I·F·S~2009

Nº 007846

B-JGT-J-QIA

PHYSICS

Paper I

Time Allowed : Three Hours | Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt questions no. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written in ENGLISH only.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Useful Constants

Electron charge (e) = $1·602 \times 10^{-19}$ C

Electron rest mass (m_e) = $9·109 \times 10^{-31}$ kg

Proton mass (m_p) = $1·672 \times 10^{-27}$ kg

Vacuum permittivity (ε_o) = $8·854 \times 10^{-12}$ farad/m

Vacuum permeability (μ_o) = $1·257 \times 10^{-6}$ henry/m

Velocity of light in free space (c) = 3×10^8 m/s

Boltzmann constant (k) = $1·38 \times 10^{-23}$ J/K

Electron volt (eV) = 1.602×10^{-19} J

Planck's constant (h) = 6.62×10^{-34} Js

Stefan's constant (σ) = 5.67×10^{-8} Wm^{-2} K^{-4}

Avogadro's constant (N) = 6.02×10^{26} kmol^{-1}

Gas constant (R) = 8.31×10^3 J kmol^{-1} K^{-1}

SECTION A

1. Answer any *four* questions from the following.
   (Each answer must not be more than 150 words long)

   $4 \times 10 = 40$

   (a) Using Euler's equations of motion for a force-free rigid body, show that the kinetic energy remains constant all along the motion of the rigid body.

   (b) The vibration of a string fixed at both ends is represented by the equation

   $$y = 2 \sin \frac{\pi x}{3} \cos 50\pi t \text{ metre.}$$

   If the above stationary wave is produced due to the superposition of two waves of same frequency, velocity and amplitude travelling in opposite directions

   $$y_1 = A \sin \frac{2\pi}{\lambda} (x - vt) \text{ and } y_2 = A \sin \frac{2\pi}{\lambda} (x + vt)$$

   (i) find the equation of the component waves and

   (ii) find the distance between two consecutive nodes of the stationary wave.

(c) Light rays of wavelength 5890 Å and 5896 Å fall on a grating having 5000 lines/cm. If a lens of focal length 100 cm is used to form spectra on a screen, find the distance between the lines in the third order.

(d) For a damped harmonic oscillator with equation

$$m\ddot{x} + \beta\dot{x} + kx = 0,$$

show that the work done against the damping force in an infinitesimal time 'dt' is equal to the loss of energy of the mass 'm' during the same time interval 'dt'.
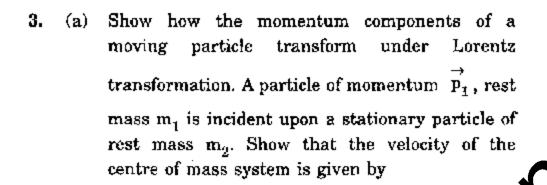
(e) Explain the mechanisms of pulse dispersion in a step index fibre.

2. (a) A moving particle of charge Ze hits a fixed charge $Z'e$. Show that the Rutherford scattering cross-section $\sigma(\theta)$ for this phenomenon can be given by

$$\sigma(\theta) = \frac{1}{4}\left(\frac{ZZ'e^2}{2E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

where θ is the scattering angle and E is the energy of the incident particle.   30

(b) Discuss the significance of the impact parameter and the scattering angle in the analysis of Rutherford scattering phenomena.   10

3. (a) Show how the momentum components of a moving particle transform under Lorentz transformation. A particle of momentum $\vec{p_1}$, rest mass m_1 is incident upon a stationary particle of rest mass m_2. Show that the velocity of the centre of mass system is given by

$$\vec{v} = \frac{\vec{p_1} c^2}{\left(p_1^2 c^2 + m_1^2 c^4\right)^{1/2} + m_2 c^2}$$

25

(b) When an external sinusoidal force is applied to a vibrating system, we have a situation like forced vibration. Show that in the steady state, the frequency of the forced vibration is the same as that of the external force.

15

4. (a) Describe the construction of a Michelson's interferometer. If the movable mirror is moved through a small distance d and the number of fringes that cross the field of view is n, then show that the wavelength of light is given by $\lambda = 2d/n$.

15

(b) Show that the dispersion D and the resolving power of a grating are given respectively by

(i) $D = \dfrac{d\theta}{d\lambda} = \dfrac{m}{d \cos \theta}$ and

(ii) $R = \dfrac{\lambda}{\Delta\lambda} = N.m,$

where d is the grating element, m is the order number and N is the total number of rulings in the grating.

10

(c) Draw the figures depicting the variation of the electric field vector with time for linearly polarized, circularly polarized and elliptically polarized light beams respectively. Describe the experimental method for detecting the state of polarization of the light beam. *15*

SECTION B

5. Answer any *four* questions from the following :
(Each answer must not be more than 150 words long)

$4 \times 10 = 40$

(a) In an RC circuit a fully charged capacitor of 1 μF discharges through a resistor of 100 kΩ. After how much time will the energy stored in the capacitor become $\frac{1}{16}$th of its initial value ?

