From the linear response theory to tipping points.

What lessons can be learnt and how far we get with applications to Earth System Models for future climate projections?

Munich, 5th March 2024

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CNR ISAC

Italian National Research Council

Outline

- **The Nobel prize in Physics 2021: a milestone;**
- **Fast vs. slow scales: the Hasselmann paradigm (just quoting…);**
- **Applications of linear response theory and asymptotic response to ESMs;**
- **The emergence of non-linearity: climate prediction, detection/attribution, tipping points;**

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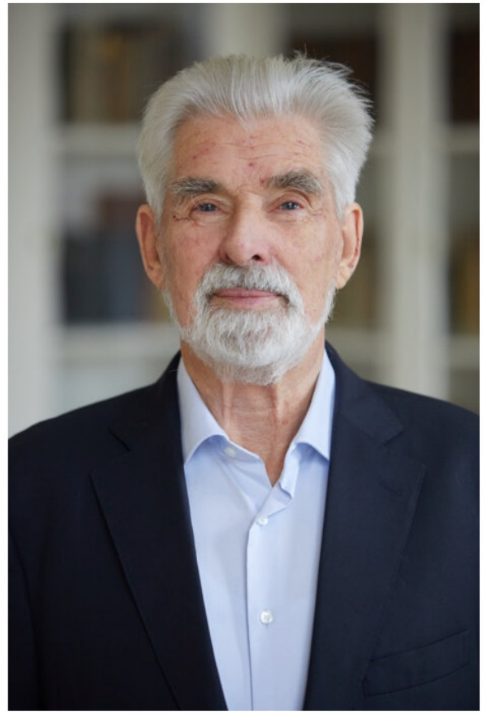
Syukuro Manabe



© Nobel Prize Outreach. Photo: Bernhard Ludewig

Klaus Hasselmann

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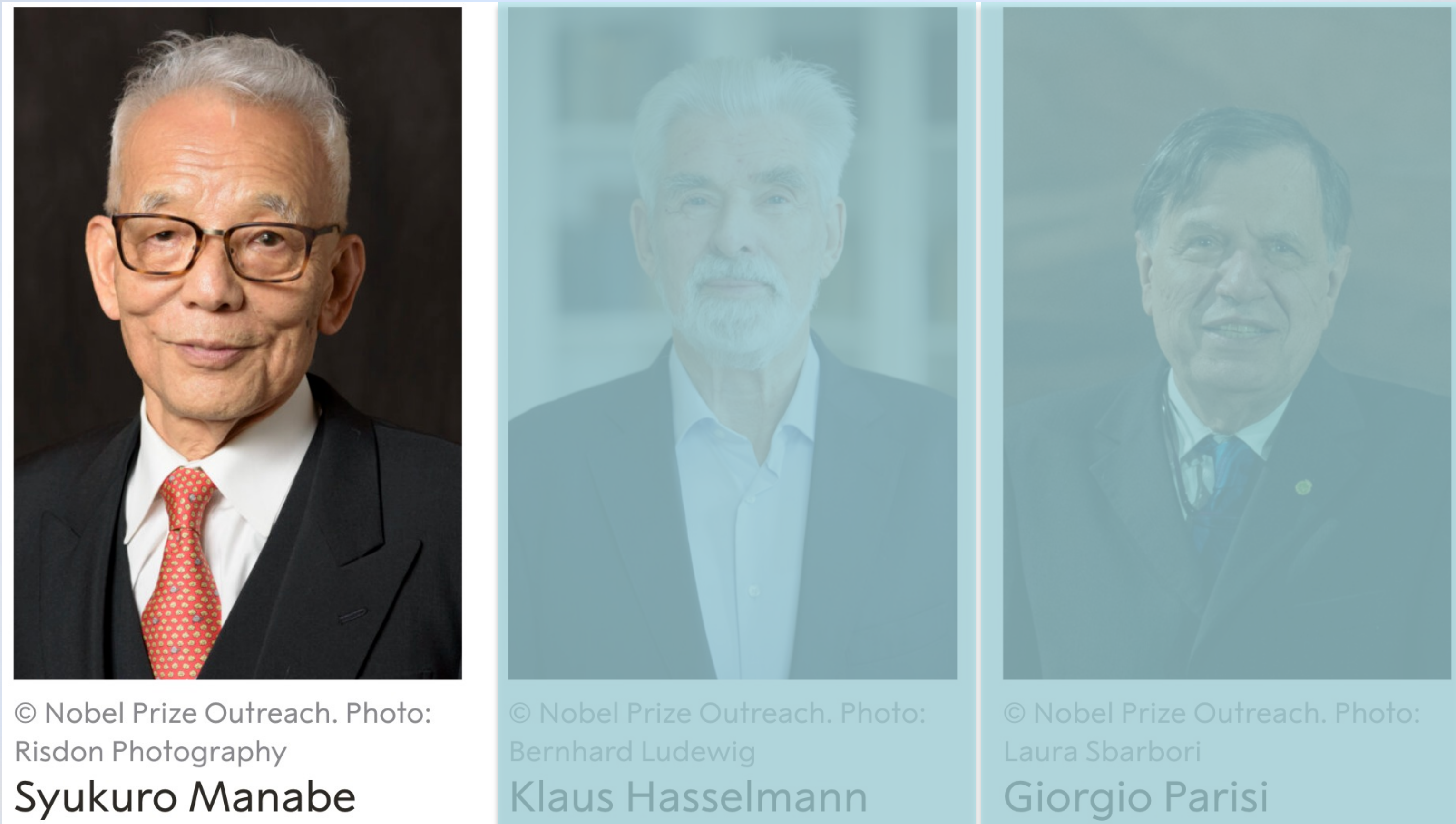
Giorgio Parisi

The Nobel Prize in Physics 2021 was awarded "for groundbreaking contributions to our understanding of complex physical systems"…



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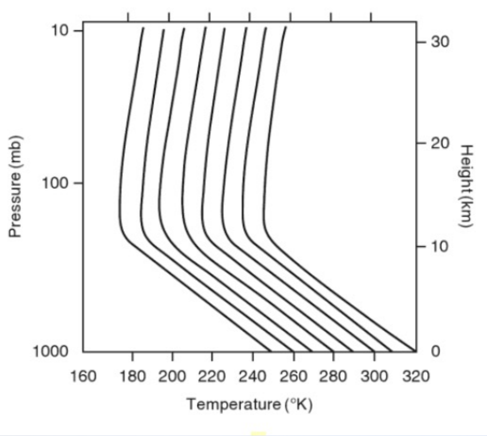
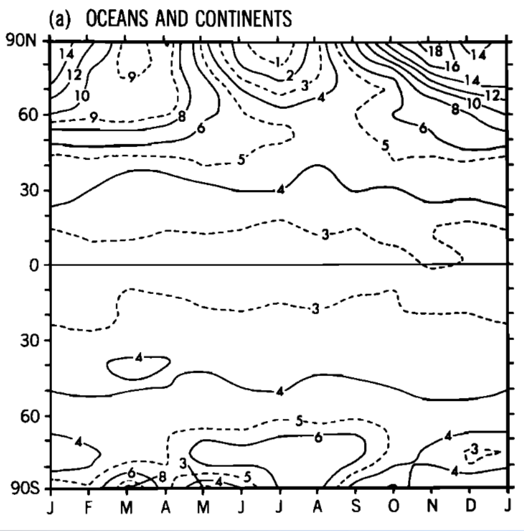
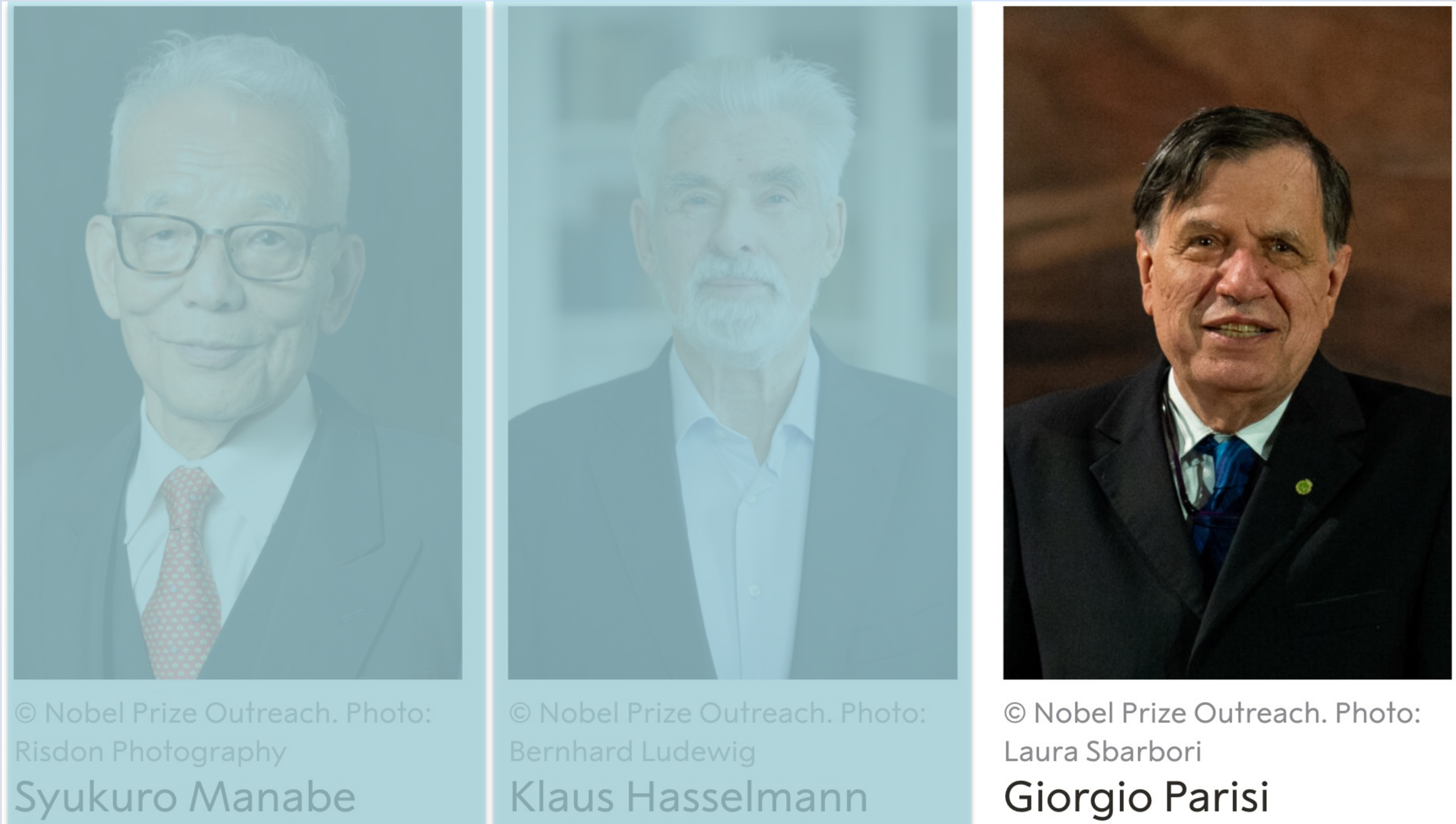
Syukuro Manabe's seminal contributions (cfr. Franzke et al. 2022):



- The "father" of numerical climate modeling (a first example of a 1-D radiative-convective model, Manabe and Wetherald, 1966);



- The first simulations of global climate response to increase in atmospheric CO2 concentrations (Manabe and Stouffer, 1980);
- The first hypotheses of abrupt climate change triggered by Atlantic Meridional Overturning Circulation (AMOC) collapse (Manabe and Stouffer, 2000);



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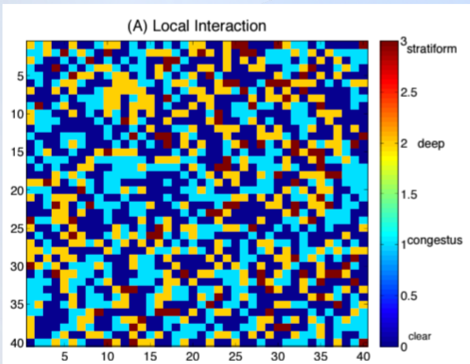
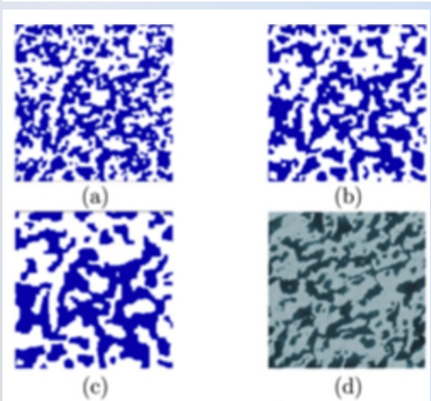
"Unexpected" implications of Giorgio Parisi's work for climate sciences:



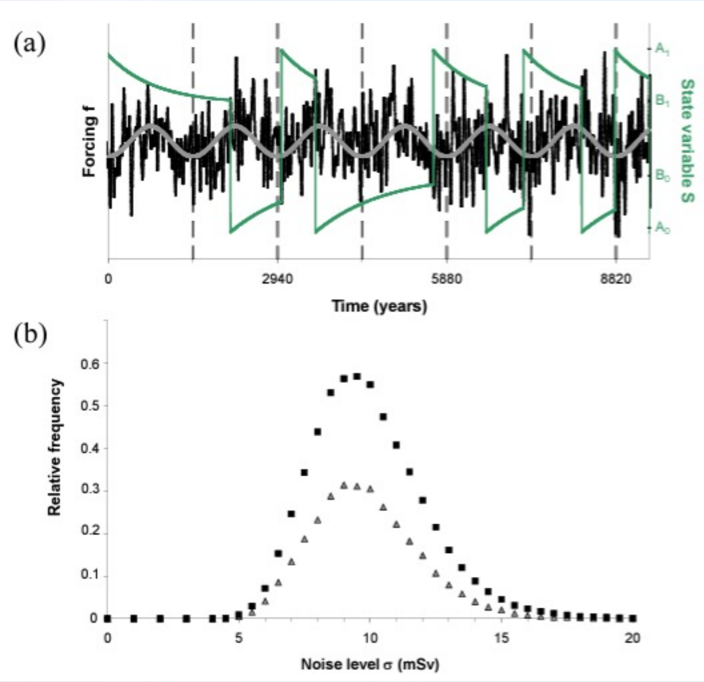
- A lattice model for self-organized tropical convection, based on Ising lattice model used to study spin-glass properties (Khouider 2014);



- A more realistic model of melt ponds over Arctic sea-ice, based on the random Ising lattice model (Ma et al. 2019);

