موضوع:

$\boxed{\text{سوال ۶}}$ ب

$x[n] = \left(\frac{1}{3}\right)^n u[n+2]$

$x[n] = 9\left(\frac{1}{3}\right)^{n+2} u[n+2]$

$F\{x[n]\} = 9 \ldots$

$F\left\{\left(\frac{1}{3}\right)^n u[n]\right\} = \dfrac{1}{1-\frac{1}{3}e^{-j\omega}} \Rightarrow$

$F\left\{\left(\frac{1}{3}\right)^{n+2} u[n+2]\right\} = \dfrac{1}{1-\frac{1}{3}e^{-j\omega}} \cdot e^{2j\omega} = \dfrac{e^{2j\omega}}{1-\frac{1}{3}e^{-j\omega}}$

۲واحد به چپ شیفت دهیم

طبق خاصیت ... $x[n-n_0] \xleftrightarrow{F} e^{-j\omega n_0} X(e^{j\omega})$

$x[n+2] \longleftrightarrow e^{2j\omega} X(e^{j\omega})$

$F\{x[n]\} = \dfrac{9e^{2j\omega}}{1-\frac{1}{3}e^{-j\omega}}$

جواب

ادامه‌ی ۶ ب

$$X(e^{jw}) = \frac{9e^{2jw}}{1-\frac{1}{3}e^{-jw}} = \frac{9(\cos(2w)+j\sin(2w))}{1-\frac{1}{3}(\cos(w)-j\sin(w))}$$

$$|X(e^{jw})| = \frac{9}{\sqrt{(1-\frac{1}{3}\cos w)^2+(\frac{1}{3}\sin w)^2}} = \boxed{\frac{9}{\sqrt{\frac{10}{9}-\frac{2}{3}\cos w}}}$$

$$\measuredangle X(e^{jw}) =$$

$$X(e^{jw}) = \frac{(1-\frac{1}{3}e^{+jw})(9e^{2jw})}{\frac{10}{9}-\frac{2}{3}\cos w}$$

$$= \frac{9e^{2jw}-3e^{3jw}}{\frac{10}{9}-\frac{2}{3}\cos w} \Rightarrow$$

$$\frac{9\cos(2w)+9j\sin(2w)-3\cos(3w)-3j\sin(3w)}{\frac{10}{9}-\frac{2}{3}\cos w}$$

ادامه ۶ ب

فاز

$$\measuredangle X(e^{jw}) = \tan^{-1}\frac{\frac{9\cos(2w)-3\cos(3w)}{\frac{10}{9}-\frac{2}{3}\cos w}}{\frac{9\sin(2w)-3\sin(3w)}{\frac{10}{9}-\frac{2}{3}\cos w}} = \boxed{\tan^{-1}\frac{9\cos(2w)-3\cos(3w)}{9\sin(2w)-3\sin(3w)}}$$

$x[n] = (\frac{1}{2})^{|n|}$ $\quad X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} (\frac{1}{2})^{|n|} e^{-j\omega n}$ (2.

$$= \sum_{n=0}^{\infty} a^{n} e^{-j\omega n} + \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n} = \frac{1}{1-ae^{-j\omega}} + \frac{ae^{j\omega}}{1-ae^{j\omega}}$$

$a=\frac{1}{2} \rightarrow$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{\frac{1}{2}e^{j\omega}}{1-\frac{1}{2}e^{j\omega}} = \frac{\cancel{1-\frac{1}{4}}\ \frac{3}{4}}{1-\cos\omega+\frac{1}{4}} =$$

$\frac{3}{4} \cdot \frac{1}{1-\cos\omega+\frac{1}{4}}$ = جواب $\quad |X(e^{j\omega})| = \frac{3}{4} \cdot \frac{1}{\sqrt{(\frac{5}{4}-\cos\omega)^2 \dots}}$

$$\frac{3}{4} \cdot \frac{1}{\frac{5}{4}-\cos\omega} \rightarrow |X(e^{j\omega})| = \frac{3}{4} \cdot \frac{1}{\sqrt{\cos^2\omega - \frac{5}{2}\cos\omega + \frac{25}{16}}}$$

اصلاحی ① (2.

$$|X(e^{j\omega})| = \frac{3}{4} \cdot \frac{1}{\sqrt{\cos^2\omega - \frac{5}{2}\cos\omega + \frac{25}{16}}}$$

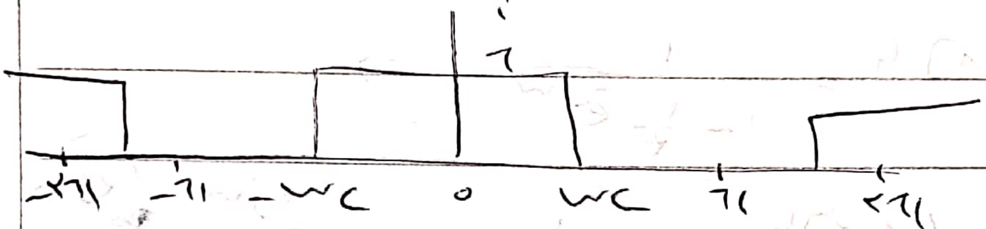
$\angle X(e^{j\omega}) = 0$. تبدیل حقیقی است

فاز ندارد

د)

$$x[n] = \frac{\sin\left(\frac{n\pi}{5}\right)\cos\left(\frac{7\pi n}{2}\right)}{n\pi}$$

می‌دانیم تبدیل فوریه $\frac{\sin\left(\frac{n\pi}{5}\right)}{n\pi}$ یک پالس متناوب است با ارتفاع ۱ و پهنای $\frac{2\pi}{5}$



$$x_1[n] = \frac{\sin \omega_c n}{\pi n} \longrightarrow \omega_c = \frac{\pi}{5} \rightarrow x_1[n] = \frac{\sin\frac{\pi n}{5}}{\pi n}$$

$$\cos\left(\frac{7\pi n}{2}\right) = \frac{1}{2}\left(e^{j\frac{7\pi n}{2}} + e^{-j\frac{7\pi n}{2}}\right) \rightarrow$$

