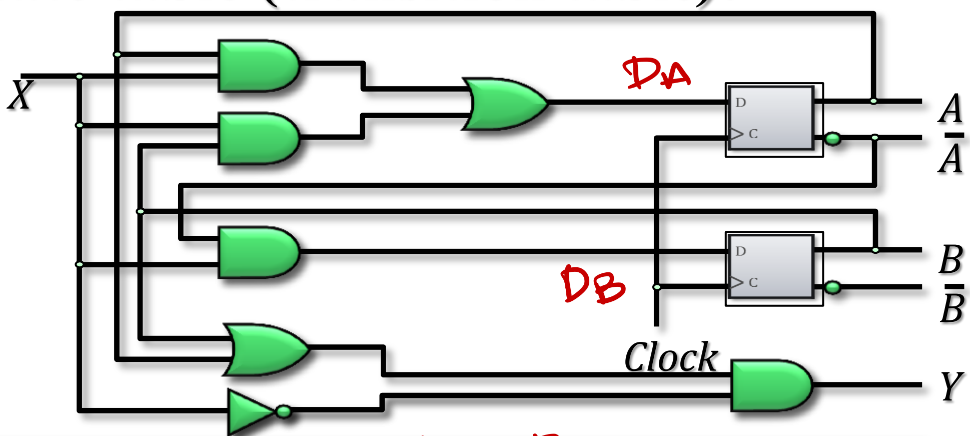
  x A(t+1) B(t+1)

- The derivation of a state table requires listing all possible binary combinations of present states and inputs.

- The next-states and the output values are then determined from the logic diagram or state equations for each possible binary combination.

State table

State table (transition table):



Present state		Input	Next state (D_A)	(D_B)	Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \overline{A}x$$
$$Y(t) = [A+B]\overline{x}$$

State table

State table (transition table):

wx 00 01 10 11

- The state transition may be conveniently expressed in a slightly different form having only three sections: present state, next state and output.

Present state	Next state				Output	
	X=0		X=1		X=0	X=1
A B	A	B	A	B		
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0

$$A(t+1) = Ax + Bx$$
$$B(t+1) = \bar{A}x$$
$$Y(t) = [A+B]\bar{x}$$

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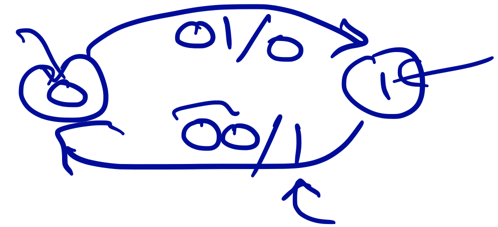
State diagram



- The information available in state tables can be represented graphically in the form of a state diagram.

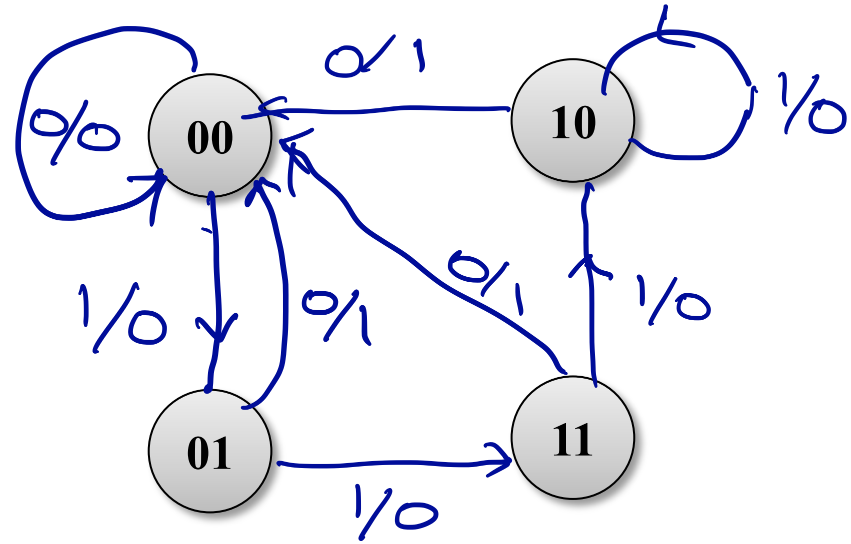
- In a state diagram, a state is indicated by a circle and the transitions between the states are represented by directed lines connecting the circles.

