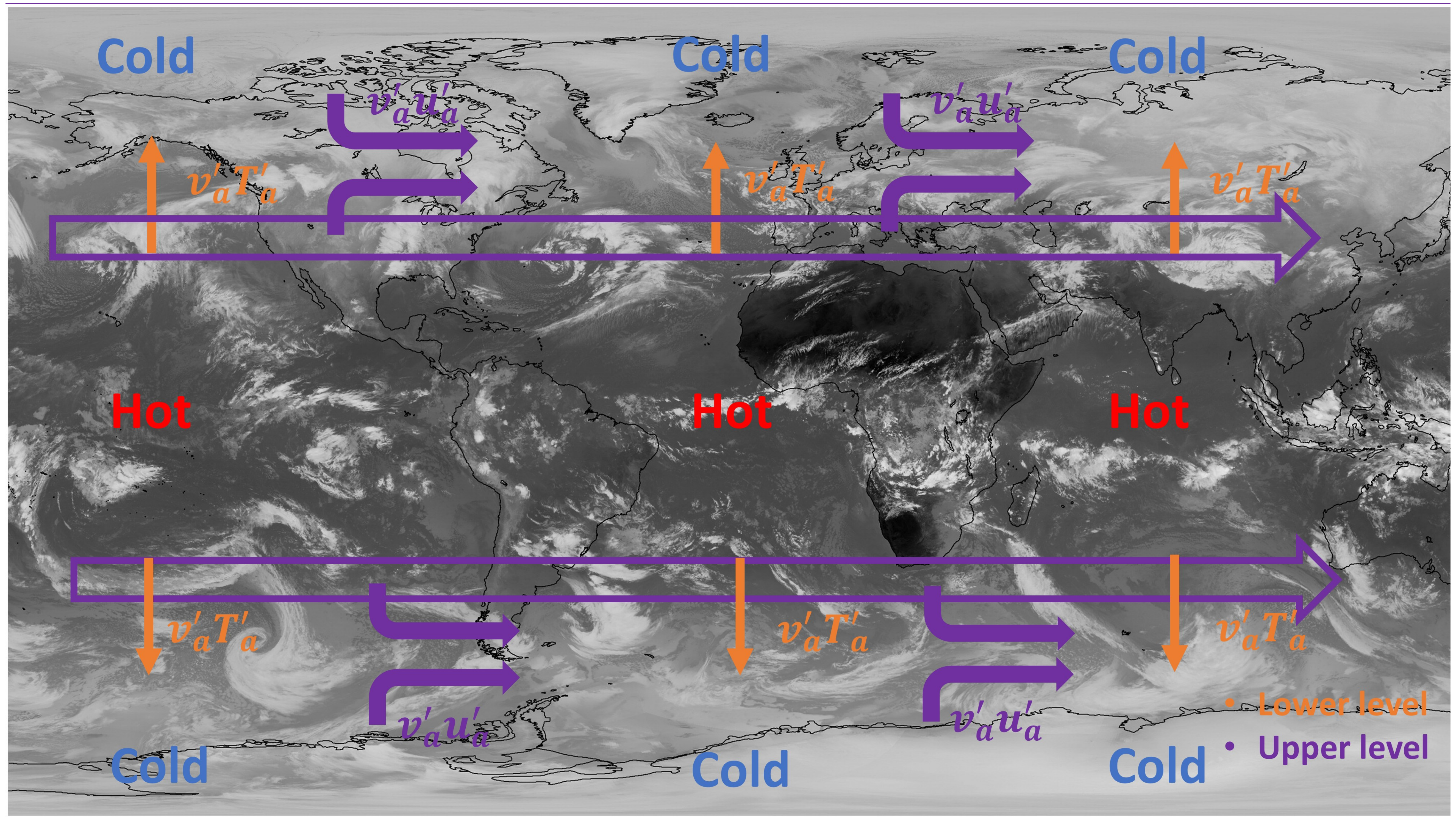
Physics of the Eddy Memory Kernel of a Baroclinic Midlatitude Atmosphere



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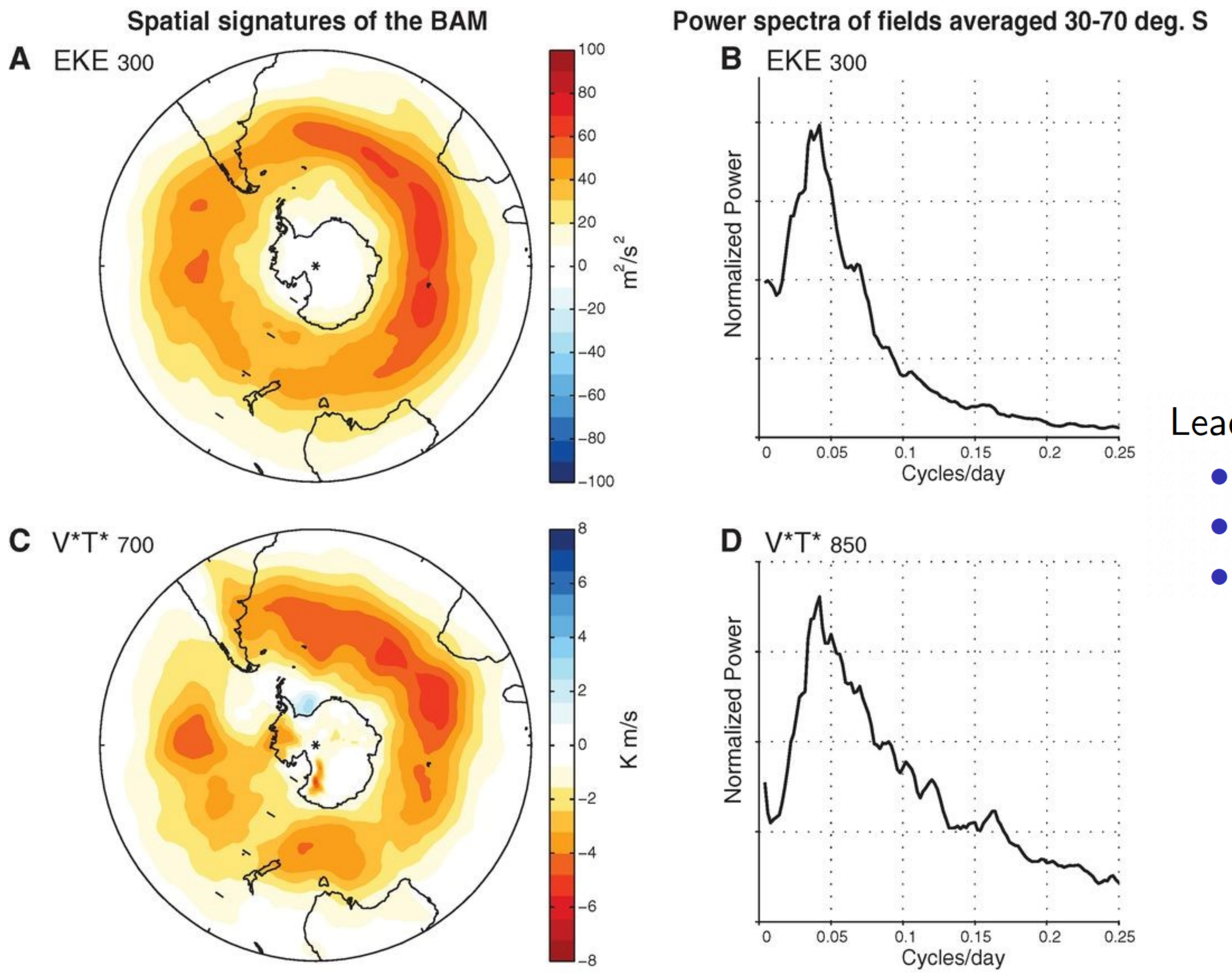
Work with Elian Vanderborght, Woosok Moon, Georgy Manucharyan, Jonathan Demaeyer

Baroclinic Annular Mode (BAM)

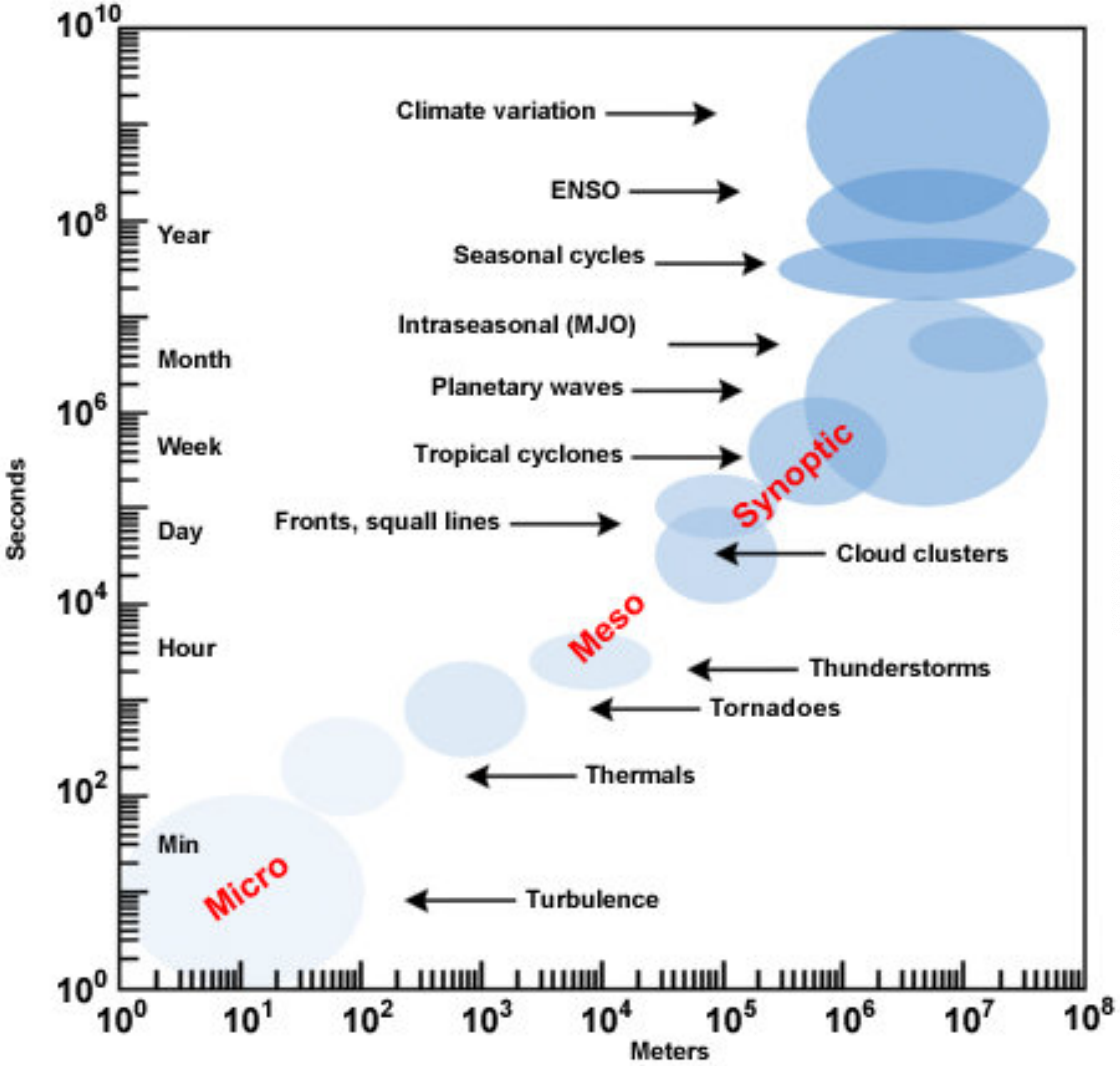


Leading pattern of Southern-Hemisphere planetary-scale variability

- **Annular mode = Zonally symmetric mode**
- Periodic (20-30 days) variability in Eddy Kinetic energy
- Driven by eddy heat fluxes

Thompson & Barnes, (2014)

Atmospheric Variability



- Intraseasonal variability: Variability on a timescale of 10 – 100 days
- Responsible forcing? Internal climate dynamics.

