بسم اللّه الرّحمن الرّحیم

دانشگاه صنعتی اصفهان ــ دانشکدهٔ مهندسی برق و کامپیوتر
(نیم‌سال تحصیلی ۴۰۰۱)

نظریهٔ زبان‌ها و ماشین‌ها

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اهمّ مطالبی که در این جلسه بیان خواهند شد:

- ☞ تعریف مفهوم تابع انتقال تعمیم‌یافته
- ☞ تعریف رسمی مفهوم پذیرش برای یک *NFA*
- ☞ اثبات قضیهٔ هم‌ارزی *NFA*ها و *DFA*ها
- ☞ بیان مثال‌ها
- ☞ بازگشت به مبحث خواص بستاری زبان‌های منظم و اثبات بسته بودن کلاس زبان‌های منظم تحت عملگرهای $\cup$، $\circ$، و $*$ با بهره‌گیری از مفهوم عدم‌قطعیت

The Extended Transition Function δ^* for a DFA

It is convenient to introduce the extended transition function $\delta^* : Q \times \Sigma^* \mapsto Q$. The second argument of δ^* is a string, rather than a single symbol, and its value gives the state the automaton will be in after reading that string.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. We define the extended transition function $\delta^* : Q \times \Sigma^* \mapsto Q$ as follows:

1. For every $q \in Q$, $\delta^*(q, \varepsilon) = q$

2. For every $q \in Q$, every $y \in \Sigma^*$, and every $\sigma \in \Sigma$, $\delta^*(q, y\sigma) = \delta(\delta^*(q, y), \sigma)$

Acceptance by a DFA

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA, and let $x \in \Sigma^*$. The string x is accepted by M if $\delta^*(q_0, x) \in F$ and is rejected by M otherwise.

The language accepted by M is the set

$$L(M) = \{x \in \Sigma^* | x \text{ is accepted by } M\}.$$

If L is a language over Σ, L is accepted by M if and only if $L = L(M)$.

The formal definition of computation for an NFA: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and w a string over the alphabet Σ. Then we say that N accepts w if we can write w as $w = y_1y_2 \cdots y_m$, where each y_i is a member of Σ_ε and a sequence of states $r_0, r_1, \cdots, r_m$ exists in Q with three conditions:

1. $r_0 = q_0$,
2. $r_{i+1} \in \delta(r_i, y_{i+1})$, for $i = 0, 1, \ldots, m-1$, and
3. $r_m \in F$.

Condition 1 says that the machine starts out in the start state. Condition 2 says that state r_{i+1} is one of the allowable next states when N is in state r_i and reading y_{i+1}. Observe that $\delta(r_i, y_{i+1})$ is the set of allowable next states and so we say that r_{i+1} is a member of that set. Finally, condition 3 says that the machine accepts its input if the last state is an accept state.

The Extended Transition Function δ^* for an NFA, and the Definition of Acceptance

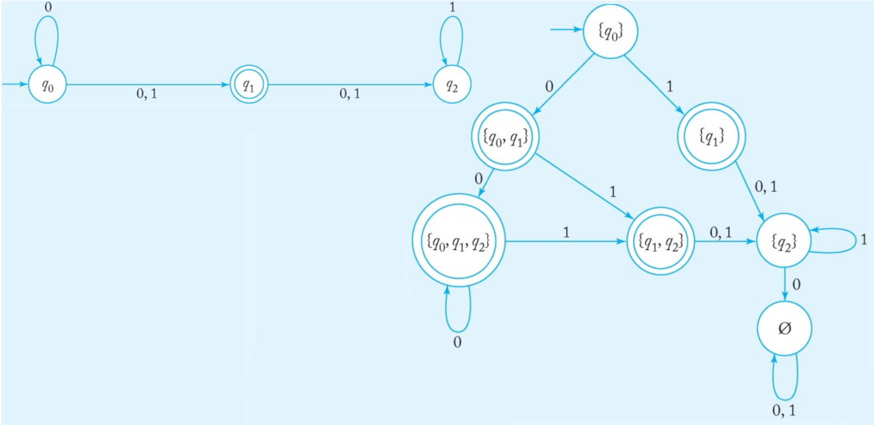
Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA. We define the extended transition function $\delta^* : Q \times \Sigma^* \mapsto 2^Q$ as follows:
1. For every $q \in Q$, $\delta^*(q, \varepsilon) = E(\{q\})$.
2. For every $q \in Q$, every $y \in \Sigma^*$, and every $\sigma \in \Sigma$,

$$\delta^*(q, y\sigma) = E\left(\bigcup \{\delta(p, \sigma) | p \in \delta^*(q, y)\}\right).$$

A string $x \in \Sigma^*$ is accepted by N if $\delta^*(q_0, x) \cap F \neq \varnothing$. The language $L(N)$ accepted by N is the set of all strings accepted by N.

