Revisiting Differential-Linear Attacks via a Boomerang Perspective

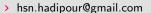


Applications to AES, Ascon, CLEFIA, SKINNY, PRESENT, KNOT, TWINE, WARP,

LBlock, Simeck, and SERPENT

Hosein Hadipour Patrick Derbez CRYPTO 2024 - California, USA

Maria Eichlseder



Research Gap and Our Contributions

- Research Gap
 - **❷** How to analytically estimate the correlation of DL distinguishers?
 - ❷ How to (efficiently) find good DL distinguishers?
- Contributions
 - igspace Generalizing the DLCT framework [Bar+19] for analytical correlation estimation
 - Introducing an efficient method to search for DL distinguishers applicable to:
 - Strongly aligned SPN primitives: AES, SKINNY
 - Weakly aligned SPN primitives: Ascon, SERPENT, KNOT, PRESENT
 - Feistel structures: CLEFIA, TWINE, LBlock, LBlock-s, WARF
 - AndRX designs: Simeck

Research Gap and Our Contributions

- # Research Gap
 - **②** How to analytically estimate the correlation of DL distinguishers?
 - **②** How to (efficiently) find good DL distinguishers?
- Contributions
 - igotimes Generalizing the DLCT framework [Bar+19] for analytical correlation estimation.
 - **♥** Introducing an efficient method to search for DL distinguishers applicable to:
 - Strongly aligned SPN primitives: AES, SKINNY
 - Weakly aligned SPN primitives: Ascon, SERPENT, KNOT, PRESENT
 - Feistel structures: CLEFIA, TWINE, LBlock, LBlock-s, WARP
 - AndRX designs: Simeck

Outline

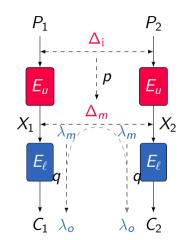
- Background
- 2 Generalized DLCT Framework
- 3 Differential-Linear Switches and Deterministic Trails
- 4 Automatic Tools to Search for DL Distinguishers
- 5 Contributions and Future Works

Background



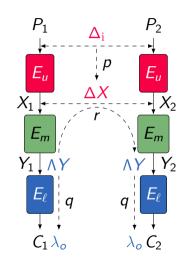
Differential-Linear (DL) Attack [LH94]

- $\blacksquare \quad \mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_u} \Delta_m) = p$
- $\blacksquare \quad \mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\mathrm{o}}) = q$
- Assumptions $(\Delta X = X_1 \oplus X_2)$:
 - 1. E_u , and E_ℓ are statistically independent
 - 2. $\mathbb{P}(\lambda_m \cdot \Delta X = 0) = 1/2$ when $\Delta X \neq \Delta_m$



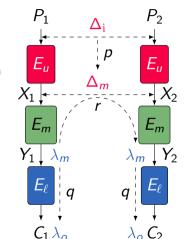
Sandwich Framework for DL Attack [BLN14; DKS14; Bar+19]

- $\blacksquare \quad \mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
- $\qquad \mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) = \sum_{\Delta X, \Lambda Y} \mathbb{P}(\Delta_{\mathrm{i}}, \Delta X) \cdot \mathbb{R}(\Delta X, \Lambda Y) \cdot \mathbb{C}^{2}(\Lambda Y, \lambda_{\mathrm{o}})$
- $\blacksquare \quad \mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_{u}} \Delta_{m}) = p$
- $\blacksquare \quad \mathbb{R}(\Delta_m, \lambda_m) = r$
- $\mathbb{C}(\lambda_0 \cdot \Delta C) \approx prg^2$



Sandwich Framework for DL Attack [BLN14; DKS14; Bar+19]

- $\blacksquare \quad \mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
- $\qquad \mathbb{C}(\lambda_{o} \cdot \Delta C) = \sum_{\Delta X, \Lambda Y} \mathbb{P}(\Delta_{i}, \Delta X) \cdot \mathbb{R}(\Delta X, \Lambda Y) \cdot \mathbb{C}^{2}(\Lambda Y, \lambda_{o})$
- $\blacksquare \quad \mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{R}(\Delta_m, \lambda_m) = r$
- $\blacksquare \quad \mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\mathrm{o}}) = q$
- $\mathbb{C}(\lambda_{0}\cdot\Delta C)\approx prq^{2}$

