

Integral Cryptanalysis of WARP based on Monomial Prediction

Hosein Hadipour Maria Eichlseder

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Motivation and Our Contributions



Motivation

- ✔ Integral analysis of WARP



Contributions

- ✔ Providing a generic SAT model for integral analysis based on monomial prediction
- ✔ Our model takes the key schedule into account
- ✔ We proposed a tool for key-recovery taking the FFT technique into account
- ✔ Thanks to our tools, we improved the integral attack of WARP by **11** rounds

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Outline

- 1 Boolean Functions and Integral Analysis
- 2 Monomial Prediction and Our SAT Model
- 3 Application of Our Modeling to Integral Analysis of WARP
- 4 Key-Recovery
- 5 Conclusion

Boolean Functions and Integral Analysis



Integral Distinguisher and The Coefficients of ANF

 $y = f(\mathbf{k}, \mathbf{x}) = \sum_{\mathbf{u} \in \mathbb{F}_2^n} \sum_{\mathbf{v} \in \mathbb{F}_2^k} a_{\mathbf{u}, \mathbf{v}} \mathbf{k}^{\mathbf{v}} \mathbf{x}^{\mathbf{u}}$

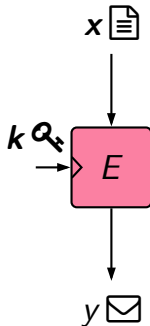
 $\mathbb{C}_{\mathbf{u}} = \{\mathbf{x} \in \mathbb{F}_2^n \mid \mathbf{x} \leq \mathbf{u}\}$

 $a_{\mathbf{u}}(\mathbf{k}) = \sum_{\mathbf{x} \leq \mathbf{u}} f(\mathbf{k}, \mathbf{x})$


 Which monomial is key-independent in the ANF?

 zero-sum: $\exists \mathbf{u}, s.t. \forall \mathbf{k} : a_{\mathbf{u}}(\mathbf{k}) = 0$

 one-sum: $\exists \mathbf{u}, s.t. \forall \mathbf{k} : a_{\mathbf{u}}(\mathbf{k}) = 1$



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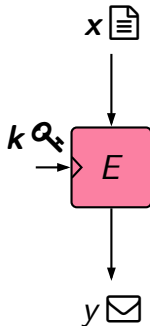
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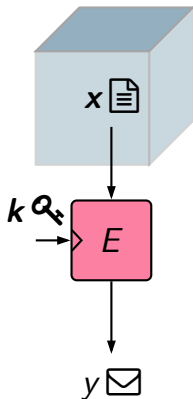
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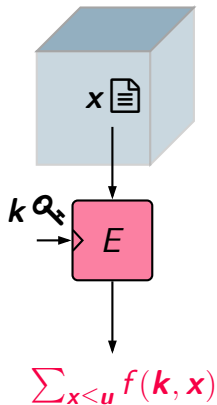
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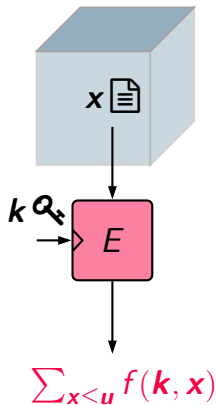
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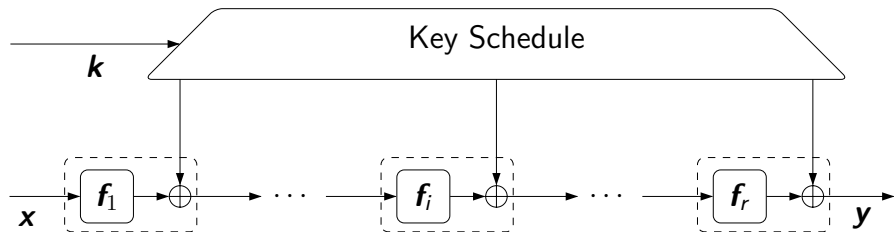
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Monomial Prediction and Our SAT Model



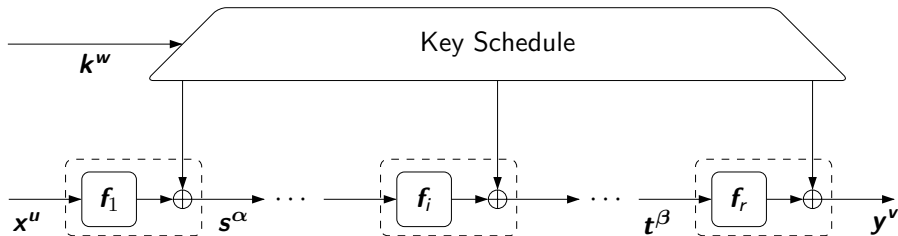
Core Idea of Monomial Prediction [Hu+20]



