INSTITUTE OF ECONOMIC STUDIES, FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE (*established 1348*)

ROBUST STATISTICS AND ECONOMETRICS

INSTITUTE OF ECONOMIC STUDIES
FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE

JAN ÁMOS VÍŠEK

Week 8

Content of lecture

1. At the beginning of any lecture let us repeat .....
   - Algorithm for *M*-estimators
   - Rousseeuw proposals of estimators and their algorithms

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   - Algorithm for LTS
   - Diagnostics by robust methods with high breakdown point
   - Algorithm for LWS

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Computing *M*-estimate of regression coefficients

We have considered the extremal problem

$$\hat{\beta}^{(GM,n,w)} = \arg\min_{\beta \in R^p} \sum_{i=1}^{n} w_i \rho \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}} \right).$$

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Write it as

$$\hat{\beta}^{(M,n)} = \arg\min_{\beta \in R^p} \sum_{i:(Y_i - X_i'\beta) \neq 0} w_i \left[\rho\left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right) \cdot \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right)^{-2}\right] \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right)^2$$

$$= \arg\min_{\beta \in R^p} \sum_{i=1}^{n} \tilde{w}_i \cdot \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right)^2$$

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$$= \arg\min_{\beta \in R^p} \sum_{i=1}^{n} \tilde{w}_i \cdot \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right)^2$$

where $\tilde{w}_i = w_i \rho\left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right) \cdot \left(\frac{Y_i - X_i'\beta}{\hat{\sigma}}\right)^{-2}$ for $i : (Y_i - X_i'\beta) \neq 0$,

otherwise $\tilde{w}_i = 0$.

Computing *M*-estimate of regression coefficients

Then

$$\hat{\beta}^{(GM,n,w)} = \left(X'\tilde{W}X\right)^{-1} X'\tilde{W}Y$$

where $\tilde{W} = \mathrm{diag}(\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n)$.

Computing *M*-estimate of regression coefficients

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And an iterative computation, starting with a "guess" of

$$\hat{\beta}^{(GM,n,w)}_{(starting)},$$

lead usually after several tens or hundreds of cycles to the desired estimate.

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Antoch, J., J. Á. Víšek (1991):
Robust estimation in linear models and its computational aspects.
Contributions to Statistics: Computational Aspects of Model Choice, Springer Verlag, (1992), ed. J. Antoch, 39 - 104.

A pursuit for highly robust estimator of regression coefficients

Prior to continuing let us make an agreement:

For any $\beta \in R^p$

$$r_i(\beta) = Y_i - X_i'\beta \qquad \text{not only} \qquad r_i(\hat{\beta}) = Y_i - X_i'\hat{\beta}$$

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Order statistics

$$r^2_{(1)}(\beta), r^2_{(2)}(\beta), \ldots, r^2_{(n)}(\beta),$$

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$$r^2_{(1)}(\beta), r^2_{(2)}(\beta), ..., r^2_{(n)}(\beta),$$

some texts alternatively employ

$$r^2_{(1:n)}(\beta), r^2_{(2:n)}(\beta), ..., r^2_{(n:n)}(\beta).$$

A pursuit for highly robust estimator of regression coefficients

Regression quantiles

Koenker,R., G. Bassett (1978): Regression quantiles.
Econometrica, 46, 33-50.

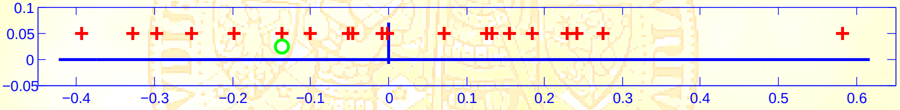
$$\hat{\beta}^{(\alpha)} = \underset{\beta \in R^p}{\arg\min} \left\{ \sum_{i=1}^{n} \left[\alpha \cdot |r_i(\beta)| \cdot I\{r_i(\beta) < 0\} + (1-\alpha) \cdot |r_i(\beta)| \cdot I\{r_i(\beta) > 0\}\right] \right\}$$

