

Improved Rectangle Attacks on SKINNY and CRAFT

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Outline

- Boomerang and Sandwich Distinguishers
- Our Method To Find Sandwich Distinguishers
- 3 BCT Framework and Our New Tools
- 4 Application to CRAFT
- 5 Application to SKINNY
- 6 Conclusion

Boomerang and Sandwich Distinguishers

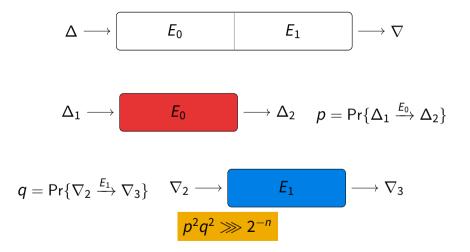


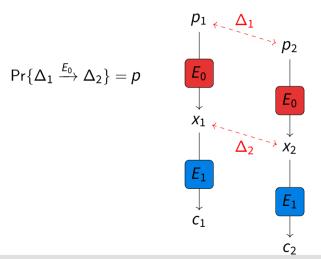
Long Weak Differentials V.S. Two Short Strong Differentials

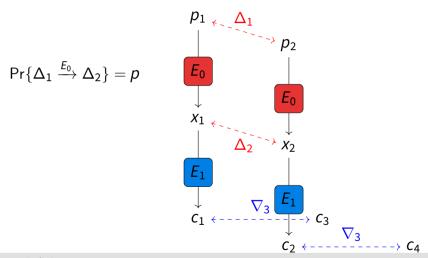
$$\Delta \longrightarrow \left[E : \mathbb{F}_2^n \to \mathbb{F}_2^n \right] \longrightarrow \nabla$$

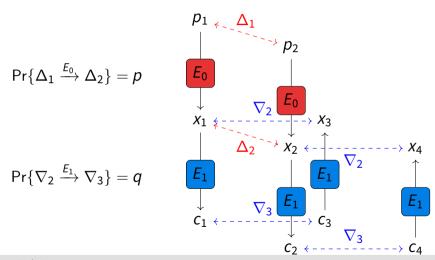
$$0 \lneq \Pr\{\Delta \xrightarrow{\mathcal{E}} \nabla\} \lll 2^{-n}$$

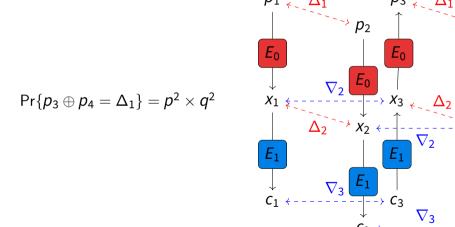
Long Weak Differentials V.S. Two Short Strong Differentials









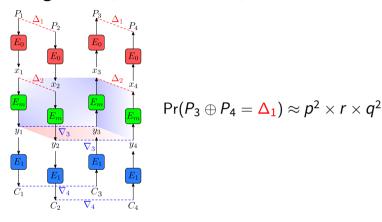


Upper and Lower Parts are Not Independent in Practice!

From the attacker's perspective:

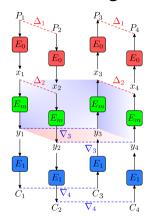
- Dependency can have a positive effect
 - Feistel Switch [Wag99]
 - Ladder Switch and S-box Switch [BK09]
- Dependency can have a negative effect
 - Inconsistency between the upper and lower trail [Mur11]

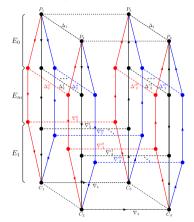
Sandwich Distinguisher [DKS10; DKS14]



$$r = r(\Delta_2, \nabla_3) = \Pr\{E_m^{-1}(E_m(x) \oplus \nabla_3) \oplus E_m^{-1}(E_m(x \oplus \Delta_2) \oplus \nabla_3) = \Delta_2\}$$

