

Summarizing what we have done.

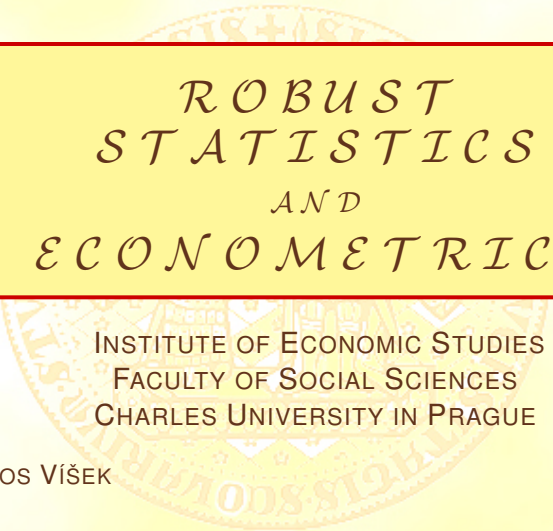
Should be the common sense employed for creating scientific theories

What about a bit more general conjecture ?



INSTITUTE OF ECONOMIC STUDIES, FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE (*established 1348*)

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ROBUST STATISTICS AND ECONOMETRICS

INSTITUTE OF ECONOMIC STUDIES
FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE

JAN ÁMOS VÍŠEK

Week 13

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Content of lecture

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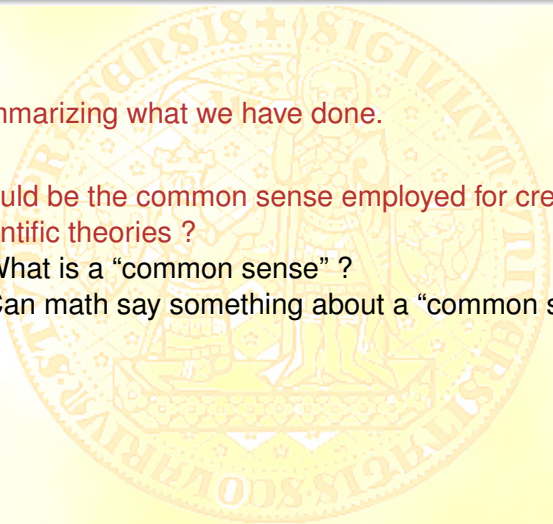


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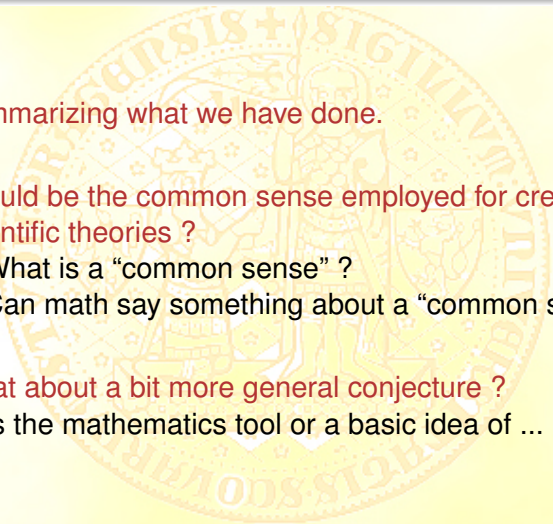
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 - 2 Should be the common sense employed for creating scientific theories ?
 - What is a “common sense” ?
 - Can math say something about a “common sense” ?

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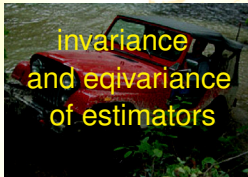
M-estimators
L-estimators
R-estimators



especially of location
and scale



later
of regression
coefficients



invariance
and equivariance
of estimators



large
breakdown
point



problems
with algorithms
and
implementation

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ESTIMATE OF
REGRESSION COEFFICIENTS BY
THE MODERN ROBUST ESTIMATORS

(assignable breakdown point,
equivariant in scale as well as in regression,
reliable algorithm and implementation)

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M-estimators for the regression framework.

The solution of the extremal problem

$$\hat{\beta}^{(M,n)} = \arg \min_{\beta \in R^p} \sum_{i=1}^n \rho(Y_i - X_i' \beta)$$

is called

*Maximum likelihood-like estimators of the regression coefficients
or M-estimators of β^0 , for short.*

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We usually adopt some basic assumptions:

Let $F(x, r)$, $x \in R^p$, $r \in R$ be a d.f. (with a density $f(x, r)$) governing the explanatory variables and disturbances in the regression model.

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Evidently this form of definition inevitably implies that $\hat{\beta}^{(M,n)}$ is not scale- and regression-equivariant.

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An advantage - on the other hand -
an easy computation of a solution, see the next slide.

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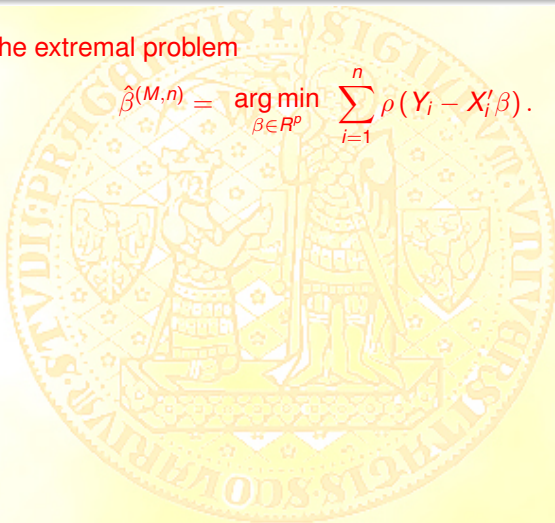
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Computing M -estimate of regression coefficients

