

Analysis of Algorithms: Assignment 1

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1 Section 1.3

1.1 Problem 1

1.1.1 b. Is this algorithm stable?

Answer: No. As you can see in the innermost block of code, if $A[i] \leq A[j]$ (aka not less than) $\text{Count}[i]$ is incremented, meaning $A[i]$ will be moved ahead of $A[j]$ when these elements are equal. This reverses their relative order, making the algorithm unstable.

1.2 c. Is it in place?

Answer: No. A new array (S) is returned from the function, thus the algorithm violates the definition of in-place.

2 Section 1.4

2.1 Problem 3

2.1.1 Create a simple algorithm for the string-matching problem

Answer: Create a list of all the substrings in your string that are of the same length as your pattern. Simultaneously create a list of integers that denote the index at which each substring begins in the parent string. Compare your pattern to each of these substrings. If the substring and the pattern match, return, or push, the corresponding starting index of the substring.

3 Section 2.1

3.1 Problem 7

Gaussian elimination, the classic algorithm for solving systems of n linear equations in n unknowns, requires about $1/3(n^3)$ multiplications, which is the algorithm's basic operation.

3.1.1 a. How much longer should you expect Gaussian elimination to work on a system of 1000 equations versus a system of 500 equations

Answer:

$$G(500) = 1/3(500^3) = 41666666.6667 \quad (1)$$

$$G(1000) = 1/3(1000^3) = 33333333.333 \quad (2)$$

$$G(1000) \div G(500) = 8 \quad (3)$$

I should expect it to take eight times longer on a system of 1000 equations versus a system of 500 equations.

3.2 Problem 8

For each of the following functions, indicate how much the functions value will change if its argument is increased fourfold.

3.2.1 a. $\log_2(n)$

Answer: $\log_2(4n) = \log_2(4) + \log_2(n) = 2 + \log_2(n)$

The value will increase by 2.

3.2.2 b. \sqrt{n}

Answer: $\sqrt{4n} = 2\sqrt{n}$

The value will double.

3.2.3 c. n

Answer: $4 * n = 4n$

The value will increase by a factor of 4.

3.2.4 d. n^2

Answer: $(4n)^2 = 4n * 4n = 16n^2$

The value will increase by a factor of 16.

3.2.5 e. n^3

Answer: $(4n)^3 = 4n * 4n * 4n = 64n^3$

The value will increase by a factor of 64.

3.2.6 f. 2^n

Answer: $2(4n) = 2^n * 2^n * 2^n * 2^n = (2^4)^n = 8^n * 2^n$

The value will increase by a factor of 8^n .

3.3 Problem 9

For each of the following pairs of functions, indicate whether the first function of each of the following pairs has a lower, same, or higher order of growth (to within a constant multiple) than the second function.

3.3.1 a. $n(n+1)$ and $2000n^2$

Answer: Same, as the highest order element in each is an n^2

3.3.2 b. $100n^2$ and $0.01n^3$

Answer: Lower, as the highest order element in the second is n^3 and only n^2 in the first. Constants do not matter.

3.3.3 c. $\log_2(n)$ and $\ln(n)$

Answer: Same, as both of these functions are on a logarithmic scale. Differing bases does not affect order of growth.

3.3.4 d. $\log_2^2(n)$ and $\log_2(n^2)$

Answer: Higher as $\log_2^2(n) = (\log_2(n))^2$ and $\log_2(n^2) = 2\log_2(n)$ meaning the first is quadratic relative to the second when placed on a logarithmic scale.

3.3.5 e. 2^n and 2^n

3.3.6 f. $(n1)!$ and $n!$

4 Section 2.2

4.1 Problem 2

5 Section 2.3

5.1 Problem 1

5.2 Problem 5

6 Section 2.4

6.1 Problem 1

6.2 Problem 4