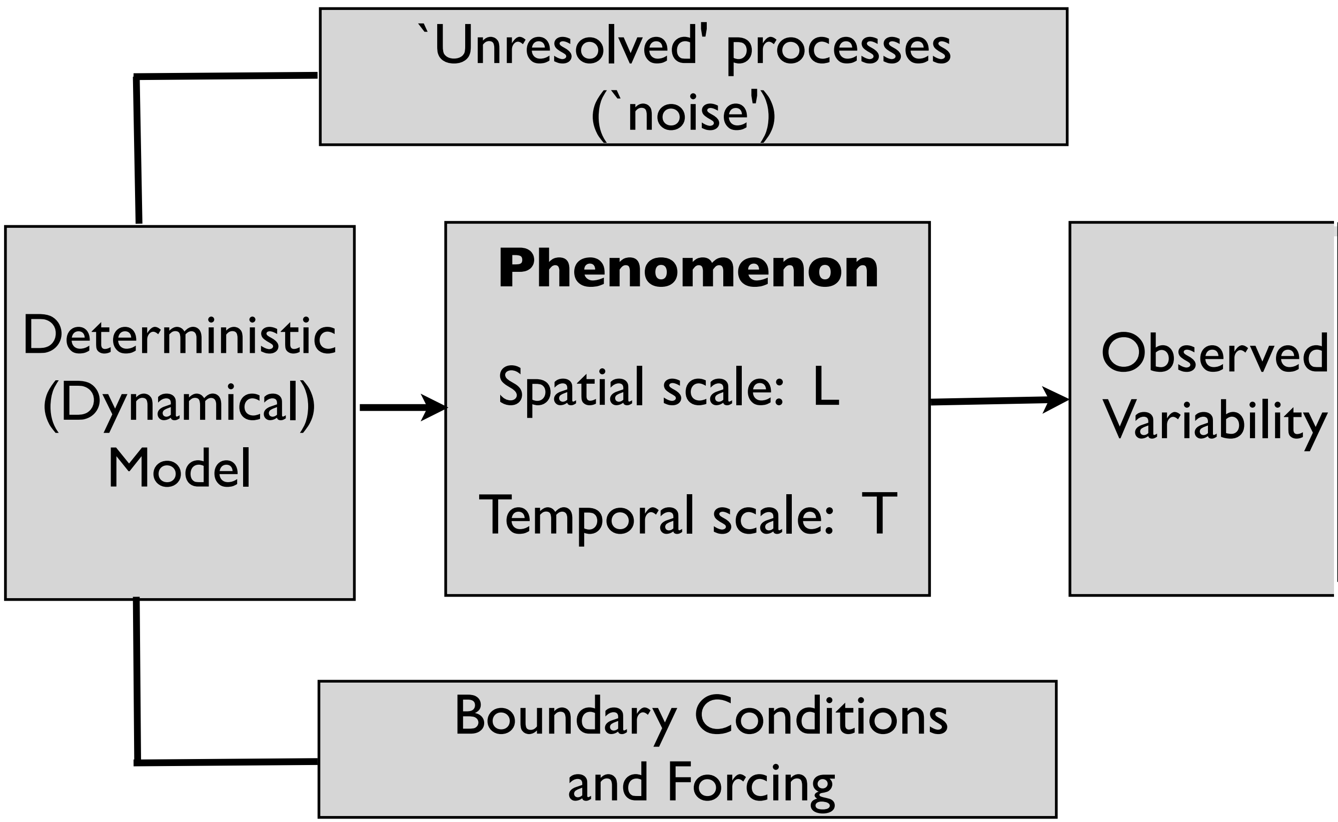
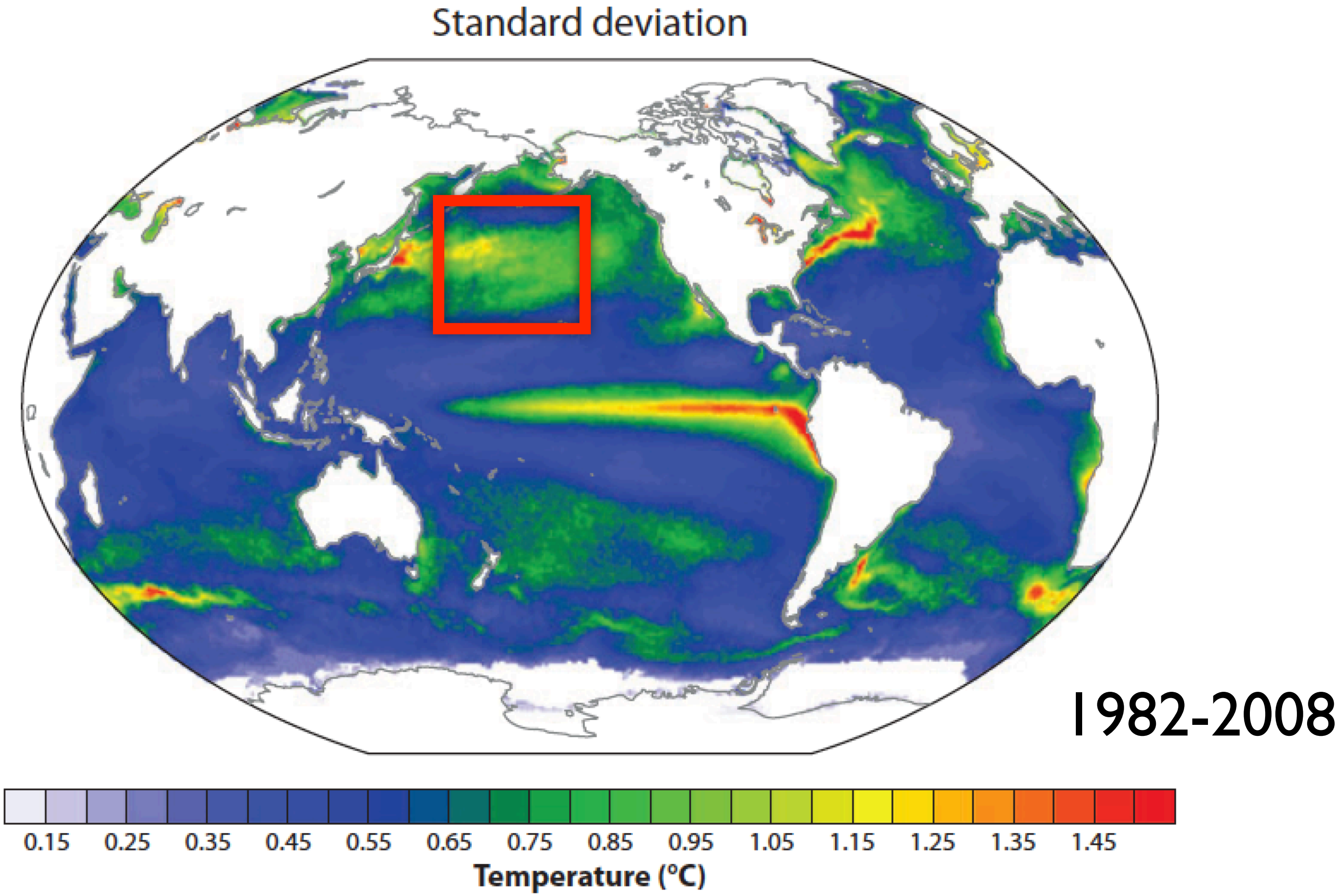
Stochastic Dynamical Modeling UniTN: 27/1/2025 - 7/2/2025

