Revisiting Differential-Linear Attacks via a Boomerang Perspective

Applications to AES, Ascon, CLEFIA, SKINNY, PRESENT, KNOT, TWINE, WARP, LBlock, Simeck, and SERPENT

Hosein Hadipour Patrick Derbez Maria Eichlseder

Lorenz Center, 22 April, 2024 - Leiden, Netherlands

› hsn.hadipour@gmail.com

SCIENCE PASSION TECHNOLOGY

Research Gap and Our Contributions

Research Gap

- How to formulate the correlation for more than one S-box layer?
- How to (efficiently) find good DL distinguishers?

Contributions

- Generalizing the DLCT framework [Bar+19] to handle multiple rounds.
- Introducing an efficient method to search for DL distinguishers applicable to:
  - Strongly aligned SPN primitives: AES, SKINNY
  - Weakly aligned SPN primitives: Ascon, SERPENT, KNOT, PRESENT
  - Feistel structures: CLEFIA, TWINE, LBlock, LBlock-s, WARP
  - AndRX designs: Simeck

Research Gap and Our Contributions

Research Gap

- How to formulate the correlation for more than one S-box layer?
- How to (efficiently) find good DL distinguishers?

Contributions

- Generalizing the DLCT framework [Bar+19] to handle multiple rounds.
- Introducing an efficient method to search for DL distinguishers applicable to:
  - Strongly aligned SPN primitives: AES, SKINNY
  - Weakly aligned SPN primitives: Ascon, SERPENT, KNOT, PRESENT
  - Feistel structures: CLEFIA, TWINE, LBlock, LBlock-s, WARP
  - AndRX designs: Simeck

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Outline

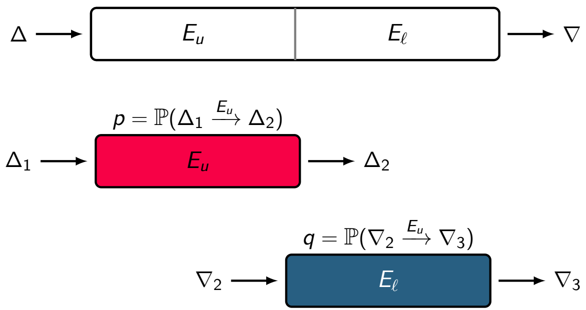
Boomerang Analysis

Boomerang Distinguishers [Wag99]

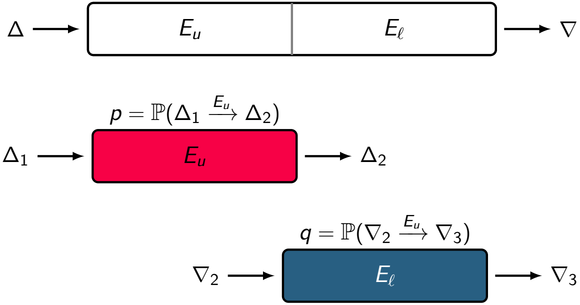
$$\Delta \longrightarrow \boxed{E : \mathbb{F}_2^n \to \mathbb{F}_2^n} \longrightarrow \nabla$$

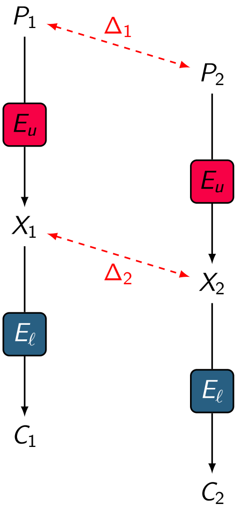
$$0 \lneq \mathbb{P}(\Delta \xrightarrow{E} \nabla) \lll 2^{-n}$$

Boomerang Distinguishers [Wag99]

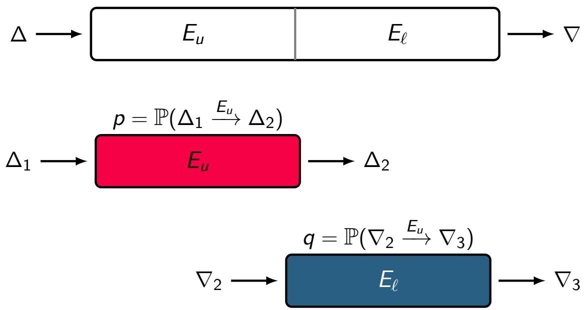


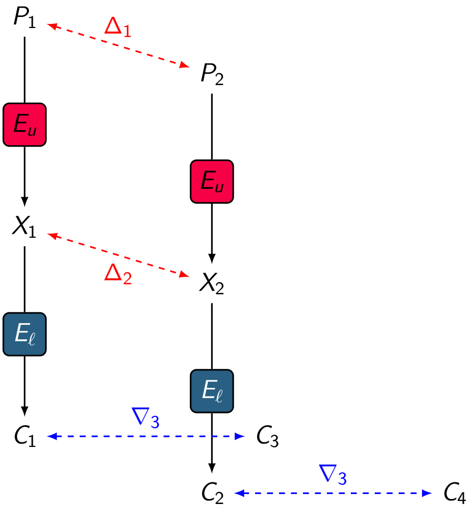
Boomerang Distinguishers [Wag99]





Boomerang Distinguishers [Wag99]



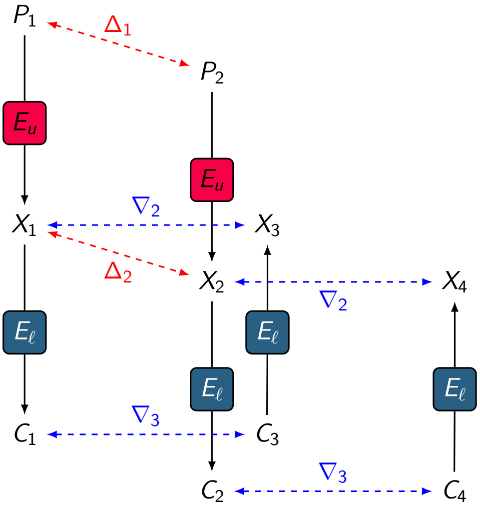


Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

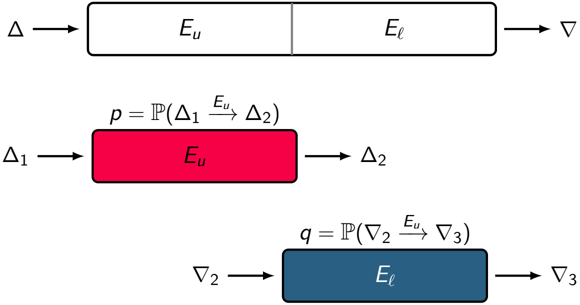
Boomerang Distinguishers [Wag99]

