O1DRO1 Decision Making under Uncertainty 2017/2018

Reinforcement Learning II.

Lecture 9

17.4.2018

Readings:

- Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction.
 MIT Press, Cambridge MA USA, 1998
- Csaba Szepesvári. Algorithms for Reinforcement Learning. Morgan & Claypool, 2010.

Reminder:

Project report (or presentation) due today: send by email. Include source files (for implementations).

Presentations: choose time slot (20min) on DRO1 web and send me email

DRO1: 1.5, 8.5 no class (holidays).

DROS: 2 cancelled lectures (28.2 and 10.4) will be 22.5 and 23.5 (usual schedule)

Mid-term test marks and correct solutions, see DRO1 web

Where are we?

Recap: do we need learning in DM?

Learning is inevitable for any intelligent agent.

Learning can help to find:

- solution that cannot be found in advance. Reasons:
 - environment is too complex
 - environment is not fully known
- decision that is gradually improving
- solution that adapts to time-varying environment

Recap: Learning in 'machine' terms

Let an agent observe a sequence of inputs: $x_1, x_2, ..., x_n$

Supervised learning: The agent is given desired outputs $y_1, y_2, ..., y_n$. Goal: to *learn how to design* the correct output y_i given input x_i .

Typical example: neural network

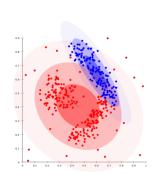
Unsupervised learning:

Agent's goal is to *build a model* of x that can be used for other tasks (reasoning, decision making, predicting, etc.)

Typical example: find patterns in data; clustering

Reinforcement learning: The agent can take actions a_1 , a_2 ,... which influence the environment state, and receives rewards (or punishments) r_1 , r_2 ,...

Agent's goal is to learn how to act to maximise long-term reward.



Recap: RL

Reinforcements used to train animals:

- Negative reinforcements (pain and hunger)
- Positive reinforcements (pleasure and food)



Reinforcement Learning

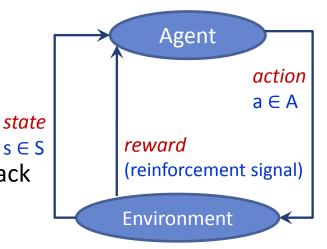
Supervised Learning





Recap: basics of RL

- Interact with a system through states and actions
- Receive rewards (reinforcement) as performance feedback



Formal RL definition: MDP with unknown transition and/or reward models:

- ⇒ agent cannot simulate interaction with environment in advance, to predict future outcomes.
- ⇒ the optimal policy is learned through **sequential interaction** and evaluative feedback.

Aim of RL: Learn an optimal policy $\pi(s)$ while interacting with the environment

Recap: Types of RL

Passive vs. Active Learning

Passive learning: the agent executes a fixed policy and tries to evaluate it Analogous to policy *evaluation* in PI

Active learning: the agent updates its policy as it learns and attempts to find an optimal (or at least good) policy

Analogous to *solving* the underlying MDP

Model-based vs. Model-free Learning

Model-based: learn transition T and reward R model (or approximated models) and use them to determine optimal policy

Model free: derive optimal policy without explicit learning the model

Note: model-free RL = indirect adaptive control; model-based RL = direct adaptive control, see Astrom, 01DR012 webpage

Recap:

- Model-based Learning
 - Adaptive DP basically learns T and R, then perform policy evaluation based on the underlying MDP (T and R)
- Model-free Learning
 - Direct Evaluation performs policy evaluation.
 - Temporal Difference (T-D) Learning performs policy evaluation
 - Q-Learning learns optimal state-action value function Q*

Temporal difference TD(0) algorithm

Model-free method that performs policy evaluation. At each time:

- Collect experience s', s, a, r
- Update $V^{\pi}(s) = V^{\pi}(s) + \alpha \Big(r(s) + \gamma V^{\pi}(s') V^{\pi}(s) \Big)$ Temporal difference

• α -is learning rate, must satisfy $\Sigma_t \alpha_t -> \infty$, $\Sigma_t (\alpha_t)^2 < \infty$

The update is stochastic variant of DP.

