01DROS
Seminar Course on Dynamic Decision Making

2017/2018

Lecture 11

2.5.2018

Uncertainty in DM. Risk. Value of Information.

Recap: Maximum Expected Utility

If there is uncertainty in action outcomes, preference ordering is not enough.

The Maximum Expected Utility (MEU) principle says that a rational agent should choose an action *a* that maximizes its expected utility in the current state s over all outputs :

$$EU(state) = \max_{a \in A} \sum_k p(output_k \mid action, state) U(output_k)$$

Utility of choice is expected utility of its outcome.

Why MEU? Action with highest EU is the *most preferred* in ordering over all possibilities.

MEU: some notes

- Utility function for a given ordering is not unique: any positive affine transformation of U induces same ordering

  normalisation in range [0,1] is common:
  0 - worst possible catastrophe; 1- best possible outcome

- Ordinal preferences “easy” to elicit (if space is small!)

  cardinal utilities difficult for humans => preference elicitation

- Multi-attribute utility functions.

  evaluate a job offer (salary, bonuses, carrier opportunities, free time, etc)
  $U(a_1,a_2,a_3,\ldots)=g[f_1(a_1), f_2(a_2), f_3(a_3),\ldots]$, g[.] – is simple function (sum)

  mutual independence of $a_1,a_2,a_3,\ldots$ => preferable to increase the value of an a_i given the rest are fixed

- Expected utility accounts for risk attitudes: always existing in preferences
  see utility of money

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Knowing:

- the current state *s* requires perception, learning, knowledge representation and inference;
- p(*output*|*action,state*) requires a complete causal model of the environment
- U(*output*) requires planning (which utilities could be obtained from the state *s*?)

Finding these can be computationally hard (often intractable) =>
(*resource-)bounded rationality* vs. perfect (full) rationality

Bounded rationality: sources

- Limited knowledge of the environment;
- Limited ability to use this knowledge;
- Limited ability to predict consequences of actions;
- Limited ability to think over possible courses of action;
- Limited ability to cope with uncertainty;
- Limited ability to decide among competing preferences.

Models of bounded rationality: models of rational choice *respecting limitations of human capacities.*

Herbert A. Simon, ‘Bounded Rationality in Social Science’

H.A. Simon
1916-2001

Prospects and lotteries

Expected utility theory measures an agent's valuation of *prospects.*

Prospects: pure prospects or lotteries.

Pure prospect (outcome) is a future event or state that occurs with certainty (e.g. buying a used car)

Lottery (also called prospects under risk) is probability distribution over events or states (e.g. bought a *good* used car).

A lottery consists of a set of prospects and assigned probabilities and

- different outcomes correspond to different gains (rewards).
- an outcome can be another lottery.

A lottery with only one outcome is a pure prospect.

How do you weight two possible actions when you are not sure what their consequences will be?

Expected monetary value (EMV)

e.g. if you bet 60$ up on ‘’6’’ (a die),

EMV of this bet is -40$:

60*1/6 +(-60)*5/6=10-50=-40

(1/6 chance of *winning* $60 and a 5/6 chance of *losing* $60).

Expected monetary value (EMV)

Example: You won \$1M prize and can gamble on it by flipping a coin. If you gamble, you will either *triple* the prize or *lose* it.

EMV of the lottery is \$1,5M, but does it have higher utility?

Let us use utility theory:

Suppose your current wealth is W

EU(*accept*) = 0.5 U(W) + 0.5U(W+3M)

EU(*decline*) = U(W+1M)

Best decision depends on the utilities of these three states:

If U(W)=5, U(W+1M) =8, U(W+3M)=10, then?

Example

What would you choose:

1. A sure gain of \$200
2. A 25% chance of winning \$1000 and a 75% chance of winning \$0.

What if (2.) was \$100K; \$300K; \$1M; p(win)= 90%; 50%; 10%;..