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Eddy-Mean Flow Interactions

1. Synoptic eddies modify the zonal-mean flow
2. Eddy characteristics depend on the structure of the zonal-mean flow
3. Eddies $\Longleftrightarrow$ zonal-mean flow = Modes of variability

Mode-decoupling on >10 days

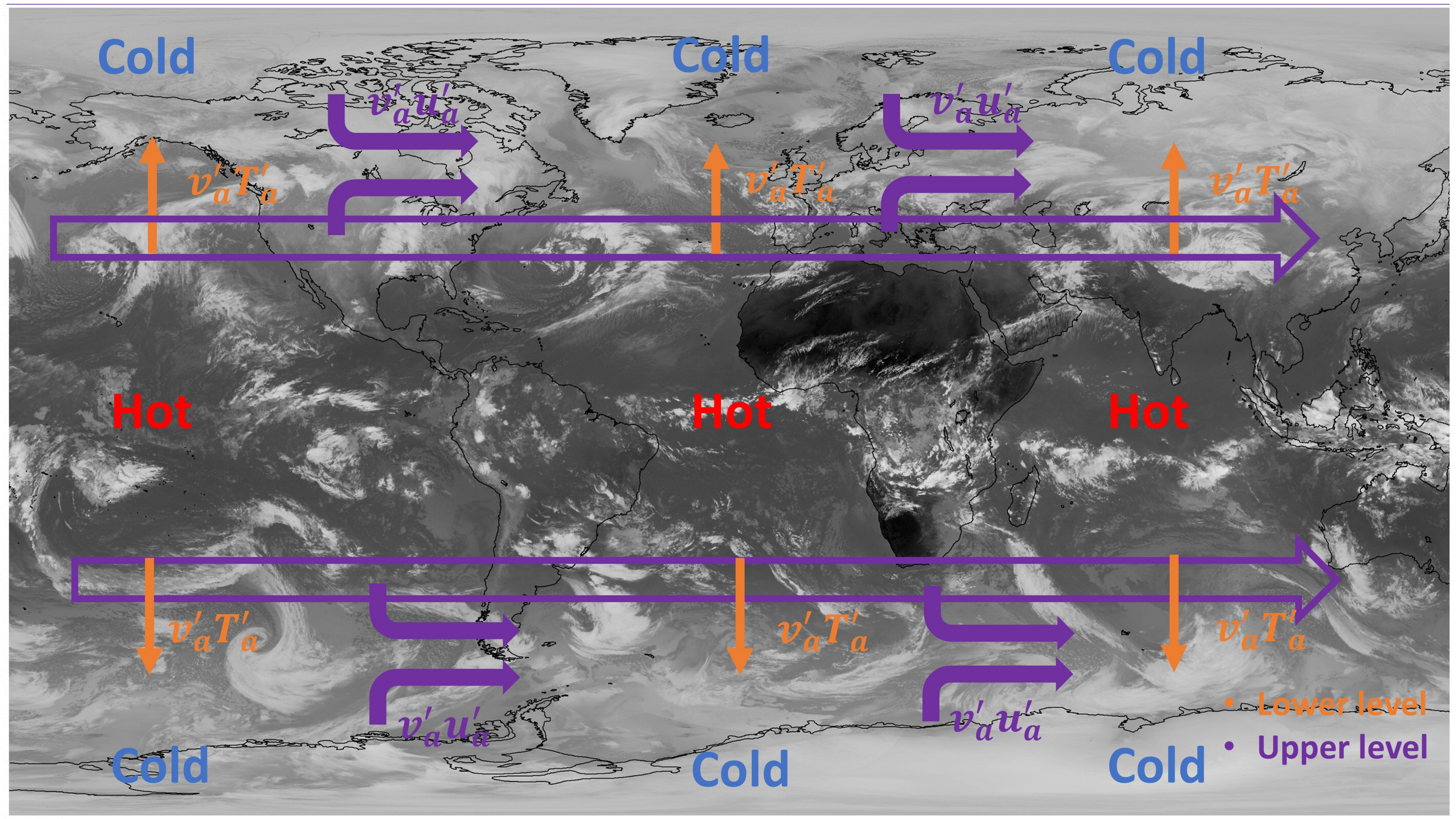
Mid-latitude modes of variability

Baroclinic modes

Barotropic modes

$v'_a T'_a$

$v'_a u'_a$



Closure Problem

Decoupled Baroclinic modes of variability should follow from **thermodynamic equation**, do they? Consider simplified Thermodynamic QG-equation. Result?

$$\partial_t \overline{T_a} + \partial_y \overline{v_a' T_a'} = \underbrace{\overline{Q} + \frac{\sigma \Delta p}{R} \overline{\omega}}_{\textbf{Thermal damping: } h_d^T (T* - T_a)} \tag{1}$$

Framework establishes connection between synoptic fluxes ($\overline{v_a' T_a'}$) and response of zonal mean temperature field ($\overline{T_a}$).
Solve?$\rightarrow$ Need for parametrization of $\overline{v_a' T_a'}$,

$$\overline{v_a' T_a'} = \mathcal{G}\{\overline{T_a}\} \tag{2}$$

What parametrization is suited?

Parameterizations

Parameterize
$\overline{v_a' T_a'}$

Fickian Diffusion
$-K \partial_y \overline{T_a}$

Finite memory
$-\alpha \int_{-\infty}^{t} e^{-\frac{t-t'}{\tau}} \partial_y \overline{T_a}(t', y) dt'$

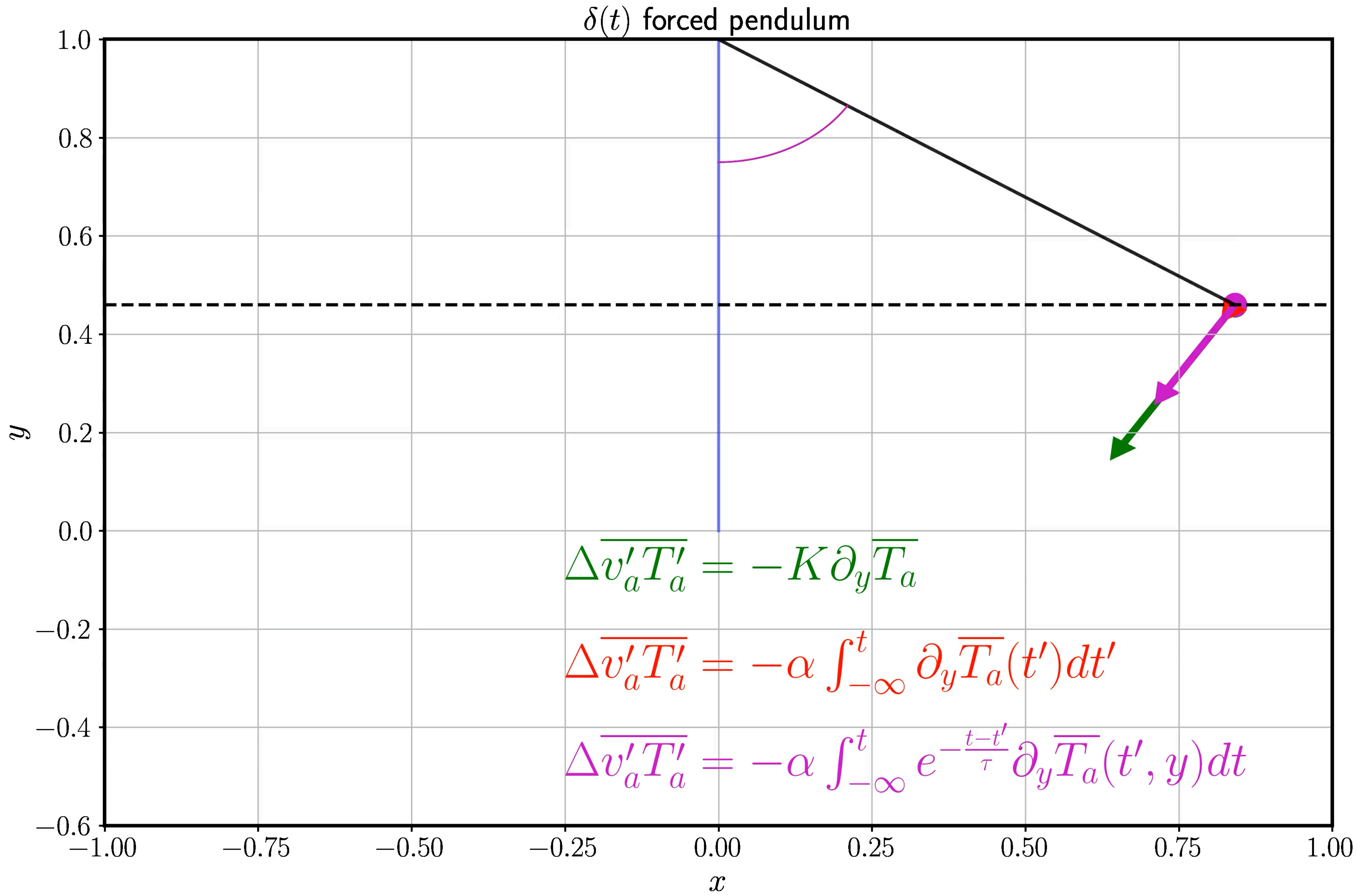
Infinite memory
$-\alpha \int_{-\infty}^{t} \partial_y \overline{T_a}(t') dt'$