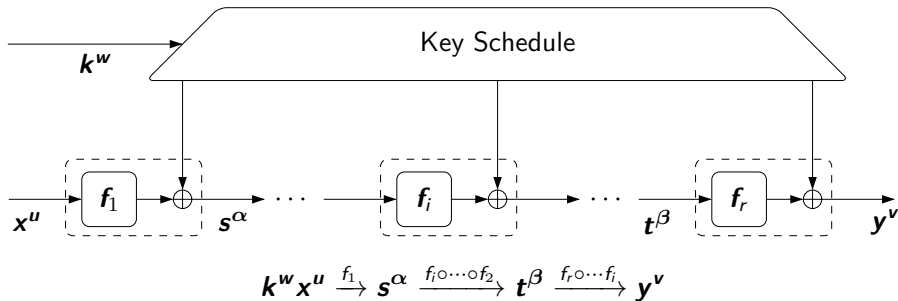
Core Idea

The absence (or presence) of a monomial in the ANF of a composite function can be checked by tracking the propagation of the given monomial through the building blocks of composite functions.

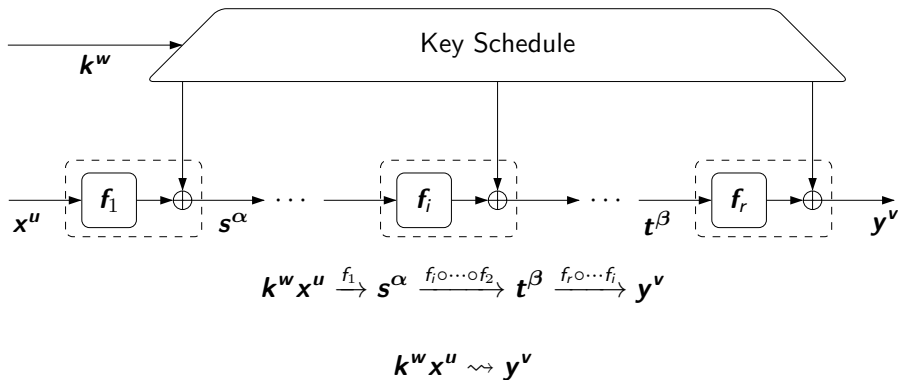
Monomial Trail and Integral Distinguisher



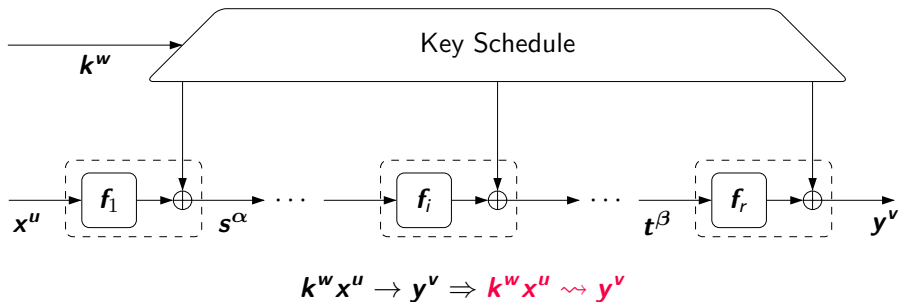
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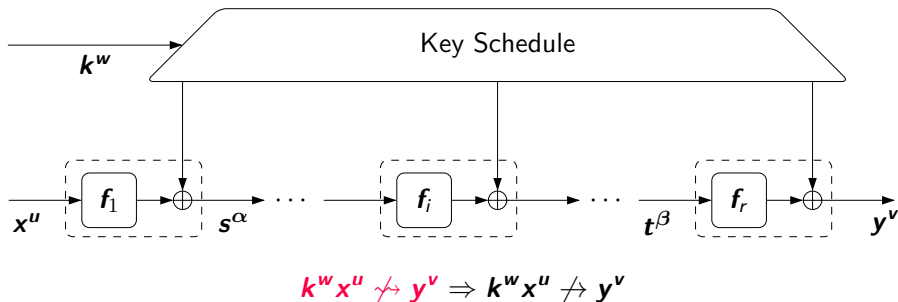
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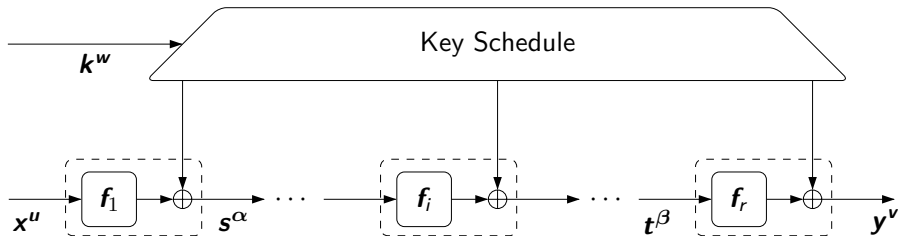
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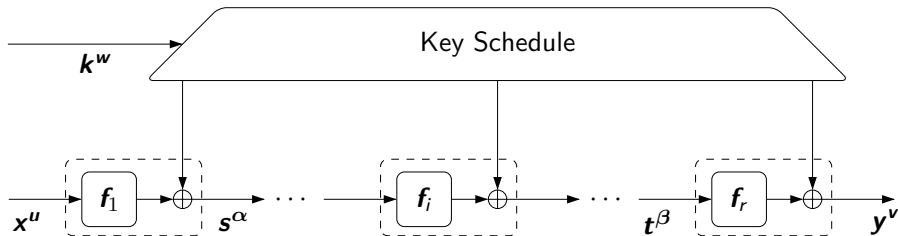
$$y^v = \sum_{u \in \mathbb{F}_2^n} \sum_{v \in \mathbb{F}_2^k} a_{u,v} k^v x^u = \sum_{u \in \mathbb{F}_2^n} a_u(k) \cdot x^u$$