- The directed lines are labeled with two binary numbers separated by a slash. The first binary number is for the input value and the second number shows the output value for the given input value during the present state which is represented by a circle from which the directed line emanates.

State diagram

- The state diagram for the following state table can be drawn as:

Present state	Next state				Output	
	X=0		X=1		X=0	X=1
A B	A	B	A	B		
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0



- The same information is conveyed by both state diagram and state table.
- State diagrams provides pictorial representation that is easier for human representation.

State and output time sequence

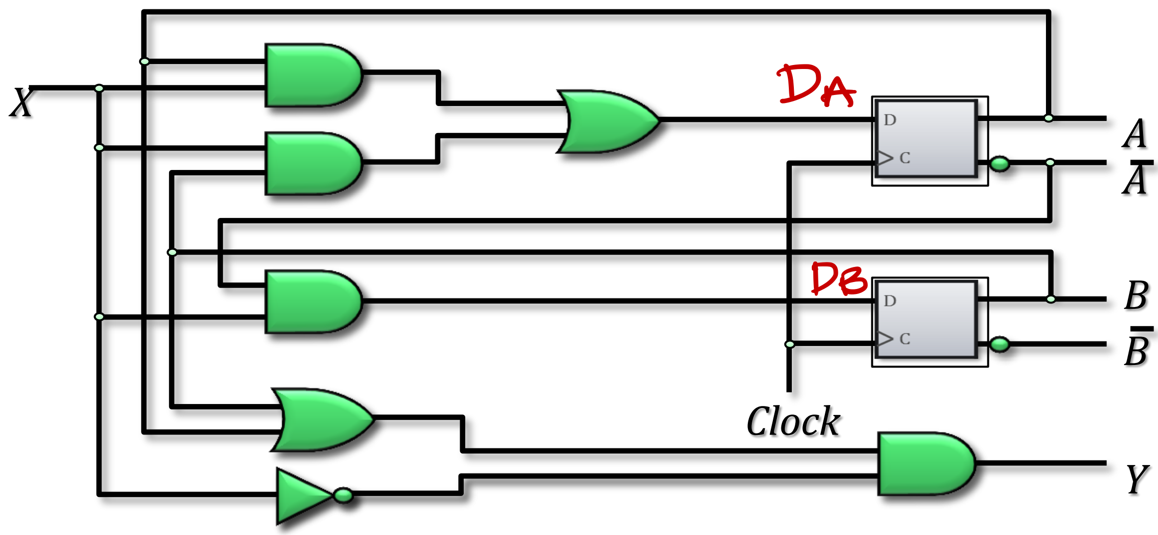
- Find state and output time sequence for input sequence of 010100110 assuming the sequential is initially in state 00

Clk										
X	0	1	0	1	0	0	1	1	0	0
A	0	0	0	0	0	0	0	0	1	0
B	0	0	1	0	1	0	0	1	1	0
Y	0	0	1	0	1	0	0	0	1	0

Present state	Next state				Output	
	X=0		X=1		X=0	X=1
A B	A	B	A	B		
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0

Flip-flop input equations

- The set of Boolean functions that describes the part of the sequential circuit that generates the inputs to flip-flops.



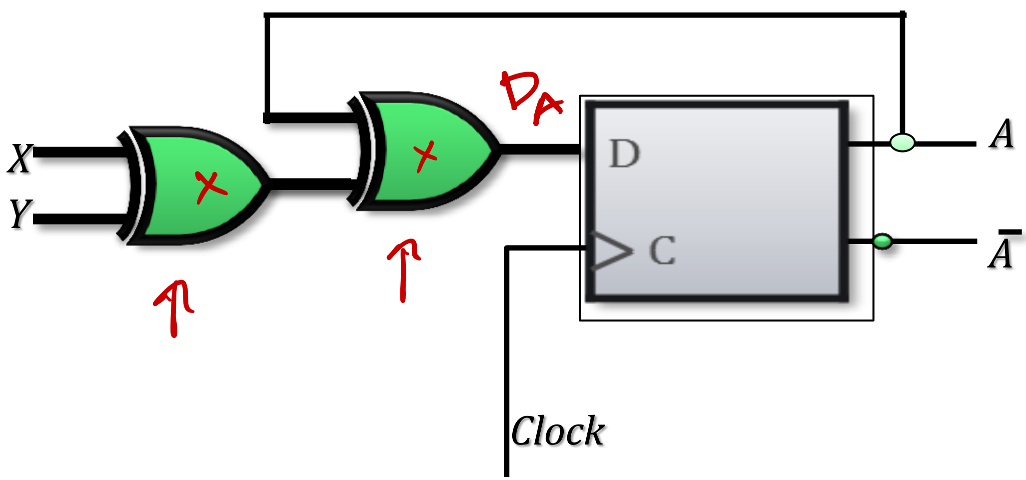
$$D_A = Ax + Bx$$
$$D_B = \bar{A}x$$
$$Y = [A + B]\bar{x}$$