Consider the extremal problem

$$\hat{\beta}^{(M,n)} = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^n \rho(Y_i - X_i' \beta).$$



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$$\hat{\beta}^{(M,n)} = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^n \rho(Y_i - X_i' \beta).$$

Write it as

$$\begin{aligned} \hat{\beta}^{(M,n)} &= \arg \min_{\beta \in \mathbb{R}^p} \sum_{\{i: Y_i - X_i' \beta \neq 0\}}^n \frac{\rho(Y_i - X_i' \beta)}{(Y_i - X_i' \beta)^2} (Y_i - X_i' \beta)^2 \\ &= \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^n w_i \cdot (Y_i - X_i' \beta)^2 \end{aligned}$$

where $w_i = \rho(Y_i - X_i' \beta) / (Y_i - X_i' \beta)^2$ if $Y_i - X_i' \beta \neq 0$, $w_i = 0$ otherwise.

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Then

$$\hat{\beta}^{(M,n)} = (X' W X)^{-1} X' W Y$$

where $W = \text{diag}(w_1, w_2, \dots, w_n)$.

Computing M -estimate of regression coefficients

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And iterative computations, starting with a preliminary “guess” of the estimates of regression coefficients, find an approximation.

Hampel's approach - characteristics of the functional T at the d.f. F

- An **overall characteristic of the functional** (the estimator) is

$$\varepsilon^* = \sup \{ \varepsilon \leq 1 : \exists K_\varepsilon \subset \Theta, K_\varepsilon \text{ compact} \}$$

$$\pi(F, G) < \varepsilon \Rightarrow G(\{T_n \in K_\varepsilon\}) \xrightarrow{n \rightarrow \infty} 1 \}$$

where $\pi(F, G)$ is the *Prokhorov metric* of $F(x)$ and $G(x)$
and T_n is an empirical counterpart to the functional T .

- ε^* is called **breakdown point**.

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What about a bit more general conjecture ?

A pursuit for highly robust estimator of regression coefficients

Let's recall:

- Breakdown point - “finite” sample version

$$x_1, x_2, \dots, x_n \Rightarrow T_n(x_1, x_2, \dots, x_n)$$

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$$x_1, x_2, \dots, x_n \Rightarrow T_n(x_1, x_2, \dots, x_n)$$

- Find maximal m_n such that for any

$$y_1, y_2, \dots, y_{m_n} \Rightarrow |T_n(x_1, x_2, \dots, x_{n-m_n}, y_1, y_2, \dots, y_{m_n})| < \infty$$

$$(0 < T_n(x_1, x_2, \dots, x_{n-m_n}, y_1, y_2, \dots, y_{m_n}) < \infty \text{ - for scale }).$$

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$$(0 < T_n(x_1, x_2, \dots, x_{n-m_n}, y_1, y_2, \dots, y_{m_n}) < \infty \text{ - for scale }).$$

- Put

$$\epsilon^* = \lim_{n \rightarrow \infty} \frac{m_n}{n}.$$

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- Breakdown point - “finite” sample version - examples

$$x_1, x_2, \dots, x_n \Rightarrow T_n(x_1, x_2, \dots, x_n) = \frac{1}{n} \sum_{i=1}^{\infty} x_i.$$

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$$\varepsilon^* = 0.$$

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- Breakdown point - “finite” sample version - examples

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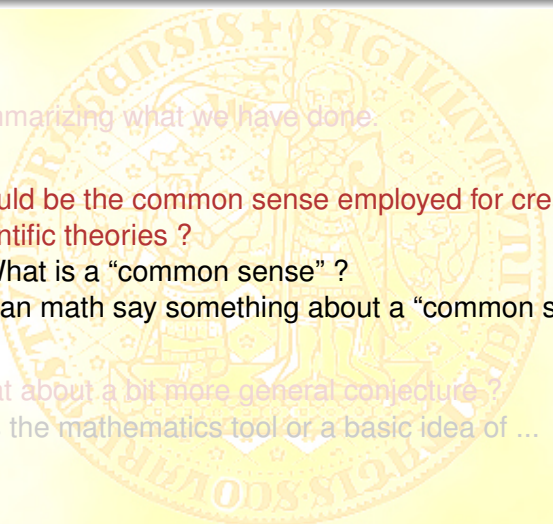
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What about a bit more general conjecture ?

What is a "common sense" ?

Can math say something about a "common sense" ?

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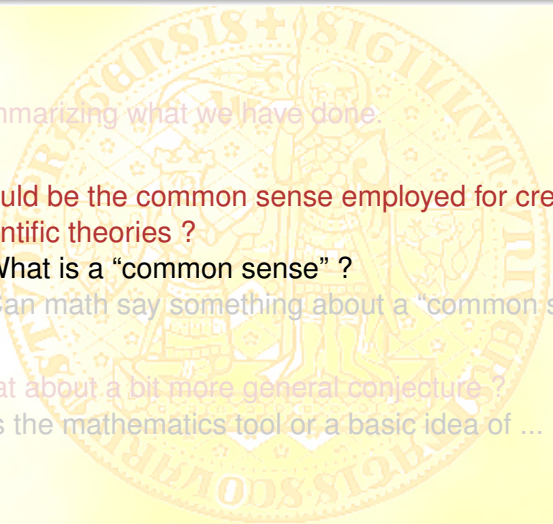
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Common sense

What is a *common sense* ?

