ENGI 9821 – DSP

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A Flexible Butterworth Window

IMPLEMENTATION & EVALUATION

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A INTRODUCTION

- **Power Spectral Density (PSD) Estimation**
  - **Important characteristics of a window used in spectral analysis**
    - **Ideal window function**
    - **Rectangular, Hanning, Hamming**
    - **Tradeoff between spectral resolution and spectral leakage**
      - Kaiser, Gaussian, and Chebyshev
  - **Butterworth filter** $|H(f)| = 1/\sqrt{1+\left(\frac{f}{f_c}\right)^{2N}},$

Ideal PSD

$$
\begin{aligned}
x(t) = {} & 0.84\cos(2\pi \cdot 52 \cdot t) \\
& + 0.8\cos(2\pi \cdot 65.5 \cdot t) \\
& + 0.3\cos(2\pi \cdot 85 \cdot t) \\
& + 1.1\cos(2\pi \cdot 105 \cdot t) \\
& + 0.35\cos(2\pi \cdot 140 \cdot t) \\
& + 0.98\cos(2\pi \cdot 159 \cdot t) \\
& + 0.6\cos(2\pi \cdot 174 \cdot t) \\
& + 0.8\cos(2\pi \cdot 190 \cdot t) \\
& + \cos(2\pi \cdot 205 \cdot t).
\end{aligned}
$$

Figure 4 - Sinusoidal Input

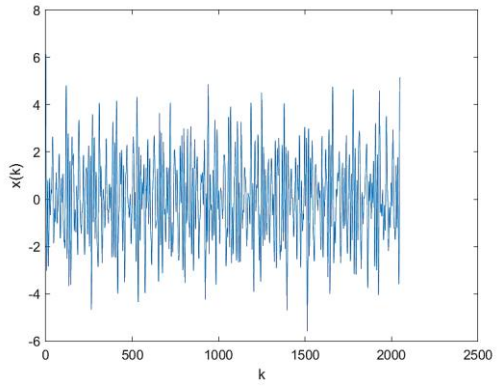
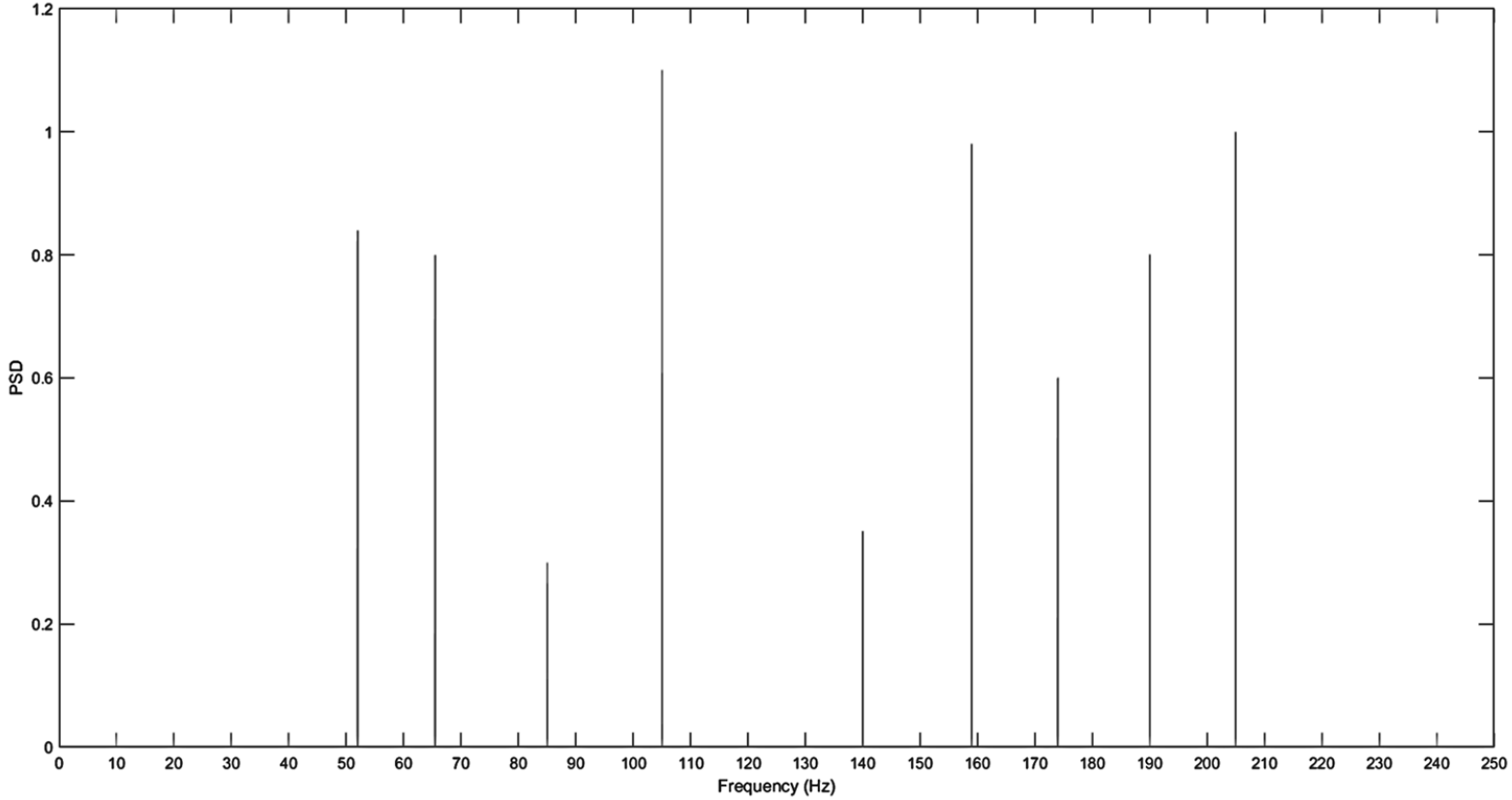
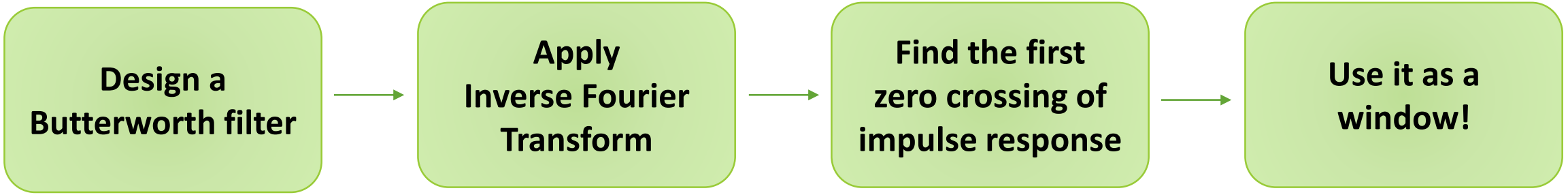


