Integral Cryptanalysis of WARP based on Monomial Prediction

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FSE 2023 - Kobe, Japan

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SCIENCE PASSION TECHNOLOGY

Motivation and Our Contributions

Motivation

- Integral analysis of WARP

Contributions

- Providing a generic SAT model for integral analysis based on monomial prediction
- Our model takes the key schedule into account
- We proposed a tool for key-recovery taking the FFT technique into account
- Thanks to our tools, we improved the integral attack of WARP by 11 rounds

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Outline

1 Boolean Functions and Integral Analysis

2 Monomial Prediction and Our SAT Model

3 Application of Our Modeling to Integral Analysis of WARP

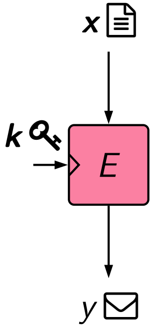
4 Key-Recovery

5 Conclusion

Boolean Functions and Integral Analysis

Integral Distinguisher and The Coefficients of ANF

- $y = f(\boldsymbol{k}, \boldsymbol{x}) = \sum_{\boldsymbol{u} \in \mathbb{F}_2^n} \sum_{\boldsymbol{v} \in \mathbb{F}_2^k} a_{\boldsymbol{u},\boldsymbol{v}} \boldsymbol{k}^{\boldsymbol{v}} \boldsymbol{x}^{\boldsymbol{u}}$
- $\mathbb{C}_{\boldsymbol{u}} = \{x \in \mathbb{F}_2^n \,|\, \boldsymbol{x} \leq \boldsymbol{u}\}$
- $a_{\boldsymbol{u}}(\boldsymbol{k}) = \sum_{\boldsymbol{x} \leq \boldsymbol{u}} f(\boldsymbol{k}, \boldsymbol{x})$
- Which monomial is key-independent in the ANF?
  - zero-sum: $\exists\, u,\ s.t. \forall \boldsymbol{k} :\ a_{\boldsymbol{u}}(\boldsymbol{k}) = 0$
  - one-sum: $\exists\, u,\ s.t. \forall \boldsymbol{k} :\ a_{\boldsymbol{u}}(\boldsymbol{k}) = 1$



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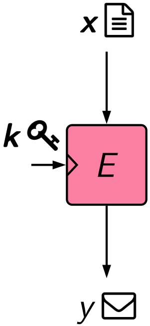

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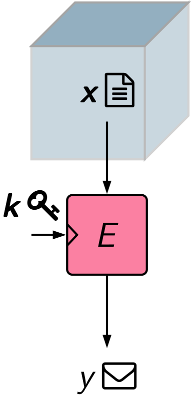

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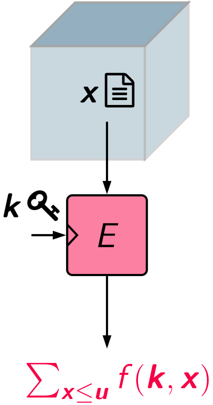

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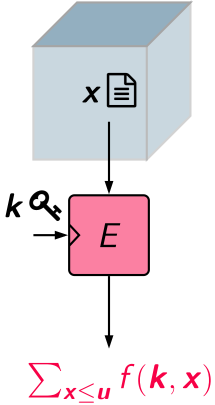

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Monomial Prediction and Our SAT Model

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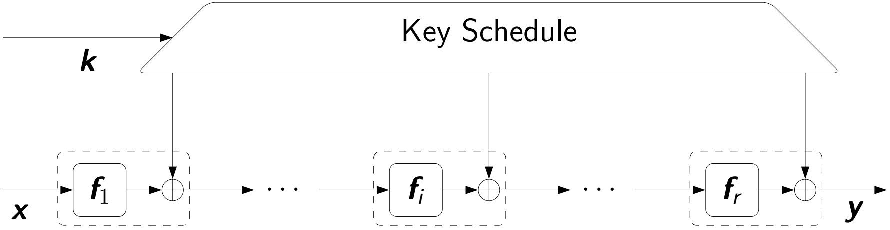
Core Idea of Monomial Prediction [Hu+20]



Core Idea

The absence (or presence) of a monomial in the ANF of a composite function can be checked by tracking the propagation of the given monomial through the building blocks of composite functions.