If α is appropriately decreased with number of times n(s) a state is visited e.g. $\alpha(s)=1/n(s)$, then $V^{\pi}(s)$ converges to correct value.

$$V^{\pi}(s) = r(s) + \gamma \sum_{s'} T(s', a, s) V^{\pi}(s')$$

more reward than expected $r(s_t) > \gamma V_{old}(s_{t+1}) - V_{old}(s_t) => \text{increase} \quad V(s_t)$ less reward than expected $r(s_t) < \gamma V_{old}(s_{t+1}) - V_{old}(s_t) => \text{decrease} \quad V(s_t)$

Q-learning (alternative TD method)

• $Q^{\pi}(s,a) = E_{\pi}[r_0 + \gamma r_1 + \gamma^2 r_2 + ... | s,a]$ is called Q-function or (state-action)-value function. Q-function is a expected total reward from taking action a at state s.

expected reinforcement of a in s and subsequent optimal choosing actions

$$Q^{opt}(s,a) = r(s,a) + \gamma \sum_{s'} T(s,a,s') \max_{a'} Q(s',a'),$$

$$V^{opt}(s) = \max_{a'} Q(s',a'), \quad \pi^{opt} = \arg\max_{a'} Q^{opt}(s',a')$$

Q-learning rule:

$$Q_{new}(s,a) = (1-\alpha)Q_{old}(s,a) + \alpha \left[r(s) + \gamma \max_{a'} Q_{old}(s',a') \right]$$

Today..

Reinforcement learning has many faces:

value iteration, policy iteration, linear programming, Q-learning, TD(), value function approximation, SARSA (State—action—reward—state—action) algorithm, Least Squares TD, Least Squares PI, policy gradient, inverse reinforcement learning, reward shaping, hierarchical reinforcement learning, inference-based methods, exploration vs. exploitation

On-policy and off-policy

Major MC assumptions (infinite sampling and exploring all possible states) are *not* realistic. Need to continually explore

- On-policy method: update Q-values based on s' and current policy action, i.e.
 assume the current (estimation) policy will be followed
- Off-policy method: update Q-values based on s' and greedy policy action (irrespectively of the real current policy)
- Give no difference if greedy policy is used
- The policy used to generate behaviour (*behaviour* policy), may be unrelated to the policy that is evaluated and improved (*estimation* policy).
- An advantage is the estimation policy may be deterministic, while the behaviour policy can continue to sample all possible actions. Agent can use a behaviour policy that is good at exploring, then infer optimal policy from that.

Types of RL algorithms

model availability	Model-based (indirect): T(s' a,s) and R(s',a,s) are known (ADP)			
	Model-free (direct): T(s' a,s) and R(s',a,s) are unknown; only data (s',a,s) are at disposal (direct evaluation, TD(), Q-learning)			
	Model-learning RL: estimate T(s' a,s) and R(s',a,s) from transition data			
interaction level	Offline: data collected in advance (Q-iteration, PI)			
	Online: Policy π^{opt} learnt by interacting with the environment (Q-learning, SARSA)			
optimal policy search	Off-policy: find Q^{opt} , use it to compute π^{opt} (Q-learning,Q-iteration)			
	On-policy: find Q^{π} , improve π and repeat (SARSA)			

RL and MDP

	MDP with known T(s' a,s) and R(s',a,s)	Unknown MDP Model- Based	Unknown MDP Model-Free
Compute V ^{opt} , Q ^{opt} , π ^{opt}	use Value Iteration (VI) or Policy Iteration (PI)	VI/PI on approximated MDP (Adaptive DP)	Q-learning
Evaluate a fixed policy π ^{opt}	Policy Evaluation (PE)	PE on approximated MDP	Value Learning

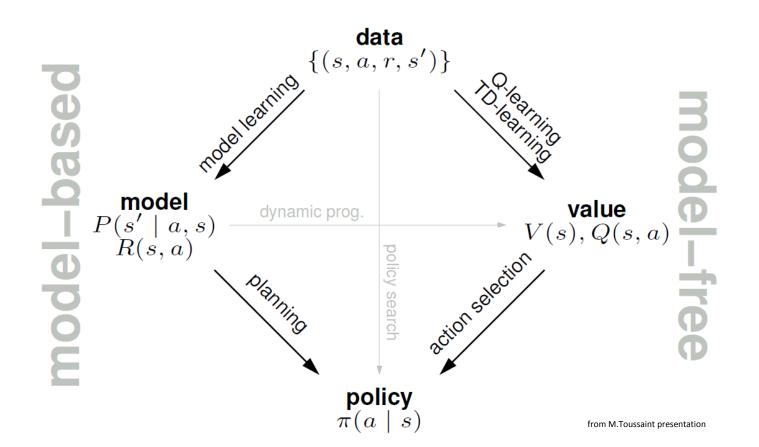
What can we learn?