Figure 5 - Ideal Power Spectral Density



from Butterworth **filter** to Butterworth **window**



B IMPLEMENTATION & RESULTS

- This algorithm was coded and implemented in MATLAB environment, and all the results and figures are obtained from computations in this platform. MATLAB's help and MATLAB Answers were frequently used for debugging the code.

- All the DTFT calculations in this implementation were carried out through the myDTFT.m script written by Dr. C. Moloney.

A Butterworth Window

Parameter	Value
F_s	2048
F_c	0.75
N	3
emp_length	0

Figure 2 - Frequency Response of Butterworth filter & Butterworth window

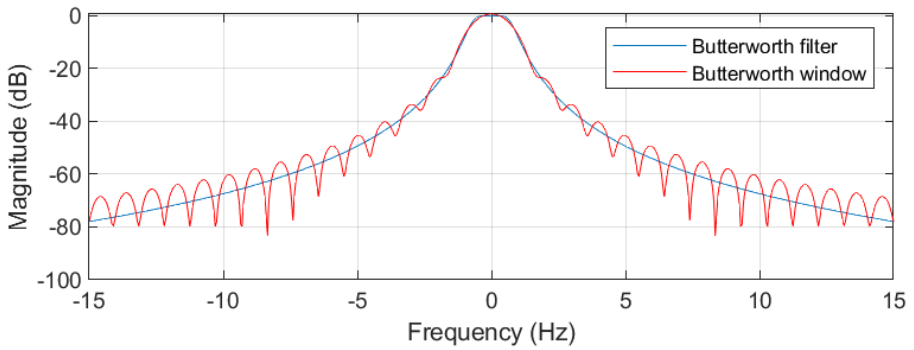


Figure 1 - Impulse Response of Butterworth filter & Butterworth window

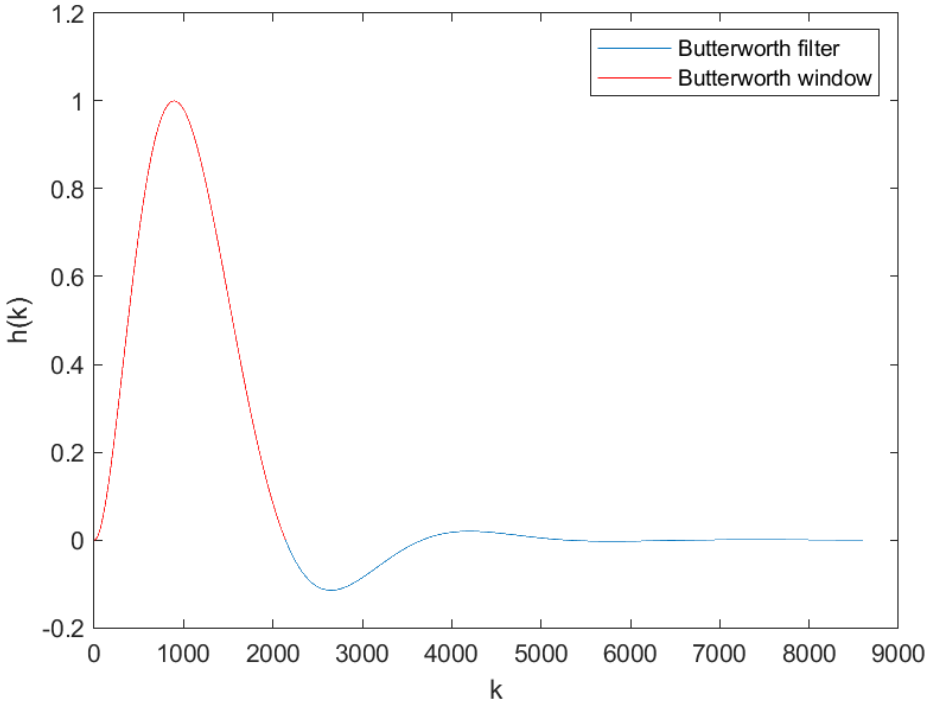
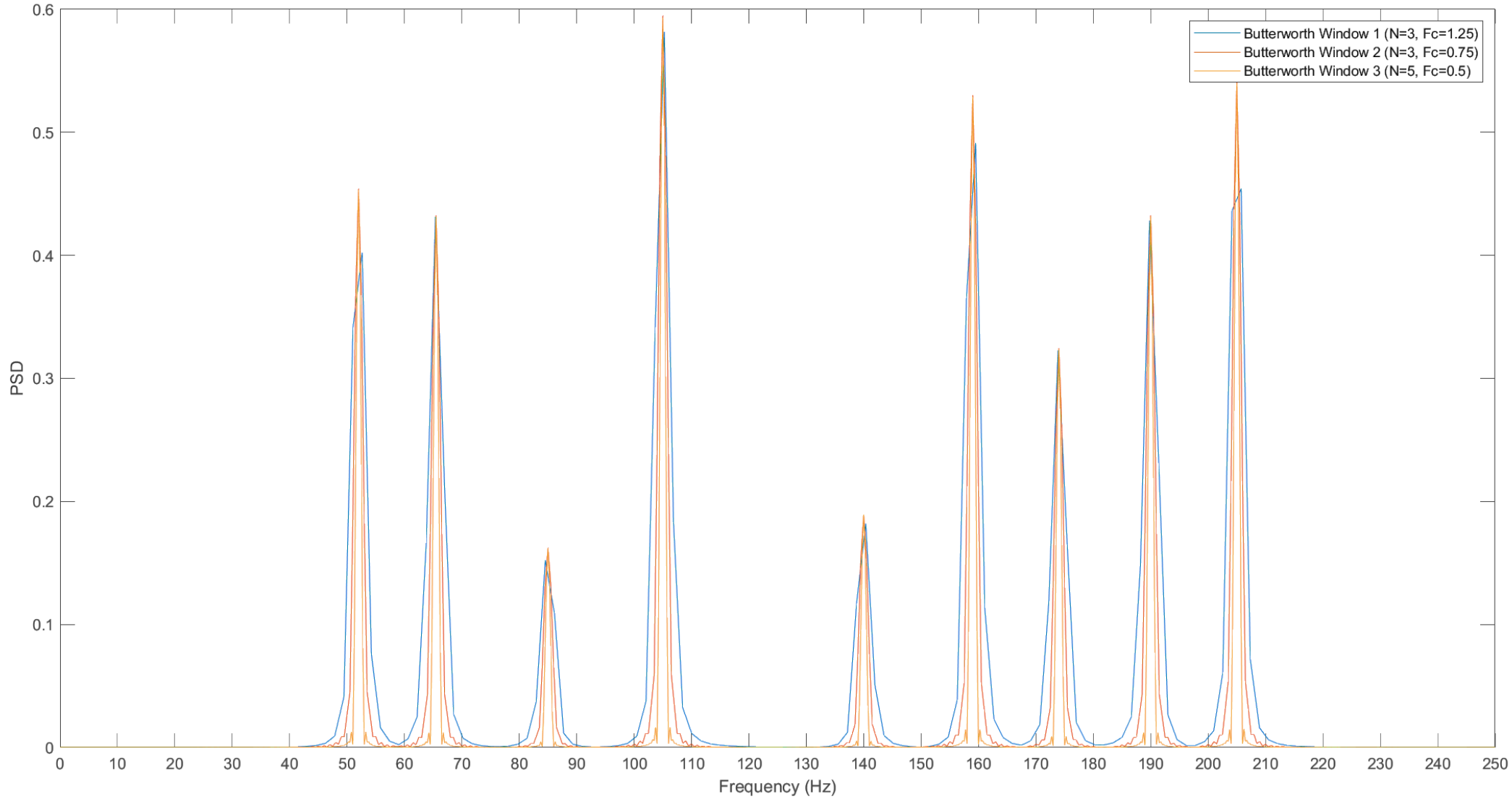