Generally,

U(EMV(lottery)) > U(lottery) , EMV - expected monetary value

Example

What would you choose:

1. A sure gain of $200 (psychology results: apprx. 80%)
2. A 25% chance of winning $1000 and a 75% chance of winning 0?

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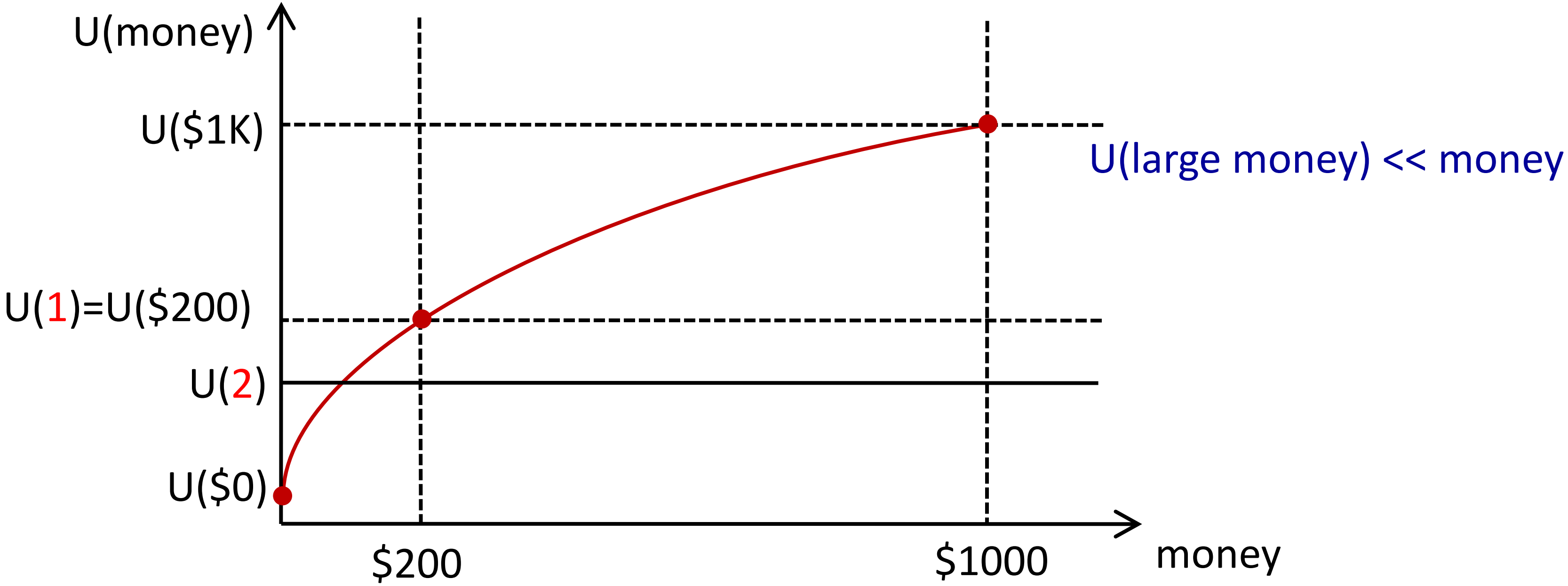
U(**2**) = 0.25 U(\$1000) + 0.75 U(\$0)

U(**1**) = U(\$200)

