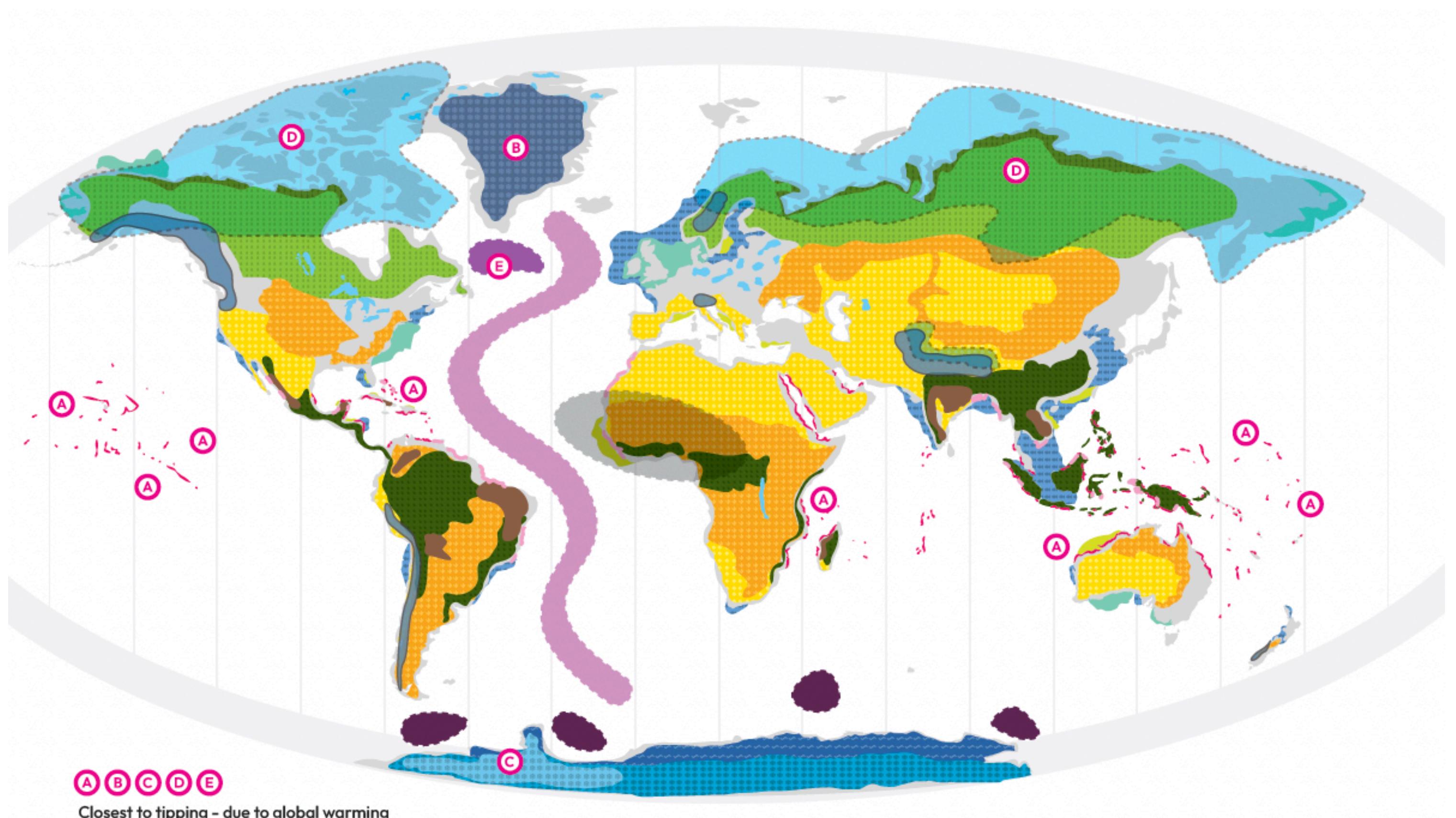


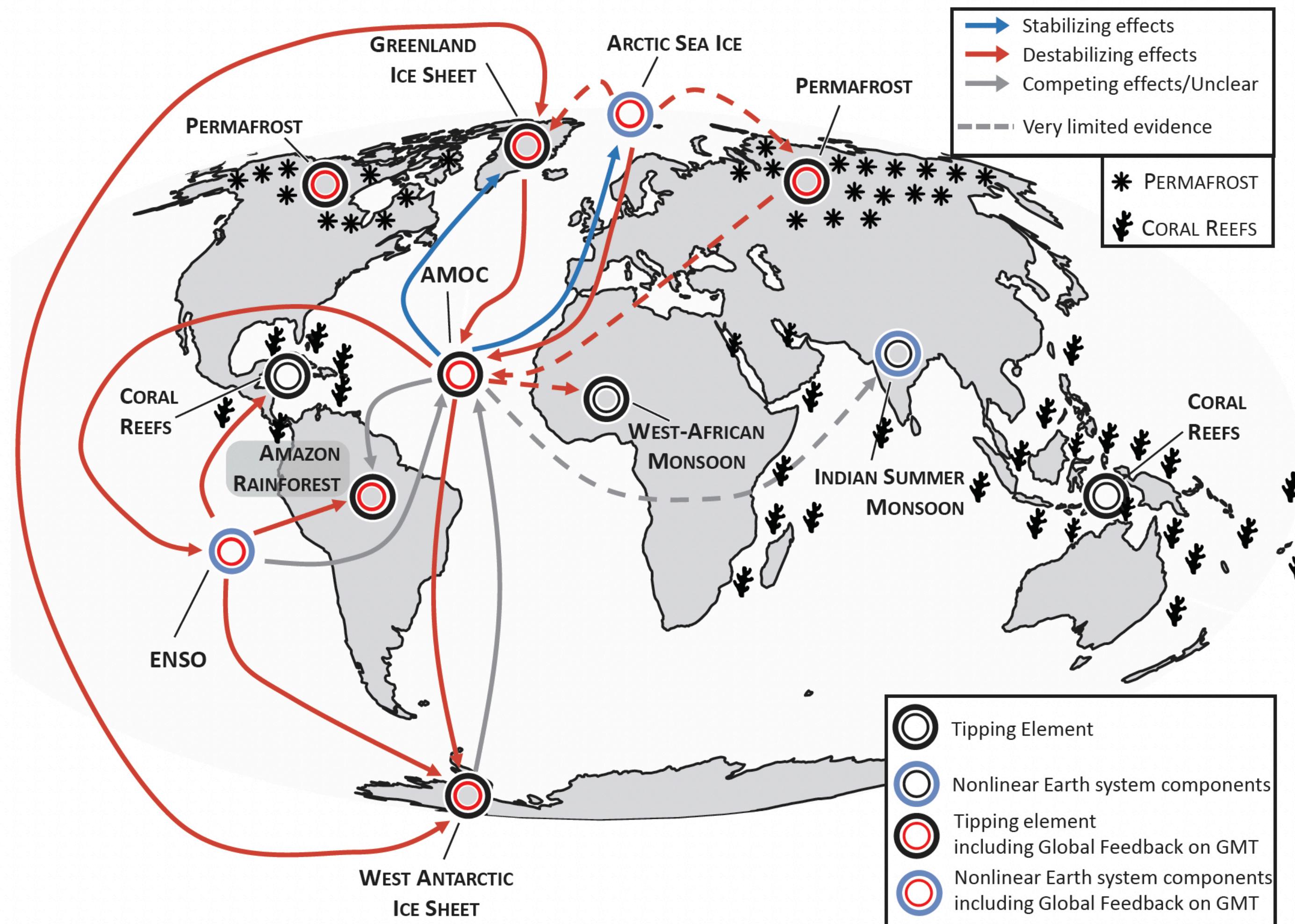
Tipping Behavior in the Climate System



Henk Dijkstra,
Department of Physics,
Utrecht University, NL
&
DICAM,
University of Trento, IT



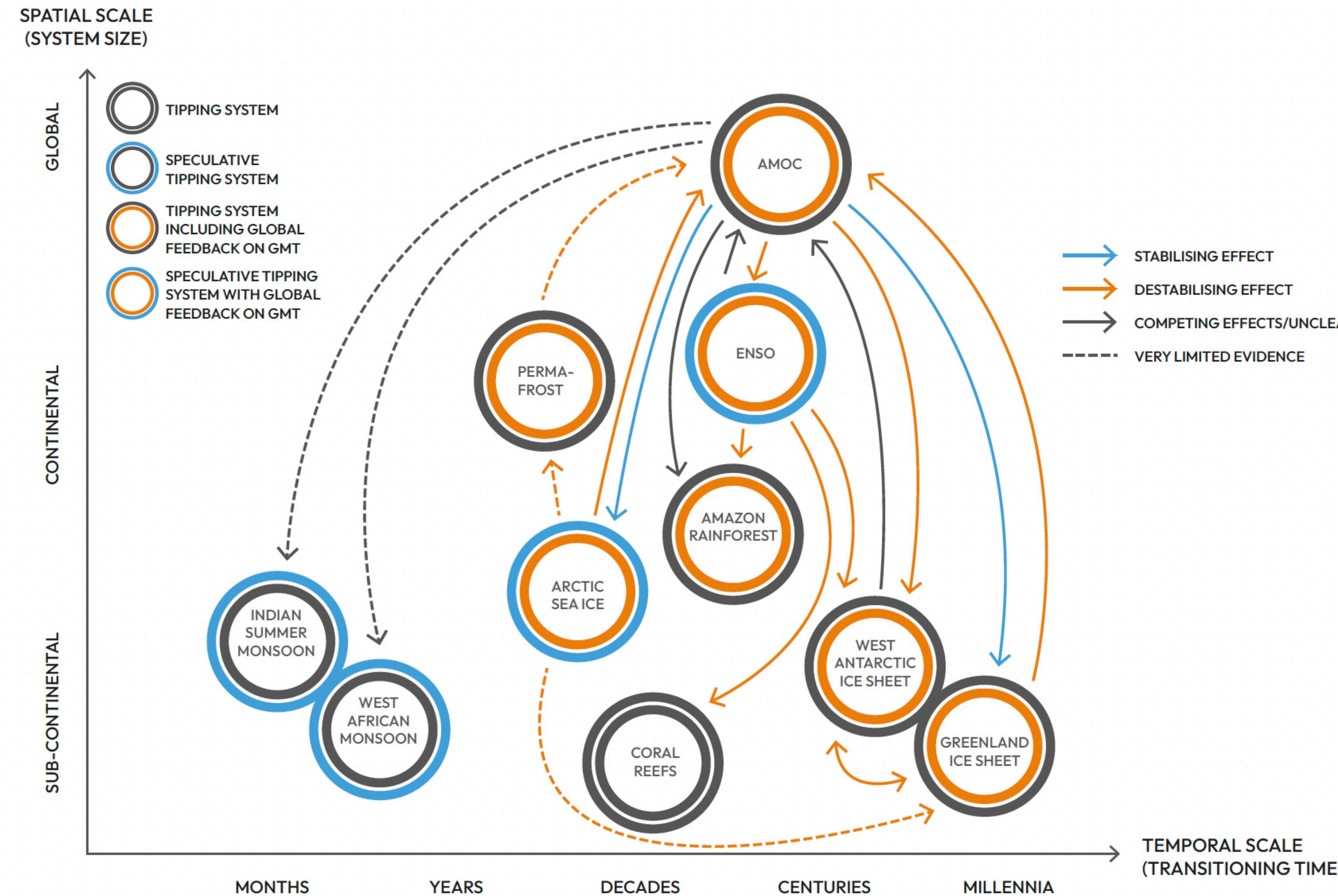
Tipping Cascades



Tipping in subsystem A (leading system) causes tipping in the subsystem B (following system)

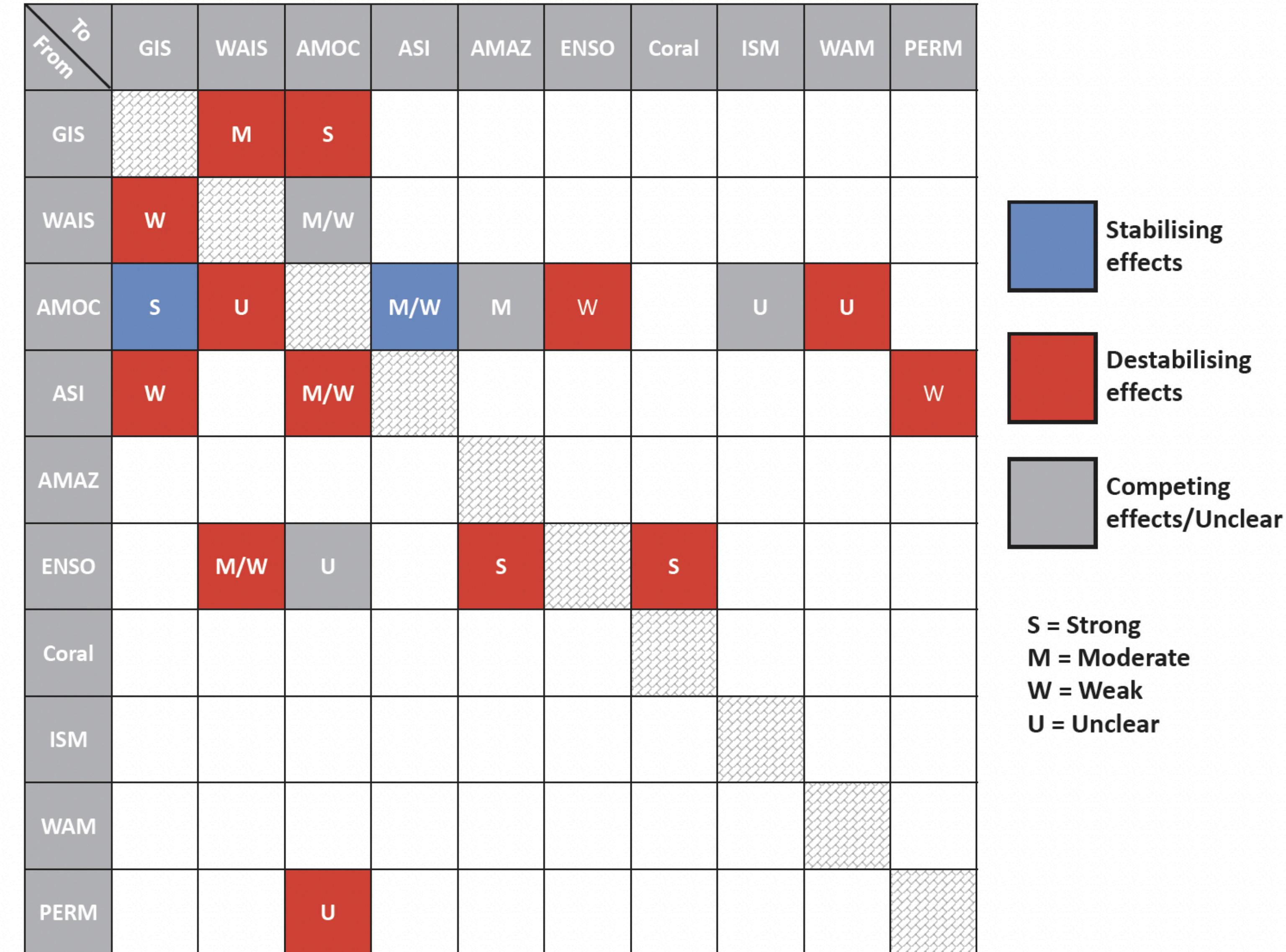


Spatial-temporal scales of coupling





Coupling of Subsystems



Cascading Tipping: Theory

Earth Syst. Dynam., 9, 1243–1260, 2018
<https://doi.org/10.5194/esd-9-1243-2018>
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Earth System
Dynamics

