

## PHYSICS

## Paper I

0000069

Time Allowed : Three Hours

Maximum Marks : 200

## QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two **Sections A and B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question / part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

## Useful Constants

Electron charge ( $e$ )	=	$1.602 \times 10^{-19}$ C
Electron rest mass ( $m_e$ )	=	$9.109 \times 10^{-31}$ kg
Proton mass ( $m_p$ )	=	$1.672 \times 10^{-27}$ kg
Vacuum permittivity ( $\epsilon_0$ )	=	$8.854 \times 10^{-12}$ farad/m
Vacuum permeability ( $\mu_0$ )	=	$1.257 \times 10^{-6}$ henry/m
Velocity of light in free space ( $c$ )	=	$3 \times 10^8$ m/s
Boltzmann constant ( $k$ )	=	$1.380 \times 10^{-23}$ J/K
Electron volt (eV)	=	$1.602 \times 10^{-19}$ J
Planck constant ( $h$ )	=	$6.626 \times 10^{-34}$ J s
Stefan constant ( $\sigma$ )	=	$5.67 \times 10^{-8}$ Wm <sup>-2</sup> K <sup>-4</sup>
Avogadro number ( $N$ )	=	$6.022 \times 10^{26}$ kmol <sup>-1</sup>
Gas constant ( $R$ )	=	$8.31 \times 10^3$ J kmol <sup>-1</sup> K <sup>-1</sup>
exp (1)	=	2.718



## SECTION 'A'

1. Answer *all* of the following : 8×5=40

1.(a) A force field is given by

$$\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$

Is it a conservative field? If so, what is the scalar potential? 2+6

1.(b) Prove that the group velocity  $V_g$  of electromagnetic waves in a dispersive medium is given by

$$V_g = \frac{c}{n + \omega \frac{dn}{d\omega}}$$

where  $c$  is the velocity of light in vacuum and  $n$  is the refractive index of the medium for the angular frequency  $\omega$  of the waves. 8

1.(c) What is the significance of the null result of Michelson-Morley experiment? In their experiment Michelson and Morley set  $l_1 = l_2 = 11$  metres. The wavelength of light used was  $5.5 \times 10^{-7}$  m. The orbital velocity of the earth is taken to be 30 kms/sec. Estimate the fringe shift to be expected. 3+5

1.(d) A diffraction grating has 5000 lines per cm. For illumination at normal incidence, determine the dispersive power of the grating in the second order spectrum in the range of wavelengths around 500 n.m. 8

1.(e) The vibrations of a string fixed at both ends is represented by the equation

$$y = 2 \sin \frac{\pi x}{3} \cos 50 \pi t \text{ (m)}$$

If the above stationary wave is produced due to the superposition of two component waves of the same frequency, velocity and amplitude travelling in opposite directions, find (i) the exact equations of the displacements associated with the vibrations of the component waves and (ii) the distance between two consecutive nodes of the stationary wave. 5+3

2.(a) What are cyclic coordinates for a system of particles? Discuss the significance of the Hamilton's principle. 5+5

Show how Lagrange's equations can be derived from Hamilton's principle. 15

2.(b) Using Euler's equations for a force free motion of a rigid body, show that the Kinetic energy remains constant throughout the motion of the rigid body. 15

3.(a) Write the components of the velocity 4-vector of a particle in an inertial frame  $S$ . How does the four velocity transform to another inertial frame  $S'$ ? 5+5

Obtain the Einstein's law of addition of velocities from the above transformations.

