

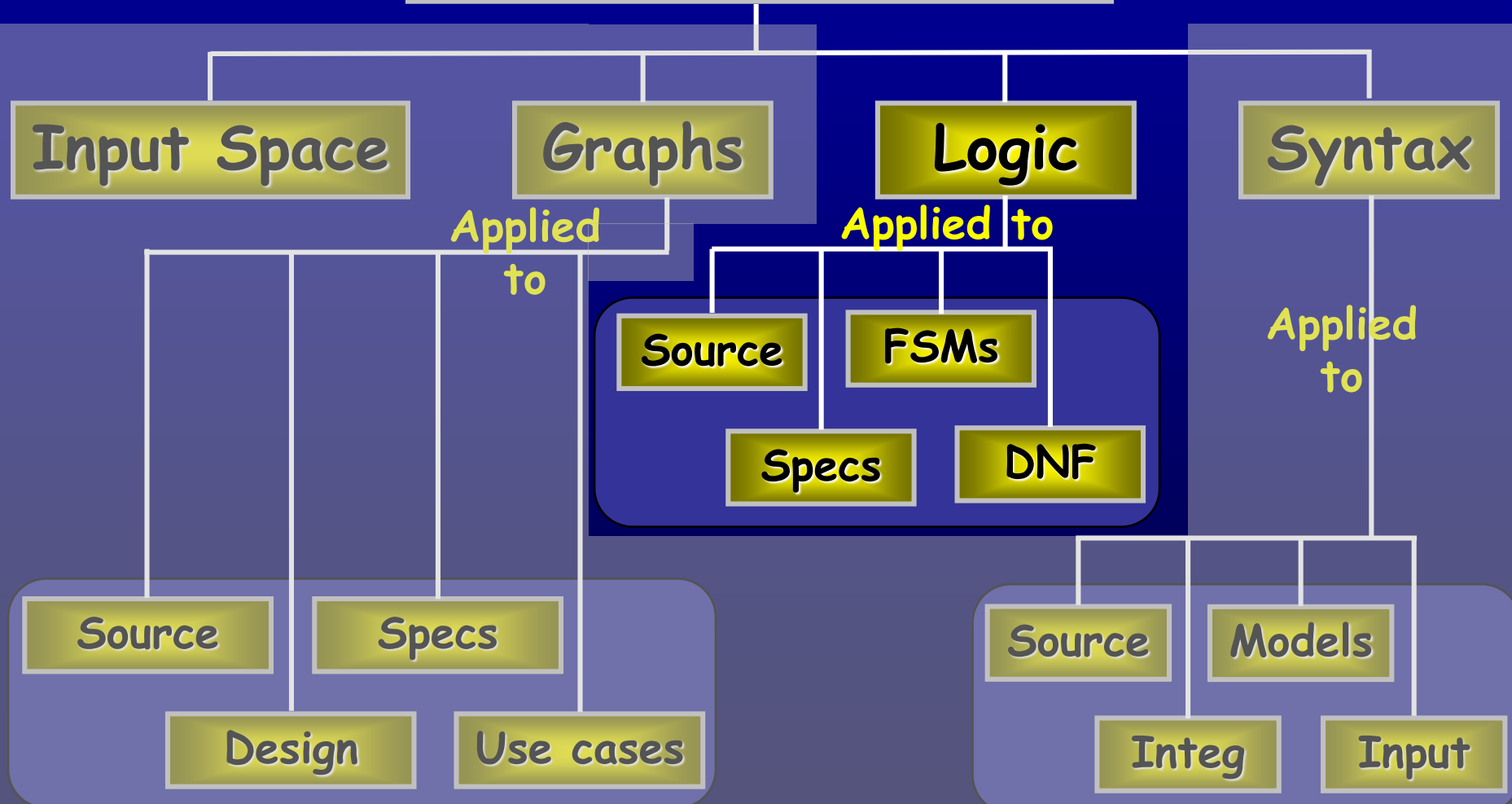
Introduction to Software Testing Chapter 8.1 Logic Coverage

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Ch. 8 : Logic Coverage

Four Structures for Modeling Software



Semantic Logic Criteria (8.1)