Open Access



Cascading transitions in the climate system

Mark M. Dekker^{1,2,3}, Anna S. von der Heydt^{1,2}, and Henk A. Dijkstra^{1,2}

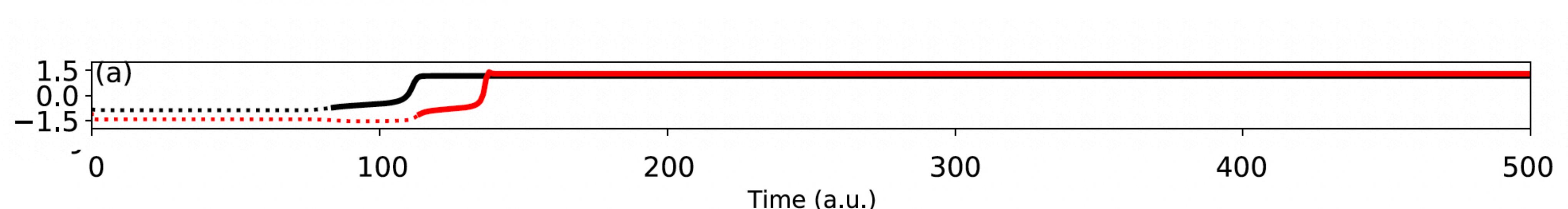
¹Institute for Marine and Atmospheric research Utrecht, Department of Physics,
Utrecht University, Utrecht, the Netherlands

²Centre for Complex Systems Studies, Utrecht University, Utrecht, the Netherlands

³Department of Information and Computing Science, Utrecht University, Utrecht, the Netherlands

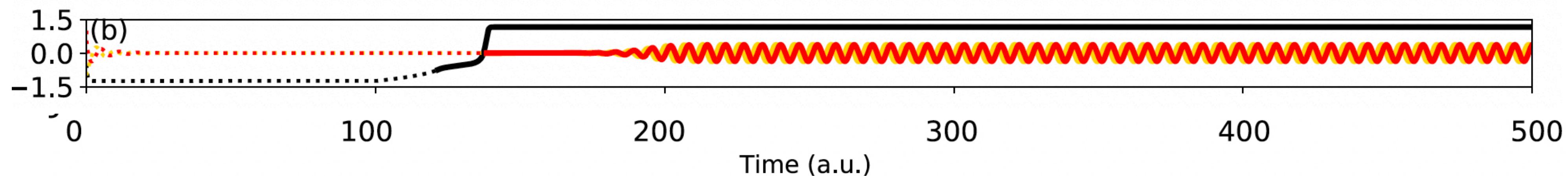
Cascading transition: two back-to back saddle nodes

$$\begin{cases} \frac{dx}{dt} = a_1 x^3 + a_2 x + \phi & a_1 = -0.5 \\ \frac{dy}{dt} = b_1 y^3 + b_2 y + \gamma(x), & a_2 = 0.5 \\ \gamma = 0.48x & b_1 = -0.5 \\ & b_2 = 1.0 \end{cases}$$



Cascading transition: saddle node - Hopf

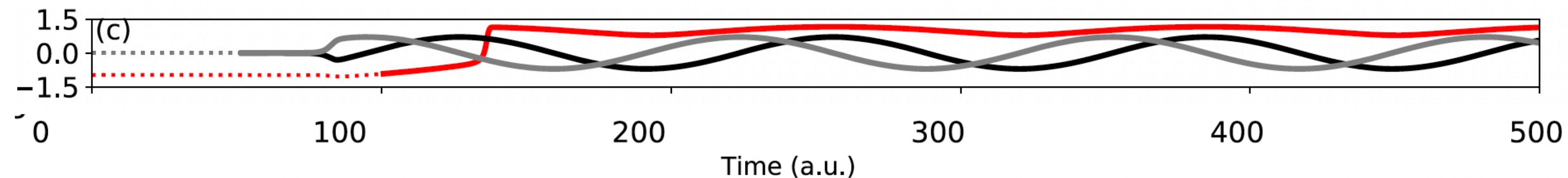
$$\begin{cases} \frac{dx}{dt} = a_1 x^3 + a_2 x + \phi \\ \frac{dy}{dt} = b_1 z + b_2 (\gamma(x) - (y^2 + z^2))y \\ \frac{dz}{dt} = c_1 y + c_2 (\gamma(x) - (y^2 + z^2))z, \end{cases} \quad \begin{aligned} a_1 &= -1 \\ a_2 &= 1 \\ b_1 &= b_2 = 1 \\ \gamma &= -0.1 + 0.12x \end{aligned}$$





Cascading transition: Hopf -saddle node

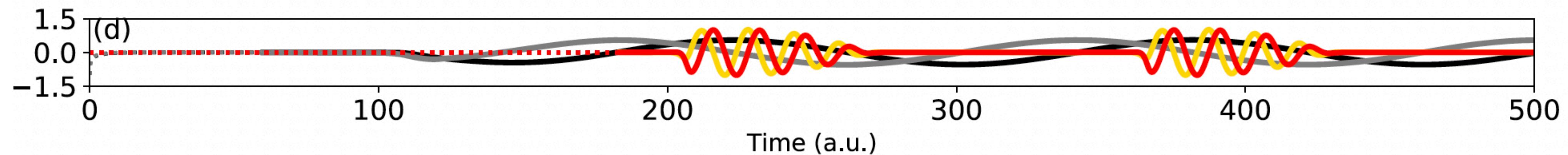
$$\begin{cases} \frac{dx}{dt} = a_1 y + a_2(\phi - (x^2 + y^2))x & a_1 = 0.05; a_2 = 1 \\ \frac{dy}{dt} = b_1 x + b_2(\phi - (x^2 + y^2))y & b_1 = -0.05; b_2 = 1 \\ \frac{dz}{dt} = c_1 z^3 + c_2 z + \gamma(x), & c_1 = -1 \\ \gamma = 0.05 + 0.5x & c_2 = 1 \end{cases}$$



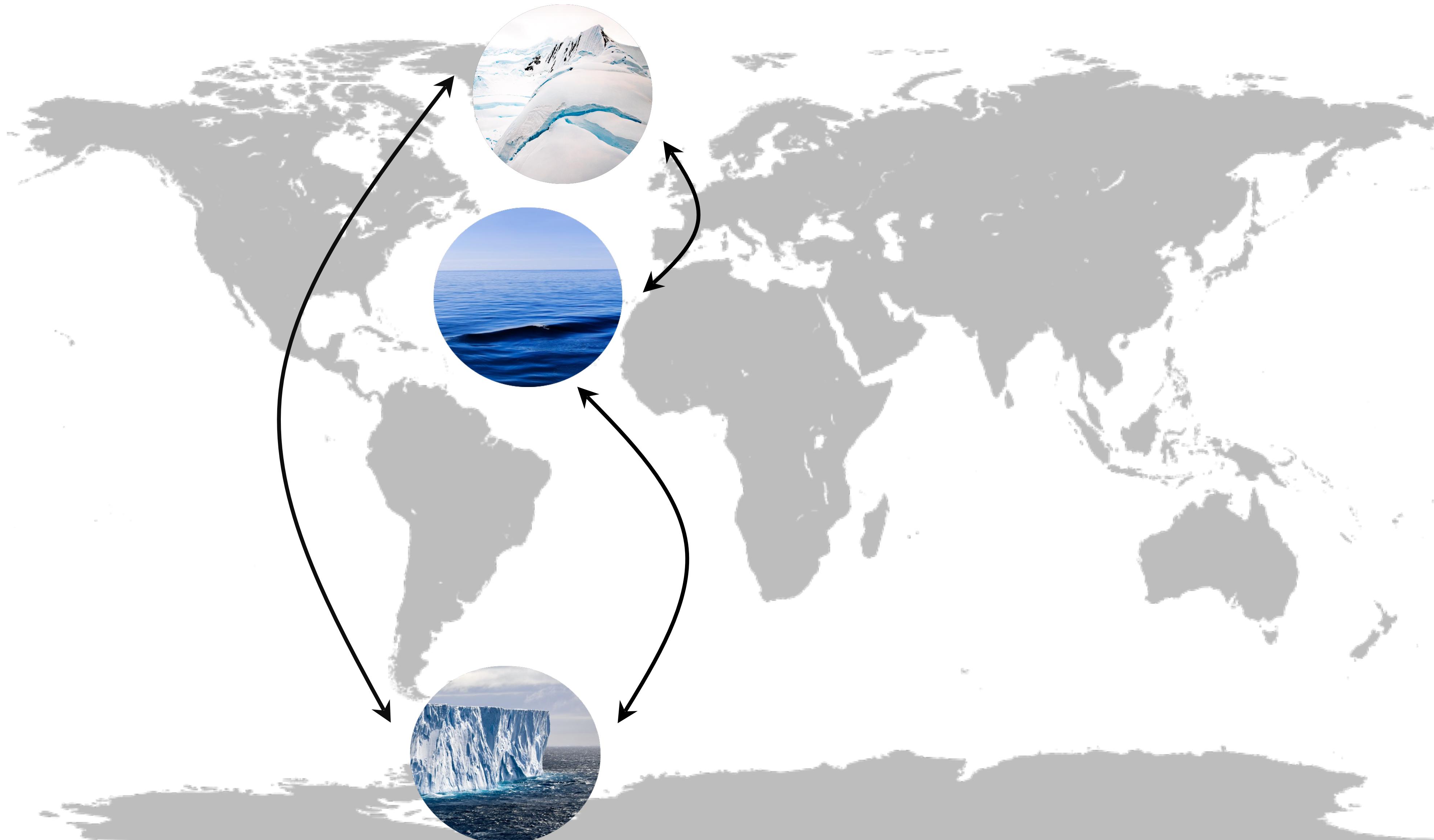
Cascading transition: *Hopf - Hopf*

$$\begin{cases} \frac{dx}{dt} = a_1 y + a_2(\phi - (x^2 + y^2))x \\ \frac{dy}{dt} = b_1 x + b_2(\phi - (x^2 + y^2))y \\ \frac{du}{dt} = c_1 v + c_2(\gamma(x) - (u^2 + v^2))u \\ \frac{dv}{dt} = d_1 u + d_2(\gamma(x) - (u^2 + v^2))v, \end{cases} \quad \begin{aligned} a_1 &= 0.04; a_2 = 2 \\ b_1 &= -0.04; b_2 = 2 \\ c_1 &= 0.4; c_2 = 1 \\ d_1 &= -0.4; d_2 = 1 \end{aligned}$$

$$\gamma = -0.05 + 2x$$



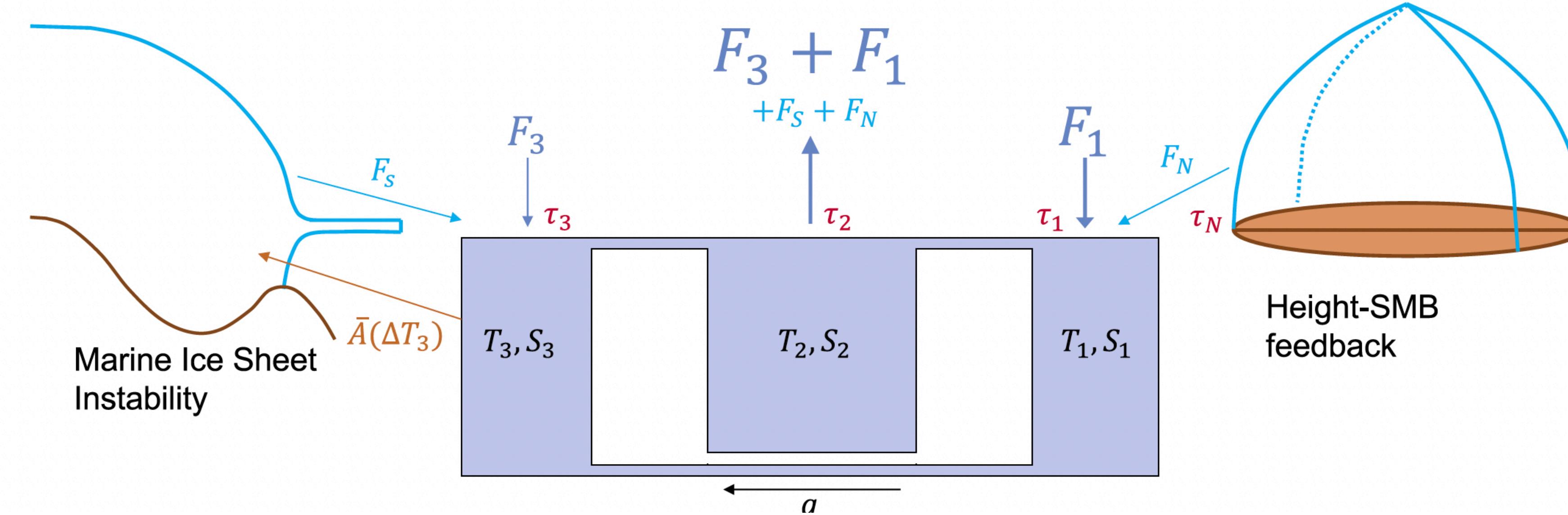
Example: AMOC-GIS-WAIS



Lenton & al. (2008)
Wunderling & al. (2024)



AMOC stabilisation through WAIS tipping



Geophysical Research Letters®

RESEARCH LETTER

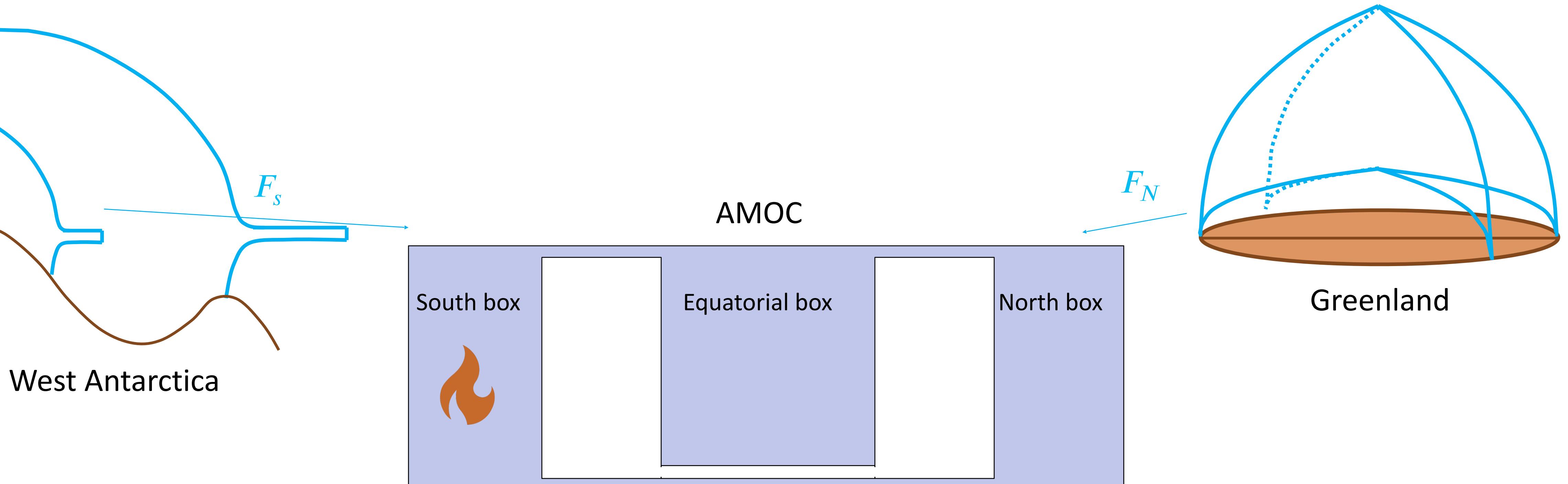
10.1029/2022GL100305

Key Points:

• A tipping point for the West Antarctic Ice Sheet (WAIS) is identified.