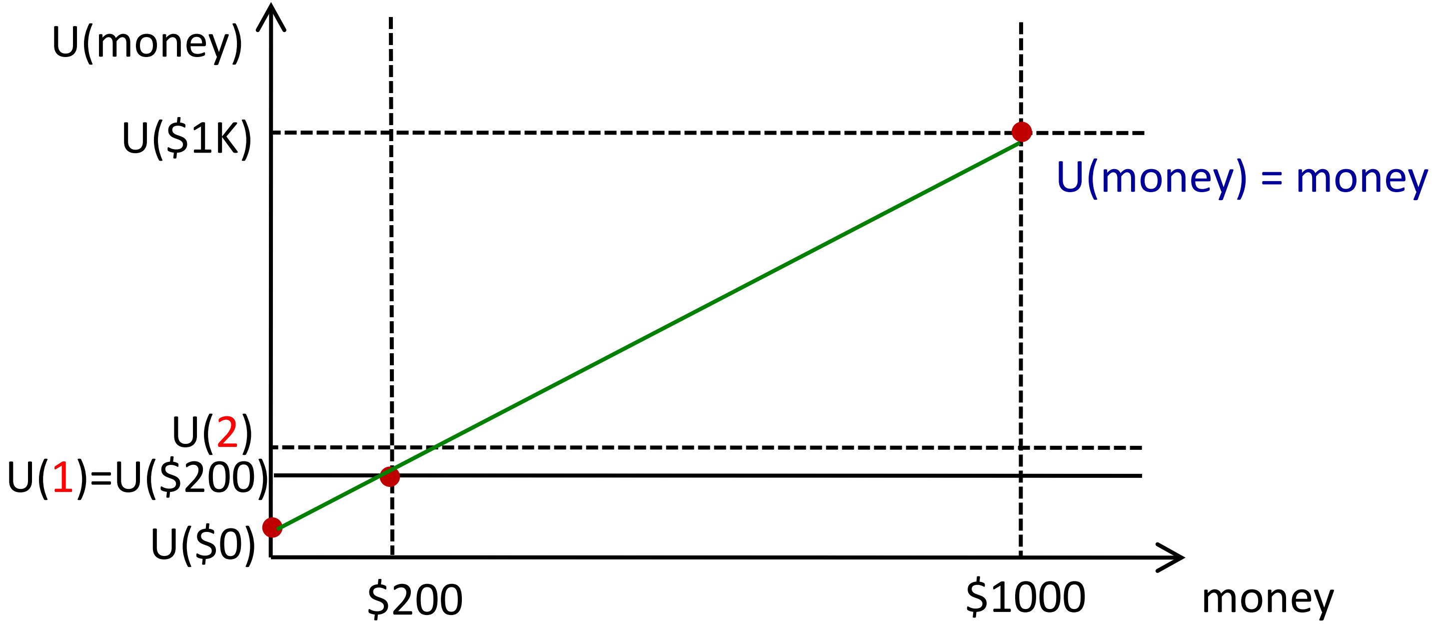
if someone selects **1** => U(**1**) > U(**2**)

Risk averse curve



Utility of money is *concave*. Agent prefers a smaller amount *for sure*, rather than lotteries with a larger expected amount of money (but *not* expected utility!). Take gambles with a *substantial* positive expected monetary payoff .Most human agents are risk averse.

Risk neutral curve



Utility of money is *linear*. Agent's expected utility for a lottery is proportional to the expected amount of money he'll earn. Risk-neutral humans are often described as "expected value" decision makers.

Modified example

What would you choose:

1. A sure gain of \$280
2. A 25% chance of winning \$1000 and a 75% chance of winning 0?

U(**2**) = 0.25 U(\$1000) + 0.75 U(\$0)