$$\hat{\beta}^{(L,n)} = \sum_{\ell=1}^{K} c_\ell \hat{\beta}^{(\alpha_\ell)}$$

$\hat{\beta}^{(\alpha)}$ is *M*- and simultaneously *L*-estimator

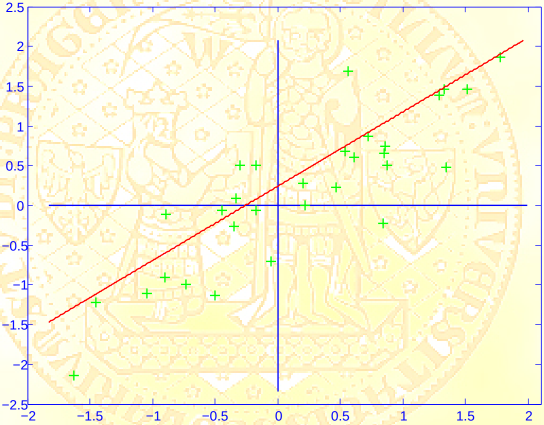
Classical quantiles

30% location quantile



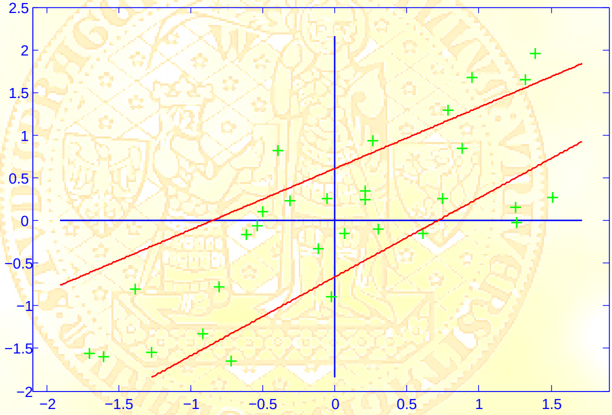
Regression quantiles

20% regression quantile



Regression quantiles

Two regression quantiles, 20% and 89%, say



A pursuit for highly robust estimator of regression coefficients

The trimmed least squares (TLS)

Ruppert, D., R. J. Carroll (1980):

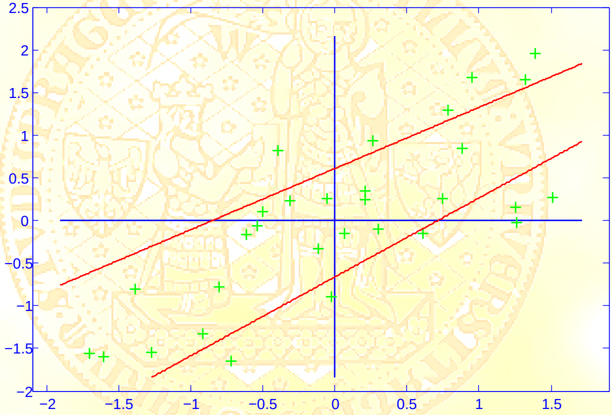
Trimmed least squares estimation in linear model.

J. Americal Statist. Ass., 75 (372), 828–838.

Trimming by $\left[x' \cdot \hat{\beta}^{(\alpha_1)}, x' \cdot \hat{\beta}^{(\alpha_2)}\right]$ $\quad 0 \leq \alpha_1 < \alpha_2 \leq 1 \quad \rightarrow \quad \hat{\beta}^{(TLS,n)_{(\alpha_1,\alpha_2)}}$

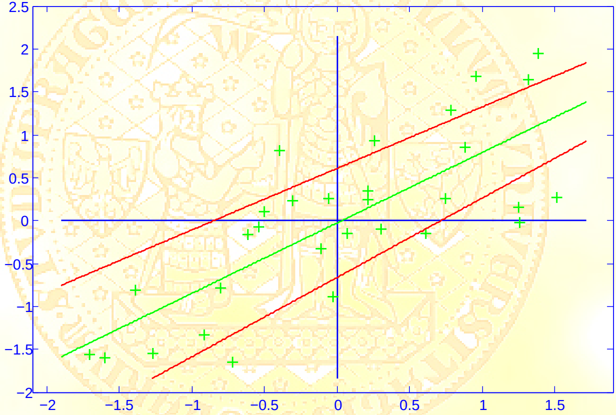
The trimmed least squares

Two regression quantiles



The trimmed least squares

Two regression quantiles with OLS for trimmed data



Content

We have studied LMS

Rousseeuw, P. J. (1983): Least median of square regression.
Journal of Amer. Statist. Association 79, pp. 871-880.

the Least Median of Squares

$$\hat{\beta}^{(LMS,n,h)} = \arg\min_{\beta \in R^p} r^2_{(h)}(\beta) \qquad \frac{n}{2} < h \leq n,$$

Many advantages - mainly

1. *breakdown point equal to* $([\frac{n-p}{2}] + 1)n^{-1}$ *if* $h = [\frac{n}{2}] + [\frac{p+1}{2}]$
2. *scale- and regression equivariant*

(without any studentization of residuals).

Main disadvantage

$$\sqrt[3]{n}\left(\hat{\beta}^{(LMS,n,h)} - \beta^0\right) = \mathcal{O}_p(1)$$

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(Cernobyl)

Peter Rousseeuw proposed the algorithm:

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3. Repeat it “10 000” times and select that model (among these “10 000”) with smallest h-th squared residual.

An improvement of the algorithm - a geometric characterization

Joss, J., A. Marazzi (1990):
Probabilistic algorithms for LMS regression.
Computational Statistics & Data Analysis 9, 123-134.

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The geometric characterization
of exact solution of LMS extremal problem:

The exact solution has at least
$p+1$ residuals of the same (absolute) value.

An improvement of the algorithm - a geometric characterization

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   and fit a regression plane to them.

