Lecture 6: Circuit Theorems

ELEC1111 Electrical and Telecommunications Engineering

UNSW AUSTRALIA

Never Stand Still

Faculty of Engineering

School of Electrical Engineering and Telecommunications

CONTENTS

INTRODUCTION

Objective:

- ✓ Use superposition theorem to determine responses in a linear circuit by adding the separate responses to each independent source.
- ✓ To transform complex circuits into simpler forms – using Thévenin or Norton equivalent circuits.
- ✓ Derive the condition for the maximum power transfer delivered by a source to a load resistor.

SUPERPOSITION

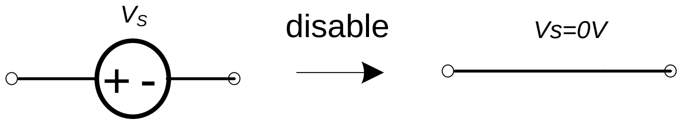
- ✓ The superposition theorem states that the *total effect of several causes acting simultaneously is equal to the sum of the effects of the individual causes acting on at a time.*

- ✓ So, the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

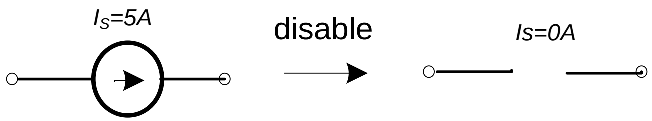
- ✓ To apply superposition to a linear circuit containing independent sources, disable all but one source and find the response due to that source. Then repeat for each source. The total response is the algebraic sum of the individual responses.

SUPERPOSITION- INDEPENDENT SOURCE DISABLED

- ✓ An independent voltage source is disabled by setting the voltage across it to zero → short circuit.

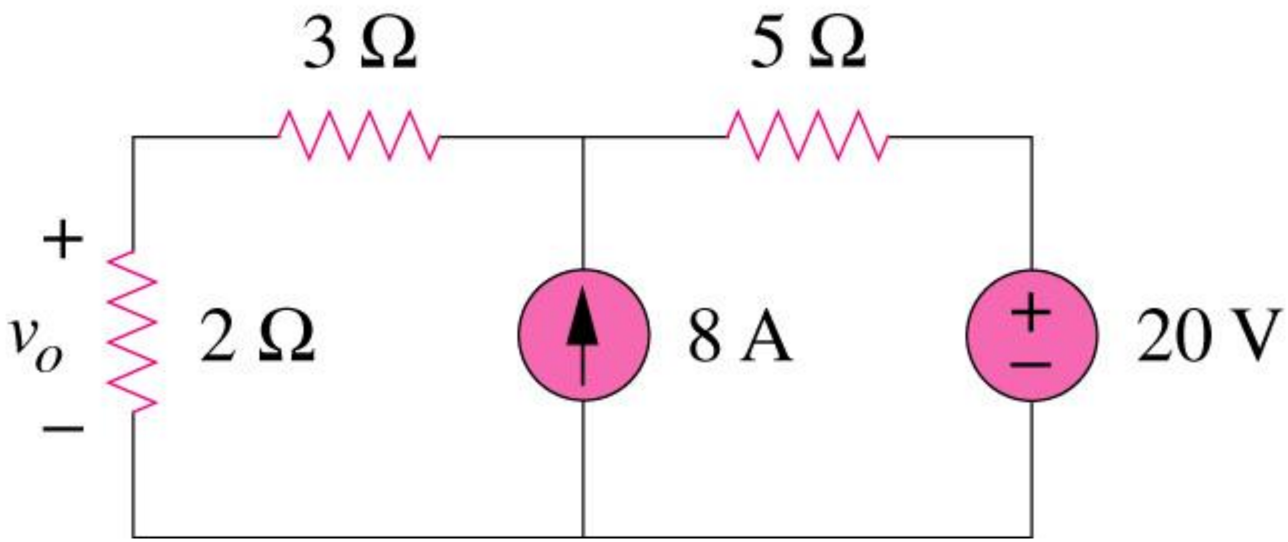


- ✓ An independent current source is disabled by setting its current to zero → open circuit.



- ✓ Dependent sources (if present), must remain active (never suppressed) during the superposition process. [Why is that?]

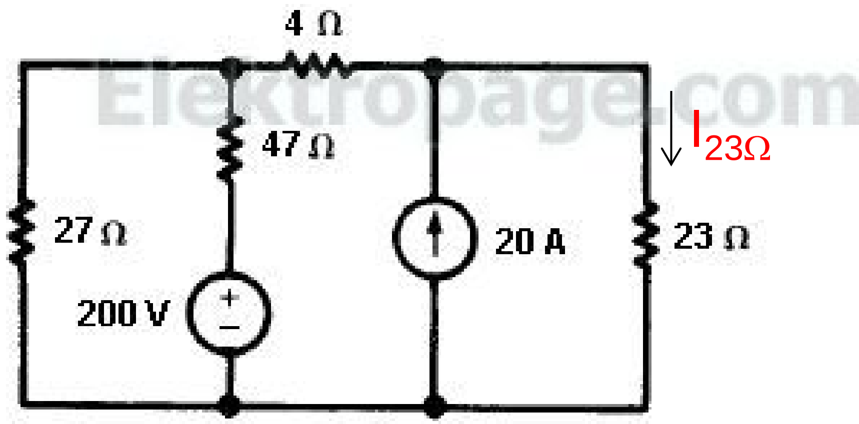
EXAMPLE 1



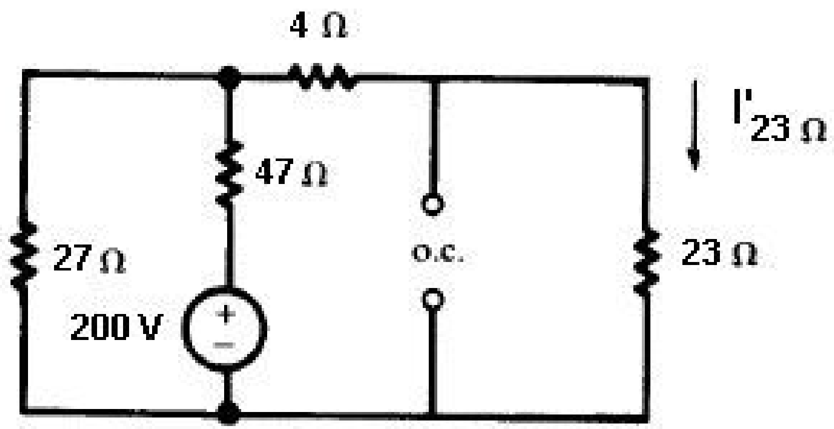
We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.

EXAMPLE 2

Find the current $I_{23\Omega}$ in the circuit (a) shown below.

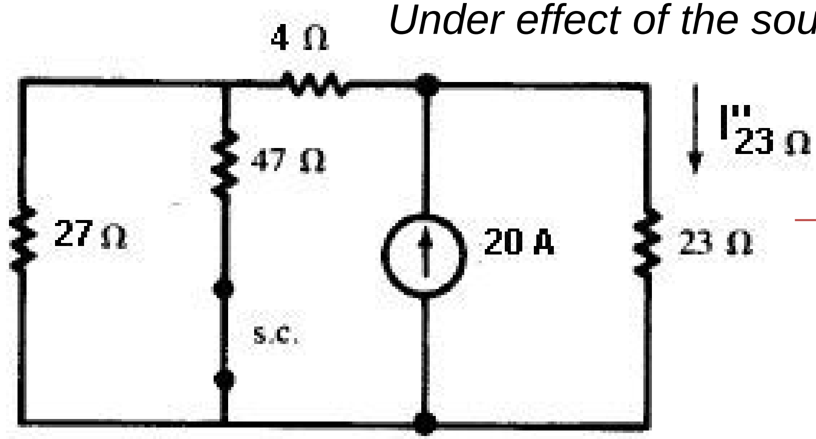


(a)



Under effect of the source 200V

Under effect of the source 20A



$I'_{23\Omega}$=1.65A

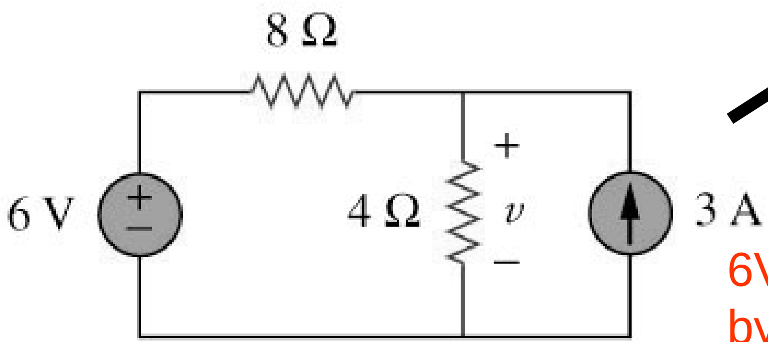
$I''_{23\Omega}$=10.4A

Superposition: $I_{23\Omega} = I'_{23\Omega} + I''_{23\Omega}$

$= 12.05\text{A}$

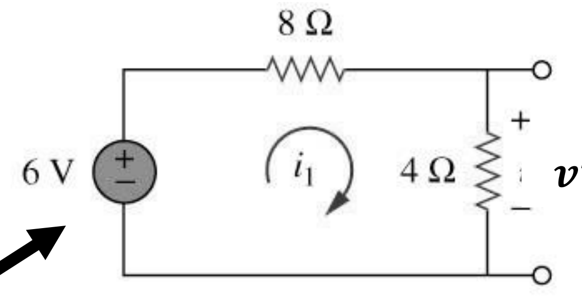
EXAMPLE 2

Find *v* in the circuit shown below.