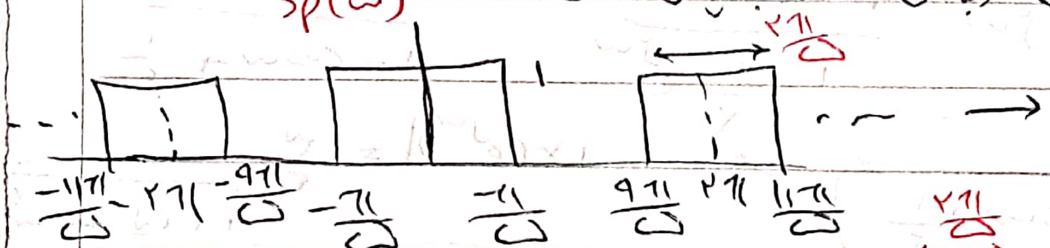
$$F\{x[n]\} = F\left\{\frac{1}{2}e^{j\frac{7\pi n}{2}}\left(\frac{\sin\left(\frac{n\pi}{5}\right)}{n\pi}\right)\right\} +$$

$$F\left\{\frac{1}{2}e^{-j\frac{7\pi n}{2}}\left(\frac{\sin\left(\frac{n\pi}{5}\right)}{n\pi}\right)\right\} \Rightarrow$$

طبق خاصیت شیفت در حوزه فرکانس $e^{j\omega_0 n}x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$

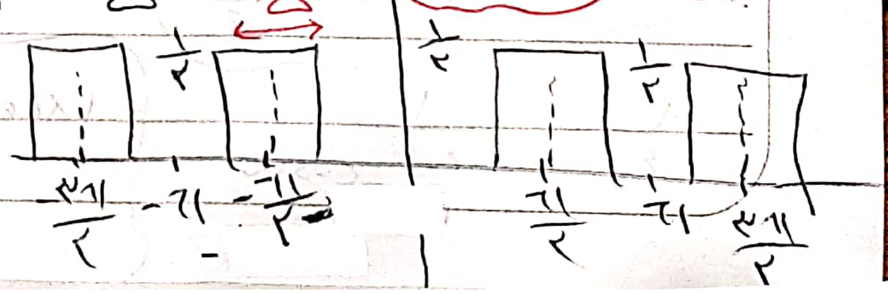
$$F\{x[n]\} = \frac{1}{2}sp\left(\omega - \frac{7\pi}{2}\right) + \frac{1}{2}sp\left(\omega + \frac{7\pi}{2}\right)$$

که $sp(\omega)$ همان دنباله‌ی متناوب پالس‌ها است



$X(e^{j\omega})$ جواب

جمع شیفت یافته‌های sp



$\measuredangle X(e^{j\omega}) = 0$ مثل الف

$|X(e^{j\omega})| = X(e^{j\omega})$

۵)
$$x[n] = \left(\frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi}\right) * \left(\frac{\sin\left(\frac{(n-8)\pi}{4}\right)}{(n-8)\pi}\right)$$

طبق خاصیت convolution در حوزه زمان

$$x[n] * y[n] \longleftrightarrow X(e^{j\omega})\,Y(e^{j\omega})$$

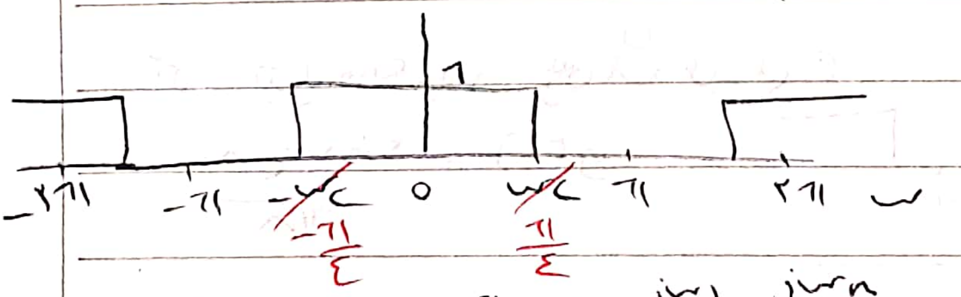
$$\to \mathcal{F}\{x[n]\} = \mathcal{F}\Big\{\underbrace{\frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi}}_{x_1[n]}\Big\} \cdot \mathcal{F}\Big\{\underbrace{\frac{\sin\left(\frac{(n-8)\pi}{4}\right)}{(n-8)\pi}}_{x_1[n-8]}\Big\}$$

اگر $x_1[n] = \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi} \to$

$$x_2[n] = x_1[n-8] = \frac{\sin\left(\frac{\pi(n-8)}{4}\right)}{\pi(n-8)}$$

پس تبدیل فوریه‌ی $x[n]$ حاصل‌ضرب تبدیل فوریه‌ی $x_1[n]$ و

تبدیل فوریه‌ی شیفت یافته‌ی آن است $\leftarrow$ $G(e^{j\omega})$



$\omega_c = \frac{\pi}{4}$

$$g(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{j\omega n}\, d\omega =$$

پس تبدیل فوریه‌ی $x_1[n]$ دنباله‌ای از پالس‌ها به $\frac{\sin\frac{\pi}{4}n}{\pi n}$

طول $\frac{2\pi}{4}$ و ارتفاع ۱ است $\leftarrow$

طبق خاصیت شیفت در حوزه زمان

$$x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$$

$$x_1[n-8] \longleftrightarrow e^{-8j\omega} X(e^{j\omega})$$

ادامه‌ی سوال ۵ (ت)

موضوع:

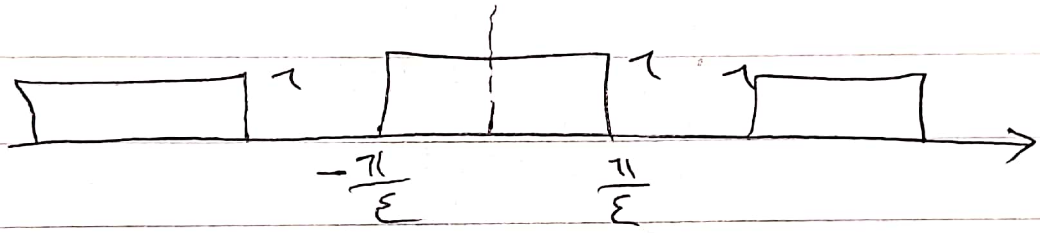
تاریخ: / /

$$X(e^{j\omega}) = e^{-8j\omega} \underbrace{X_1^2(e^{j\omega})}_{\text{توان دوم تبدیل فوریه پالس}} = \boxed{e^{-8j\omega} X_1(e^{j\omega})}$$

$$|X(e^{j\omega})| = |e^{-8j\omega} X_1(e^{j\omega})| = |X_1(e^{j\omega})|$$

$|X(e^{j\omega})|$

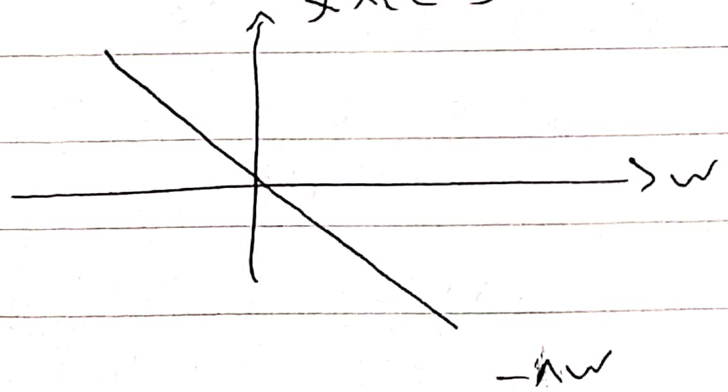
اندازه



$\angle X(e^{j\omega}) = -8\omega$ $\qquad$ $\angle X_1(e^{j\omega}) = 0$

پس سیگنال حقیقی الف ← فاز ندارد ولی وقتی در $e^{-8j\omega}$ ضرب شود ←

فازش به اندازه‌ی -8ω می‌شود ← $\angle X(e^{j\omega})$



سوال ۲۷ — الف

$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\angle X(e^{j\omega})}$

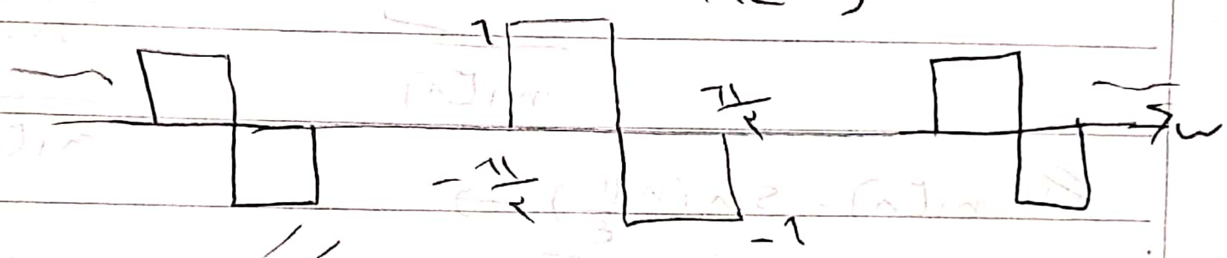
حاصل ضرب اندازه و فاز سیگنال

$X(e^{j\omega}) = 6 \cdot e^{j\times 0} = 6 \rightarrow$ در بازه $(-\frac{\pi}{2} \text{ تا } 0)$

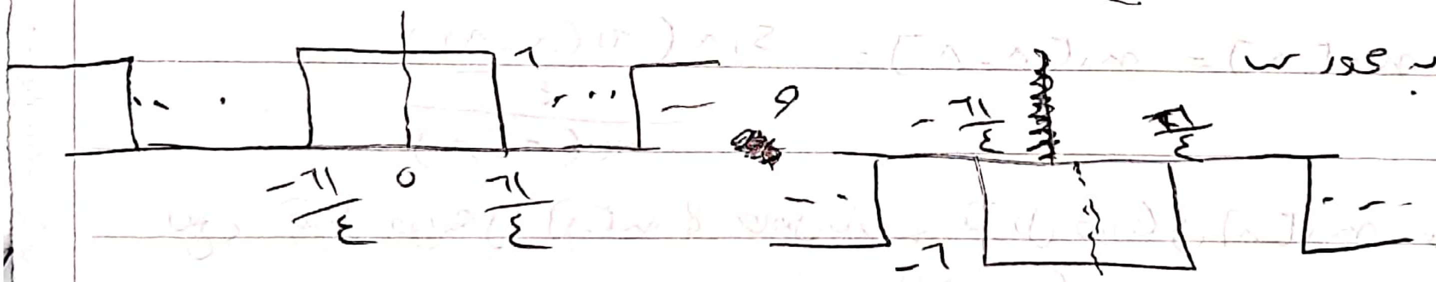
$X(e^{j\omega}) = 6 \cdot e^{j-\pi} = \rightarrow (0 \text{ تا } \frac{\pi}{2})$

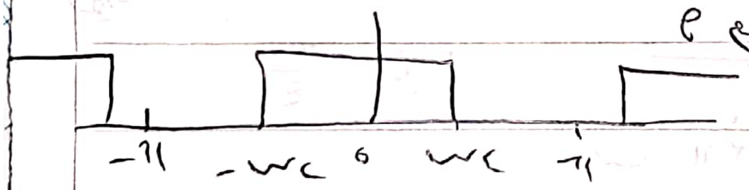
$e^{-j\pi} = \cos(-\pi) + j\cancel{\sin(-\pi)} = -6$

$X(e^{j\omega}) = -6$



$X(e^{j\omega})$ حاصل G از جمع ۲ پالس متفاوت به رنگ جدا که یکی از آن دو G وارون قرینه نسبت به محور ω





اگر تبدیل فوریه به صورت پالس باشد می‌دانیم

$g[n] = \frac{\sin \omega_c n}{\pi n}$

$\boxed{g[n] = \frac{\sin(\frac{\pi}{4} n)}{\pi n}}$ ← اگر $\omega_c = \frac{\pi}{4}$ باشد

$X(e^{j\omega}) = G(e^{j(\omega + \frac{\pi}{4})}) - G(e^{j(\omega - \frac{\pi}{4})})$