Instantaneous response
$\Delta\varphi = 0^\circ$

Delayed response
$\Delta\varphi = \arctan(\omega\tau)$

Quadrature phase shift
$\Delta\varphi = 90^\circ$

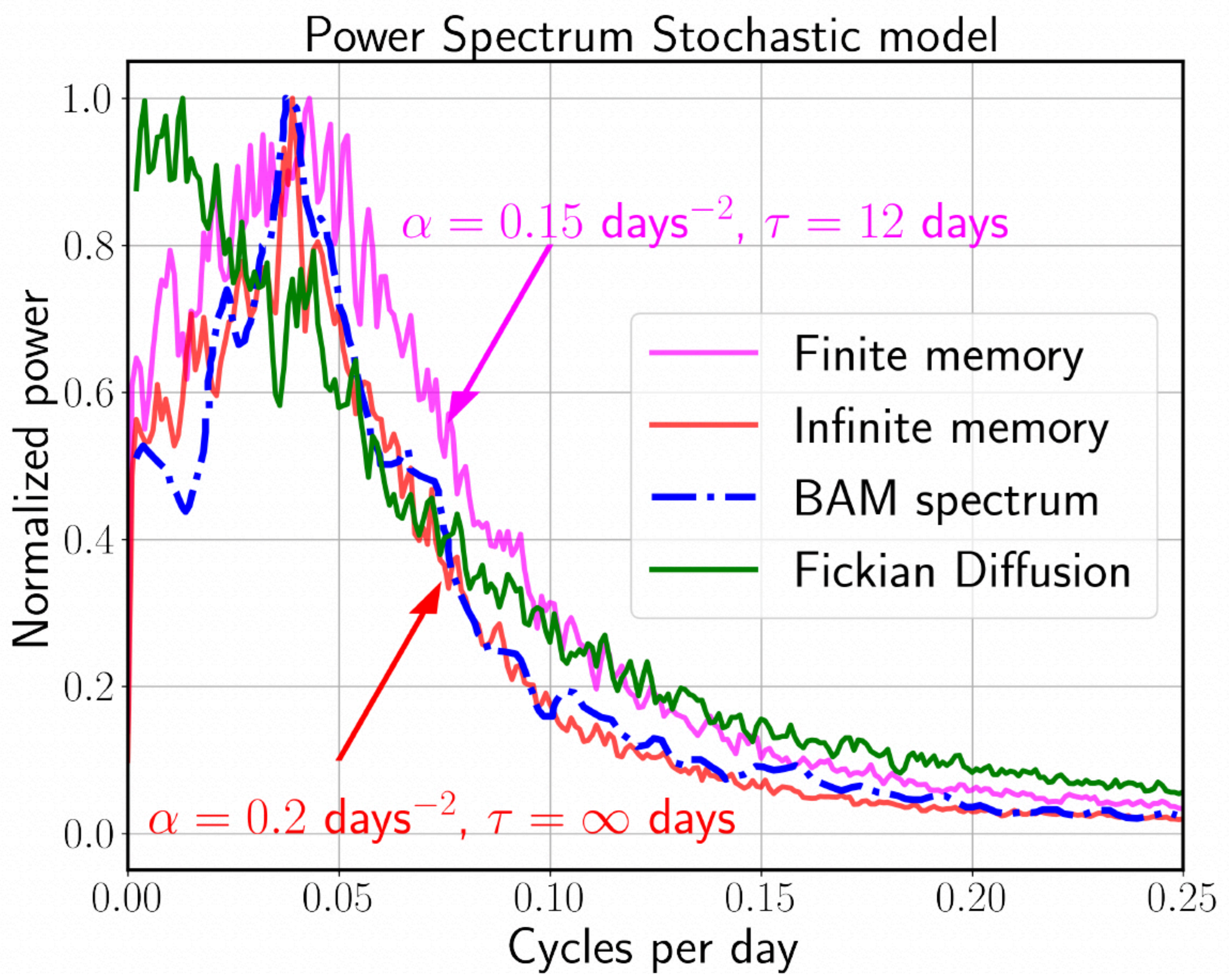
Effect of Memory



Parameterization for the BAM

Treat radiation as stochastic process,

$$\underbrace{\partial_t \overline{T_a} + \partial_y \overline{v_a' T_a'} = h_d^T (T^* - T_a)}_{\textbf{SDE}} \quad \textbf{where} \quad T^* dt = \sigma dW^t \qquad (3)$$



Challenge

'Memory' concept to explain baroclinic low-frequency variability.

Thompson et al. 2014
$\tau \to \infty$

Moon et al. 2021
$\tau = 4$ days

What is missing?

- Are we lucky, or **is there indeed memory**? And if so, over what **timescale** and **Why**?

Need to generalize,

$$\overline{v'_a T'_a}(t,y) = \int_{-\infty}^{t} G^{X}_{\phi,\Phi}(t-t',y)\,\partial_y \overline{T_a}(t',y)\,dt' \tag{4}$$

Here, $G^{X}_{\phi,\Phi}$ is unknown **proxy Green's function**.

Assume memory kernel shape

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RESEARCH ARTICLE

Eddy memory as an explanation of intraseasonal periodic behaviour in baroclinic eddies

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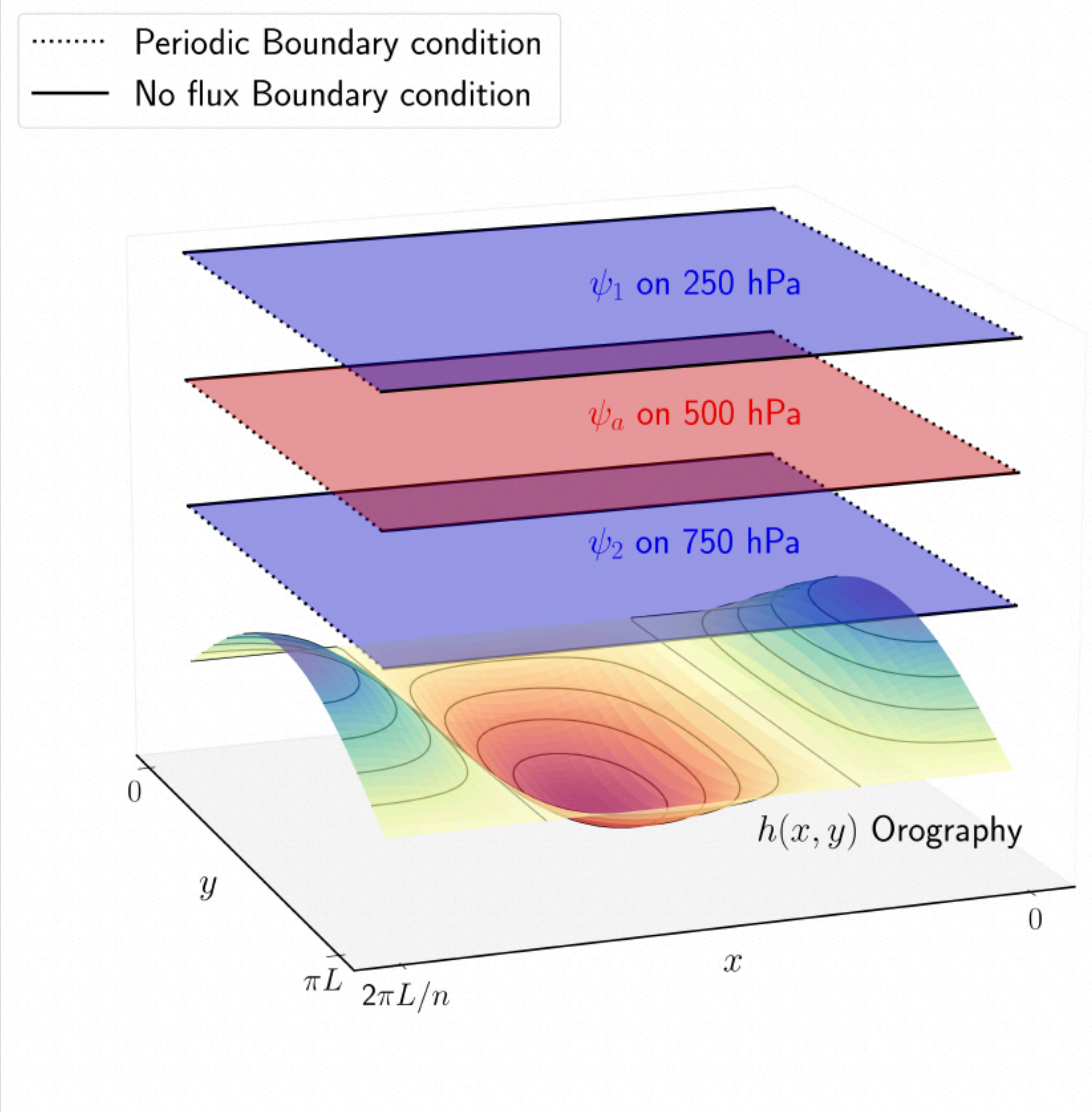
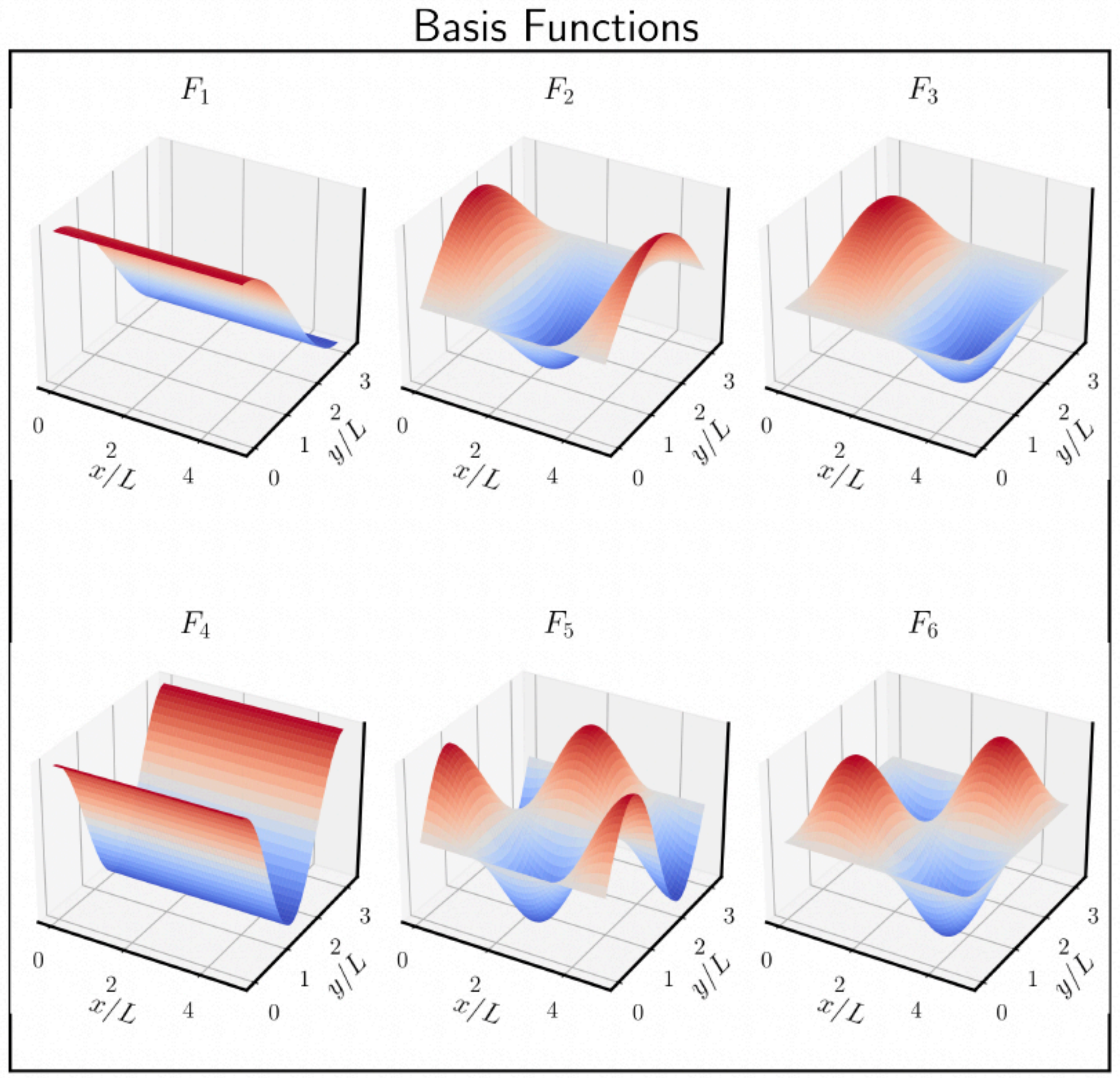
QGS model