- Logic expressions show up in many situations
- Covering logic expressions is required by the US Federal Aviation Administration for safety critical software
- Logical expressions can come from many sources
 - Decisions in programs
 - FSMs and statecharts
 - Requirements
- Tests are intended to choose some subset of the total number of truth assignments to the expressions

Logic Predicates and Clauses

- A *predicate* is an expression that evaluates to a **boolean** value
- Predicates can contain
 - **boolean variables**
 - non-boolean variables that contain **>, <, ==, >=, <=, !=**
 - boolean **function** calls
- Internal structure is created by logical operators
 - \neg – the *negation* operator
 - \wedge – the *and* operator
 - \vee – the *or* operator
 - \rightarrow – the *implication* operator
 - \oplus – the *exclusive or* operator
 - \leftrightarrow – the *equivalence* operator
- A *clause* is a predicate with no logical operators

Example and Facts

- $(a < b) \vee f(z) \wedge D \wedge (m \geq n * o)$ has four clauses:
 - $(a < b)$ – relational expression
 - $f(z)$ – boolean-valued function
 - D – boolean variable
 - $(m \geq n * o)$ – relational expression
- Most predicates have **few clauses**
 - 88.5% have 1 clause
 - 9.5% have 2 clauses
 - 1.35% have 3 clauses
 - Only 0.65% have 4 or more !
- **Sources** of predicates
 - Decisions in **programs**
 - Guards in **finite state machines**
 - Decisions in **UML** activity graphs
 - **Requirements**, both formal and informal
 - **SQL** queries

*from a study of 63 open
source programs,
>400,000 predicates*

Translating from English

- “I am interested in SWE 637 and CS 652”
- *course = swe637* **OR** *course = cs652*

Humans have trouble translating from English to logic

- “If you leave before 6:30 AM, take Braddock to 495, if you leave after 7:00 AM, take Prosperity to 50, then 50 to 495”
- $(time < 6:30 \rightarrow path = Braddock) \wedge (time > 7:00 \rightarrow path = Prosperity)$
- Hmm ... this is incomplete !
- $(time < 6:30 \rightarrow path = Braddock) \wedge (time \geq 6:30 \rightarrow path = Prosperity)$

Logic Coverage Criteria (8.1.1)

- We use predicates in testing as follows :
 - Developing a model of the software as one or more predicates
 - Requiring tests to satisfy some combination of clauses
- Abbreviations:
 - P is the set of predicates
 - p is a single predicate in P
 - C is the set of clauses in P
 - C_p is the set of clauses in predicate p
 - c is a single clause in C

Predicate and Clause Coverage

- The first (and simplest) two criteria require that each predicate and each clause be evaluated to both true and false

Predicate Coverage (PC) : For each p in P , TR contains two requirements: p evaluates to true, and p evaluates to false.

- When predicates come from conditions on edges, this is equivalent to edge coverage
- PC does not evaluate all the clauses, so ...

Clause Coverage (CC) : For each c in C , TR contains two requirements: c evaluates to true, and c evaluates to false.

