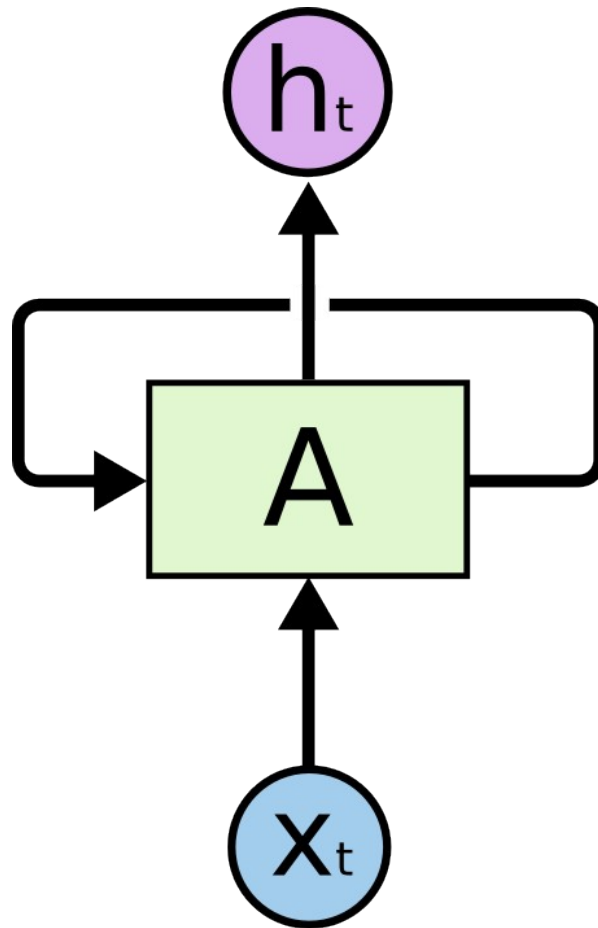
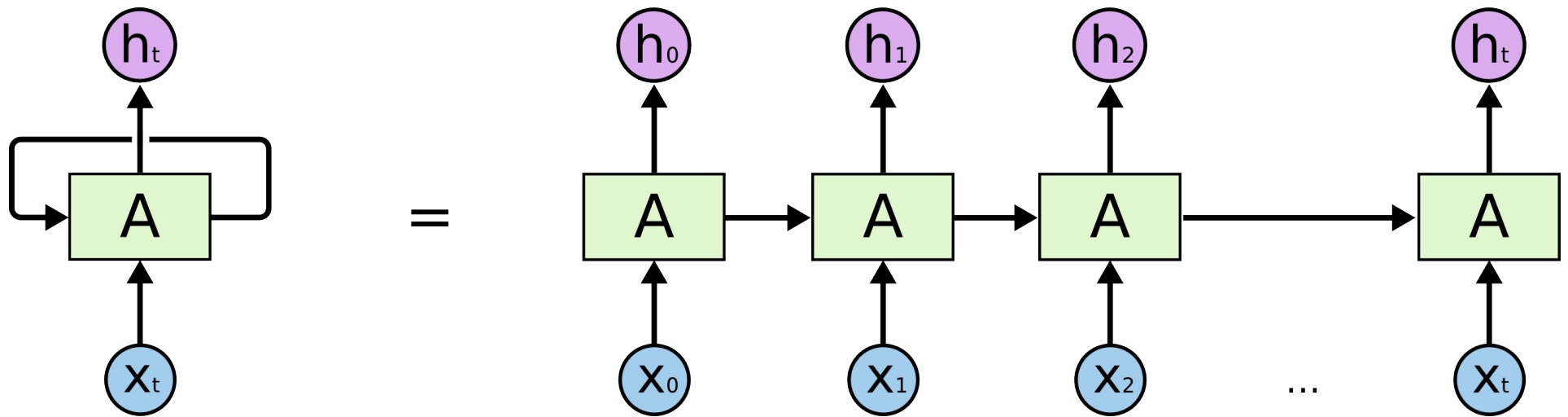