$$p = \mathbb{P}(\Delta_1 \xrightarrow{E_u} \Delta_2)$$

$$q = \mathbb{P}(\nabla_2 \xrightarrow{E_u} \nabla_3)$$



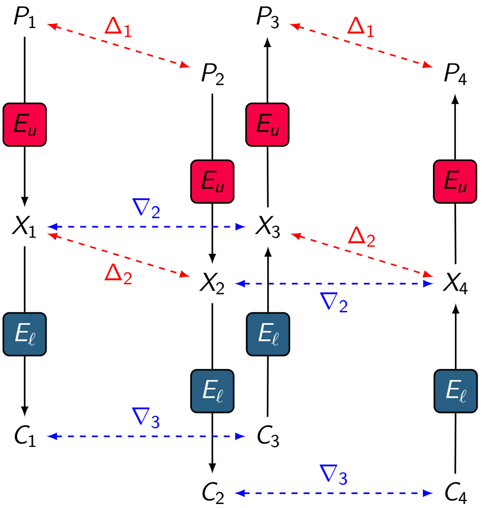
Boomerang Distinguishers [Wag99]



$$p = \mathbb{P}(\Delta_1 \xrightarrow{E_u} \Delta_2)$$

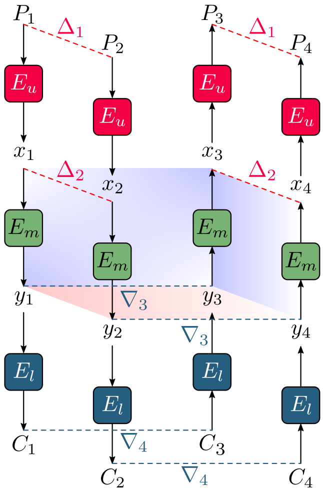
$$q = \mathbb{P}(\nabla_2 \xrightarrow{E_u} \nabla_3)$$

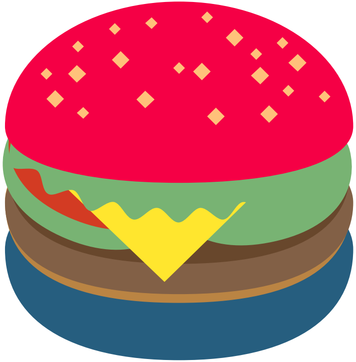
$$\boxed{\mathbb{P}(P_3 \oplus P_4 = \Delta_1) = p^2 q^2}$$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

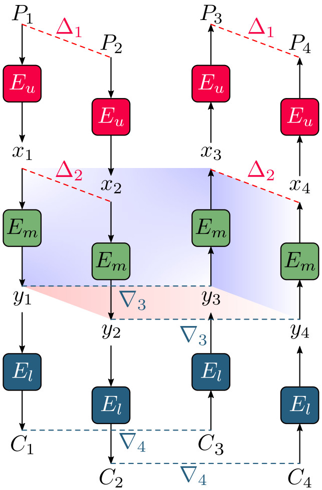
Sandwiching the Differentials! [DKS10; DKS14]





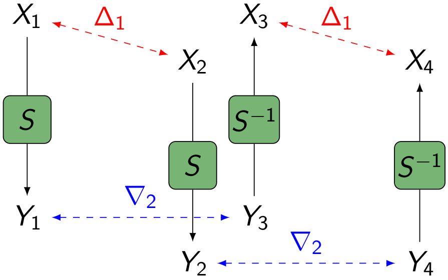
Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

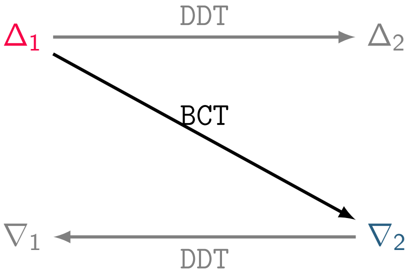
Sandwiching the Differentials! [DKS10; DKS14]



$$\mathbb{P}(P_3 \oplus P_4 = \Delta_1) \approx p^2 \times r \times q^2$$
$$r = \mathbb{P}(\Delta_2 \rightleftarrows \nabla_3)$$

Boomerang Connectivity Table (BCT) [Cid+18]

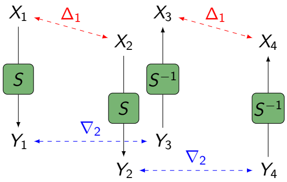




$$\mathtt{BCT}(\Delta_1, \nabla_2) \coloneqq \#\{X \in \mathbb{F}_2^n \mid S^{-1}(S(X) \oplus \nabla_2) \oplus S^{-1}(S(X \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}$$

$$\mathbb{P}(\Delta_1 \rightleftarrows \nabla_2) = 2^{-n} \cdot \mathtt{BCT}(\Delta_1, \nabla_2)$$

Generalized BCT Framework (GBCT) - I

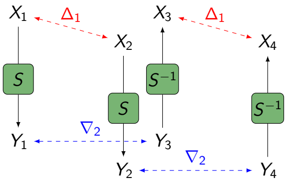


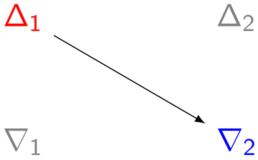
$$\Delta_1 \longrightarrow \Delta_2$$

$$\nabla_1 \longleftarrow \nabla_2$$

- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathtt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2)$
- $\mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\mathtt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

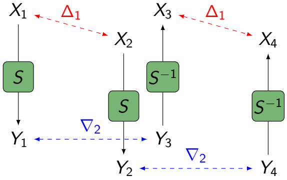
Generalized BCT Framework (GBCT) - I

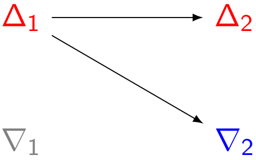




- $\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \texttt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \texttt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2)$
- $\texttt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\texttt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\texttt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework (GBCT) - I

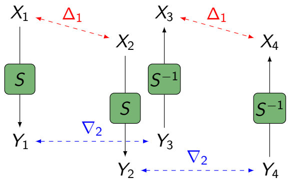


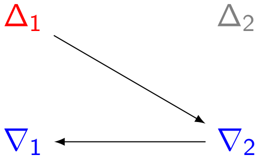


- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathtt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2)$
- $\mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\mathtt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

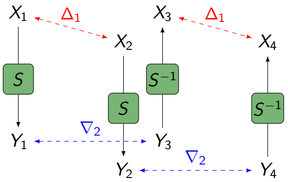
Generalized BCT Framework (GBCT) - I





- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathtt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2)$
- $\mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\mathtt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework (GBCT) - I



$$\begin{tikzcd} \Delta_1 \arrow[r] \arrow[dr] & \Delta_2 \\ \nabla_1 & \nabla_2 \arrow[l] \end{tikzcd}$$

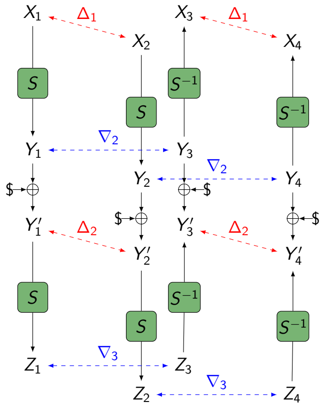
- $\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \texttt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \texttt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2)$
- $\texttt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\texttt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\texttt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Generalized BCT Framework (GBCT) - II

- Double Boomerang Connectivity Table (DBCT) [HB21]

$$\begin{tikzcd} \Delta_1 \arrow[r] \arrow[dr] & \Delta_2 \arrow[dr] & \\ & \nabla_2^* \arrow[r] & \nabla_3 \end{tikzcd}$$



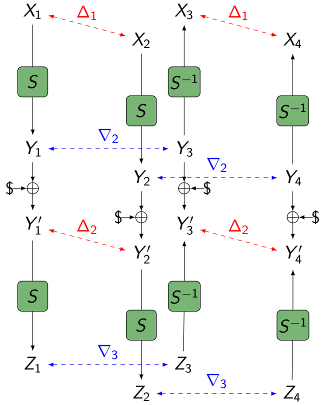
- $\mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$
- $\mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$.
- $\mathtt{DBCT}(\Delta_1, \nabla_3) = \sum_{\Delta_2} \mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3)$.