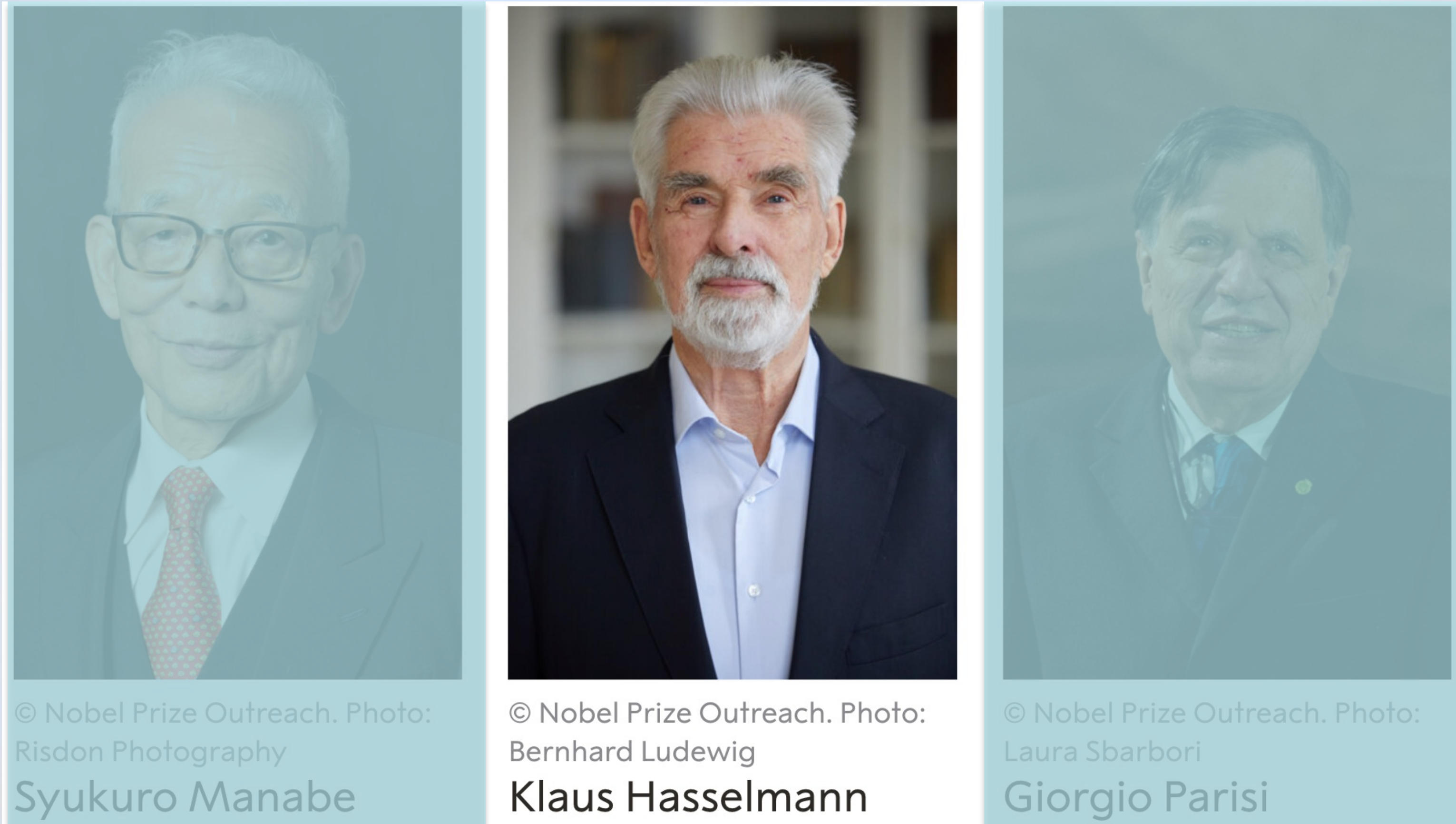
… to Giorgio Parisi "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales".

Less "Unexpected" implications of Giorgio Parisi's work for climate sciences:



- Stochastic resonance as a way to interpret aspects of paleoclimatic variability (e.g. 150k Milankovitch cycle; Benzi et al. 1982; the onset of D-O events; Braun et al. 2007);

- A multifractal theory of 3D turbulence (cfr. Lovejoy and Schertzer 2018);



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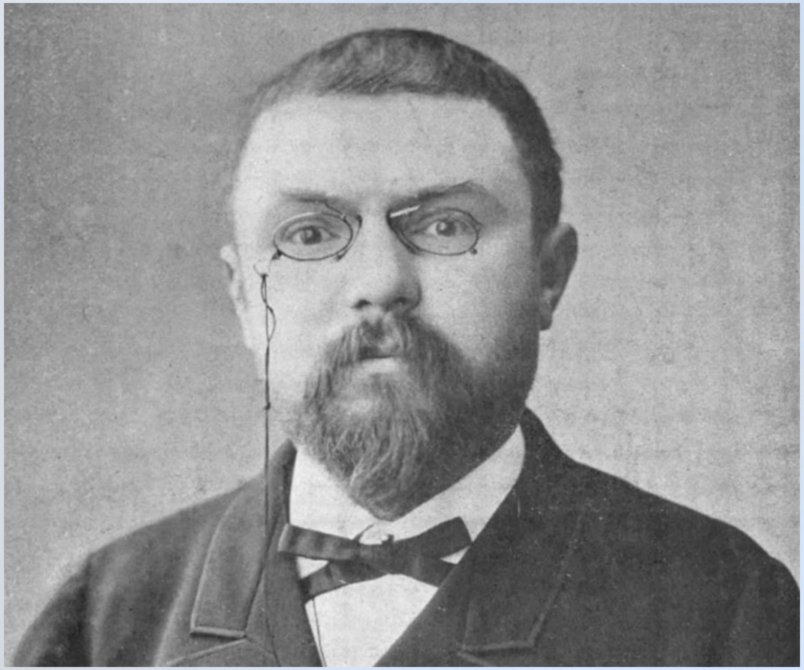
The wise man said…

*Why have **meteorologists** such difficulty in **predicting** the weather with any certainty? Why is it that showers and even storms seem to come **by chance** […]? We see that great disturbances are generally produced in regions where the atmosphere is in **unstable** equilibrium. The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed **somewhere**, but exactly where they are not in a position to say. […]*
*If they had been aware of this […] they could have known it beforehand, but the **observations** were neither sufficiently **comprehensive** nor sufficiently **precise**…*



(Henri Poincaré,
Science et Methode, 1908)

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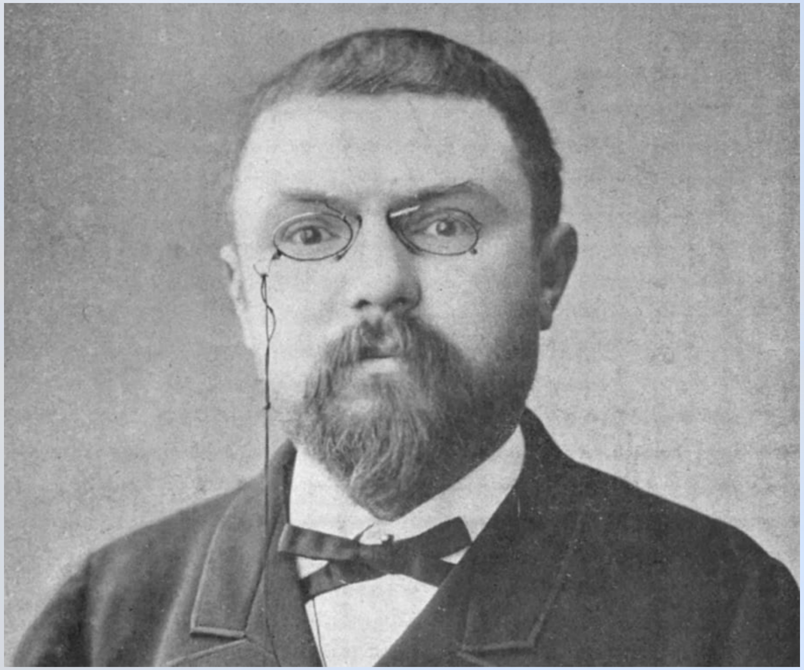


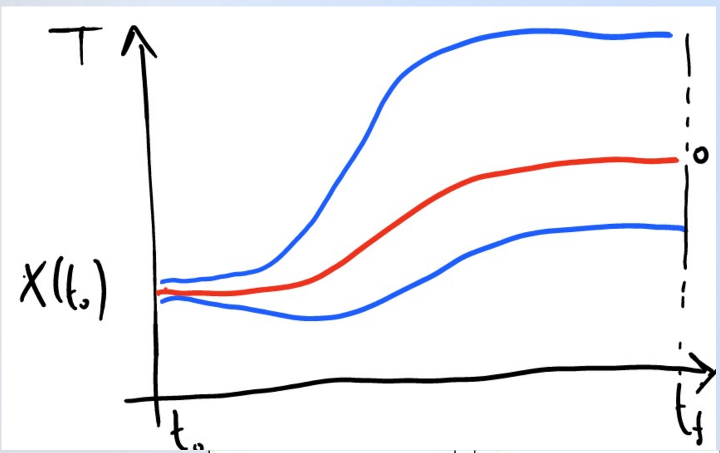
(Henri Poincaré,
Science et Methode, 1908)

The road to "**unpredictability**":
- Intrinsic (non-linear) instability;
- Incomplete initial conditions;

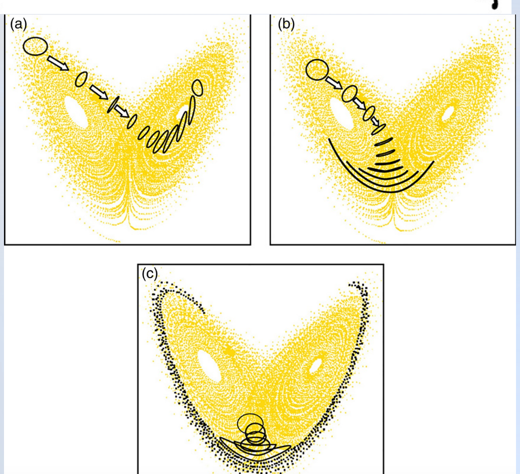
—> **chaos**

Predictability

Predictability of first kind: the aim is to address the deterministic evolution of the system, given a sampling of observed initial conditions. This is what people do in Numerical Weather Prediction (NWP).



Issue: there is no “perfect” initial condition (cfr. Poincaré). Different sampling (perturbations) lead to radically different trajectories.

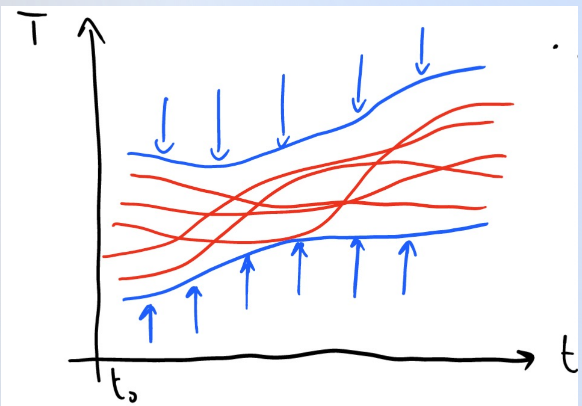


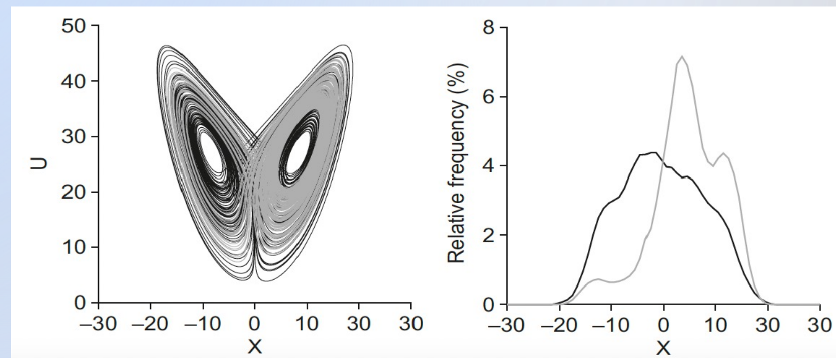
Possible workaround: ensemble prediction systems (EPS); sampling of predictability.

(Palmer 1993)

Predictability

Predictability of second kind: for larger time scales, the modeled sensitivity to boundary conditions is more relevant than to initial conditions. People are interested in the asymptotic behavior of the system when subjected to an external forcing.

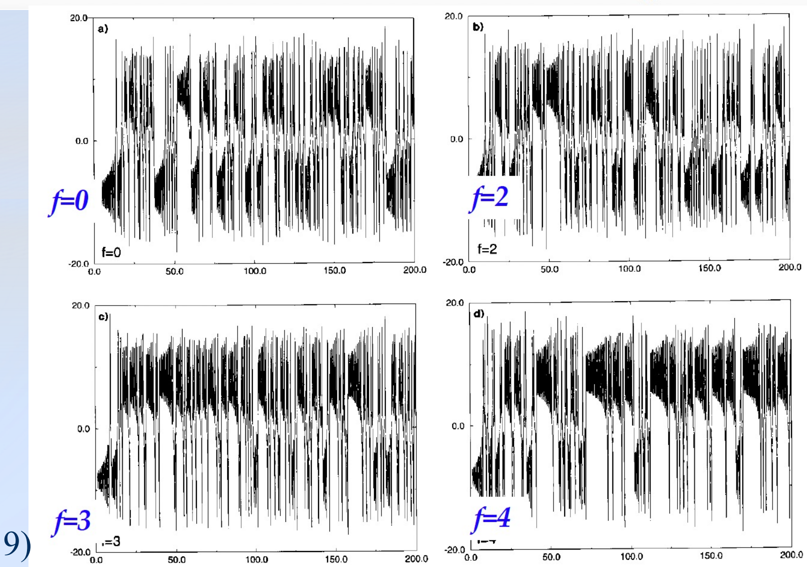




$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y + f$$

$$\frac{dz}{dt} = xy - \beta z + f$$



(Weisheimer 2019)

Slow vs. fast systems: the Hasselmann paradigm

$$\mathbf{z} = (x_1, x_2, \dots) = (\mathbf{x}, \mathbf{y})$$

The two subsystems have very different timescales for their evolution, with:

$$\tau_y \gg \tau_x$$

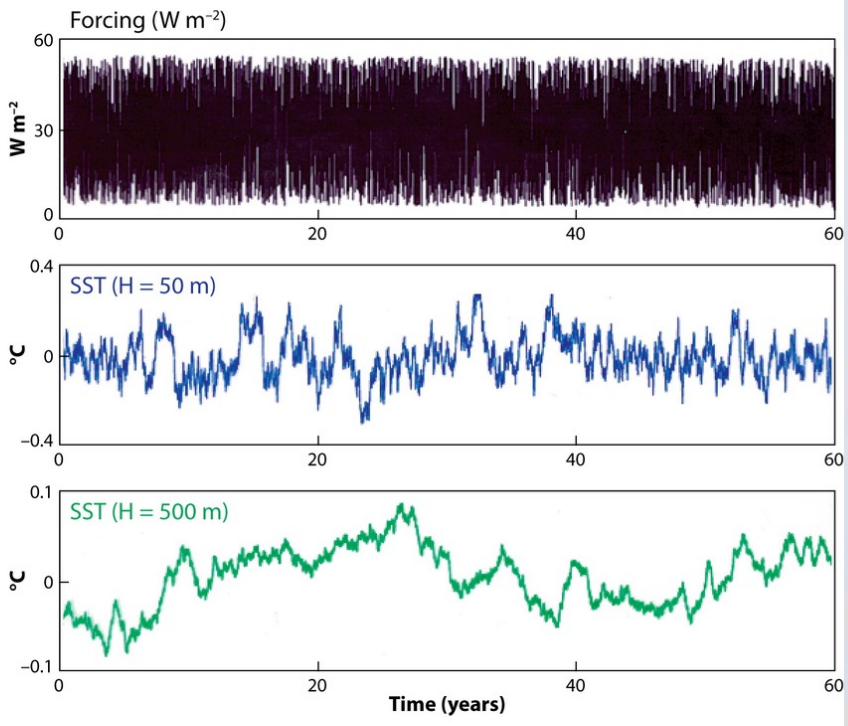
The evolution of the two subsystems is written as:

$$\frac{d\mathbf{y}}{dt} = u(\mathbf{x}, \mathbf{y})$$

$$\frac{d\mathbf{x}}{dt} = \frac{1}{\delta} v(\mathbf{x}, \mathbf{y})$$

where the fluctuations of the "slow" subsystem are described probabilistically by a Fokker-Planck equation (**random walk**), or equivalently:

$$d\mathbf{x} = \mathbf{F}(\mathbf{x})dt + \boldsymbol{\Sigma}(\mathbf{x})dW_t$$



Deser C, et al. 2010.
Annu. Rev. Mar. Sci. 2:115–43

EUREKA!