Understanding LSTM Networks

Recurrent Neural Networks



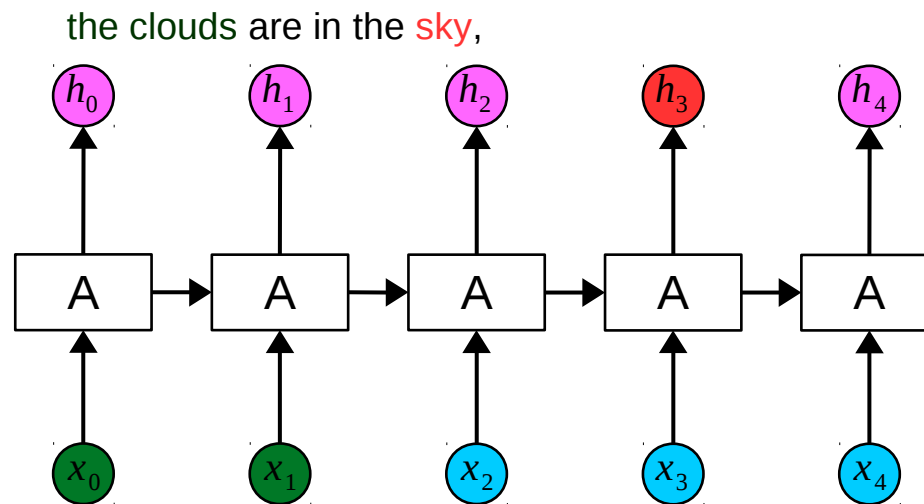
An unrolled recurrent neural network



The Problem of Long-Term Dependencies

RNN short-term dependencies

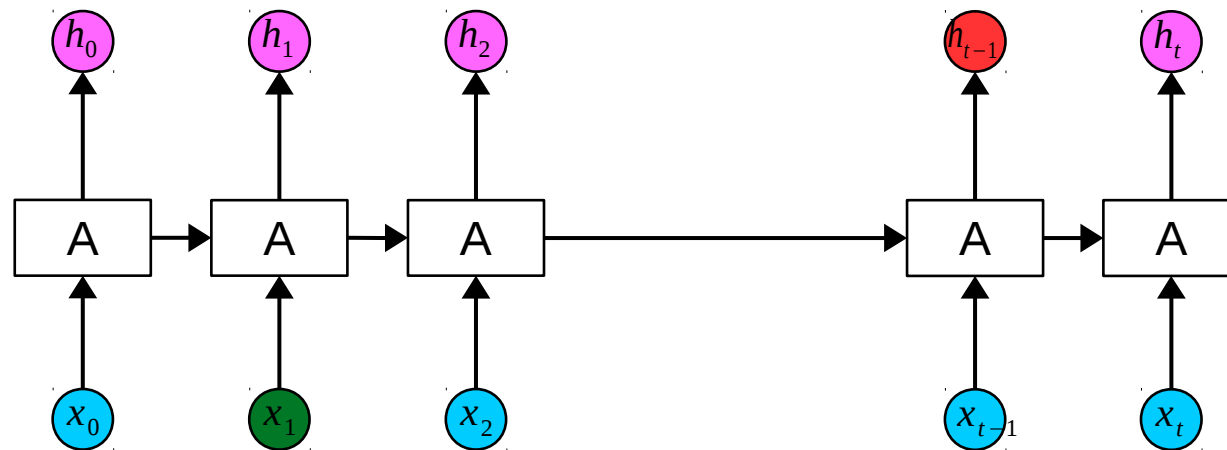
Language model trying to predict the next word based on the previous ones



RNN long-term dependencies

Language model trying to predict the next word based on the previous ones

I grew up in India... I speak fluent Hindi.



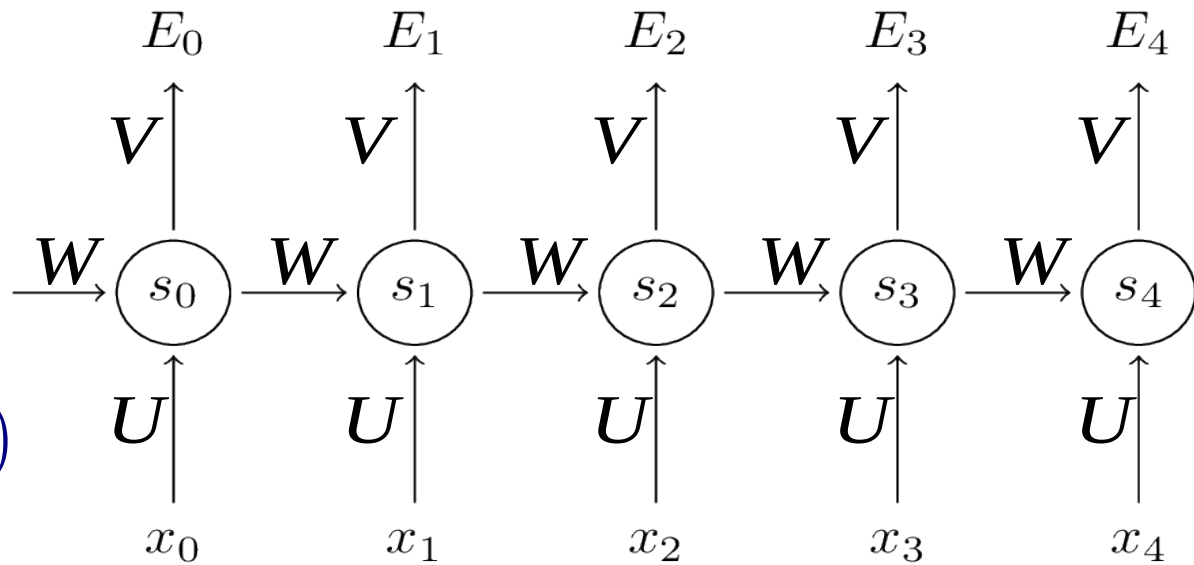
Backpropagation Through Time (BPTT)