Predicate Coverage Example

$$((a < b) \vee D) \wedge (m \geq n * o)$$

predicate coverage

Predicate = true

$a = 5, b = 10, D = \text{true}, m = 1, n = 1, o = 1$

$= (5 < 10) \vee \text{true} \wedge (1 \geq 1 * 1)$

$= \text{true} \vee \text{true} \wedge \text{TRUE}$

$= \text{true}$

Predicate = false

$a = 10, b = 5, D = \text{false}, m = 1, n = 1, o = 1$

$= (10 < 5) \vee \text{false} \wedge (1 \geq 1 * 1)$

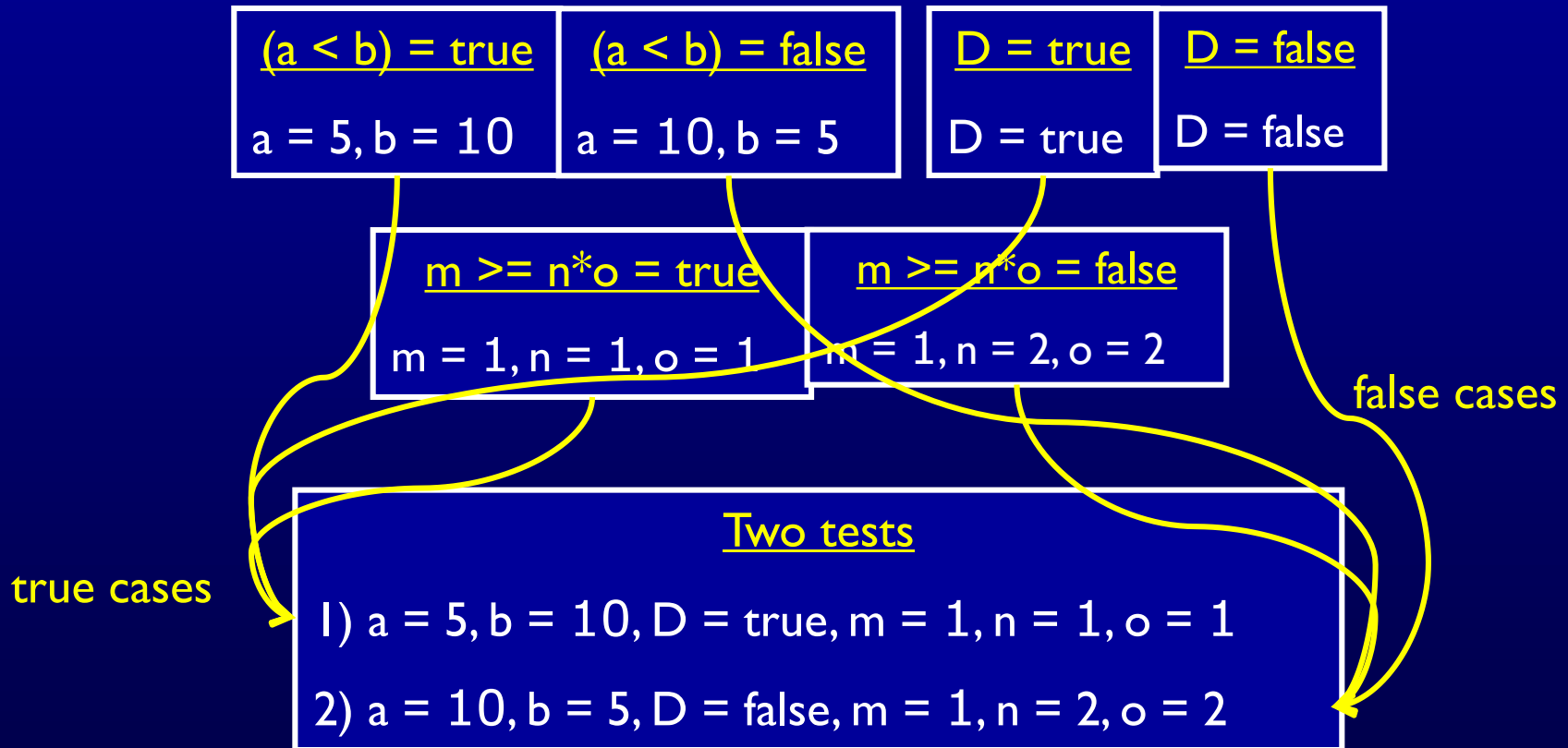
$= \text{false} \vee \text{false} \wedge \text{TRUE}$

$= \text{false}$

Clause Coverage Example

$$((a < b) \vee D) \wedge (m \geq n * o)$$

Clause coverage



Problems with PC and CC

- PC does not **fully exercise** all the clauses, especially in the presence of short circuit evaluation
- CC does not always **ensure PC**
 - That is, we can satisfy CC without causing the predicate to be both true and false
 - This is definitely not what we want !
- The simplest solution is to test **all combinations** ...

Combinatorial Coverage

- CoC requires every possible combination
- Sometimes called Multiple Condition Coverage

Combinatorial Coverage (CoC) : For each p in P , TR has test requirements for the clauses in C_p to evaluate to each possible combination of truth values.

