Extreme Events in the Climate System

April 28th 2022

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Thanks: T. Bodai, D. Faranda, V.M. Galfi, F. Ragone, J. Wouters, M. Zahid

TIPPING POINTS IN THE EARTH SYSTEM

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Initial Remarks

- Theory of climate: unfinished business; endeavor across mathematics, physics, geosciences
- Interplay between climate response & variability:
    - Anthropogenic climate change
    - Paleoclimate → Life
    - Planetary Science → Habitability
- Nonequilibrium system: No free lunch via standard FDT
- Multiscale properties: need for coarse graining
- Time-dependent system: pullback attractor
- Strike a balance between rigour and realism

REVIEWS OF MODERN PHYSICS, VOLUME 92, JULY–SEPTEMBER 2020

The physics of climate variability and climate change

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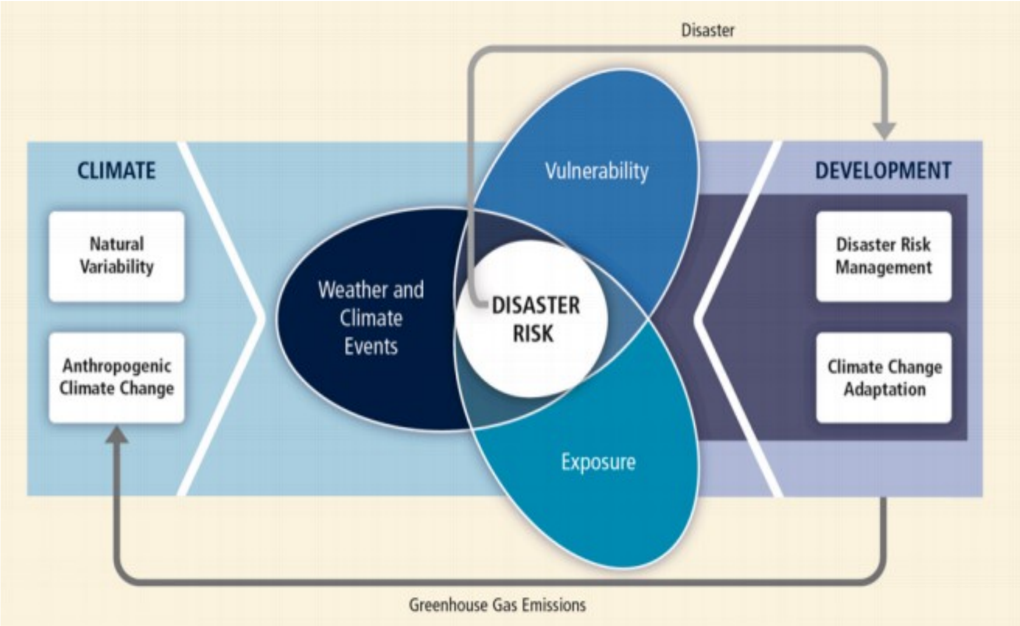
M. Ghil and V. Lucarini,
Rev. Mod. Phys. 92 035002 (2020)

(published 31 July 2020)

The climate is a forced, dissipative, nonlinear, complex, and heterogeneous system that is out of thermodynamic equilibrium. The system exhibits natural variability on many scales of motion, in time as well as space, and it is subject to various external forcings, natural as well as anthropogenic. This review covers the observational evidence on climate phenomena and the governing equations of planetary-scale flow and presents the key concept of a hierarchy of models for use in the climate sciences. Recent advances in the application of dynamical systems theory, on the one hand, and nonequilibrium statistical physics, on the other hand, are brought together for the first time and shown to complement each other in helping understand and predict the system's behavior. These complementary points of view permit a self-consistent handling of subgrid-scale phenomena as stochastic processes, as well as a unified handling of natural climate variability and forced climate change, along with a treatment of the crucial issues of climate sensitivity, response, and predictability.

DOI: 10.1103/RevModPhys.92.035002

Why is it Important to Study Extremes?



From 1970 to 2012, 8,835 events, 1.94 million deaths, and US$ 2.4 trillion of economic losses due to disasters like:

“droughts, floods, windstorms, tropical cyclones, storm surges, extreme temperatures, landslides and wildfires, or by health epidemics and insect infestations directly linked to meteorological and hydrological conditions” (WMO, 2014)

… plus, they are very interesting in terms of basic science

Onset:

Slow

Fast

Natural Hazards

Some recent examples 2011-2015 From WMO

Event	Affected area	Date	Assessed impact
Drought	Australia	2012–2015	Severe rainfall deficit
	Brazil	2012–2015	Severe restriction on water supply in São Paulo
	East Africa, in particular western Somalia and eastern Kenya	2010–2012	Estimated 258 000 excess mortality in Somalia; 13 million people in need of humanitarian assistance
	South-western United States	2011–2015	More than US$ 60 billion of economic losses
	Southern Africa, in particular Angola, Namibia and the North West province of South Africa	2013–2015	18 million people in need of humanitarian assistance
Drought combined with forest fires	South-East Asia and western Pacific	2015	Over 500 000 reported instances of respiratory illness in Indonesia and neighboring countries; 34 directly attributed deaths
Extreme cold	China	January–February 2011	Equivalent of US$ 1.8 billion of economic losses
	Eastern and central United States and southern Canada	2013/2014 and 2014/2015 winters	Prolonged below freezing conditions and frequent snowfalls
	Europe	February 2012	Coldest February in three decades recorded in several countries
Flooding and flash floods	Australia	December 2010–February 2011	Several US$ billion of economic losses in Queensland
	Brazil	January 2011	More than 900 deaths due to flash floods and landslides
	Central Europe	May–June 2013	Most intense and extended flooding in the Danube and Elbe river catchment since at least 1950
	India	June 2013	Over 5 800 deaths, 8 due largely to landslides
	Pakistan	September 2012	5 million people affected, 460 000 homes damaged or destroyed
	South-East Asia (Thailand, Lao People's Democratic Republic and Cambodia)	2011	Over 800 deaths; more than US$ 40 billion of economic losses
Heatwaves and extreme heat	Australia	2012/2013 and 2013/2014 summers	Record temperatures in many places, reaching 45.8 °C in Sydney in January 2013
	East Asia (eastern China, Republic of Korea, western Japan)	July–August 2013	More than 41 people died in China
	India and Pakistan	May–June 2015	Over 4 100 deaths
	West and Central Europe	June–August 2015	Very hot summer, extended hot spells of days with more than 30 °C in several places
Tropical cyclones	Hurricane *Sandy*, Caribbean and United States	October 2012	233 deaths; US$ 67 billion of economic losses in the United States
	Typhoon *Haiyan* (*Yolanda*), Philippines	November 2013	Over 7 800 deaths
	Tropical cyclone *Patricia*, west coast of Mexico	October 2015	Most intense cyclone ever recorded in the western hemisphere

From an Extreme Event to its Impacts

- **Risk = hazard x vulnerability**

- **Risk**: threat to humans and the things we value

- **Hazard**: feature of the natural event

- **Vulnerability** made of three components
- **Exposure – Resistance – Adaptive Capacity**

- *Analysis of the impacts of extremes (risks) is a truly multidisciplinary business*

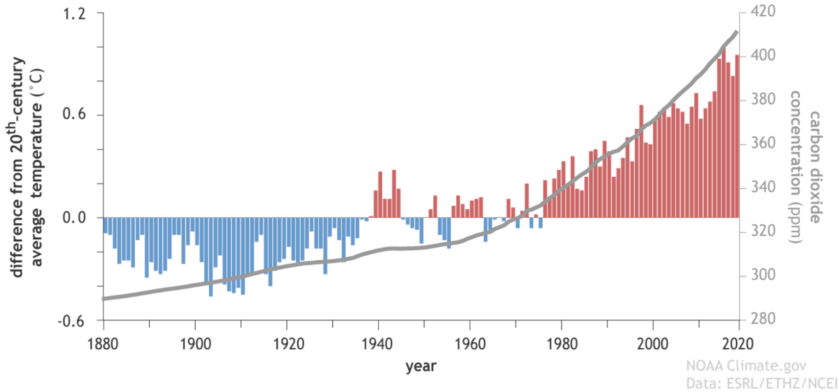
Why the Guardian is changing the language it uses about the environment

From now, house style guide recommends terms such as 'climate crisis' and 'global heating'

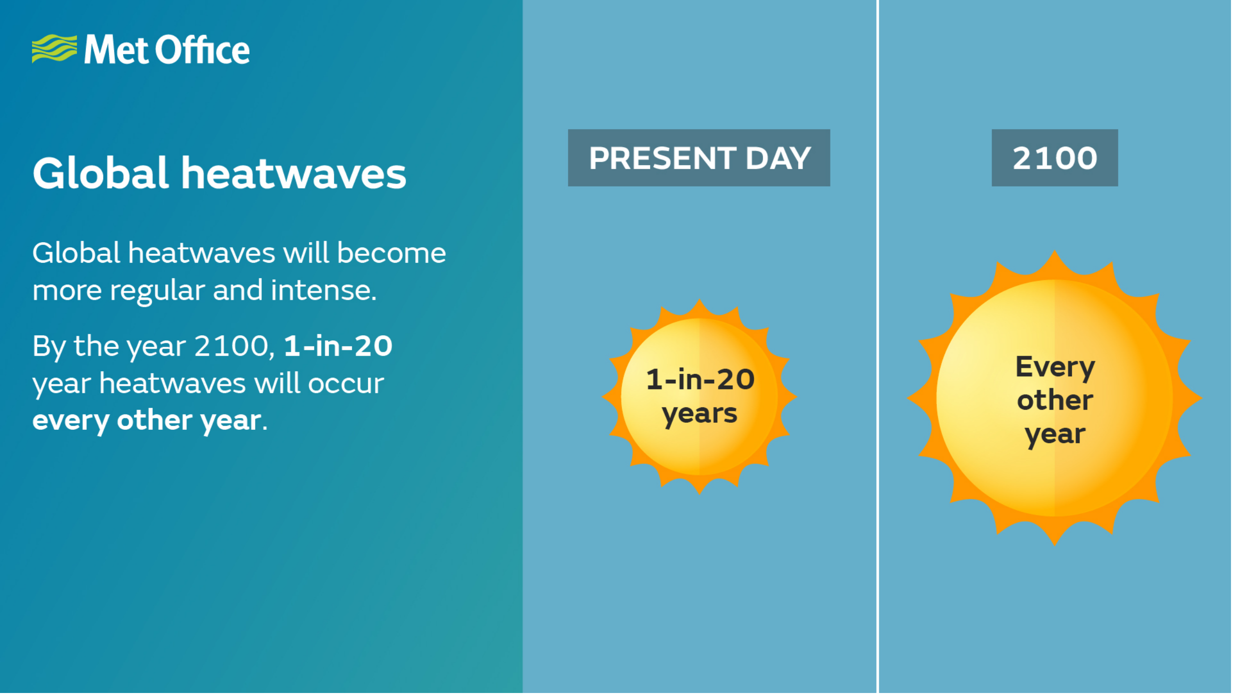
Climate crisis: world is at its
at least 12,000 years - study

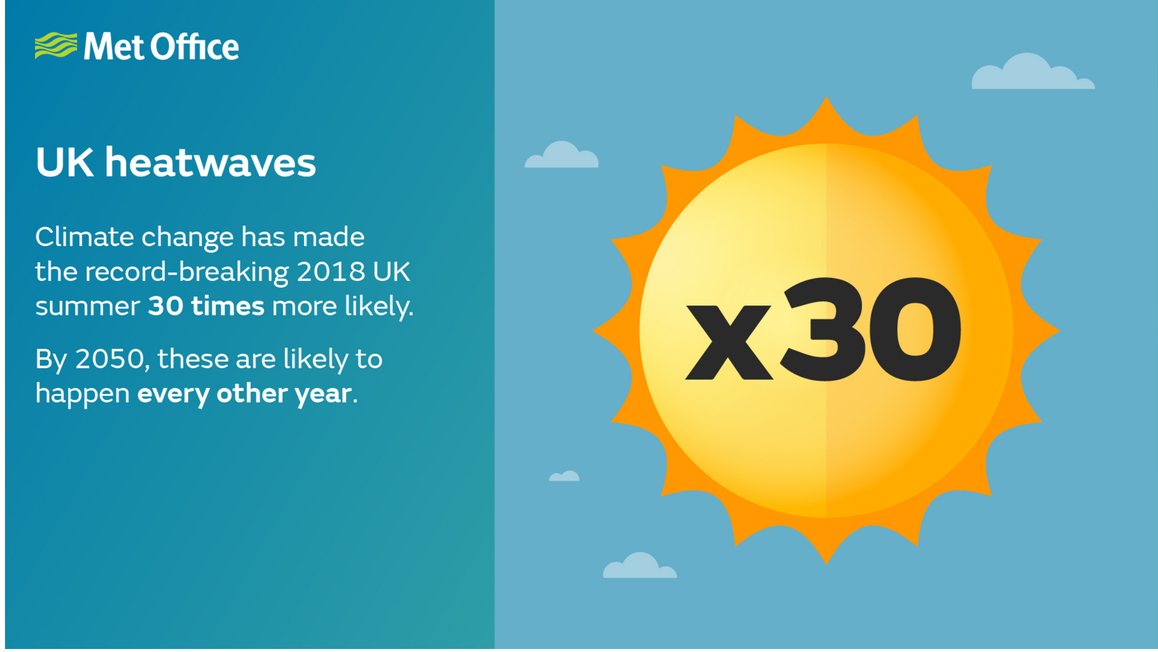
Japan / Climate crisis 'likely cause'
early cherry blossom

Atmospheric carbon dioxide and Earth's surface temperature (1880-2019)



Climate Narrative is Changing





Why the Guardian is changing the language it uses about the environment

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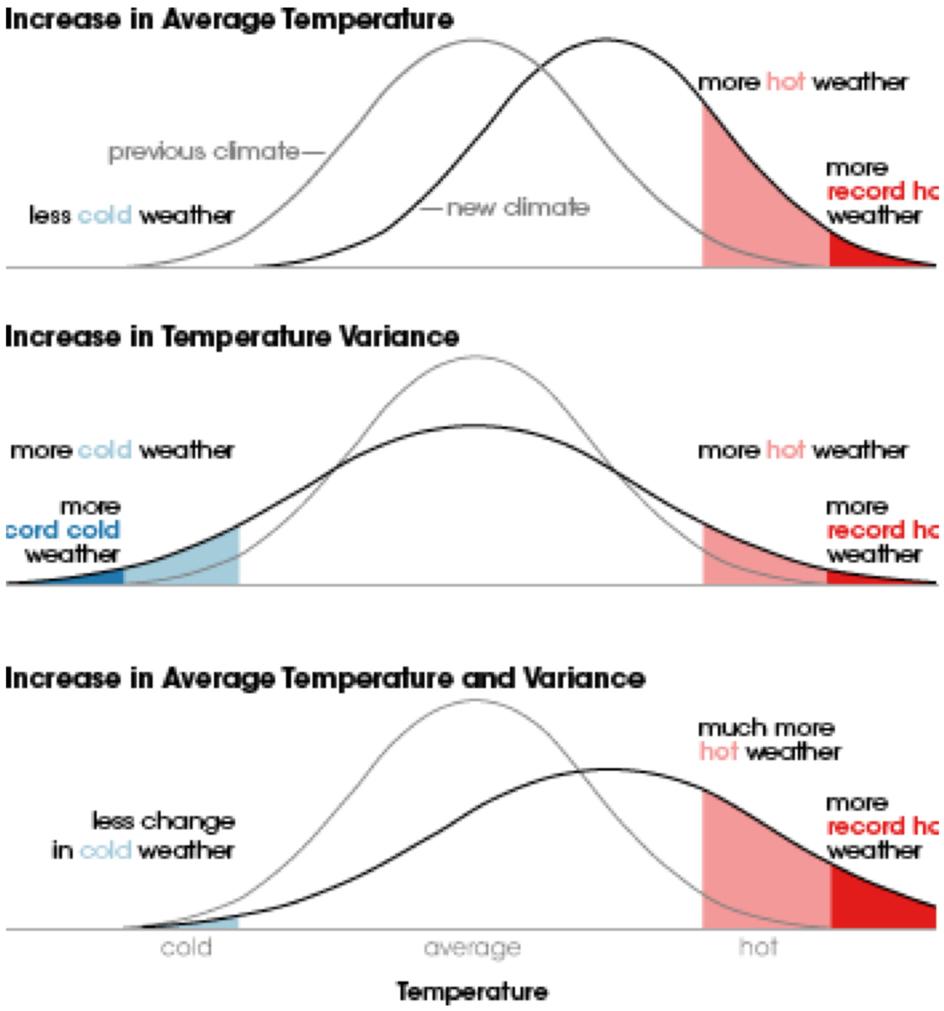
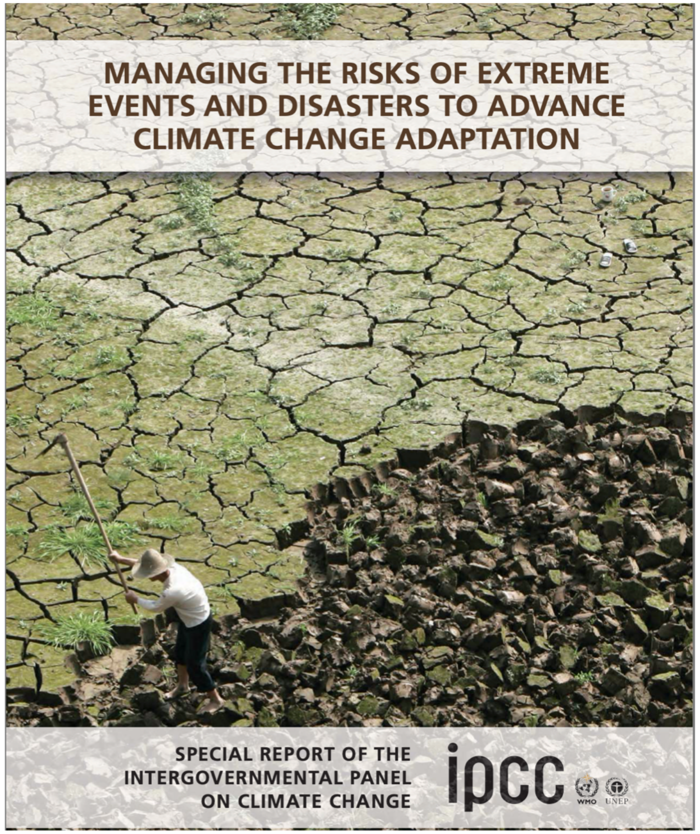
Climate Change

Climate Crisis

Japan / Climate crisis 'likely cause' early cherry blossom

IPCC view: Temperature Extremes and Climate Change



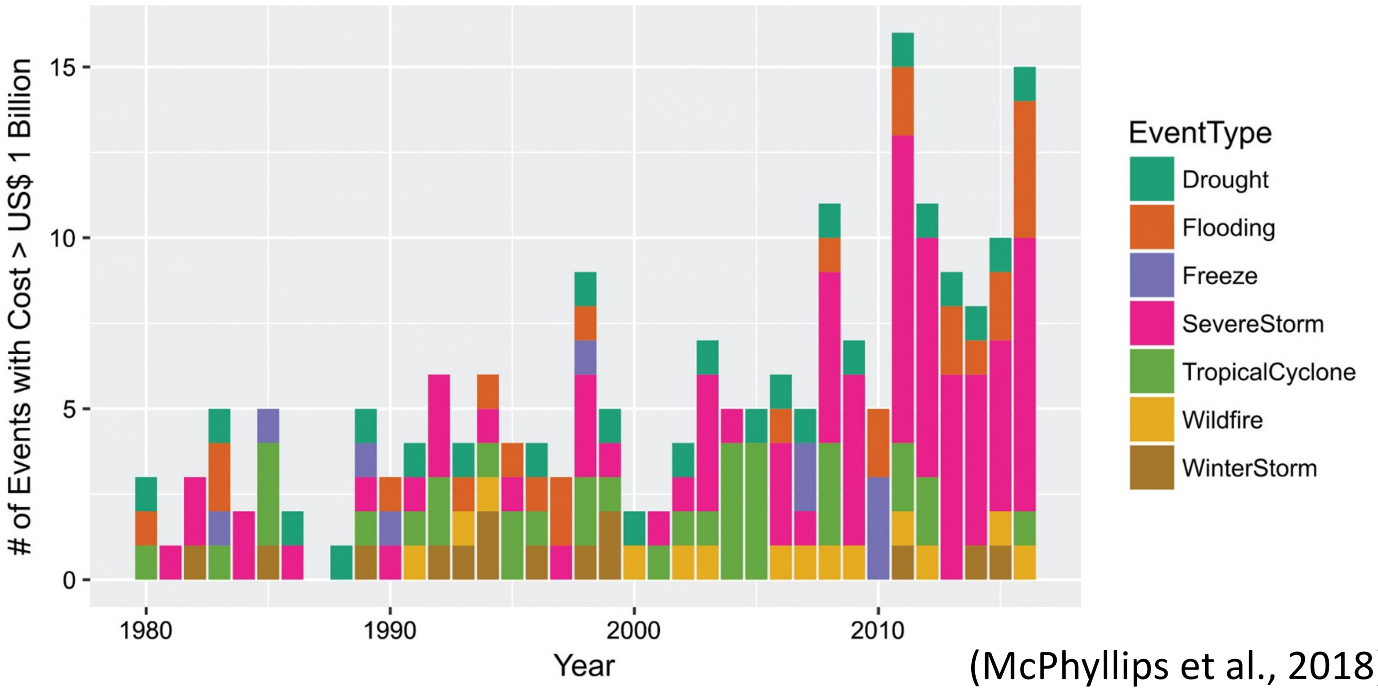


Basic tenet: associate all moments to the first and the second
.. but this applies if we are considering a **Gaussian** case!

How will (the Impact of) Extremes Change in a Changing Climate?

- More than 50% of the world's population lives in cities, many of them are located in coastal areas (McPhyllips et al., 2018).
- More vulnerable to: coastal storms, sea-level rise, extreme heatwaves, earthquakes, cyclones, coastal flooding….

Change in exposure
& vulnerability
+
Change in frequency
& magnitude
of certain events



(McPhyllips et al., 2018)

First Step towards Understanding Extremes: Definitions

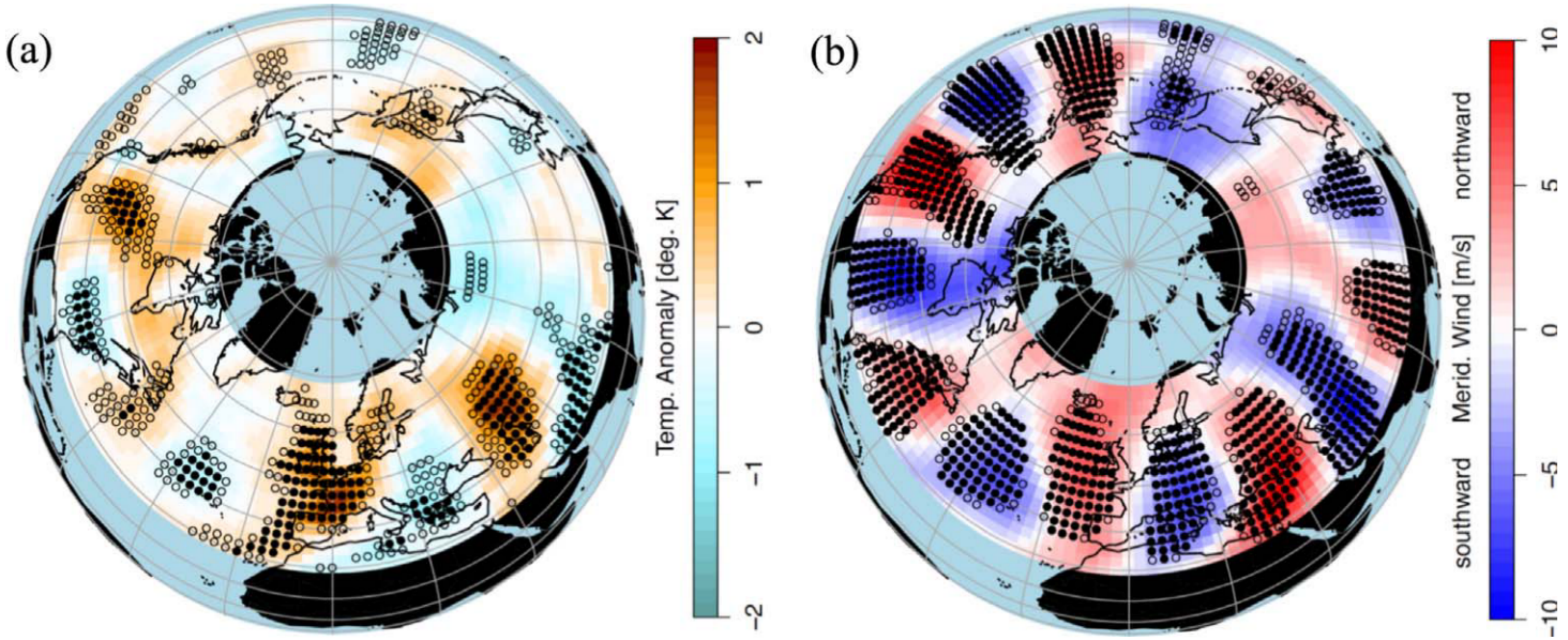
- **Extreme** as something **very rare**
- **Extreme** as something **very large**
- **Extreme** as something with **very high impact**

- “a lack of coherence exists in what constitutes and defines an extreme event across these fields, which impedes our ability to holistically understand and manage these events.” (McPhyllips et al., 2018)

- “Despite the efforts of numeral stakeholders’ to make a consensus about precise definition of extreme weather events like a first step which leads to its adequate response, success in that actions is still missing. Confirmation for this statement can be found in the work of the Task Team on the Definition of Extreme Weather and Climate Events..” of the WMO (Radović and Iglesias, 2019)

- Considering only heatwaves: „… it seems that almost, if not every climatological study that looks at heatwaves uses a different metric.” (Perkins, 2015)

Spatial Extension – Simultaneous events

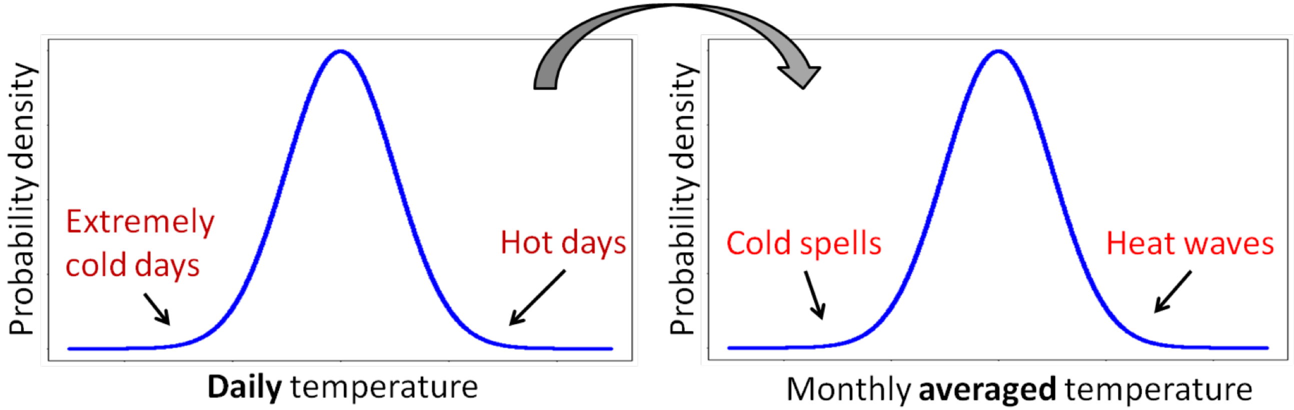


Composites of temperature anomalies and meridional wind anomalies during weeks of high wave-7 amplitudes. (Kornhuber et al., 2019)

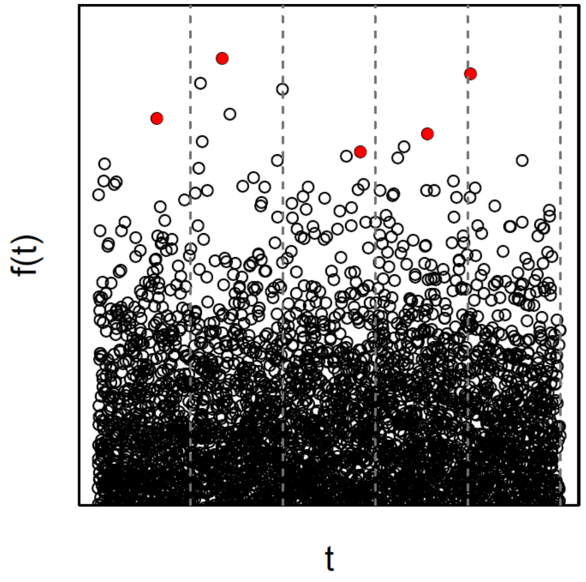
How to Study Extreme Events?

I. We are interested at instantaneous extremes: Extreme Value Theory (EVT)
- Provides limit laws for probabilities of extremes.
- Problem in case of persistent events: extremes are correlated.

II. We are interested at persistent extremes: Large Deviation Theory (LDT)
- Provides limit laws for probabilities of averages.
- Persistent events can be represented by extremes of averages
- … There is more. Connection between extremes and dynamics
- These theories are very powerful. No need to resort to spherical cows
- We can treat observational and model-generated data.



EVT V1 - Block Maxima: Extremes as Rare Events



$$M_m = \max\{X_1, \ldots, X_m\}$$

where $X_1, \ldots, X_m$ are independent, identically distributed (i.i.d. random variables with common distribution function F.

If there exist sequences of constants $a_m > 0$ and b_m such that

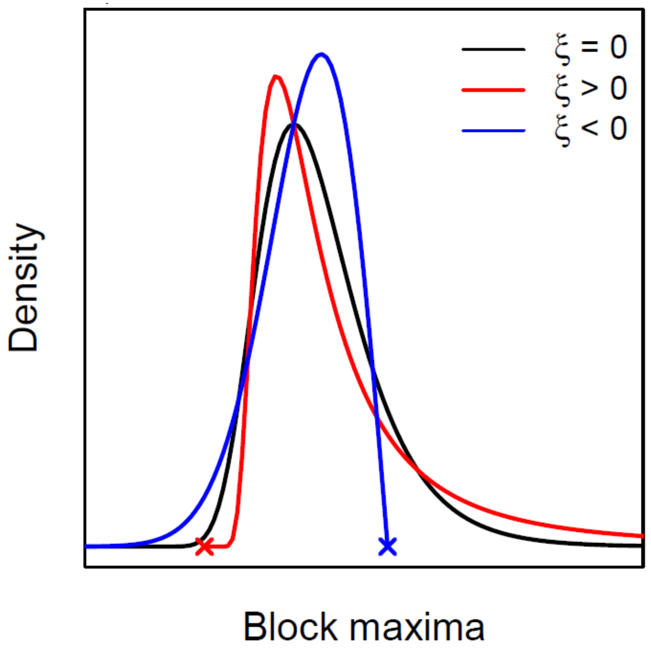
$$\Pr(M_m - b_m)/a_m \leq z \to G(z) \text{ as } m \to \infty$$

for a non-degenerate distribution funcion G,
then G is a member of the Generalized Extreme Value (GEV) family

Gnedenko (1943)

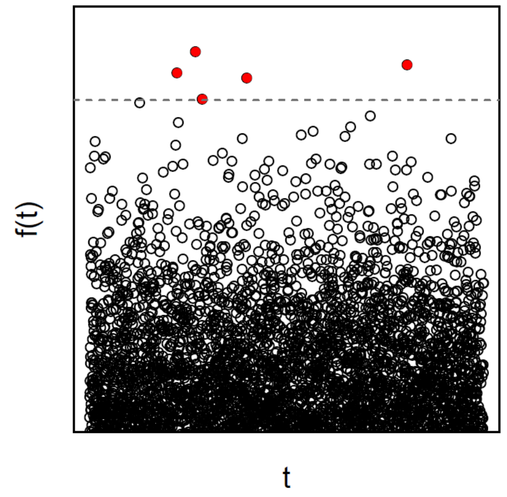
$$G(z) = \exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{1/\xi}\right\}$$

Gumbel: $\xi = 0$
Frechet: $\xi > 0$
Weibull: $\xi < 0$



with $\{z : 1 + \xi(z-\mu)/\sigma > 0\}$, $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$.

EVT V2: Peaks-Over-Threshold: Extremes as very Large



We regard as extreme events those of the X_i that exceed some high thershold u.

$$\Pr\{X > u + y | X > u\} = \frac{1 - F(u+y)}{1 - F(u)}, \qquad y > 0$$

Suppose F satisfies (1). Then, for large enough u, the distribution function of $(X - u)$, conditional on $X > u$, is approximately

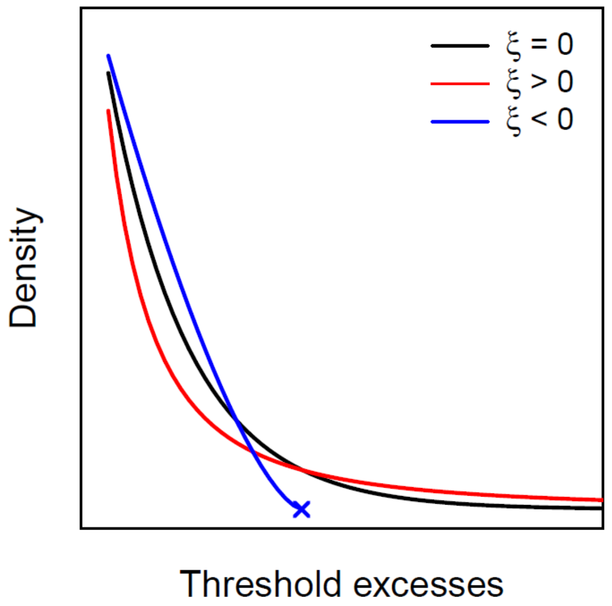
Shape p.
Scale p.

$$H(y) \approx 1 - \left(1 + \frac{\xi y}{\tilde{\sigma}}\right)^{-1/\xi}$$

defined on $y : y > 0$ and $(1 + \xi y / \tilde{\sigma}) > 0$, where

$$\tilde{\sigma} = \sigma + \xi(u - \mu),$$

and H is a member of the Generalized Pareto Ditribution (GPD) family.