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Generalized BCT Framework (GBCT) - II

- Double Boomerang Connectivity Table (DBCT) [HB21]

$$\begin{tikzcd} \Delta_1 \arrow[r] \arrow[dr] & \Delta_2^* \arrow[dr] & \\ & \nabla_2 \arrow[r] & \nabla_3 \end{tikzcd}$$



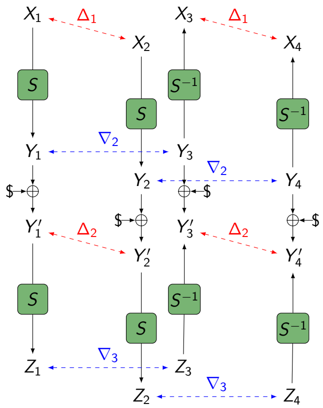
- $\mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$
- $\mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$.
- $\mathtt{DBCT}(\Delta_1, \nabla_3) = \sum_{\Delta_2} \mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3)$.

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Generalized BCT Framework (GBCT) - II

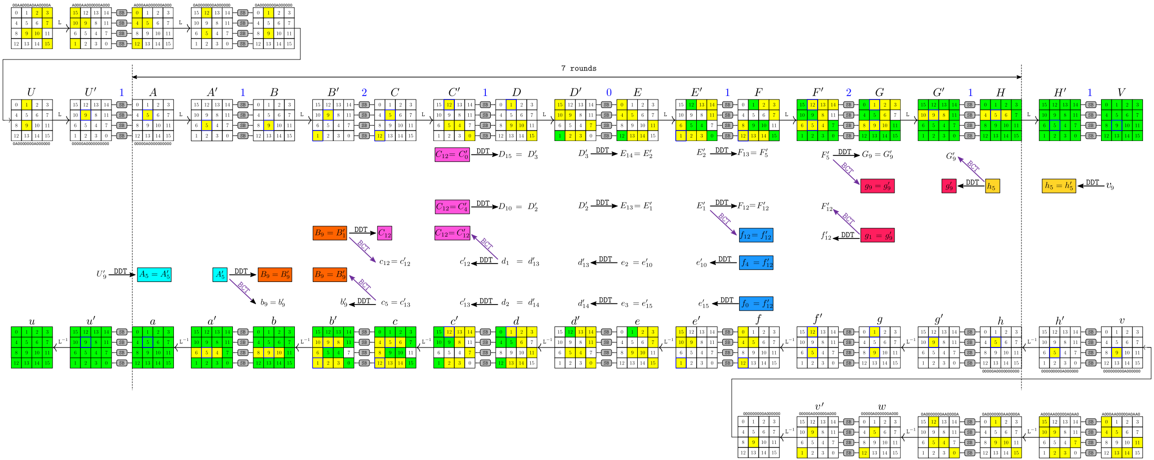
- Double Boomerang Connectivity Table (DBCT) [HB21]

$$
\begin{tikzcd}
\color{red}{\Delta_1} \arrow[r] \arrow[dr] & \color{red}{\Delta_2^*} \arrow[dr] & \\
& \color{blue}{\nabla_2^*} \arrow[r] & \color{blue}{\nabla_3}
\end{tikzcd}
$$



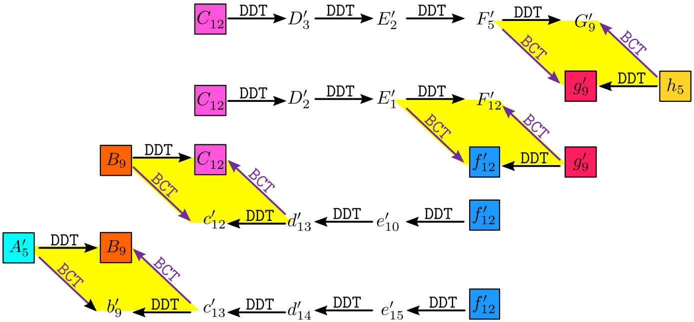
- $\mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$
- $\mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$.
- $\mathtt{DBCT}(\Delta_1, \nabla_3) = \sum_{\Delta_2} \mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3)$.

Application of GBCT [HB21]



Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Application of GBCT [HB21]



$$\mathtt{DBCT}_{\text{total}} = \mathtt{DBCT}^{\vdash}(A_5, B_9, c_5) \cdot \mathtt{DBCT}^{\vdash}(B_9, C_{12}, d_1) \cdot \mathtt{DBCT}^{\dashv}(E'_1, f'_{12}, g'_9) \cdot \mathtt{DBCT}^{\dashv}(F'_5, g'_9, h_5)$$

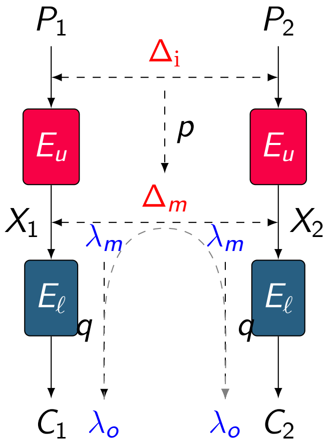
$$\Pr_{\text{total}} = \Pr(d_1 \xleftarrow{2\ \mathtt{DDT}} f'_{12}) \cdot \Pr(c_5 \xleftarrow{3\ \mathtt{DDT}} f'_{12}) \cdot \Pr(C_{12} \xrightarrow{2\ \mathtt{DDT}} E'_1) \cdot \Pr(C_{12} \xrightarrow{3\ \mathtt{DDT}} F'_5)$$

$$r = 2^{-8 \cdot n} \cdot \sum_{B_9} \sum_{C_{12}} \sum_{g'_9} \sum_{f'_{12}} \sum_{c_5} \sum_{d_1} \sum_{E'_1} \sum_{F'_5} \mathtt{DBCT}_{\text{total}} \cdot \Pr_{\text{total}}.$$

Differential-Linear Cryptanalysis

Differential-Linear (DL) Attack [LH94]

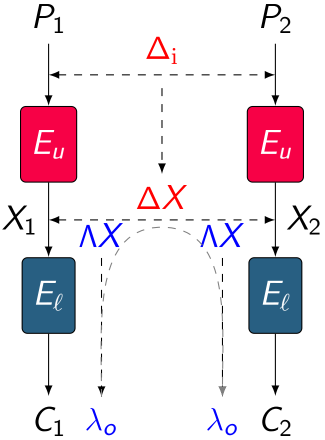
- $\mathbb{P}(\Delta_\mathrm{i} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_\mathrm{o}) = q$
- Assumptions ($\Delta X = X_1 \oplus X_2$):
  1. E_u, and E_ℓ are statistically independent
  2. $\mathbb{P}\left(\lambda_m \cdot \Delta X = 0\right) = 1/2$ when $\Delta X \neq \Delta_m$
- $\mathbb{C}\left(\lambda_\mathrm{o} \cdot C_1 \oplus \lambda_\mathrm{o} \cdot C_2\right) = (-1)^{\lambda_m \cdot \Delta_m} \cdot pq^2 = \pm pq^2$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Differential-Linear Attack Revisited [BLN14; BLN17]