An improvement of the algorithm - a geometric characterization

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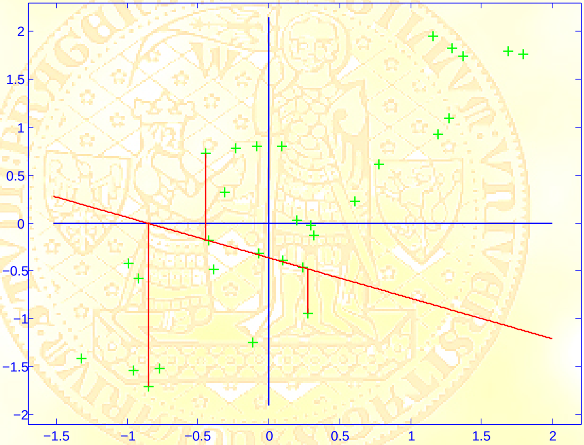
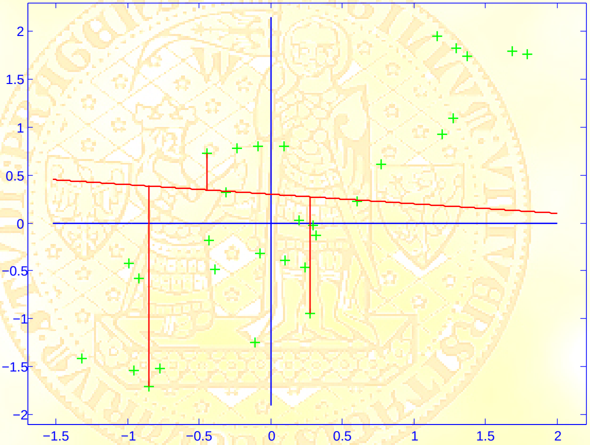
A geometric characterization

Unlucky selection of starting points



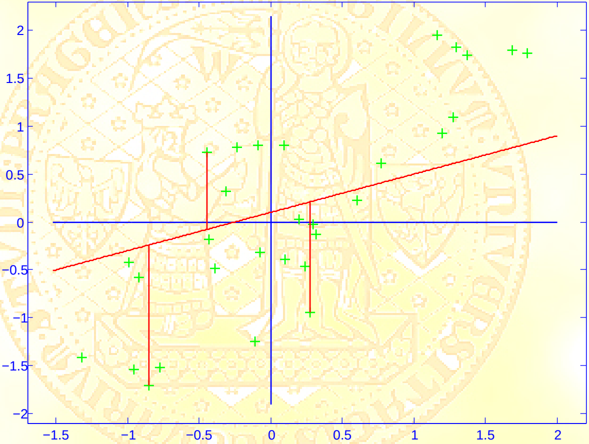
A geometric characterization

Starting shifting and spinning the line



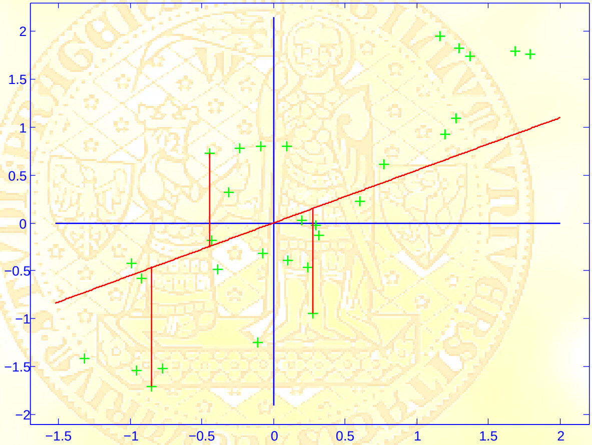
A geometric characterization

Continuing shifting and spinning the line



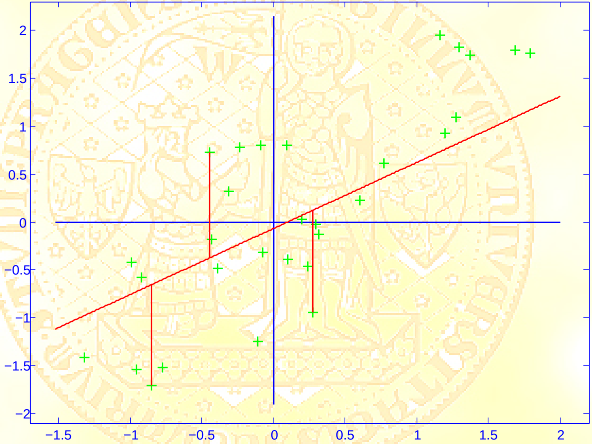
A geometric characterization

Nearly reaching the geometric characterization



A geometric characterization

Reaching the geometric characterization



A shock and frustration - Engine Knock Data

Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
The American Statistician 46, 79–83.

Engine Knock Data $(n = 16, p = 4, h = 11)$

c	x_1	x_2	x_3	x_4	y
1	13.3	13.9	31	697	84.4
2	13.3	14.1	30	697	84.1
3	13.4	15.2	32	700	88.4
4	12.7	13.8	31	669	84.2
⋮	⋮	⋮	⋮	⋮	⋮
14	12.7	16.1	35	649	93.0
15	12.9	15.1	36	721	93.3
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x_1 is spark timing x_2 air/fuel ratio
x_3 intake temperature x_4 exhaust temperature
y engine knock number

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In fact they worked with two data sets.

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Let's call these data "Correct".

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In fact they worked with two data sets.

