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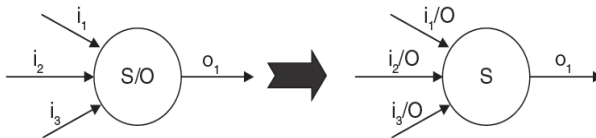


Fig. 3.31 *Process of Conversion of a Moore Machine to a Mealy*

Step III: Repeat these steps for each of the states. By this process, the Mealy machine-equivalent Moore machine is constructed. The process is described in Fig.3.31.

Consider the following examples.

Example 3.21 Convert the following Moore machine into an equivalent Mealy machine as given in Fig. 3.32.

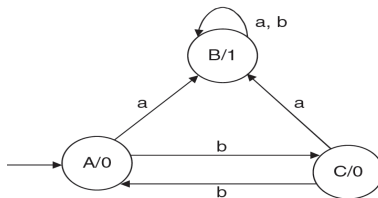


Fig. 3.32

Example 3.21 Convert the following Moore machine into an equivalent Mealy machine as given in Fig. 3.32.

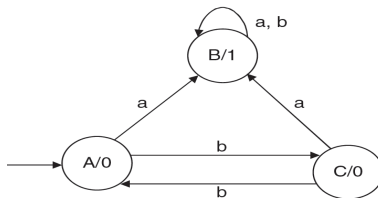


Fig. 3.32

Solution:

1. In this machine, A is the beginning state. So, start from A. For A, there are two incoming arcs, from C to A with input b and another in the form of the start-state indication with no input. State A is labelled with output 0. As the start-state indication contains no input, it is useless and, therefore, keep it as it is.

Modify the label of the incoming edge from C to B including the output of state A. So, the label of the incoming state is C to A with label $b/0$.

2. State B is labelled with output 1. The incoming edges to the state B are from A to B with input a, B to B with inputs a and b, and C to B with input a.

Modify the labels of the incoming edges including the output of state B. So, the labels of the incoming states become A to B with label $a/1$, B to B with labels $a/1$ and $b/1$, and C to B with label $a/1$.

3. State C is labelled with output 0. There is only one incoming edge to this state, from A to C with input b. The modified label is $b/0$.

The converted Mealy machine is given in Fig. 3.33.

2. State B is labelled with output 1. The incoming edges to the state B are from A to B with input a, B to B with inputs a and b, and C to B with input a.

Modify the labels of the incoming edges including the output of state B. So, the labels of the incoming states become A to B with label $a/1$, B to B with labels $a/1$ and $b/1$, and C to B with label $a/1$.

3. State C is labelled with output 0. There is only one incoming edge to this state, from A to C with input b. The modified label is $b/0$.

The converted Mealy machine is given in Fig. 3.33.

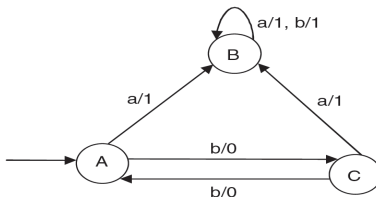


Fig. 3.33

Example 3.22 Convert the following Moore machine into an equivalent Mealy machine as given in Fig. 3.34.

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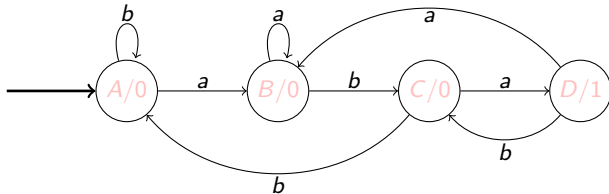


Fig. 3.34

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Solution: State A is labelled with output 0. The incoming edges to this state are C to A for input b, A to A for input b, and one as start-state indication with no input. Ignore the edge with start-state indication.

Modify the labels of the incoming edges by including the output of state A. So, the modified labels become A to A with label $b/0$ and C to A with label $b/0$.

State B is labelled with output 0. The incoming edges to this state are A to B for input a, B to B for input a, and D to B for input a. The modified labels are A to B with label $a/0$, B to B with label $a/0$, and D to B with label $a/0$.

State C is labelled with output 0. The incoming edges to this state are B to C for input b, and D to C for input b. The modified labels are B to C with label $b/0$ and D to C with label $b/0$.

State D is labelled with output 1. The incoming edge to this state is C to D for input a. The modified label is from C to D with label a/1.

The Mealy machine equivalent to the given Moore machine is given in Fig. 3.35.

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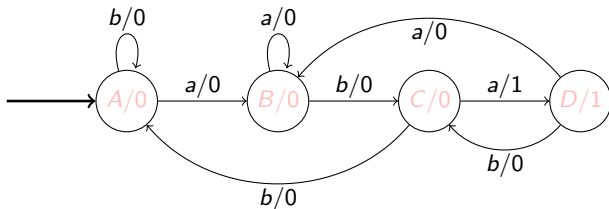


Fig. 3.35

State D is labelled with output 1. The incoming edge to this state is C to D for input a. The modified label is from C to D with label a/1.