AMOC Stabilization Under the Interaction With Tipping Polar Ice Sheets

S. Sinet^{1,2} , A. S. von der Heydt^{1,2} , and H. A. Dijkstra^{1,2}

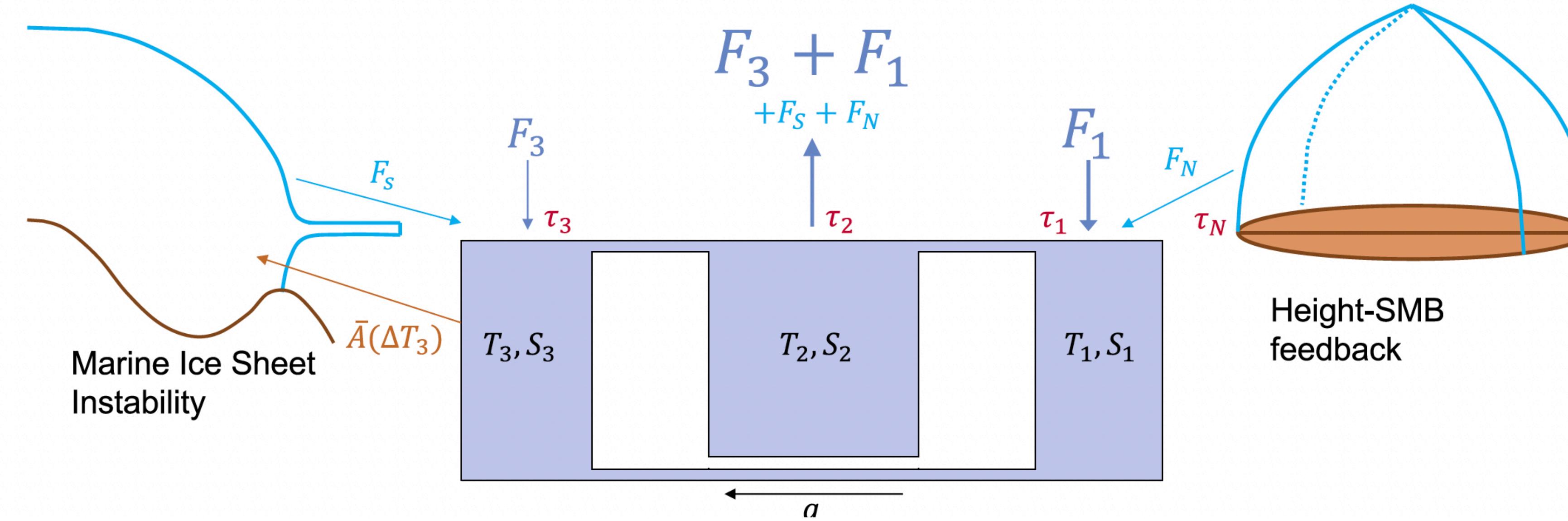


No Collapse !

Sinet & al. (2023a)



Model

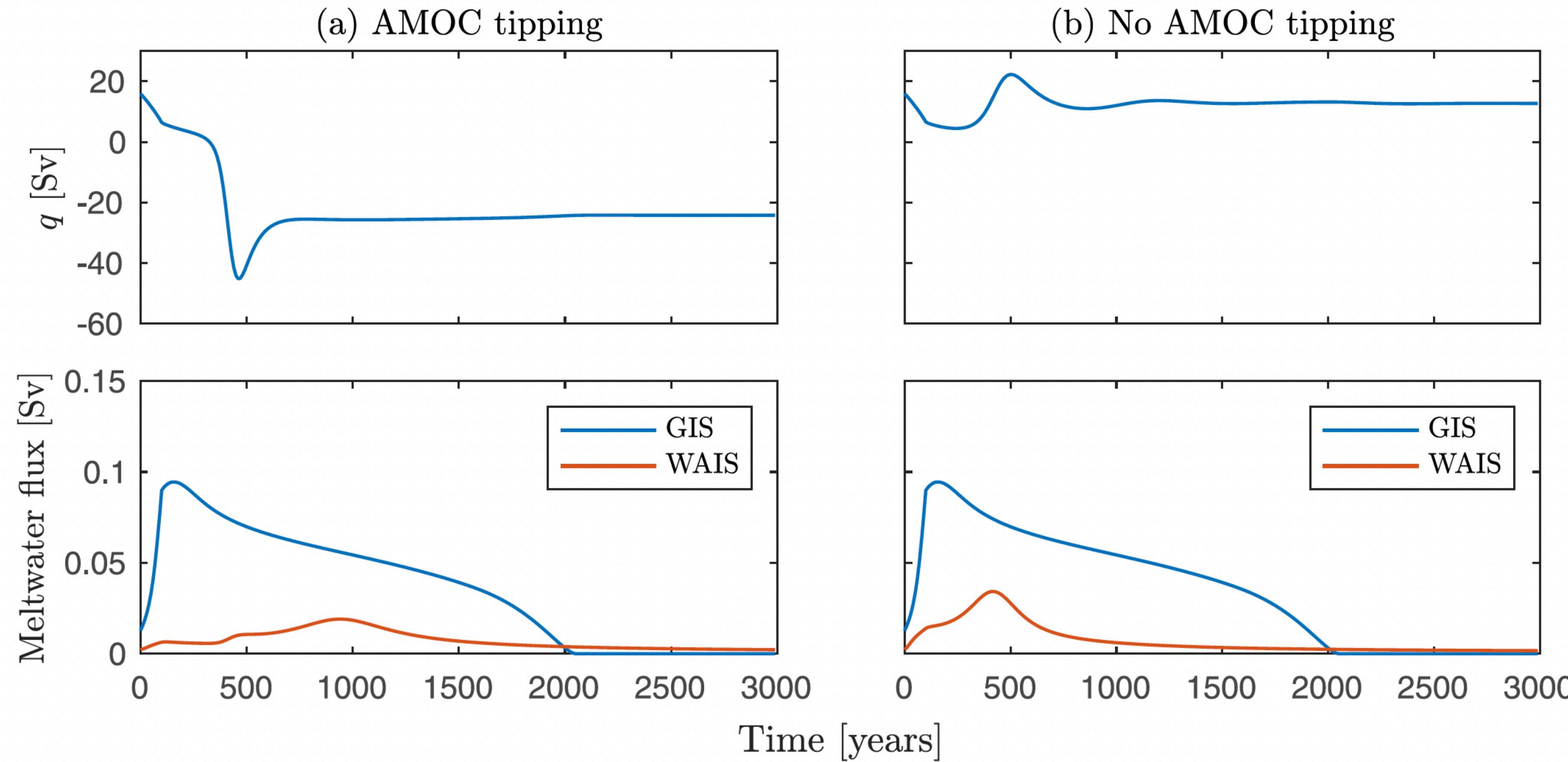


$$\bar{A}(\Delta T_3) = \frac{\bar{A}^0}{T_3^0} [T_3^0 + c_S \Delta T_3].$$

$$\tau_j(t) = \tau_j(0) + \gamma_j \frac{\Delta \tau_2}{100} t.$$



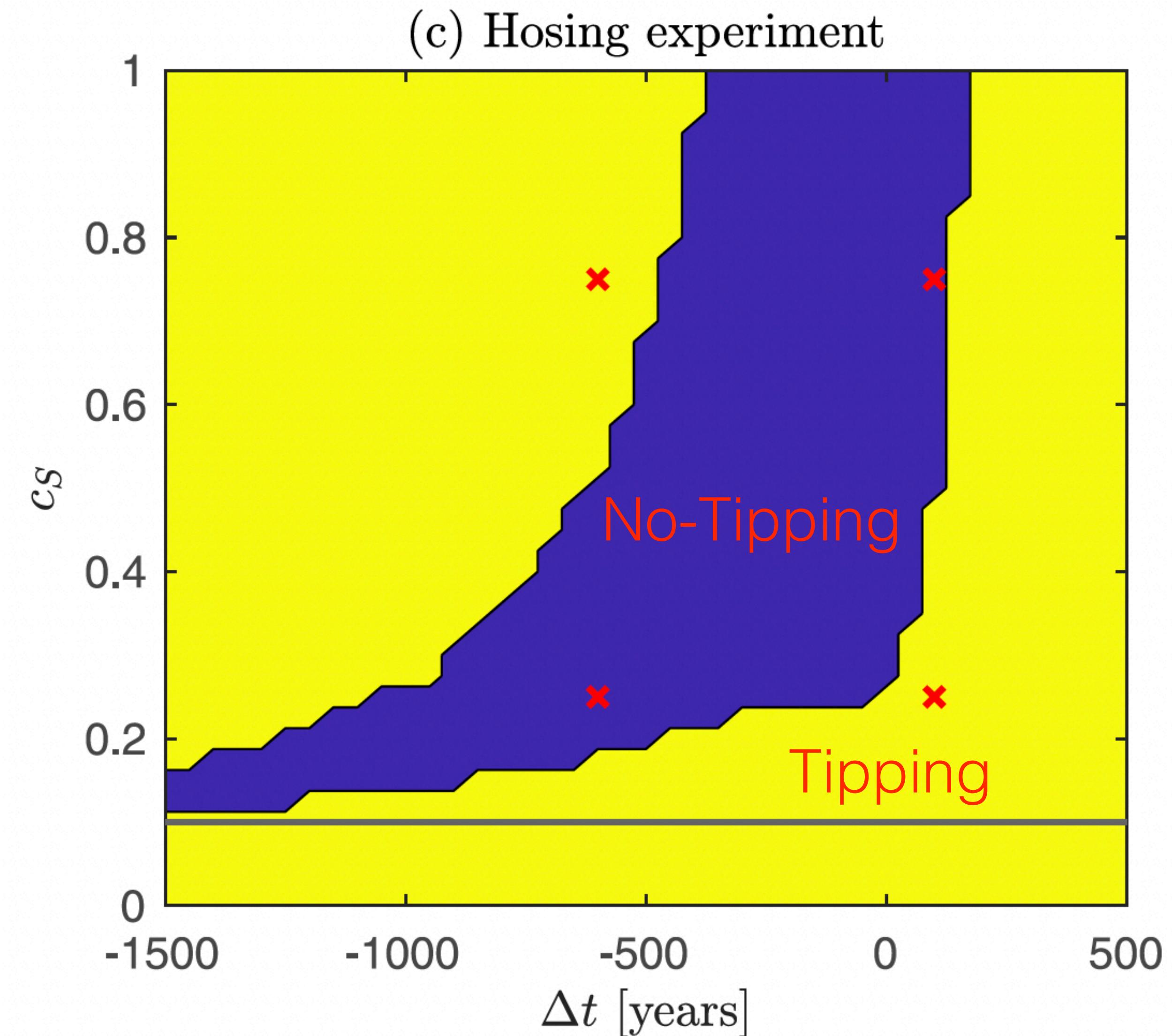
Result



(a) $c_s = 0.2$ and (b) $c_s = 0.8$.



AMOC stabilisation



Time delay between GIS and WAIS tipping

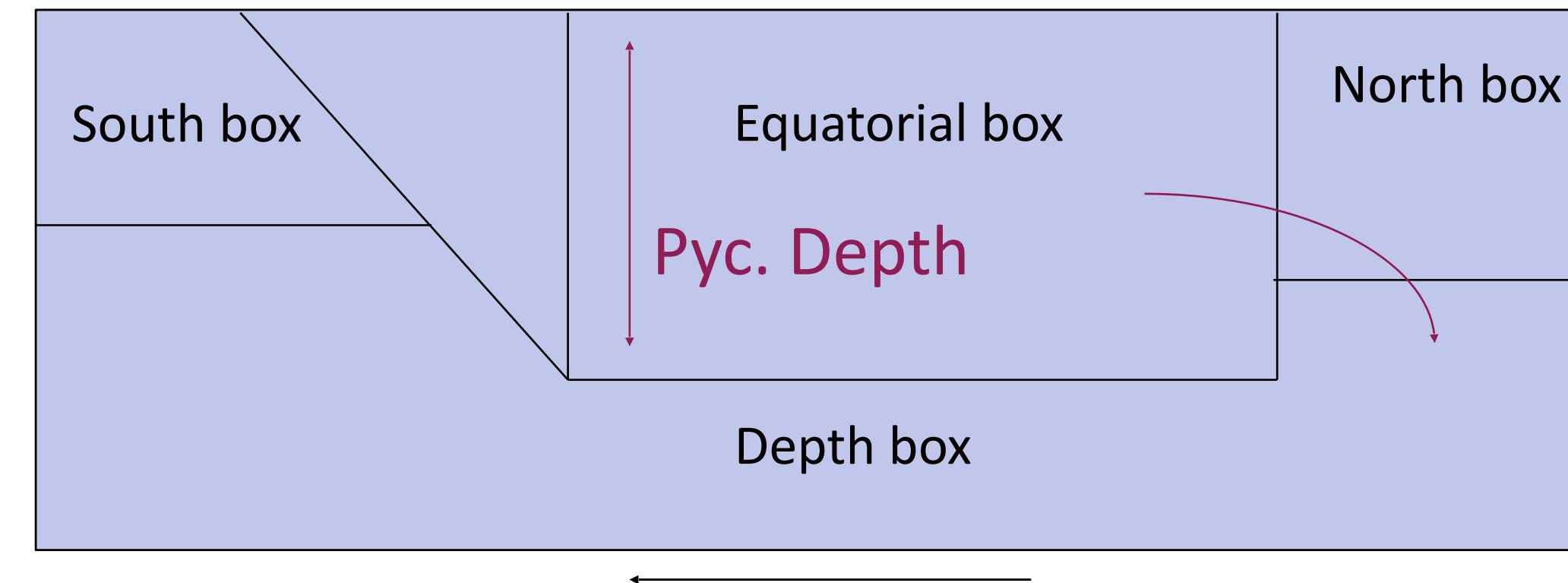
Detailed Explanation

<https://doi.org/10.5194/egusphere-2023-2661>
Preprint. Discussion started: 6 December 2023
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AMOC Stability Amid Tipping Ice Sheets: The Crucial Role of Rate and Noise

Sacha Sinet^{1,2}, Peter Ashwin³, Anna S. von der Heydt^{1,2}, and Henk A. Dijkstra^{1,2}



I. No warming

II. Better AMOC model

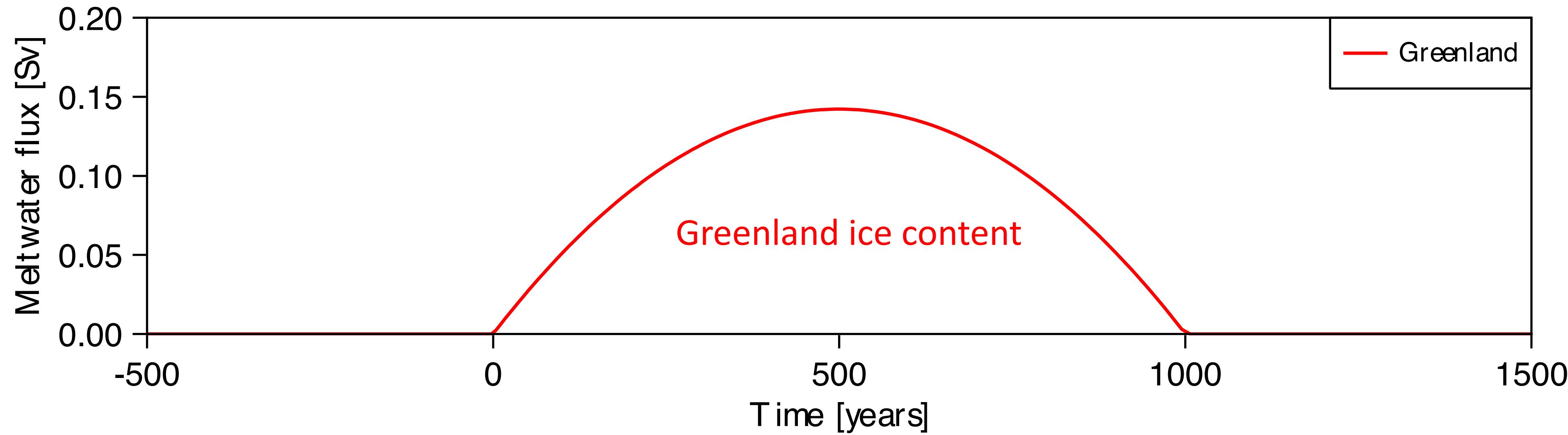
Cimatoribus & al. (2014)