Let us consider some examples:

- 1 It is extremely unhealthy to sleep in boots. Every time when I wake up and

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Let us consider some examples:

- ① It is extremely unhealthy to sleep in boots. Every time when I wake up and
- ② A man walks over a garden, back and forth, and strikes by one stick to over stick. Another man asks him: "*What are you doing, Sir ?*"
"*I frighten the elephants.*"

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Should be the common sense employed for creating scientific theories

What about a bit more general conjecture ?

What is a "common sense" ?

Can math say something about a "common sense" ?

Common sense - can we say what it consists of ?

What is an *innumeracy* ?

1 Zeno's paradoxes (born approximately 490 BC)

Achilles (Discobolus) runs 100m and the tortoise also runs but it starts at 99m. When Achilles arrives at 99m the tortoise is already about a half of meter next to the end of 100m. When Achilles arrives at 99.5m, the tortoise

(Zeno's race course - Lecture notes from the University of Washington)

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2 An information in the journal SAN FRANCISCO MONITOR (1995):
*Every year since 1950, the number
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Compute with me:

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Common sense - a vague expression for ...

Illiteracy - innumeracy

1 1950 - 1 child



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- 1 1950 - 1 child
- 2 1951 - 2 children, 1952 - 4 children,
1953 - 8 children, 1960 - 1024 children, etc.



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(more than 4x total population of US)

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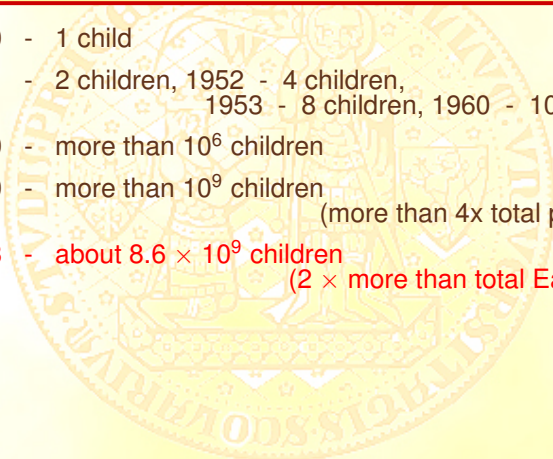
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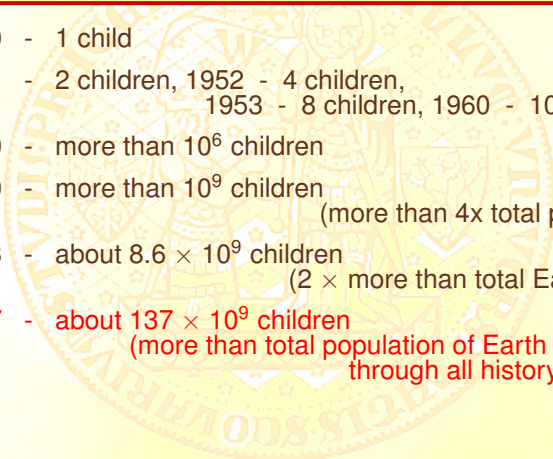
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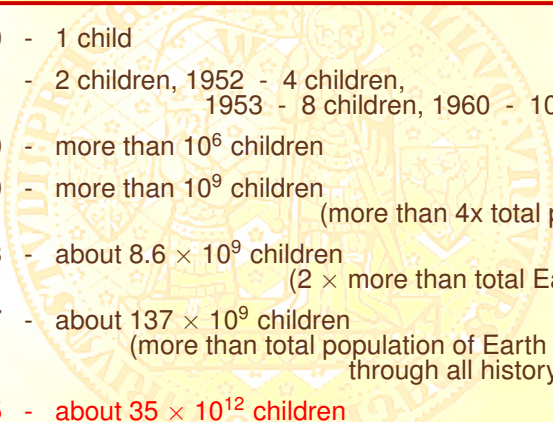
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 - ⑦ 1995 - about 35×10^{12} children
(number encountered rarely outside the astronomy)

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Can we rely on common sense?

Innumneracy - undersyanding ?

- 1 Original information of the Children's Defense Fund

(in 1995):

*Since 1950, the number of American children
gunned down every year has doubled.*

(But in San Francisco Monitor it was written:

*Every year since 1950, the number
of American children gunned down has doubled.)*

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- 2 This is example, how much the order of words is important even in everyday-life-sentence.
- 3 You have already learnt in mathematics that in definitions the order of words is (usually) crucial.

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What is a "common sense" ?

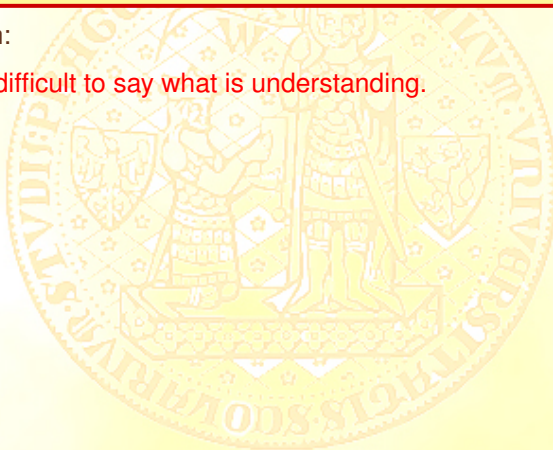
Can math say something about a "common sense" ?

Can we rely on common sense?

What is an *understanding* ?

Conclusion:

- 1 It is difficult to say what is understanding.



Summarizing what we have done.

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- ③ There is something, say *common sense* (*zdravý rozum*), which is to be "satisfied", if we want to claim that something is an explanation of something.
- ④ If somebody says that he/she understands to something, he/she is to be able to create an explanation in similar situations (of course, we never know it surely).

