# Lecture 3: Basic Circuit Laws

ELEC1111 Electrical and Telecommunications Engineering

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Never Stand Still

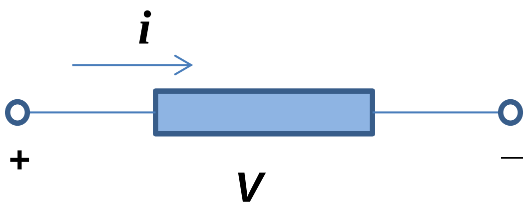
Faculty of Engineering

School of Electrical Engineering and Telecommunications

# CONTENTS

# 1. CIRCUIT ELEMENTS

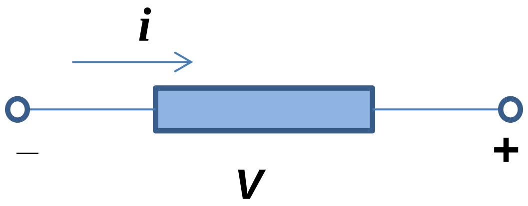
- **Passive element** – absorbs energy, so that the total energy delivered to it from the rest of the circuit is non-negative.




$w = \int_{-\infty}^{t} v.i \, d\tau \geq 0$ for all values of $t$

A resistor is a passive element

- **Active element** – capable of supplying energy.



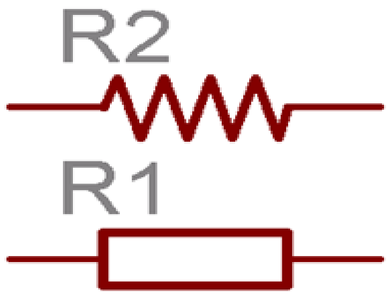

$w = \int_{-\infty}^{t} v.i \, d\tau > 0$ for at least one value of $t$

e.g. batteries and generators

# 1. CIRCUIT ELEMENTS

## Resistors

- **Resistance –** physical property of a circuit element that impedes the flow of charge. The element is a **resistor**.



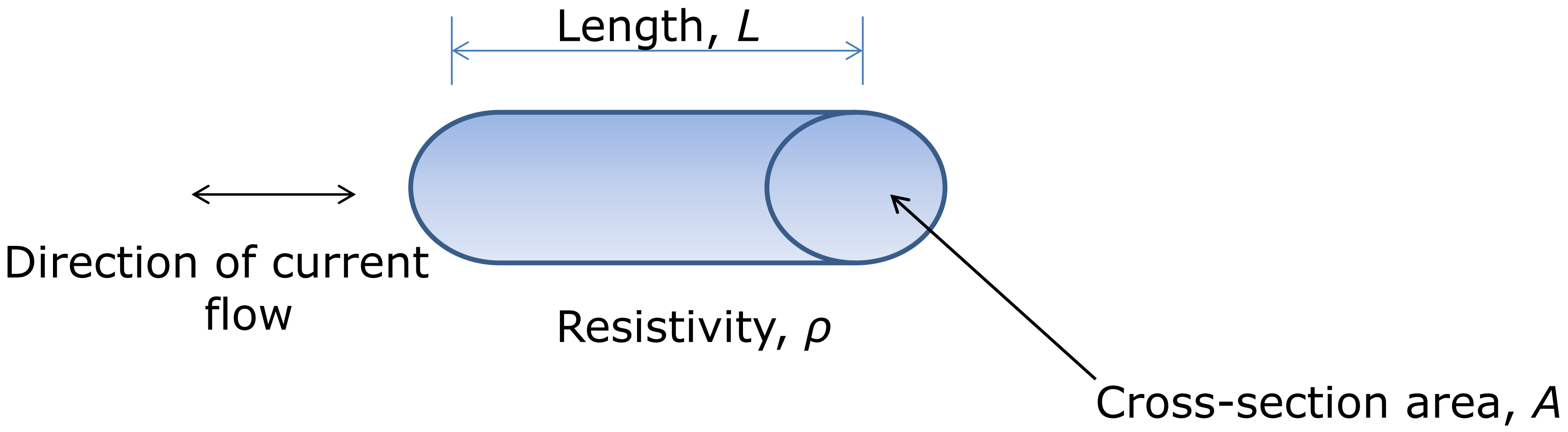

- **Resistivity –** a measure of the ability to resist the flow of charge by a certain material.

  **Copper –** a good conductor, low resistivity
  **Polystyrene –** a good insulator, high resistivity
  **Silicon -** a good semi-conductor, intermediate resistivity

# 1. CIRCUIT ELEMENTS

## Resistors

- ✓ Resistance of a conducting wire of uniform cross-section:




$$R = \frac{\rho L}{A}$$

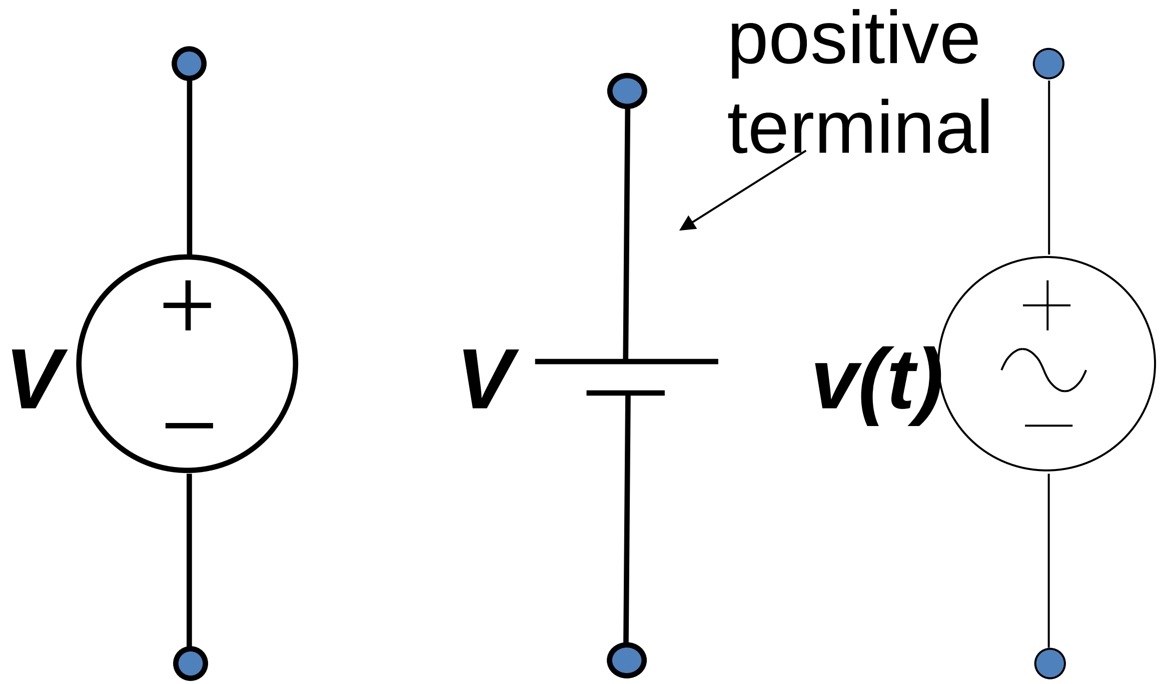
# 1. CIRCUIT ELEMENTS

## Independent voltage source

- ✓ Provides a specified voltage which is **independent of current through it** or any other circuit variables.
- ✓ It is an **ideal source,** used to approximate a practical voltage source.
- ✓ No practical source is truly independent.

# 1. CIRCUIT ELEMENTS

## Symbols




(DC) voltage source

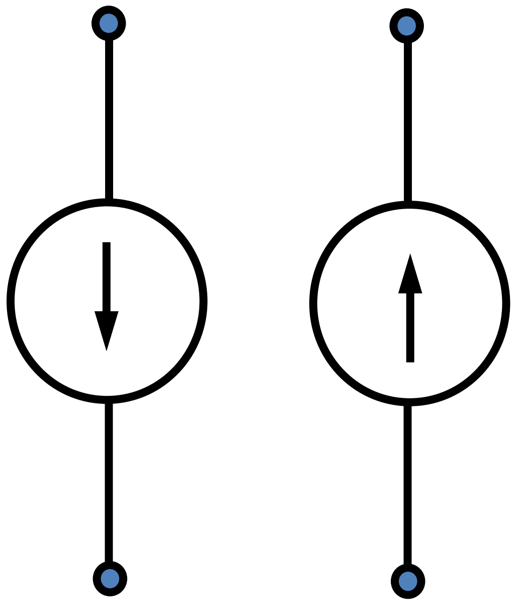
Battery

AC voltage source

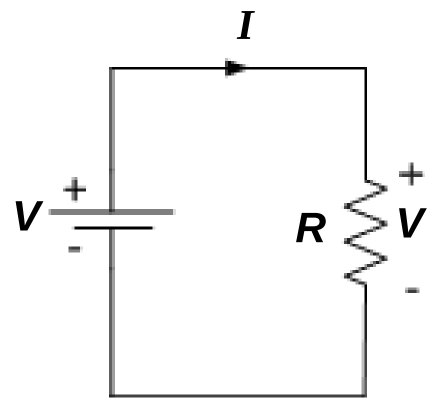
# 1. CIRCUIT ELEMENTS

## Independent current source

- ✓ It provides a specified current which is independent of voltage across the source element or any other circuit variables.



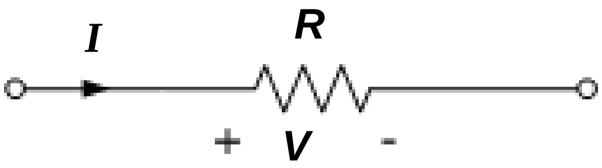
# 1. CIRCUIT ELEMENTS



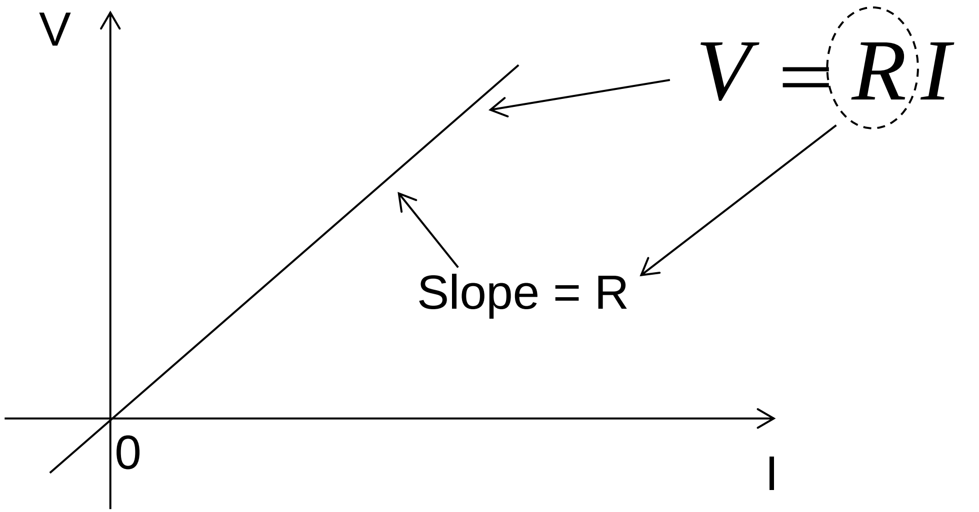

- Circuit Elements: Battery, resistor, conducting wire.
- Circuit variables: Voltage *V*, current *I*, resistor *R*.

