How to avoid dangerous climate change?

- Determining efficiently what happens under different scenario's

- Evaluate effects of action choices

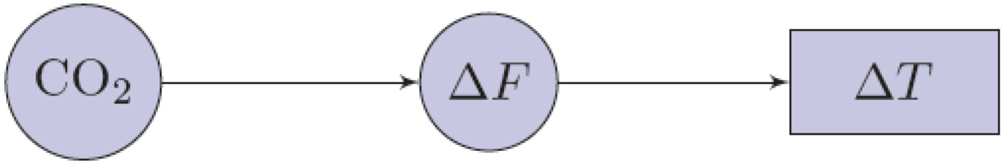
Problems

1. Climate is complex $\Rightarrow$ chaos, randomness
2. $\Rightarrow$ Detailed modeling is expensive!
3. Climate is uncertain $\Rightarrow$ Lack of knowledge/data

Therefore: Need a model that

1. is simple and 'cheap' $\Rightarrow$ can be evaluated many times
2. is accurate $\Rightarrow$ reconstructs CMIP5 GMST
3. is forced by emissions $\Rightarrow$ climate is forced by fluxes
4. gives information on risk and uncertainty

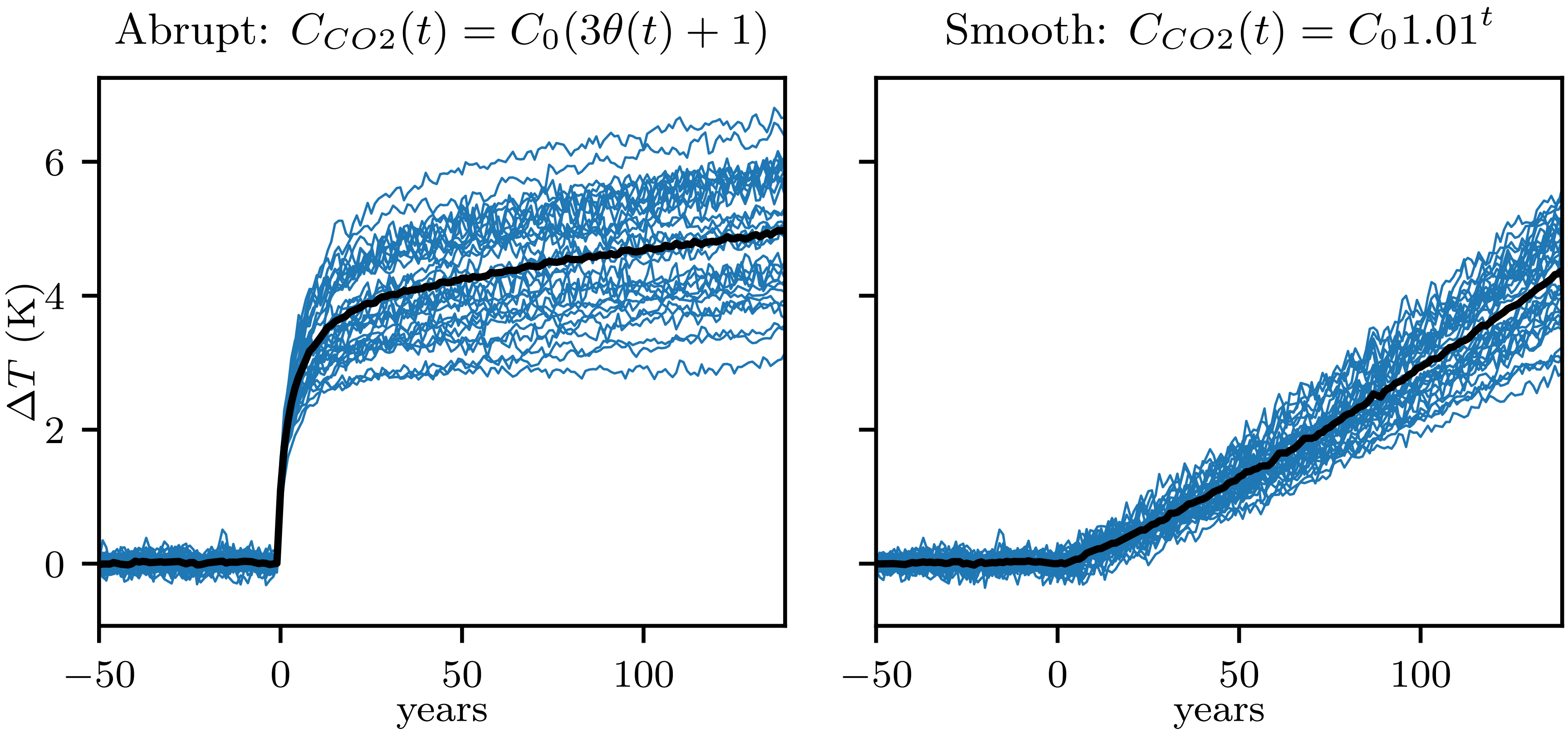
Linear Response Theory (LRT)