Utrecht University

Quasi-Geostropic-Spectral-model

- Two layer Quasi-Geostrophic model
- Defined on $\beta-$plane and coupled with orography $h(x, y)$
- Diagnostic fields projected on orthogonal basis-functions
- Nonlinear PDEs $\rightarrow$ Nonlinear ODEs: $\frac{d\boldsymbol{\eta}}{dt} = F(t, \boldsymbol{\eta})$

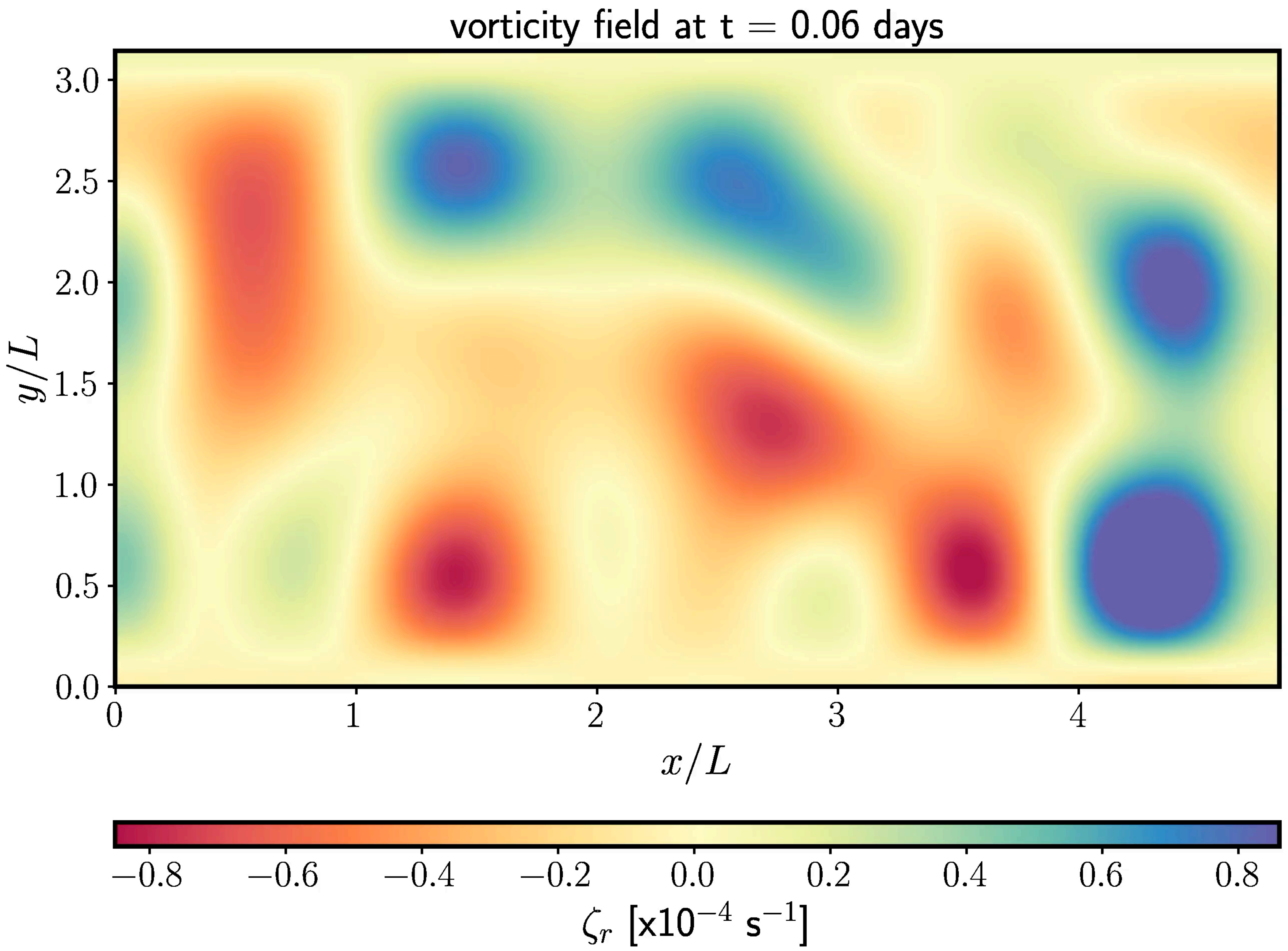




QGS model: typical results

Configuration with **36 different basis functions**

$$\partial_t T_a + \underbrace{J(\psi_a, T_a)}_{\textbf{Baroclinic instability}} = \underbrace{h_d(T^* - T_a)}_{\textbf{Forcing towards } T^*} \rightarrow \textbf{Geostrophic Turbulence}$$



Use of Linear Response Theory

How do we determine the proxy Green's function in a **chaotic nonlinear** model?

Step 1: **Linear response theory** (LRT), applies for Axiom A dynamical systems $\frac{d\boldsymbol{\eta}}{dt} = F(t, \boldsymbol{\eta})$

$\rightarrow$ Average response to small forcing of any observable may be **approximated as linear** eventhough the **dynamical system is nonlinear**

Consider observable $\langle \Xi \rangle$, in phase space $\boldsymbol{\eta}$, under-forcing pattern $\boldsymbol{g} = \epsilon \boldsymbol{X}(\boldsymbol{\eta})\Theta(t)$, may be approximated as,

$$\Delta \langle \Xi \rangle^{\boldsymbol{g}}_{\boldsymbol{\eta}}(t) = \int_{-\infty}^{t} G^{\boldsymbol{X}}_{\Xi}(t - t')\Theta(t')dt' = (G^{\boldsymbol{X}}_{\Xi} * \Theta)(t) \tag{5}$$

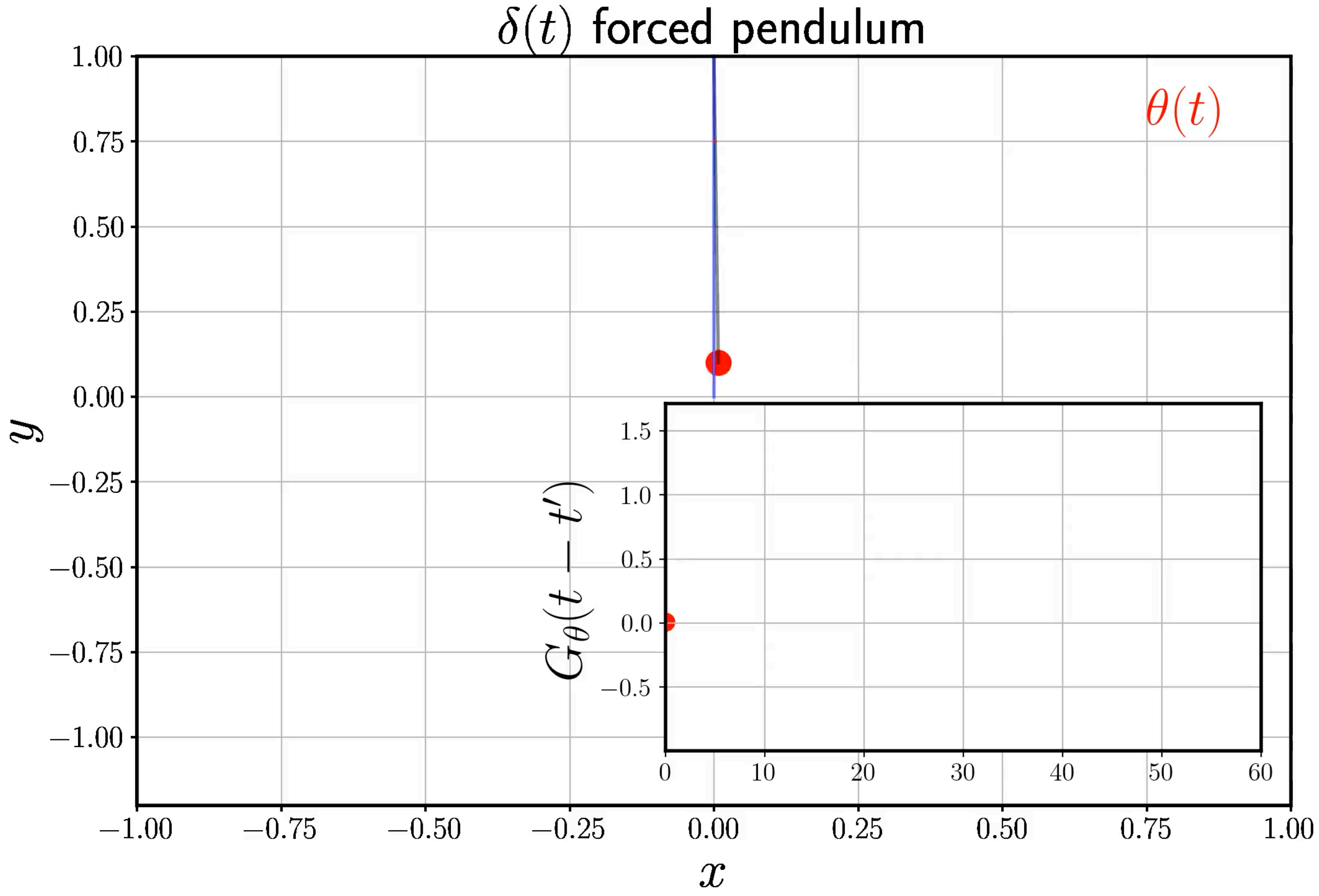
Here $G^{\boldsymbol{X}}_{\Xi}(t - t')$ is the **linear Green's function**, only defined for $t - t' > 0 \rightarrow$ **Causal**.

