

PHYSICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully
before attempting questions**

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Useful Constants

Electron charge (e)	$= 1.602 \times 10^{-19} \text{ C}$
Electron rest mass (m_e)	$= 9.109 \times 10^{-31} \text{ kg}$
Proton mass (m_p)	$= 1.672 \times 10^{-27} \text{ kg}$
Vacuum permittivity (ϵ_0)	$= 8.854 \times 10^{-12} \text{ farad/m}$
Vacuum permeability (μ_0)	$= 1.257 \times 10^{-6} \text{ henry/m}$
Velocity of light in free space (c)	$= 3 \times 10^8 \text{ m/s}$
Boltzmann constant (k)	$= 1.380 \times 10^{-23} \text{ J/K}$
Electronvolt (eV)	$= 1.602 \times 10^{-19} \text{ J}$
Planck constant (h)	$= 6.626 \times 10^{-34} \text{ J s}$
Stefan constant (σ)	$= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Avogadro number (N)	$= 6.022 \times 10^{26} \text{ kmol}^{-1}$
Gas constant (R)	$= 8.31 \times 10^3 \text{ J kmol}^{-1} \text{ K}^{-1}$
exp (1)	$= 2.718$

SECTION—A

1. Answer **all** of the following :

8×5=40

(a) The Lagrangian for a particle has the form

$$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - U(r)$$

Obtain its form in plane polar coordinates and then get expression for the Hamiltonian using appropriate transformation.

(b) Discuss that the motion of a particle in a central field gets restricted to a plane provided its angular momentum is nonzero. What will happen if the angular momentum is zero?

(c) Find the velocity and momentum of a particle having rest mass m_0 and kinetic energy equal to two times of its rest mass energy.

(d) In Young's double-slit experiment, a thin mica sheet (refractive index = 1.5) is introduced in the path of one of the beams. If the central fringe gets shifted by 0.2 cm, calculate the thickness of the mica sheet. Assume separation between the two slits (d) and between slit and source (D) as $d = 0.1$ cm and $D = 50$ cm.

(e) Explain physically why dispersion is diminished in multimode parabolic-index fiber than in step-index fiber.

2. (a) Obtain the equation of motion of a simple pendulum using Lagrangian formalism. Hence, obtain expression for the angular frequency of a simple harmonic oscillator. 15

(b) A particle of mass m is moving with the velocity \vec{v} on the surface of the earth. Discuss the nature of its motion in both the hemispheres due to the Coriolis force $\vec{F} = -2m\vec{\omega} \times \vec{v}$. 15

(c) Solve Euler's equation of motion for a rigid body to show that the torque-free motion of a spherical top is a uniform rotation about an axis fixed in space. 10

3. (a) Describe briefly the Michelson-Morley experiment and discuss the significance of its null result in the context of the special theory of relativity. 15

(b) Viewing ionosphere as a dielectric medium of refractive index $\mu = \sqrt{1 - \omega_p^2 / \omega^2}$, ω_p being known as the plasma frequency, determine the group velocity of a radio wave of frequency $\omega = \sqrt{2}\omega_p$. 10

(c) Explain, with neat diagrams, what you mean by monochromatic spherical aberration with reference to paraxial focus, marginal focus and circle of least confusion. 15

4. (a) With a planoconvex lens of radius of curvature R resting on a plane glass surface, Newton's rings are observed with incident light of wavelength λ . Find the radii of m th order dark rings. What will be the colour of the innermost ring with white light illumination? 10

- (b) Deduce an expression for resolving power of a diffraction grating, explaining its meaning. What will be the value of the number of slits in the grating to resolve D_1 and D_2 lines of sodium in the first order, assuming λ and $\Delta\lambda$ respectively as 600 nm and 0.6 nm? 15

- (c) Explain the refractive index distribution for a graded index optical fiber given as

$$n^2(r) = n_1^2 \left[1 - 2\delta \left(\frac{r}{a} \right)^q \right], r < a$$

$$= n_1^2 [1 - 2\delta] = n_2^2, r > a$$

with usual meaning of symbols. Sketch and name the above profiles with justification for $q = 1, 2$ and ∞ . 15

SECTION—B

5. Answer **all** of the following :

8×5=40

- (a) Verify whether the electric potential $V = 15x^2yz - 5y^3z$ satisfies Laplace's equation or not.
- (b) In a simple AC circuit involving only a resistor $R = 50 \Omega$ and a voltage source V , find the linear frequency of the generator if $V = 0.5 V_m$ (V_m is peak e.m.f.) at time $t = \frac{1}{720}$ s (assuming $V = 0$ at $t = 0$).
- (c) Which of the Maxwell equations imply that there are no magnetic monopoles? Explain how the equations would get modified if magnetic monopoles would exist.
- (d) Discuss the consequence following from Joule's free expansion experiments in the context of the internal energy of an ideal gas.
- (e) Plot the Fermi distribution function versus energy at temperatures $T = 0$ and $T > 0$. Explain the nature of the former curve on the basis of the Pauli principle.
6. (a) The electric potential of a grounded conducting sphere of radius a in a uniform electric field $E = E_0 \hat{z}$ is given as

$$\phi(r, \theta) = -E_0 r \left[1 - \left(\frac{a}{r} \right)^3 \right] \cos \theta$$

Find the expression for the surface charge density on the sphere. 15

- (b) Using Ampere's law, derive the magnetic field of a toroid (N turns each carrying current I) of inner radius a and outer radius b at a distance r midway between a and b . 10

- (c) A plane conducting circular wire loop lies perpendicular to a uniform magnetic field B and its area $S(t)$ is changed as $S(t) = S_0(1 - \alpha t)$, where $0 < t < \frac{1}{\alpha}$ (S_0 and α are constants). The wire has resistance per unit length $\rho \Omega \text{ m}^{-1}$. Find the induced current through the wire.

If the current in a certain coil varies at a rate of 50 As^{-1} , the induced e.m.f. is $V = 20$ volts. What is the inductance of the coil? 10+5

7. (a) Prove that the work done on the charges by the electromagnetic force is equal to decrease in energy stored in the field, less the energy that flowed out through the surface. 20+5
Explain what you mean by the Poynting vector.

- (b) Discuss the physical significance of Planck's radiation law in the context of emergence of New Physics. 5+10
If the blackbody energy density $u(\omega)$ has a functional dependence like $u(\omega) \propto \omega^3 (e^x - 1)^{-1}$, where $x = \frac{\hbar \omega}{kT}$, derive Wien's displacement law and comment on its importance.

8. (a) Start from the equation

$$TdS = C_P dT - T \left(\frac{\partial V}{\partial T} \right)_P dP$$

and get the relation

$$\left(\frac{\partial C_P}{\partial P} \right)_T = -T \left(\frac{\partial^2 V}{\partial T^2} \right)_P$$

Hence, show that the third law of thermodynamics requires for the coefficient of thermal expansion of any substance to vanish at $T = 0$. 5+15

- (b) Derive an expression for the total number of particles N in an ideal Bose gas at any temperature T . Hence, obtain the relation

$$N_0 = N \left[1 - \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \right]$$

for the number of particles N_0 in the ground state at $T < T_0$, the Bose-Einstein condensation temperature, and draw N_0 versus T curve. 10+10