Here comes the **Nobel-worth** idea!

- A clear scale separation between fast and slow time evolution is found:
    1. Weather (the "atmosphere")
    2. Climate (the "ocean")

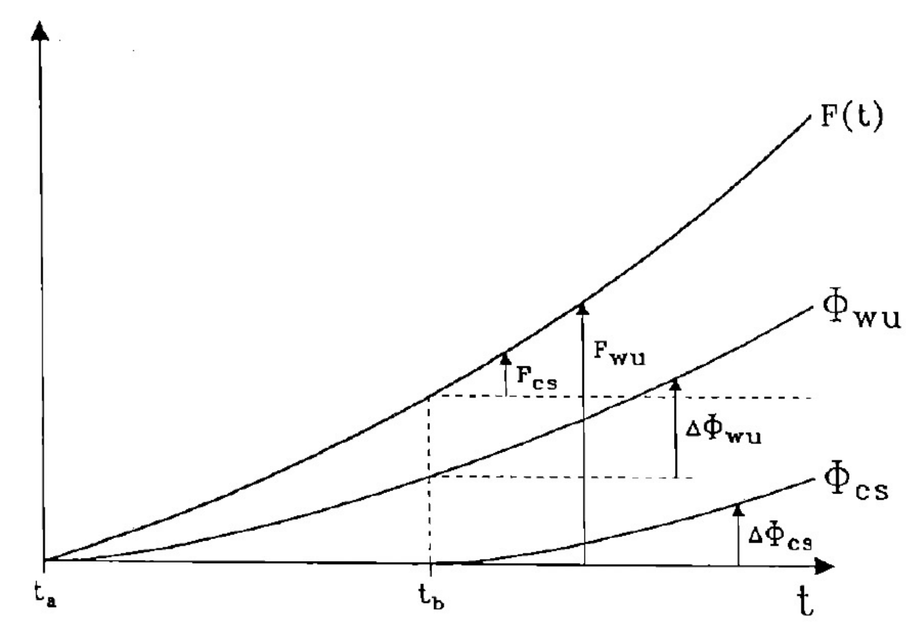
- The probabilistic evolution of the fast system is described in terms of a Brownian motion;

- The system might be or not be "deterministic". What matters in the long term is the evolution of its statistics!

A very practical problem: the cold-start error

In the old days of the MHz CPUs, running a coupled atmosphere-ocean model for several decades was really challenging!

Modelers started figuring up how they could run future climate projections without waiting for the model to go through the historical build-up in CO2 concentrations, starting from pre-industrial conditions (1800s);



(Hasselmann et al. 1992)

t_a: 1820 —> "warm-up" experiment
t_b: 1985 —> "cold-start" experiment

Depending on the starting time, the system is featuring different climatic change (no matter what CO2 evolution is prescribed as forcing). Why?

Hasselmann's paradigm was instrumental in understanding this problem!

The Hasselmann's feedback equation

When forced, the Hasselmann's model of slow climate fluctuations with stochastic "weather" noise, behaves like an exponential **relaxation process**, with timescale coincident with the one of the slow subsystem, governed by a feedback equation:

$$\frac{dT}{dt} = -\lambda T + \alpha F$$

Where λ and α are constants
F is a forcing dependent on time

In which the solution is given **analytically** as:

$$\Delta T(t) = -\alpha \int F(u) e^{(t-u)/\tau} du$$

With $\tau = 1/\lambda$ the timescale of the relaxation process

The evolution from time t_b for a linear forcing like $F(t) = F_1 \gamma [t - t_a]$ and $F_1 = \frac{1}{ln2}$ reduces to:

$$\Delta T(t) = \tau F_1 \gamma \alpha \left[t - t_b - \tau (1 - e^{-\frac{(t-t_b)}{\tau}}) \right]$$

$$\delta T(t) = -R(t - t_b) K(t_b - t_a)$$

$$K(t_b - t_a) = F_1 [1 - e^{-\frac{t_b - t_a}{\tau}}]$$

The different slopes of the two evolutions

Meaning that the two solutions with different initial times will have two different asymptotic behaviors!

A special case for Ruelle's response theory (RRT)

Hasselmann's ideas can be generalized using a rigorous mathematical framework:

For any observable (temperature, precipitation…) Φ, it can be shown (e.g. Ruelle 1998a,b, Ruelle 2009) that:

$$\langle \Phi(t) \rangle = \langle \Phi \rangle_0 + \sum_n \langle \Phi \rangle^{(n)}(t)$$

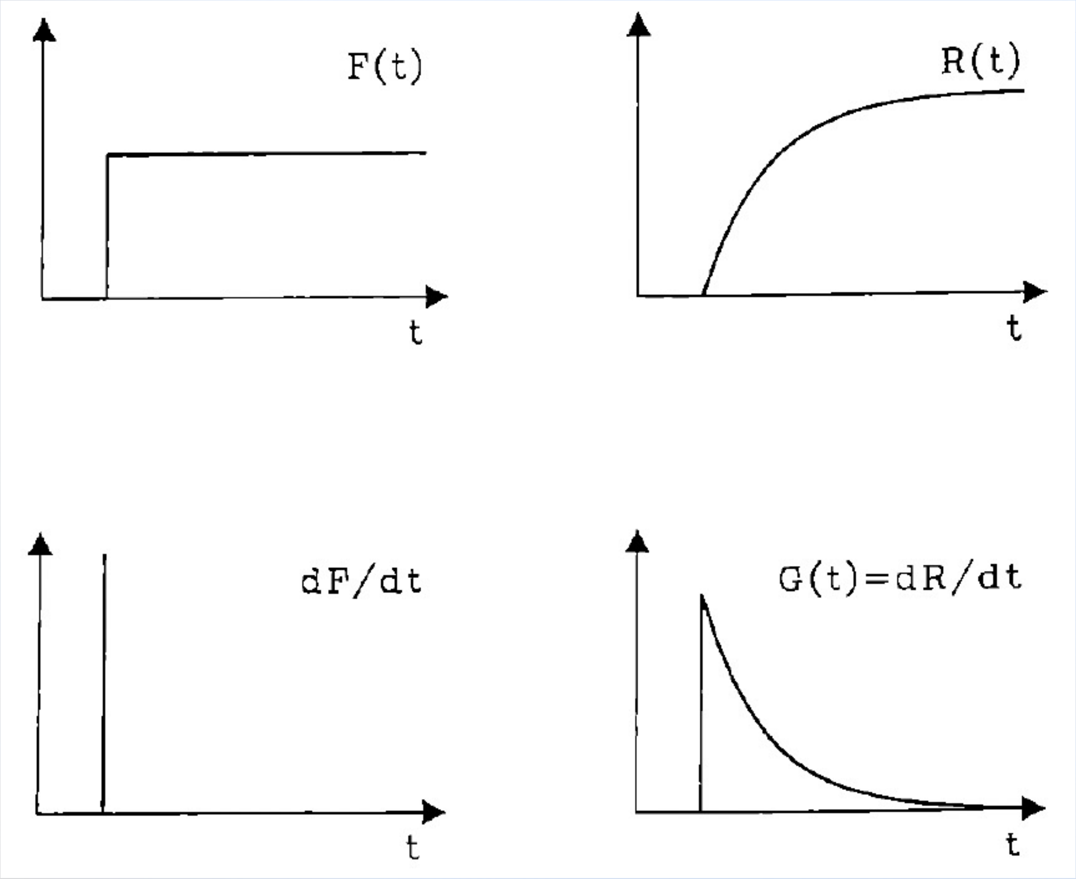
With the first order term of this expansion being called "linear term" and defined as:

$$\langle \Phi \rangle^{(1)}(t) = \int du G_{\Phi}^{(1)}(t-u) F(u)$$

G is the so-called "first-order Green's function" and it is the time derivative of R, the linear response function.

To predict the linear evolution of the observable Φ, we need to know G or R!

A very basic algorithm



(Hasselmann et al. 1992)

1. For an instantaneous step forcing, the convolution is trivial and the Green's function is written as:

$$G_{\Phi}^{(1)}(t) = \frac{1}{F} * \frac{d\Phi^{(1)}(t)}{dt} = \frac{dR(t)}{dt}$$

Which is the same as solving the linear feedback equation, whose solution is:

$$R(t) = \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$$

2. Once we know $G_{\Phi}^{(1)}(t)$, we can multiply it by any other forcing F(t) and predict the evolution of the observable Φ in response to such forcing!

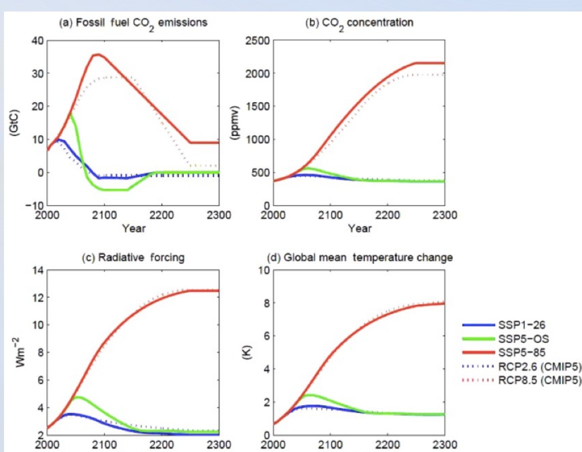
Hasselmann and the linear response theory: 30 years later

A clear scale timescale separation is an idea that is usefully exploited even for the analysis of projections in state-of-the-art Earth System Models:

Experiment short name	CMIP6 label	Experiment description	Forcing methods	Start year	End year	Minimum no. years per simulation	Major purpose
DECK experiments							
AMIP	*amip*	Observed SSTs and SICs prescribed	All; CO_2 concentration prescribed	1979	2014	36	Evaluation, variability
Pre-industrial control	*piControl* or *esm-piControl*	Coupled atmosphere–ocean pre-industrial control	CO_2 concentration prescribed or calculated	n/a	n/a	500	Evaluation, unforced variability
Abrupt quadrupling of CO_2 concentration	*abrupt-4×CO2*	CO_2 abruptly quadrupled and then held constant	CO_2 concentration prescribed	n/a	n/a	150	Climate sensitivity, feedback, fast responses
1 % yr^{-1} CO_2 concentration increase	*1pctCO2*	CO_2 prescribed to increase at 1 % yr^{-1}	CO_2 concentration prescribed	n/a	n/a	150	Climate sensitivity, feedback, idealized benchmark
CMIP6 historical simulation							
Past ~ 1.5 centuries	*historical* or *esm-hist*	Simulation of the recent past	All; CO_2 concentration prescribed or calculated	1850	2014	165	Evaluation

CMIP6 DECK experiments: net zero forcing, stabilization, ramp-up, historical scenarios.

Required by all models;



CMIP6 future projections: based on «shared socio-economic pathways».

Guide impact studies;

Hasselmann and the linear response theory: 30 years later

Model	N. hours per sim. year	Energy per month	N. hours per month	Joule per sim. year
CM2.6 S	2.12×10^5	3.14×10^{12}	5.64×10^7	1.17×10^{10}
CM2.6 T	1.81×10^5	3.14×10^{12}	5.64×10^7	1.00×10^{10}
CM2.5 T	14 327	3.14×10^{12}	5.64×10^7	7.99×10^8
FLOR T	5844	3.14×10^{12}	$5.6 \times 10^7 4$	3.26×10^8
CM3 T	2974	3.14×10^{12}	5.64×10^7	1.66×10^8
ESM2G S	279	1.97×10^{12}	3.02×10^7	1.82×10^7
ESM2G T	235	1.97×10^{12}	3.02×10^7	1.53×10^7
CM4H T	7729	1.68×10^{12}	3.47×10^7	3.75×10^8
CM4L T	3277	1.68×10^{12}	3.47×10^7	1.59×10^8
ESM4L T	5340	1.68×10^{12}	3.47×10^7	2.59×10^8
ARPEGE5-NEMO T	5190	5.92×10^{12}	5.88×10^7	5.22×10^8
EC-Earth3.2 T	12 126	1.62×10^{12}	3.86×10^7	5.08×10^8
EC-Earth3.2 S	21 481	1.62×10^{12}	3.86×10^7	8.99×10^8
CESM1.2.2-NEMO T	59 100	9.00×10^{12}	6.76×10^7	7.87×10^9
MPI-ESM1 T	3363	1.30×10^{12}	2.65×10^7	1.64×10^8
NorESM1 S	1369	1.41×10^{12}	1.64×10^7	1.18×10^8
IPSL-CM6-LR S	2166	5.92×10^{12}	5.88×10^7	2.18×10^8
HadGEM3-GC2 T	6504	4.27×10^{12}	8.5×10^7	3.27×10^8

(Balaji et al., 2017)

Computational resources – CMIP6

CMIP3: 17 institutes (groups) and 25 models (40 TB)
- Total years simulated: 70,000
- Individual models simulated 500 to 8,400 years (mean: 2,800 yr)
- Individual groups simulated on average 70,000/17 = 4,100 yr

x50

CMIP5: 26 institutes (groups) and 60 models (2 PB)
- Total years simulated: 330,000 (estimate Oct 2014)
- Individual models simulated on average 330,000/60 = 5,500 yr
- Individual groups estimated on average 330,000/26 = 13,000 yr

→ 4.3 Mio files

CMIP6: length of simulations similar to CMIP5, but higher resolution models, larger ensemble sizes, more diverse experiment structure

→ Factor of 20: 36 PB in 86 Mio files
→ Factor of 50: 90 PB in 215 Mio files

→ Careful planning required to determine subset of CMIP6 model output to best serve WHOI community

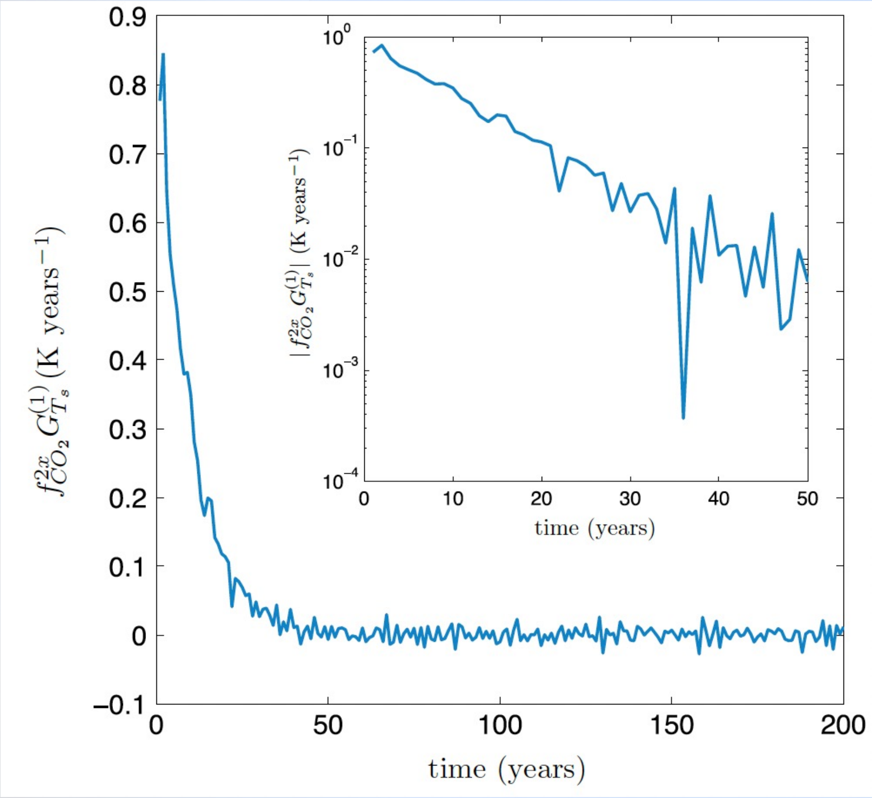
According to Taylor & Balaji (2015) and Denvil (2015)