RNN forward pass

$$s_t = \tanh(Ux_t + Ws_{t-1})$$

$$\hat{y}_t = \text{softmax}(Vs_t)$$

$$E(y, \hat{y}) = \sum_t E_t(y_t, \hat{y}_t)$$



Backpropagation Through Time

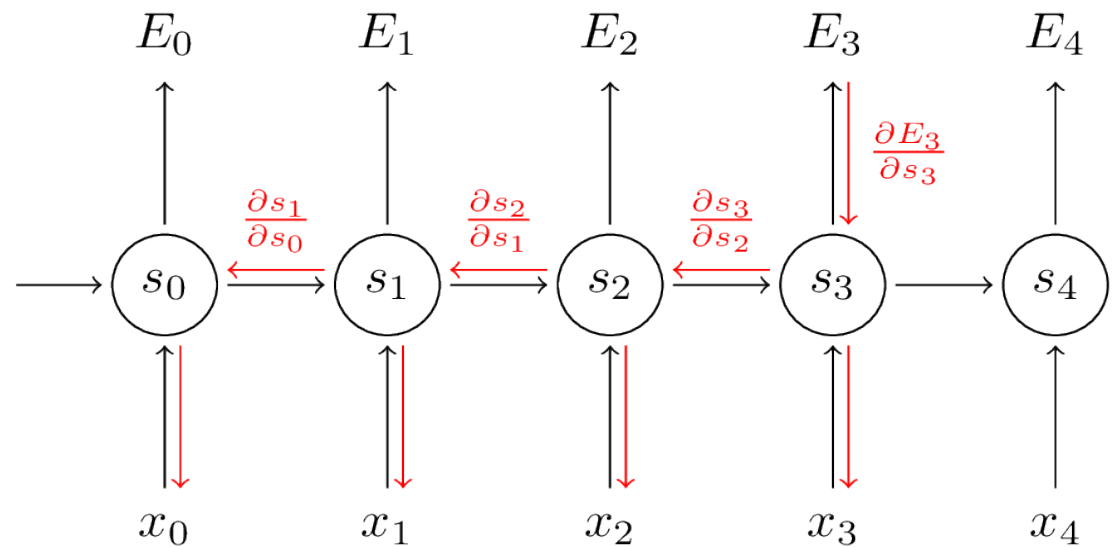
$$\frac{\partial E}{\partial \mathbf{W}} = \sum_t \frac{\partial E_t}{\partial \mathbf{W}}$$

$$\frac{\partial E_3}{\partial \mathbf{W}} = \frac{\partial E_3}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial s_3} \frac{\partial s_3}{\partial \mathbf{W}}$$

But $s_3 = \tanh(Ux_t + Ws_2)$

S₃ depends on s₂, which depends on W and s₁, and so on.

$$\frac{\partial E_3}{\partial \mathbf{W}} = \sum_{k=0}^3 \frac{\partial E_3}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial s_3} \frac{\partial s_3}{\partial s_k} \frac{\partial s_k}{\partial \mathbf{W}}$$