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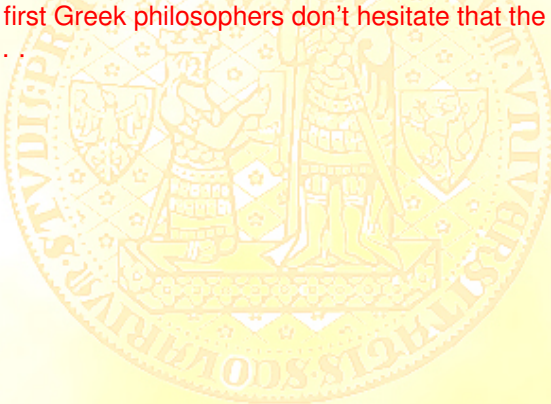
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The *common sense* - is it reliable or not?

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 - elliptic trajectories of planets.

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The *common sense* (continued)

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- ④ Bernard Bolzano (5.10.1781 - 18.12.1848)
 - two century latter

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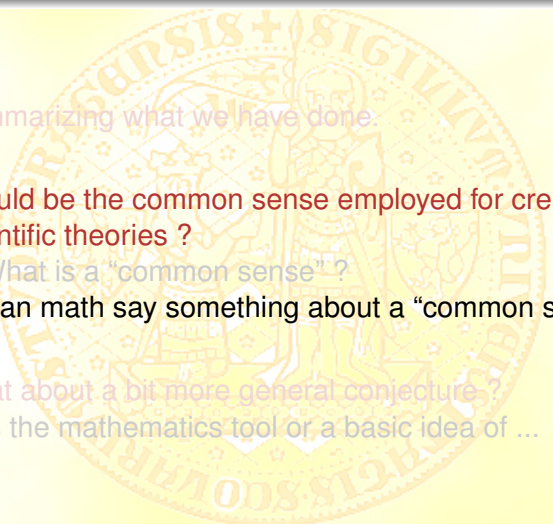
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In fact:

"Figures may not lie, but liars figure."

Joel Best: *Damned Lies and Statistics*,
University of California Press,
Berkeley, 2001.

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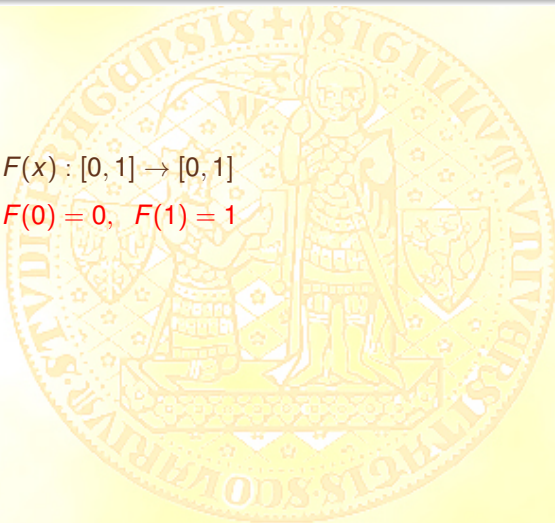
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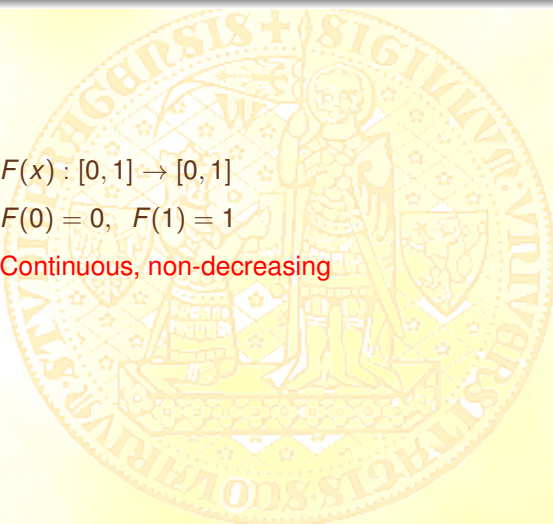
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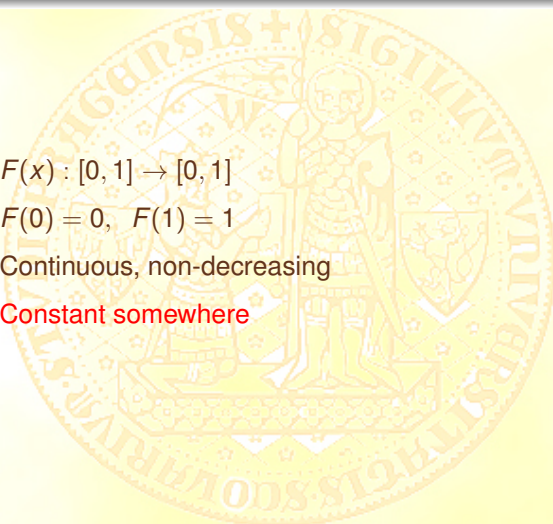
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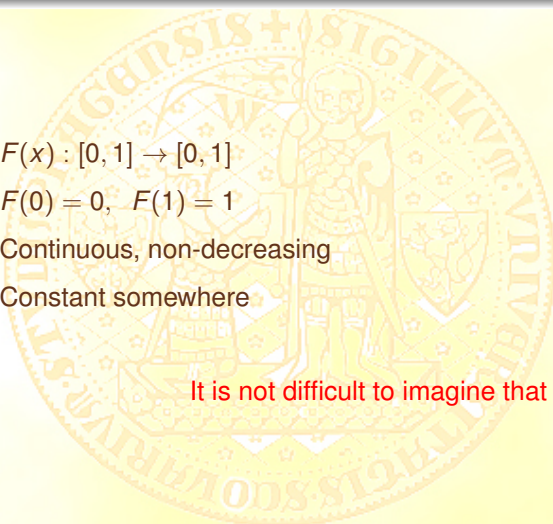
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It is not difficult to imagine that it is possible !