	$a < b$	D	$m \geq n * o$	$((a < b) \vee D) \wedge (m \geq n * o)$
1	T	T	T	T
2	T	T	F	F
3	T	F	T	T
4	T	F	F	F
5	F	T	T	T
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

Combinatorial Coverage

- This is simple, neat, clean, and comprehensive ...
- But quite **expensive!**
- 2^N tests, where N is the number of clauses
 - Impractical for predicates with more than 3 or 4 clauses
- The literature has lots of suggestions – some confusing
- The general idea is simple:

Test each clause independently from the other clauses

- Getting the details right is hard
- What exactly does “independently” mean ?
- The book presents this idea as “*making clauses active*” ...

Active Clauses (8.1.2)

- Clause coverage has a **weakness** : The values do not always make a difference
- Consider the first test for **clause coverage**, which caused each clause to be true:
 - $(5 < 10) \vee \text{true} \wedge (1 \geq 1 * 1)$
- Only the first clause **counts** !
- To really test the results of a clause, the clause should be the **determining factor** in the value of the predicate

Determination :	A clause c_i in predicate p , called the <i>major clause</i> , determines p if and only if the values of the remaining <i>minor clauses</i> c_j are such that changing c_i changes the value of p
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- This is considered to **make the clause active**

Determining Predicates

$$\underline{P = A \vee B}$$

if $B = \text{true}$, p is always true.

so if $B = \text{false}$, A determines p .

if $A = \text{false}$, B determines p .

$$\underline{P = A \wedge B}$$

if $B = \text{false}$, p is always false.

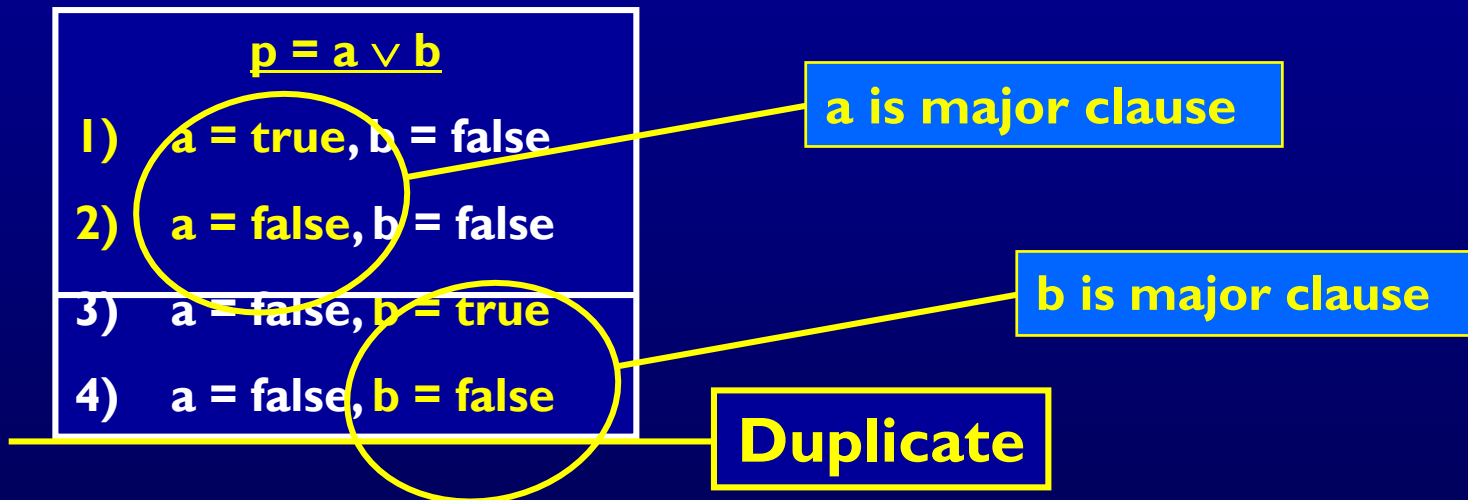
so if $B = \text{true}$, A determines p .

if $A = \text{true}$, B determines p .