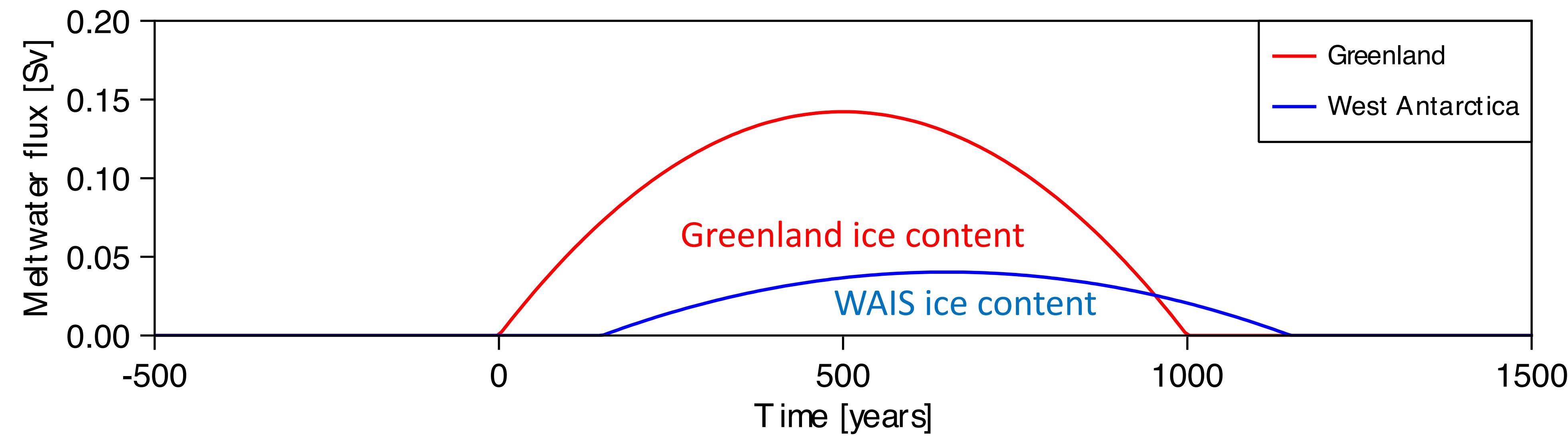
III. No dynamical Ice Sheet

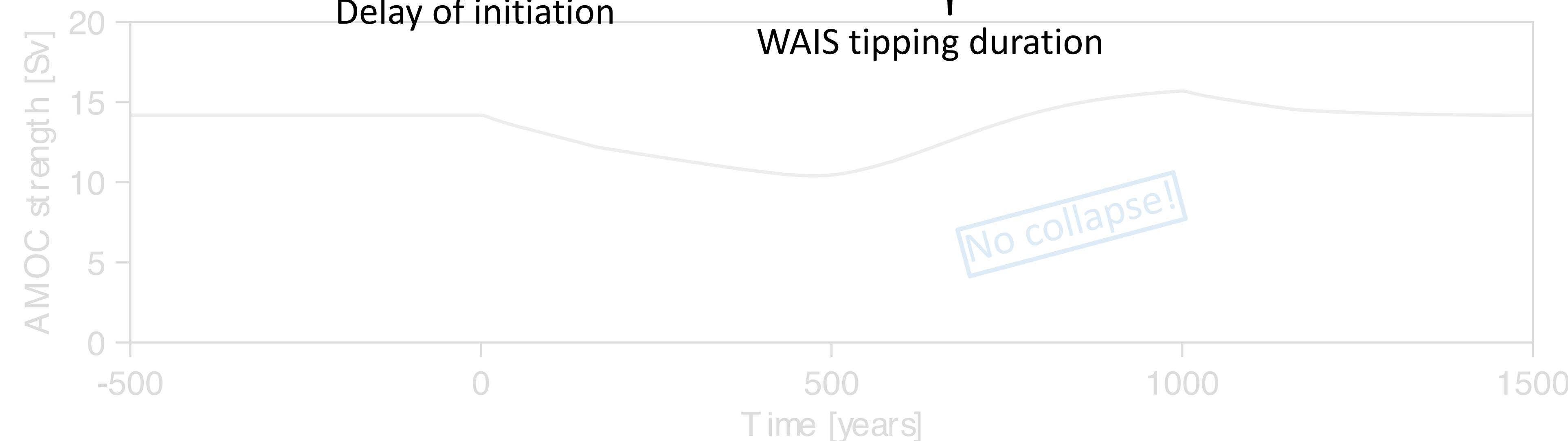
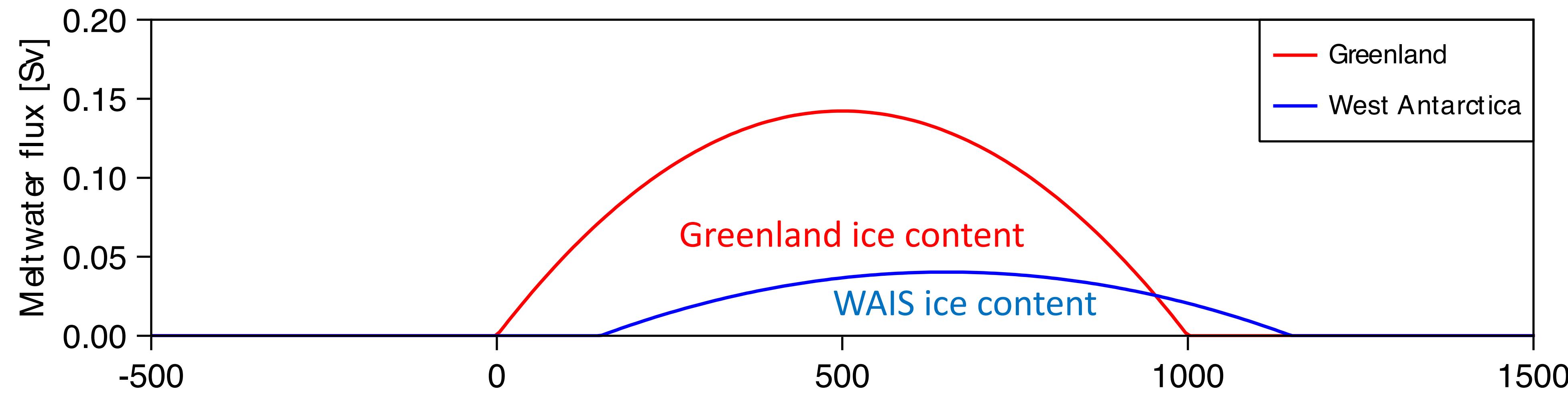
Only parametrization

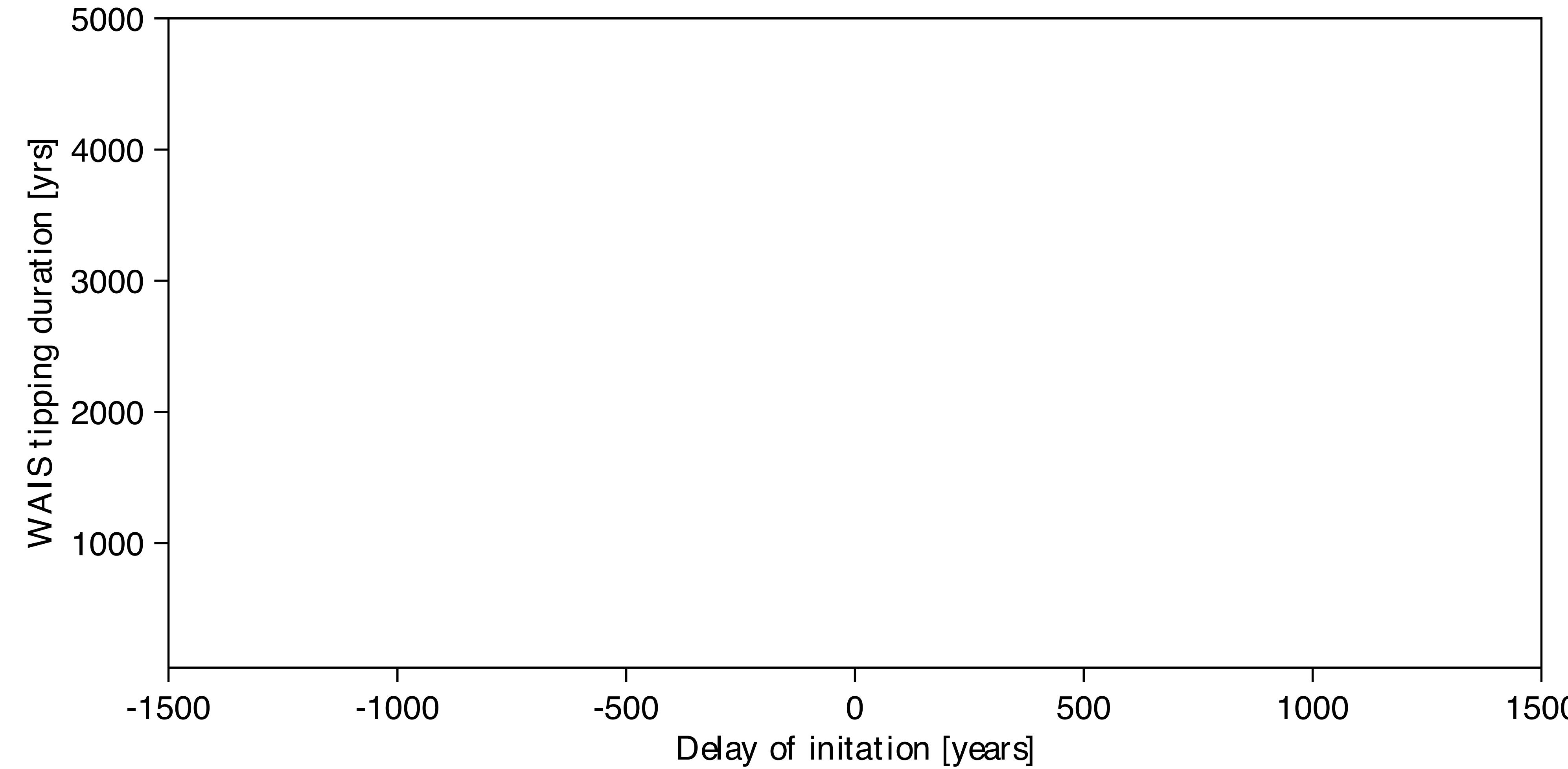
$$F_N(t; P_N) = \begin{cases} -\frac{6V_N}{P_N^3}t(t - P_N) & \text{if } 0 < t < P_N, \\ 0 & \text{otherwise,} \end{cases}$$

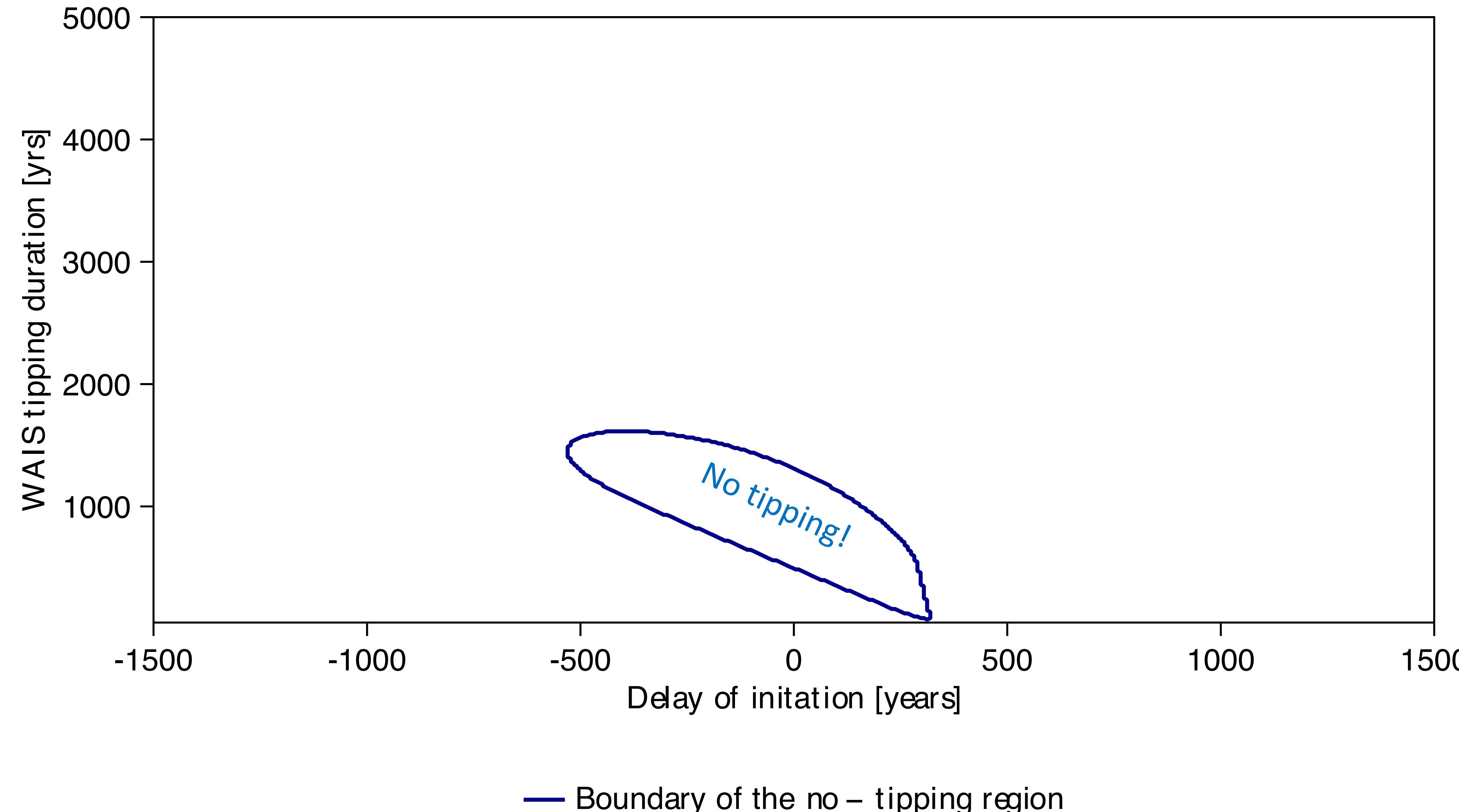
$$F_S(t; P_S, \Delta t) = \begin{cases} -\frac{1}{4} \frac{6V_S}{P_S^3}(t - \Delta t)(t - \Delta t - P_S) & \text{if } 0 < t - \Delta t < P_S, \\ 0 & \text{otherwise,} \end{cases}$$

