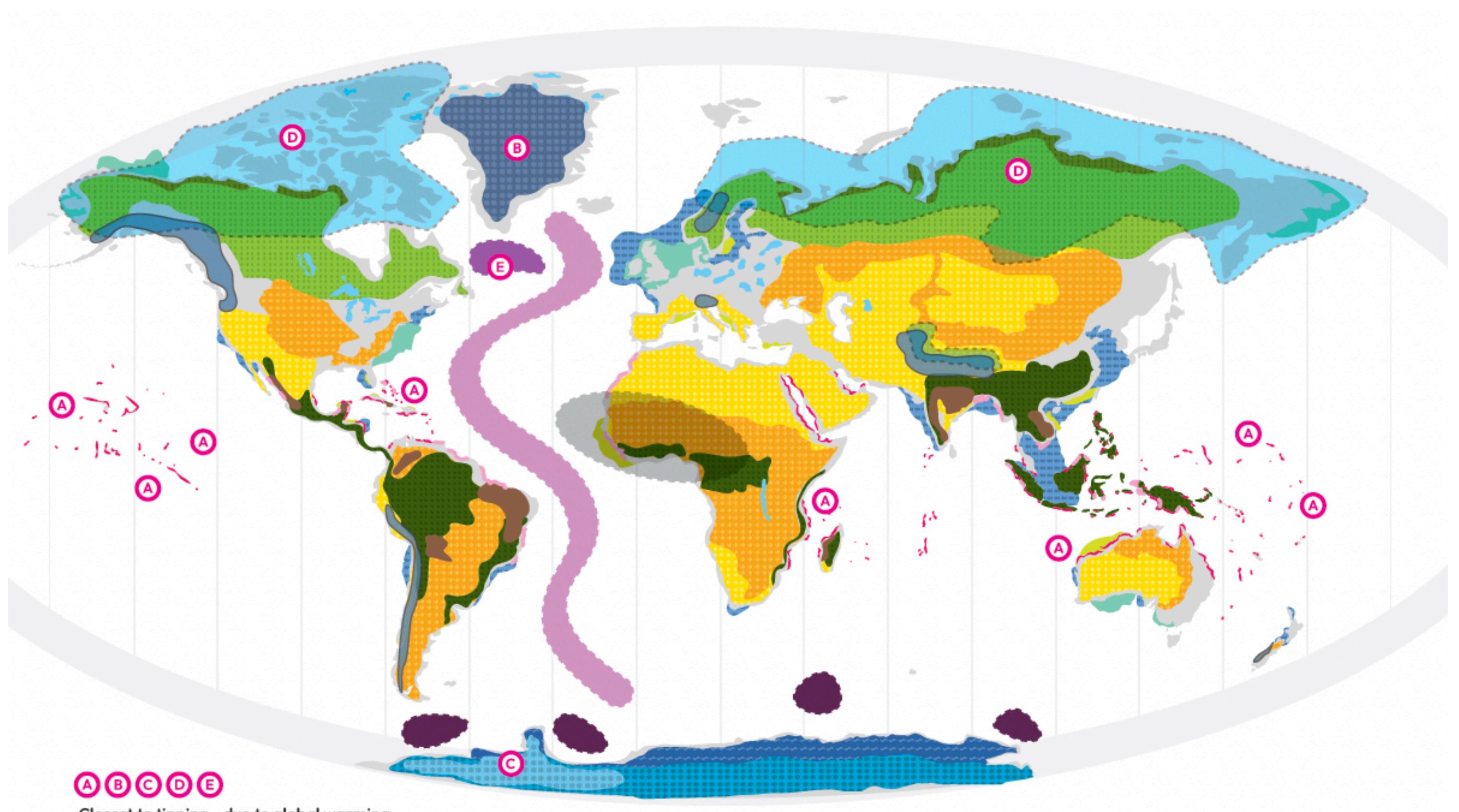


# Tipping Behavior in the Climate System



Henk Dijkstra,  
Department of Physics,  
Utrecht University  
&  
DICAM,  
University of Trento

# Overview of Course

Date	10:00-10:45	11:00-11:45	12:00-12:45
Mo 5-2	General Intro Tipping Elements Lenton et al., Section 0 + 1.1	Dynamical Systems + Bifurcation induced TB Strogatz, Chapters 1 + 2	Exercises
We 7-2	Oceans Lenton et al., Section 1.4	Noise induced TB Dijkstra, Chapter 10	Exercises
Fr. 9-2	Cryosphere Lenton et al., Section 1.2	Early Warning Signals of TB Lenton, Section 0 + Dijkstra, Chapter 5	Exercises
Mo 12-2	Atmosphere Lenton et al., Section 1.4	Overshoot + Rate induced TB Lenton, Section 1.6	Exercises
We 14-2	Climate Impact of TB Lenton et al., Section 1.5	Cascading TB Lenton, Section 1.5	Exercises + Question Time
Fr 16-2	Exam	Exam	Exam

1. T. M. Lenton et al. (eds), 2023, The Global Tipping Points Report 2023. University of Exeter, Exeter, UK.
2. Strogatz, S, 2015, Nonlinear Dynamics and Chaos, Westview Press, USA.
3. Dijkstra, H.A., 2013, Nonlinear Climate Dynamics, Cambridge University Press, UK.

Grading: Exam (100%)



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# Material



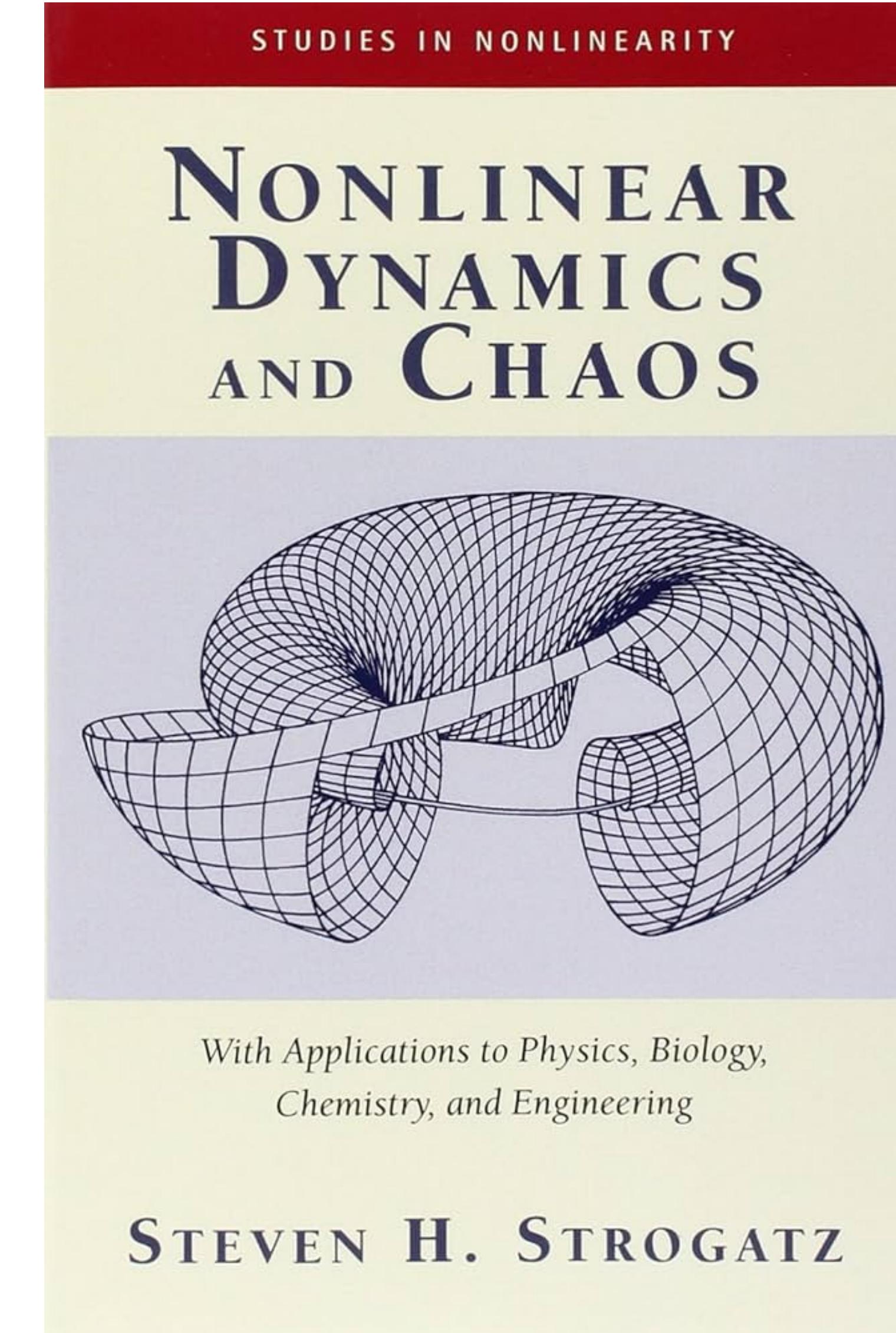
## Global Tipping Points

Report 2023

Led by:



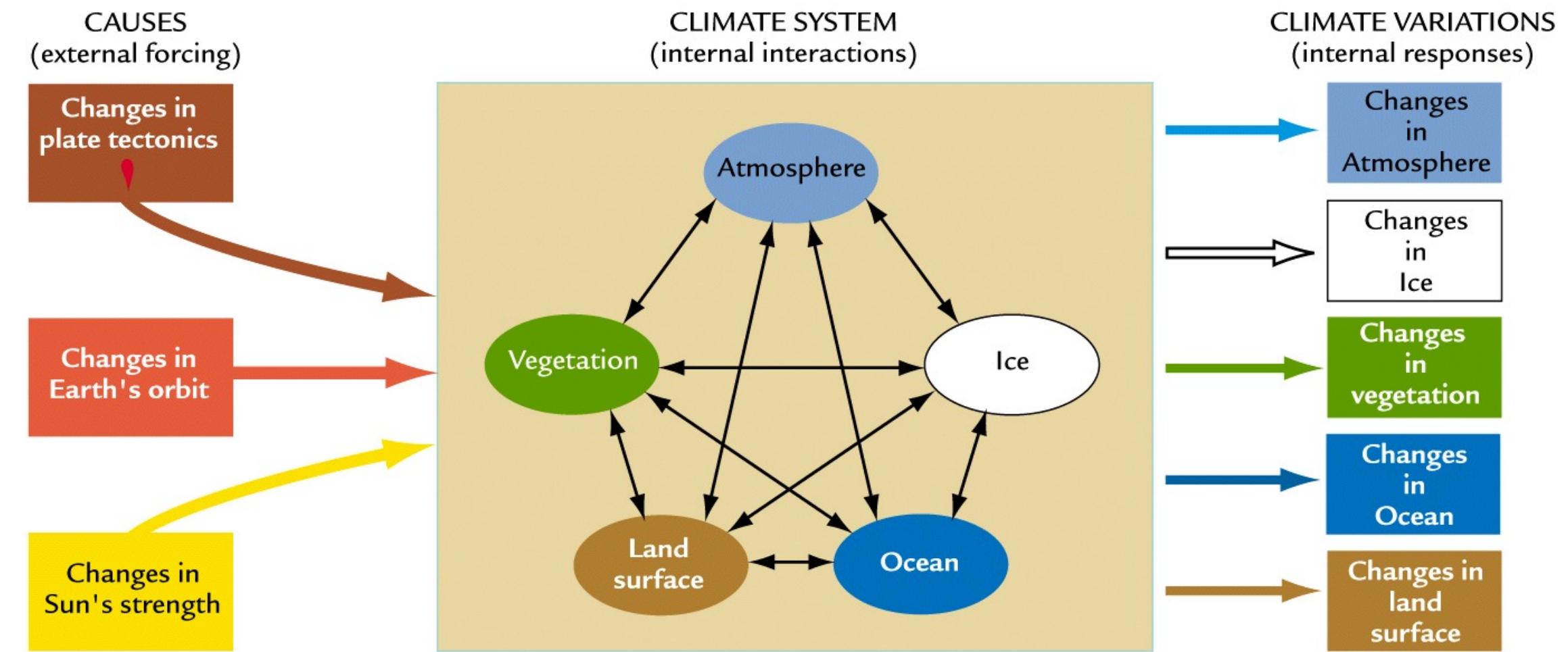
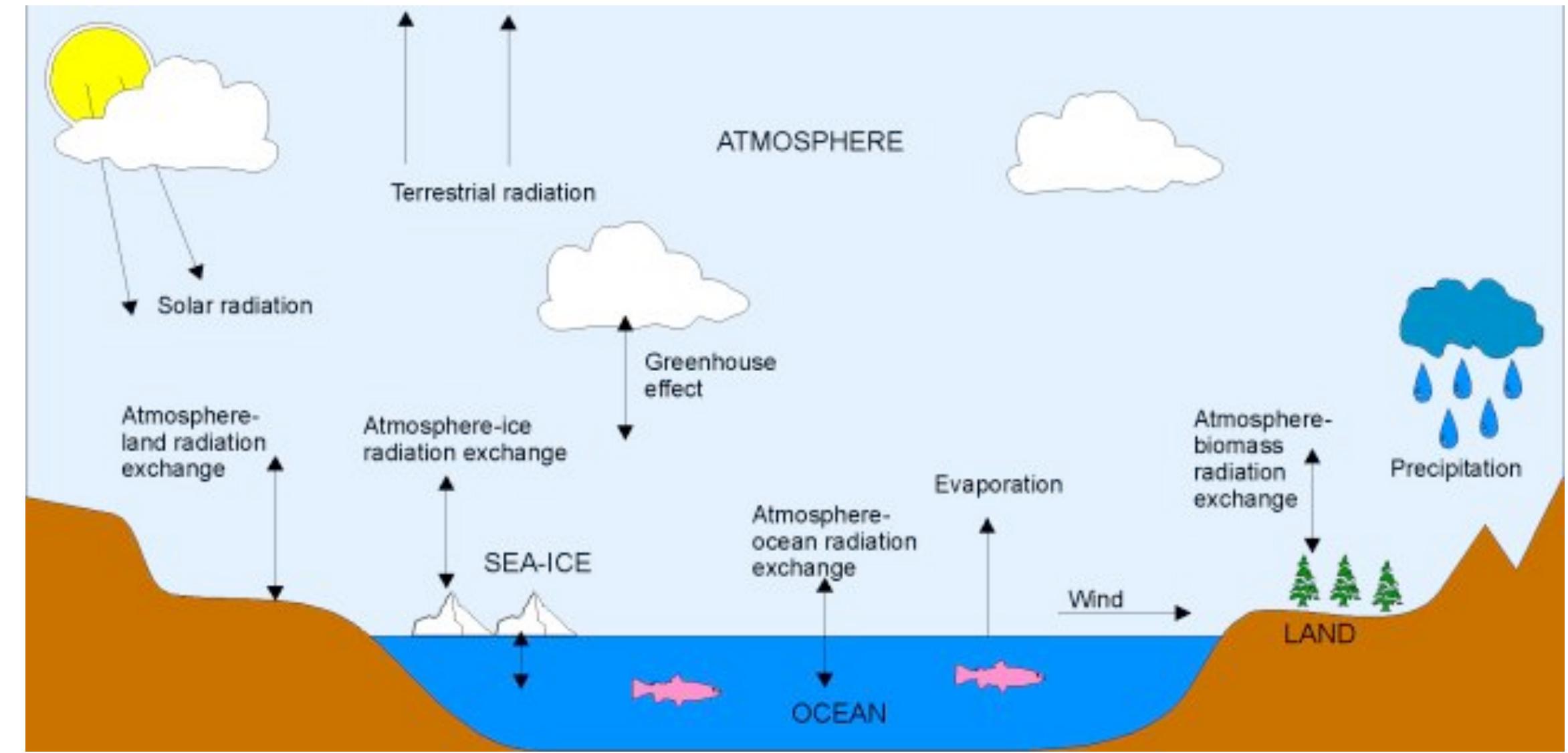
Funded by:





# Climate System

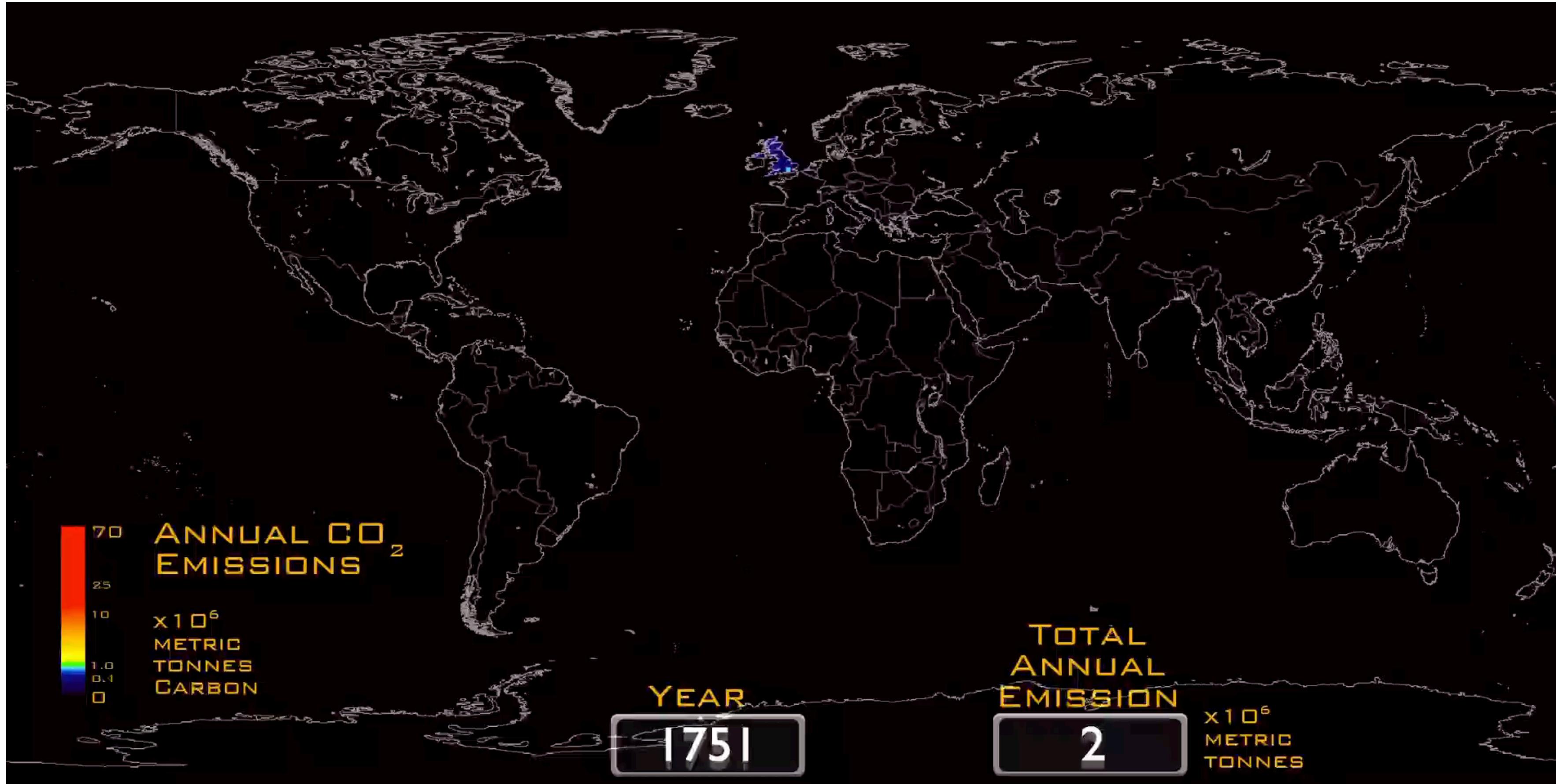
**Complex:** Multi-scale interactions between different components



**Complex:**  
Turbulent  
Flows  
(Ocean  
and  
Atmosphere)



# Anthropogenic Climate Change



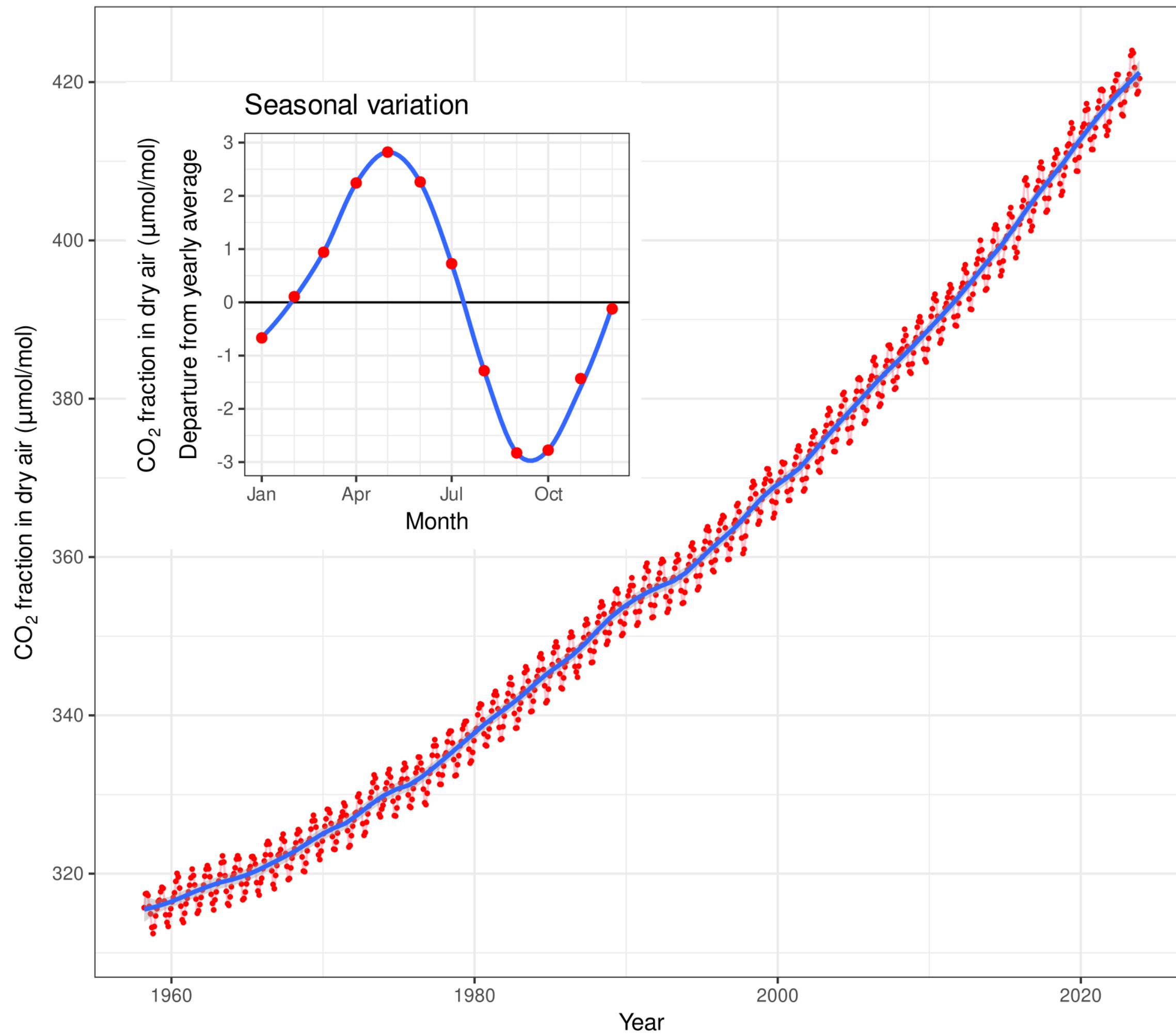
From 1870 to 2014, cumulative carbon emissions totaled about 545 GtC. Emissions were partitioned among the atmosphere (approx. 230 GtC or 42%), ocean (approx. 155 GtC or 28%) and the land (approx. 160 GtC or 29%).