Intermezzo: Green's functions

Consider a linear Differential operator, $\mathcal{L}^{\boldsymbol{X}}\{\Delta\langle\Xi\rangle^{\boldsymbol{g}}_{\boldsymbol{\eta}}(t)\} = \Theta(t)$
The linear Green's function is defined as,

$$\mathcal{L}^{\boldsymbol{X}}\{G^{\boldsymbol{X}}_{\underline{\Xi}}(t-t')\} = \delta(t-t') \tag{6}$$

Example? Hit pendulum with hammer at $t'=0$. Result



Proxy Response Theory

How do we determine the proxy Green's fucntion in a chaotic model?
Step 1: **Linear response theory** (LRT).

$\rightarrow$ Determine Linear Green's function of $\Delta\langle\partial_y \overline{T_a}\rangle^{\boldsymbol{g}}_{\boldsymbol{\eta}}$ ($=\Delta\langle\phi\rangle$, for short) and $\Delta\langle\overline{v_a' T_a'}\rangle^{\boldsymbol{g}}_{\boldsymbol{\eta}}$ ($=\Delta\langle\Phi\rangle$)$\rightarrow$ $G^{\boldsymbol{X}}_{\phi}(t,y)$ and $G^{\boldsymbol{X}}_{\Phi}(t,y)$

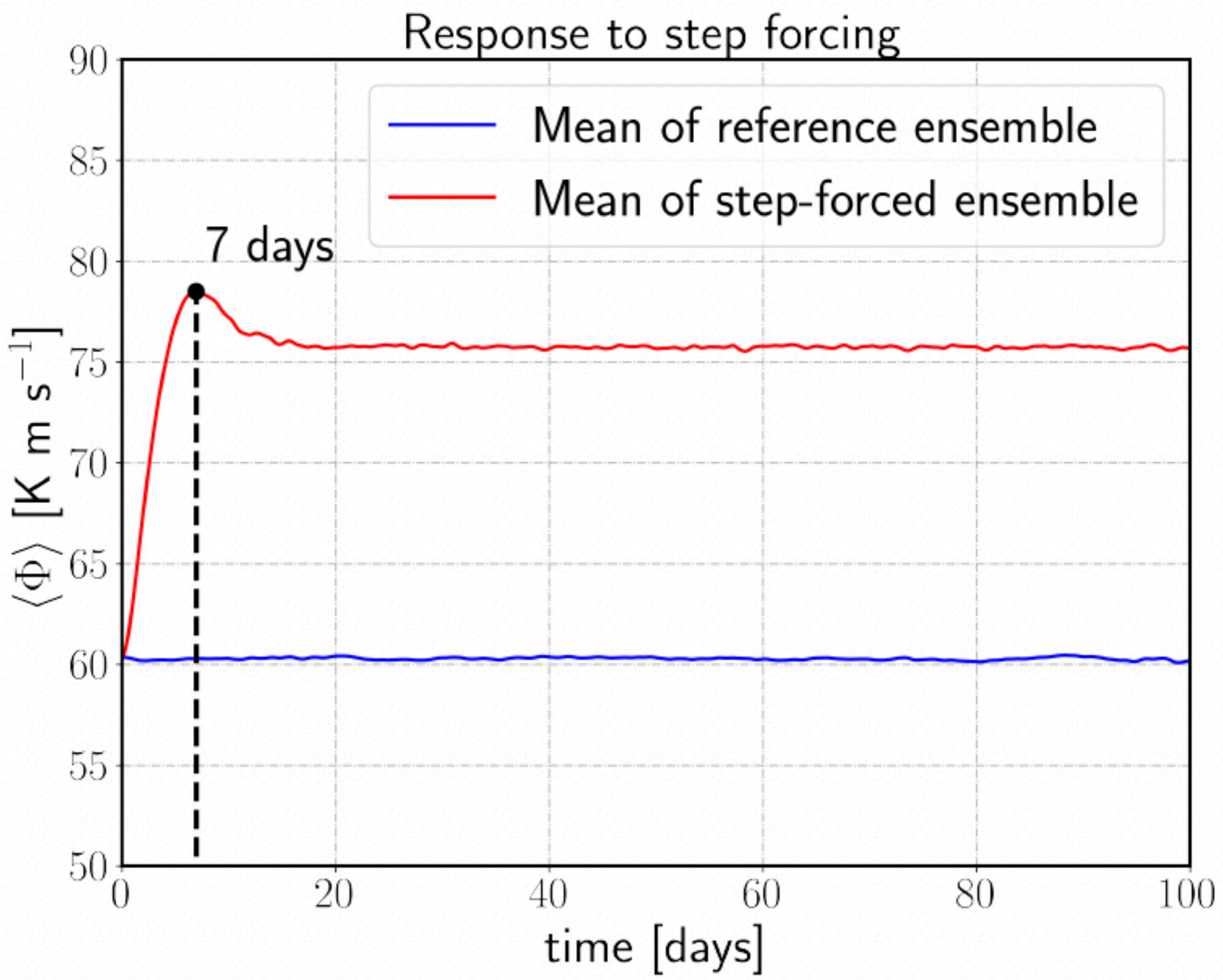
Step 2: **Proxy response theory** (PRT), how do we relate $\Delta\langle\phi\rangle$ to $\Delta\langle\Phi\rangle$?

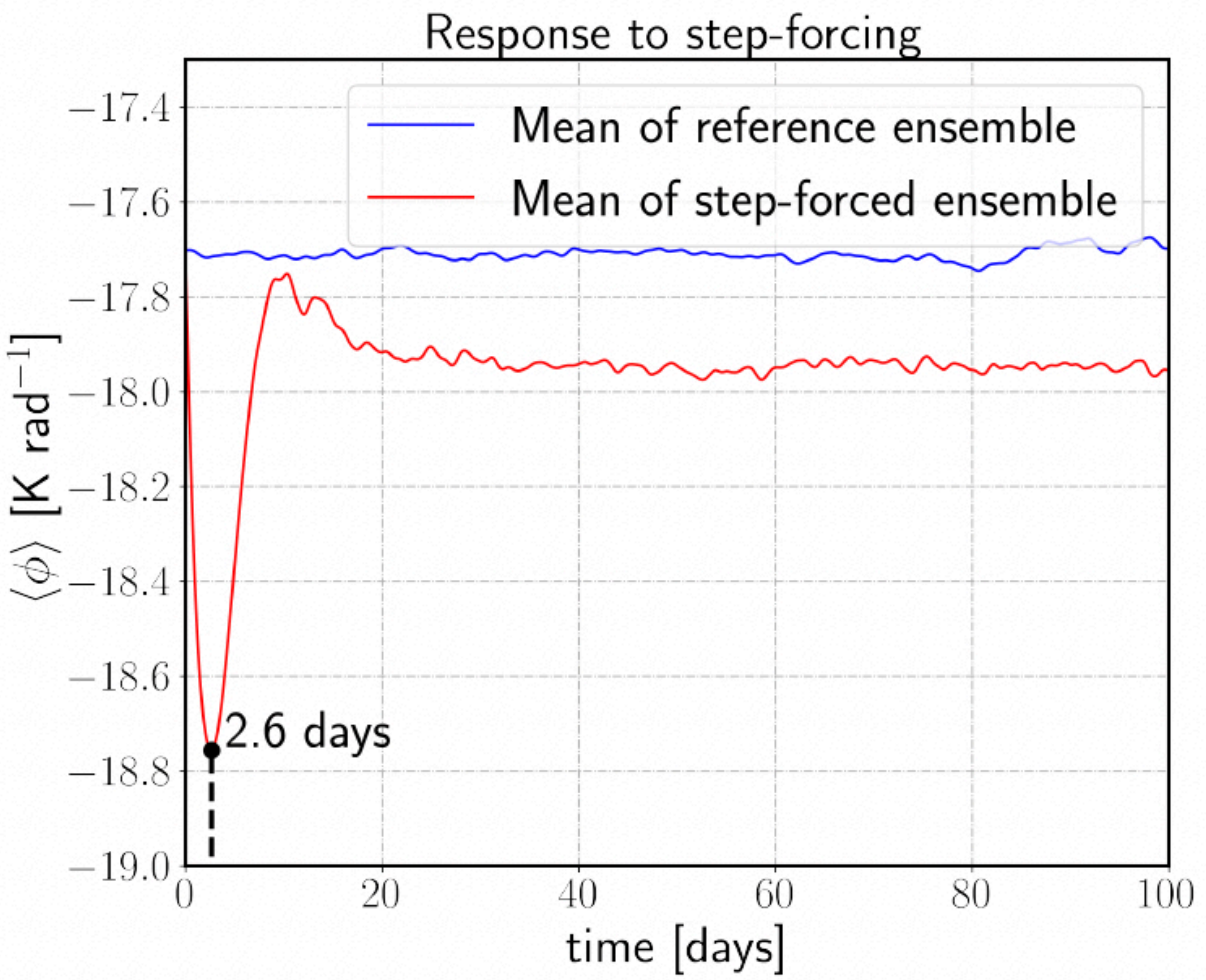
$$\Delta\langle\Phi\rangle = \int_{-\infty}^{t} G^{\mathrm{X}}_{\phi,\Phi}(t-t',y)\Delta\langle\phi\rangle(t',y)dt' \qquad (7)$$

Here $G^{\mathrm{X}}_{\phi,\Phi}(t,y)$ follows from $G^{\mathrm{X}}_{\phi}(t,y)$ and $G^{\mathrm{X}}_{\Phi}(t,y)$. Meaning?

