Finding the Impossible: Automated Search for Full Impossible-Differential, Zero-Correlation, and Integral Attacks

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TU Graz
SCIENCE PASSION TECHNOLOGY

Research Gap and Our Contributions

Research gap

- Lack of automatic tool to find full ID/ZC, and integral attacks

Contributions

- Introduced a new CP-based method to find ID/ZC, and integral distinguishers
- Our CP model can be extended to an efficient unified model for key recovery
- Found improved attacks for SKINNY, CRAFT, SKINNYee, and SKINNYe-v2

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Part of Our Result

Cipher	#R	Time	Data	Mem.	Attack	Setting / Model	Ref.
SKINNY-64-192	23	$2^{155.60}$	$2^{73.20}$	2^{138}	Int	180,SK / CP,CT	[Ank+19]
	26	2^{172}	2^{61}	2^{172}	Int	180,SK / CP,CT	This paper
SKINNY-64-128	18	2^{126}	$2^{62.68}$	2^{64}	ZC	STK / KP	[SMB18]
	19	$2^{119.12}$	$2^{62.89}$	2^{49}	ZC	STK / KP	This paper
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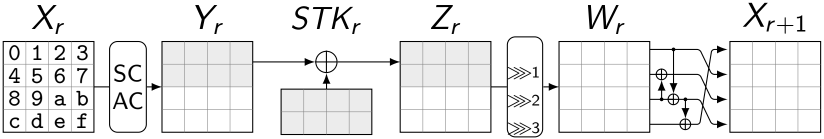
	19	$2^{219.23}$	$2^{117.86}$	2^{208}	ID	STK / CP	This paper
SKINNY-64-64	14	2^{62}	$2^{62.58}$	2^{64}	ZC	STK / KP	[SMB18]
	16	$2^{62.71}$	$2^{61.35}$	$2^{37.80}$	ZC	STK / KP	This paper
CRAFT	**20**	$2^{120.43}$	$2^{62.89}$	2^{49}	ZC	STK / KP	This paper
	21	$2^{106.53}$	$2^{60.99}$	2^{100}	ID	STK / CP	This paper

Outline

Background and the Research Gap

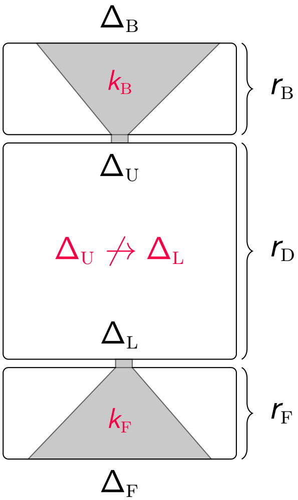
SKINNY Family of Tweakable Block Ciphers [Bei+16]



- Introduced in CRYPTO 2016 [Bei+16]
- It has 6 main variants: SKINNY-n-$z \cdot n$, where $n \in \{64, 128\}$, and $z \in \{1, 2, 3\}$
- ISO/IEC 18033-7: SKINNY-64-192, SKINNY-128-256, SKINNY-128-384

Impossible Differential Attack [BBS99; Knu98]

- Find an impossible-differential $\Delta_U \not\rightarrow \Delta_L$
- Build a key-recovery attack
  - Create a pool of pairs satisfying (Δ_B, Δ_F)
  - For all $k \in k_B \cup k_F$:
    - If a pair suggests (Δ_U, Δ_L), discard k
  - Brute force the remaining key candidates



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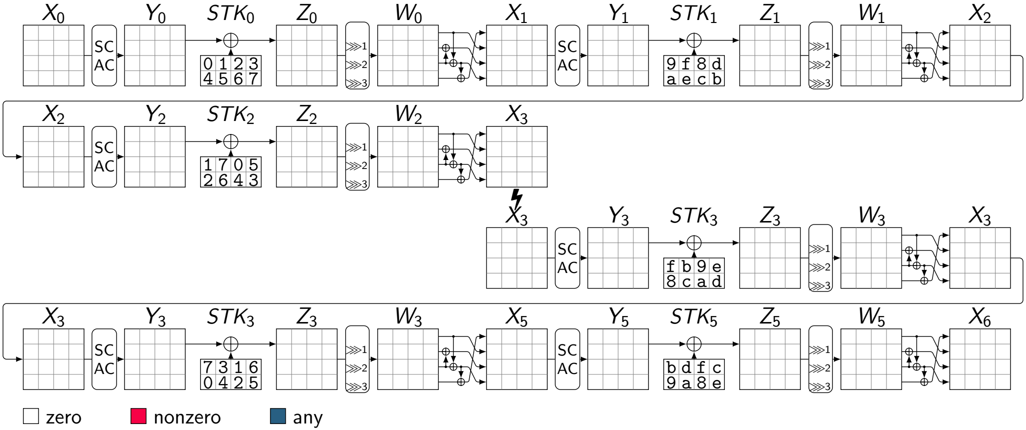
Miss-in-the-Middle Technique [BBS99]



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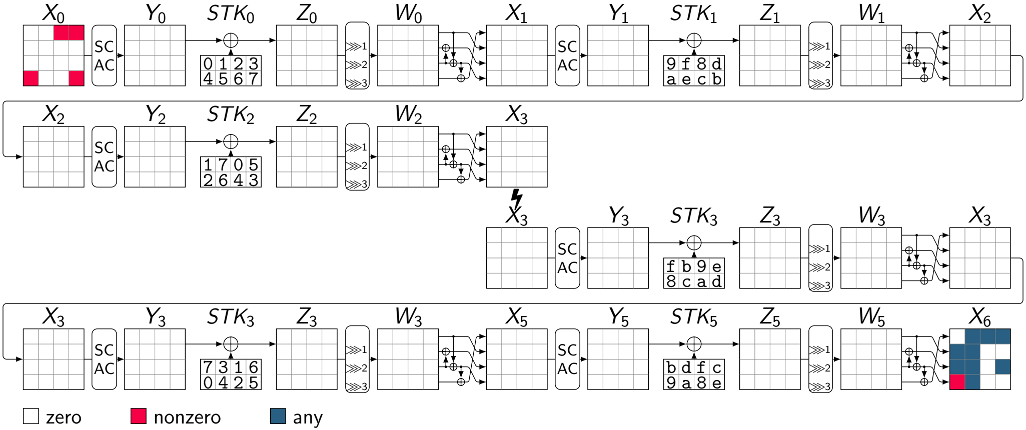

