

Black Scholes

$$\text{Stock price} = S_0 = \$40$$

$$\text{Strike Price} = K = \$45$$

$$\text{Time (in years)} = T = 4/12 = 0.33$$

$$\text{Risk free rate} = r = 0.03$$

$$\text{Standard deviation} = \sigma = 0.4$$

N \rightarrow Normal distribution

$$C_0 = S_0 N(d_1) - K e^{-rt} N(d_2)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2) \times T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$d_1 = \frac{\ln(40/45) + (0.03 + (0.4)^2/2) \times 0.33}{0.4 \times \sqrt{0.33}}$$

$$= \frac{\ln(0.8889) + (0.03 + 0.08) \times 0.33}{0.4 \times 0.5745}$$

$$= \frac{-0.1178 + 0.0363}{0.2298}$$

$$= \frac{-0.0815}{0.2298}$$

$$\therefore d_1 = -0.3547$$

$$\begin{aligned}
 d_2 &= d_1 - \sigma \sqrt{t} \\
 &= -0.3547 - (0.4 \times 0.33) \\
 &= -0.3547 - 0.132
 \end{aligned}$$

$$d_2 = -0.4867$$

Substitute in the Call option formulae

$$C_0 = S_0 N(d_1) - K e^{-rt} N(d_2)$$

$$N(d_1) = N(-0.3547)$$

$$= 1 - N(0.3547)$$

$$= 1 - 0.6388$$

$$= 0.3614$$

$$N(d_2) = N(-0.4867)$$

$$= 1 - N(0.4867)$$

$$= 1 - 0.6868$$

$$= 0.3132$$

$$C_0 = (40 \times 0.3614) - 45 \times e^{-0.03(0.33)} \times 0.3132$$

$$= 14.456 - 45 \times e^{-0.0099} \times 0.3132$$

$$= 14.456 - 45 \times 0.9901 \times 0.3132$$

$$= \cancel{14.456} - \cancel{13.9544} \quad 14.456 - 13.9545$$

$$C_0 = 0.5015$$