

Project Lorenz-Reservoir Computing

The well-known toy model for atmospheric variability is the Lorenz 1963 model given by the set of ODEs,

$$\frac{dx}{dt} = s(y - x) \quad (1a)$$

$$\frac{dy}{dt} = rx - y - xz \quad (1b)$$

$$\frac{dz}{dt} = xy - bz \quad (1c)$$

This system is, for standard values of parameters $s = 10$, $r = 28$ and $b = 2.667$, a chaotic system with sensitivity to initial conditions. In this project, we will look at the skill of Reservoir Computer (RC) based predictions, using the Python Notebook provided.

- (i) Generate the required data by using a time step $\Delta t = 0.02$ and integrating over the time interval $[-100, 25]$. First use the interval $[-100, 0]$ as training data and the interval $[0, 25]$ as ‘truth’.
- (ii) Use initial hyperparameter values $d = 300$, $< k > = 6$, $\rho = 1.2$, $\sigma = 0.1$ and $\beta = 0$ (see Notes_RC.pdf). Train the RC using the training data set and generate a RC prediction \mathbf{x}_R of the ‘truth’ \mathbf{x} over the test interval $[0, 25]$. Set a tolerance error

$$\epsilon = ||\mathbf{x} - \mathbf{x}_R||^2$$

and determine the time t_e when $\epsilon > 0.1$.

- (iii) How could one determine the Lyapunov exponents of the Lorenz system, using the RC approach (you do not have to do this, but you may)?
- (iv) Study the effect of the hyperparameter ρ on t_e and try to explain the behavior.
- (v) To which of the hyperparameters is t_e most sensitive? How could one optimize the values of these hyperparameters to maximize t_e ?