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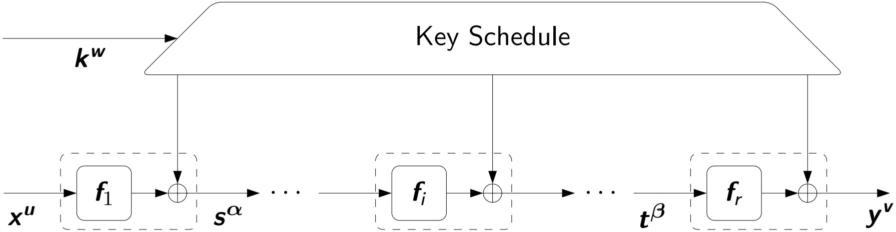
Monomial Trail and Integral Distinguisher



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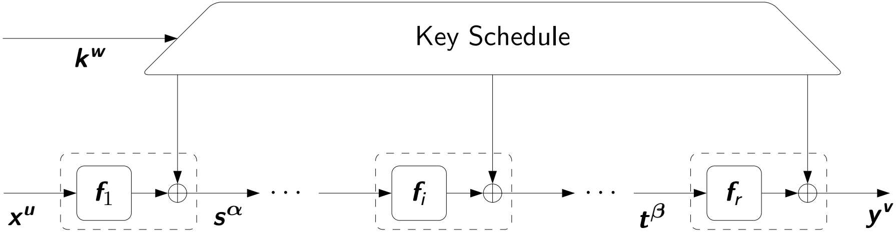

$$\boldsymbol{k}^{\boldsymbol{w}}\boldsymbol{x}^{\boldsymbol{u}} \xrightarrow{f_1} \boldsymbol{s}^{\boldsymbol{\alpha}} \xrightarrow{f_1} \cdots \xrightarrow{f_i} \cdots \xrightarrow{f_{r-1}} \boldsymbol{t}^{\boldsymbol{\beta}} \xrightarrow{f_r} \boldsymbol{y}^{\boldsymbol{v}}$$

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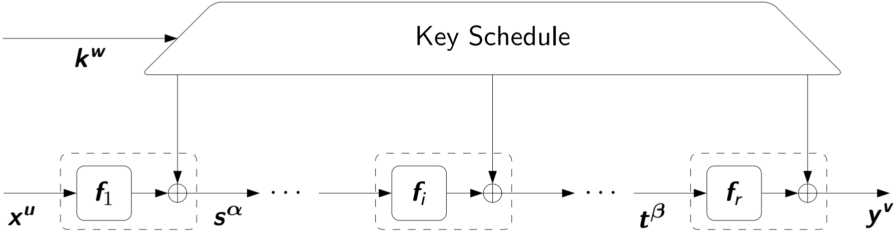

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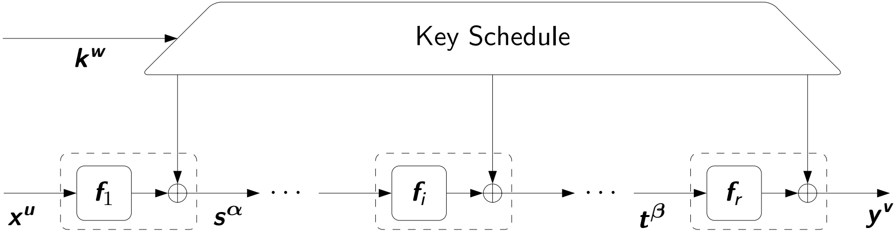

$$\boldsymbol{k}^{\boldsymbol{w}}\boldsymbol{x}^{\boldsymbol{u}} \rightarrow \boldsymbol{y}^{\boldsymbol{v}} \Rightarrow \boldsymbol{k}^{\boldsymbol{w}}\boldsymbol{x}^{\boldsymbol{u}} \rightsquigarrow \boldsymbol{y}^{\boldsymbol{v}}$$

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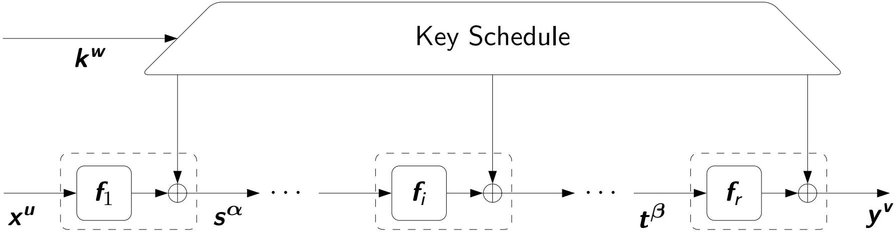

$$\boldsymbol{k}^{\boldsymbol{w}}\boldsymbol{x}^{\boldsymbol{u}} \not\rightsquigarrow \boldsymbol{y}^{\boldsymbol{v}} \Rightarrow \boldsymbol{k}^{\boldsymbol{w}}\boldsymbol{x}^{\boldsymbol{u}} \not\rightarrow \boldsymbol{y}^{\boldsymbol{v}}$$