- Generalizes parametrization
- Cautious with **causality**
- If defined for $t > 0$, **eddies have memory**
- If defined on $t = 0$, **eddies feel effect instantly (singular)**

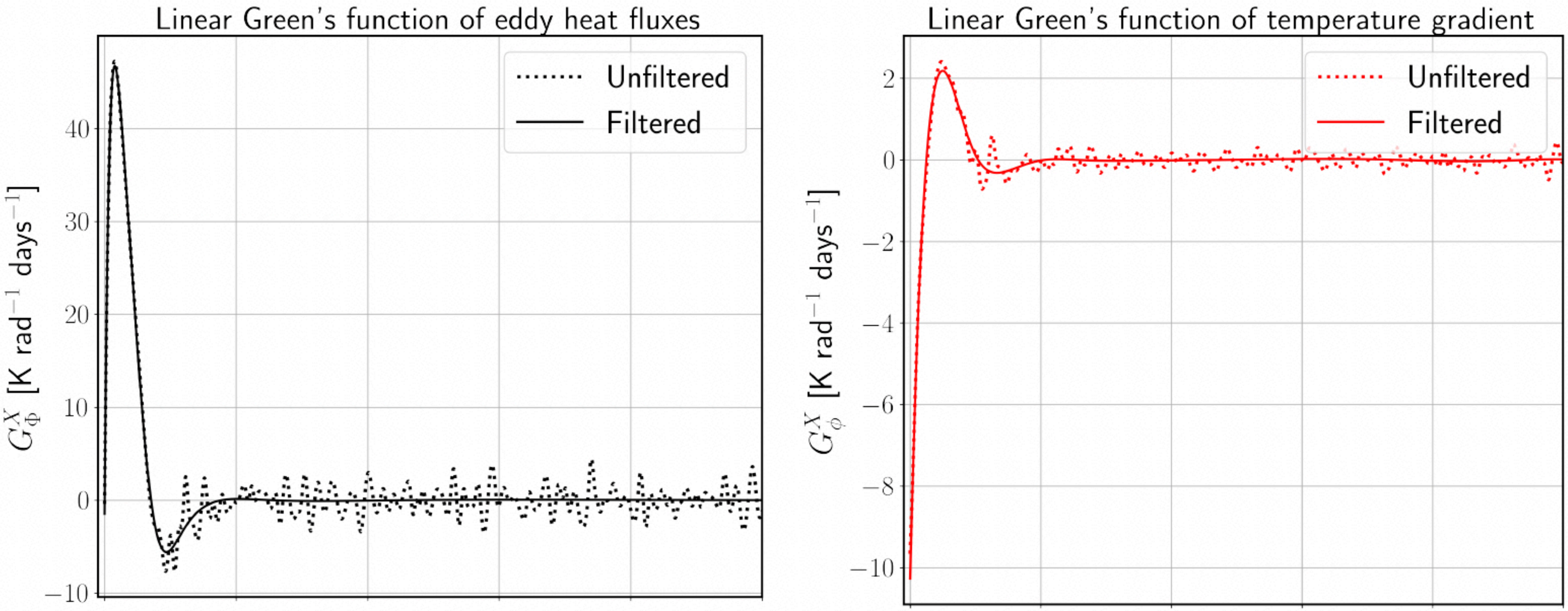
Results Step forcing: $T_f^* = \mathcal{H}(t)(1+a)T_r^*$



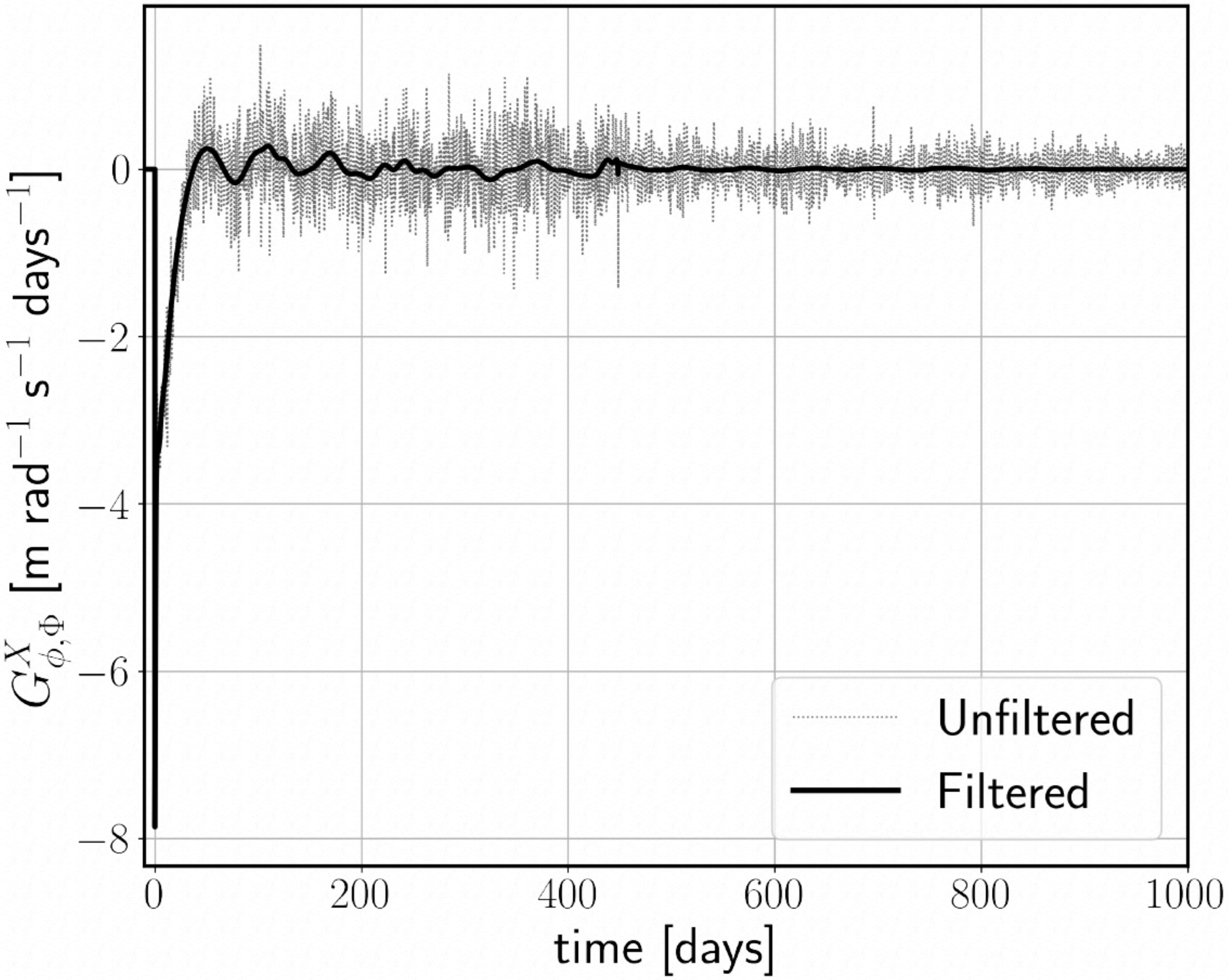


$$G_{\Phi}^{\mathbf{X}} = \frac{1}{a}\frac{d\Delta\langle\Phi\rangle}{dt} \quad \text{and} \quad G_{\phi}^{\mathbf{X}} = \frac{1}{a}\frac{d\Delta\langle\phi\rangle}{dt}$$

Results Green's functions



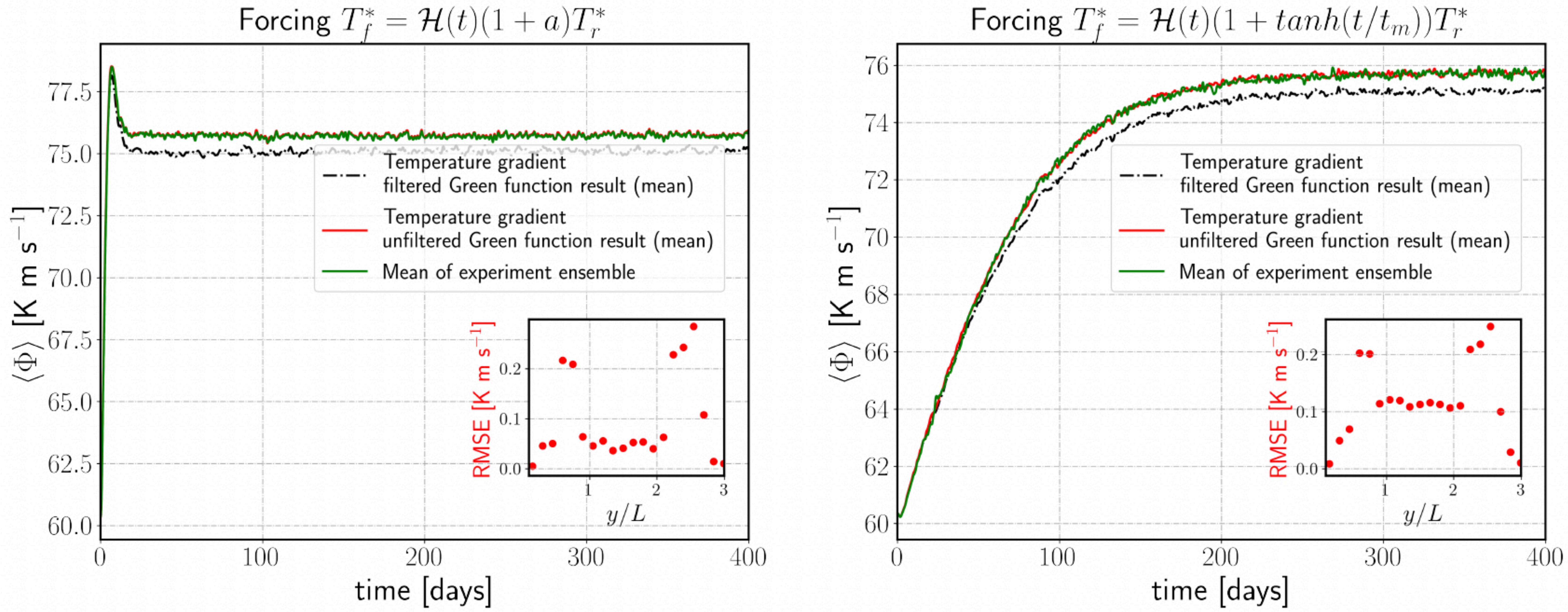
Results proxy Green's function



1. $G^{\boldsymbol{X}}_{\phi,\Phi}$ defined for $t > 0 \rightarrow$ **Memory** up to 500 days!!
2. Corresponding memory timescale is **finite** $\tau \sim 12$days!
3. $G^{\boldsymbol{X}}_{\phi,\Phi} = 0$ for $t = 0 \rightarrow$ **Non-Singular Relation!**

Results proxy Green's function

Can $\Delta\langle\phi\rangle$ predict $\Delta\langle\Phi\rangle \rightarrow$ test by $(G^{\mathbf{X}}_{\phi,\Phi} * \Delta\langle\phi\rangle)(t,y)$ under two different temporal forcings

