INSTITUTE OF ECONOMIC STUDIES, FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE (*established 1348*)

ROBUST STATISTICS AND ECONOMETRICS

INSTITUTE OF ECONOMIC STUDIES
FACULTY OF SOCIAL SCIENCES
CHARLES UNIVERSITY IN PRAGUE

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Week 13

Content of lecture

1. Summarizing what we have done.
2. Should be the common sense employed for creating scientific theories ?
   - What is a “common sense” ?
   - Can math say something about a “common sense” ?
3. What about a bit more general conjecture ?
   - Is the mathematics tool or a basic idea of ... ?

How were the robust statistics and econometrics built up?





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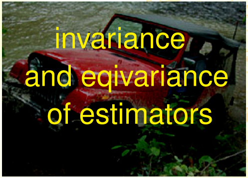

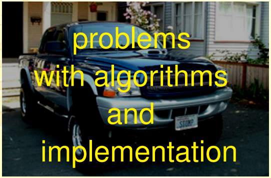

How were the robust statistics and econometrics built up?

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ESTIMATE OF REGRESSION COEFFICIENTS BY THE MODERN ROBUST ESTIMATORS

(assignable breakdown point,
equivariant in scale as well as in regression,
reliable algorithm and implementation)

M-estimators for the regression framework.

The solution of the extremal problem

$$\hat{\beta}^{(M,n)} = \arg\min_{\beta \in R^p} \sum_{i=1}^{n} \rho\,(Y_i - X_i'\beta)$$

is called

Maximum likelihood-like estimators of the regression coefficients

or M-estimators of β^0, for short.

We usually adopt some basic assumptions:

Let $F(x,r), x \in R^p, r \in R$ be a d.f. (with a density $f(x,r)$) governing the explanatory variables and disturbances in the regression model.

Evidently this form of definition inevitably implies that $\hat{\beta}^{(M,n)}$ is not scale- and regression-equivariant.

An advantage - on the other hand -

an easy computation of a solution, see the next slide.

Computing M-estimate of regression coefficients

Consider the extremal problem

$$\hat{\beta}^{(M,n)} = \arg\min_{\beta \in R^p} \sum_{i=1}^{n} \rho\left(Y_i - X_i'\beta\right).$$

Write it as

$$\hat{\beta}^{(M,n)} = \arg\min_{\beta \in R^p} \sum_{\{i: Y_i - X_i'\beta \neq 0\}}^{n} \frac{\rho\left(Y_i - X_i'\beta\right)}{\left(Y_i - X_i'\beta\right)^2} \left(Y_i - X_i'\beta\right)^2$$

$$= \arg\min_{\beta \in R^p} \sum_{i=1}^{n} w_i \cdot \left(Y_i - X_i'\beta\right)^2$$

where $w_i = \rho\left(Y_i - X_i'\beta\right) / \left(Y_i - X_i'\beta\right)^2$ if $Y_i - X_i'\beta \neq 0$, $w_i = 0$ otherwise.

Then

$$\hat{\beta}^{(M,n)} = \left(X'WX\right)^{-1} X'WY$$

where $W = \mathrm{diag}(w_1, w_2, ..., w_n)$.

And iterative computations, starting with a preliminary “guess” of the estimates of regression coefficients, find an approximation.

Hampel's approach - characteristics of the functional T at the d. f. F

- An overall characteristic of the functional (the estimator) is

$$\varepsilon^* = \sup\Big\{\varepsilon \le 1 : \exists K_\varepsilon \subset \Theta, \quad K_\varepsilon \ \ \textit{compact}$$

$$\pi(F, G) < \varepsilon \ \Rightarrow \ G(\{T_n \in K_\varepsilon\}) \underset{n\to\infty}{\longrightarrow} 1\Big\}$$

where $\pi(F, G)$ is the *Prokhorov metric* of $F(x)$ and $G(x)$ and T_n is an empirical counterpart to the functional T.

- ε^* is called *breakdown point*.