10



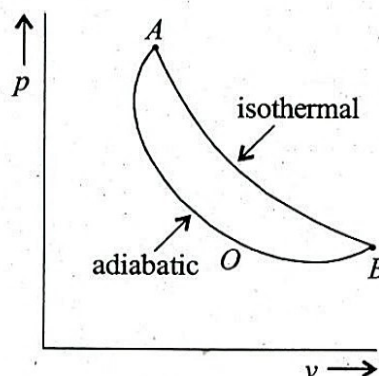
- 3.(b) State Huygen's principle. Using this principle and suitable diagram, show that this principle could lead to Snell's law of refraction. 20
- 4.(a) Discuss the principles of Fresnel's half period zone and explain how these are used in the construction of a zone plate. Show how the zone plate has several foci. 20
- 4.(b) Plane polarized light of wavelength 6000 Å is incident on a thin quartz plate cut with faces parallel to the optic axis. Given,  $\mu_o = 1.544$  and  $\mu_e = 1.553$ , calculate the following quantities :
- (i) The ratio of the intensities of the ordinary and the extraordinary light if the planes of vibration of two incident lights make an angle of  $30^\circ$  with the optic axis. 10
- (ii) The minimum thickness of the plate for which ordinary and the extraordinary waves will combine to produce plane polarized light. 10

## SECTION 'B'

5. Answer *all* of the following : 8×5=40
- 5.(a) An inductor  $L = 0.1 \text{ H}$  in series with a resistor  $R = 20 \Omega$  is connected across 230 V, 50 Hz a.c. voltage. Determine the following :
- (i) inductive reactance (ii) impedance of the circuit (iii) current in the circuit (iv) voltage across  $R$  (v) voltage across  $L$  (vi) phase angle between the current and voltage (vii) power factor (viii) power consumed in the circuit 8
- 5.(b) What is the volume density of charge in a region of space where electrostatic potential is given by
- $$V = a - b(x^2 + y^2) - c \log(x^2 + y^2),$$
- where  $a, b, c$  are constants. 8
- 5.(c) If the magnetic field  $\vec{B}$ , at a point with position vector  $\vec{r}$  is uniform, show that the corresponding vector potential  $\vec{A}(\vec{r})$  is given by
- $$\vec{A}(\vec{r}) = -\frac{1}{2} [\vec{r} \times \vec{B}]$$
- 8
- 5.(d) A point charge of  $q$  is held above a grounded conducting plane located at  $z = 0$ . If the position of the charge is  $(0, 0, d)$ , obtain an expression for the induced charge density on the plane as a function of the coordinates  $x$  and  $y$ . 8
- 5.(e) A system of non-interacting fermions enclosed in a volume,  $V$ , is at  $T = 0^\circ \text{K}$ . Find an expression for the internal energy,  $U$ , of the system. 8



- 6.(a) Distinguish between diamagnetic, paramagnetic and ferromagnetic substances. 5
- 6.(b) (i) Show that the work done per unit volume of a ferromagnetic substance per one complete cycle of hysteresis is given by  $W = \oint \vec{H} \cdot d\vec{B}$  15  
(ii) Further, show that the value of  $W$  is equal to the area of  $B-H$  loop. Which material is preferred for the core of a transformer and why? 10
- 6.(c) Show that for any electromagnetic wave propagating in free space, the total average energy per unit volume is  $\frac{1}{2} \epsilon_0 E_{max}^2$ ; where  $E_{max}$  is the amplitude of the electric field associated with the electromagnetic wave. 10
- 7.(a) (i) Derive Planck's law of Black body radiation. 10  
(ii) Obtain its limiting forms for (i) very low frequency and (ii) very high frequency. 10
- 7.(b) Show that Maxwell's equations of electrodynamics are invariant under Lorentz transformations. 20
- 8.(a) A quasistatic isothermal and quasistatic adiabatic intersect in a  $p-v$  diagram at more than one point as shown in diagram. Does it imply violation of 2nd law of thermodynamics? Give reasons for your answer.



1+9

- 8.(b) Write down the expression for the Bose-Einstein distribution function and explain the meaning of the symbols used. 10
- 8.(c) The mean speed of the molecules of an ideal monatomic gas when it contracts (or expands) adiabatically depends on the pressure according to the law  $v = Kp^n$ , where  $K$  is a constant. Find  $n$ . 10
- 8.(d) Derive the expression for the specific heat of a solid based on Einstein's theory. Obtain the limiting form of the specific heat at very low temperature. 10