Task 1:
Consider the relation schema R(A, B, C, D, E, F) and the following three FDs:

FD1: {A} → {B, C}
FD2: {C} → {A, D}
FD3: {D, E} → {F}

Use the Armstrong rules to derive each of the following two FDs. In both cases, describe the derivation process step by step (i.e., which rule did you apply to which FDs).

a) {C} → {B}   b) {A, E} → {F}

(a)
FD4: {C} → {A} (Decomposition of FD2)
FD5: {A} → {B} (Decomposition of FD1)
FD6: {C} → {B} (Transitivity of FD4 and FD5)

(b)
FD7: {A, E} → {B, C, E} (Augmentation of FD1 with E)
FD8: {A, E} → {C} (Decomposition of FD7)
FD9: {A, E} → {A, D} (Transitivity of FD8 and FD2)
FD10: {A, E} → {A, D, E} (Augmentation of FD9 with E)
FD11: {A, E} → {D, E} (Decomposition of FD10)
FD12: {A, E} → {F} (Transitivity of FD11 and FD3)

Task 2:
For the aforementioned relation schema with its functional dependencies, compute the attribute closure X+ for each of the following two sets of attributes.

a) X = {A} b) X = {C, E}

(a)
X+ = {A, B, C, D}

(b)
X+ = {C, E, A, D, F, B}

Task 3

Consider the relation schema R(A, B, C, D, E, F) with the following FDs

FD1: {A, B} → {C, D, E, F}

FD2: {E} → {F}

FD3: {D} → {B}

a) Determine the candidate key(s) for R.
b) Note that R is not in BCNF. Which FD(s) violate the BCNF condition?
c) Decompose R into a set of BCNF relations, and describe the process step by step (don't forget to determine the FDs and the candidate key(s) for all of the relation schemas along the way).

(a)
Assumptions:

- A must be part of each CK (does **not** exist in RHS)
- C and F cannot be part of any CK (exist in RHS but **not** in LHS)

{A}+ = {A}

A itself cannot be a CK. We look further for sets of two attributes in which we have A included but not C and F:
{A, B}+ = {A, B, C, D, E, F} so it is a CK
{A, D}+ = {A, D, B, C, E, F} so it is a CK
{A, E}+ = {A, E, F}

CKs : {A, B} and {A, D}

(b)

FD2 and FD3

(c)

We choose FD2 which violates the BCNF and create a smaller relation named R1 as follows:

R1(E, F) is associated with FD2 and CK: {E}

The other smaller relation named R2 should contain all original attributes except those which appear in RHS of FD2 namely F.

R2(A, B, C, D, E) is associated with FD3 and FD4: {A, B} → {C, D, E} which is decomposed by FD1.

Computing CK(s):
Assumptions:

- A must be part of each CK (does **not** exist in RHS)
- C and E cannot be part of any CK (exist in RHS but **not** in LHS)

{A}+ = {A} is not a super key and thus not a CK either.

{A, B}+ = {A, B, C, D, E } hence this is a CK.

{A, D}+ = {A, D, B, C, E} hence this is a CK.

Hence CKs : {A, B}, {A, D}

Now R1 is in BCNF but R2 is not in BCNF because of FD3. Attribute D itself is not a superkey for relation R2. So we need to decompose it again and create two smaller relations R2A and R2B same as last one:

R2A(D, B) is associated with FD3. CK: {D}

R2B(A, C, D, E) only with trivial FDs, CK: {A, C, D, E}

Task 4

Consider the relation schema R(A, B, C, D, E) with the following FDs

FD1: {A, B, C} → {D, E}

FD2: {B, C, D} → {A, E}

FD3: {C} → {D}

a) Show that R is not in BCNF.
b) Decompose R into a set of BCNF relations (describe the process step by step).

(a)

To show that R is not in BCNF we need to find CK(s) of R.

Assumption:

- B and C must be part of each CK (does **not** exist in RHS)
- E cannot be part of any CK (exist in RHS but **not** in LHS)

{B}+ = {B}

{C}+ = {C, D}

B and C cannot be CK because their closure does not include all attributes of R.

{B, C}+ = {B, C, D, A, E} so it is a CK

Hence {A, B, C} and {B, C, D} are superkeys because their closure contains all attributes of R and therefore only FD3 violates the BCNF because {C} itself is not a superkey.

(b)

We decompose R into two smaller relations named R1 and R2 based on FD3 as follows:

R1(C, D) is associated with FD3. CK:{C}

For R2 we derive the following FDs:

FD4: {A,B,C} → {E} (Decomposition of FD1 with D)

FD5: {B, C} →{ D, B, C} (Augmentation rule of FD3 with {B,C})

FD6: {B, C} →{A, E} (Transitivity rule of FD2 and FD5)

R2(A, B, C, E) is associated with derived FD4 and FD6. CK: {A, B, C}

Now we can compute the CK(s):

Assumption:

- B and C must be part of each CK (does **not** exist in RHS)
- E cannot be part of any CK (exist in RHS but **not** in LHS)

FD4: {A,B,C} → {E}

FD6: {B, C} →{A, E}

{B, C}+ = {B, C, A, E} hence this is a Candidate key

{A, B, C}+ = {A, B, C, E} hence this is a super key

Now both R1 and R2 are in BCNF so we stop the process of decomposition here.