PHYSICS

PAPER—II

Time Allowed : Three Hours | Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

Useful Constants :

Mass of proton	=	$1 \cdot 673 \times 10^{-27}$ kg
Mass of neutron	=	$1 \cdot 675 \times 10^{-27}$ kg
Mass of electron	=	$9 \cdot 11 \times 10^{-31}$ kg
Planck constant	=	$6 \cdot 626 \times 10^{-34}$ J s
Boltzmann constant	=	$1 \cdot 380 \times 10^{-23}$ J K^{-1}
Bohr magneton (μ_B)	=	$9 \cdot 273 \times 10^{-24}$ A m^2
Nuclear magneton (μ_N)	=	$5 \cdot 051 \times 10^{-27}$ J T^{-1} (A m^2)
Electronic charge	=	$1 \cdot 602 \times 10^{-19}$ C
Atomic mass unit (u)	=	$1 \cdot 660 \times 10^{-27}$ kg
	=	931 MeV

g_s^p	=	$5 \cdot 5855\ \mu_N$	$m(p)$	=	$1 \cdot 00727$ u
$m(n)$	=	$1 \cdot 00866$ u	$m(^4_2\text{He})$	=	$4 \cdot 002603$ u
$m(^{12}_6\text{C})$	=	$12 \cdot 00000$ u	$m(^{87}_{38}\text{Sr})$	=	$86 \cdot 908893$ u
$m(^2_1\text{H})$	=	$2 \cdot 014022$ u	$m(^3_1\text{H})$	=	$3 \cdot 0160500$ u
$m(^{16}_8\text{O})$	=	$15 \cdot 999$ u			
$\hbar$	=	$1 \cdot 05 \times 10^{-34}$ J s			
$\hbar c$	=	197 eV nm			

SECTION—A

1\. (a) The lifetime of a given atom in an excited state is 10^{-8} s. It comes to the ground state by emitting a photon of wavelength 5800 Å. Find the energy uncertainty and wavelength uncertainty of the photon. 8

(b) An electron is confined to an infinite well whose width L (= 100 pm) is roughly the size of an atom.

(i) What are the energies of four least energetic quantum states?

(ii) What energy must be imparted to the electron to raise it from a state with $n = 12$ to a higher energy state with $n = 25$?

8

(c) (i) Find the lowest energy of an electron confined in a three-dimensional cubic box of each side 1 Å.

(ii) Find the temperature at which the average energy of the molecule would be equal to the lowest energy of the electron. [Given : $k = 1{\cdot}38 \times 10^{-23}$ J/K]

4+4=8

(d) The quantum numbers of the two optical electrons in a two-valence electron atom are

$$n_1 = 6, \quad l_1 = 3, \quad s_1 = \frac{1}{2}$$
$$n_2 = 5, \quad l_2 = 1, \quad s_2 = \frac{1}{2}$$

Assuming j-j coupling, find the possible values of J. 8

(e) Describe the basic principle of nuclear magnetic resonance (NMR) and mention its different applications. 8

2\. (a) Derive an approximate expression for the transmission coefficient for a rectangular potential barrier for which $\frac{a}{\hbar}\sqrt{2m(V_0 - E)} \gg 1$. 15

(b) Distinguish between normal and anomalous Zeeman effects. Explain the Zeeman pattern of the resonance (D_1, D_2) lines of sodium. 15

(c) Explain the mechanism of phosphorescence emission of radiation. Distinguish between fluorescence and phosphorescence. 10

3\. (a) The components of the angular momenta $\vec{J}_1$ and $\vec{J}_2$ satisfy commutation rules. Show that the components of the sum $\vec{J} = \vec{J}_1 + \vec{J}_2$ also satisfy the commutation rules. Whether the difference $\vec{J}_1 - \vec{J}_2$ satisfies an angular momentum? 10

(b) Find the values of the following :

(i) $\vec{L} \times \vec{L}$ (ii) $L_+ L_-$ (iii) $[L_z, L_+]$

where $\vec{L}$ is the angular momentum operator. L_+ and L_- are raising and lowering operators respectively. 15

(c) Discuss the main features of the vibrational and the rotational Raman spectra of diatomic molecules. How is it used to explain the structure of a molecule? 15

4. (a) A one-dimensional oscillator of mass $m = 10^{-20}$ kg oscillates under a force constant $k = 10^{-4}$ N/m.

(i) Evaluate the zero-point energy.

(ii) Calculate the classical amplitude for which the oscillator can have this energy.

(iii) What is the energy for the second excited state?