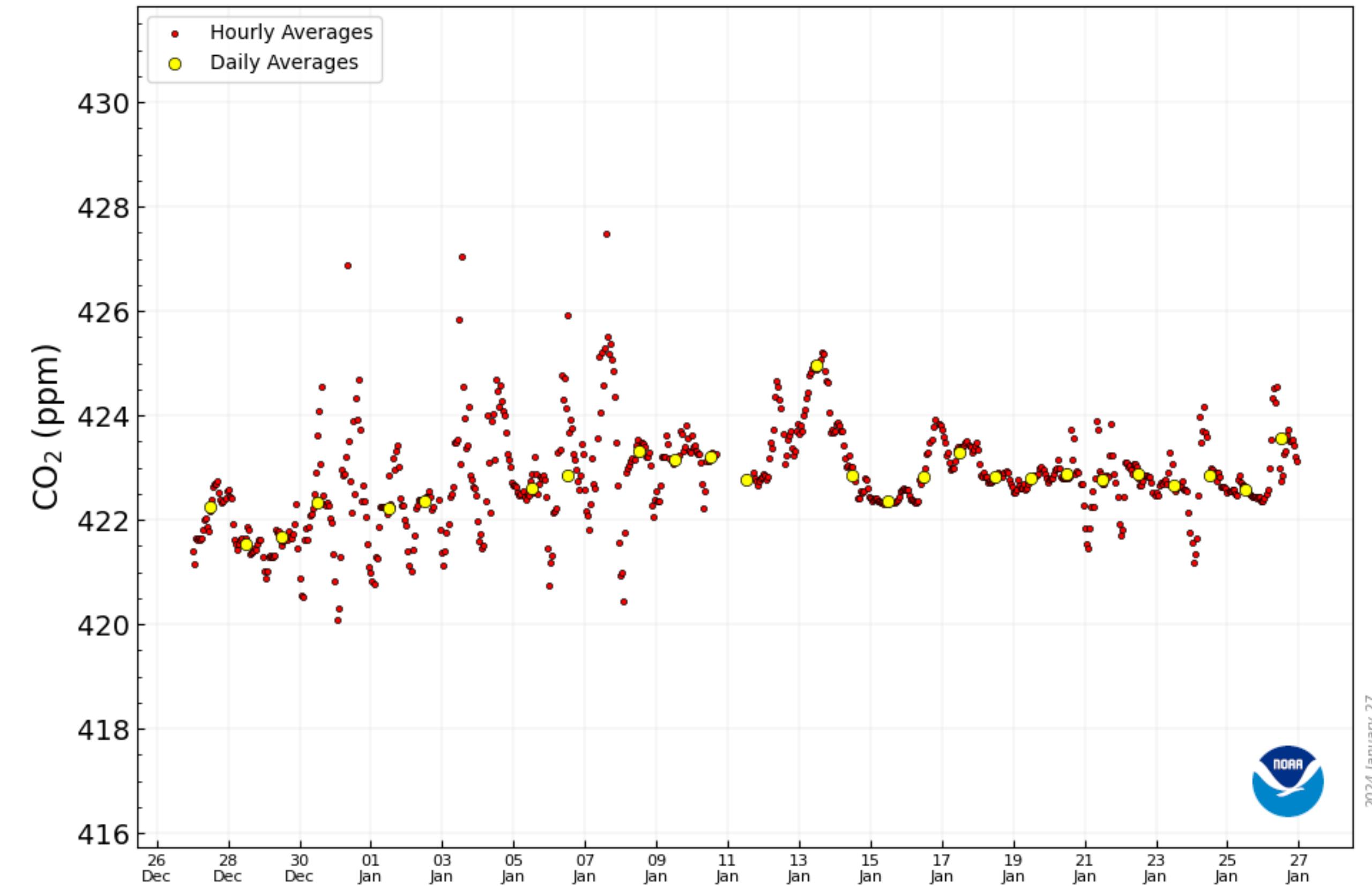
# Changes in atmospheric CO<sub>2</sub>

Monthly mean CO<sub>2</sub> concentration

Mauna Loa 1958-2023

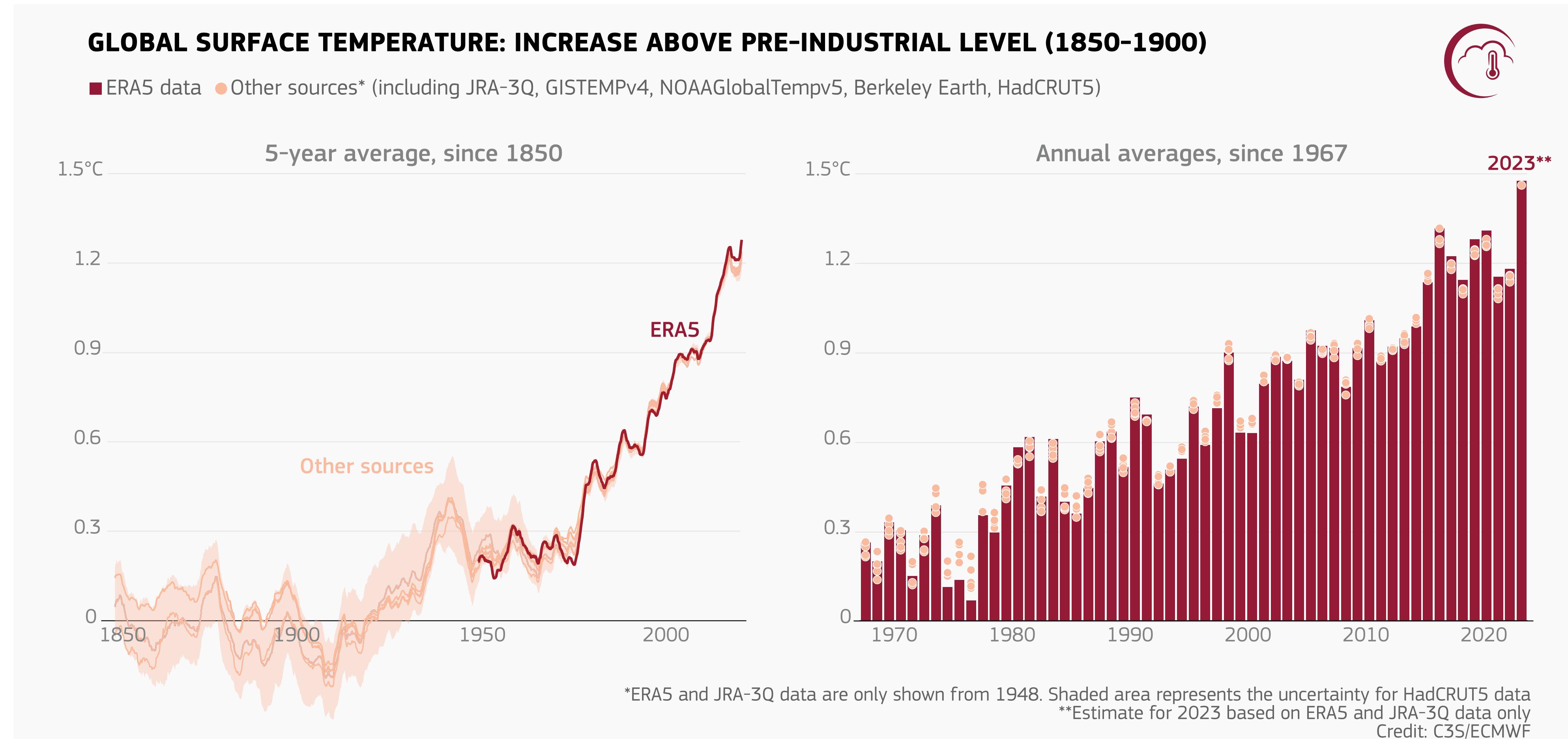


Mauna Loa Carbon Dioxide





# Global Mean surface temperature (GMST)

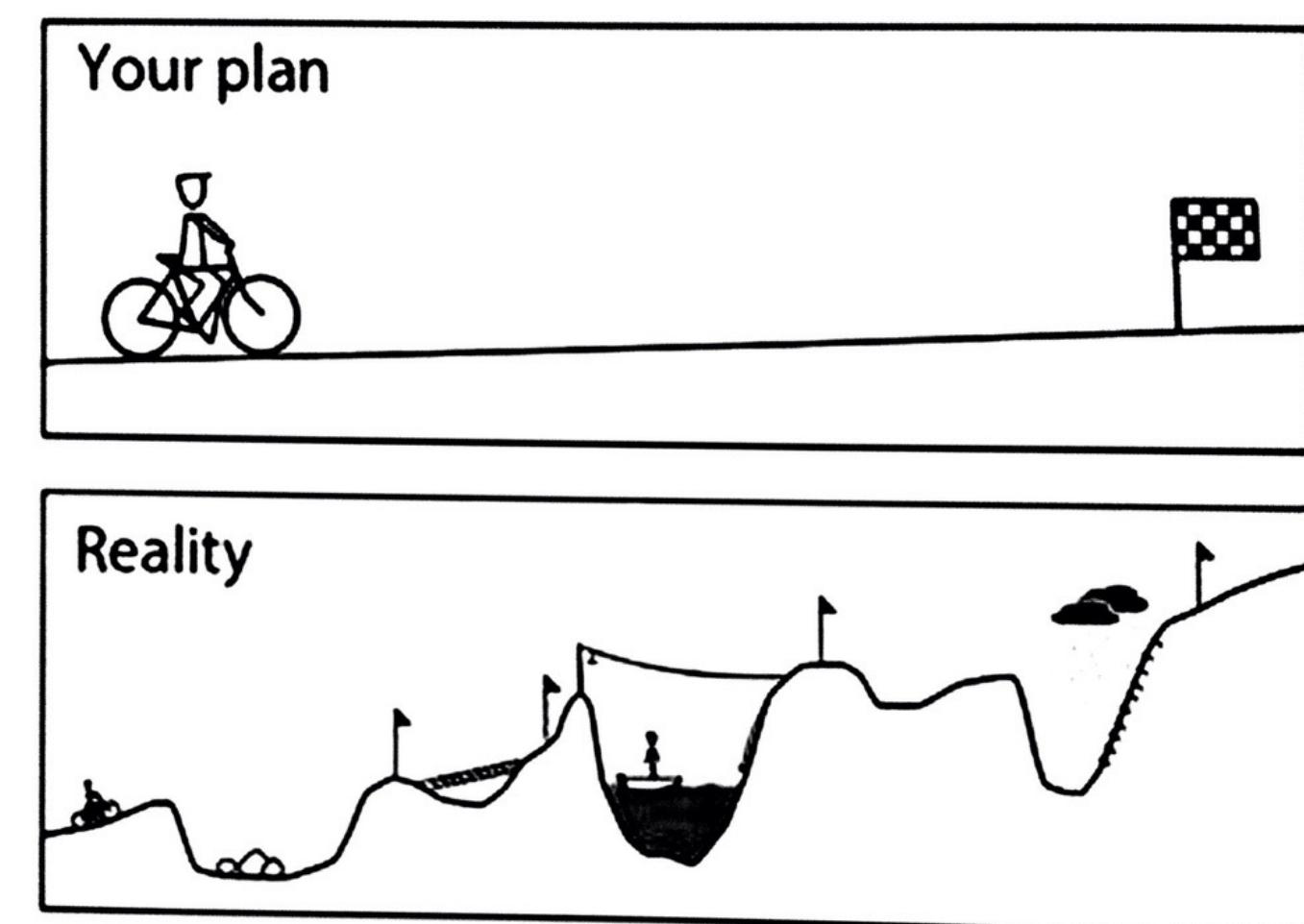


# Main questions

1. How warm is it going to be in 2100?

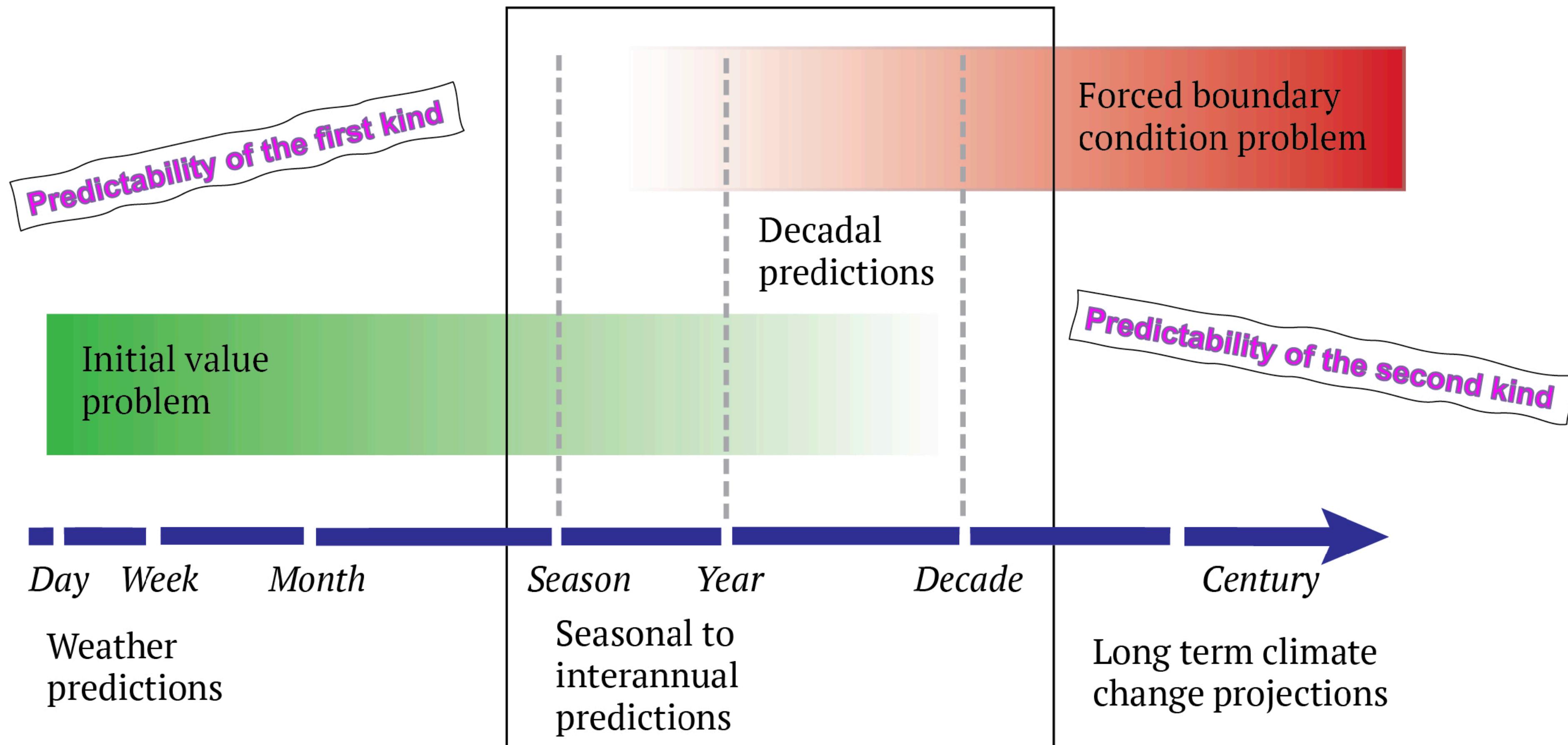