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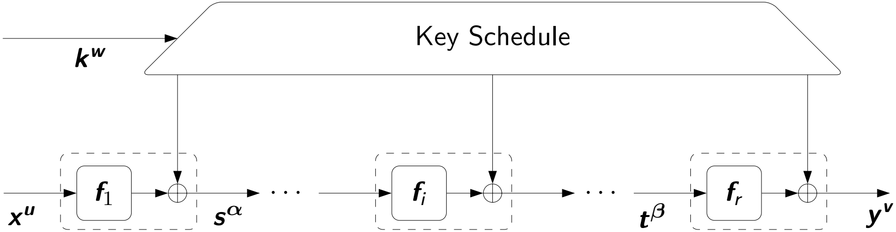

$$y^v = \sum_{u \in \mathbb{F}_2^n} \sum_{v \in \mathbb{F}_2^k} a_{u,v} k^v x^u = \sum_{u \in \mathbb{F}_2^n} a_u(k) \cdot x^u$$

From Monomial Trails to Integral Distinguisher

- If $\exists u$ s.t. $k^w x^u \not\rightsquigarrow y^v$ for all $w \in \mathbb{F}_2^k$ then $a_u(k) = 0$ (zero-sum)
- If $\exists u$ s.t. $k^w x^u \not\rightsquigarrow y^v$ for all $w \in \mathbb{F}_2^k \setminus \{0\}$ then $a_u(k) = \texttt{constant}$ (zero/one-sum)

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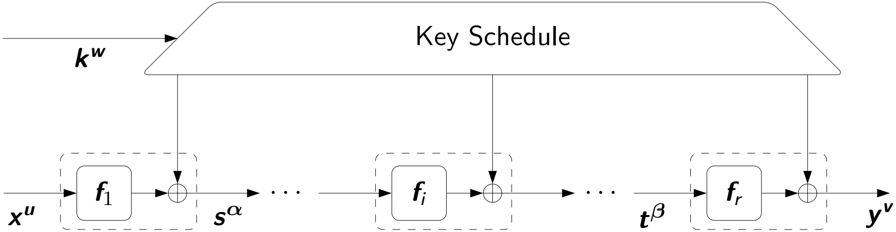

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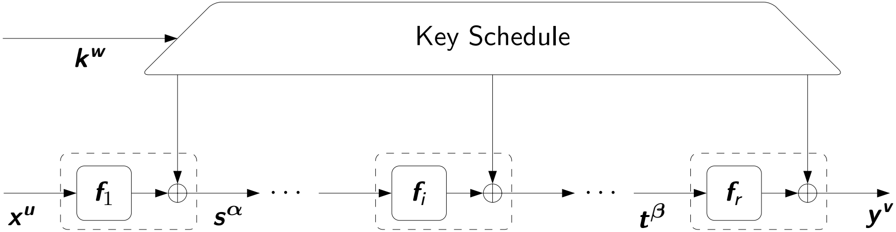

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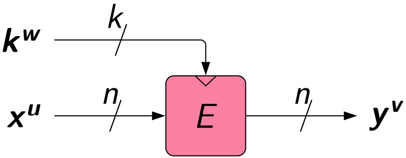
From Monomial Prediction to SAT Problem



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- Model the propagation of monomial trails through the building blocks by a CNF clause
- Main variables are the monomial exponents, i.e., $u, w, v, \ldots$ not $x, k, y, \ldots$
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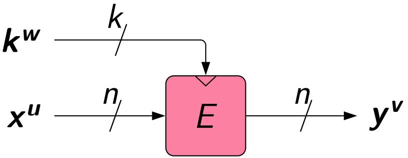

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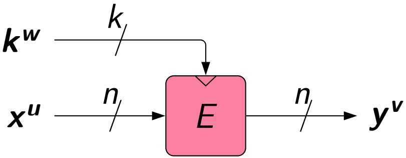

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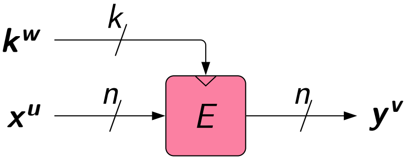

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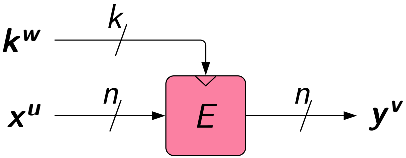

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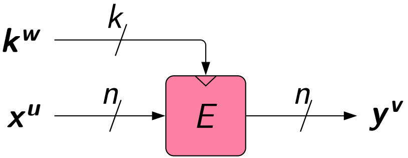

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Monomial Prediction Table ($\texttt{MPT}$)

- Let $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ be an m-bit to n-bit vectorial Boolean function. Then $\texttt{MPT}(\boldsymbol{u}, \boldsymbol{v}) = 1$ if $\boldsymbol{x}^{\boldsymbol{u}} \xrightarrow{f} \boldsymbol{y}^{\boldsymbol{v}}$, and $\texttt{MPT}(\boldsymbol{u}, \boldsymbol{v}) = 0$ otherwise.