- The three equations provide the necessary information for drawing the logic diagram of the sequential circuit.
- The symbol D_A specifies a D flip-flop labeled A and D_B is for another D flip-flop labeled B.
- The flip-flop equations specify the combination circuit that drives the flip-flops and the type of flip-flops used.

Analysis with D flip-flops

- Steps in analyzing clocked sequential circuits with D flip-flops:
  - Find the flip-flop input equations.
  - The state equations are the same as the flip-flop input equations for D flip-flops.
  - Derive the state table from the state equations.
  - Draw the state diagram from the state table

Example:

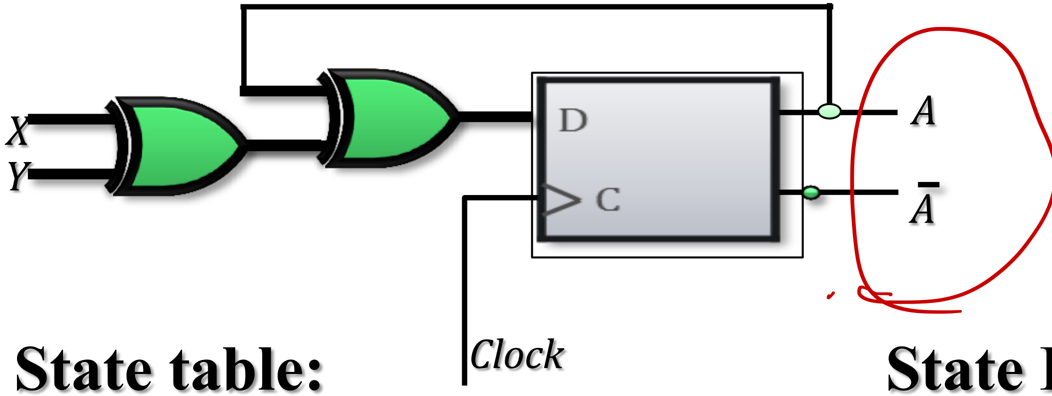


$D_A = X \oplus Y \oplus A$

$A(t+1) = D_A$

$= X \oplus Y \oplus A$

Analysis with D flip-flops

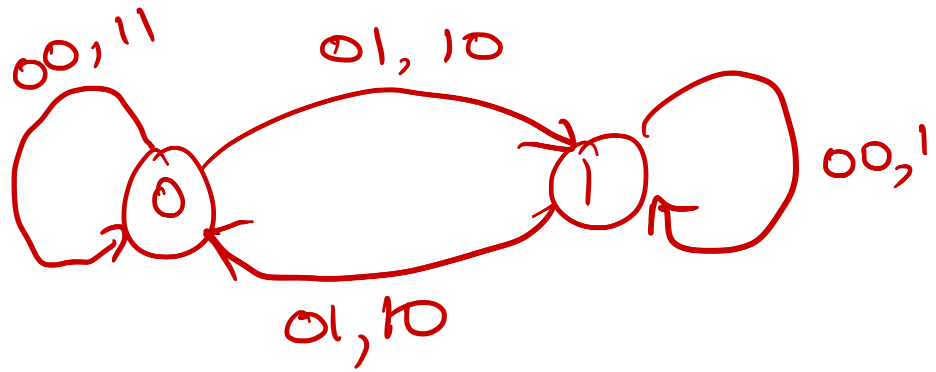


$A(t+1) = X \oplus Y \oplus A$

- **State table:**

Present state	Input		Next state
A	**X**	**Y**	**A**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