2. Are the changes going to be  
'smooth' or 'bumpy'?



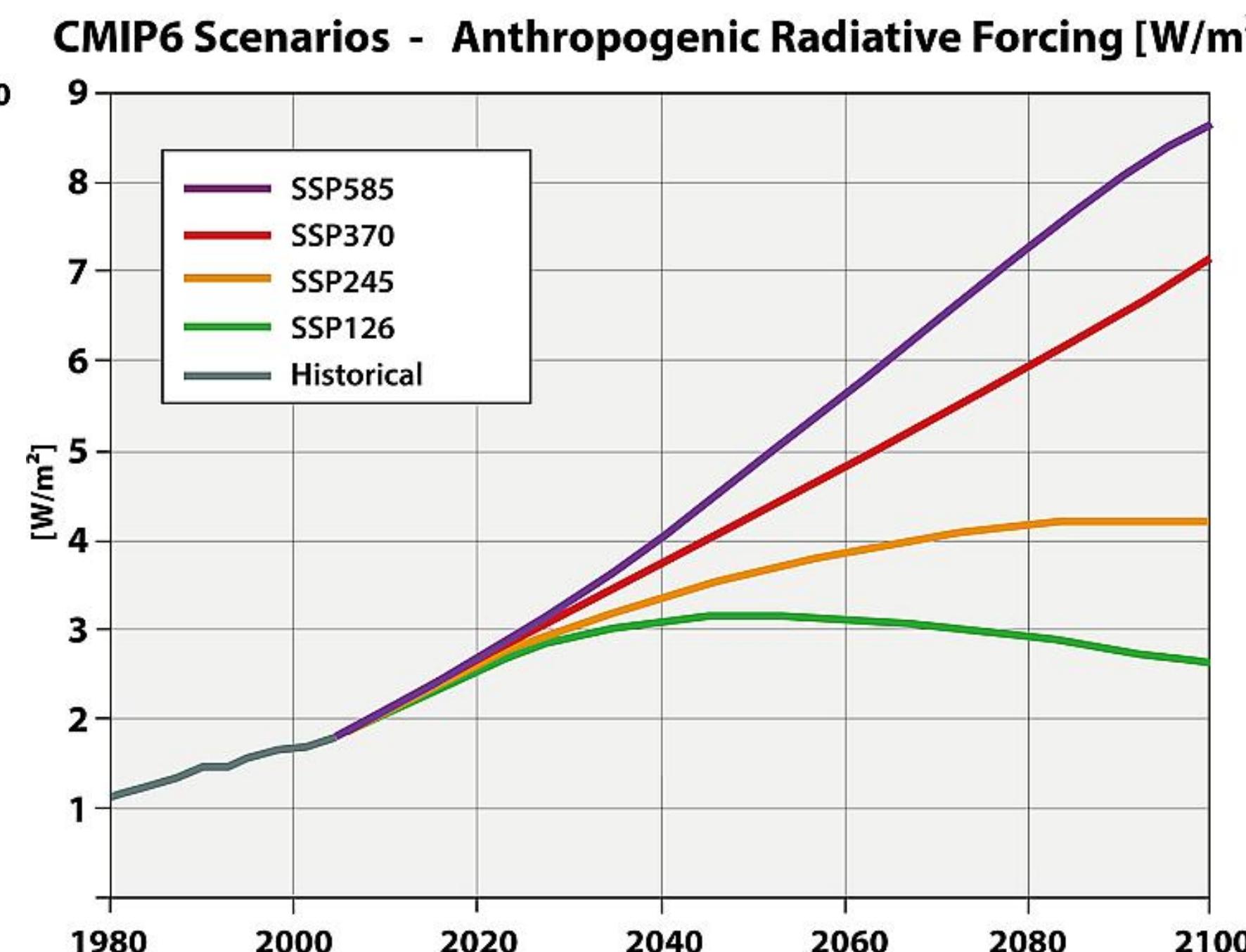
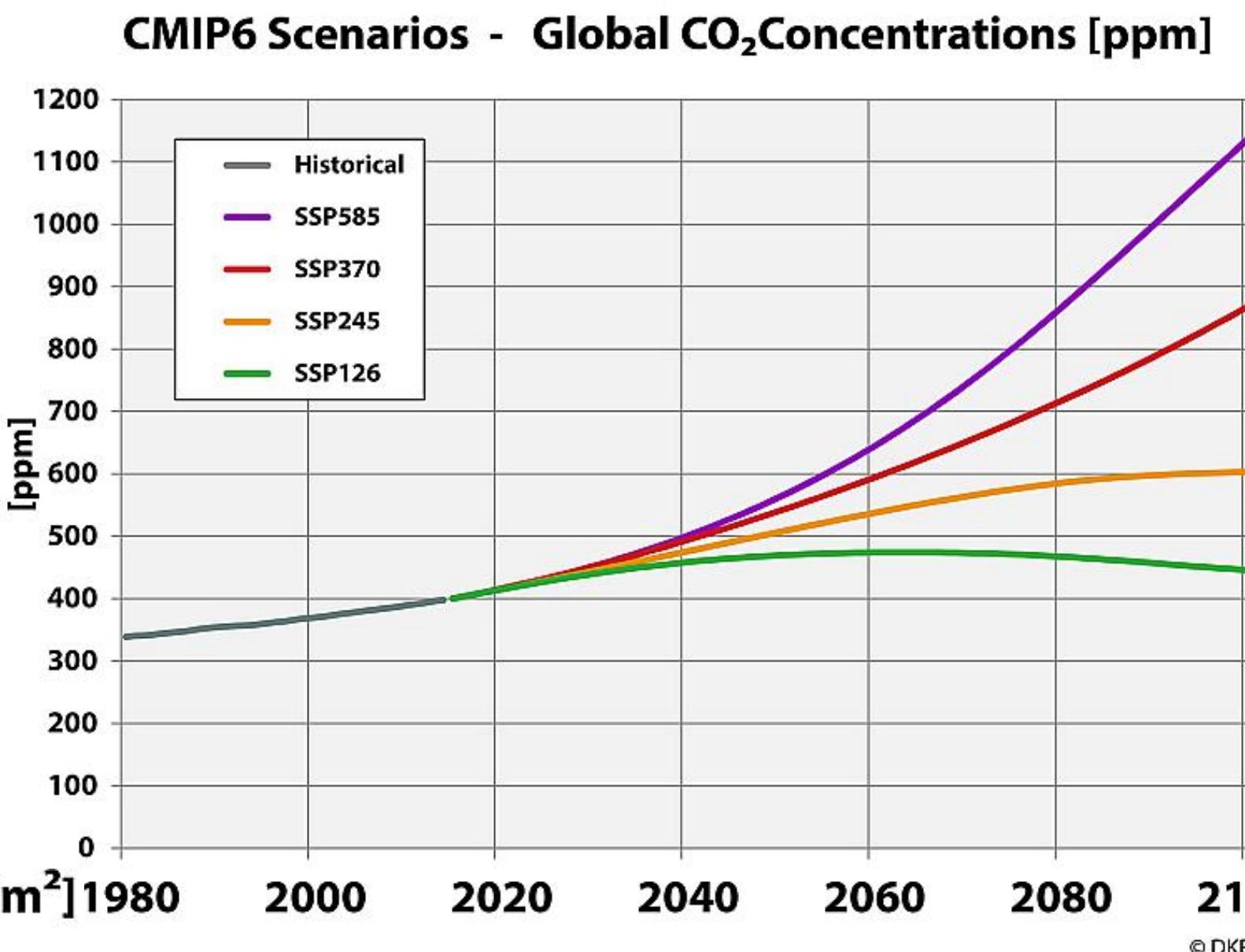
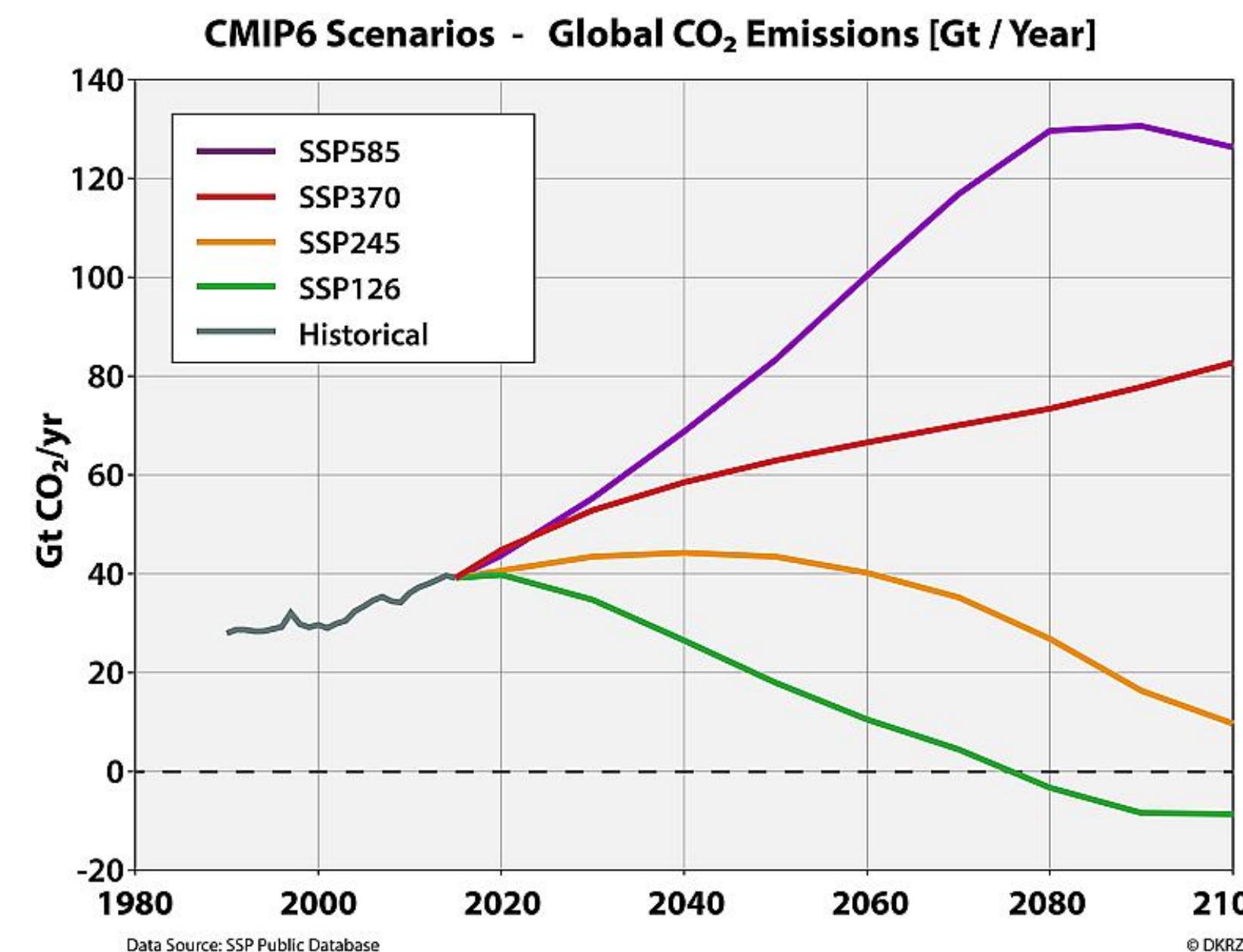


# Future Climate Change



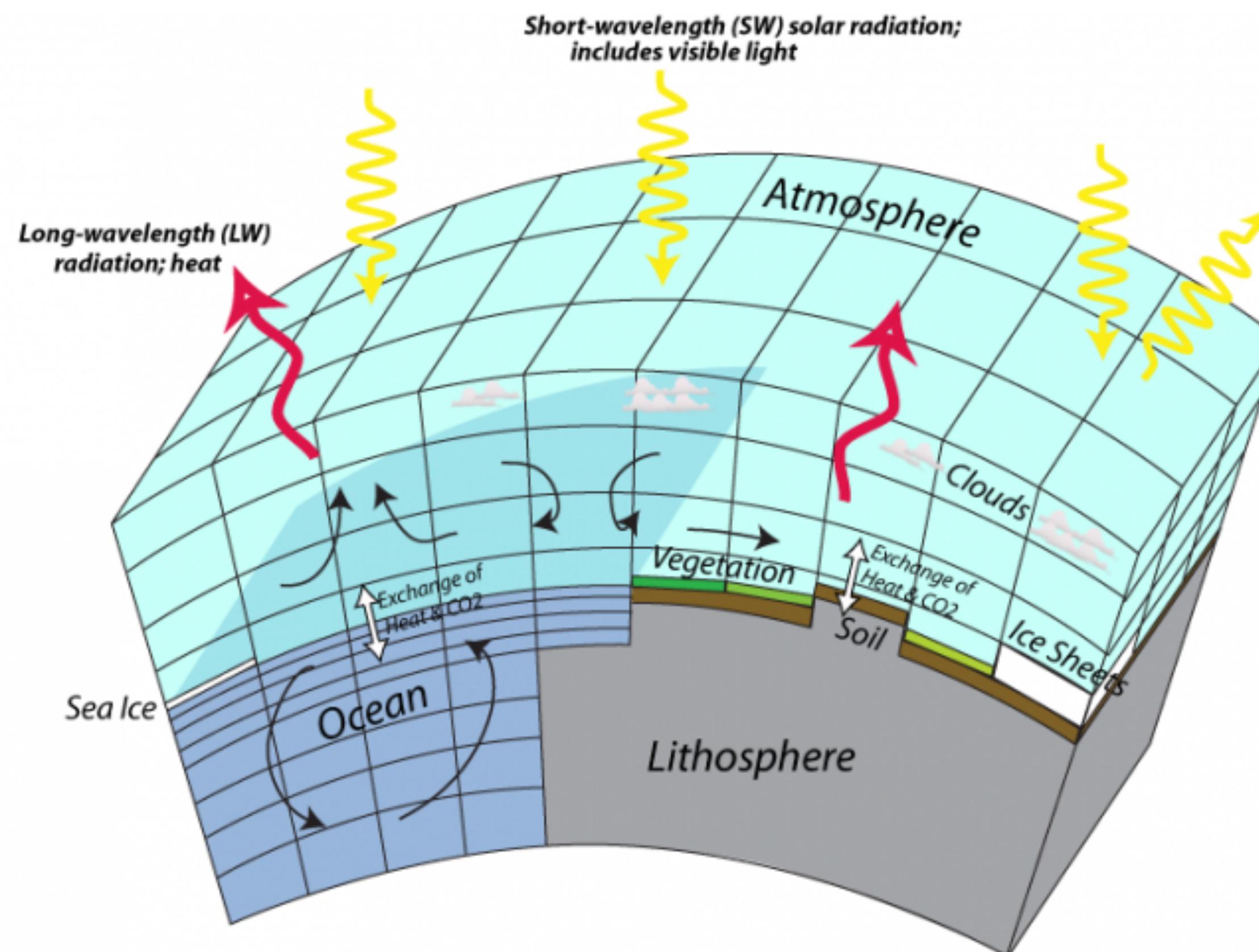


# Emission Scenarios



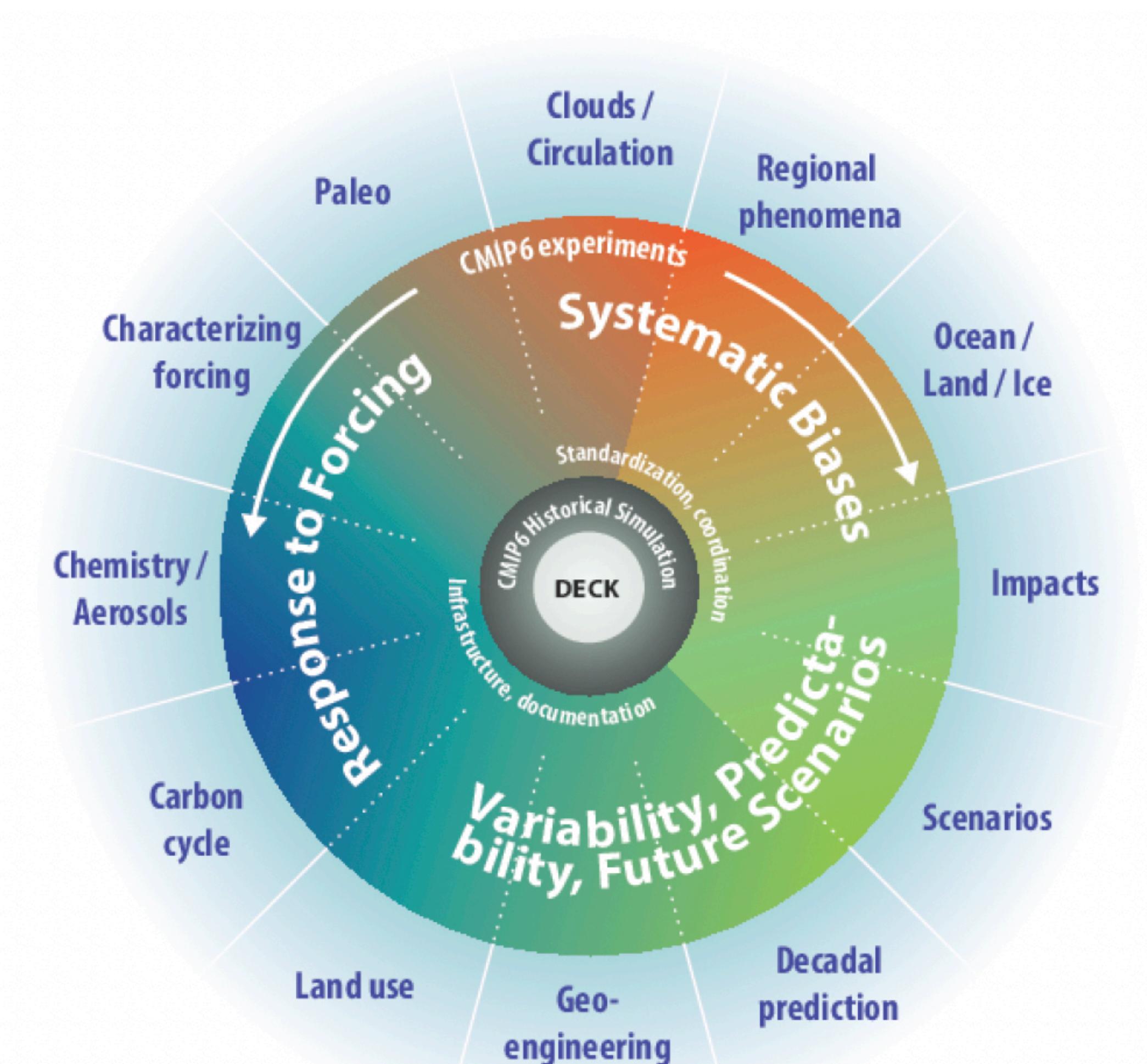


# Climate Models



Typical horizontal resolution:  
ocean, sea-ice: 0.5-1 degree  
atmosphere, land: 1-2 degree

