

# PHYSICS

## Paper - I

Time Allowed : **Three Hours**Maximum Marks : **200**

### Question Paper Specific Instructions

**Please read each of the following instructions carefully before attempting questions :**

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

### Useful Constants :

Electron charge ( $e$ )	$= 1.602 \times 10^{-19} \text{ C}$
Electron rest mass ( $m_e$ )	$= 9.109 \times 10^{-31} \text{ kg}$
Proton mass ( $m_p$ )	$= 1.672 \times 10^{-27} \text{ kg}$
Vacuum permittivity ( $\epsilon_0$ )	$= 8.854 \times 10^{-12} \text{ farad/m}$
Vacuum permeability ( $\mu_0$ )	$= 1.257 \times 10^{-6} \text{ henry/m}$
Velocity of light in free space ( $c$ )	$= 3 \times 10^8 \text{ m/s}$
Boltzmann constant ( $k$ )	$= 1.380 \times 10^{-23} \text{ J/K}$
Electron volt (eV)	$= 1.602 \times 10^{-19} \text{ J}$
Planck constant ( $h$ )	$= 6.626 \times 10^{-34} \text{ Js}$
Stefan constant ( $\sigma$ )	$= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$
Avogadro number ( $N$ )	$= 6.022 \times 10^{26} \text{ kmol}^{-1}$
Gas constant ( $R$ )	$= 8.31 \times 10^3 \text{ J kmol}^{-1} \text{ K}^{-1}$
exp (1)	$= 2.718$

## SECTION A

**Q1. Answer the following :**

**8×5=40**

- (a) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a (\theta - \sin \theta)$$

$$y = a (1 + \cos \theta) \text{ with } 0 \leq \theta \leq 2\pi.$$

Find the Lagrangian and equation of motion.

8

- (b) Show that the relativistic invariance laws of conservation of momentum lead to the concepts of variation of mass with velocity and mass energy equivalence.

8

- (c) A parallel beam of light of wavelength 5890 Å is incident at an angle of 30° on a plane transmission grating with 15000 lines/inch. Find the highest order of spectrum that can be observed.

8

- (d) Discuss absorption loss in an optical fibre comparing and contrasting the intrinsic and extrinsic absorption mechanisms.

8

- (e) Although the principle of operation of a basic LASER is based upon two energy levels, why does one need a 3-level or a 4-level scheme to achieve satisfactory lasing ? Explain your answer with special reference to a Ruby-laser.

8

**Q2. (a)** Discuss the mechanics of a system of point particles with special emphasis on the conservation theorems. How can we extend the results to a system with continuous mass distribution ?

10

- (b) (i) State and prove Hamilton's principle and use it to prove that the shortest distance between two points in space is a straight line joining them.

- (ii) Use Hamiltonian mechanics to find the differential equation for planetary motion, moving under force  $f(r) = -\frac{k}{r^2}$  and prove that the areal velocity is constant.

8+7=15

- (c) (i) What is Holography ?

- (ii) Show with simple diagrams, how a hologram is written and read using a laser.

- (iii) Mention some important applications of holography.

3+8+4=15



- Q3.** (a) State the fundamental postulates of Einstein's special theory of relativity. Deduce Lorentz transformation equation and discuss how this accounts for the phenomenon of length contraction. 10
- (b) Discuss the properties of Cornu spiral. Show that the spiral can be used to obtain the intensity distribution in the Fresnel's diffraction pattern due to a straight edge. 10
- (c) (i) Using the concept of spontaneous and stimulated emission of radiation, obtain the relation between Einstein's A and B coefficients.
- (ii) What is the physical significance of Einstein's A coefficient ?
- (iii) Justify why lasing action is much more difficult at X-ray frequency than in case of infrared frequency spectrum. 10+5+5=20
- Q4.** (a) (i) In a Michelson's interferometer, 100 fringes cross the field of view when the movable mirror is displaced through  $2.894 \times 10^{-3}$  cm. Calculate the wavelength of the monochromatic source of light.
- (ii) A shift of 200 fringes is observed when the movable mirror of a Fabry-Pérot interferometer is shifted by 0.0298 mm. Calculate the wavelength of the incident radiation. 8+7=15
- (b) State and explain Fermat's principle of extremum path and use the same to deduce the laws of reflection and refraction of light. 10
- (c) (i) Explain the reason for pulse broadening due to intermodal and material dispersion. Deduce the relation of pulse broadening for intermodal dispersion in optical fiber.
- (ii) A step index fiber in air has a numerical aperture of 0.16, a core refractive index of 1.45 and a core diameter of 60  $\mu\text{m}$ . Determine the normalized frequency for the fiber when light at a wavelength of 0.8  $\mu\text{m}$  is transmitted. Also estimate the number of guided modes propagating in the fiber. 10+5=15

