MA1691 - Summer Project 2020

Student number - 1923114

Task 1(b)

The penultimate digit of the student number is u=1 which is an odd number, therefore we will work with Network 1. The first thing that needs to be done is to construct the adjacency matrix A.

$$A=\begin{pmatrix}0&0&1&0&0&1\\1&0&1&0&0&0\\0&1&0&1&0&0\\0&0&0&0&0&0\\0&0&1&1&0&1\\1&1&1&1&1&0\end{pmatrix}$$

Now that we the adjacency matrix we can modify the script direct1.m so that it produces the vector of limiting probabilities for Network 1. Since the script is written in a way that it works for matrices of any size, the only thing that needs to be changed in the script is the matrix A. Once we have the adjacency matrix for Network 1 in the script `task1a.m` we can run it to obtain the vector of limiting probabilities p.

$$p=\begin{pmatrix}0.2121\\0.1212\\0.0606\\-0.0000\\0.2121\\0.3939\end{pmatrix}$$

Based on these probabilities the node that is first in rank is node 6 (Matrices) which has a limiting probability 0.3939. If we look at Network 1 we realize that this is no surprise since all other topics have a dependency on Matrices.

The next 2 ranking nodes are node 1 (Determinants) and node 5 (Vectors). Since the two nodes have identical limiting probabilities of value 0.2121, I arbitrarily chose node 1 to be second in rank and node 5 to be third in rank. Looking again at Network 1 we see that the ranking of these nodes is not as intuitive as for node 6. They have a higher probability then nodes 2, 3 and 4 because node 6 which is highest in rank has a dependency on them. What is less obvious is why they have equal probabilities since the in-degree of node 5 is 3, while the in-degree of node 1 is 2. The reason for this is that the dependency of node 4 on node 5 does not add to the probability of node 5, since the limiting probability of node 4 is 0.

Task 2 (b)

As instructed I added 4 more nodes to Network 1 as well as 14 more edges. The resulting network can be found the file Network-Task2(a).pdf. In the network, the nodes and edges that are coloured black were already part of Network 1 and the ones that are coloured red are the ones that I added.

The adjacency matrix A was also constructed and can be seen below.

Node 7 has dependencies on nodes 2, 3 and 6 because eigenvalues, systems of linear equations and vectors can be used to solve systems of differential equations.

$$A = \begin{pmatrix} 0&0&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&1&1&0&1 \\ 0&1&0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&0&1&0&1&1&1 \\ 1&1&1&1&1&0&1&1&0&1 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \end{pmatrix}$$

Between nodes 8 and 5 have dependencies on each other because Hilbert spaces are vector spaces, which make use of vector but can also be used to solve problems with vectors.
Node 8 has a dependency on node 2 because eigenvalues are used in some of the Hilbert spaces applications, like the Sturm–Liouville theory and Fourier analysis. The dependency of node 8 on node 6 is explained by the fact that the Hilbert space generalizes the spectral decomposition of a matrix.
Finally for node 8, it has a dependency of node 7 because differential calculus can be used in Hilbert spaces for some applications, an example is again the Sturm–Liouville theory.

$$p = \begin{pmatrix} 0.1168 \\ 0.1385 \\ 0.0764 \\ -0.0000 \\ 0.1707 \\ 0.2931 \\ 0.0213 \\ 0.0854 \\ -0.0000 \\ 0.0977 \end{pmatrix}$$

Node 9 only has a dependency on node 5 because vectors are often used in algebraic geometry.
Nodes 10 and 6 have a mutual dependency because matrix decomposition is part of the broader subject “Matrices”. Node 10 also has dependencies on nodes 2 and 5 primarily because of one type of matrix decomposition (eigendecomposition) which makes use of eigenvalues and eigenvectors.
The new matrix can now replace the matrix in the script `task1a.m` to create the script `task2c.m` and after running the new script get a new vector of limiting probabilities p. In the new network node 6 is still first in rank, but with a smaller limiting probability of 0.2931. However, the second and third ranking nodes have changed due to the addition of new dependencies.
The second-ranking node is now node 5 (Vectors) with a smaller limiting probability than before, which is now 0.1707.
The third-ranking node is node 2 (eigenvalues) which has a larger limiting probability than it had in Network 1, now being equal to 0.1385. The increase is due to the fact that three out of the four new nodes have dependencies on node 2.