Figure 6 - PSD using various Butterworth windows



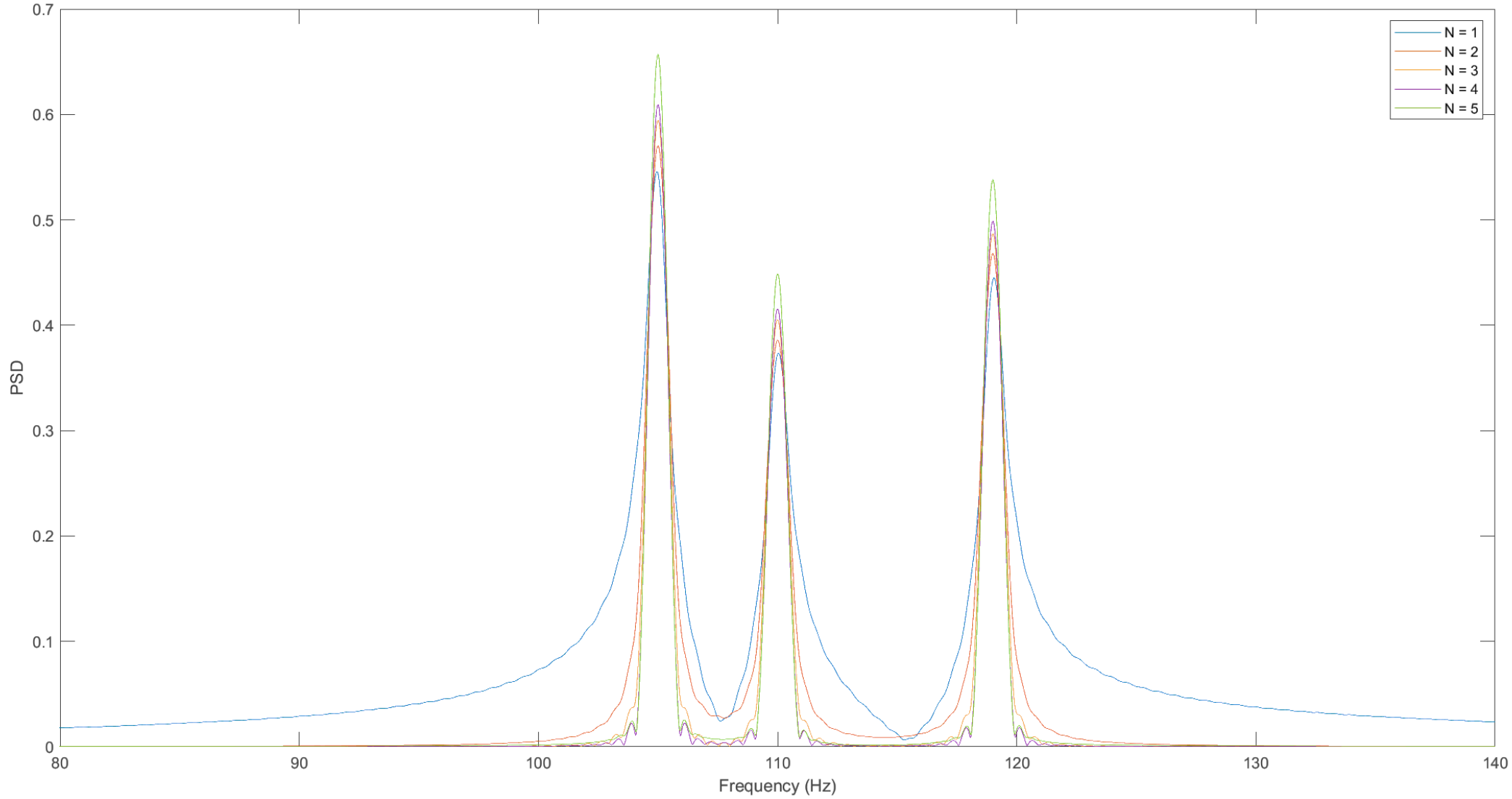
A “Flexible” Butterworth Window

- **Effect of Changing the Order (N)**
- **Effect of Changing the Cut-off frequency (Fc)**
- **Effect of Changing the sampling Frequency (Fs)**