(b) Suppose a cavity of volume V contains black body radiation in equilibrium with the walls of the cavity at a temperature T. For a reversible adiabatic change of volume show that

$$VT^3 = \text{constant.}$$

If the initial temperature is 2000 K and the volume is increased from 10 cm³ to 1250 cm³, reversibly and adiabatically, what would be the final temperature of the radiation ?

(c) Consider a system of two particles and three quantum states. Discuss how the particles will distribute themselves among the three states if they obey

(i) Maxwell – Boltzmann statistics

(ii) Bose – Einstein statistics

(iii) Fermi – Dirac statistics.

Ignore the spin degree of freedom.

(d) Write a short note on Saha's ionization formula and discuss its applications.

(e) Calculate the radius of an oxygen molecule, if its coefficient of viscosity at $15°$ C is $19 \cdot 6 \times 10^{-6}$ Pl (Pl = Poiseuille) and the mean speed of the oxygen molecules is 436 ms^{-1}.

6. (a) What is the Method of Images ? What are the necessary conditions which must be satisfied in order to apply the image method ? Show that a point charge placed at a distance above a perfect conducting plane of infinite extent can be replaced by itself, its image and an equipotential surface in place of the conducting plane. 20

(b) State Poynting's theorem for a combined system of charges and electromagnetic fields in the form of continuity equation. Explain the physical significance of each term in the equation. 10

(c) Determine the mutual inductance of two coplanar concentric circular loops of radii 1 m and 2 m. (Usual assumptions may be used, where required, to solve the problem). 10

7. (a) Derive Planck's law of black-body radiation. Obtain its limiting forms for (i) very low frequencies (ii) very high frequencies. 20

(b) Consider a system of N non-interacting bosons, occupying a volume V, at a temperature T. Derive an expression for n(E), the average number of bosons occupying the energy state E. 16

(c) What do you understand by Bose – Einstein condensation ? 4

8. (a) Derive an expression for the specific heat of a solid based on Einstein's theory. Obtain the limiting form of the specific heat at very low temperatures. *16*

(b) For diamond, the Einstein temperature, $\theta_E = 1450$ K. Calculate the specific heat of diamond at T = 290 K. *4*

(c) For a system of N non-interacting fermions, enclosed in a volume V, at T = 0, find an expression for (i) the internal energy U, (ii) the pressure P. *15*

(d) Metallic silver has a density of 10.5×10^3 kg m^{-3} and its atomic weight is 107. Taking one free electron per silver atom, calculate the pressure of the electron gas in silver at T = 0. *5*

PHYSICS

Paper—II

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and THREE of the remaining questions, selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written in ENGLISH only.

Assume suitable data, if considered necessary and indicate the same clearly.

List of Useful Constants

Mass of proton $= 1 \cdot 673 \times 10^{-27}$ kg

Mass of neutron $= 1 \cdot 675 \times 10^{-27}$ kg

Mass of electron $= 9 \cdot 11 \times 10^{-31}$ kg

Planck's constant $= 6 \cdot 626 \times 10^{-34}$ J-s

Boltzmann constant $= 1 \cdot 380 \times 10^{-23}$ J-K^{-1}

Bohr magneton $= 9 \cdot 273 \times 10^{-24}$ A-m^2

Electronic charge $= 1 \cdot 602 \times 10^{-19}$ C

Atomic mass unit (u) $= 1 \cdot 660 \times 10^{-27}$ kg

$= 931$ MeV

Velocity of light in
vacuum, c $= 3 \times 10^8$ m-s^{-1}

$$m(_1^1H) \qquad = 1 \cdot 007825 \text{ u}$$

$$m(_1^2H) \qquad = 2 \cdot 014102 \text{ u}$$

$$m(_1^3H) \qquad = 3 \cdot 016049 \text{ u}$$

$$m(_6^{12}C) \qquad = 12 \cdot 00000 \text{ u}$$

$$m(_{10}^{20}Ne) \qquad = 19 \cdot 992439 \text{ u}$$

$$m(_2^4He) \qquad = 4 \cdot 002603 \text{ u}$$

$$m(_0^1n) \qquad = 1 \cdot 008665 \text{ u}$$

$$m(e) \qquad = 0 \cdot 000549 \text{ u}$$

$$m(_{62}^{152}Sm) \qquad = 151 \cdot 919756 \text{ u}$$

$$m(_{63}^{152}Eu) \qquad = 151 \cdot 919749 \text{ u}$$

$$m(_{64}^{152}Gd) \qquad = 151 \cdot 919794 \text{ u}$$

$$m(^{35}Cl) \qquad = 35 \cdot 450 \text{ u}$$

Unless otherwise indicated, symbols and notations have usual meanings.