(Ummenhofer, 2019)

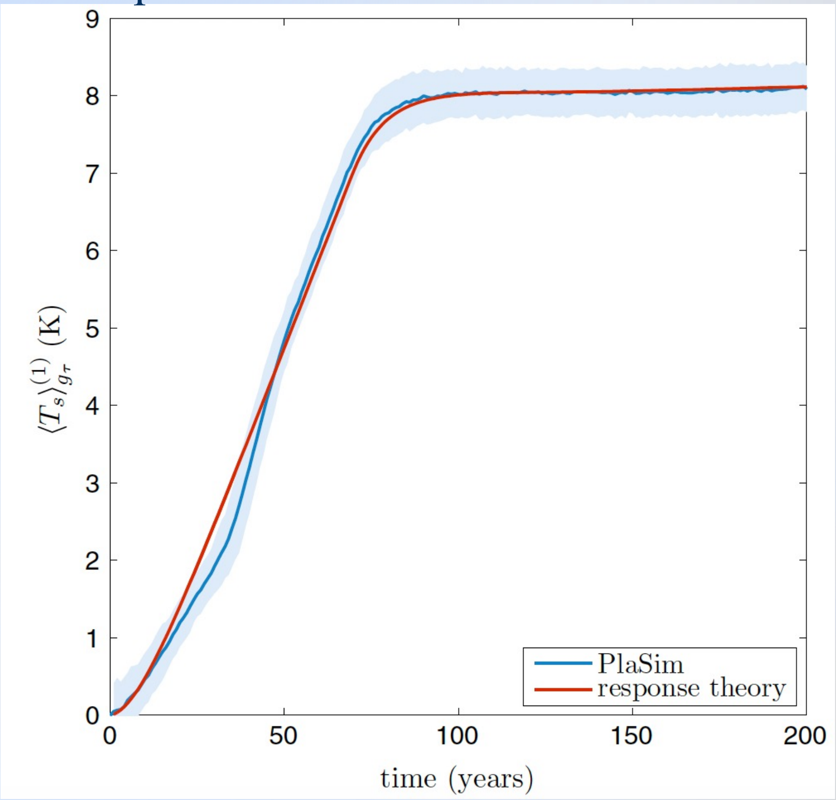
Each model simulation hopefully comes with an ensemble of realizations, crucial to sample internal variability (Deser et al. 2012)

An idealized case with an atmospheric-only GCM

Green's function (G) for T2m



Predicted (red) and modeled (blue) response to 1%CO2 increase



(Ragone et al. 2015)

A practical implementation with a Coupled GCM

- Model version: MPI-ESM-CR v1.2 - ECHAM6 (T31L31) + MPIOM (GR30L40);
- 2 ensembles: 20 members for each ensemble with same initial conditions;
- Integration period: 2000 years for the predictor, 1000 years for the predictand;
- Observables: T2m, AMOC at 26N and ACC at the Drake passage;

PREDICTOR

A step increase in CO_2 concentrations at time t=0 until 2x the preindustrial value (0-2000 yrs);

PREDICTAND

A ramp function experiment:

- CO_2 linear increase by 1% until doubling (0-70 yrs);
- Stationary CO_2 with 2x the preindustrial value (70-1000 yrs);

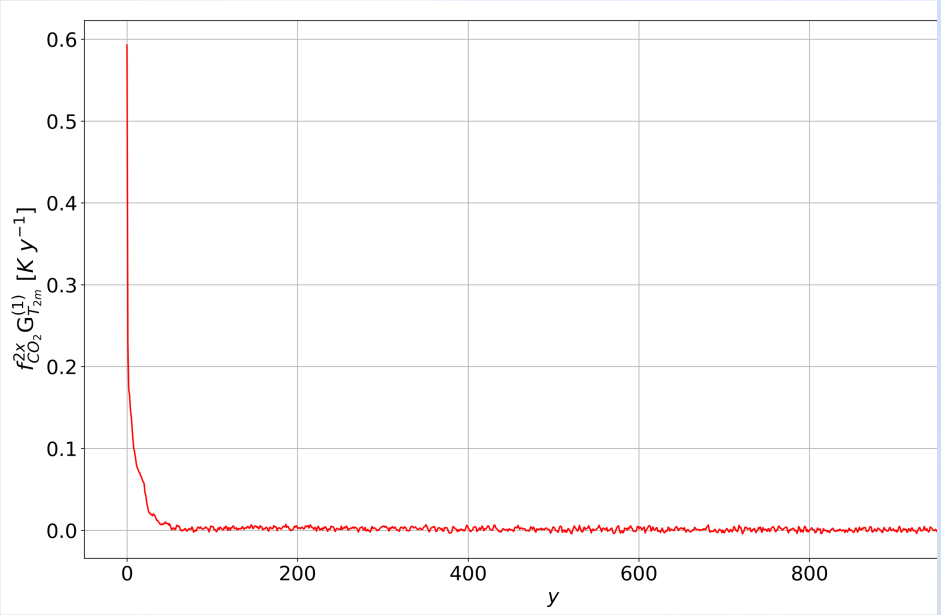
1. We consider the comfortable situation of a step-like forcing (cfr. Eq. 8)

2. We compute the Green’s function from an ensemble of model realizations

3. We use the convolution to predict the response to a transient forcing of the same amplitude

A practical implementation with a Coupled GCM

Green's function (G) for T2m



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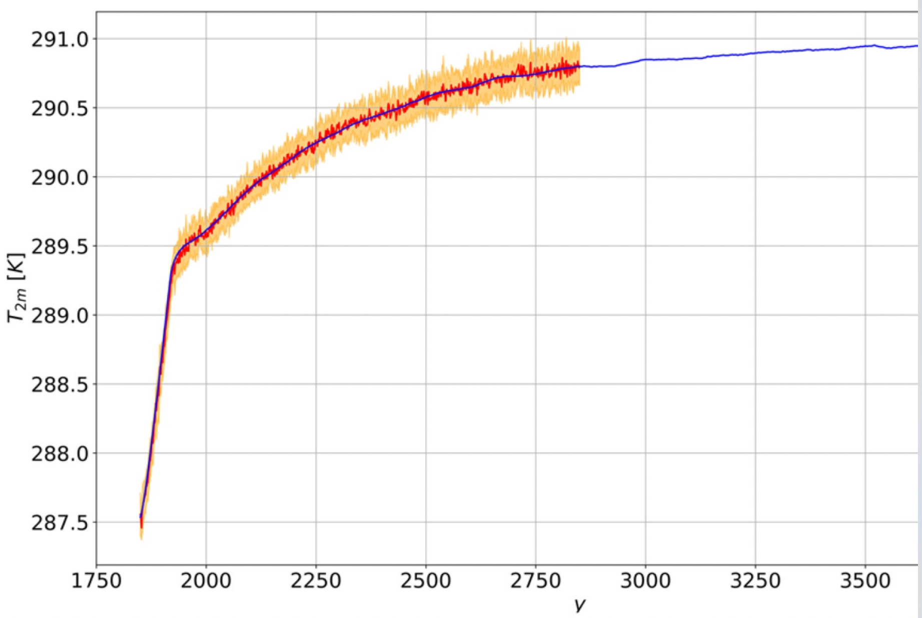
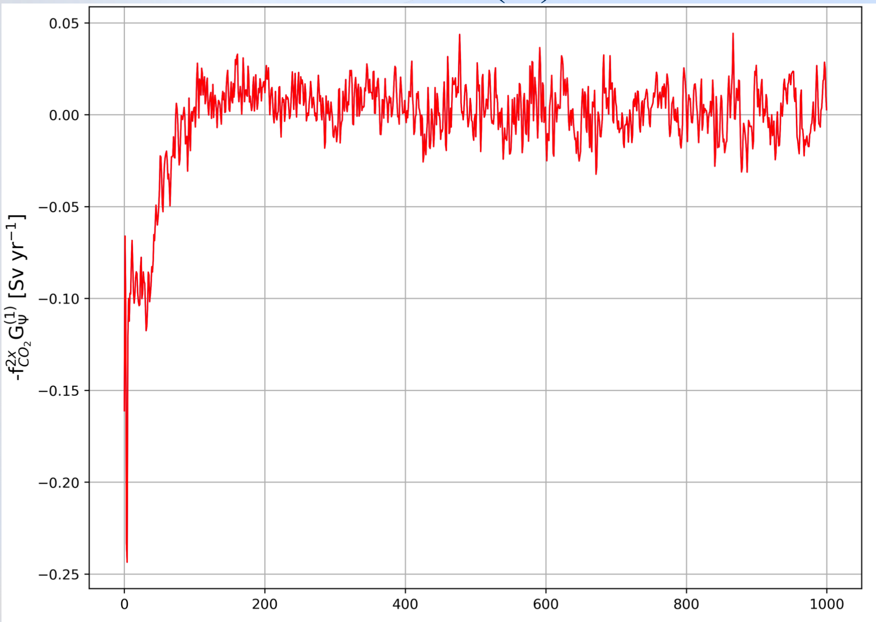


(Lembo et al. 2020)

A practical implementation with a Coupled GCM

Green's function (G) for AMOC



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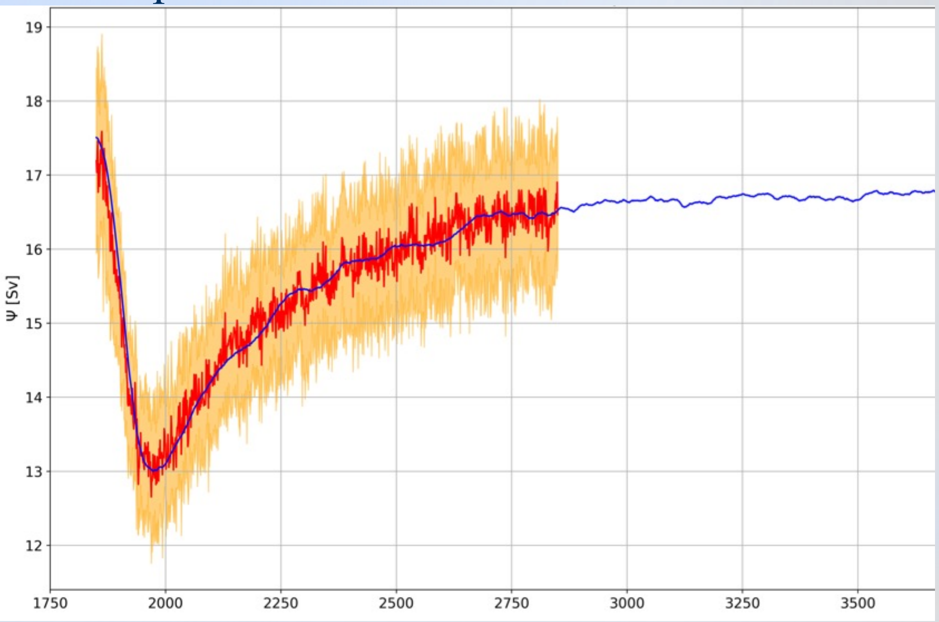
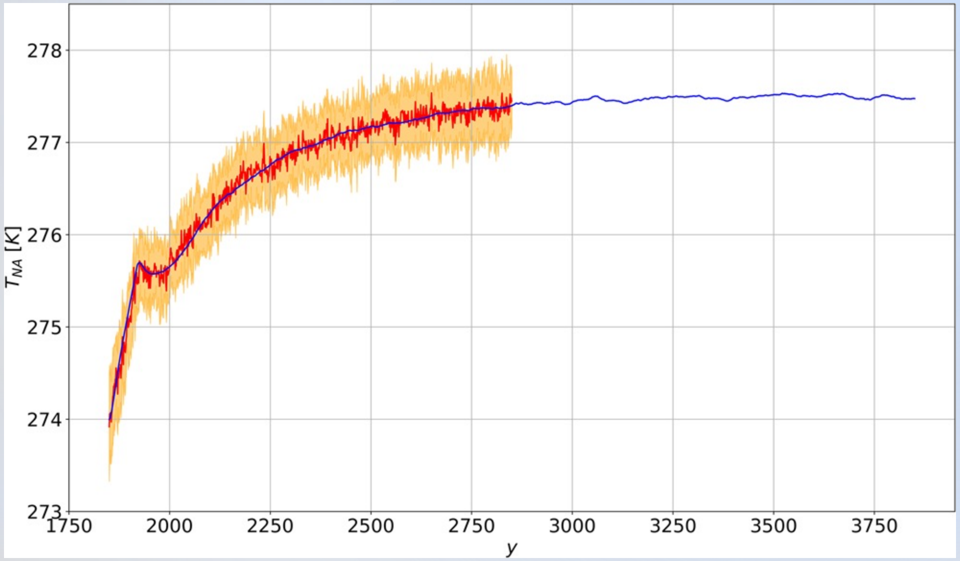


(Lembo et al. 2020)

A practical implementation with a Coupled GCM

North Atlantic mean temperature



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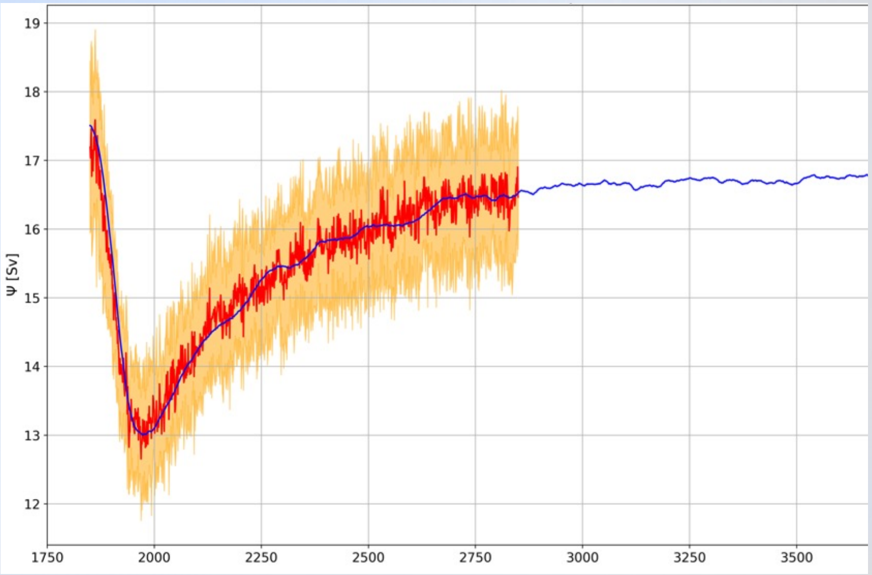
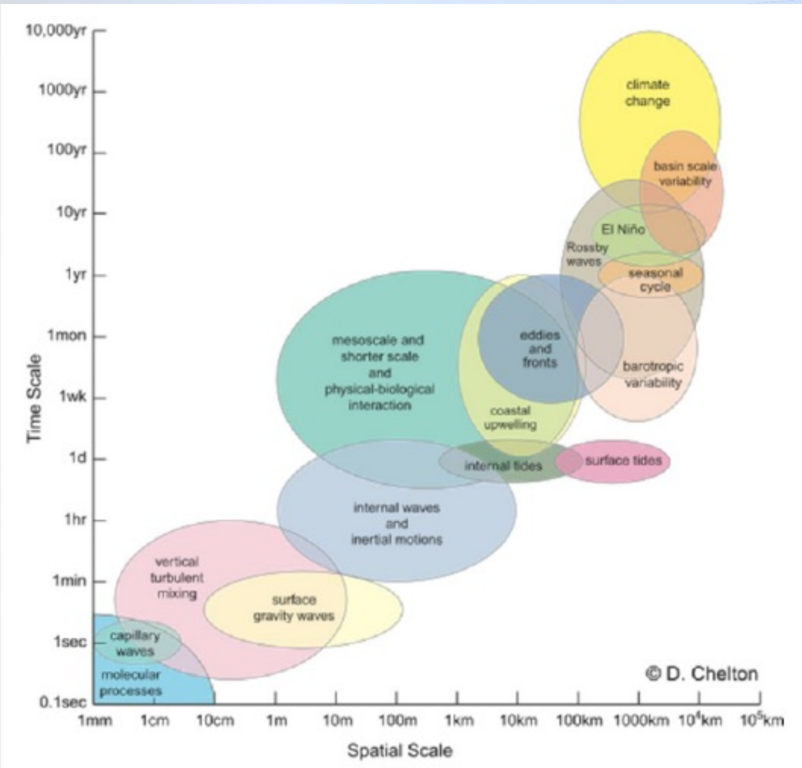