We have a sequence of data:

(s,a,r,s')=(state, action, immediate reward, next state)

- Model-based RL
 - learn to predict next state, i.e. estimate T(s'|s, a)
 - learn to predict immediate reward, i.e. estimate p(r|s,a)
- Model-free RL
 - learn to predict value of state V(s) or value of action state pair V(s, a)
- Direct policy search
 - performs policy evaluation

Learning in MDP



Model-Based RL

- Learn the model empirically via growing experience (s,a,r,s')
 - Collect outcomes (s, a) for sufficient time (so-called learning phase)
 - Fit transition model T'(s' | a,s) and estimate reward R'(s,a) for instance by using empirical observed distributions (or update some prior beliefs)

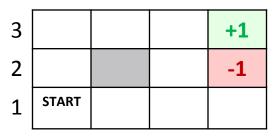
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T'(s'|a,s)=\#(s',a,s)/\#(s,a) (discrete case)
```

 Solve the MDP with the learned approximate model (S, A, T', R') as if the learned model were correct (using standard VI with the estimated T')

Example: Adaptive/Approximate DP (learn transitions and rewards from observations then update the values of the states)

For continuous case: Least Squares Value Iteration, Stochastic Optimal Control

Grid example: passive ADP





Three training sequences of (state, action, reward):

$$(1,1)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$$

$$(1,1)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$$

$$(1,1)_{-0.4} \rightarrow (2,1)_{-0.4} \rightarrow (3,1)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (4,2)_{-1}$$

$$p((1,2)|(1,3),r)=1/3$$

$$p((2,3)|(1,3),r)=2/3$$

Substitute in $V^{\pi}(s) = R(s, a) + \gamma \sum_{s'} T'(s' | \pi(s), s) V^{\pi}(s')$

Adaptive Dynamic Programming

Utilities of neighboring states are mutually constrained, Bellman equation:

$$V(s) = R(s) + \gamma \Sigma_{s'}T'(s'|a,s) V(s')$$

- Estimate T'(s'|a,s) from the frequency with which s' is reached when executing a in s.
- Can use VI: initialize utilities based on the rewards and update all values based on the above equation.
- Can be intractable given a big state space.

Model-free RL

No need to learn the transition model and reward function

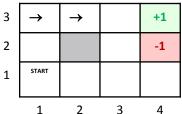
Common approaches:

- Direct evaluation
- Repeatedly execute the policy
- Value of the state s = the average sum of discounted rewards accumulated from s onwards (over all times the state s was visited)

Note: easy; corrupt info about state connections; long time to learn

- Temporal Difference Learning
 Learning from every experience, update V and/or Q any transition
- Q-Learning

Direct evaluation (model-free)



- Developed in the late 1950's in the adaptive control theory.
- Rule: keep a running average of rewards for each state.

For each training sequence, compute the reward-to-go for each state in the sequence and update the utilities.

$$(1,1)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$$

0.72 0.76 0.84 0.88 0.92 0.96 1.0

$$V(1.2)=(0.76+0.84)/2=0.8$$

$$V(1.3)=(0.8+0.88)/2=0.84$$

Model-free RL

No need to learn the transition model and reward function

Common approaches:

- Direct evaluation
- Repeatedly execute the policy
- Value of the state s = the average sum of discounted rewards accumulated from s onwards (over all times the state s was visited)

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Q-Learning a Temporal Difference Learning
 Learning from every experience, update V and/or Q any transition

Model-free: Q-learning

Given new data (s,a,r,s'), compute an average without knowing T() and R()