| | What is known | What is needed |
|---|---|---|
| Case 1 | Battery voltage, *V* & resistor value, *R* | Current, *I* |
| Case 2 | Battery voltage, *V* & current, *I* | Resistor, *R* |
| Case 3 | Current, *I* & resistor, *R* | Battery voltage, *V* |

# 2. OHM'S LAW




$$\boxed{V \propto I}$$




So the voltage across a resistor, *V*, is directly proportional to the current, *I*, flowing through the resistor.

$$\boxed{V = IR}$$

$$or \quad I = \frac{V}{R} \quad or \quad R = \frac{V}{I}$$

# 2. OHM'S LAW

- ✓ Ohm's law can also be written as

$$i = G.v$$

  - here, *G* denotes the **conductance** in siemens (S) or mhos

$$G \triangleq \frac{1}{R}$$

- ✓ Power delivered to a resistor

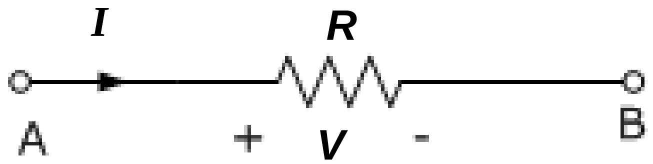
$$p = vi = v.\left(\frac{v}{R}\right) \longrightarrow p = \frac{v^2}{R}$$

$$p = vi = (iR).i \longrightarrow p = i^2R$$

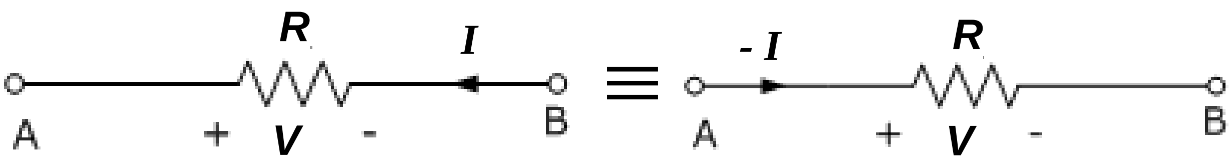
- ✓ What about energy absorbed by a resistor? – a passive element.
- ✓ Example: energy delivery by car battery with lights on for 4 hours?

# 2. OHM'S LAW – POLARITY CONVENTION

Current flows in the direction from high potential (voltage) to low potential (voltage).