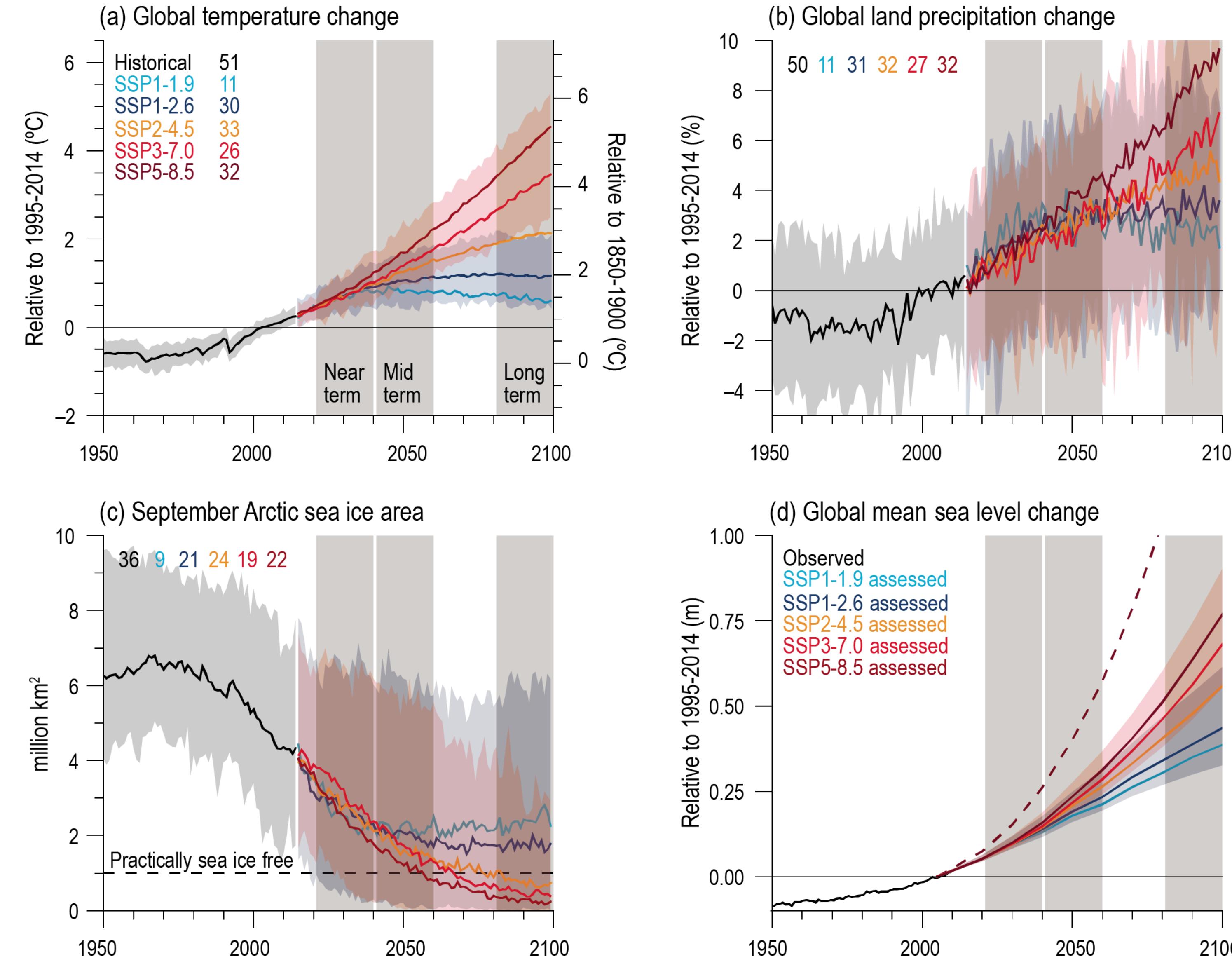
Eyring et al. (2016)



~ 20 modeling groups  
~ 50 different models  
Ensemble simulations



# Future Climate Change

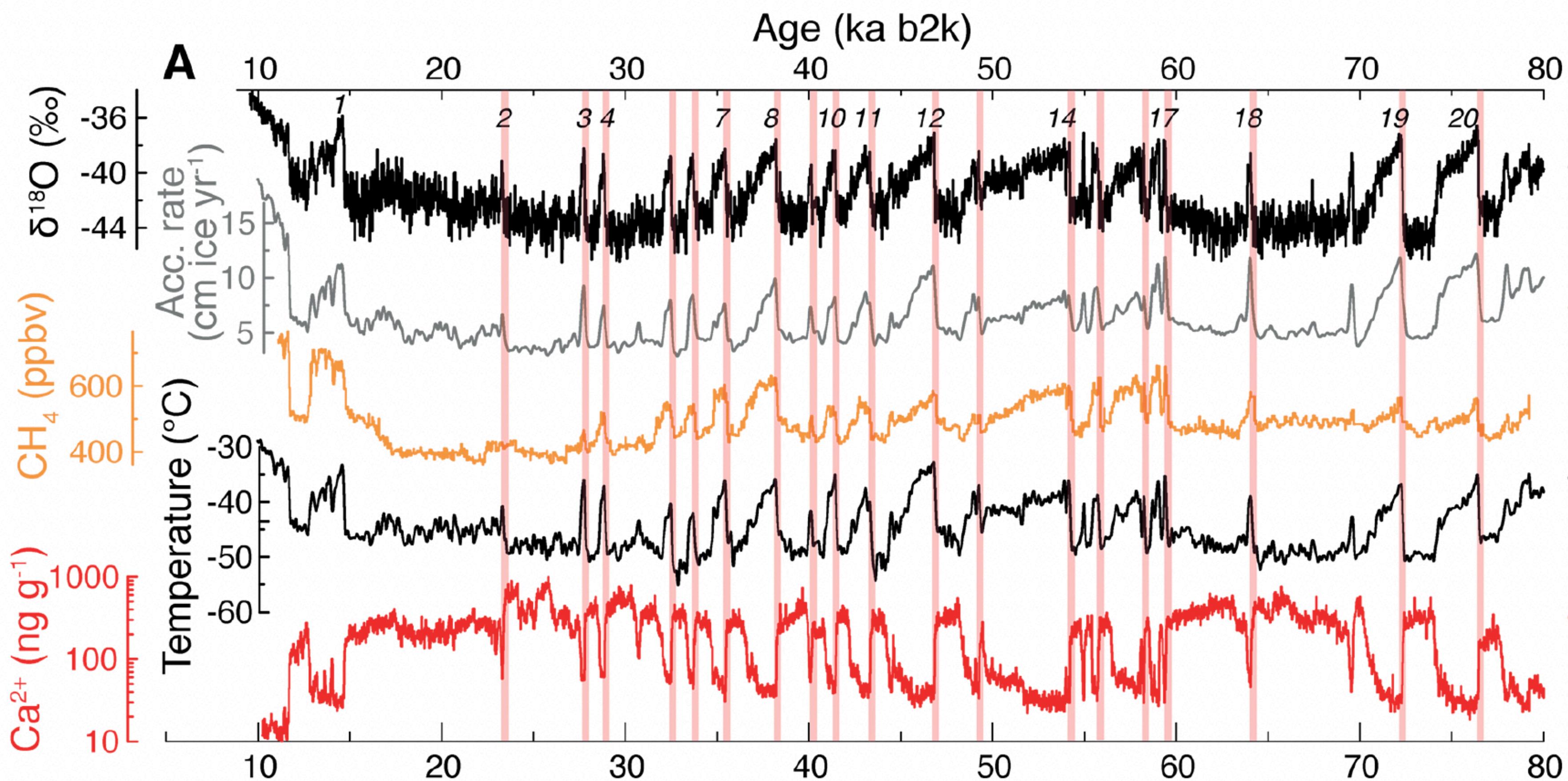


How warm is  
it going to be  
in 2100?

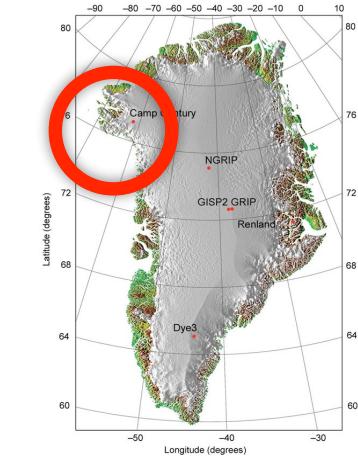
+1.5-5.5  $^{\circ}\text{C}$   
wrt pre-industrial



# Why worry about ‘bumpy’ future climate change?



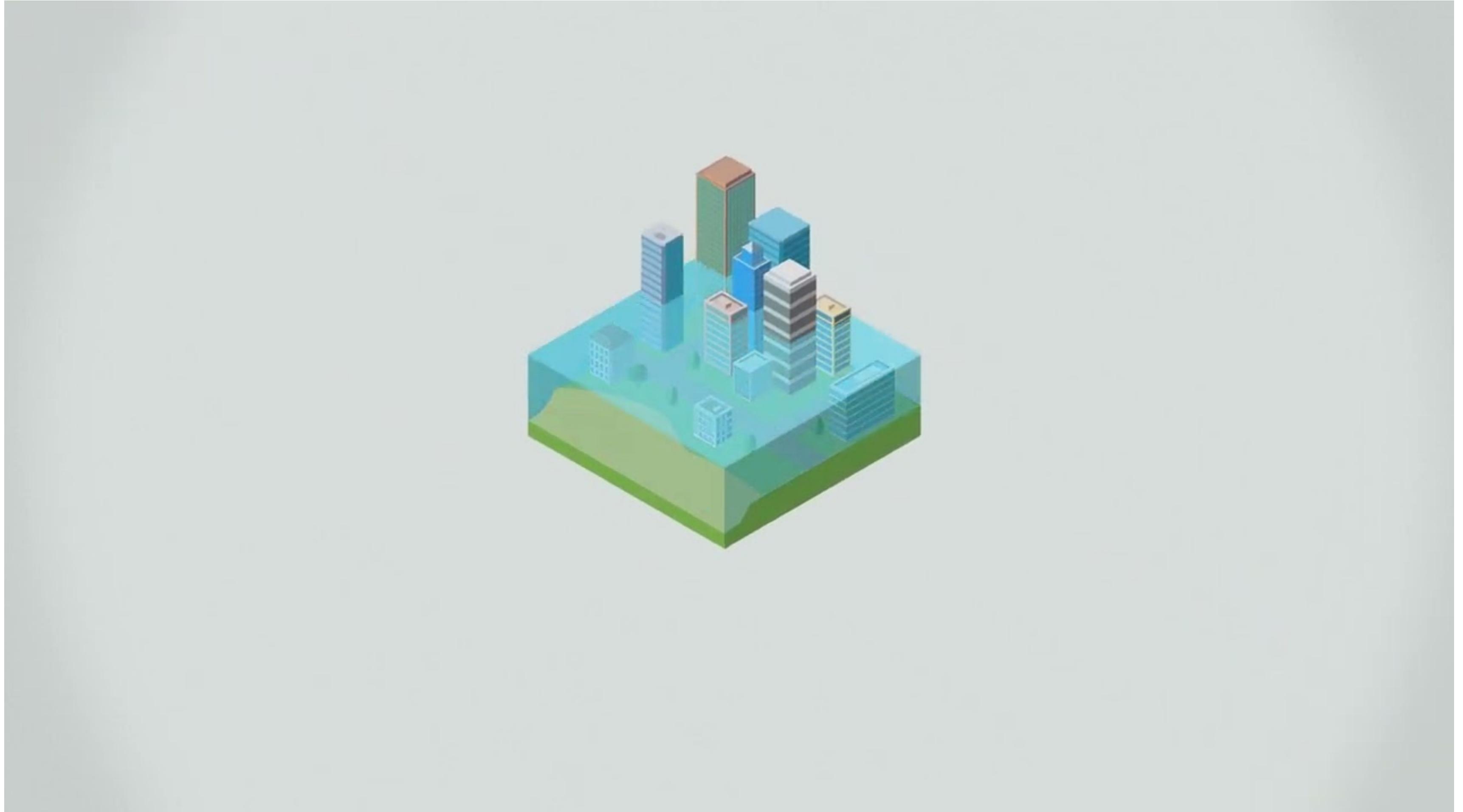
Proxy data  
(geological past)