هم تعریف این اسلاید و هم تعریف اسلاید قبل، هردو معتبر هستند.

An NFA and its equivalent DFA: Example 1



Equivalence of DFAs & NFAs

Every NFA has an equivalent DFA. This means that, for every language $A \subseteq \Sigma^*$ accepted by an NFA $N = (Q, \Sigma, \delta, q_0, F)$, there is a DFA $M = (Q', \Sigma, \delta', q_0', F')$ that also accepts A.

Proof: Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing some language A. We construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ recognizing A.

☞ *Before doing the full construction, let's first consider the easier case wherein N has no ε arrows. Later we take the ε arrows into account.*

1. $Q' = \mathcal{P}(Q)$. Every state of M is a set of states of N. Recall that $\mathcal{P}(Q)$ is the set of subsets of Q.

2. For $R \in Q'$ and $a \in \Sigma$, let

$$\delta'(R, a) = \{q \in Q | q \in \delta(r, a) \text{ for some } r \in R\}.$$

If R is a state of M, it is also a set of states of N. When M reads *a symbol a in state R, it shows where a takes each state in R. Because each state may go to a set of states, we take the union of all these sets. Another way to write this expression is*

$$\delta'(R,a) = \bigcup_{r \in R} \delta(r,a).$$

3. $q'_0 = \{q_0\}$. M starts in the state corresponding to the collection containing just the start state of N.

4. $F' = \{R \in Q' | R$ contains an accept state of $N\}$. The machine M accepts if one of the possible states that N could be in at this point is an accept state.

Now we need to consider the ε arrows.

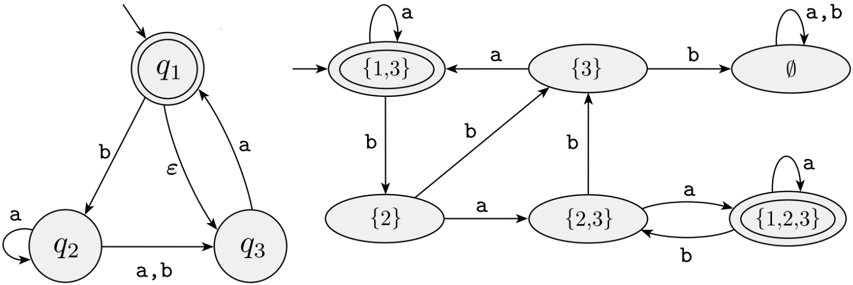
☞ *We modify the transition function of M to place additional fingers on all states that can be reached by going along ε arrows after every step. Replacing $\delta(r, a)$ by $E(\delta(r, a))$ achieves this effect. Thus*

$$\delta'(R, a) = \{q \in Q | q \in E(\delta(r, a)) \text{ for some } r \in R\} = \bigcup_{r \in R} E(\delta(r, a)).$$

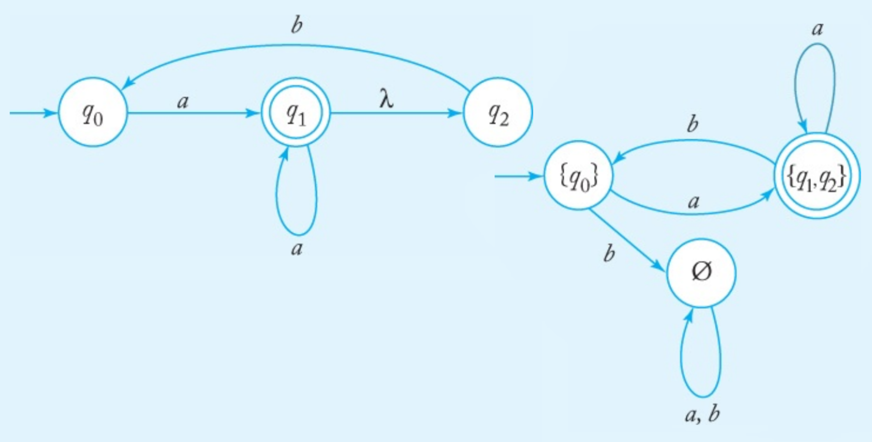
☞ *Additionally, we need to modify the start state of M to move the fingers initially to all possible states that can be reached from the start state of N along the ε arrows. Changing q_0' to be $E(\{q_0\})$ achieves this effect.*