The Mealy machine equivalent to the given Moore machine is given in Fig. 3.35.

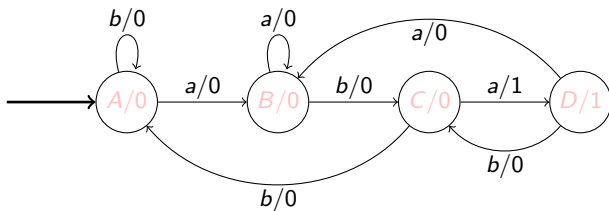


Fig. 3.35

3.13.2.2 Mealy Machine to Moore Machine

For a given Mealy machine M_C , there is an equivalent Moore machine M_o .

These can be constructed in several steps.

Step I: In a Mealy machine, the output depends on the present state and the present input. So, in the transactional diagram of a Mealy machine, the transactional edges are labelled with input as well as output. For a state, there two types of edges, namely, incoming edges and outgoing edges. For incoming edges, it may happen that the output differs for two incoming edges like the example shown in Fig. 3.36.

Step I: In a Mealy machine, the output depends on the present state and the present input. So, in the transactional diagram of a Mealy machine, the transactional edges are labelled with input as well as output. For a state, there are two types of edges, namely, incoming edges and outgoing edges. For incoming edges, it may happen that the output differs for two incoming edges like the example shown in Fig. 3.36.

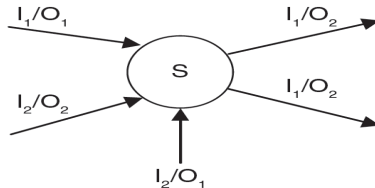


Fig. 3.36

In the transitional diagram for state S , the incoming edges are labelled as I_1/O_1 , I_2/O_2 , and I_2/O_1 , and the outgoing edges are labelled as I_1/O_2 and I_2/O_2 . For state S for incoming edges, we get two types of output O_1 and O_2 . The state S is divided into n number of parts, where $n = \text{number}$ of different outputs for the incoming edges to the state. The output edges are repeated for all the divided states. The transitional diagram for this case is shown in Fig. 3.37.

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Step II: If a state has a loop like Fig. 3.38, that state also needs to be divided as shown in Fig. 3.39.

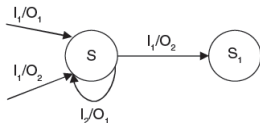


Fig. 3.38

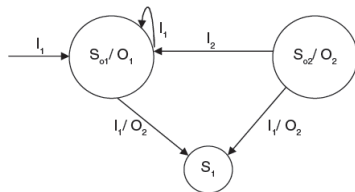


Fig. 3.39

[Here, for input I_1 , the output is O_1 as well as O_2 . So, the state S needs to be divided. S has a loop with input I_2 and output O_1 .]. The state S will be divided into two states S_{01} and S_{02} .

As the loop for input I_2 is labelled with output O_1 , there will be a loop on state S_{O1} . But this loop is not possible on S_{O2} , because it produces the output O_2 . So, there will be a transition from S_{O2} to S_{O1} with input label I_2 .

Step III: Repeat the steps I and II. By this, the Moore machine equivalent to the Mealy machine is constructed.

The following examples (Examples 3.23 and 3.24) describe the process.

Example 3.23 Convert the following Mealy machine to an equivalent Moore machine as given in Fig. 3.40.

As the loop for input I_2 is labelled with output O_1 , there will be a loop on state S_{O1} . But this loop is not possible on S_{O2} , because it produces the output O_2 . So, there will be a transition from S_{O2} to S_{O1} with input label I_2 .

Step III: Repeat the steps I and II. By this, the Moore machine equivalent to the Mealy machine is constructed.

The following examples (Examples 3.23 and 3.24) describe the process.

Example 3.23 Convert the following Mealy machine to an equivalent Moore machine as given in Fig. 3.40.

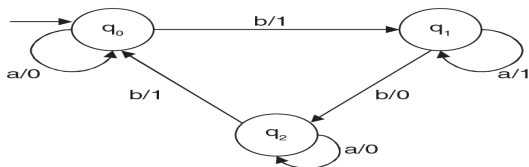


Fig. 3.40

Solution: For state q_0 , there are two incoming states, q_0 to q_0 with label $a/0$ and q_2 to q_0 with label $b/1$. Two incoming edges are labelled with two different outputs 0 and 1. So, the state q_0 needs to be divided into two states as q_00 and q_01 . A loop for input 'a' on q_00 is constructed. A transition is made from q_01 to q_00 with input label 'a'. From both the states, transitions are made to the state q_1 with label $b/1$. The modified machine is given in Fig. 3.41.