Utrecht  
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# Tipping Behavior



## Global Tipping Points

Report 2023

Led by:  
 University  
of Exeter | Global Systems  
Institute

Funded by:  
 BEZOS  
EARTH  
FUND

# Mathematical Basis of Tipping Behavior

Dynamical System:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t, \lambda)$$

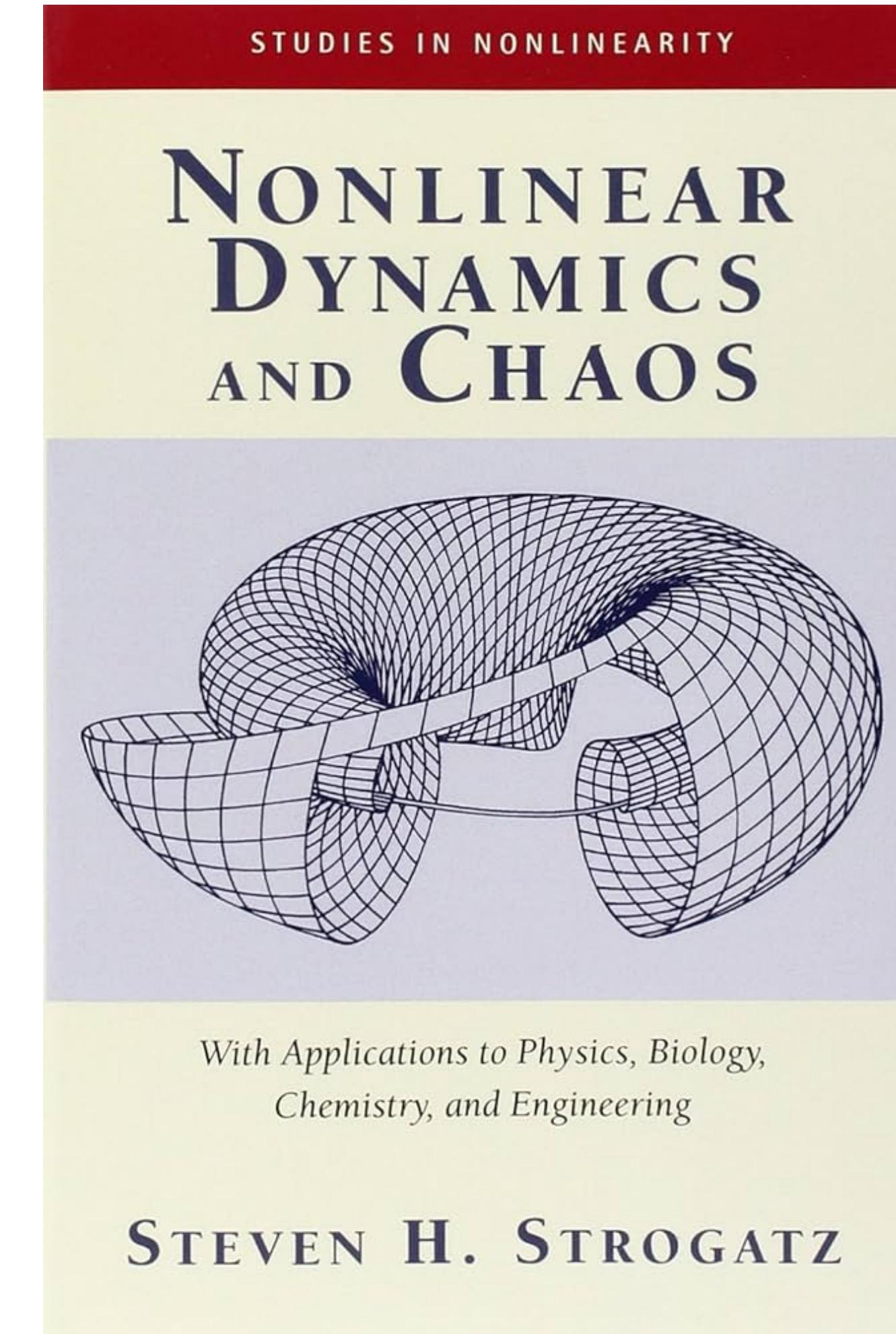
state vector  $\mathbf{x} \in \mathbb{R}^d$ ;  $d$ : dimension

$t$ : time       $\lambda$ : parameter

vector field  $\mathbf{f}(\mathbf{x}, t, \lambda)$

non-autonomous  $\mathbf{f}(\mathbf{x}, t, \lambda)$

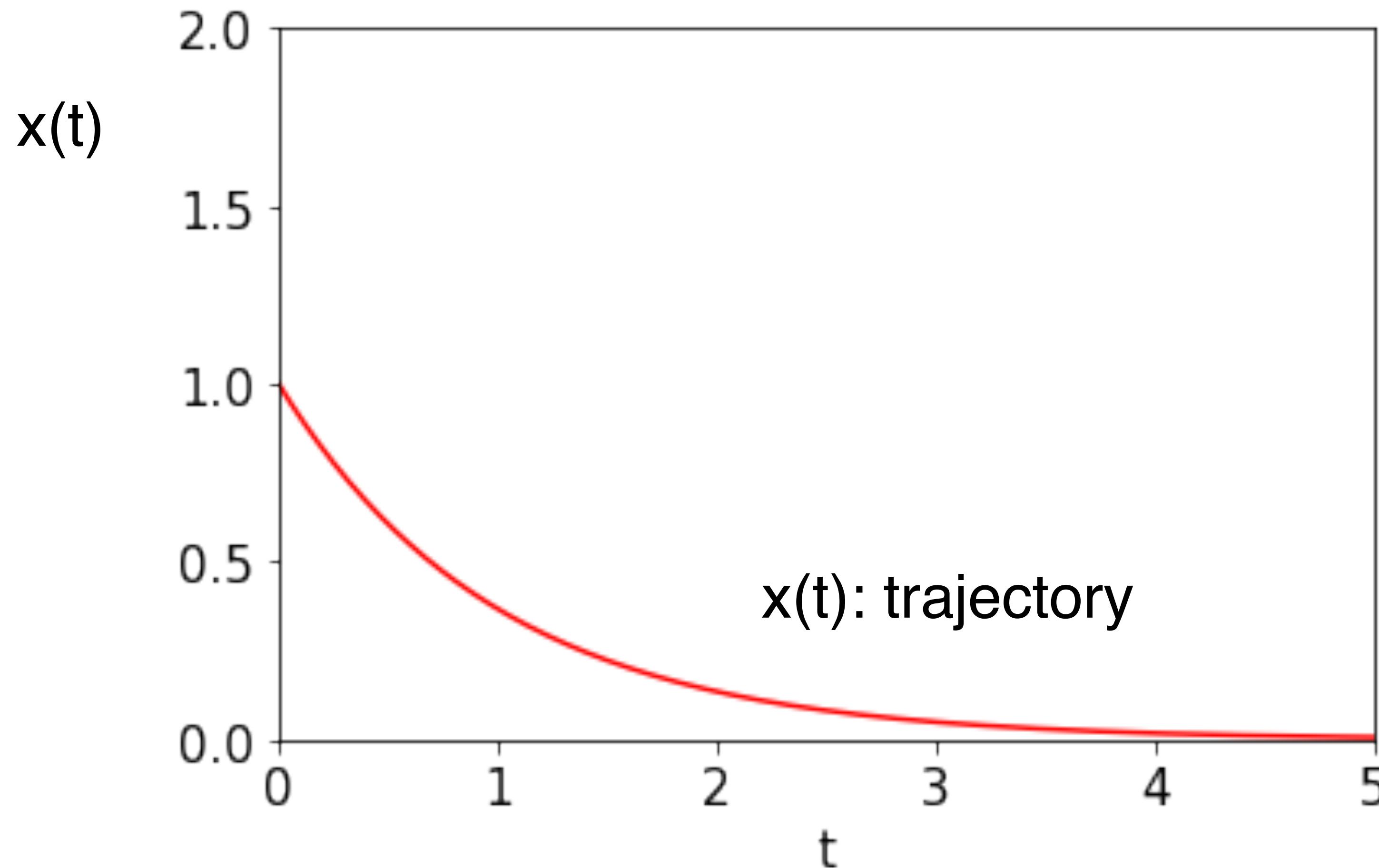
autonomous  $\mathbf{f}(\mathbf{x}, \lambda)$