طبق خاصیت شیفت در حوزه فرکانس $e^{j\omega_0 n} x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$

اینجا اگر $\omega_0 = -\frac{\pi}{4}$ ← $\mathcal{F}^{-1}\{G(e^{j(\omega+\frac{\pi}{4})})\} = \frac{e^{-j\frac{\pi}{4}n} \sin(n\frac{\pi}{4})}{n\pi}$

سوال ۲ ـ (الف)

$\mathcal{F}\{G(e^{j(\omega-\frac{\pi}{4})})\} =$ ← اگر $\omega_c = \frac{\pi}{4}$

$$e^{j\frac{\pi}{4}n}\,\frac{\sin(\frac{n\pi}{4})}{n\pi}$$

پس $\mathcal{F}\{x[n]\} = e^{-j\frac{\pi}{4}n}\,\frac{\sin(\frac{n\pi}{4})}{n\pi}$

$$- e^{j\frac{\pi}{4}n}\,\frac{\sin(\frac{n\pi}{4})}{n\pi} =$$

$$-\underbrace{(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n})}_{-2j\sin(\frac{n\pi}{4})}\cdot\frac{\sin(\frac{n\pi}{4})}{n\pi} = \frac{-2j\sin(\frac{n\pi}{4})^2}{n\pi}$$

جواب

(ب)

$$X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

$$x[n] = \mathcal{F}^{-1}\{X(e^{j\omega})\} = \mathcal{F}^{-1}\left\{\frac{1-\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{4}e^{-j\omega})}\right\}$$

$$X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{4}e^{-j\omega})} = \frac{A}{(1-\frac{1}{2}e^{-j\omega})} + \frac{B}{(1+\frac{1}{4}e^{-j\omega})}$$

$$A = (1-\frac{1}{2}e^{-j\omega})X(e^{j\omega})\Big|_{e^{-j\omega}=2} \quad \frac{1-\cancel{\frac{2}{3}}\ \frac{1}{3}}{\cancel{1+\frac{1}{2}}\ \frac{3}{2}} = \frac{1}{3}\times\frac{2}{3} = \frac{2}{9}$$

$$B = (1+\frac{1}{4}e^{-j\omega})X(e^{j\omega})\Big|_{e^{-j\omega}=-4} \quad \frac{1+\frac{4}{3}}{\cancel{1+2}\ 3} = \frac{7}{9}$$

$$\rightarrow X(e^{j\omega}) = \frac{\frac{2}{9}}{(1-\frac{1}{2}e^{-j\omega})} + \frac{\frac{7}{9}}{1+\frac{1}{4}e^{-j\omega}}$$

Scanned by CamScanner

ادامه‌ی سوال ۲ ـ ب

$$f^{-1}\left\{ \frac{2}{9} \cdot \frac{1}{1-\frac{1}{2} e^{-jw}} \right\} +$$

$$f^{-1}\left\{ \frac{7}{9} \cdot \frac{1}{1+\frac{1}{4} e^{-jw}} \right\} = \frac{2}{9}\left(\frac{1}{2}\right)^{n} u[n] + \frac{7}{9}\left(-\frac{1}{4}\right)^{n} u[n]$$

جواب

(2.

$$X(e^{jw}) = \cos^2(w) + \sin^2(3w)$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})\, e^{jwn}\, dw =$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\cos^2 w + \sin^2(3w)\right) e^{jwn}\, dw =$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\frac{1+\cos 2w}{2} + \frac{1-\cos 6w}{2}\right) e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jwn} + \frac{1}{8\pi}\int_{-\pi}^{\pi} \left(e^{j2w} + e^{-j2w}\right) e^{jwn}\, dw$$

۲ واحد به راست ‌ ۲ واحد به چپ ‌ شیفت

$$-\frac{1}{8\pi}\int_{-\pi}^{\pi} \left(e^{j6w} + e^{-j6w}\right) e^{jwn}\, dw$$

۶ واحد به چپ

۶ واحد به راست

طبق خاصیت خطی بودن انتگرال و رابطه‌ی اثبات شده

$$\frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn}\, dw = \frac{\sin w_c n}{\pi n}$$

طبق خاصیت شیفت در زمان

$$x[n-n_0] \longleftrightarrow e^{-jwn_0} X(e^{jw})$$

$$\frac{\sin \pi n}{\pi n} + \frac{\sin(n+2)\pi}{4\pi(n+2)} + \frac{\sin(n-2)\pi}{4\pi(n-2)}$$

$$- \frac{\sin(n+6)\pi}{4\pi(n+6)} - \frac{\sin(n-6)\pi}{4\pi(n-6)}$$

جواب 6

موضوع: تاریخ: / /

$$X(e^{j\omega}) = \sum_{K=-\infty}^{+\infty} (-1)^K \delta(\omega - \frac{K\pi}{2})$$ ادامه سوال ۲ ⑦

$$x[n] = F^{-1}\{X(e^{j\omega})\} = F^{-1}\{\sum_{K=-\infty}^{+\infty} (e^{j\pi})^K \delta(\omega - \frac{\pi}{2}K)\}$$

$\omega - \frac{\pi}{2}K = 0 \rightarrow$ به جای K، $\frac{2\omega}{\pi}$ می‌گذاریم $\leftarrow K = \frac{2\omega}{\pi}$

$$= F^{-1}\{\sum_{K=-\infty}^{+\infty} e^{j2\omega} \delta(\omega - \frac{\pi}{2}K)\} =$$

$$F^{-1}\{e^{j2\omega} (\sum_{K=-\infty}^{+\infty} \delta(\omega - \frac{\pi}{2}K))\} =$$

طبق رابطه‌ی convolution

$$x[n] * y[n] \longleftrightarrow X(e^{j\omega}) Y(e^{j\omega})$$

$$\sum_{K=-\infty}^{+\infty} \delta[n - KN] \overset{F}{\longleftrightarrow} \frac{2\pi}{N} \sum_{K=-\infty}^{+\infty} \delta(\omega - \frac{2\pi K}{N})$$ و می‌دانیم