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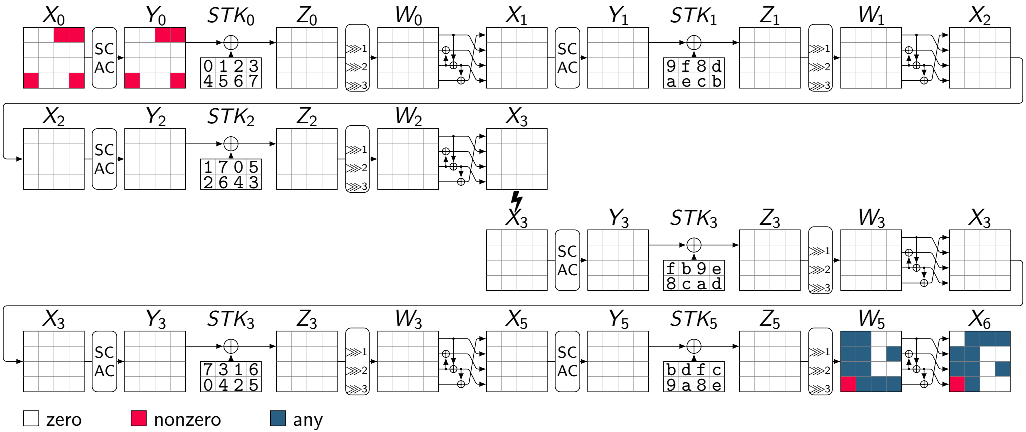

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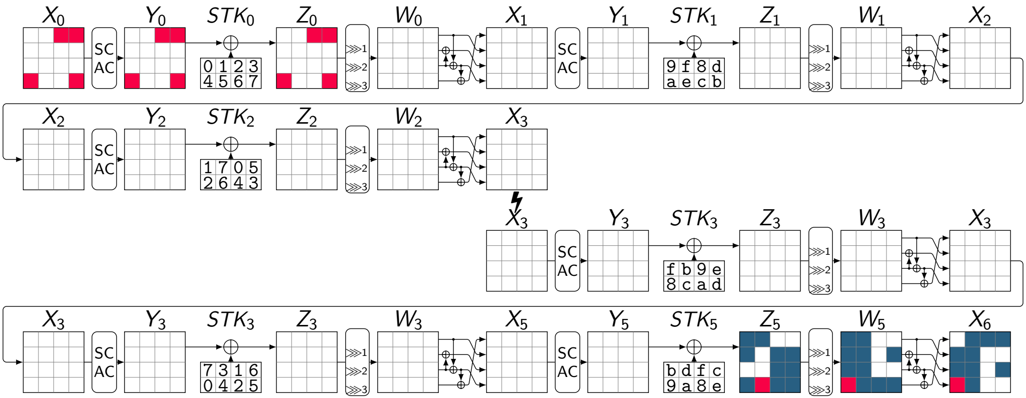

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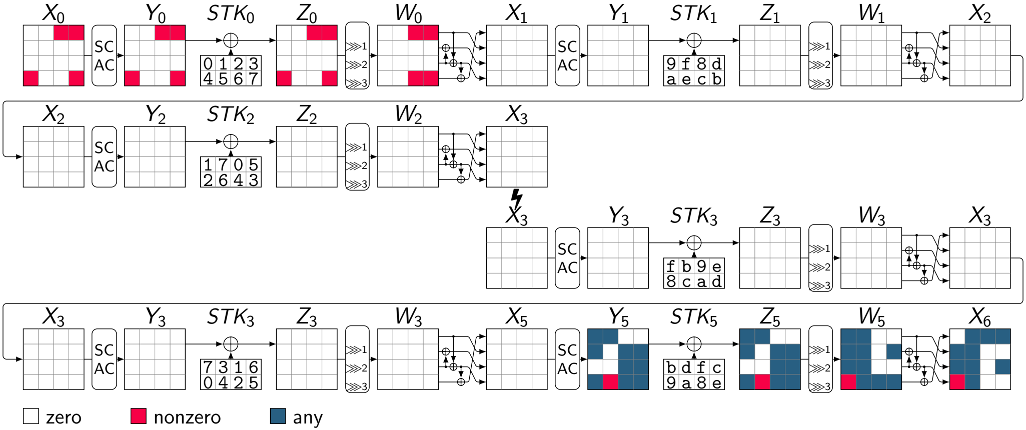