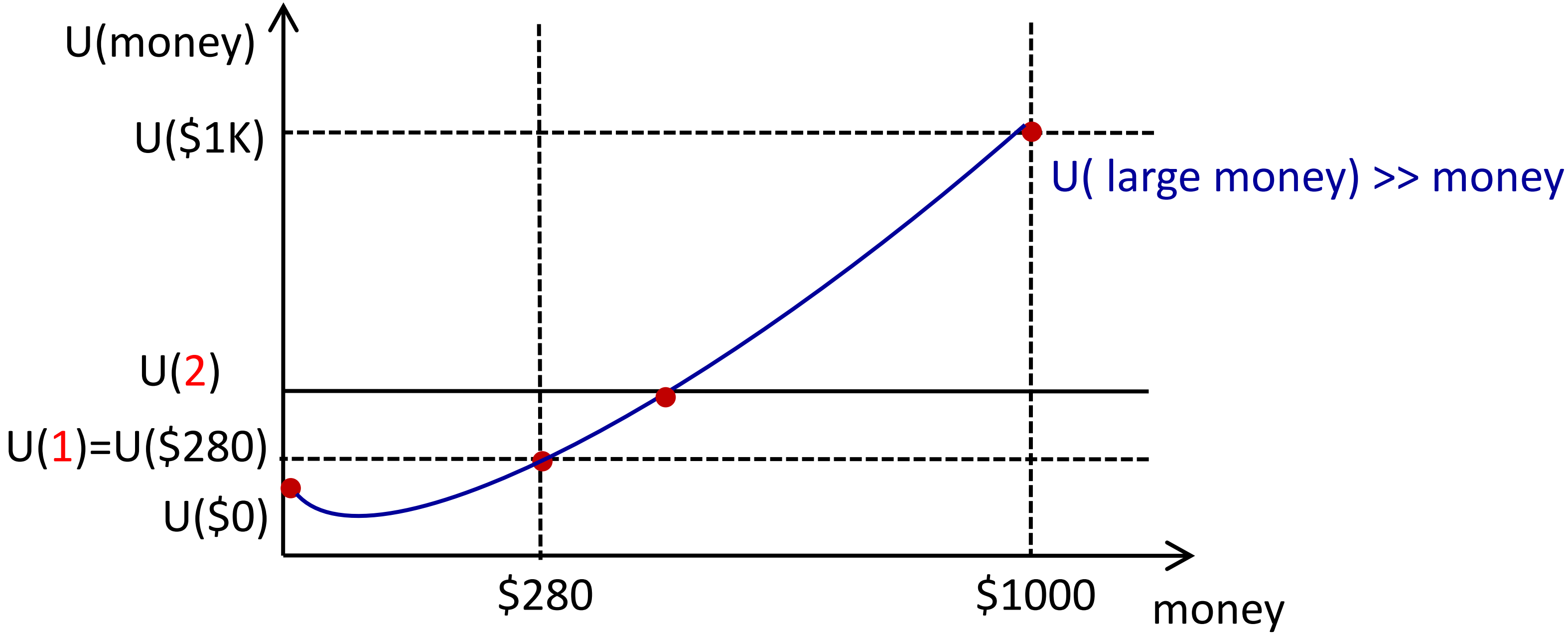
U(**1**) = U(\$280)

The utility of **2** remains ¼ of the way between the utility of \$0 and U(\$1000).

Expected monetary value of **2** (\$250) is lower than **1** (\$280).

Agent prefers **2,** i.e U(**1**) < U(**2**).

Risk seeking curve



Utility of money is *convex*. Agent prefers a *riskier* situation to a sure one. U(**1**) < U(**2**). Such a preference curve is called "*risk seeking*". E.g. agents in bad circumstances when the prospect of winning is worth trying even under the risk of almost certain loss.

Example

What would you choose:

1. A sure loss of $700
2. A 75% chance of losing $1000 and a 25% chance of losing 0?

Risk attitude in losses is the same (*optional homework*)

Summary of risk attitudes

Risk premium : how much of EMV will I give up to remove risk of losing?

- Risk averse: agent has positive risk premium; U(money) is concave
  Decreasing utility for money. *Will* buy insurance. Gamble with substantial positive expected monetary payoff.
- Risk neutral: agent has zero risk premium; U(money) is linear.
- Risk seeking: agent has negative risk premium; U(money)is convex. Increasing utility for money. Will *not* buy insurance. Participate in gamble having negative expected monetary payoff if there is a chance for high payoff.
- Most people are risk averse – explains insurance
- Regardless of risk attitude the utility function can always be approximated by a straight line over a small range of monetary outcome.

St. Peterburg Paradox (author: Nicolaus Bernoulli)

How much would you pay to play the following game?