- **Goal** : Find tests for each clause when the clause determines the value of the predicate
- This is formalized in a **family of criteria** that have subtle, but very important, differences

Active Clause Coverage

Active Clause Coverage (ACC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i determines p . TR has two requirements for each c_i : c_i evaluates to true and c_i evaluates to false.



- This is a form of **MCDC**, which is required by the FAA for safety critical software
- **Ambiguity** : Do the minor clauses have to have the **same values** when the major clause is true and false?

Resolving the Ambiguity

$$p = a \vee (b \wedge c)$$

Major clause : **a**

a = true, b = false, c = true

a = false, b = false, **c = false**

Is this allowed ?

- This question caused **confusion** among testers for years
- Considering this carefully leads to **three** separate criteria :
 - Minor clauses **do not** need to be the same
 - Minor clauses **do** need to be the same
 - Minor clauses **force the predicate** to become both true and false

General Active Clause Coverage

General Active Clause Coverage (GACC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i determines p . TR has two requirements for each c_i : c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j do not need to be the same when c_i is true as when c_i is false, that is, $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$ for all c_j OR $c_j(c_i = \text{true}) \neq c_j(c_i = \text{false})$ for all c_j .

- This is **complicated** !
- It is possible to satisfy GACC **without satisfying** predicate coverage
- We **really want** to cause predicates to be both true and false

Restricted Active Clause Coverage

Restricted Active Clause Coverage (RACC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i determines p . TR has two requirements for each c_i : c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j must be the same when c_i is true as when c_i is false, that is, it is required that $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$ for all c_j .

- This has been a common interpretation by aviation developers
- RACC often leads to infeasible test requirements
- There is no logical reason for such a restriction

Correlated Active Clause Coverage

Correlated Active Clause Coverage (CACC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i determines p . TR has two requirements for each c_i : c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j must cause p to be true for one value of the major clause c_i and false for the other, that is, it is required that $p(c_i = \text{true}) \neq p(c_i = \text{false})$.

- A more recent interpretation
- Implicitly allows minor clauses to have different values
- Explicitly satisfies (**subsumes**) predicate coverage

CACC and RACC

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

major clause

$P_a : b = \text{true} \text{ or } c = \text{true}$

CACC can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7 – a total of nine pairs

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

RACC can only be satisfied by row pairs (1, 5), (2, 6), or (3, 7)

Only three pairs

Inactive Clause Coverage (8.1.3)

- The active clause coverage criteria ensure that “major” clauses **do affect** the predicates
- Inactive clause coverage takes the opposite approach – major clauses **do not affect** the predicates

Inactive Clause Coverage (ICC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i **does not** determine p . TR has **four** requirements for each c_i : (1) c_i evaluates to true with p true, (2) c_i evaluates to false with p true, (3) c_i evaluates to true with p false, and (4) c_i evaluates to false with p false.

General and Restricted ICC

- Unlike ACC, the notion of correlation is not relevant
 - c_i does not determine p , so cannot correlate with p
- Predicate coverage is always guaranteed

General Inactive Clause Coverage (GICC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i does not determine p . The values chosen for the minor clauses c_j do not need to be the same when c_i is true as when c_i is false, that is, $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$ for all c_j OR $c_j(c_i = \text{true}) \neq c_j(c_i = \text{false})$ for all c_j .

Restricted Inactive Clause Coverage (RICC) : For each p in P and each major clause c_i in C_p , choose minor clauses $c_j, j \neq i$, so that c_i does not determine p . The values chosen for the minor clauses c_j must be the same when c_i is true as when c_i is false, that is, it is required that $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$ for all c_j .

