Tutorial 3: February 9, 12:00-12:45

1. Red noise or AR(1) process

The red noise process X at times t_n is defined as

$$X_{n+1} = \alpha X_n + Z_n$$

where $Z_n \sim N(0, \sigma^2)$ and $\alpha \in (0, 1)$.

a. Show that the variance of X is given by

$$Var[X] = \frac{\sigma^2}{1 - \alpha^2}$$

b. Show that the autocorrelation of X_t is given by

$$\rho_n(X) = \alpha^n$$

2. EWS of AMOC tipping

Notebook: Tutorial9-2_2.ipynb

We consider again the reduced AMOC model

$$dY_t = (F(t) - Y_t(1 + \mu(1 - Y_t)^2))dt + (\sigma_0 + \sigma_1 Y_t)dW_t$$

over the time interval $[0, T]$ with now $F(t) = F_0 + \Delta F t/T$.

a.
Take $\sigma_1 = 0$, $F_0 = 1.1$ and $\Delta F = 0.3$ such that the final value of F is beyond the multi-stable regime. Do the classical EWSs (variance, autocorrelation) show a hit (correct flag for tipping) and how does this depend on σ_0?

b.
Next study the case $\sigma_1 = 0$, $F_0 = 0.6$ and $\Delta F = 0.3$. What is the problem here with the classical EWSs? What is the reason this happens?

c.
Next study the case $\sigma_1 = 0.1$ such that we have state-dependent noise. Consider both cases for $F(t)$ as in a. and b. Identify any problems with the classical EWSs in these cases.