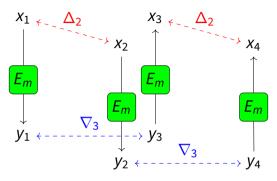
Sandwich Distinguisher [DKS10; DKS14]





$$\Pr\left(P_3 \oplus P_4 = \Delta_1\right) = \sum_{\Delta_2, \Delta_3', \nabla_3, \nabla_3'} p_{\Delta_2} \times p_{\Delta_2'} \times r(\Delta_2, \Delta_2', \nabla_3, \nabla_3') \times q_{\nabla_3} \times q_{\nabla_3'}$$

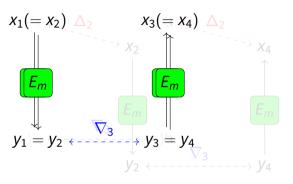
Ladder Switch



$$r = r(\Delta_2, \nabla_3) = \Pr\{E_m^{-1}(E_m(x) \oplus \nabla_3) \oplus E_m^{-1}(E_m(x \oplus \Delta_2) \oplus \nabla_3) = \Delta_2\}$$

$$\nabla_3 = 0 \Longrightarrow r = r(\Delta_2, 0) = 1$$

Ladder Switch

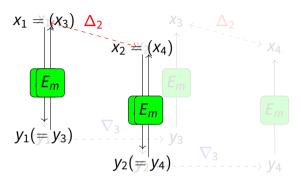


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Ladder Switch



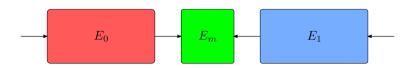
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Effective Parameters in p^2q^2r for SPN Ciphers

- \bigcirc p is mostly determined by the number of active S-boxes in E_0
- Θ q is mostly determined by the number of active S-boxes in E_1
- \odot r is mostly determined by the number of common active S-boxes in E_m
- \triangle Active S-boxes in E_0, E_1 are more expensive than common active S-boxes in E_m



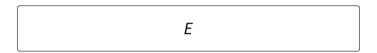


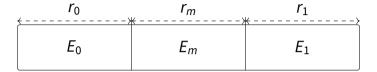
- Find the truncated upper and lower trails minimizing
 - number of active S-boxes in outer parts
 - and number of common active S-boxes in the middle part
- Instantiate the discovered truncated trails with concrete differential trails
- \bigcirc Compute p, q and r to derive the entire probability, i.e., p^2q^2r

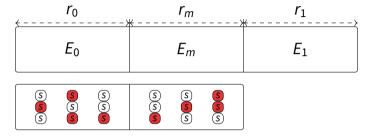
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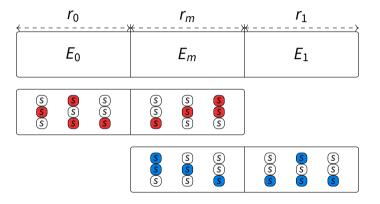
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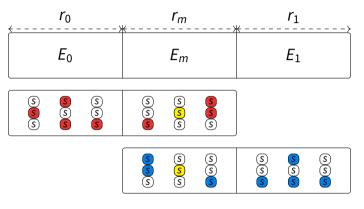
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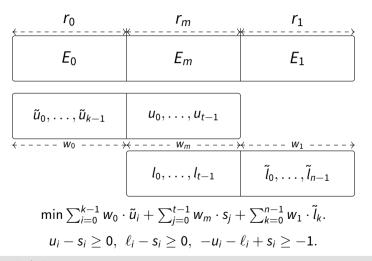




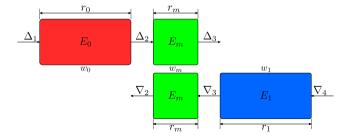




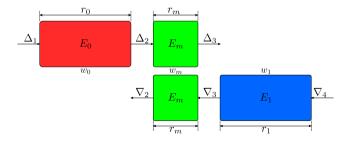
$$u_i - s_i \ge 0, \ \ell_i - s_i \ge 0, \ -u_i - \ell_i + s_i \ge -1$$