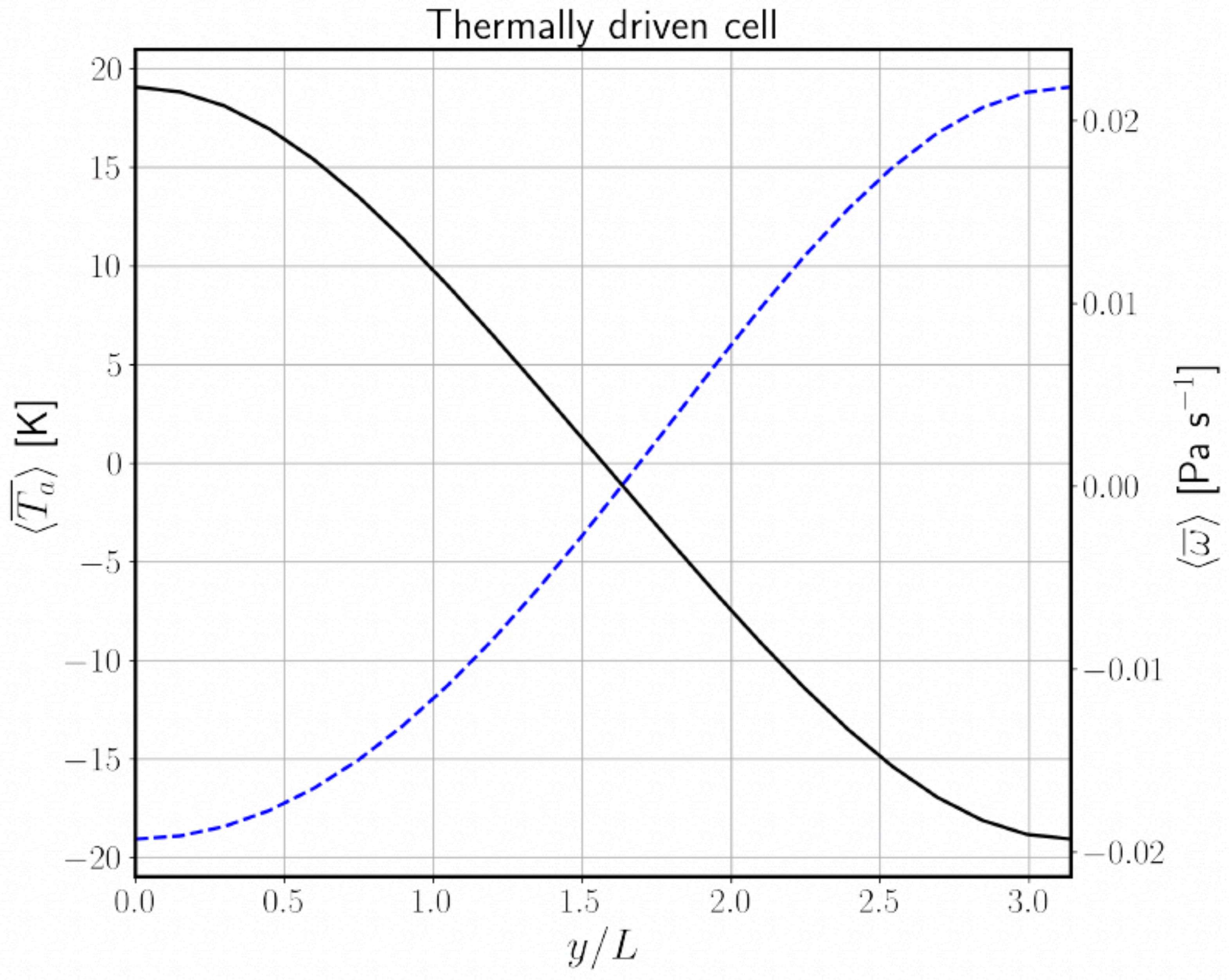


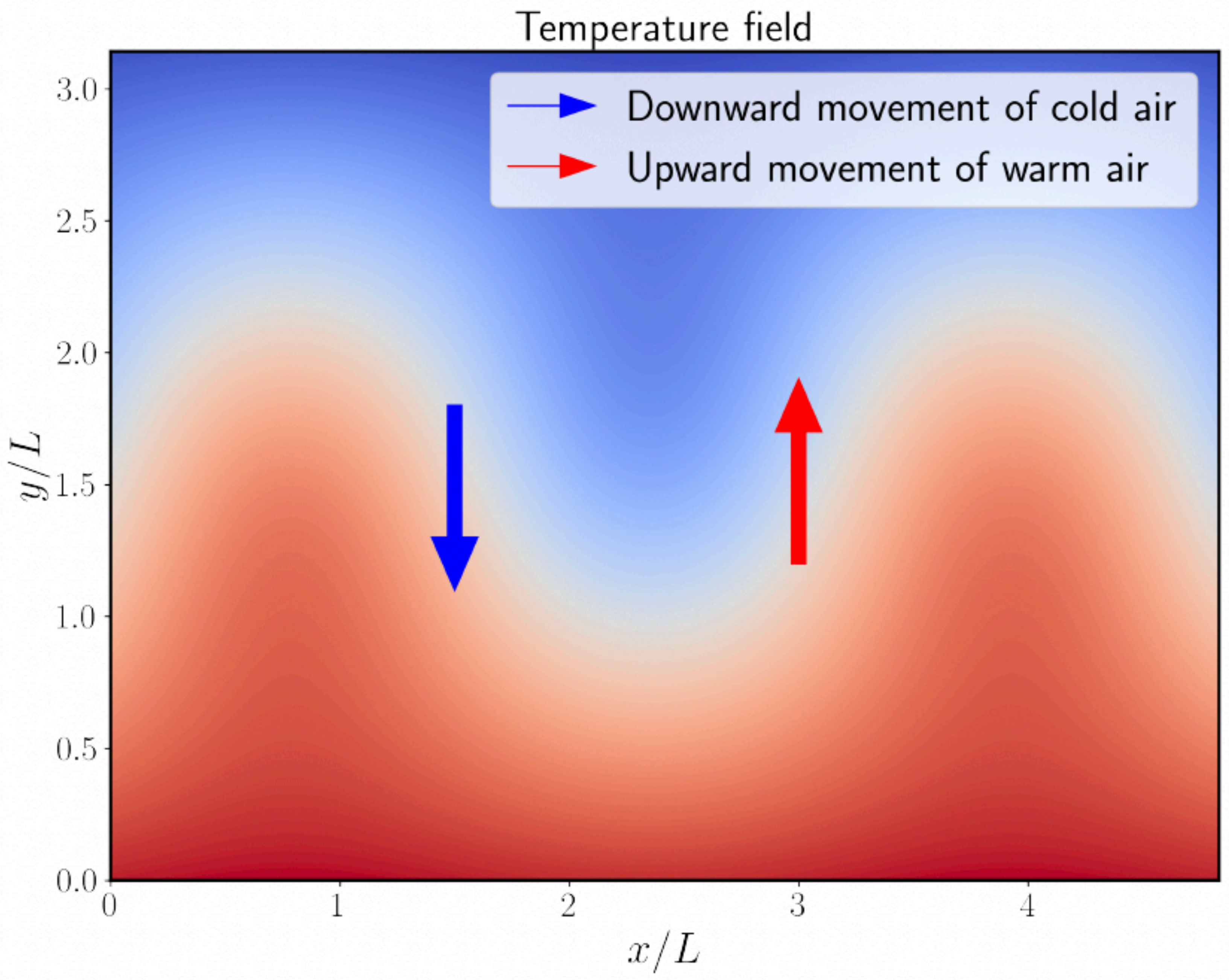
- Temperature gradient good causal predictor→Parametrization is sensible
- Finite memory is crucial in parametrization
- Discrepancies are due to non-causality on short timescales

Physics of Memory: Energy Analysis

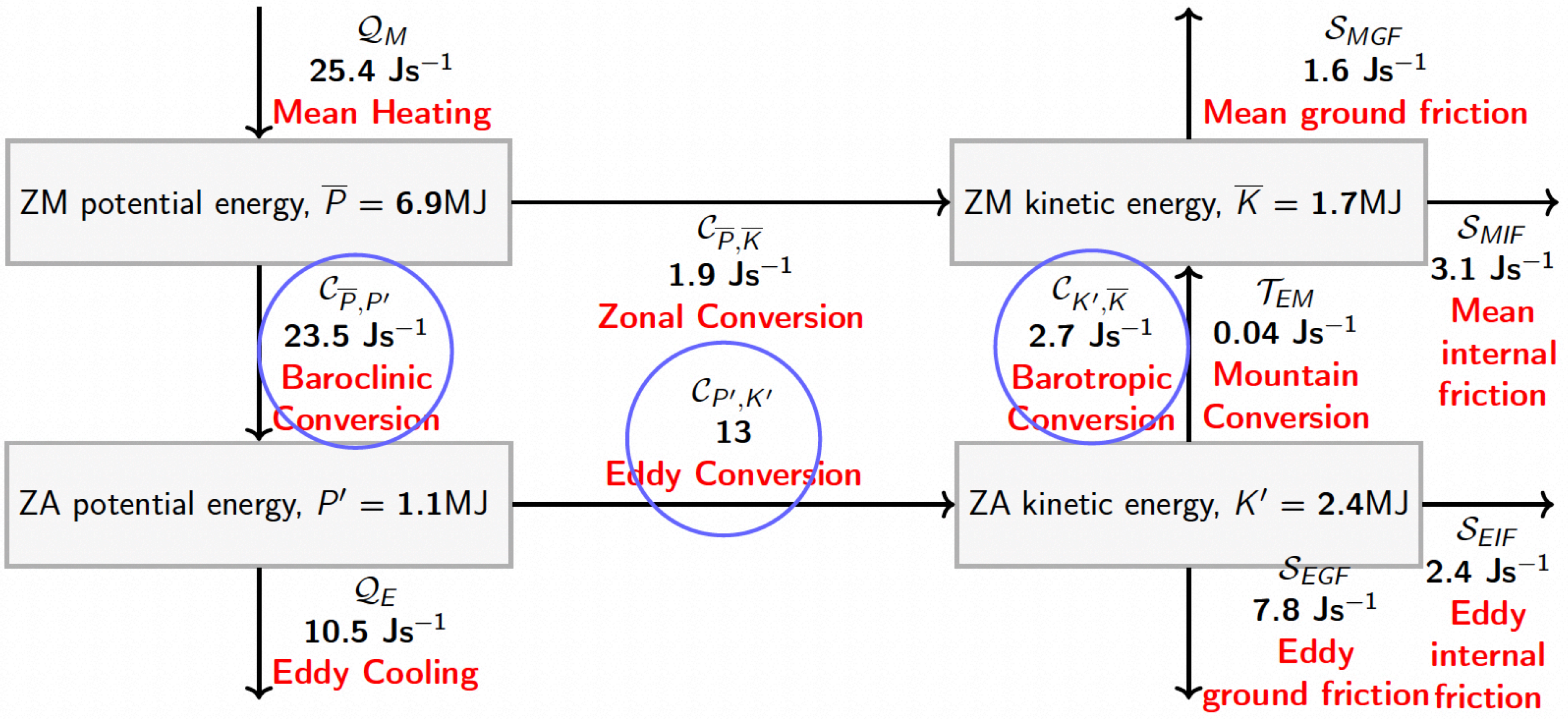
Why is memory sensible?$\rightarrow$**Lorenz Energy Cycle** (LEC):