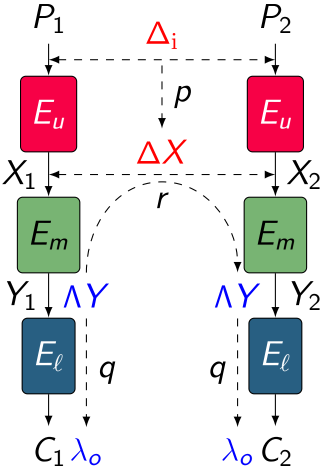
- $\mathbb{C}(\Lambda X \xrightarrow{E_\ell} \lambda_o) = \mathbb{C}(\Lambda X, \lambda_o)$
- Assumptions:
  1. E_u, and E_ℓ are statistically independent
- $\mathbb{C}(\lambda_o \cdot C_1 \oplus \lambda_o \cdot C_2) = \sum_{\Delta X, \Lambda X} \mathbb{C}(\Lambda X \cdot \Delta X) \cdot \mathbb{C}^2(\Lambda X, \lambda_o)$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder

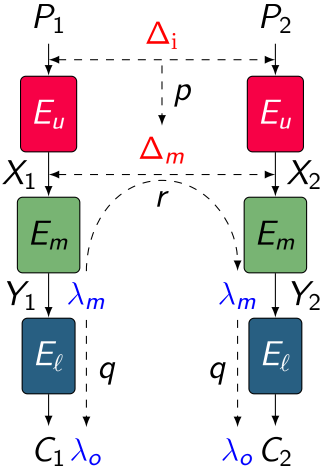
Sandwich Framework for DL Attack [DKS14; Bar+19]

- $\mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) = \sum_{\Delta X, \Lambda Y} \mathbb{P}(\Delta_{\mathrm{i}}, \Delta X) \cdot \mathbb{R}(\Delta X, \Lambda Y) \cdot \mathbb{C}^2(\Lambda Y, \lambda_{\mathrm{o}})$
- $\mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{R}(\Delta_m, \lambda_m) = r$
- $\mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\mathrm{o}}) = q$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) \approx prq^2$



Sandwich Framework for DL Attack [DKS14; Bar+19]

- $\mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) = \sum_{\Delta X, \Lambda Y} \mathbb{P}(\Delta_{\mathrm{i}}, \Delta X) \cdot \mathbb{R}(\Delta X, \Lambda Y) \cdot \mathbb{C}^2(\Lambda Y, \lambda_{\mathrm{o}})$
- $\mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{R}(\Delta_m, \lambda_m) = r$
- $\mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\mathrm{o}}) = q$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) \approx prq^2$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Differential-Linear Connectivity Table ($\mathtt{DLCT}$)

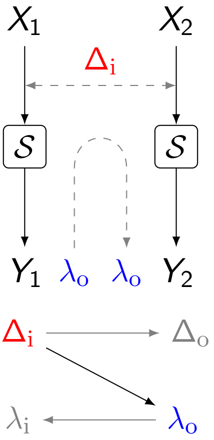
Differential-Linear Connectivity Table ($\mathtt{DLCT}$) [Bar+19]

For a vectorial Boolean function $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$, the $\mathtt{DLCT}$ of S is a $2^n \times 2^m$ table whose rows correspond to the input difference Δ_{i} to S and whose columns correspond to the output mask λ_{o} of S. The entry at index $(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$ is

$$\mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = |\mathtt{DLCT}_0(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})| - |\mathtt{DLCT}_1(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})|,$$

where $\mathtt{DLCT}_b(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \{x \in \mathbb{F}_2^n : \ \lambda_{\mathrm{o}} \cdot S(x) \oplus \lambda_{\mathrm{o}} \cdot S(x \oplus \Delta_{\mathrm{i}}) = b\}$.

$$\mathbb{C}_{\mathtt{DLCT}}\left(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}\right) = 2^{-n} \cdot \mathtt{DLCT}\left(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}\right)$$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Generalized DLCT Framework

Upper Differential-Linear Connectivity Table (UDLCT)

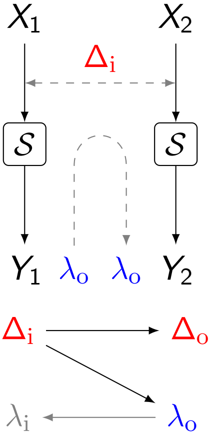
Upper Differential-Linear Connectivity Table (UDLCT)

For a vectorial Boolean function $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$, the UDLCT of S is a $2^n \times 2^n \times 2^m$ table. The entry at index $(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o})$ is

$$\mathtt{UDLCT}(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o}) = |\mathtt{UDLCT}_0(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o})| - |\mathtt{UDLCT}_1(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o})|,$$

where $\mathtt{UDLCT}_b(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o}) = \{x \in \mathbb{F}_2^n : \; S(x) \oplus S(x \oplus \Delta_\mathrm{i}) = \Delta_\mathrm{o} \text{ and } \lambda_\mathrm{o} \cdot \Delta_\mathrm{o} = b\}$.

$$\mathbb{C}_{\mathtt{UDLCT}}\left(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o}\right) = 2^{-n} \cdot \mathtt{UDLCT}\left(\Delta_\mathrm{i}, \Delta_\mathrm{o}, \lambda_\mathrm{o}\right)$$



Lower Differential-Linear Connectivity Table ($\mathtt{LDLCT}$)

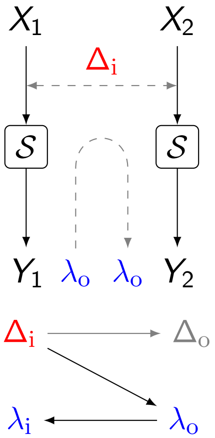
Lower Differential-Linear Connectivity Table ($\mathtt{LDLCT}$)

For a vectorial Boolean function $S : \mathbb{F}_2^n \to \mathbb{F}_2^m$, the $\mathtt{LDLCT}$ of S is a $2^n \times 2^m \times 2^m$ table. The entry at index $(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_o)$ is

$$\mathtt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = |\mathtt{LDLCT}_0(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})| - |\mathtt{LDLCT}_1(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})|,$$

where $\mathtt{LDLCT}_b(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \{x \in \mathbb{F}_2^n : \lambda_{\mathrm{o}} \cdot S(x) = \lambda_{\mathrm{o}} \cdot S(x \oplus \Delta_{\mathrm{i}}) \text{ and } \lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} = b\}$.

$$\mathbb{C}_{\mathtt{LDLCT}}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \mathtt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$$



Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Extended Differential-Linear Connectivity Table ($\mathtt{EDLCT}$)