Stochastic Dynamical Systems Framework

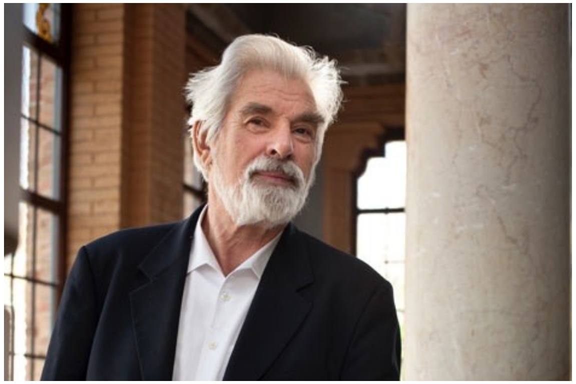


Sea surface temperature variability



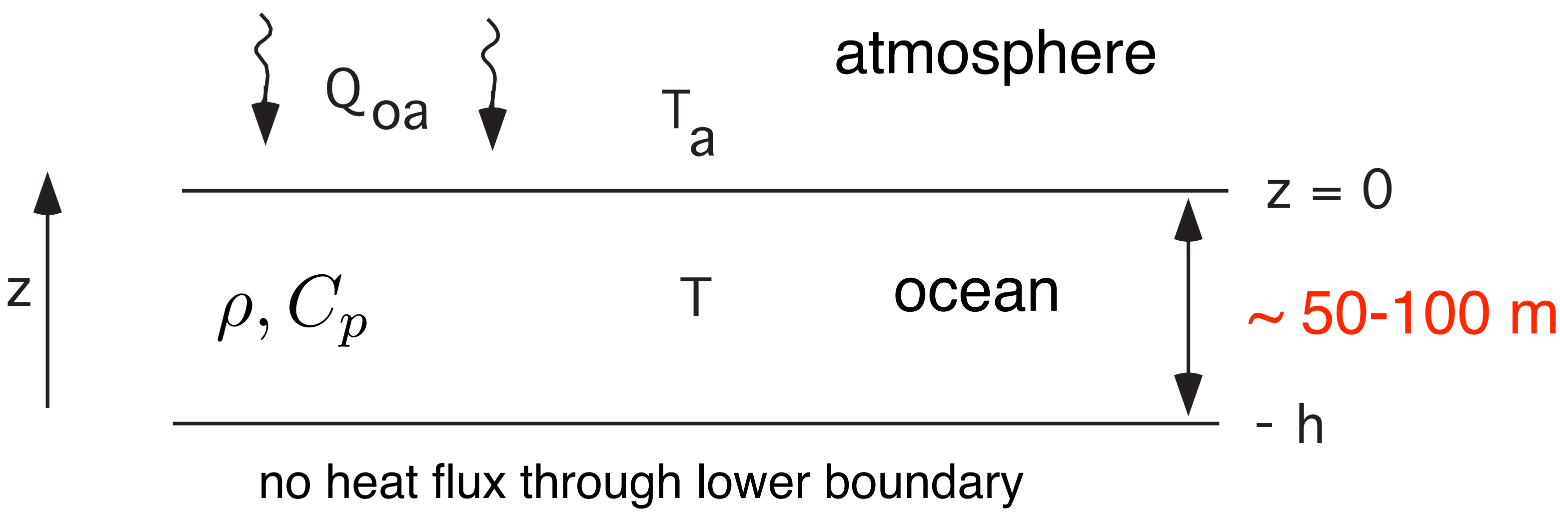
Deser et al., Ann. Rev. Mar. Sc. (2010)

Hasselmann approach (1976)



Klaus Hasselmann (1931-)

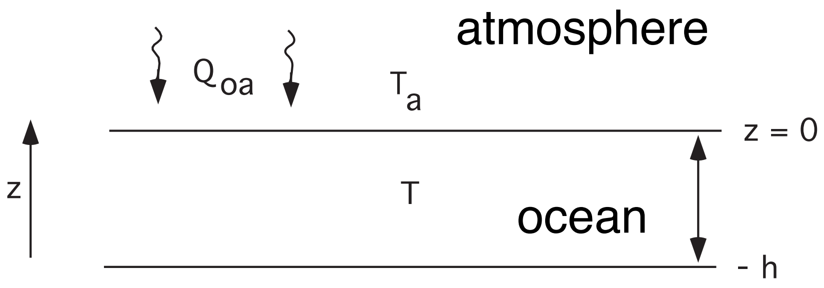
Hasselmann K. (1976), "Stochastic climate models, Part 1: Theory", *Tellus*, 28: 473-485.