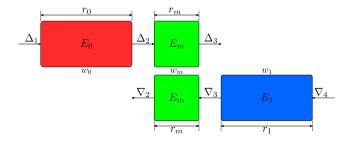
- → We Instantiate the first and last parts with concrete bit-wise differentials.
- → To compute p, q and r we fix the differences at only four positions
- $lack \Delta$ Our distinguishers do not rely on differential characteristics for E_0, E_1, E_m



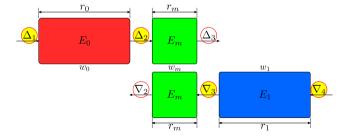
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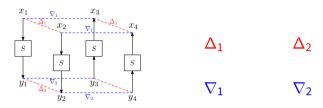


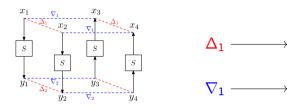
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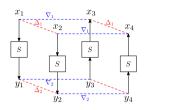
BCT Framework And Our New Tools

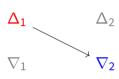




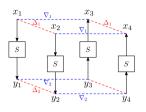


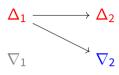
- $\mathcal{X}_{\mathrm{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathrm{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathrm{BCT}}(\Delta_1, \nabla_2) \ [\mathsf{Cid} + 18]$





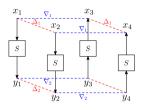
- $\mathcal{X}_{\text{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \text{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\text{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathrm{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathrm{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathrm{BCT}}(\Delta_1, \nabla_2) \ [\mathsf{Cid} + 18]$

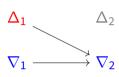




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- $\qquad \qquad \mathsf{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$

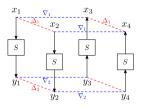
BCT Framework

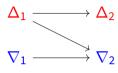




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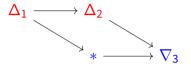
BCT Framework





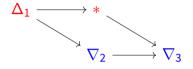
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Double Boomerang Connectivity Table (DBCT)



- igotagraphi DBCT $^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} ext{UBCT}(\Delta_1, \nabla_2, \Delta_2) \cdot ext{LBCT}(\Delta_2, \nabla_3, \nabla_2)$
- $igotimes ext{DBCT}^\dashv(\Delta_1,
 abla_2,
 abla_3) = \sum_{\Delta_2} ext{UBCT}(\Delta_1,
 abla_2, \Delta_2) \cdot ext{LBCT}(\Delta_2,
 abla_3,
 abla_2).$
- $igotag{}$ DBCT $(\Delta_1, \nabla_3) = \sum_{\Delta_2} \mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3)$

Double Boomerang Connectivity Table (DBCT)



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Double Boomerang Connectivity Table (DBCT)

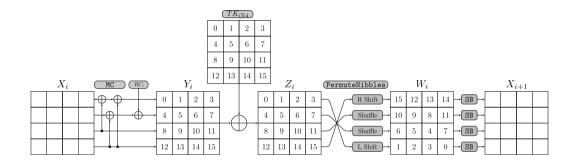
$$\Delta_1 \longrightarrow * \\ * \longrightarrow \nabla_3$$

- igotagraphi DBCT $^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} ext{UBCT}(\Delta_1, \nabla_2, \Delta_2) \cdot ext{LBCT}(\Delta_2, \nabla_3, \nabla_2)$
- igotagraphi DBCT $^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \text{UBCT}(\Delta_1, \nabla_2, \Delta_2) \cdot \text{LBCT}(\Delta_2, \nabla_3, \nabla_2).$

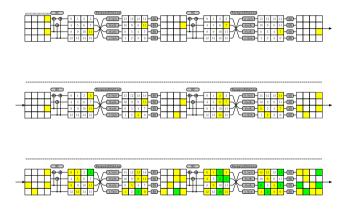
Application to CRAFT



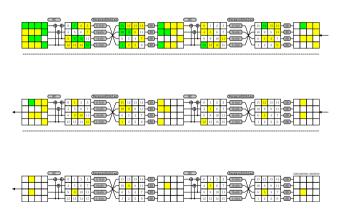
CRAFT [Bei+19]