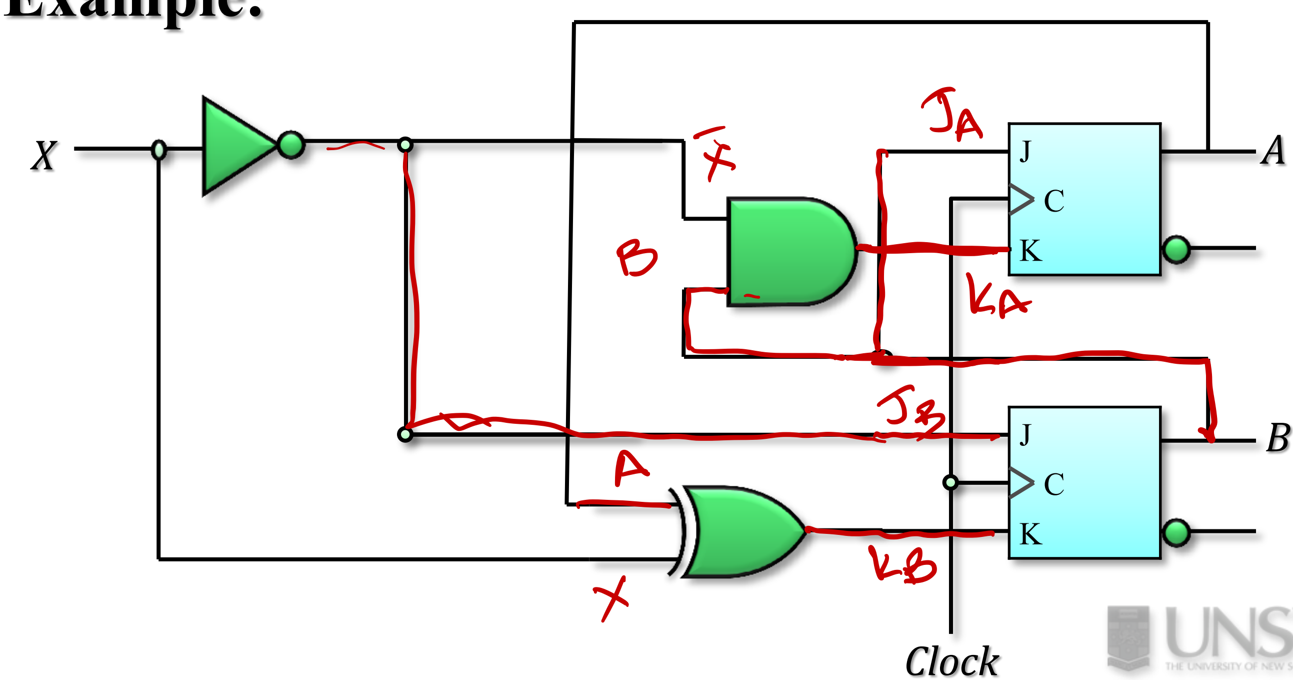
State Diagram:



Analysis with JK flip-flops

- Steps in analyzing clocked sequential circuits with JK flip-flops:
  - Find the flip-flop input equations in terms of present state and inputs.
  - List the binary values of each input equation.
  - Use the JK flip-flop characteristic table to determine the next states in the state table.
  - Use K-map to obtain minimized state equations

Example:



$J_A = B$

$K_A = B\bar{x}$

$J_B = \bar{x}$

$K_B = A \oplus x$

Analysis with JK flip-flops

- **Flip-flop input equations:**

$J_A = B$ $\qquad$ $K_A = B\bar{x}$

$J_B = \bar{x}$ $\qquad$ $K_B = A \oplus x$

$Q(t+1) = J\bar{Q} + \bar{K}Q$

$A(t+1) = J_A\bar{Q} + \bar{K}_A Q$

Present state		Input	Next state		Flip-Flop inputs			
A	B	X	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	0	0	0	0	0	0	1
0	1	1	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	0	1	0	0	0	0	0
1	1	1	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

A \ Bx	00	01	11	10
0	0	0	1	1
1	1	1	1	0

$A(t+1) = \bar{A}B + A\bar{B} + Ax$

A \ Bx	00	01	11	10
0	1	0	0	1
1	1	0	1	0

$B(t+1) = \bar{B}\bar{x} + \bar{A}\bar{x} + AxB$

Analysis with JK flip-flops

- Steps in analyzing clocked sequential circuits with JK flip-flops:
  - Find the flip-flop input equations in terms of present state and inputs.
  - Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.
  - Use the corresponding state equations to determine the next-values in the state table and state diagrams