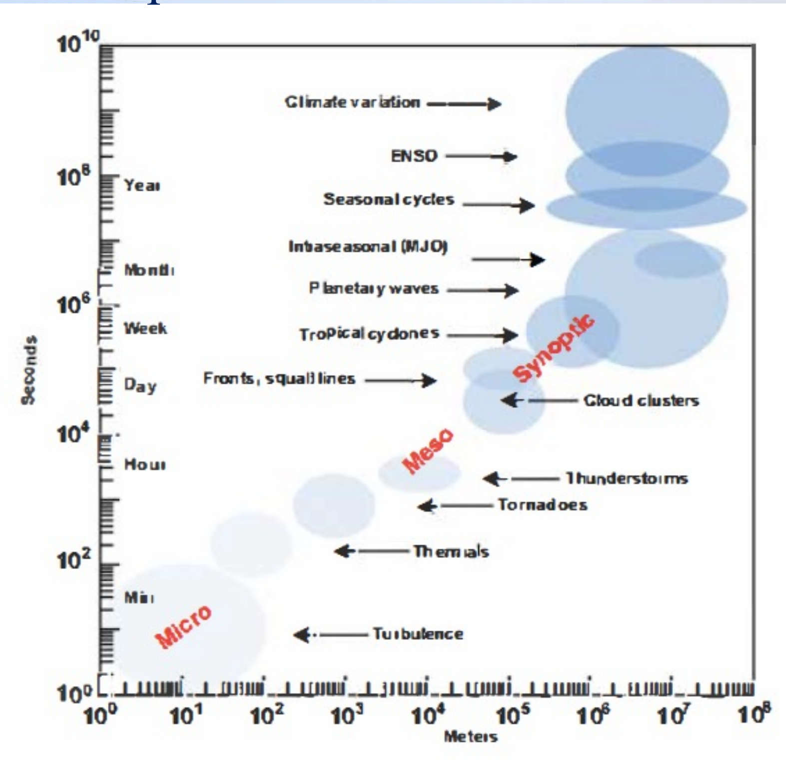
(Lembo et al. 2020)

The multi-scale nature of the climate system

Oceans



Atmosphere

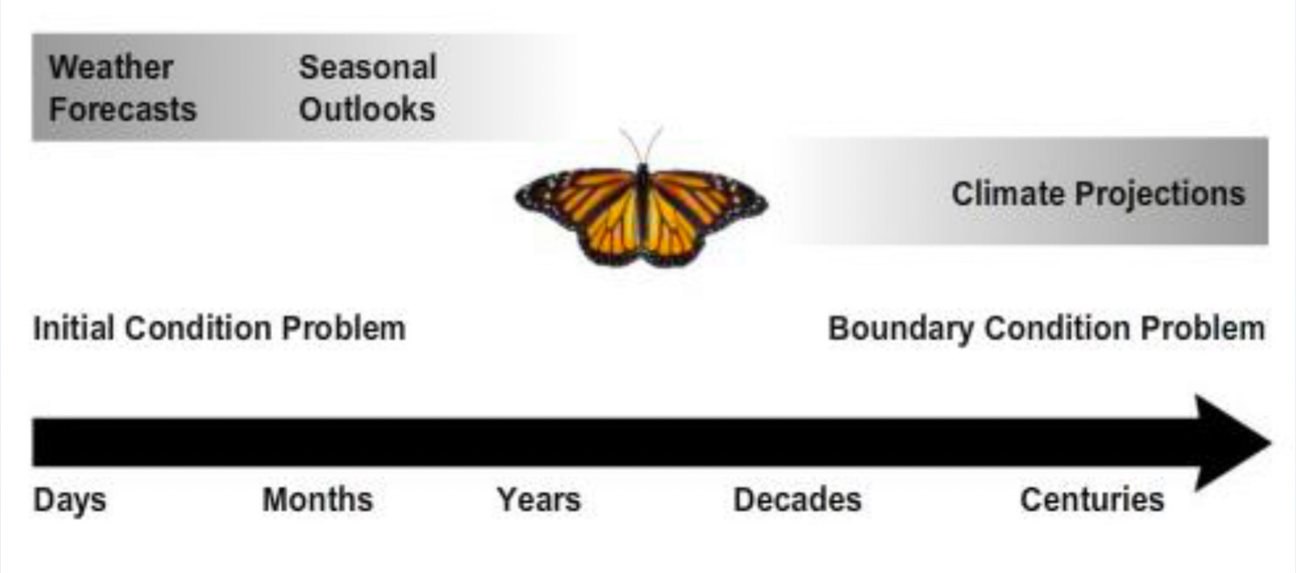


(Ghil and Lucarini 2019)

1) The assumption of strong timescale separation is somehow too restrictive;
2) Stochastic «fast» scales influence «slow» scales in a non-linear way;
3) High-dimensional chaos generates stochastic processes that affect the dynamics of interest in the system;

The limits of linear response

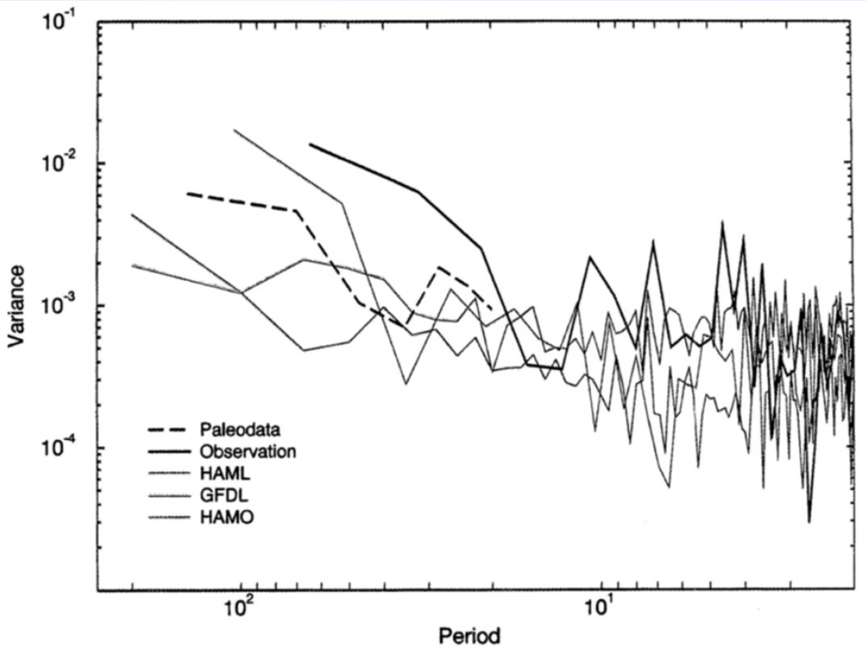
1. Climate predictions: we are beyond weather forecasting, but we still care about the butterfly in Amazonia…



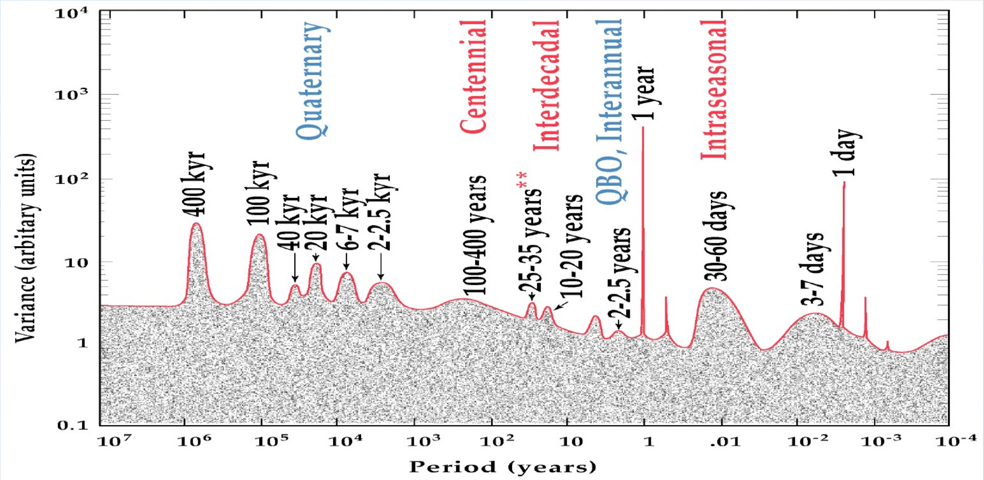
(Weisheimer 2019)

The limits of linear response

2. Detection and attribution: the forcing is time-dependent, we need to disentangle the forced signal from natural variability. In other words, what does natural variability look like?



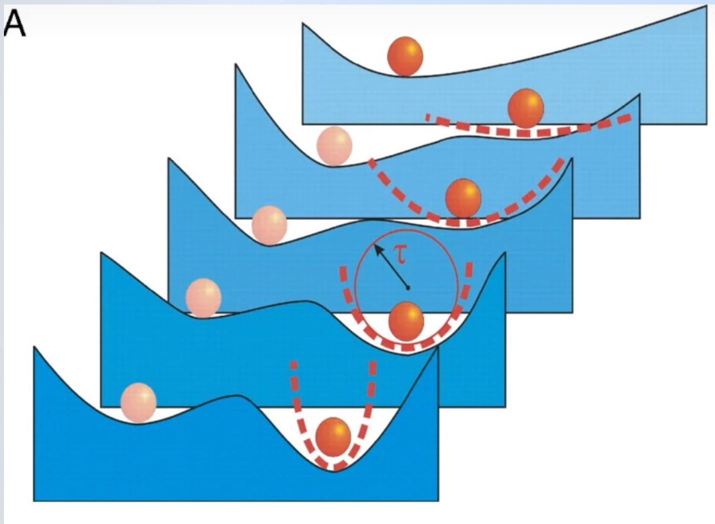
(Hegerl et al. 1996)

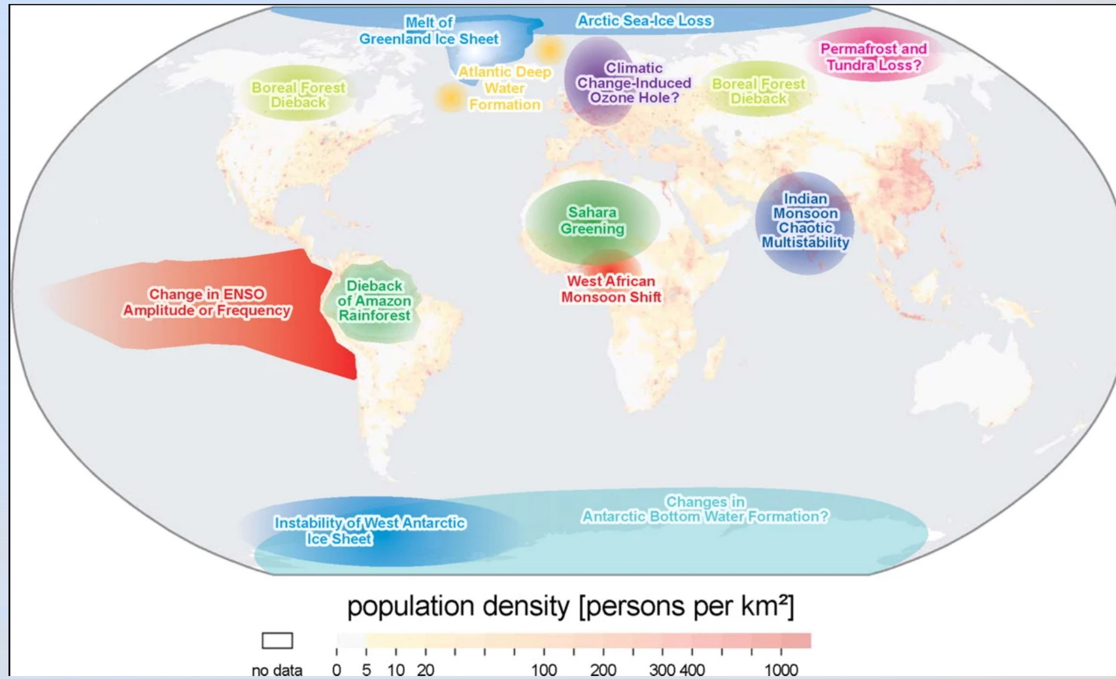


(Ghil 2001)

The limits of linear response

3. Climatic tipping points: aka the catastrophe of the asymptotic response!

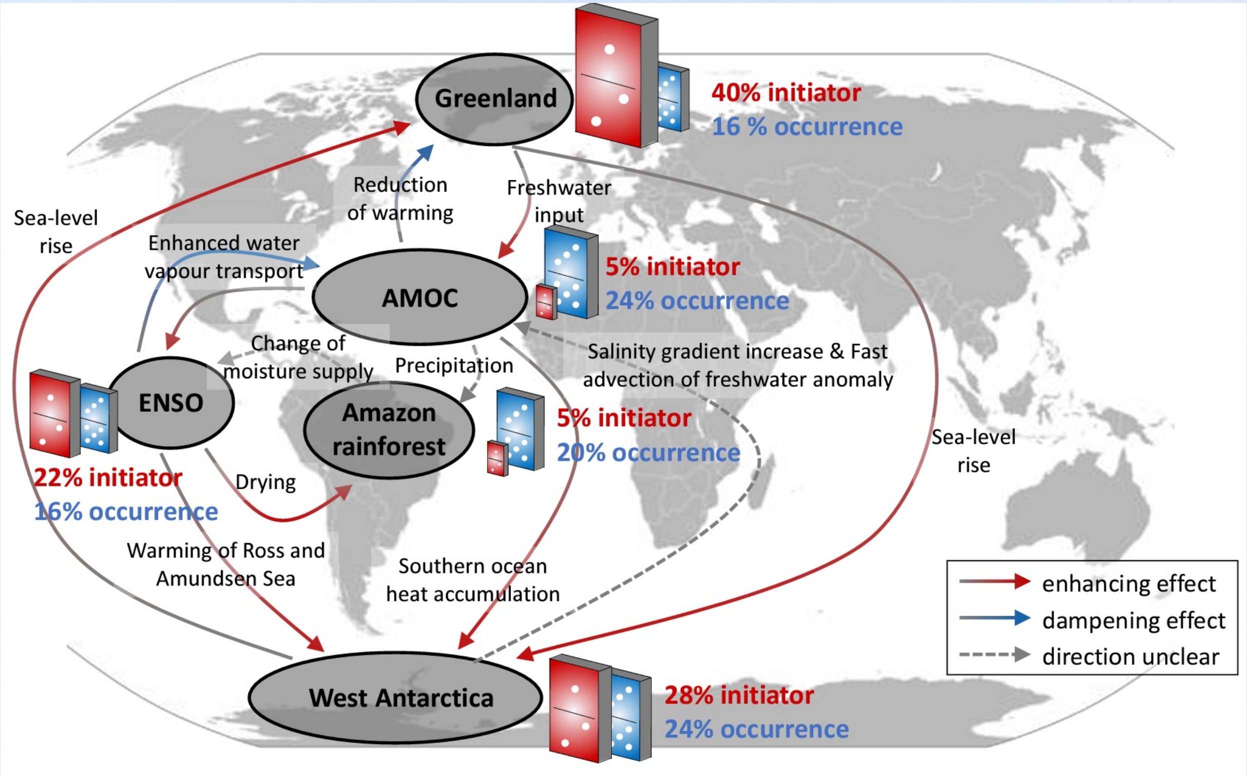




The basin of attraction of the system is changed by an external forcing until the system suddenly finds itself in a different mean state!