A pursuit for highly robust estimator of regression coefficients

Let's recall:

- *Breakdown point* - "finite" sample version

$$x_1, x_2, \ldots, x_n \quad \Rightarrow \quad T_n(x_1, x_2, \ldots, x_n)$$

- Find maximal m_n such that for any

$$y_1, y_2, \ldots, y_{m_n} \quad \Rightarrow \quad |T_n(x_1, x_2, \ldots, x_{n-m_n}, y_1, y_2, \ldots, y_{m_n})| < \infty$$

($0 < T_n(x_1, x_2, \ldots, x_{n-m_n}, y_1, y_2, \ldots, y_{m_n}) < \infty$ - for scale).

- Put

$$\varepsilon^* = \lim_{n \to \infty} \frac{m_n}{n}.$$

A pursuit for highly robust estimator of regression coefficients

Hampel's approach - characteristics of the functional T at the d. f. F

- *Breakdown point* - "finite" sample version - examples

$$x_1, x_2, ..., x_n \quad \Rightarrow \quad T_n(x_1, x_2, ..., x_n) = \frac{1}{n} \sum_{i=1}^{\infty} x_i.$$

- Maximal m_n such that for any
  $y_1, y_2, ..., y_{m_n} \quad \Rightarrow \quad |T_n(x_1, x_2, ..., x_{n-m_n}, y_1, y_2, ..., y_{m_n})| < \infty$
  is zero,
- hence

$$\varepsilon^* = 0.$$

A pursuit for highly robust estimator of regression coefficients

Hampel's approach - characteristics of the functional T at the d. f. F

- *Breakdown point* - "finite" sample version - examples

$$x_1, x_2, ..., x_n \quad \Rightarrow \quad T_n(x_1, x_2, ..., x_n) = med\{x_1, x_2, ..., x_n\}.$$

- Maximal m_n such that for any

$$y_1, y_2, ..., y_{m_n} \quad \Rightarrow \quad |T_n(x_1, x_2, ..., x_{n-m_n}, y_1, y_2, ..., y_{m_n})| < \infty$$

is $\frac{n}{2}$,

- hence

$$\varepsilon^* = \frac{1}{2}.$$

Common sense

What is a *common sense* ?

Let us consider some examples:

1. It is extremely unhealthy to sleep in boots. Every time when I wake up and .... .

2. A man walks over a garden, back and forth, and strikes by one stick to over stick. Another man askes him: *“What are you doing, Sir ?” “I frighten the elephants.”* ....

Common sense - can we say what it consists of ?

What is an *innumeracy* ?

1. Zeno’s paradoxes(*born approximately* 490 *BC*)

Achilles(*Discobolus*) runs 100m and the tortoise also runs but it starts at 99m. When Achilles arrives at 99m the tortoise is already about a half of meter next to the end of 100m. When Achilles arrives at 99.5m, the tortoise ...... (Zeno’s race course - Lecture notes from the University of Washington)

2. An information in the journal SAN FRANCISCO MONITOR (1995):

Every year since 1950, the number of American children gunned down has doubled.

Compute with me:

Common sense - a vague expression for ...

Illiteracy - innumeracy

1. 1950 - 1 child
2. 1951 - 2 children, 1952 - 4 children, 1953 - 8 children, 1960 - 1024 children, etc.
3. 1970 - more than 10^6 children
4. 1980 - more than 10^9 children (more than 4x total population of US)
5. 1983 - about 8.6×10^9 children ($2 \times$ more than total Earth's population)
6. 1987 - about 137×10^9 children (more than total population of Earth through all history = cca 110×10^9)
7. 1995 - about 35×10^{12} children (number encountered rarely outside the astronomy)

Can we rely on common sense?

Innumneracy - undersyanding ?

1. Original information of the Children’s Defense Fund (in 1995):

   Since 1950, the number of American children gunned down every year has doubled.

   (But in San Francisco Monitor it was written:
   Every year since 1950, the number of American children gunned down has doubled.)

2. This is example, how much the order of words is important even in everyday-life-sentence.

3. You have already learnt in mathematics that in definitions the order of words is (usually) crucial.

Can we rely on common sense?

What is an *understanding* ?

Conclusion:

1. It is difficult to say what is understanding.
2. Nevertheless there are different levels of understanding requiring different level of abstraction (or fantasy ?).
3. There is something, say *common sense (zdravý rozum)*, which is to be “satisfied”, if we want to claim that *something is an explanation of something*.
4. If somebody says that he/she understands to something, he/she is to be able to create an explanation in similar situations (of course, we never know it surely).

