Notes

7/2/24

Conceptual AMOC model

Stommel (1961)

- equations, ~~se~~ see slides

- towards dimensionless equations:

(i) $\Delta T = T_e - T_p$

$$\rightarrow \begin{cases} \dfrac{d\Delta T}{dt} = -\dfrac{1}{t_r}\left(\Delta T - \theta\right) - Q(\Delta \rho)\,\Delta T \\ \Delta S = S_e - S_p \\ \dfrac{d\Delta S}{dt} = \dfrac{F_s}{H} S_e - Q(\Delta \rho)\,\Delta S \end{cases}$$

$$\tilde{t} = t / t_d$$

(ii) $\Delta T = x\,\theta$, $\Delta S = y \dfrac{\alpha_T}{\alpha_S}\,\theta$

$$\rightarrow Q(\Delta \rho) = \frac{1}{t_d} + \frac{q}{\rho_0^2 V} (\Delta \rho)^2$$

$$\Delta \rho = \rho_e - \rho_p =$$

$$= -\alpha_T \Delta T + \alpha_S \Delta S$$

$$= -\alpha_T \theta x + \alpha_T \theta y$$

$$= \theta \alpha_T (y - x) .$$

$$\Rightarrow \frac{\cancel{\theta}}{t_d} \dot{x} = -\frac{1}{t_r} (x \cancel{\theta} - \overset{1}{\cancel{\theta}}) +$$

$$+ \left(\frac{1}{t_d} + \frac{q}{V} \theta^2 \alpha_T^2 (y - x)^2 \right) *$$

$$* \; x \cancel{\theta}$$

$$\longrightarrow \dot{x} = -\alpha (x - 1) + x \; *$$

$$* \left(1 + \mu^2 (y - x)^2 \right)$$

$$\frac{\frac{\alpha_T}{\alpha_S}\theta}{t_d}\,\dot{y} = \frac{F_S}{H}S_0 -$$

$$- \left(\frac{1}{t_d} + \frac{\mu^2}{t_d}\left(y - x\right)^2\right)\frac{\alpha_T}{\alpha_S}\theta\, y$$

$$\rightarrow \quad \dot{y} = F - \left(1 + \mu^2\left(y - x\right)^2\right) y$$

where $\boxed{\alpha} = \dfrac{t_d}{t_r}$

$$\boxed{\mu^2} = \frac{9\,\theta^2\alpha_T^2\,t_d}{V}$$

$$\boxed{F} = \frac{F_S}{H}S_0 \cdot \frac{t_d\,\alpha_S}{\alpha_T\,\theta}$$

<u>Values</u> : slide

$\alpha = 3\cancel{8}6$

$\mu^2 = 6.2$

$F = 1.1$

Finally

$$\begin{cases} \dot{x} = -\alpha(x-1) - x\left(1+\mu^2(x-y)^2\right) \\ \dot{y} = F - y\left(1+\mu^2(x-y)^2\right) \end{cases}$$

Control parameter : F