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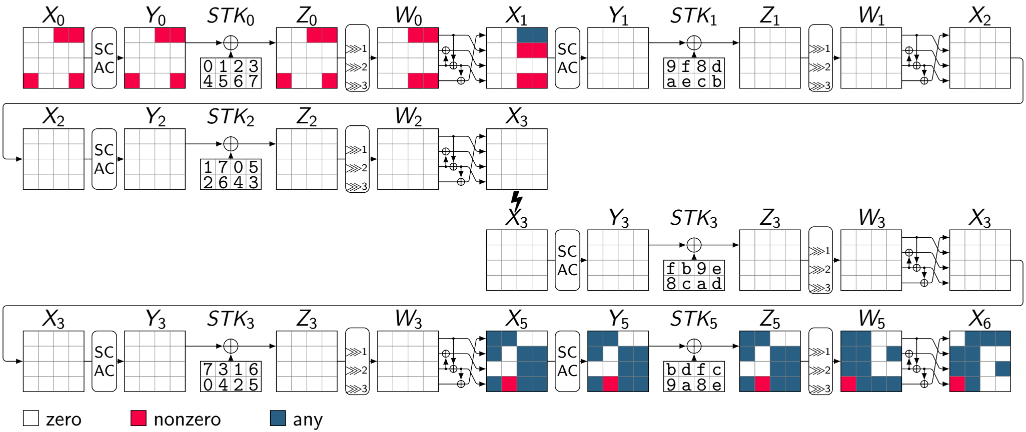

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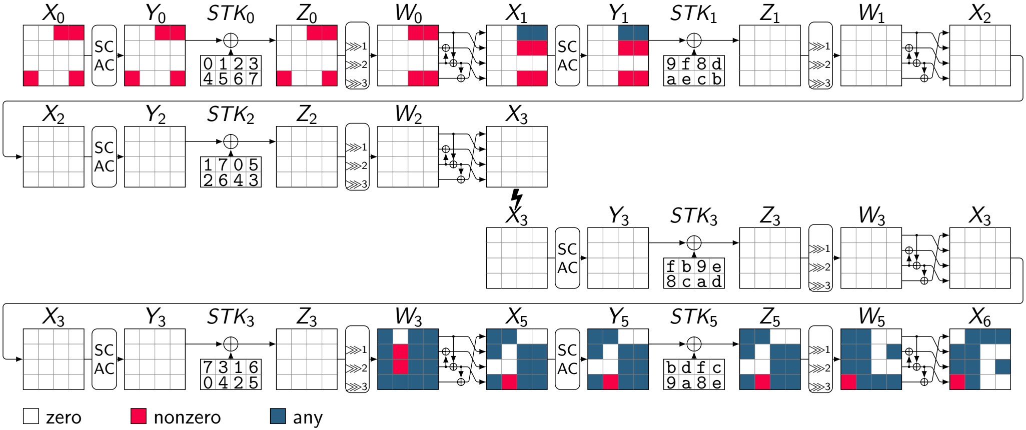

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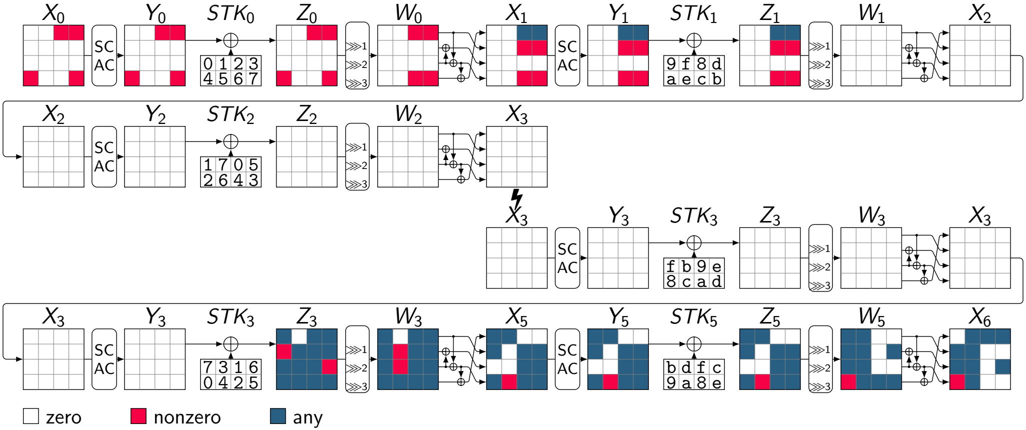

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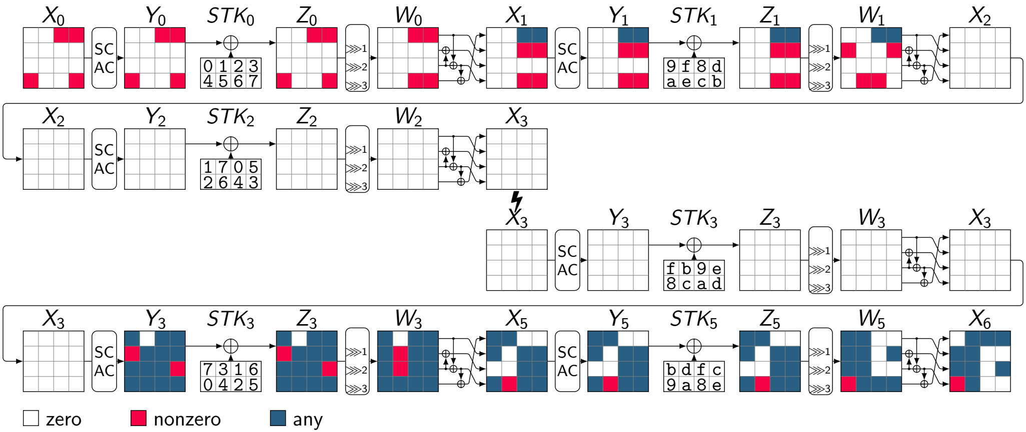