```
Q^{\text{new}}(s, a) = (1-\alpha) Q^{\text{old}}(s, a) + \alpha [r + \gamma \max_{a'} Q^{\text{old}}(s', a')]
= Q^{\text{old}}(s, a) + \alpha [r - Q^{\text{old}}(s, a) + \gamma \max_{a'} Q^{\text{old}}(s', a')],
```

Reinforcement

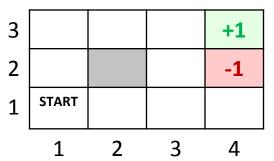
- more reward than expected $r > Q^{old}(s,a) \gamma \max_{a'} Q^{old}(s',a') \Rightarrow increase Q(s,a)$
- less reward than expected $r < Q^{old}(s,a) \gamma \max_{a'} Q^{old}(s',a') \Rightarrow decrease Q(s,a)$

Notes:

Q-learning is off-policy as agent estimates Q (s, a) while executing π

Q-Learning is the first provably convergent direct adaptive optimal control algorithm Automatically focuses on the proper part of the state space, have to explore enough More efficient using *eligibility traces* (V may depend on sequence of states; *all* values where you've been recently are updated)

Grid example





Non-deterministic actions (transition model and reward function are unknown to the agent)

Every state except of terminal states has reward -0.04; action = {left, right, up} Given policy π . Follow the policy for many epochs

Three training sequences of (state, action, reward):

$$(1,1)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$$

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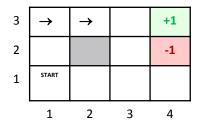
$$(1,1)_{-0.4} \rightarrow (2,1)_{-0.4} \rightarrow (3,1)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (4,2)_{-1}$$
Oldron, lecture slides 2017/2018, T.V. Guy

Q-learning

For each s and a initialise Q(s,a) (can be 0 or random). Observe current state s. Make a loop:

- select a and receive immediate reward r
- observe new state s'
- Update Q(s,a) =Q(s,a)+ α (r(s,a)+ γ max_a, Q(s',a') Q(s,a))
- set s=s'

In grid example (r=0 for non-terminal states)



$$Q((1,3),right) = Q((1,3),right) + \alpha(r((1,3)) + \gamma \max_{a'} Q((2,3),a') - Q((1,3),right))$$

Approximate RL

Q-learning needs all to keep all Q-values => not realistic as too many states

- to experience all of them
- to store the values in memory

To avoid this problem => generalisation is used, i.e. knowledge gained on a small training data is transferred to similar situations.

- Use feature-based representation with feature function reflecting important property of the state
- Approximate $Q(s,a) = \sum_i w_i f_i(s,a)$, then update weights of important/active features rather than Q(s,a)

$$w_i^{\text{new}} = w_i^{\text{old}} + \alpha \left[r - w^{\text{old}}(s, a) + \gamma \max_{a'} w_i^{\text{old}}(s', a') \right] f_i(s, a)$$

Note: Often very effective, helps RL scale up to very large MDPs, but convergence is *not* guaranteed

Model-free RL

No need to learn the transition model and reward function

Common approaches:

- Direct evaluation
- Repeatedly execute the policy
- Value of the state s = the average sum of discounted rewards accumulated from s onwards (over all times the state s was visited)

Note: easy; corrupt info about state connections; long time to learn

Q-Learning a Temporal Difference Learning
 Learning from every experience, update V and/or Q any transition

Model-free: SARSA (TD-learning of Q(s,a))

```
Given new data (s,a,r,s',a'), where a' = \pi(s')

Q^{\text{new}}(s,a) = (1-\alpha) Q^{\text{old}}(s,a) + \alpha [r(s',a,s) + \gamma Q^{\text{old}}(s',a')]