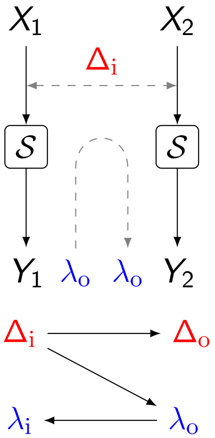
Extended Differential-Linear Connectivity Table ($\mathtt{EDLCT}$)

For a vectorial Boolean function $S : \mathbb{F}_2^n \to \mathbb{F}_2^m$, the $\mathtt{EDLCT}$ of S is a $2^n \times 2^n \times 2^m \times 2^m$ table. The entry at index $(\Delta_i, \Delta_o, \lambda_i, \lambda_o)$ is

$$\mathtt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = |\mathtt{EDLCT}_0(\Delta_{\mathrm{i}},\Delta_{\mathrm{o}},\lambda_{\mathrm{i}},\lambda_{\mathrm{o}})| - |\mathtt{EDLCT}_1(\Delta_{\mathrm{i}},\Delta_{\mathrm{o}},\lambda_{\mathrm{i}},\lambda_{\mathrm{o}})|,$$

where $\mathtt{EDLCT}_b(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \{x \in \mathbb{F}_2^n : \ S(x) \oplus S(x \oplus \Delta_{\mathrm{i}}) = \Delta_{\mathrm{o}} \text{ and } \lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}} = b\}$.

$$\mathbb{C}_{\mathtt{EDLCT}}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \mathtt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$$

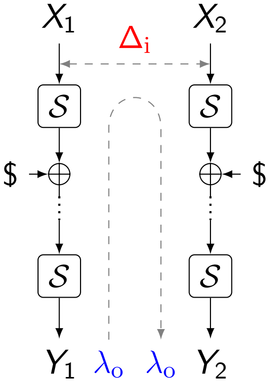


Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Double Differential-Linear Connectivity Table (DDLCT)

$$
\begin{tikzcd}
\Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_m \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\
\lambda_{\mathrm{i}} \arrow[r] & \lambda_m \arrow[r] & \lambda_{\mathrm{o}}
\end{tikzcd}
$$

$$\mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_m} \sum_{\lambda_m} \mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_m, \lambda_m\right) \cdot \mathtt{LDLCT}\left(\Delta_m, \lambda_m, \lambda_{\mathrm{o}}\right)$$



Generalized DLCT Framework (GBCT)

- How to formulate the correaltion for more than 1 round?

$$\begin{tikzcd} \Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\ \lambda_{\mathrm{i}} \arrow[r] & \lambda_{\mathrm{o}} \end{tikzcd}$$

$\mathtt{DLCT}\,(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$

$$\begin{tikzcd} \Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\ \lambda_{\mathrm{i}} \arrow[r] & \lambda_{\mathrm{o}} \end{tikzcd}$$

$\mathtt{UDLCT}\,(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}})$

$$\begin{tikzcd} \Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\ \lambda_{\mathrm{i}} \arrow[r] & \lambda_{\mathrm{o}} \end{tikzcd}$$

$\mathtt{LDLCT}\,(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$

$$\begin{tikzcd} \Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\ \lambda_{\mathrm{i}} \arrow[r] & \lambda_{\mathrm{o}} \end{tikzcd}$$

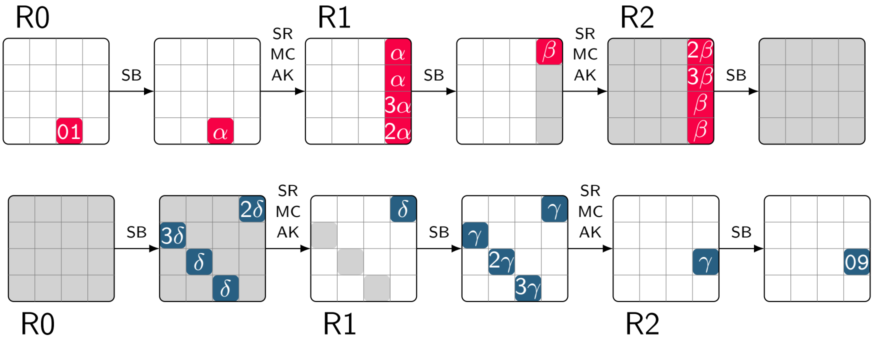
$\mathtt{EDLCT}\,(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$

$$\begin{tikzcd} \Delta_{\mathrm{i}} \arrow[r] \arrow[dr] & \Delta_m \arrow[r] \arrow[dr] & \Delta_{\mathrm{o}} \\ \lambda_{\mathrm{i}} \arrow[r] & \lambda_m \arrow[r] & \lambda_{\mathrm{o}} \end{tikzcd}$$

$\mathtt{DDLCT}\,(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$

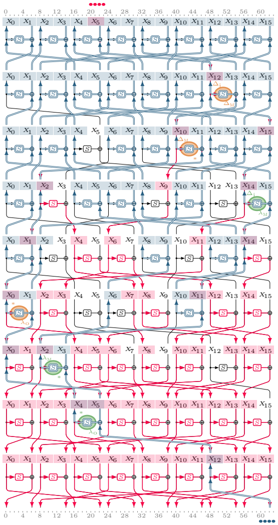
Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Application of the Generalized DLCT Tables - AES (– differential – linear)



$$\sum_{\alpha,\beta,\gamma,\delta} \mathbb{C}_{\mathtt{UDLCT}}(1,\alpha,\delta)\cdot\mathbb{C}_{\mathtt{EDLCT}}(\alpha,\beta,\delta,\gamma)\cdot\mathbb{C}_{\mathtt{LDLCT}}(\beta,\gamma,9) = -2^{-7.94}$$

Application of the Generalized DLCT Tables - TWINE (– differential – linear)



$$\mathbb{C}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_m} \mathbb{P}_{\mathtt{DDT}}(\Delta_{\mathrm{i}}, \Delta_m) \cdot \mathbb{C}_{\mathtt{DDLCT}}(\Delta_m, \lambda_{\mathrm{o}})$$

$$= \sum_{\lambda_m} \mathbb{C}_{\mathtt{DDLCT}}(\Delta_{\mathrm{i}}, \lambda_m) \cdot \mathbb{C}^2_{\mathtt{LAT}}(\lambda_m, \lambda_{\mathrm{o}}).$$

$$\mathbb{C}_{tot}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \mathbb{C}^2(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}).$$

Input/Output Differences/Linear-mask	Formula	Exp. Correlation
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xb4}, \mathtt{0x67})$	$-2^{-7.66}$	$-2^{-7.64}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x02}, \mathtt{0x02})$	$-2^{-7.92}$	$-2^{-7.93}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x55}, \mathtt{0x55})$	$-2^{-7.99}$	$-2^{-7.98}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xbf}, \mathtt{0xef})$	$-2^{-8.05}$	$-2^{-8.06}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xfe}, \mathtt{0x06})$	$-2^{-8.26}$	$-2^{-8.25}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x4b}, \mathtt{0x1a})$	$-2^{-8.43}$	$-2^{-8.44}$

Differential-Linear Switches and Deterministic Trails

Cell-Wise and Bit-Wise Switches

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	**-16**	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	**-16**	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	**-16**	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	**-16**	0	0	0	**-16**	0	0	0	**16**	0	0	0
9	16	-8	0	-8	**16**	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	**16**	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	**-16**	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