Ocean Mixed-layer Model

Equation:

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2}$$



Boundary conditions:

$$z = 0 : K \frac{\partial T}{\partial z} = Q_{oa}$$

$$z = -h : \frac{\partial T}{\partial z} = 0$$

Solution

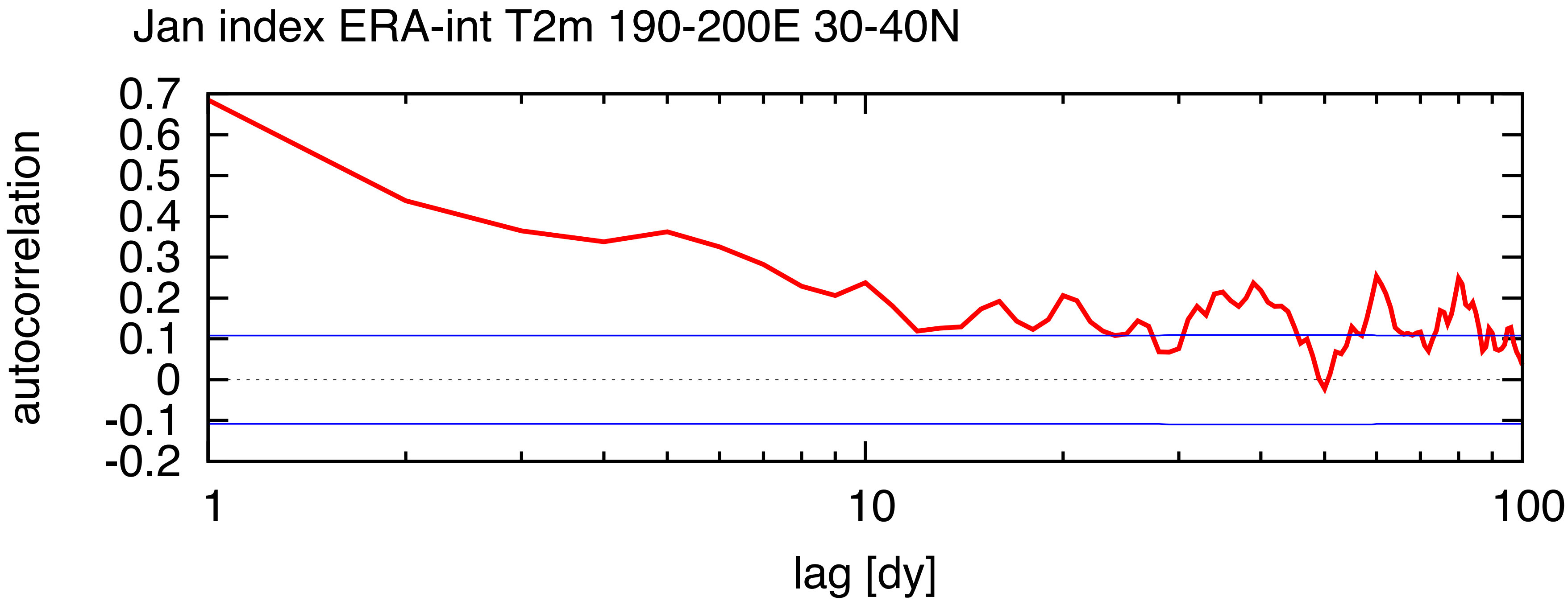
Use: $$\bar{T} = \frac{1}{h}\int_{-h}^{0} T \, dz$$ and $$Q_{oa} = \alpha(T_a - \bar{T})$$

$$\bar{T} =< \bar{T} > +\tilde{T} \qquad T_a =< T_a > +\tilde{T}_a$$

Result: $$\boxed{\frac{d\tilde{T}}{dt} = \frac{\alpha}{\rho C_p h}(\tilde{T}_a - \tilde{T})}$$