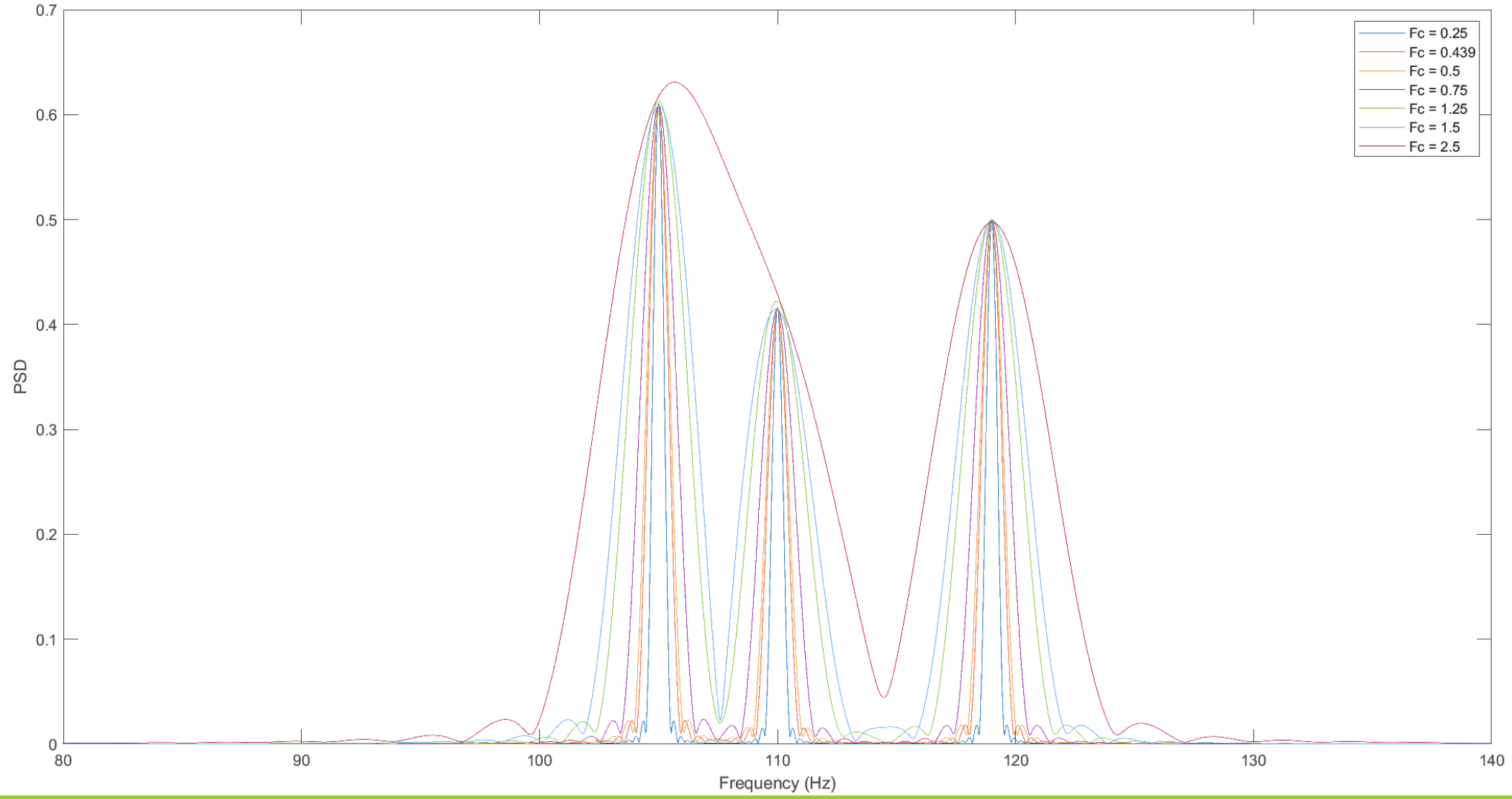
- Considered Sinusoidal signal:

$$x_2(t)=1.1\cos 210\pi t+0.75\cos 220\pi t+0.9\cos 238\pi t$$

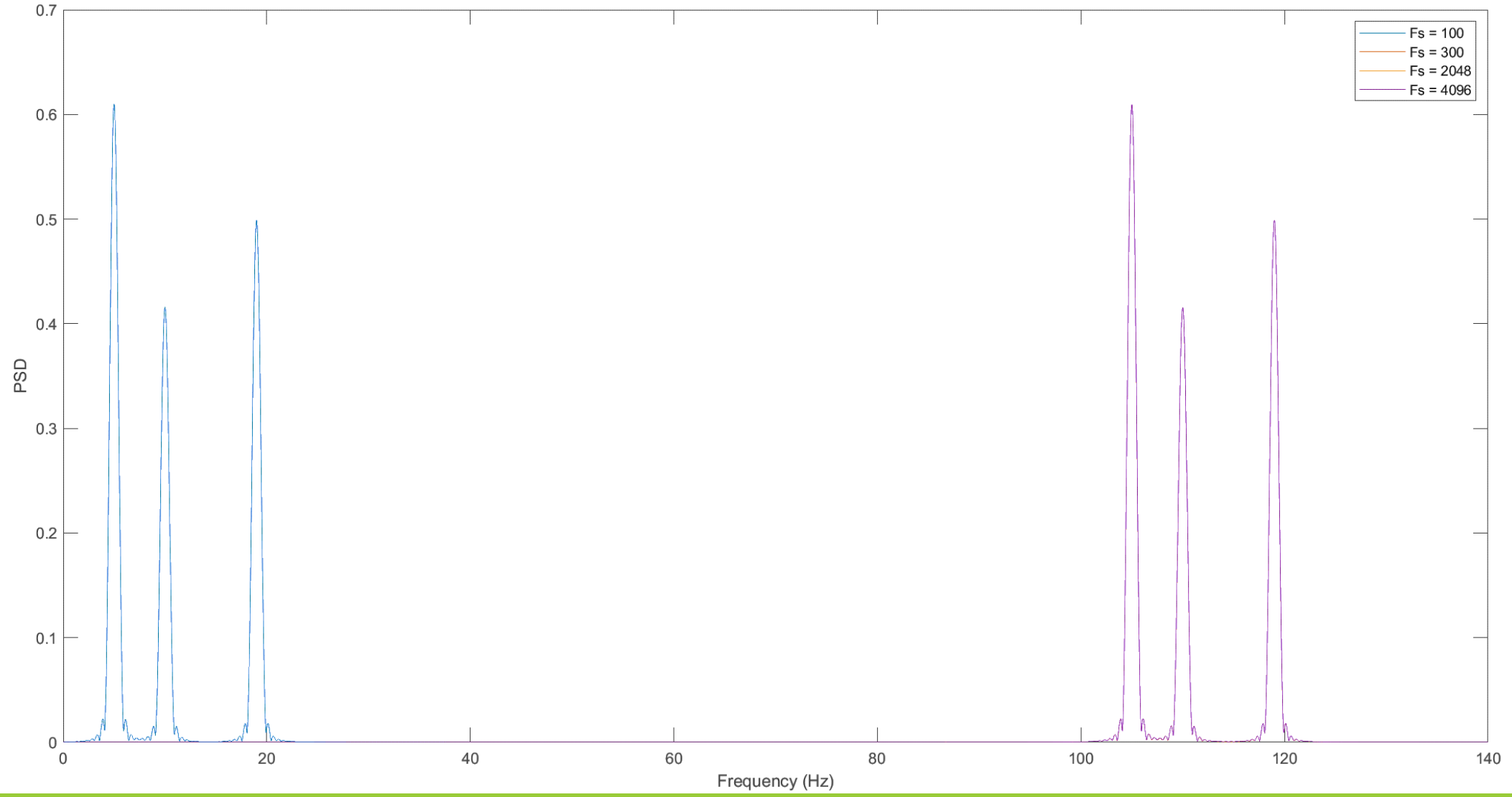
Effect of Changing the Order **(N)**