$\leftarrow N = 4$

$$\frac{2}{\pi} F^{-1}\{e^{j2\omega}\} * F^{-1}\{\frac{2}{\pi} \sum_{K=-\infty}^{+\infty} \delta(\omega - \frac{\pi}{2}K)\} =$$

$$\frac{2}{\pi}\delta[n+2] * \sum_{K=-\infty}^{+\infty} \delta[n - 4K] = \frac{2}{\pi} \sum_{K=-\infty}^{+\infty} \delta[n+2-4K]$$

جواب

سوال ۳

$$y[n] = x^*[-n+1]$$

$$F\{x[n+1]\} = e^{j\omega} X(e^{j\omega}) \rightarrow F\{x[-n+1]\} = e^{-j\omega} X(e^{-j\omega})$$

$$F\{x^*[-n+1]\} = e^{j\omega} X^*(e^{j\omega})$$

جواب : $\boxed{e^{j\omega} X^*(e^{j\omega})}$

موضوع: ____ تاریخ: / /

سوال ۳ (ب)

$$y[n] = \frac{x[n] + x^*[-n]}{2}$$

$x^*[n] \xleftrightarrow{F} X^*(e^{-j\omega}) \longrightarrow F\{x^*[-n]\} = X^*(e^{j\omega})$

و $x[-n] \xleftrightarrow{F} X(e^{-j\omega})$

$$Y(e^{j\omega}) = \boxed{\frac{1}{2}\left(X(e^{j\omega}) + X^*(e^{j\omega})\right)}$$

جواب

ج)

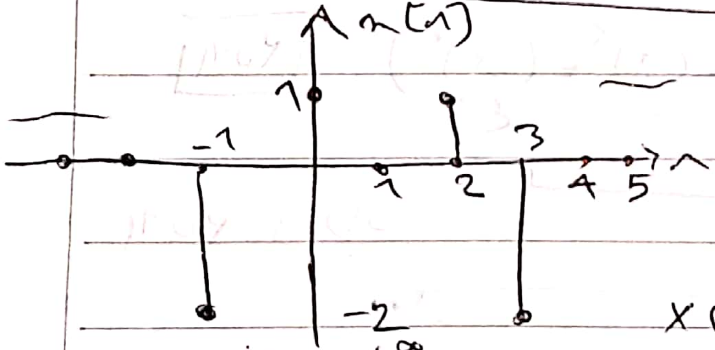
$$y[n] = x\left[-\frac{n}{2}\right]$$

$$x_{(2)}[n] = \begin{cases} x[\frac{n}{2}] & n = 2k \\ 0 & n \neq 2k \end{cases} \xleftrightarrow{F} X(e^{2j\omega}) \longrightarrow$$

و $x[-n] \xleftrightarrow{F} X(e^{-j\omega})$

$F\{x[-\frac{n}{2}]\} = \boxed{X(e^{-2j\omega})}$ جواب

سوال ۴



الف) $X(e^{j0}) = ?$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n} \Rightarrow$$

$$X(e^{j0}) = \sum_{n=-\infty}^{+\infty} x[n] = -2+1+1-2 = -2$$

جواب الف

$\boxed{e^{-j\omega} \quad \pm\pi}$

جواب ب) $\measuredangle X(e^{j\omega})$ ؟ $x[n]$ را می‌توان سیگنال حقیقی و زوجی مثل
$\hat{x}[n]$ دانست که به اندازه‌ی ۱ واحد به راست منتقل شده ← چون تبدیل فوریه
$\hat{X}(e^{j\omega})$ حقیقی و زوج است ← غیر از فاز اصلی $\pm\pi$ که آن هم به دلیل
$\hat{X}(e^{j\omega})$ منفی، فاز دیگری ندارد ← فاز به خاطر عمل انتقال بوده

$$x(e^{j\omega}) = F\{\hat{x}[n-1]\} = e^{-j\omega}\hat{X}(e^{j\omega}) \Rightarrow \measuredangle X(e^{j\omega}) = \left(e^{-j\omega} \pm\pi\right)$$

ج.

$$\int_{-\pi}^{\pi} X(e^{jw})\, e^{jw}\, dw$$

چون $x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw})\, e^{jwn}\, dw$

اگر $\boxed{n=1}$ باشد $\int_{-\pi}^{\pi} X(e^{jw})\, e^{jw}\, dw = 2\pi x[1]$

$\longrightarrow 2\pi x[1]$ جواب ج

د)

$$\int_{-\pi}^{\pi} |X(e^{jw})|^2\, dw$$

طبق رابطه پارسوال e

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{jw})|^2\, dw$$

پس $\int_{-\pi}^{\pi} |X(e^{jw})|^2\, dw = 2\pi \sum_{n=-\infty}^{+\infty} |x[n]|^2$

$$= 2\pi \underbrace{\left(\underset{4}{\cancel{(-2)^2}} + (1)^2 + (1)^2 + \underset{4}{\cancel{(-2)^2}} \right)}_{10} = \boxed{20\pi}$$

20π د جواب

ه)