Let’s call these data “Damaged”.

x_1 is spark timing x_2 air/fuel ratio
x_3 intake temperature x_4 exhaust temperature
y engine knock number

A shock and frustration

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⋮	⋮	⋮	⋮	⋮	⋮
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16	12.7	15.9	37	696	93.1

Let's verify that
$h = [\frac{n}{2}] + [\frac{p+1}{2}] = 11.$

x_1 is spark timing x_2 air/fuel ratio
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An example of the real data, indicating a high sensitivity of the estimator with high breakdown point (LMS) to the shift of one observation.

x_1 is spark timing $\quad$ x_2 air/fuel ratio
x_3 intake temperature $\quad$ x_4 exhaust temperature
y engine knock number

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Engine Knock Data ($n = 16$, $p = 4$, $h = 11$)

c	x_1	x_2	x_3	x_4	y

The values of $\hat{\beta}^{(LMS,n,h)}$ by "elemental" algorithm !
(still included in some packages - see the next slide)

Data	Interc.	SPARK	AIR	INTK	EXHS.
Correct data ($x_{22} = 14.1$)	30.08	0.21	2.90	0.56	-0.01
Damaged data($x_{22} = 15.1$)	-86.50	4.59	1.21	1.47	0.07

x_1 is spark timing x_2 air/fuel ratio
x_3 intake temperature x_4 exhaust temperature
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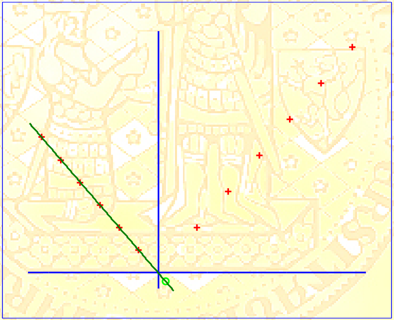
An (academic) explanation by a shift of “inlier”

SENSITIVITY OF ANY HIGH-BREAKDOWN-POINT ESTIMATOR
TO A SMALL CHANGE OF DATA

Model for the majority of data



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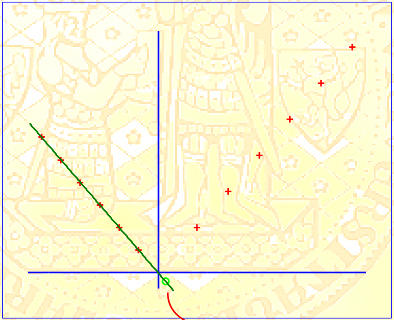
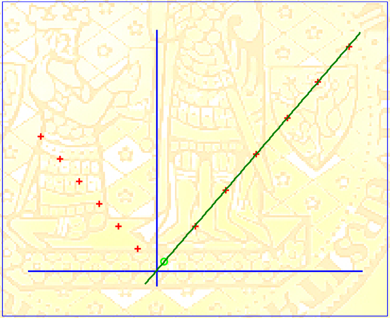


We are going to shift up this point “ o ”.

An (academic) explanation by a shift of "inlier"

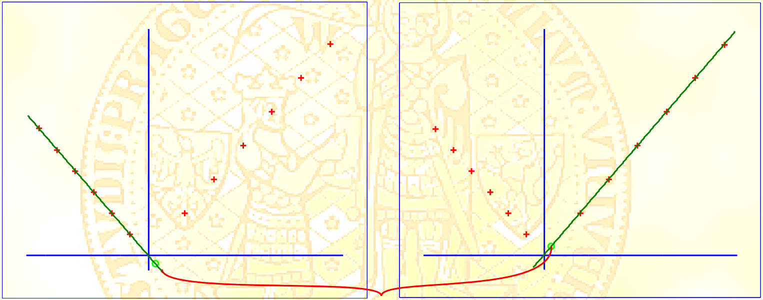
SENSITIVITY OF ANY HIGH-BREAKDOWN-POINT ESTIMATOR
TO A SMALL CHANGE OF DATA

Again model for the majority of data



An (academic) explanation by a shift of "inlier"

In both cases the model is for the majority of data



Notice: *The closer* the point (" o ") is to the y-axe,
the smaller shift causes the "switch" of the model.

Content

Content

A substantial improvement of the algorithm

- an employment of simplex method

Boček, P., P. Lachout (1993):
Linear programming approach to LMS-estimation.
Memorial volume of Comput. Statist. & Data Analysis 19(1995), 129 - 134.

A description is a bit complicated - it requires
to be familiar with a dual form of simplex method.

Boček-Lachout algorithm

First of all, the algorithm gave:

1. much smaller 11^{th} squared residual than the algorithm used by Hettmansperger & Sheather,

	11^{th} order statistics	
Method	PRO-*LMS*	Bo-La-*LMS*
Correct data ($x_{22} = 14.1$)	0.322	0.227
Damaged data ($x_{22} = 15.1$)	0.573	0.451

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Method	PRO-*LMS*	Bo-La-*LMS*
Correct data ($x_{22} = 14.1$)	0.322	0.227
Damaged data ($x_{22} = 15.1$)	0.573	0.451

2. in a much shorter time.

Boček-Lachout algorithm

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15	12.9	15.1	36	721	93.3
16	12.7	15.9	37	696	93.1

The value of $\hat{\beta}^{(LMS,n,h)}$ by Boček-Lachout algorithm.