Balkema and de Haan (1974), Pickands (1975)

GPD vs GEV

- Extremes should be extremes, but:
- The two points of view lead to different candidates for extremes

- Nonetheless:
- Pickands (1975)-Balkema-De Haan (1974) theorem: two representations are equivalent: 1-to-1 correspondence between GEV & GPD parameters

- True if i.i.d. and (weakly) correlated stochastic variables (Leadbetter 1983)
- Long-term correlation: no EVT at all
- Strong short-term correlation: clusters, but EVT applies

$$G^{\theta}(z) = \exp\left\{-\theta\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]\right\} = \exp\left\{-\left[1+\xi\left(\frac{z-\mu^*}{\sigma^*}\right)\right]\right\}$$

θ: extremal index $\quad \theta = (\text{limiting mean cluster size})^{-1}$

Fitting Methods

Assuming a random sample (X_1, X_2, …, X_n) the logarithm of the maximum likelihood for a GPD with shape parameter ξ and scale parameter σ is:

$$l(\sigma,\xi;X) = -n\log\sigma - \left(1+\xi\right)\sum_{i=1}^{n}\log\left(1+\frac{kX_i}{\xi\sigma}\right).$$

Maximum Likelihood Estimates are obtained by **maximizing** it

Defining the density as: $$f_{GPD}\left(z;\xi,\sigma\right) = \frac{d}{dz}F_{GPD}\left(z;\xi,\sigma\right)$$

The first **two moments** can be **easily related to** ξ and σ:

$$\int_0^{uep} dz\ z\, f_{GPD}\left(z;\xi,\sigma\right) \approx \frac{1}{N}\sum_{j=1}^{N}X_j = \mu_1 = \frac{\sigma}{1-\xi}$$

$$\int_0^{uep} dz\ z^2\, f_{GPD}\left(z;\xi,\sigma\right) \approx \frac{1}{N}\sum_{j=1}^{N}X_j^2 = \mu_2 = \frac{2\sigma^2}{\left(1-\xi\right)\left(1-2\xi\right)}$$

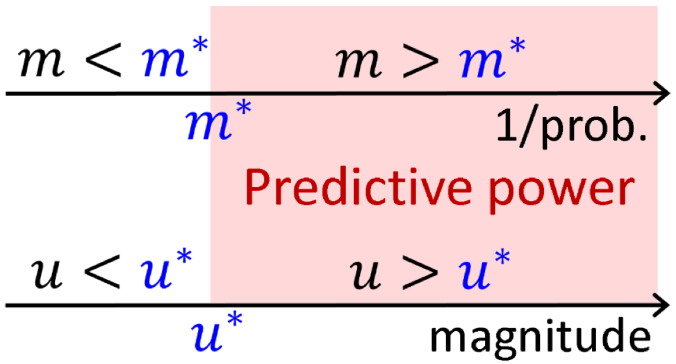
Moments Estimates are based upon inverting these expressions

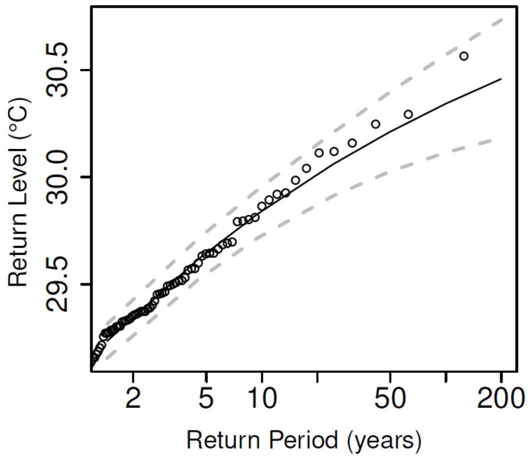
We Apply a Limit Law to Finite-Size Data

Convergence to the limit needs to be verified in order to apply the method properly.

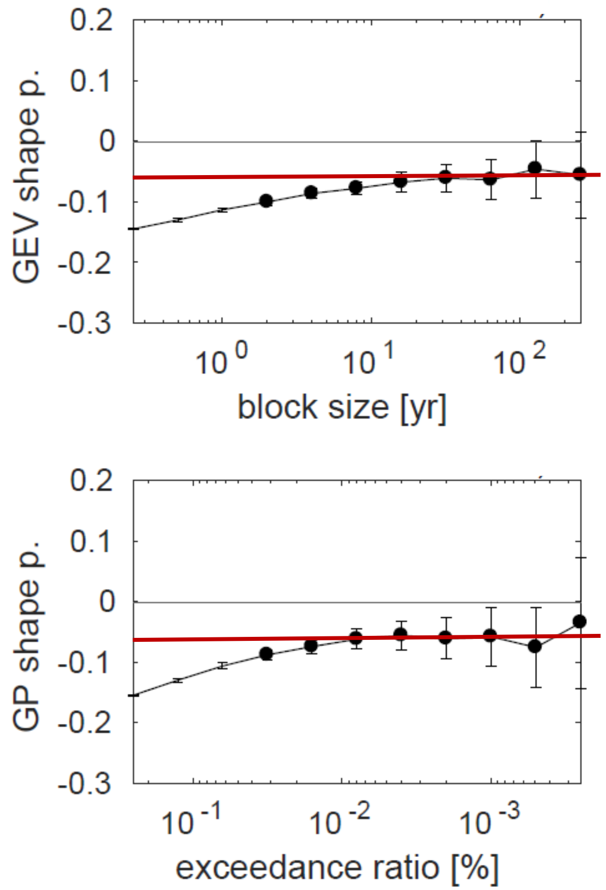
Choosing the right block size m or threshold u requires a balance between bias and uncertainty.

If we reach asymptotic levels, we gain predictive power in a statistical sense.





(Zahid et al 2017)



(Galfi et al., 2017)

Zahid et al., ESD 8, 1263-1278 (2017)

Sindh Extremes

You can apply it in reality: Study Area – Sindh, Pakistan

The New York Times

ASIA PACIFIC

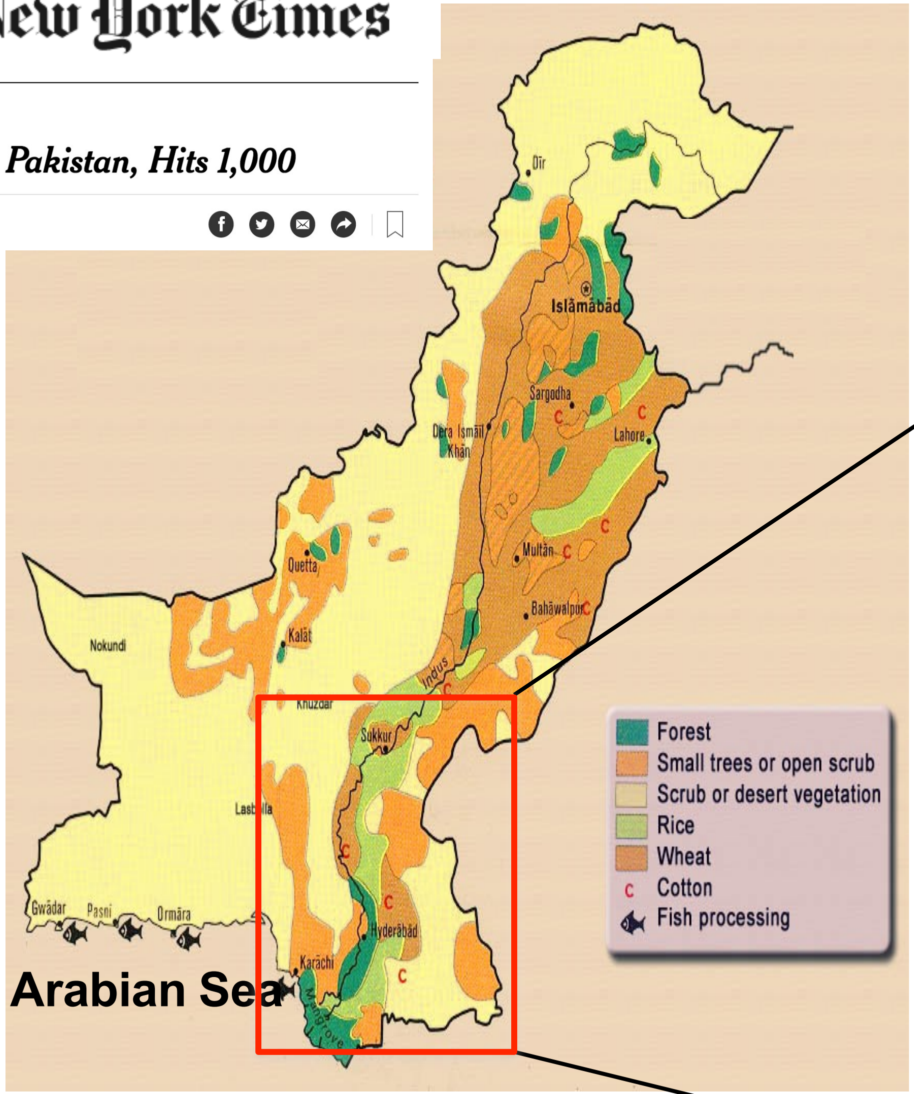
Death Toll From Heat Wave in Karachi, Pakistan, Hits 1,000

By SABA IMTIAZ and ZIA ur-REHMAN JUNE 25, 2015

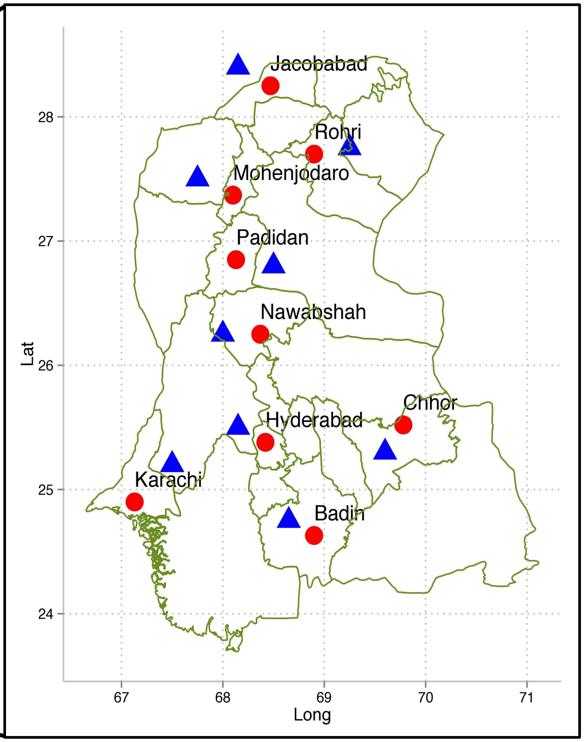
CNN Regions | U.S. Politics | Money | Entertainment | Tech | Sport | Travel | Style | Features | Video International Edition + menu

Record heat wave kills hundreds in Pakistan

Temperatures in Pakistan reached 44.8 degrees Celsius or 113 degrees Fahrenheit, the highest recorded temperature in 15 years. CNN's Pedram Javaheri reports. Source: CNN



(NDMC, 2009)



Name
Sindh Province

Coordinates
23.5 – 28.5° N
66.5 – 71.1°E

Area
140,914 km^2

Population
55.52 Million

Urban 49%,
Rural 51%

Important Crops
- Cotton
- Rice
- Wheat
- Sugarcane
- Dates
- Banana
- Mango

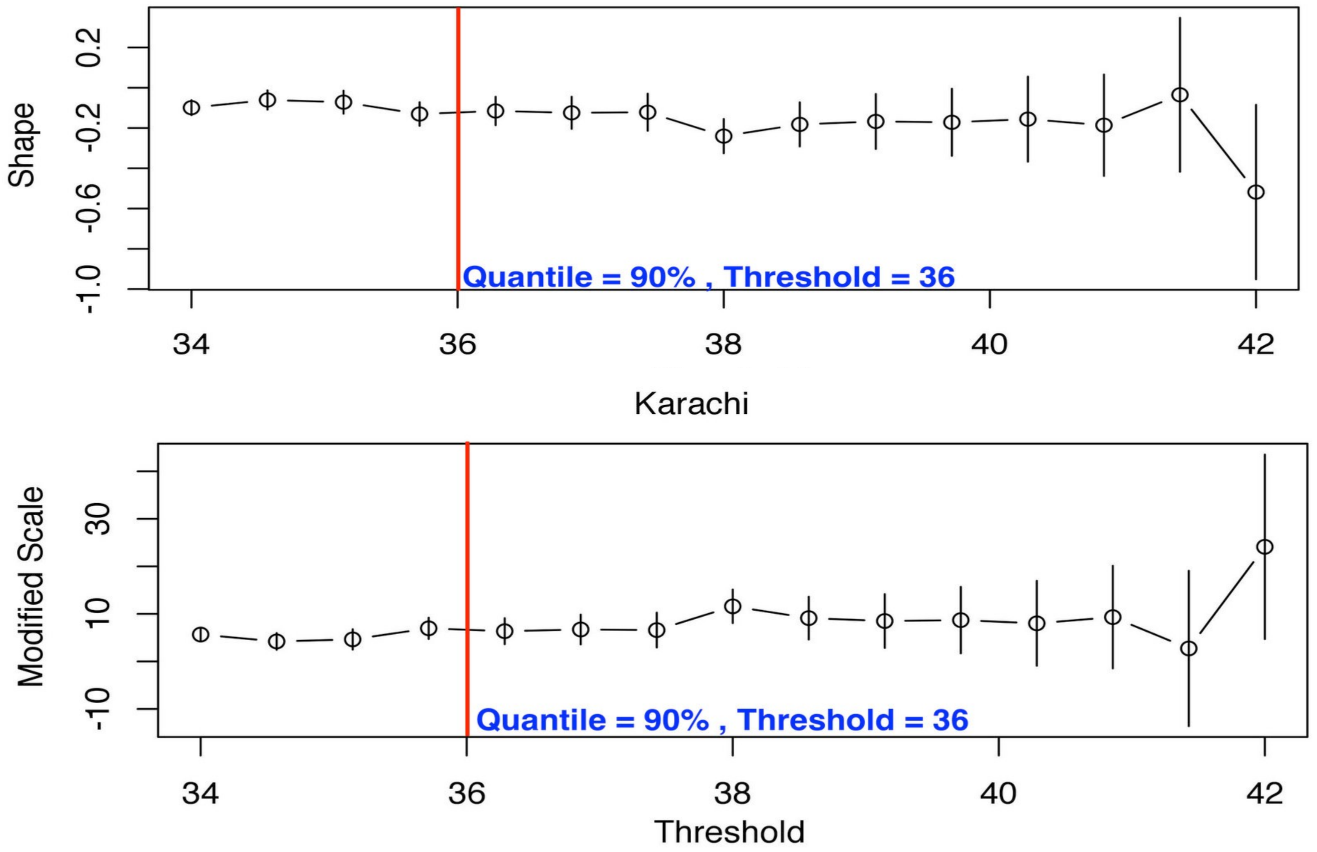
Important Activities
- Fish processing
- Ports

Threshold Selection

- Higher threshold ➔ closer to asymptotic regime BUT higher uncertainty (few data)
- How to find the optimal threshold?

The shape parameter ξ and the mofiied scale parameter: $\sigma_{mod}=\sigma - \xi u$ do not depend on the threshold, if we are in asymptotic regime.

- We compare the estimates of ξ and σ_{mod} for different thresholds… Karachi



<u>THIS DEFINES WHAT IS A TRUE EXTREME</u>

Absolute Maxima of T_{max}

Absolute maxima is calculated through the following formula;

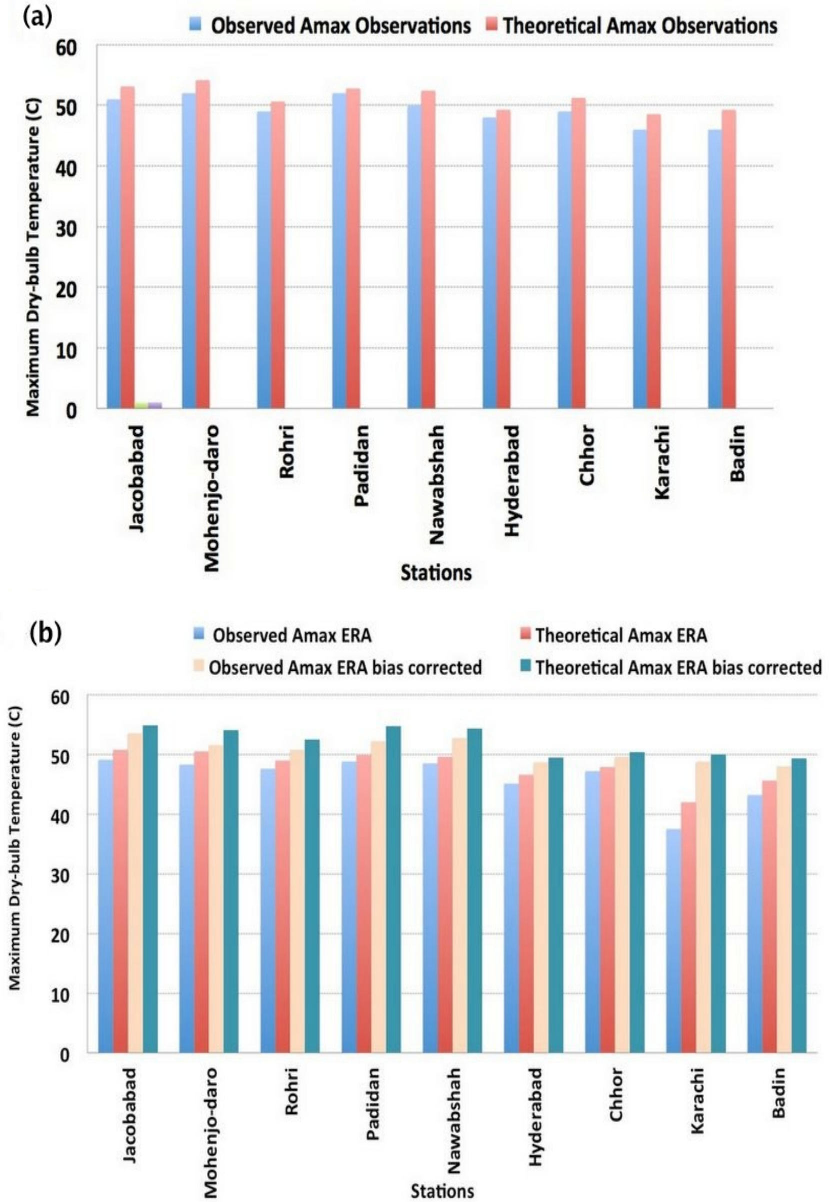
$$A_{max} = u - \sigma/\xi$$

Where,

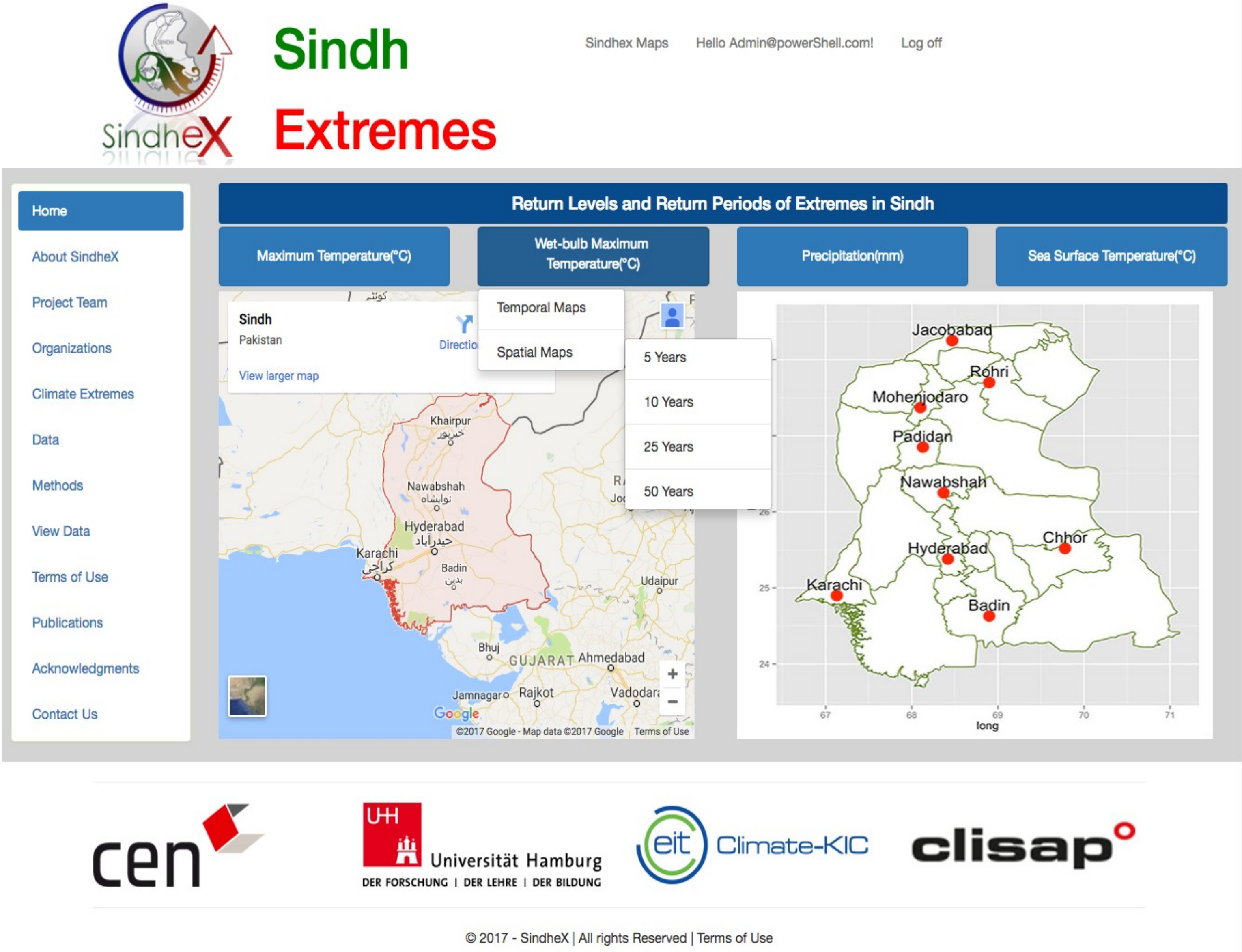
A_{max} = Absolute Maxima
u = Threshold
σ = Scale parameter
ξ = Shape parameter

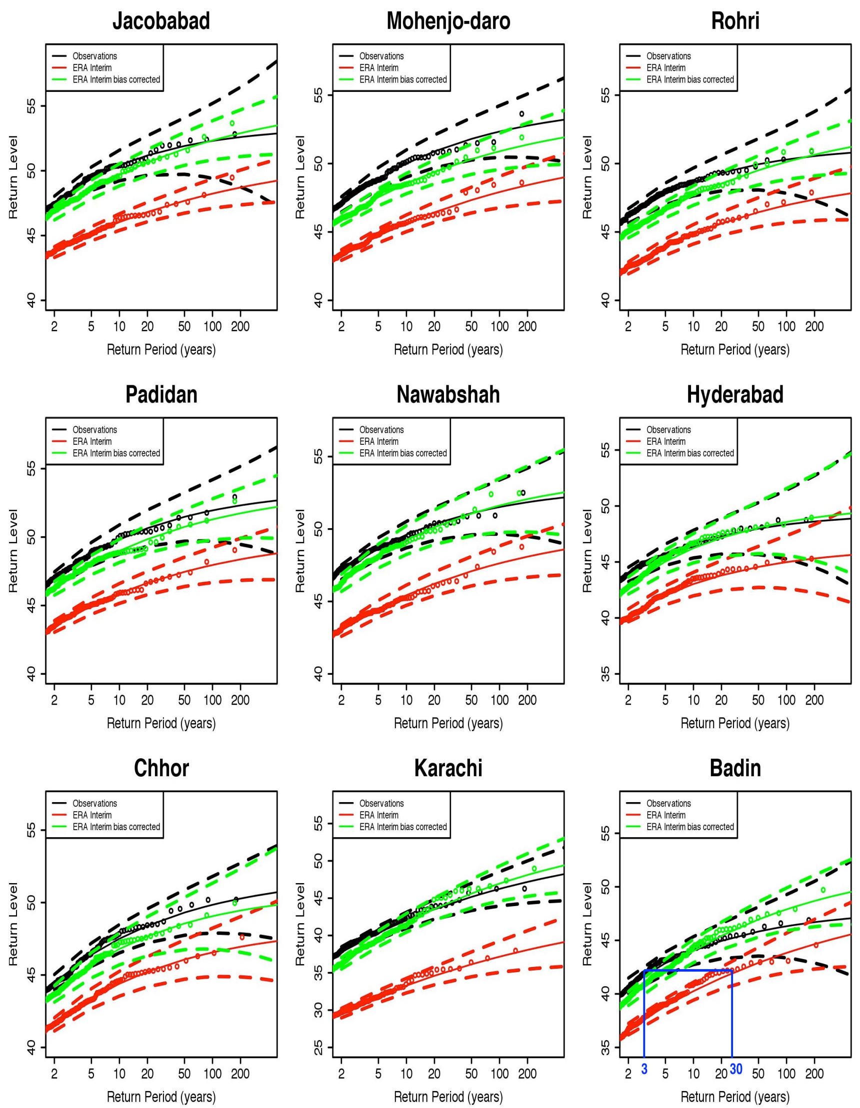
(Lucarini et al., 2014)

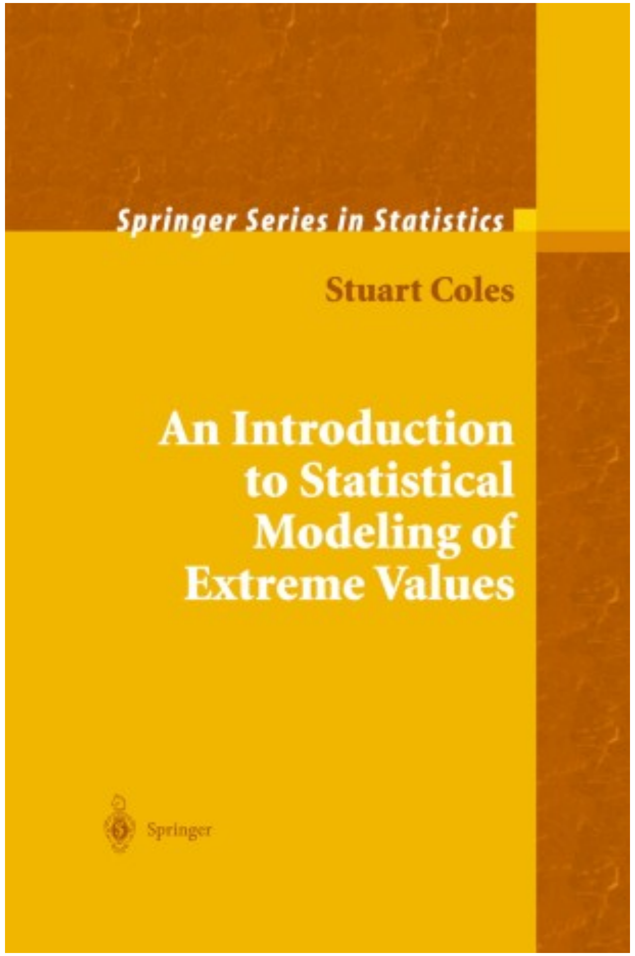
- Absolute maxima estimate the maximum values
- Theoretical upper limits predicted by the GPD fit



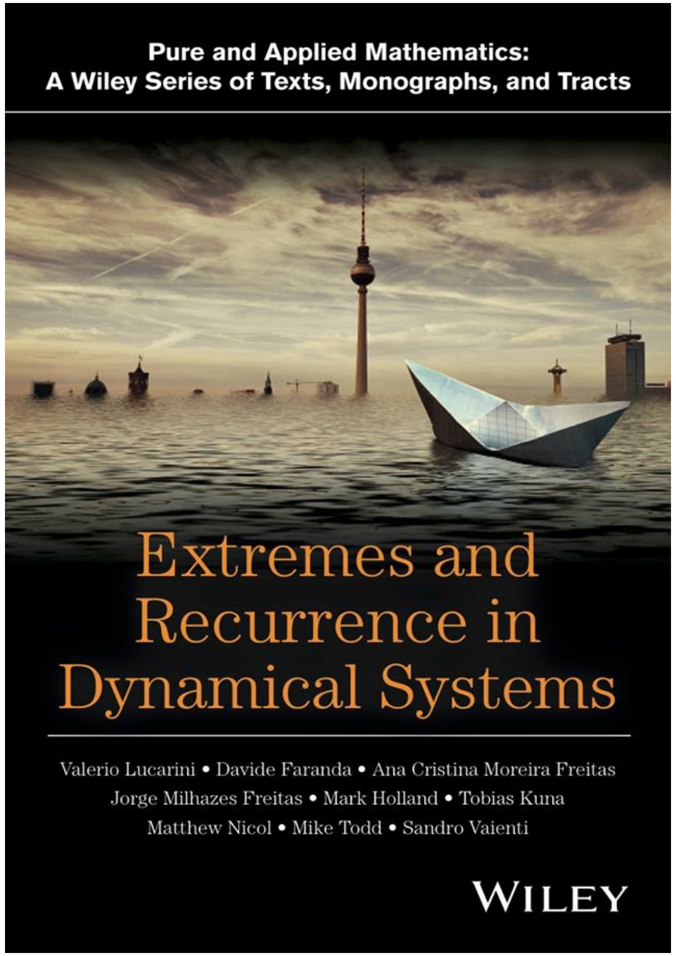
Return Levels of T_{max}







Dynamics



Recurrence: Better Understanding of Atmospheric Predictability (Faranda, Messori..)

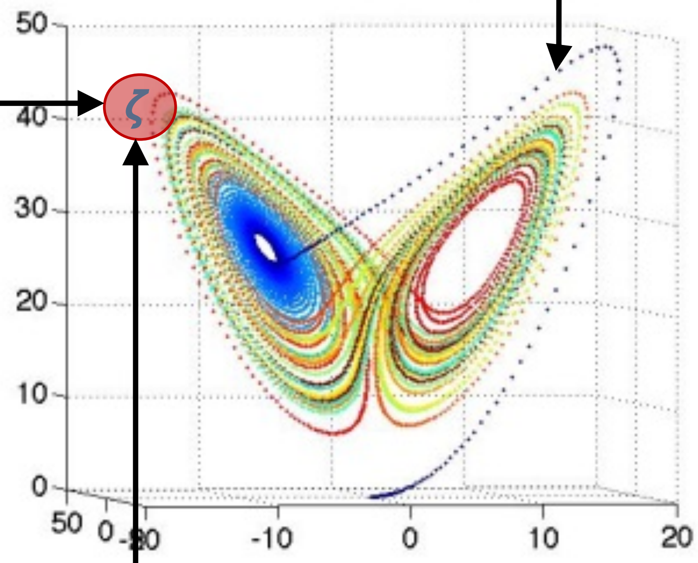
Recurrence and Extremes

1) In a chaotic dynamical system, take a trajectory of the system: $f^m(x)$

2) Rare events are recurrences of a state ζ:

$$\boxed{X_m(x) = g(\mathrm{dist}(f^m(x), \zeta))}$$

3) Then, chose observables such that the maxima of *g* correspond to the minima of the distances with respect to ζ.