15

(b) The normalized eigenfunction for the ground state of hydrogen atom is

$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

Calculate the expectation value of the radius vector r of the electron in the ground state. 10

(c) The fine-structure lines of CN band at 3883·4 Å can be represented by the following equation :

$$\nu = 25798 + 3 \cdot 85m + 0 \cdot 068m^2 \text{ cm}^{-1}$$

Calculate the separation between the null-line and the band-head, and state the direction of degradation of the band. 15

SECTION—B

5. (a) (i) The radius of a nucleus is given to be 3·46 fermi. Estimate the mass number A of the nucleus and identify the nucleus.

(ii) What is Bohr magneton? Find out the ratio of nuclear magnetic moment to Bohr magneton.

5+3=8

(b) A reactor is developing nuclear energy at a rate of $3 \cdot 2 \times 10^{7}$ watts. How many atoms of U^{235} undergo fission per second? How many kg of U^{235} would be used up in 1000 hours of operation? Assume that an average energy of 200 MeV is released per fission. Take Avogadro number as 6×10^{23} and $1 \text{ MeV} = 1 \cdot 6 \times 10^{-13}$ joule. 8

(c) What do you mean by strength of interaction among the elementary particles? Calculate the dimensionless coupling constants of gravitational and electromagnetic interactions. 8

(d) (i) Diamond and silicon have similar band structure. Why is diamond an insulator while silicon a semiconductor?

(ii) An insulator has an optical absorption which occurs for all wavelengths lesser than 1400 Å. Find the width of the forbidden energy band for the insulator.

3+5=8

(e) (i) Prove the following using De Morgan's theorems :

$$A \cdot B + C \cdot D = \overline{\overline{A \cdot B} \cdot \overline{C \cdot D}}$$

(ii) Simplify the following expression :

$$Y = A \cdot B \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot \overline{C} \cdot \overline{D} + B \cdot \overline{C} \cdot D$$

4+4=8

6. (a) Write the semi-empirical mass formula. Establish the relation $A \simeq 2Z$, where A is mass number and Z is atomic number of the light nuclei, using this semi-empirical mass formula. [Given : good fitting coefficients for Coulomb energy $(a_C) = 0 \cdot 71$ MeV, for asymmetry energy $(a_n) = 22 \cdot 7$ MeV] 15

(b) (i) Show that pion decay, muon decay and pair production conserve lepton numbers L_e and L_μ.

(ii) Which of the following reactions is/are possible? Justify with reason :

$$\overline{\nu}_e + p = n + \mu^{+}$$
$$\overline{\nu}_e + p \rightarrow n + e^{+}$$

9+6=15

(c) An n-p-n transistor with $\alpha = 0 \cdot 98$ is operated in the CB configuration. If the emitter current is 3 mA and the reverse saturation current is $I_{CO} = 10\ \mu\text{A}$, what are the base current and the collector current? 10

7. (a) Explain how the nuclear shell model predicts the existence of magic numbers. Using shell model, predict the spin parity or characteristics of ground state of ${}^{15}_{8}O$, ${}^{16}_{8}O$ and ${}^{17}_{8}O$. 10

(b) (i) Distinguish between d.c. Josephson effect and a.c. Josephson effect. Show that Josephson effect exhibits the quantum interference phenomenon on macroscopic scale.

(ii) The critical magnetic field of superconducting niobium is 1×10^5 A/m at 8 K and 2×10^5 A/m at 0 K. Calculate the critical transition temperature (T_c) of niobium.

10+5=15

(c) State the basic conditions for oscillations in a feedback amplifier. What are the primary requirements to obtain steady oscillations at a fixed frequency? 15

8. (a) (i) What is Q-value and how does it help to study the kinematics of a nuclear reaction?

(ii) Prove that ^{252}Cf can undergo spontaneous fission as under :

$$^{252}\text{Cf} = {}^{98}\text{Zr} + {}^{145}\text{Ce} + 9\,{}^{1}n$$

Given :

$$M(^{252}\text{Cf}) = 252\cdot081621 \text{ a.m.u.}$$
$$M(^{98}\text{Zr}) = 97\cdot912735 \text{ a.m.u.}$$
$$M(^{145}\text{Ce}) = 144\cdot917230 \text{ a.m.u.}$$
$$M(^{1}n) = 1\cdot008665 \text{ a.m.u.}$$

5+5=10

(b) What is Meissner effect? Show that the Maxwell's equation is in contradiction to the Meissner effect. What are the inferences from this contradiction? 15

(c) Describe briefly the functions of the basic segments of a microprocessor. Draw a block diagram of the registers in the 8085 microprocessor. 15

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