Data	Interc.	SPARK	AIR	INTK	EXHS.
Correct data ($x_{22} = 14.1$)	30.04	0.14	3.08	0.46	-0.01
Damaged data($x_{22} = 15.1$)	48.38	-0.73	3.36	0.23	-0.01

x_1 is spark timing x_2 air/fuel ratio
x_3 intake temperature x_4 exhaust temperature
y engine knock number

Boček-Lachout algorithm

Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
The American Statistician 46, 79–83.

Engine Knock Data $(n = 16, p = 4, h = 11)$

c	x_1	x_2	x_3	x_4	y

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Data	Interc.	SPARK	AIR	INTK	EXHS.
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The difference between these two models is much lower.

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The difference between these two models is much lower.
So, the effect announced by H-S was a consequence of the bad algorithm.

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x_3 intake temperature $\quad$ x_4 exhaust temperature
y engine knock number

Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
The American Statistician 46, 79–83.

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Data	Interc.	SPARK	AIR	INTK	EXHS.
Correct data ($x_{22} = 14.1$)	30.04	0.14	3.08	0.46	-0.01

BUT THIS CONCLUSION - ALTHOUGH TRUE - WAS MISLEADING.

x_1 is spark timing x_2 air/fuel ratio
x_3 intake temperature x_4 exhaust temperature
y engine knock number

We have seen: A shock and frustration

Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
The American Statistician 46, 79–83.

Engine Knock Data $(n = 16, p = 4, h = 11)$

c	x_1	x_2	x_3	x_4	y
1	13.3	13.9	31	697	84.4
					84.1
					88.4
					84.2
⋮	⋮	⋮	⋮	⋮	⋮
14	12.7	16.1	35	649	93.0
15	12.9	15.1	36	721	93.3
16	12.7	15.9	37	696	93.1

Let's verify once again that
$h = [\frac{n}{2}] + [\frac{p+1}{2}] = 11.$

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x_3 intake temperature x_4 exhaust temperature
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The American Statistician 46, 79–83.

Engine Knock Data ($n = 16$, $p = 4$, $h = 11$)

c	x_1	x_2	x_3	x_4	y
1	13.3	13.9	31	697	84.4
2	13.3	14.1	30	697	84.1
3	13.4	15.2	32	700	88.4
4	12.7	13.8	31	669	84.2
⋮	⋮	⋮	⋮	⋮	⋮
14	12.7	16.1	35	649	93.0
15	12.9	15.1	36	721	93.3
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Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
The American Statistician 46, 79–83.

Engine Knock Data $(n = 16, p = 4, h = 11)$

Realize that $\binom{16}{11} = 4368$, so that we can compute $\hat{\beta}^{(LTS,16,11)}$ exactly, just computing $\hat{\beta}^{(OLS,11)}$ for all subsamples of size 11 and select the "best" one.

⋮	⋮	⋮	⋮	⋮	⋮
14	12.7	16.1	35	649	93.0
15	12.9	15.1	36	721	93.3
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This is the exact value of $\hat{\beta}^{(LTS,n,h)}$!

Data	Interc.	SPARK	AIR	INTK	EXHS.
Correct data ($x_{22} = 14.1$)	35.11	-0.028	2.949	0.477	-0.009
Damaged data ($x_{22} = 15.1$)	-88.7	4.72	1.06	1.57	0.068

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Hettmansperger, T. P., S. J. Sheather (1992):
A Cautionary Note on the Method of Least Median Squares.
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x_1 is spark timing x_2 air/fuel ratio

Víšek, J.Á (1994): A cautionary note on the method of Least Median of Squares reconsidered.
Transactions of the Twelfth Prague Conference 1994, 254 - 259.

Correct data

Engine Knock Data (Air/Fuel 14.1, $n = 16$, $p = 4$)
(p is dimension of data including intercept)

Method	PRO-*LMS*	Bo-La-*LMS*	Exact *LTS*	Iterative *LTS*
11^{th} order stat.	0.3221	0.22783	0.3092	0.3092
Sum of squares	0.4239	0.3575	0.2707	0.2707

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Method	PRO-*LMS*	Bo-La-*LMS*	Exact *LTS*	Iterative *LTS*
11^{th} order stat.	0.5729	0.4506	0.5392	0.5392
Sum of squares	1.0481	1.432	0.7283	0.7283

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Another benchmark

Stackloss Data ($n = 21$, $p = 4$)
(p is dimension of data including intercept)

Brownlee, K.A. (1965):
Statistical Theory and Methodology in Science and Engineering, Wiley, NY.
Rousseew, P. J., A. M. Leroy (1987):
Robust Regression and Outlier Dectection, Wiley, NY.

Operational data of a plant for the oxidation of ammonia to nitric acid.

X1 - Air Flow X2 - Temperature X3 - Acid Concentration Y - Stackloss

Case	X1	X2	X3	Y
1	80	27	89	42
2	80	27	88	37
3	75	25	90	37
4	62	24	87	28
5	62	22	87	18
6	62	23	87	18
7	62	24	93	19

Case	X1	X2	X3	Y
8	62	24	93	20
9	58	23	87	15
10	58	18	80	14
11	58	18	89	14
12	58	17	88	13
13	58	18	82	11
14	58	19	93	12

Case	X1	X2	X3	Y
15	50	18	89	8
16	50	18	86	7
17	50	19	72	8
18	50	19	79	8
19	50	20	80	9
20	56	20	82	15
21	70	20	91	15

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Rousseew, P. J., A. M. Leroy (1987):
Robust Regression and Outlier Dectection, Wiley, NY.