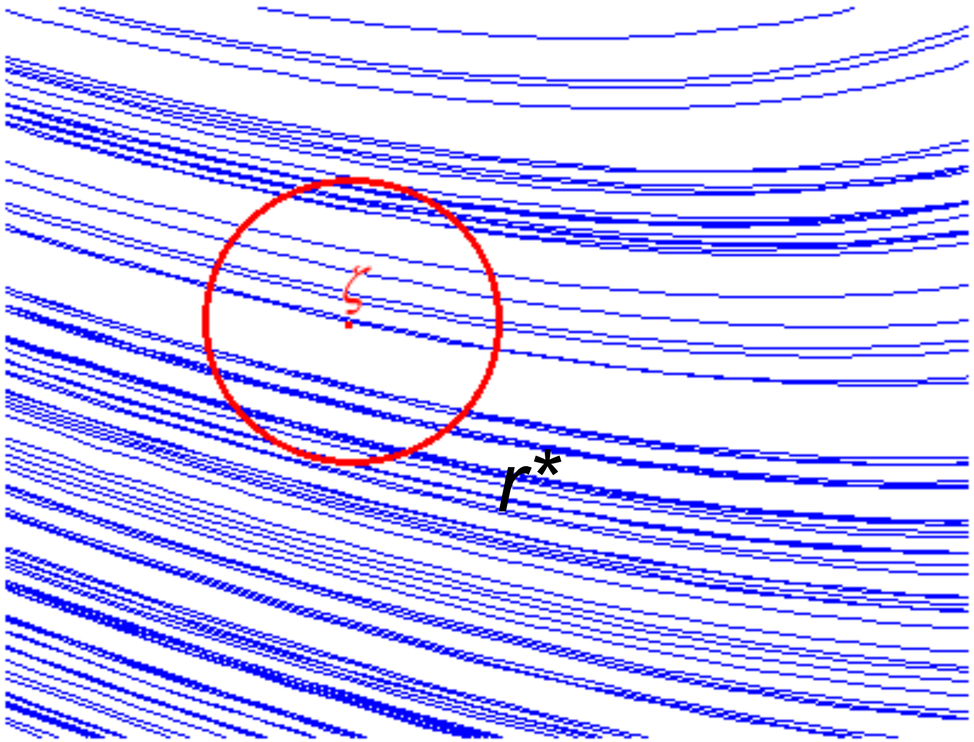


Example:

$$\boxed{g_1(x) = -\log(\mathrm{dist}(x, \zeta)))}$$

Courtesy of Faranda

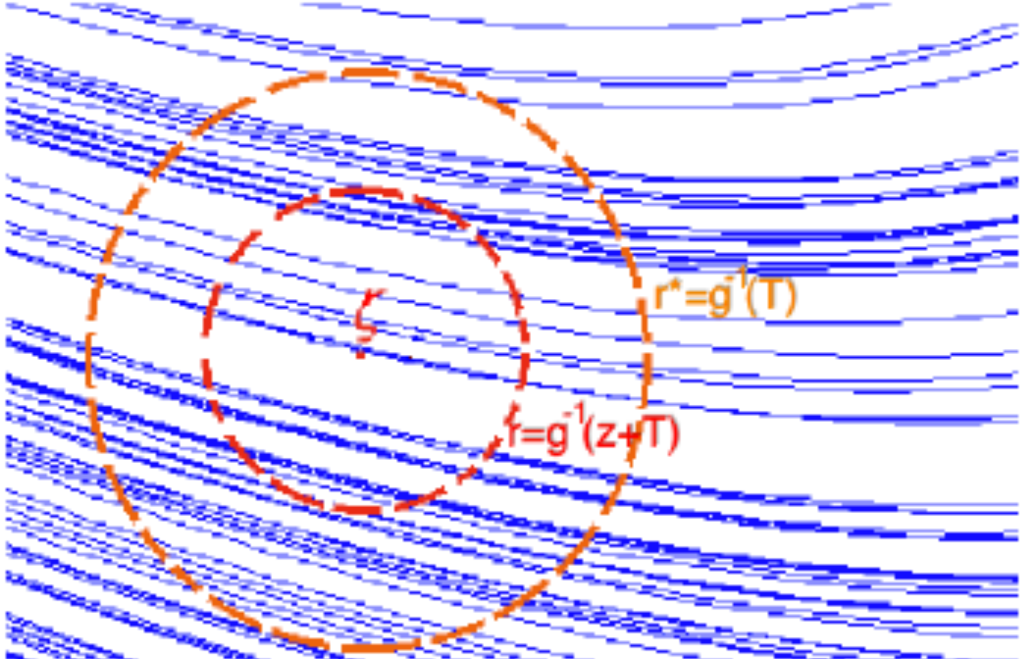
Looking at extremes is like using magnifying glass



- A threshold T corresponds to a radius $r^* = g^{-1}(T)$

General EVLs for distance observables

- Function $g = g(r)$ of the distance $r = dist(x, \zeta)$
- g is monotonically decreasing $\rightarrow$ a threshold T is a radius $r^* \rightarrow$
- An above-threshold event is a close recurrence
- Conditional probability $\rightarrow$ Complementary cumulative function $H_{g,T} = 1 - F_{g,T}$



$$\mathbb{P}(g(\mathrm{dist}(x,\zeta)) < z+T | g(\mathrm{dist}(x,\zeta)) < T) =$$

$$= \frac{\mathbb{P}(g(\mathrm{dist}(x,\zeta)) < z+T)}{\mathbb{P}(g(\mathrm{dist}(x,\zeta))r < T)} =$$

$$= H_{g,T}(z) \equiv \frac{\mu(B_{g^{-1}(z+T)}(\zeta))}{\mu(B_{g^{-1}(T)}(\zeta))}$$

Generalized Pareto Distributions

- We assume a local scaling of the measure of the attractor:

$$\lim_{r \to 0} \frac{\log \nu(B_r(\zeta))}{\log r} = D(\zeta), \text{ for } \zeta \text{ chosen } \nu - \text{a.e.}$$

Functional form of the $g_i's$:

$$g_1(r) = -\log(r)$$
$$g_2(r) = r^{-\beta}$$
$$g_3(r) = C - r^{\beta}$$

$$F_{g_i,T}(z) = GPD_{\xi}\left(\frac{z}{\sigma}\right)$$

- g_1-type observable:

$$\sigma = \frac{1}{D} \qquad \xi = 0;$$

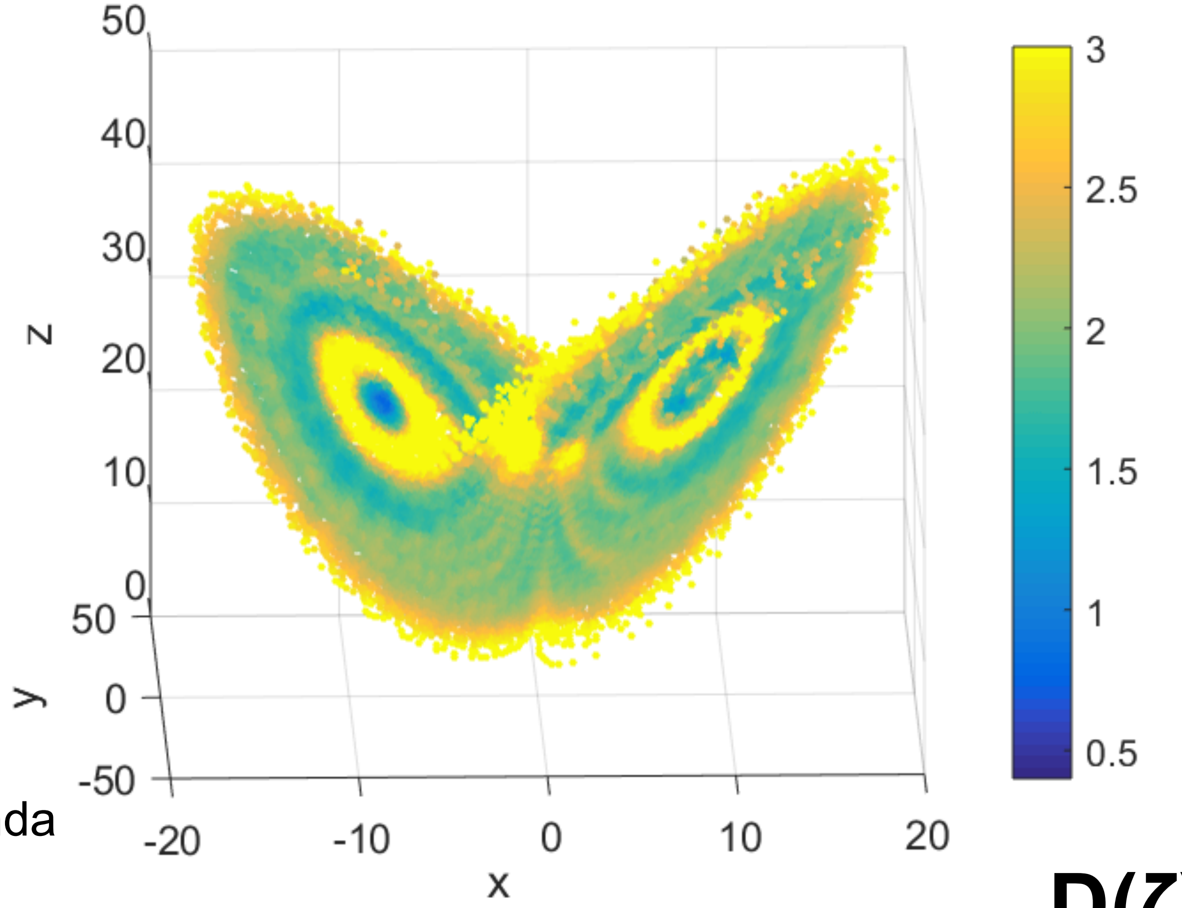
- g_2-type observable:

$$\sigma = \frac{T\beta}{D} \qquad \xi = \frac{\beta}{D};$$

- g_3-type observable:

$$\sigma = \frac{(C-T)\beta}{D} \qquad \xi = -\frac{\beta}{D}.$$

Local Dimension and Lorenz Attractor



Courtesy of Faranda

D(ζ)

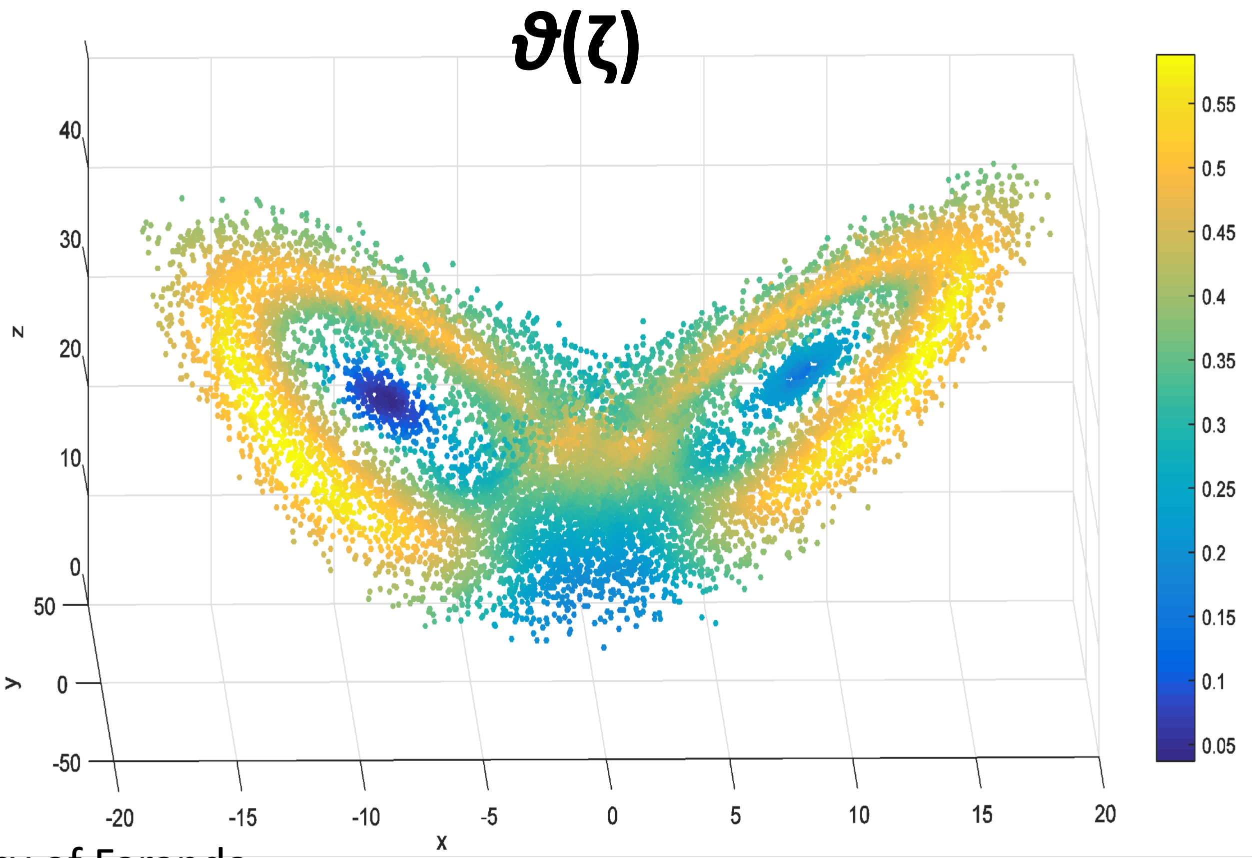
$$\boxed{g_1(x) = -\log(\mathrm{dist}(x, \zeta))}$$

Expectation < D >=<1/σ > = 2.06 agrees with classical value

High dimension – low predictability

Courtesy of Faranda

Persistence of Trajectories



More Unstable/ Less Persistent

More Stable/ More Persistent

Courtesy of Faranda

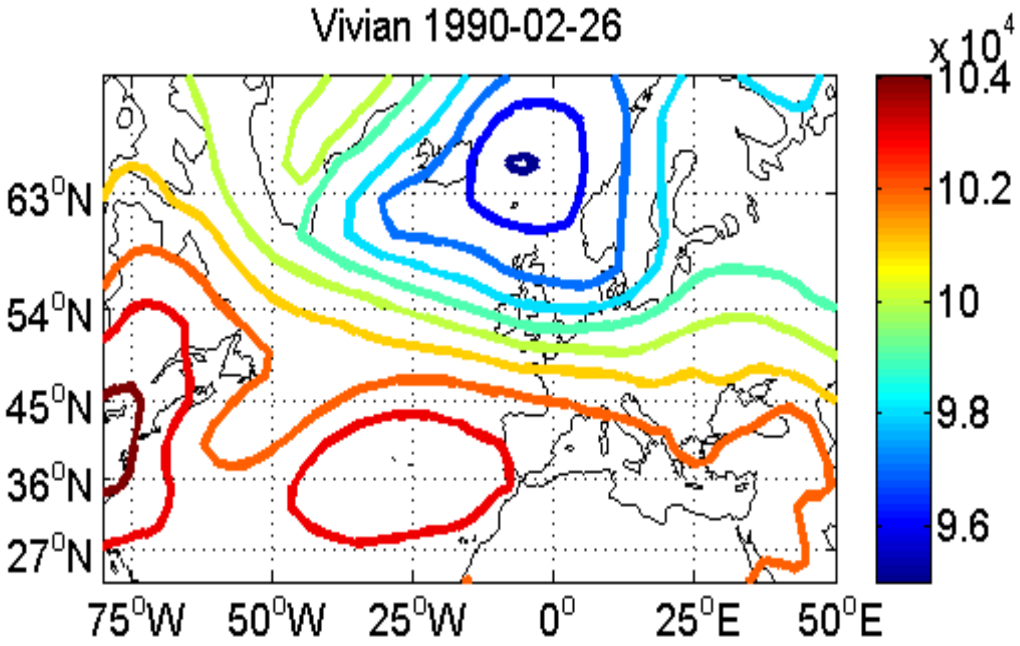
Θ is a good proxy for **unstable fixed points of the system**

Θ is close t**o zero** for **weakly repelling points – large clustering, long persistence**

Real Time Analysis N. Atlantic – SLP

Characterize the predictability of Atmospheric Fields

- *How recurrent?*
- *How rare?*
- *How persistent?*
- *How predictable?*



Courtesy of Faranda

Courtesy of Faranda

- Dataset: **NCEP**
- one day = one point in the we construct the distance **r** between such day and all the others in the record
- For each day one estimates **d** and **θ** using EVT.

- **the higher d, the more unpredictable is the atmospheric circulation**

- **the lower θ the more stable is the atmospheric circulation**

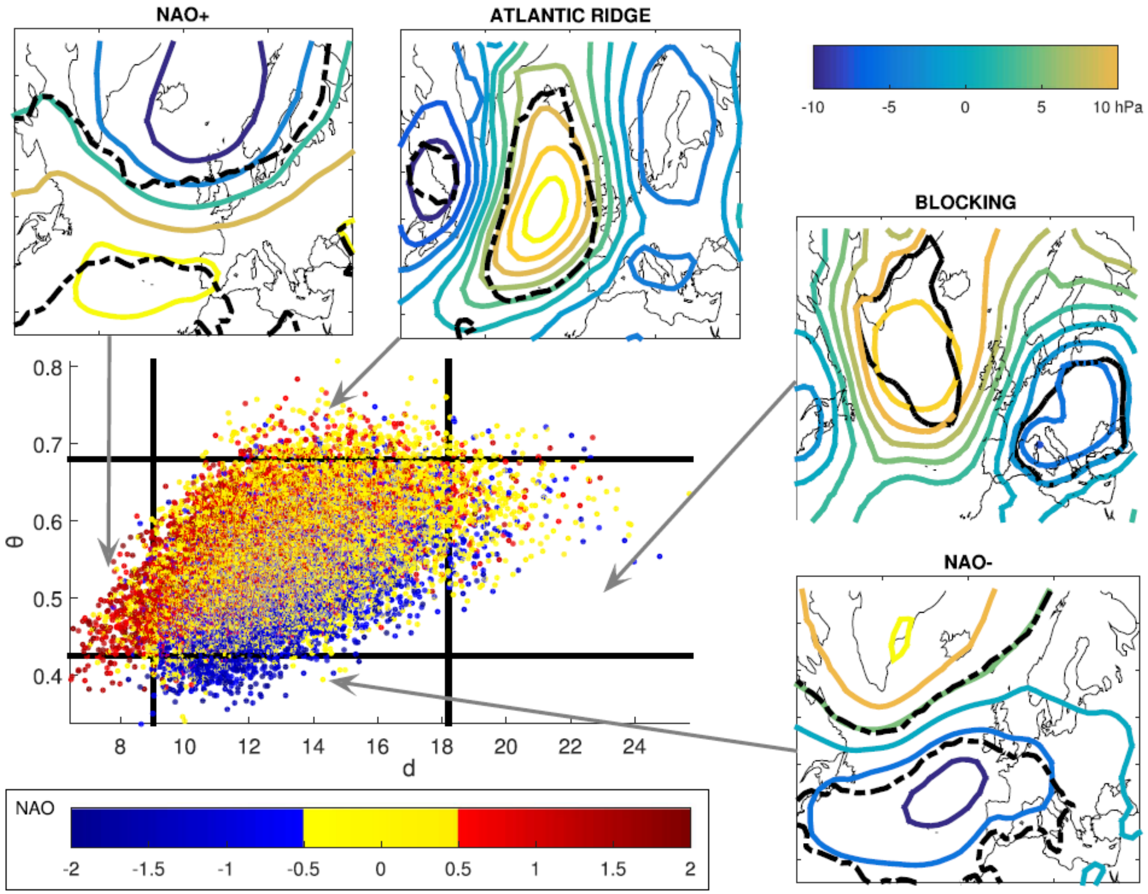
Dynamical Systems Metrics based on EVT Help us Understanding Atmospheric Dynamics

Shape parameter ξ is related to the local dimension D of the attractor at x_0.

The extremal index θ represents the persistence of the system near to x_0

Large D (remember Kaplan-Yorke formula!) associated with higher instability

Many applications in complex systems



(Faranda et al., 2017)

Physical Observables

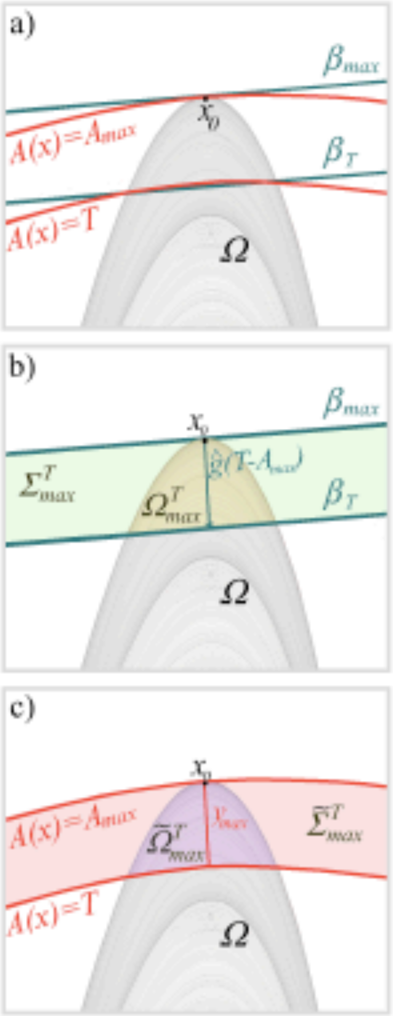
Distance observables are not very *physical*

- .... unless one looks at recurrences of the orbits $\rightarrow$ *weather analogues*
- Example: we want to look at *energy-like* observables of the form $\langle \vec{x}, M\vec{x} \rangle$ where $\vec{x}$ is a point in the phase space, M is a matrix, and $\langle \bullet, \bullet \rangle$ is the scalar product
- In Fluid Dynamics, many physical observables can be written in this form!
- More in general, we want to study the extremes of $A : \vec{x} \in \mathbb{R}^n \rightarrow A(\vec{x}) \in \mathbb{R}$.
- Difference *wrt* previous case: **Geometry of the portion of attractor where extremes are realized**

Above-threshold events for Physical Observables

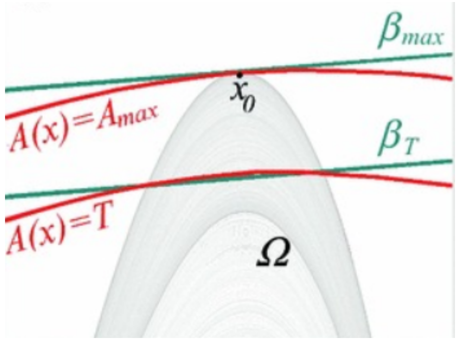
Construction

- $A(x)$ has isolated max A_{max} in $x_0 \in \Omega$
- β_{max} is tangent to $A(x) = A_{max}$ in x_0
- $\hat{g}$ is parallel to $\nabla A(x)|_{x=x_0} \neq 0$
- Important: $\hat{g}$ is orthogonal to unstable and neutral manifolds in x_0!
- u and n mans. tangent to β_{max}!
- $A(x) = T$ is threshold, β_T is $\beta_{max} \to \hat{g}$
- Extremes of $A(x)$ are in $\tilde{\Omega}^T_{max}$...



Extremes of Physical Observables

- Extremes of general physical observables are located in rarely visited regions of the phase space.



Shape parameter ξ is related to attractor dimension along the stable, unstable and neutral manifolds.

(Holland et al. 2012, Lucarini et al., 2014)

$$\xi = -\frac{1}{\delta} \qquad \delta = d_s + (d_u + d_n)/2$$

d_u: number of pos. Lyapunov exp.
d_n: number of Lyap. Exp. = 0
$d_s = d_{KY} - d_u - d_n$

ξ does not depend on the chosen observable.
ξ is negative and close to 0 in a high dimensional system.

Hindawi
Complexity
Volume 2017, Article ID 5340858, 20 pages
https://doi.org/10.1155/2017/5340858

WILEY | Hindawi

Research Article

Convergence of Extreme Value Statistics in a Two-Layer Quasi-Geostrophic Atmospheric Model

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Received 14 April 2017; Accepted 10 July 2017; Published 6 September 2017

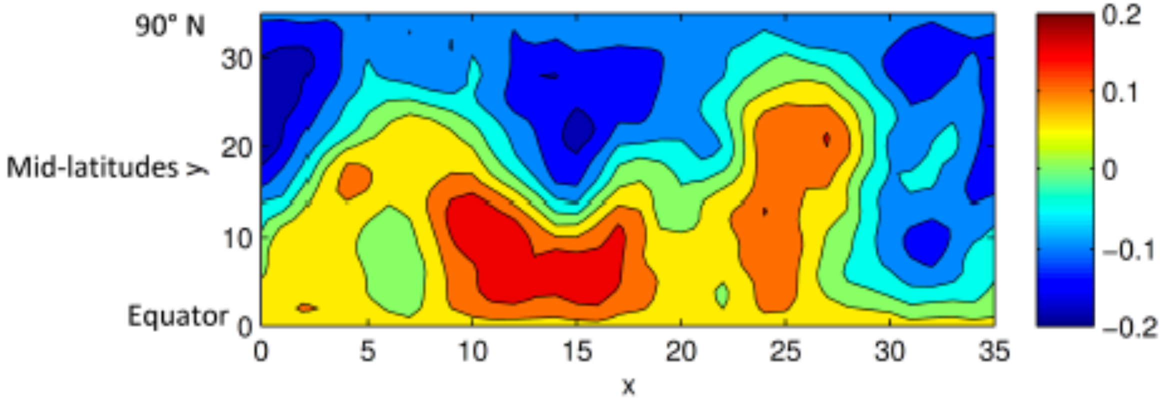
Academic Editor: Gabriele Messori

V. M. Gálfi, T. Bódai, and V. Lucarini, *Complexity, 2017*

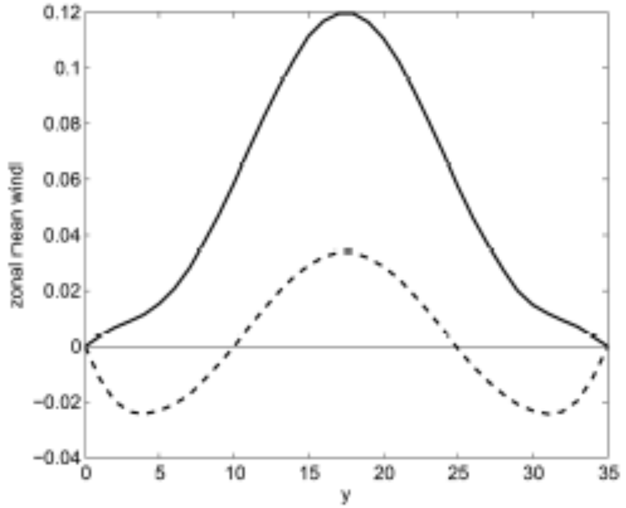
The QG model

Quasi-geostrophic 2-layer atmospheric spectral model

Periodic, rectangular β-channel



Upper layer stream function

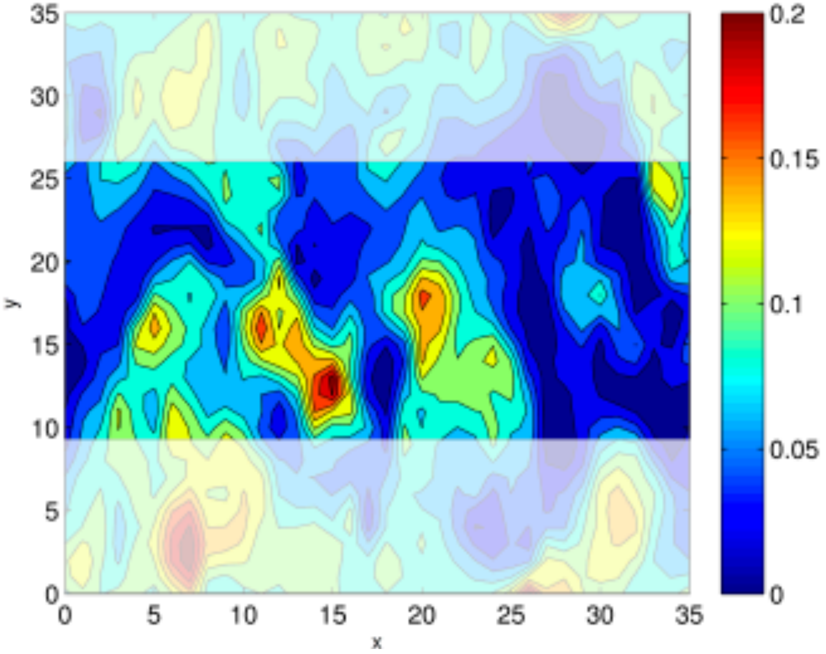


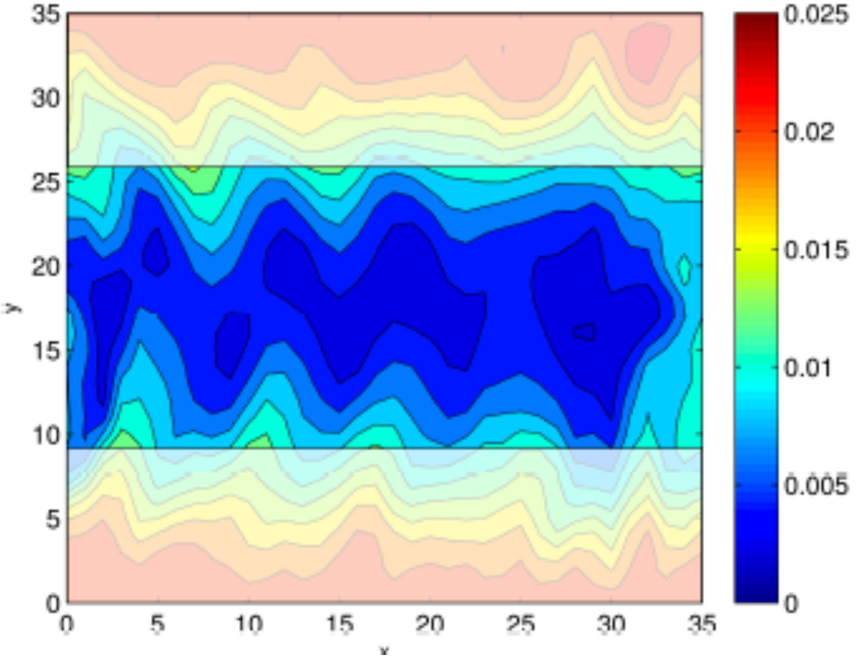
Forcing: Newtonian cooling $r_R(T_e - T)$ with $T_e(y) = \frac{\Delta T}{2} \cos \frac{\pi y}{L_y}$
Spectral resolution: 16×16, phase space dimension: 1056
Grid point resolution: 36×36

Simulations with strong and weak forcing

Forcing: $T_e(y) = \frac{\Delta T}{2} \cos \frac{\pi y}{L_y}$, ΔT: forced meridional temp. difference

Strong forcing	Weak forcing
$\Delta T = 133K$	$\Delta T = 40K$
$\xi_\delta = -0.0021$	$\xi_\delta = -0.03$
$d_{KY} = 585.95$	$d_{KY} = 39.31$
# pos. Lyap. Exp.: 222	# pos. Lyap. Exp.: 17





Main questions

MLE:

local

– latitude 9: $\xi_{e,9}(n)$

– .

– .

– .

– latitude 26: $\xi_{e,26}(n)$

$\rightarrow$

We look at local and zonal kinetic plus potential energy

ξ_δ, for $n \rightarrow \infty$

Theoretical shape parameter

zonal

– latitude 9: $\xi_{e_z,9}(n)$

– .

– .

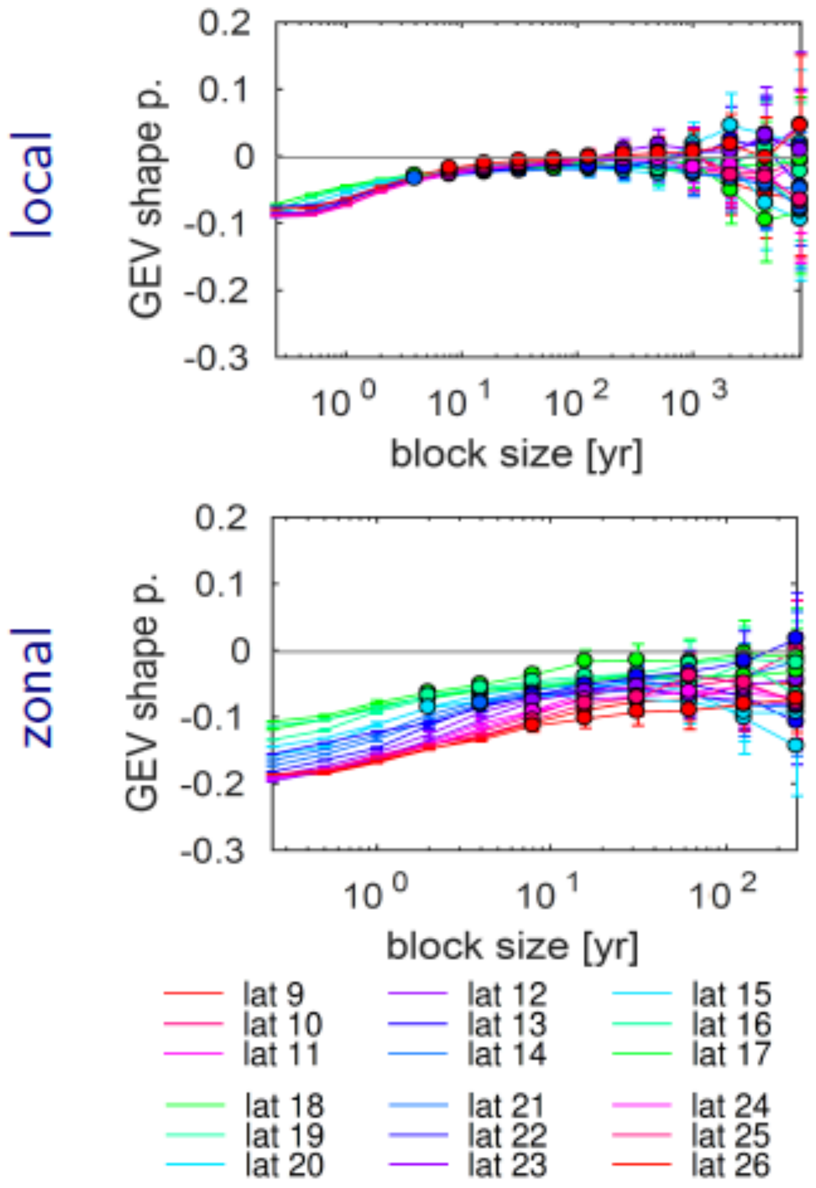
– .

– latitude 26: $\xi_{e_z,26}(n)$

Can we detect universal properties of extremes based on our finite time series?

Differences between strong and weak forcing

Strong forcing

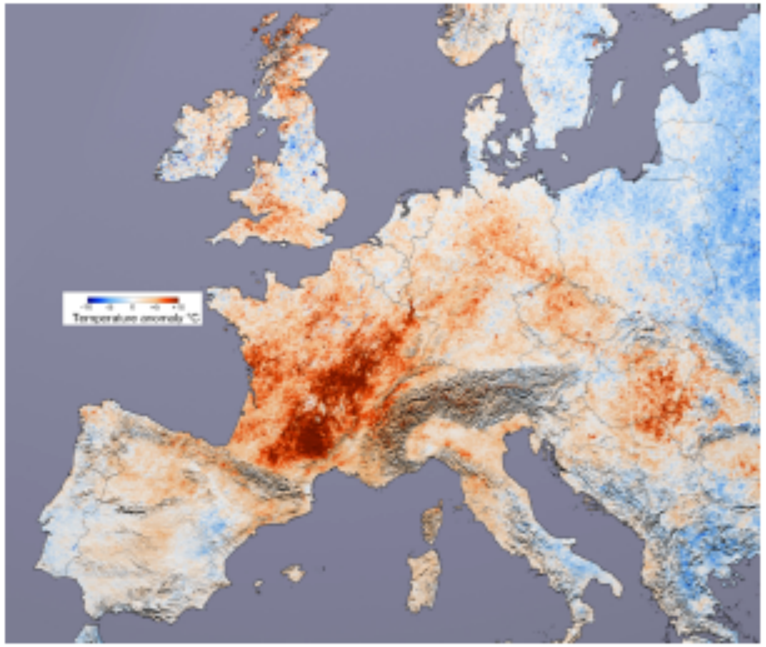


- Universality of extremes on asymptotic level.
- Differences for finite block sizes.

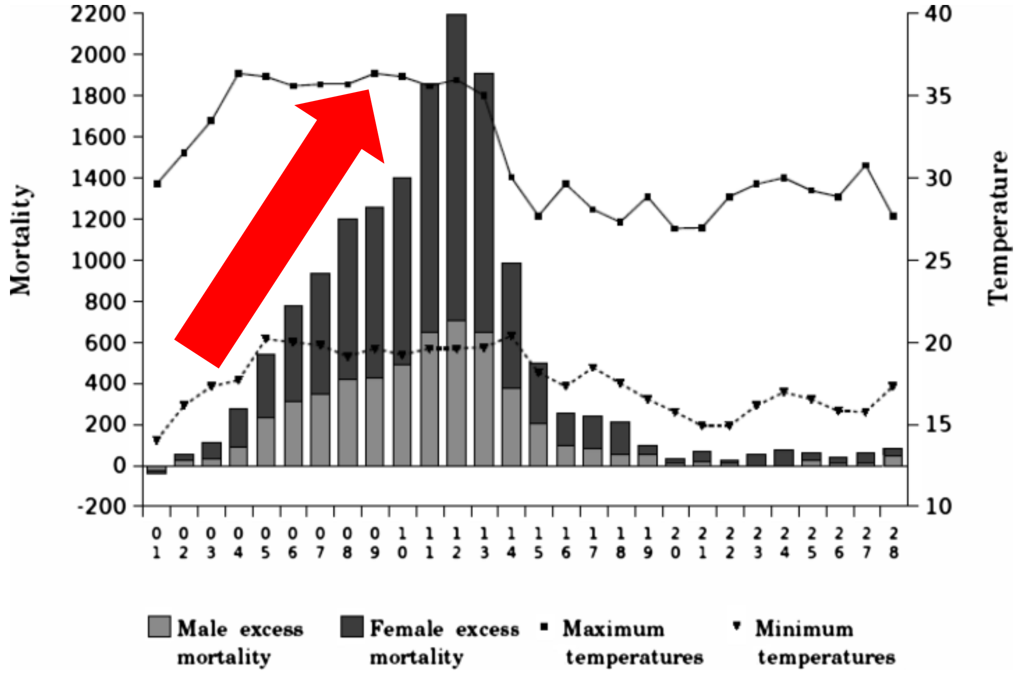
The speed of convergence to the asymptotic level is not universal!