The Vanishing Gradient Problem

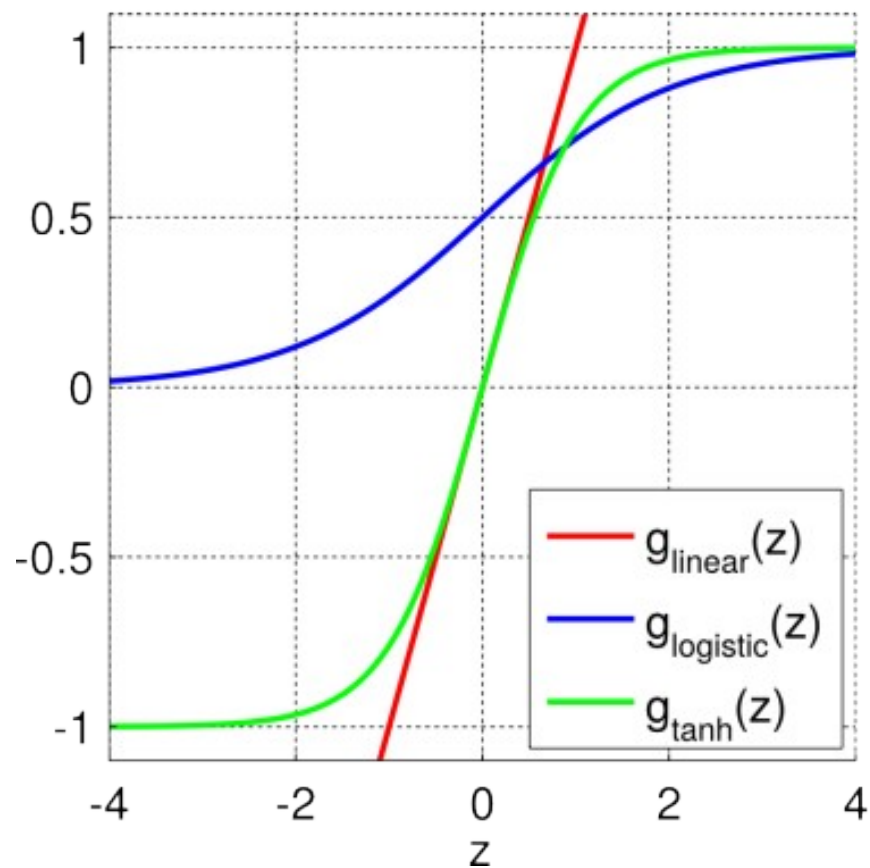
$$\frac{\partial E_3}{\partial \mathbf{W}} = \sum_{k=0}^3 \frac{\partial E_3}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial s_3} \frac{\partial s_3}{\partial s_k} \frac{\partial s_k}{\partial \mathbf{W}}$$

$$\frac{\partial E_3}{\partial \mathbf{W}} = \sum_{k=0}^3 \frac{\partial E_3}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial s_3} \left(\prod_{j=k+1}^3 \frac{\partial s_j}{\partial s_{j-1}} \right) \frac{\partial s_k}{\partial \mathbf{W}}$$

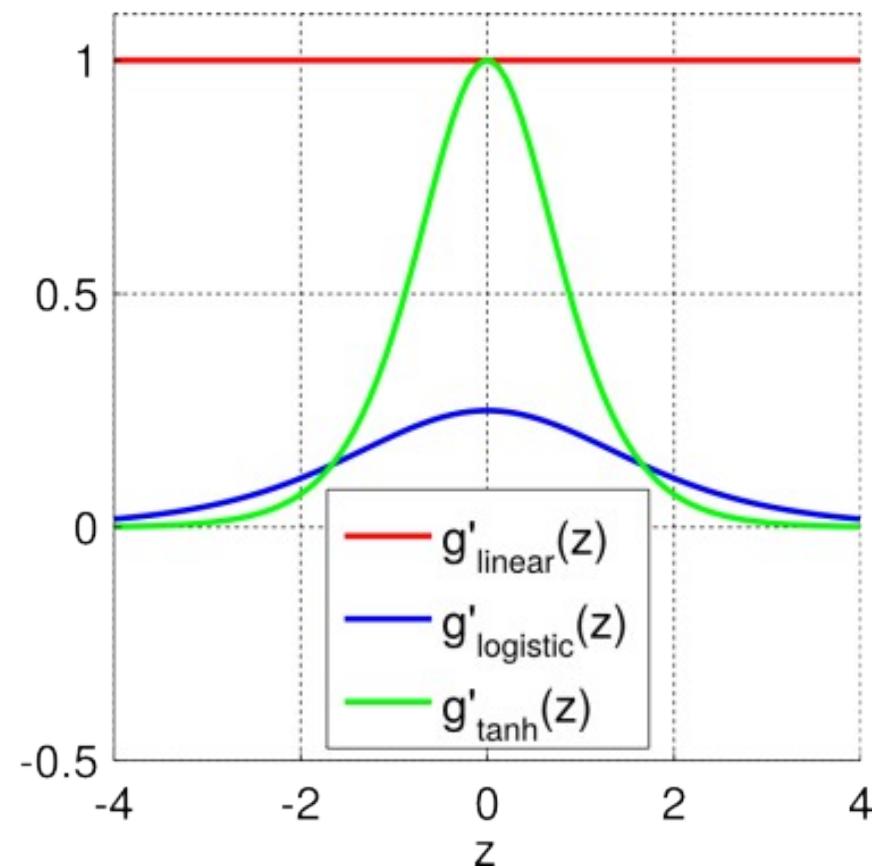
- Derivative of a vector w.r.t a vector is a matrix called jacobian
- 2-norm of the above Jacobian matrix has an upper bound of 1
- **tanh** maps all values into a range between -1 and 1, and the **derivative is bounded by 1**
- With multiple matrix multiplications, gradient values **shrink exponentially**
- Gradient contributions from “far away” steps become zero
- Depending on activation functions and network parameters, gradients could **explode** instead of vanishing