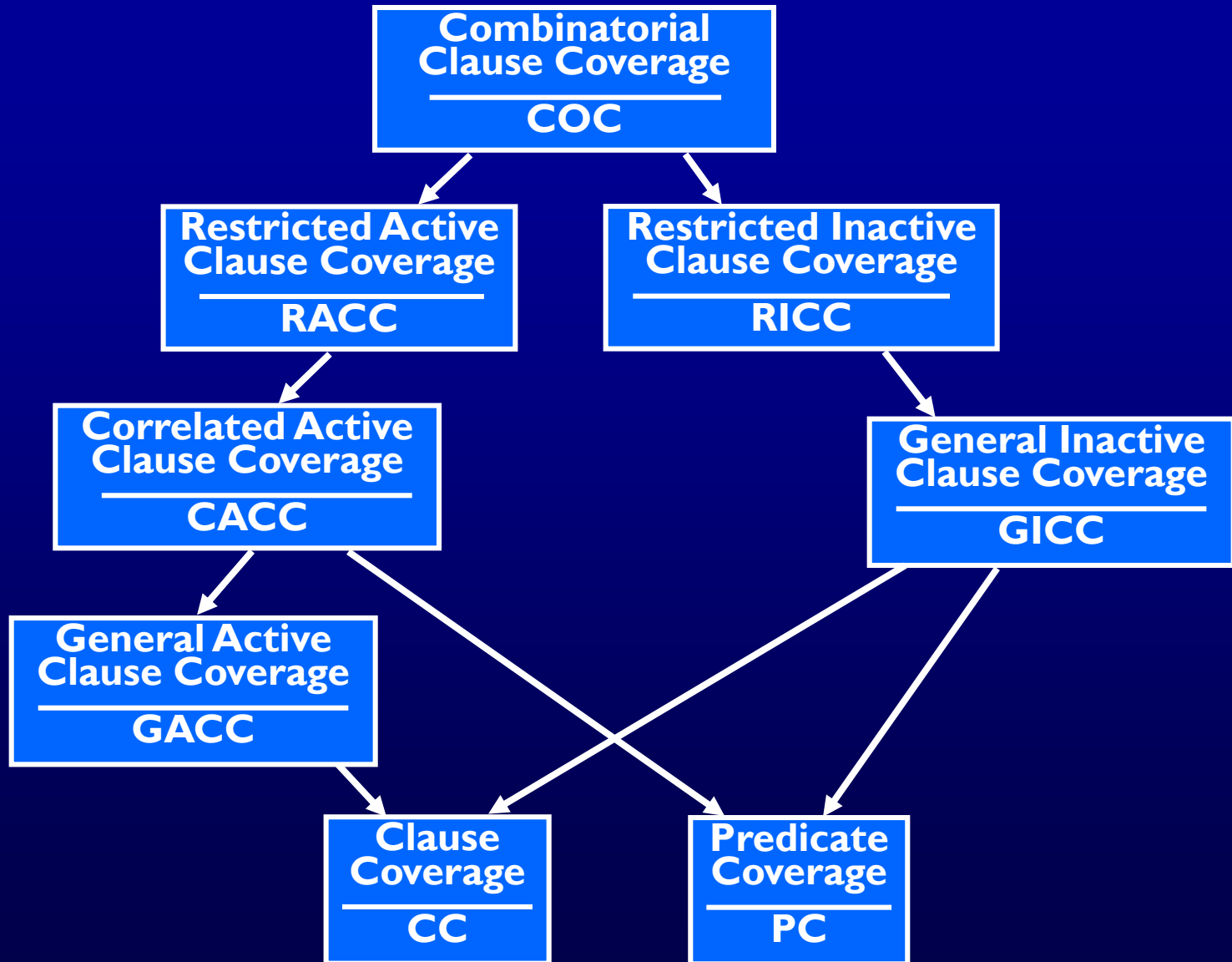
Infeasibility & Subsumption (8.1.4)

- Consider the predicate:

$$(a > b \wedge b > c) \vee c > a$$

- $(a > b) = \text{true}, (b > c) = \text{true}, (c > a) = \text{true}$ is infeasible
- As with graph-based criteria, infeasible test requirements have to be **recognized** and **ignored**
- Recognizing infeasible test requirements is hard, and in general, **undecidable**
- Software testing is **inexact** — engineering, not science

Logic Criteria Subsumption



Making Clauses Determine a Predicate

(8.1.5)