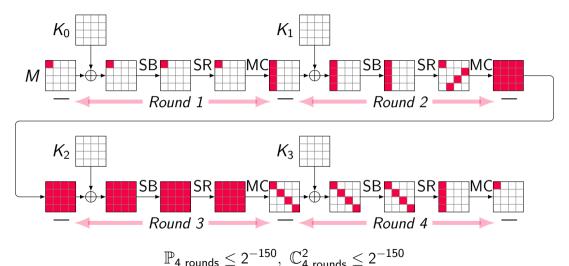


Differential-Linear Connectivity Table (DLCT) [Bar+19]



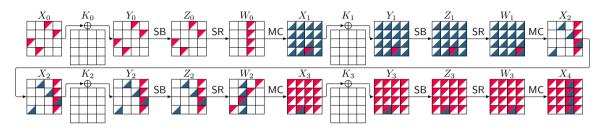
$$\begin{aligned} \mathtt{DLCT}_b(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) &= \{x \in \mathbb{F}_2^n: \ \lambda_{\mathrm{o}} \cdot S(x) \oplus \lambda_{\mathrm{o}} \cdot S(x \oplus \Delta_{\mathrm{i}}) = b\} \\ \mathtt{DLCT}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) &= |\mathtt{DLCT}_0(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}})| - |\mathtt{DLCT}_1(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}})| \\ \mathbb{C}_{\mathtt{DLCT}}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) &= 2^{-n} \cdot \mathtt{DLCT}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) \end{aligned}$$

Security of AES Against Differential/Linear Attacks



<u>Hosein Hadipour</u>, Patrick Derbez, Maria Eichlseder CRYPTO 2024 - California, USA

A 4-round DL Distinguisher for AES



$$r_u = 1, r_m = 3, r_\ell = 0, \ p = 2^{-24.00}, \ r = 2^{-7.66}, q^2 = 1, \ \mathbb{C} = prq^2 = 2^{-31.66}$$

2^{63.32} v.s. 2¹⁵⁰

Generalized DLCT Framework



Upper Differential-Linear Connectivity Table (UDLCT)



$$\begin{split} \text{UDLCT}_b(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) &= \{x \in \mathbb{F}_2^n: \ S(x) \oplus S(x \oplus \Delta_{\mathrm{i}}) = \Delta_{\mathrm{o}} \ \text{and} \ \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}} = b\} \\ \\ \text{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) &= |\text{UDLCT}_0(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}})| - |\text{UDLCT}_1(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}})| \\ \\ \mathbb{C}_{\text{UDLCT}}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) &= 2^{-n} \cdot \text{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) \end{split}$$

Lower Differential-Linear Connectivity Table (LDLCT)



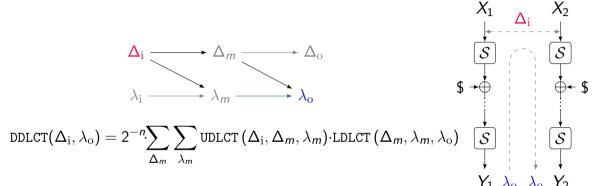
$$\begin{split} \text{LDLCT}_b(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= \{x \in \mathbb{F}_2^n : \ \lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot S(x) \oplus \lambda_{\mathrm{o}} \cdot S(x \oplus \Delta_{\mathrm{i}}) = b\} \\ \text{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= |\text{LDLCT}_0(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})| - |\text{LDLCT}_1(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})| \\ \mathbb{C}_{\text{LDLCT}}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= 2^{-n} \cdot \text{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) \end{split}$$

Extended Differential-Linear Connectivity Table (EDLCT)



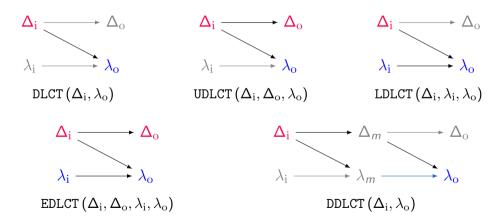
$$\begin{split} \mathtt{EDLCT}_b(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= \{x \in \mathbb{F}_2^n: \ S(x) \oplus S(x \oplus \Delta_{\mathrm{i}}) = \Delta_{\mathrm{o}} \ \mathsf{and} \ \lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}} = b\} \\ &\quad \mathtt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= |\mathtt{EDLCT}_0(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})| - |\mathtt{EDLCT}_1(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})| \\ &\quad \mathbb{C}_{\mathtt{EDLCT}}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \mathtt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) \end{split}$$

Double Differential-Linear Connectivity Table (DDLCT)

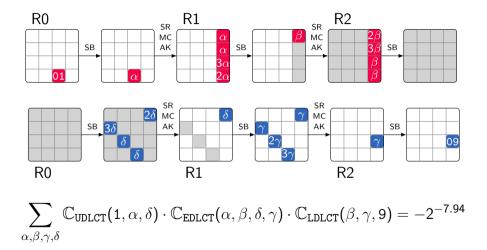


Generalized DLCT Framework (GBCT)

How to formulate the correlation for more than 1 round?



Application of the Generalized DLCT Tables - AES (- differential - linear)



Application of the Generalized DLCT Tables - TWINE (- differential - linear)



$$\begin{split} \mathbb{C}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) &= \sum_{\Delta_{m}} \mathbb{P}_{\mathtt{DDT}}(\Delta_{\mathrm{i}},\Delta_{m}) \cdot \mathbb{C}_{\mathtt{DDLCT}}\left(\Delta_{m},\lambda_{\mathrm{o}}\right) \\ &= \sum_{\lambda_{m}} \mathbb{C}_{\mathtt{DDLCT}}\left(\Delta_{\mathrm{i}},\lambda_{m}\right) \cdot \mathbb{C}_{\mathtt{LAT}}^{2}\left(\lambda_{m},\lambda_{\mathrm{o}}\right). \\ \mathbb{C}_{tot}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}) &= \mathbb{C}^{2}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}). \end{split}$$

$Input/Output\ Differences/Linear-mask$	Formula	Exp. Correlation
$(\Delta_{ m i},\lambda_{ m o})=$ (0xb4, 0x67)	$-2^{-7.66}$	$-2^{-7.64}$
$(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}})=(\texttt{0x02},\texttt{0x02})$	$-2^{-7.92}$	$-2^{-7.93}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\texttt{0x55}, \texttt{0x55})$	$-2^{-7.99}$	$-2^{-7.98}$
$(\Delta_{\rm i}, \lambda_{\rm o}) = (\texttt{Oxbf}, \texttt{Oxef})$	$-2^{-8.05}$	$-2^{-8.06}$
$(\Delta_{ m i},\lambda_{ m o})=$ (0xfe,0x06)	$-2^{-8.26}$	$-2^{-8.25}$
$(\Delta_{ m i},\lambda_{ m o})=$ (0x4b,0x1a)	$-2^{-8.43}$	$-2^{-8.44}$

Differential-Linear Switches and Deterministic Trails



Cell-Wise and Bit-Wise Switches

X	0	1	2	3	4	5	6	7	8	9	а	b	С	d	е	f
S(x)	4	0	а	7	b	е	1	d	9	f	6	8	5	2	С	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	С	d	е	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
С	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
е	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

- Cell-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}},0) = \mathtt{DLCT}(0,\lambda_{\mathrm{o}}) = 2^n$ for all $\Delta_{\mathrm{i}},\lambda_{\mathrm{o}}$
 - Bit-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}},\lambda_{\mathrm{o}})=\pm 2^{n} \text{ for } \Delta_{\mathrm{i}},\lambda_{\mathrm{o}}\neq 0$
 - Example: $\mathbb{C}(9,4) = \frac{16}{16}$

Bit-Wise Switches and Deterministic Trails

X		1														
S(x)	4	0	a	7	b	е	1	d	9	f	6	8	5	2	С	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	С	d	е	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
С	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
е	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

$$\Delta_{\mathrm{i}} = (0,0,0,1) \xrightarrow{S} \Delta_{\mathrm{o}} = (?,1,?,?)$$

$$\Delta_{\rm i} = (0,1,0,0) \xrightarrow{\mathcal{S}} \Delta_{\rm o} = (1,?,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,0) \stackrel{\mathcal{S}}{
ightarrow} \Delta_{\mathrm{o}} = (1,1,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,1) \stackrel{\mathcal{S}}{\rightarrow} \Delta_{\mathrm{o}} = (?,0,?,?)$$

$$\Delta_i = (1, 1, 0, 0) \xrightarrow{S} \Delta_0 = (0, ?, ?, ?)$$

$$\lambda_{i} = (1, ?, ?, 1) \stackrel{S}{\leftarrow} \lambda_{0} = (0, 1, 0, 0)$$

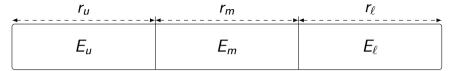
$$\lambda_{\rm i} = (1, 1, ?, ?) \stackrel{S}{\leftarrow} \lambda_{\rm o} = (1, 0, 0, 0)$$

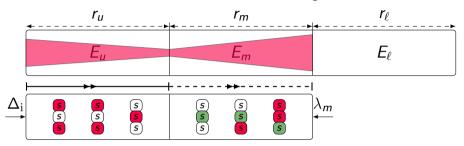
$$\lambda_{\rm i} = (0,?,?,?) \stackrel{S}{\leftarrow} \lambda_{\rm o} = (1,1,0,0)$$

Automatic Tools to Search for DL Distinguishers

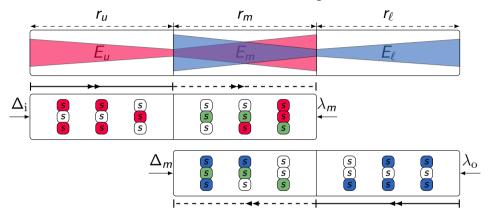


E

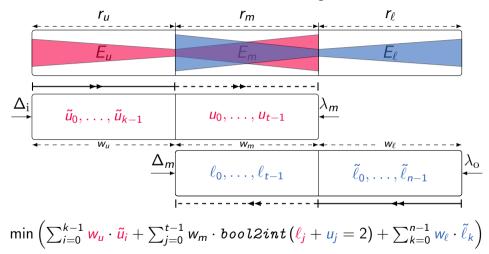




- differentially active S-box
 linearly active S-box
 common active S-box



- differentially active S-box
 linearly active S-box
 common active S-box



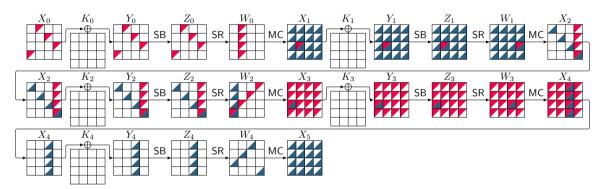
Hosein Hadipour, Patrick Derbez, Maria Eichlseder CRYPTO 2024 - California, USA

Usage of Our Tool

python3 attack.py -RU 6 -RM 10 -RL 6

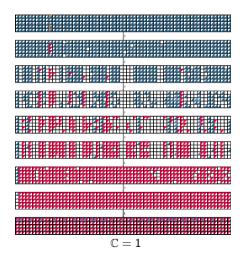
r_u	r _m	r_ℓ
E _u	E _m	E_ℓ

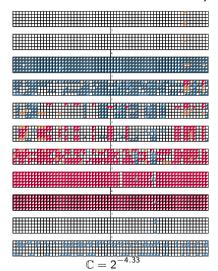
Results: A 5-round DL Distinguisher for AES



$$r_0 = 1, r_m = 3, r_1 = 1, p = 2^{-24.00}, r = 2^{-7.66}, q^2 = 2^{-24.00}, prq^2 = 2^{-55.66}$$