The *common sense* - is it reliable or not?
Let us consider some examples of development of it:

1. Very first Greek philosophers don’t hesitate that the Earth is a plate and .... .
2. Ptolemaios (100 - po 170)
   - Earth is a space body but fixed (center of Space)
   and he created geocentric (planet) system.
   He observed that the trajectories of planets are not cirles, so .... .
3. Nicolas Copernikus (1473 - 1543)
   - heliocentric system with circle trajectories of planet.
4. Johanes Kepler (1571 - 1630; assistant of Tycho de Brahe, 1571 - 1630)
   - elliptic trajectories of planets.

The *common sense* (continued)

Let us consider some examples:

1. Giordano Bruno (1547 - 1600) - non-heliocentric system of the Space.
2. Galileo Galilei (1564 - 1642)
   - guessed that the Earth is rotating but he can’t to prove it.
3. Rodrigo de Arriaga (1592 - 1667)
   - ideas of Arabic scientist brought 1625 to Prague
   (“to see Prague, to listen to Arriaga and die”).
4. Bernard Bolzano (5.10.1781 - 18.12.1848)
   - two century latter .... .

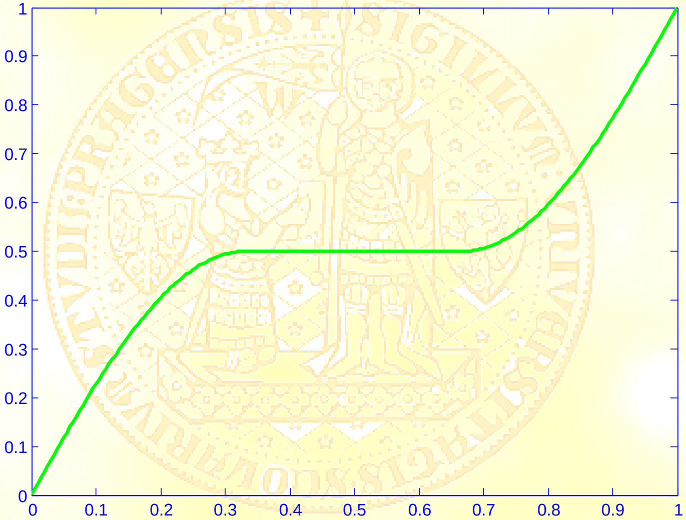
In fact:

“Figures may not lie, but liars figure.”

Joel Best: *Damned Lies and Statistics*,
University of California Press,
Berkeley, 2001.

1. $F(x) : [0,1] \to [0,1]$
2. $F(0) = 0, \quad F(1) = 1$
3. Continuous, non-decreasing
4. Constant somewhere

It is not difficult to imagine that it is *possible* !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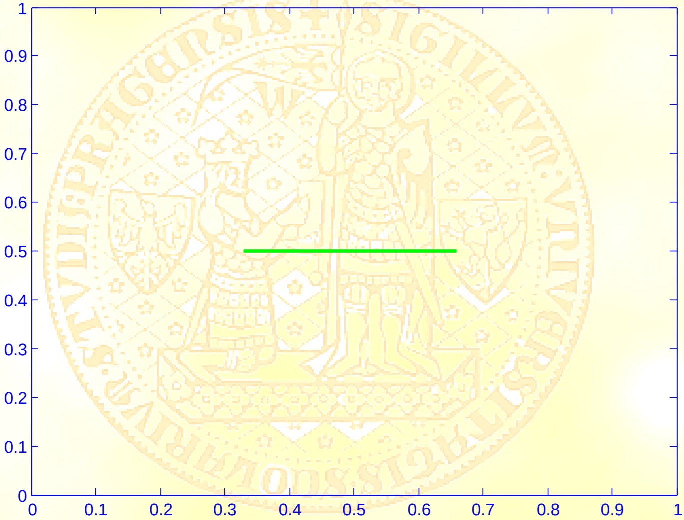
1. $F(x) : [0,1] \to [0,1]$
2. $F(0) = 0, \quad F(1) = 1$
3. Continuous, non-decreasing
4. Constant on intervals of common length 1
   (i.e. $F(x)$ is almost everywhere constant.)