Persistent Extremes

Persistent extreme events: persistence in time ⇔ extension in space

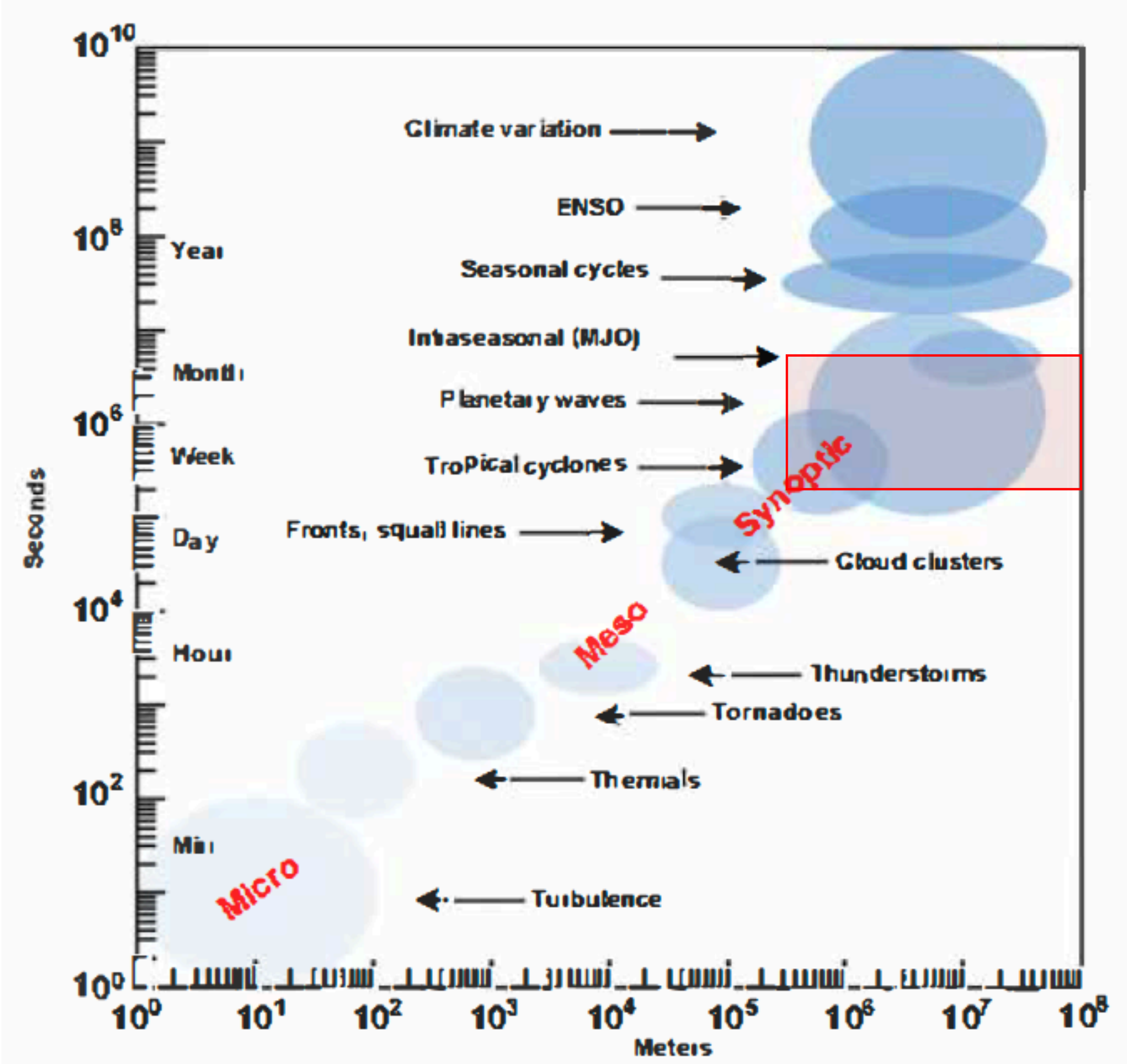


Summer 2003 Heatwave



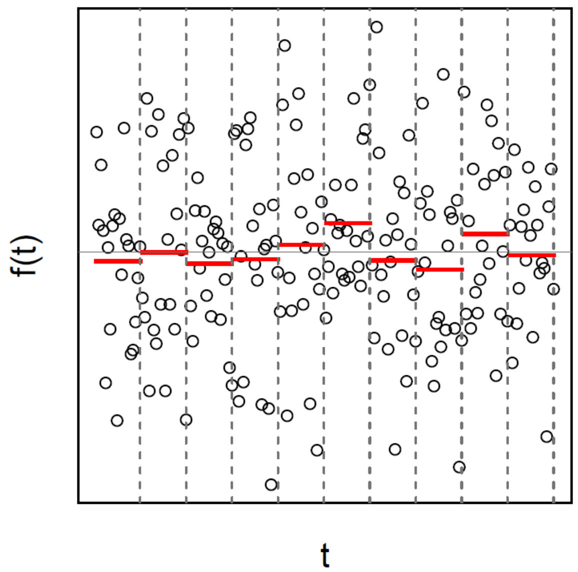
Excess death in summer 2003 in France
Poumadere (2005)

Spatial and Temporal Scales



Limit Laws for Averages over Large Averaging Blocks

- Random variables $X_1, X_2, \ldots$
- $\rightarrow$ averages over blocks of length n: $A_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

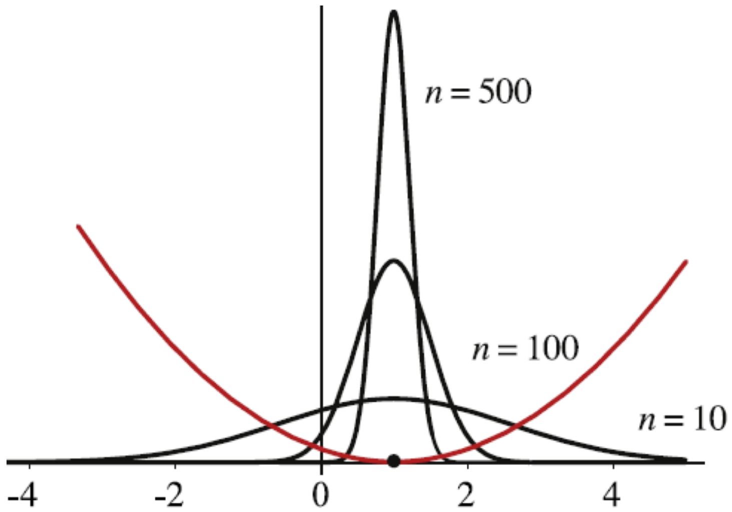


Large deviation principle:

for $n \rightarrow \infty$, the probability of A_n:

$$p(A_n = a) \approx e^{-nI(a)}$$

rate function I(a): speed of decay



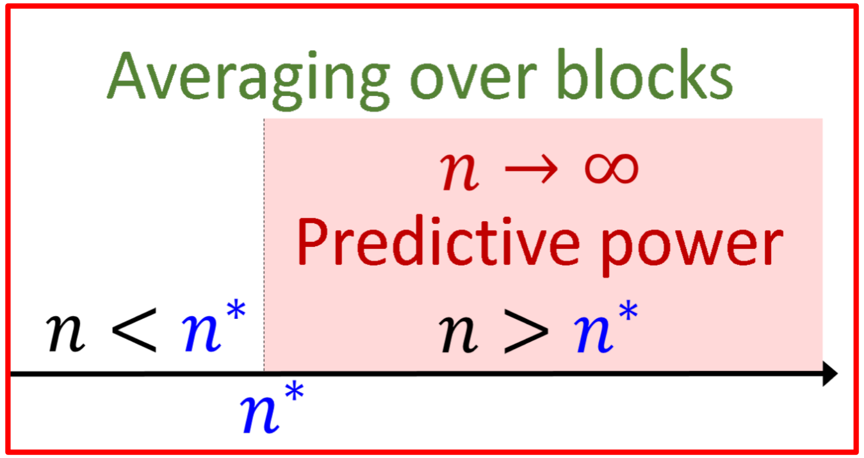
- As n grows $\rightarrow$ more peaked around <A>
- Law of large number as a special case
- $\sigma \rightarrow \sigma/\sqrt{n}$ as a special case

Large Deviation Theory

1)based on a **limit law**

$\Rightarrow$ the limit $n \to \infty$ has to be reached,
i.e. n has to be large enough $n > n^*$
$\Rightarrow$ we find a LDP

$\Rightarrow$ predictive power: results valid for every $n > n^*$



- Most likely among the unlikely events
- Leaves out freak events

<u>Problem: assumption of independent random variables</u>

BUT correlations in a physical systems

Solution: rescale time via integrated autocorrelation time τ

$$I_n(a) = -\frac{1}{n/\tau} \ln p(A_n = a) \qquad \tau = 1 + 2\sum_{l=1}^{n_l} c(l)$$

The PUMA Model

The **P**ortable **U**niversity **M**odel of the **A**tmosphere

https://www.mi.uni-hamburg.de/en/arbeitsgruppen/theoretische-meteorologie/modelle/plasim.html

- *primitive* equations on the sphere
- *hydrostatic* approximation
- *simple parametrisations* for friction (Rayleigh friction), diffusion (hyperdiffusion), and diabatic heating (Newtonian cooling).
- Zonally-symmetric boundary conditions, no orography
- *Basically, a dynamical core*

Forcing of the model: Newtonian cooling

ΔT - meridional temperature difference

ΔT large $\Rightarrow$ system chaotic $\Rightarrow$ weak correlations

Simulations with PUMA

Simulations:

- 10000 years,
- no orography, no annual cycle
- symmetric N-S forcing
- T42 resolution, 10 vertical levels.

Temperature field

- in the lowest level (960 hPa).

High dimensional chaos

- LTD will likely work (Chaotic hypothesis etc).

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Journal of Statistical Mechanics: Theory and Experiment

An IOP and SISSA journal

PAPER: Interdisciplinary statistical mechanics

A large deviation theory-based analysis of heat waves and cold spells in a simplified model of the general circulation of the atmosphere

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Received 22 July 2018
Accepted for publication 14 January 2019
Published 19 March 2019

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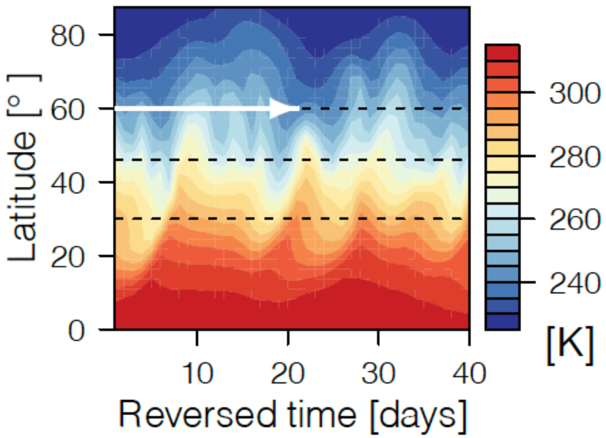
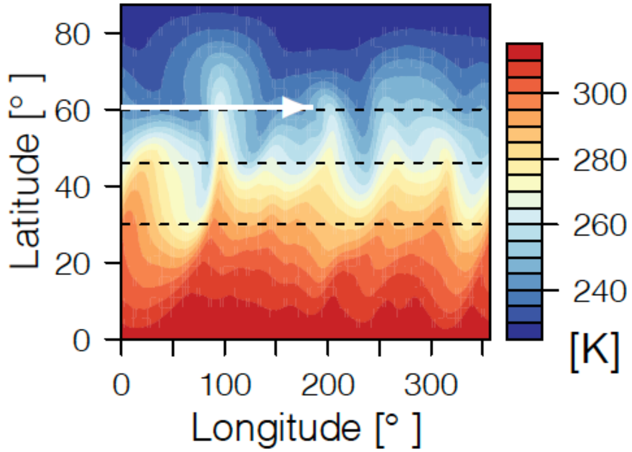
Online at stacks.iop.org/JSTAT/2019/033404
https://doi.org/10.1088/1742-5468/ab02e8

Persistent Temperature Extremes in PUMA

Average in time



Average in space



persistent events

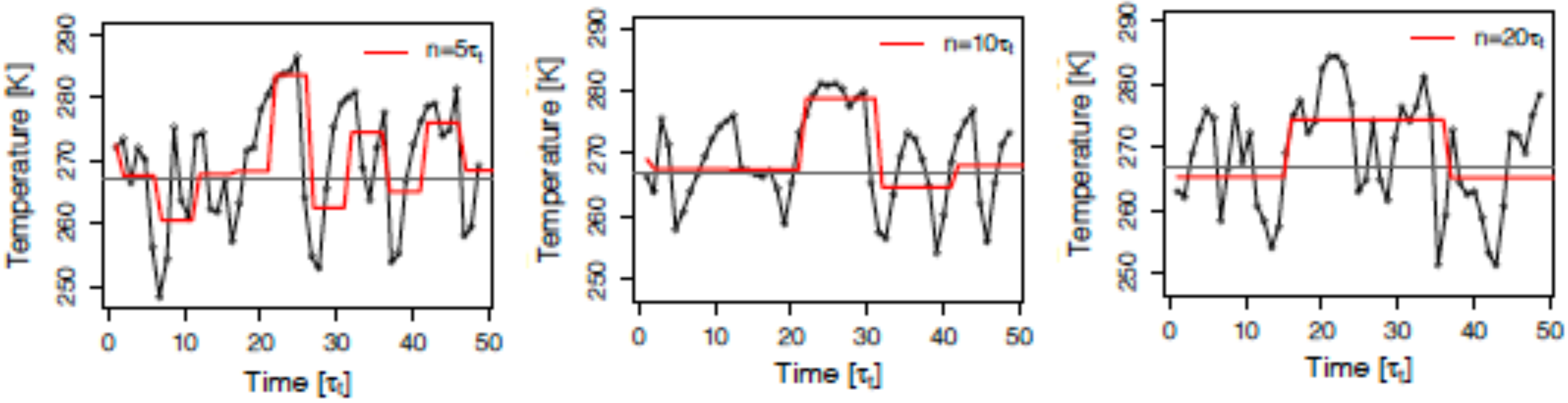
- **temporal averages** A_{n_t} for increasing $n_t = 1\tau_t, 5\tau_t, 10\tau_t, \ldots$
- temporal rate function estimates $\Leftrightarrow$ $I_{n_t} = \frac{\ln p(A_{n_t})}{n_t/\tau_t}$

- **spatial averages** A_{n_x} for increasing $n_x = 1\tau_x, 5\tau_x, 10\tau_x, \ldots$
- spatial rate function estimates $I_{n_x} = \frac{\ln p(A_{n_x})}{n_x/\tau_x}$

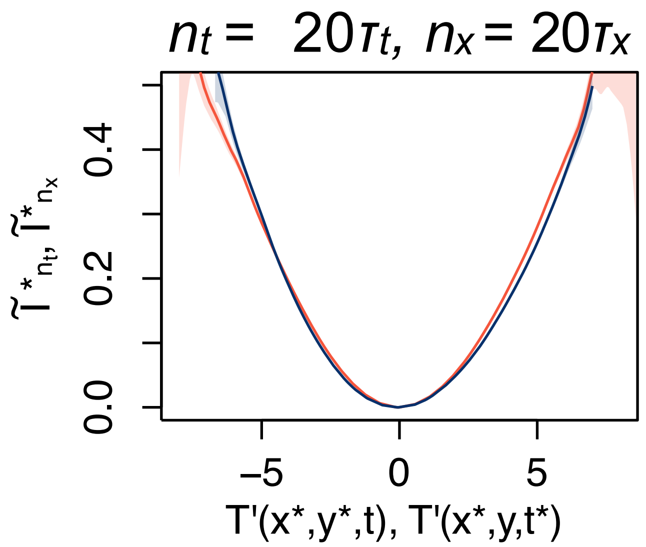
Questions

1) What is the connection between temporal and spatial averages?
2) How to study persistent events based on LDT?

Long Averages ⇔ Persistent Events



Temporal and Spatial Large Deviations - Universality



2) **Temporal averages ⇔ spatial averages**

- re-normalised rate functions based on the integrated auto-correlation.

$$I_{n_t} = \frac{\ln p(A_{n_t})}{n_t/\tau_t},\ I_{n_x} = \frac{\ln p(A_{n_x})}{n_x/\tau_x}$$

⇒ $I_{n_t} = I_{n_x}$ ⇒ Universal function I_n.

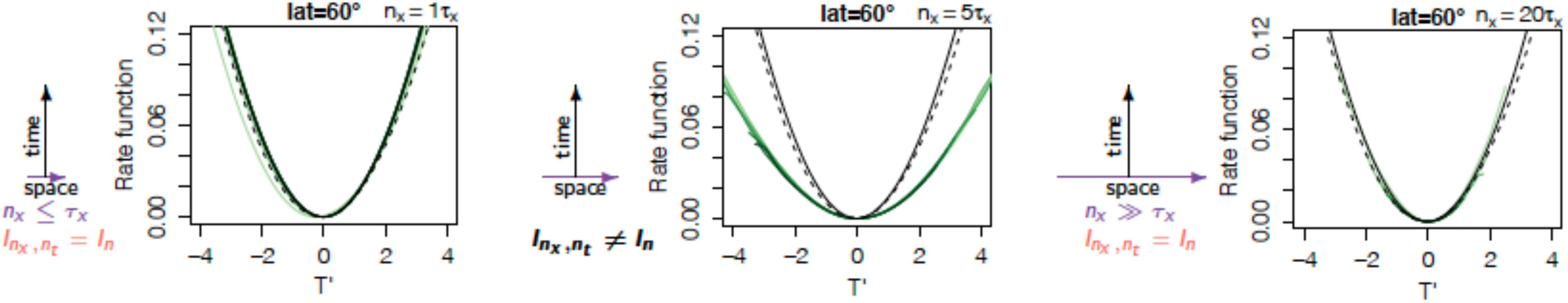
⇒ Time series ⇒ probabilities of spatial averages (and vice-versa)

Universality of temporal and spatial averages

<u>We have predictive power for events lasting more than n*</u>

<u>We are somewhat lucky – only negligible deviations from Central Limit Theorem</u>

Spatio-temporal Large Deviations - Persistence



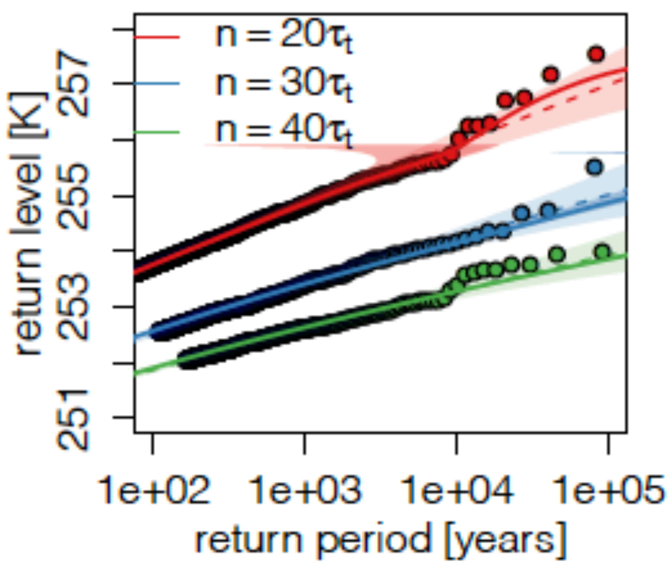
3) On intermediate spatial scales:

- the **spatio-temporal** renorm. rate function **differs from the universal function** I .
  $\Leftarrow$ spatial correlations.

- We “capture” persistent events by **1.** averaging on intermediate spatial scales and **2.** obtaining a LDP in time
- Persistent extremes: large deviations connected to a high degree of spatio-temporal organisation of the system on specific scales.

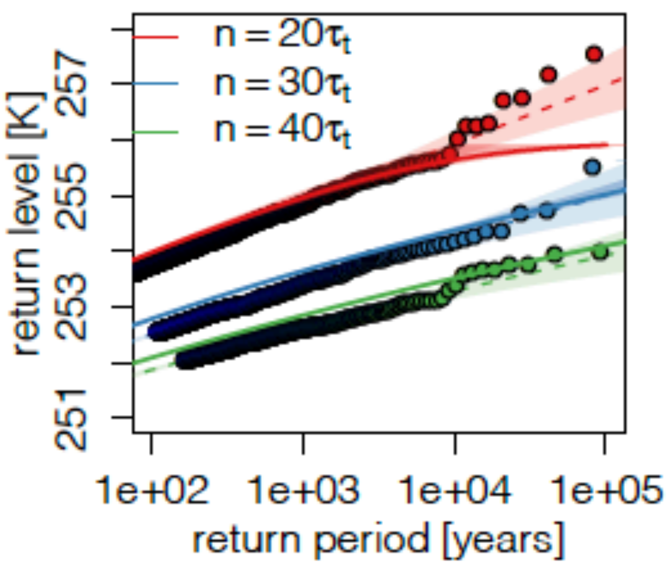
Organised structures become more apparent

Return Level Plots Based on Rate Functions



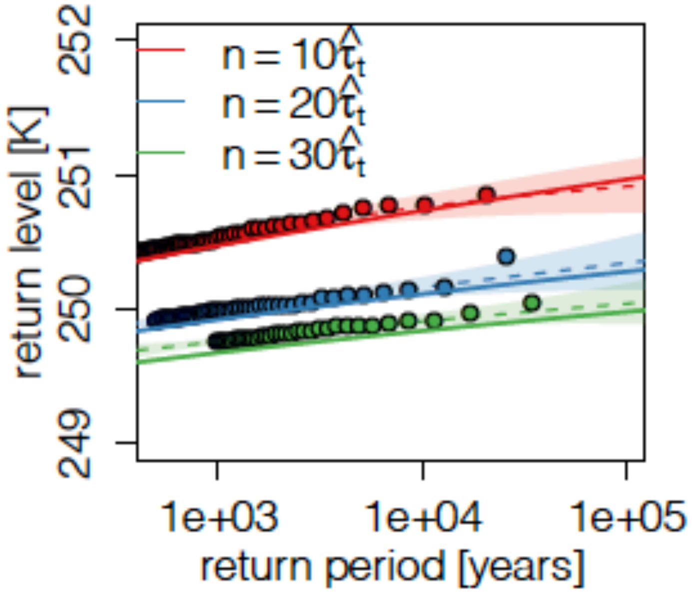
$I_{n_t=20\tau_t} \Rightarrow$ return periods for $n_t = 20\tau_t, 30\tau_t, 40\tau_t$

LDP in time



$I_{n_x=20\tau_x} \Rightarrow$ return periods for $n_t = 20\tau_t, 30\tau_t, 40\tau_t$

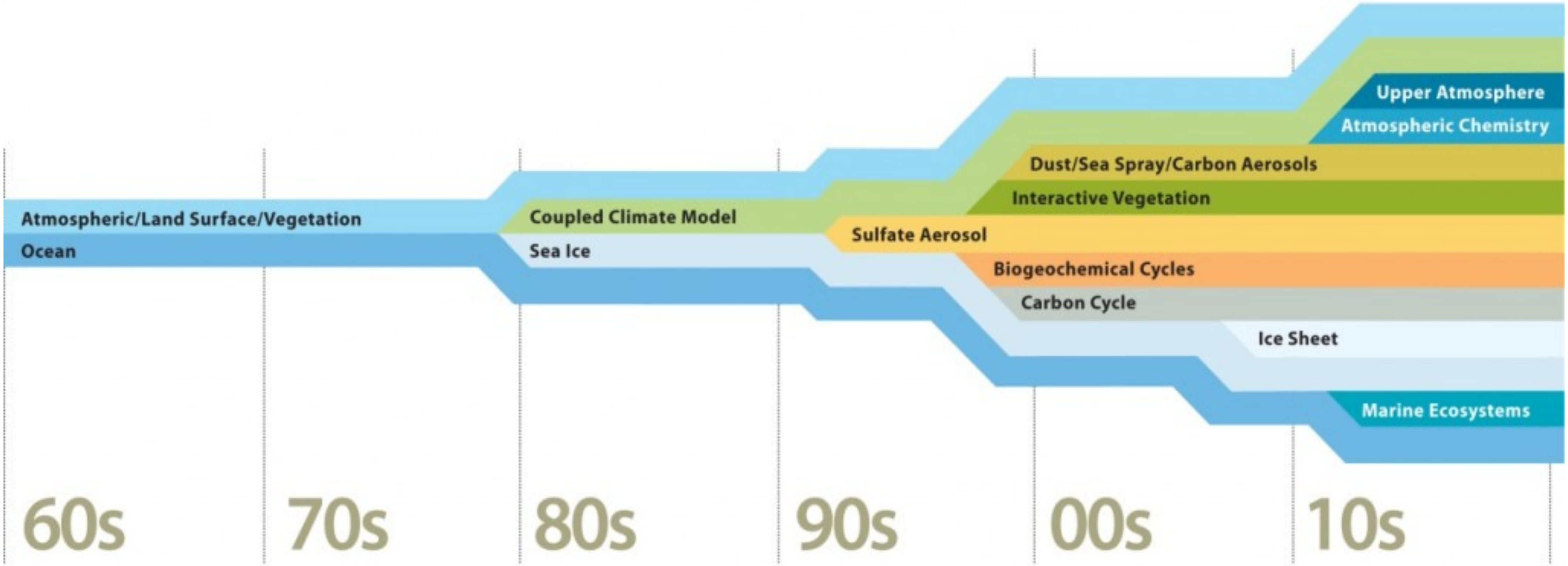
Equivalence $I_{n_t} = I_{n_x}$



$I_{n_t=20\tau_t} \Rightarrow$ return periods for $n_{xt} = 20\tau_{xt}, 30\tau_{xt}, 40\tau_{xt}$

Equivalence $I_{n_t} = I_{n_x t}$

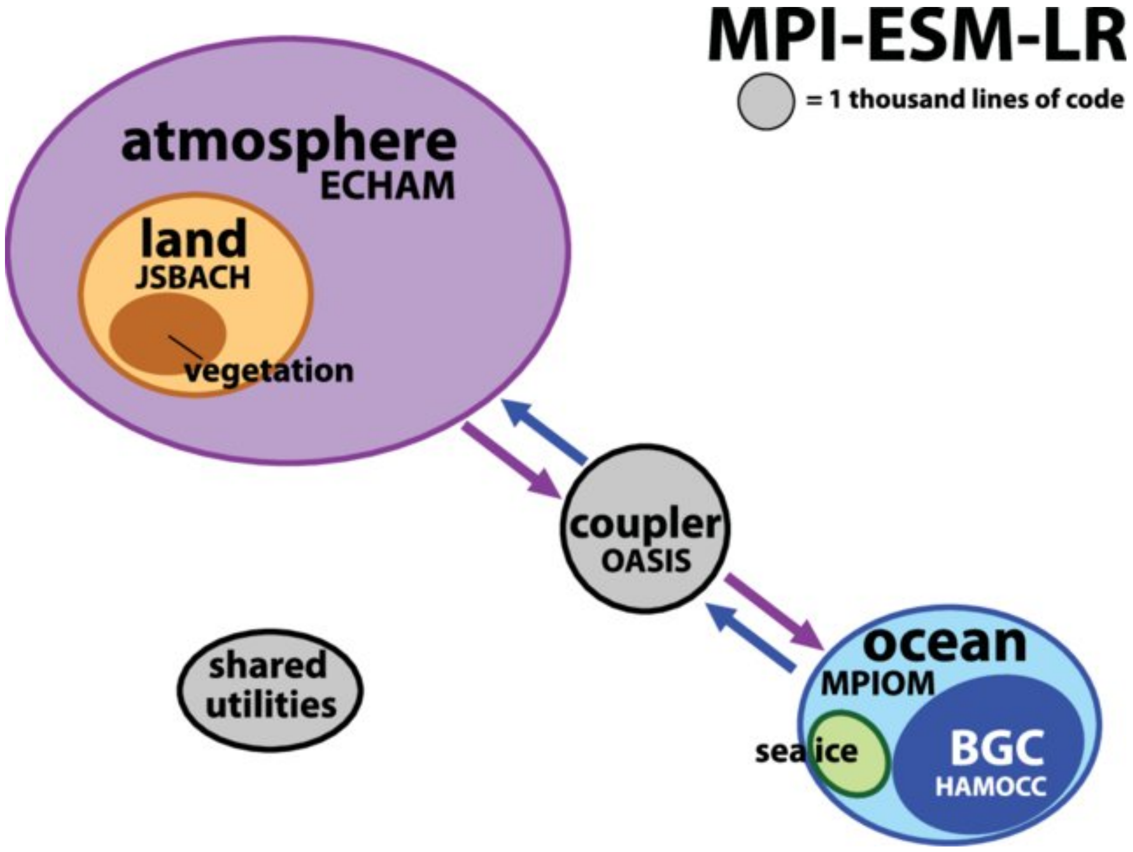
The Long Journey of Climate Models

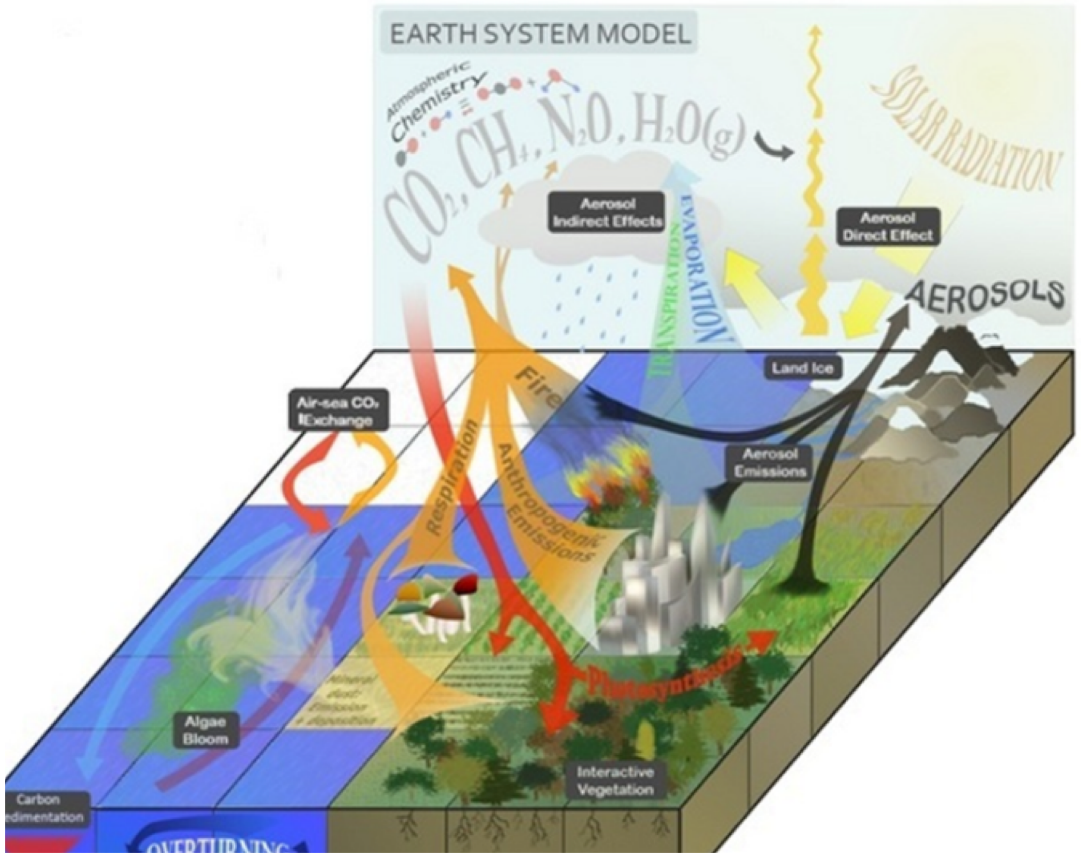


From here.. → To here..

Out of the Comfort Zone

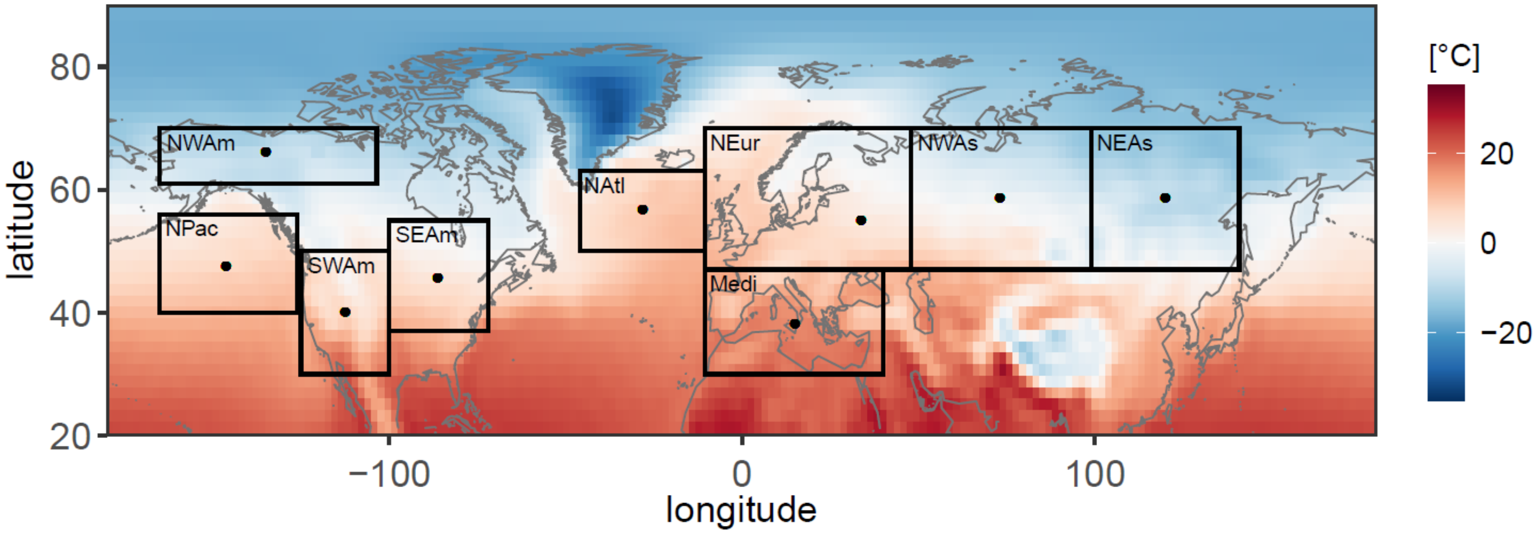
We study persistent T fluctuations in a full ESM
<u>Advantages</u>: We can hope to learn something about the real world
<u>Disadvantages</u>: Things becomes substantially more complicated





CMIP6 Control and 4xCO_2 MPI-ESM-LR Runs

- 1000 years pre-industrial control run
- 140 years abrupt 4xCO_2 simulations
- We select 9 regions on the NH - inspired by Giorgi (2006).



