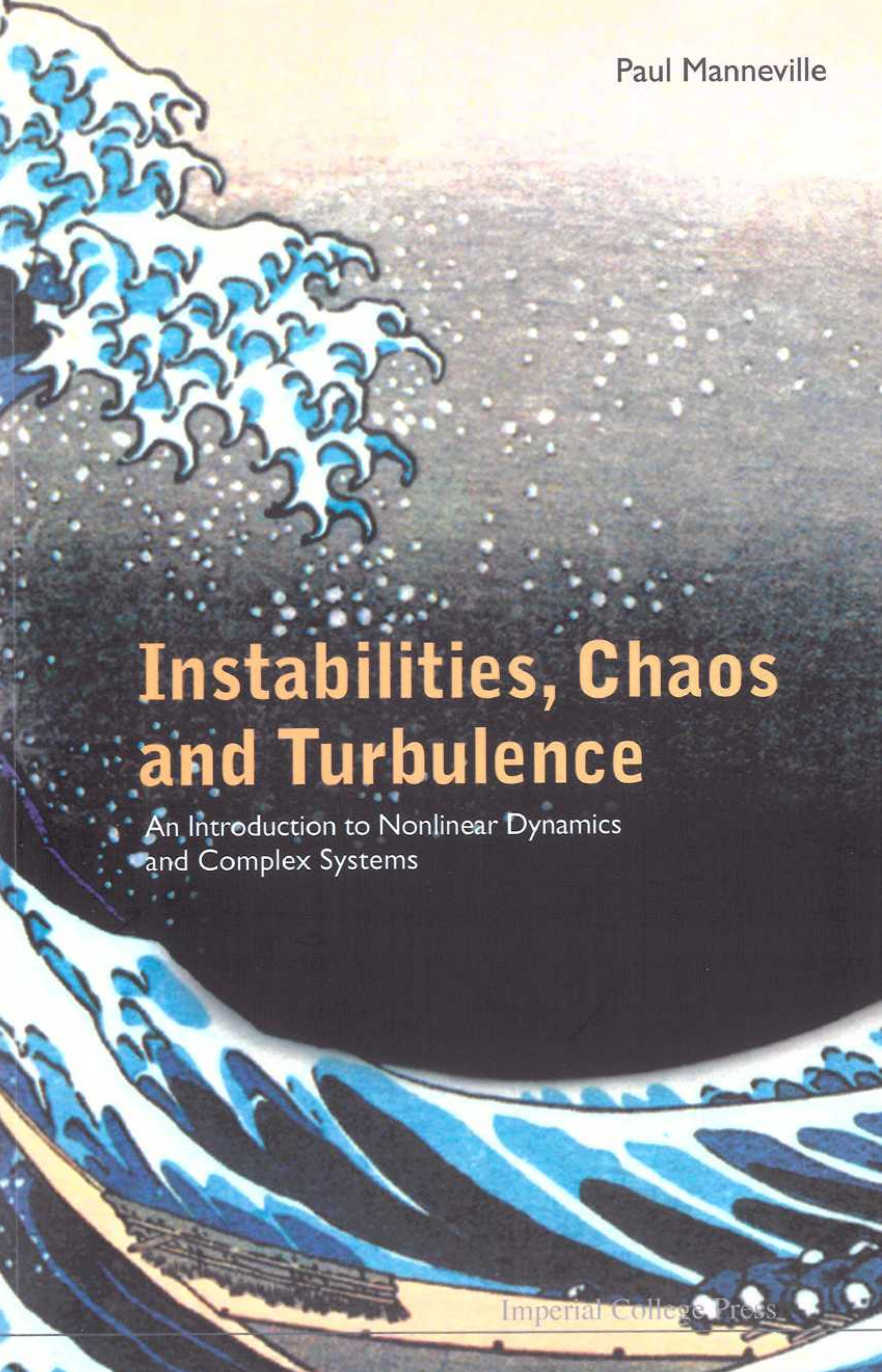

Paul Manneville

Instabilities, Chaos and Turbulence

An Introduction to Nonlinear Dynamics and Complex Systems

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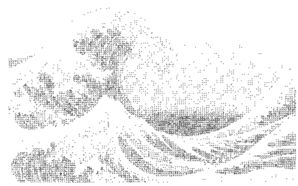

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INSTABILITIES, CHAOS AND TURBULENCE
An Introduction to Nonlinear Dynamics and Complex Systems

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To Jacqueline,

Jean-Baptiste, Sébastien, Alexis, and Claire.