For previous example:

J_A= B $\qquad K_A = B\overline{X}$

$J_B = \overline{X} \qquad K_B = \overline{A \oplus x}$

Characteristic equation for JK flip-flop:

$$A(t+1) = J_A\overline{A} + \overline{K_A}\mathrm{A}$$

$$\mathrm{B}(t+1) = J_B\overline{B} + \overline{K_B}B$$

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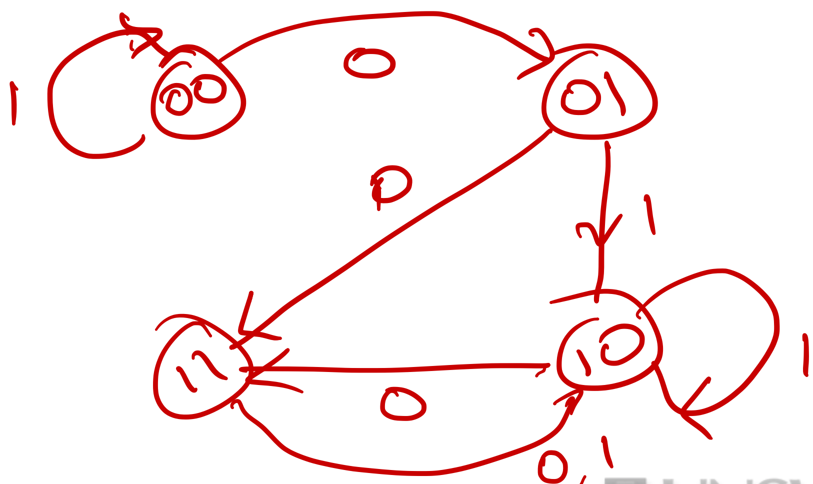
Analysis with JK flip-flops

State Table:

Present state			Next State				
			X=0		X=1		
A	B		A	B	A	B	
0	0		0	1	0	0	
0	1		1	1	1	0	
1	0		1	1	1	0	
1	1		1	0	1	0	

State Diagram:



Analysis with T flip-flops

- The same procedure as JK flip-flop is to be followed:
  - Find the flip-flop input equations in terms of present state and inputs.
  - List the binary values of each input equation.
  - Use the T flip-flop characteristic table to determine the next states in the state table.
  - Use K-map to obtain minimized state equations

OR

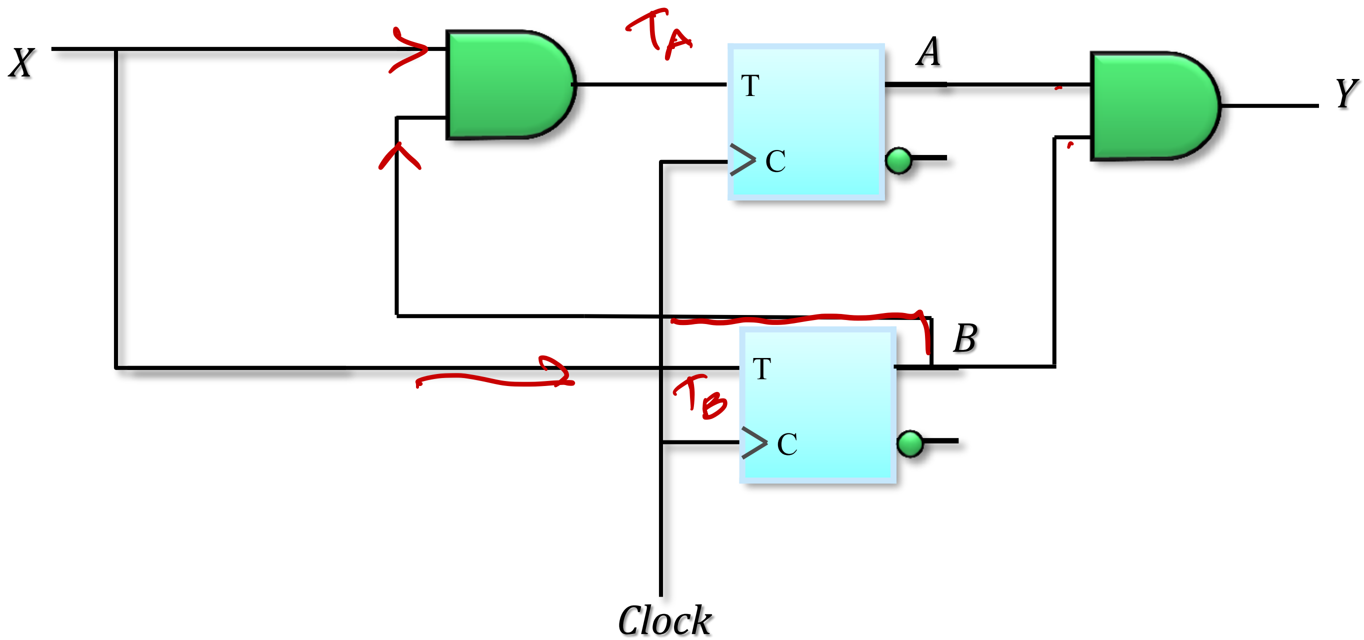
  - Find the flip-flop input equations in terms of present state and inputs.
  - Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.
  - Use the corresponding state equations to determine the next-values in the state table and state diagrams