Effect of Changing the Cut-off frequency **(Fc)**



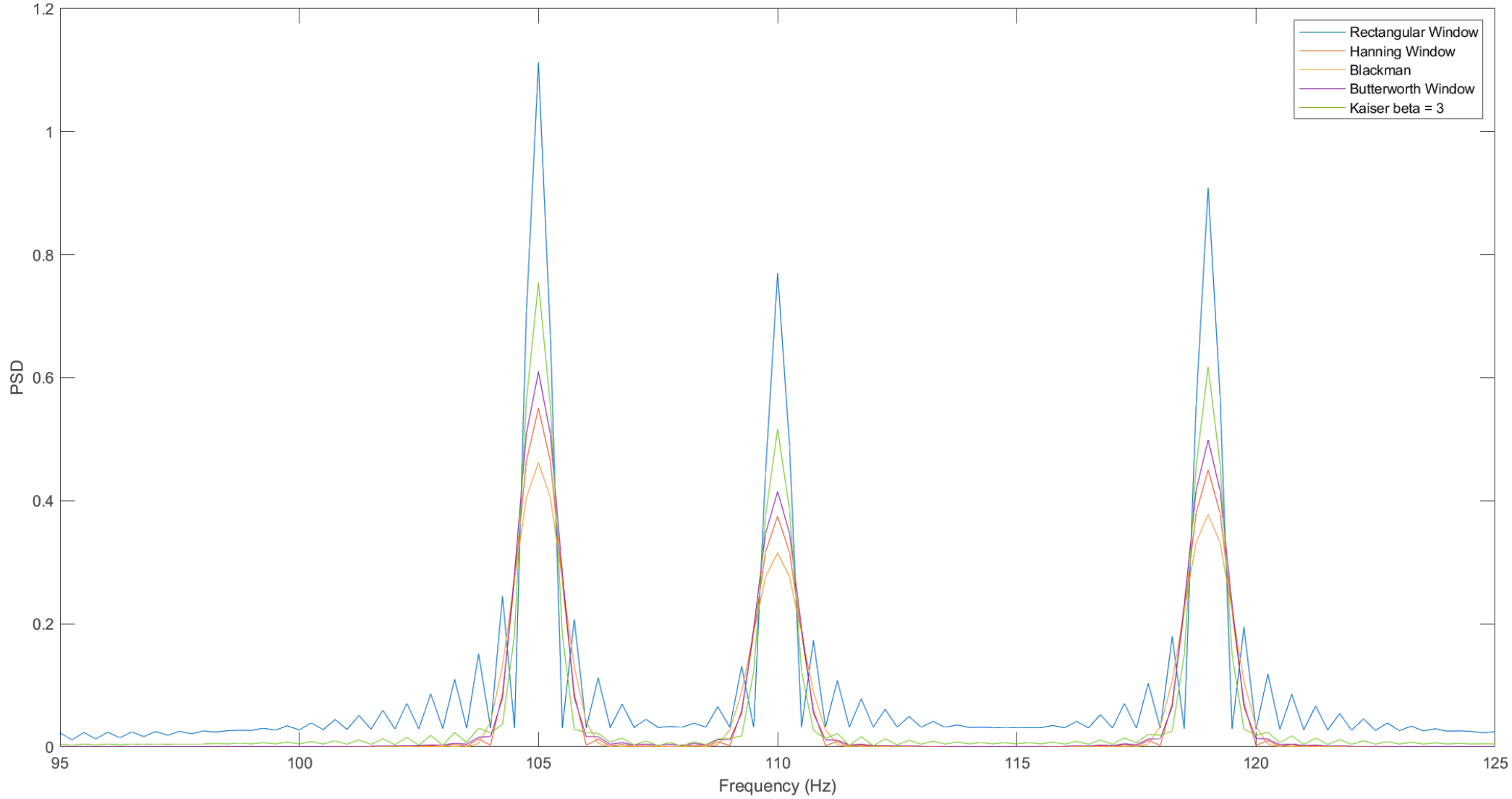
Effect of Changing the sampling Frequency **(Fs)**