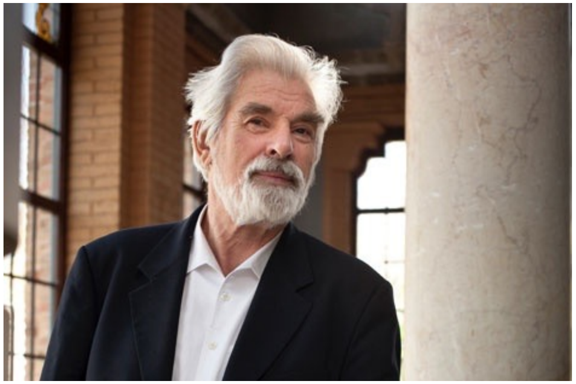
$$\gamma = \frac{\alpha}{\rho C_p h}$$ ~ 1/(100 days)

Example: Autocorrelation
Pacific atmospheric surface temperatures



Decorrelation time scale atmospheric forcing << ocean damping time scale

The Hasselmann (1976) stochastic climate model



Hasselmann K. (1976), "Stochastic climate models, Part 1: Theory", *Tellus*, 28: 473-485.

$$\boxed{\frac{d\tilde{T}}{dt} = -\gamma\tilde{T} + \sigma\xi} \qquad \gamma = \frac{\alpha}{\rho C_p h}$$

