PHYSICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

Useful Constants

Electron charge (e)	$= 1 \cdot 602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9 \cdot 109 \times 10^{-31}$ kg
Proton mass (m_p)	$= 1 \cdot 672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8 \cdot 854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1 \cdot 257 \times 10^{-6}$ henry/m
Velocity of light in free space (c)	$= 3 \times 10^{8}$ m/s
Boltzmann constant (k)	$= 1 \cdot 380 \times 10^{-23}$ J/K
Electronvolt (eV)	$= 1 \cdot 602 \times 10^{-19}$ J
Planck constant (h)	$= 6 \cdot 626 \times 10^{-34}$ J s
Stefan constant (σ)	$= 5 \cdot 67 \times 10^{-8}$ W m^{-2} K^{-4}
Avogadro number (N_A)	$= 6 \cdot 022 \times 10^{26}$ kmol^{-1}
Gas constant (R)	$= 8 \cdot 31 \times 10^{3}$ J kmol^{-1} K^{-1}
exp (1)	$= 2 \cdot 718$

SECTION—A

1. (a) Suppose the position of a particle of mass m on the xy-plane is

$$\bar{r} = (x, y, z) = (r\cos\phi, \, r\sin\phi, \, 0)$$

Here r and ϕ are functions of time t. Find l_z, the z-component of $\vec{l}$, where $\vec{l}$ is the angular momentum of the particle. Further, show that the areal velocity is equal to the angular momentum divided by $2m$. Is areal velocity constant?

(b) Explain the rotation of a free rigid body and show that a rigid body rotating in any manner about a fixed point has two constants of motion, L^2 and T. Here $\vec{L}$ is angular momentum and T is kinetic energy. **8**

(c) Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is $0 \cdot 7c$. Determine the velocity of each spaceship as measured by the stationary observer on the Earth. **8**

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(d) Write an expression for the profile ($t = 0$) of a harmonic wave moving in the +x-direction such that at $x = 0$, $\psi = 10$; at $x = \lambda/6$, $\psi = 20$ and at $x = 5\lambda/12$, $\psi = 0$. (The notations carry usual meanings)

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(e) What do you understand by quarter wave plate? How can it be used to convert a linearly polarized wave into a circularly polarized wave?

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2. *(a)* A rocket of mass $m_1 + m_2$ is launched with a velocity whose horizontal and vertical components are u_x and u_y. At the highest point in its path, the rocket explodes into two parts of masses m_1 and m_2 that separate in a horizontal direction in the original plane of motion. Show that the fragments strike the ground at a distance apart given by

$$D = \frac{u_y}{g}\sqrt{\frac{2(m_1+m_2)K}{m_1 m_2}}$$

where K is the kinetic energy produced by the explosion. (Neglect air resistance, the mass of explosive, any spinning motion of the fragments and assume that g is constant)

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(b) *(i)* Explain Coriolis force. Compare the qualitative effects of the Coriolis forces in the two geographical hemispheres of the Earth for rivers flowing east.

(ii) With schematic, explain Foucault's pendulum.

7

(c) Derive Euler-Lagrange equations of motion from Hamilton's principle for any bilateral holonomic system having no non-potential forces and n degree of freedom. Can it be applied to non-holonomic systems?

18

3. *(a)* A rod of length L_0 moves with speed v along the horizontal direction. The rod makes an angle θ_0 with respect to the x'-axis.

(i) Determine the length of the rod as measured by a stationary observer.

(ii) Determine the angle θ the rod makes with the x-axis.

(v is comparable to c)

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(b) Describe Michelson-Morley experiment with the help of a diagram. Discuss why it is considered as the 'most famous failed experiment'.