3A is disabled: replace by open-circuit

$v' = 2$ V

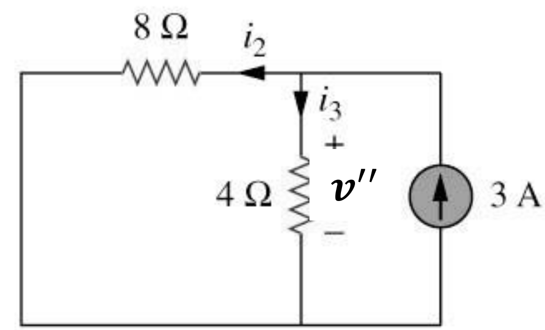


(a)

6V is disabled: replace by short-circuit

$i_3 = 2$ A

$v'' = 8$ V

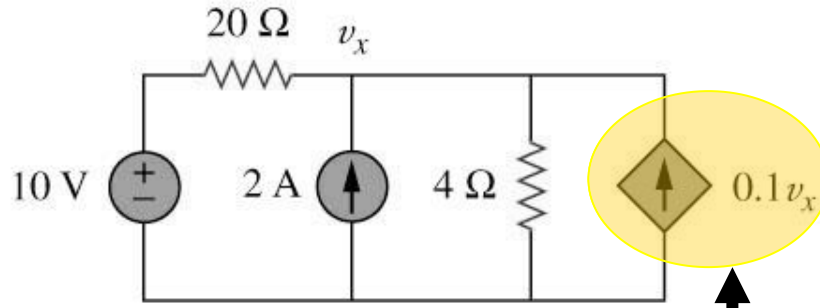


(b)

$$\boldsymbol{v = v' + v'' = 10\ \mathrm{V}}$$

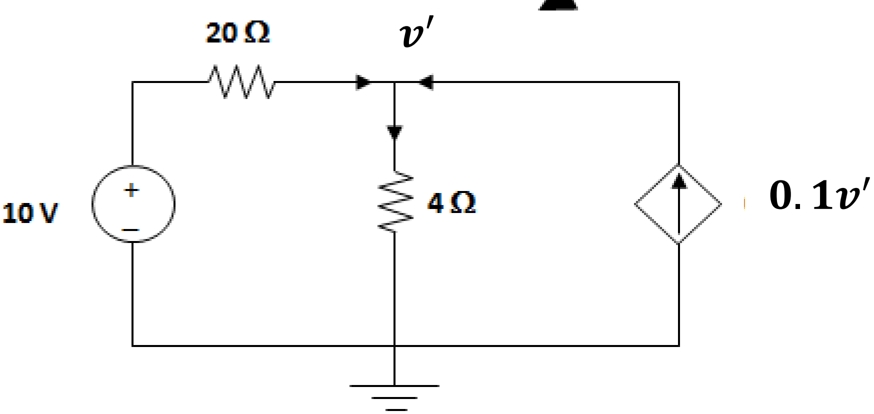
EXAMPLE 3

Find v_x in the circuit below. (v_x = 12.5V)

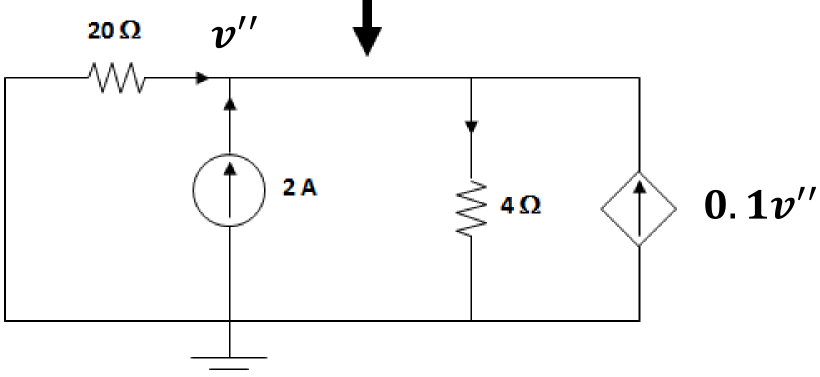


Dependent source keep unchanged

2A is discarded by open-circuit



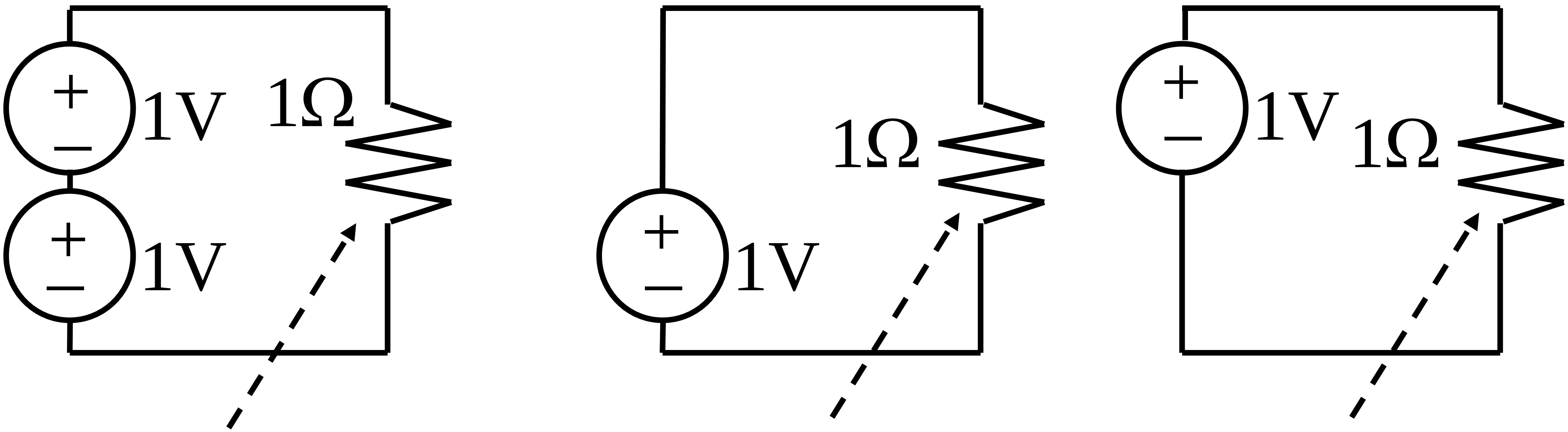
10V is discarded by open-circuit



Superposition: $v_x=v'+v''$

SUPERPOSITION DOES NOT WORK WITH POWER

- ✓ Remember that superposition is applicable only to linear responses. E.g. power is a nonlinear response.



4W delivered to resistor $\neq$ 1W delivered to resistor + 1W delivered to resistor

STEPS TO APPLY SUPERPOSITION

1. Disable (turn off or supress) all <u>independent source</u> except one source. Find the output (voltage or current) due to that active source

2. Repeat step 1 for each of the other independent sources

3. Find the total contribution by adding algebraically all the contributions due to the individual independent sources

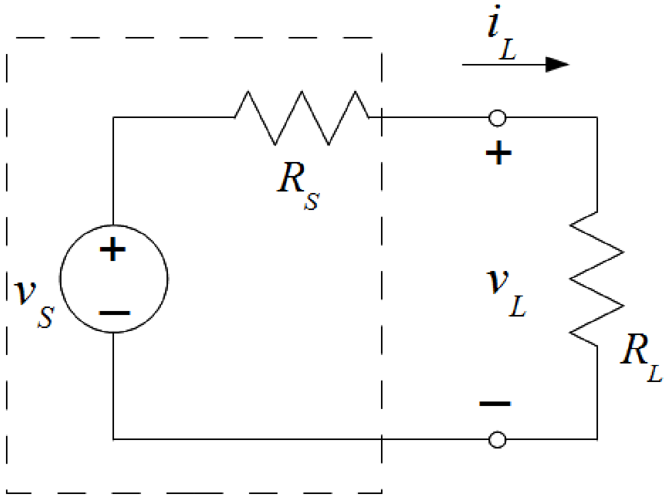
<u>Notice</u>:

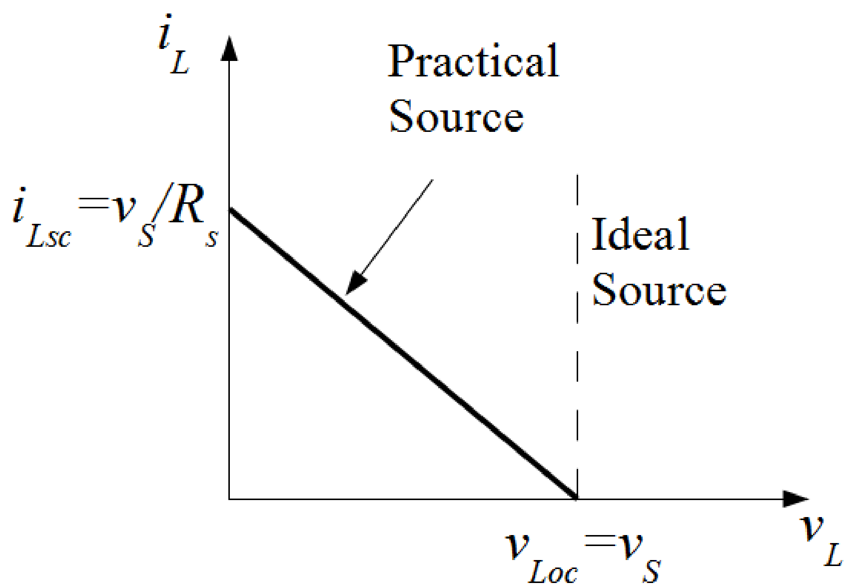
Analyse one complicated circuit having N independent sources	$\xrightarrow{\textit{Superposition}}$	Analyse N simpler circuits, each has only 1 independent source

SOURCE TRANSFORMATION

Motivation - Practical voltage sources

- ✓ For more accurate description of the behaviour of a real device (e.g. battery), we need to account for the lowering of its terminal voltage when large currents are drawn from it.
- ✓ A more appropriate model includes a *resistor in series with an ideal independent voltage source*