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x	$S(x)$
0	c
1	a
2	d
3	3
4	e
5	b
6	f
7	7
8	8
9	9
a	1
b	5
c	0
d	2
e	4
f	6

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6	f
7	7
8	8
9	9
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b	5
c	0
d	2
e	4
f	6

$\boldsymbol{u} \backslash \boldsymbol{v}$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	1	·	·	·	1	·	·	·	1	·	·	·	1	·	·	·
1	·	·	1	·	1	·	·	·	·	·	1	·	1	·	·	·
2	·	1	·	·	·	1	·	·	·	1	·	·	·	1	·	·
3	·	·	·	1	·	1	·	·	1	1	1	·	·	1	·	·
4	·	·	1	·	·	·	1	·	·	·	1	·	·	·	1	·
5	·	1	1	1	·	·	1	·	·	1	1	1	·	·	1	·
6	·	·	·	1	·	·	·	1	·	·	·	1	·	·	·	1
7	·	1	·	·	1	1	1	·	·	1	·	·	·	·	·	1
8	·	·	·	·	1	·	·	·	·	·	·	·	1	·	·	·
9	·	1	1	·	1	·	·	·	·	1	1	·	1	·	·	·
a	·	·	·	·	·	1	·	·	1	1	·	·	·	1	·	·
b	·	1	·	1	1	·	·	·	1	·	1	·	·	1	·	·
c	·	·	1	·	·	·	1	·	1	·	1	·	·	·	1	·
d	·	·	·	1	·	·	1	·	·	·	1	1	·	·	1	·
e	·	1	·	1	1	·	·	1	1	·	·	1	·	·	·	1
f	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	1

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x	$S(x)$
0	c
1	a
2	d
3	3
4	e
5	b
6	f
7	7
8	8
9	9
a	1
b	5
c	0
d	2
e	4
f	6

$$
\begin{array}{lll}
(u_2 \vee \neg v_1 \vee \neg v_3) & \wedge (\neg u_1 \vee \neg v_0 \vee \neg v_1 \vee v_2) & \wedge (\neg u_0 \vee \neg u_1 \vee \neg u_2 \vee \neg v_2 \vee v_3) \\
\wedge (u_2 \vee u_3 \vee \neg v_3) & \wedge (\neg u_0 \vee \neg u_1 \vee \neg u_3 \vee v_2) & \wedge (\neg u_0 \vee \neg u_3 \vee v_0 \vee \neg v_1 \vee \neg v_3) \\
\wedge (u_1 \vee \neg v_1 \vee \neg v_2) & \wedge (\neg u_1 \vee u_2 \vee v_0 \vee v_2 \vee v_3) & \wedge (\neg u_0 \vee \neg u_1 \vee \neg u_3 \vee v_0 \vee v_1 \vee v_3) \\
\wedge (u_1 \vee u_3 \vee \neg v_2) & \wedge (u_2 \vee \neg u_3 \vee v_1 \vee v_2 \vee v_3) & \wedge (\neg u_0 \vee \neg u_2 \vee \neg u_3 \vee \neg v_0 \vee v_1 \vee \neg v_3) \\
\wedge (u_0 \vee \neg u_2 \vee u_3 \vee v_3) & \wedge (u_1 \vee \neg v_0 \vee \neg v_2 \vee \neg v_3) & \wedge (\neg u_1 \vee \neg u_2 \vee \neg u_3 \vee v_1 \vee \neg v_2) \\
\wedge (u_0 \vee \neg u_1 \vee u_3 \vee v_2) & \wedge (\neg u_0 \vee u_1 \vee u_3 \vee v_0 \vee v_1) & \wedge (\neg u_1 \vee \neg u_2 \vee \neg u_3 \vee v_1 \vee v_3) \\
\wedge (\neg u_2 \vee v_0 \vee v_1 \vee v_3) & \wedge (\neg u_1 \vee u_3 \vee \neg v_0 \vee v_2 \vee \neg v_3) & \wedge (u_0 \vee u_1 \vee \neg u_3 \vee v_0 \vee v_1 \vee v_2) \\
\wedge (u_0 \vee u_1 \vee u_2 \vee \neg v_3) & \wedge (u_0 \vee u_1 \vee \neg u_2 \vee \neg v_1 \vee v_3) & \wedge (\neg u_3 \vee v_0 \vee \neg v_1 \vee \neg v_2 \vee \neg v_3) \\
\wedge (u_1 \vee u_2 \vee \neg v_2 \vee \neg v_3) & \wedge (u_1 \vee \neg u_2 \vee u_3 \vee \neg v_1 \vee v_3) & \wedge (\neg u_0 \vee u_1 \vee u_2 \vee v_1 \vee v_2 \vee v_3) \\
\wedge (\neg u_2 \vee \neg v_0 \vee \neg v_1 \vee v_3) & \wedge (\neg u_1 \vee u_3 \vee \neg v_1 \vee v_2 \vee \neg v_3). &
\end{array}
$$