For all the other states q_1 and q_2 the outputs for all the incoming edges are 1 and 0 respectively. So there is no need to divide the states. The final Moore machine is as given in Fig. 3.42.

Solution: For state q_0 , there are two incoming states, q_0 to q_0 with label $a/0$ and q_2 to q_0 with label $b/1$. Two incoming edges are labelled with two different outputs 0 and 1. So, the state q_0 needs to be divided into two states as q_{00} and q_{01} . A loop for input 'a' on q_{00} is constructed. A transition is made from q_{01} to q_{00} with input label 'a'. From both the states, transitions are made to the state q_1 with label $b/1$. The modified machine is given in Fig. 3.41.

For all the other states q_1 and q_2 the outputs for all the incoming edges are 1 and 0 respectively. So there is no need to divide the states. The final Moore machine is as given in Fig. 3.42.

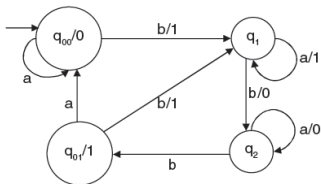


Fig. 3.41

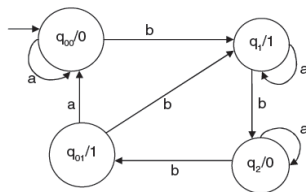


Fig. 3.42

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Example 3.23 Convert the following Mealy machine as given in Fig. 3.43 to an equivalent Moore machine.

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Example 3.23 Convert the following Mealy machine as given in Fig. 3.43 to an equivalent Moore machine.

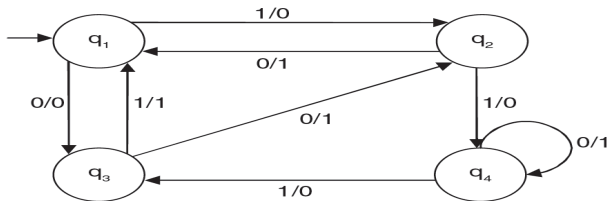


Fig. 3.43

Solution: The machine contains four states. Let us start from the state q_1 . The incoming edges to this state are from q_2 to q_1 with label 0/1 and from q_3 to q_1 with label 1/1. There is no difference in the outputs of the incoming edges to this state and, therefore, in the constructing Moore machine, the output for this state is 1. The modified machine is as shown in Fig. 3.44.

Solution: The machine contains four states. Let us start from the state q_1 . The incoming edges to this state are from q_2 to q_1 with label 0/1 and from q_3 to q_1 with label 1/1. There is no difference in the outputs of the incoming edges to this state and, therefore, in the constructing Moore machine, the output for this state is 1. The modified machine is as shown in Fig. 3.44.

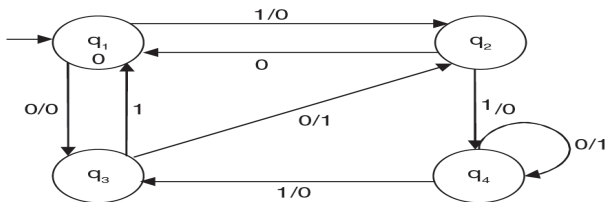
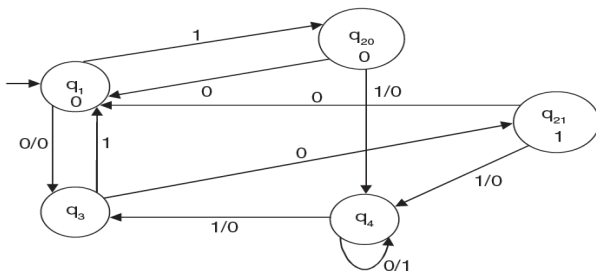


Fig. 3.44

Consider the state q_2 . This state contains two incoming edges: from q_1 to q_2 with label $1/0$ and q_3 to q_2 with label $0/1$. We get two different outputs for the two incoming edges (q_1 to q_2 output 0, q_3 to q_2 output 1). So, the state q_2 is divided into two, namely, q_20 and q_21 . The outgoing edges are duplicated for both the states generated from q_2 as shown in Fig. 3.45.

**Fig. 3.45**

Consider the state q_3 . The incoming edges to this state are from q_1 to q_3 with label 0/0 and from q_4 to q_3 with label 1/0. There is no difference in the outputs of the incoming edges to this state and, therefore, in the constructing Moore machine, the output for this state is 0. The modified machine is given in Fig. 3.46.

Consider the state q_4 . This state contains three incoming edges, from q_2 to q_4 with label 1/0, from q_2 to q_4 with label 1/0, and from q_4 to q_4 with label 0/1. We get two different outputs for the three incoming edges (q_2 to q_4 output 0, q_2 to q_4 output 0, and q_4 to q_4 output 1). So, the state q_4 will be divided into two, namely, q_4 0 and q_4 1.

The modified Moore machine is given in Fig. 3.47.