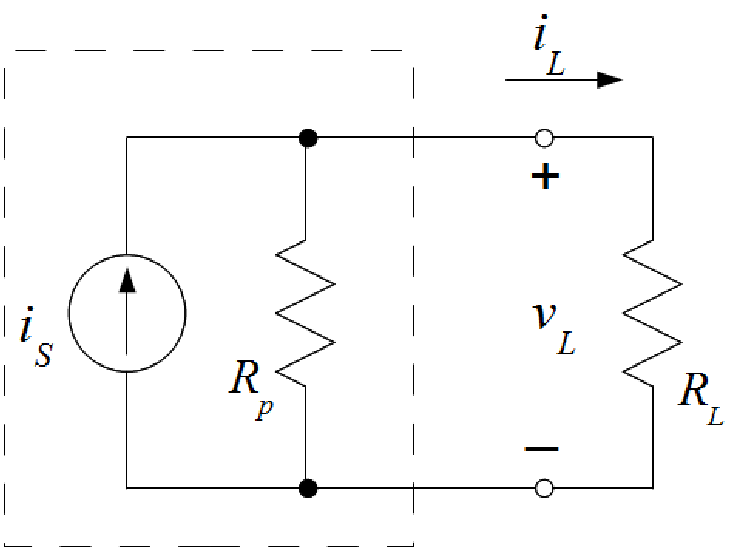


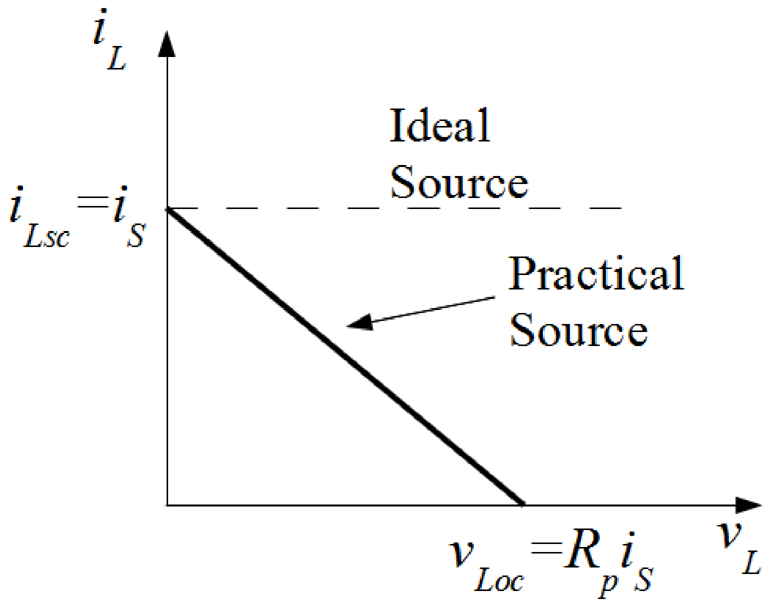


SOURCE TRANSFORMATION

Motivation - Practical current sources

✓ A more appropriate model includes a *resistor in parallel with an ideal independent current source*





SOURCE TRANSFORMATION

Equivalent practical sources

- ✓ Two practical sources are equivalent if, when connected to the same independent load R_L, they produce identical values of v_L and i_L.

- ✓ For practical voltage and current sources, the voltage across the load resistor for these circuits is:

$$v_L = v_s \frac{R_L}{R_L + R_s}$$

and

$$v_L = \left[i_s \frac{R_p}{R_L + R_p} \right] R_L$$

Electrically equivalent if:

$$R_s = R_p$$

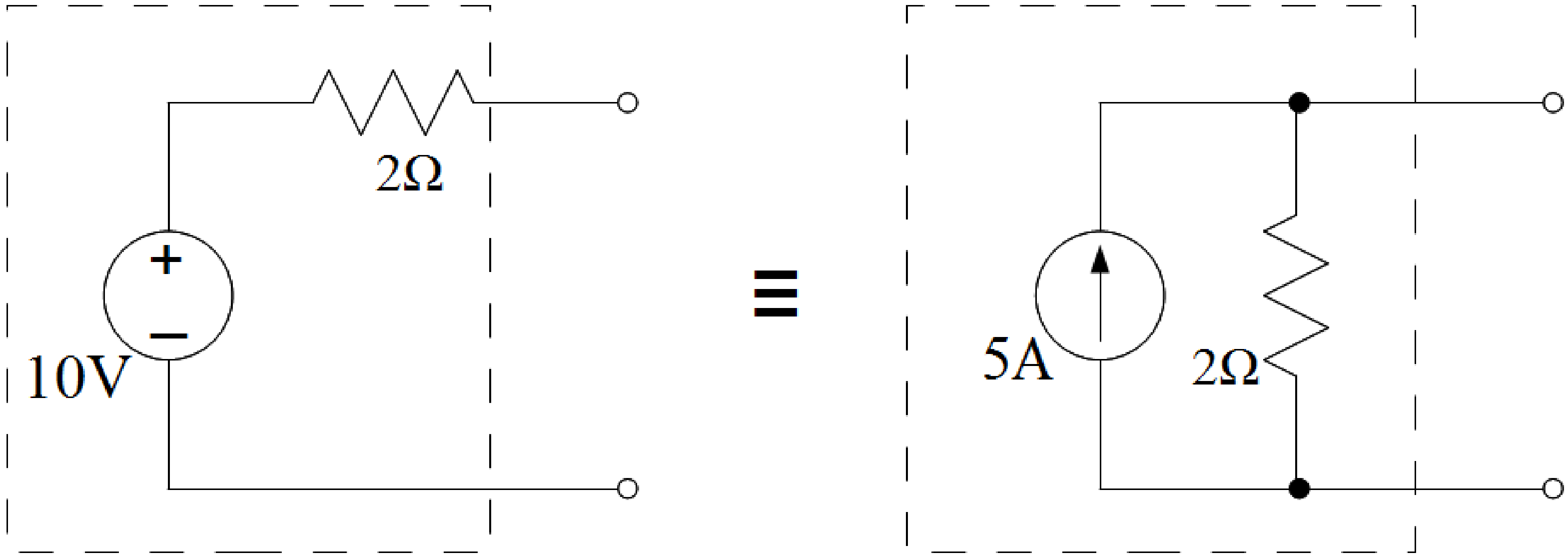
and

$$v_s = R_p i_s = R_s i_s$$

SOURCE TRANSFORMATION

- ✓ So we can easily transform one source into another while retaining the terminal characteristics of the original source.

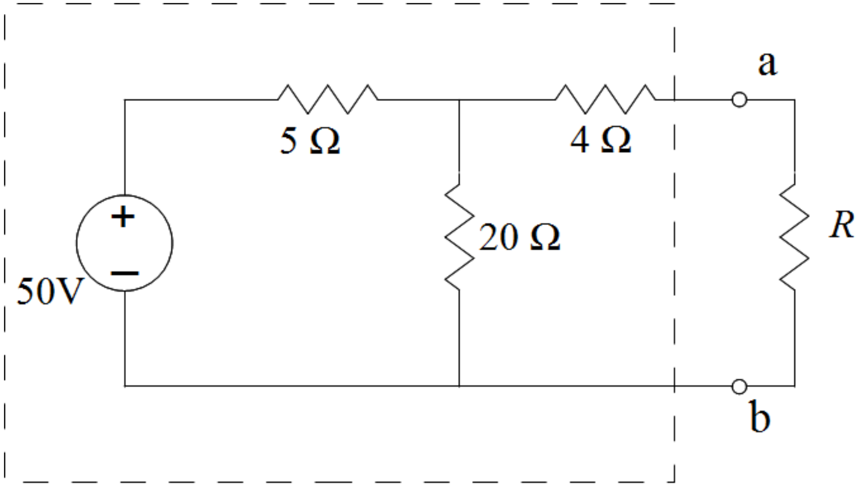
- ✓ For a circuit with a mixture of voltage and current sources, we can convert all sources into one type before applying the mesh or nodal analysis.

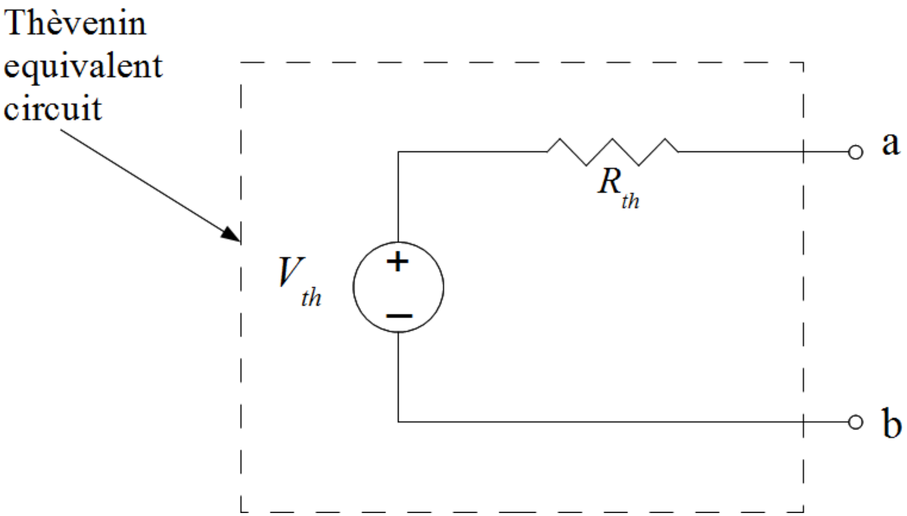


THEVENIN’S THEOREM

- ✓ Aim – to reduce part of a circuit to an equivalent source and a single element.

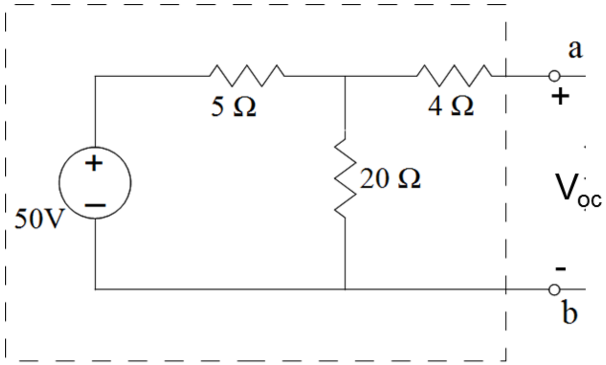
- ✓ Thévenin’s Theorem - *Any circuit of sources and resistances with an identified terminal pair can be replaced by an equivalent circuit which consists of a voltage source* V_{th} *in series with a resistance* R_{th}.

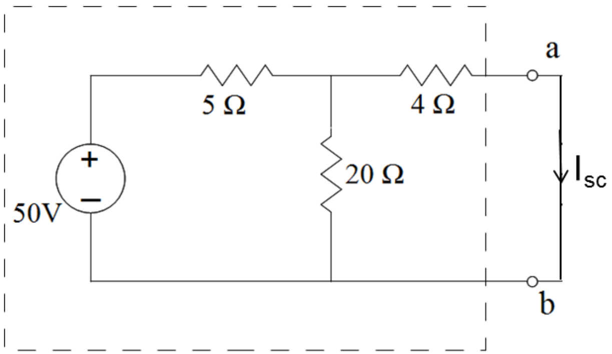




THEVENIN'S THEOREM

- ✓ V_{th} is the open-circuit voltage (often referred to as v_{oc}) at the two terminals and R_{th} is the ratio of the open-circuit voltage to the short-circuit current at the terminal pair.





