Tutorial 1: February 5, 12:00-12:45

1. Hopf Bifurcation

Notebook: Tutorial5-2_1.ipynb

We consider first the two-dimensional dynamical system I

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

with $A_{11} = \lambda$, $A_{12} = 1$, $A_{21} = -1$, $A_{22} = \lambda$, where λ is a real number.
a.
Plot the phase portraits for $\lambda = -1, \lambda = 0$ and $\lambda = 1$.

b.
For which values of λ is the origin a stable fixed point?

Next, we study the dynamical system II

$$\frac{dx}{dt} = \lambda x - y - x(x^2 + y^2)$$

$$\frac{dy}{dt} = \lambda y + x - x(x^2 + y^2)$$

c.
What is the relation between the dynamical systems I and II?

d.
Determine the phase portraits of system II for $\lambda = -1, \lambda = 0$ and $\lambda = 1$ and describe what new dynamical behaviour occurs when λ crosses zero.

2. Back-to-back saddle-node bifurcation

Notebook: Tutorial5-2_2.ipynb

In this exercise, we consider the dynamical system

$$\frac{dx}{dt} = ax^3 + bx + \phi$$

where $a < 0$ and ϕ is considered as a control parameter.

a.
Show that this system has multiple (real valued) equilibria if and only if $b > 0$ and

$$|\phi| < \sqrt{\frac{-4b^3}{27a}}$$

b.
If the conditions under a. are satisfied, determine the positions ϕ of the two saddle-node bifurcations.

c.
Plot the bifurcation diagram versus ϕ for $a = -0.5$ and $b = 0.5$.

Next, let $\phi = \phi_0 + \alpha t$ be time dependent.

d. For $a = -0.5$, $b = 0.5$ and $\phi_0 = -0.5$, study the behaviour of trajectories for several well-chosen values of α. Describe and explain the behaviour found.