$$\int_{-\pi}^{\pi} X^2(e^{jw})\, dw$$

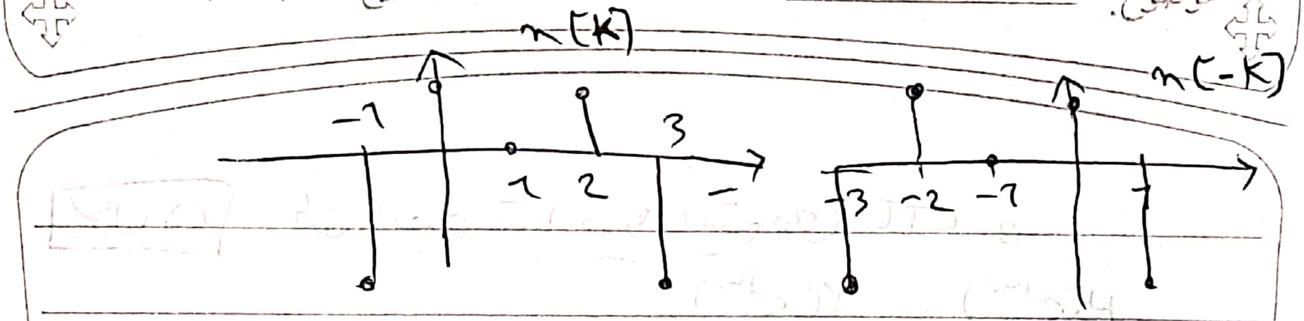
طبق خاصیت convolution

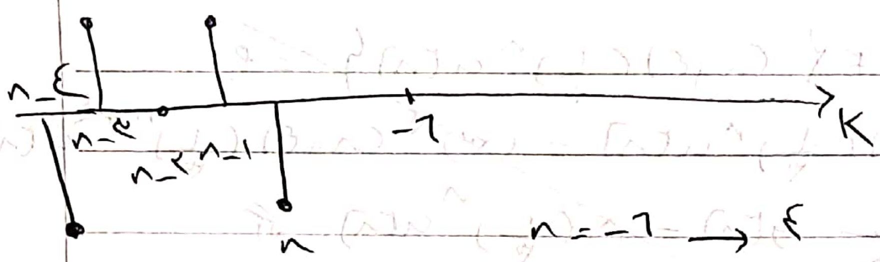
$$F\{x[n] * x[n]\} = X^2(e^{jw})$$

پس اگر $x[n]$ را با خودش کانوالو کنیم e

$$y[n] = x[n] * x[n] \rightarrow y[n] = x[0]x[n] + x[1]x[n-1]$$

موضوع: تاریخ: / /





$n=-1 \rightarrow 4$

$n=0 \rightarrow -2+-2=-4$

$n=1 \rightarrow 1$

$n=2 \rightarrow -2-2=-4$

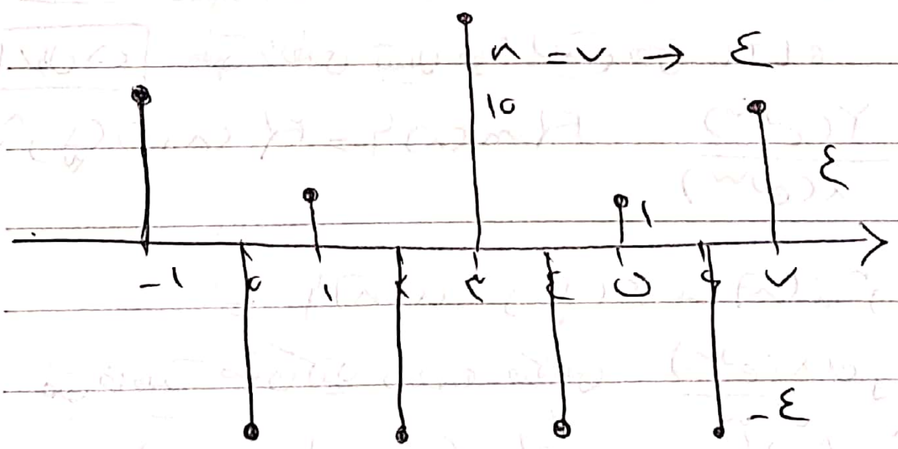
$n=3 \rightarrow 4+1+1+4=10$

$n=4 \rightarrow -2-2=-4$

$n=5 \rightarrow 1$

$n=6 \rightarrow -2-2=-4$

$n=7 \rightarrow 4$



$x[n]*x[n] = h[n]$

$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})\, e^{jwn}\, dw$ طبق رابطه

اگر $n=0$

$h[n] = \frac{1}{2\pi}\int_{2\pi} \underbrace{X^2(e^{jw})}_{G(e^{jw})}\, dw \rightarrow 2\pi h[0] = \int_{2\pi} G(e^{jw})\, dw$

-8π

جواب

$\boxed{-8\pi} = 2\pi h[0] \leftarrow$ الذ $x[n]*x[n]$ همان $h[n]$

$\boxed{\text{سوال ث}}$ طبق رابطه ی تبدیل فوریه سیستم های LTI؛

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \qquad F\{x[n]\} = F\{(n+4)(\tfrac{1}{3})^n u[n]\}$$

$$= F\{n(\tfrac{1}{3})^n u[n] + 4(\tfrac{1}{3})^n u[n]\} =$$

$$F\{n\,x[n]\} = j\frac{dX(e^{j\omega})}{d\omega}$$ طبق خاصیت مشتق گیری در حوزه فرکانس

پس $$F\{n(\tfrac{1}{3})^n u[n]\} = j\frac{d}{d\omega}\left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)$$

$$= j\left(\frac{-\frac{1}{3}je^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})^2}\right) = \frac{\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})^2}$$

$$F\{4(\tfrac{1}{3})^n u[n]\} = 4 \cdot \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

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ادامه‌ی سوال د

$$X(e^{j\omega}) = \frac{e^{-j\omega}}{3(1-\frac{1}{3}e^{-j\omega})^2} + \frac{1}{4(1-\frac{1}{3}e^{-j\omega})}$$

$$y[n] = (\frac{1}{4})^n u[n] \rightarrow Y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$\rightarrow H(e^{j\omega}) = \frac{\frac{1}{1-\frac{1}{4}e^{-j\omega}}}{X(e^{j\omega})}$$

$$X(e^{j\omega}) = \frac{4e^{-j\omega} + 3(1-\frac{1}{3}e^{-j\omega})}{12(1-\frac{1}{3}e^{-j\omega})^2} = \frac{\cancel{3}(1+e^{-j\omega})}{\cancel{12}(1-\frac{1}{3}e^{-j\omega})^2} \qquad \frac{\cancel{3 + 3e^{-j\omega}}}{4}$$

$$= \frac{1+e^{-j\omega}}{4(1-\frac{1}{3}e^{-j\omega})^2}$$

$$\rightarrow H(e^{j\omega}) = \frac{4(1-\frac{1}{3}e^{-j\omega})^2}{(1+e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})} =$$