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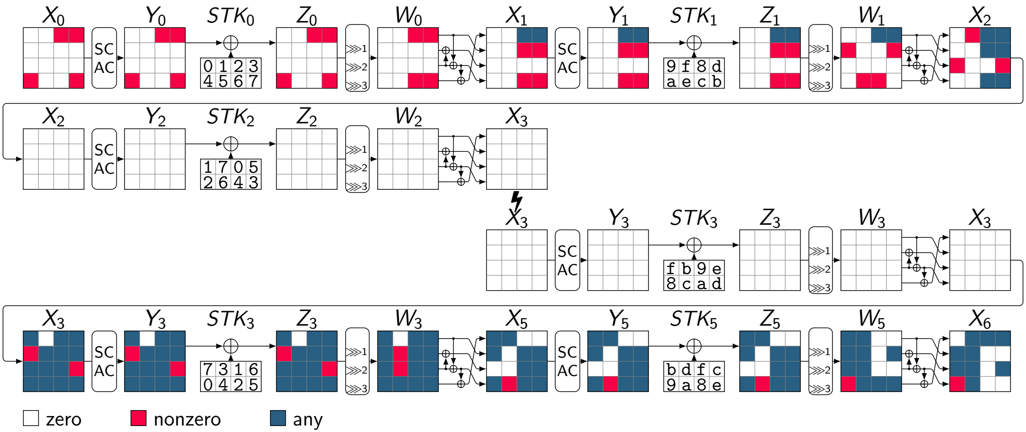

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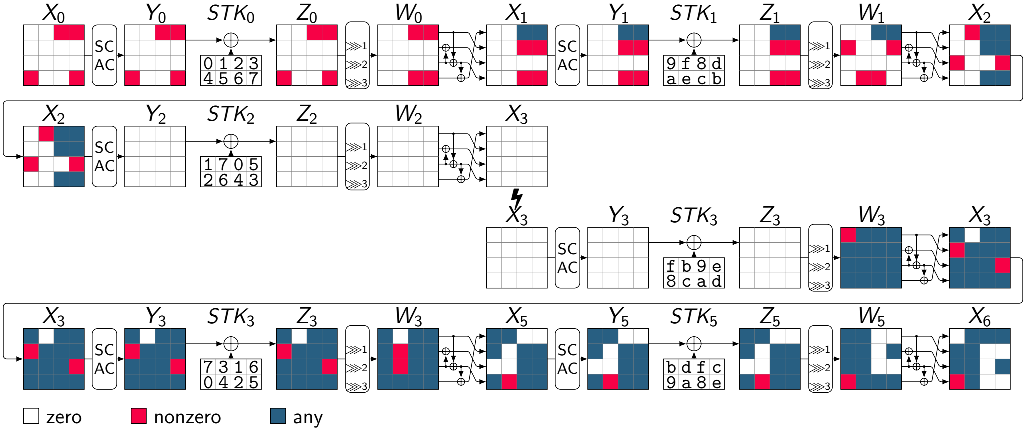

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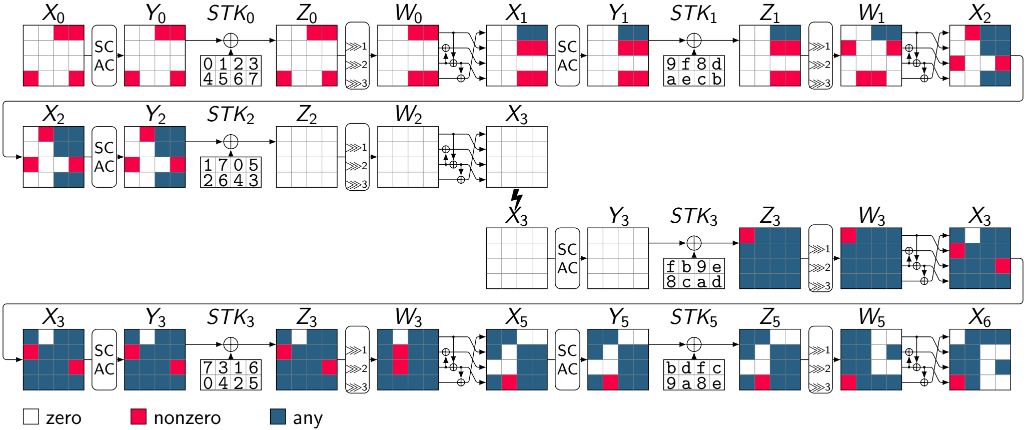

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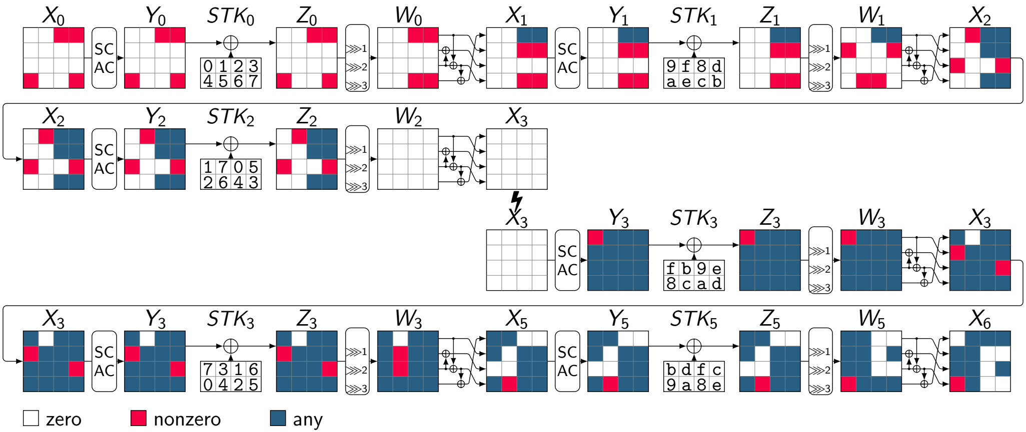

