CSP lecture 25/26 – Problem Set 1

$\mathbb{A} = (A; R_1, R_2, \ldots)$ is called *a relational structure* if

- A is a set, called *domain*,
- $R_1, R_2, \ldots$ are *relations* on A, i.e. $R_i \subseteq A^{n_i}$ for some finite arity $n_i \geq 1$.

Definition: CSP($\mathbb{A}$)
Given a list of constraints $R_i(x_{i_1}, \ldots, x_{i_r})$, $R_j(x_{j_1}, \ldots, x_{j_s})$, $R_k(x_{k_1}, \ldots, x_{k_t}), \ldots$ **Decide** whether they are satisfiable.

Consider the following relations on $\{0, 1\}$:

- $C_i := \{i\}$, for $i \in \{0, 1\}$
- $R := \{(0, 0), (1, 1)\}$
- $N := \{(0, 1), (1, 0)\}$
- $S_{ij} := \{0, 1\}^2 \setminus \{(i, j)\}$, for $i, j \in \{0, 1\}$
- $H := \{0, 1\}^3 \setminus \{(1, 1, 0)\}$
- $G_1 := \{(0, 0, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0)\}$, $G_2 := \{(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)\}$

Problem 1. Find a polynomial–time algorithm for CSP($\mathbb{A}$), where

1. $\mathbb{A} = (\{0, 1\}; R)$
2. $\mathbb{A} = (\{0, 1\}; R, C_0, C_1)$
3. $\mathbb{A} = (\{0, 1\}; S_{10})$
4. $\mathbb{A} = (\{0, 1\}; S_{10}, C_0, C_1)$
5. $\mathbb{A} = (\{0, 1\}; S_{01}, S_{10}, C_0, C_1)$
6. $\mathbb{A} = (\{0, 1\}; N)$
7. $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1)$
8. $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1, S_{00}, S_{01}, S_{10}, S_{11})$
9. $\mathbb{A} = (\{0, 1\}; \text{all unary and binary relations})$

Problem 2. Find a polynomial–time algorithm for CSP($\{0, 1\}; H, C_0, C_1$).

Problem 3. Find a polynomial–time algorithm for CSP($\{0, 1\}; C_0, C_1, G_1, G_2$).

Problem 4. Find a polynomial–time algorithm for CSP($\mathbb{Q}; <$).

Problem 5. Prove that CSP($\mathbb{Q}; <$) $\neq$ CSP($\mathbb{A}$), for every finite relational structure $\mathbb{A} = (A; R)$.