ادامه‌ی سوال (۵) (الف)

$$\frac{4\left(1-\frac{1}{3}e^{-j\omega}\right)^2}{\left(1+e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)} \xrightarrow{\text{اگر } e^{-j\omega}=z} \frac{4\left(1+\frac{1}{9}z^2-\frac{2}{3}z\right)}{1-\frac{1}{4}z+z-\frac{1}{4}z^2} =$$

$$\frac{3}{4}z$$

$$\frac{\frac{4}{9}\left(z^2-6z+9\right)}{-\frac{1}{4}\left(z^2-3z-4\right)} = \frac{-16}{9}\,\frac{z^2-3z-4-3z+13}{z^2-3z-4} =$$

$$(z-4)(z+1)$$

$$\frac{-16}{9}\left(1+\left(\frac{-3z+13}{z^2-3z-4}\right)\right) \to \frac{-3z+13}{z^2-3z-4} = \frac{A}{z-4}+\frac{B}{z+1}$$

$$A = (z-4)\left(\frac{-3z+13}{z^2-3z-4}\right)\Big|_{z=4} = \frac{1}{5}$$

$$B = \Big|_{z=-1} = \frac{16}{-5} = \boxed{\frac{-16}{5}}$$

$$H(e^{j\omega}) = \frac{-16}{9} - \frac{16}{9}\left(\frac{1}{5}\cdot\frac{1}{z-4} - \frac{16}{5}\cdot\frac{1}{z+1}\right) =$$

$$-\frac{16}{9} - \frac{16}{45}\cdot\frac{1}{e^{-j\omega}-4} + \frac{256}{45}\cdot\frac{1}{\cancel{z}+1}$$

$$-\frac{16}{9} + \frac{64}{45}\cdot\frac{1}{1-\frac{e^{-j\omega}}{4}} + \frac{256}{45}\cdot\frac{1}{1+e^{-j\omega}} =$$

$$h[n] = -\frac{16}{9}\delta[n] + \frac{64}{45}\left(\frac{1}{4}\right)^n u[n] + \frac{256}{45}(-1)^n u[n]$$

جواب الف

اگر $z=e^{-j\omega}$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \to \frac{-16}{9}\cdot\frac{z^2-6z+9}{z^2-3z-4} = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$-16e^{-2j\omega}X(e^{j\omega}) + 96e^{-j\omega}X(e^{j\omega}) - 144X(e^{j\omega}) =$$

$$9e^{-2j\omega}Y(e^{j\omega}) - 27e^{-j\omega}Y(e^{j\omega}) - 36Y(e^{j\omega}) \to$$

از دو طرف تبدیل فوریه وارون بگیریم

$$-16x[n-2] + 96x[n-1] - 144x[n] = 9y[n-2] - 27y[n-1] - 36y[n]$$

جواب ب

ARSH

$y[n] = \delta[n] (-\frac{1}{3})^n u[n]$ ج. ورودی ؟

$Y(e^{j\omega}) = 1 - \frac{1}{1+\frac{1}{3}e^{-j\omega}}$ $H(e^{j\omega}) =$

$H(e^{j\omega})X(e^{j\omega}) = Y(e^{j\omega}) \rightarrow \frac{4(1-\frac{1}{3}e^{-j\omega})^2}{(1+e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$

$\rightarrow X(e^{j\omega}) = \frac{\frac{1}{3}e^{-j\omega}}{1+\frac{1}{3}e^{-j\omega}} \cdot \frac{(1+e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}{4(1-\frac{1}{3}e^{-j\omega})^2}$

اگر $e^{-j\omega} = s$

$X(s) = \frac{\cancel{\frac{1}{3}}s}{1+\cancel{\frac{1}{3}}s}\left(\frac{-16}{9}\right)\left(\frac{s^2-6s+9}{s^2-3s-4}\right) = \frac{-16}{9}\cdot\frac{s}{s+3}\cdot\frac{s^2-6s+9}{s^2-3s-4}$

$\frac{\frac{3+s}{3}}{} \quad \frac{s}{3+s}$

$= \frac{-16}{9}\,\frac{s^3-6s^2+9s}{s^3-3s^2-4s+3s^2-9s-12}$

$s^3-13s-12$

$= \frac{-16}{9}\left(\frac{s^3-13s-12 \; -6s^2+24s+12}{s^3-13s-12}\right) = \frac{-16}{9}\left(1-\frac{6(s^2-4s-2)}{s^3-13s-12}\right)$

$= \frac{-16}{9}\left(1-\frac{6(s^2-4s-2)}{s^3-13s-12}\right)$, $\frac{s^2-4s-2}{s^3-13s-12} = ?$

$$x(n) = 1 + \cos\left(\frac{2\pi n}{3}\right) = 1 + \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\frac{2\pi n}{3}} \quad (1)$$

$$y(n) = 2 + \sin\left(\frac{2\pi n}{3}\right) = 3 + \frac{1}{2j} e^{j\frac{2\pi n}{3}} - \frac{1}{2j} e^{-j\frac{2\pi n}{3}}$$

$$e^{j\omega_0 n} \longrightarrow H(\omega_0) e^{j\omega_0 n}$$

پس $H(0) = 2 \quad H\left(\frac{2\pi}{3}\right) = \frac{1}{j} \quad H\left(-\frac{2\pi}{3}\right) = \frac{-1}{j}$

$$\rightarrow H(\omega) = 2 \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k) + \frac{1}{j} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) - \frac{1}{j} \sum_{k=-\infty}^{+\infty} \delta\left(\omega + \frac{2\pi}{3} - 2\pi k\right)$$