- Finding values for minor clauses c_j is easy for simple predicates
- But how to find values for more complicated predicates ?
- Definitional approach:
 - $p_{c=true}$ is predicate p with every occurrence of c replaced by **true**
 - $p_{c=false}$ is predicate p with every occurrence of c replaced by **false**
- To find values for the minor clauses, connect $p_{c=true}$ and $p_{c=false}$ with exclusive OR
$$p_c = p_{c=true} \oplus p_{c=false}$$
- After solving, p_c describes exactly the values needed for c to determine p

Examples

$$\underline{p = a \vee b}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \vee b) \text{ XOR } (\text{false} \vee b) \\ &= \text{true} \text{ XOR } b \\ &= \neg b \end{aligned}$$

$$\underline{p = a \wedge b}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \wedge b) \oplus (\text{false} \wedge b) \\ &= b \oplus \text{false} \\ &= b \end{aligned}$$

$$\underline{p = a \vee (b \wedge c)}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \vee (b \wedge c)) \oplus (\text{false} \vee (b \wedge c)) \\ &= \text{true} \oplus (b \wedge c) \\ &= \neg (b \wedge c) \\ &= \neg b \vee \neg c \end{aligned}$$

- “**NOT b \vee NOT c** ” means either b or c can be false
- **RACC** requires the same choice for both values of a , **CACC** does not

XOR Identity Rules

Exclusive-OR (*xor*, \oplus) means both cannot be true

That is, $A \text{ xor } B$ means

“A or B is true, but not both”

$$\begin{aligned} p &= A \oplus A \wedge b \\ &= A \wedge \neg b \end{aligned}$$

$$\begin{aligned} p &= A \oplus A \vee b \\ &= \neg A \wedge b \end{aligned}$$

with fewer symbols ...

$$\begin{aligned} p &= A \text{ xor } (A \text{ and } b) \\ &= A \text{ and } !b \end{aligned}$$

$$\begin{aligned} p &= A \text{ xor } (A \text{ or } b) \\ &= !A \text{ and } b \end{aligned}$$

Repeated Variables

- The definitions in this chapter yield the same tests no matter how the predicate is expressed
- $(a \vee b) \wedge (c \vee b) == (a \wedge c) \vee b$
- $(a \wedge b) \vee (b \wedge c) \vee (a \wedge c)$
 - Only has 8 possible tests, not 64
- Use the simplest form of the predicate, and ignore contradictory truth table assignments

A More Subtle Example

$$\underline{p = (a \wedge b) \vee (a \wedge \neg b)}$$

$$\begin{aligned} p_a &= p_{a=\text{true}} \oplus p_{a=\text{false}} \\ &= ((\text{true} \wedge b) \vee (\text{true} \wedge \neg b)) \oplus ((\text{false} \wedge b) \vee (\text{false} \wedge \neg b)) \\ &= (b \vee \neg b) \oplus \text{false} \\ &= \text{true} \oplus \text{false} \\ &= \text{true} \end{aligned}$$

$$\underline{p = (a \wedge b) \vee (a \wedge \neg b)}$$

$$\begin{aligned} p_b &= p_{b=\text{true}} \oplus p_{b=\text{false}} \\ &= ((a \wedge \text{true}) \vee (a \wedge \neg \text{true})) \oplus ((a \wedge \text{false}) \vee (a \wedge \neg \text{false})) \\ &= (a \vee \text{false}) \oplus (\text{false} \vee a) \\ &= a \oplus a \\ &= \text{false} \end{aligned}$$

- **a** always determines the value of this predicate
- **b** never determines the value – **b** is **irrelevant** !

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler











• Example

*b & c
different
to determine
value of p*

*A pair of
rows also
determine
value of p*

*For a and
b, only one
pair of rows
determine
value of p*

Likewise, for clause c, only one pair, TFT and TFF, cause c to determine the value of p

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

In sum, three separate pairs of rows can cause a to determine the value of p, and only one pair each for b and c

Logic Coverage Summary

- Predicates are often **very simple**—in practice, most have less than 3 clauses
 - In fact, most predicates only have one clause !
 - With only clause, PC is enough
 - With 2 or 3 clauses, CoC is practical
 - Advantages of ACC and ICC criteria significant for large predicates
 - CoC is impractical for predicates with many clauses
- **Control software** often has many complicated predicates, with lots of clauses