Tutorial 2: February 7, 12:00-12:45

1. Bifurcation diagram AMOC model

Notebook: Tutorial7-2_1.ipynb

We consider the reduced AMOC model

$$\frac{dy}{dt} = F - y(1 + \mu^2(1-y)^2)$$

with $\mu^2 = 6.25$ and F is the dimensionless freshwater flux.

a.
Use pyDSTool to compute directly the bifurcation diagram of this model, with F as a control parameter. Verify the positions of the saddle-node bifurcations through analytical calculations.

b.
Use an ensemble of simulations (you can do this pyDSTool) to demonstrate that the unstable steady state (in the multi-stable regime) is indeed the boundary separating the initial conditions which will approach one of the two attractors.

2. Stochastic resonance

Notebook: Tutorial7-2_2.ipynb

In this exercise, we consider the stochastic dynamical system

$$dY_t = (F(t) - Y_t(1 + \mu(1-Y_t)^2))dt + \sigma dW_t$$

where $F(t)$ is now in general a time dependent function and σ is the variance of the noise (represented by the Wiener process W_t).

a.
Take $F = 1.1$ constant and study the behaviour of the model solutions for different $\sigma = 0.0, 0.1, 0.2$ and 0.5. Make an estimate of the noise induced transition probability within a time $T = 100$ as a function of σ.

b.
Next take $F(t) = F_0 + A\sin\omega t$, where $F_0 = 1.1$ and A is the amplitude of the period variation. Take $\omega = 1.0$ and $A = 1.0$. Study the behavior of the model solutions versus σ and investigate any special behaviour.