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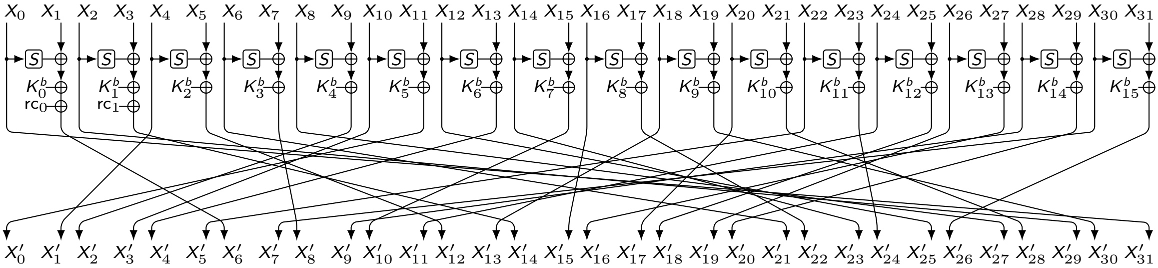
Application of Our Modeling to Integral Analysis of WARP

WARP[Ban+20]

- Proposed in SAC 2020 [Ban+20] as the lightweight alternative of `AES-128`
- 128-bit block/key size, and 41 rounds (40.5 rounds)
- Splits 128-bit K into two halves $K^{(0)}||K^{(1)}$ and uses $K^{(r-1 \mod 2)}$ in the rth round



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22-round Integral Distinguisher for WARP

The best previous integral distinguisher: 20 rounds [Ban+20]

$$(2) \xrightarrow{\text{22 rounds}} (\underline{20, 21, 22, 23}, \ 118, \ \underline{60, 61, 62, 63}),$$

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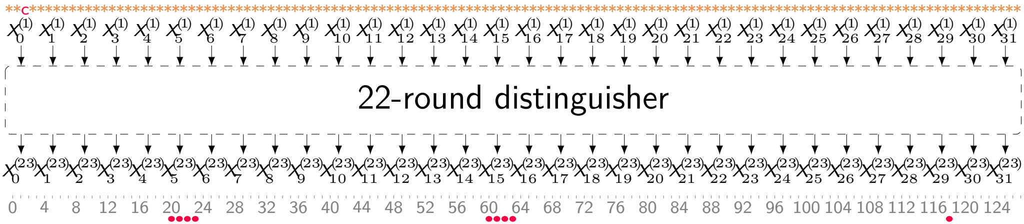

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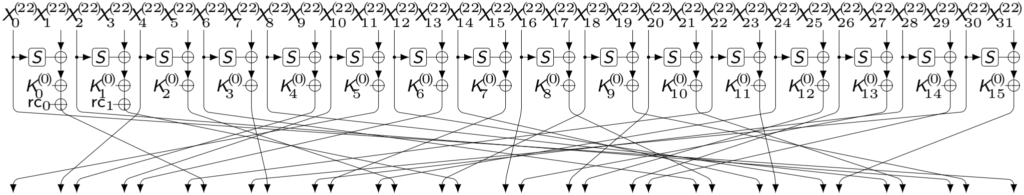
FSE 2023 - Kobe, Japan

23-round Integral Distinguisher for WARP

- Any r-round integral distinguisher of WARP can be extended by 1 round



- $\sum_{\mathbb{C}} X_4^{(22)} = \sum_{\mathbb{C}} X_1^{(23)}$

- $\sum_{\mathbb{C}} X_{11}^{(22)} = \sum_{\mathbb{C}} \left(S(X_4^{(23)}) \oplus X_0^{(23)}\right) \oplus \sum_{\mathbb{C}} K_i^{(b)}$

