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Problem Set #1 **Date: 08/18/2011**

1 Two Missing Numbers

Description

This problem is similar to the missing number problem you saw before.

You are given a natural number N where $3 \leq N \leq 100$. Then you are also given N distinct natural numbers ranged between 1 and $N + 2$, inclusive. You want to find the two missing numbers.

Input Format

You are first given N, and then you are given N numbers as described above.

Output Format

You output the two missing numbers in order.

Sample Input 1

```
5
5 7 1 4 2
```

Sample Output 1

```
3 6
```

Sample Input 2

```
8
1 2 3 4 7 8 9 10
```

Sample Output 2

```
5 6
```

2 Caesar Cipher

Description

From Wikipedia

In cryptography, a Caesar cipher is one of the simplest known encryption techniques. It is a type of substitution cipher in which each letter in the plain-text is replaced by a letter *some fixed number of positions* down the alphabet. For example, with a shift of 3, A would be replaced by D, B would become E, and so on. The method is named after Julius Caesar, who used it to communicate with his generals.

The transformation can be represented by aligning two alphabets; the cipher alphabet is the plain alphabet rotated left or right by some number of positions. For instance, here is a Caesar cipher using a left rotation of three places (the shift parameter, here 3, is used as the key):

```
Plain:    ABCDEFGHIJKLMNOPQRSTUVWXYZ
Cipher:   DEFGHIJKLMNOPQRSTUVWXYZABC
```

When encrypting, a person looks up each letter of the message in the ”plain” line and writes down the corresponding letter in the ”cipher” line. Deciphering is done in reverse.

```
Ciphertext: WKH TXLFN EURZQ IRA MXPSV RYHU WKH ODCB GRJ
Plaintext:  the quick brown fox jumps over the lazy dog
```

Input Format

You are given a string which may be a cipher-text or plain-text. The given string consists only of upper-case alphabets. Then, you are given an integer that denotes the shift that was used in encryption or should be used in decryption. This integer ranges between 1 and 25, inclusive.

Output Format

You first output the encrypted string using the given shift. Next, you output the decrypted string using the given shift.

Sample Input 1

```
GRJ
3
```

Sample Output 1

```
JUM
DOG
```

Sample Input 2

```
ABCDE
1
```

Sample Output 2

```
BCDEF
ZABCD
```

3 Fibonacci Numbers

Description

In Mathematics, the Fibonacci numbers are defined as follows:

$$F_n = F_{n-1} + F_{n-2}$$

with seed values $F_0 = 0$ and $F_1 = 1$.

Given some number n, you want to compute the value of F_n.

Input Format

You are given an integer n where $0 \leq n \leq 1,000$.

Output Format

Output the value of F_n. Since F_n could become really large, you should output F_n modulo 100,007 (that is the remainder of F_n divided by 100,007).

Sample Input 1

```
5
```

Sample Output 1

```
5
```

Sample Input 2

```
12
```

Sample Output 2

```
144
```

4 Flower Shop

Description

You are the owner of a really big flower shop that delivers flowers to customers. On a particular day, you decided to have the following promotional event.

You ask N customers to write down the highest price they are willing to pay, provided that you deliver the most beautiful flower bouquet in time. After collecting the N numbers (the prices), you will decide the border line price p such that any customer who is willing to pay p or higher will pay p dollars and receive the bouquet, but all other customers who are willing to pay strictly less than p will NOT pay and will NOT receive any bouquets.

As a businessman, you want to maximize profit by choosing the optimal value p. You want to write a program that finds such p for you.

Input Format

You are first given the number of customers, N where $2 \leq N \leq 20$. Then, you are given N natural numbers $(m_1, m_2, \ldots, m_N)$ where m_i represents the maximum amount of money that customer i is willing to pay where $1 \leq i \leq N$. You can assume that $1 \leq m_i \leq 100$ for all m_i where $1 \leq i \leq N$.

Output Format

You first output the optimal price p, and then output your maximized profit. If there are multiple p values that maximize your profit, you may output any such p.

Sample Input 1

```
5
2 3 2 3 3
```

Sample Output 1

```
2 10
```

Note 1

There is no point for you to set $p > 3$ as your profit will be zero. If you set $2 < p \leq 3$, then you can only collect money from customer $2, 4$, and 5, which makes your profit equal to $3p$ dollars; obviously, you will set $p = 3$ in this case to make 9 dollars. If you set $p = 2$, you will collect 2 dollars from all five customers to obtain 10 dollars of profit (the optimal solution). If you set $p < 2$, you will collect p dollars from all customers, but it will always be less than 10 dollars.

Hence, you print 210 to denote that you chose the price to be 2 and your maximized profit was 10.

Sample Input 2

```
5
100 100 10 10 10
```

Sample Output 2

```
100 200
```