ECS 170 Problem Set #2 Hadi B

1) a) $h(n) \le h^*(n)$ where $h^*(n)$ is the true cost from n to the goal. Since each h_i is given to be admissible, we have $h_i(n) \le h^*(n)$ for all i. Taking the maximum over all h_i's: $h_{max}(n) = \max h_i(n) \le h^*(n)$ thus h_{max} is admissible b/c it does not exceed the true cost $h^*(n)$

b) $h_1(n) \ge h_2(n)$ for all n which means for every heuristic $h_i(n)$, we have $h_{max}(n) \ge h_i(n)$. Since h_{max} is always greater than or equal to each individual h_i, it dominates all the other heuristic in the ensemble

2) a) the misplaced tiles heuristic $h_{misplaced}(n)$ simply counts the number of tiles that are in the wrong position compared to the goal state. Since each misplaced tile must be moved at least once to its correct position, this heuristic never overestimates the number of moves needed

b) Each tile must be moved at least its manhattan distance to reach its goal position, and since tiles can only move one step at a time, this heuristic never overestimates the true cost, thus, $h_{manhattan}(n) \le h^*(n)$, proving that $h_{manhattan}$ is also admissible,

c) the misplaced tiles heuristic only counts the number of tiles that are in the wrong position but does not consider how far they are from their correct position. On the other hand, the manhattan distance heuristic provides a more informed estimate by summing the actual number of moves needed for each tile. Since $h_{manhattan}(n) \ge h_{misplaced}(n)$ for all n and is often strictly greater, it dominates the misplaced tiles heuristic, making it a more accurate and efficient heuristic in guiding search algorithms like A^*.

3) a) A heuristic is better if it provides a more accurate estimate of the actual cost

- The misplaced tiles heuristic only counts the number of tiles that are not in their correct position, w/o considering how far they are from the goal.
- the relaxed 8-puzzle heuristic allows any tile to swap with the blank space, meaning that the number of swaps required to reach the goal is considered.
- Since this heuristic considers the number of tile moves needed to reach the goal, it is at least as large as (and often larger than) the misplaced tiles heuristic

Thus this heuristic dominates the misplaced tiles heuristic b/c:

$h_{relaxed}(n) \geq h_{misplaced}(n)$ for all states n

b) Example initial state: $\begin{bmatrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & \_ & 5 \end{bmatrix}$

Manhattan distance: each tile's manhattan distance to its goal state is summed up this equals X

Relaxed puzzle: Since any tile can swap w/ the blank, the number of swaps required may be closer to the true cost, This equals Y and Y > X, giving a better estimate

c) get goal state
set misplaced tiles to []

identify misplaced tiles and append
swaps = 0

process misplaced tiles to estimate the minimum swaps needed
while misplaced tiles is not empty, popping when at correct position

return swaps