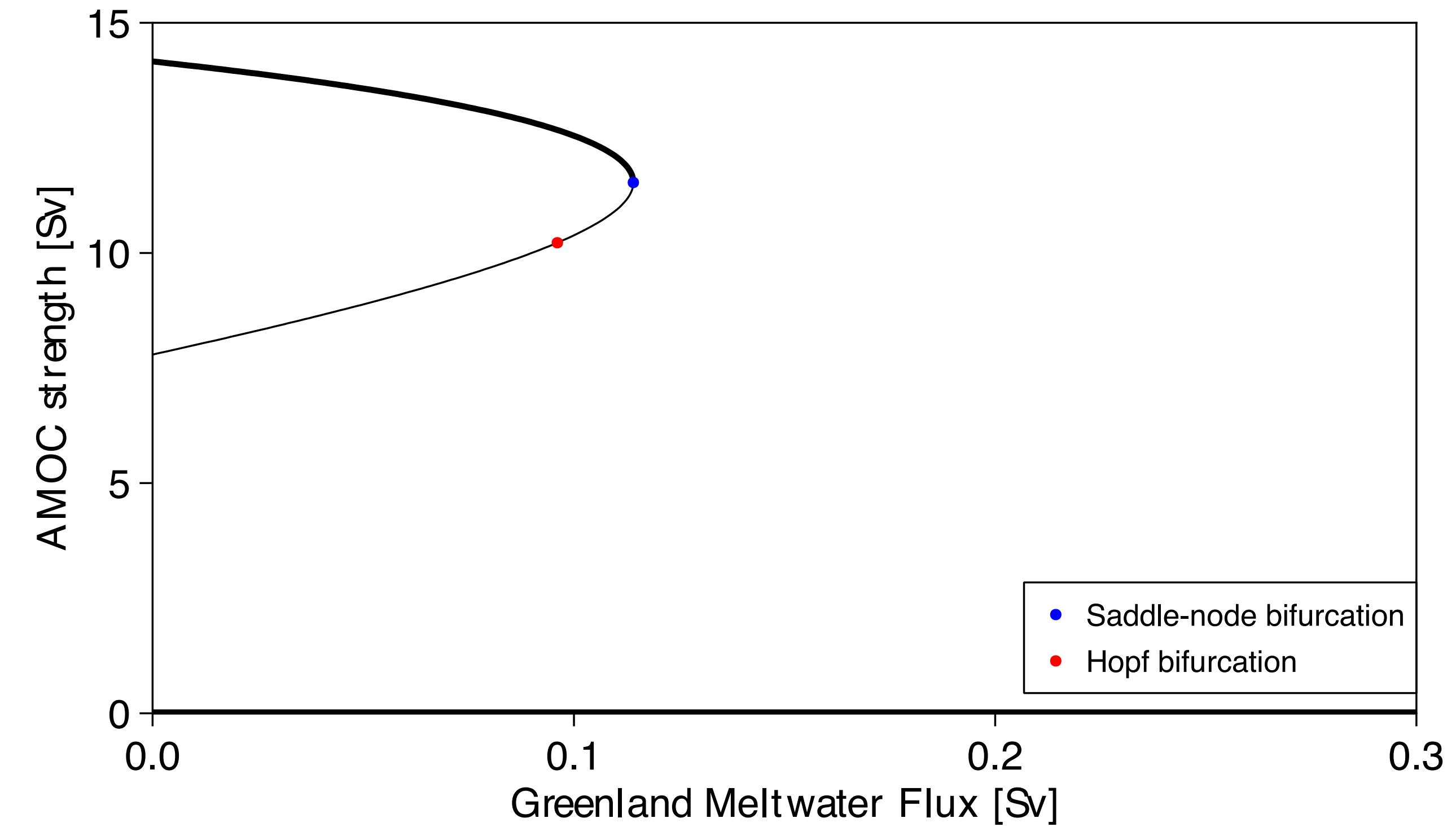
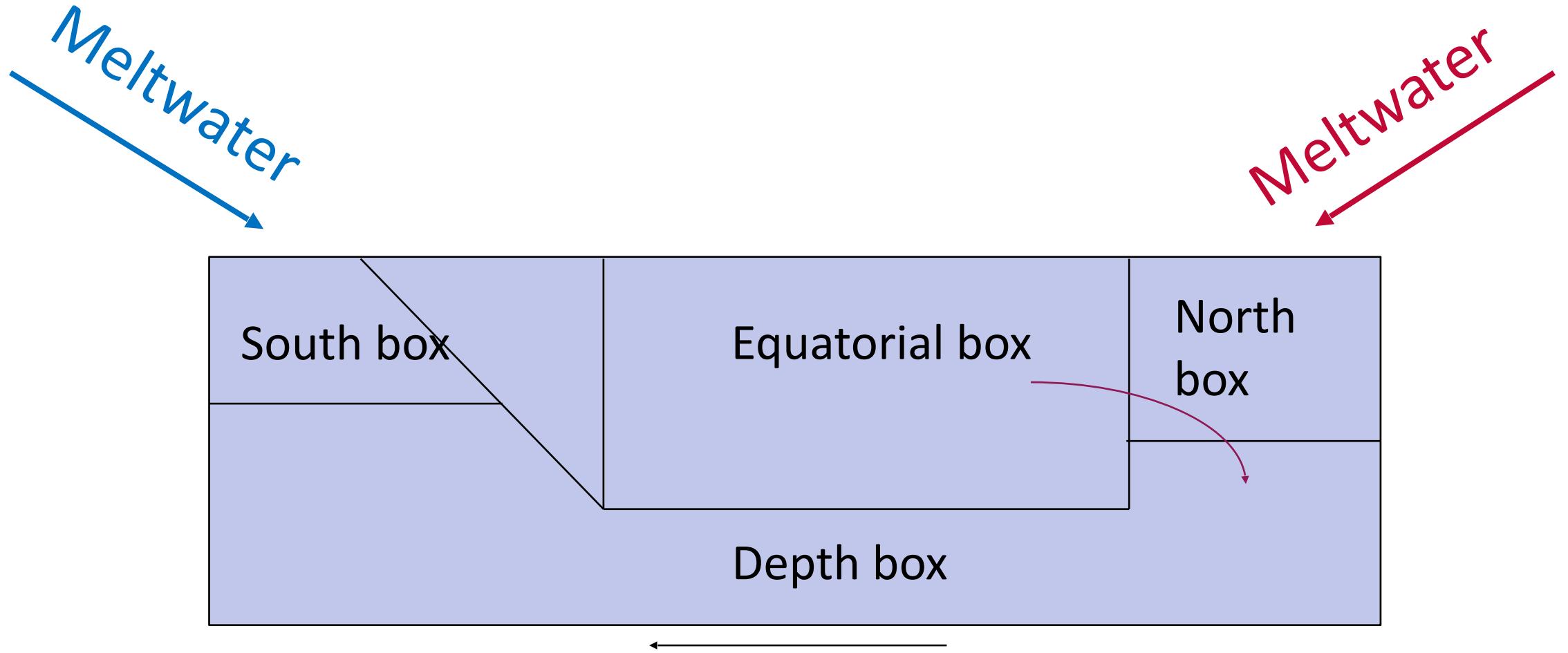


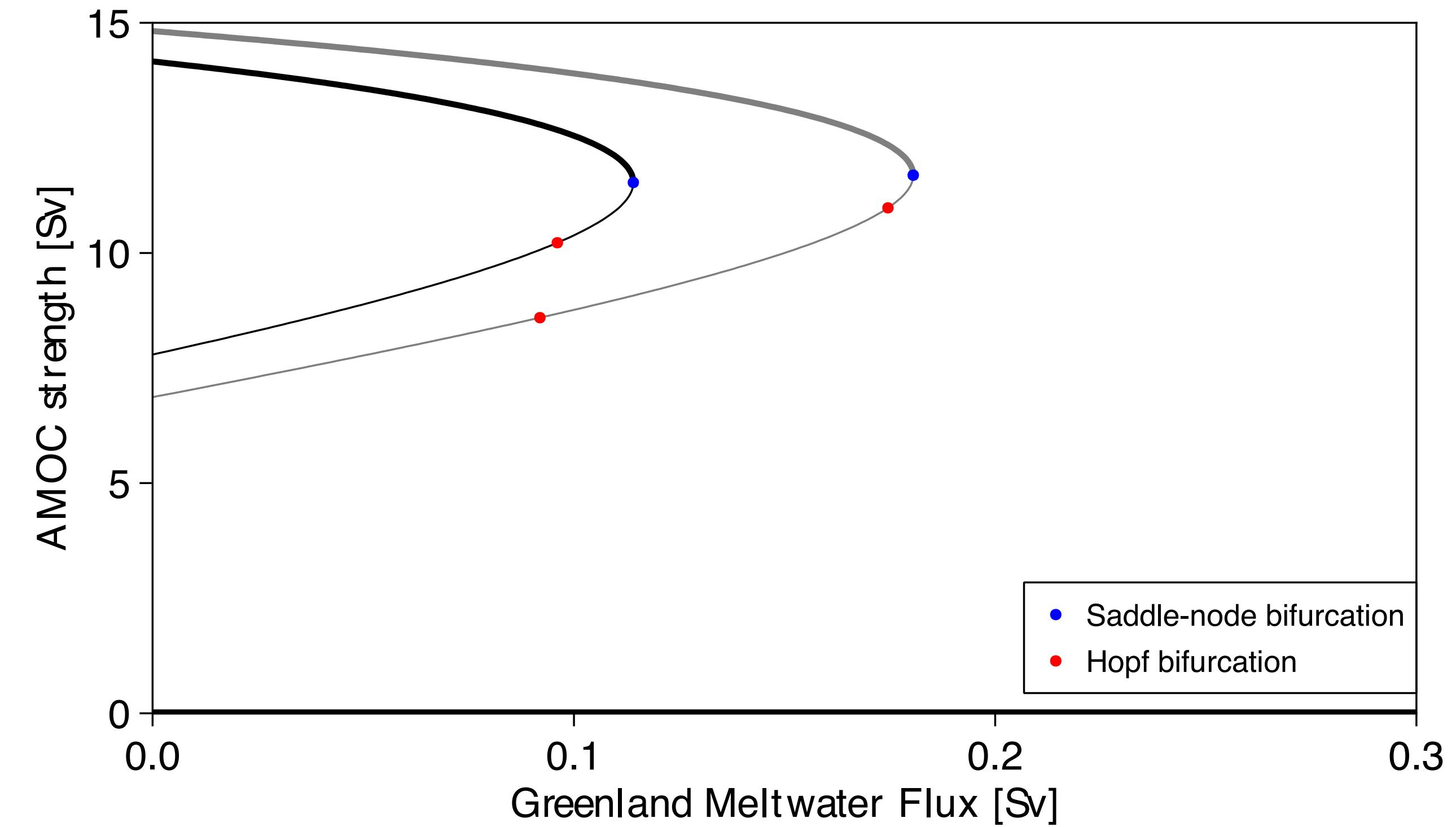
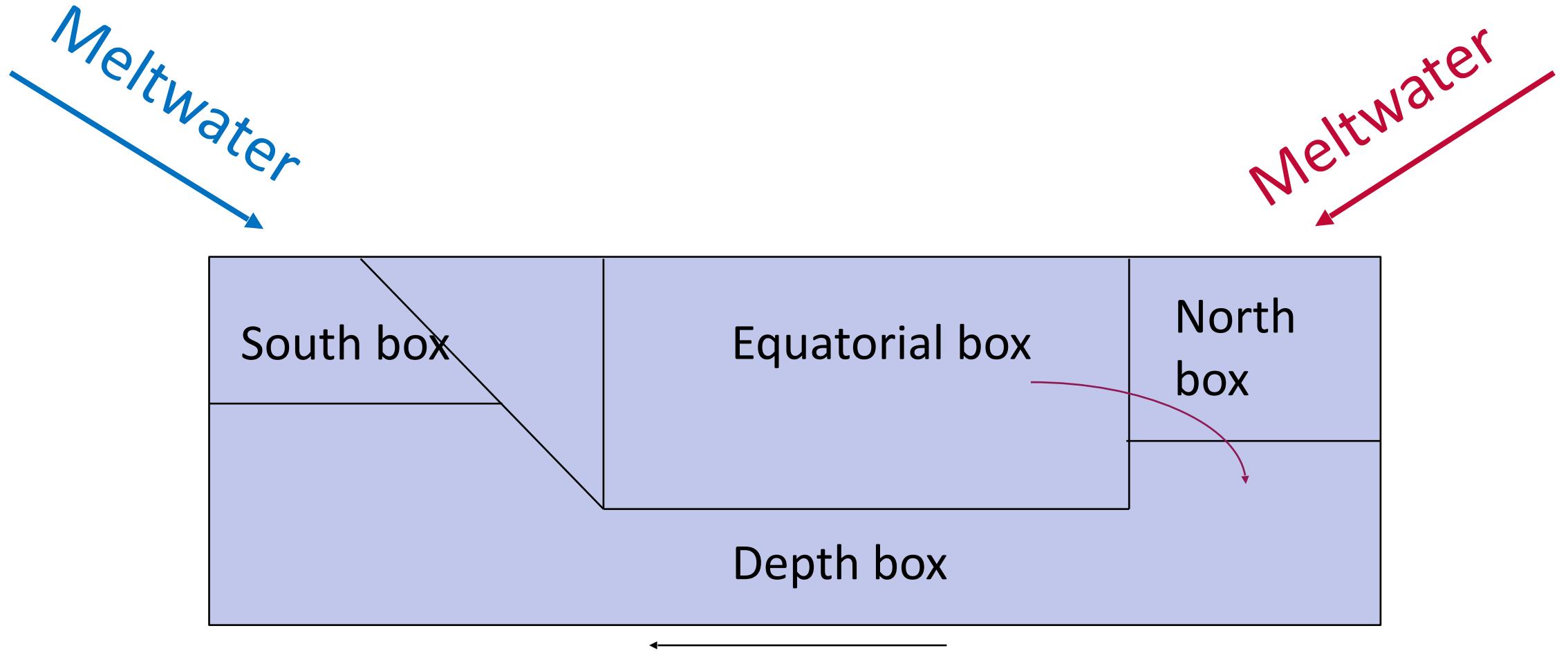