Analysis with T flip-flops

T=0 $Q(t+1) = Q(t)$

T=1 $Q(t+1) = \overline{Q}(t)$

Example:



Flip-flop input equations:

$$T_A = Bx$$

$$T_B = x$$

Output equation:

$$Y = AB$$

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Analysis with T flip-flops

$T = 0 \quad Q(t+1) = Q(t)$

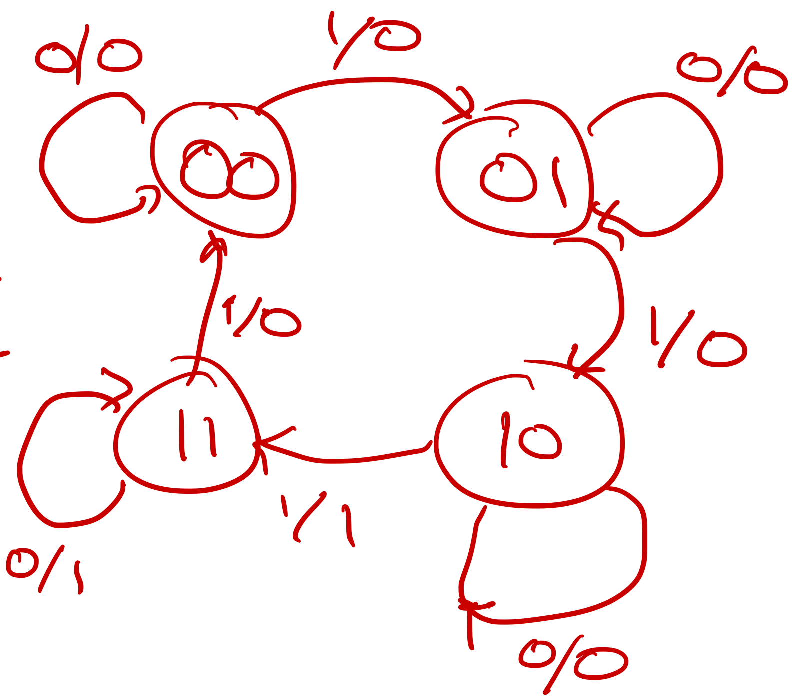
$\;\;= 1 \quad Q(t+1) = \overline{Q}(t)$

Present state		Input	Next state		Flip-Flop inputs		
A	B	X	A	B	T_A	T_B	Y
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	0	1	0	0	0
0	1	1	1	0	1	1	0
1	0	0	1	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	0	0	1
1	1	1	0	0	1	1	0

$T_A = xB$

$T_B = x$

$Y = AB$



Analysis with T flip-flops

For previous example:

$T_A = xB$ $\cancel{K_A = B\bar{x}}$

$T_B = x$

Characteristic equation for T flip-flop:

$$A(t+1) = T_A \oplus A$$

$$B(t+1) = T_B \oplus B$$

Analysis with T flip-flops

State Table:

Present state			Next State				Output state	
			X=0		X=1		X=0	X=1
A	B		A	B	A	B	Y	Y
0	0		0	0	0	1	0	0
0	1		0	1	1	0	0	0
1	0		1	0	1	1	0	0
1	1		1	1	0	0	1	1

State Diagram: