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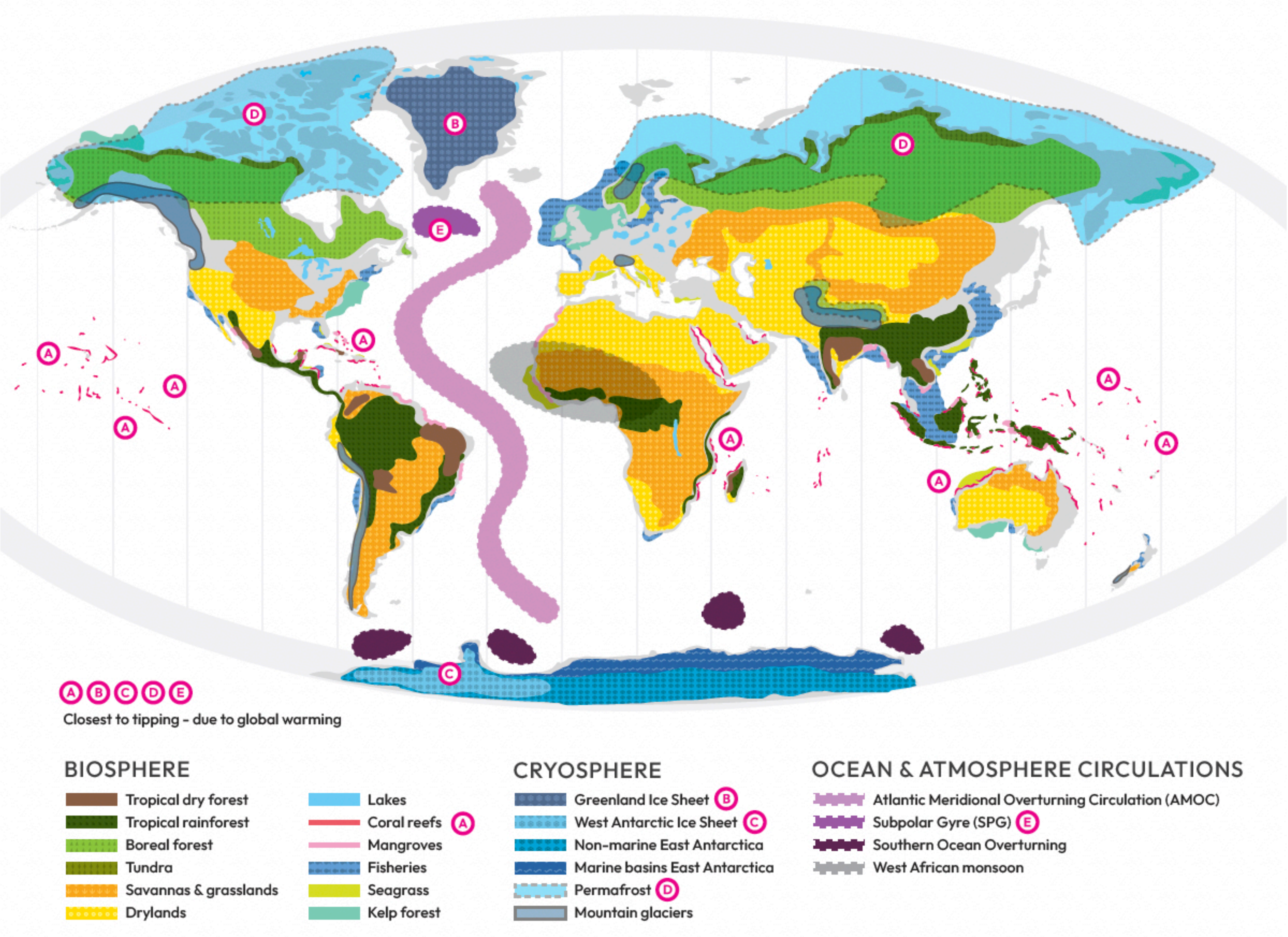
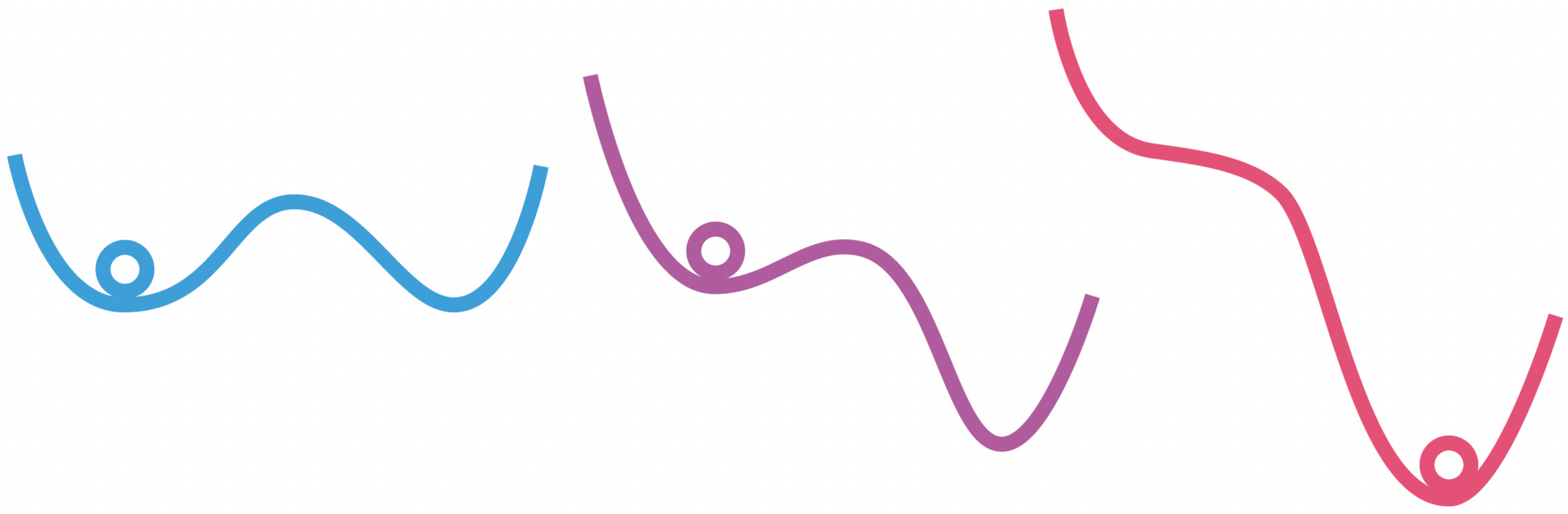
Tipping Behavior in the Climate System



Henk Dijkstra,
Department of Physics,
Utrecht University, NL
&
DICAM,
University of Trento, IT

Tipping Point



A tipping point occurs when change in part a system becomes self-perpetuating beyond a threshold, leading to substantial, widespread, frequently abrupt and often irreversible impact.

Tipping Elements

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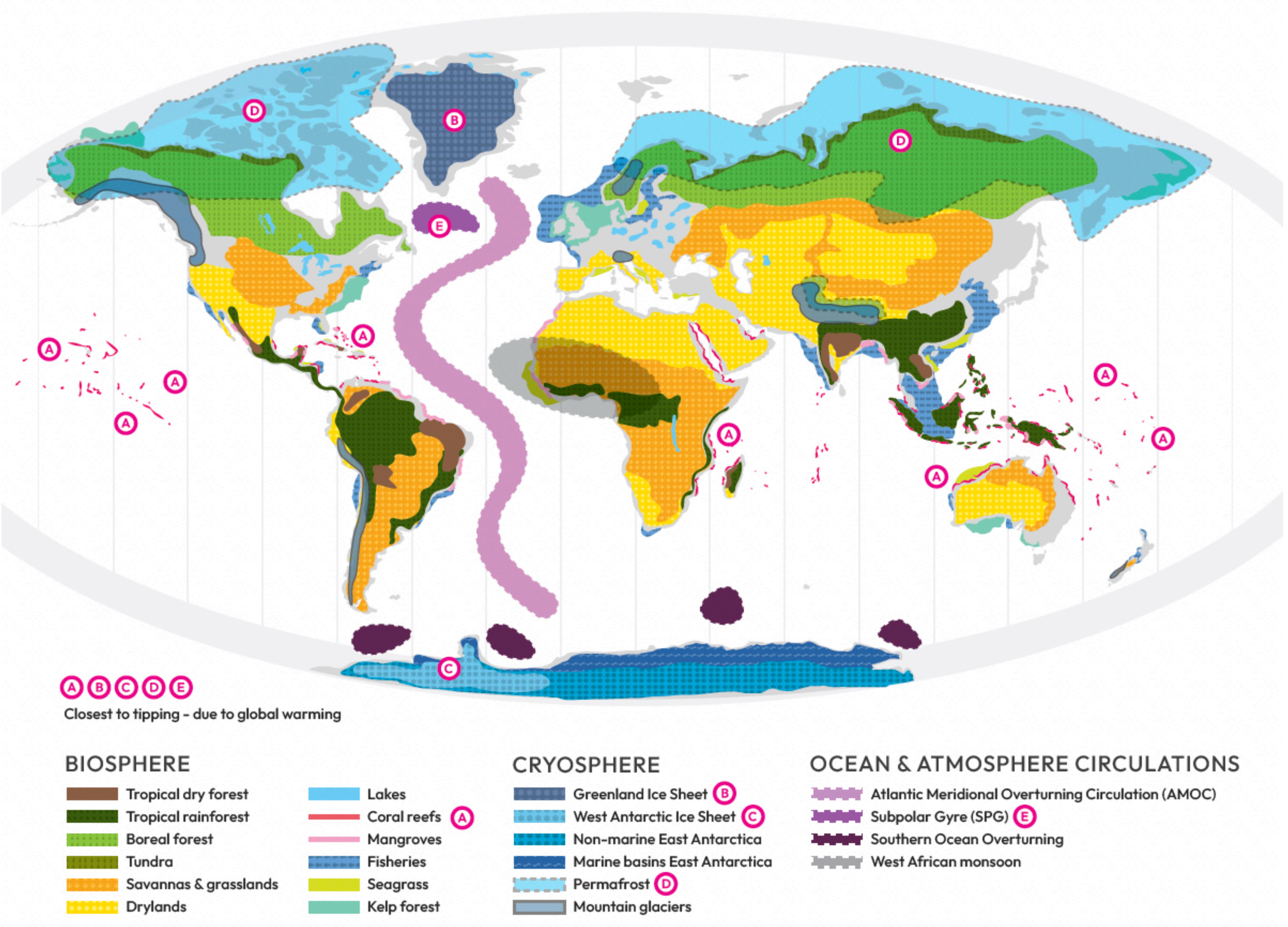
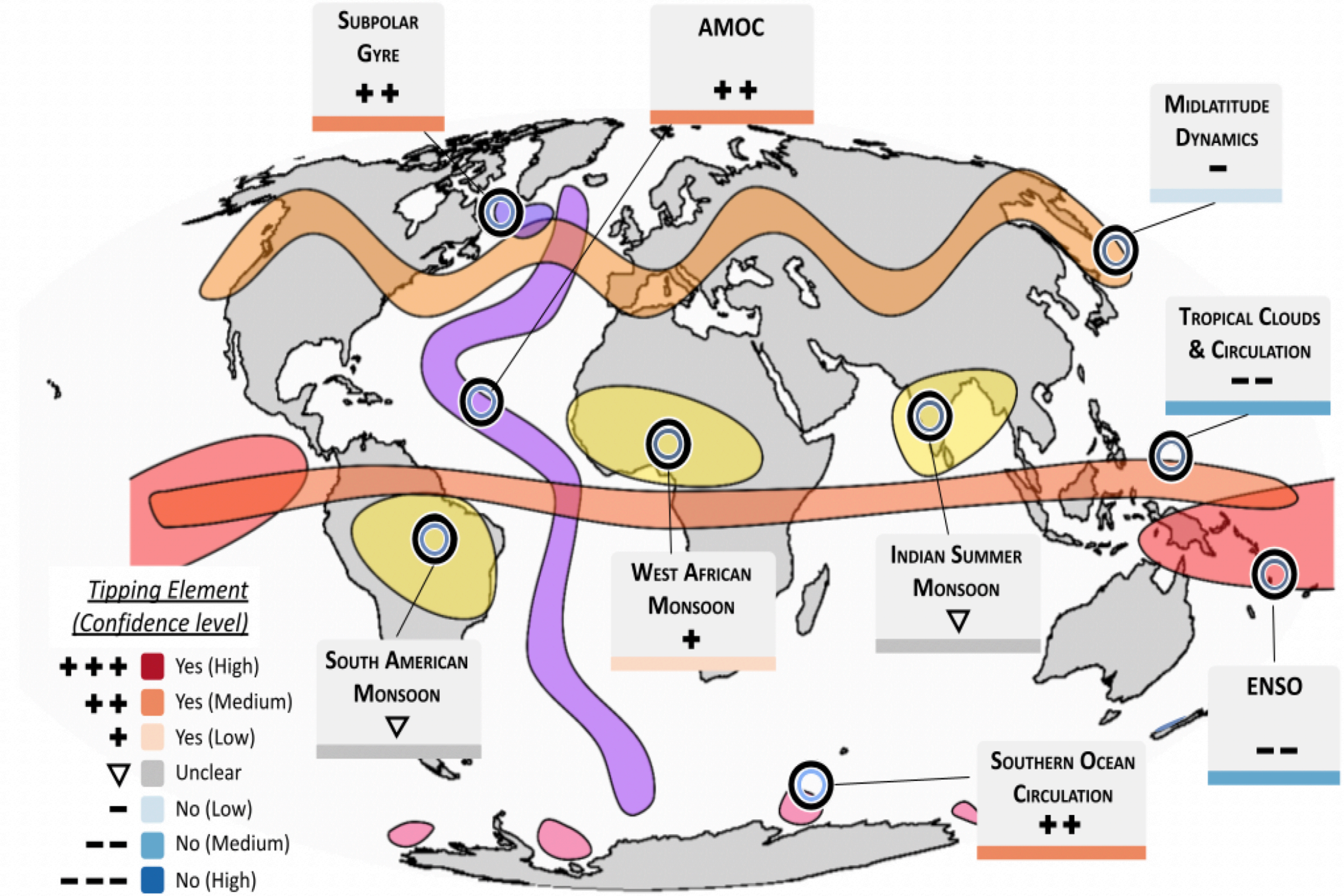
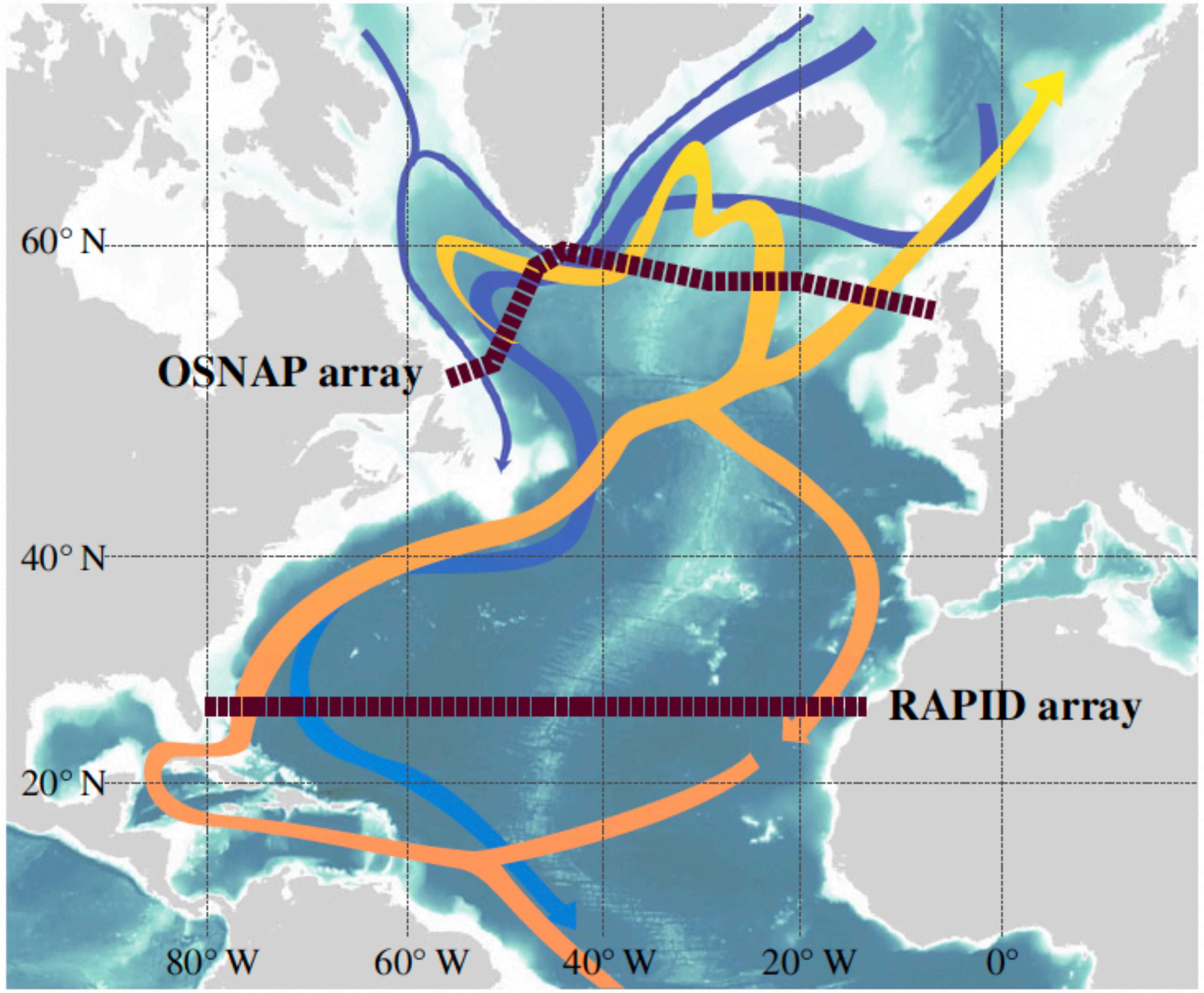
Climate subsystems
that can cross
a tipping point



Tipping Elements: Ocean-atmosphere circulation

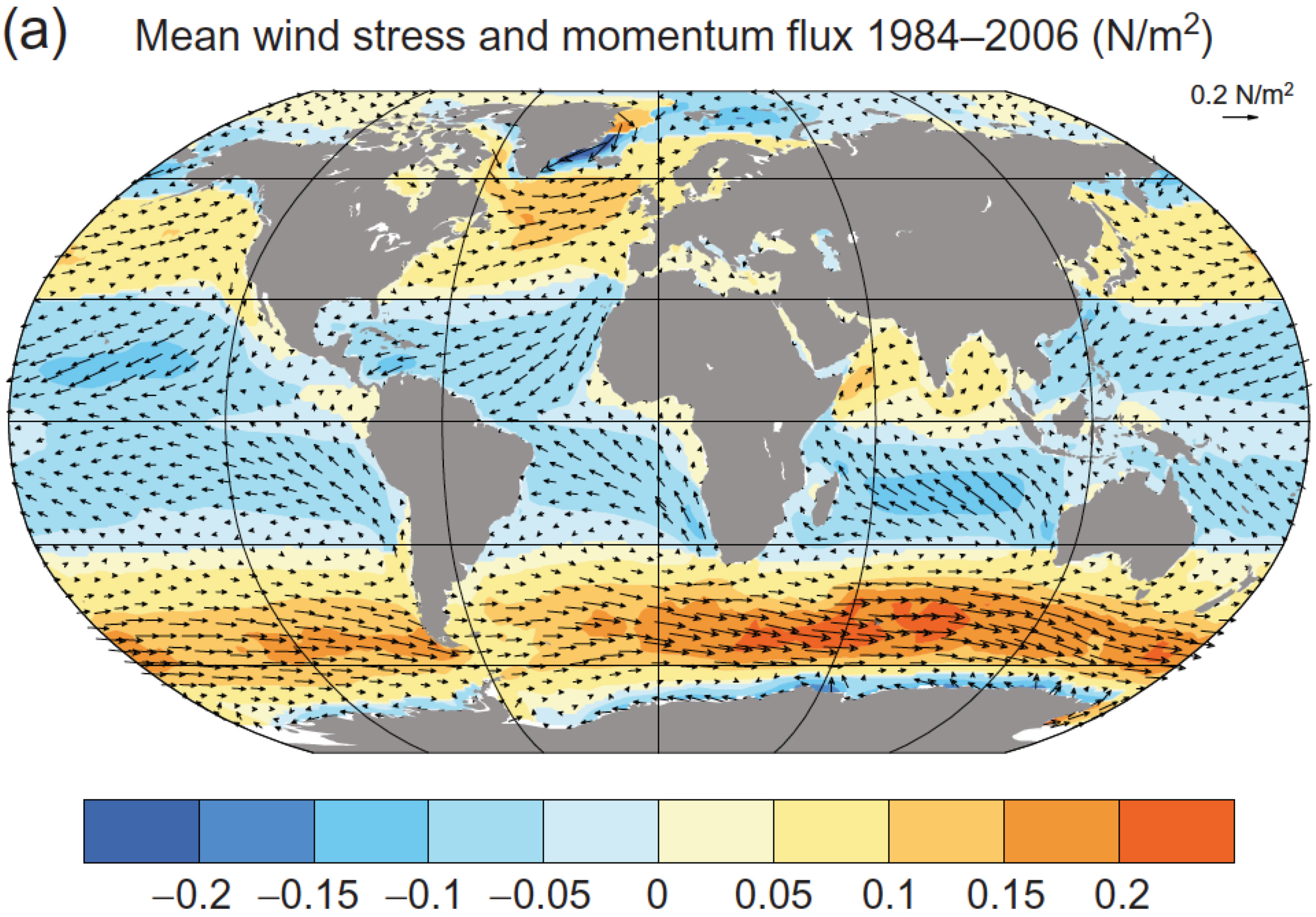


Tipping Element: Atlantic Meridional Ocean Circulation



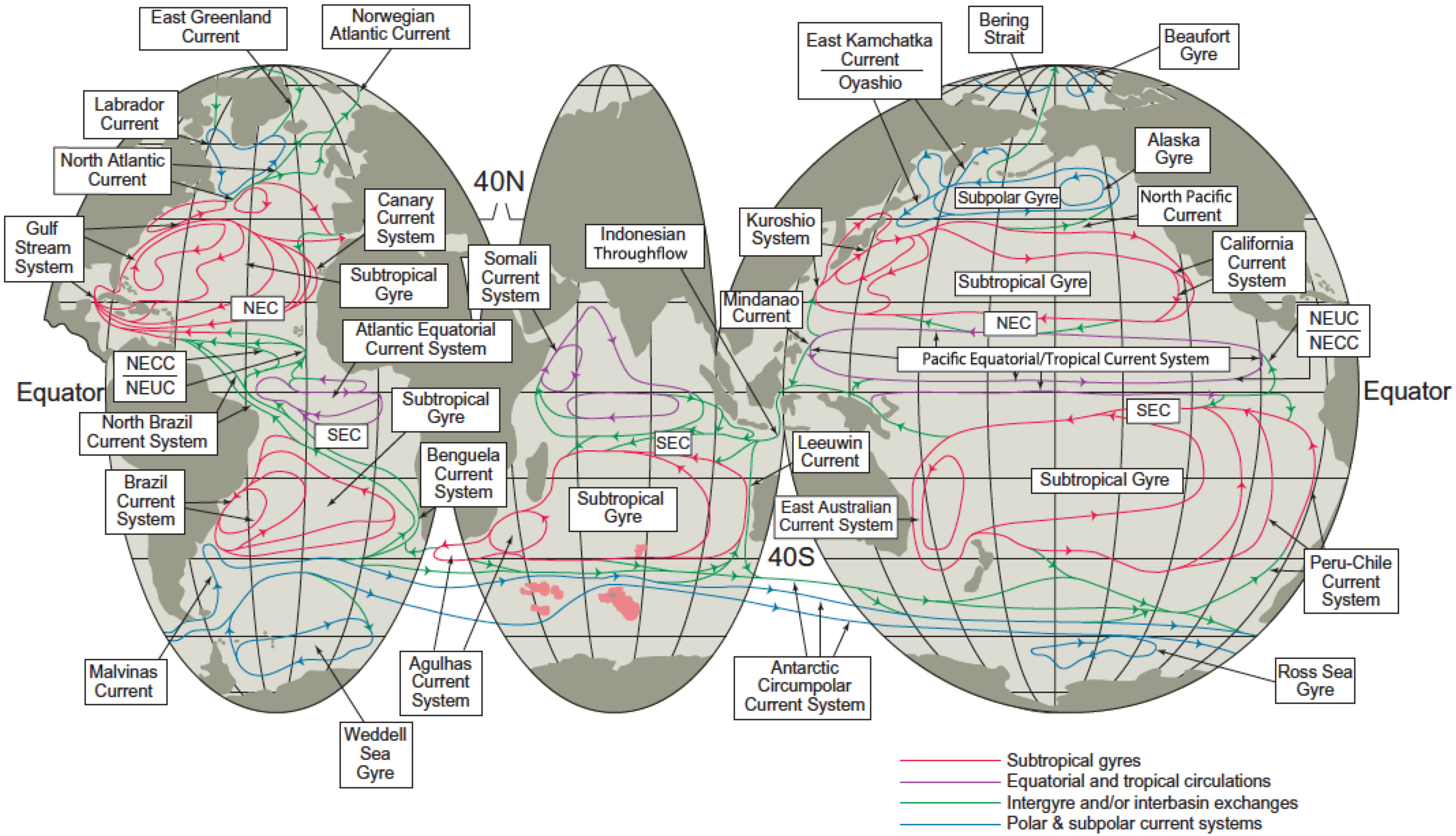
Srokosz et al. (2023)

Wind stress

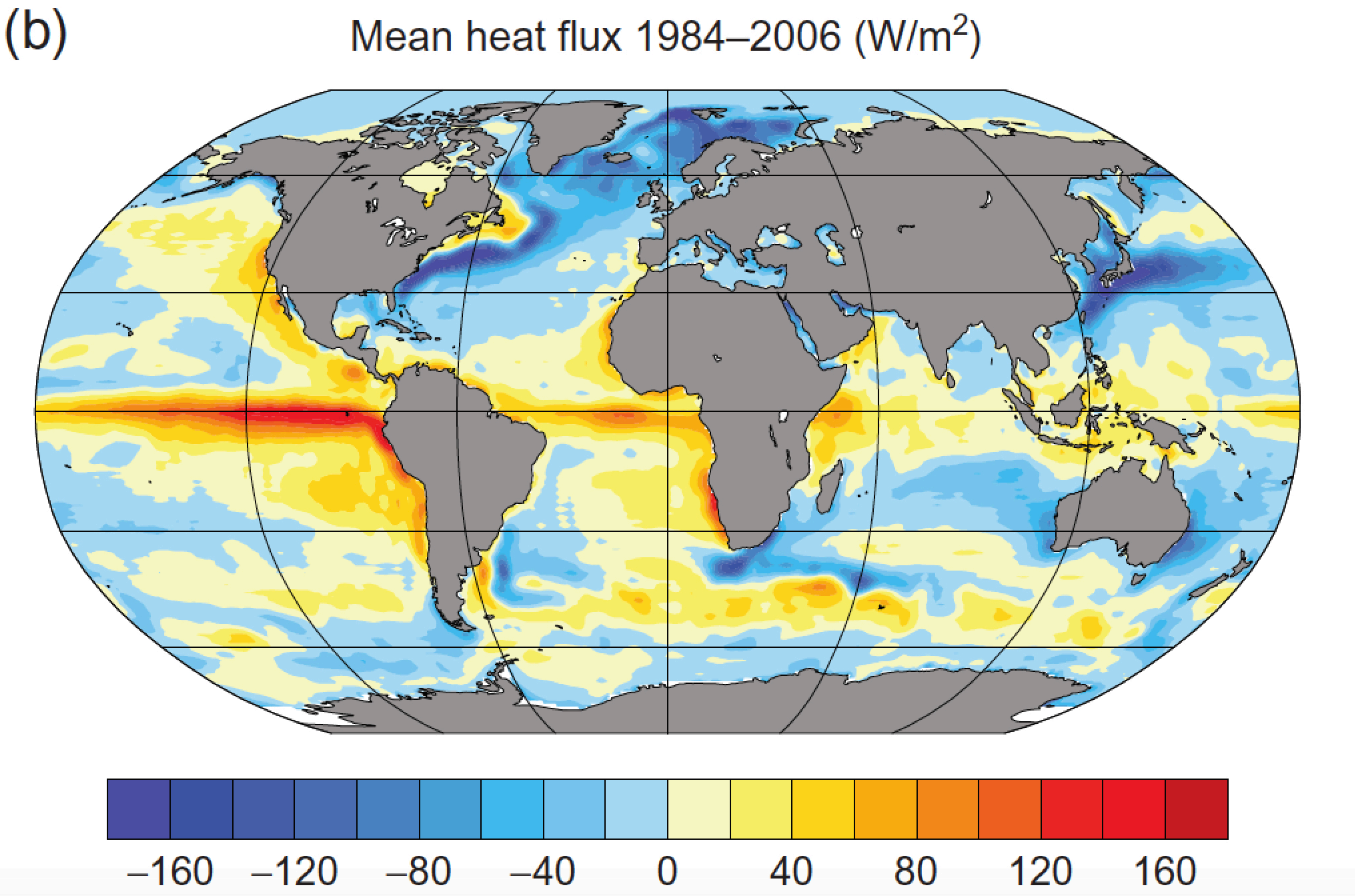


(a) Mean wind stress and momentum flux 1984–2006 (N/m²)
0.2 N/m²
–0.2
–0.15
–0.1
–0.05
0
0.05
0.1
0.15
0.2

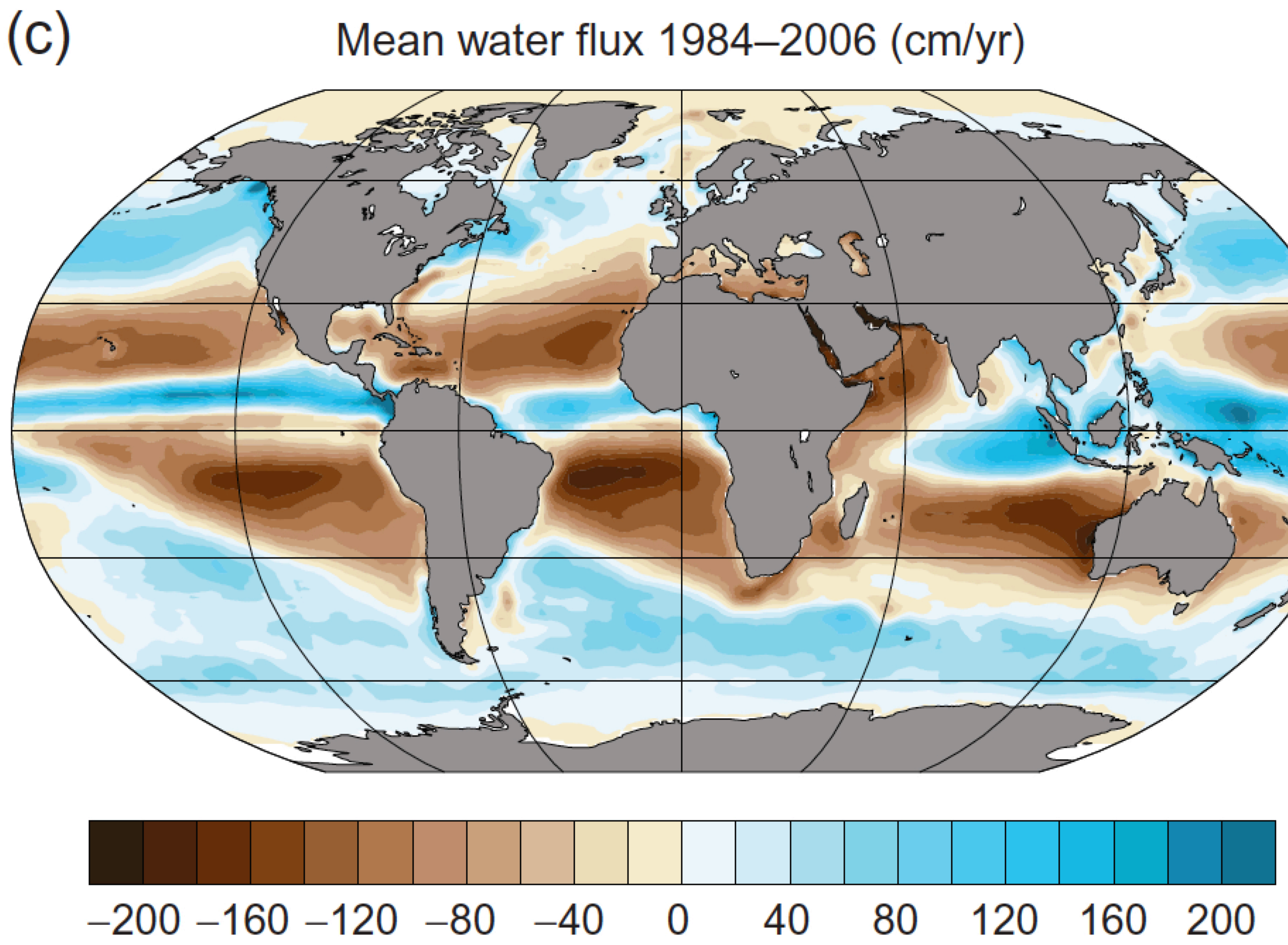
Global Ocean Circulation



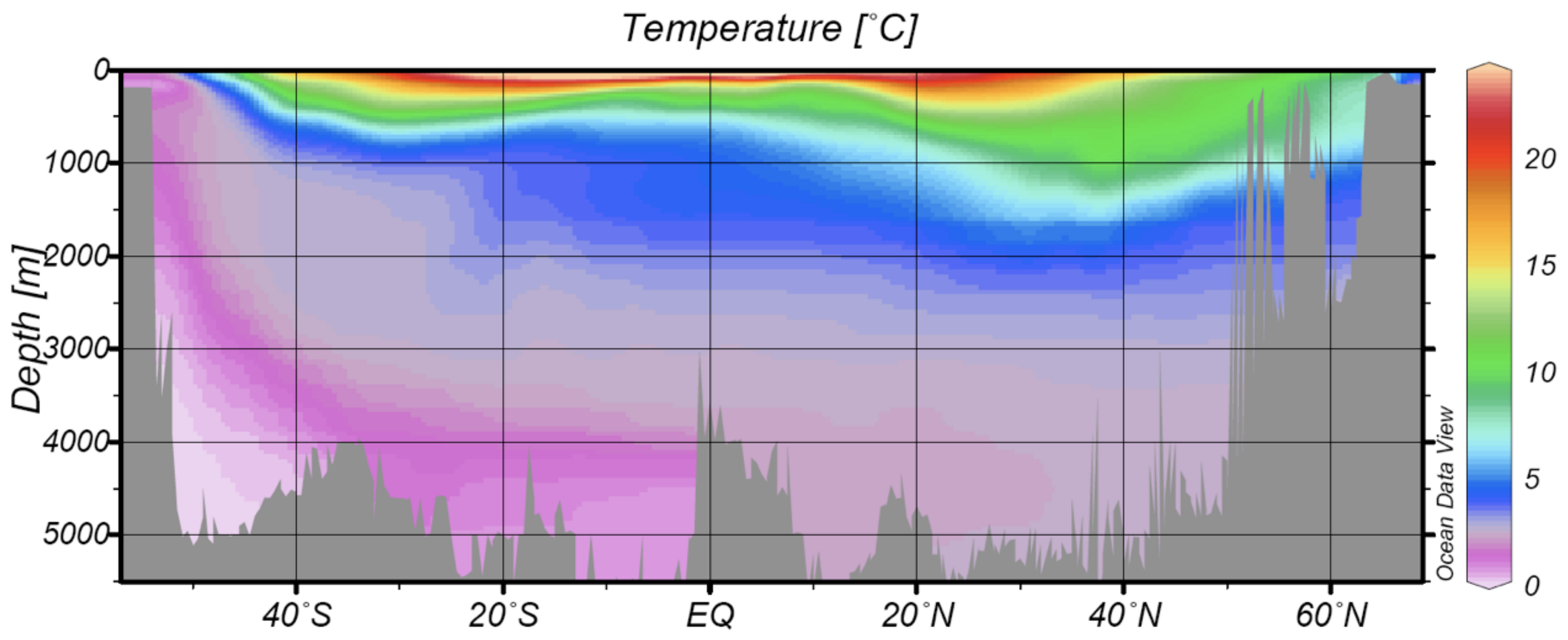
Heat Flux

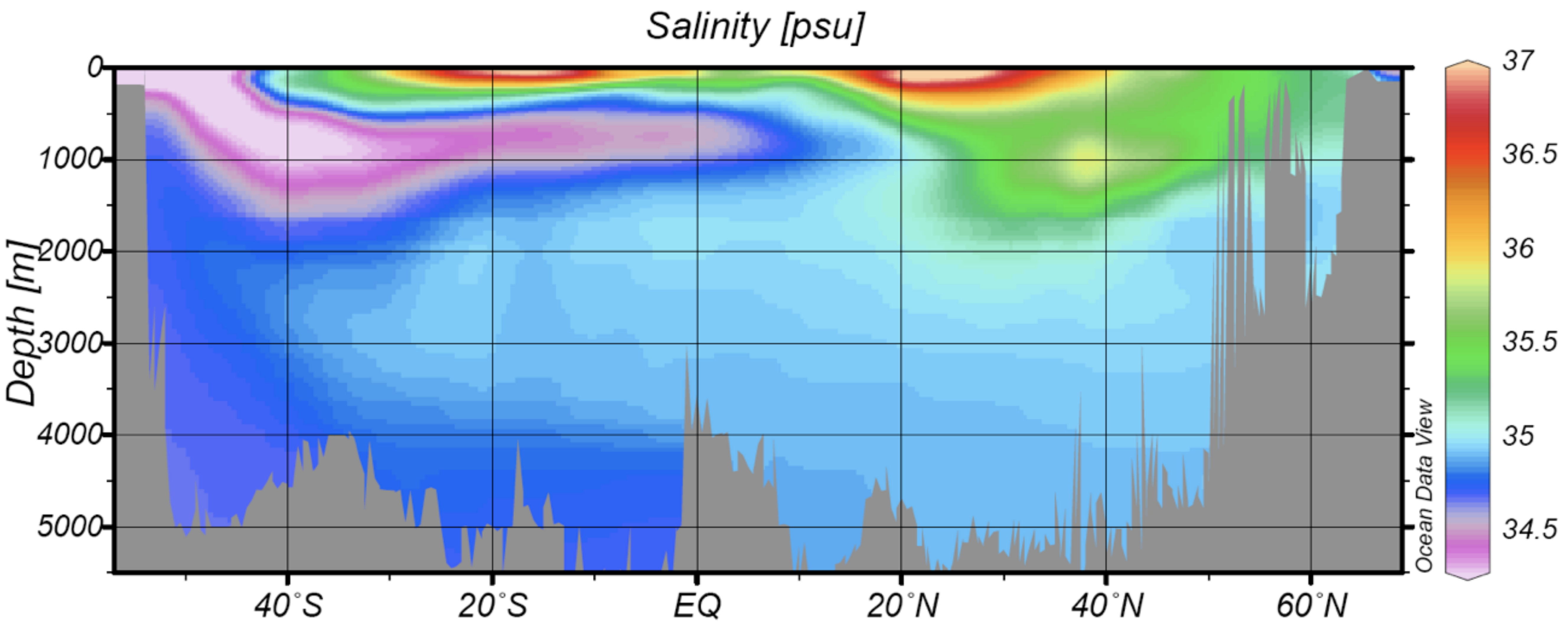


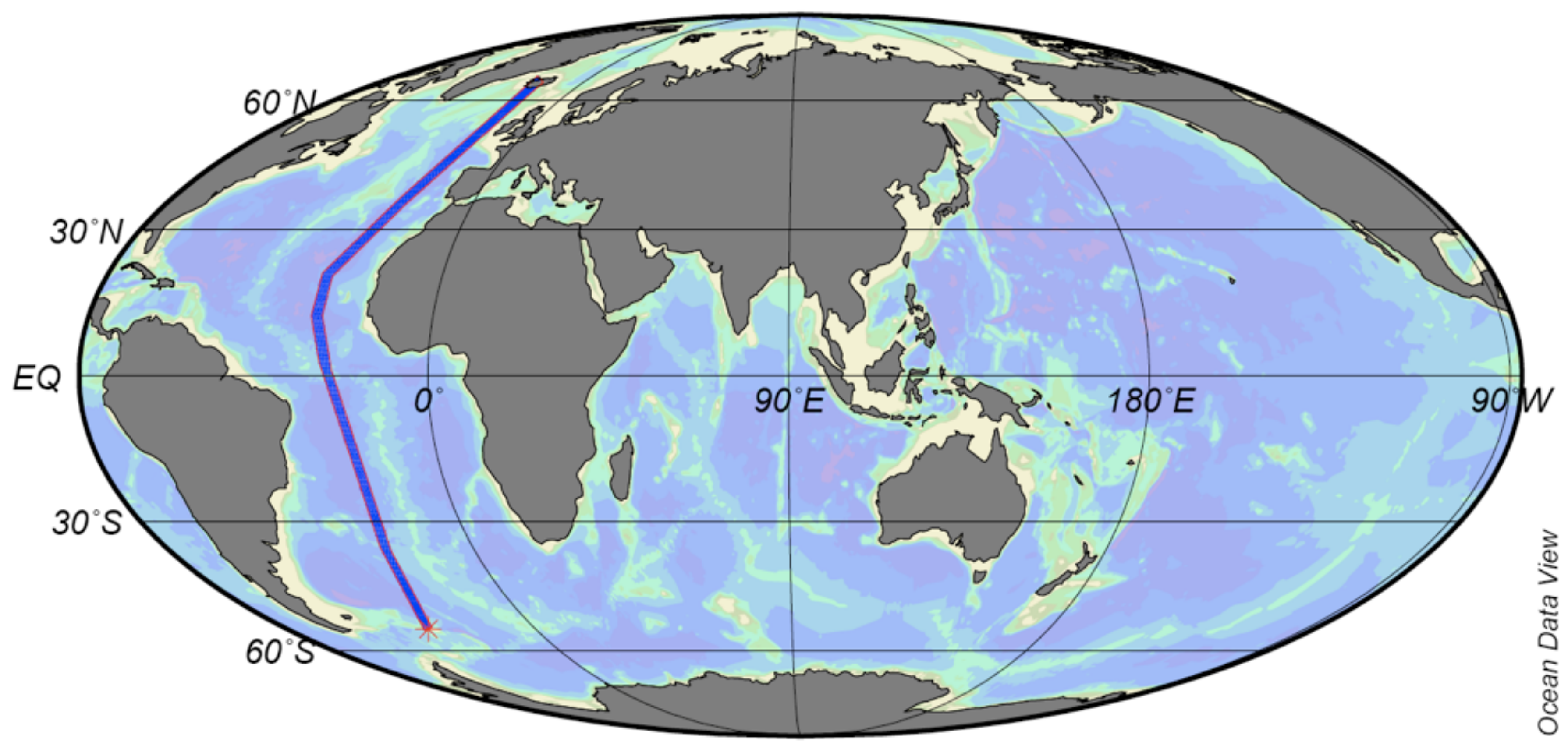
Freshwater Flux



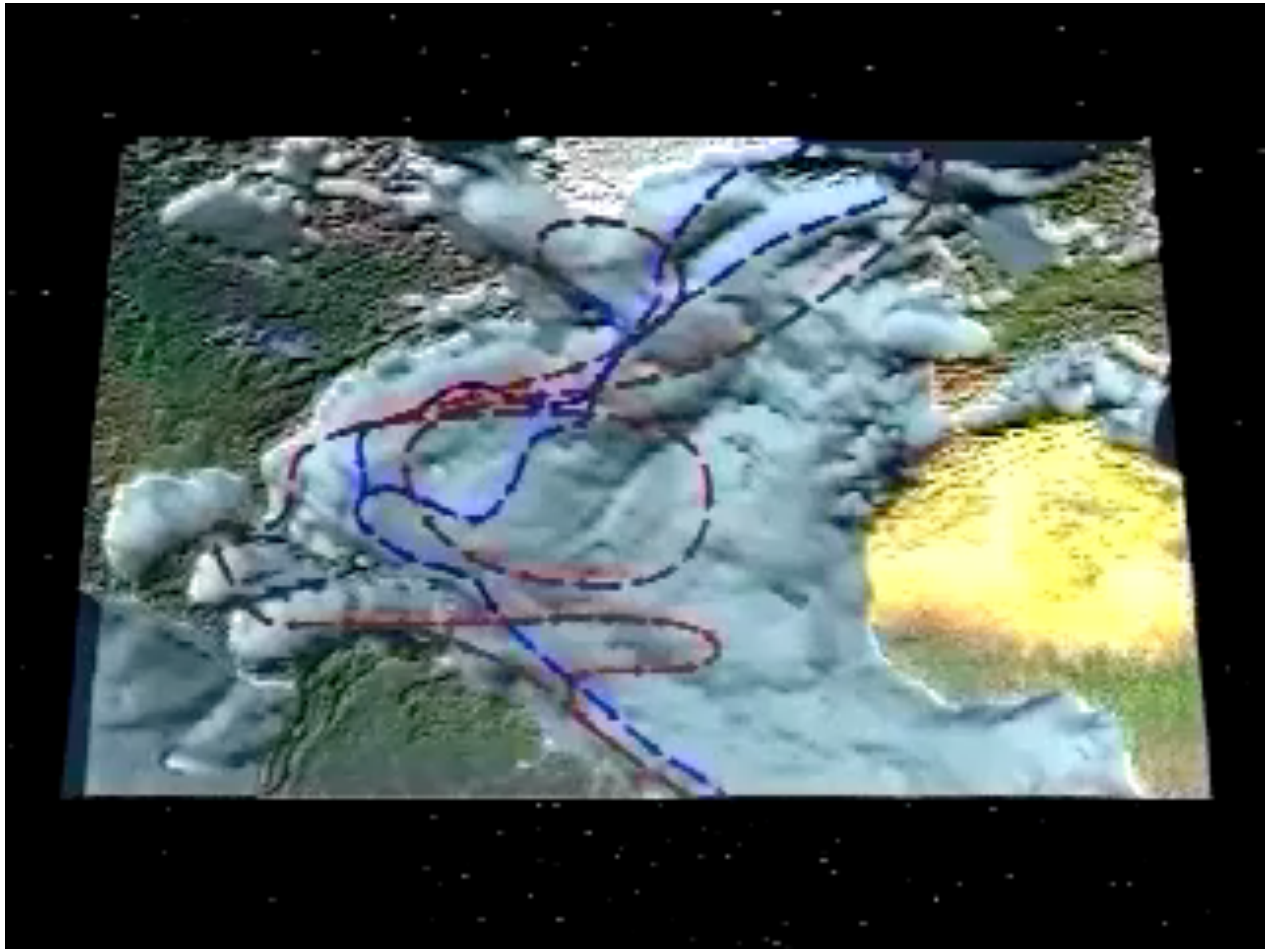
Atlantic Section Profiles





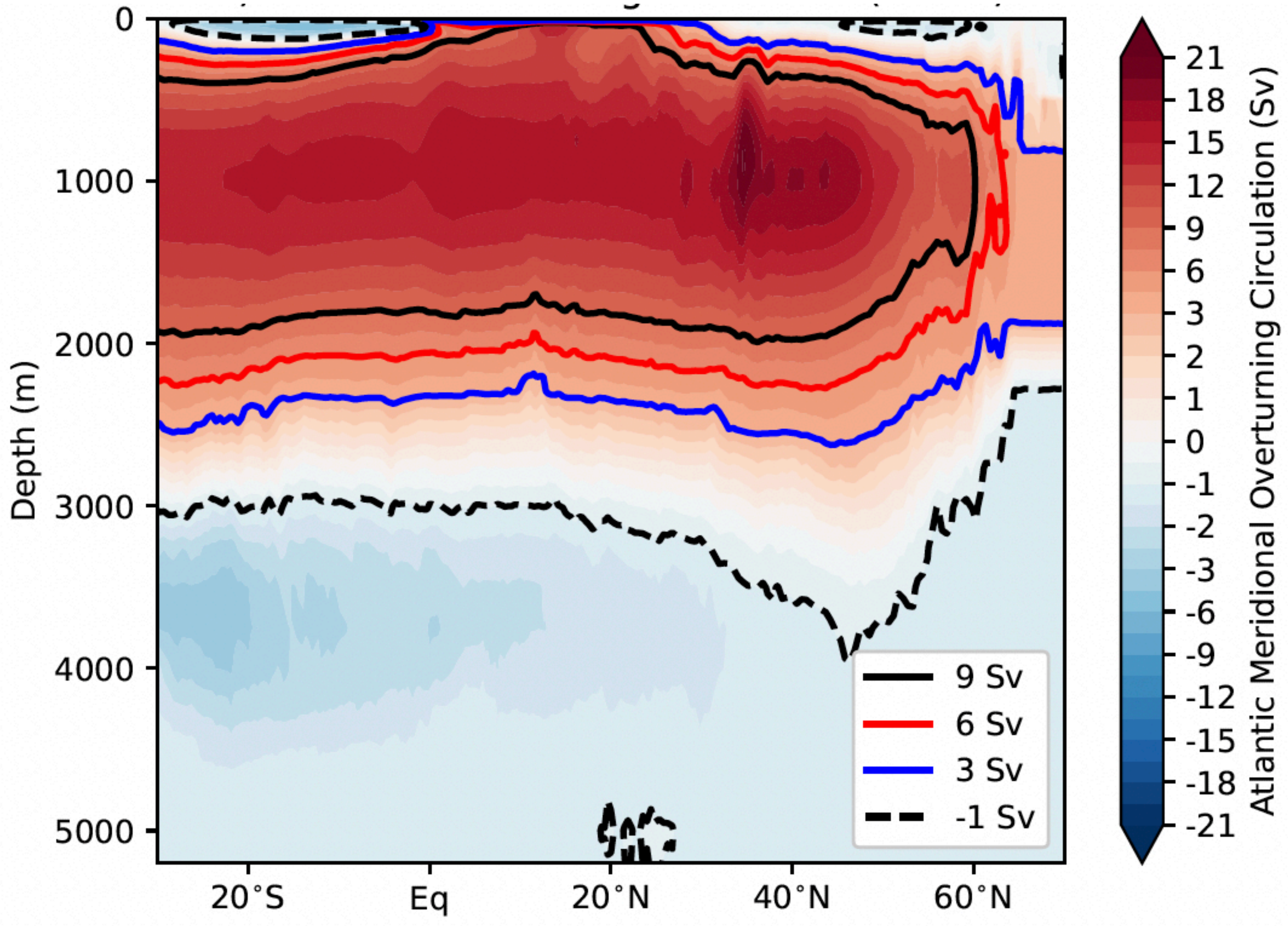


Atlantic Meridional Overturning Circulation (AMOC)



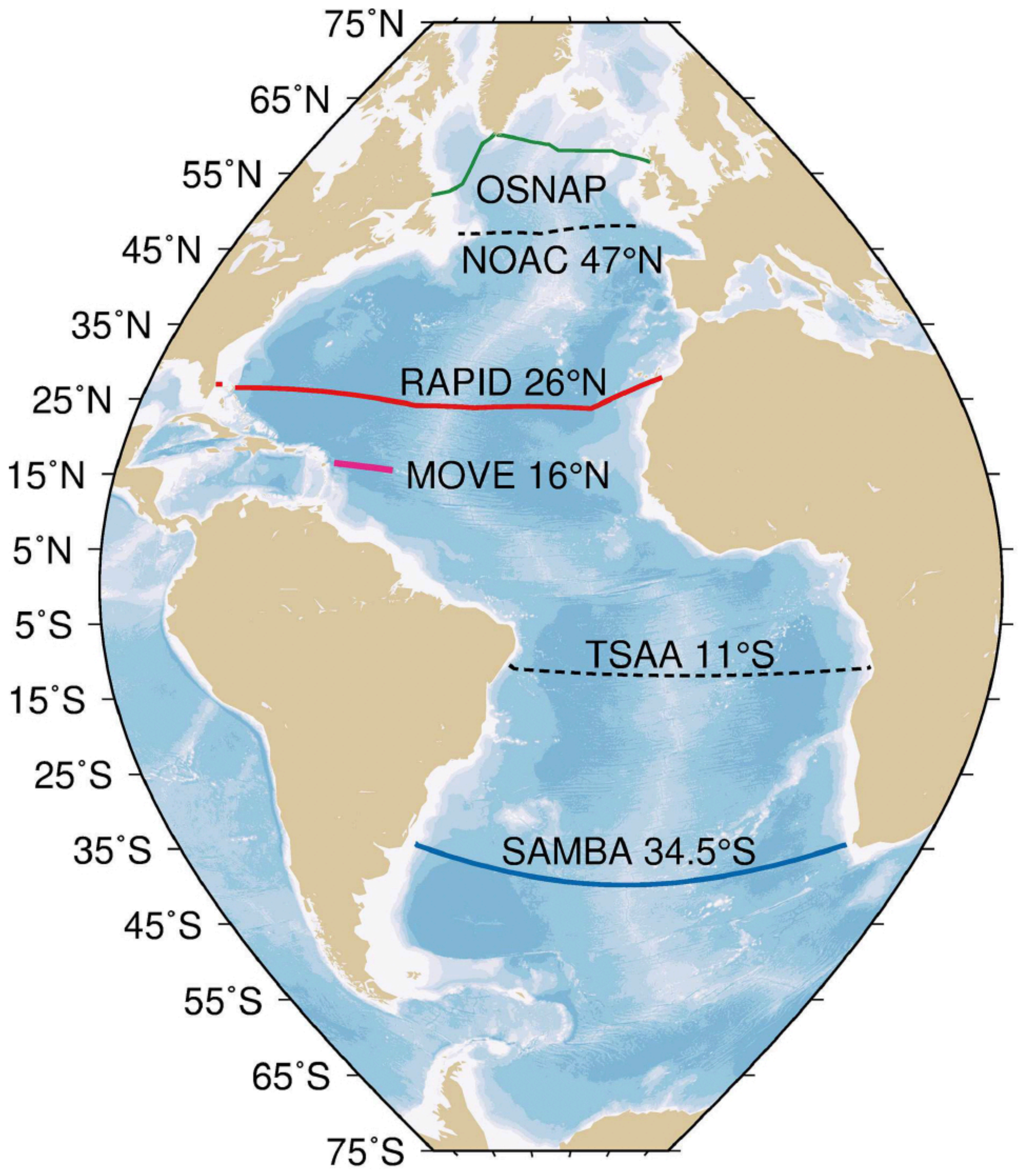
courtesy: WHOI

(Model) AMOC stream function

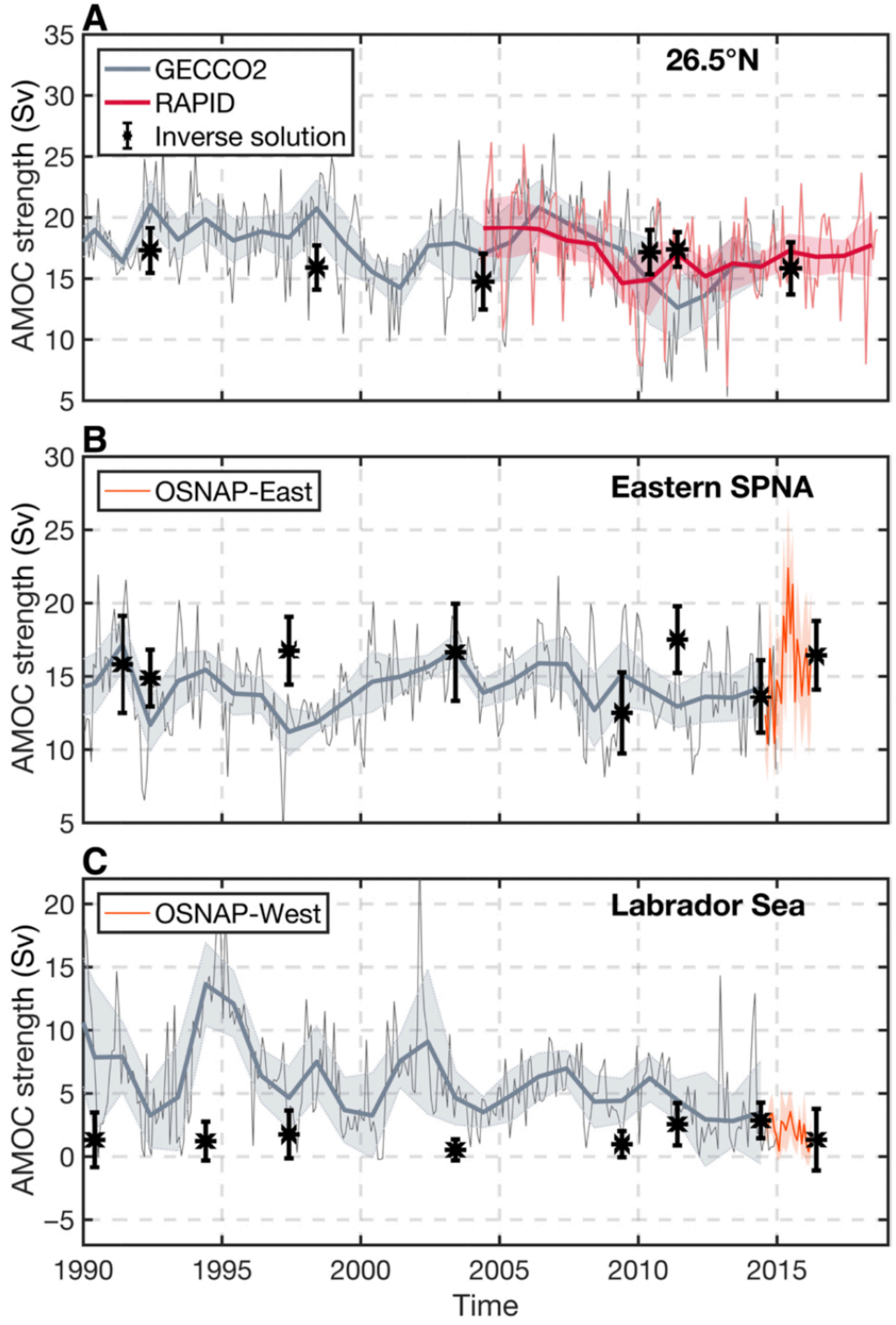


Unit: 1 Sv = 10^6 m^3/s
(Amazon river: 0.1 Sv)

AMOC observations



Frajka-Williams et al. (2019)



Fu et al. (2020)

Recent (Alarming) Papers

ARTICLES
https://doi.org/10.1038/s41558-021-01097-4
nature climate change
Check for updates

Observation-based early-warning signals for a collapse of the Atlantic Meridional Overturning Circulation

Niklas Boers[1,2,3]

CLIMATE AND ENVIRONMENT

A critical ocean system may be heading for collapse due to climate change, study finds

'The consequences of a collapse would likely be far-reaching'

By Sarah Kaplan

August 5, 2021 at 11:01 a.m. EDT
The Washington Post

nature communications

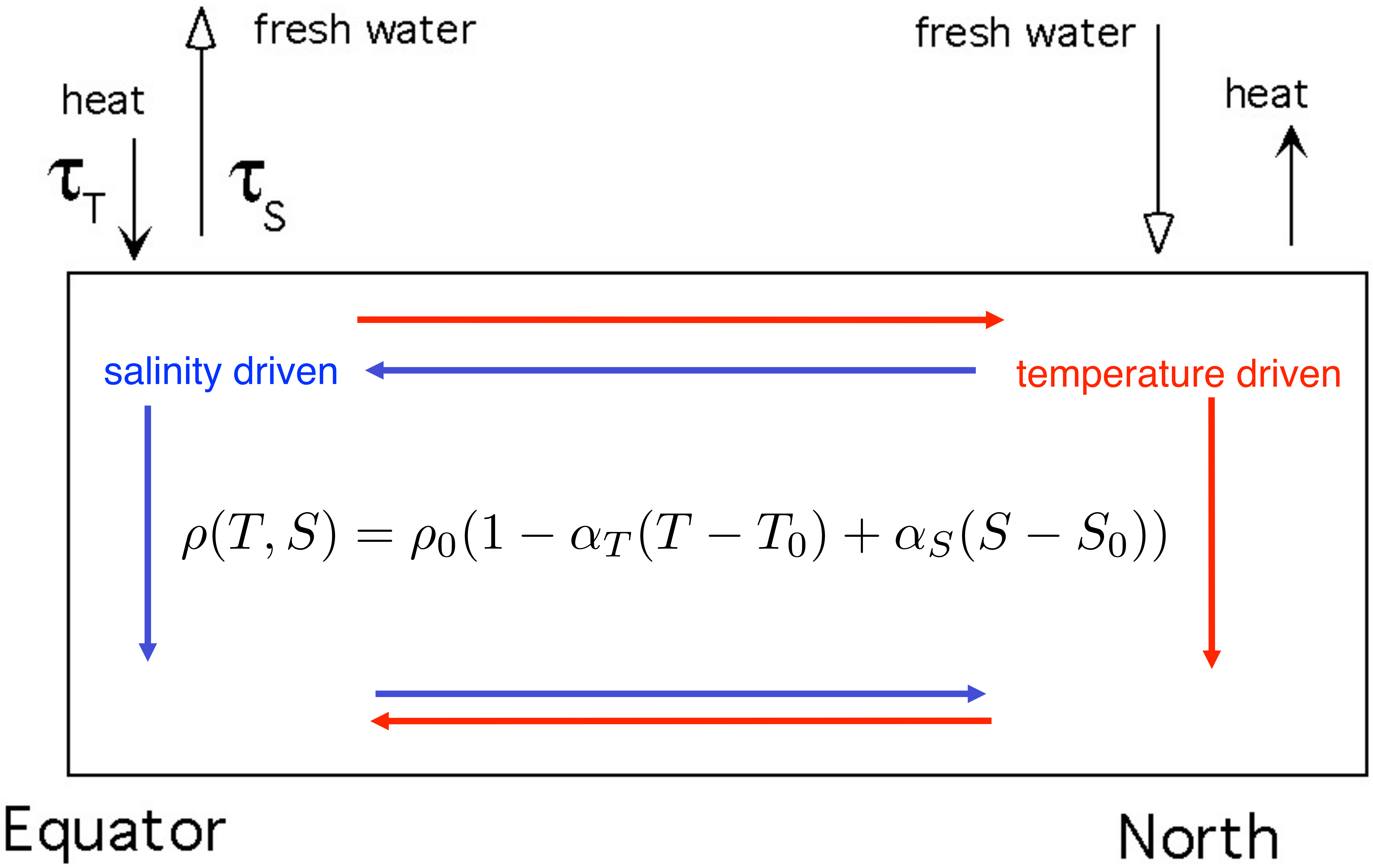
Article
https://doi.org/10.1038/s41467-023-39810-w

Warning of a forthcoming collapse of the Atlantic meridional overturning circulation

Received: 3 March 2023
Accepted: 29 June 2023

Peter Ditlevsen[1,3] & Susanne Ditlevsen[2,3]

Physics of Potential AMOC collapse



$$\rho(T,S) = \rho_0(1 - \alpha_T(T - T_0) + \alpha_S(S - S_0))$$

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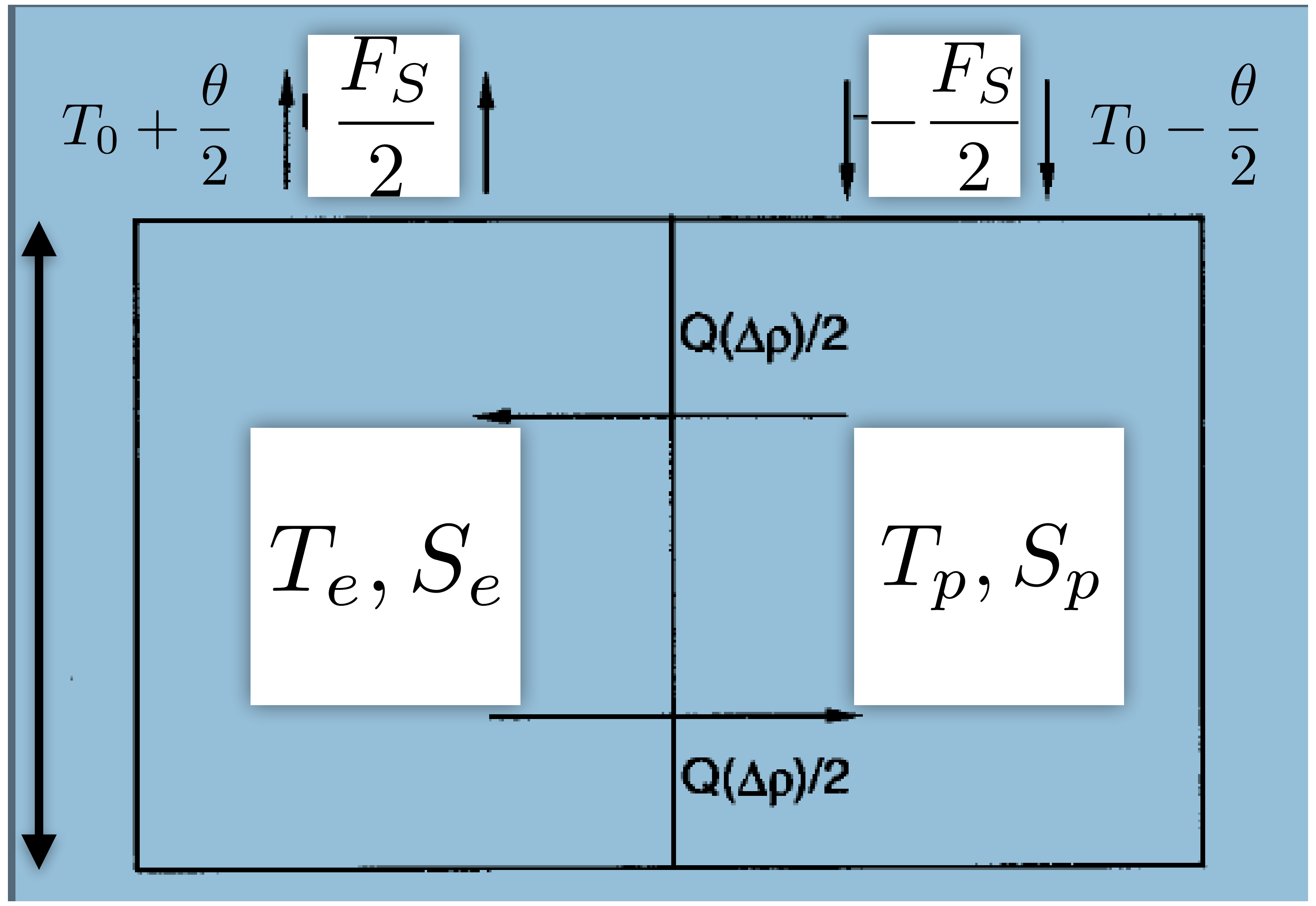
AMOC Tipping: North Atlantic Model

$$\frac{dT_e}{dt} = -\frac{1}{t_r}(T_e - (T_0 + \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_e - T_p),$$
$$\frac{dT_p}{dt} = -\frac{1}{t_r}(T_p - (T_0 - \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_p - T_e),$$
$$\frac{dS_e}{dt} = \frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_e - S_p),$$
$$\frac{dS_p}{dt} = -\frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_p - S_e),$$

Stommel, Tellus, (1961)



Equator Pole

$$\rho = \rho_0(1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)),$$

$$Q(\Delta\rho) = \frac{1}{t_d} + \frac{q}{\rho_0^2 V}(\Delta\rho)^2,$$

Cessi, JPO, (1994)

Dimensionless model

$$\frac{dx}{dt} = -\alpha(x-1) - x(1+\mu^2(x-y)^2),$$
$$\frac{dy}{dt} = F - y(1+\mu^2(x-y)^2),$$

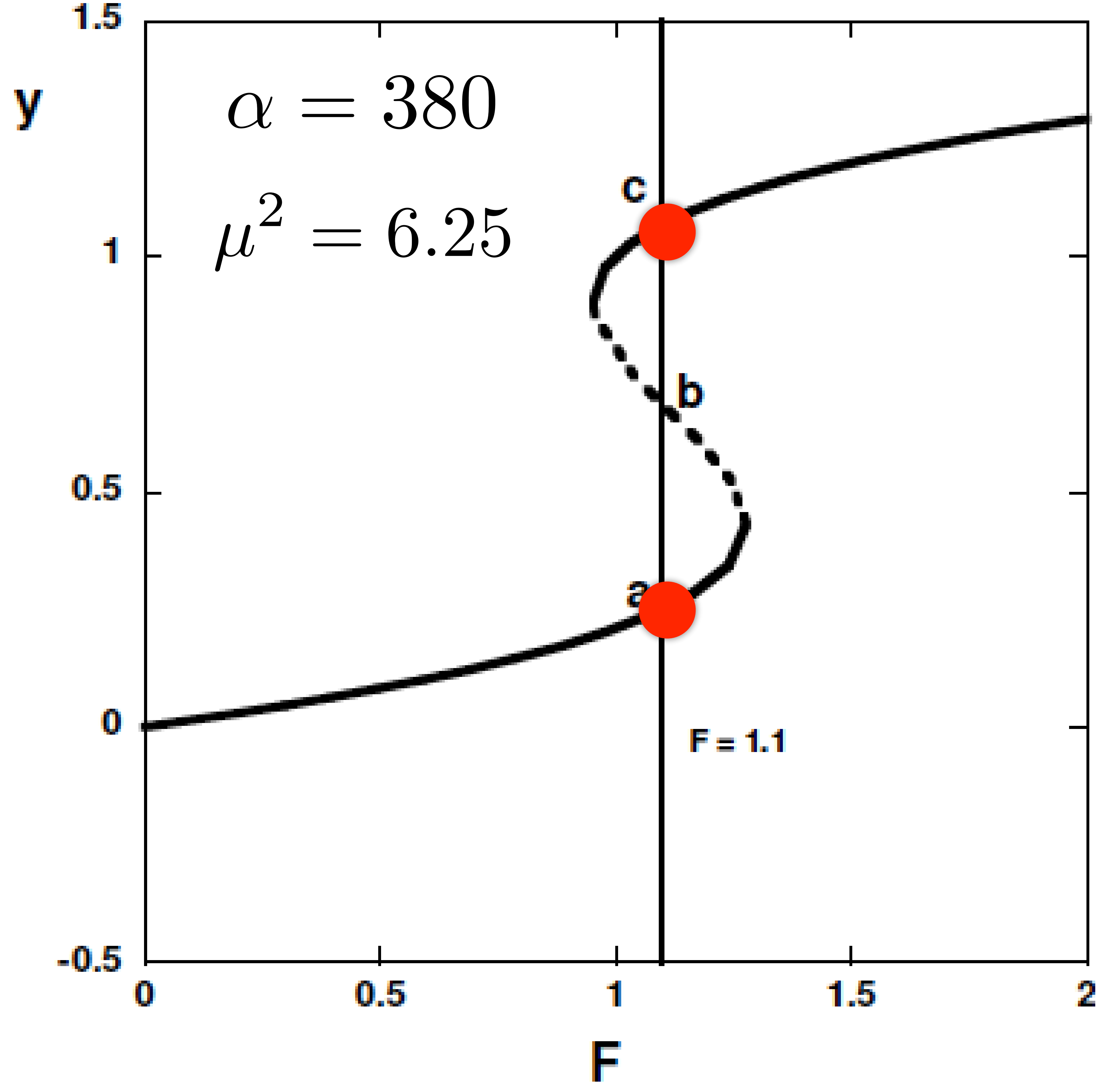
Parameter	Meaning	Value	Unit
t_r	temperature relaxation time scale	25	days
H	mean ocean depth	4500	m
t_d	diffusion time scale	180	years
t_a	advective time scale	29	years
q	transport coefficient	1.92×10^{12}	$m^3\ s^{-1}$
V	ocean volume	$300 \times 4.5 \times 8200$	km^3
α_T	thermal expansion coefficient	10^{-4}	K^{-1}
α_S	haline contraction coefficient	$7.6 \quad 10^{-4}$	-
S_0	reference salinity	35	$g\ kg^{-1}$
θ	meridional temperature difference	25	K

$$\mu^2 = \frac{q t_d (\alpha_T \theta)^2}{V}$$

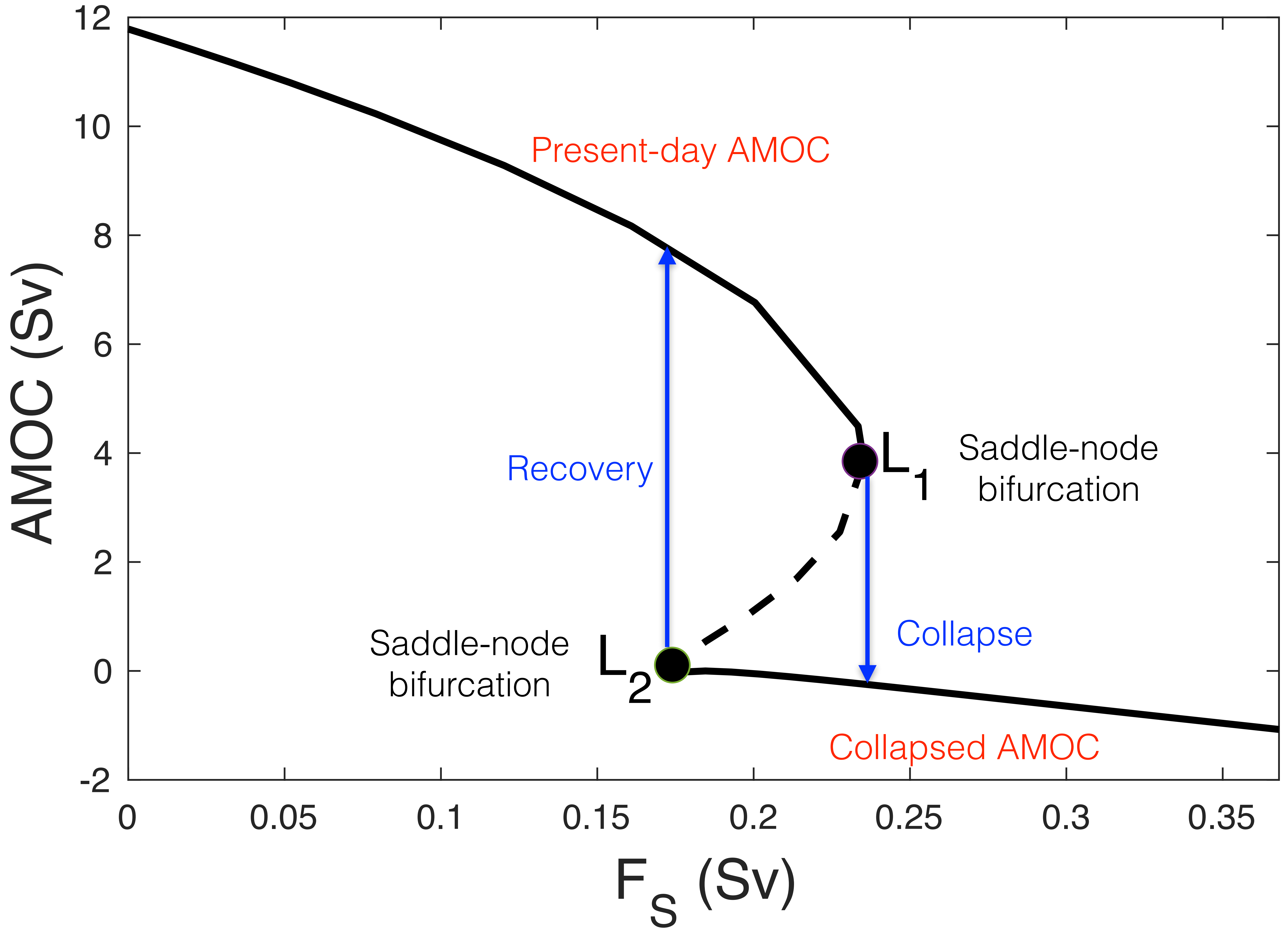
$$\alpha = t_d / t_r$$

$$F = \frac{\alpha_S S_0 t_d}{\alpha_T \theta H} F_S.$$

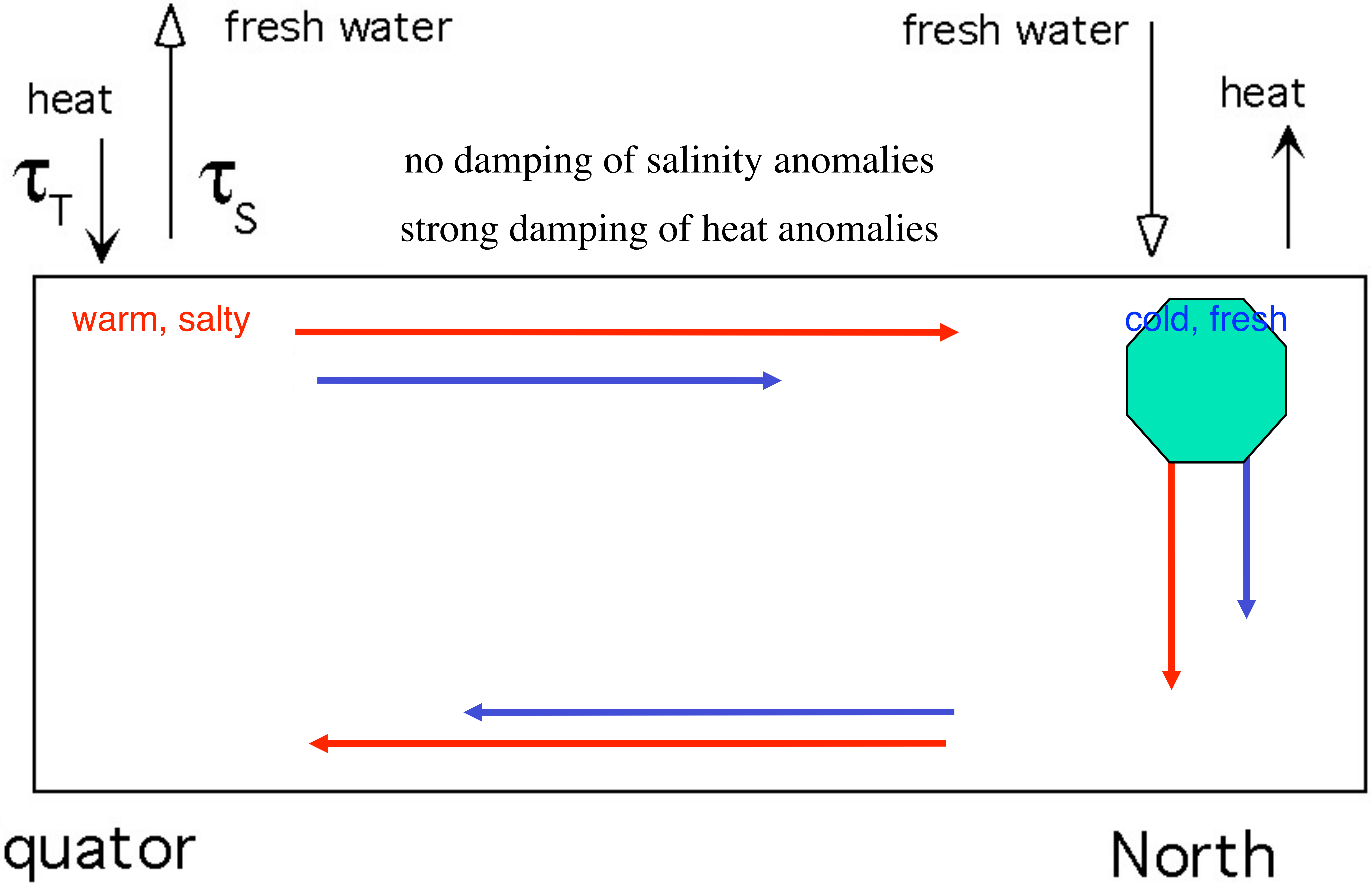
Bifurcation Diagram



Bifurcation Diagram (dimensional)



Salt advection feedback

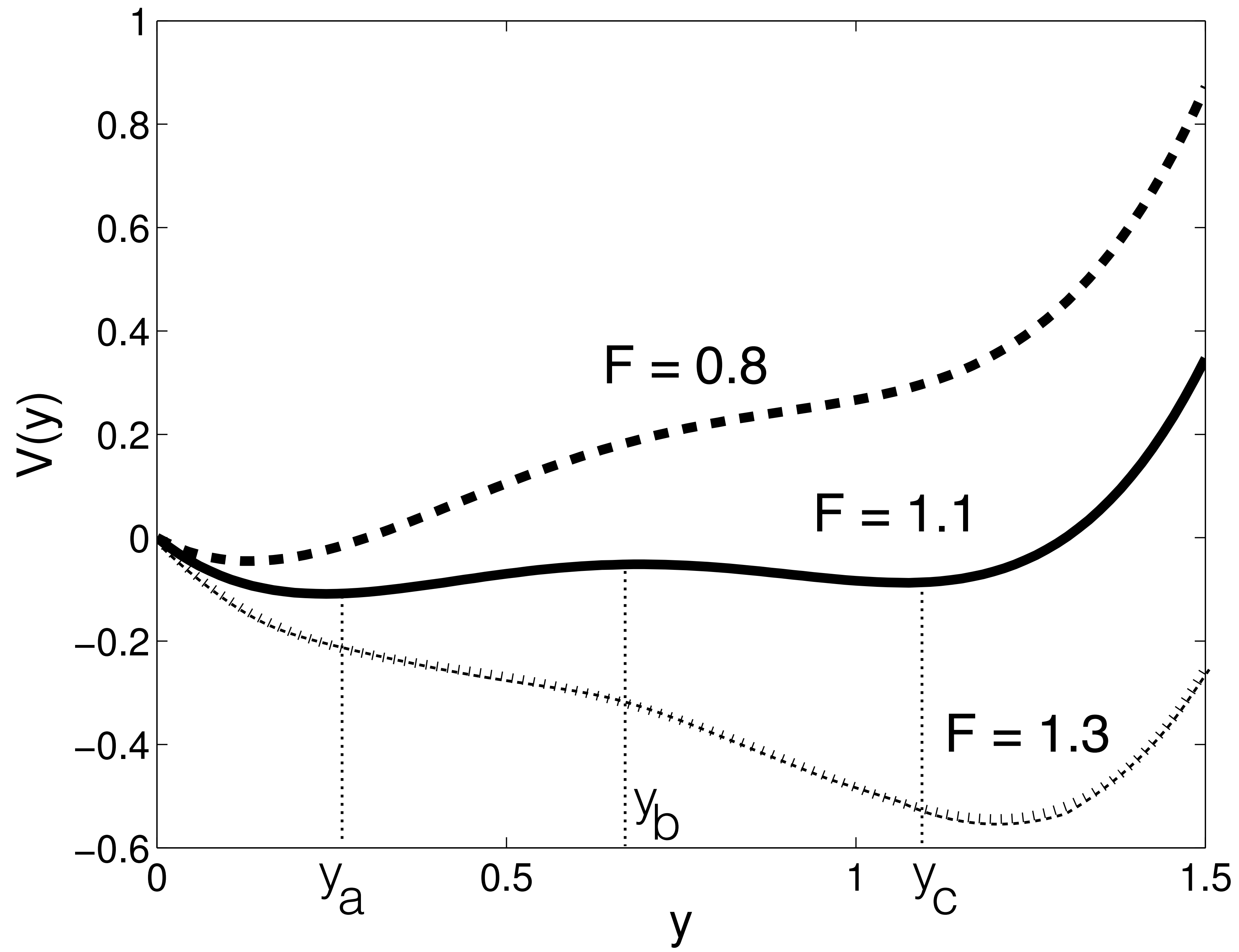


Main mechanism of AMOC collapse

Model reduction

$$\frac{dx}{dt} = -\alpha(x-1) - x(1+\mu^2(x-y)^2),$$
$$\frac{dy}{dt} = F - y(1+\mu^2(x-y)^2),$$

$$\alpha \to \infty \quad : \qquad \dot{y} = -V'(y) \; ; \; V(y) = -(Fy - \frac{y^2}{2} - \mu^2(\frac{y^2}{2} - \frac{2y^3}{3} + \frac{y^4}{4}))$$



Example of a Gradient System

Effects of Noise

$$\frac{dy}{dt} = -V'(y) + \sigma\xi$$

Freshwater noise product

White noise:

$$E[\xi(t)] = 0$$

$$E[\xi(t)\xi(s)] = \delta(t-s)$$

Stochastic Differential Equations

$$\frac{dy}{dt} = -V'(y) + \sigma\xi$$

Stochastic process: $X_t = y$

Wiener process: W_t $N(0,t)$ distributed

$$E[(dW_t)^2] = dt$$

$$\boxed{\begin{aligned} dX_t &= f(X_t)dt + \sigma dW_t \\ X_0 &= X(0) \end{aligned}}$$