(Lenton et al. 2008)

Tipping points are the real elephant in the room of climate change?



(Wunderling et al. 2021)

Tipping points are not only suddenly changing the response of the system in local elements, but also have global effects, cascading on each other!

Thank you for the attention!

v.lembo@isac.cnr.it

What is climate?

“Following Aristoteles ‘definitio fit per genum proximum et differentiam specificam’, we can describe the climate system as a dynamical system consisting of thermodynamically open subsystems, being externally [forced], dissipative, and high-dimensional.”

(M. Claußen, formerly MPI-M director, now retired)

A minimal, non rigorous glossary…

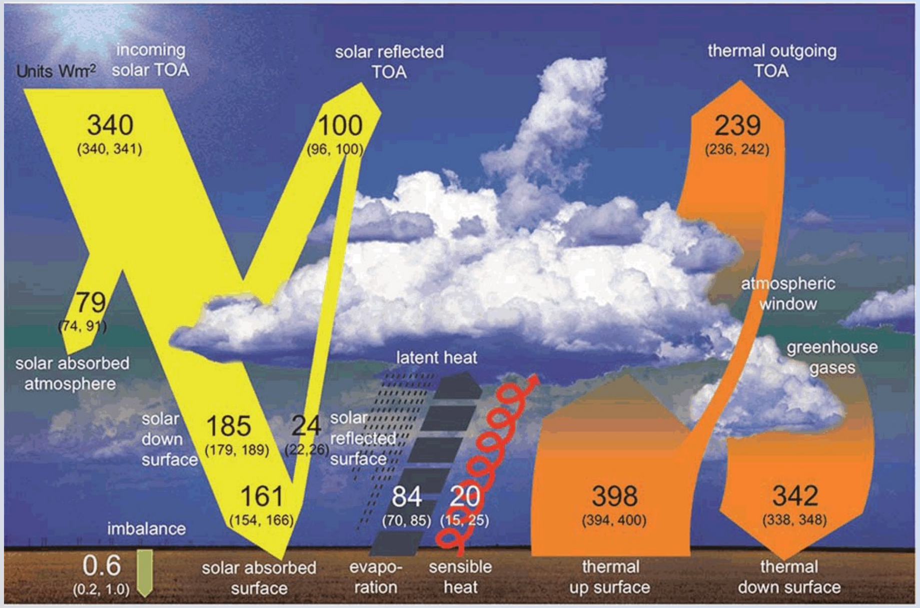
- **Deterministic:** a system described exactly by a set of differential equations. Knowing exactly the initial conditions, we could describe the evolution of such system;

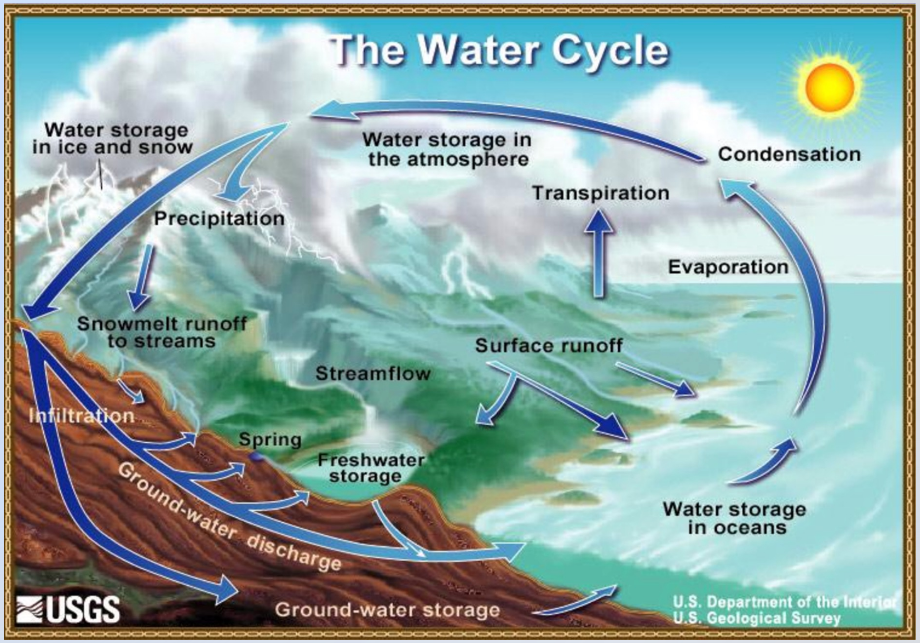
- **Stochastic:** a system, or a process, that can be described statistically by a "random" probability distribution function (e.g. random walk, red/white/multiplicative noise…);

- **Chaotic:** a system in which the non-linear growth of perturbations lead to qualitatively divergent solutions after a finite amount of time;

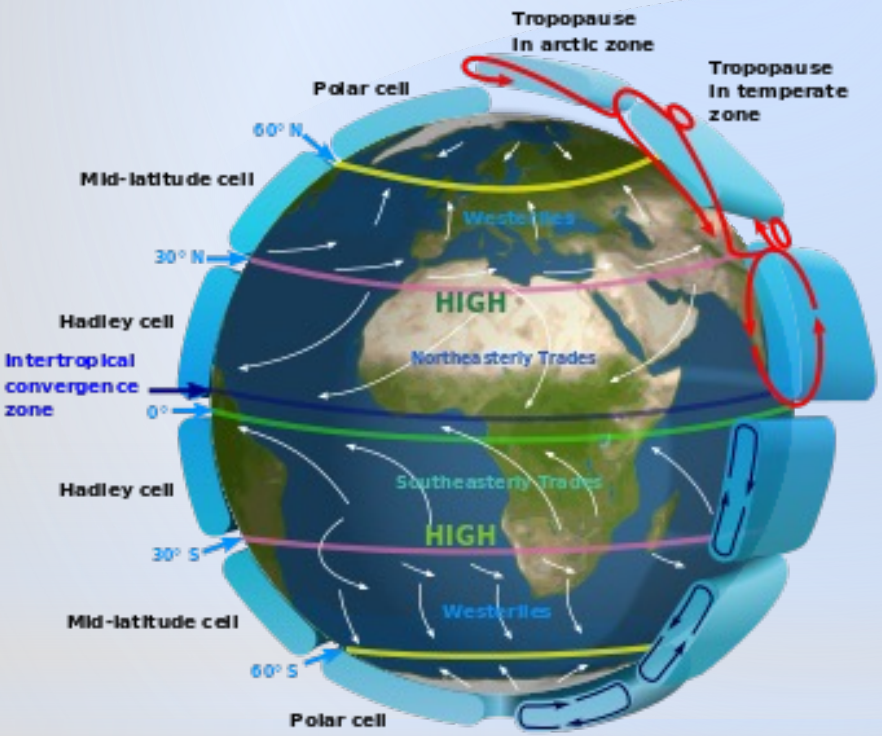
- **Autonomous:** a system, or a model describing it under certain assumptions, in which there is no explicit dependence on time (e.g. a periodic system);

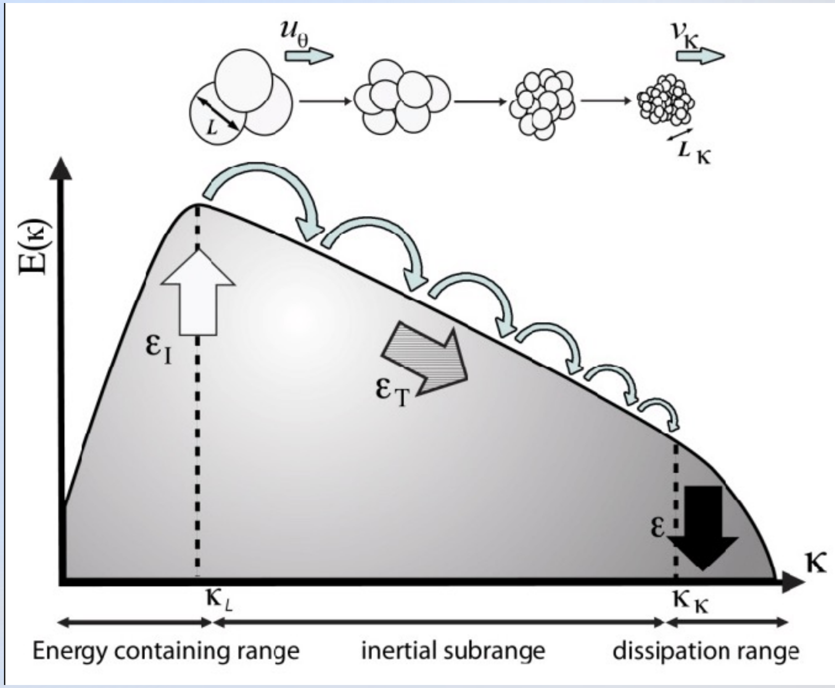
“Externally forced, thermodynamically open subsystems”





"Dissipative"





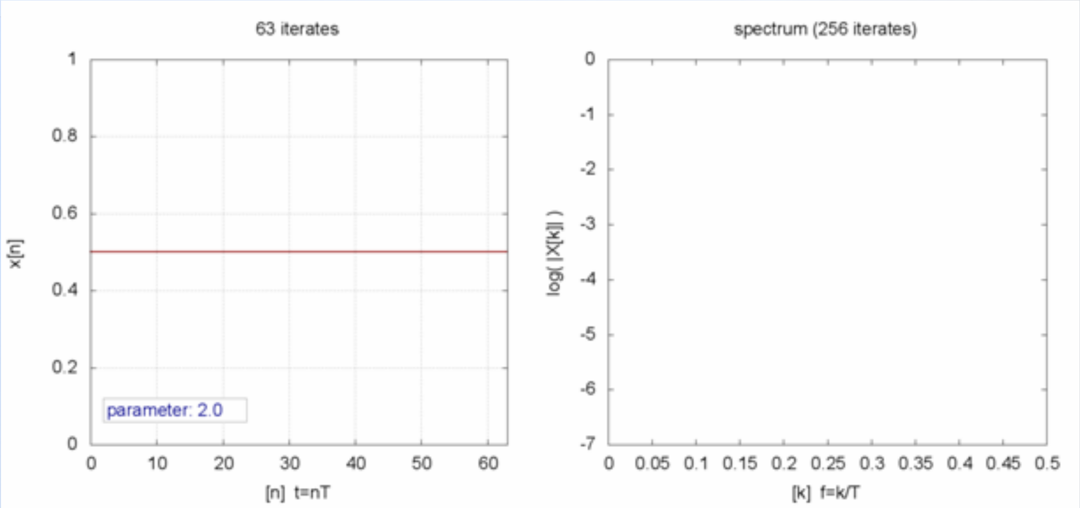
"High-dimensional"?

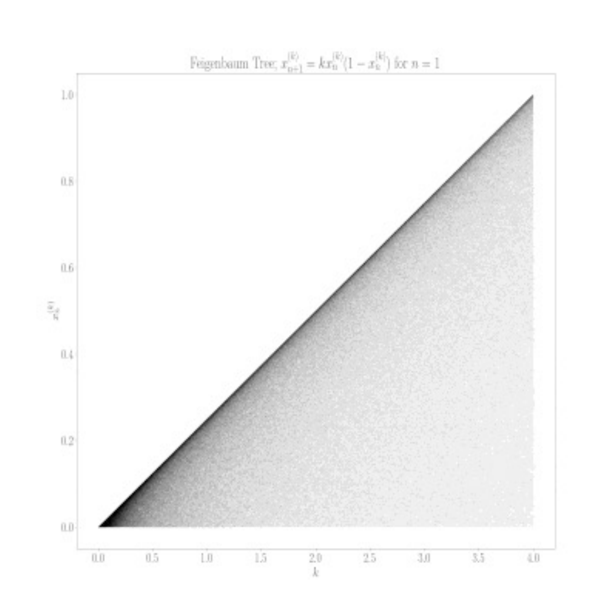
"Martin, wait a second… Is "high dimensional" really a defining feature?"

The logistic map

A classical demographic model:

$$x_{n+1} = r x_n (1 - x_n)$$





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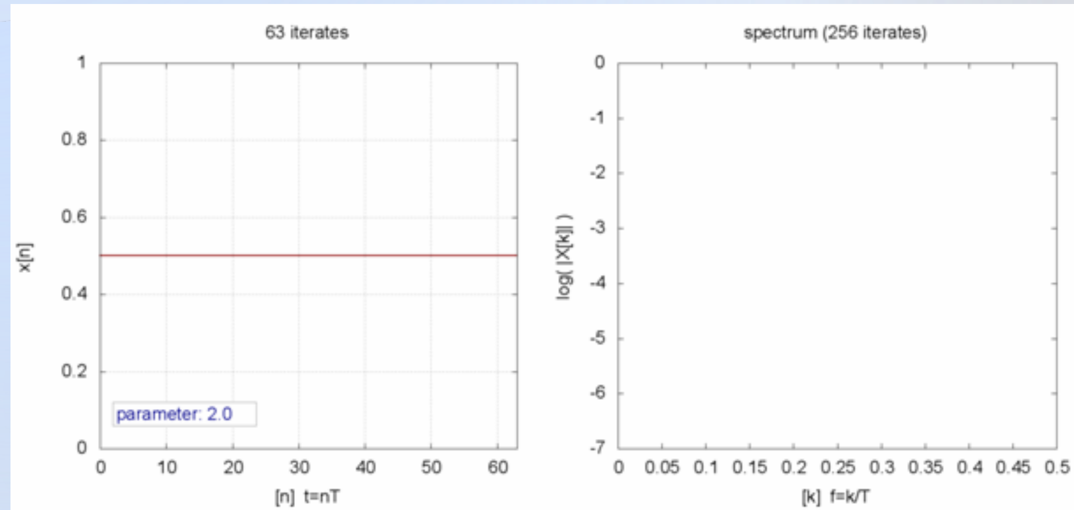
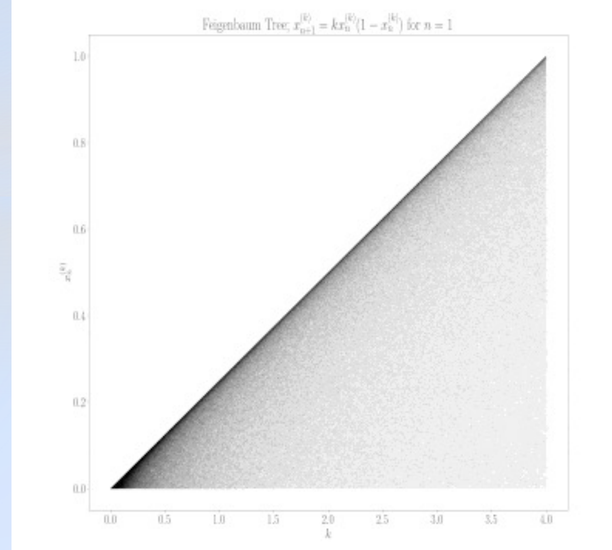


Shows that non-linear dynamics emerges for systems with very small degrees of freedom!