# Example: linear system, $d = 1$

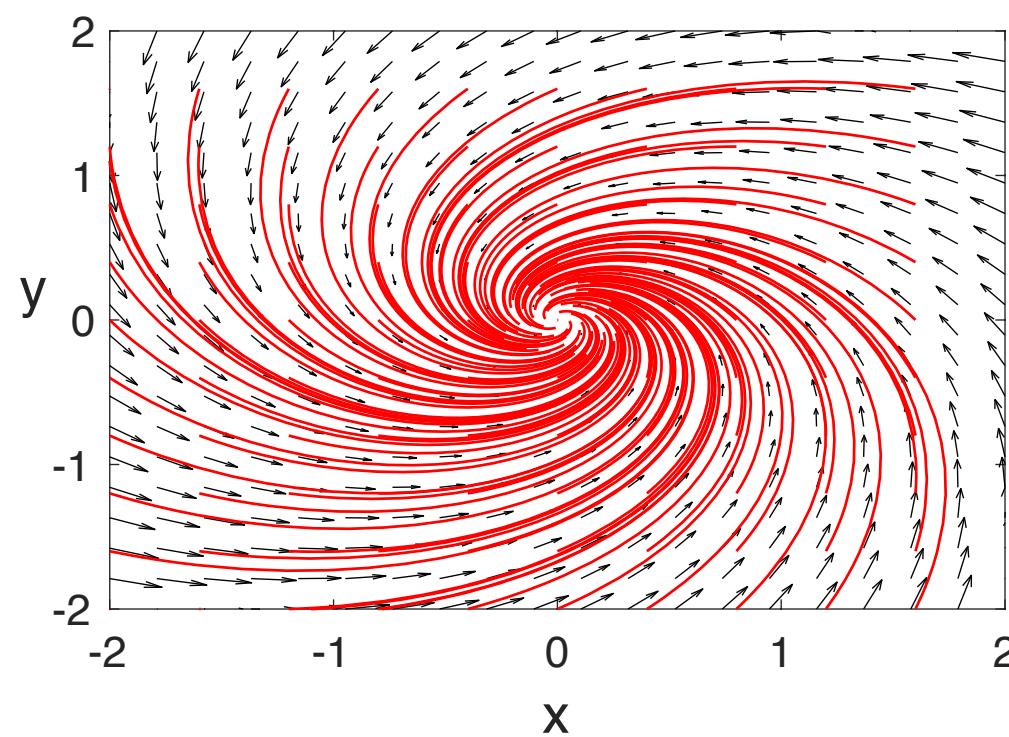
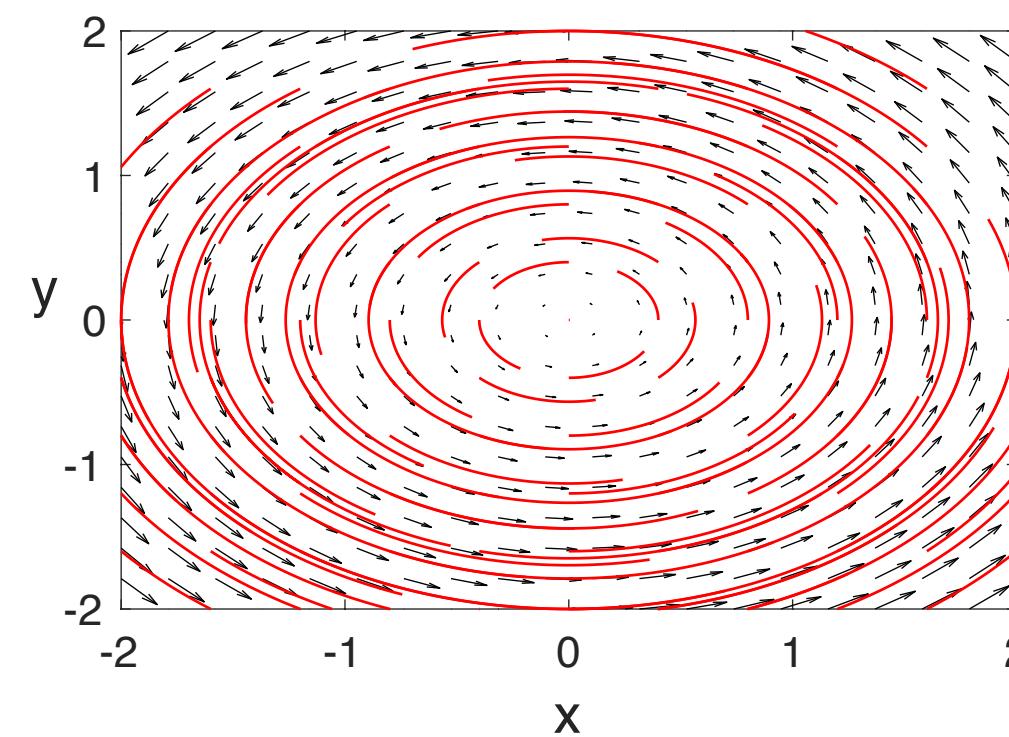
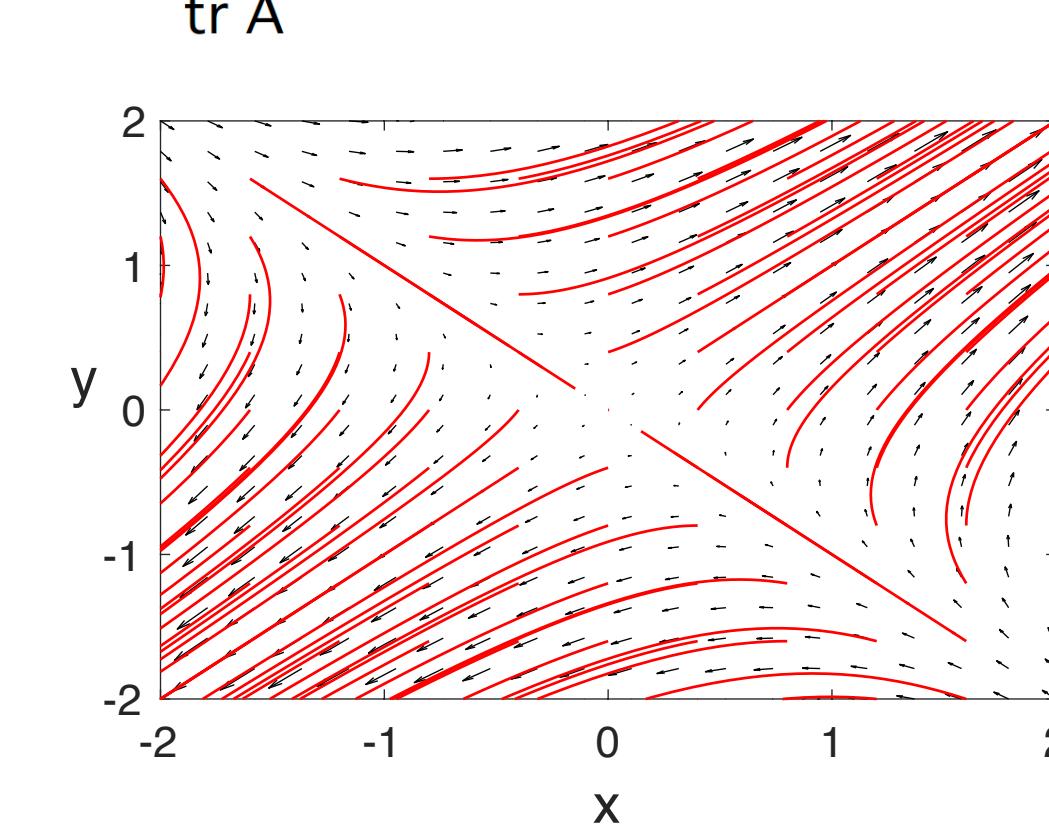
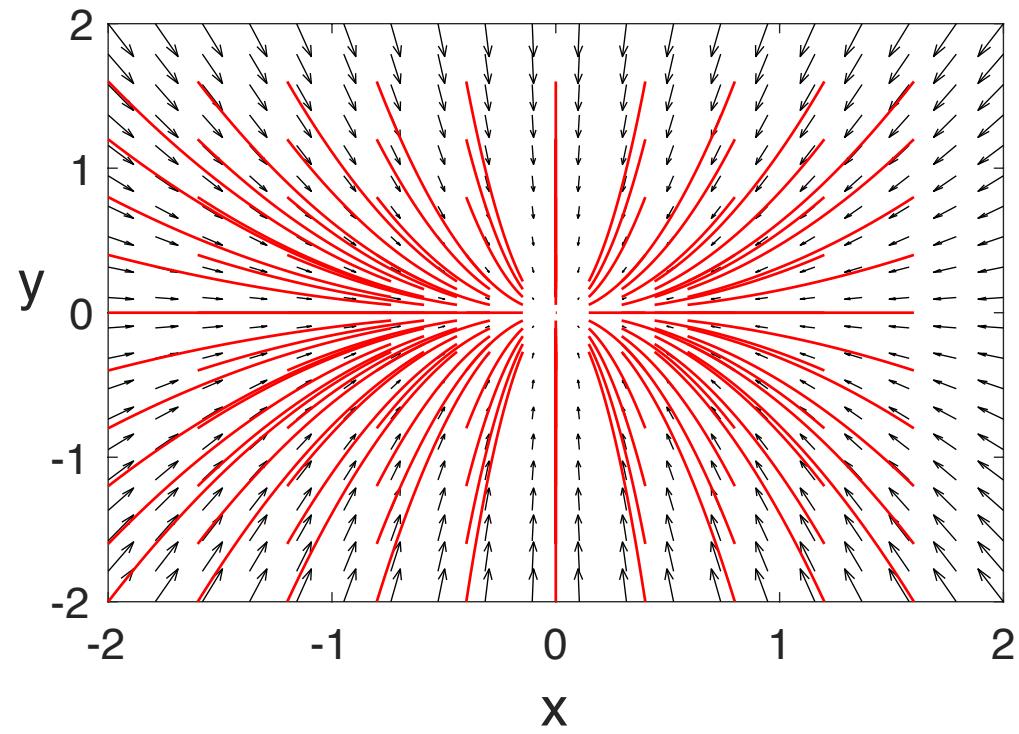
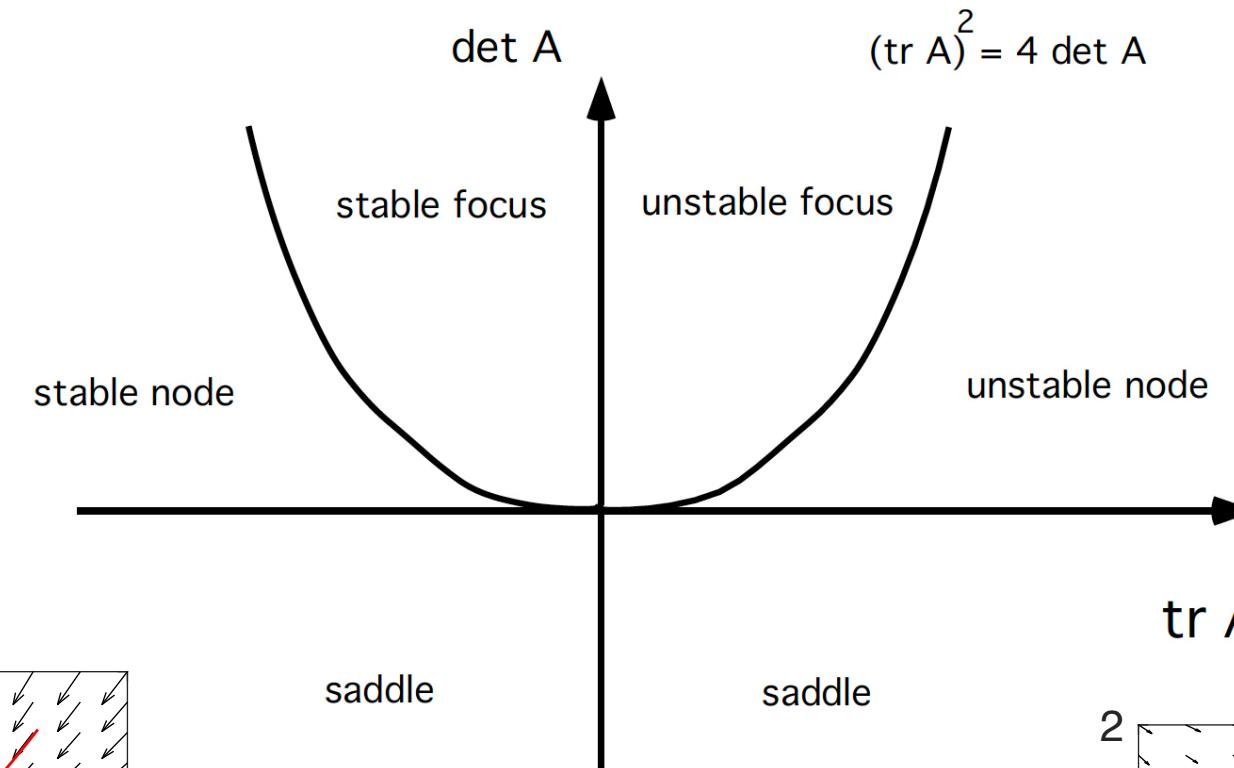
$$\frac{dx}{dt} = -x$$



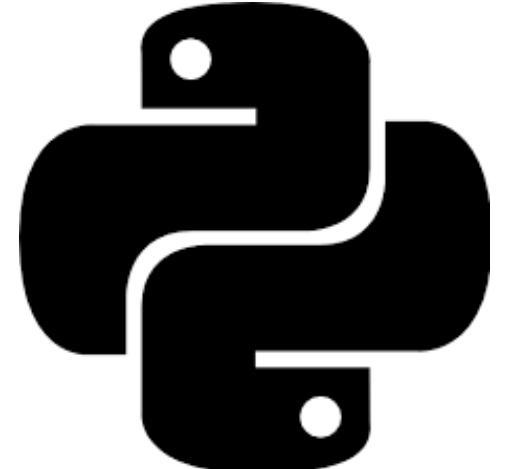


# Example: linear systems, $d = 2$

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$



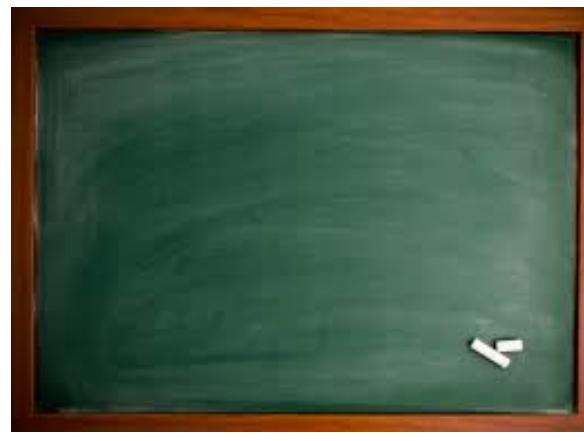
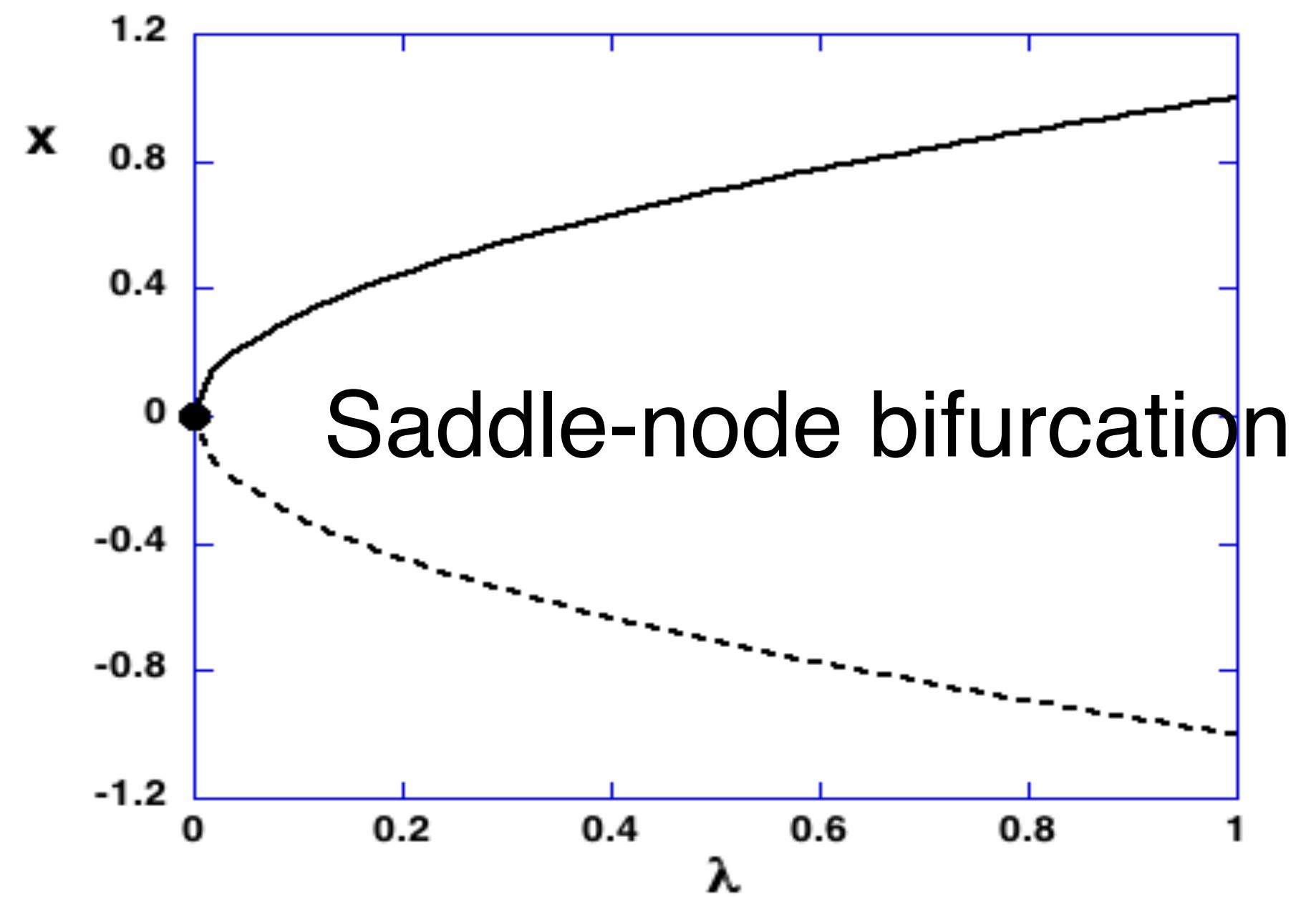
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$