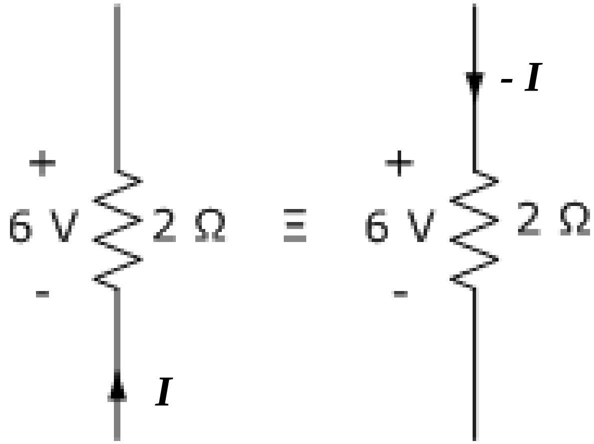

$V = V_{AB} = V_A - V_B = IR \Rightarrow V = IR$




$V = V_{AB} = V_A - V_B = -IR \Rightarrow V = -IR$

**Actual (physical) current flow same in both cases**

Example



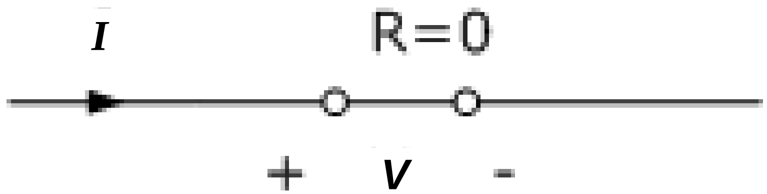

$6 = -2I$

$I = -3\ A$

# 2. OHM'S LAW – SHORT & OPEN CIRCUIT

## SHORT CIRCUIT

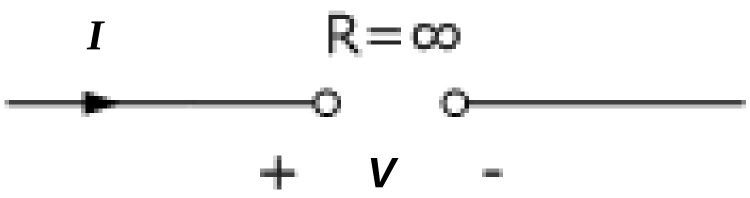
As $R \to 0$




$$V = IR = 0$$

## OPEN CIRCUIT

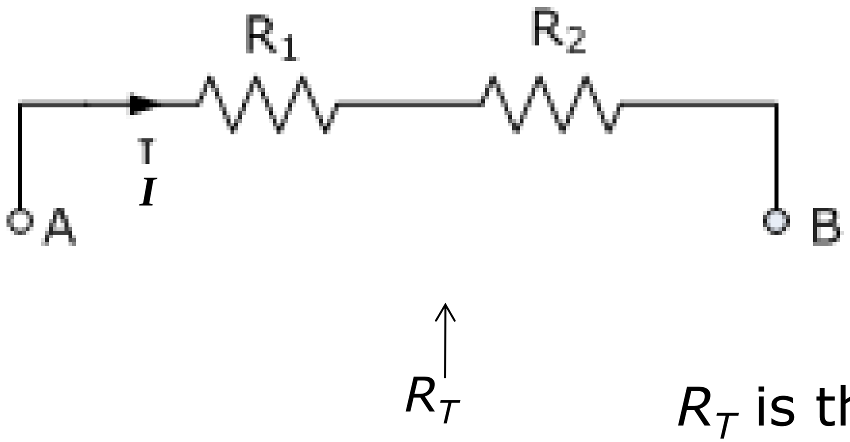
As $R \to \infty$




$$I = \frac{V}{R} = \lim_{R \to \infty} \frac{V}{R} = 0$$

# 3. SERIES AND PARALLEL RESISTORS

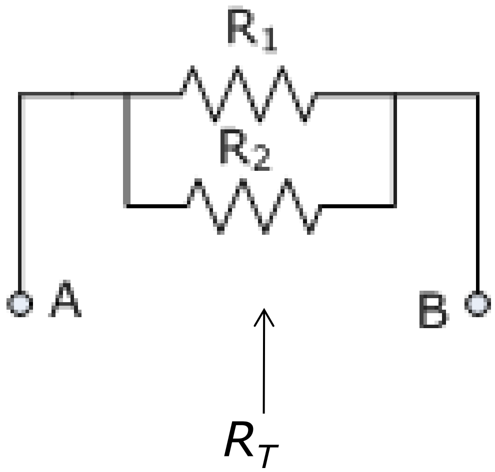
Series: *Must have the same current*




$$R_T = R_1 + R_2$$

$R_T$ is the equivalent resistance looking into the terminals A & B

Parallel *Must have the same voltage*



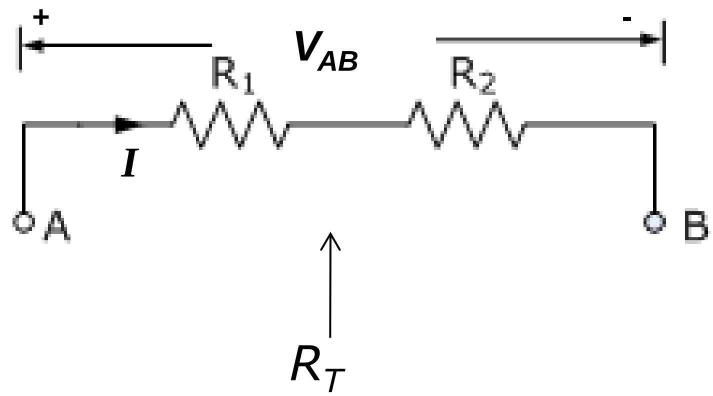

$$R_T = R_1 \parallel R_2$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow R_T = \frac{R_1 R_2}{R_1 + R_2}$$

# 3. SERIES AND PARALLEL RESISTORS

## PROOF FOR SERIES RESISTORS




$$IR_1 + IR_2 = V_{AB}$$

$$\therefore \qquad I = \frac{V_{AB}}{R_1 + R_2}$$

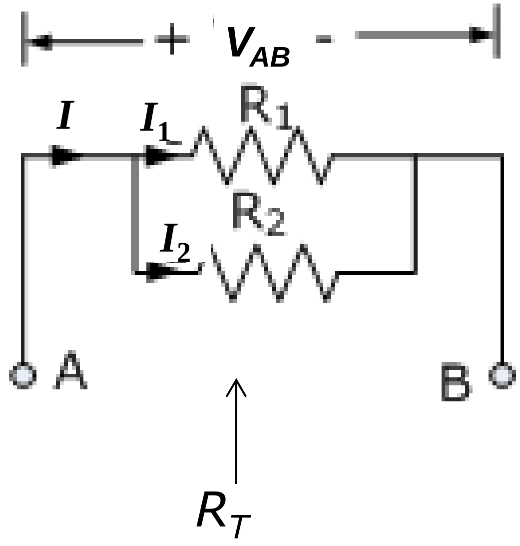
$$\therefore \qquad \frac{V_{AB}}{I} = R_1 + R_2 = R_T$$

So $\boxed{R_T = R_1 + R_2}$

$V_{AB} = V_A - V_B$
$V_{BA} = V_B - V_A$
$V_{AB} = -V_{BA}$

# 3. SERIES AND PARALLEL RESISTORS

## PROOF FOR PARALLEL RESISTORS




We have

$$I = I_1 + I_2$$

$$V_{AB} = I_1 R_1 \Rightarrow I_1 = \frac{V_{AB}}{R_1}$$

$$V_{AB} = I_2 R_2 \Rightarrow I_2 = \frac{V_{AB}}{R_2}$$

But

$$I = I_1 + I_2 = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2}$$

$V_{AB}$ = Potential difference between A & B.