Method	PRO-*LMS*	Bo-La-*LMS*	Exact *LTS*	Iterative *LTS*
12^{th} order stat.	0.6640	0.5321	0.7014	0.7014
Sum of squares	2.4441	1.9358	1.6371	1.6371

Another benchmark

Demographical Data $(n = 49, p = 7)$

Gunst, R. F., and Mason, R. L. (1980):
Regression Analysis and Its Application: A Data-Oriented Approach.
New York: Marcel Dekker.

Chatterjee, S., Hadi, A. S. (1988):
Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

1. X1 - Infant deaths per 1000 live birth

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1. X1 - Infant deaths per 1000 live birth
2. X2 - Number of inhabitants per physician

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1. X1 - Infant deaths per 1000 live birth
2. X2 - Number of inhabitants per physician
3. X3 - Population per square kilometer

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2. X2 - Number of inhabitants per physician
3. X3 - Population per square kilometer
4. X4 - Population per 1000 hectares of agricultural land

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1. X1 - Infant deaths per 1000 live birth
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5. X5 - Percentage literate of population aged 15 years and over

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2. X2 - Number of inhabitants per physician
3. X3 - Population per square kilometer
4. X4 - Population per 1000 hectares of agricultural land
5. X5 - Percentage literate of population aged 15 years and over
6. X6 - Number of students enrolled in higher education per 100000 population

Another benchmark

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5. X5 - Percentage literate of population aged 15 years and over
6. X6 - Number of students enrolled in higher education per 100000 population
7. Y - Gross national product per capita in 1957 in $ (U.S.)

Content

An algorith for LTS

LTS - ALGORITHM

A

Find the plane through $p+1$ randomly selected observations.

Evaluate squared residuals of all observations and order them increasingly. Then sum up the h smallest squares of residuals and the sum denote $S(\hat{\beta}_{present})$.

Is $S(\hat{\beta}_{present})$ less than $S(\hat{\beta}_{past})$?

no → B

yes

Establish *new* $\hat{\beta}_{present}$ just applying *OLS* on the h observations with the smallest squared residuals.

An algorithm for LTS

LTS - ALGORITHM$_{(continued)}$

B

Was ℓ-times found the same model with minimal value of $S(\beta)$?

yes

no

Was already k-times repeated outer cycle ?

no → A

yes

As $\hat{\beta}^{(LTS,n,w)}$ we will assume $\beta \in R^p$ for which the functional $S(\beta)$ attained - through just described iterations - minimal value.

Another benchmark

Demographical Data ($n = 49$, $p = 7$)

Gunst, R. F., and Mason, R. L. (1980):
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New York: Marcel Dekker.

Chatterjee, S., Hadi, A. S. (1988):
Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

Method	PRO-*LMS*	Bo-La-*LMS*	Iterative *LTS*
28^{th} order stat.	131.50	95.38	104.20
Sum of squares	134260	132340	64159

Another benchmark

Educational Data $(n = 50, p = 4)$

Rousseeuw, P. J., Leroy, A. M. (1987):
Robust Regression and Outlier Detection. New York: J.Wiley & Sons.

Chatterjee, S., Hadi, A. S. (1988):
Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

1. X1 - Number of residents (per 1000) residing in urban areas in 1970

Another benchmark

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1. X1 - Number of residents (per 1000) residing in urban areas in 1970
2. X2 - Personal income per capita in 1973
(i. e. sum of personal incomes divided by number of inhabitants)

Another benchmark

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Robust Regression and Outlier Detection. New York: J.Wiley & Sons.

Chatterjee, S., Hadi, A. S. (1988):
Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

1. X1 - Number of residents (per 1000) residing in urban areas in 1970
2. X2 - Personal income per capita in 1973
   (i. e. sum of personal incomes divided by number of inhabitants)
3. X3 - Number of residents per thousand under 18 years of age in 1974

Another benchmark

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Rousseeuw, P. J., Leroy, A. M. (1987):
Robust Regression and Outlier Detection. New York: J.Wiley & Sons.

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Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

1. X1 - Number of residents (per 1000) residing in urban areas in 1970
2. X2 - Personal income per capita in 1973
(i. e. sum of personal incomes divided by number of inhabitants)
3. X3 - Number of residents per thousand under 18 years of age in 1974
4. Y - Education expenditure for public education per capita in 1975

Another benchmark

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Rousseeuw, P. J., Leroy, A. M. (1987):
Robust Regression and Outlier Detection. New York: J.Wiley & Sons.

Chatterjee, S., Hadi, A. S. (1988):
Sensitivity Analysis in Linear Regression. New York: J. Wiley & Sons.

Method	PRO-*LMS*	Bo-La-*LMS*	Iterative *LTS*
27^{th} order stat.	19.3562	16.63511	19.0378
Sum of squares	3605.5	3728.6	3414.5

Content

Diagnostics by LTS

THE PROBLEM IS HOW LARGE h WE SHOULD SELECT FOR LTS.