Results: Application to Ascon-p(active difference unknown difference active mask unknown mask)





Contributions and Future Works



Contributions and Future Works

- Contributions
 - We generalized the DLCT framework from one S-box layer to multiple rounds
 - We proposed an automatic tool for finding optimum DL distinguishers
 - We applied our tool to almost any design paradigm
- Future works
 - A Extending the application of our tool to other primitives, e.g., ARX
 - A Extending our tool to a unified model for finding complete attack (key recovery)
 - : https://github.com/hadipourh/DL
 - : https://ia.cr/2024/255

Bibliography I

- [Bar+19] Achiya Bar-On et al. DLCT: A New Tool for Differential-Linear Cryptanalysis. EUROCRYPT 2019. Vol. 11476. LNCS. Springer, 2019, pp. 313–342. DOI: 10.1007/978-3-030-17653-2_11.
- [BLN14] Céline Blondeau, Gregor Leander, and Kaisa Nyberg. **Differential-Linear**Cryptanalysis Revisited. FSE 2014. Ed. by Carlos Cid and Christian Rechberger. Vol. 8540. LNCS. Springer, 2014, pp. 411–430. DOI: 10.1007/978-3-662-46706-0_21.
- [DIK08] Orr Dunkelman, Sebastiaan Indesteege, and Nathan Keller. A Differential-Linear Attack on 12-Round Serpent. INDOCRYPT 2008. Ed. by Dipanwita Roy Chowdhury, Vincent Rijmen, and Abhijit Das. Vol. 5365. LNCS. Springer, 2008, pp. 308–321. DOI: 10.1007/978-3-540-89754-5_24.

Bibliography II

- [DKS14] Orr Dunkelman, Nathan Keller, and Adi Shamir. A Practical-Time Related-Key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony. J. Cryptol. 27.4 (2014), pp. 824–849. DOI: 10.1007/s00145-013-9154-9.
- [HNE22] Hosein Hadipour, Marcel Nageler, and Maria Eichlseder. Throwing Boomerangs into Feistel Structures Application to CLEFIA, WARP, LBlock, LBlock-s and TWINE. *IACR Trans. Symmetric Cryptol.* 2022.3 (2022), pp. 271–302. DOI: 10.46586/TOSC.V2022.I3.271–302.
- [LH94] Susan K. Langford and Martin E. Hellman. Differential-Linear Cryptanalysis. CRYPTO '94. Vol. 839. Springer, 1994, pp. 17–25. DOI: 10.1007/3-540-48658-5_3.

Bibliography III

[ZWH24] Yanyan Zhou, Senpeng Wang, and Bin Hu. MILP/MIQCP-Based Fully Automatic Method of Searching for Differential-Linear Distinguishers for SIMON-Like Ciphers. *IET Information Security* 2024 (2024). DOI: 10.1049/2024/8315115.

Properties of Generalized DLCT Tables - I

- DLCT $(\Delta_{i}, \lambda_{o}) = \sum_{\Delta_{o}} \text{UDLCT}(\Delta_{i}, \Delta_{o}, \lambda_{o})$
- $\quad \quad \text{UDLCT}(\Delta_{\rm i}, \Delta_{\rm o}, \lambda_{\rm o}) = (-1)^{\Delta_{\rm o} \cdot \lambda_{\rm o}} \text{DDT}(\Delta_{\rm i}, \Delta_{\rm o})$
- $\blacksquare \quad \mathsf{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\Delta_{\mathrm{i}} \cdot \lambda_{\mathrm{i}}} \mathsf{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$
- $\qquad \text{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}}} \text{DDT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}})$
- LDLCT $(\Delta_i, \lambda_i, \lambda_o) = \sum_{\Delta_o} \mathtt{EDLCT}(\Delta_i, \Delta_o, \lambda_i, \lambda_o)$
- $\sum_{\Lambda_i} \text{LDLCT}(\Delta_i, \lambda_i, \lambda_o) = \text{LAT}^2(\lambda_i, \lambda_o)$