So $\therefore$

$$\frac{I}{V_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_T}$$

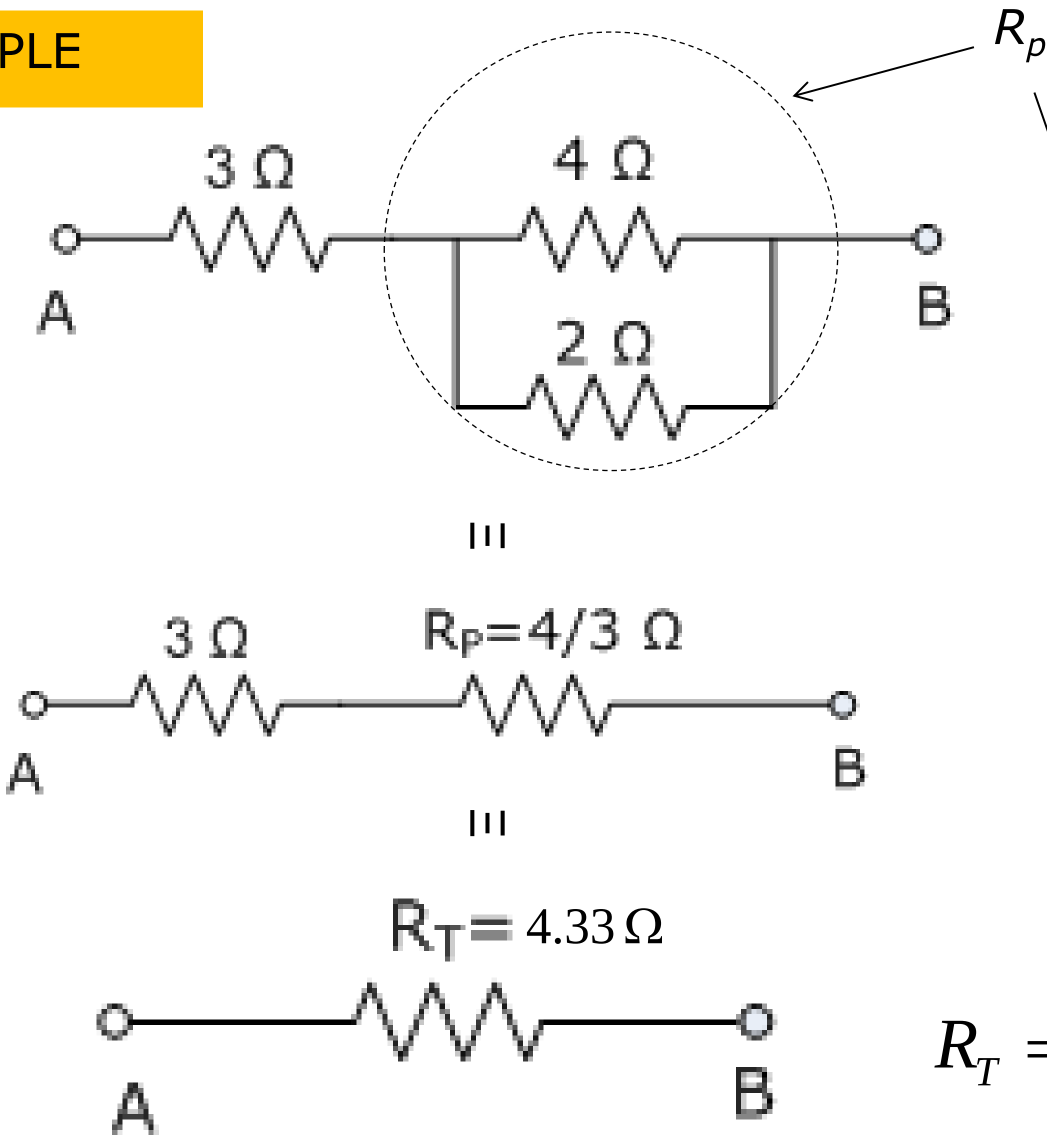
That is

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}}$$

$$\boxed{\Rightarrow R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

# 3. SERIES AND PARALLEL RESISTORS

EXAMPLE




$$R_P = 4\,\Omega \,||\, 2\,\Omega$$

$$\frac{1}{R_P} = \frac{1}{4} + \frac{1}{2}$$

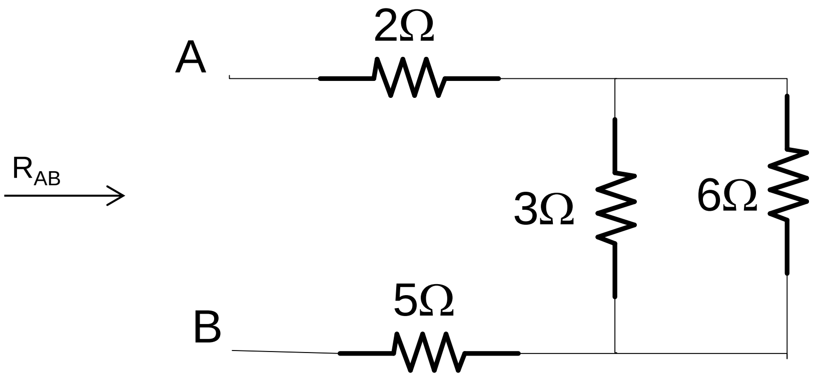
$$R_P = \frac{4 \times 2}{4+2} \approx 1.33\Omega$$

$$R_T = R_{AB} = 3 + \frac{4}{3} \approx 4.33\,\Omega$$

# 3. SERIES AND PARALLEL RESISTORS

EXAMPLE

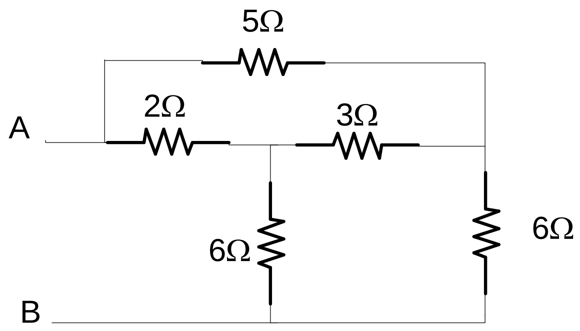
Find the equivalent resistor between A and B?



# 3. SERIES AND PARALLEL RESISTORS

EXAMPLE

Find the equivalent resistor between A and B?