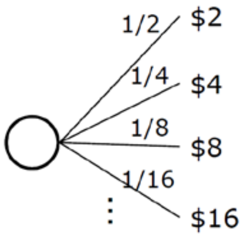
- A coin is tossed until it falls heads. If it occurs on the k-th toss you get $\$2^k$

Why paradox?

- The game has an expected money value of infinity (1/2*2 is 1; 1/4*4 is 1; etc). Thus expected win for repeated play is an *infinite* amount of money.
- However most people do not want to pay more than \$4-\$20 to play it.

n	prob.	payoff	EV
1	$\frac{1}{2}$	2	1
2	$\frac{1}{4}$	4	1
3	$\frac{1}{8}$	8	1
⋮	⋮	⋮	⋮
N	$\frac{1}{2^n}$	2^n	1

doesn't contradict utility theory



Expected value = 1 + 1 + 1 + ... = ∞

Example: Value of information

You consider buying a used SW that costs \$100. There is a prior probability 0.7 that SW is good and will simplify your work (gain \$500). There is a probability 0.3 that SW is not suitable in which case it will have no positive effect.

What is the value of knowing whether SW suits your tasks before buying it?

Example: Value of information

- Expected utility *given* information equals [0.7(500-100)+0.3(0)]
- Expected utility not given information [0.7(500-100)+0.3(0-100)]
- Value of Information is thus

  [0.7(500-100)+0.3(0)] - [0.7(500-100)+0.3(0-100)] = 280 - 250 = $30

Value of Perfect Information

The general case: let the exact evidence can be obtained about some random variable *X*. The agent's current knowledge is *K*.

The value of the current best action *A* is defined by:

$$EU(A|K) = \max_A \sum_i p(output_i(A)|A,K)U(output_i(A))$$

With info *X* the value of the new best action will be:

$$EU(A_X|K,X) = \max_A \sum_i p(output_i(A)|A,K,X)U(output_i(A))$$

X is unknown and random thus we must average over all possible values using our belief (VPI - value of perfect information):

$$VPI(X) = \sum_j p(X_j|K)EU(A_{X_j}|K,X_j) - EU(A|K)$$

VPI – cost that agent willing to pay in order to gain *perfect* information

EXAMPLE : TAXI SERVICE (TS)

TS NEEDS AN APPLICATION

EXPECTED PROFIT $200

BUT: 80% – ONLY ONE ADDITIONAL OPERATOR NEEDED (-$50)

20% – 5 operators needed (-$250)

EXPERT CAN DECIDE, BUT HE IS COSTLY.

WHAT COST OF EXPERT TS CAN AFFORD?

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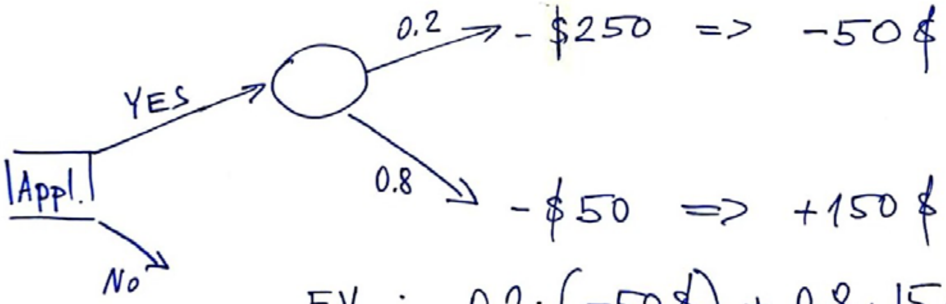
1. ASSUME : RISK-NEUTRAL => $U(\$100) = \$100$.

2.