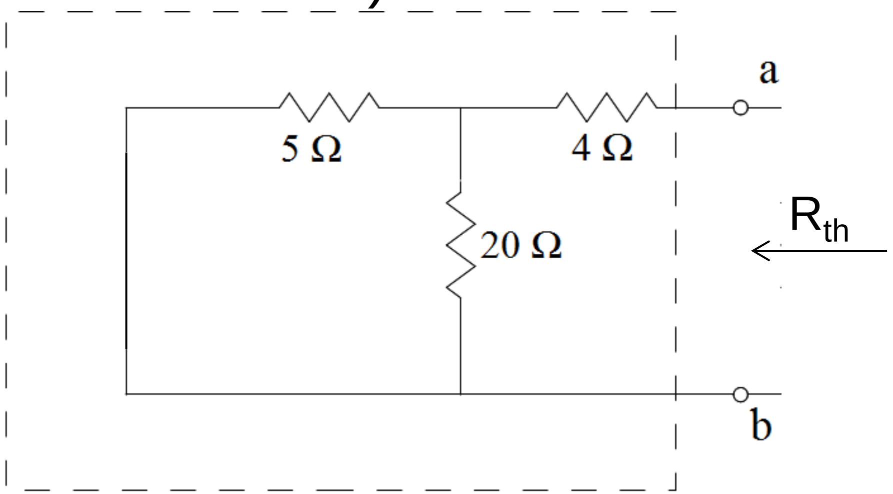
$$\boldsymbol{V_{th} = V_{oc}}$$

$$\boldsymbol{R_{th} = \frac{V_{oc}}{I_{sc}}}$$

- ✓ V_{oc} is the open-circuit voltage at the terminal pair
- ✓ I_{sc} *is the* short-circuit current at the terminal pair.

THEVENIN'S THEOREM

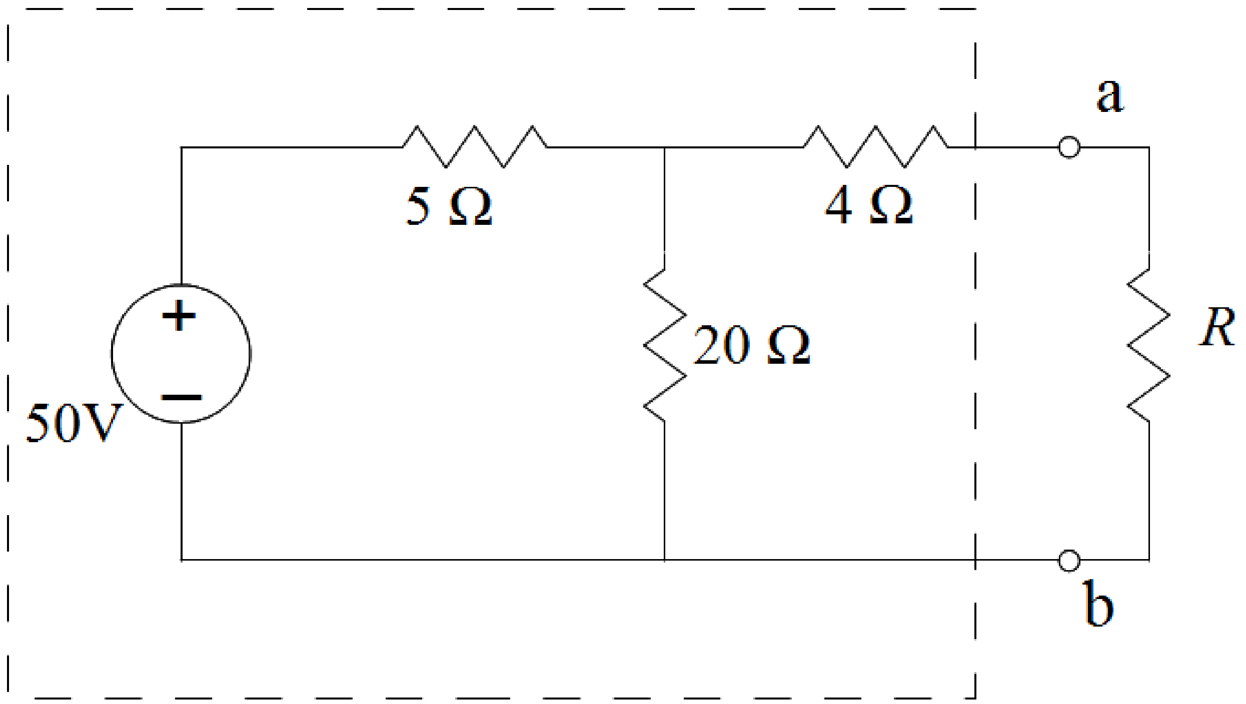
- ✓ Alternative method for finding R_{th}:
  Calculate the equivalent resistance between the terminal pair from simple circuit reduction using series/parallel rules with independent sources disabled (**cannot be used for dependent sources**).



- ✓ When determining a Thévenin equivalent circuit containing dependent sources, ensure you do not separate the source from its control variable.

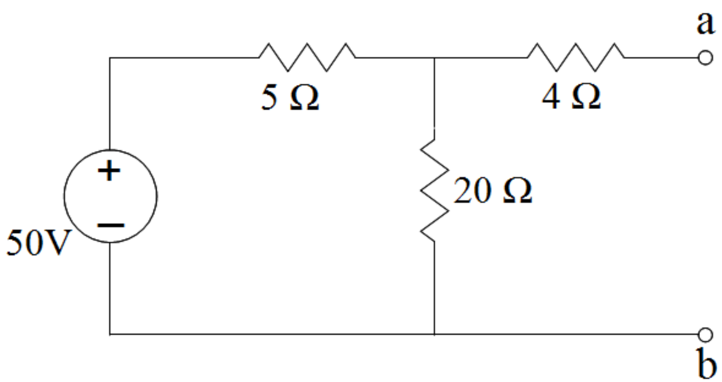
THEVENIN'S THEOREM - EXAMPLE

- ✓ Replace the circuit in the box with its Thévenin equivalent circuit.



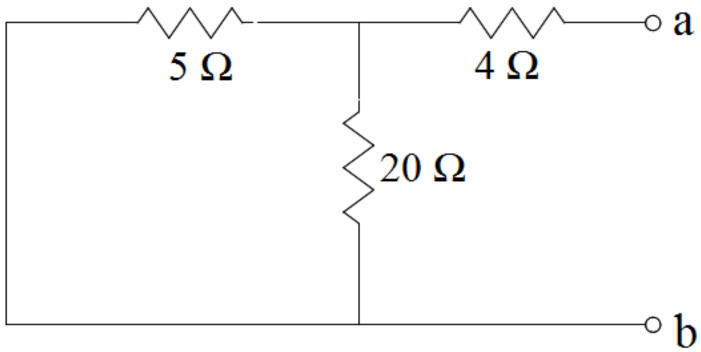
THEVENIN'S THEOREM - EXAMPLE

- ✓ Thévenin voltage ≡ open-circuit voltage.



$$V_{th} = \frac{20}{5+20} \times 50 = 40V$$

- ✓ Thévenin resistance ≡ equivalent resistance looking into terminal *a*-*b* with source disabled.



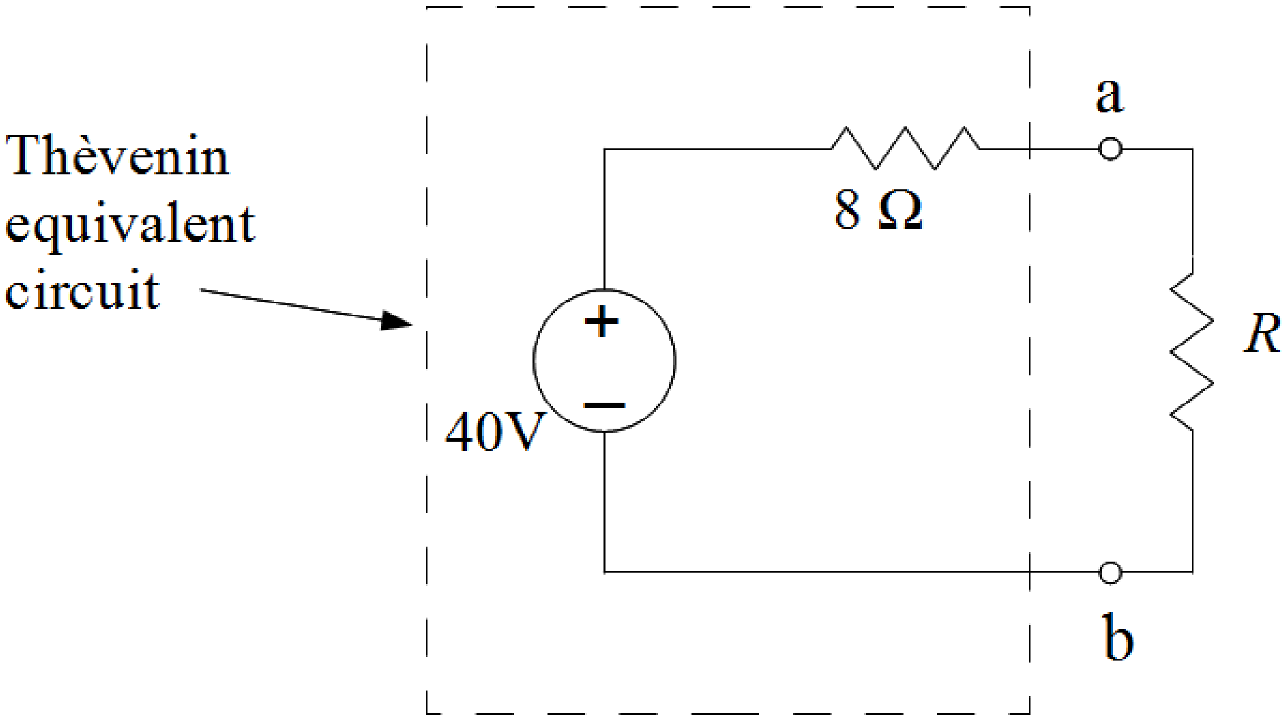
$$R_{th} = 4 + (5||20) = 8\Omega$$

THEVENIN'S THEOREM - EXAMPLE

- ✓ Thévenin equivalent circuit:

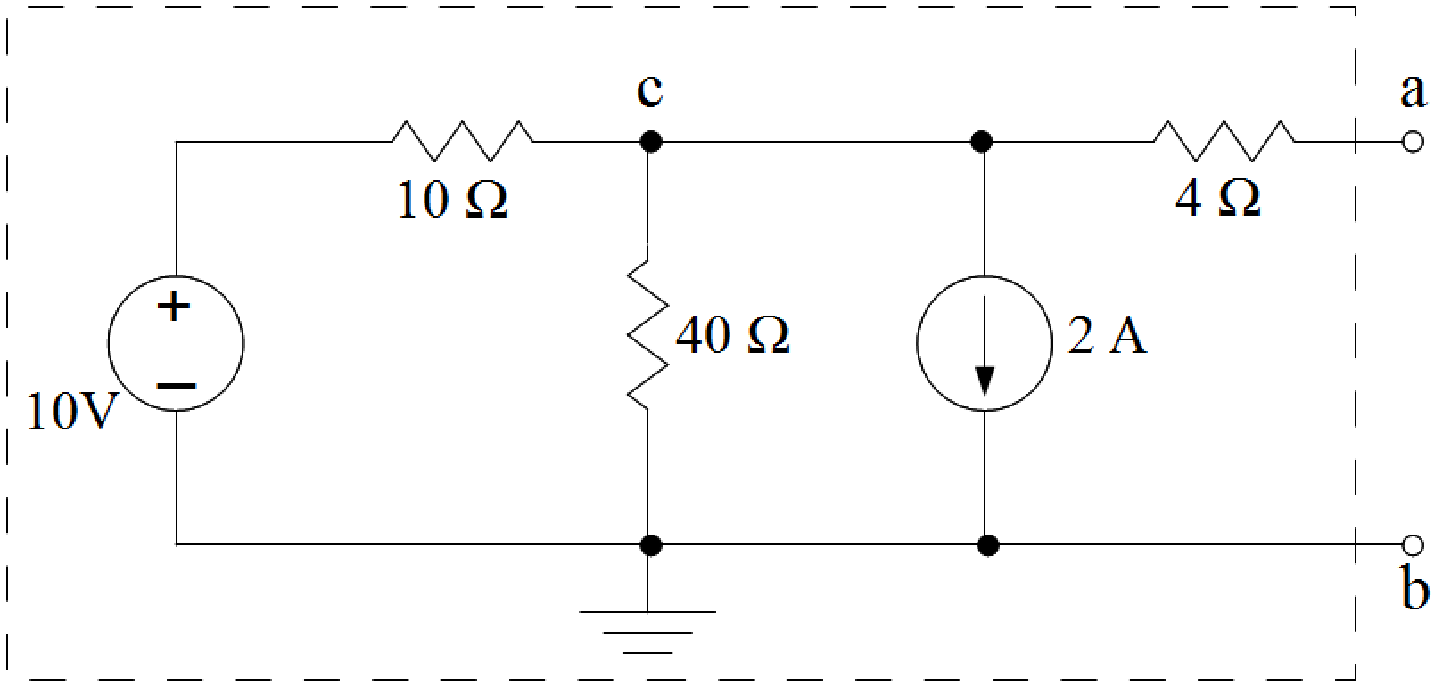
$$V_{th} = 40V$$

$$R_{th} = 8\Omega$$



THEVENIN'S THEOREM - EXAMPLE

✓ Replace the circuit in the box with its Thévenin equivalent circuit.



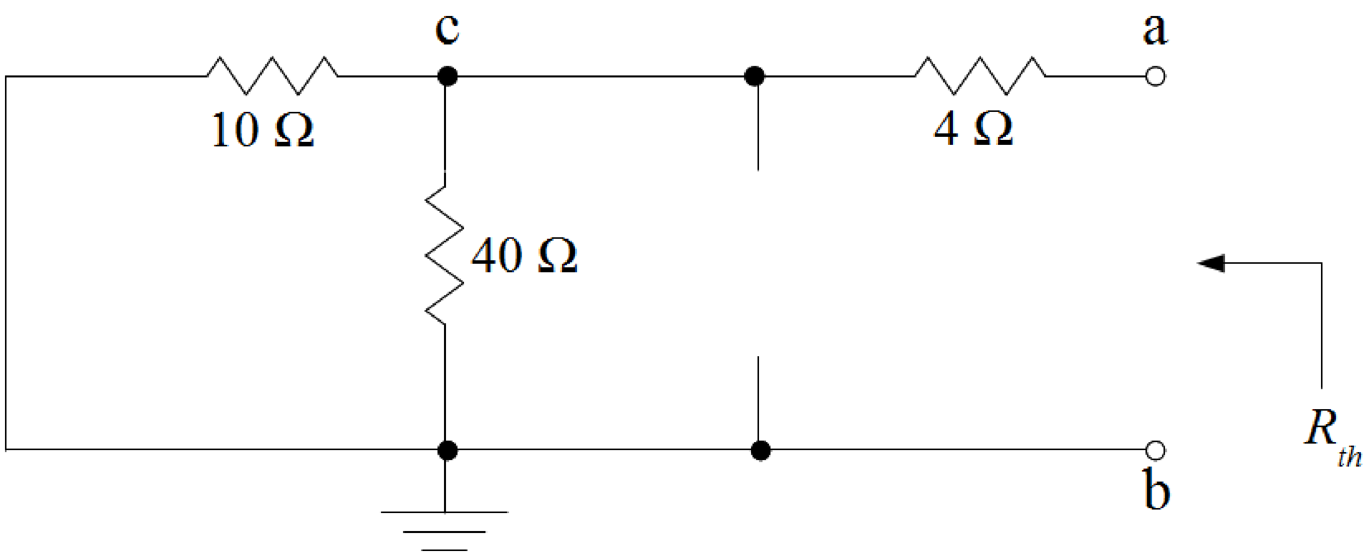
THEVENIN'S THEOREM - EXAMPLE

- ✓ For open-circuit voltage v_{oc}, use nodal analysis:

$$\frac{v_c - 10}{10} + \frac{v_c}{40} + 2 = 0 \Rightarrow V_{th} = v_c = -8V$$

- ✓ For R_{th},

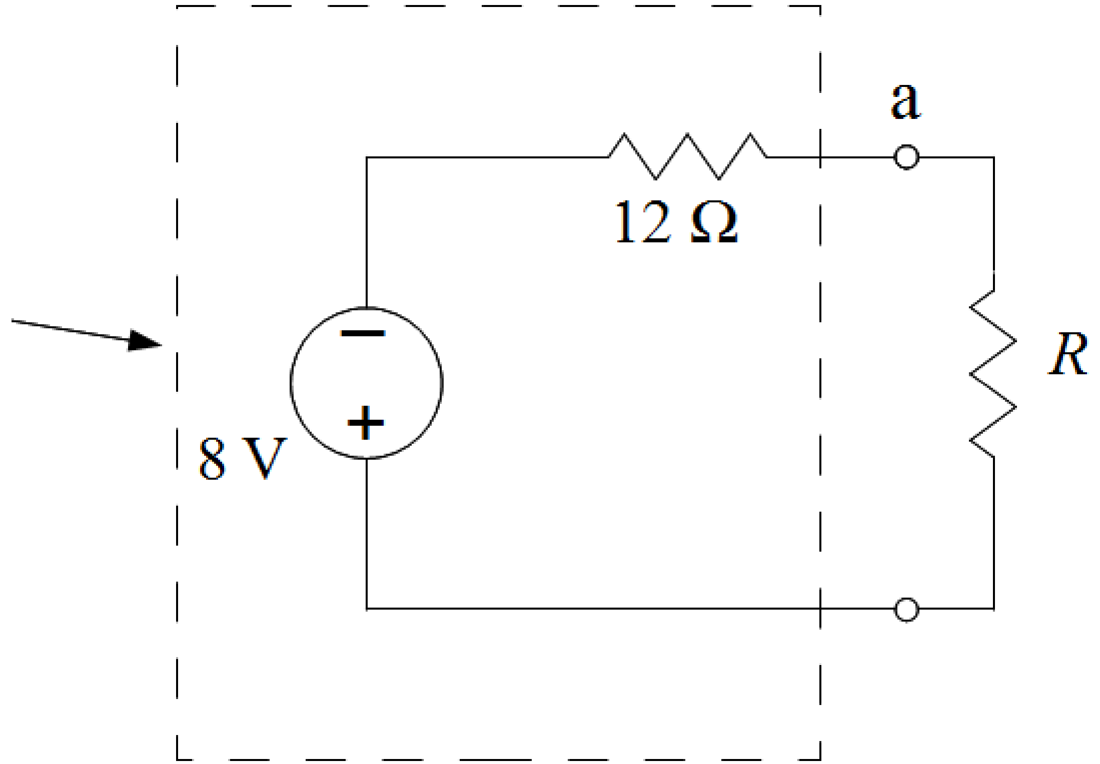
$$R_{th} = 4 + (10||40) = 12\Omega$$



THEVENIN'S THEOREM - EXAMPLE

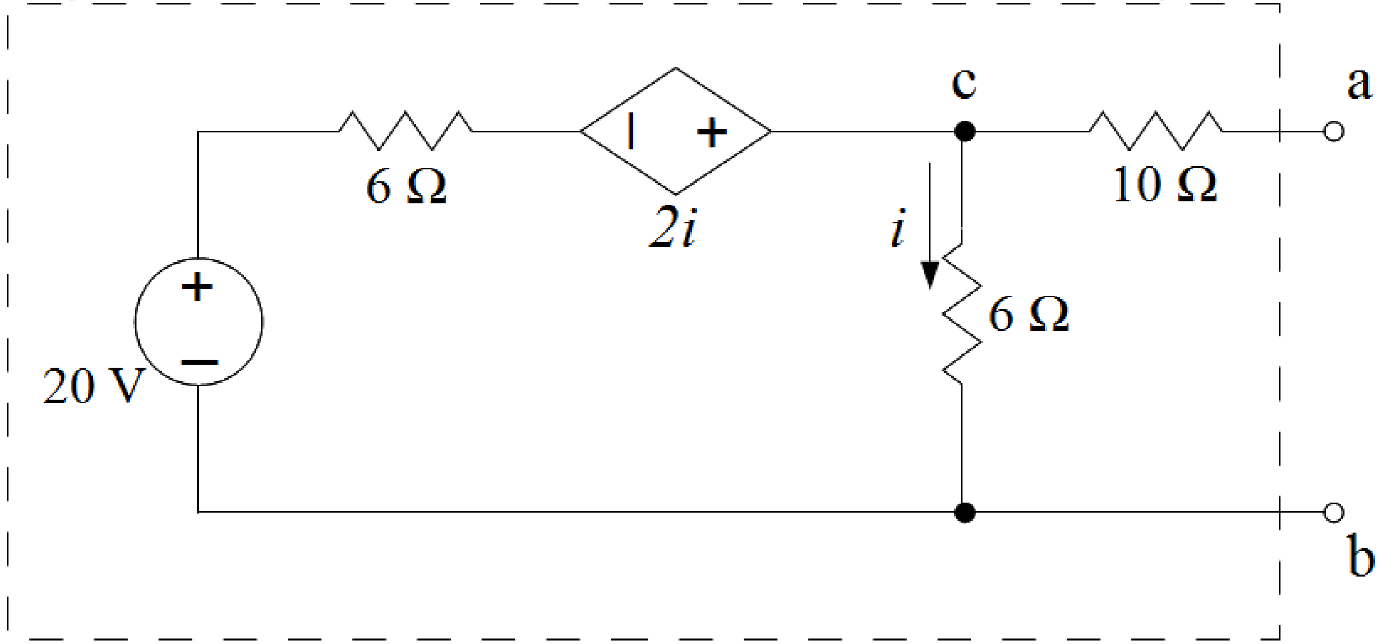
✓ Thévenin equivalent circuit:

Thévenin equivalent circuit



THEVENIN’S THEOREM - EXAMPLE

- Replace the circuit in the box with its Thévenin equivalent circuit.