white noise

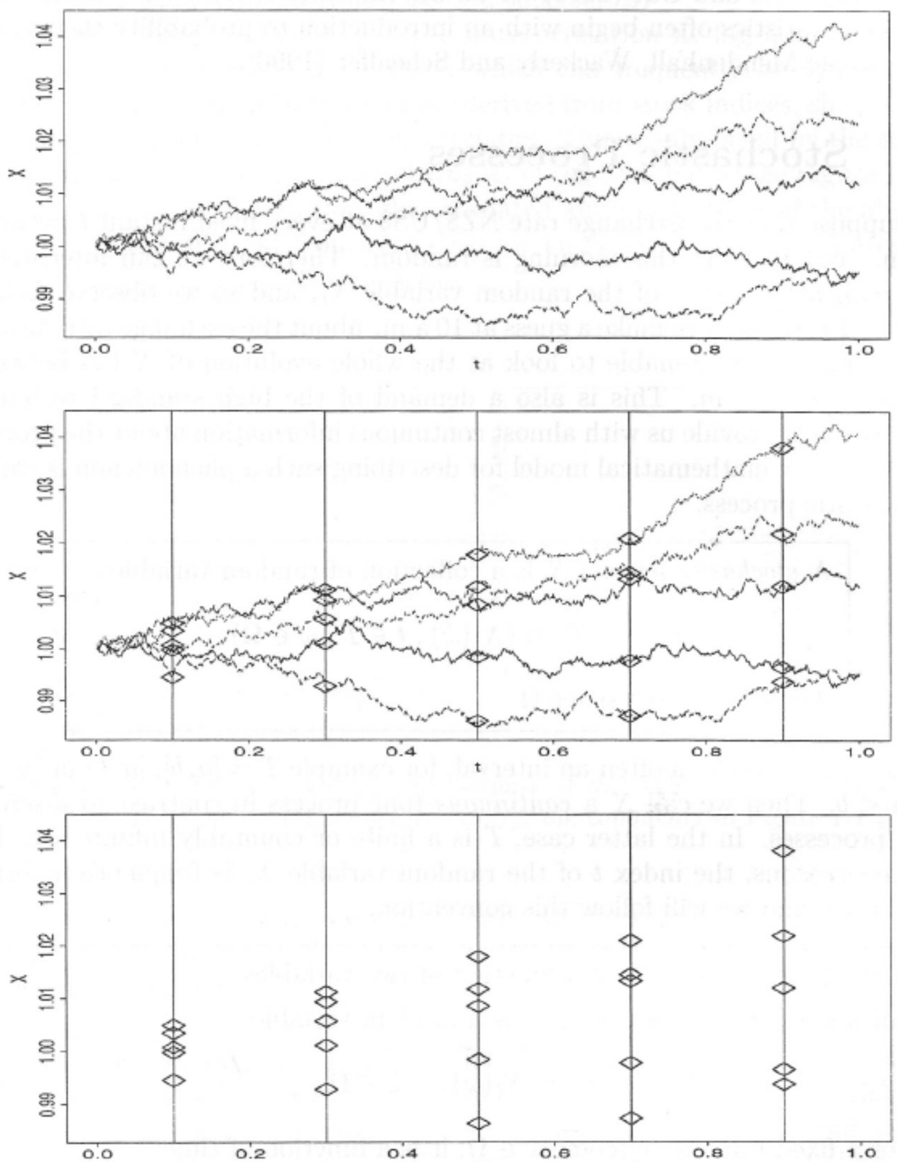
$$E[\xi(t)] = 0$$

$$E[\xi(t)\xi(s)] = \delta(t-s)$$

"The choice between a deterministic and a stochastic formulation of the equations .... [is] dictated by convenience" Lorenz (1987)