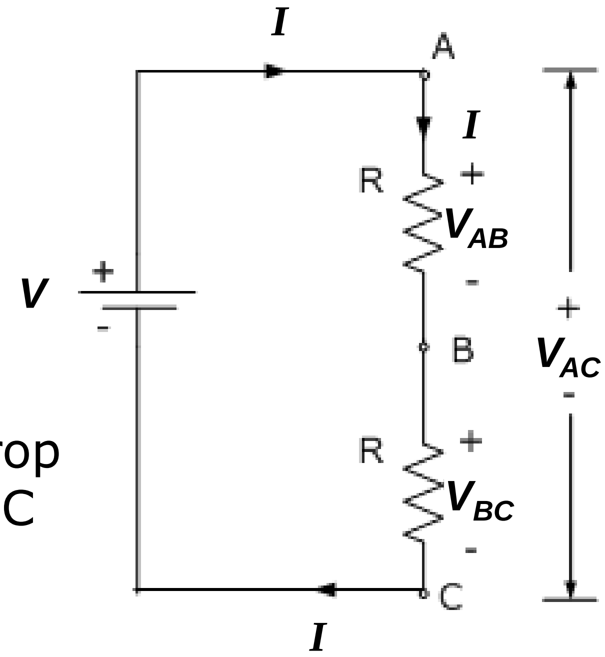



Note: Resistors can be connected neither in series nor in parallel

# 4. VOLTAGE DIVISION

The same current, $I$, flows through both resistors.

The voltage drop across each resistor is the same (because both resistors have the same resistance).
=> $V_{AB}=V_{BC}=IR$




Voltage drop from A to C = Voltage drop from A to B + Voltage drop from B to C

$$V_{AC} = V_{AB} + V_{BC}$$

$$V_{AC} = IR + IR = I(2R)$$

But $V_{AC} = V$

$$\therefore V_{AB} = IR = \frac{V}{2R} \times R = 0.5V$$

$$\therefore I = \frac{V}{2R}$$

Similarly $V_{BC} = 0.5V$

# 4. VOLTAGE DIVISION

What if $R_1 \neq R_2$ ?

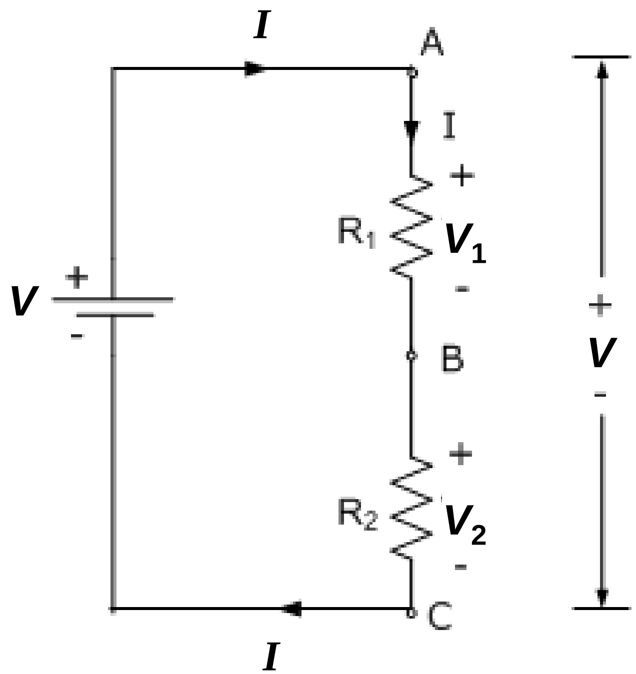
Again, $V = V_1 + V_2$

$$= IR_1 + IR_2 = I(R_1 + R_2)$$

So, $I = \dfrac{V}{R_1 + R_2}$

and $V_1 = IR_1 = \dfrac{V}{R_1 + R_2} R_1 = \left( \dfrac{R_1}{R_1 + R_2} \right) V$

similarly $V_2 = \left( \dfrac{R_2}{R_1 + R_2} \right) V$



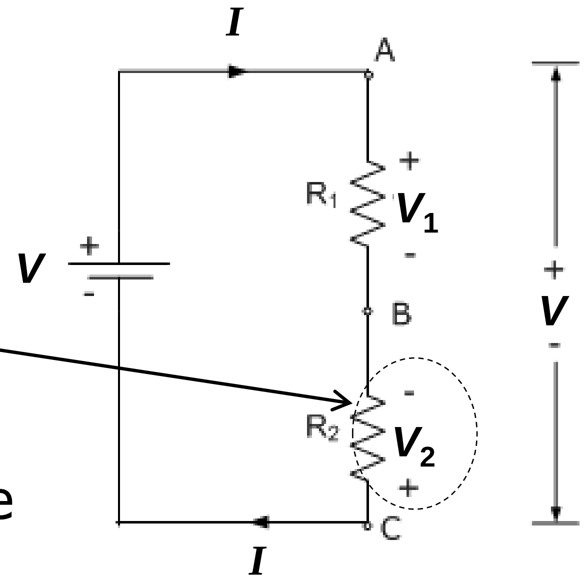

(Equivalent resistance in circuit is $R_1$+$R_2$ i.e. in series)

E.g. $R_1$=1Ω, $R_2$=4Ω $\Rightarrow V_1 = 1/5 = 0.2V, \quad V_2 = 4/5 = 0.8V$

# 4. VOLTAGE DIVISION

**POLARITY CONVENTION REMINDER**

Polarity on voltage **label** $V_2$ reversed




$V$ = Voltage difference across $R_1$ from A to B + Voltage difference across $R_2$ from B to C