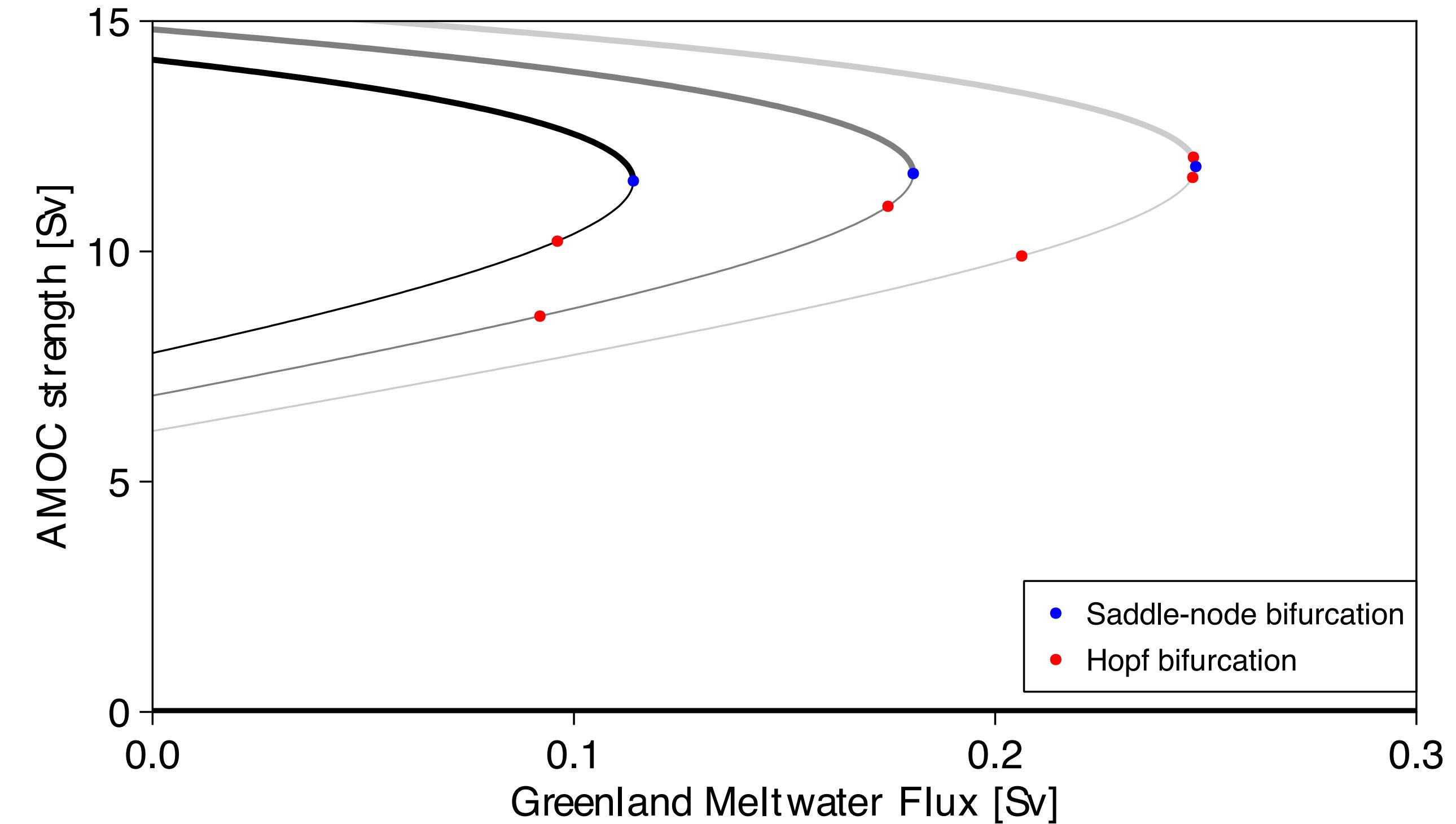
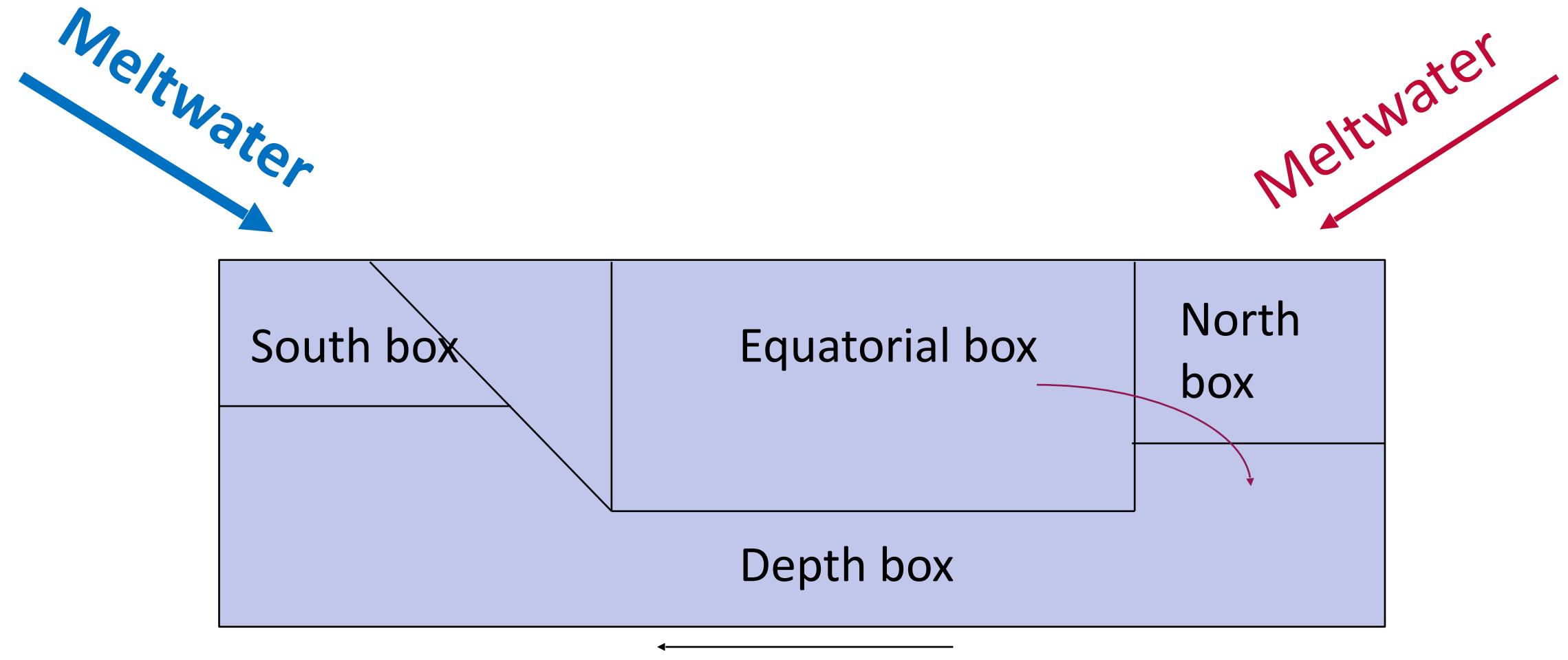








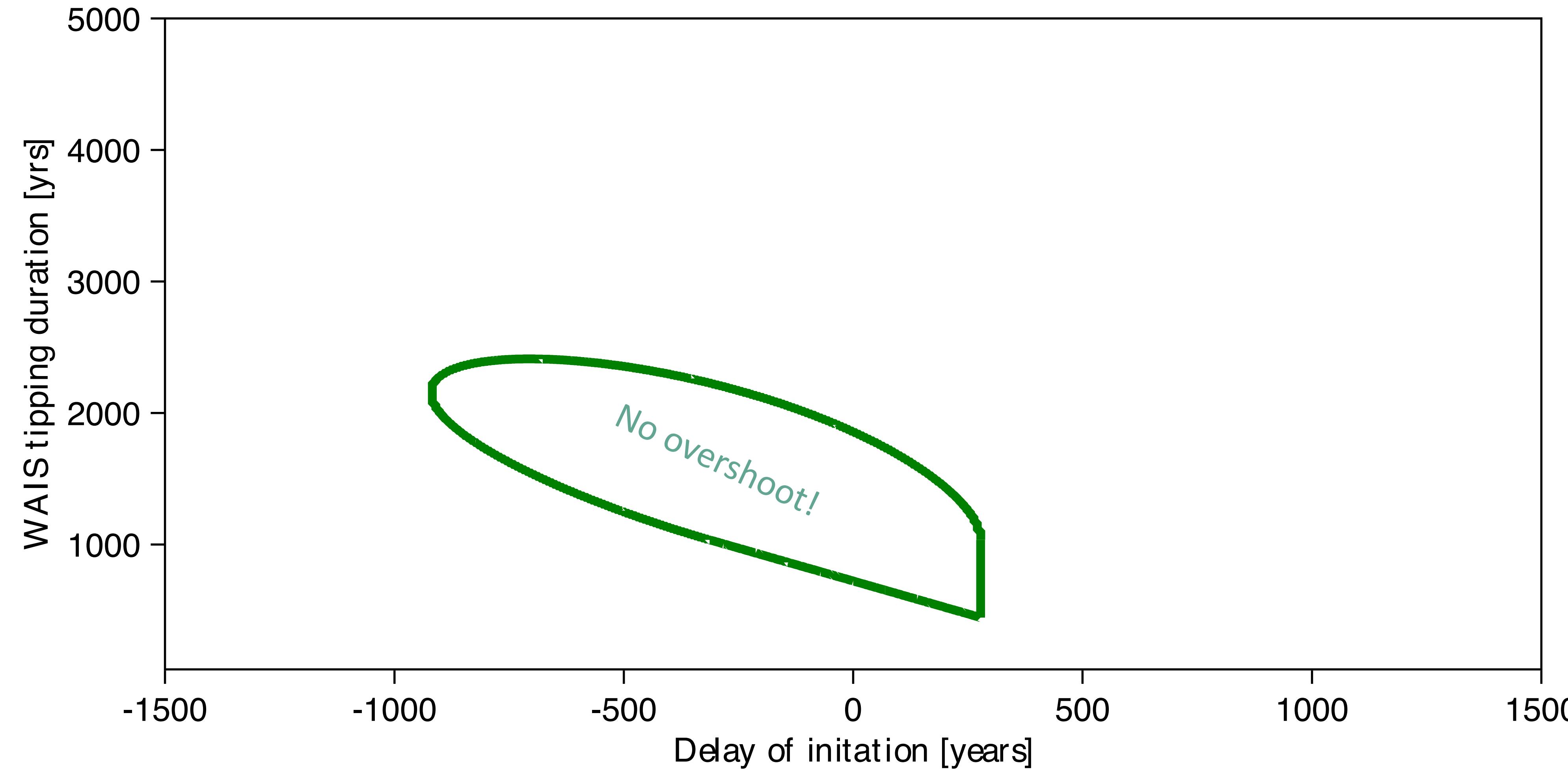




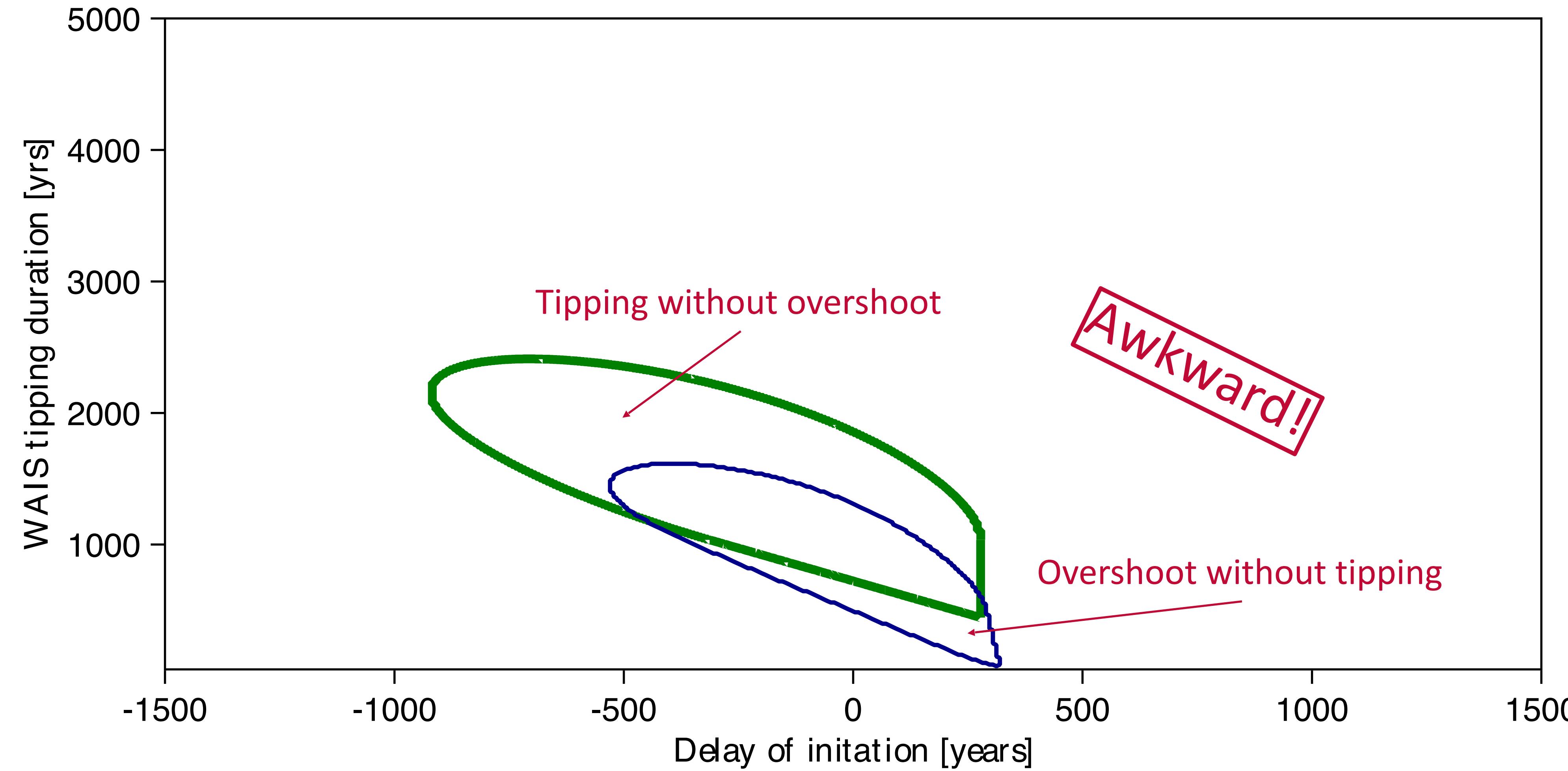
$$F_N(t) - \delta F_S(t) < F_{N,c},$$

No overshoot of saddle-node if at all time:

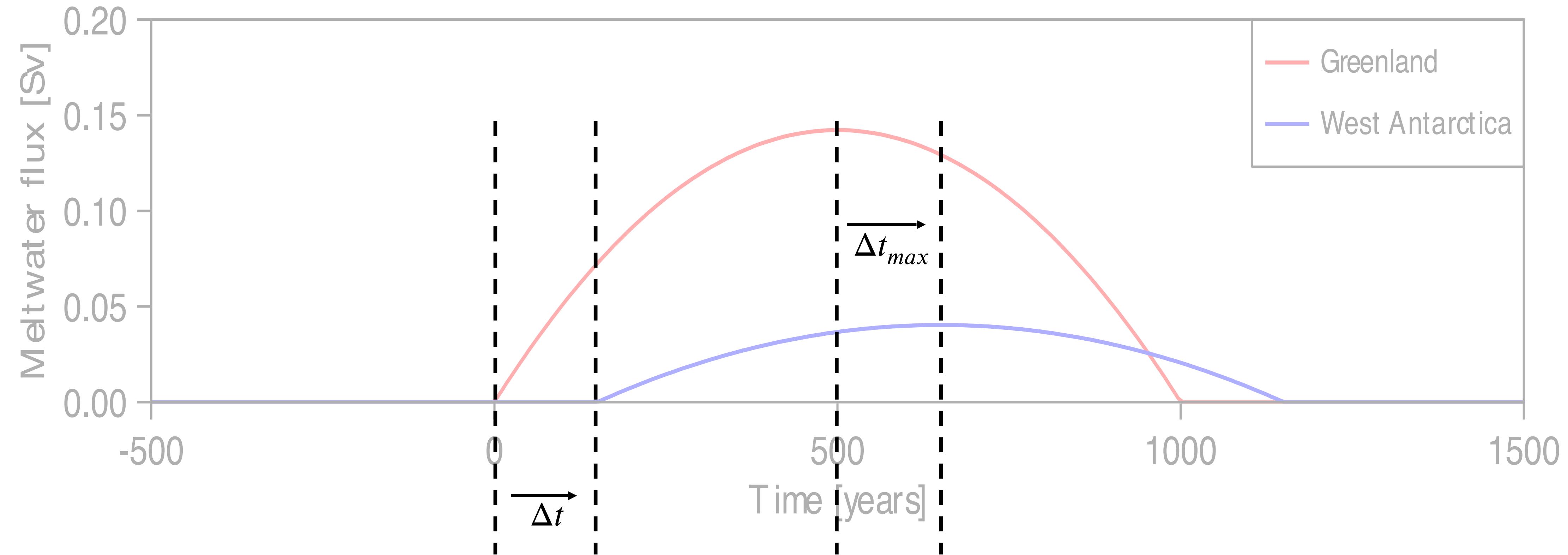
$$F_{N,c} = 0.11 ; \delta = 1.68$$

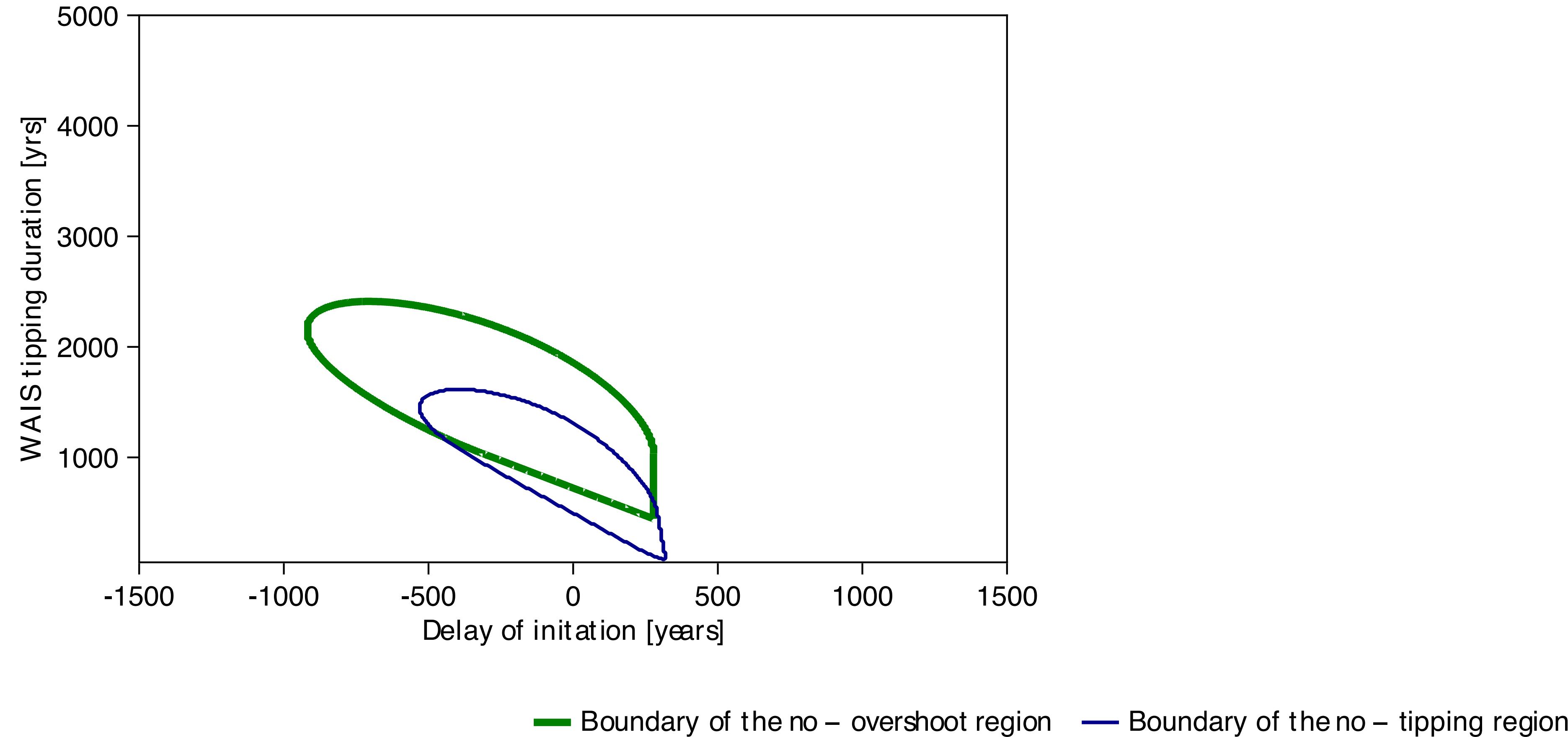


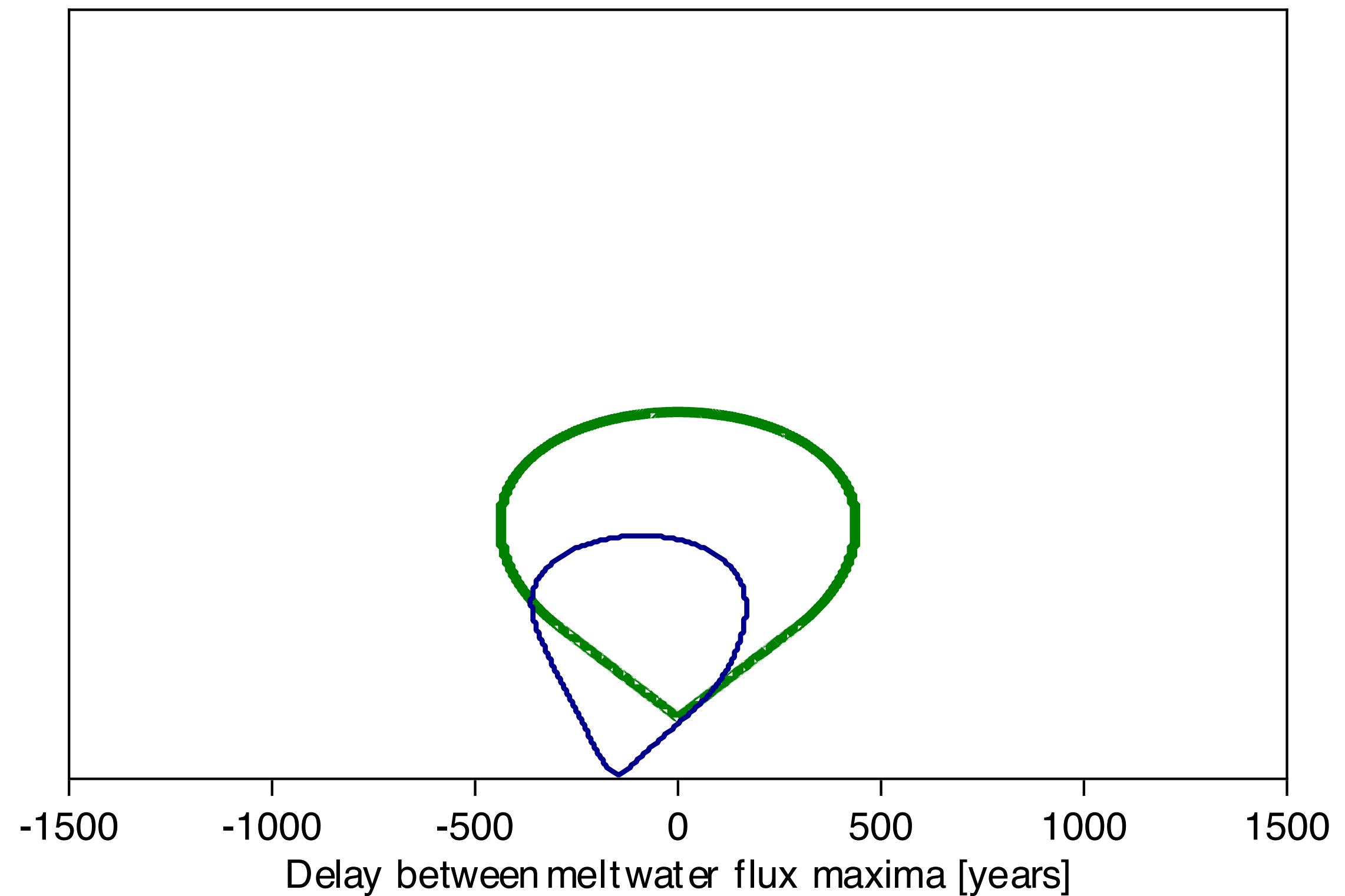
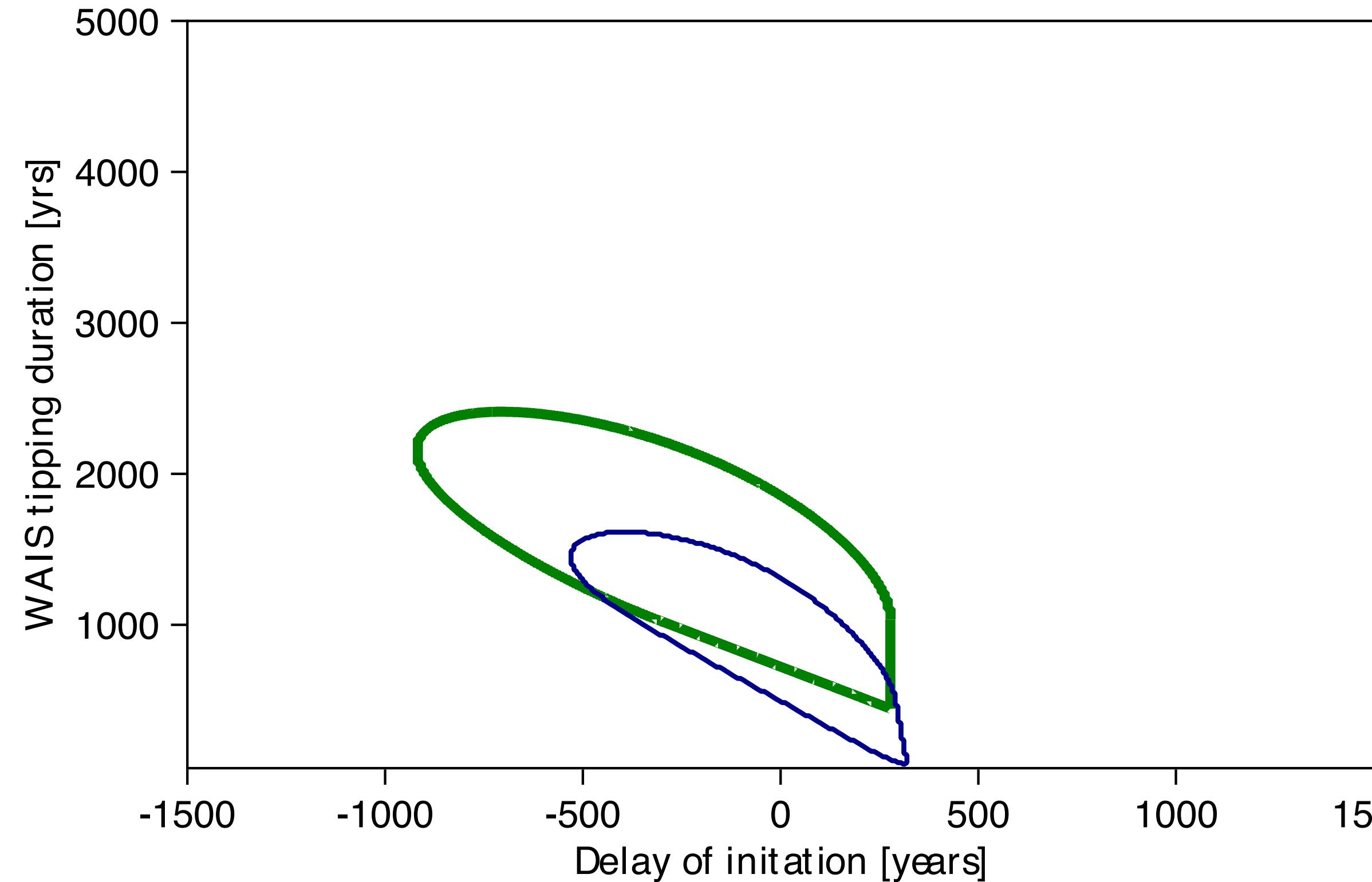
— Boundary of the no – overshoot region — Boundary of the no – tipping region



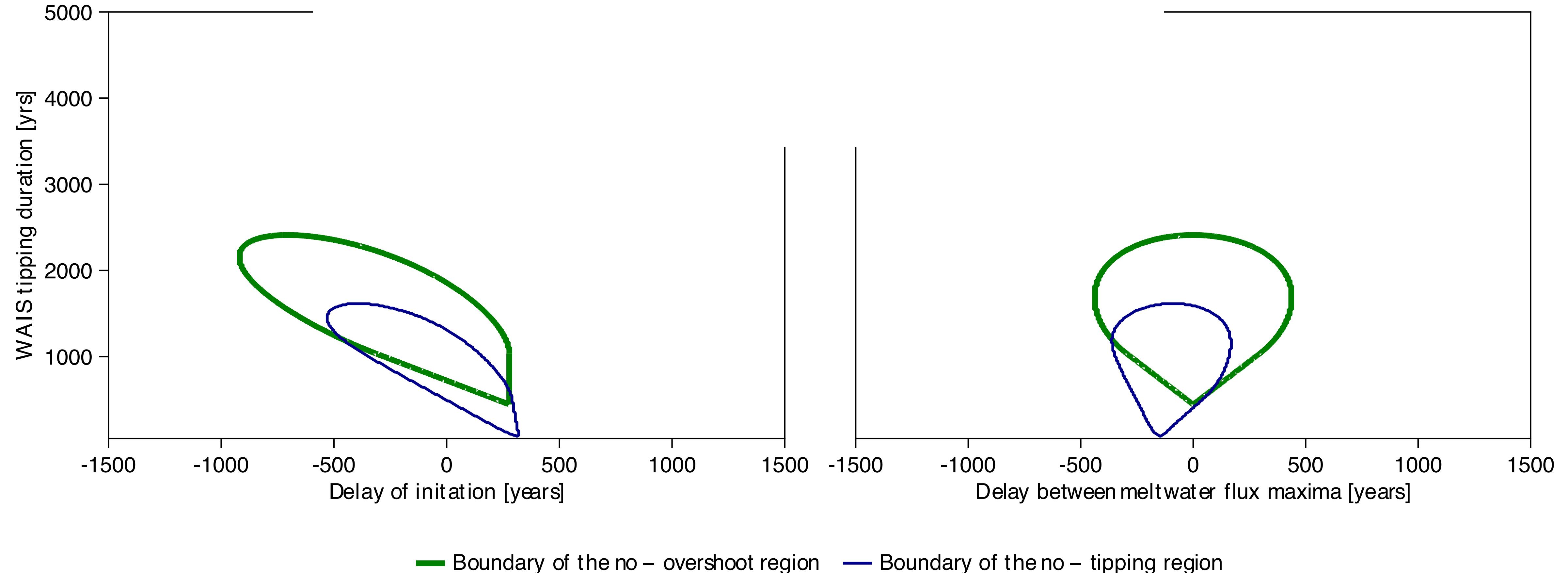
— Boundary of the no – overshoot region — Boundary of the no – tipping region