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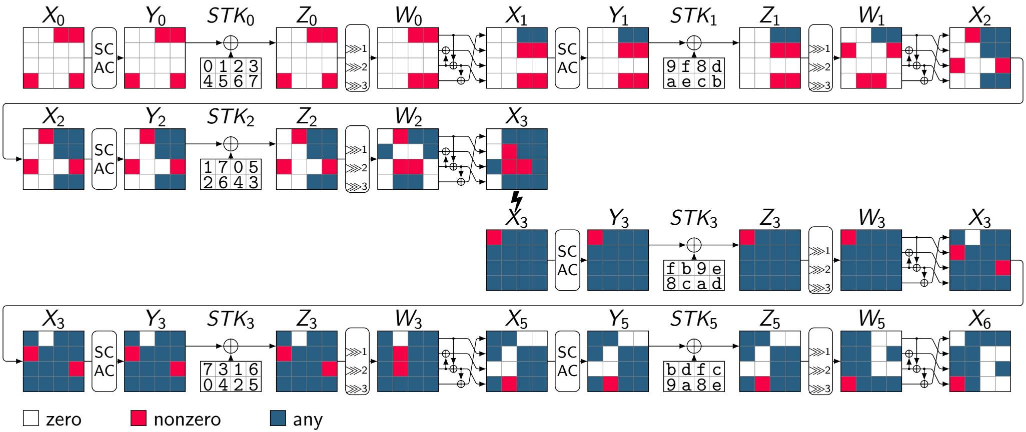
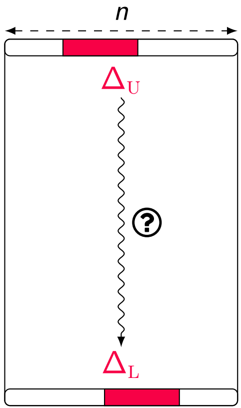


Previous Tools for ID/ZC, and Integral Attacks

- Tools based on dedicated algorithms:
  - CRYPTO 2016 ($\mathcal{DC}$-MITM, ID) [DF16]
- Tools based on general purpose solvers:
  - Eprint 2016 (ID) [Cui+16]
  - ASIACRYPT 2016 (Integral) [Xia+16]
  - EUROCRYPT 2017 (ID, ZC) [ST17]
  - ToSC 2017 (ID, ZC) [Sun+17]
  - ToSC 2020 (ID, ZC) [Sun+20]



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Our Method to Search for Distinguishers

Our Method to Search for ID/ZC and Integral Distinguishers

$$E$$

- $CSP_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$
- $CSP_{\mathrm{L}}(\Delta_{\mathrm{L}}, \Delta'_{\mathrm{L}})$
- $CSP_{\mathrm{M}}(\Delta'_{\mathrm{U}}, \Delta'_{\mathrm{L}})$

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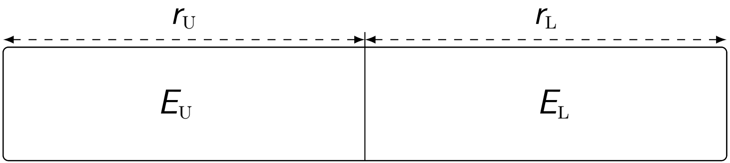

- $CSP_U(\Delta_U, \Delta'_U)$
- $CSP_L(\Delta_L, \Delta'_L)$
- $CSP_M(\Delta'_U, \Delta'_L)$

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- $CSP_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$
- $CSP_{\mathrm{L}}(\Delta_{\mathrm{L}}, \Delta'_{\mathrm{L}})$
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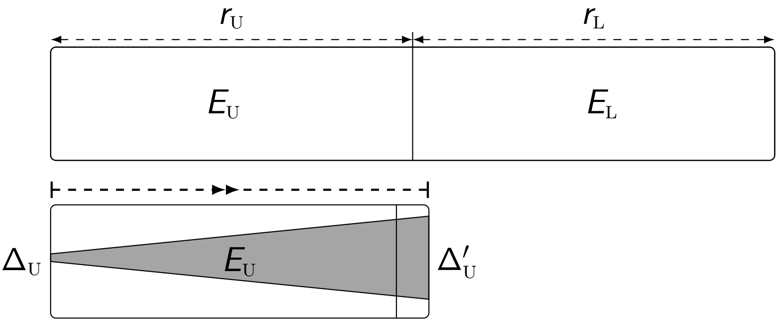

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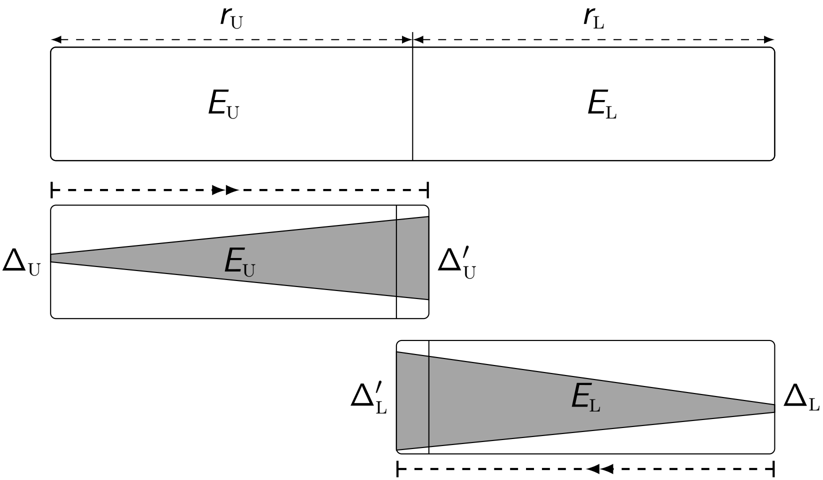