The common sense says that something like it is not *possible* !

By the way, Sir Karl Popper said:

If we can’t imagine something in an alternative way
than we are used to it, it is not the proof that it is impossible,
but it is a lack of our fantasy !

So, let us demonstrate that in the case of just required function
it is really a lack of fantasy!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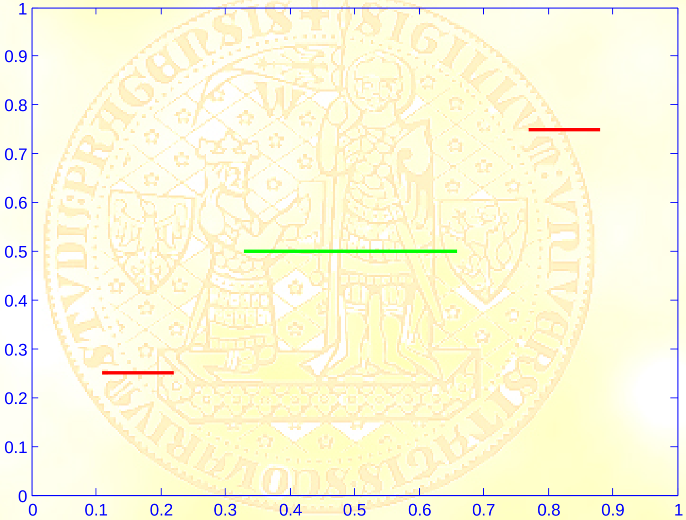
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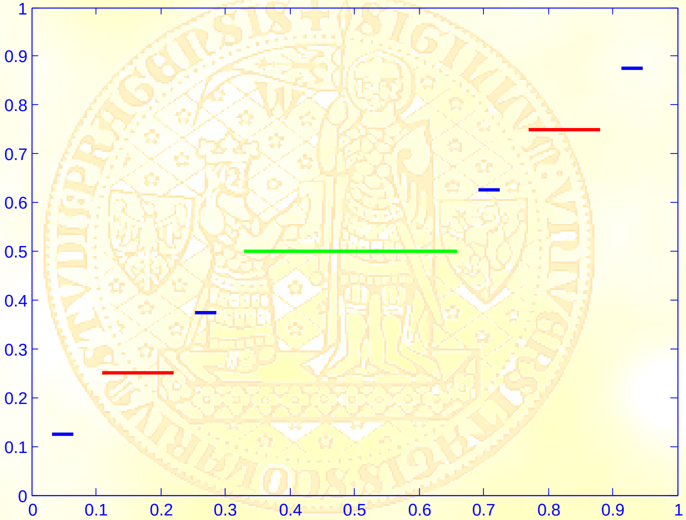
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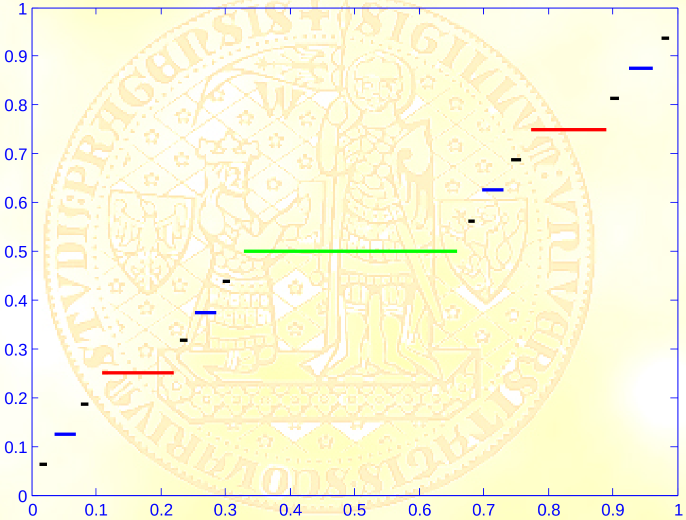
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And we may continue, the function is completed after infinite (countable) number of steps.