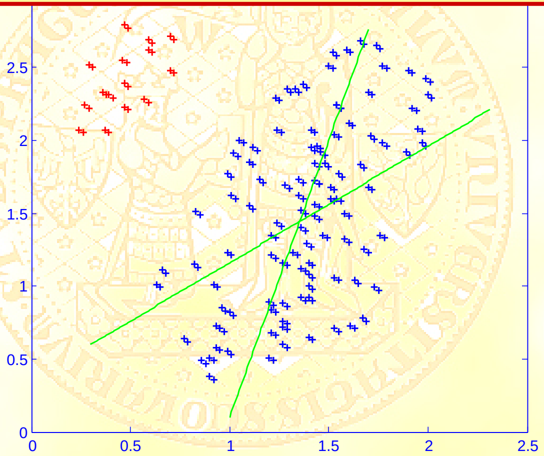
Diagnostics by LTS

THE PROBLEM IS HOW LARGE h WE SHOULD SELECT FOR LTS.

We may start with $h \approx \frac{n}{2}$ and increase it step by step. It works as follows.

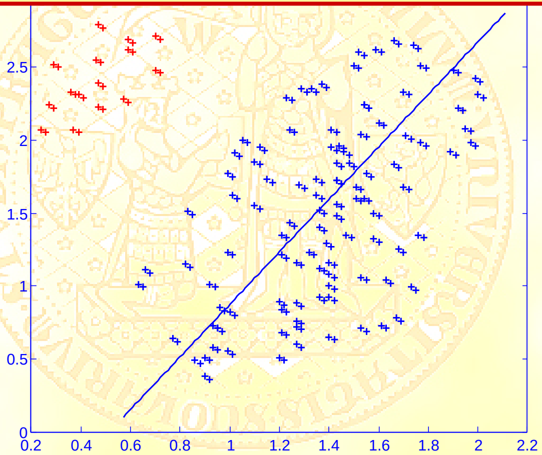
Diagnostics by LTS

FOR $h << k$ WE OBTAIN ONE OF GREEN LINES,
AND ESTIMATES OF COEFFS (ETC.) MODESTLY VARY.



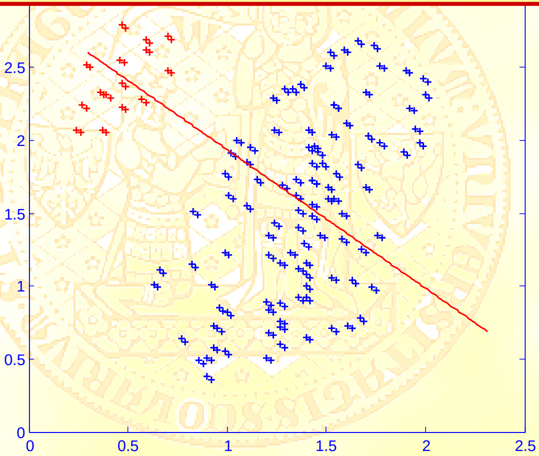
Diagnostics by LTS

FOR $h \leq k$ BUT NEAR TO k WE OBTAIN BLUE LINE,
POPULATIONS ARE NESTED
AND ESTIMATES OF COEFFS (ETC.) ARE STABLE.



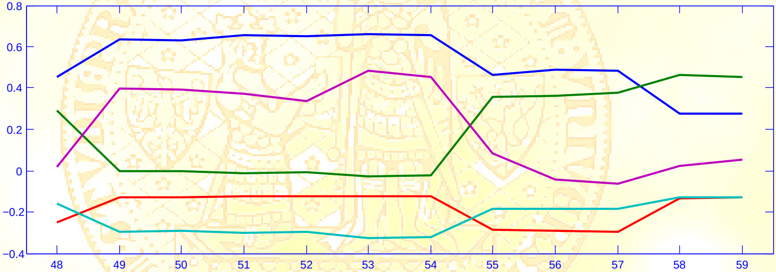
Diagnostics by LTS

FOR $h > k$ WE OBTAIN RED LINE
AND ESTIMATES OF COEFFS (ETC.) SIGNIFICANTLY CHANGED.



Diagnostics by LTS

ANALYSIS OF THE EXPORT FROM THE CZECH REPUBLIC TO EU IN 1994 BY MEANS OF THE *least trimmed squares*.

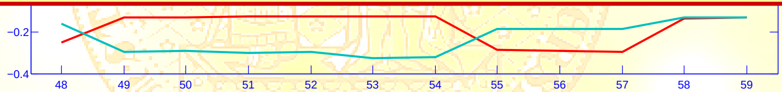


The development of the estimates of regression coefficients. The blue curve represents $\hat{\beta}_1^{(LTS,n,h)}$ (down-scaled by $\frac{1}{10}$), the purple one is $\hat{\beta}_8^{(LTS,n,h)}$, the green is $\hat{\beta}_3^{(LTS,n,h)}$, the red is $\hat{\beta}_4^{(LTS,n,h)}$ and the light blue (the lowest curve) is $\hat{\beta}_6^{(LTS,n,h)}$ (down-scaled again by $\frac{1}{10}$). There is an evident break at 54.