- Cell-wise switches: $\mathrm{DLCT}(\Delta_{\mathrm{i}}, 0) = \mathrm{DLCT}(0, \lambda_{\mathrm{o}}) = 2^n$ for all $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}$
- Bit-wise switches: $\mathrm{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \pm 2^n$ for $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}} \neq 0$
  - Example: $\mathbb{C}(9, 4) = \frac{16}{16}$

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Cell-Wise and Bit-Wise Switches

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

- Cell-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, 0) = \mathtt{DLCT}(0, \lambda_{\mathrm{o}}) = 2^n$ for all $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}$
- Bit-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \pm 2^n$ for $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}} \neq 0$
  - Example: $\mathbb{C}(9, 4) = \frac{16}{16}$

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Cell-Wise and Bit-Wise Switches

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

- Cell-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, 0) = \mathtt{DLCT}(0, \lambda_{\mathrm{o}}) = 2^n$ for all $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}$
- Bit-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \pm 2^n$ for $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}} \neq 0$
  - Example: $\mathbb{C}\,(9, 4) = \frac{16}{16}$

Deterministic Bit-Wise Differential Trails (Forward)

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

$$\Delta_i = (1,0,0,0) \xrightarrow{S} \Delta_o = (1,1,?,?)$$

$$\Delta_i = (1,0,0,1) \xrightarrow{S} \Delta_o = (?,0,?,?)$$

$$\Delta_i = (1,1,0,0) \xrightarrow{S} \Delta_o = (0,?,?,?)$$

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Deterministic Bit-Wise Linear Trails (Backward)

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

$$\lambda_{\mathrm{i}} = (1, ?, ?, 1) \xleftarrow{S} \lambda_{\mathrm{o}} = (0, 1, 0, 0)$$

$$\lambda_{\mathrm{i}} = (1, 1, ?, ?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1, 0, 0, 0)$$

$$\lambda_{\mathrm{i}} = (0, ?, ?, ?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1, 1, 0, 0)$$

Bit-Wise Switches and Deterministic Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

$$\Delta_{\mathrm{i}} = (0,0,0,1) \xrightarrow{S} \Delta_{\mathrm{o}} = (?,1,?,?)$$

$$\Delta_{\mathrm{i}} = (0,1,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (1,?,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (1,1,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,1) \xrightarrow{S} \Delta_{\mathrm{o}} = (?,0,?,?)$$

$$\Delta_{\mathrm{i}} = (1,1,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (0,?,?,?)$$

$$\lambda_{\mathrm{i}} = (1,?,?,1) \xleftarrow{S} \lambda_{\mathrm{o}} = (0,1,0,0)$$

$$\lambda_{\mathrm{i}} = (1,1,?,?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1,0,0,0)$$

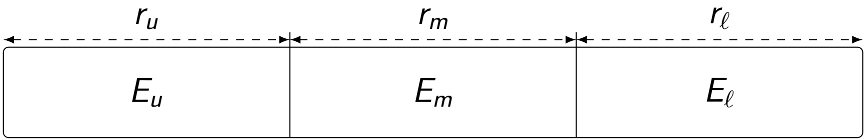
$$\lambda_{\mathrm{i}} = (0,?,?,?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1,1,0,0)$$

Automatic Tools to Search for DL Distinguishers

Overview of Our Method to Search for Distinguishers

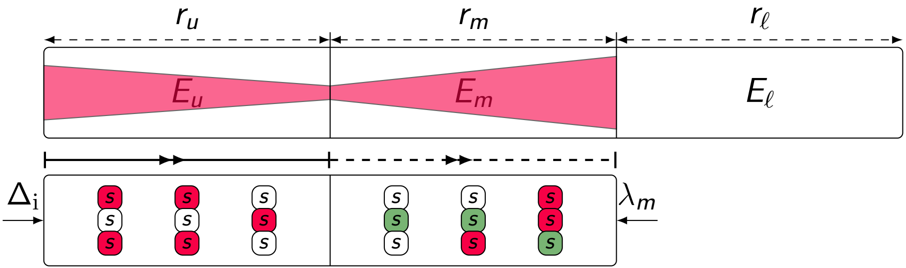
E

Overview of Our Method to Search for Distinguishers



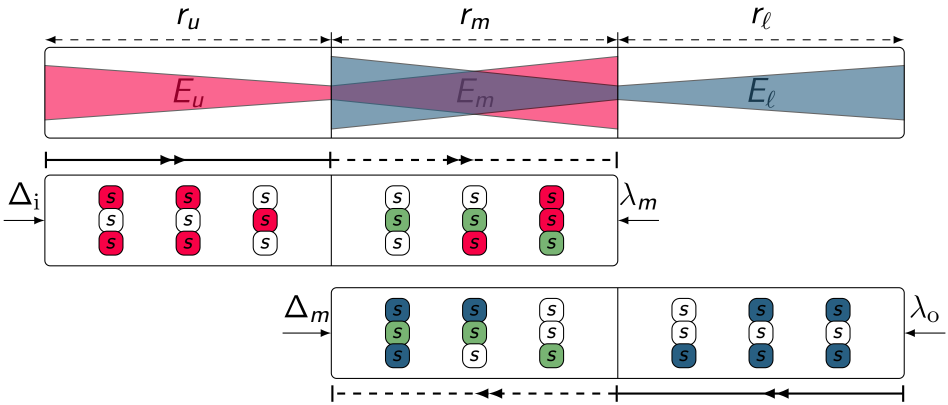
Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Overview of Our Method to Search for Distinguishers



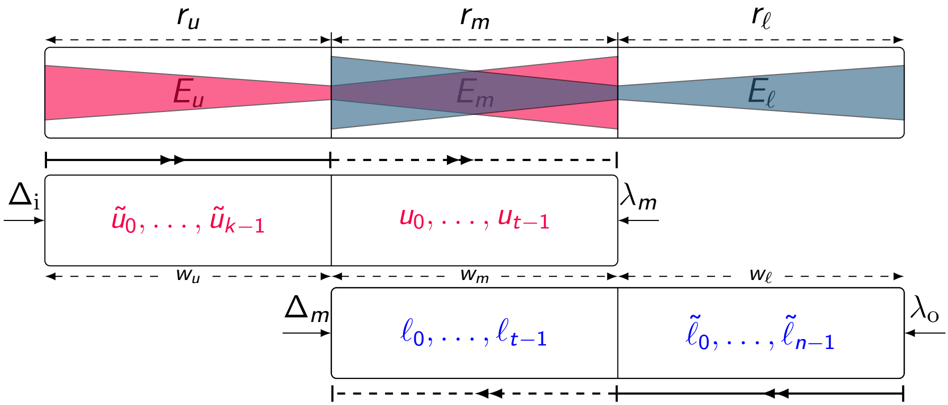
differentially active S-box linearly active S-box common active S-box