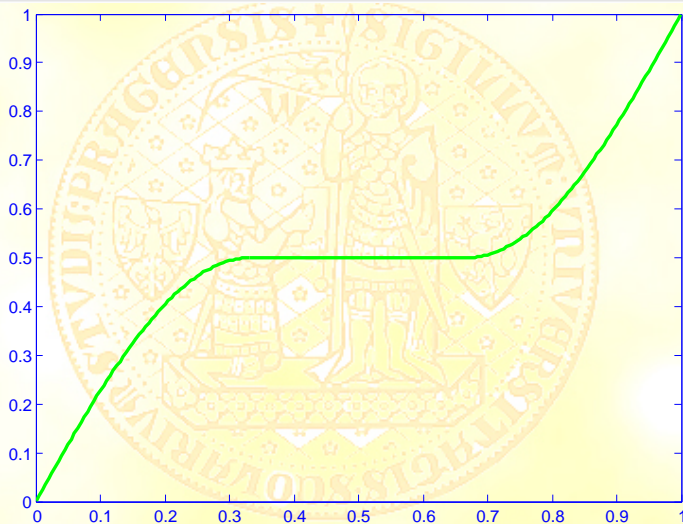
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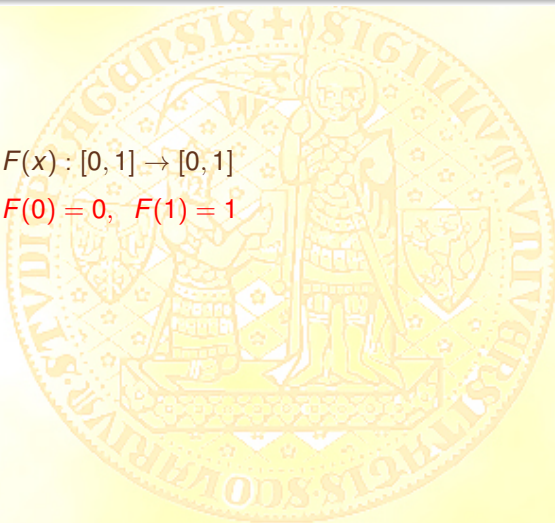
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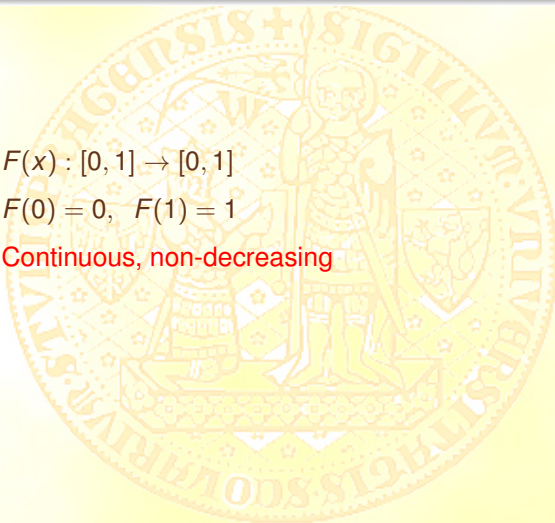
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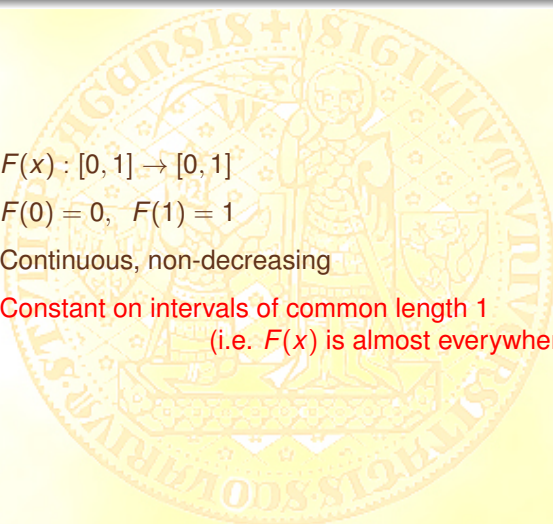
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(i.e. $F(x)$ is almost everywhere constant.)

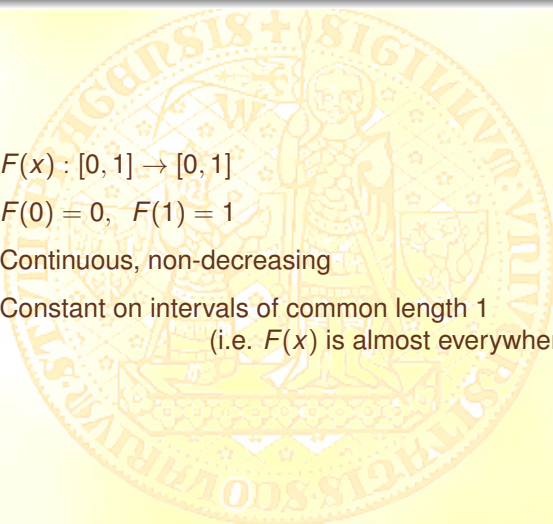
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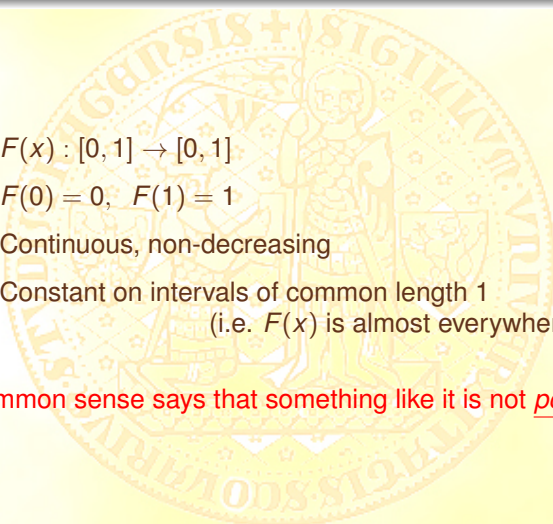
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The common sense says that something like it is not possible !