Butterworth VS Commonly used

Window_type	N	Fc	bandwidth	sidelobe_attenuation	window_length
Rectangular	0	0	0.84766	13.262	2140
Hanning	0	0	1.3779	31.555	2140
Kaiser beta=2	0	0	0.9502	18.946	2140
Kaiser beta=4	0	0	1.1475	32.601	2140
Blackman	0	0	1.5723	62.235	2140
Butterworth	2	0.439	0.81738	17.591	3299
Butterworth	3	0.439	0.74219	26.259	3655
Butterworth	4	0.439	0.68701	29.8	4157
Butterworth	4	0.439	0.68701	29.8	4157
Butterworth	4	1.5	2.3477	29.785	1217
Butterworth	4	2.5	3.9141	29.791	730

Figure 10 - PSD estimation using common windows and the novel Butterworth window



Butterworth window length

[TABLE 1] SUITABLE LENGTHS OF BUTTERWORTH WINDOWS (F_S = SAMPLING FREQUENCY, F_C = CUTOFF FREQUENCY).

FILTER ORDER, *N*	WINDOW LENGTH (SAMPLES)
1	$\left\lfloor 0.660 \cdot \frac{f_s}{f_c} \right\rfloor$
2	$\left\lfloor 0.705 \cdot \frac{f_s}{f_c} \right\rfloor$
3	$\left\lfloor 0.784 \cdot \frac{f_s}{f_c} \right\rfloor$
4	$\left\lfloor 0.890 \cdot \frac{f_s}{f_c} \right\rfloor$
5	$\left\lfloor 1.005 \cdot \frac{f_s}{f_c} \right\rfloor$

Window_specification	empirical_length	zero_crossing_length
N = 1, Fc = 1.000, Fs = 2048	1351	0
N = 2, Fc = 1.500, Fs = 2048	962	966
N = 3, Fc = 2.500, Fs = 4096	1284	1284
N = 4, Fc = 0.439, Fs = 4096	8303	8308
N = 5, Fc = 1.500, Fs = 1024	686	686

* Table 1 from Tae Hyun Yoon, Eon Kyeong Joo, “A Flexible Window Function for Spectral Analysis”, 2010

C CONCLUSION

PROs and CONs of the Butterworth windows

- **Tradeoff issue**
- **Length issues**
  - Estimation
  - Application
- **Not symmetrical**
- **Computational efficiency** $f_c = \frac{Kf_s}{2^{M+1}},$

D APPLICATION

Figure 12 - PSD estimation for EEG signals using Hanning window and the Butterworth window