Using LRT one can determine the response to any forcing!

$$\Delta T^{(1)}_{\Delta F}(t) = \int_0^t G_T(t')\Delta F(t-t')\ dt' \qquad (2)$$

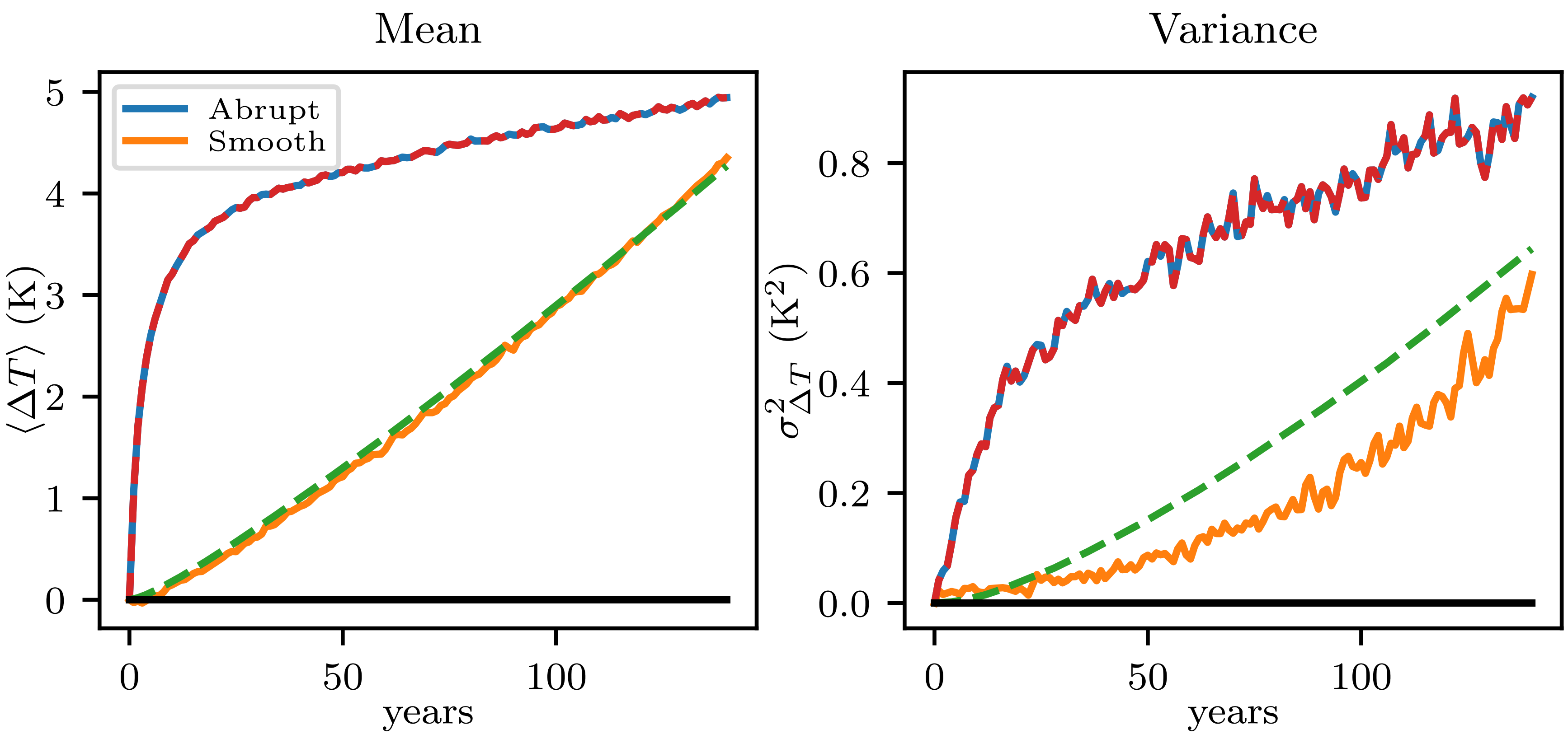
CMIP5 simulations



Abrupt forcing

Smooth forcing

Results Linear Response Theory

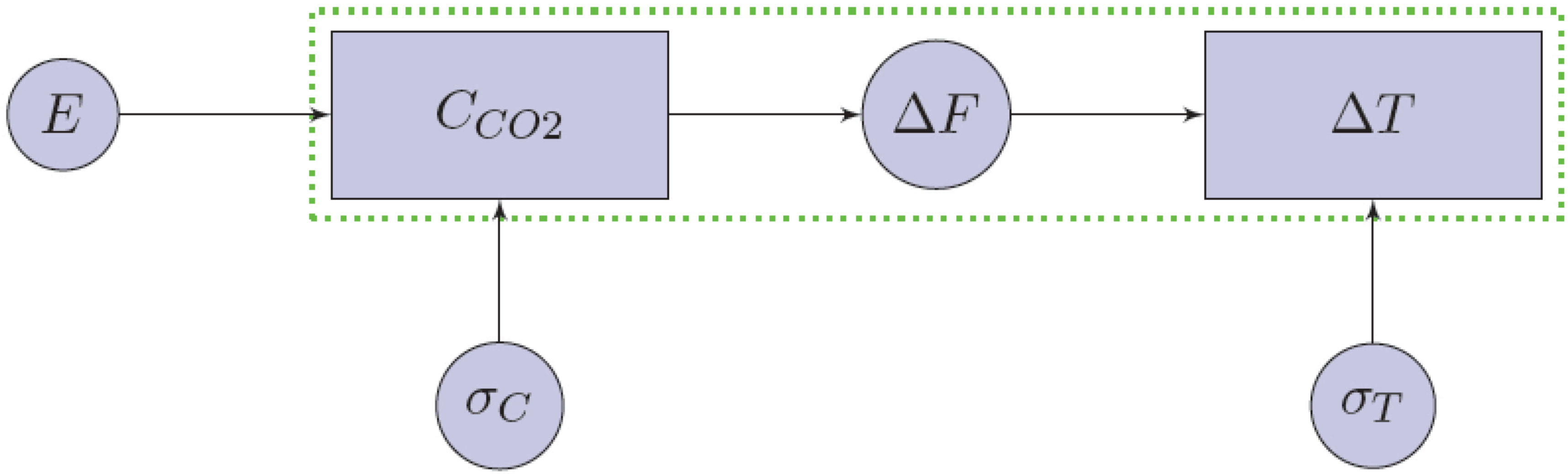


solid: CMIP5

dashed: LRT

From LRT to State Space Model

- Up to now: determine ΔT from CO_2 concentrations
- Climate is forced by *fluxes*
- Relate CO_2 emissions to concentrations
- Risk, uncertainty: introduce stochasticity



Coupling a Carbon Model

- Carbon Model (Joos et al., 2013):

$$G_{CO2}(t) = a_0 + \sum_{i=1}^{3} a_i e^{-\frac{t}{\tau_i}} \tag{4}$$

- Full Reponse Function Model

$$C_{CO2}(t) = C_{CO2,0} + \int_0^t G_{CO2}(\tau)\ E_{CO2}(t-\tau)\ d\tau \tag{5}$$