= Q^{\text{old}}(s,a) + \alpha [r(s',a,s) + \gamma Q^{\text{old}}(s',a') - Q^{\text{old}}(s,a)],
```

Reinforcement

- more reward than expected $r > Q^{old}(s,a) \gamma Q^{old}(s',a') \Rightarrow increase Q(s,a)$
- less reward than expected $r < Q^{old}(s,a) \gamma Q^{old}(s',a') \Rightarrow decrease Q(s,a)$

TD update adjusts Q estimate to agree with Bellman equation

Typical parameter values:

- discount factor $\gamma > 0.9$
- learning rate α < 0.5 ($\alpha \in (0, 1]$) or decrease with time
- exploration probability $\varepsilon \approx 0.1$ or decrease with time
- decay rate for eligibility traces $\lambda \in [0.5, 0.9]$ ($\lambda \in [0, 1]$)

Exploration-Exploitation: examples

call a taxi service you know or try a new one





select your favourite pub/club or try a new

 play a move in a game you know as the best or play experimental (risky) move



• 'classical' example of E2 problem: multi-armed bandit



E2: problem or dilemma?

To ensure convergence, RL methods need to sample all actions at every state sufficiently often

- Execute best estimated action, i.e action with the highest value => exploit
 - Note: the learned model can never be the real one => suboptimal results
- Try an action with lower estimated value (or random action) for which we may gather more useful knowledge => explore
 - Agent can learn the very precise model
 - It can be of use if some parts of model are never used

Problem:

- The best long-term strategy may involve short-term sacrices
- Gather enough information to make the best overall decisions

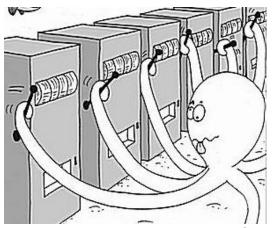
Balance exploitation-exploration is important.



Multi armed bandit: example of E2

- A known set of m actions (# of arms)
- $R(r, a) = Pr(r \mid a)$ unknown probability distribution over rewards
- Agent selects action (arm) $a_t \in A$, bandit generates a reward r_t
- Goal: maximize $\Sigma_t r_t$
- Rewards of not chosen actions are unknown

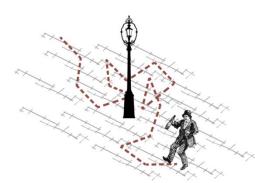
To solve the n-armed bandit problem: you must explore a variety of actions and exploit the best actions



Tradeoff: immediate vs. long-term profit

- Problem: choosing actions with the highest expected utility ignores their contribution to learning.
 - A random-walk agent learns faster but never uses that knowledge.
- A greedy agent learns very slowly and acts based on current, inaccurate knowledge.

Exploitation- Exploration tradeoff crucial for performance of online RL.



Some principles solving the E2 dilemma

Naive exploration

Adding noise to greedy policy (e.g. ε -greedy: choose best $a^* = \operatorname{argmax}_a Q(s,a)$ with probability $1/[\varepsilon(n-1)]$, n — is a number of trials, otherwise execute random action) or even simpler: execute random action with probability ε , and best with $(1-\varepsilon)$

- Optimistic initialisation
 Assume the best until proven otherwise, e.g. initialise Q(a) to high value
- Optimism in the Face of Uncertainty
 Prefer actions with uncertain values, e.g. more uncertainty about an action value=>
 more important to explore that action
- Information State Search
 Search (ahead) incorporating value of information
- .. and many other heuristic and theoretically sound variants.

E2 dilemma: value of info

If we know value of information, we can trade-off exploration and exploitation optimally

- Exploration gains information, but how to quantify the value of this info?
- How much reward does an agent want to give up to get that info?
- Uncertain situations imply higher information gains
- Exploring uncertain situations more is reasonable

Optimal exploration strategy: utopia or not?

What does optimal mean? Try to formalize exploration as POMDP (policy learning is trivial for bandits)

- Use goodness measure of exploration step: measure regret of the total mistake: regret=expected reward of best action actual reward of taken action
 Keep track of average reward for each action; exploit: take a=argmin_a regret(a)
 But: optimal exploration has higher regret than optimal exploitation
- Bayesian model identification (see Lecture 7) + explicitly reason about value of information (small grid problems)
- In large or infinite MDP intractable, optimal methods do not work ② Way out: solve using small-scale methods

Other common exploration methods

■ Boltzmann exploration: execute each a with p(a) =e^{Q(s,a)/T}/Σ_ae^{Q(s,a)/T} parameter T→∞ means all actions equally likely, T→0 means a* will be chosen with near certainty (states with lower energy will always have a higher probability of being occupied)

 Confidence-based methods: use statistical tests to estimate a confidence bounds on estimated Q-values, then choose actions based on mean values but with an "exploration bonus" if high uncertainty (e.g., large upper confidence bound)