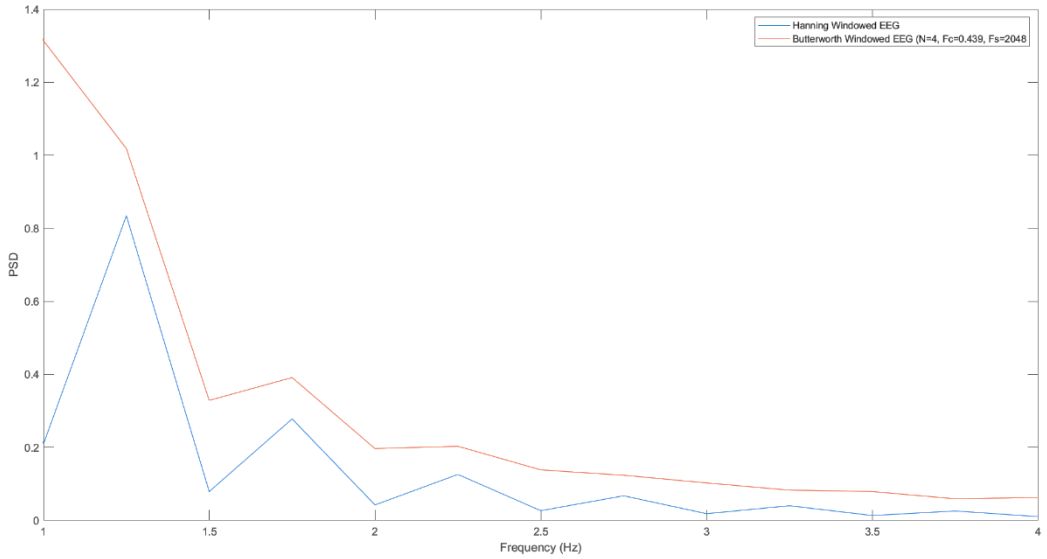


Figure 13 - PSD estimation for EEG signals using Hanning window and the Butterworth window

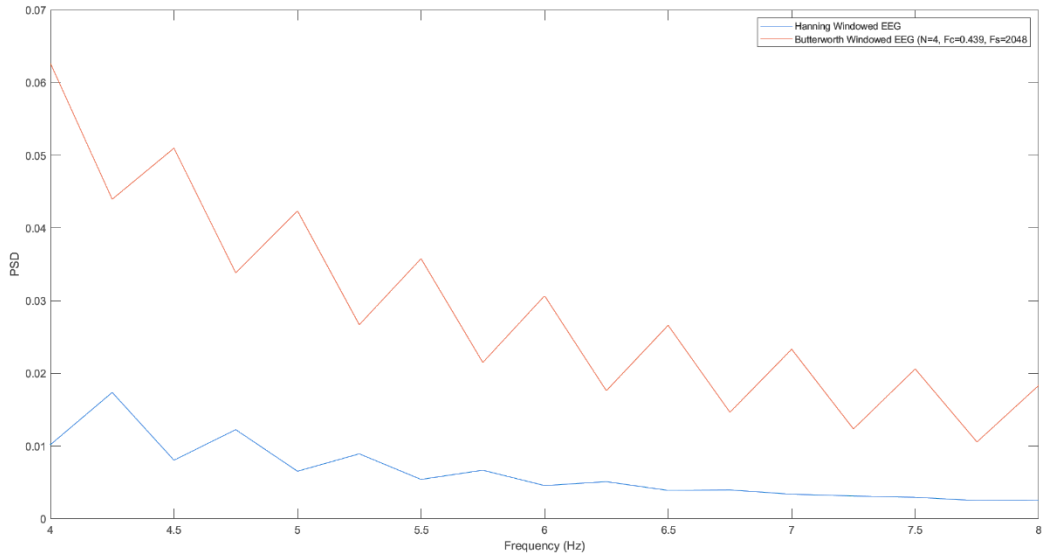


Figure 14 - PSD estimation for EEG signals using Hanning window and the Butterworth window

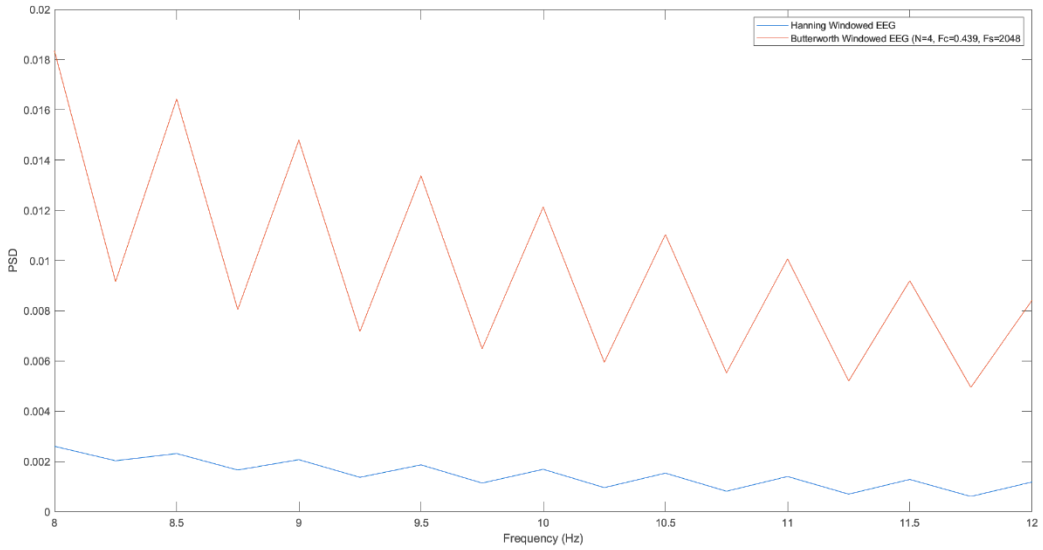


Figure 15 - PSD estimation for EEG signals using Hanning window and the Butterworth window