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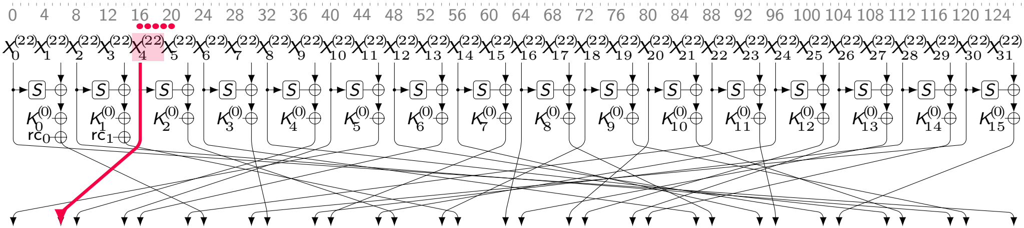

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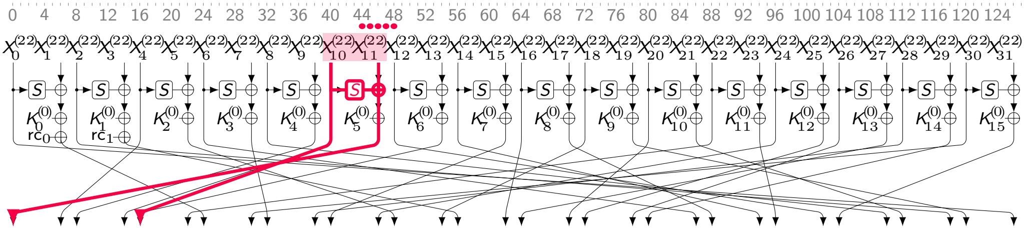

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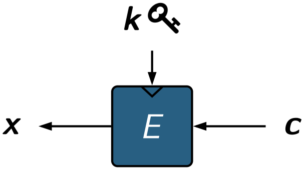
Key-Recovery

Naive Approach v.s. FFT Technique [TA14]



- Naive approach:
  - $\sum \boldsymbol{x} = \sum_{c \in \mathbb{C}} f(\boldsymbol{k}, \boldsymbol{c})$
  - $T_{tot} = 2^{|\mathbf{k}|}|\mathbb{C}|$, where $\mathbb{C} = \mathbf{2}^{|\mathbf{k}|}$
  - $T_{tot} = 2^{2|\boldsymbol{k}|}$
- FFT technique:
  - $\sum \boldsymbol{x} = \sum_{c \in \mathbb{C}} F(\boldsymbol{k} \oplus \boldsymbol{c})$
  - $T_{tot} = 4 \cdot |\boldsymbol{k}| \cdot 2^{|\boldsymbol{k}|}$

Naive Approach v.s. FFT Technique [TA14]

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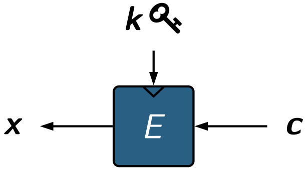

- FFT technique:
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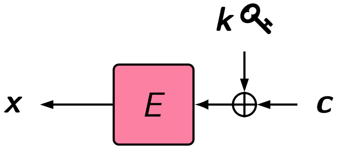

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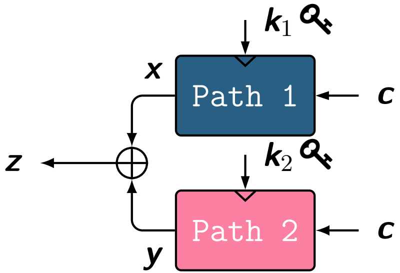
MitM [SW12]

- Naive approach:
  - $\boldsymbol{x} = F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{c})$
  - $T = |\mathbb{C}| \cdot 2^{|\boldsymbol{k}_1 \cup \boldsymbol{k}_2|}$
- MitM:
  - $\boldsymbol{x} = g(\boldsymbol{k}_1, \boldsymbol{c}),\ \boldsymbol{y} = h(\boldsymbol{k}_2, \boldsymbol{c})$
  - $T = |\mathbb{C}| \cdot 2^{|\boldsymbol{k}_1|} + |\mathbb{C}| \cdot 2^{|\boldsymbol{k}_2|}$



$$\sum \boldsymbol{z} = 0$$

MitM [SW12]

Naive approach:

- $\boldsymbol{x} = F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{c})$
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MitM:

- $\boldsymbol{x} = g(\boldsymbol{k}_1, \boldsymbol{c}),\ \boldsymbol{y} = h(\boldsymbol{k}_2, \boldsymbol{c})$
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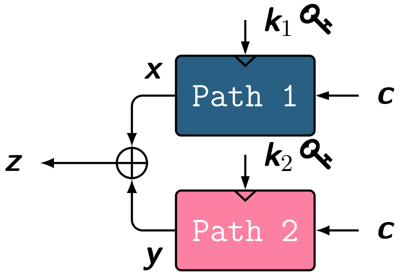

$$\sum \boldsymbol{z} = 0 \iff \sum \boldsymbol{x} = \sum \boldsymbol{y}$$

Overall View of Our Key-Recovery Tool

1- Assume that $\boldsymbol{x} = \boldsymbol{y} \oplus \boldsymbol{z}$ and $\sum \boldsymbol{x} = 0$