From Monomial Trails to Integral Distinguisher

🧨 If $\exists u$ s.t. $k^w x^u \not\rightarrow y^v$ for all $w \in \mathbb{F}_2^k$ then $a_u(k) = 0$ (zero-sum)

🧨 If $\exists u$ s.t. $k^w x^u \not\rightarrow y^v$ for all $w \in \mathbb{F}_2^k \setminus \{0\}$ then $a_u(k) = \text{constant}$ (zero/one-sum)

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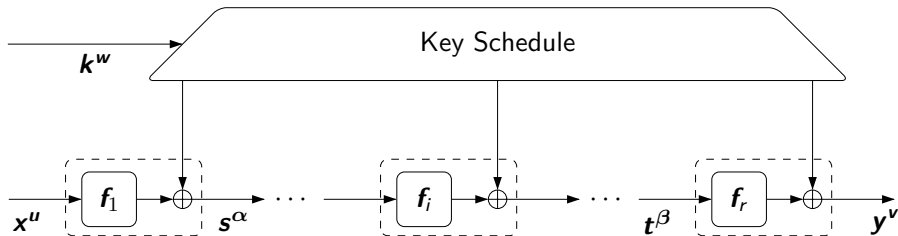
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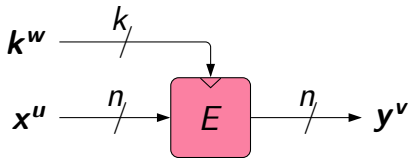


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🔗 Model the propagation of monomial trails through the building blocks by a CNF clause

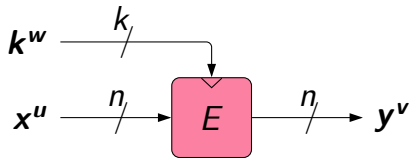
🔧 Main variables are the monomial exponents, i.e., u, w, v, \dots not x, k, y, \dots

⚓ Fix u to a certain vector and set v to e_i (w should be a free variable but non-zero)

🏠 Any possible solution of the model is a monomial trail from $k^w x^u$ to y^v

🚩 If the model is impossible, then $k^w x^u \not\rightsquigarrow y^v$ for all $w \in \mathbb{F}_2^k$, and $a_u(\mathbf{k}) = \text{constant}$

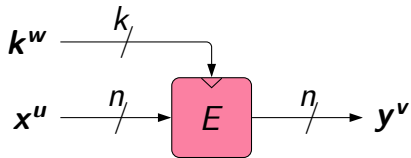
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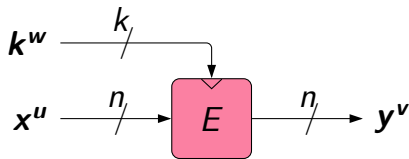
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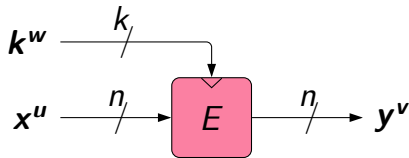
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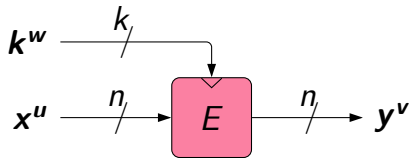
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Monomial Prediction Table (MPT)

- Let $\mathbf{y} = \mathbf{f}(\mathbf{x})$ be an m -bit to n -bit vectorial Boolean function. Then $\text{MPT}(\mathbf{u}, \mathbf{v}) = 1$ if $\mathbf{x}^{\mathbf{u}} \xrightarrow{\mathbf{f}} \mathbf{y}^{\mathbf{v}}$, and $\text{MPT}(\mathbf{u}, \mathbf{v}) = 0$ otherwise.