Stochastic process

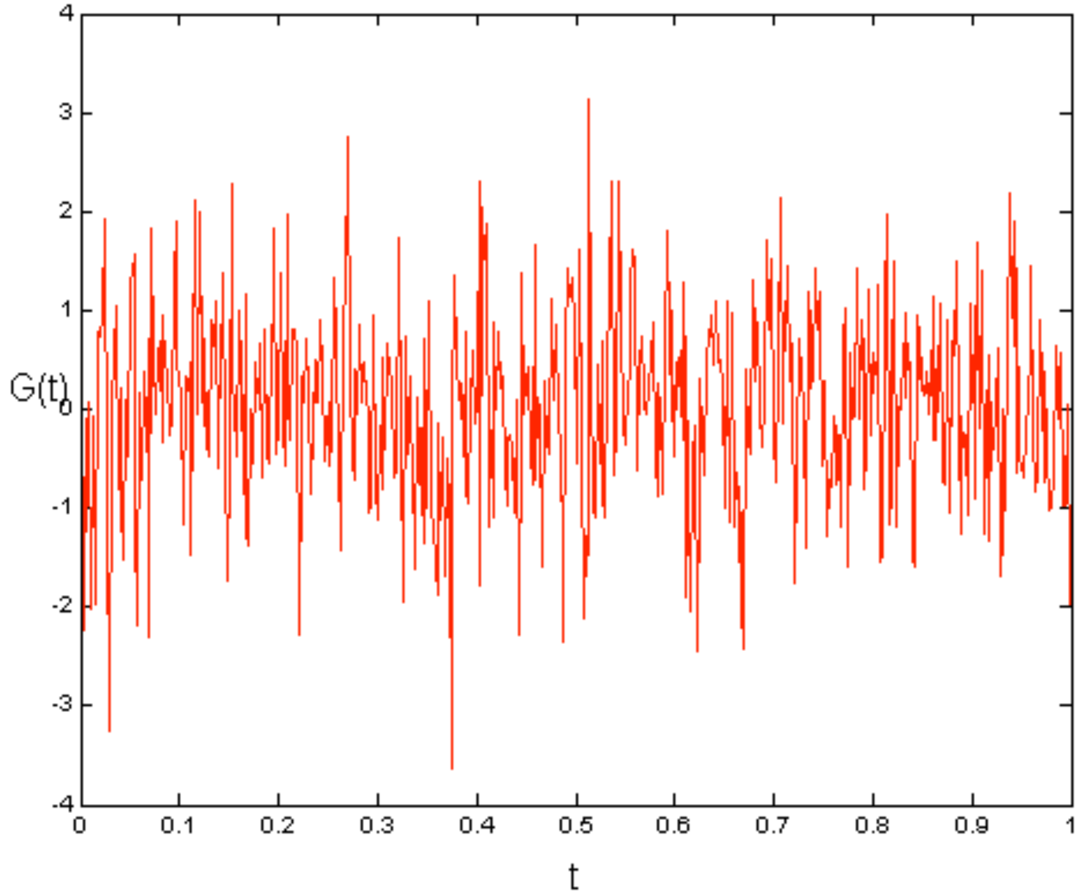
$X_t(\bar{\omega})$



$X_{\bar{t}}(\omega)$

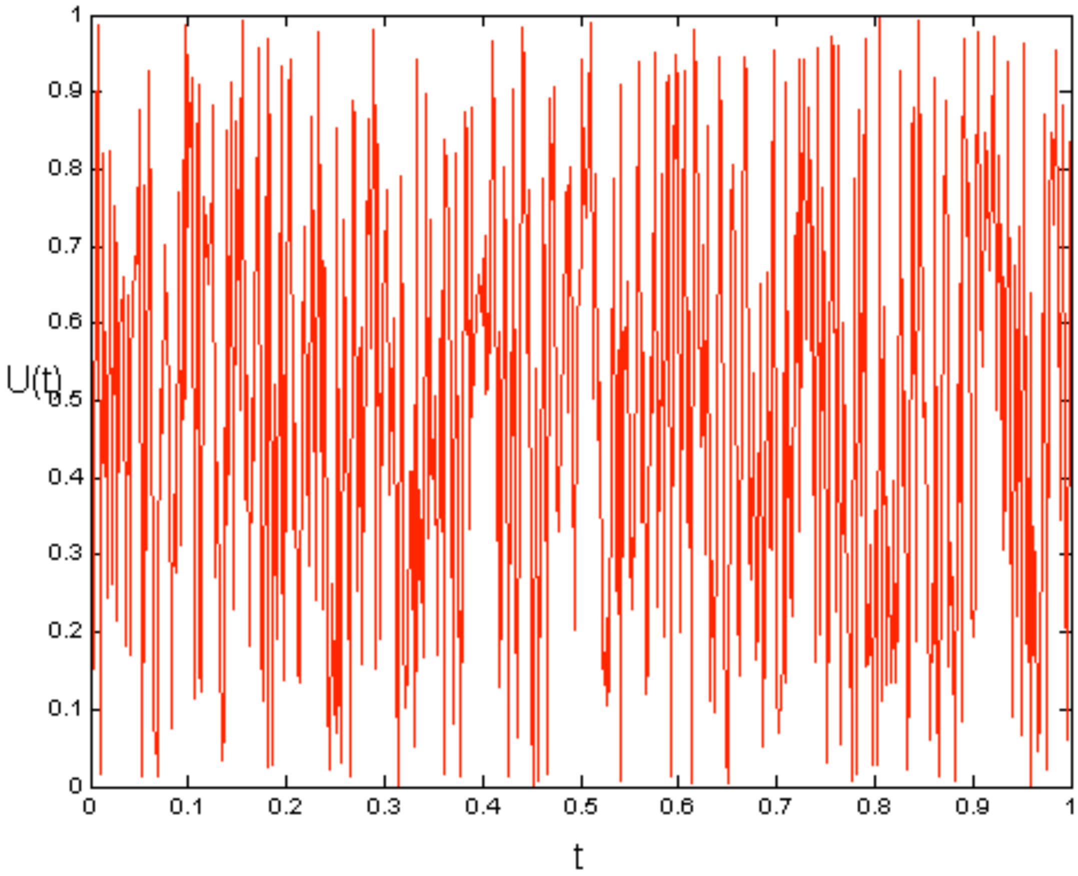
Gaussian process

$$X_t \ iid \ N(0,1)$$

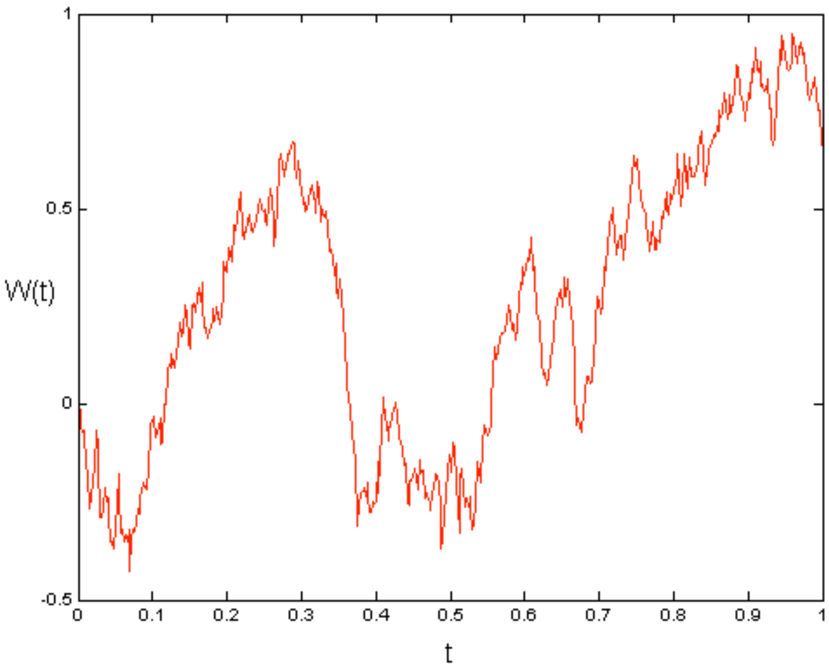


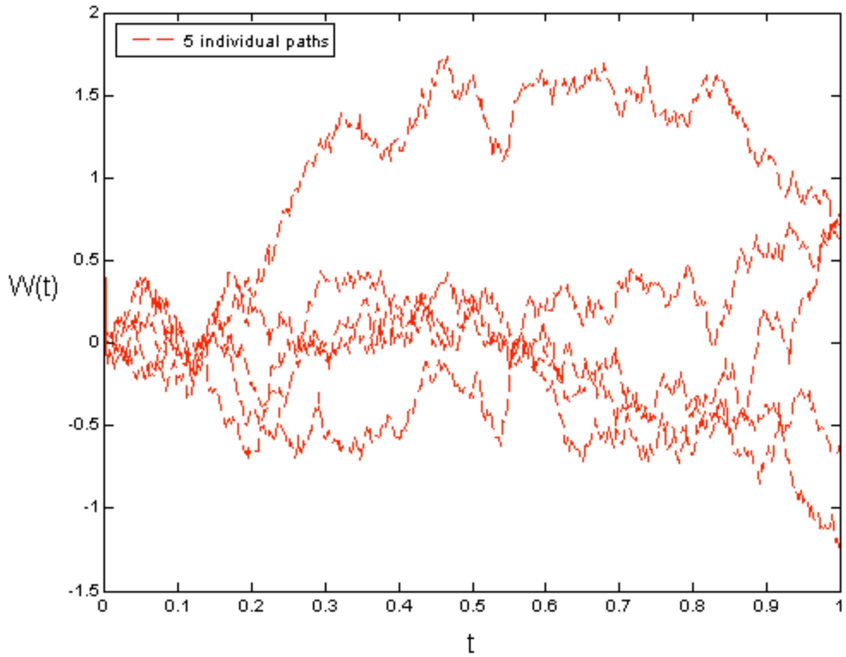
Uniform process

$$X_t \; iid \; U(0,1)$$



Wiener process





Numerical solution of SDEs

$$X(t) = X(0) + \int_0^t f(X(s))ds + \int_0^t g(X(s))dW(s)$$

$$\tau_j = j\Delta t, j = 0, \ldots, n \text{ on } [0, T]$$

$$\Delta t = T/n$$

Gisiro Maruyama (1916-1986)

Euler-Maruyama scheme:

$$X_j - X_{j-1} = f(X_{j-1})\Delta t + g(X_{j-1})(W(\tau_j) - W(\tau_{j-1}))$$