*Before the **tipping points** ('10s)*
(Breaking Bad)

*.. and after the **hockey stick** ('90s)*
(Sex & the City)

the "thing" was

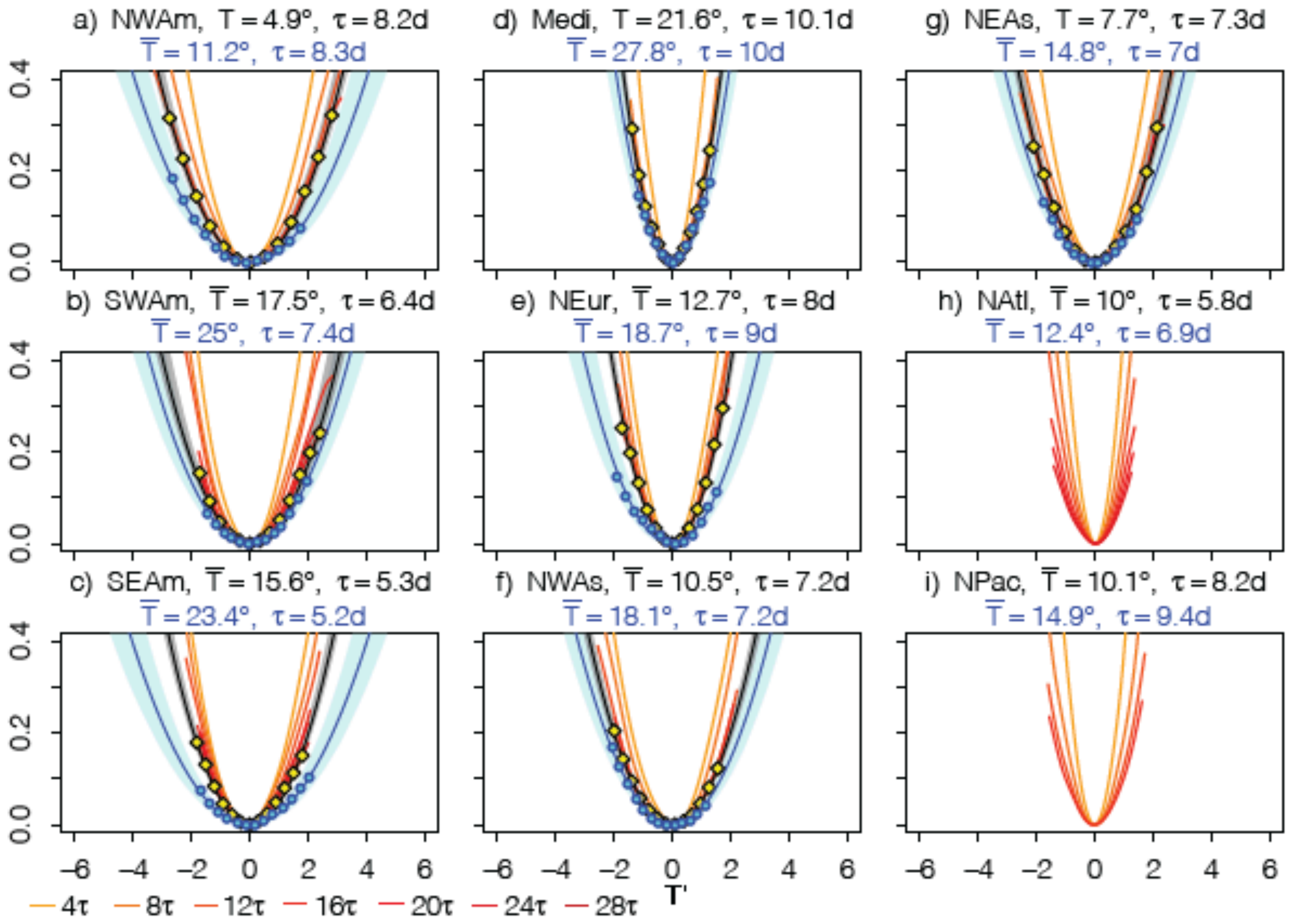
the climatic hotspots ('00s)
(Lost)

We eliminate the seasonal cycle by subtracting the long term daily mean.

We define an extended summer (160 days) and winter (105 days)

Rigorous, universal climatology of heat waves and cold spells

Summer Rate Functions Converge over Land



- Rate functions estimates:

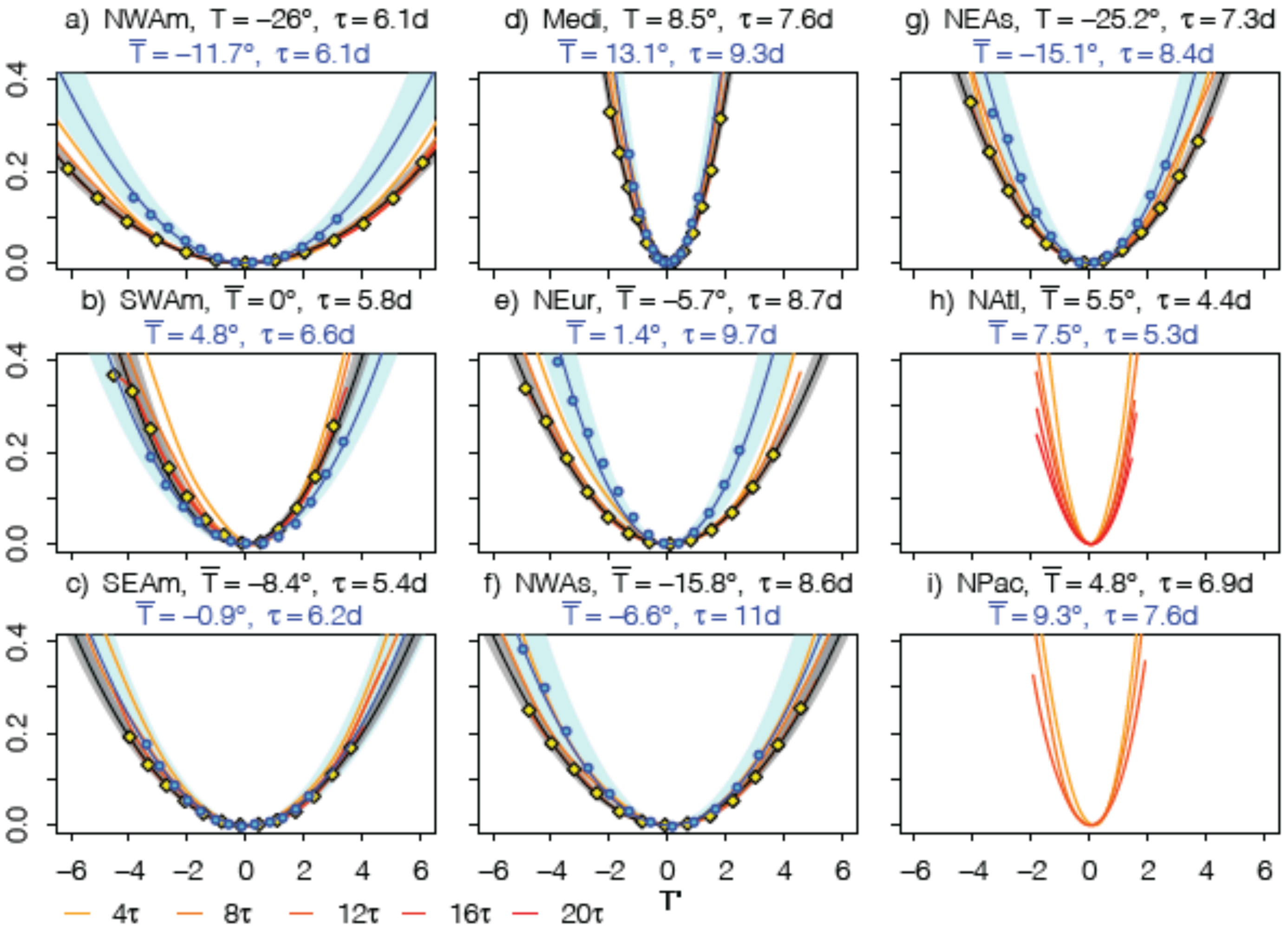
$$I_n(a) = -\frac{1}{n/\tau} \ln p(A_n = a)$$

- Convergence over land regions for
  $$n \geq n^* \approx 12 - 16\,\tau$$
- No convergence over oceanic regions because of long-term memory
- Within the range of interest, no observed deviations from CLT

Impact of Climate Change

larger deviations with longer duration on a warmer mean

Winter rate Functions are Flatter



- The deviations from the mean are larger in winter over land
- Weather variability is much larger
- Clear sign of continental effects
- Within the range of interest, no observed deviations from CLT

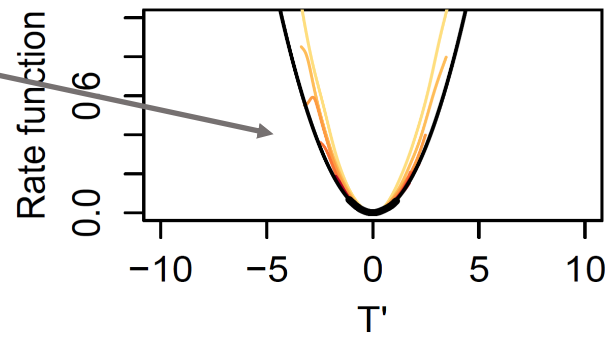
Impact of Climate Change

smaller deviations on a warmer mean

Idea: approximate the probability of high-impact persistent temperature fluctuation using CLT

- Rate functions are approximately quadratic (even if original statistics not Gaussian!) we can approximate the probability of heat waves lasting longer than $10\ \tau$

$$\log\left(p(T_n = a)\right) \approx -n I_T(a), \quad I_T(a) = \frac{a^2}{2\tau_T \sigma_T^2}$$

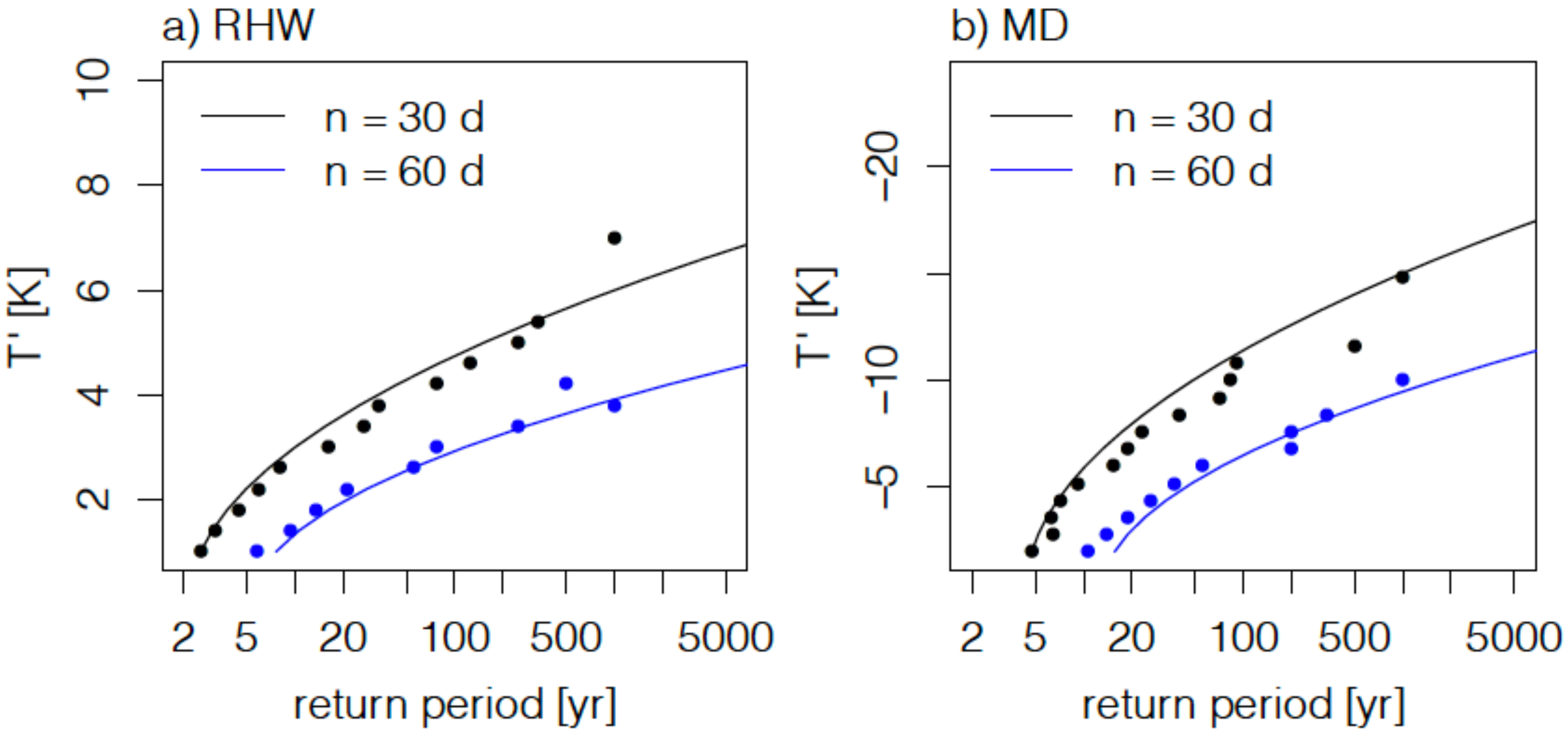


We obtain predictability for larger $a \rightarrow a'$ and longer events $n \rightarrow n'$.

$$\boxed{p(T_{n'} = a')} \approx \boxed{p(T_n = a)} \exp\left(\frac{na^2 - n'a'^2}{2\tau_T \sigma_T^2}\right)$$

Predicting the return times

$$p(T_{n'} = a') \approx p(T_n = a) \exp\left(\frac{na^2 - n'a'^2}{2\tau_T \sigma_T^2}\right)$$



Final Part: Getting the Events Right

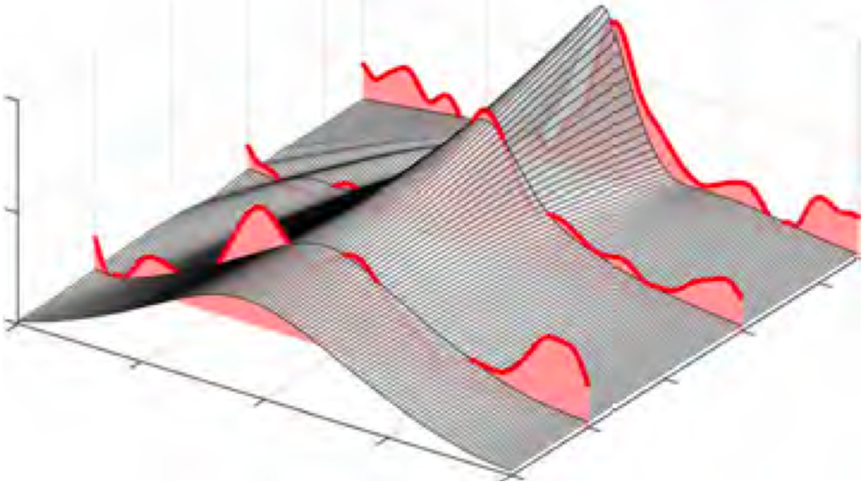
$$\cancel{dX_t^\varepsilon = b(X_t^\varepsilon)\,dt + \sqrt{\varepsilon}\sigma\,dW_t}$$

Dematteis et al. 2018, 2019; Grafke and Vanden-Eijden. 2019

Dynamical System

$$\partial_t u = b(u,\theta), \quad u(t=0) = u_0(\theta) \qquad \mu(\theta)$$

Rogue waves as fluctuations on top of instanton



Target

$$P_T(z) = P(F(\theta) \geq z), F(\theta) = \begin{cases} f(u(T,\theta)) \\ \int_0^T f(u(t,\theta))\,dt \\ \max\limits_{0\leq t\leq T} f(u(t,\theta)). \end{cases}$$

$$S(\eta) = \log \mathbb{E} \exp\langle \eta, \theta\rangle = \log \int_\Omega \exp\langle \eta, \theta\rangle d\mu(\theta)$$

Rate function

$$I(\theta) = \max_\eta(\langle \eta, \theta\rangle - S(\eta))$$

$$P_T(z) \asymp \exp\left(-\min_{\theta\in\Omega(z)} I(\theta)\right) \quad \Omega(z) = \{\theta \in \Omega | F(\theta) \geq z\}$$

Dominating Point

$$\theta^*(z) = \mathrm{argmin}_{\theta\in\Omega(z)} I(\theta)$$

Most Likely Trajectory

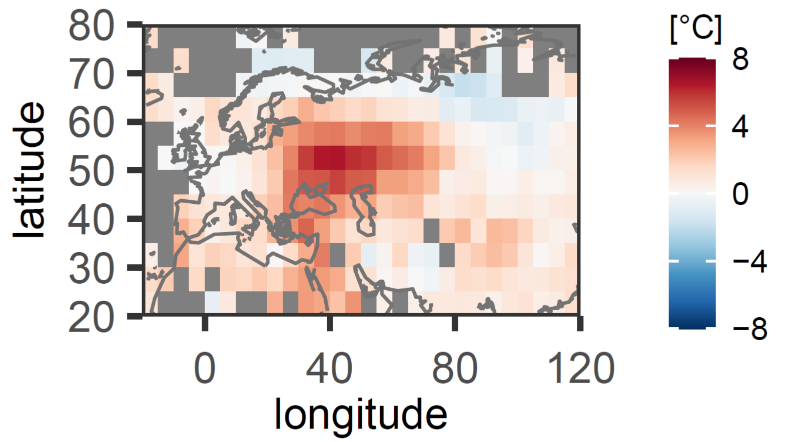
$$u(\theta^*)$$

Fingerprinting Persistent Extremes

Two High-impact Persistent Events in 2010…

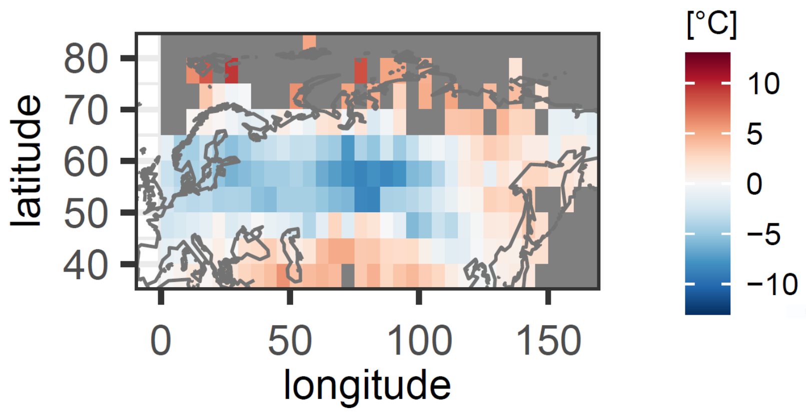
Russian (Eurasian) heat wave

- Area of more than 2 million km² affected
- Death toll of 55,000
- More than 1 million ha of burned areas



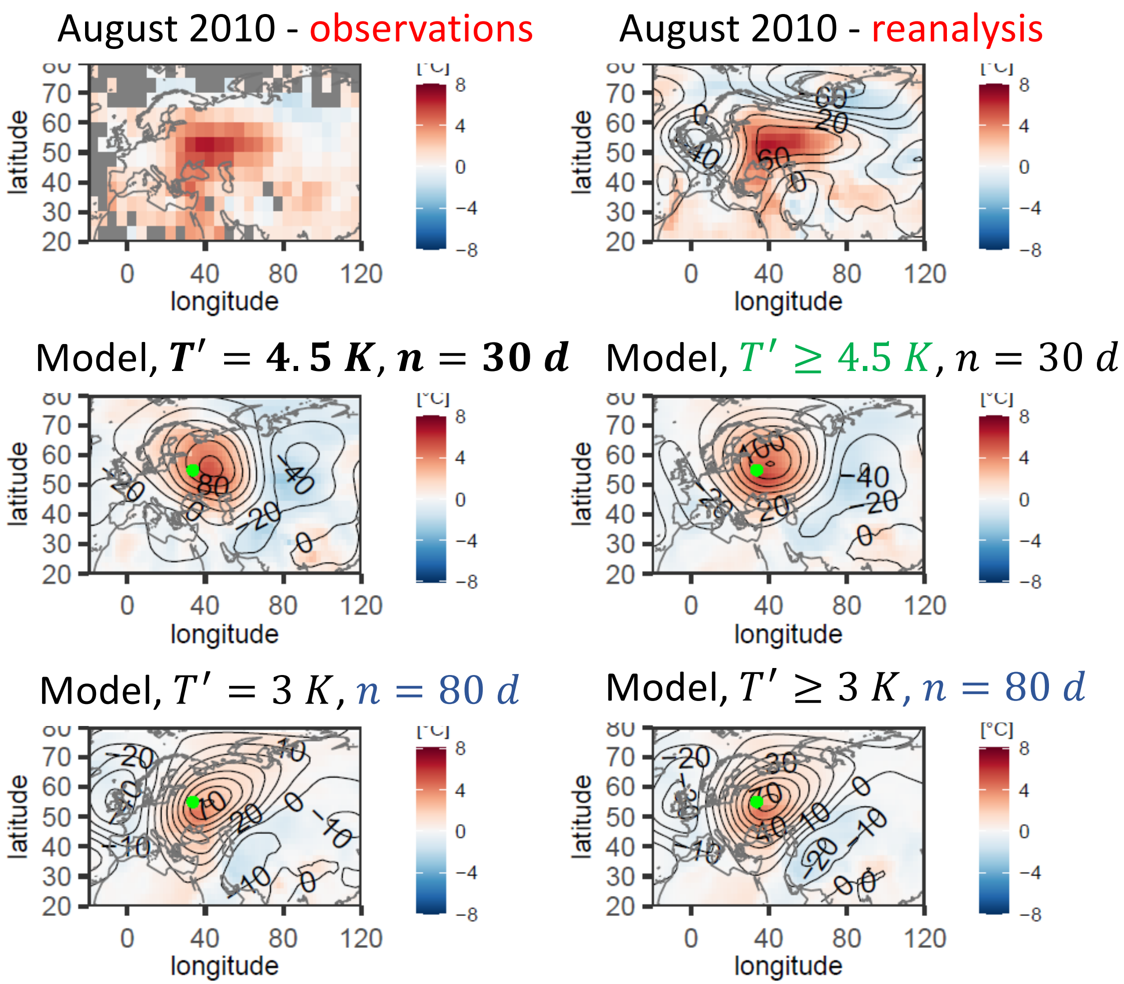
Mongolian Dzud (harsh winter conditions)

- Killed 10 million cattle or 20%
- Devastating economic impact
- Huge longitudinal extent
- Note: Dzud events historically responsible for migrations of nomadic populations



Are they really exceptional, freak events? In which sense?

The anomaly patterns during the 2010 RHW look similar to the ones corresponding to large deviations



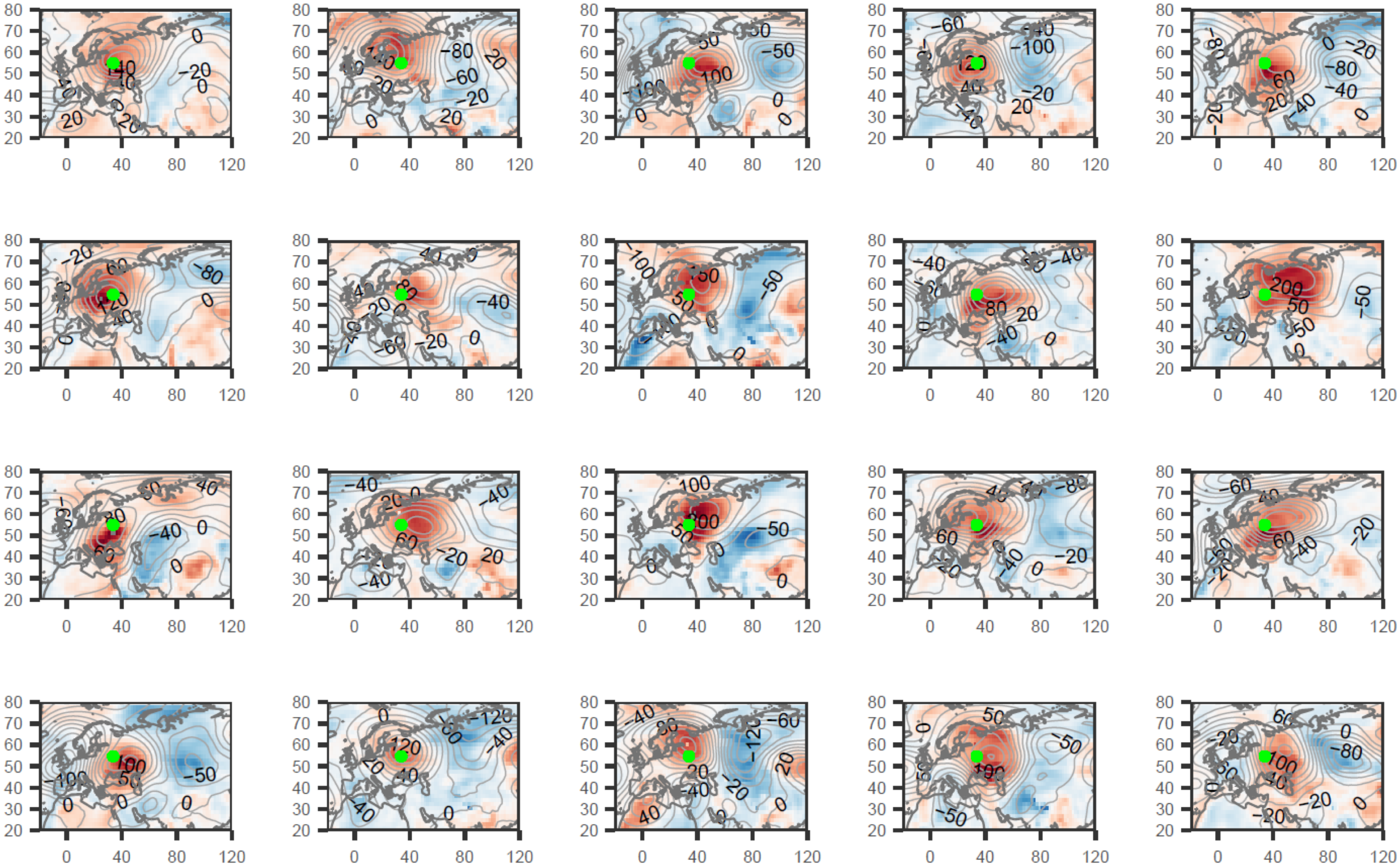
We select deviations of **4.5** over **30 days**. Good agreement between:

1) the mean of the related ST and 500 hPa GPH anomalies with
2) observations and reanalysis.

The pattern is stable in case of larger and longer events
- universal character.

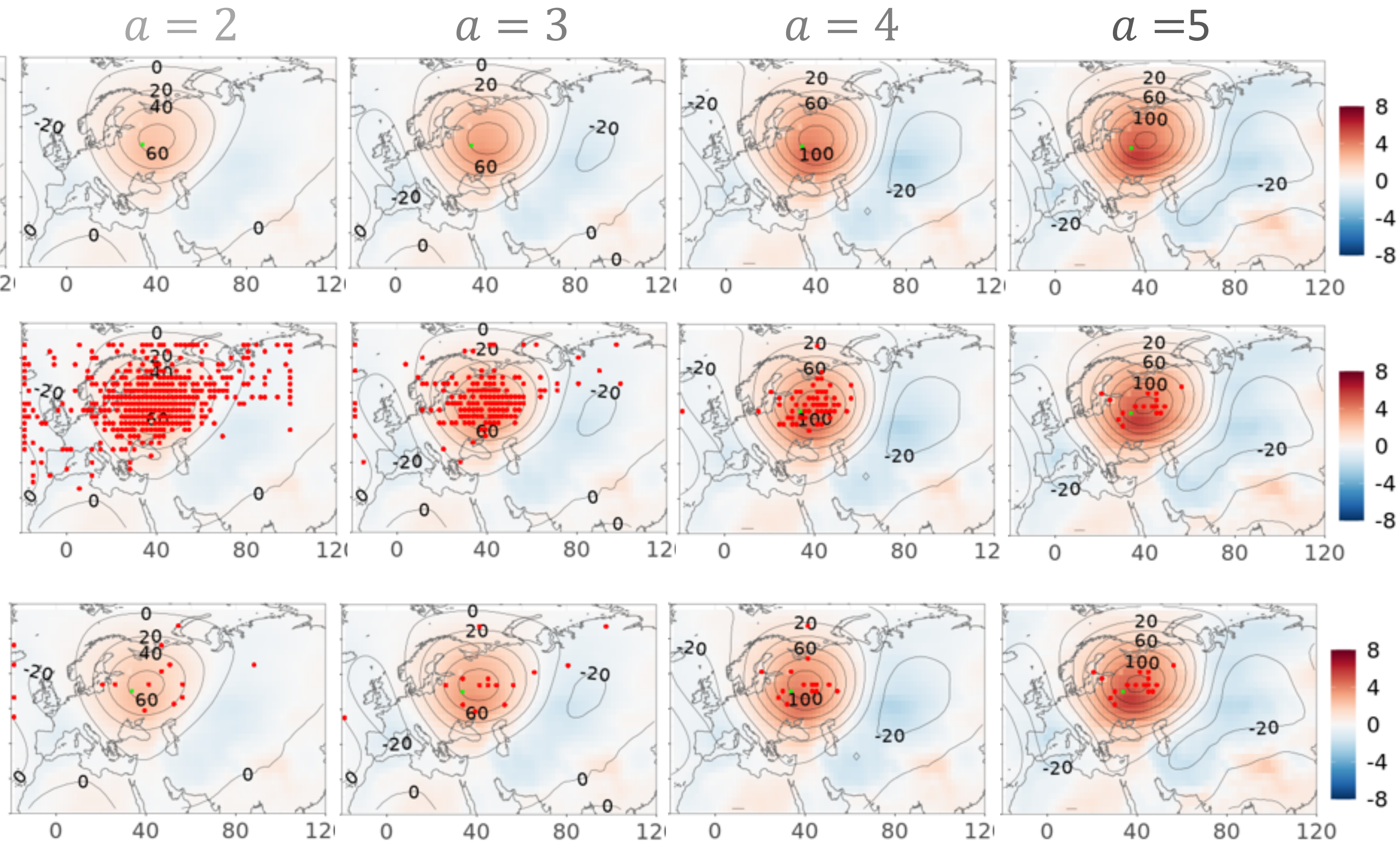
Russian Blocking is “typical” (Dole et al. 2011)

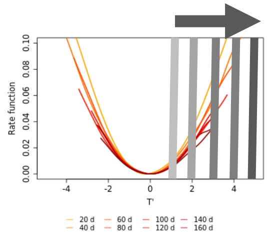
20 Pseudo 2010 Russian Heat Events



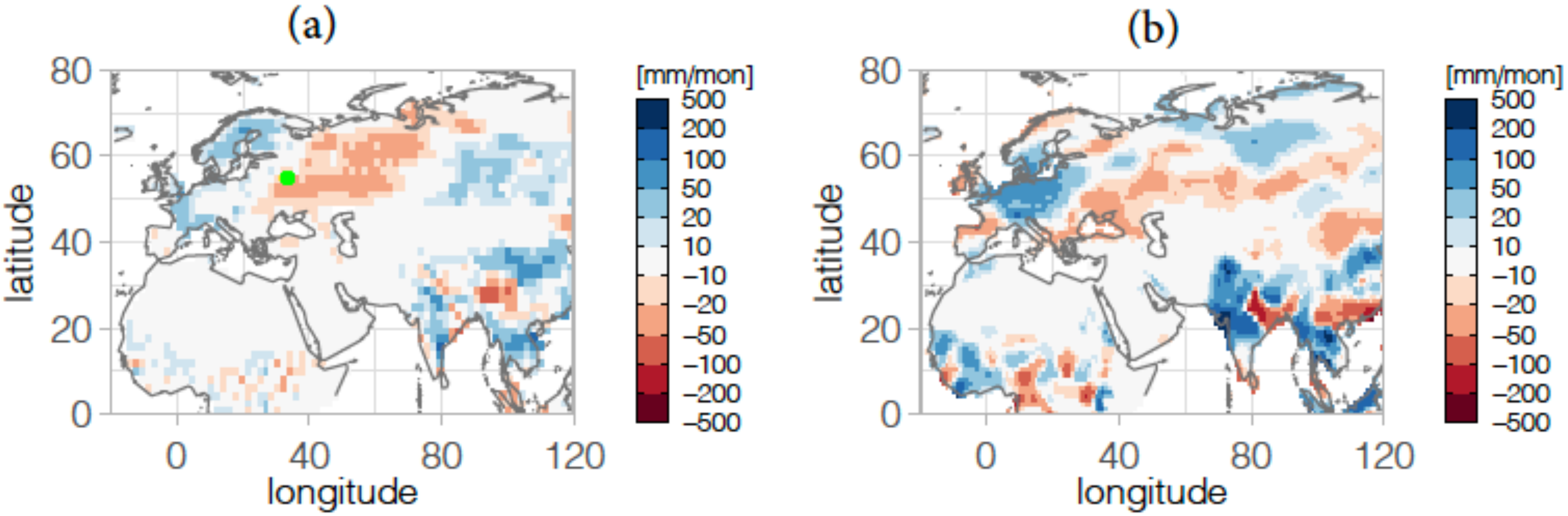
Typical spatial pattern for heatwaves

$n = 30$ days





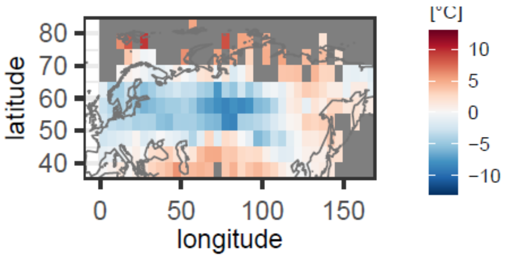
2010 RHW Large Scale Precipitations Pattern



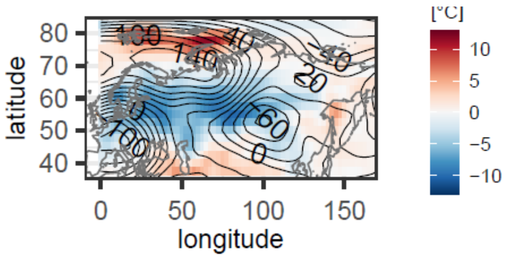
This shows that the LDT-based selection pattern captures something very real about

Same for the 2010 Mongolian Dzud!