## SECTION B

Q

**Q5. Answer the following :**

**8×5=40**

- (a) In a one-dimensional device, the charge density is given by

$$\rho_V = \rho_0 \frac{x}{a}.$$

If  $E = 0$  at  $x = 0$  and  $V = 0$  at  $x = a$ ,

find  $V$  and  $E$  using Laplace equation of electrostatics.

8

- (b) State and explain the Biot-Savart law. Derive an expression for the magnetic field at a point due to an infinitely long straight current carrying conductor.

8

- (c) Write down the four Maxwell's equations and explain the contribution of Maxwell in the development of these equations.

8

- (d) Prove the thermodynamic relation :

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

and hence show that

$$\frac{dP}{dt} = \frac{L}{T(V_2 - V_1)};$$

all the terms have their usual meanings.

8

- (e) Describe neutron star on the basis of Fermi-Dirac statistics and obtain the condition of critical mass for a neutron star.

8

G

- Q6.** (a) Use the method of electric images to find the electric field on the surface of a grounded conducting sphere.

10

- (b) (i) State Faraday's law of electromagnetic induction and prove that it can be expressed in the following vector form :

$$\text{Curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

with  $\vec{E}$  and  $\vec{B}$  being the electric and magnetic fields.

- (ii) A coil of 10 turns has dimension  $9 \text{ cm} \times 7 \text{ cm}$ . It rotates at the rate of  $15\pi \text{ rad/sec}$  in a uniform field whose flux density is  $0.6 \text{ weber/m}^2$ . What is the maximum e.m.f. induced in the coil ?

10+5=15



- (c) How does one explain the observed spectrum of black-body radiation using Planck's quantum hypothesis ? State and obtain Wien's displacement law. Also explain the important features of this law. 15

**Q7.** (a) (i) Using Maxwell's equations, obtain the relation

$$\frac{1}{c} \frac{\partial}{\partial t} \left( \frac{E^2 + B^2}{2} \right) + \nabla \cdot (\vec{E} \times \vec{B}) = 0$$

- (ii) What is Poynting vector ? Deduce Poynting theorem for the flow of energy in an electromagnetic field. 5+10=15

(b) Discuss the reflection and refraction of plane electromagnetic waves at plane dielectric boundaries for normal incidence and also find the reflection and transmission coefficients. 15

(c) What do you understand by spontaneous magnetization below Curie temperature ? Explain with an appropriate diagram, the occurrence of a hysteresis loop in a ferromagnetic material. 10

**Q8.** (a) State Maxwell's distribution law of molecular speeds. Draw and explain a curve between  $n(c)$  and  $c$  in a gas at a given temperature  $T$ , where  $n(c) dc$  is the number of molecules having speed between  $c$  and  $c + dc$ . Discuss the effect of  $T$  and mass  $m$  of the molecule on the nature of the curve. 15

(b) (i) Define and explain the significance of the quality factor of an electrical machine. 5+5=10

(ii) Discuss in brief, the working principle of a transformer.

(c) Derive the mathematical expression for the total energy of a degenerate Fermi gas at a temperature  $T$  and calculate the specific heat of the Fermi gas at this temperature. 15