EV : $0.2 \cdot (-50\$) + 0.8 \cdot 150\$ = \$110$

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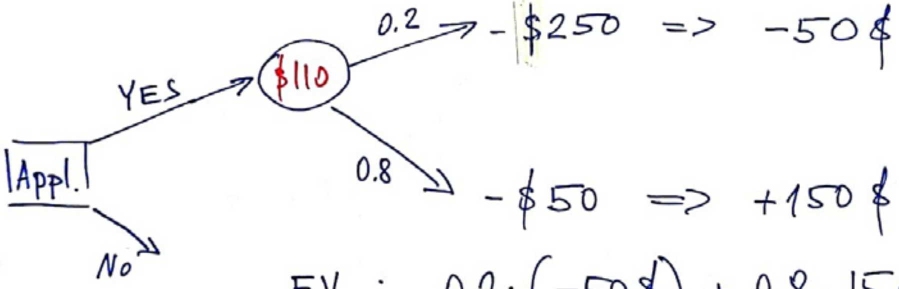

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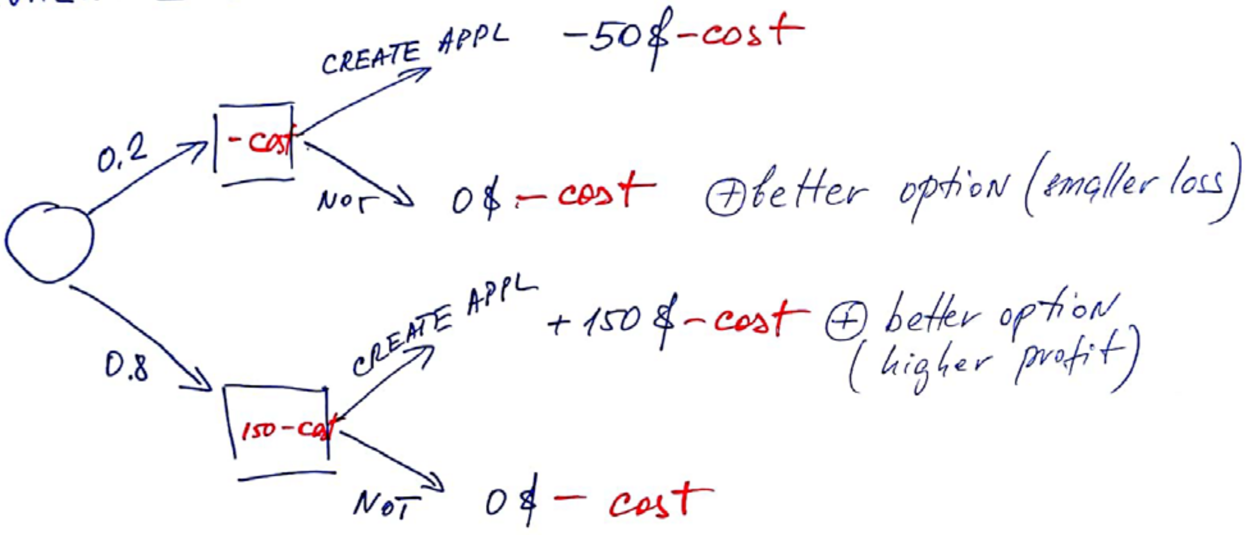
3. Let 'cost' - is "price of the Expert".

EVALUATE UNCERTAINTY WHAT CONCEQUENCES OF Expert's INFO.



$$0.2 \cdot (-cost) + 0.8\,(150 - cost) = 120 - cost$$ (EXPECTED VALUE WITH EXPERT)

WITHOUT EXPERT \$110 $\Rightarrow$ EXPERT SHOULD NOT COST $> \$10$.

OTHERWISE EXPERT'S INFO BRINGS NO good

IN CASE IF COST = 10\$ THE SITUATION is THE SAME AS ORIGINAL ONE.

Additional (optional) readings

- L. Savage. *The Foundations of Statistics. Wiley, NY, 1954.*
- R. L. Keeney and H. Raiffa. *Decisions with Multiple Objectives: Preferences and Value Trade-offs. Wiley, NY, 1976.*
- Peter C. Fishburn. *Utility Theory for Decision Making. Wiley, New York, 1970.*