Diagnostics by LTS

ANALYSIS OF THE EXPORT FROM THE CZECH REPUBLIC TO EU IN 1994
BY MEANS OF THE *least trimmed squares*.

Beňáček, V., J. Á Víšek (2002):
Impacts of the EU opening-up on small open economy:
Czech exports and imports.
In *Karadeloglou P. (ed.): Enlarging the EU - The Trade Balance Effects Palgrave/Macmillan, New York, 2002, 3 - 29.*



The development of the estimates of regression coefficients. The blue curve represents $\hat{\beta}_1^{(LTS,n,h)}$ (down-scaled by $\frac{1}{10}$), the purple one is $\hat{\beta}_8^{(LTS,n,h)}$, the green is $\hat{\beta}_3^{(LTS,n,h)}$, the red is $\hat{\beta}_4^{(LTS,n,h)}$ and the light blue (the lowest curve) is $\hat{\beta}_6^{(LTS,n,h)}$ (down-scaled again by $\frac{1}{10}$). There is an evident break at 54.

Diagnostics by LTS

ANALYSIS OF THE EXPORT FROM THE CZECH REPUBLIC TO EU IN 1994 BY MEANS OF THE *least trimmed squares*.

Beňáček, V., J. Á Víšek (2002):
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green is $\hat{\beta}_3^{(LTS,n,h)}$, the red is $\hat{\beta}_4^{(LTS,n,h)}$ and the light blue (the lowest curve) is $\hat{\beta}_6^{(LTS,n,h)}$ (down-scaled again by $\frac{1}{10}$). There is an evident break at 54.

ANALYSIS OF THE EXPORT FROM THE CZECH REPUBLIC TO EU IN 1994 BY MEANS OF THE *least trimmed squares*

has found:

MAIN SUBGROUP

with number of industries 54 and model

$$\frac{X_\ell}{S_\ell} = 4.64 - 0.032 \cdot \frac{US_\ell}{VA_\ell} - 0.022 \cdot \frac{HS_\ell}{VA_\ell} - 0.124 \cdot \frac{K_\ell}{VA_\ell} + 1.035 \cdot CR_\ell$$

$$-3.199 \cdot TFPW_\ell + 1.048 \cdot BAL_\ell + 0.452 \cdot DP_\ell + \varepsilon_\ell$$

X_ℓ	-	*export from i-th industry,*
US_ℓ	-	*number of university-passed employees in the i-th industry,*
HS_ℓ	-	*nuber of high school-passed employees in the i-th industry,*
VA_ℓ	-	*value added in the i-th industry,*
K_ℓ	-	*capital in the i-th industry,*
CR_ℓ	-	*percentage of market occupied by 3 largest producers,*
$TFPW_\ell$	-	*by wages normed productvity in the i-th industry,*
Bal_ℓ	-	*Balasa index in the i-th industry,,*
DP_ℓ	-	*cost discontinuity in 1993 in the i-th industry*

with coefficient of determination 0.97 and stable submodels

ANALYSIS OF THE EXPORT FROM THE CZECH REPUBLIC TO EU IN 1994 BY MEANS OF THE *least trimmed squares*

has found:

COMPLEMENTARY SUBGROUP

with number of industries 33 and model

$$\frac{X_\ell}{S_\ell} = -0.634 + 0.089 \cdot \frac{US_\ell}{VA_\ell} + 0.235 \cdot \frac{HS_\ell}{VA_\ell} + 0.249 \cdot \frac{K_\ell}{VA_\ell} + 1.174 \cdot CR_\ell$$
$$+0.690 \cdot TFPW_\ell + 2.691 \cdot BAL_\ell - 0.051 \cdot DP_\ell + \varepsilon_\ell$$

X_ℓ	-	*export from i-th industry,*
US_ℓ	-	*number of university-passed employees in the i-th industry,*
HS_ℓ	-	*nuber of high school-passed employees in the i-th industry,*
VA_ℓ	-	*value added in the i-th industry,*
K_ℓ	-	*capital in the i-th industry,*
CR_ℓ	-	*percentage of market occupied by 3 largest producers,*
$TFPW_\ell$	-	*by wages normed productvity in the i-th industry,*
Bal_ℓ	-	*Balasa index in the i-th industry„*
DP_ℓ	-	*cost discontinuity in 1993 in the i-th industry*

with coefficient of determination 0.93 and stable submodels

Content

An algorith for LWS

LWS - ALGORITHM

A

Find the plane through $p+1$ randomly selected observations.

Evaluate squared residuals of all observations. Then sum up the products of the weights and of the order statistics of squared residuals and the sum denote $S(\hat{\beta}_{present})$.

Is $S(\hat{\beta}_{present})$ less than $S(\hat{\beta}_{past})$?

no → B

yes

Establish *new* $\hat{\beta}_{present}$ just applying *WLS* on the reordered observations (reoredered according to the squared residuals).

An algorith for LWS

LWS - ALGORITHM(*continued*)

B

Was ℓ-times found the same model with minimal value of $S(\beta)$?

yes

no

Was already k-times repeated outer cycle ?

no

A

yes

As $\hat{\beta}^{(LTS,n,w)}$ we will assume $\beta \in R^p$ for which the functional $S(\beta)$ attained - through just described iterations - minimal value.

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