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x	$S(x)$
0	c
1	a
2	d
3	3
4	e
5	b
6	f
7	7
8	8
9	9
a	1
b	5
c	0
d	2
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$\mathbf{u} \setminus \mathbf{v}$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	1	.	.	.	1	.	.	.	1	.	.	.	1	.	.	.
1	.	.	1	.	1	1	.	1	.	.	.
2	.	1	.	.	.	1	.	.	.	1	.	.	.	1	.	.
3	.	.	.	1	.	1	.	.	1	1	1	.	.	1	.	.
4	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	.
5	.	1	1	1	.	.	1	.	.	1	1	1	.	.	1	.
6	.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1
7	.	1	.	.	1	1	1	.	.	1	1
8	1	1	.	.	.
9	.	1	1	.	1	1	1	.	1	.	.	.
a	1	.	.	1	1	.	.	.	1	.	.
b	.	1	.	1	1	.	.	.	1	.	1	.	.	1	.	.
c	.	.	1	.	.	.	1	.	1	.	1	.	.	.	1	.
d	.	.	.	1	.	.	1	.	.	.	1	1	.	.	1	.
e	.	1	.	1	1	.	.	1	1	.	.	1	.	.	.	1
f	1

Monomial Prediction Table (MPT)

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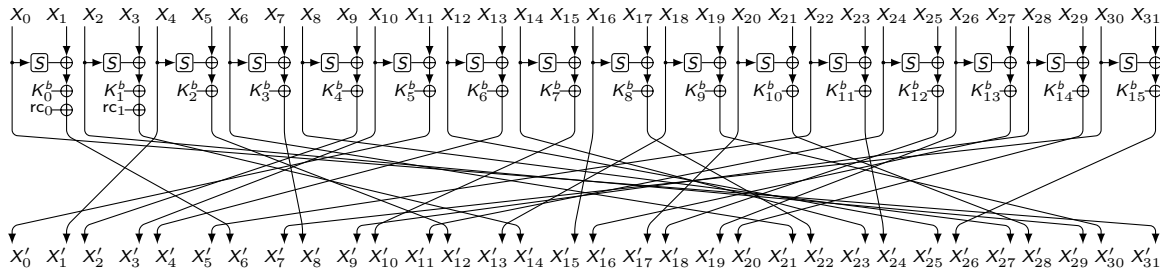
x	$S(x)$			
0	c			
1	a	$(u_2 \vee \neg v_1 \vee \neg v_3)$	$\wedge (\neg u_1 \vee \neg v_0 \vee \neg v_1 \vee v_2)$	$\wedge (\neg u_0 \vee \neg u_1 \vee \neg u_2 \vee \neg v_2 \vee v_3)$
2	d	$\wedge (u_2 \vee u_3 \vee \neg v_3)$	$\wedge (\neg u_0 \vee \neg u_1 \vee \neg u_3 \vee v_2)$	$\wedge (\neg u_0 \vee \neg u_3 \vee v_0 \vee \neg v_1 \vee \neg v_3)$
3	3	$\wedge (u_1 \vee \neg v_1 \vee \neg v_2)$	$\wedge (\neg u_1 \vee u_2 \vee v_0 \vee v_2 \vee v_3)$	$\wedge (\neg u_0 \vee \neg u_1 \vee \neg u_3 \vee v_0 \vee v_1 \vee v_3)$
4	e	$\wedge (u_1 \vee u_3 \vee \neg v_2)$	$\wedge (u_2 \vee \neg u_3 \vee v_1 \vee v_2 \vee v_3)$	$\wedge (\neg u_0 \vee \neg u_2 \vee \neg u_3 \vee \neg v_0 \vee v_1 \vee \neg v_3)$
5	b	$\wedge (u_0 \vee \neg u_2 \vee u_3 \vee v_3)$	$\wedge (u_1 \vee \neg v_0 \vee \neg v_2 \vee \neg v_3)$	$\wedge (\neg u_1 \vee \neg u_2 \vee \neg u_3 \vee v_1 \vee \neg v_2)$
6	f	$\wedge (u_0 \vee \neg u_1 \vee u_3 \vee v_2)$	$\wedge (\neg u_0 \vee u_1 \vee u_3 \vee v_0 \vee v_1)$	$\wedge (\neg u_1 \vee \neg u_2 \vee \neg u_3 \vee v_1 \vee v_3)$
7	7	$\wedge (\neg u_2 \vee v_0 \vee v_1 \vee v_3)$	$\wedge (\neg u_1 \vee u_3 \vee \neg v_0 \vee v_2 \vee \neg v_3)$	$\wedge (u_0 \vee u_1 \vee \neg u_3 \vee v_0 \vee v_1 \vee v_2)$
8	8	$\wedge (u_0 \vee u_1 \vee u_2 \vee \neg v_3)$	$\wedge (u_0 \vee u_1 \vee \neg u_2 \vee \neg v_1 \vee v_3)$	$\wedge (\neg u_3 \vee v_0 \vee \neg v_1 \vee \neg v_2 \vee \neg v_3)$
9	9	$\wedge (u_1 \vee u_2 \vee \neg v_2 \vee \neg v_3)$	$\wedge (u_1 \vee \neg u_2 \vee u_3 \vee \neg v_1 \vee v_3)$	$\wedge (\neg u_0 \vee u_1 \vee u_2 \vee v_1 \vee v_2 \vee v_3)$
a	1			
b	5			
c	0	$\wedge (\neg u_2 \vee \neg v_0 \vee \neg v_1 \vee v_3)$	$\wedge (\neg u_1 \vee u_3 \vee \neg v_1 \vee v_2 \vee \neg v_3)$	
d	2			
e	4			
f	6			