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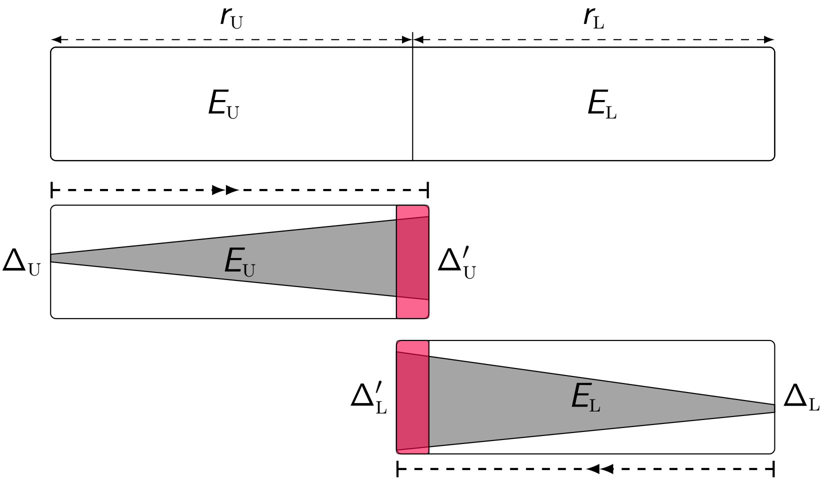

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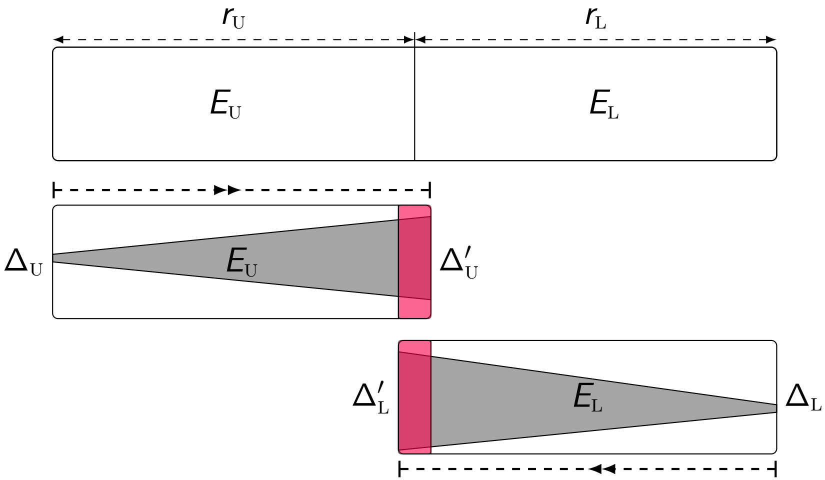

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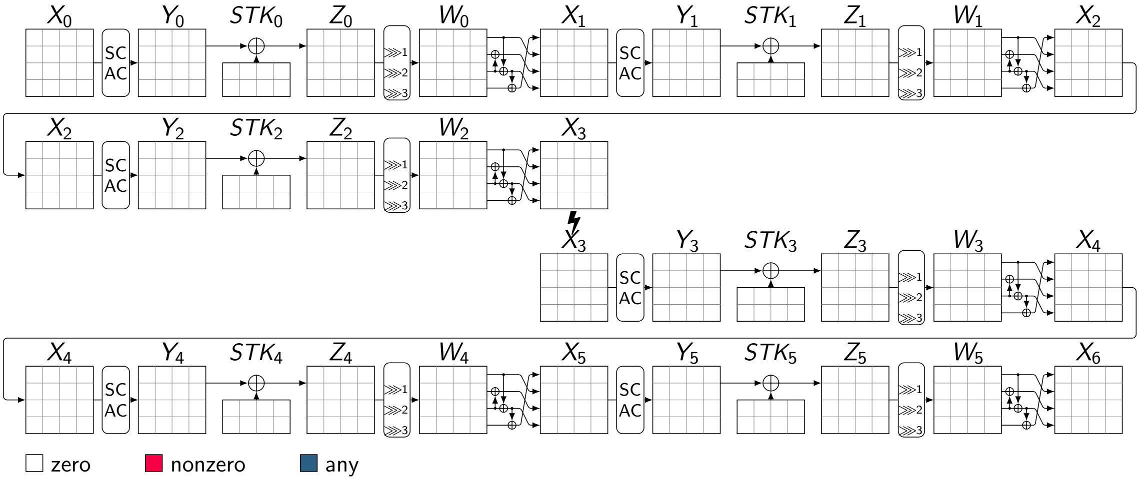
A Basic Example



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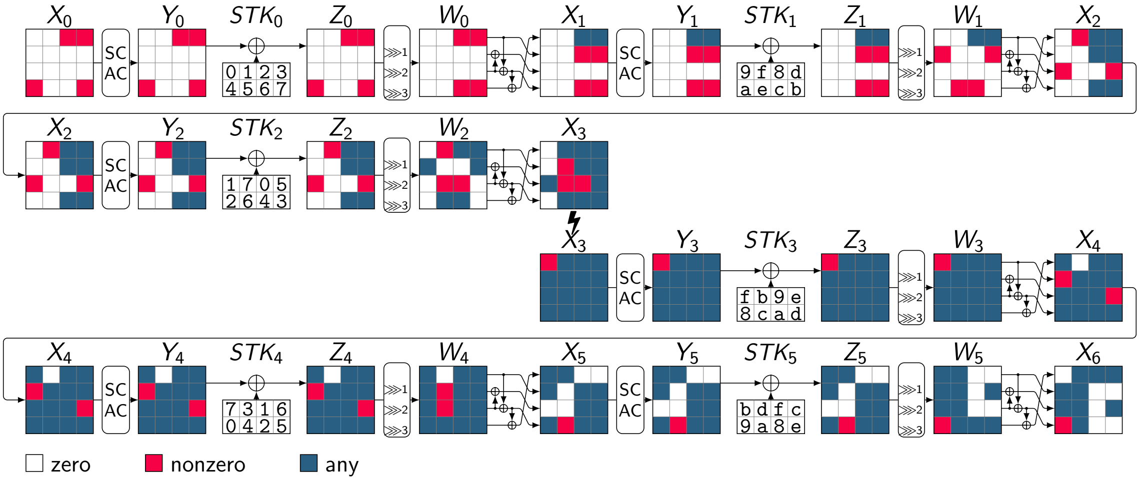