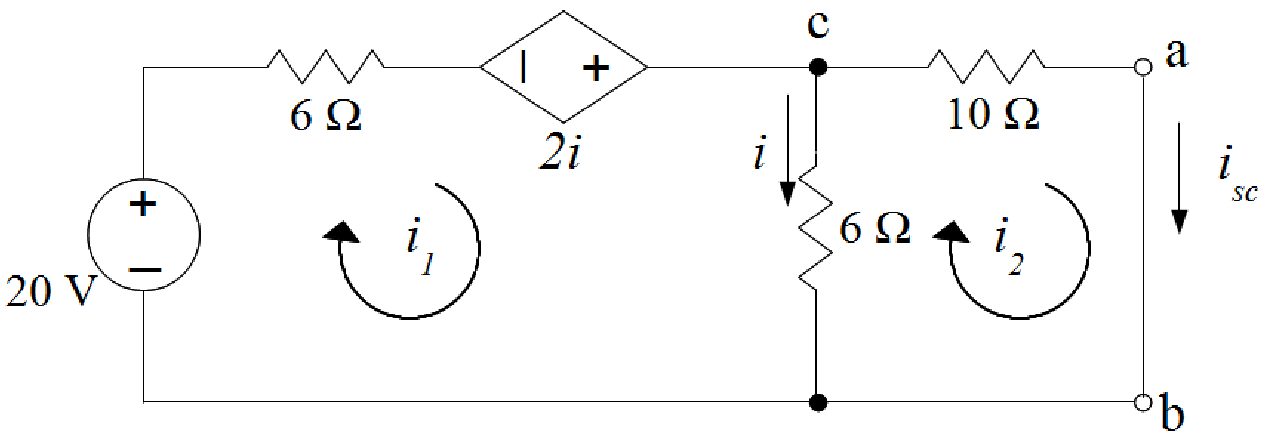
- For open-circuit voltage V_{th}, voltage across terminals a-b is equal to voltage across terminals c-b ➔ apply KVL:

$$20 - 6i + 2i - 6i = 0 \quad \Rightarrow \quad i = 2A \quad \Rightarrow \quad V_{th} = 12\,V$$

THEVENIN'S THEOREM - EXAMPLE

- ✓ To determine i_{sc}, short-circuit output terminals, and use mesh equations:

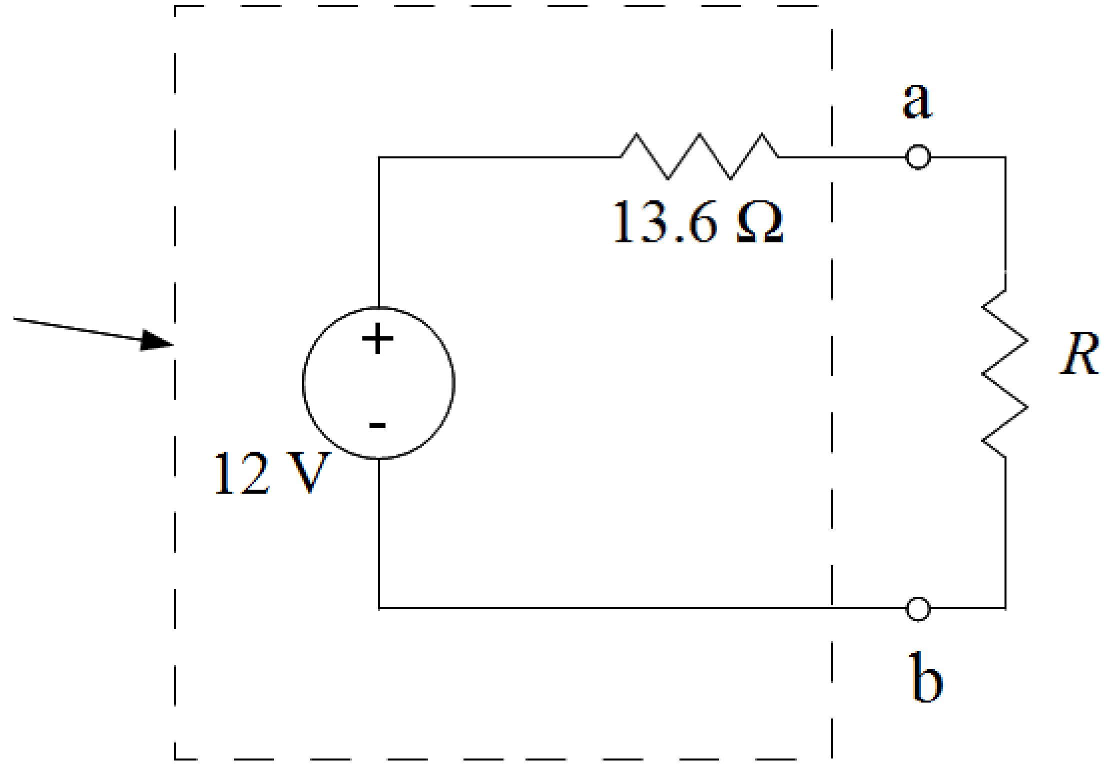
$$i_{sc} = i_2 = \frac{120}{136} \quad \Rightarrow \quad R_{th} = \frac{V_{th}}{i_{sc}} = 13.6\Omega$$



THEVENIN'S THEOREM - EXAMPLE

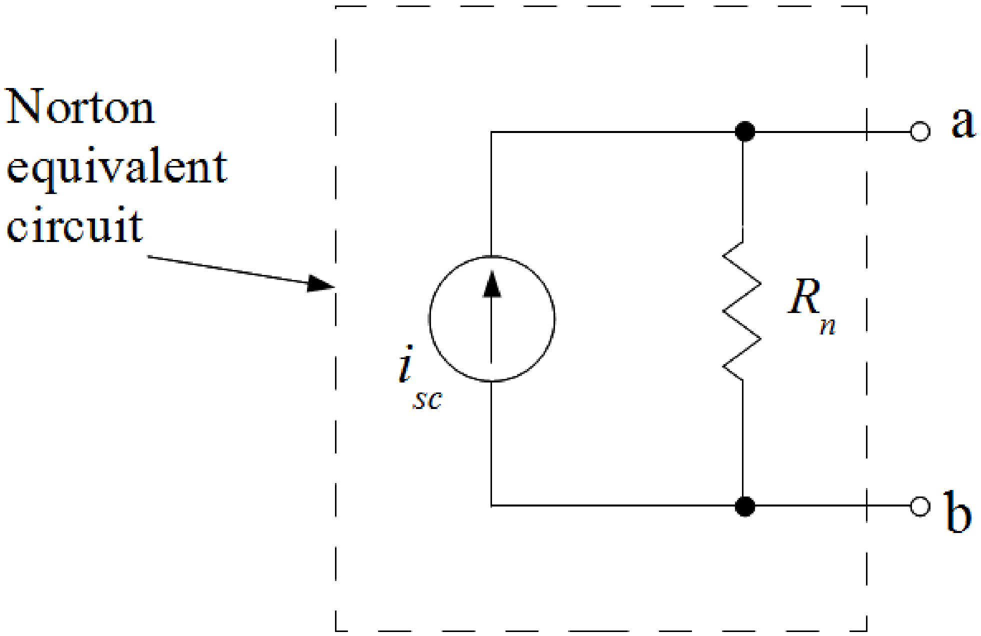
- ✓ Thévenin equivalent circuit:

Thévenin equivalent circuit



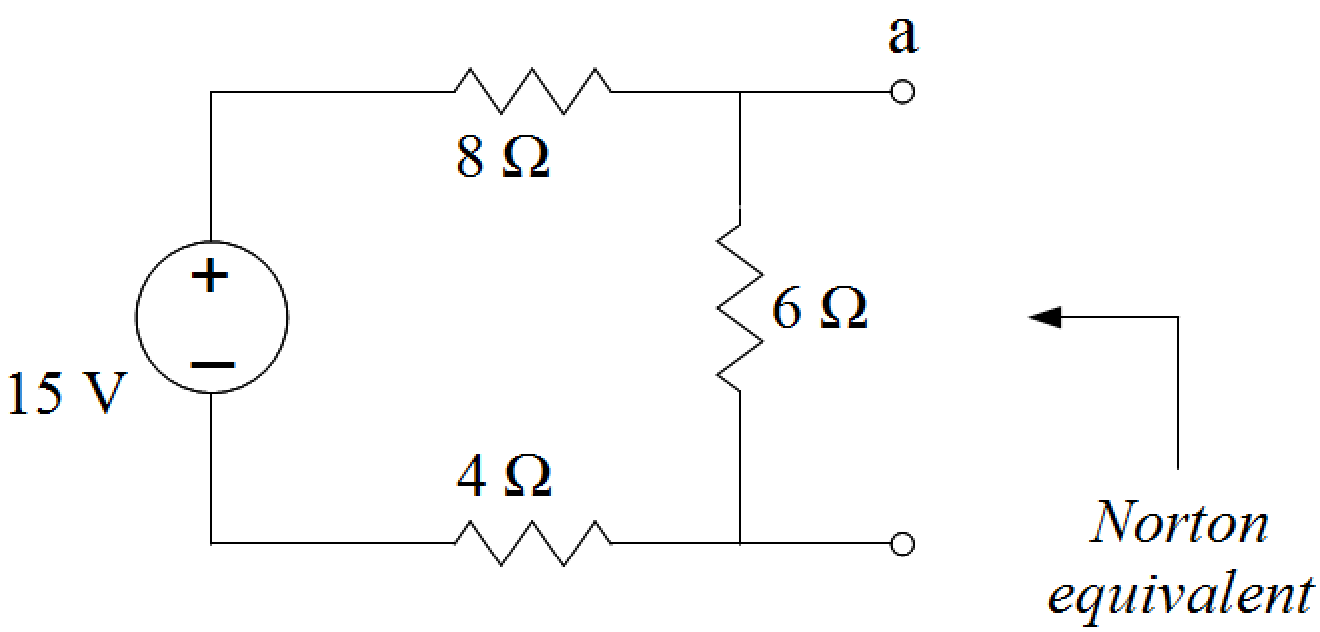
NORTON'S EQUIVALENT CIRCUIT

- ✓ Dual of the Thévenin equivalent circuit.
- ✓ Norton's Theorem – *Any circuit of sources and resistances with an identified terminal pair can be replaced by an equivalent circuit which consists of an ideal current source i_{sc} in parallel with a conductance $G_n = \frac{1}{R_n}$.*
- ✓ i_{sc} is the short-circuit current (often referred to as i_n) at the two terminals and $G_n = \frac{1}{R_n}$ is the ratio of the short-circuit current to the open-circuit voltage at the terminal pair.