Preface

During the last quarter of the Twentieth Century, the study of nonlinear and complex systems has experienced unprecedented development. In the following pages, we present a first approach to standard results and recent advances in this field, applied to phenomena at our scale in our surrounding world. The pretext chosen is mainly instabilities in out-of-equilibrium systems, and more specifically the transition to turbulence in flowing fluids. It should however be clear that the methods used are of fully general use.

The book is based on lecture notes for a short optional course given to second-year students in an engineering school, École Nationale Supérieure de Techniques Avancées, in Paris. This school is devoted to the training of high level engineers in fields including applied mathematics, mechanics and hydrodynamics, electronics,..., oceanography, and management. At the time of the course, students have not yet chosen their specialty, so the course has to be sufficiently general and without too specific requisites. Accordingly, the book should be of interest to nearly any science-oriented undergraduate student and, potentially, to everybody wanting to learn about recent advances in the field of applied nonlinear dynamics. Technicalities are not completely avoided but they are explained as simply as possible using heuristic arguments and specific worked examples, while openings on different topics can be gained by solving exercises at the end of each chapter using the same methods as those explained in the text.

At first, the problem of *chaos* that one has to face very early in this field may seem abstract and difficult. Even if the treatment of examples is not complete, the reader should get a concrete and operational mastery of concepts and techniques to be used from them. As far as the difficulty is concerned, our aim is to transmit the knowledge rather informally and without full mathematical rigor. With respect to mathematics and physics, only basic understanding is required, at the level of what is currently known after one or two years of undergraduate training. In mathematics, this does not go further than elementary algebraic calculus, basic notions of linear algebra and ordinary differential calculus. As far as physics is concerned, it should suffice to follow one's intuition and to admit a few fundamental equations without discussion. So, adaptation of the approach to any other field of interest should thus be envisaged without excessive anxiety.

A first brief chapter situates the context of the study, that of evolutionary problems involving a specific independent variable called *time*, distinguishing *discrete* systems governed by finite sets of ordinary differential equations and *continuous* media described by partial differential equations. It serves as an introduction to the rest of the book, explaining in particular that continuous media driven out of equilibrium may experience instabilities inducing structures that further break down, leading to turbulence.

The second chapter is devoted to a preliminary study of dynamical systems *with a small number of degrees of freedom*. The archetype of such systems is the oscillator which serves to introduce the first manifestations of nonlinear effects, *e.g.* the occurrence of self-oscillations or the relation between amplitude and frequency.

The way to complex behavior is then apparently left at a too early stage, before the occurrence of chaos. In Chapter 3 we indeed turn to a specific but particularly simple and intuitive physical problem, the stability of a fluid layer heated from below and entering a convection regime. The first part of the chapter is devoted to the analysis of the instability mechanism and an approximate determination of the threshold. In the second part, a description is given of the "death" of the so-formed *dissipative structure* (pattern of convection rolls) and of the steps toward turbulence.

After this detour, we come back to the mathematics of the transition within the dynamical systems framework. A preliminary step of the reduction of the dynamics has to be performed, resting on the distinction between *driving* and *enslaved* modes, and on elimination of the latter modes. The emergence of complexity is then analyzed as a result of the increase of the number of driving modes. This is done in two steps. The first one constitutes the last part of Chapter 4 where we introduce *scenarios* of transition to *temporal chaos* and present some of the tools used to identify it and measure its amount from an empirical point of view. However, as already observed when studying convection, *confinement* effects play an important role, and all that precedes relates to the case when they are strong enough to freeze the spatial structure of the modes. Otherwise, scenarios relevant to *extended* systems, the second step, involve large scale modulations and *spatio-temporal chaos*, both introduced in Chapter 5.

In common language, the third element of the title, *turbulence*, refers to the irregular, highly fluctuating, behavior of most of the flows surrounding us (the opposite situation of so-called laminar flows is rather exceptional). This problem is tackled in two steps. Instability and transition of *open flows* is examined in Chapter 6. By contrast with systems considered in Chapter 3, where the fluid remained confined to an enclosure, it now circulates from upstream to downstream, the consequences of which will be sketchily discussed. In Chapter 7, we consider *developed turbulence*, again along two paths. First we analyze the different scales, from the largest where energy is injected eddies by instability mechanisms to the smallest where it is consumed *via* viscous dissipation. In a second instance, we turn to the statistical problem of predicting the average properties of a given turbulence flow, of utmost practical interest for an engineer.