Activation function

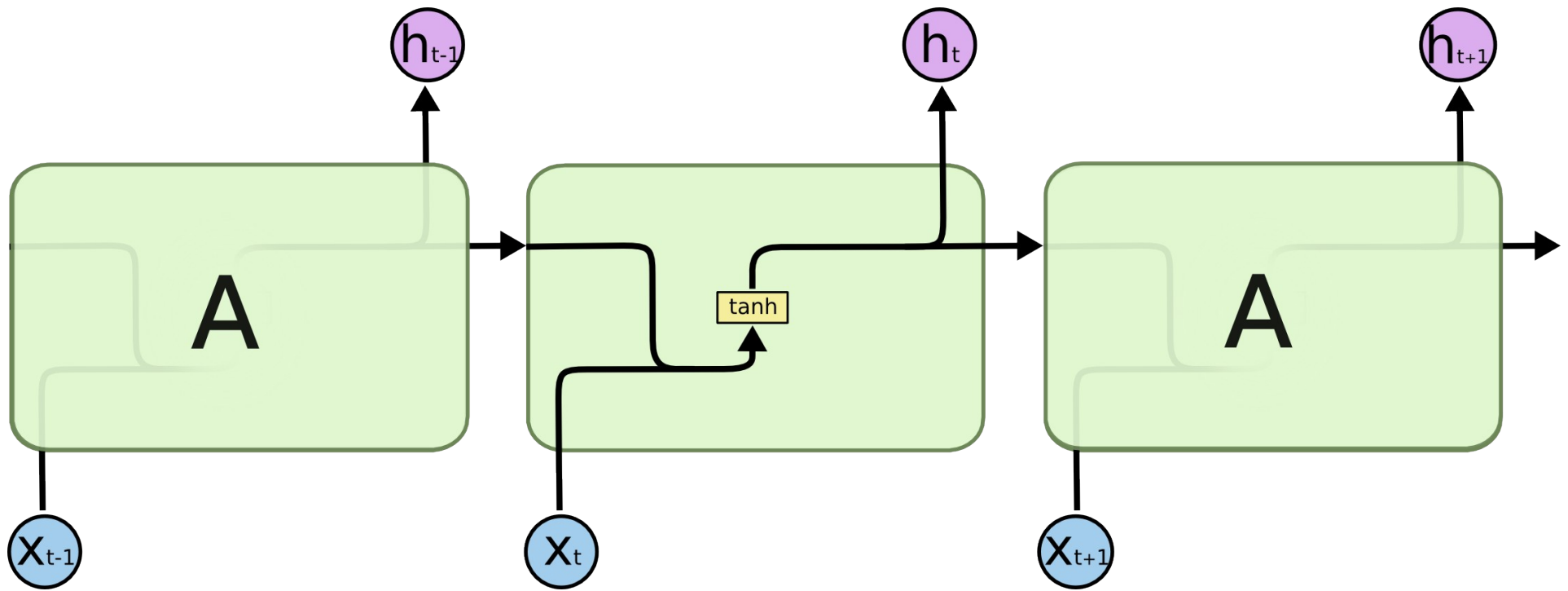
Some Common Activation Functions



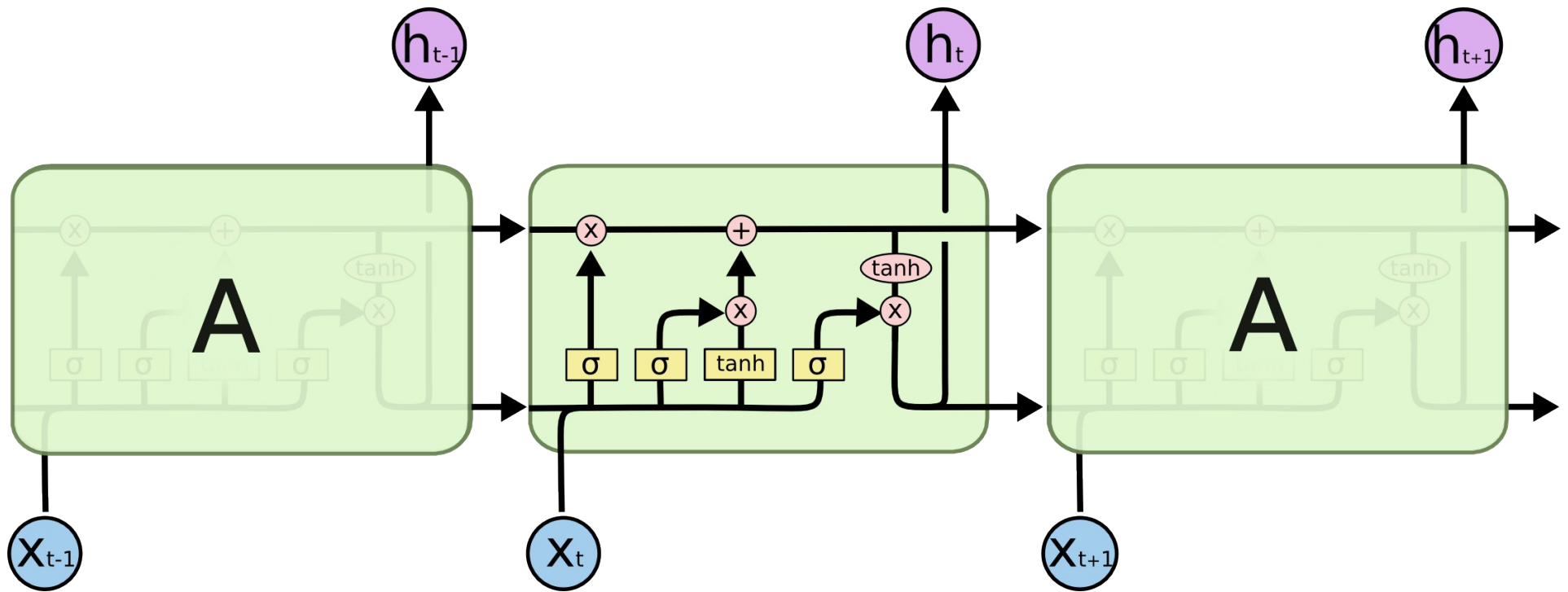
Activation Function Derivatives



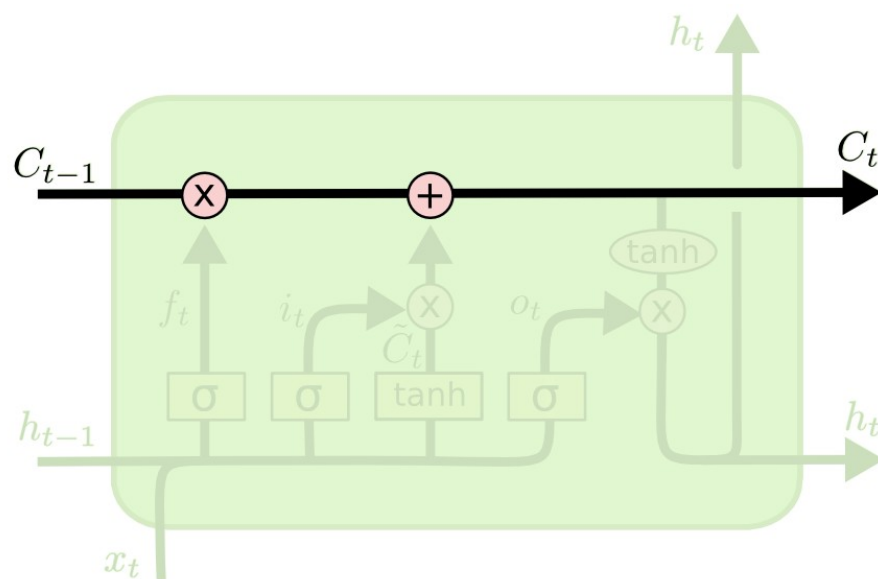
Standard RNN



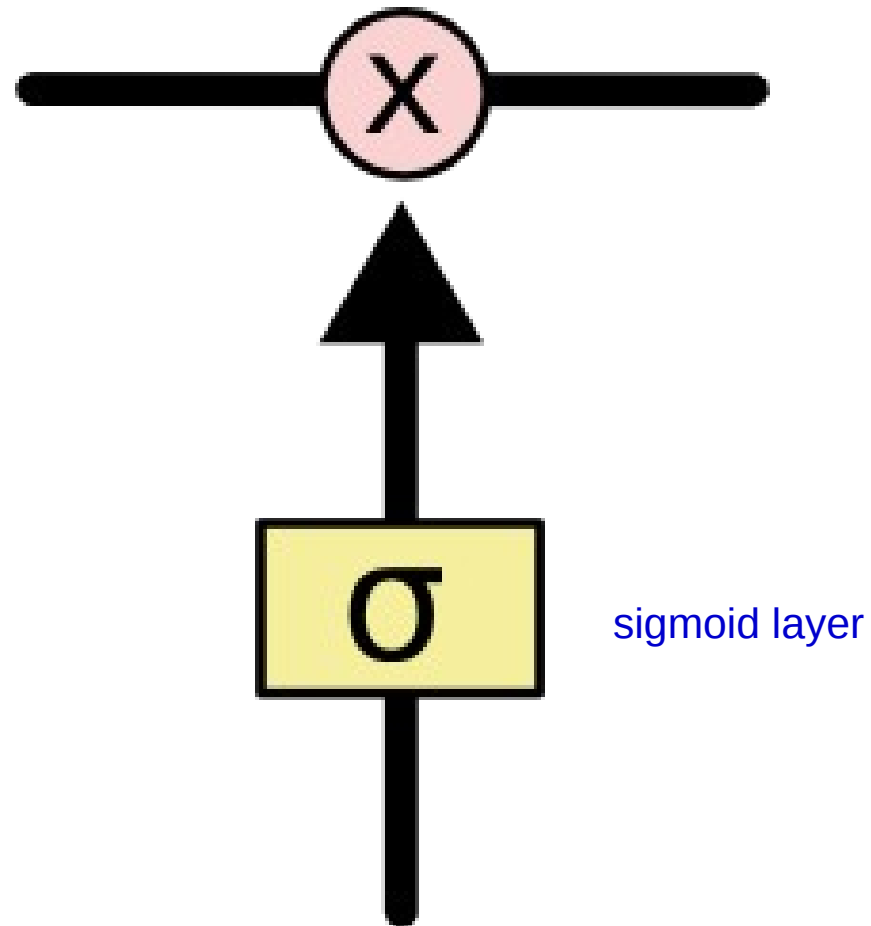
LSTM with four interacting layers



The cell state

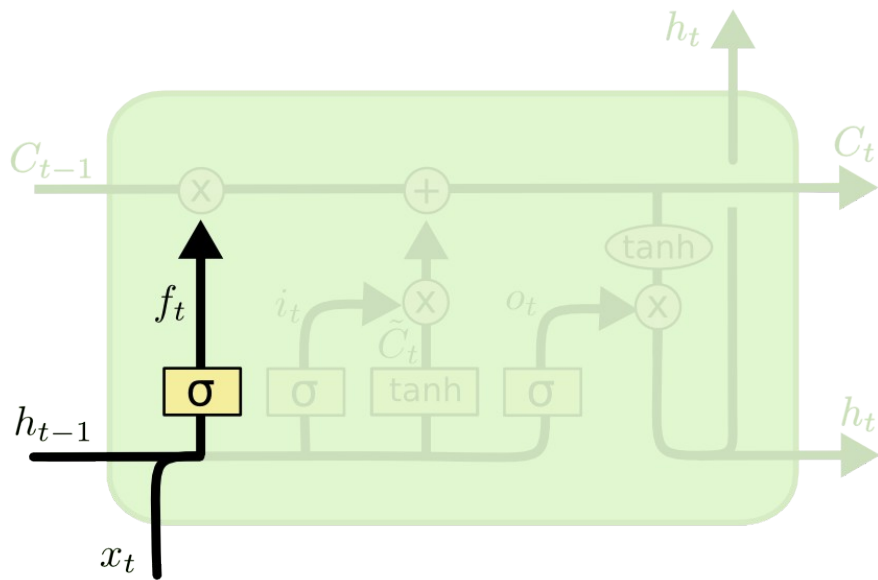


Gates