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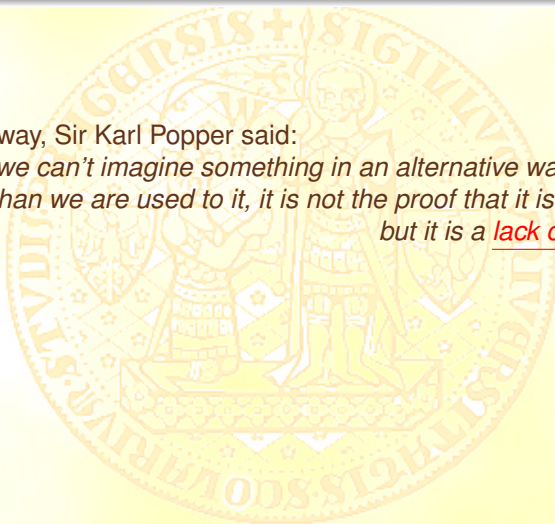
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By the way, Sir Karl Popper said:

*If we can't imagine something in an alternative way
than we are used to it, it is not the proof that it is impossible,
but it is a lack of our fantasy !*



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So, let us demonstrate that in the case of just required function
it is really a lack of fantasy!

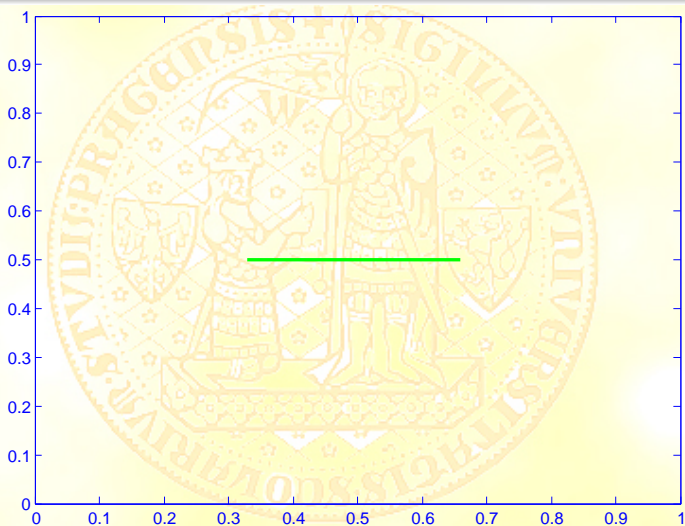
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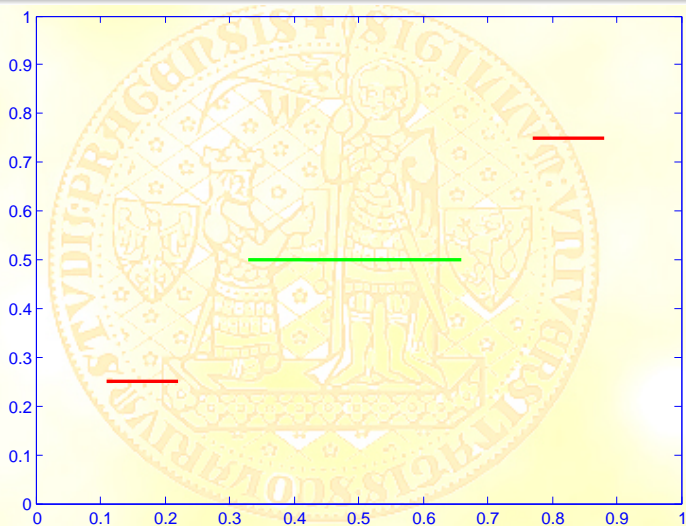
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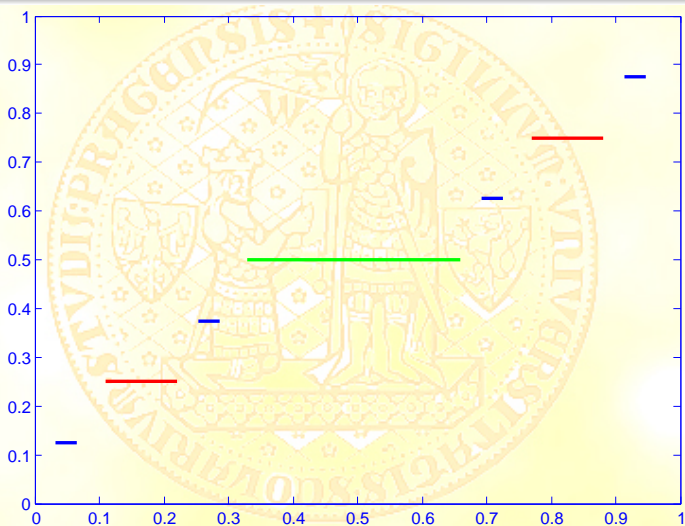
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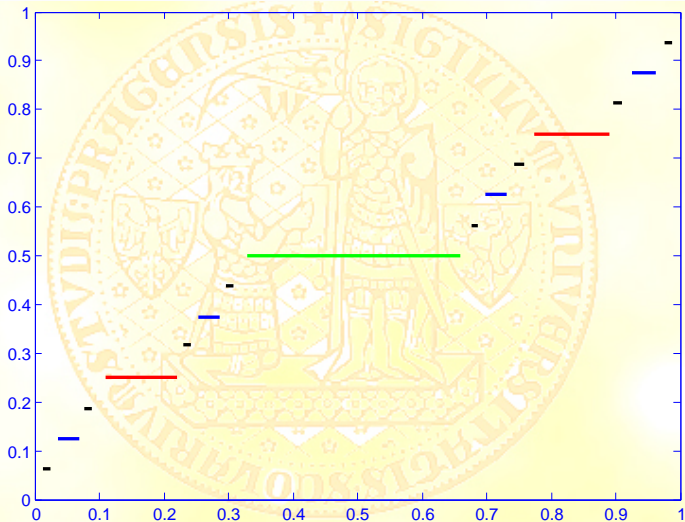
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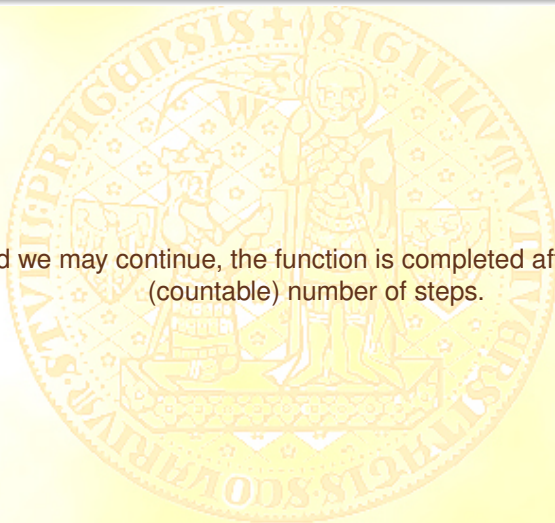
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And we may continue, the function is completed after infinite
(countable) number of steps.