Numerical Solution of SDEs

$$X(t) = X(0) + \int_0^t f(X(s))ds + \int_0^t g(X(s))dW(s)$$

$$\tau_j = j\Delta t, j = 0, \ldots, n \text{ on } [0, T]$$

$$\Delta t = T/n$$

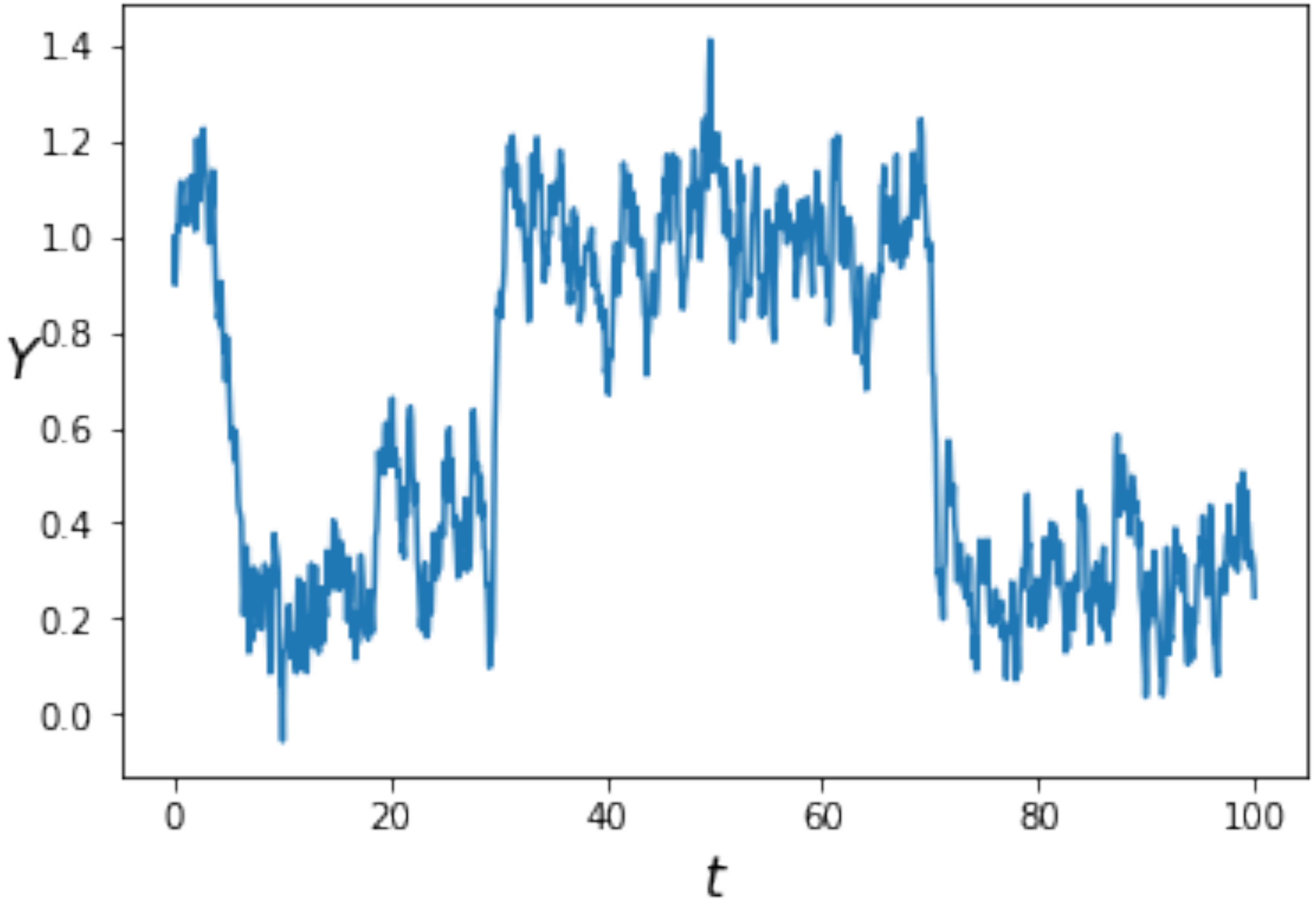
Gisiro Maruyama (1916-1986)

Euler-Maruyama scheme:

$$X_j - X_{j-1} = f(X_{j-1})\Delta t + g(X_{j-1})(W(\tau_j) - W(\tau_{j-1}))$$

Effects of Noise

$$\frac{dy}{dt} = -V'(y) + \sigma\xi$$

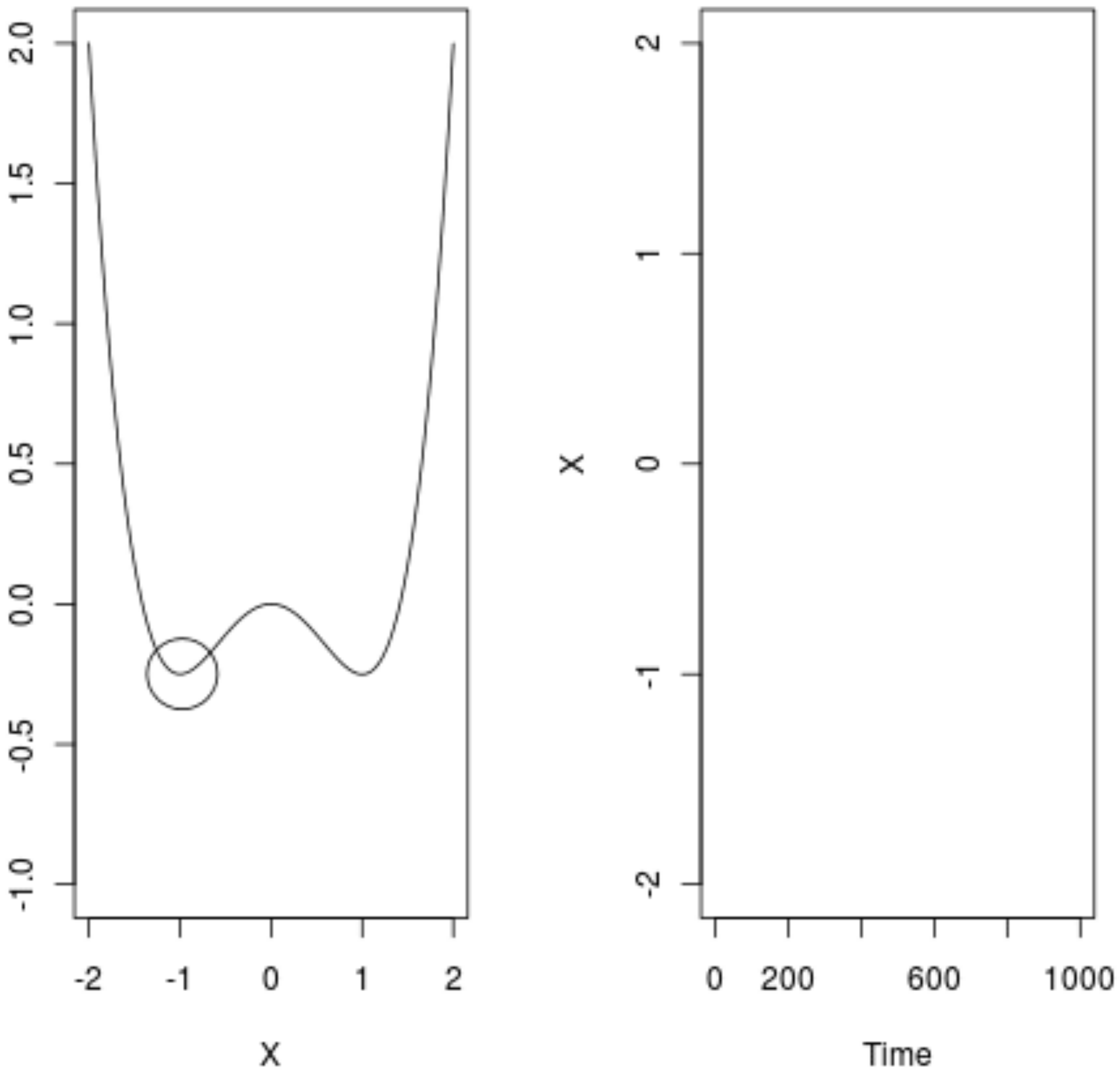


$$E[\xi(t)] = 0$$

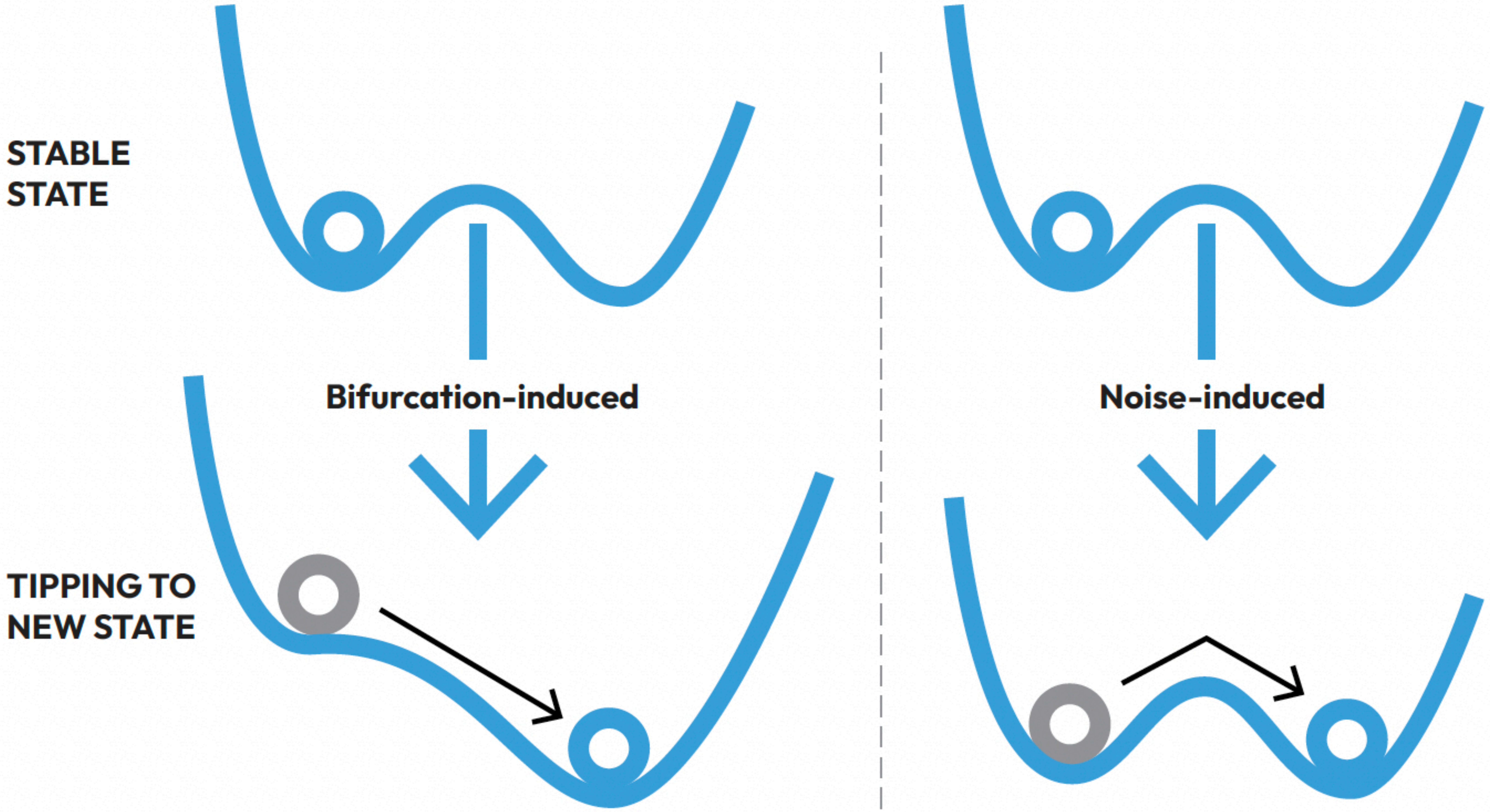
$$E[\xi(t)\xi(s)] = \delta(t-s)$$

White noise

Noise induced Tipping Behavior



Types of Tipping



Bifurcation induced transition

Noise induced transition

Exercises!