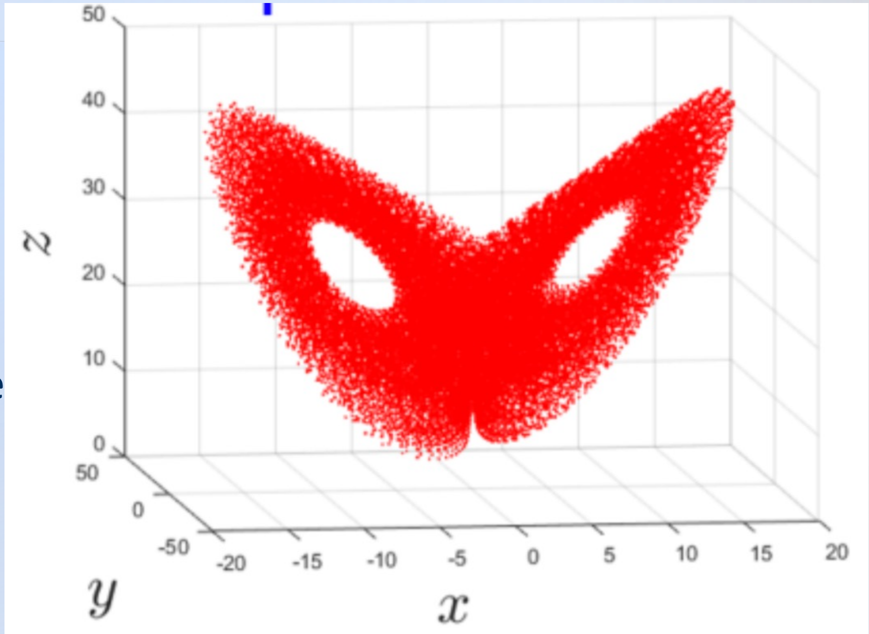
Lorenz 63 model - deterministic chaos out of convection!

A minimal model for convection:

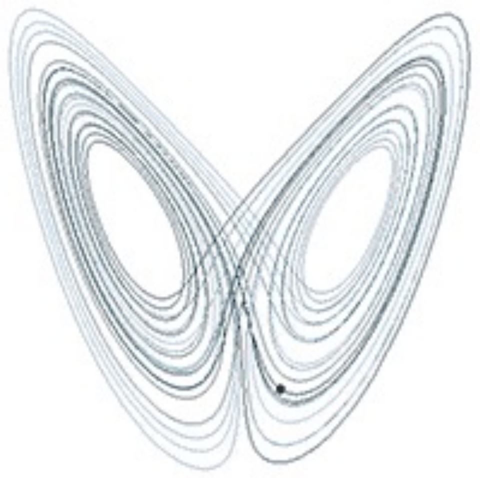
$$\frac{dx}{dt} = \sigma(y - x)$$ Rate of convection

$$\frac{dy}{dt} = x(\rho - z) - y$$ Horizontal temperature

$$\frac{dz}{dt} = xy - \beta z$$ Vertical temperature



A 3-degrees of freedom model relevant to the dynamics of the atmosphere shows a nonlinear behavior!



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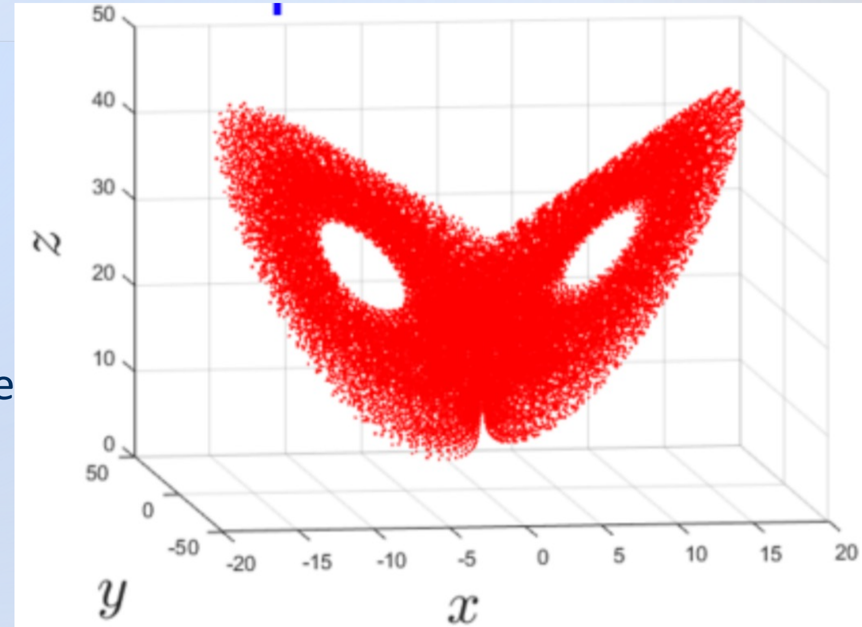
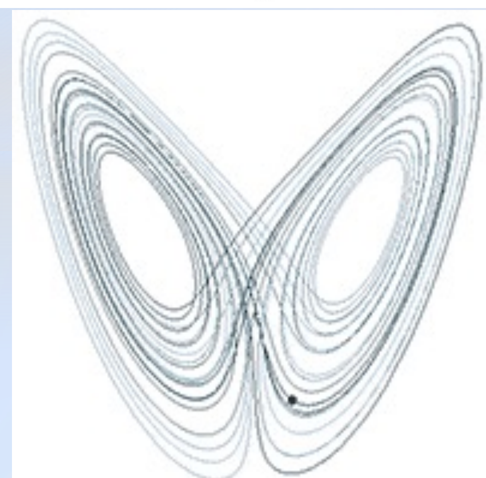
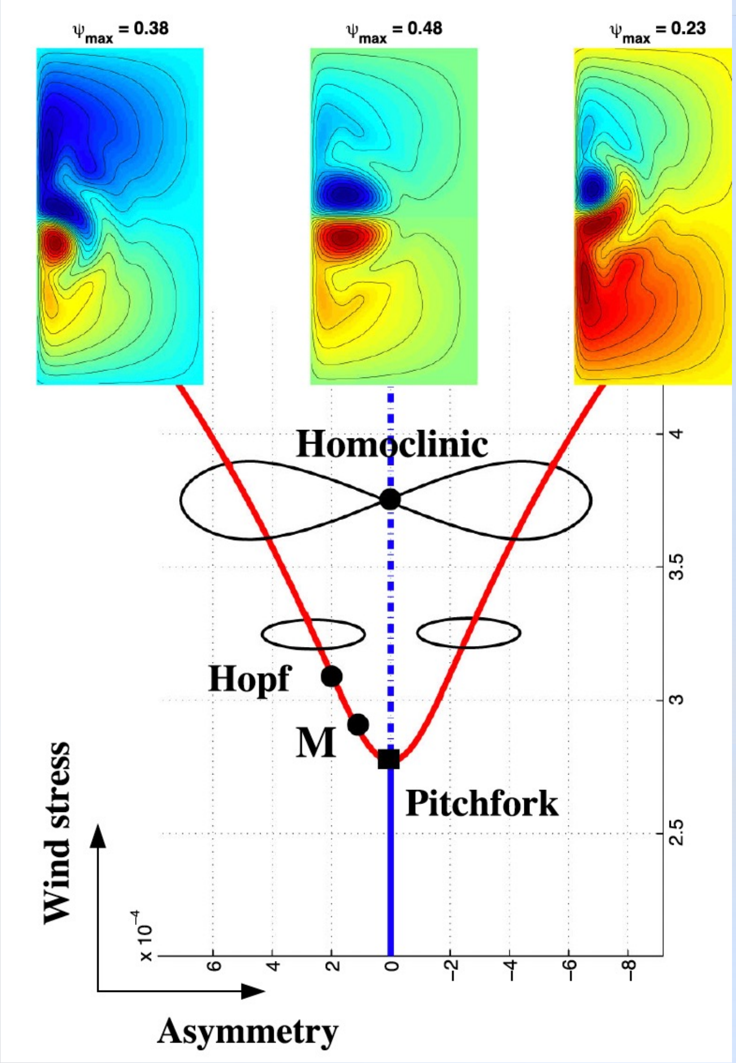


The system is ”deterministic”, “autonomous”, but trajectories diverge exponentially:



DETERMINISTIC CHAOS!

Routes to chaos



How does chaos possibly arise in a (anti)symmetric autonomous system?

Ex.: a double-gyre quasi-geostrophic oceanic model (Simonnet et al. 2005)

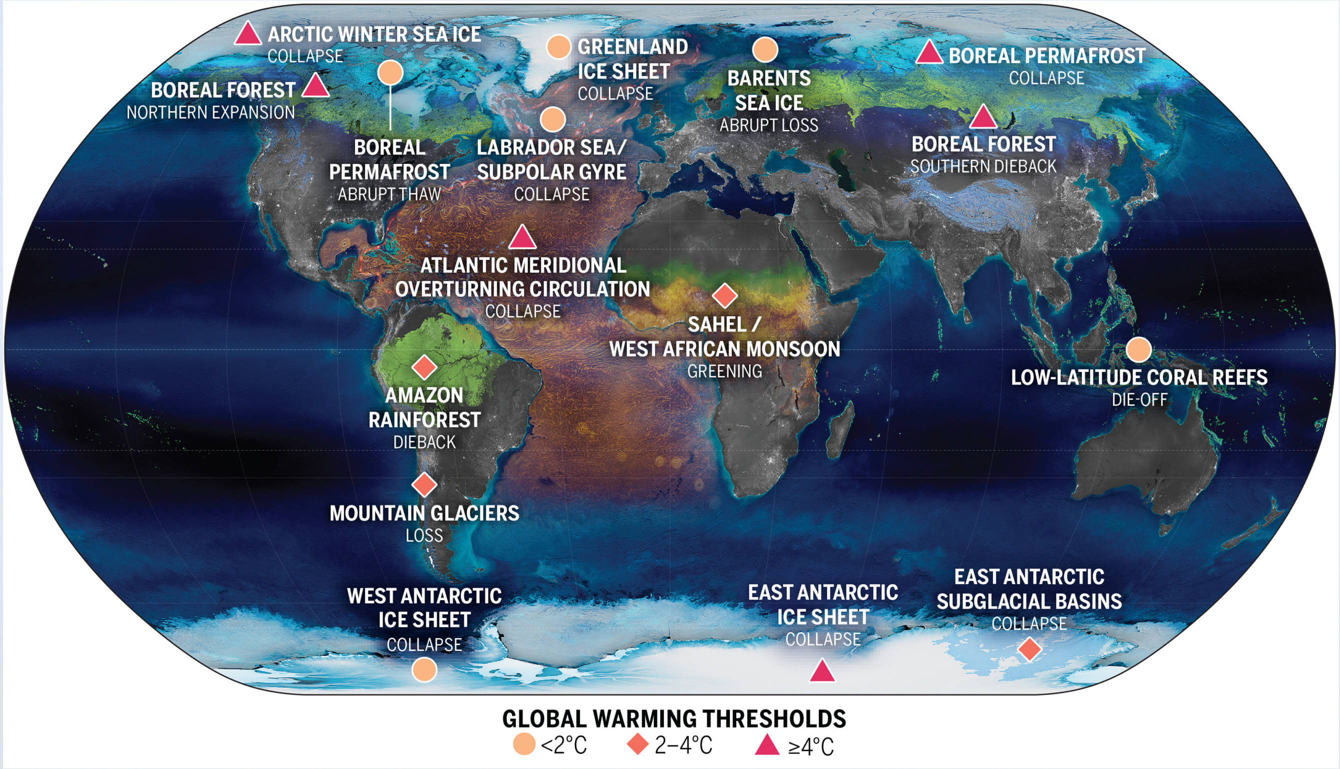
- Infinitely dimensional
- One steady solution if the friction is weak;

Increasing the wind stress:

- A pitchfork bifurcation
- A Hopf bifurcation (local)
- A homoclinic bifurcation (global)

3 successive bifurcations lead to chaotic solution (general property, e.g. Ruelle-Takens-Newhouse scenario)

Tipping points are the real elephant in the room of climate change?

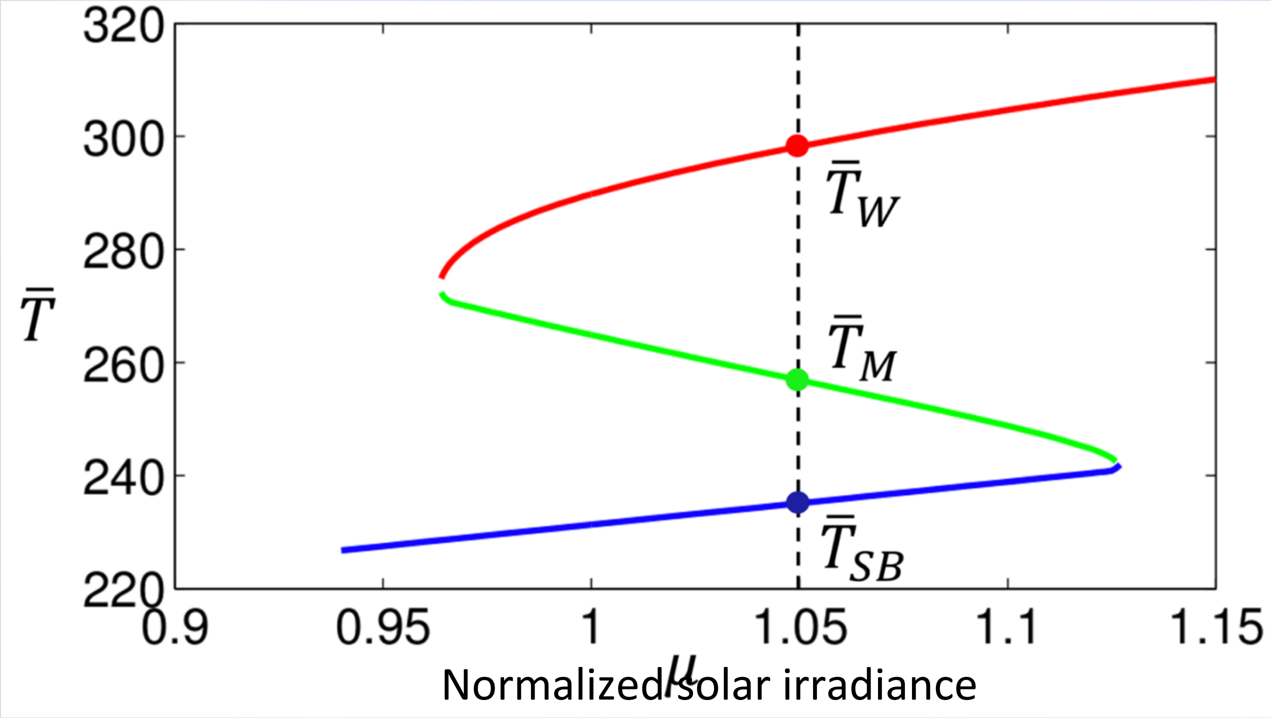


(McKay et al. 2022)

Several tipping elements are prone to abrupt transitions for different levels of global warming. Some of them are very close to be reached.