Conclusion:

1. Common sense is a collection of preconceptions, prejudices (against something), legends, common beliefs, scientific folklore, consequences of dogmas, etc.

   (Vladimir Iljic Uljanov (Lenin) 1870 - 1924)

and hence

when we want to build up a correct theory
(of anything, e. g. statistics, econometrics, macro or micro)

we cannot rely on common sense, intuition etc. *but on formal logic.*

Thales of Miletus (624 - 546 BC):

“We can prove anything, especially if we rely on words.”

Is mathematics only a tool or .... ?

The mathematics is assumed to be language:

"While it has many features of a language - mainly notation that looks like Egyptian hieroglyphics to nonmathematicians - there must be more than mere language going on. We still have to account for amazing success of mathematics as a description of physical reality. The precision of so many theories of physical reality may hint at a deeper truth, that mathematics is a major structural foundation of our universe."

Alexander Keewatin Dewdney: *Beyond Reason*.
(professor of computer science at the University of Western Ontario

What about a main feature of science ?

Generality

Scientific knowledge is about generality. One could say that the more general a successful theory is, the more "scientific" it has become. E.g. in biology the two greatest discovery - the Darwin-Wallace theory of evolution and the Watson-Crick discovery of DNA - are its the most general and most mathematical parts. And of course they are the most important.

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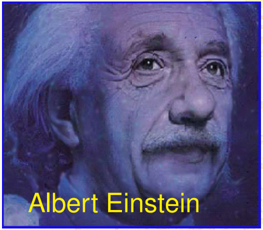
Why is a man admired by people?



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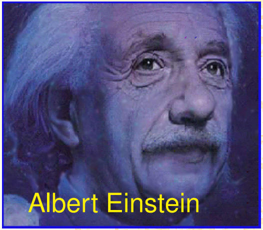

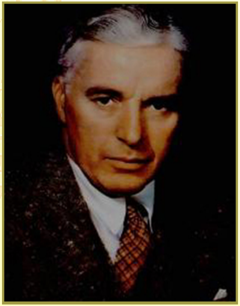

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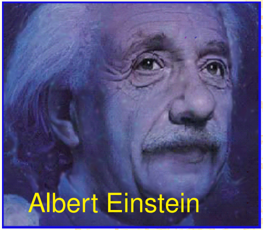

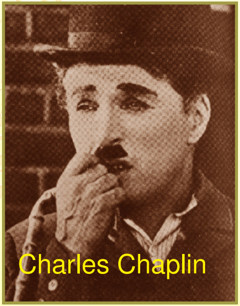

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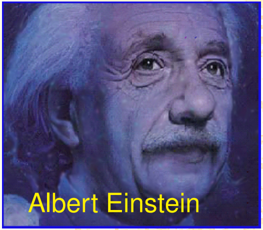
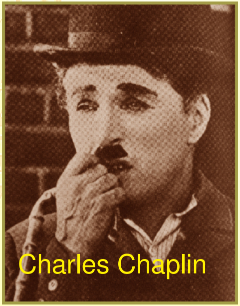


Dear friend,
I'm fascinated
as the people understand You
and as they admire You.



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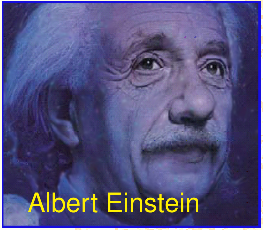

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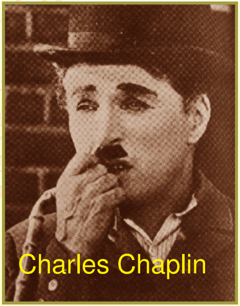
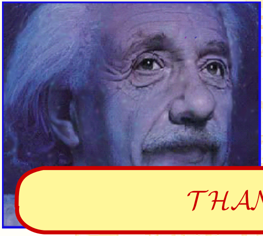


Dear friend,
I'm fascinated
that although the people don't
understand You at all,
they admire You.

Why is a man admired by people?



Dear friend,
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as the people understand You
and as they admire You.

THANKS FOR ATTENTION

Dear friend,
I'm fascinated
that although the people don't
understand You at all,
they admire You.