Application of Our Modeling to Integral Analysis of WARP



WARP[Ban+20]

- ➡ Proposed in SAC 2020 [Ban+20] as the lightweight alternative of AES-128
- ➡ 128-bit block/key size, and 41 rounds (40.5 rounds)
- ➡ Splits 128-bit K into two halves $K^{(0)} || K^{(1)}$ and uses $K^{(r-1 \bmod 2)}$ in the r th round



22-round Integral Distinguisher for WARP

The best previous integral distinguisher: 20 rounds [Ban+20]

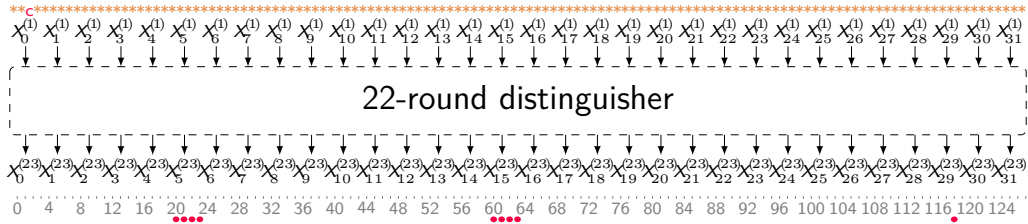
$$(2) \xrightarrow{22 \text{ rounds}} (\underline{20, 21, 22, 23}, 118, \underline{60, 61, 62, 63}),$$



22-round Integral Distinguisher for WARP

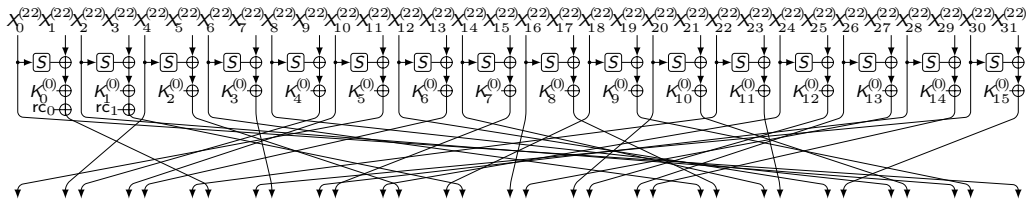
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23-round Integral Distinguisher for WARP