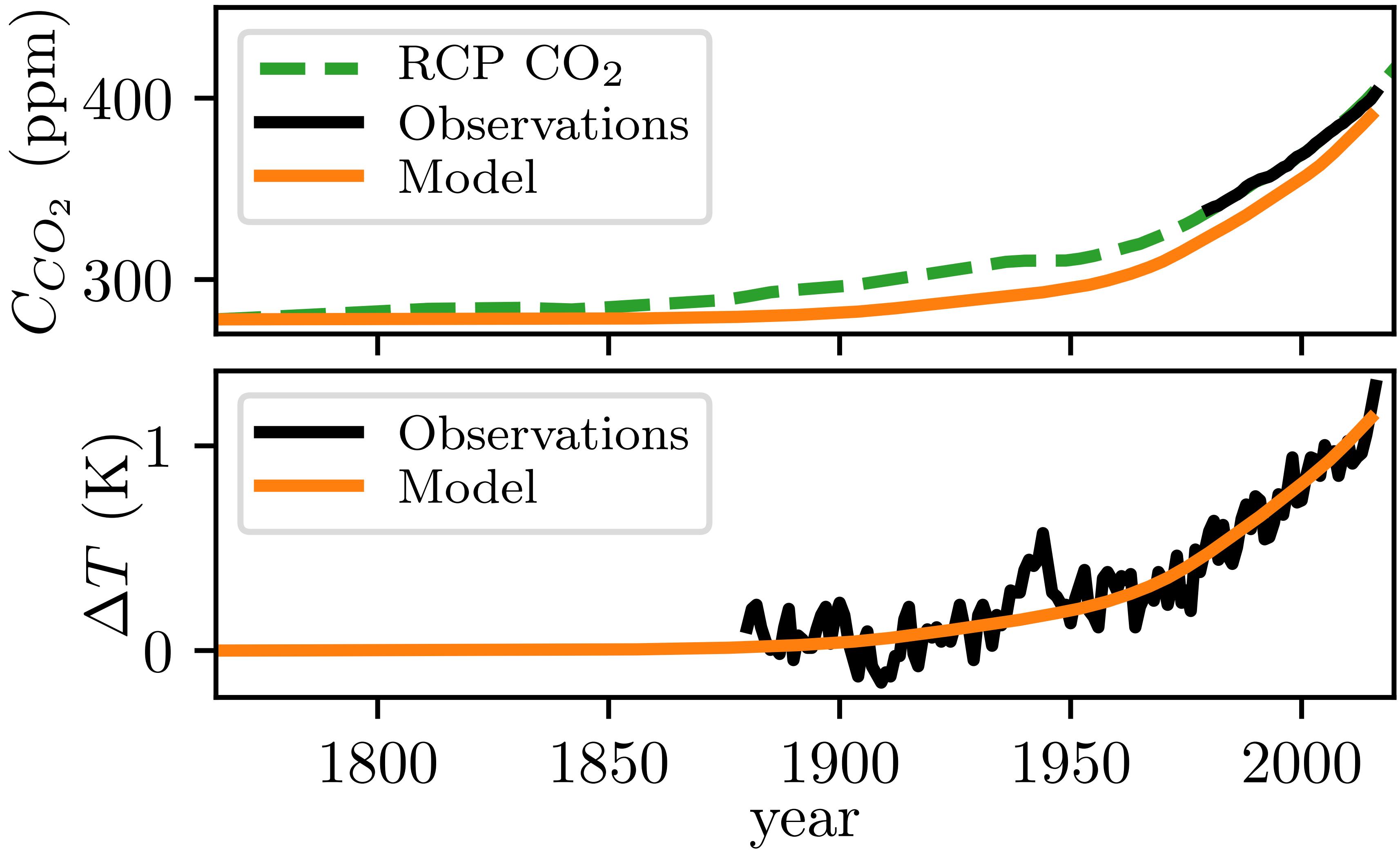
$$\Delta F_{CO2} = A\ \alpha_{CO2} \ln(C/C_0) \tag{6}$$

$$\Delta T(t) = \int_0^t G_T(\tau) \Delta F_{CO2}(t-\tau) d\tau \tag{7}$$

- We also find:

$$G_T(t) = \sum_{i=0}^{2} b_i e^{-t/\tau_{bi}} \tag{8}$$

Carbon Model Performance



Stochastic State Space Model

Carbon

$$dC_P = a_0 E dt$$

$$dC_1 = \left(a_1 E - \frac{1}{\tau_1} C_1\right) dt$$

$$dC_2 = \left(a_2 E - \frac{1}{\tau_2} C_2\right) dt + \sigma_{C2}\, dW_t$$

$$dC_3 = \left(a_3 E - \frac{1}{\tau_3} C_3\right) dt$$

$$C_{CO2} = C_P + \sum_{i=1}^{3} C_i$$

Temperature

$$\Delta F = A\, \alpha \ln(C_{CO2}/C_0)$$

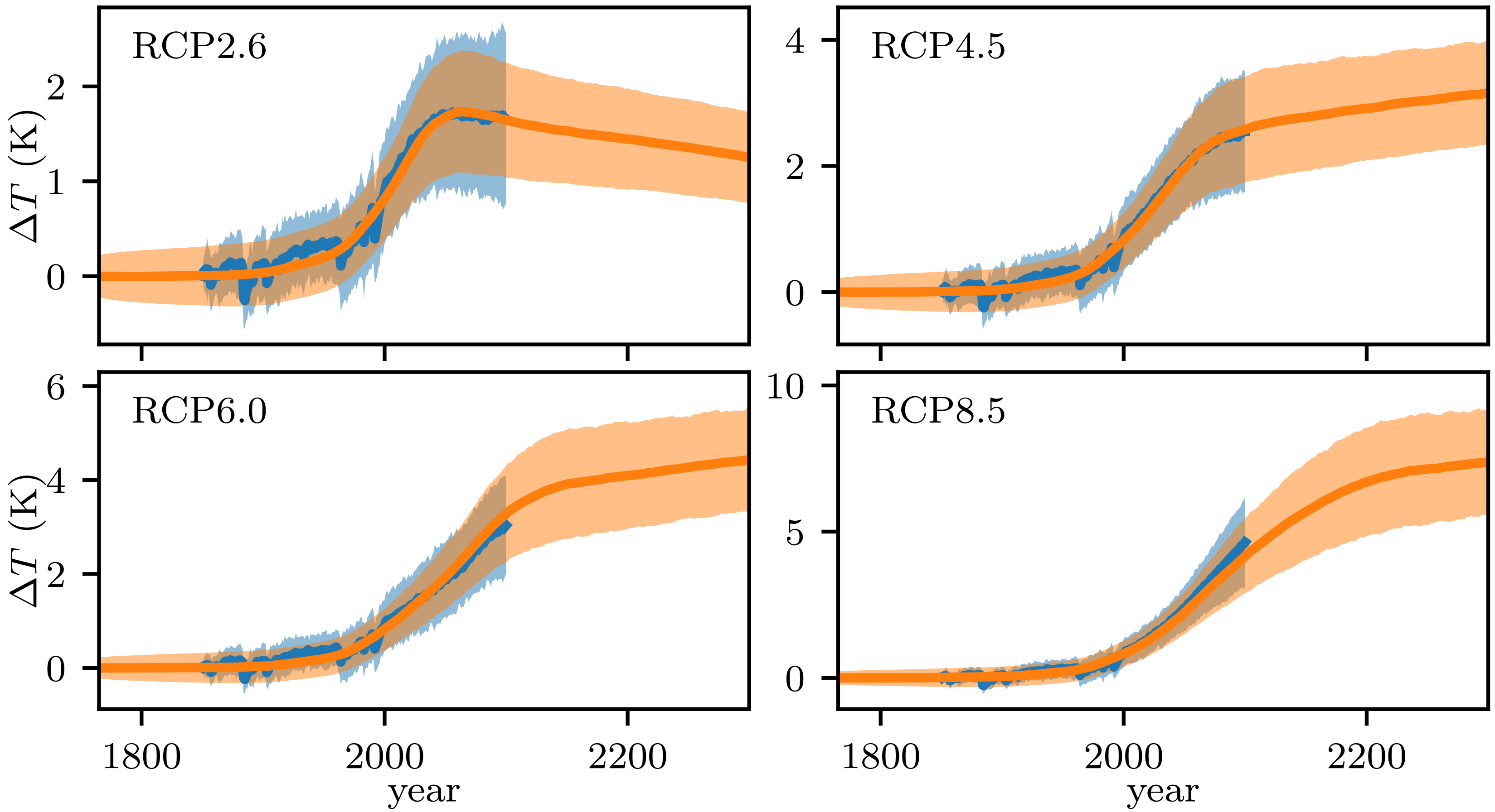
$$d\Delta T_0 = \left(b_0 \Delta F - \frac{1}{\tau_{b0}} \Delta T_0\right) dt + \sigma_{T0}\, dW_t$$

$$d\Delta T_1 = \left(b_1 \Delta F - \frac{1}{\tau_{b1}} \Delta T_1\right) dt$$

$$d\Delta T_2 = \left(b_2 \Delta F - \frac{1}{\tau_{b2}} \Delta T_2\right) dt + \sigma_{T2} \Delta T_2\, dW_t$$

$$\Delta T = \sum_{i=0}^{2} \Delta T_i$$

Results: RCP reconstruction



2 standard deviation envelopes

Results: Probability density functions