Chapter 8 recapitulates the results and opens the perspective toward a complex dynamical system of contemporary interest, the climate of the Earth, and the problem of understanding/modeling its past and present trends.

A first appendix is devoted to a summary of linear algebra results that are useful throughout the book. As far as the understanding of nonlinear phenomena is concerned, the recourse to computers has been of considerable help at different levels. This is the reason why a second appendix, introducing hands-on computer sessions, is devoted to elementary methods that can be developed, with sufficient common sense and no superfluous specialization, to extract useful information from numerical simulations of simple, even simplistic, but well-designed generic models of nonlinear dynamics and pattern formation. The course is completed by laboratory sessions on topics, the theory of which is considered in some exercises.

Palaiseau, June 2004.

Notations.

Here are some indications about the conventions that, with as few exceptions as possible, we will be using throughout. First, upper-case bold letters will denote points in the spaces that will be considered, *e.g.* $\mathbf{X} \in \mathbb{X}$, $\mathbb{X}$ will most often be a real vector space $\mathbb{R}^d$. Components will be $X_1, X_2, ..., X_d$, collectively denoted $\{X_i\}$. We will also possibly use the bold and normal upper-case Greek letters in the same context.

For operators working in these spaces, we will use the bold 'cal' TEX font, *e.g.* $\boldsymbol{\mathcal{F}}$, and for their components the normal 'cal' font with indices, *e.g.* $\mathcal{F}_i$, hence $\mathbf{Y} = \boldsymbol{\mathcal{F}}(\mathbf{X})$ and $Y_j = \mathcal{F}_j(\{X_i\})$, $i, j = 1, \ldots, d$.

In the case of a linear operator, *e.g.* $\boldsymbol{\mathcal{L}}$, we will rather write $\mathbf{Y} = \boldsymbol{\mathcal{L}}\mathbf{X}$. In general, we will not distinguish the operator from the matrix that represents it in a given basis, $\boldsymbol{\mathcal{L}}$ will denote, with little ambiguity, either the operator or the matrix with elements $l_{jj'}$. The above equation would then read in developed form: $Y_j = \sum_i l_{ji} X_i$. Some elements of linear algebra are recalled in Appendix A.

With respect to differentiation, for the ordinary derivative with respect to some variable U we will use $\mathrm{d}(...)/\mathrm{d}U$. With respect to time t, we usually prefer dots surmounting the variable, one for each differentiation order: $\mathrm{d}X/\mathrm{d}t \equiv \dot{X}$ and $\mathrm{d}^2X/\mathrm{d}t^2 \equiv \ddot{X}$. A short-hand notation for the partial derivative with respect to variable X, $\partial(...)/\partial X$, will be $\partial_X(...)$.

Points in physical space will usually be noted $\mathbf{x}$ with coordinates x, y, z, corresponding unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$, and partial derivatives $\partial_x, \partial_y, \partial_z$. In the same way, a velocity vector will be noted $\mathbf{V}$ or $\mathbf{v}$ with components V_x, V_y, V_z or v_x, v_y, v_z. On some occasions we will rather take x_j, v_j, and $\partial_j \boxminus \partial_{x_j}$, $j = 1, 2, 3$ allowing us to use the Einstein convention of implicit sum over repeated indices.

The complex conjugate of a complex number $Z = Z_\mathrm{r} + iZ_\mathrm{i}$, $\mathcal{R}e(Z) = Z_\mathrm{r}$, $\mathcal{I}m(Z) = Z_\mathrm{i}$, will be noted as Z^*.

For quantities that play the role of control parameters, we will *not* follow the convention used in fluid mechanics to take two letters, *e.g.* 'Re' for the Reynolds number of a given flow, 'Ra' for the Rayleigh number in convection, etc., but a simple ordinary letter, upper or lower case, most usually R or r, to stress the fact that control parameters are ordinary variables (ambiguities will be raised on a case-by-case basis).

Numerical illustrations have most often been obtained using MATLAB. We will occasionally use some of its conventions.

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