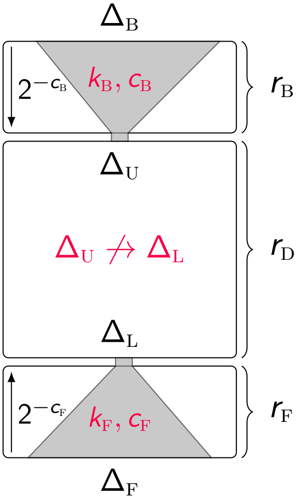
The Advantages of Our Method to Search for Distinguishers

- Based on satisfiability of the CP model
- Any feasible solutions of our CP model is a distinguisher
- We do not fix the input/output of distinguisher
- Extendable to a unified model for key-recovery
  - Find a distinguisher optimized for key-recovery
  - Taking some key-recovery techniques into account, e.g., MitM, and key bridging

Our Unified CP Model for Key-Recovery

Complexity Analysis of ID Attack [Bou+18; BNS14]

- Number of required pairs: N
- Pair generation: $T_0 = N 2^{n+1-|\Delta_{\mathrm{B}}|-|\Delta_{\mathrm{F}}|}$
- Guess-and-filter:
  - $T_1 + T_2 = N + 2^{|k_{\mathrm{B}} \cup k_{\mathrm{F}}|} \frac{N}{2^{c_{\mathrm{B}}+c_{\mathrm{F}}}}$
  - $P = \left(1 - 2^{-(c_{\mathrm{B}}+c_{\mathrm{F}})}\right)^N$
- Exhaustive search: $T_3 = P 2^k$
- $T_{tot} = (T_0 + (T_1 + T_2) C_{E'} + T_3) C_E$



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Overall View of Our CP Model for Key-Recovery

- Model the distinguisher for E_{D} $(\Delta_{\mathrm{U}}, \Delta_{\mathrm{F}})$
- Model the filters in E_{B}, and E_{F} $(c_{\mathrm{B}}, c_{\mathrm{F}}, \Delta_{\mathrm{B}}, \Delta_{\mathrm{F}})$
- Model the guess-and-determine in E_{B}, and E_{F}
- Model the key bridging
  - Encode $|k_{\mathrm{B}} \cup k_{\mathrm{F}}|$
- Model the complexity formulas

Δ_{U}

E_{D}

Δ_{L}

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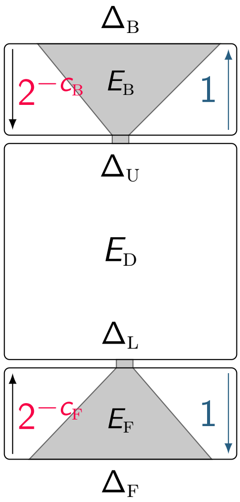

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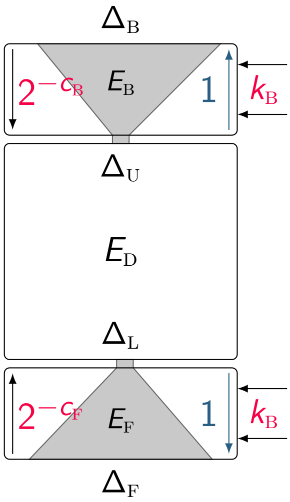

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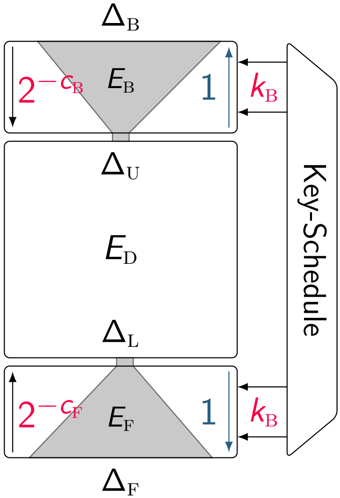

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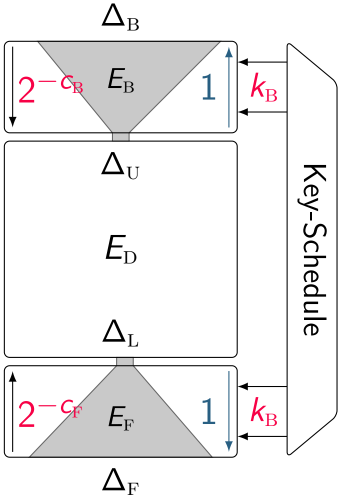

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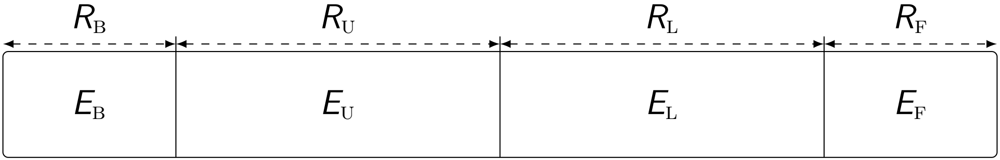
Usage of Our Tool

