FNSPE CTU

DYNAMIC DECISION MAKING

SEMINAR PAPER

Minority Game
From the Dynamic Decision Making Perspective

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1 Introduction

In the following seminar paper the Minority Game Paper is going to be discussed. In the next section we will define the general mathematical model of the Minority Game. Afterwards, agent Q-learning and Roth-Erev learning will be introduced. Within the scope of this work the implementation was performed and its results will be presented in some section. The system MG describes originates from the El Farol Bar Problem which was introduced by the economist W.B. Arthur [ref], it goes as follows: every Thursday the population of Santa Fe has a desire to visit the bar - if 60+ % of people come to the bar, then it is considered to be overcrowded, so it is no fun there, if less, then it is fun there and those who stayed at home are at a loss. Therefore, it can be easily seen, that the minority wins in games of El Farol Bar Problem type, that is the reason why does the model has such name. Nowadays this model is used frequently in Finance, Network Analysis, even Biology, etc.

2 Mathematical Model of the Minority Game

Firstly, consider an odd $N = 2k - 1$, $k = 1, 2, \ldots$, number of agents participating in the game. At each time step $t = 1, 2, \ldots$ each agent has to make a decision whether to perform an action $+1$ (e.g. go to the bar, sell an asset on the market) or -1 (e.g. stay at home, buy an asset on the market). Formally designated $\forall i \in \{1, 2, \ldots, N\}$:

$$a_i(t) = \pm 1 \tag{1}$$

In order to study game dynamics a special parameter called *total action* was introduced:

$$A(t) = \sum_{i=1}^{N} a_i(t), \forall t \in \{1, 2, \ldots\}, \tag{2}$$

which is basically the sum of actions performed by every agent at a given game round.

After each game round the outcome is disclosed to each of the agents: as soon as the round is finished, everyone gets to know what the winning action was. This action $W(t+1)$ ($t+1$ is used to specify, that this information is available at a time step $t+1$) is determined by a simple rule:

- $A(t) > 0 \implies$ the action -1 was victorious, $W(t+1) = -1$;
- $A(t) < 0 \implies$ the action $+1$ was victorious, $W(t+1) = 1$;
- $A(t) = 0$ will never occur due to oddness of N.

In other words, $W(t+1) = -\text{sign}\big(A(t)\big)$.

Minority Game is a system of agents with bounded memory: each agent remembers $m \in \{1, 2, \ldots\}$ recent game outcomes. In accordance with its memory. This *recent history* can be represented two different ways:

- As an m-tuple of most recent game outcomes: $\bar{\mu}(t) = \big(W(t-m+1), W(t-m+2), \ldots, W(t)\big)$;
- As a single decimal number $\mu(t)$ with binary representation equal to $\bar{\mu}(t)$.[1]

It can be easily seen, that in total 2^m possible recent histories exist, therefore, $\mu(t) \in \{1, 2, \ldots, 2^m - 1\}$.

At the beginning of the game each agent gets a fixed number of strategies. For any fixed m a *strategy* is a mapping $\pi : (0, 1, \ldots, 2^m - 1) \mapsto \{-1, 1\}^{2^m}$ (all possible outcomes are mapped to respective actions, e.g. see Table 1). The construction of such mapping implies, that agents are able to get from 0 to 2^{2^m} strategies. The action/decision of agent i with respect to recent history $\mu(t)$ using the strategy π will be denoted as $\pi_i(\mu(t))$, e.g. if agent i uses the strategy from Table 1 when recent history is 101, then $\pi_i(5) = -1$.

Not the question arises: how to model the process of decision making? In the following section we will discuss the original learning mechanism and some other.

[1]E.g.: let $m = 3$, then for $\bar{\mu}(t) = (-1, -1, 1)$ its decimal representation (obtained by applying the transformation $-1 \to 0$) is $\mu(t) = 1$, because $001_2 = 1_{10}$.

Recent History, $\bar{\mu}(t)$	$\mu(t)$	Action/Decision
000	0	-1
001	1	$+1$
010	2	$+1$
011	3	-1
100	4	$+1$
101	5	-1
110	6	$+1$
111	7	$+1$

Table 1: Strategy example for $m = 3$.

3 Learning in the Minority Game

To begin, we will try to describe the Minority Game in a more formal way.

References