Any r -round integral distinguisher of WARP can be extended by 1 round

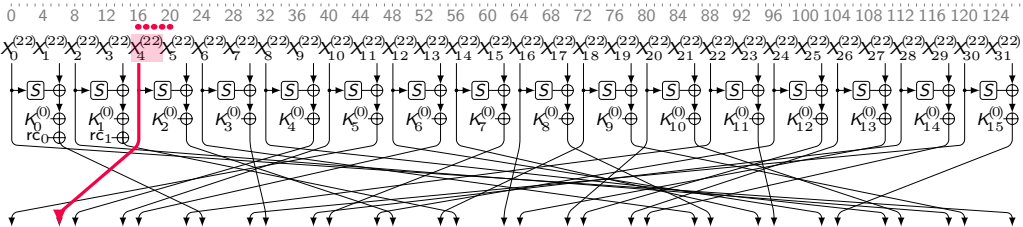


$$\sum_{\mathbf{c}} x_4^{(22)} = \sum_{\mathbf{c}} x_1^{(23)}$$

$$\sum_{\mathbf{c}} x_{11}^{(22)} = \sum_{\mathbf{c}} \left(s(x_4^{(23)}) \oplus x_0^{(23)} \right) \oplus \sum_{\mathbf{c}} K_i^{(b)}$$

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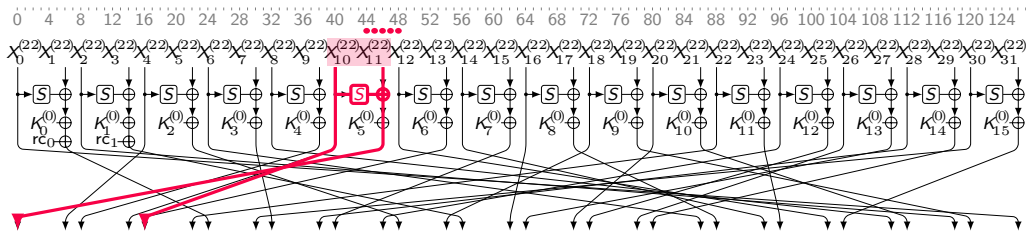


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Key-Recovery



Naive Approach v.s. FFT Technique [TA14]

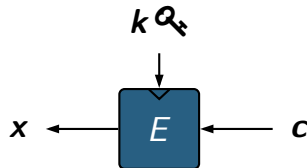


Naive approach:

✔ $\sum \mathbf{x} = \sum_{c \in \mathbb{C}} f(\mathbf{k}, \mathbf{c})$

✔ $T_{tot} = 2^{|\mathbf{k}|} |\mathbb{C}|$, where $\mathbb{C} = 2^{|\mathbf{k}|}$

✔ $T_{tot} = 2^{2|\mathbf{k}|}$



FFT technique:

✔ $\sum \mathbf{x} = \sum_{c \in \mathbb{C}} F(\mathbf{k} \oplus \mathbf{c})$

✔ $T_{tot} = 4 \cdot |\mathbf{k}| \cdot 2^{|\mathbf{k}|}$

Naive Approach v.s. FFT Technique [TA14]



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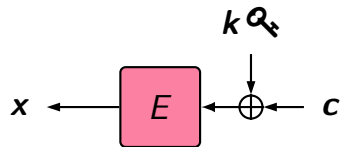
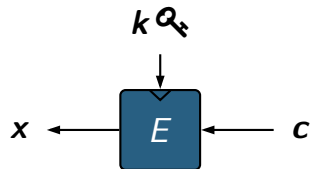
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MitM [SW12]



Naive approach:

✔ $x = F(k_1, k_2, c)$

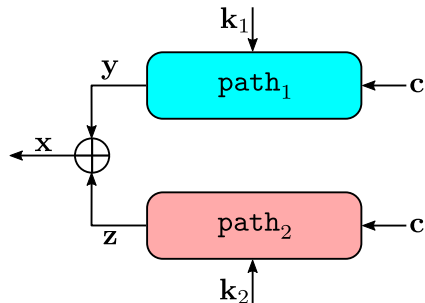
✔ $T = 2^{|k_1 \cup k_2|}$



MitM:

✔ $y = F(k_1, c), z = g(k_2, c)$

✔ $T = 2^{|k_1|} + 2^{|k_2|}$



$$\sum x = 0$$

MitM [SW12]



Naive approach:

✔ $x = F(k_1, k_2, c)$

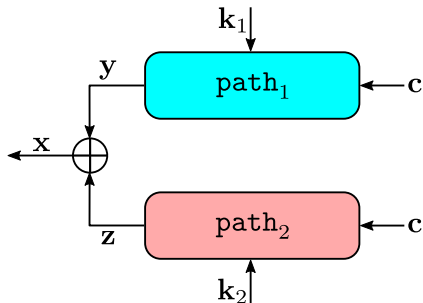
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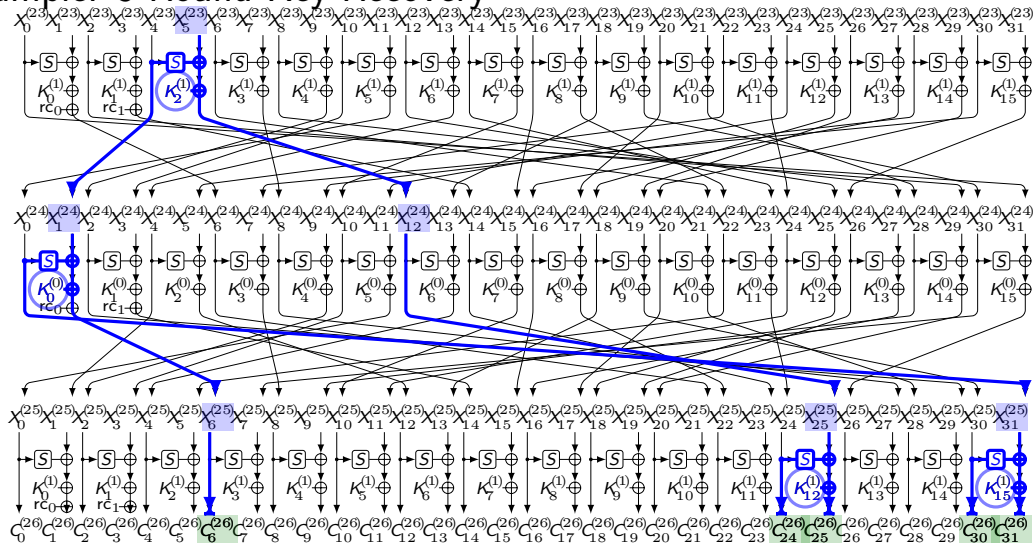


$$\sum x = 0 \iff \sum y = \sum z$$

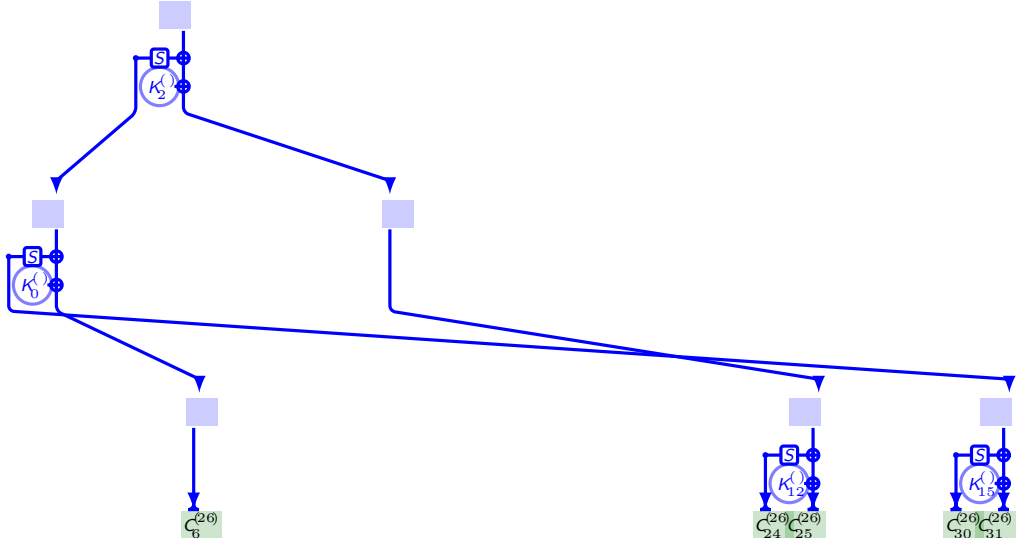
Overall View of Our Key-Recovery Tool

- 1- Assume that $\mathbf{x} = \mathbf{y} \oplus \mathbf{z}$ and $\sum \mathbf{x} = 0$
- 2- For each path, i.e., \mathbf{y} , and \mathbf{z} :
 - Build the graph of dependencies: $\mathbf{y} = f(\mathbf{k}, \mathbf{c})$
 - Simplify the dependency graph: reform $f(\mathbf{k}, \mathbf{c})$ to $F(\tilde{\mathbf{k}} \oplus \tilde{\mathbf{c}})$
 - Use FFT to compute the list $[\sum \mathbf{y} \mid \tilde{\mathbf{k}} = 0, \dots, 2^{|\mathbf{k}|-1}]$
- 3- Compare the two lists to find candidates for the involved key bits
- 4- Brute force the remaining keys to find the correct key