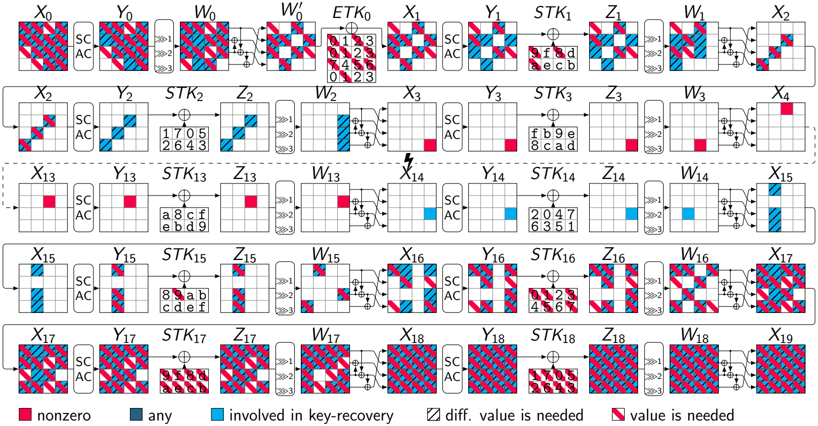
```
python3 attack.py -RB 4 -RU 10 -RL 6 -RF 7
```



- We use MiniZinc [Net+07] to create our CP models
- We use Gurobi [Gur22] and OrTools [PF] as the CP solvers
- Our tool can find the results in a few seconds running on a regular laptop

Example: 19-round ID Attack on SKINNY-n-$2n$

- $|k_{\mathrm{B}} \cup k_{\mathrm{F}}| = 26 \cdot c$
- $c_{\mathrm{B}} = 6 \cdot c$
- $c_{\mathrm{F}} = 15 \cdot c$
- $\Delta_{\mathrm{B}} = 7 \cdot c$
- $\Delta_{\mathrm{F}} = 16 \cdot c$
- $c \in \{4, 8\}$



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Part of Our Improved Results for SKINNY

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Detecting Flaws in The Previous Attacks Using our Automatic Tools

Invalid Attacks on SKINNY

Cipher	Attack	#R	Setting / Model	Ref.	Flaw
SKINNY-n-n	ID	18	STK / CP	[TAY17]	KR
SKINNY-n-$2n$	ID	20	STK / CP	[TAY17]	KR
	ZC/Int †	22	SK / CP, CT	[ZCW22]	Dist
SKINNY-n-$3n$	ID	22	STK / CP	[TAY17]	KR
	ZC/Int †	26	SK / CP, CT	[ZCW22]	Dist

Conclusion

Contributions and Future Works

- Contributions
  - Introduced efficient unified model for finding full ID/ZC/integral attacks
  - Found improved attacks for SKINNY, CRAFT, SKINNYee, and SKINNYe-v2
- Future works
  - Applying our method to other ciphers, e.g., AES, MANTIS, QARMA, etc
  - Creating the bit-oriented version of our method
  - Applying our approach in other CP models, e.g., division property
  - Improving the key-recovery part of our CP models for ZC and integral attacks

: https://github.com/hadipourh/zero

: https://ia.cr/2022/1147

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Zero-Correlation Attack and Its Relation to Integral Attack

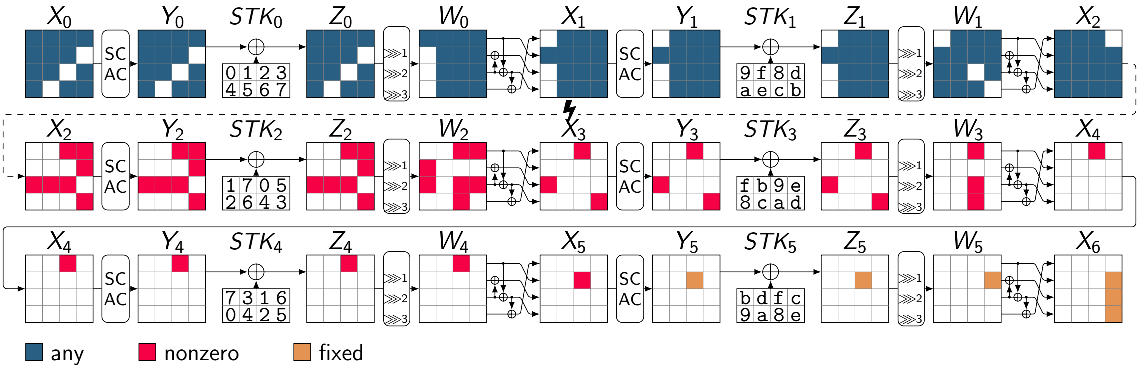
- ZC is the dual of ID in the context of linear cryptanalysis [BR14]
- Multidimensional ZC attack (ASIACRYPT 2012 [Bog+12])

Link Between ZC and Integral Attack [Sun+15]

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ be a vectorial Boolean function. Assume A is a subspace of $\mathbb{F}_2^n$ and $\beta \in \mathbb{F}_2^n \setminus \{0\}$ such that (α, β) is a ZC approximation for any $\alpha \in A$. Then, for any $\lambda \in \mathbb{F}_2^n$, $\langle \beta, F(x + \lambda) \rangle$ is balanced over the set

$$A^{\perp} = \{x \in \mathbb{F}_2^n \mid \forall\, \alpha \in A : \langle \alpha, x \rangle = 0\}.$$

Example: Conversion of ZC Distinguisher to Integral Distinguisher



- $X_0[7, 10, 13]$ takes all possible values and the remaining cells take a fixed value
- $X_6[7] \oplus X_6[11] \oplus X_6[15]$ is balanced

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