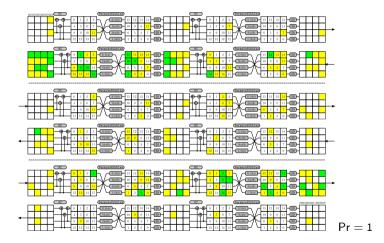
A 6-round ST Deterministic Distinguisher for CRAFT



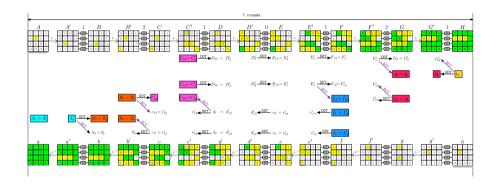
A 6-round ST Deterministic Distinguisher for CRAFT



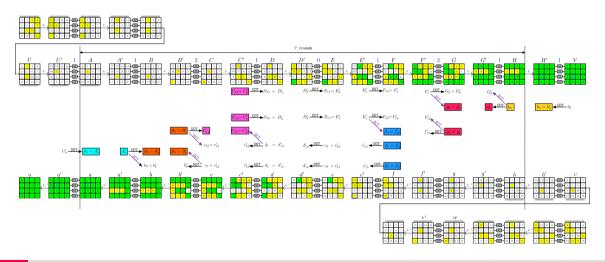
A 6-round ST Deterministic Distinguisher for CRAFT



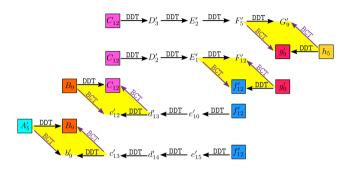
A 7-round Distinguisher (Extendable up to 14 rounds)



A 7-round Distinguisher (Extendable up to 14 rounds)



A 7-round Distinguisher (Extendable up to 14 rounds)



$$\begin{split} \text{DBCT}_{\text{total}} &= \text{DBCT}^{\vdash}(A_{5}, B_{9}, c_{5}) \cdot \text{DBCT}^{\vdash}(B_{9}, C_{12}, d_{1}) \cdot \text{DBCT}^{\dashv}(E'_{1}, f'_{12}, g'_{9}) \cdot \text{DBCT}^{\dashv}(F'_{5}, g'_{9}, h_{5}) \\ \text{Pr}_{\text{total}} &= \text{Pr}(d_{1} \xleftarrow{2 \text{ DDT}} f'_{12}) \cdot \text{Pr}(c_{5} \xleftarrow{3 \text{ DDT}} f'_{12}) \cdot \text{Pr}(C_{12} \xrightarrow{2 \text{ DDT}} E'_{1}) \cdot \text{Pr}(C_{12} \xrightarrow{3 \text{ DDT}} F'_{5}) \\ r &= 2^{-8 \cdot n} \cdot \sum_{B_{9}} \sum_{C_{12}} \sum_{g'_{9}} \sum_{f'_{12}} \sum_{c_{5}} \sum_{d_{1}} \sum_{E'_{1}} \sum_{F'_{5}} \text{DBCT}_{\text{total}} \cdot \text{Pr}_{\text{total}}. \end{split}$$

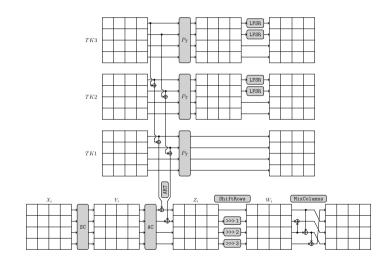
Summary of Our Distinguishers for CRAFT

Distinguisher Type	# Rounds	Probability	Reference
ST-Differential	9	2-40.20	
	10	$2^{-44.89}$	
	11	$2^{-49.79}$	[[]]
	12	$2^{-54.48}$	[Had+19]
	13	$2^{-59.13}$	
	14	$2^{-63.80}$	
ST-Boomerang	6	1	
	7	2 -4	
	8	2^{-8}	
	9	2 ^{-14.76}	This Dance
	10	2 ^{-19.83}	This Paper
	11	2 ^{-24.90}	
	12	2 ^{-34.89}	
	13	$2^{-44.89}$	
	14	$2^{-55.85}$	