Section—A

1. Answer any *four* of the following :

(a) The wavelength and frequency of a guided wave are related by

$$\lambda = c / \sqrt{v^2 - v_0^2}$$

Express the wave's group velocity v_g in terms of c and its phase velocity v_p.   10

(b) What is the minimum angle that the angular momentum vector may make with the z-axis, in the case of $l = 3$? How many levels are possible? Show the angle in a vector diagram. **10**

(c) An H ^{35}Cl molecule is found to have a fundamental vibration at $2885 \cdot 9 \text{ cm}^{-1}$. Its internuclear distance changes by $0 \cdot 05$ nm. What will be the change in potential energy? **10**

(d) Calculate the Zeeman energy splitting of the $2P_{1/2}$ and $2S_{1/2}$ states in sodium for a magnetic field of 2 T. **10**

(e) If $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ are Pauli spin matrices, prove the following relationship : **10**

$$\sin(\hat{\sigma}_x \phi) = \hat{\sigma}_x \sin\phi$$

2. (a) The wave function of a quantum particle is $\psi(x) = A e^{i(\alpha x - \alpha^2 \hbar / 2m)}$. Find $\langle p_x \rangle$. **10**

(b) For the $E < U_0$ potential barrier, obtain expression for the probability that the particles will be transmitted resonantly.

$$U(x) = \begin{cases} 0 & x < 0, \ x > L \\ U_0 & 0 < x < L \end{cases}$$ **20**

(c) If an electron is incident on a barrier potential $0 \cdot 100$ eV and width 15 nm in the problem (b) above, calculate the transmission probability, if its energy is $0 \cdot 060$ eV. **10**

3. (a) Prove that $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ commutes with all the three components of angular momentum operator $\hat{L}$. 20

(b) Discuss briefly the Stern-Gerlach experiment with the help of a diagram. Analyse the outcome of this experiment. 20

4. (a) Discuss the number of fundamental vibrations of water molecule. 15

(b) Frequency separation between adjacent lines in the rotational spectrum of $^{35}Cl\,^{19}F$ is measured as 11.2 GHz. Calculate the interatomic spacing. 15

(c) The NMR spectrum of CH_3I taken in a 60 MHz spectrometer gave a strong signal at 130 Hz and chemical shift 2.16 ppm. Obtain the absorption frequency if the spectrum were to be made in a 90 MHz spectrometer. What would be the chemical shift in the new spectrum? 10

Section—B

5. Answer any *four* of the following :

(a) (i) What is the Q-value of the reaction $^{152}Eu\,(n,\,p)^{152}Sm$? 5

(ii) What type of weak interaction can occur for ^{152}Eu ? 5

(b) Lead shows superconductivity at $7 \cdot 19$ K for zero applied magnetic field. When magnetic field of $0 \cdot 08$ T is applied, superconductivity will not take place at any temperature. Calculate the applied magnetic field that will stop superconductivity at $2 \cdot 0$ K. 10

(c) What is internal conversion? Show that the internal conversion coefficient depends upon the atomic number of the nucleus.

(d) Indicate with reasons, why each of the following high-energy reactions/particle decays is either allowed or forbidden : 10

 (i) $\pi^- + p \rightarrow \pi^0 + n$

 (ii) $\pi^0 \rightarrow \gamma + \gamma + \gamma$

 (iii) $\pi^0 \rightarrow \gamma + \gamma$

 (iv) $\pi^+ \rightarrow \mu^+ + \nu_\mu$

 (v) $\pi^+ \rightarrow \mu^+ + \bar{\nu}_\mu$

(e) Write down the quark composition of the following : 10

 (i) Neutron (n)

 (ii) Proton (p)

 (iii) Sigma-plus (Σ^+)

 (iv) Kaon-zero (K^0)

 (v) Pion-zero (π^0)

6. *(a)* Derive an expression for the recoil energy loss in the emission of gamma ray. What is natural line width? Explain. 10

(b) What is Mössbauer effect? Define the Mössbauer parameters—isomer shift, quadrupole splitting and magnetic splitting. 20

(c) For atomic transitions, resonance fluorescence and absorption are normal, but in nuclear gamma decay this does not occur. Why? Explain with a typical illustrative example. 10

7. *(a)* Discuss Grand Unified Theory schemes for Strong, Weak and Electromagnetic interactions. What are experimentally verifiable predictions of such schemes? 20

(b) Explain Kronig-Penney model for the motion of an electron in a periodic one-dimensional square well potential. Show from the energy band diagram that the materials can be classified as conductor, semiconductor and insulator. 20

8. *(a)* Calculate the resistivity of silicon having atomic mass 28, and having density of $2 \cdot 42 \times 10^3$ kg-m^{-3} and a valence of 2. Assume that the Fermi energy for

silicon is 2 eV. (Assume electron mean free path to be 100 times the interatomic spacing.)　　　　20

(b) Discuss the construction and operation of Enhancement MOSFET and Depletion MOSFET. Give their characteristics. How do they differ from JFET ?　　　　20

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