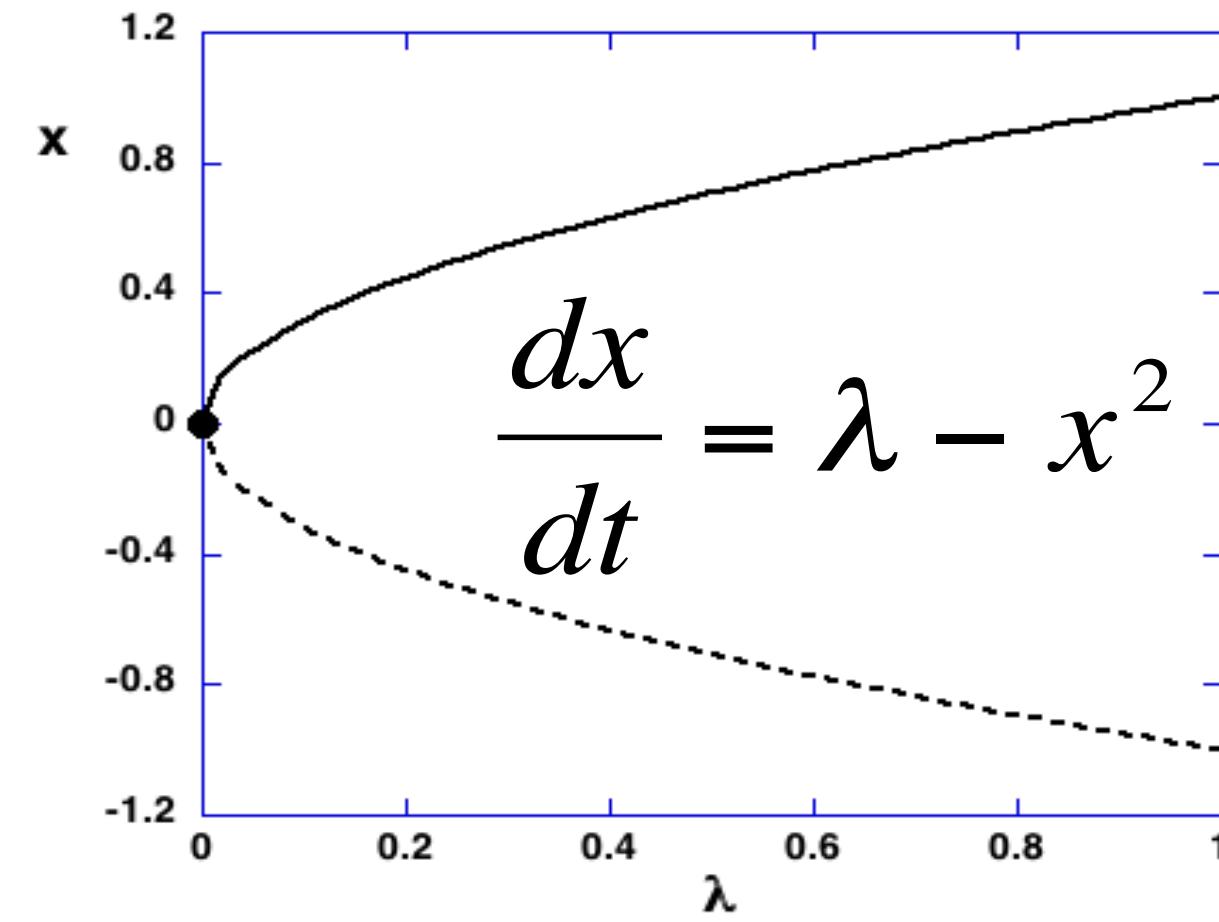
# *Example: nonlinear system, d = 1*

$$\frac{dx}{dt} = \lambda - x^2$$

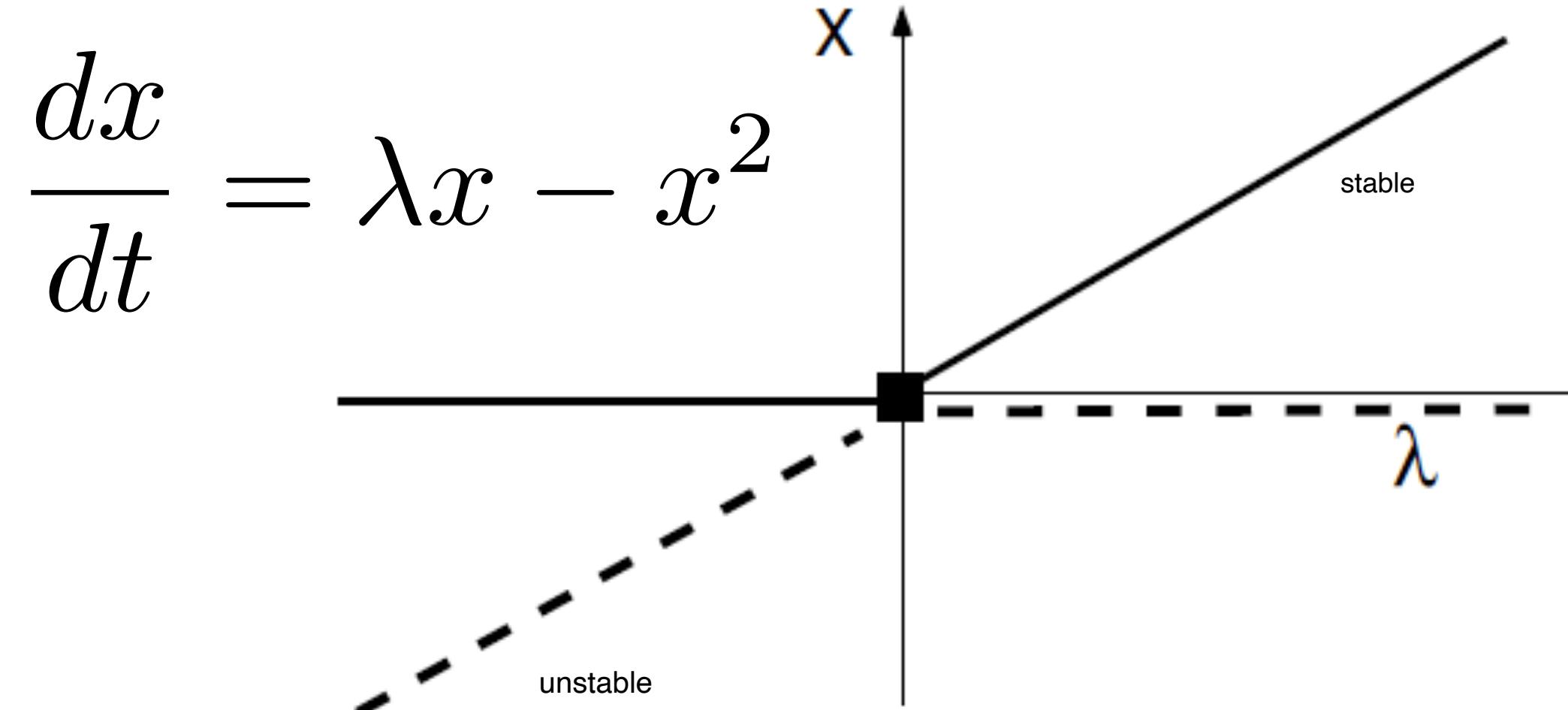
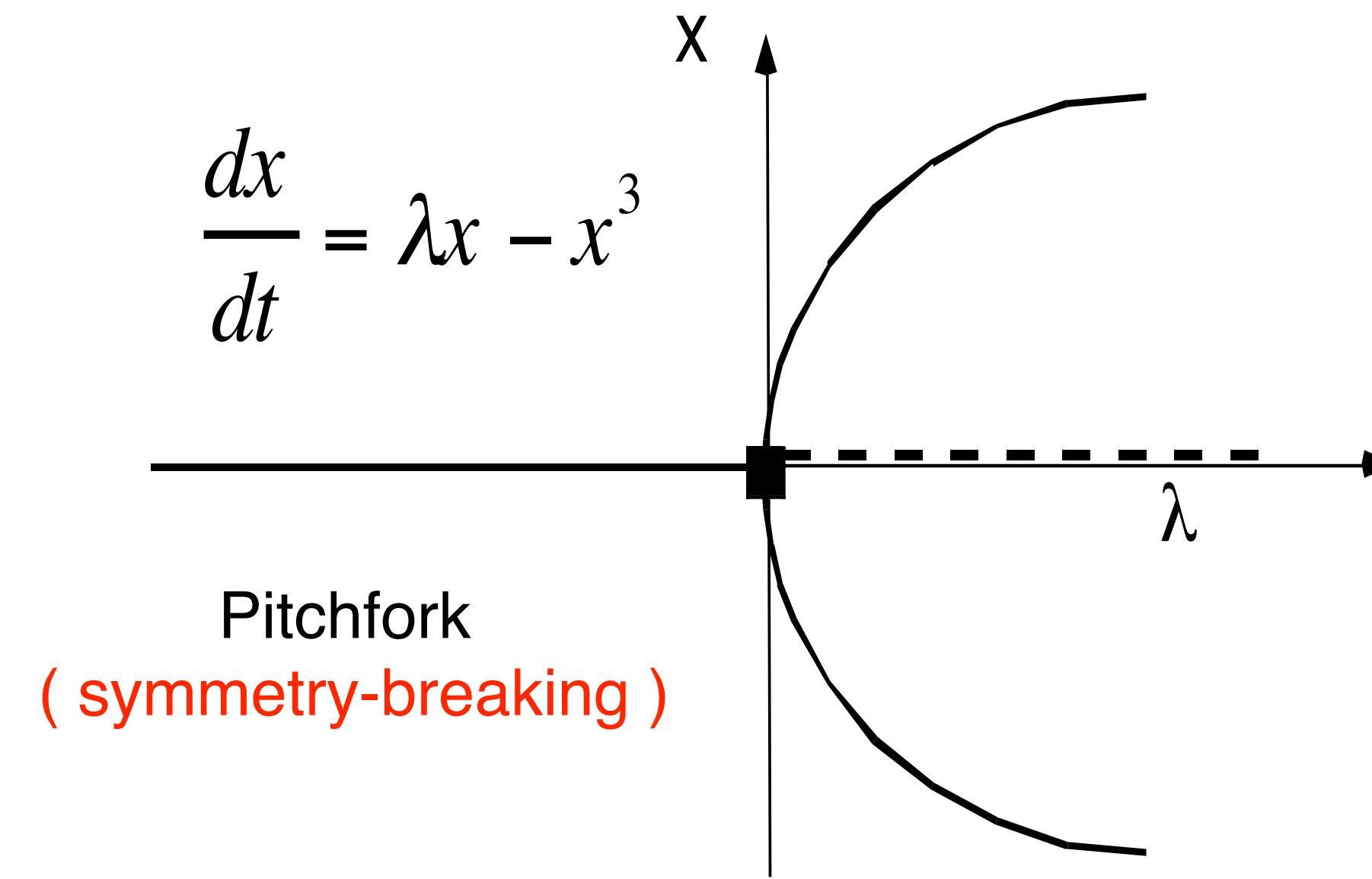




# Elementary bifurcations, $d = 1$



Saddle-node ( non-existence )

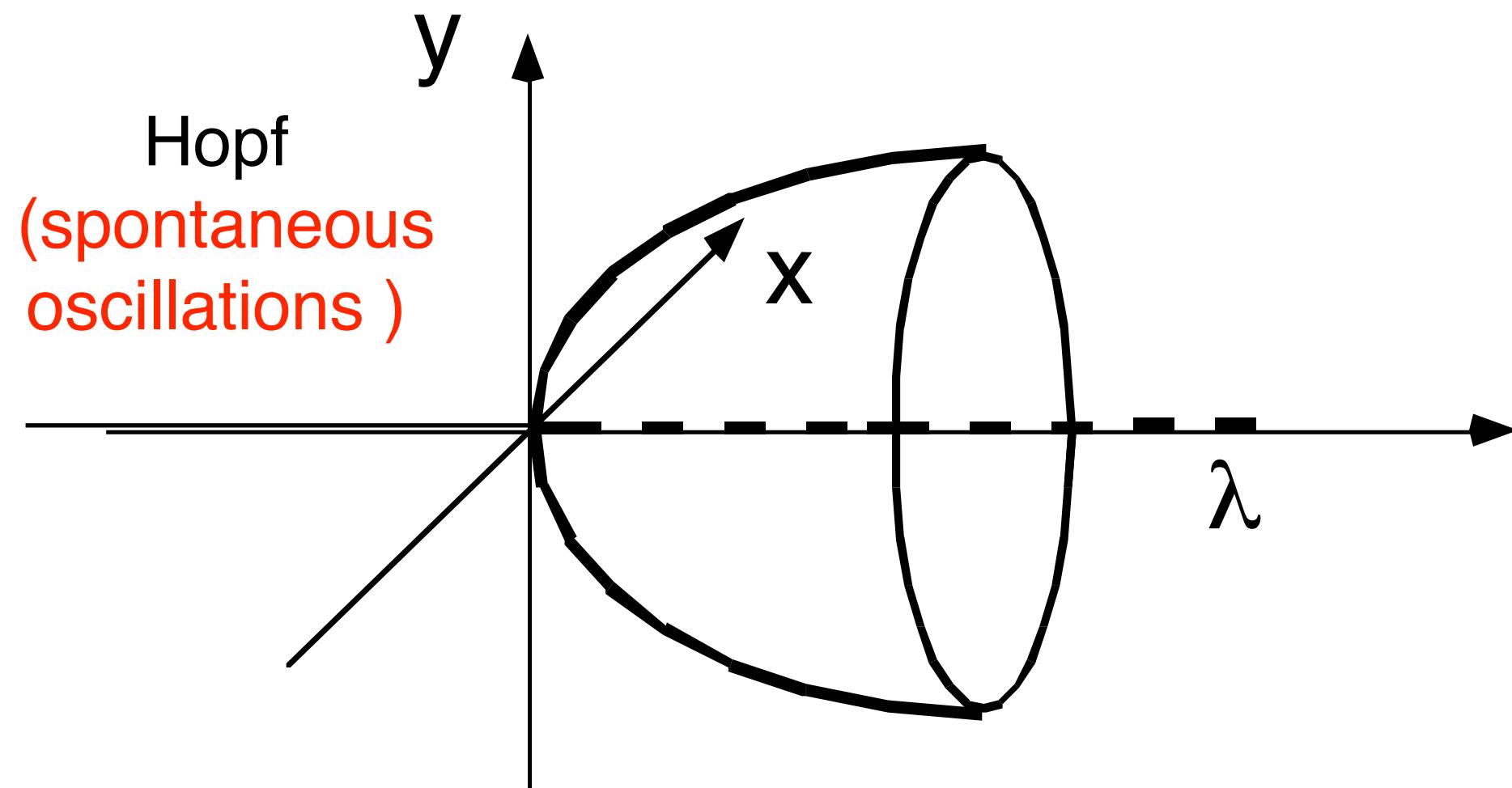


Transcritical ( exchange of stability )





# Elementary bifurcations, $d = 2$



$$\dot{x} = \lambda x - \omega y - x(x^2 + y^2)$$

$$\dot{y} = \lambda y + \omega x - y(x^2 + y^2)$$





# *Exercises!*