Overview of Our Method to Search for Distinguishers



differentially active S-box | linearly active S-box | common active S-box

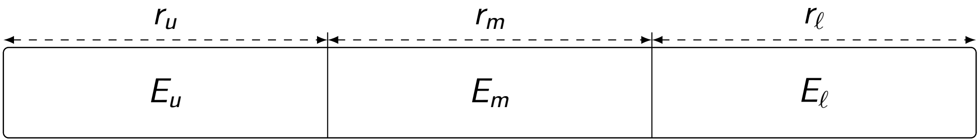
Overview of Our Method to Search for Distinguishers



$$\min \left(\sum_{i=0}^{k-1} w_u \cdot \tilde{u}_i + \sum_{j=0}^{t-1} w_m \cdot \mathtt{bool2int}\left(\ell_j + u_j = 2\right) + \sum_{k=0}^{n-1} w_\ell \cdot \tilde{\ell}_k \right)$$

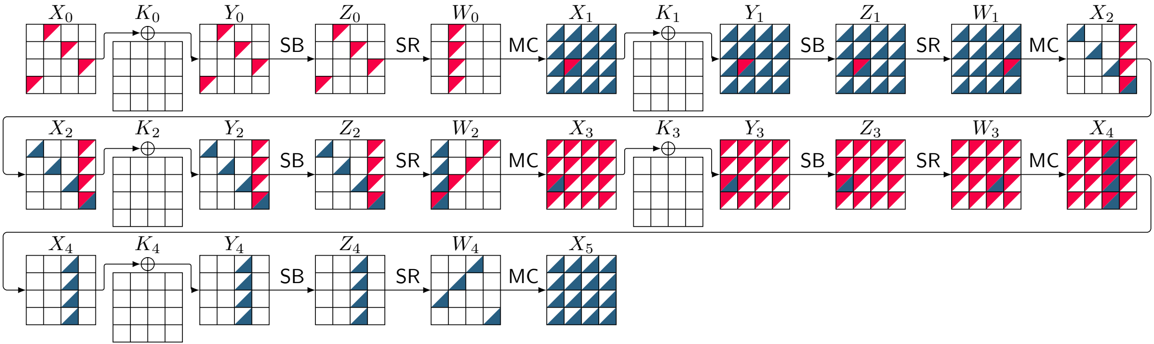
Usage of Our Tool

```
python3 attack.py -RU 6 -RM 10 -RL 6
```



Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Example: A 5-round DL Distinguisher for AES



$$r_0 = 1, r_m = 3, r_1 = 1, \ p = 2^{-24.00}, r = 2^{-7.66}, \ q^2 = 2^{-24.00}, \ prq^2 = 2^{-55.66}$$

ΔX_0	`001c00000000e200000000dfb3000000`	ΔX_1	`0000000000000000f700000000000000`
ΓX_4	`00000000000000006700000000000000`	ΓX_5	`21d3814d93b1ef228e923507f67383fd`

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands

Example: Distinguishers for up to 17 Rounds of TWINE

- Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	$2^{3.20}$	1	$2^{3.20}$
13	$2^{34.32}$	$2^{27.16}$	$2^{7.16}$
14	$2^{42.25}$	$2^{31.28}$	$2^{10.97}$
15	$2^{51.03}$	$2^{38.98}$	$2^{12.05}$
16	$2^{58.04}$	$2^{47.28}$	$2^{10.76}$
17	-	$2^{59.24}$	-

Example: Distinguishers for up to 17 Rounds of LBlock

- Comparing the data complexity of best boomerang and DL distinguishers

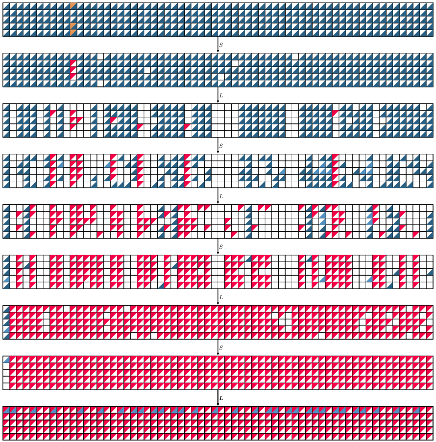
# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	$2^{2.97}$	1	$2^{2.97}$
13	$2^{30.28}$	$2^{23.78}$	$2^{6.50}$
14	$2^{38.86}$	$2^{30.34}$	$2^{8.52}$
15	$2^{46.90}$	$2^{38.26}$	$2^{8.64}$
16	$2^{57.16}$	$2^{46.26}$	$2^{10.90}$
17	-	$2^{58.30}$	-

Example: Distinguishers for up to 8 Rounds of CLEFIA

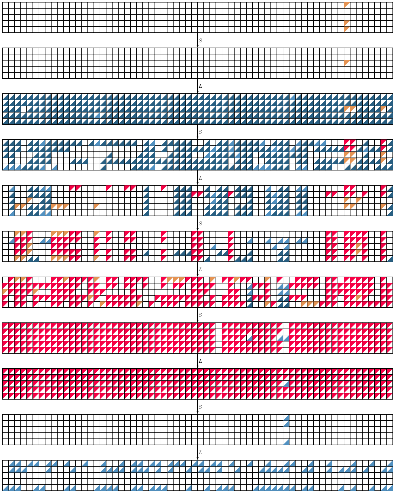
- Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
3	1	1	1
4	$2^{6.32}$	1	$2^{6.32}$
5	$2^{12.26}$	$2^{5.36}$	$2^{6.90}$
6	$2^{22.45}$	$2^{14.14}$	$2^{8.31}$
7	$2^{32.67}$	$2^{23.50}$	$2^{9.17}$
8	$2^{76.03}$	$2^{66.86}$	$2^{9.17}$

Application to Ascon-p(active difference unknown difference active mask unknown mask)



$\mathbb{C} = 1$



$\mathbb{C} = 2^{-4.33}$

Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Application to SERPENT

- 💻: Experimentally verified

Cipher	#R	$\mathbb{C}$	💻	Ref.
SERPENT	3	$\mathbf{2^{-0.68}}$	✓	This work
	4	$2^{-12.75}$		[DIK08]
	4	$\mathbf{2^{-5.54}}$	✓	This work
	5	$2^{-16.75}$		[DIK08]
	5	$\mathbf{2^{-11.10}}$	✓	This work
	8	$2^{-39.18}$		This work
	9	$2^{-56.50}$		[DIK08]
	9	$\mathbf{2^{-50.95}}$		This work

Application to Simeck

- 💻: Experimentally verified

Cipher	#R	$\mathbb{C}$	💻	Ref.
	7	$\mathbf{1}$	✓	This work
Simeck-32	14	$2^{-16.63}$		[ZWH24]
	14	$\mathbf{2^{-13.92}}$	✓	This work

Cipher	#R	$\mathbb{C}$	💻	Ref.
	8	$\mathbf{1}$	✓	This work
	17	$2^{-22.37}$		[ZWH24]
	17	$\mathbf{2^{-13.89}}$	✓	This work
Simeck-48	18	$2^{-24.75}$		[ZWH24]
	18	$\mathbf{2^{-15.89}}$		This work
	19	$\mathbf{2^{-17.89}}$		This work
	20	$\mathbf{2^{-21.89}}$		This work

Cipher	#R	$\mathbb{C}$	💻	Ref.
	10	$\mathbf{1}$	✓	This work
	24	$2^{-38.13}$		[ZWH24]
	24	$\mathbf{2^{-25.14}}$		This work
Simeck-64	25	$2^{-41.04}$		[ZWH24]
	25	$\mathbf{2^{-27.14}}$		This work
	26	$\mathbf{2^{-30.35}}$		This work

Contributions and Future Works

Contributions and Future Works

- Contributions
  - We generalized the DLCT framework from one S-box layer to multiple rounds
  - We proposed an automatic tool for finding optimum DL distinguishers
  - We applied our tool to almost any design paradigm
- Future works
  - Extedning the application of our tool to other primitives, e.g., ARX
  - Extending our tool to a unified model for finding complete attack (key recovery)

: https://ia.cr/2024/255

Hosein Hadipour, Patrick Derbez, Maria Eichlseder

Bibliography I

[Bar+19] Achiya Bar-On et al. **DLCT: A New Tool for Differential-Linear Cryptanalysis**. EUROCRYPT 2019. Vol. 11476. LNCS. Springer, 2019, pp. 313–342. DOI: 10.1007/978-3-030-17653-2_11.