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(c) *(i)* Obtain an expression for the displacement of the damped harmonic oscillator where the damping force is proportional to the velocity. Discuss the effect of the damping on the displacement and frequency of the oscillator. Discuss the difference physically between overdamped and critically damped by taking example of a pendulum. 8

(ii) A particle of mass 3 moves along the *x*-axis attracted towards the origin by a force whose magnitude is numerically equal to $12x$. The particle is also subjected to damping force whose magnitude is numerically equal to 12 times the instantaneous speed. If it is initially at rest at $x = 10$, find the position and the velocity of the particle at any time. 7

4. *(a)* Consider the formation of Newton's rings when two closely spaced wavelengths are present, for example, the D_1 and D_2 lines of sodium ($\lambda_1 = 5890$ Å and $\lambda_2 = 5896$ Å). What will be the effect of the presence of these two wavelengths as the lens is gradually moved away from the plate? What will happen if the sodium lamp is replaced by a white source? (The radius of curvature of the lens, $R = 100$ cm) 20

(b) A glass chamber 25 mm long filled with air is positioned in front of one of the secondary sources in Young's experiment. The air is removed from the chamber and a test gas is entered in its stead. On comparing the fringe system corresponding to air with that of the test gas, it is found that the entire interference pattern on the screen was displaced by 21 bright bands towards gas -containing chamber. Given that $\lambda = 656 \cdot 2816$ nm for which the refractive index of air is $n_a = 1 \cdot 000276$. Determine the refractive index of the gas (n_g). 10

(c) *(i)* Describe the wave given by the expression

$$E = \hat{j} E_0 \sin\left(\frac{2\pi x}{\lambda} - \omega t\right) - \hat{k} E_0 \sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$$

where $\hat{j}$ and $\hat{k}$ are unit basis vectors in Cartesian coordinates. 5

(ii) Write an expression for linearly polarized harmonic plane wave of scalar amplitude E_0, propagating along a line in the *xy*-plane at 45° to the *x*-axis and having the *xy*-plane as its plane of vibration. 5

5. *(a)* 10 g of water at 60 °C is mixed with 30 g of water at 20 °C. Will the entropy of the system increase or decrease? Calculate the change.

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(b) Write down the distribution law obeyed by electron gas and apply the same to derive Richardson-Dushman equation.

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(c) In what way Maxwell's concept towards electric field and magnetic field is different from Faraday's concept? Derive magnitude of displacement current in a parallel-plate capacitor and account it for the continuous path for the charges across the capacitor.

(d) Calculate the magnitude of Poynting vector at the surface of the Sun. Given that the power radiated by the Sun $= 3\cdot8\times10^{26}$ watts and the radius of the Sun $= 7\times10^8$ m.

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(e) Derive Poisson's equation. Under what conditions it takes the form of Laplace's equation? If ϕ_1, ϕ_2, $\cdots$, ϕ_n are solutions of Laplace's equation, then show that

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$$\phi = \lambda_1\phi_1 + \lambda_2\phi_2 + \cdots + \lambda_n\phi_n$$

is also a solution. Here λs are arbitrary constants.

6. *(a)* Show that the energy density in electrostatic field is given as

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$$u = \frac{1}{2}\overline{D}\cdot E$$

Further, show that the dielectric constant is a symmetric tensor except when the field energy vanishes identically.

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(b) What are ferromagnetic materials? By suitable figure, illustrate magnetic domain and magnetization curve for a ferromagnetic material.

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(c) What is a transformer? Is it for a.c. or d.c.? With a suitable diagram and representative symbol, explain the construction and necessary terms used in a transformer.

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7. *(a)* Write the equation for total scattering cross-section for electromagnetic waves. Under what condition the scattering is known as Rayleigh scattering? Write the importance of Rayleigh scattering in nature.

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(b) Define a perfect blackbody. State Kirchhoff's law of radiation. What are the limitations of Kirchhoff's law?   15

(c) Each square meter of the Sun's surface radiates energy at the rate of $6 \cdot 3 \times 10^7$ J/m^2/s and Stefan constant is $5 \cdot 669 \times 10^{-8}$ W/m^2/K^4. Find the temperature of the Sun's surface. Assume that the Stefan's law applies to the radiation.   10

8. (a) Deduce Clausius-Clapeyron equations based on reversible cycle. Show that the specific heat of steam is negative. What is the significance of negative specific heat?   15

(b) There are g cells of energy ε. Show that the number n of bosons of energy ε distributed among these cells is given by

$$n = \frac{g}{e^{(\varepsilon - \mu)/kT} - 1}$$

What is μ and how will you find it?   15

(c) Calculate (i) the internal energy of the electron gas per unit volume and (ii) the molar specific heat at constant volume for sodium at 100 K containing one free electron per atom. Given that the density of sodium = $0 \cdot 97 \times 10^3$ kg m^{-3} and the atomic weight of sodium = 23.   10

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