than the states with higher energy)

Exploration and Q-learning

Q-learning converges to optimal Q-values even if the agent acting suboptimally and if

- Every state is visited infinitely often (due to exploration)
- The action selection becomes greedy as time approaches infinity
- The learning rate a is decreased fast enough but not too fast
- At early stages of learning, estimations can be unrealistic low
- In the early phase of search agent is more willing to explore.

Bayesian RL

- dealing with uncertainty, where 'classic RL' does not.
- includes modelling the transition-function (value-function, policy, reward) probabilistically.

Bayesian RL:

- solves E2 by planning in belief space.
- is generally computationally intractable, but approximations exist
- efficiently chooses samples to learn from
- suitable when sample cost is high.

Bayesian Reinforcement Learning

MDP is unknown, but can be learned based on experience

- Let MDP be parameterised as $T(s'|s,a) = \theta_{s',a,s}$. Then having experience $(s_1,a_1,...,s_t,a_t)$ the posterior $b(\theta)=p(\theta|s_1,a_1,...,s_t,a_t)$ can be estimated.
- given a posterior belief b about MDP we plan to maximise policy value in this distribution of MDPs:

$$V^{opt}(s,a) = \max_{a} \left[R(s,a) + \gamma \sum_{s'} \int_{\theta} T(s'|,a,s,\theta) b(\theta) d\theta V^{opt}(s',b') \right]$$

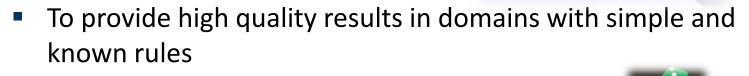
details see for instance freely available: M. Ghavamzadeh et al. *Bayesian Reinforcement Learning: A Survey* Foundations and Trends in Machine Learning Vol.8, No.5-6 359–483 2015

Bayesian RL

- A prior distribution over model parameters is available
- Updates distribution based on observed transitions
- Chooses actions with greatest expected long-term value
- Has no exploration-exploitation dilemma as solves it optimally
- Difficult to work with large POMDP, with high-dimensional continuous state space => tractability & efficiency

What can RL do well now?

- To learn from human-expert training behaviour .
 drive a car
- To learn simple abilities (skills) with noisy observations (having rich experience at disposal)
 robot-manipulator



play a game





What remains to be a challenge?

- Not clear what the reward function should be
- Not clear what the role of prediction should be
- Speed of RL is very far from humans
- Transfer learning in RL remains to be an open problem
- E2
- Generalisation over state and actions
- Partial observability
- Multiple-agents
- Non-stationary reward and transition models
- Proof of convergence, suboptimality conditions (convergence and optimality are difficult to achieve when state spaces are large)

• . . .

Beyond RL

Inverse RL: learning reward function from experience
 => preference elicitation

- Transfer learning: transfer knowledge between different examples/domains
- Meta-learning: learning to learn

Transfer learning

- use experience from one set (type) of problems (source domain) for faster learning and better performance in a new task (target domain)
- The more diversity we observe in source domain, the richer knowledge we transfer => randomisation
- Ensures that the differences are functionally irrelevant, it is not granted
- Tools: physical rules => models (model-based RL); policies; learning methods

Meta-learning

- Closely related to multi-task learning
- Use past experience to find more efficient deep RL algorithm
- Learning to learn how to:
 - explore more efficiently
 - avoid actions that bring no reward
 - acquire the proper features more quickly

hard optimisation problem; works well in smaller tasks.

Problem with large MDPs:

RL can solve large problems (e.g. backgammon 1020 states; computer Go 10170 states), but

- too many states and/or actions to store (limited memory)
- to learn the value of each state individually needs lot of timeWay out:
- generalise observed states to yet unobserved
- estimate V or Q with function approximation:
 - linear combinations of features
 - neural networks
 - nearest neighbour
 - wavelet-based, ...

Deep RL: what is it?

RL uses neural networks to approximate:

- Policies (select next action)
- Value functions (measure goodness of states or state-action pairs)
- Models (predict next states and rewards)

What has proven to be a challenge in Deep RL?

- Human able to learn quickly. Compare to them Deep RL is slow
- Transfer learning (use past knowledge/experience) in deep RL is not covered
- Form and importance of reward

Deep RL is good now (beginning 2018)

- in domains with simple and fixed rules (Go, ATARI)
- to learn from human-expert behaviour, i.e. learn from imitating (robots, driving, etc)
- to learn simple abilities from rich experience (robots)

Picture sources https://dir.indiamart.com/, http://gigabotics.com, https://github.com/