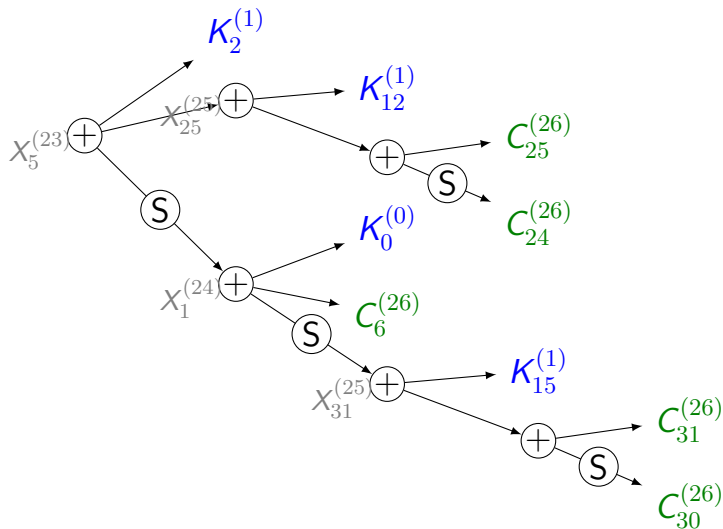
Example: 3-Round Key Recovery



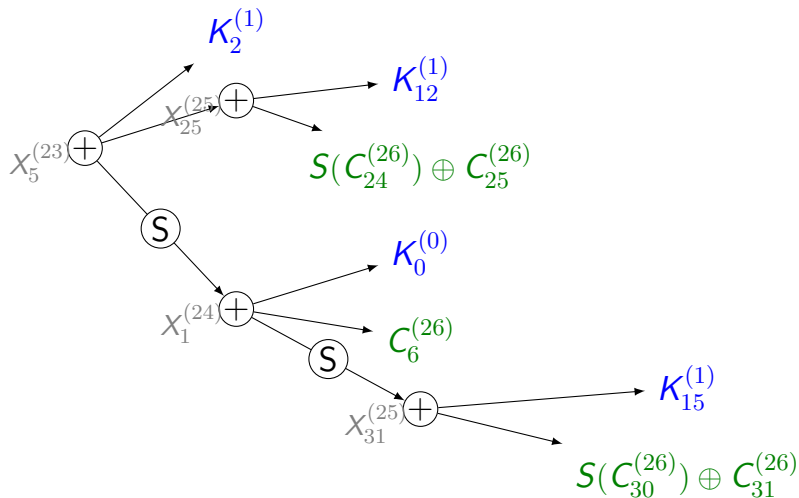
Example: 3-Round Key Recovery



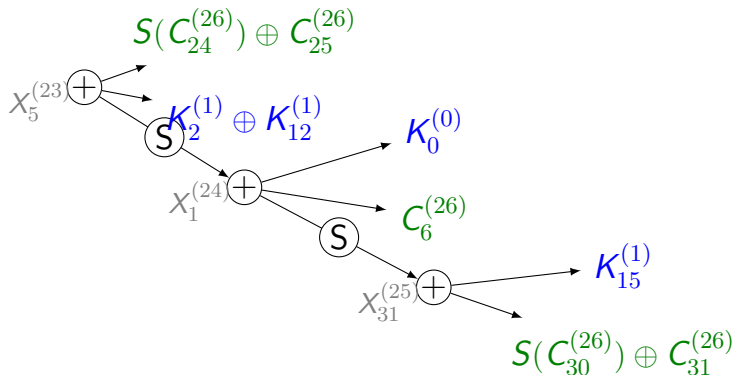
Example: Dependency Graph



Example: Dependency Graph



Example: Dependency Graph



Summary of Our Result

#R	Data	Time	Memory	Attack	Reference
32	2^{127}	2^{127}	2^{108}	Integral	This paper
21	2^{124}	-	-	Integral	[Ban+20]
18	$2^{104.62}$	-	-	Differential	[TB22]
21	-	-	-	Impossible diff.	[Ban+20]
21	2^{113}	2^{113}	2^{72}	Differential	[KY21]
23	$2^{106.62}$	$2^{106.62}$	$2^{106.62}$	Differential	[TB22]
24	$2^{126.06}$	$2^{125.18}$	$2^{127.06}$	Rectangle	[TB22]

Conclusion



Contributions

- ✔ We provided a SAT model for integral analysis based on Monomial prediction
- ✔ Our modeling is generic and can be applied to other (binary field) block ciphers
- ✔ We proposed a tool for key-recovery taking the FFT technique into account
- 💎 Overall, we improved the integral attack of WARP by **11** rounds

Thanks for your attention!

<https://github.com/hadipourh/mpt>

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