$$p = \begin{pmatrix} 0.1053 \\ 0.1305 \\ 0.0879 \\ 0.0207 \\ 0.1677 \\ 0.2542 \\ 0.0380 \\ 0.0872 \\ 0.0207 \\ 0.0879 \end{pmatrix}$$

Task 3(b)
Based on my student number the value of v is 4, which makes α equal to 0.19. Running the script `task3a.m` with this value of α gives, as a result, a new vector of limiting probabilities p.
The first ranking node is still node 6, with a slightly smaller limiting probability of 0.2542. The second-ranking node is now node 5 with a limiting probability of

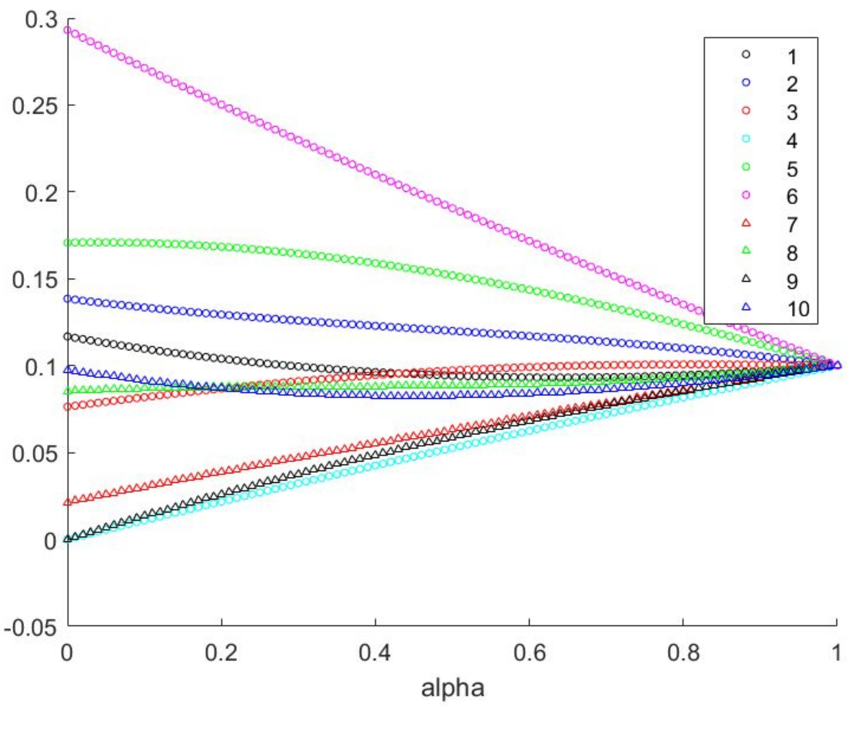
0.1677. The third-ranking node is node 2 with 0.1305 as the new value for the limiting probability.

All of the values in the vector of limiting probabilities have changed compared to the result in Task 2(c). The first thing that can be observed is that we no longer have any nodes with a limiting probability of 0. The second observation is that all of the larger values of limiting probabilities have decreased slightly, while the smaller values have increased.

Task 4(b)

In order to generate the plot, I needed to create a figure that will record all of the plotting commands. Once that was done I needed to calculate the vector of limiting probabilities with values of α ranging between 0 and 1. I chose to do this by using a for-loop using a variable i that takes values from 0 to 100 and dividing this value by 100 to get the value of α. For each vector of limiting probabilities that were calculated the values of the vector fere plotted in the figure using the plot command.

The resulting plot shows much more clearly what we have observed in Task 3(b), which is that the larger probabilities decrease and the smaller probabilities increase as the value of α increases. What is new is the fact that we can now see that all of the probabilities converge to the same value of 0.1 when the value of α becomes 1. This is because, as the value of α increases, the probability to randomly jump from one node to another increases and the impact of dependencies decreases. When α gets the value 1 there is an equal chance to jump from one node to any other node, so after a number of jumps, the surfer has an equal probability to be at any node, which is equal to 1 divided by the number of nodes.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