Early warning signals of an approaching tipping point

An energy balance model (Budyko-Sellers) shows that two stable states are possible for the climate system



Warm chaotic state

"Snowball" Earth

(Adapted from Bodai et al. 2015)

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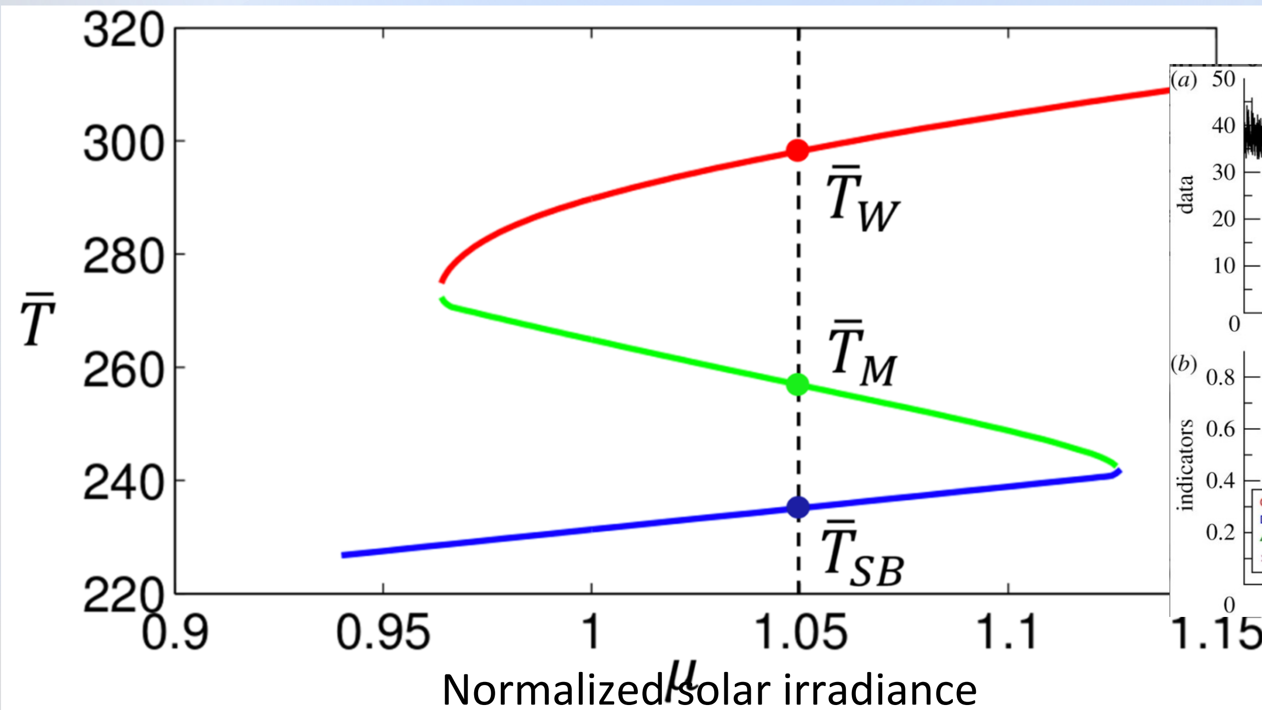
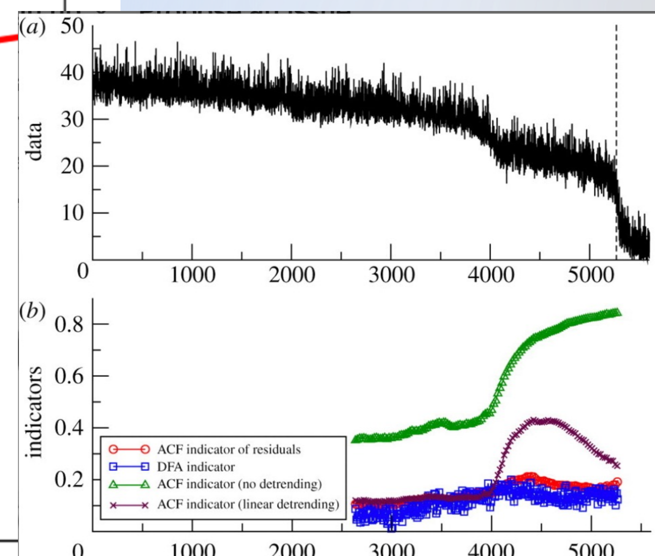
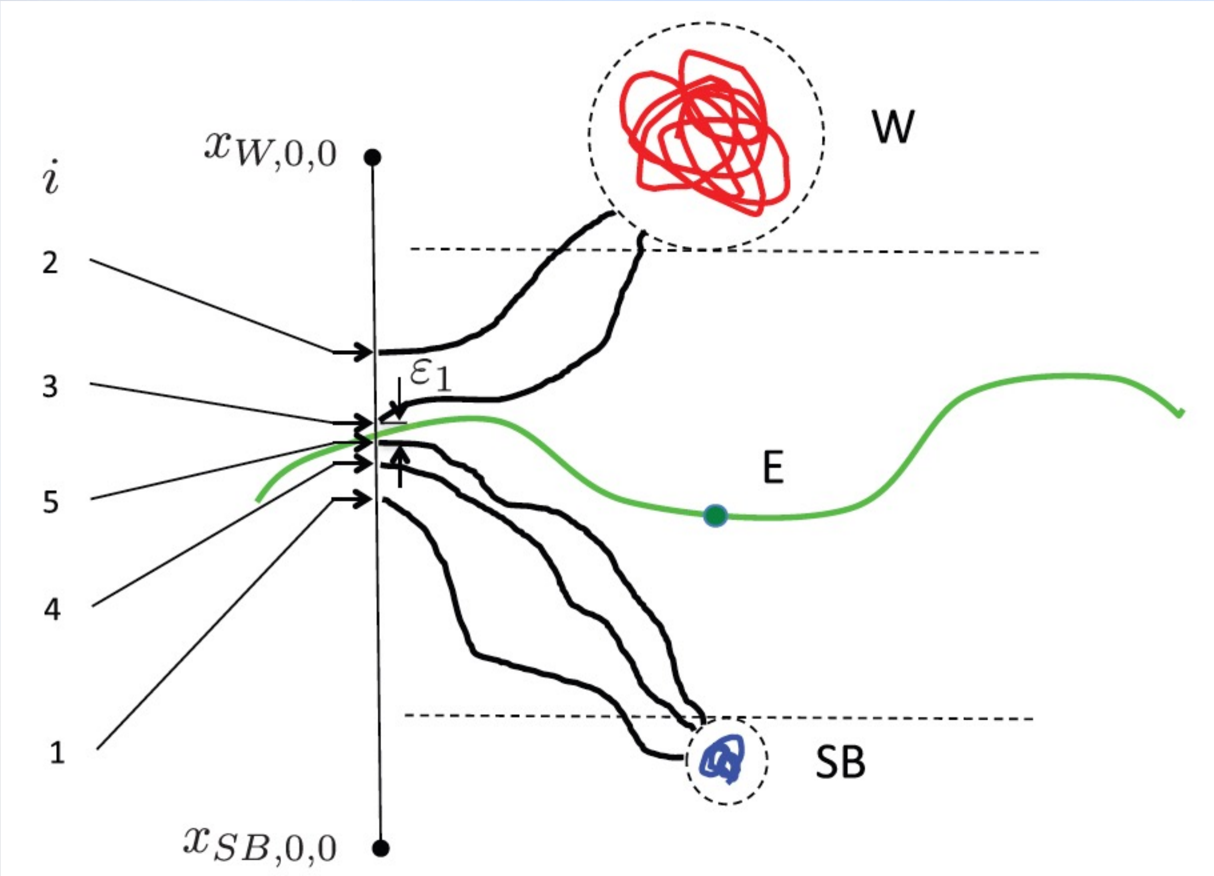




(Lenton et al. 2012)

When a system approaches a critical transition:
- the decay of autocorrelation becomes slower;
- The memory of the system becomes larger;

Melancholia states: a journey through troubled waters

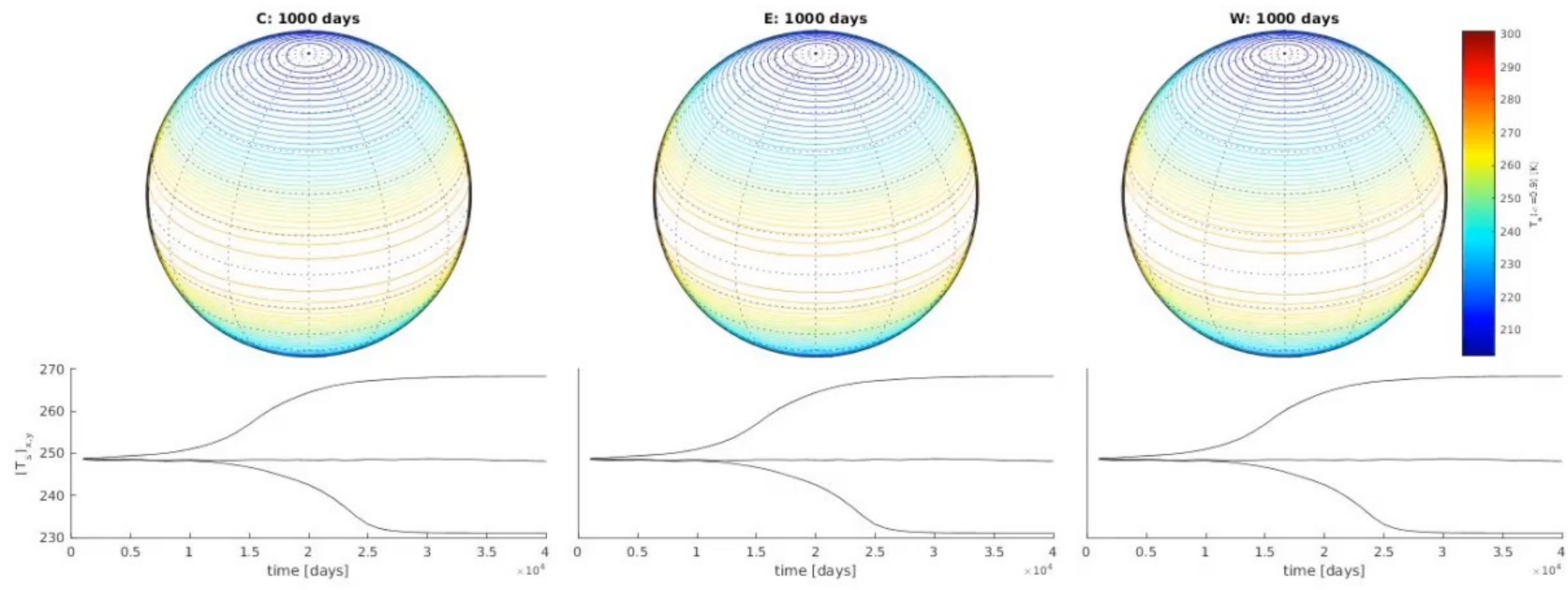


(Bodai et al. 2015)

The state describing the "unstable branch" of the diagram. A small perturbation of the state can abruptly lead to the warm state branch or the snowball earth branch.

It shows what crossing the tipping point would look like!

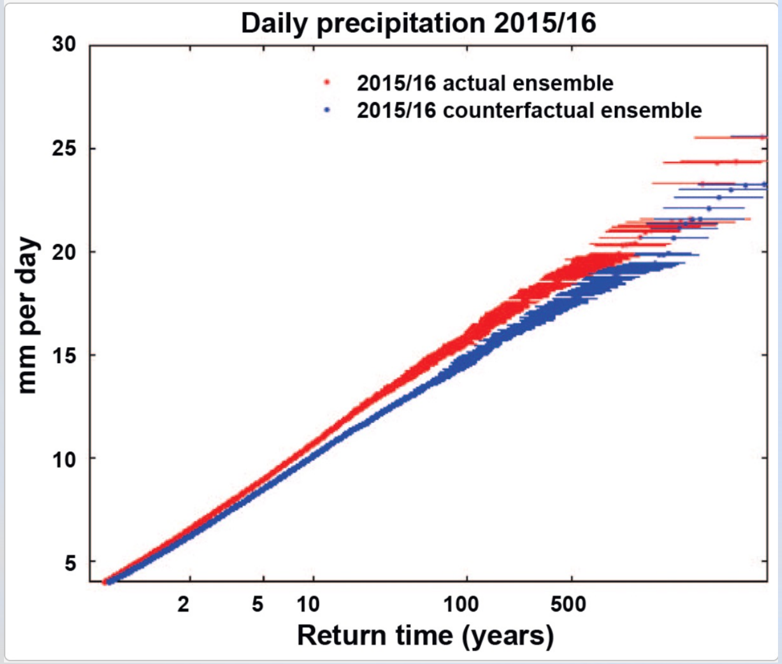
Melancholia states: a journey through troubled waters



(Bodai et al. 2015)

Climate surprises: low probability high impact events

Changes in extreme events probability are not necessarily related to median state change and require a different approach



- Philip et al. 2020: a protocol for extreme event attribution at operational level;

Issue: we make assumptions about the shape of the distribution.

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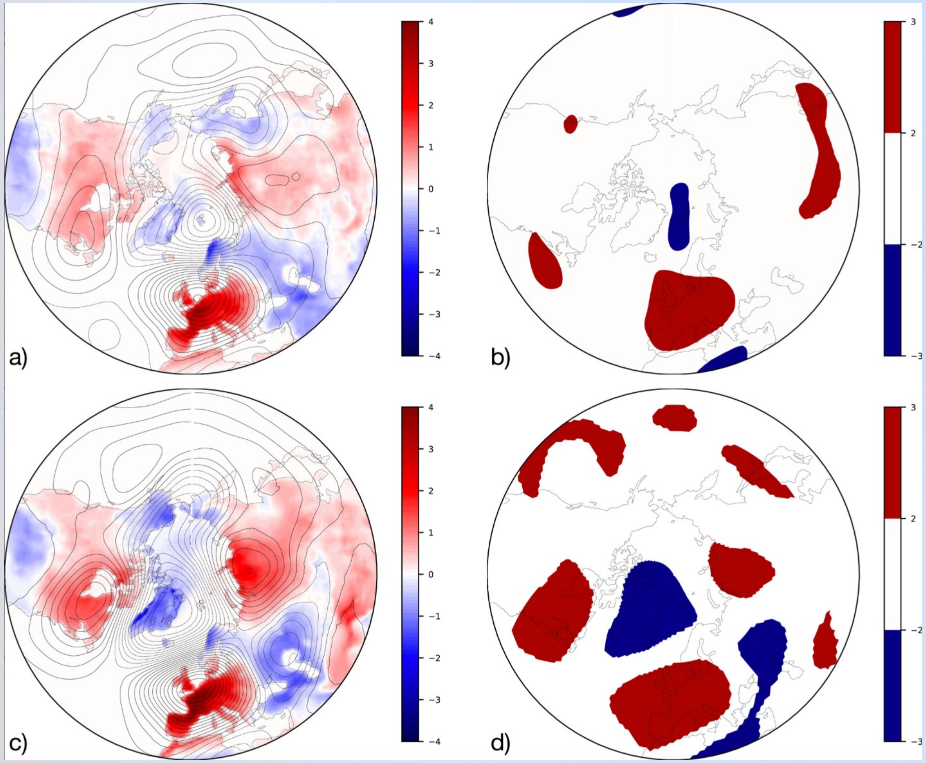

- Philip et al. 2020: a protocol for extreme event attribution at operational level;

Issue: we make assumptions about the shape of the distribution.

- Yiou 2014, Ragone and Bouchet 2018, Yiou and Jezequel 2020, Galfi et al. 2021: a rare event algorithm based on large deviation theory;

Issue: we need to perform a lot of numerical simulations;