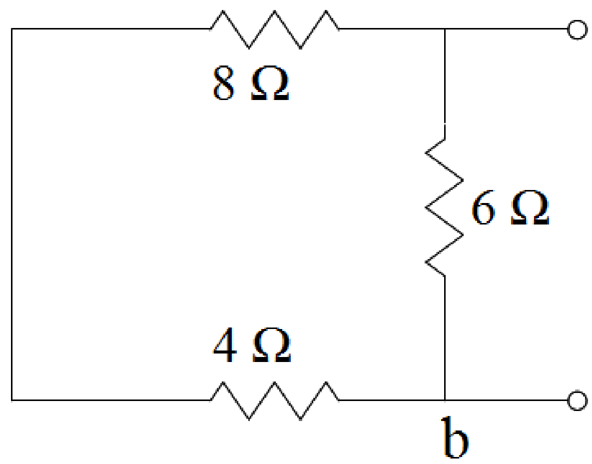
NORTON'S EQUIVALENT CIRCUIT - EXAMPLE

- ✓ Replace the circuit in the box with its Norton equivalent circuit:

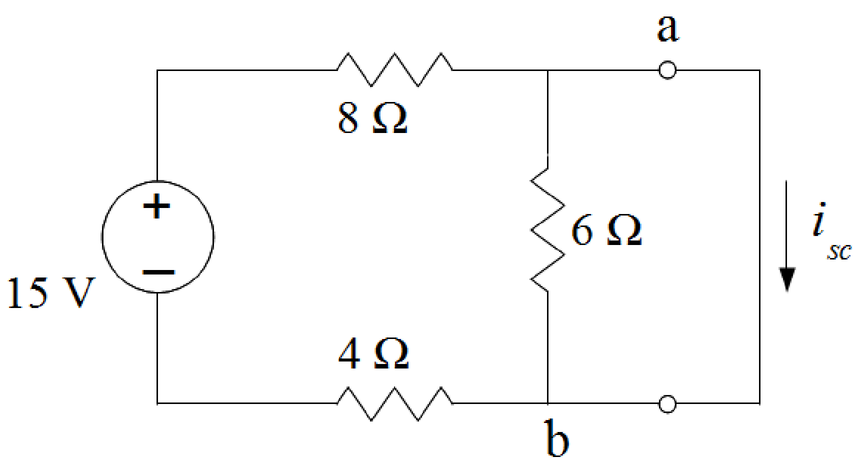


- ✓ R_n found by deactivating the source and finding equivalent resistance.

NORTON'S EQUIVALENT CIRCUIT - EXAMPLE

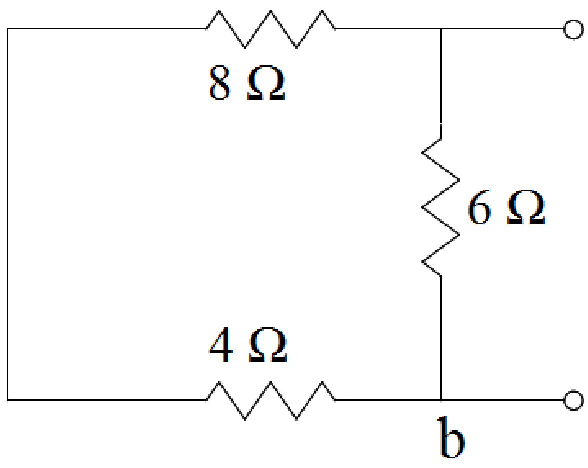


$R_n =$

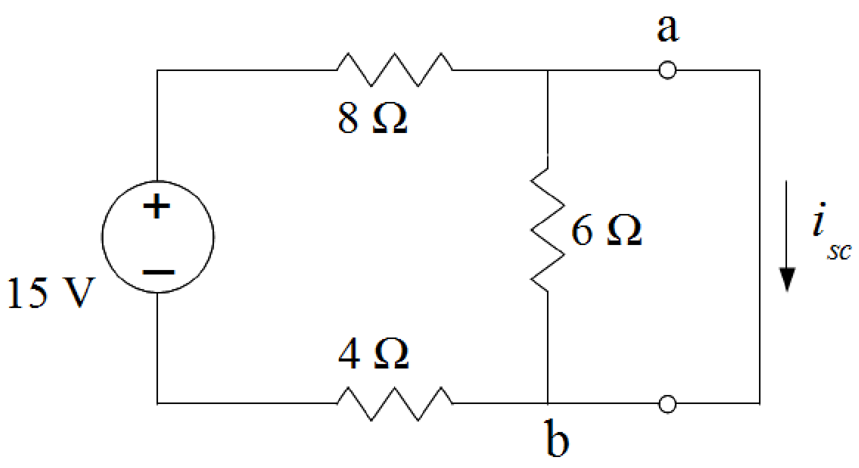


$i_n = i_{sc} =$

NORTON'S EQUIVALENT CIRCUIT - EXAMPLE



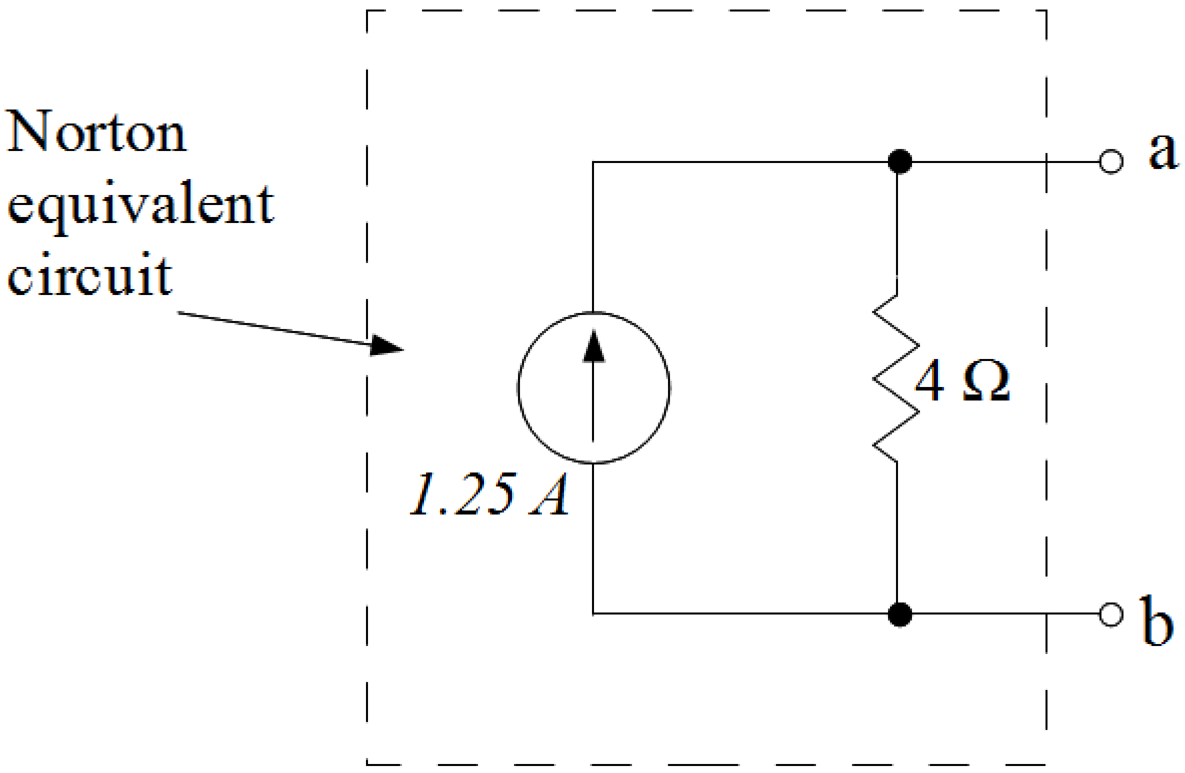
$$R_n = 6||(8+4) = 4\Omega$$



$$i_n = i_{sc} = \frac{15}{8+4} = 1.25A$$

NORTON'S EQUIVALENT CIRCUIT - EXAMPLE

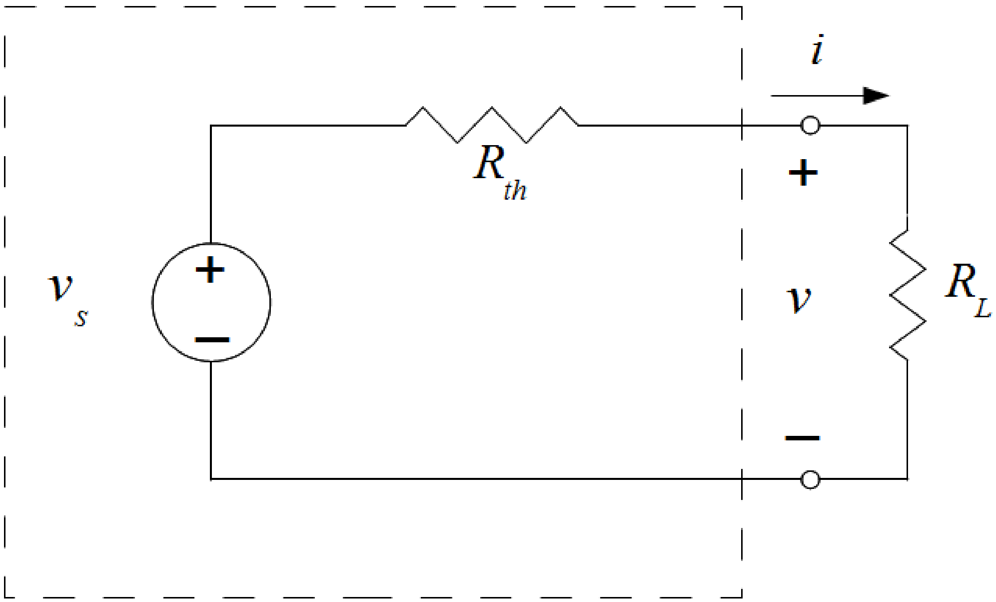
✓ Norton equivalent circuit:



MAXIMUM POWER TRANSFER

- ✓ The function of many circuits is to act as a power source and thus the objective is to deliver maximum power to a load resistor R_L.

