Maths

$y = \sqrt{((x+6)^2 + 25)} + \sqrt{((x-6)^2 + 121)}$

Soln

Let $w = \sqrt{(x+6)^2 + 25}$

$a = (x+6)^2 + 25$

$da/dx = 2(x+6)$

$u = \sqrt{a}$

$du/da = 1/2\sqrt{a}$

$\frac{du}{dx} = \frac{da}{dx} \times \frac{du}{dx} \Rightarrow \cancel{2}(x+6) = \frac{1}{\cancel{2}\sqrt{a}}$

$\frac{du}{dx} = \frac{x+6}{\sqrt{(x+6)^2 + 25}}$ — ① *

Also $u = \sqrt{((x-6)^2 + 121)}$

$b = (x-6)^2 + 121$; $db/dx = 2(x-6)$

$\frac{du}{dx} = \frac{db}{dx} \times \frac{du}{db} \Rightarrow \frac{x-6}{\sqrt{(x-6)^2 + 121}}$ — ② *

$\therefore \frac{dy}{dx} = \frac{x+6}{\sqrt{(x+6)^2 + 25}} + \frac{x-6}{\sqrt{(x-6)^2 + 121}}$

For stationary Points ; $dy/dx = 0$

$\frac{x+6}{\sqrt{(x+6)^2 + 25}} + \frac{x-6}{\sqrt{(x-6)^2 + 121}} = 0$

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$$\frac{(x+6)\sqrt{(x-6)^2+121} + (x-6)\sqrt{(x+6)^2+25}}{\sqrt{[(x+6)^2+25]\cdot[(x-6)^2+121]}} = 0$$

$$x+6\sqrt{(x-6)^2+121} + (x-6)\sqrt{(x+6)^2+25} = 0$$

$$x+6\sqrt{(x-6)^2+121} = -(x-6)\sqrt{(x+6)^2+25}$$

$$(x+6)^2\left[(x-6)^2+121\right] = +(x-6)^2\left[(x+6)^2+25\right]$$

$$(x^2+12x+36)\left[x^2-12x+157\right] = (x^2-12x+36)\left[x^2+12x+61\right]$$

$$x^4 - \cancel{12x^3} + 157x^2 + \cancel{12x^3} - 144x^2 + 1844x + 36x^2 - 432x - 5652 = x^4 + \cancel{12x^3}$$

$$+61x^3 - \cancel{12x^3} - \cancel{144x^2} - 732x + 36x^2 + 432x + 2196$$

Collect like terms

$$96x^2 + 1752x - 7848 = 0$$

$$12x^2 + 219x - 981 = 0$$

$$4x^2 + 73x - 327 = 0$$

Solving quadratically

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad ; a = 4, b = 73, c = -327$$

$$\frac{-73 \pm \sqrt{73^2 - (4 \times 4 \times -327)}}{2 \times 4}$$

$$\frac{-73 \pm \sqrt{5329 + 5232}}{8}$$

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$$\frac{-73 \pm \sqrt{10561}}{8}$$

$$x = \frac{-73 + 102.7667}{8} \text{ or } \frac{-73 - 102.7667}{8}$$

$$x = \frac{29.7667}{8} \text{ or } \frac{-175.7667}{8}$$

$x = 3.72$ or $x = -21.97$

For second derivative

$$\frac{d^2y}{dx^2} = \frac{x+6}{\sqrt{(x+6)^2+25}} + \frac{x-6}{\sqrt{(x-6)^2+121}}$$

Using quotient rule $\rightarrow \dfrac{Vdu - Udv}{V^2}$

for $\dfrac{x+6}{\sqrt{(x+6)^2+25}}$, $u = x+6$; $du = 1$

$V = \sqrt{(x+6)^2+25}$; $dv = \dfrac{x+6}{\sqrt{(x+6)^2+25}}$ -> from (1)

$$\frac{\left(\sqrt{(x+6)^2+25} \cdot 1\right) - x+6 \cdot \dfrac{x+6}{\sqrt{(x+6)^2+25}}}{\sqrt{(x+6)^2+25}}.$$

$$\frac{\left(\sqrt{(x+6)^2+25}\right)^{\cancel{2}} - (x+6)^2}{\left(\sqrt{(x+6)^2+25}\right)^2 \sqrt{(x+6)^2+25}} \Rightarrow \frac{(x+6)^2+25-(x+6)^2}{\left(\sqrt{(x+6)^2+25}\right)^3}$$

$$\frac{25}{[(x+6)^2+25]^{3/2}}$$

Also for $\dfrac{x-6}{\sqrt{(x-6)^2+121}}$; $u = x-6$; $du = 1$

$v = \sqrt{(x-6)^2+121}$; $dv = \dfrac{x-6}{\sqrt{(x-6)^2+121}}$ from (2)

$$\frac{\sqrt{(x-6)^2+121}\cdot 1 - x-6 \cdot \dfrac{x-6}{\sqrt{(x-6)^2+121}}}{\left(\sqrt{(x-6)^2+121}\right)^2}$$

$$\frac{\left(\sqrt{(x-6)^2+121}\right)^{\cancel{2}} - (x-6)^2}{((x-6)^2+121)^{1/2}\cdot\sqrt{(x-6)^2+121}}$$

$$\frac{\cancel{(x-6)^2}+121-\cancel{(x-6)^2}}{[(x-6)^2+121]^{3/2}} \Rightarrow \frac{121}{[(x-6)^2+121]^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{25}{[(x+6)^2+25]^{3/2}} + \frac{121}{[(x-6)^2+121]^{3/2}}$$

$$F''(3.72) = \frac{25}{[(3.72+6)^2+25]^{3/2}} + \frac{121}{[(3.72-6)^2+121]^{3/2}}$$

$$= 0.0191 + 0.0853$$

$$F''(3.72) = 0.1044$$

$$F''(-21.97) = \frac{25}{[(-21.97)^2+25]^{3/2}} + \frac{121}{[(-21.97)^2+121]^{3/2}}$$

$F''(-21.97) = 0.1053 + 0.0045$

$= 0.0098$

Therefore

$\left.\frac{d^2y}{dx^2}\right|_{x=3.72} \Rightarrow 0.1044 > 0$

$\left.\frac{d^2y}{dx^2}\right|_{x=-21.97} = 0.0098$

Hence, the minimum point at $x = 3.72$ & $x = -21.97$