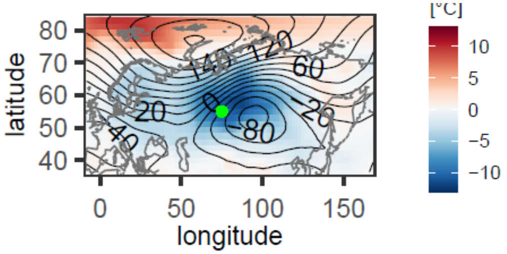
January 2010 - observations



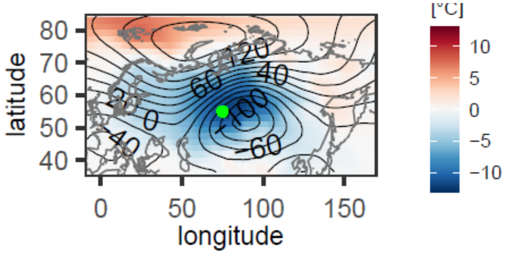
January 2010 - reanalysis



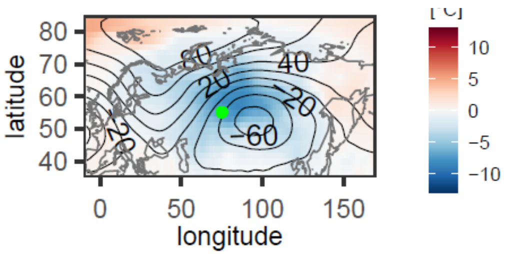
Model, $\boldsymbol{T' = -10.5\ K, n = 30\ d}$



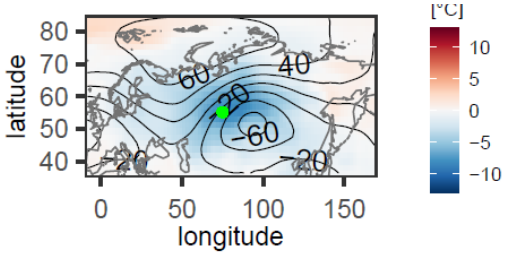
Model, $T' \leq -10.5\ K, n = 30\ d$



Model, $T' = -7.4\ K, n = 80\ d$



Model, $T' \leq -7.4\ K, n = 80\ d$

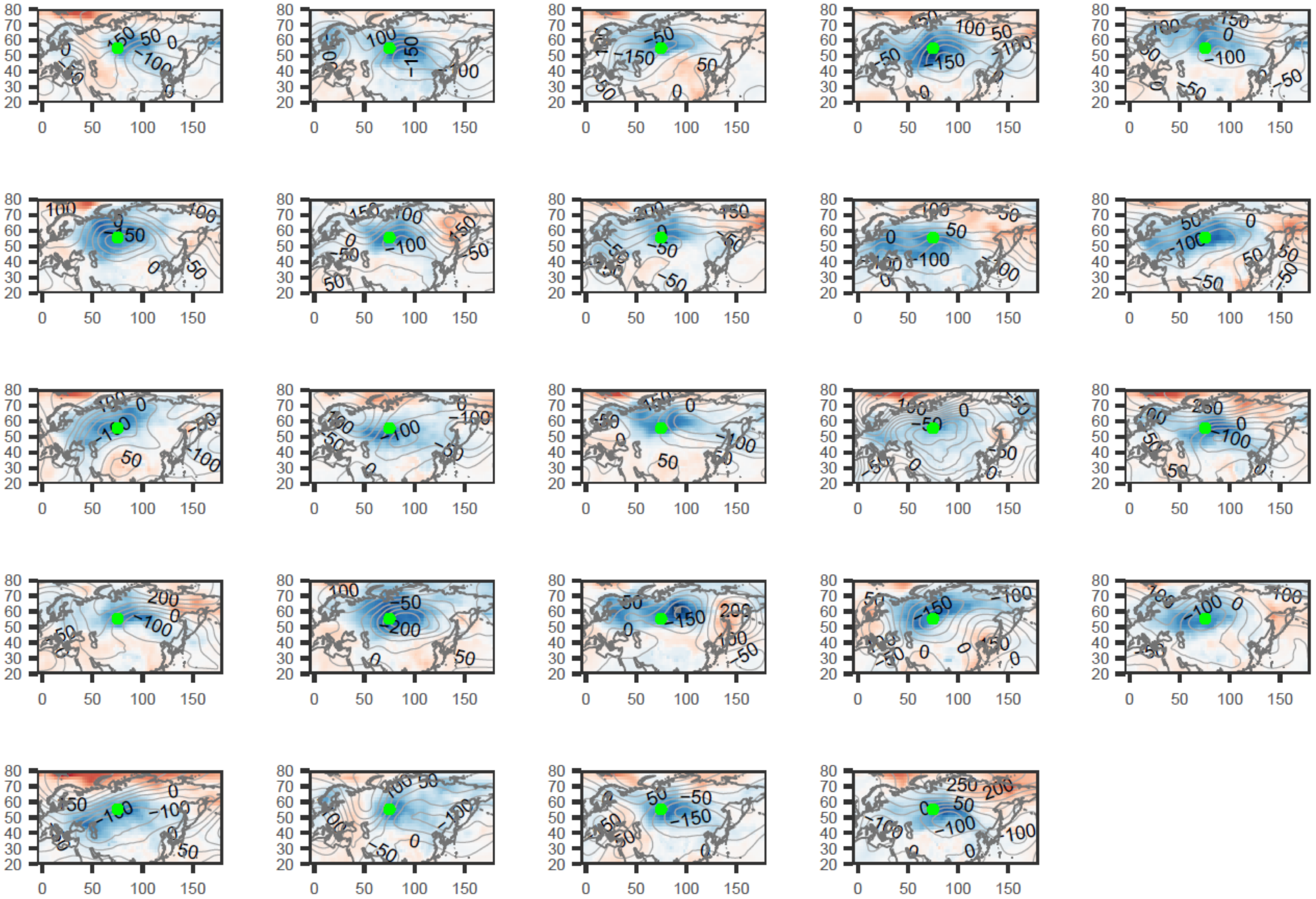


We select deviations of **-10.5** over **30 days**. Good agreement between:

1) the mean of the related ST and 500 hPa GPH anomalies with
2) observations and reanalysis.

The pattern is stable in case of larger and longer events.
- universal character

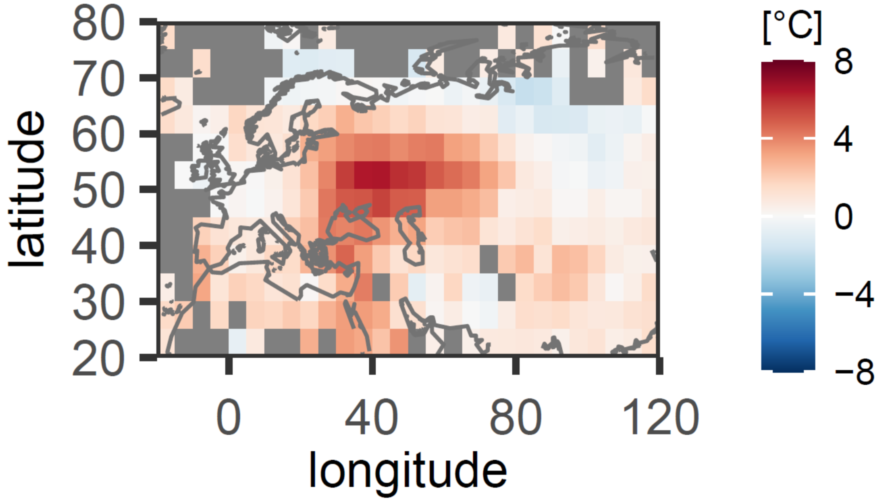
20 Pseudo 2010 Mongolia Dzuds

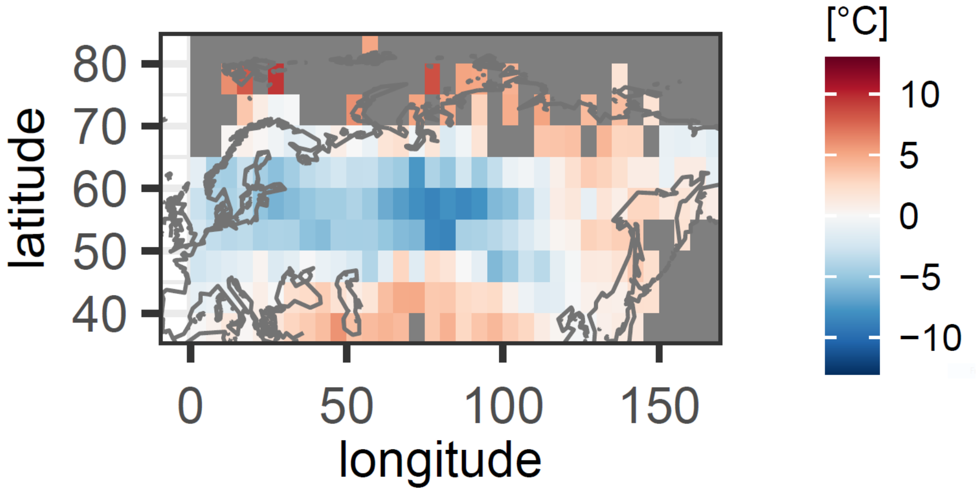


Based on large deviation theory,

we obtain the probability of long persistent events over land regions,

we find typical anomaly patterns related to persistent temperature events.





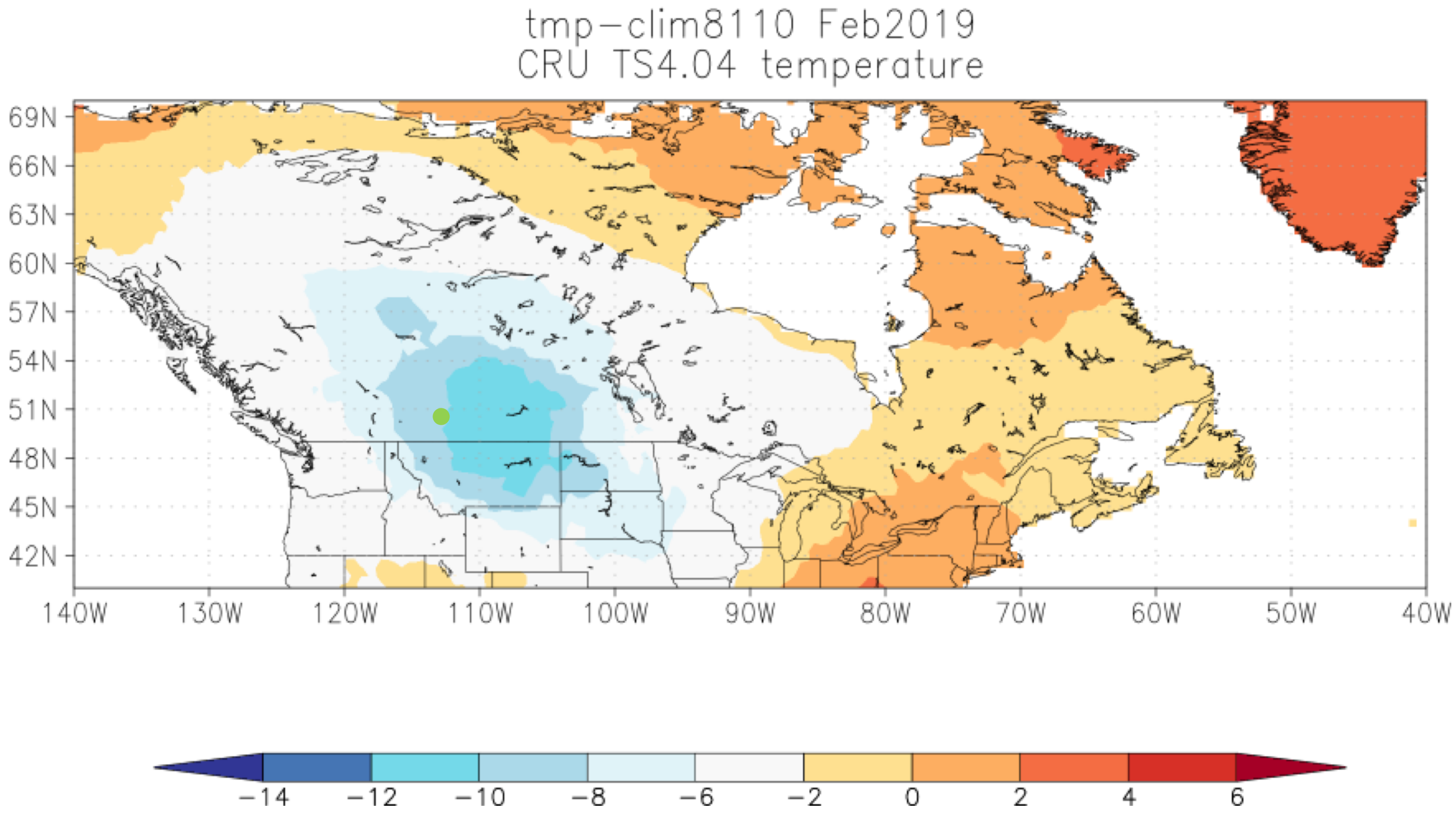
Are these freak event? Are they exceptional?

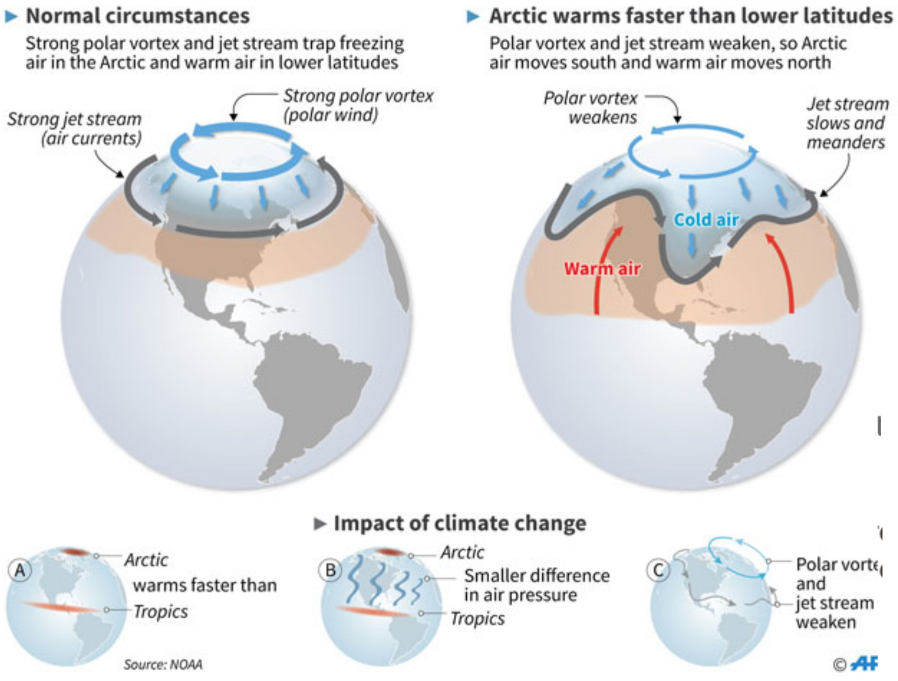
Not really. They are encoded in the natural variability

Climate Change will likely make them more and less likely, respectively

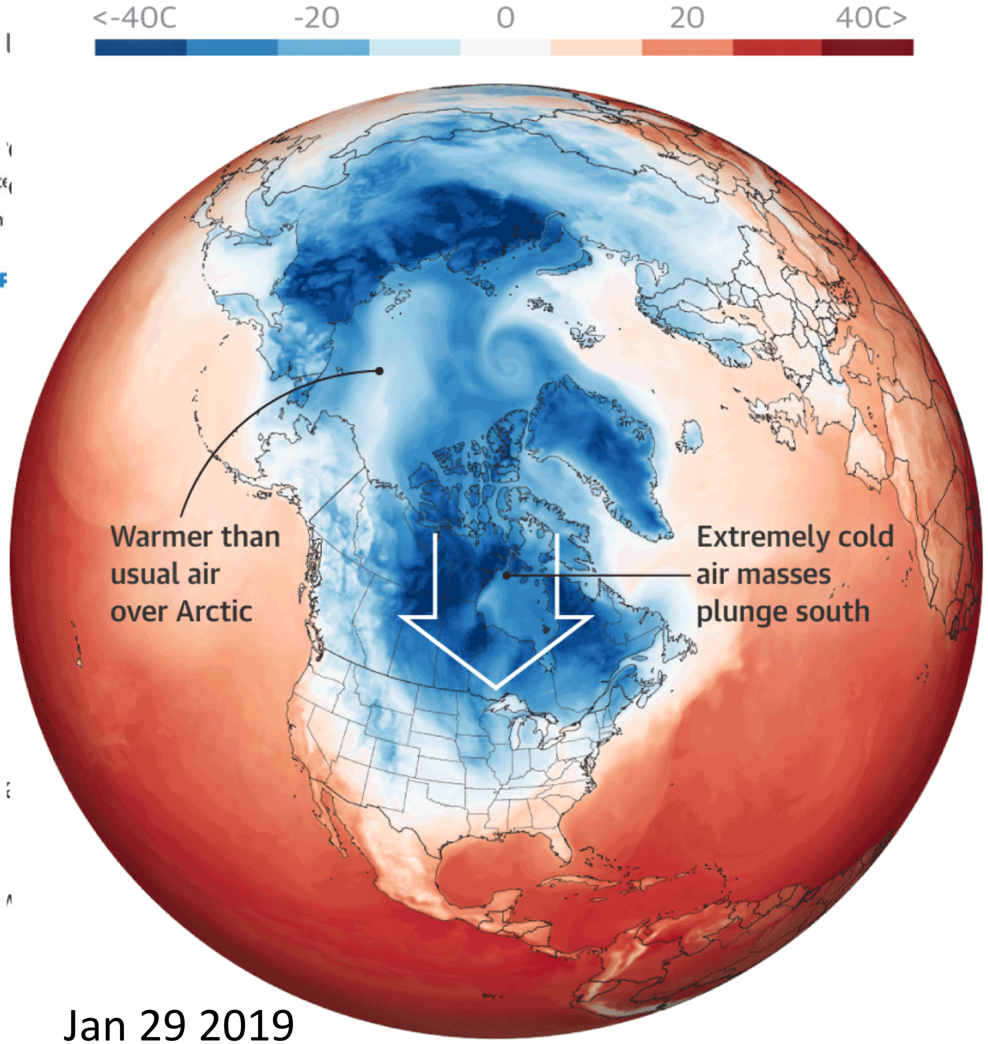
Galfi and Lucarini (2020), Fingerprinting Heatwaves and Cold Spells and Assessing Their Response to Climate Change using Large Deviation Theory, in review, Phys. Rev. Lett. (2020)

2019 Cold Spell – N. America



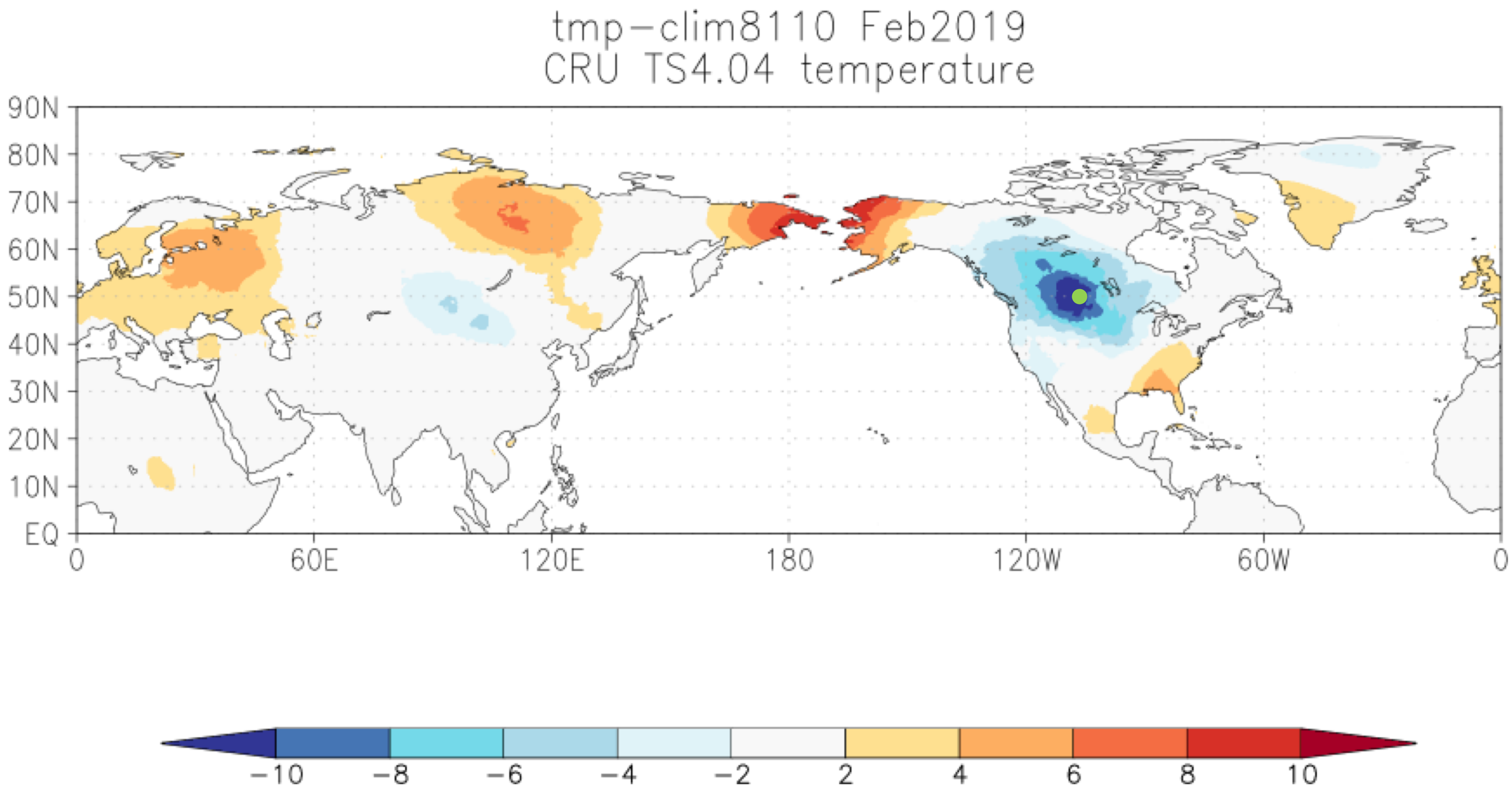


Polar Vortex 2019

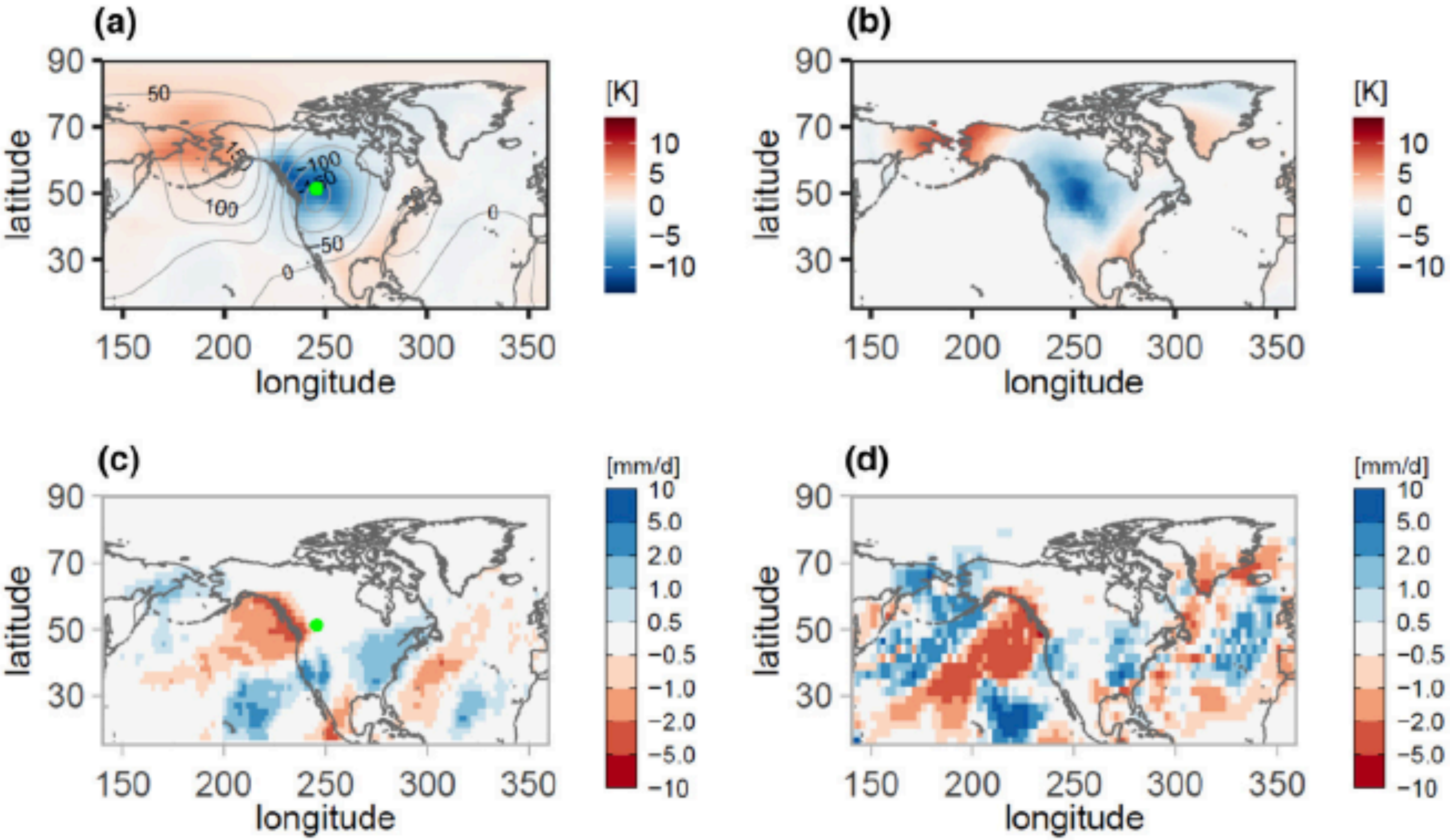


Jan 29 2019

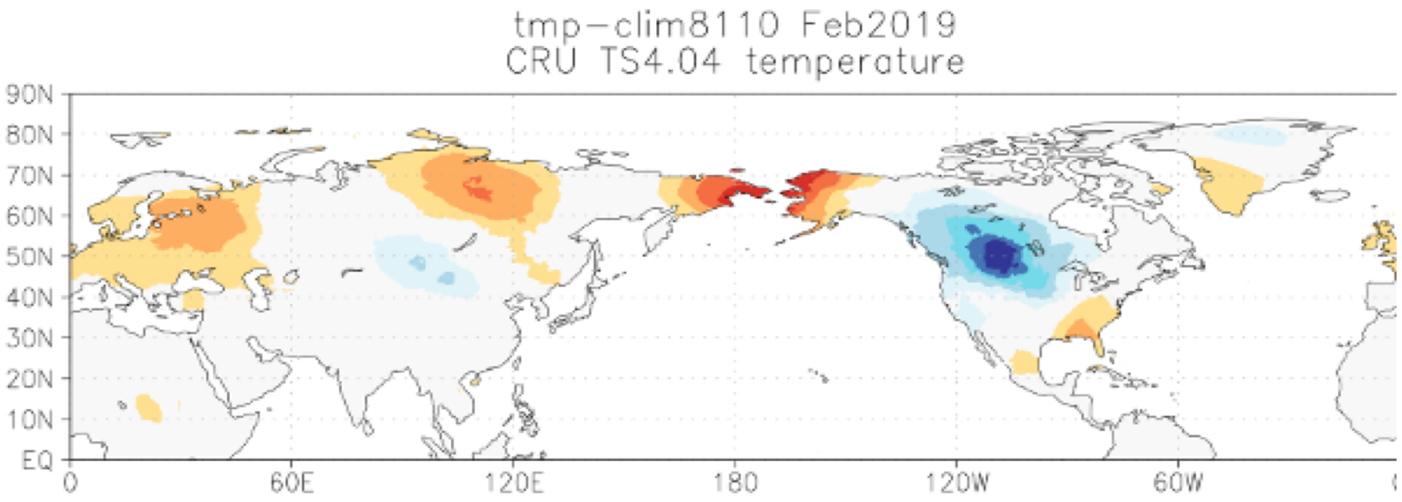
Global Features!



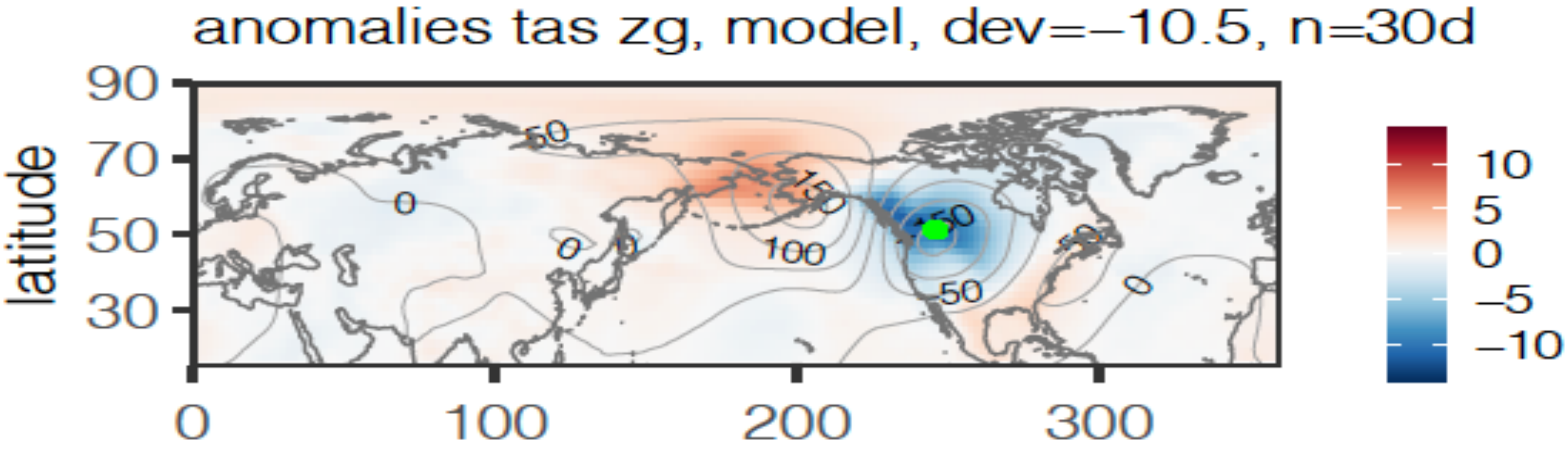
February 2019 Cold Spell - Calgary



Data

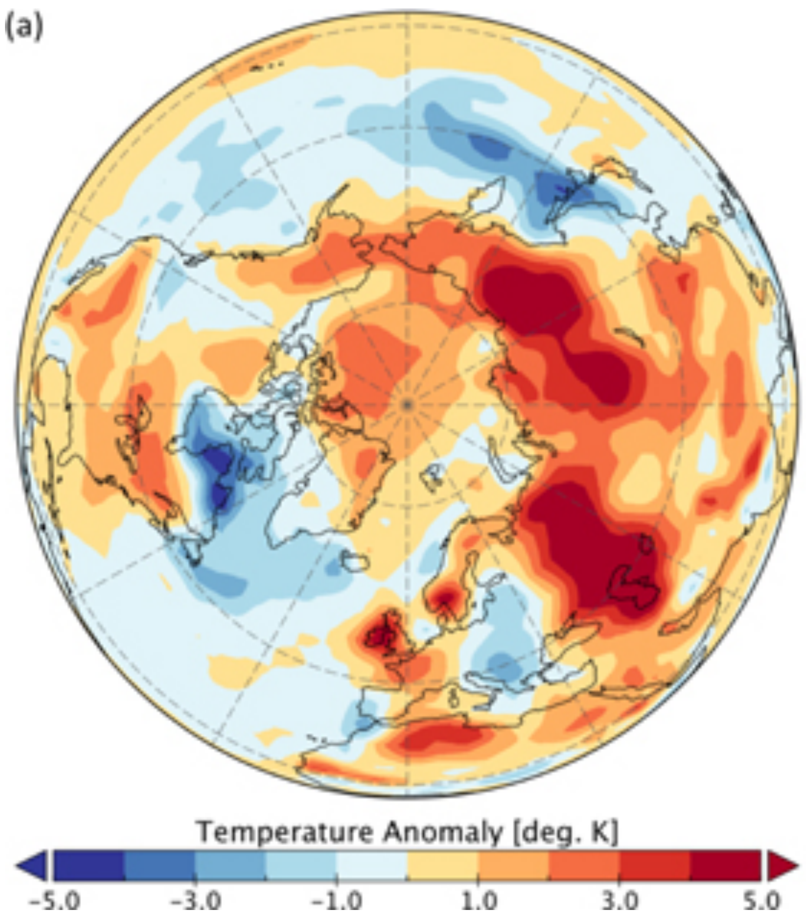

tmp−clim8110 Feb2019
CRU TS4.04 temperature
90N
80N
70N
60N
50N
40N
30N
20N
10N
EQ
0
60E
120E
180
120W
60W

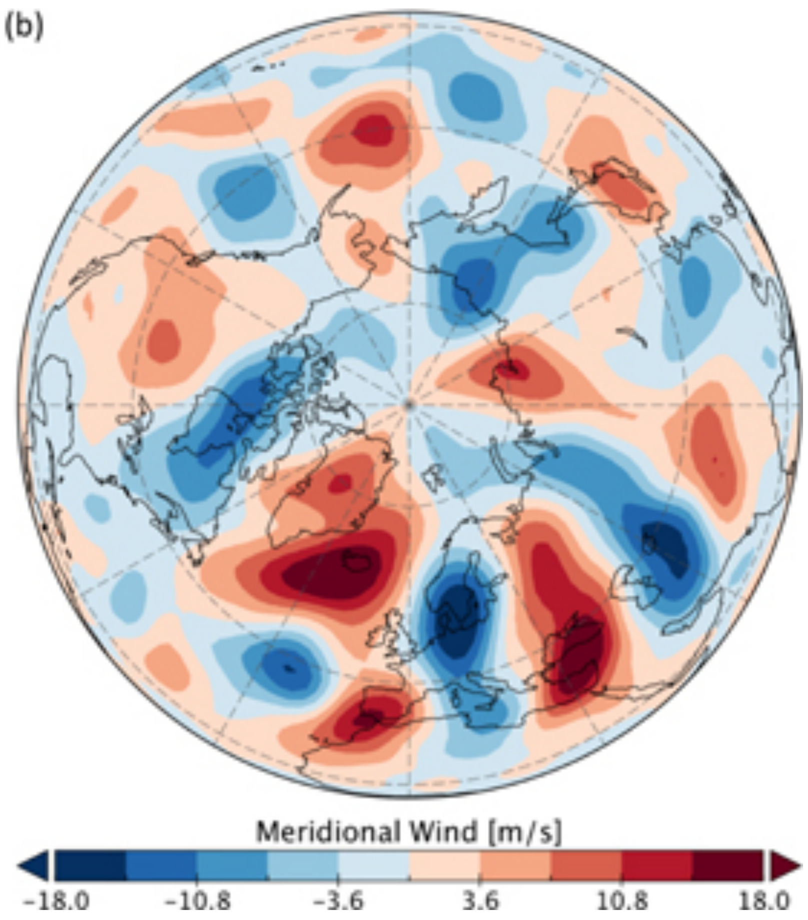
Model


anomalies tas zg, model, dev=−10.5, n=30d
latitude
90
70
50
30
0
100
200
300
50
0
150
100
−150
50
10
5
0
−5
−10