$= \quad V_1 \quad + \quad (-V_2)$

a drop of $V_1$

An increase of $V_2$ = a drop of $(-V_2)$

$$V_1 = IR_1 \quad \text{and} \quad V_2 = -IR_2$$

So $\quad V = IR_1 - (-IR_2) = I(R_1 + R_2)$

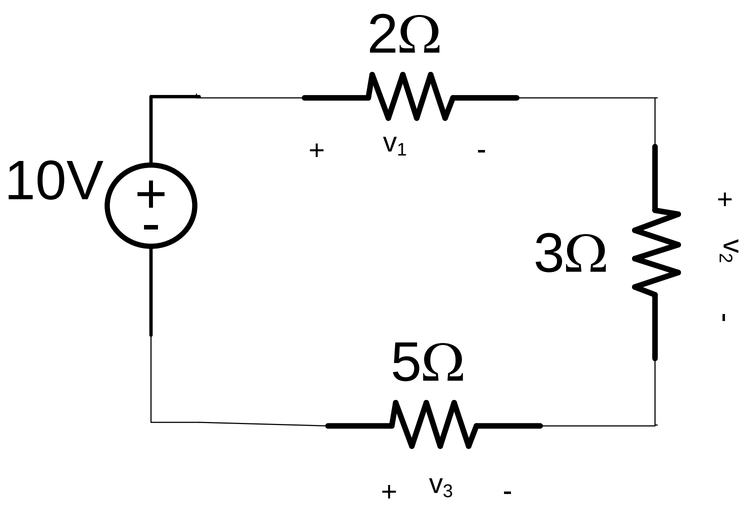
$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V$$

$$V_2 = -\left(\frac{R_2}{R_1 + R_2}\right)V$$

# 4. VOLTAGE DIVISION

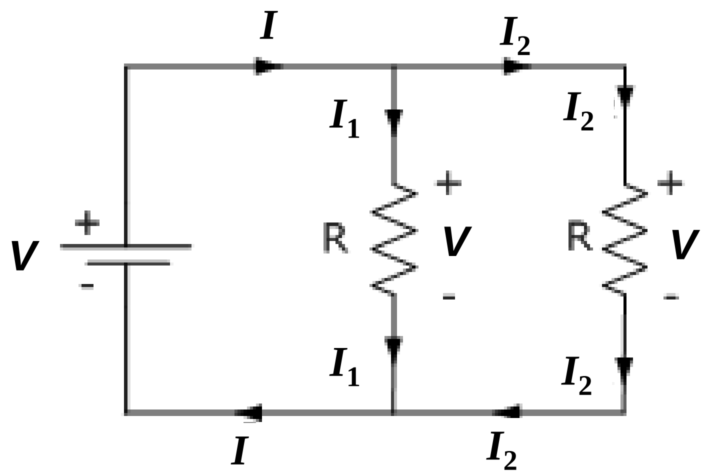
**Example**

Find the value of the voltage $V_1$, $V_2$ and $V_3$



# 5. CURRENT DIVISION

The same voltage, V, exists across both resistors.
Current through each resistor is the same (because both resistors have the same resistance)
=> $I_1=I_2=V/R$




Total current, $I$ = Current through first resistor, $I_1$ + Current through second resistor, $I_2$

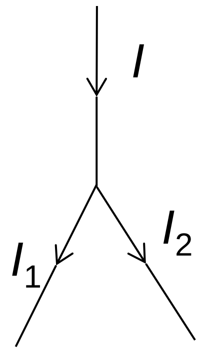
i.e.
$$I = I_1 + I_2$$
$$= \frac{V}{R} + \frac{V}{R} = V\left(\frac{1}{R} + \frac{1}{R}\right)$$

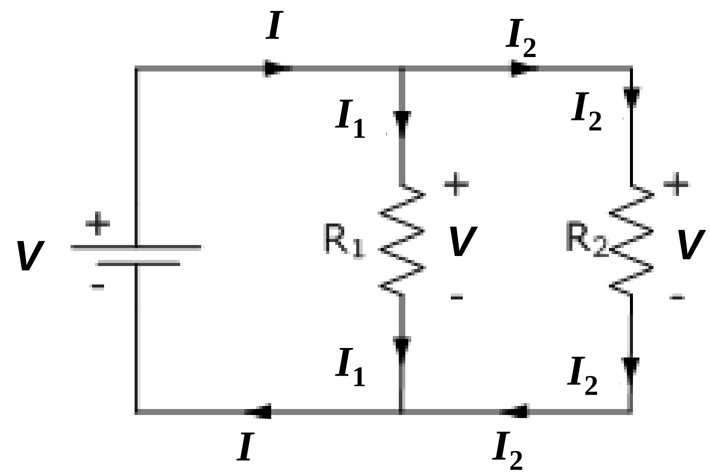
(Equivalent resistance in circuit is $R/2$, i.e. in parallel)

So $I = \dfrac{V}{R/2}$ or $\dfrac{V}{R} = \dfrac{I}{2}$ and $I_1 = I_2 = \dfrac{V}{R} = \dfrac{I}{2}$

# 5. CURRENT DIVISION

What if $R_1 \neq R_2$ ?




Again,

$$I = I_1 + I_2$$

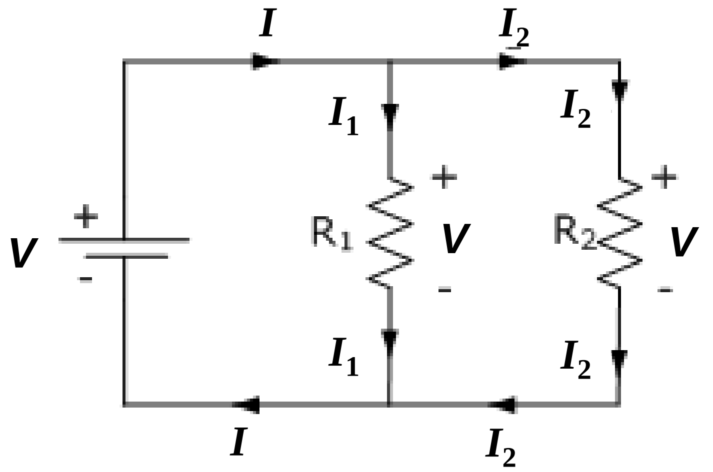
$$= \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{V}{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