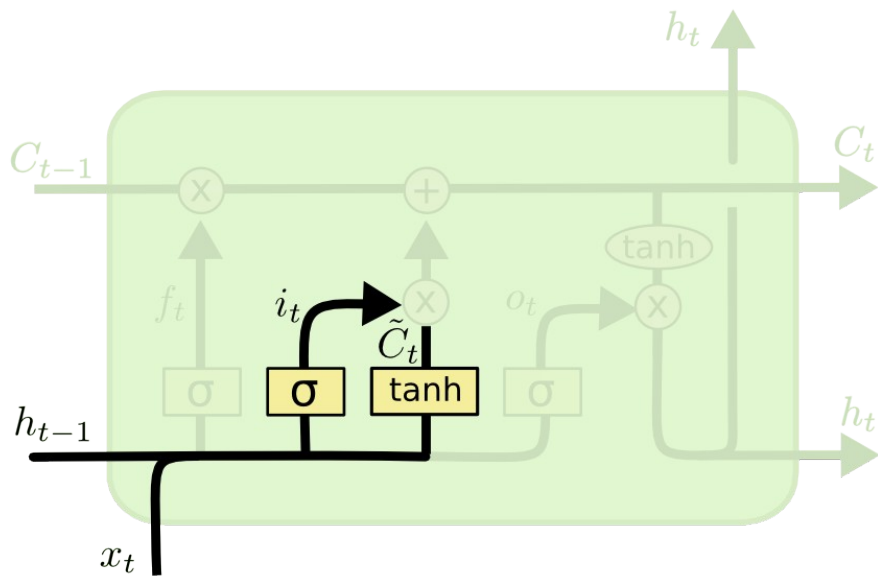
Step-by-Step LSTM Walk Through

Forget gate layer



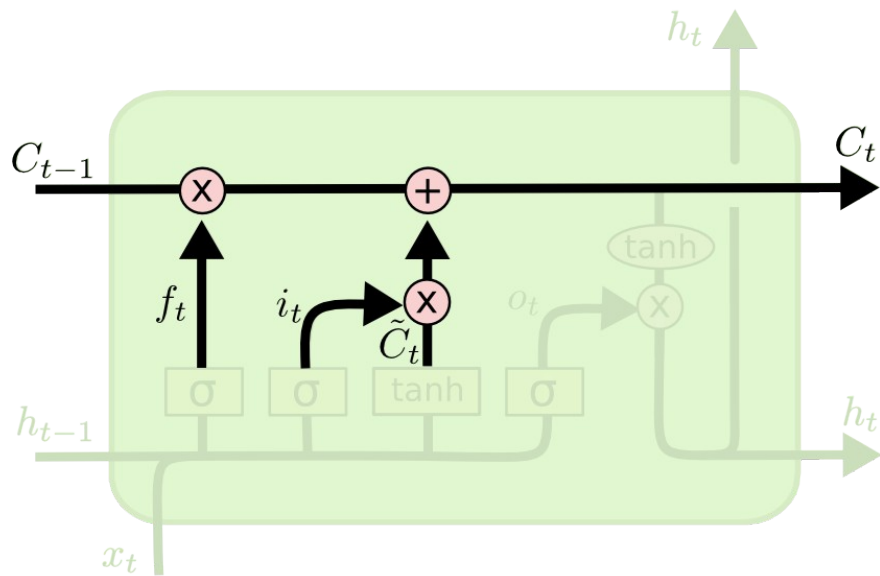
$$f_t = \sigma (W_f \cdot [h_{t-1}, x_t] + b_f)$$

Input gate layer



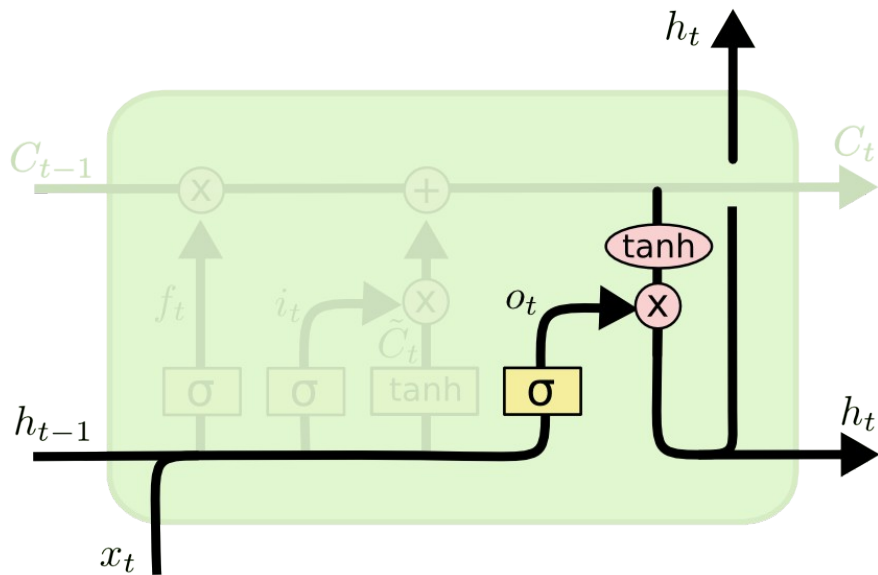
$$i_t = \sigma (W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

The current state



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Output layer



$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

Reference

- <http://colah.github.io/posts/2015-08-Understanding-LSTMs/>
- <http://www.wildml.com/>
- <http://nikhilbuduma.com/2015/01/11/a-deep-dive-into-recurrent-neural-networks/>
- <http://deeplearning.net/tutorial/lstm.html>
- <https://theclevermachine.files.wordpress.com/2014/09/act-funs.png>
- <http://blog terminal.com/demistifying-long-short-term-memory-lstm-recurrent-neural-networks/>
- A Critical Review of Recurrent Neural Networks for Sequence Learning, Zachary C. Lipton, John Berkowitz
- Long Short-Term Memory, Hochreiter, Sepp and Schmidhuber, Jurgen, 1997
- Gers, F. A.; Schmidhuber, J. & Cummins, F. A. (2000), 'Learning to Forget: Continual Prediction with LSTM.', Neural Computation 12 (10) , 2451-2471 .