Basic Project: Stochastic Models for Pacific SST and SSH variability

In this project, you will practice with the analysis of observations and with several conceptual stochastic models to explain certain aspects of these data. Use the Jupyter notebooks provided on GitHub and make a new Jupyter notebook for all computations below.

Part A, Deadline Report: Friday 7/2 at 13:00

(i) Read in the two time series of the corresponding data file for this project (same data set as we used in class). The data consists of sea surface height (SSH) and sea surface temperature (SST) somewhere in the Pacific Ocean between 1993 – 2018. Plot both the SSH and SST anomalies (w.r.t. to the time mean).

(ii) Remove the seasonal cycle and long-term trend. Determine the probability density function (PDF) of the time series, calculate the Fourier spectra and determine the power law coefficient of the spectral power decay at high frequency. Do this for both SST and SSH.

Consider the following stochastic differential equation

$$dX_t = -\gamma X_t dt + \sigma dW_t$$

with X_0 given and real parameters γ and σ.

(iii) Provide a method to estimate the parameters γ and σ for both SSH and SST and determine their 'best' values.

(iv) For these best estimates, compute a realization of the stochastic model and determine the equilibrium PDF. Compare this PDF for the estimated parameters with the PDF of SSH and SST (as in (ii)). Discuss the differences between SST and SSH data.

Part B

Consider next the following stochastic differential equation

$$dX_t = \lambda X_t dt + (\mu + \nu X_t) dW_t$$

with X_0 given and real parameters λ, μ and ν. This stochastic model is called CAM (Correlated Additive-Multiplicative) noise.

(i) Determine the analytical equilibrium PDF solution of the Fokker-Planck equation associated with this stochastic differential equation.

(ii) Determine the variance associated with this probability density function and plot it as a function of ν.

(iii) Provide a method to estimate the parameters λ, μ and ν for the observations for SST and SSH and determine their values. Give a physical argument why these parameters are different for SST and SSH.

(iv) Discuss the differences in probability density functions between the two different cases $\nu = 0$ as in part A and $\nu \neq 0$ in part B.