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Conclusion:

- 1 Common sense is a collection of preconceptions, prejudices (against something), legends, common beliefs, scientific folklore, consequences of dogmas, etc.
(Vladimir Iljic Uljanov (Lenin) 1870 - 1924)

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when we want to build up a correct theory
(of anything, e. g. statistics, econometrics, macro or micro)

we cannot rely on common sense, intuition etc.

but on formal logic.

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Thales of Miletus (624 - 546 BC):

"We can prove anything, especially if we rely on words."

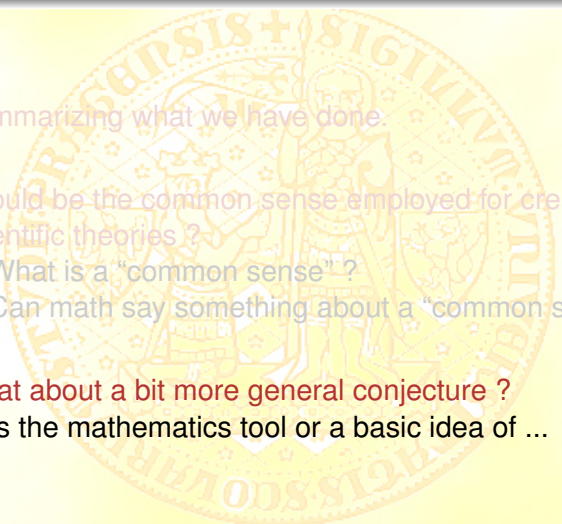
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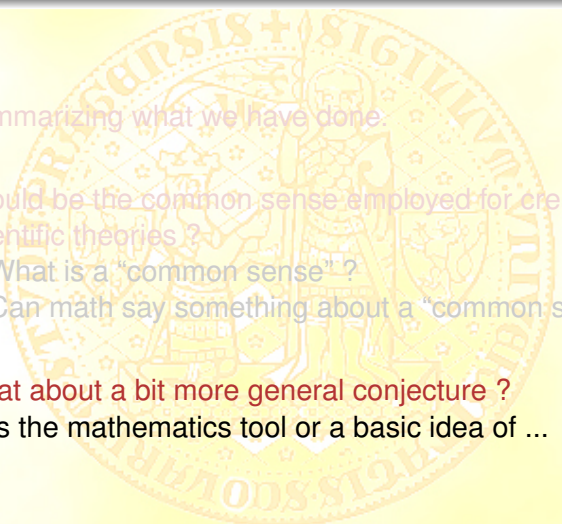
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Is mathematics only a tool or ?

The mathematics is assumed to be language:

“While it has many features of a language - mainly notation that looks like Egyptian hieroglyphics to nonmathematicians - there must be more than mere language going on. We still have to account for amazing success of mathematics as a description of physical reality. The precision of so many theories of physical reality may hint at a deeper truth, that mathematics is a major structural foundation of our universe.”

Alexander Keewatin Dewdney: *Beyond Reason*.

(professor of computer science at the University of Western Ontario)

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What about a main feature of science ?

Generality

Scientific knowledge is about generality. One could say that the more general a successful theory is, the more “scientific” it has become. E.g. in biology the two greatest discovery - the Darwin-Wallace theory of evolution and the Watson-Crick discovery of DNA - are its the most general and most mathematical parts. And of course they are the most important.