☞ *We have now completed the construction of the DFA M that simulates the NFA N. The construction of M obviously works correctly. At every step in the computation of M on an input, it clearly enters a state that corresponds to the subset of states that N could be in at that point. Thus our proof is complete.*

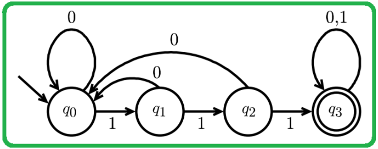
An NFA and its equivalent DFA: Example 2

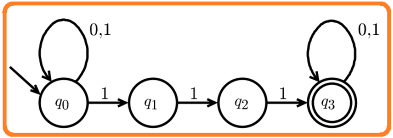


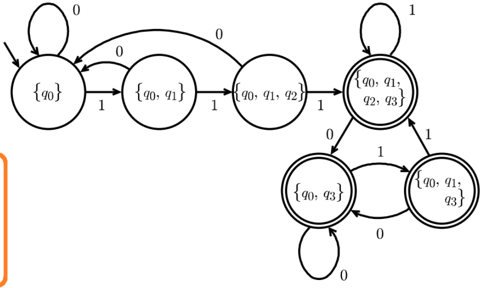
An NFA and its equivalent DFA: Example 3



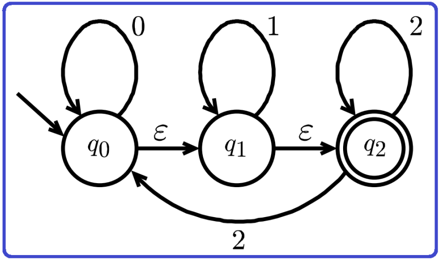
An NFA and its equivalent DFA: Example 4

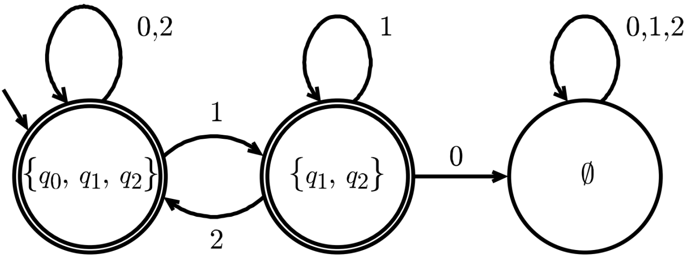






An NFA and its equivalent DFA: Example 5



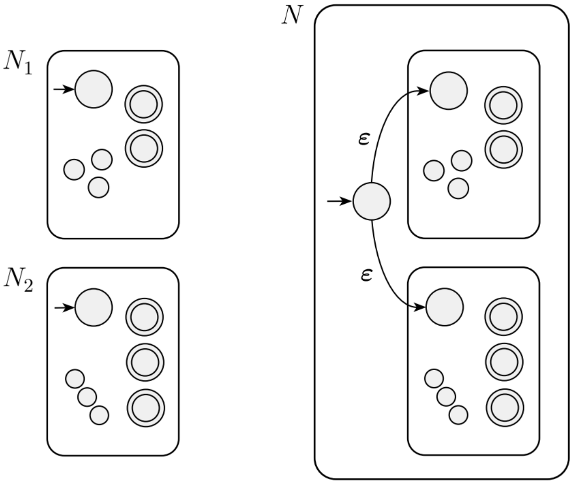


Now we return to the closure of the class of regular languages under the operations $\cup$, $\circ$, and $*$. Our aim is to prove that the union, concatenation, and star of regular languages are still regular.

The use of nondeterminism makes the proofs much easier.

First, let's consider again closure under union. Earlier we proved closure under union by simulating deterministically both machines simultaneously via a Cartesian product construction. We now give a new proof to illustrate the technique of nondeterminism. Reviewing the first proof may be worthwhile to see how much easier and more intuitive the new proof is.

The class of regular languages is closed under the union operation.