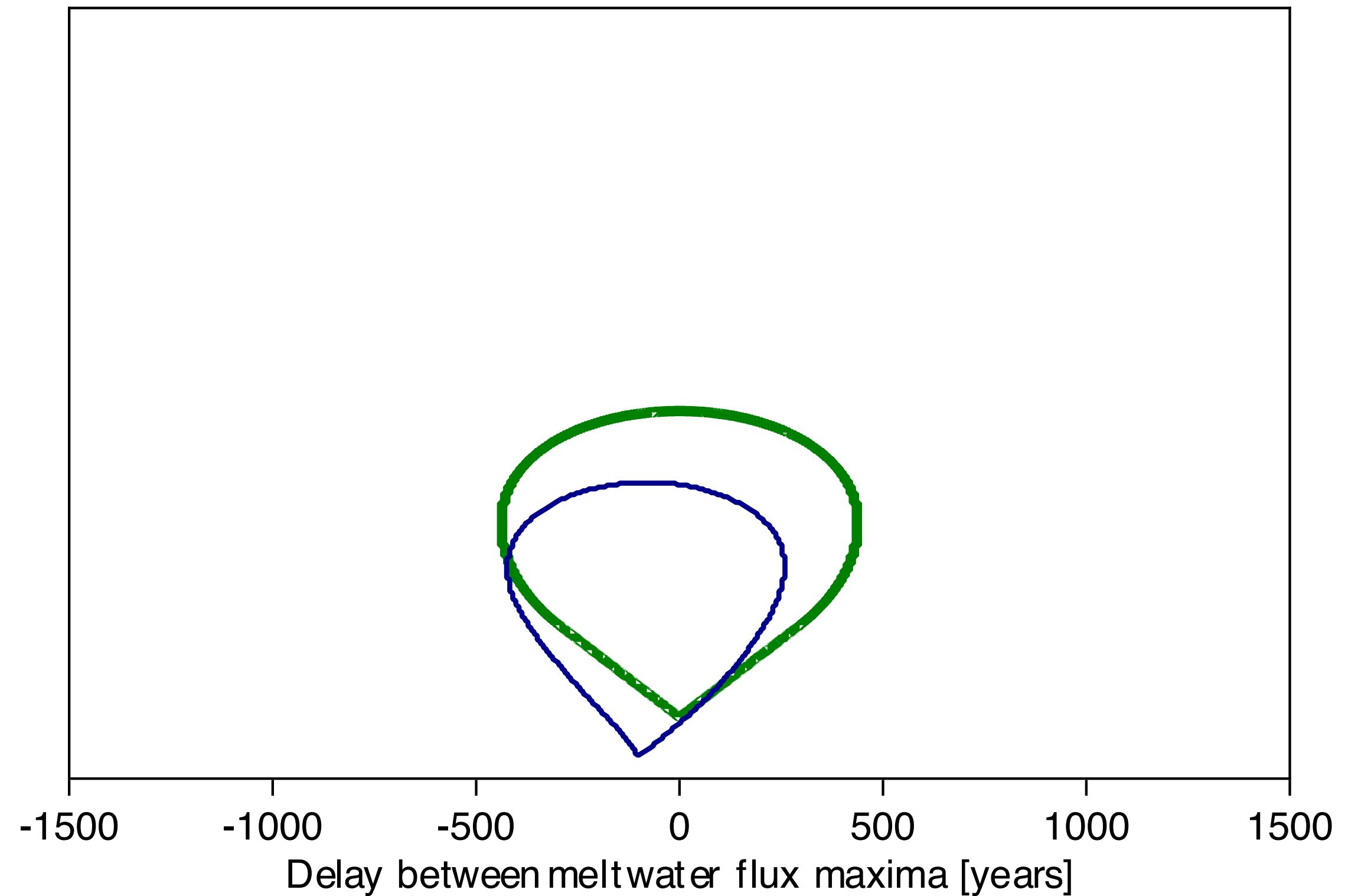
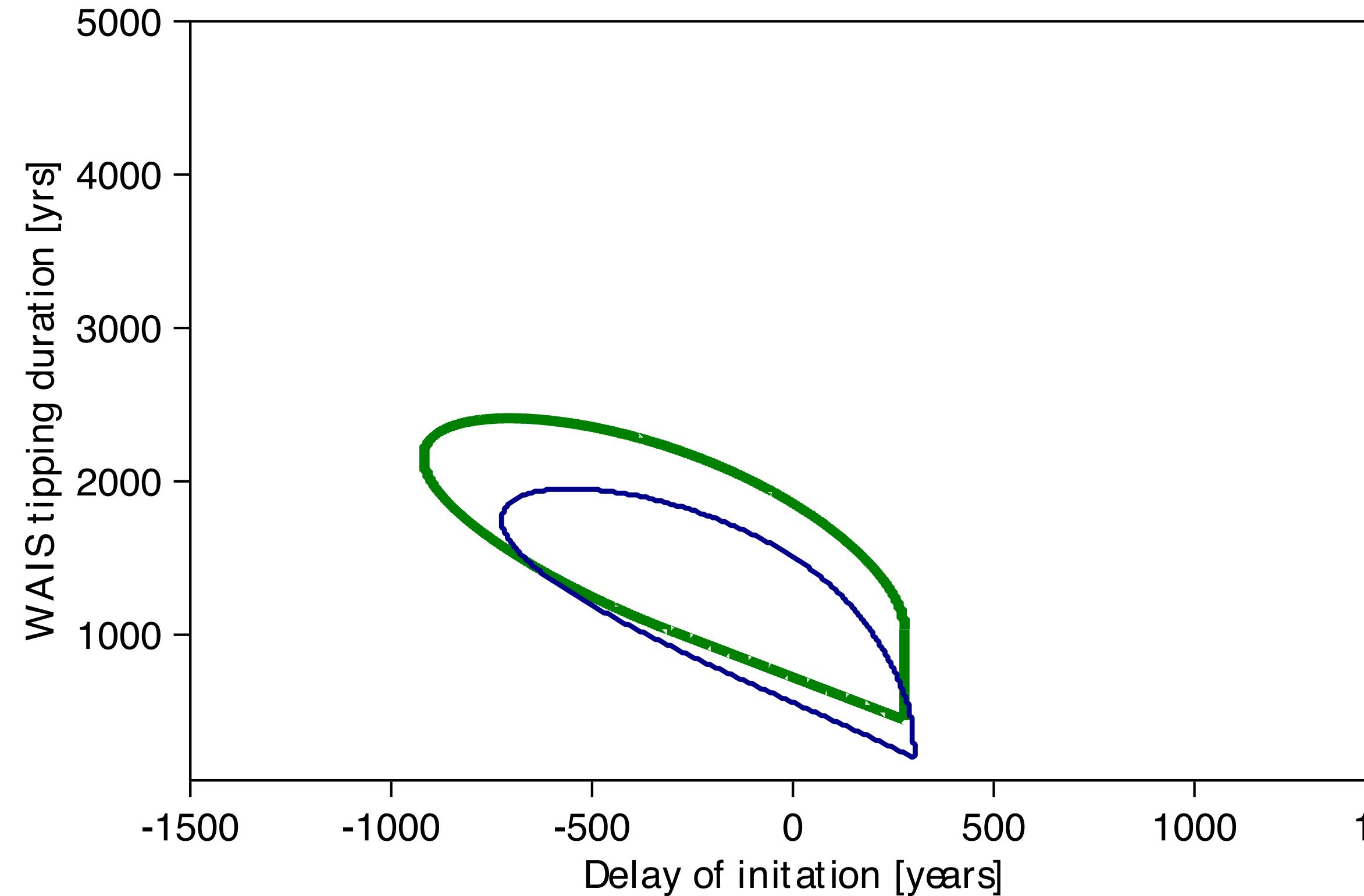




— Boundary of the no – overshoot region — Boundary of the no – tipping region



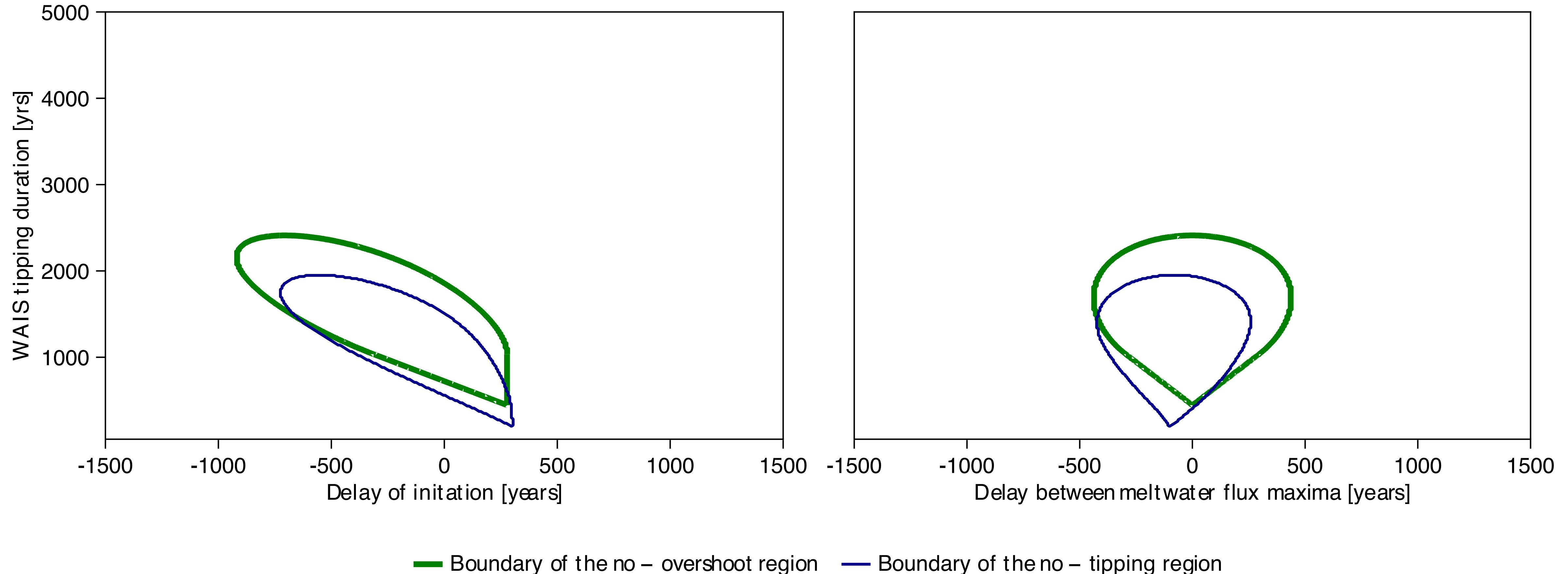
Rate-induced Effects



— Boundary of the no – overshoot region — Boundary of the no – tipping region

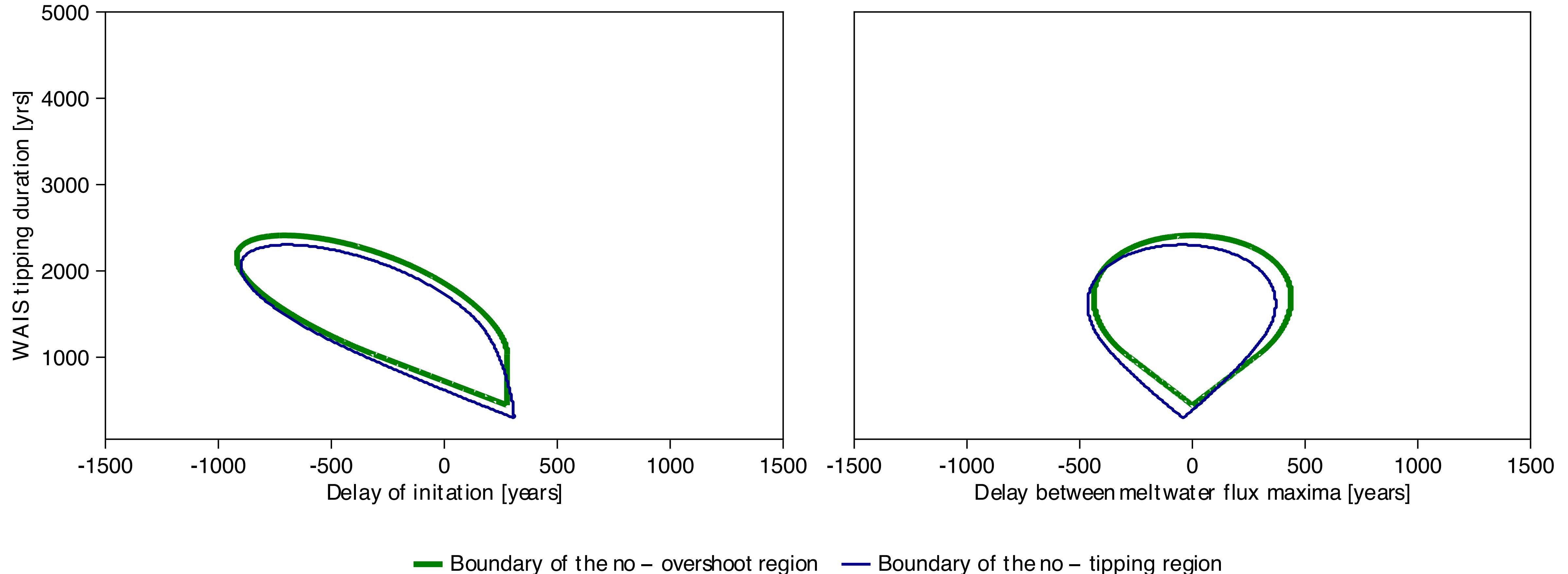
Let's slow down forcing rate

Forcing(t) → Forcing ($t/2$)



Let's slow down forcing rate

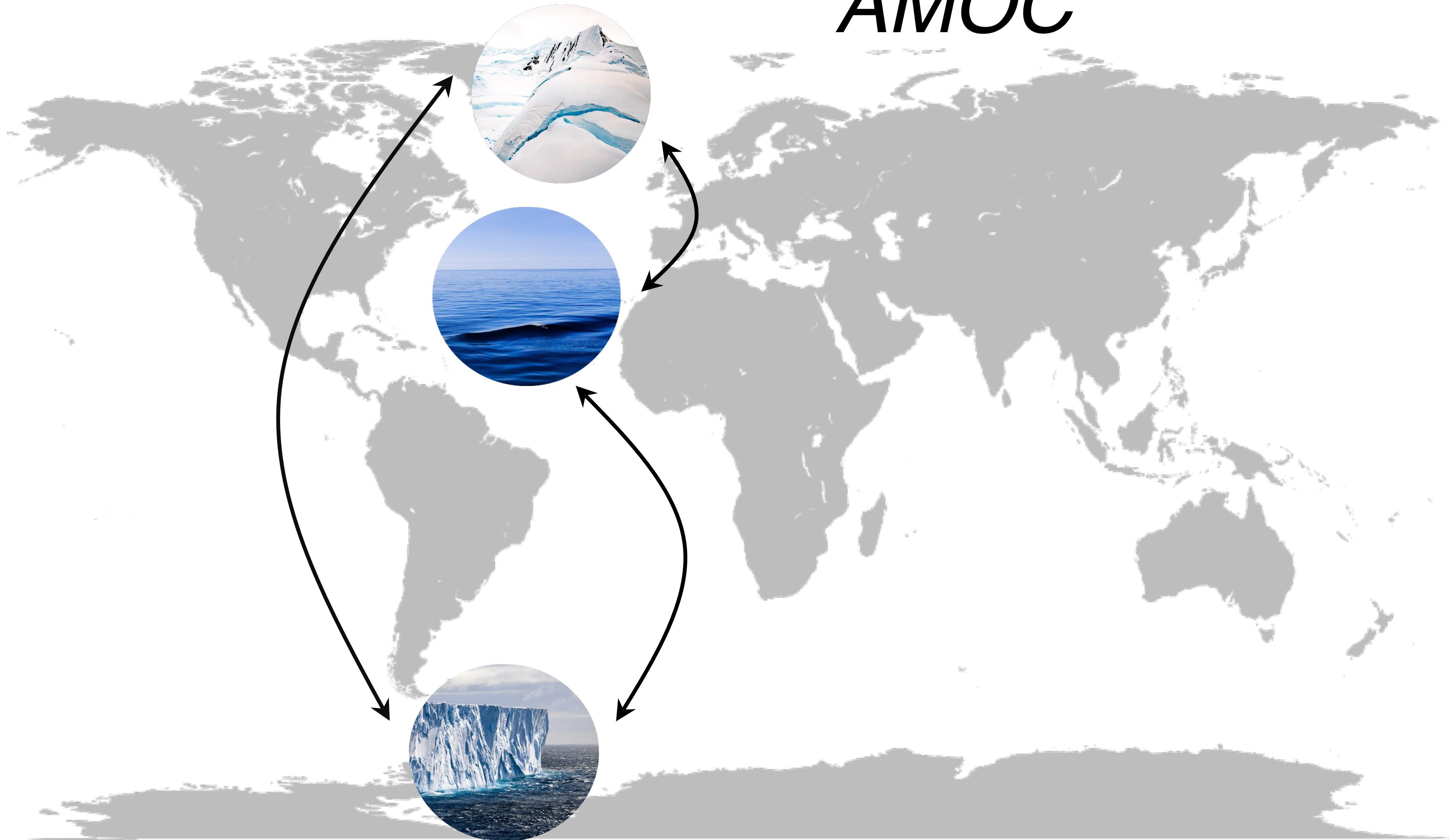
Forcing(t) \rightarrow Forcing ($t/4$)



Let's slow down forcing rate

Forcing(t) \rightarrow Forcing ($t/4$)

Main point: WAIS collapse can stabilize the AMOC





Exercises!