Alexander Keewatin Dewdney: *Beyond Reason*.
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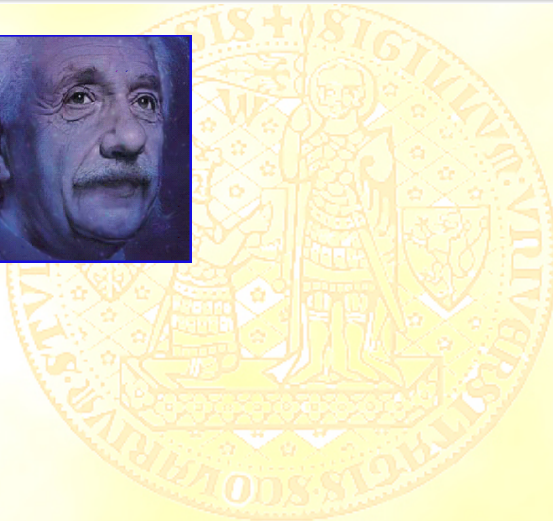
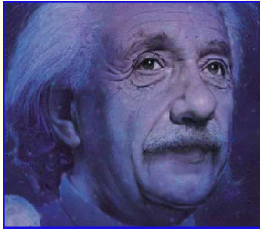
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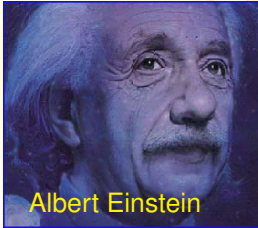
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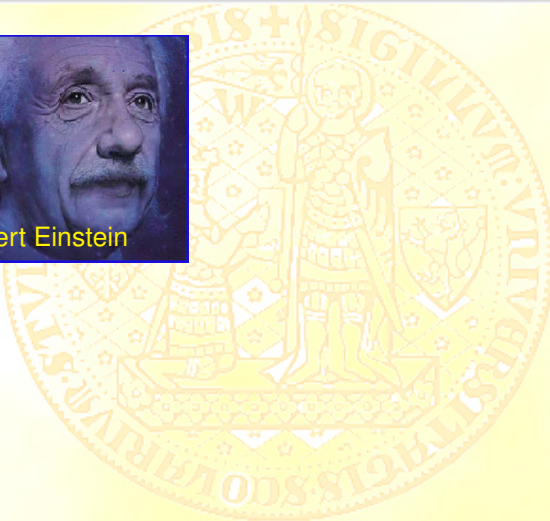
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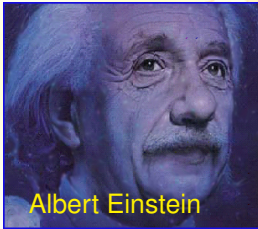
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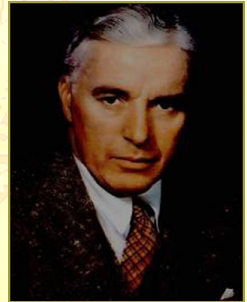
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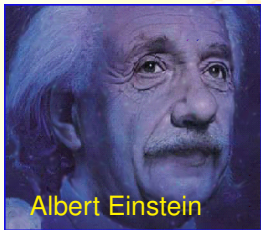
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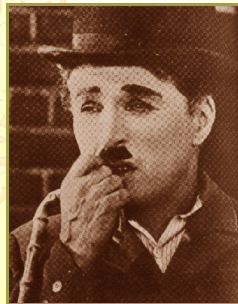
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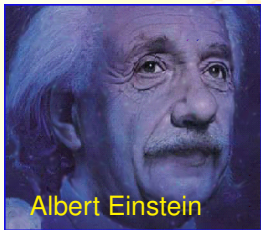
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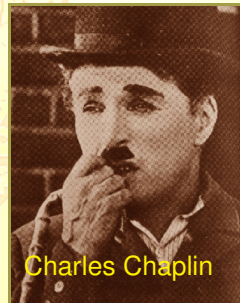
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Charles Chaplin

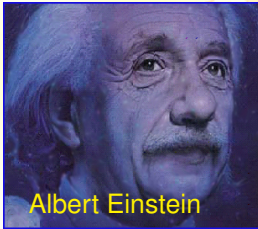
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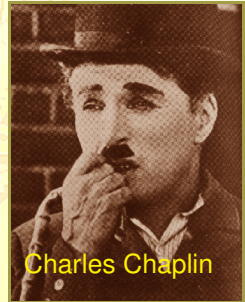
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Dear friend,
I'm fascinated
as the people understand You
and as they admire You.



Charles Chaplin

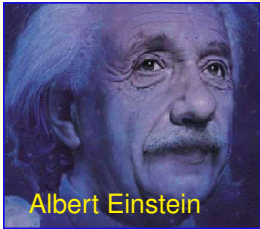
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What about a bit more general conjecture ?

Is the mathematics tool or a basic idea of ... ?

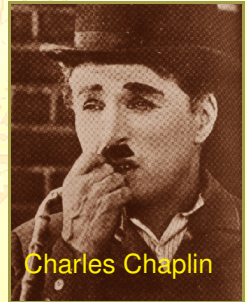
Why is a man admired by people?



Albert Einstein

Dear friend,
I'm fascinated
as the people understand You
and as they admire You.

Dear friend,
I'm fascinated
that although the people don't
understand You at all,
they admire You.



Charles Chaplin

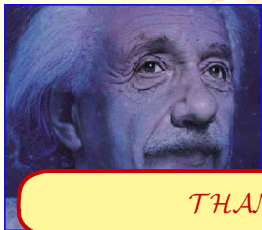
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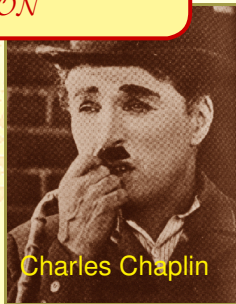
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THANKS FOR ATTENTION

Dear friend,
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Charles Chaplin