Properties of Generalized DLCT Tables - II

 $\qquad \mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \sum_{\Delta_{m}} \sum_{\lambda_{m}} \mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_{m}, \lambda_{m}\right) \cdot \mathtt{LDLCT}\left(\Delta_{m}, \lambda_{m}, \lambda_{\mathrm{o}}\right)$

$$\begin{split} \text{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= \sum_{\Delta_{m}} \text{DDT}(\Delta_{\mathrm{i}}, \Delta_{m}) \cdot \text{DLCT}(\Delta_{m}, \lambda_{\mathrm{o}}) \\ &= 2^{-n} \sum_{\lambda_{-}} \text{DLCT}(\Delta_{\mathrm{i}}, \lambda_{m}) \cdot \text{LAT}^{2}(\lambda_{m}, \lambda_{\mathrm{o}}). \end{split}$$

Example: Distinguishers for up to 17 Rounds of TWINE

Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	2 ^{3.20}	1	$2^{3.20}$
13	2 ^{34.32}	$2^{27.16}$	$2^{7.16}$
14	2 ^{42.25}	$2^{31.28}$	$2^{10.97}$
15	2 ^{51.03}	$2^{38.98}$	$2^{12.05}$
16	2 ^{58.04}	2 ^{47.28}	$2^{10.76}$
17	-	2 ^{59.24}	-

Results: Distinguishers for up to 17 Rounds of LBlock

Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	$2^{2.97}$	1	$2^{2.97}$
13	2 ^{30.28}	2 ^{23.78}	$2^{6.50}$
14	2 ^{38.86}	2 ^{30.34}	$2^{8.52}$
15	2 ^{46.90}	2 ^{38.26}	$2^{8.64}$
16	2 ^{57.16}	2 ^{46.26}	$2^{10.90}$
17	-	2 ^{58.30}	-

Results: Distinguishers for up to 8 Rounds of CLEFIA

Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
3	1	1	1
4	$2^{6.32}$	1	$2^{6.32}$
5	$2^{12.26}$	2 ^{5.36}	$2^{6.90}$
6	2 ^{22.45}	2 ^{14.14}	$2^{8.31}$
7	2 ^{32.67}	2 ^{23.50}	$2^{9.17}$
8	2 ^{76.03}	2 ^{66.86}	$2^{9.17}$

Results: Application to SERPENT

■ □: Experimentally verified

Cipher	#R	\mathbb{C}		Ref.
	3	$2^{-0.68}$	√	This work
	4	$2^{-12.75}$		[DIK08]
	4	$2^{-5.54}$	\checkmark	This work
CEDDENT	5	$2^{-16.75}$		[DIK08]
SERPENT	5	$2^{-11.10}$	\checkmark	This work
	8	$2^{-39.18}$		This work
	9	$2^{-56.50}$		[DIK08]
	9	$2^{-50.95}$		This work

Results: Application to Simeck

■ □: Experimentally verified

Cipher	#R	\mathbb{C}		Ref.
	7	1	✓	This work
Simeck-32	14	$2^{-16.63}$		[ZWH24]
	14	$2^{-13.92}$	\checkmark	This work

Cipher	#R	\mathbb{C}		Ref.
	8	1	√	This work
	17	$2^{-22.37}$		[ZWH24]
	17	$2^{-13.89}$	\checkmark	This work
Simeck-48	18	$2^{-24.75}$		[ZWH24]
	18	$2^{-15.89}$		This work
	19	$2^{-17.89}$		This work
	20	$2^{-21.89}$		This work

Cipher	#R	\mathbb{C}		Ref.
Simeck-64	10	1	√	This work
	24	$2^{-38.13}$		[ZWH24]
	24	$2^{-25.14}$		This work
	25	$2^{-41.04}$		[ZWH24]
	25	$2^{-27.14}$		This work
	26	$2^{-30.35}$		This work