2- For each path, i.e., $\boldsymbol{y}$, and $\boldsymbol{z}$:

- Build the graph of dependencies: $\boldsymbol{y} = f(\boldsymbol{k}, \boldsymbol{c})$
- Simplify the dependency graph: reform $f(\boldsymbol{k}, \boldsymbol{c})$ to $F(\tilde{\boldsymbol{k}} \oplus \tilde{\boldsymbol{c}})$
- Use FFT to compute the list $[\sum \boldsymbol{y} \mid \tilde{\boldsymbol{k}} = 0, \ldots, 2^{|\boldsymbol{k}|-1}]$

3- Compare the two lists to find candidates for the involved key bits

4- Brute force the remaining keys to find the correct key

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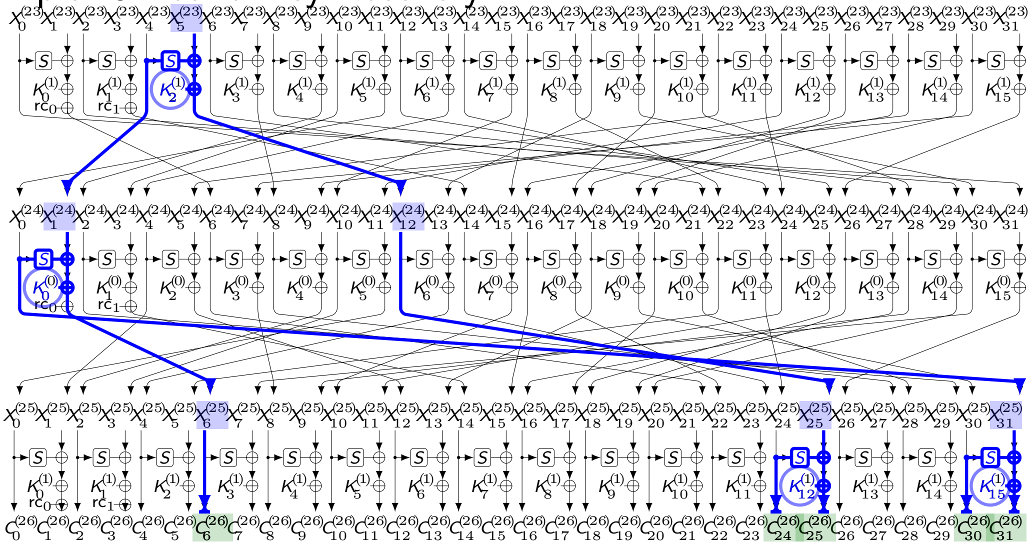
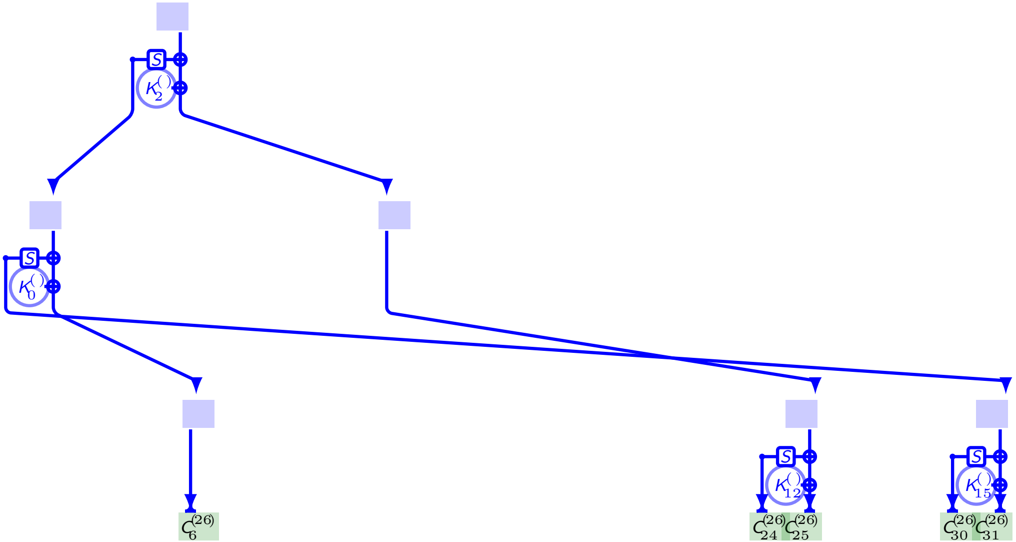
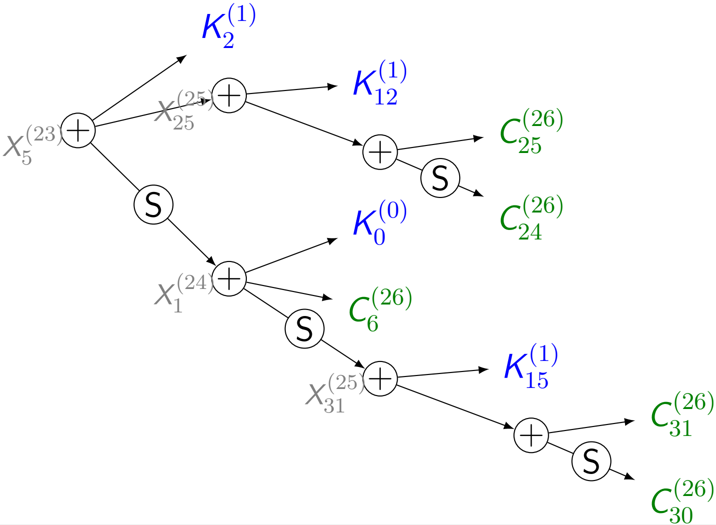
Example: 3-Round Key Recovery