$$h(n) = \mathcal{F}^{-1}\{H(e^{j\omega})\}$$

پاسخ سیستم به $x(n) = \sum_{m=-\infty}^{+\infty} \delta[n - 3m] \leftarrow$

$$y(n) = \sum_{m=-\infty}^{+\infty} h(n - 3m)$$

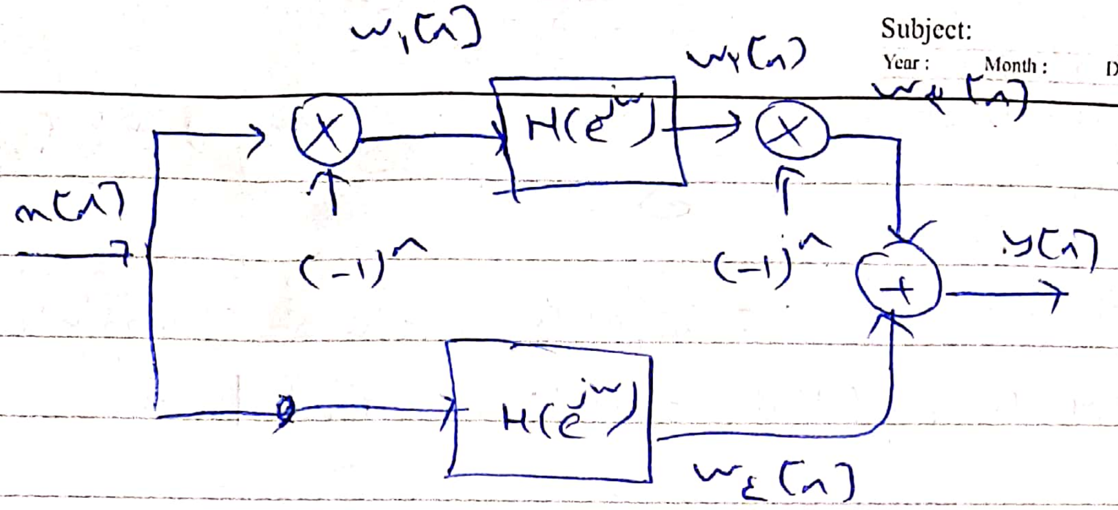
$$h(n) = \frac{1}{\pi} + \frac{1}{j} e^{j\frac{2\pi}{3}n} - \frac{1}{j} e^{-j\frac{2\pi}{3}} = \frac{1}{\pi} + 2\sin\left(\frac{2\pi n}{3}\right)$$

$$h(n) = \frac{1}{\pi} + 2\sin\left(\frac{2\pi n}{3}\right) \rightarrow y(n) = \sum_{m=-\infty}^{+\infty} \frac{1}{\pi} + 2\sin\left(\frac{2\pi}{3}(n - 3m)\right)$$

(1) جواب

Subject:
Year : Month : Day : ( )

روش ٤



$(-1)^n = e^{j\pi n} \rightarrow w_1[n] = e^{j\pi n} x[n] \rightarrow$

$W_1(e^{j\omega}) = X(e^{j(\omega-\pi)})$ ①

② $W_2(e^{j\omega}) = H(e^{j\omega})\, X(e^{j(\omega-\pi)})$ طبق خاصیت Convolution در زمان

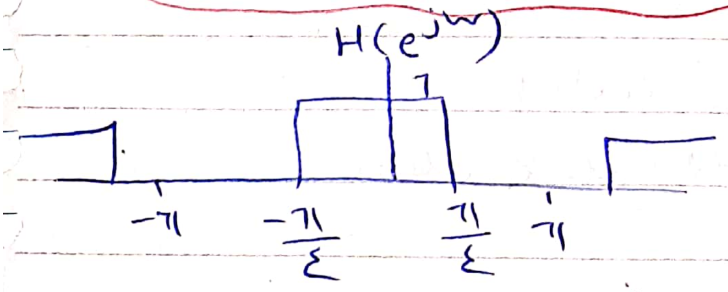
③ $w_3[n] = e^{j\pi n} w_2[n] \Rightarrow W_3(e^{j\omega}) = W_2(e^{j(\omega-\pi)})$

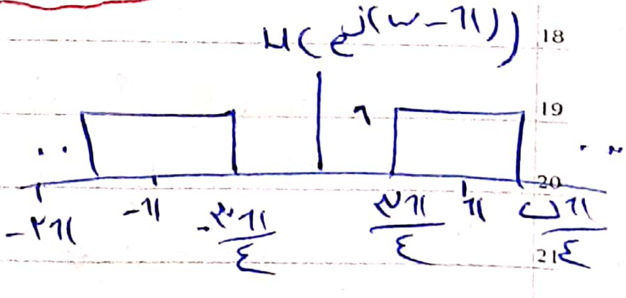
$\rightarrow W_3(e^{j\omega}) = H(e^{j(\omega-\pi)})\, X(e^{j(\omega-2\pi)}) =$
$H(e^{j(\omega-\pi)})\, X(e^{j\omega})$

④ $W_4(e^{j\omega}) = H(e^{j\omega}) \, . \, X(e^{j\omega})$

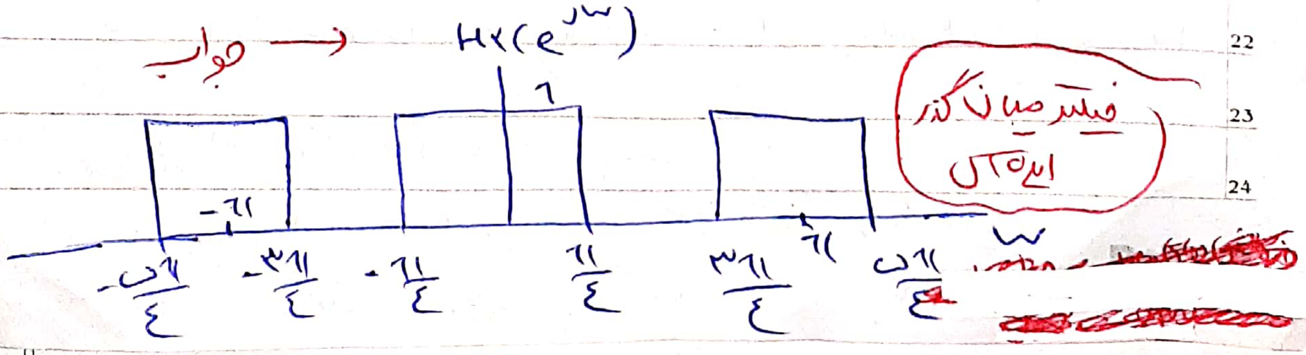
⑤ $Y(e^{j\omega}) = W_3(e^{j\omega}) + W_4(e^{j\omega}) =$
$X(e^{j\omega})[H(e^{j\omega}) + H(e^{j(\omega-\pi)})]$

$\rightarrow H_p(e^{j\omega}) = H(e^{j\omega}) + H(e^{j(\omega-\pi)})$





جواب →



فیلتر میان گذر ایده‌آل