$R_T$

(Equivalent resistance in circuit is $\frac{R_1 R_2}{R_1 + R_2}\Omega$, i.e. in parallel)

# 5. CURRENT DIVISION




$$V = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$So \quad I_1 = \frac{V}{R_1} = \left(\frac{R_2}{R_1 + R_2}\right) I$$

$$I_2 = \frac{V}{R_2} = \left(\frac{R_1}{R_1 + R_2}\right) I$$

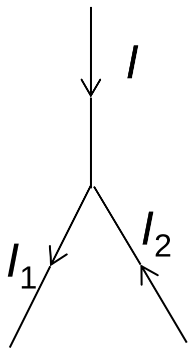
E.g. $R_1 = 1\,\Omega, \quad R_2 = 4\,\Omega$

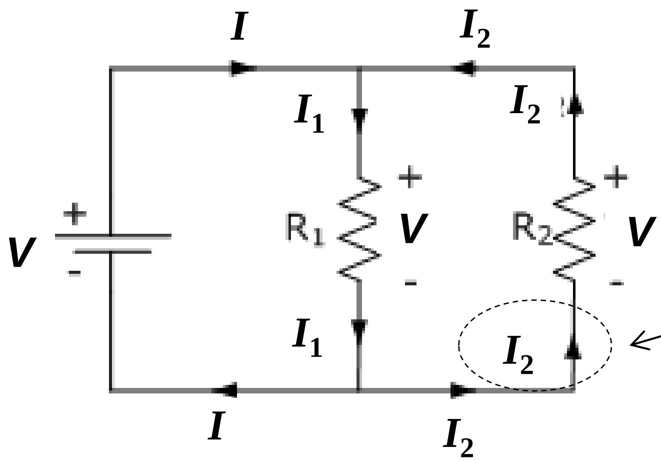
$$I_1 = V, \quad I_2 = \frac{V}{4}, \quad I = 1.25 \times V$$

Equivalent resistance in circuit = 1 Ω //4 Ω =0.8Ω

# 5. CURRENT DIVISION

## POLARITY CONVENTION AGAIN




Direction of current **label** is reversed

Total current, $I$ = Current "down" through $R_1$ + Current "down" through $R_2$ $= I_1 + \left(-I_2\right)$

$$V = I_1 R_1 = \left(-I_2 R_2\right) \Rightarrow I_1 = \frac{V}{R_1},\ I_2 = -\frac{V}{R_2}$$

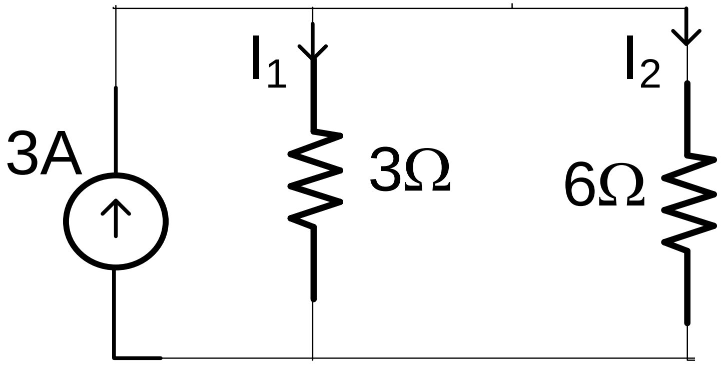
$$So,\quad I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I,\quad I_2 = -\left(\frac{R_1}{R_1 + R_2}\right) I$$

# 5. CURRENT DIVISION

Example

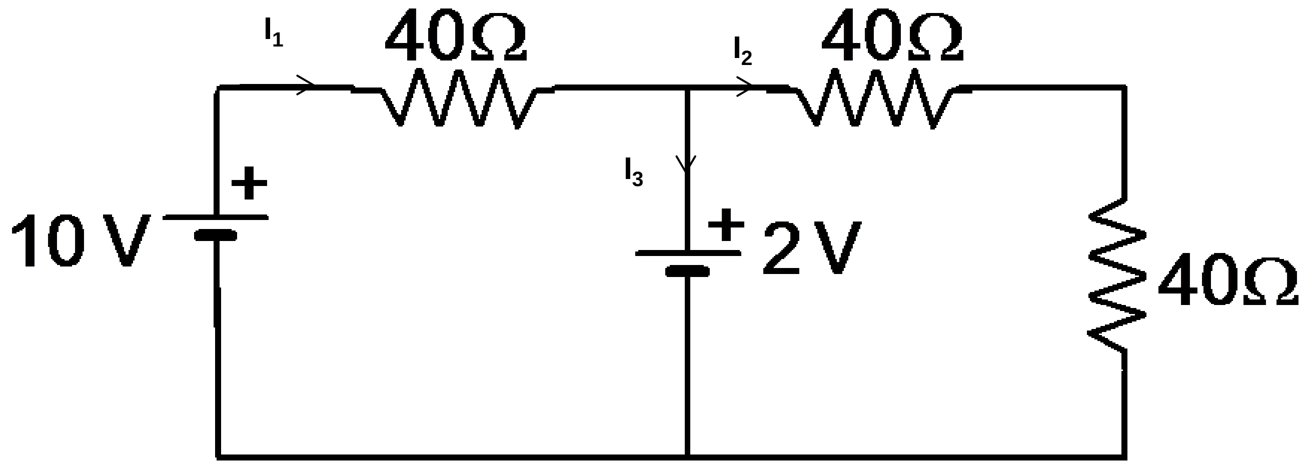
Find the value of the current $I_1$ and $I_2$



# 5. CURRENT DIVISION

Example

Analyze the below circuit to find the value of currents $I_1$, $I_2$, $I_3$




Need to apply more advanced circuit analysis methods