2018 Summer Extremes

- **Wavenumber 7 Standing Rossby Wave (Konrhuber et al. Env. Res. Lett. 2018)**

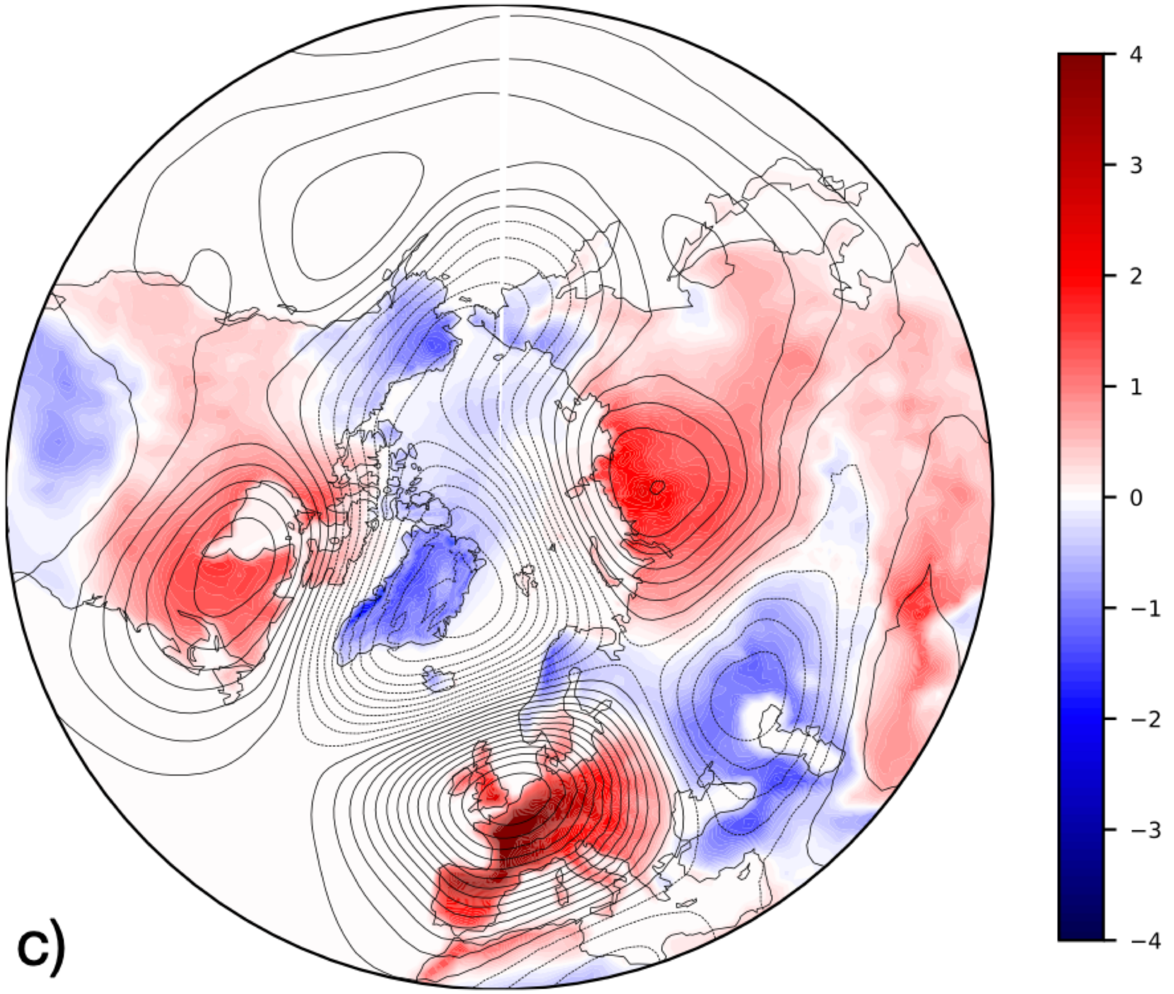




Seasonal Scale Heatwaves

- Large scale structures emerge when looking at local temperature extremes

Scandinavian Heatwave, Wavenumber 3

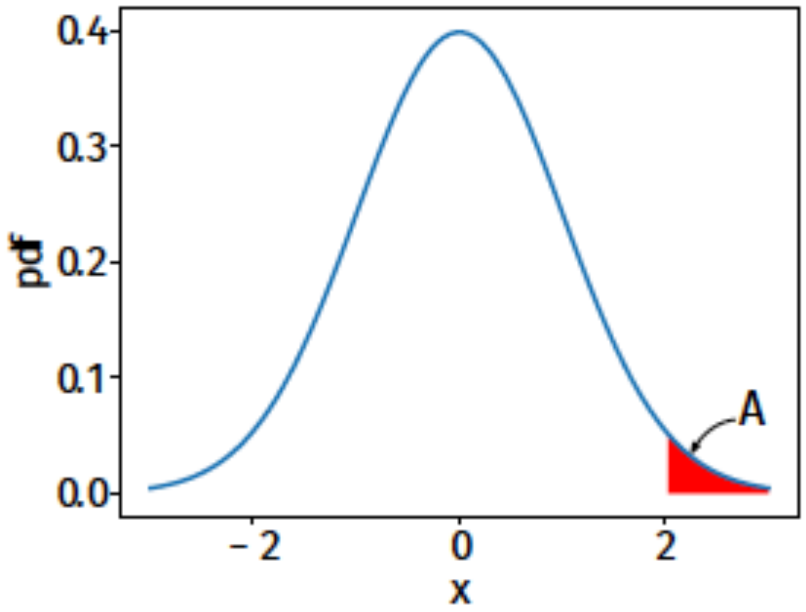


Ragone and Bouchet, GRL 2021

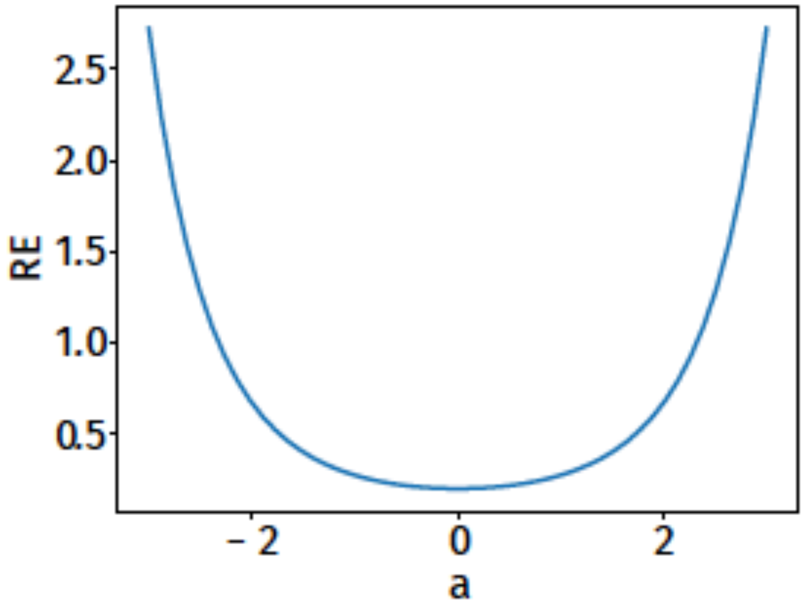
Courtesy of J. Wouters

Extremes are hard to sample

For a rare event $A = \{X > a\}$, brute force Monte Carlo sampling $\frac{1}{N}\sum_{i=1}^{N} \mathbb{1}_A(X_i)$ has a relative error $RE \sim \frac{1}{\sqrt{N\mathbb{P}(A)}}$



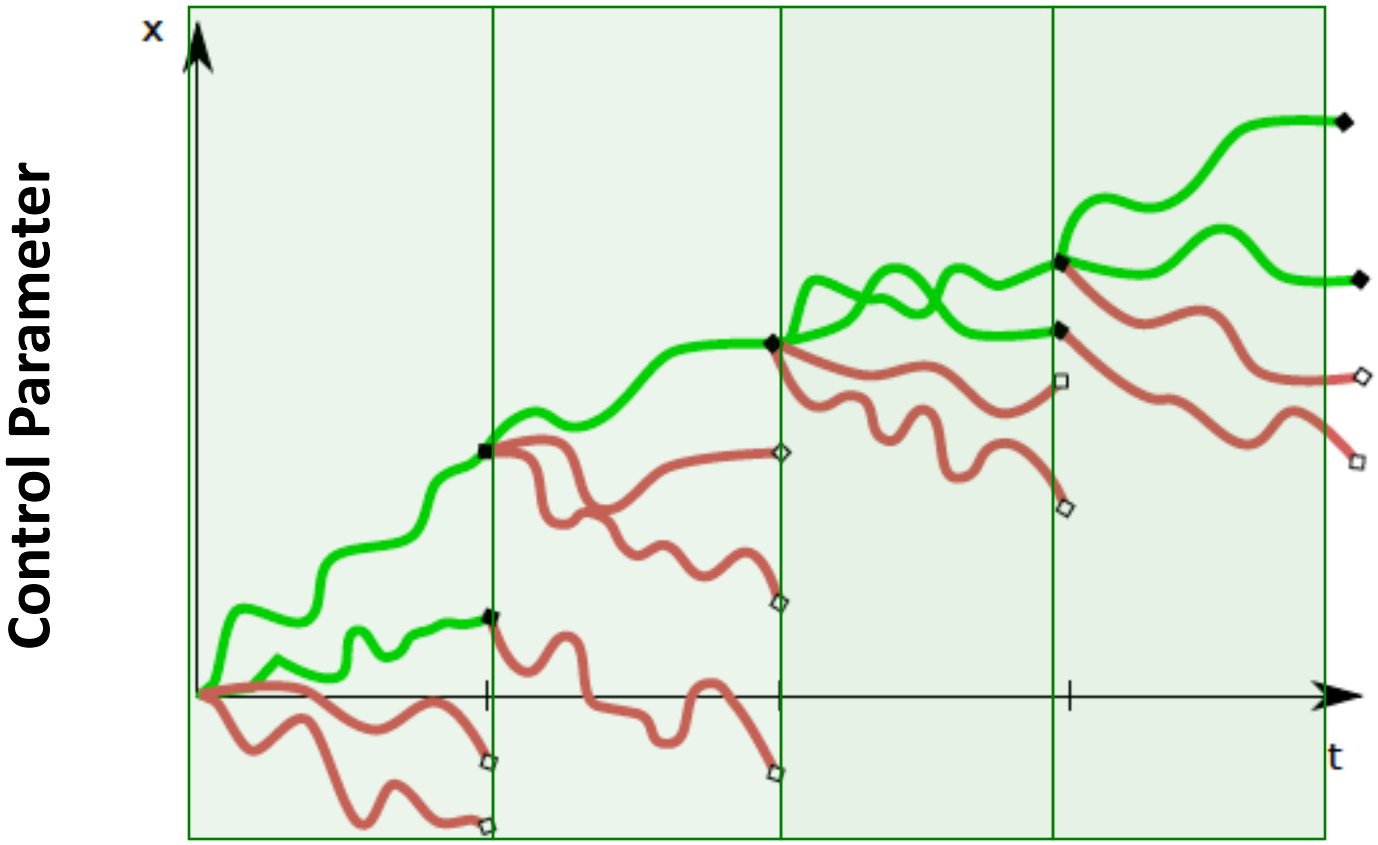
Probability distribution



MC relative error

We need smarter ways to study extremes!

Genetic Algorithms to telescope on extremes



Courtesy of
J. Wouters

- Similar to Metropolis: one runs many parallel simulation ..
- Selects & clones only the offsprings going “in the right direction”
- Objective function W for killing offsprings

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Storms corresponds to minima of local dimension

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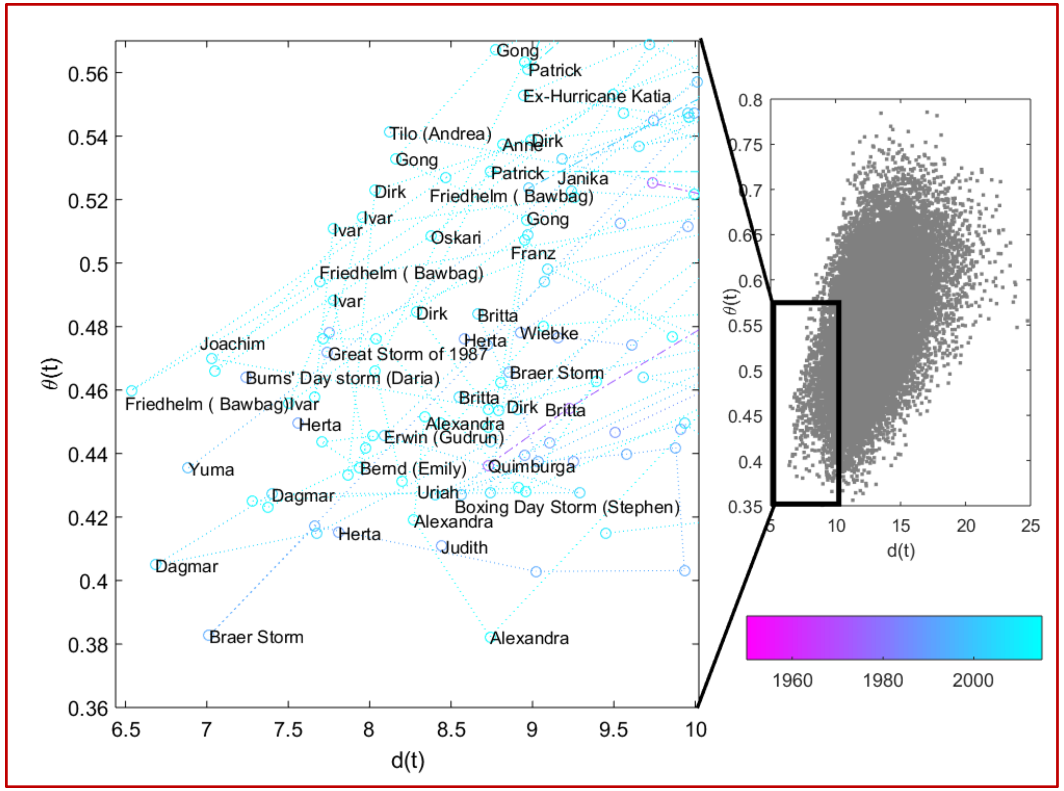
European windstorm

From Wikipedia, the free encyclopedia

This article is about storms crossing the Atlantic Ocean eastward and striking Europe. It is not to be confused with storms arising w Europe.

European windstorm is a name given to the strongest extratropical cyclones which occur across the continent of Europe.[2] They form as cyclonic windstorms associated with areas of low atmospheric pressure that track across the North Atlantic Ocean towards western Europe. They are most common in the autumn and winter months. On average, the month when most windstorms form is January.

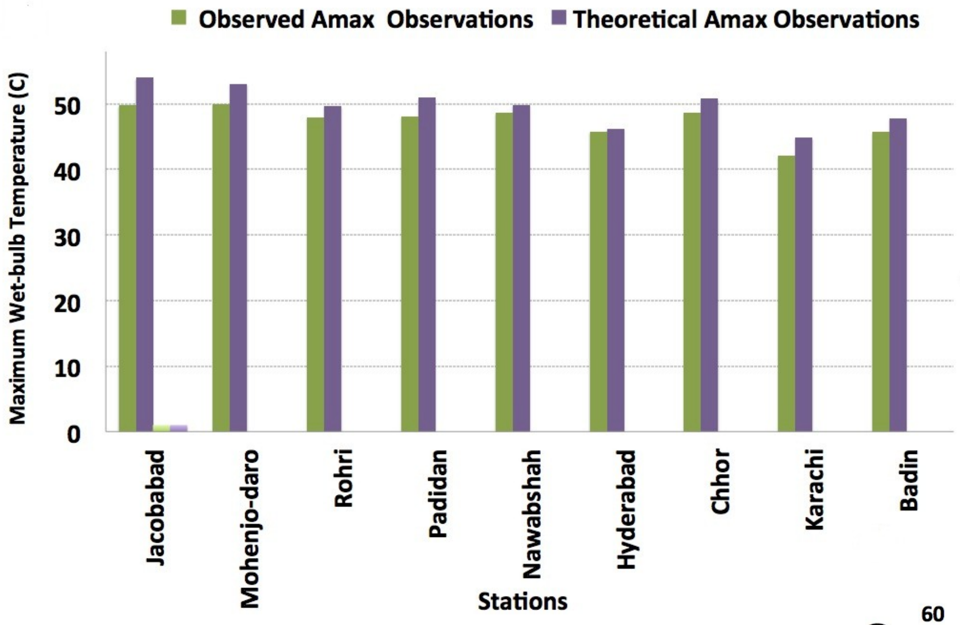


Storms matching the <u>minima</u> of the instantaneous dimensions.

The instantaneous dimensions *d* (x-axis) and persistence *θ* (y-axis) for the selected historical storms are plotted along with the storms' names and years of occurrence (colourscale). Repeated names indicate storms which persisted for several days. The inset shows the full distribution of *(d,θ)* values. The black lines delimit the phase-space region in which the selected storms lie.

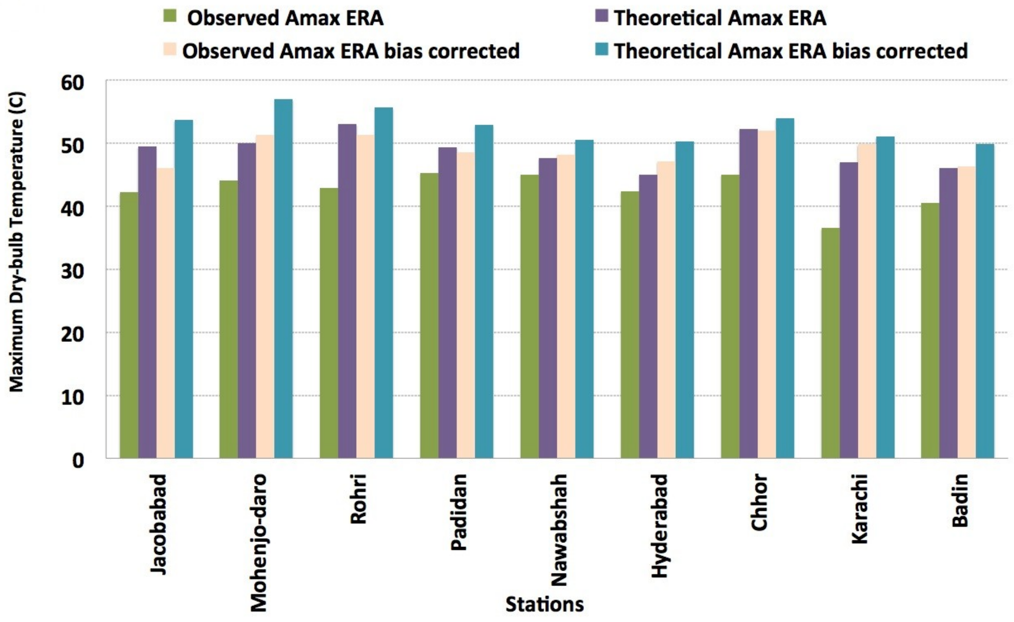
Courtesy of D. Faranda

Absolute Maxima of TW_{max}

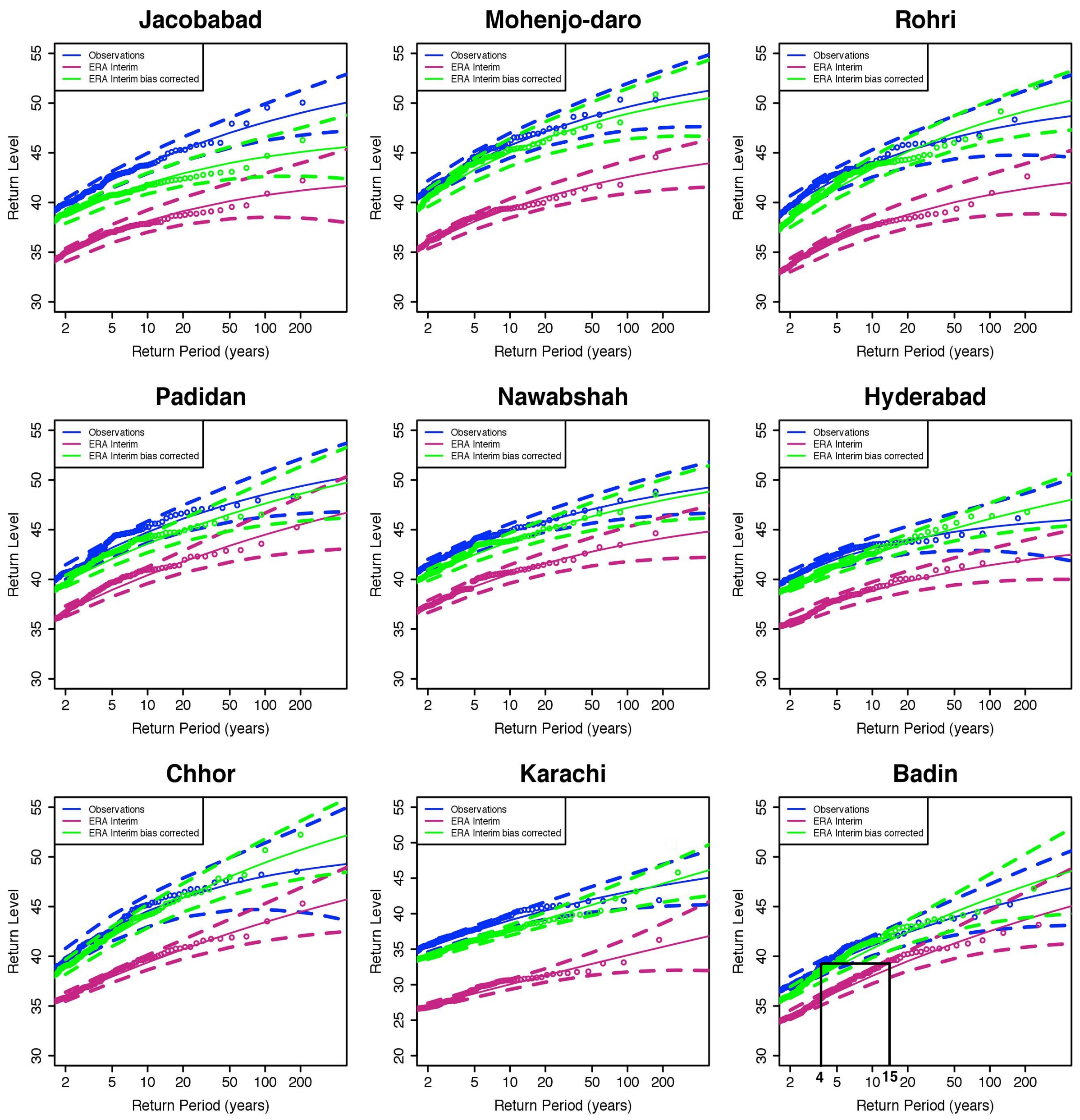


Observations based Absolute maxima of Wet-bulb Temperature TW_{max}

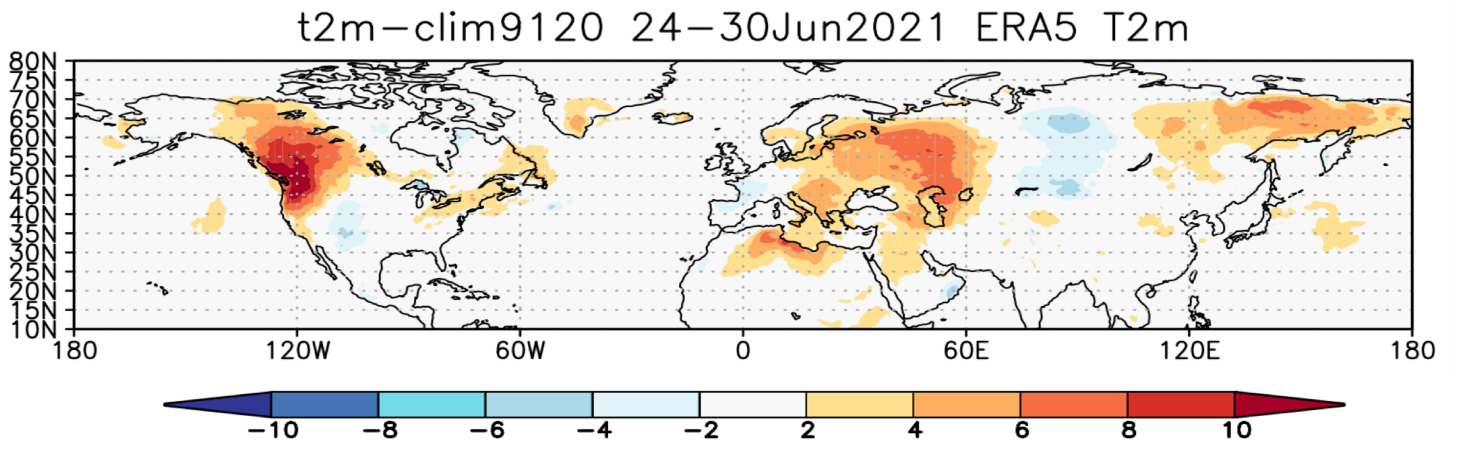
ERA Interim based Absolute maxima of Wet-bulb Temperature TW_{max}

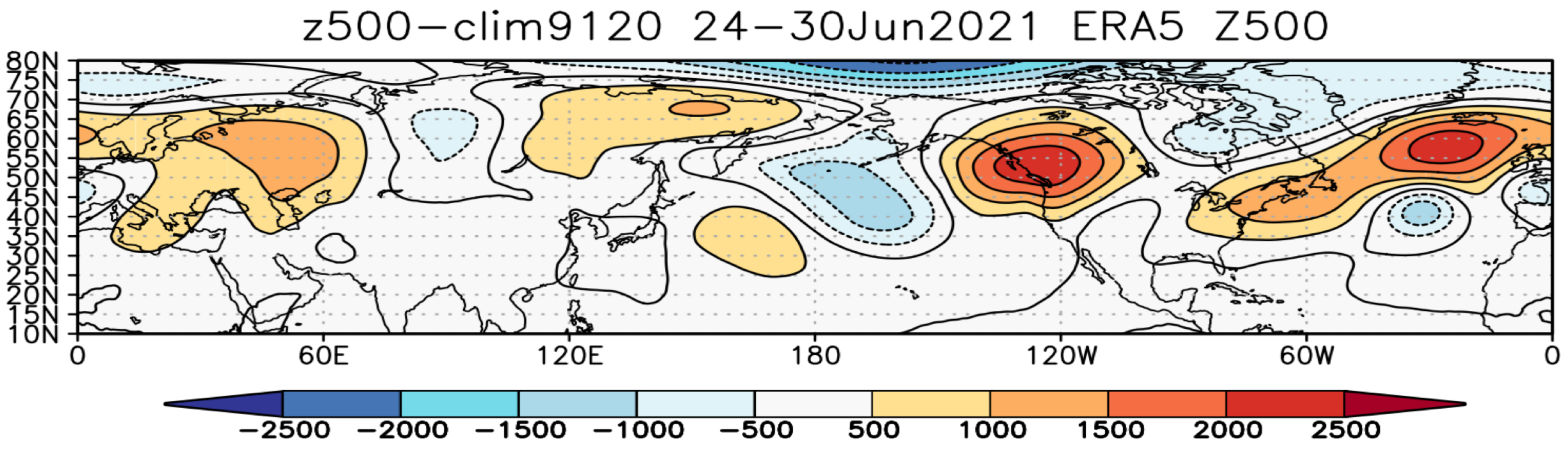


Return Levels of TW_{max}



2021 Heatwave in North America



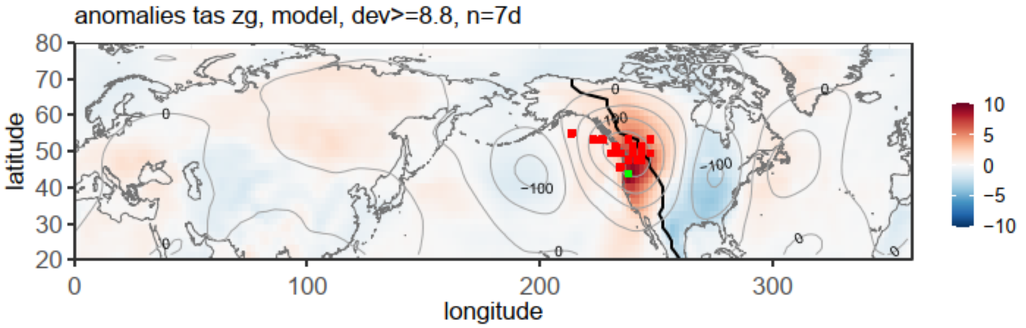


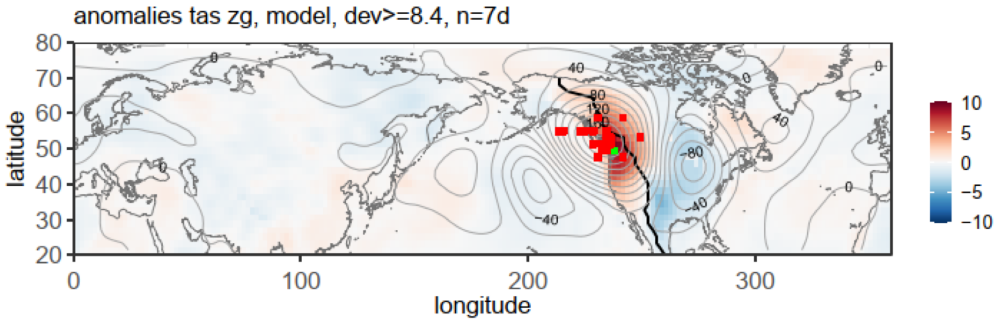
Sensitivity to the choice of the target region

Lat = 44 N

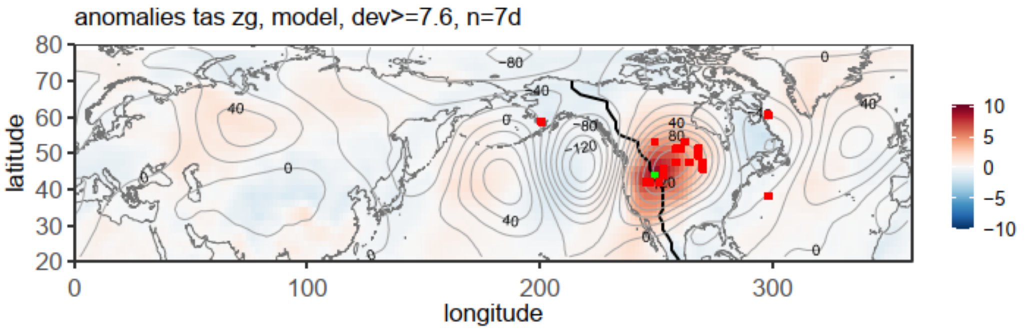
Lat = 49 N

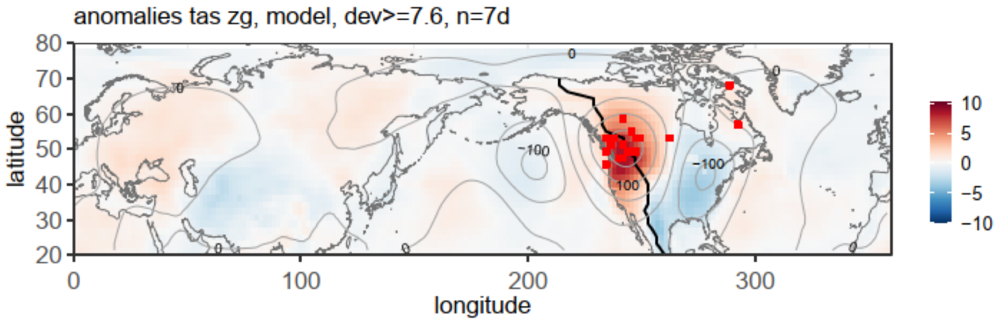
Peak Coast

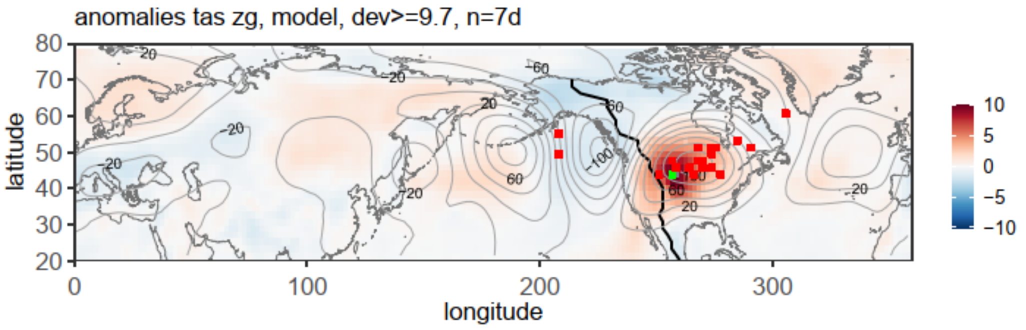


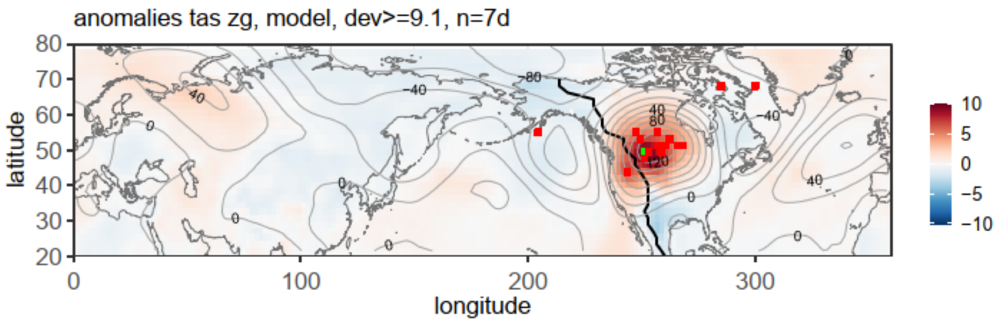


Continent





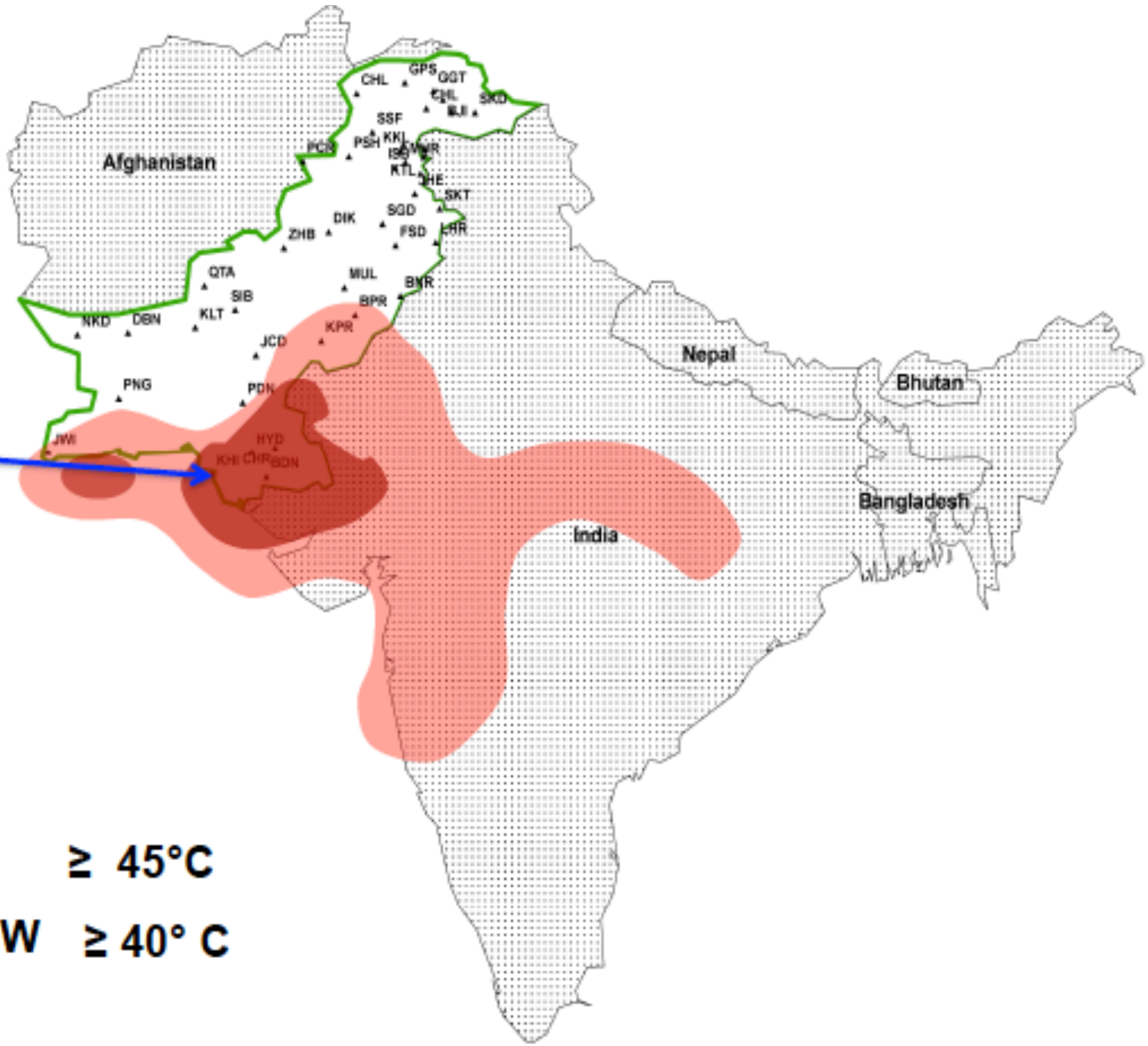




2015 Heat Wave in South Asia

Heat Waves 2015
- 1400 deaths
- 14000 hospitalized

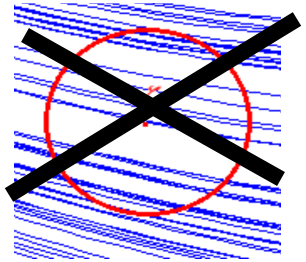
Highest fatalities occurred in Karachi >900 deaths



Severe HW ≥ 45°C

Moderate HW ≥ 40° C

Not intersection of a ball with a fractal! Scaling will be different



- Exceedance $Z = A(x) - T$ is $\propto$ distance btw x and β_T
- Complementary cumulative function comes from conditional probability
- How does $\nu(\Omega_{A_{max}}^{T+Z})$ scale with Z?