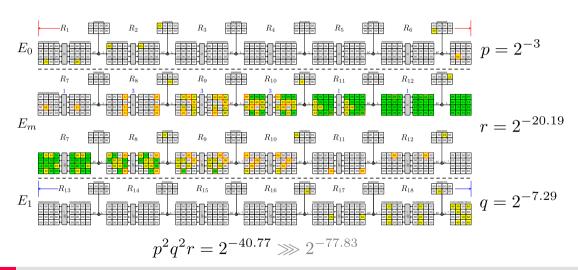
Application to SKINNY



SKINNY [Bei+16]



18-round Practical Sandwich Distinguisher for SKINNY-128-256



Summary of Our Distinguishers for SKINNY

			Probability		
Version	n	#Rounds	Our Distinguisher	[SQH19]	
SKINNY-n-2n	64	17	2 ^{-26.54} (II)	$2^{-29.78}$	
		18	2 ^{-37.90} (II)	$2^{-45.14}$	
		19	2 ^{-51.08} (II)	$2^{-65.62}$	
	128	18	2 ^{-40.77} (II)	$2^{-77.83}$	
		19	2 ^{-58.33} (II)	$2^{-97.53}$	
		20	2 ^{-85.31} (I)	$2^{-128.65}$	
		21	2 ^{-114.07} (II)	$2^{-171.77}$	
SKINNY-n-3n	64	22	2 ^{-38.84} (I)	$2^{-42.98}$	
		23	2 ^{-52.84} (I)	$2^{-67.36}$	
	128	22	2 ^{-40.57} (I)	$2^{-48.30}$	
		23	2^{-56.47} (I)	$2^{-75.86}$	
		24	2 ^{-87.39} (I)	$2^{-107.86}$	
		25	2 ^{-116.59} (I)	$2^{-141.66}$	

Summary of Our Key Recovery Attacks

Scheme	#rounds	Data	Memory	Time	Attack	P_s	Reference
SKINNY-64-128	23/36	2 ^{60.54}	2 ^{60.9}	$2^{120.7}$	Rectangle	0.977	This paper
SKINNY-64-192	29/40	2 ^{61.42}	2 ⁸⁰	2 ¹⁷⁸	Rectangle	0.977	This paper
SKINNY-128-256	24/48	$2^{125.21}$	2 ^{125.54}	2 ^{209.85}	Rectangle	0.977	This paper
SKINNY-128-384	30/56	2 ^{125.29}	2 ^{125.8}	2 ^{361.68}	Rectangle	0.977	This paper
CRAFT	18/32	2 ^{60.92}	2 ⁸⁴	$2^{101.7}$	Rectangle	0.977	This paper
SKINNY-64-128	23/36	2 ^{62.47}	2 ¹²⁴	2 ^{125.91}	Impossible	1	[LGS17]
SKINNY-64-192	27/40	2 ^{63.5}	2 ⁸⁰	2 ^{165.5}	Rectangle	0.916	[LGS17]
SKINNY-128-256	23/48	2124.47	2 ²⁴⁸	2 ^{251.47}	Impossible	1	[LGS17]
SKINNY-128-384	28/56	2 ¹²²	2122.32	2 ^{315.25}	Rectangle	0.8315	[Zha+20]

Conclusion



Our Main Contributions

- ❷ We introduced a heuristic method to search for sandwich distinguishers
- We introduced new tools in BCT framework (DBCT, ...)
- ❷ We significantly improved the rectangle attacks on SKINNY and CRAFT

Thanks for your attention!

https://github.com/hadipourh/Boomerang

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