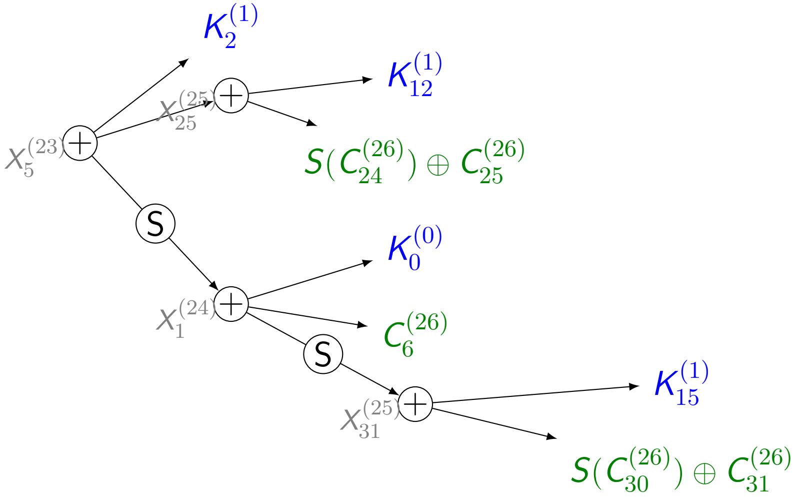
Example: 3-Round Key Recovery



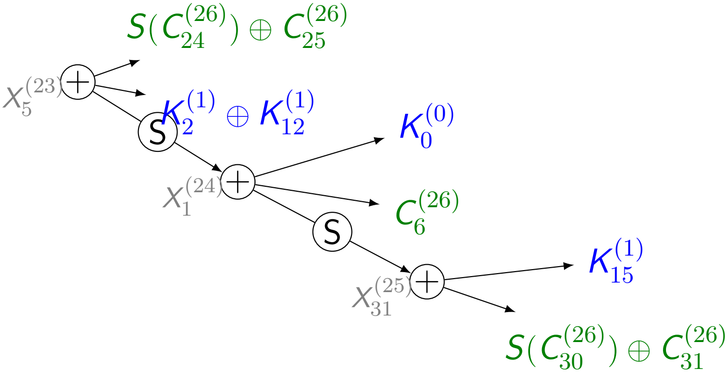
Example: Dependency Graph



Example: Dependency Graph



Example: Dependency Graph



Summary of Our Result

#R	Data	Time	Memory	Attack	Reference
32	2^{127}	2^{127}	2^{108}	Integral	This paper
21	2^{124}	-	-	Integral	[Ban+20]
18	$2^{104.62}$	-	-	Differential	[TB22]
21	-	-	-	Impossible diff.	[Ban+20]
21	2^{113}	2^{113}	2^{72}	Differential	[KY21]
23	$2^{106.62}$	$2^{106.62}$	$2^{106.62}$	Differential	[TB22]
24	$2^{126.06}$	$2^{125.18}$	$2^{127.06}$	Rectangle	[TB22]

Conclusion

Contributions

- We provided a SAT model for integral analysis based on Monomial prediction
- Our modeling is generic and can be applied to other (binary field) block ciphers
- We proposed a tool for key-recovery taking the FFT technique into account
- Overall, we improved the integral attack of `WARP` by **11** rounds

Thanks for your attention!

https://github.com/hadipourh/mpt

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