[BLN14] Céline Blondeau, Gregor Leander, and Kaisa Nyberg. **Differential-Linear Cryptanalysis Revisited**. FSE 2014. Ed. by Carlos Cid and Christian Rechberger. Vol. 8540. LNCS. Springer, 2014, pp. 411–430. DOI: 10.1007/978-3-662-46706-0_21.

[BLN17] Céline Blondeau, Gregor Leander, and Kaisa Nyberg. **Differential-Linear Cryptanalysis Revisited**. *J. Cryptol.* 30.3 (2017), pp. 859–888. DOI: 10.1007/s00145-016-9237-5.

Bibliography II

[Bou+20] Hamid Boukerrou et al. **On the Feistel Counterpart of the Boomerang Connectivity Table Introduction and Analysis of the FBCT**. *IACR Trans. Symmetric Cryptol.* 2020.1 (2020), pp. 331–362. DOI: 10.13154/TOSC.V2020.I1.331-362.

[Cid+18] Carlos Cid et al. **Boomerang Connectivity Table: A New Cryptanalysis Tool**. EUROCRYPT 2018. Ed. by Jesper Buus Nielsen and Vincent Rijmen. Vol. 10821. LNCS. Springer, 2018, pp. 683–714. DOI: 10.1007/978-3-319-78375-8_22.

[DDV20] Stéphanie Delaune, Patrick Derbez, and Mathieu Vavrille. **Catching the Fastest Boomerangs Application to SKINNY**. *IACR Trans. Symmetric Cryptol.* 2020.4 (2020), pp. 104–129. DOI: 10.46586/TOSC.V2020.I4.104-129.

Bibliography III

[DIK08] Orr Dunkelman, Sebastiaan Indesteege, and Nathan Keller. **A Differential-Linear Attack on 12-Round Serpent**. INDOCRYPT 2008. Ed. by Dipanwita Roy Chowdhury, Vincent Rijmen, and Abhijit Das. Vol. 5365. LNCS. Springer, 2008, pp. 308–321. DOI: 10.1007/978-3-540-89754-5_24.

[DKS10] Orr Dunkelman, Nathan Keller, and Adi Shamir. **A Practical-Time Related-Key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony**. CRYPTO. Vol. 6223. LNCS. Springer, 2010, pp. 393–410. DOI: 10.1007/978-3-642-14623-7_21.

[DKS14] Orr Dunkelman, Nathan Keller, and Adi Shamir. **A Practical-Time Related-Key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony**. *J. Cryptol.* 27.4 (2014), pp. 824–849. DOI: 10.1007/s00145-013-9154-9.

Bibliography IV

[HB21] Hosein Hadipour and Nasour Bagheri. **Improved Rectangle Attacks on SKINNY and CRAFT**. *IACR Trans. Symmetric Cryptol.* 2021.2 (2021), pp. 140–198. DOI: 10.46586/TOSC.V2021.I2.140-198.

[HNE22] Hosein Hadipour, Marcel Nageler, and Maria Eichlseder. **Throwing Boomerangs into Feistel Structures Application to CLEFIA, WARP, LBlock, LBlock-s and TWINE**. *IACR Trans. Symmetric Cryptol.* 2022.3 (2022), pp. 271–302. DOI: 10.46586/TOSC.V2022.I3.271-302.

[LH94] Susan K. Langford and Martin E. Hellman. **Differential-Linear Cryptanalysis**. CRYPTO '94. Vol. 839. Springer, 1994, pp. 17–25. DOI: 10.1007/3-540-48658-5_3.

Bibliography V

[SQH19] Ling Song, Xianrui Qin, and Lei Hu. **Boomerang Connectivity Table Revisited. Application to SKINNY and AES**. *IACR Trans. Symmetric Cryptol.* 2019.1 (2019), pp. 118–141. DOI: 10.13154/TOSC.V2019.I1.118-141. URL: https://doi.org/10.13154/tosc.v2019.i1.118-141.

[Wag99] David A. Wagner. **The Boomerang Attack**. FSE. Vol. 1636. LNCS. Springer, 1999, pp. 156–170. DOI: 10.1007/3-540-48519-8_12.

[WP19] Haoyang Wang and Thomas Peyrin. **Boomerang Switch in Multiple Rounds. Application to AES Variants and Deoxys**. *IACR Trans. Symmetric Cryptol.* 2019.1 (2019), pp. 142–169. DOI: 10.13154/TOSC.V2019.I1.142-169.

Bibliography VI

[ZWH24] Yanyan Zhou, Senpeng Wang, and Bin Hu. **MILP/MIQCP-Based Fully Automatic Method of Searching for Differential-Linear Distinguishers for SIMON-Like Ciphers**. *IET Information Security* 2024 (2024). DOI: 10.1049/2024/8315115.

Properties of Generalized $\texttt{DLCT}$ Tables - I

- $\texttt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_{\mathrm{o}}} \texttt{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}})$
- $\texttt{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) = (-1)^{\Delta_{\mathrm{o}} \cdot \lambda_{\mathrm{o}}} \texttt{DDT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}})$
- $\texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\Delta_{\mathrm{i}} \cdot \lambda_{\mathrm{i}}} \texttt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$
- $\texttt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}}} \texttt{DDT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}})$
- $\texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_{\mathrm{o}}} \texttt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$
- $\sum_{\Delta_{\mathrm{i}}} \texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \texttt{LAT}^2(\lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$

Properties of Generalized DLCT Tables - II

- $\mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_m} \sum_{\lambda_m} \mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_m, \lambda_m\right) \cdot \mathtt{LDLCT}\left(\Delta_m, \lambda_m, \lambda_{\mathrm{o}}\right)$

$$\begin{aligned} \mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= \sum_{\Delta_m} \mathtt{DDT}(\Delta_{\mathrm{i}}, \Delta_m) \cdot \mathtt{DLCT}(\Delta_m, \lambda_{\mathrm{o}}) \\ &= 2^{-n} \sum_{\lambda_m} \mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_m) \cdot \mathtt{LAT}^2(\lambda_m, \lambda_{\mathrm{o}}). \end{aligned}$$

Hosein Hadipour, Patrick Derbez, Maria Eichlseder
Lorenz Center, 22 April, 2024 - Leiden, Netherlands