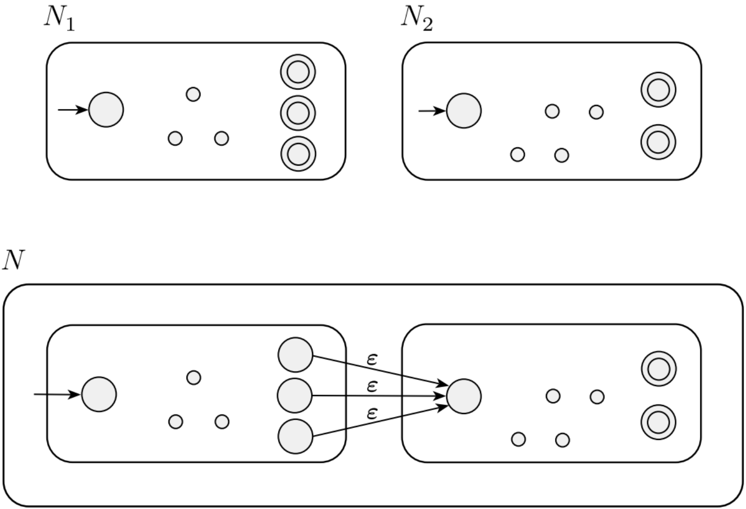
Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$.
   The states of N are all the states of N_1 and N_2, with the addition of a new start state q_0.
2. The state q_0 is the start state of N.
3. The set of accept states $F = F_1 \cup F_2$.
   The accept states of N are all the accept states of N_1 and N_2. That way, N accepts if either N_1 accepts or N_2 accepts.
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$

The class of regular languages is closed under the concatenation operation.



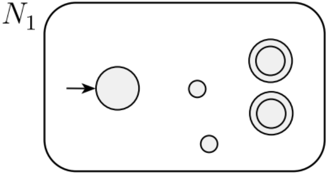
Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

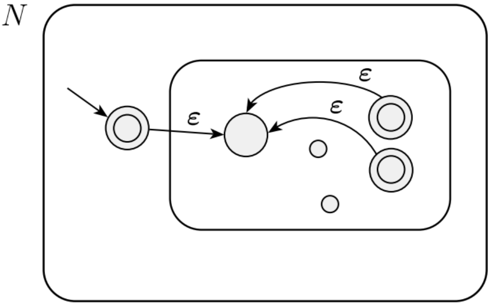
Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$.

1. $Q = Q_1 \cup Q_2$.
   The states of N are all the states of N_1 and N_2.
2. The state q_1 is the same as the start state of N_1.
3. The accept states F_2 are the same as the accept states of N_2.
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q, a) & q \in F_1 \text{ and } a \neq \boldsymbol{\varepsilon} \\ \delta_1(q, a) \cup \{q_2\} & q \in F_1 \text{ and } a = \boldsymbol{\varepsilon} \\ \delta_2(q, a) & q \in Q_2. \end{cases}$$

The class of regular languages is closed under the star operation.



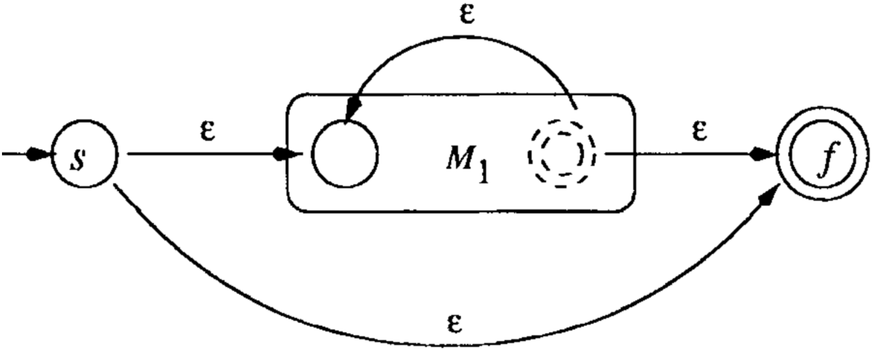


PROOF Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*.

1. $Q = \{q_0\} \cup Q_1$.
   The states of N are the states of N_1 plus a new start state.
2. The state q_0 is the new start state.
3. $F = \{q_0\} \cup F_1$.
   The accept states are the old accept states plus the new start state.
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$

یک شیوهٔ ساخت معتبر دیگر:



تمرین ۱.۱۵ کتاب: دربارهٔ یک ایدهٔ ساخت بد

1.15 Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.[7] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a. The states of N are the states of N_1.

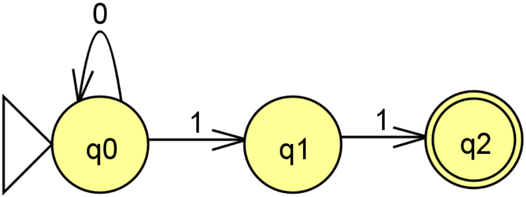
b. The start state of N is the same as the start state of N_1.

c. $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d. Define δ so that for any $q \in Q_1$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \boldsymbol{\varepsilon} \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \boldsymbol{\varepsilon}. \end{cases}$$

(Suggestion: Show this construction graphically, as in Figure 1.50.)



A bad idea (Exercise 1.15):