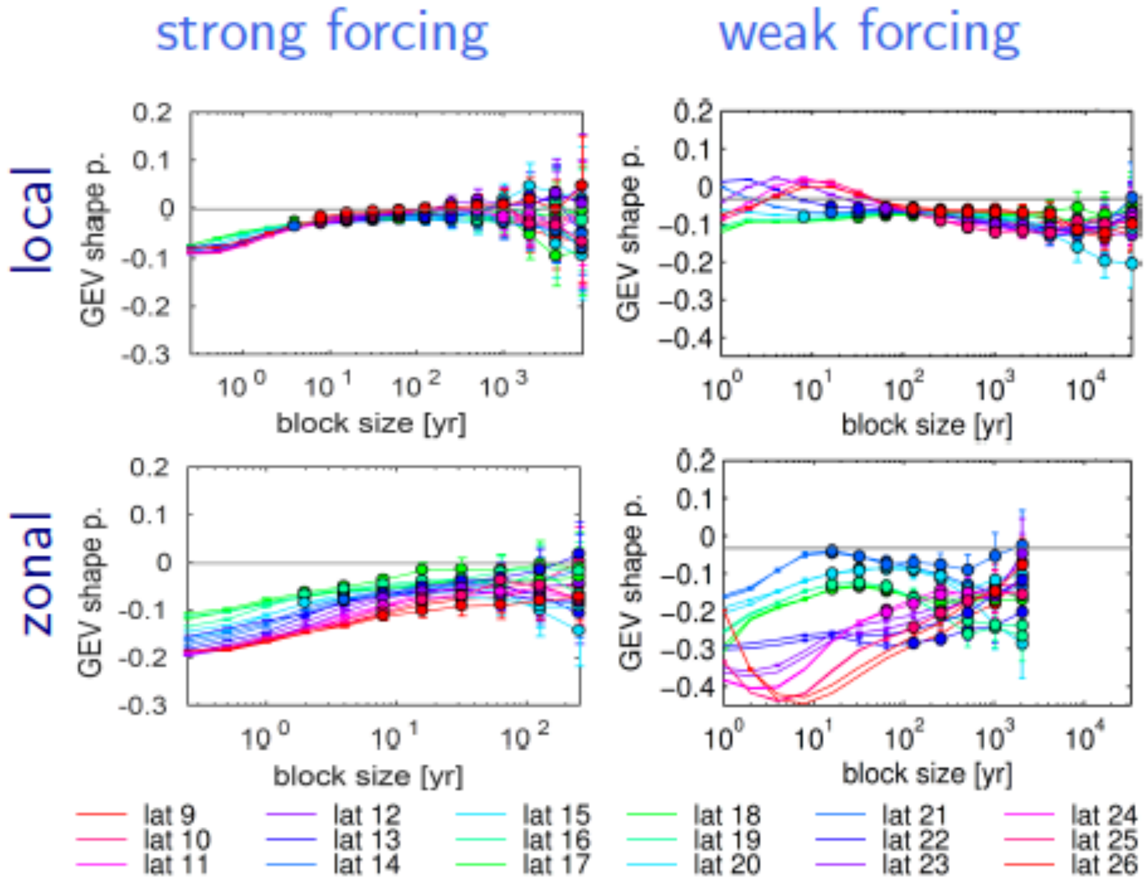
We define the exceedances for the points $x \in \Omega_{A_{max}}^T$ as $z = A(x) - T$. An exceedance z corresponds geometrically to a distance $y = \mathrm{dist}(x, \beta_T) = z/|\nabla A|_{x=x_0}|$ from β_T and a distance $k = \mathrm{dist}(x, \beta_{max}) = y_{max} - y = (A_{max} - T)/|\nabla A|_{x=x_0}| - z/|\nabla A|_{x=x_0}|$ from β_{max}.

Therefore, $P(z > Z|z > 0) = P(y > Y|y > 0)$, where $Y = Z/|\nabla A|_{x=x_0}|$. We have that $P(y > Y|y > 0) = P(y > Y)/P(y > 0)$. In terms of the invariant measure of the system, we have that the probability $H_T(Z)$ of observing an exceedance of at least Z given that an exceedence occurs is given by:

$$H_T(Z) \equiv \frac{\nu(\Omega_{A_{max}}^{T+Z})}{\nu(\Omega_{A_{max}}^{T})}. \tag{5}$$

where we have used the ergodicity of the system. Obviously, the value of the previous expression is 1 if $Z = 0$. The expression contained in Eq. (5) monotonically decreases with Z (as $\Omega_{A_{max}}^{T+Z_2} \subset \Omega_{A_{max}}^{T+Z_1}$ if $Z_1 < Z_2$) and vanishes when $Z = A_{max} - T$.

Main results

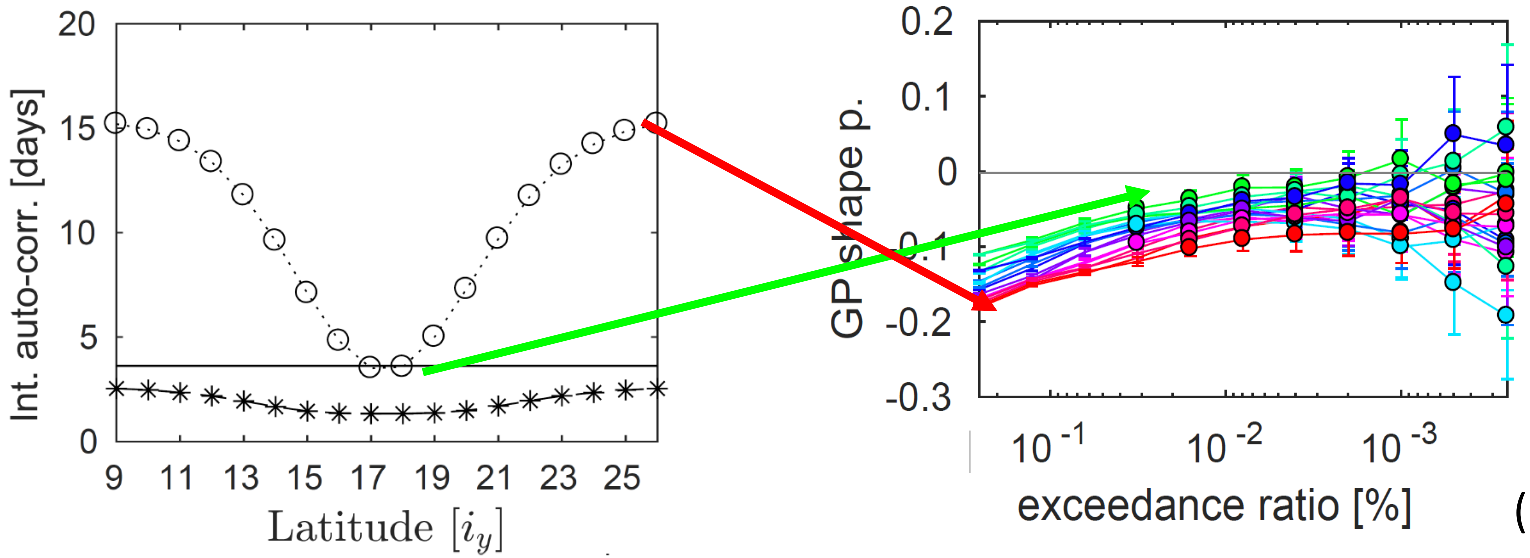


- Strong forcing
  - monotonic convergence to ξ_δ
  - convergence slower for zonal observables
- Weak forcing
  - no convergence to ξ_δ, non-monotonic $\xi(n)$ values
  - even worse for zonal observables

Results look more promising for

- strong forcing
- local observables

Correlations reduce the number of uncorrelated data, reaching asymptotic levels is more difficult



(Galfi et al., 2017)

$\xi(u)$ and $\xi(m)$ are modified by the presence of correlations.
Larger thresholds or larger block sizes are needed to reach convergence.

Extreme Value Theory in a Time-Dependent Setting

- One can use in principle use Response Theory to predict the change in the number of events above threshold as well as the EVT parameters.

Pragmatic approaches.

1) Prepare the data before applying EVT.
   - remove seasonal cycle
   - remove trends
2) Use standard EVT models + statistical modelling

$$Z_t \sim \mathrm{GEV}(\mu(t), \sigma(t), \xi)$$

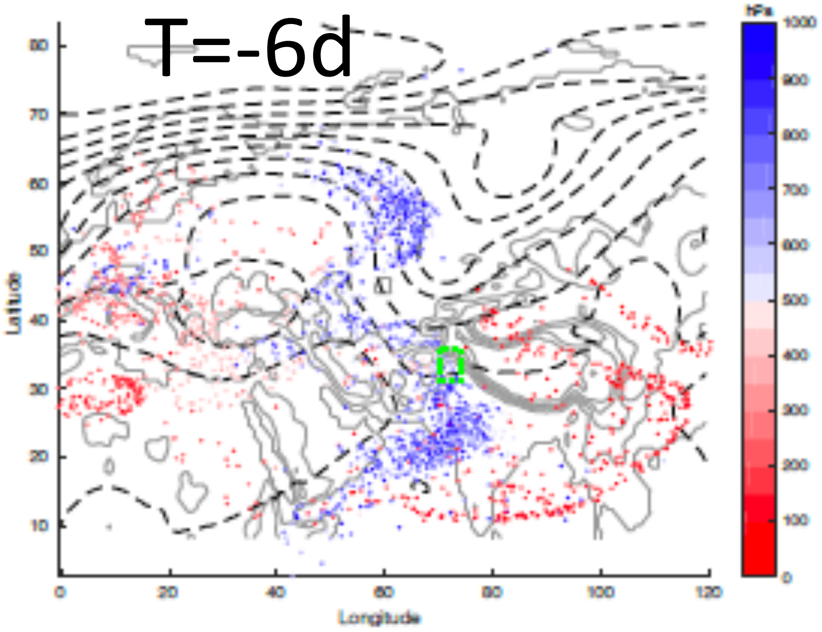
$$\mu(t) = \beta_0 + \beta_1 t$$

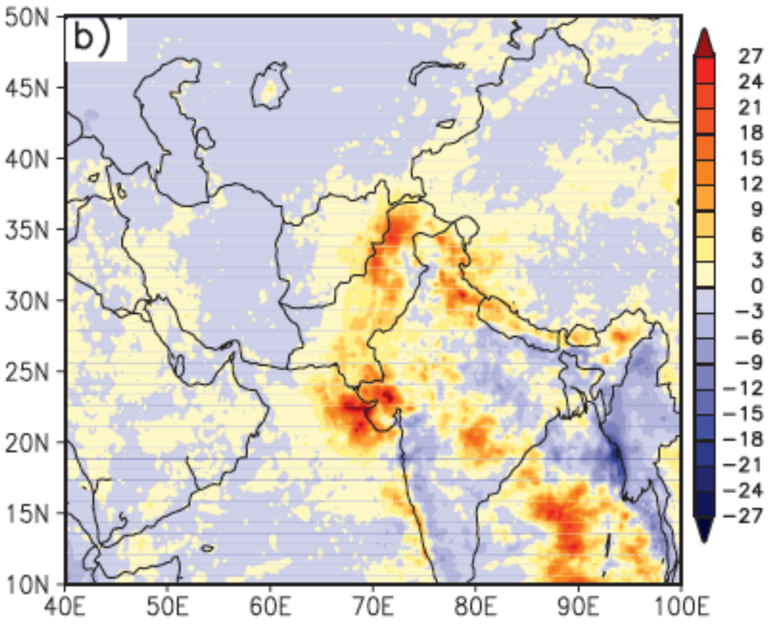
$$\mu(t) = \beta_0 + \beta_1 t + \beta_2 t^2$$

$$\mu(t) = \beta_0 + \beta_1 \mathrm{SOI}(t)$$

Lagrangian Trajectories (Backtracking)

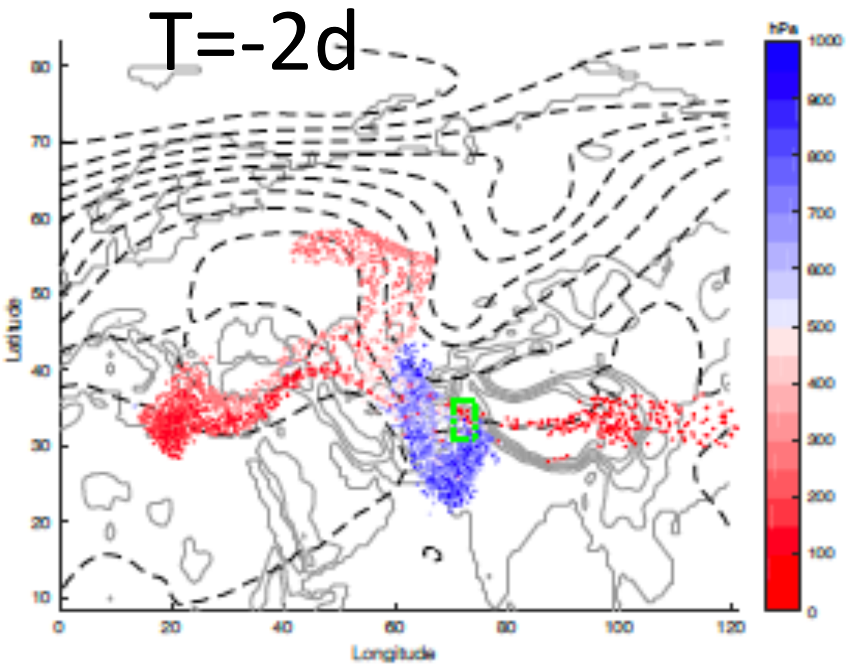
- Some of the precipitating water comes from Central Eurasia thanks to the blocking, joins on Arabian sea moisture
- See also Hunt et al. MWR (2018)





Lau and Kim JHM (2011)

- Boschi and L. Atmosphere (2019)



•Question

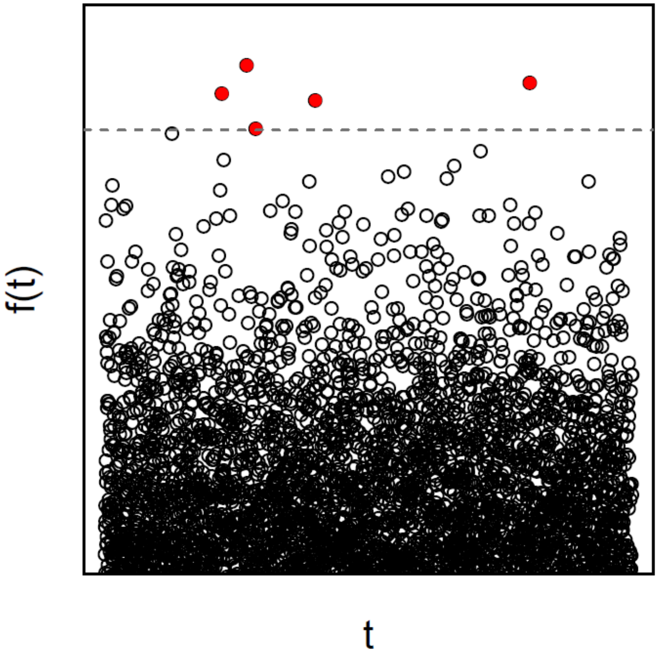
What’s the highest air temperature a human body can sustain without dying?

EVT V2: Peaks-Over-Threshold: Extremes as very Large

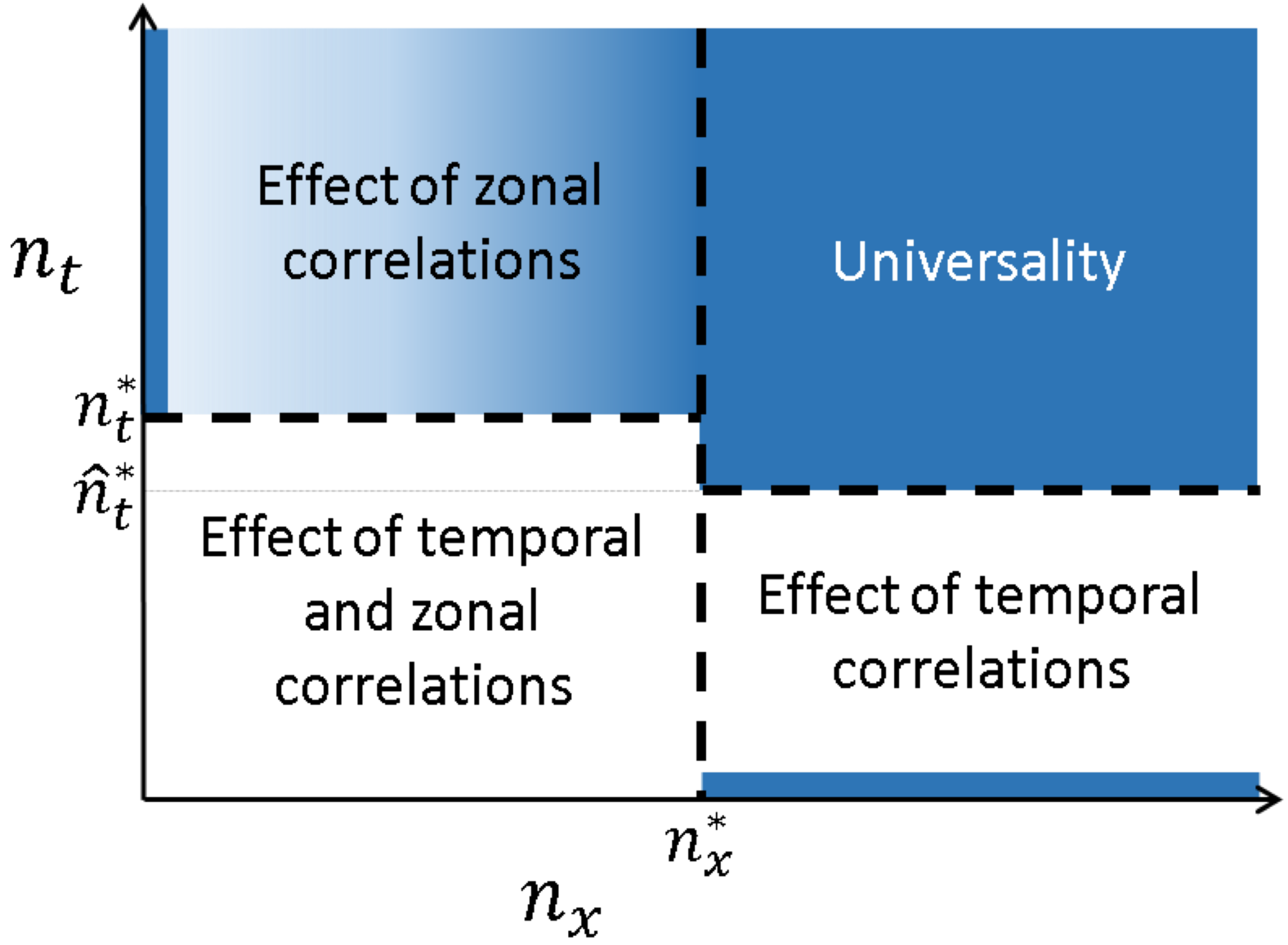
Let $X_1, ..., X_m$ be a sequence of i.i.d. random variables with common distribution function F.
We regard as extreme events those of the X_i that exceed some high thershold u.

$$\Pr\{X > u + y | X > u\} = \frac{1 - F(u+y)}{1 - F(u)}, \qquad y > 0$$

unknown



Universality vs. Persistence



Emergence of GEV

Following the BM procedure we obtain the corresponding results

Choice of the observable functions

1. $\mathbf{g_1}(\mathbf{dist}(f^m(\mathbf{x}), \zeta)) = -\log(\mathbf{dist}(f^m(\mathbf{x}), \zeta)) \rightarrow$ **type 1** dist

   At first order: $\xi = 0$, $\sigma = \frac{1}{D(\zeta)}$, $\mu = \frac{\log(m)}{D(\zeta)}$

2. $\mathbf{g_2}(\mathbf{dist}(f^m(\mathbf{x}), \zeta)) = (\mathbf{dist}(f^m(\mathbf{x}), \zeta))^{-1/\alpha} \rightarrow$ **type 2** dist. $\alpha > 0$, $\alpha \in R$

   At the first order: $\xi = \frac{1}{\alpha D(\zeta)}$, $\sigma = m^{\frac{1}{\alpha D(\zeta)}}$, $\mu = m^{\frac{1}{\alpha D(\zeta)}}$

3. $\mathbf{g_3}(\mathbf{dist}(f^m(\mathbf{x}), \zeta)) = \mathbf{C} - (\mathbf{dist}(f^m(\mathbf{x}), \zeta))^{1/\alpha} \rightarrow$ **type 3** dist. $C \in R$.

   At first order: $\xi = -\frac{1}{\alpha D(\zeta)}$, $\sigma = m^{\frac{-1}{\alpha D(\zeta)}}$, $\mu = C$.

- Asymptotically we get a GEV distribution whose parameters **depend on the local dimension $D(\zeta)$ of a ball centered on ζ.**

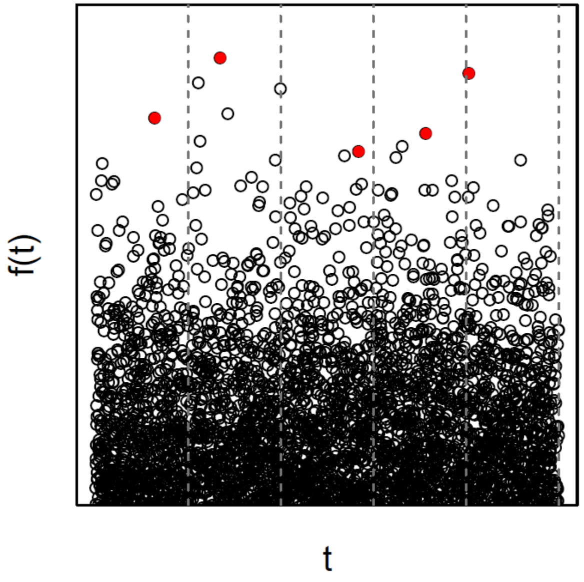
EVT V1 - Block Maxima: Extremes as Rare Events

$$M_m = \max\{X_1, ..., X_m\}$$

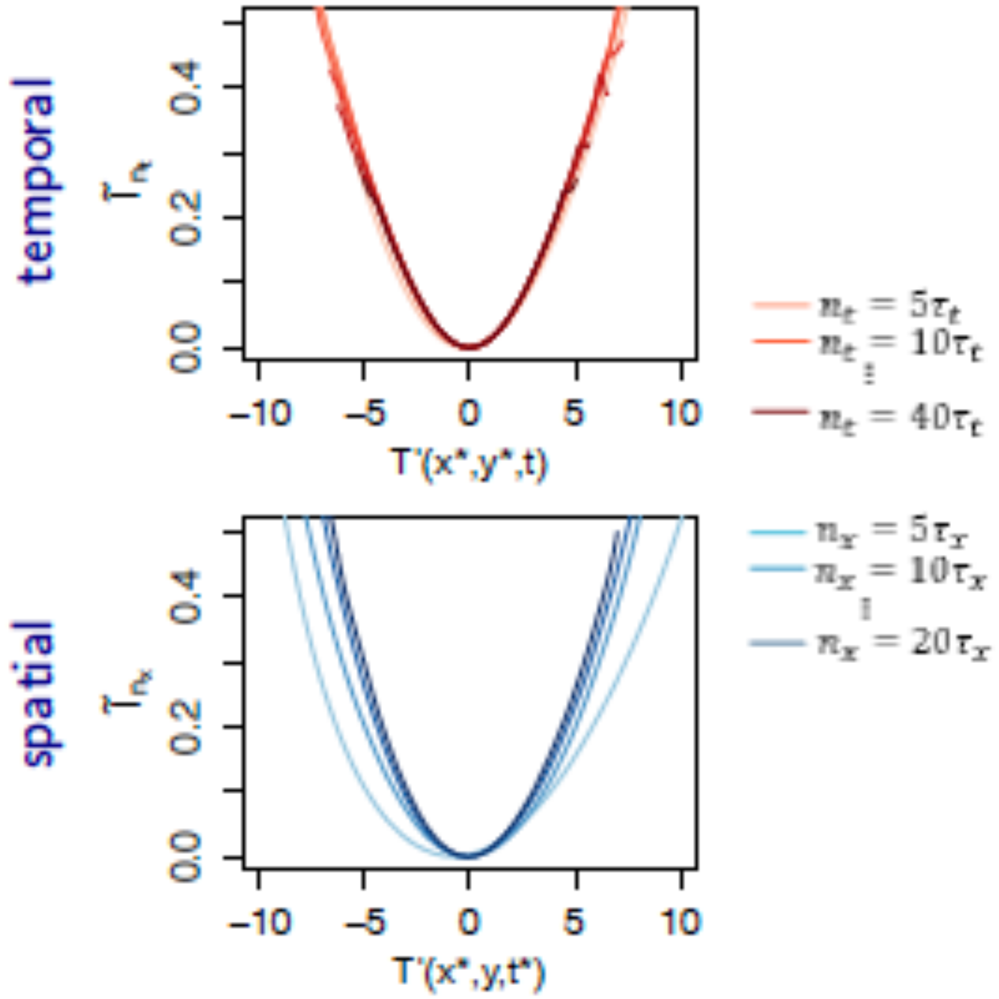
where $X_1, ..., X_m$ are independent, identically distributed (i.i.d. random variables with common distribution function F.

$$\begin{aligned}\Pr\{M_m \le z\} &= \Pr\{X_1 \le z, ..., X_m \le z\} \\ &= \Pr\{X_1 \le z\} \times ... \times \Pr\{X_m \le z\} \\ &= \{F(z)\}^m\end{aligned}$$

unknown



Temporal and Spatial Large Deviations



- 1) **Convergence** of I_{n_t} and I_{n_x} $\Rightarrow$ LDP
  - $\Rightarrow$ optimal averaging length $n^* \approx 20\tau$

    asymptotic regime for $n > n^*$:
  - predictive power for averages over $n > n^*$

Standard LDT for stochastic fieds

$$dX_t^\varepsilon = b(X_t^\varepsilon)\,dt + \sqrt{\varepsilon}\sigma\,dW_t$$

Target path $\phi(t)$

$$P\{\sup_{t\in[0,T]} \|X_t^\varepsilon - \phi(t)\| < \delta\} \asymp \exp\left(-\varepsilon^{-1} S_T(\phi)\right)$$

Rate function/Action functional

$$S_T(\phi) = \begin{cases} \int_0^T L(\phi,\dot{\phi})\,dt & \text{if the integral converges,} \\ \infty & \text{otherwise.} \end{cases}$$

Lagrangian

$$L(\phi,\dot{\phi}) = \tfrac{1}{2}\|\dot{\phi} - b(\phi)\|_a^2$$

Instanton

$$\phi^*(t) = \underset{\phi\in\mathscr{C}}{\operatorname{argmin}}\, S_T(\phi)$$

If T goes to infinity one can introduce quasi-potential …

Deadly Heat Wave Creates Havoc Across South Asia

By **Climate Nexus**

The New York Times

ASIA PACIFIC

Death Toll From Heat Wave in Karachi, Pakistan, Hits 1,000

By SABA IMTIAZ and ZIA ur-REHMAN JUNE 25, 2015

Deaths 1,200 as Karachi wilts under heat

June 24, 2015/ 34 Comments

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Record heat wave kills hundreds in Pakistan

Temperatures in Pakistan reached 44.8 degrees Celsius or 113 degrees Fahrenheit, the highest recorded temperature in 15 years. CNN's Pedram Javaheri reports. Source: CNN

General EVLs for distance observables

- We assume a local scaling of the measure of the attractor:

$$\lim_{r \to 0} \frac{\log \nu(B_r(\zeta))}{\log r} = D(\zeta), \text{ for } \zeta \text{ chosen } \nu - \text{a.e.}$$

- $\nu(B_l(\zeta) \sim l^{D(\zeta)} \to$ Equation for $H_{g,T} = 1 - F_{g,T}$:

$$H_{g,T}(z) \sim \left(\frac{g^{-1}(z+T)}{g^{-1}(T)} \right)^D$$

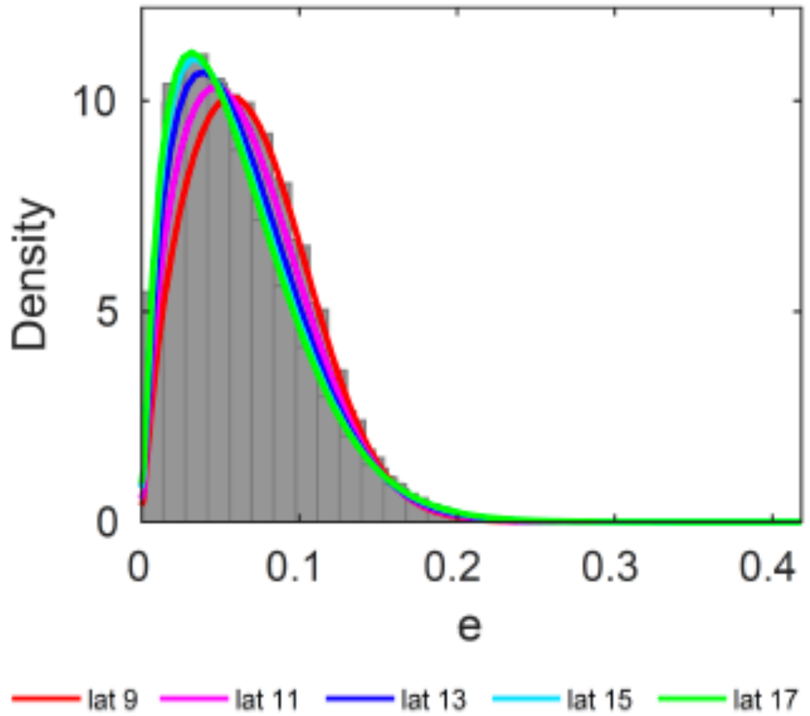
- Axiom A: $D(\zeta) = D = d_H \simeq d_{KY} \Rightarrow$ mixing: local $\to$ global

Strong forcing

Why is the convergence so slow?

ξ_δ very near 0 $\Rightarrow$ Gumbel distribution.

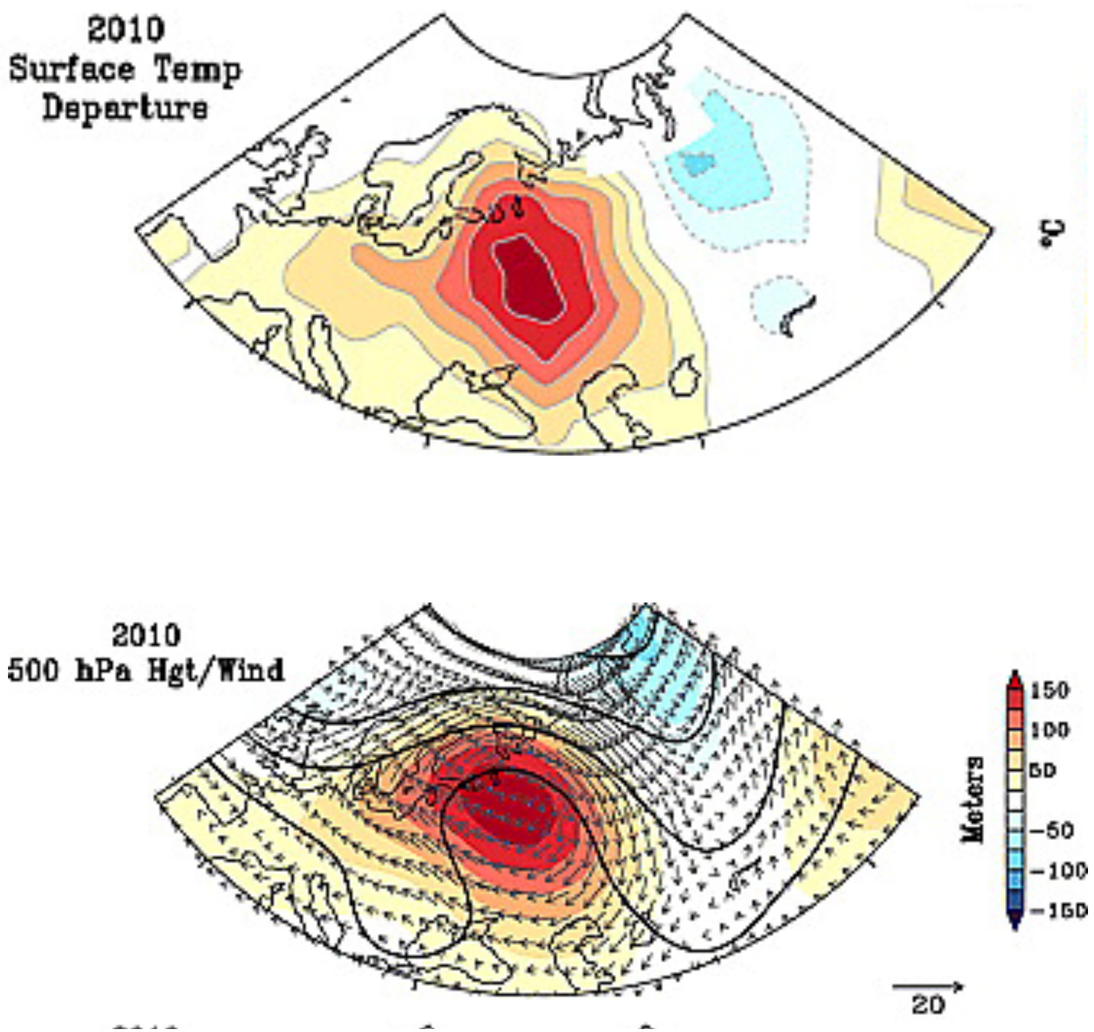
Absolute maximum
(Lucarini et al., 2014)
$A_{max} = \sigma^* / - \xi_\delta$
$A_{max} \approx 12.5 \approx 200\times < e >$
$A_{max} \approx 20\times$ largest extremes



Gumbel distribution: statistical model of reference for physical extremes in high-dimensional chaotic systems.

2010 Russian Heatwave

- Associated with a powerful, long-lasting blocking
- Combined extremes: forest fires (locally)
- Remote connections: closely linked to floods in Pakistan



Dole et al. 2011