- ✓ Maximum Power Transfer theorem – *the source delivers the maximum power to the load when the load resistance is equal to the Thévenin resistance of the source.*



MAXIMUM POWER TRANSFER

- ✓ Power delivered to load R_L:

$$p = i^2 R_L \qquad\qquad i = \frac{v_s}{R_{th} + R_L}$$

$$p = \left(\frac{v_s}{R_{th} + R_L}\right)^2 R_L$$

- ✓ For a given source, v_s and R_{th} are fixed. Thus power is a function of R_L. So maximum power occurs when:

$$\frac{dp}{dR_L} = 0 \qquad\qquad \frac{d^2p}{dR_L{}^2} < 0$$

MAXIMUM POWER TRANSFER

- ✓ Solve:

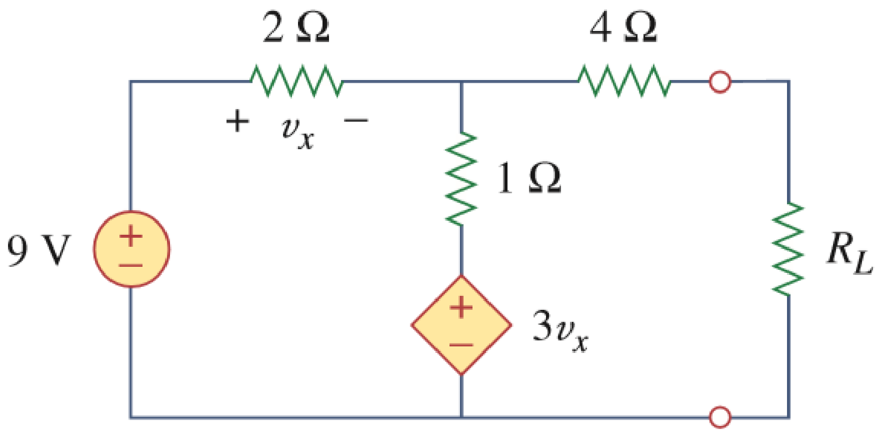
$$R_L = R_{th}$$

- ✓ So maximum power is:

$$p = \frac{{v_s}^2}{4R_L}$$

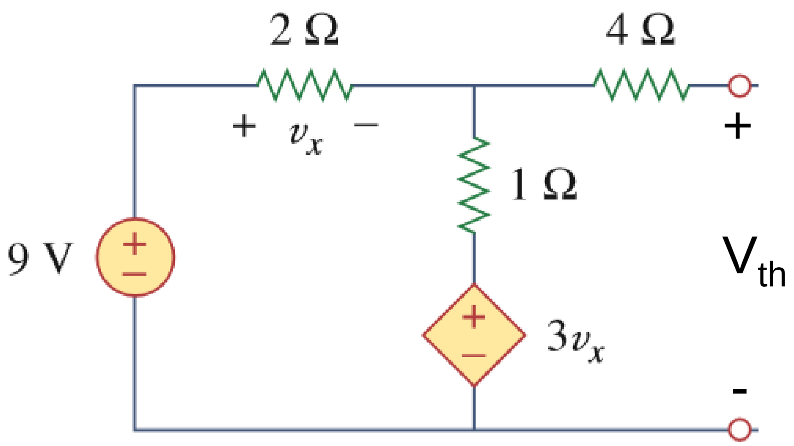
EXAMPLE 3

- ✓ Determine the value of R_L that will draw the maximum power from the rest of the following circuit. Calculate the maximum power.



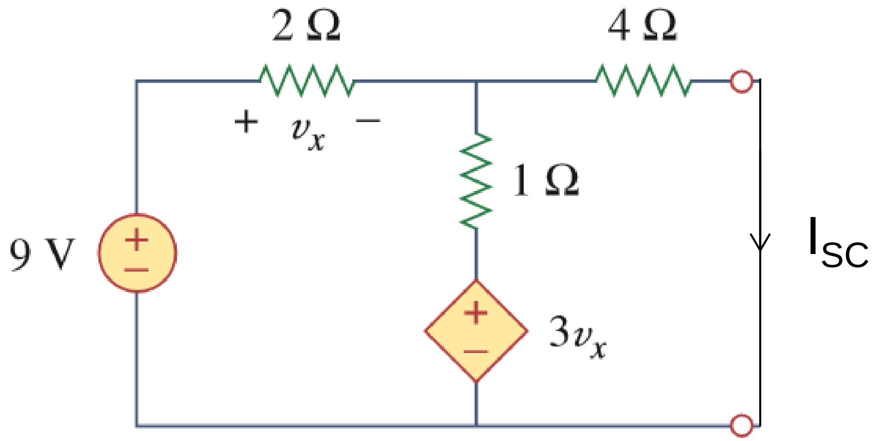
EXAMPLE 3 (cont.)

- ✓ Open circuit to find Thevenin voltate



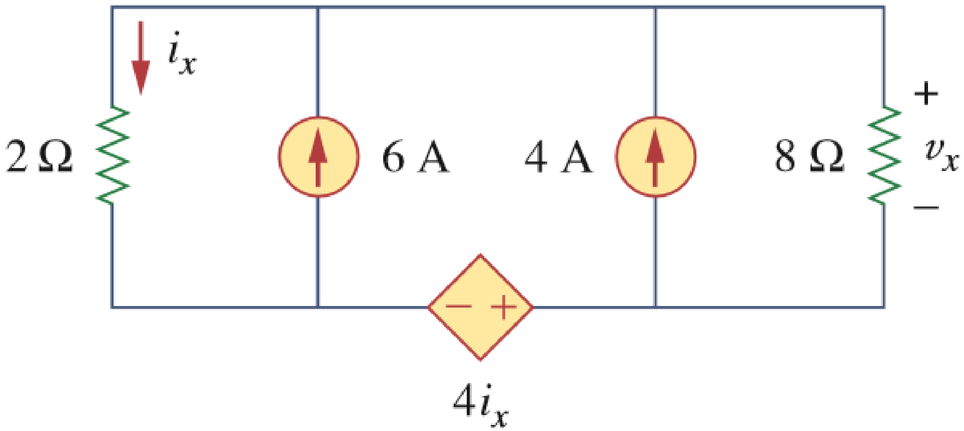
EXAMPLE 3 (cont.)

- ✓ Shorted circuit to find shorted circuit current I_{SC}



EXAMPLE 4

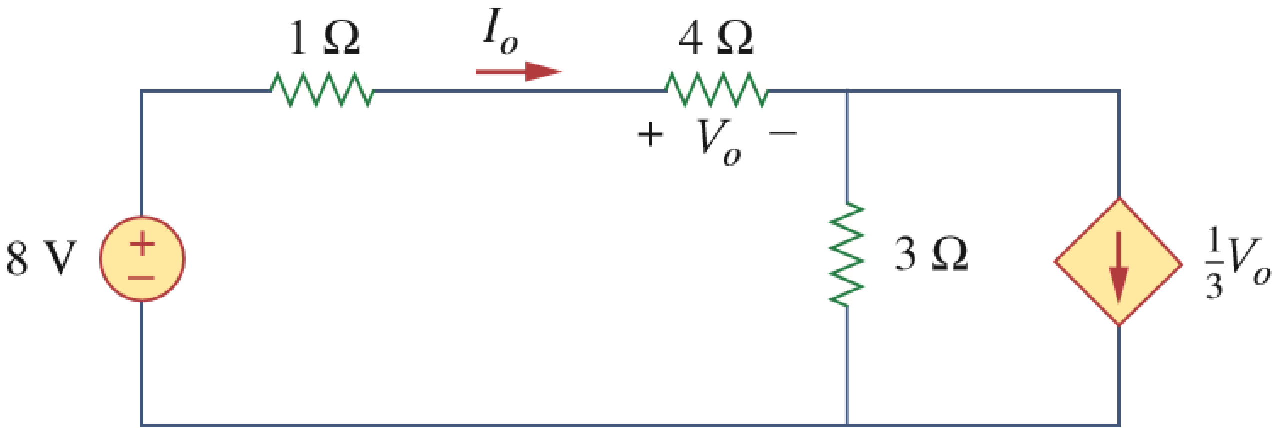
- ✓ Use superposition to find the voltage v_x in the below circuit



Ans: *Vx=-26.67 V*

EXAMPLE 5

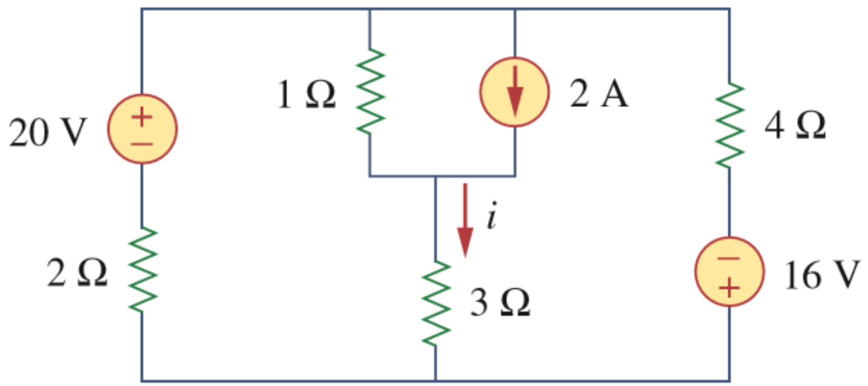
- ✓ Use source transformation to find I_o in the following circuit



$I_o = 2\,A$

EXAMPLE 6

- ✓ For the following circuit, use superposition to find *i*



Ans: *i=1.875 A*