- $K = \overline{K} + K'$ and $P = \overline{P} + P'$ $\rightarrow$Evolution equation
- Energy may be lost, gained or converted from one reservoir to another (X to Y, by $\mathcal{C}_{X,Y}$)
- Dominant pathways (note $\overline{P}$ $\propto$centre of mass height$\propto$Stability)
    1. $\overline{P} \rightarrow_{\mathcal{C}_{\overline{P},\overline{K}}} \overline{K}$: Thermally direct circulation cell
    2. $\overline{P} \rightarrow_{\mathcal{C}_{\overline{P},P'}} P' \rightarrow_{\mathcal{C}_{P',K'}} K' \rightarrow_{\mathcal{C}_{K',\overline{K}}} \overline{K}$: Growth and Decay of **waves**





Lorenz Energy Cycle



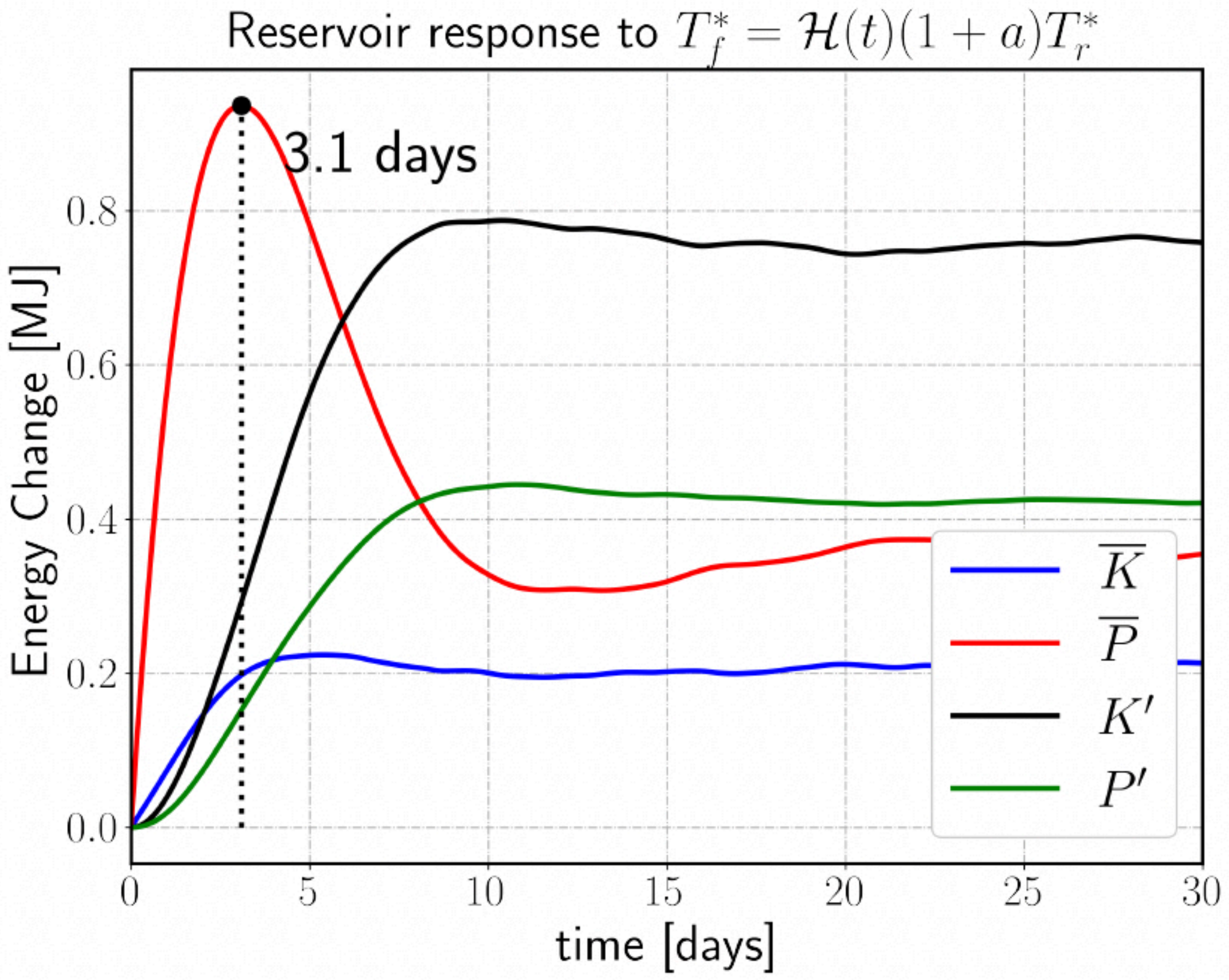
Growth of waves: $\overline{P} \rightarrow_{\mathcal{C}_{\overline{P},P'}} P' \rightarrow_{\mathcal{C}_{P',K'}} K'$: $\overline{v_a' T_a'} \longrightarrow$ **Dominant**

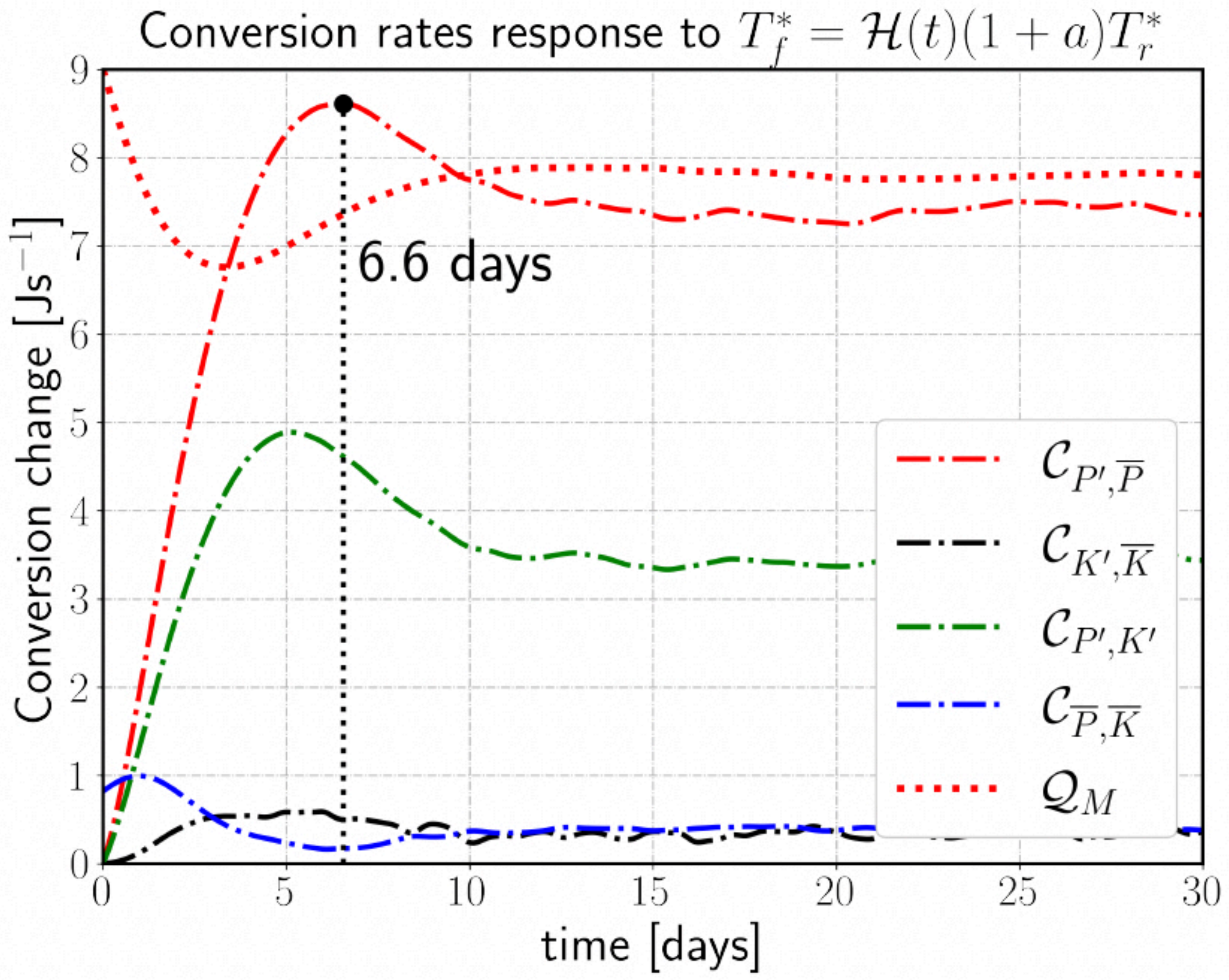
Decay of waves: $K' \rightarrow_{\mathcal{C}_{K',\overline{K}}} \overline{K}$: $\overline{v_a' u_a'} \longrightarrow$ **Neglible**

Analysis: Step Function Response

Model internal dynamics driven by growing baroclinic waves!

- QGS-model prefect testing ground for decoupled baroclinic modes
- Baroclinic modes are driven by eddy heat fluxes that have memory!
- Response $\mathcal{C}_{\overline{P},P'}(t - \Delta t) \propto \overline{P}(t)$
- Timelag proportional to memory timescale
- Physics? Instabilities require a finite time to adapt to changed stability





Summary & Conclusions

Utrecht University

1. $\Delta\langle\Phi\rangle$ has **finite memory** of $\Delta\langle\phi\rangle \to \tau \sim 12$ days.
2. Using $\Delta\langle\phi\rangle$ to predict $\Delta\langle\Phi\rangle$ is sensible for timescales > 2 days.
3. Variability in QG-model is driven by heat fluxes$\to$Baroclinic modes!
4. Proxy Green's function contains interaction of growing baroclinic waves with zonal mean state.
5. **Memory?** Growing baroclinic waves require finite time to adapt accordingly to the zonal mean stability of the atmosphere.

Implications?

1. Annular decoupled baroclinic modes likely follow from memory effects$\to$***Baroclinic annular memory modes***

Further Reading

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Physics of the Eddy Memory Kernel of a Baroclinic Midlatitude Atmosphere

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