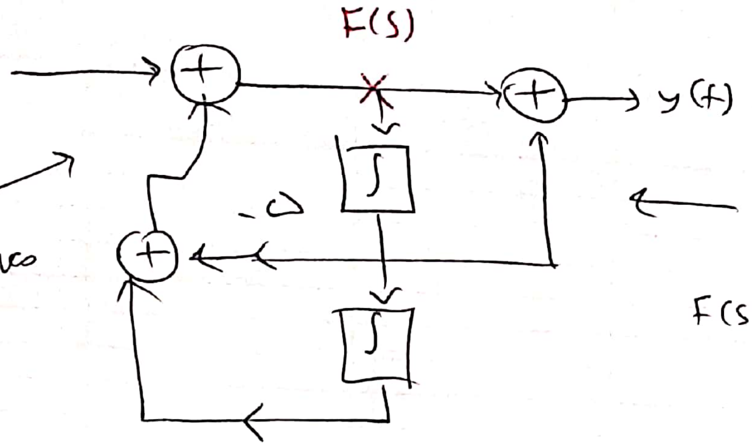
(سوال ۴)



$F(s) + \frac{1}{s} F(s) = Y(s)$

$-\frac{5}{s} F(s) + a \frac{1}{s^2} F(s) + X(s) = F(s)$

$Y(s) = (1+\frac{1}{s}) F(s) \rightarrow \frac{Y(s)}{X(s)} = \frac{1+\frac{1}{s}}{1+\frac{5}{s}-\frac{a}{s^2}} = \frac{s+1}{s+5-\frac{a}{s}}$

$X(s) = (1+\frac{5}{s}-\frac{a}{s^2}) F(s)$

$= \frac{s^2+s}{s^2+5s-a} = H(s)$

$\rightarrow \frac{Y(s)}{X(s)} = \frac{s^2+s}{s^2+5s-a}$ (الف)

حوزه زمان

$x''(t) + x'(t) = y''(t) + 5y'(t) - ay(t)$

(ب)

$x(t) = e^t \rightarrow y(t) = \frac{1}{6} e^t \qquad e^{s_0 t} \rightarrow H(s_0) e^{s_0 t} \rightarrow$

$H(s_0)\big|_{s_0=1} = \frac{1}{6}$

$\frac{s^2+s}{s^2+5s-a}\Big|_{s=1} = \frac{1}{6} \longrightarrow \frac{2}{6-a} = \frac{1}{6} \rightarrow 12 = 6-a \rightarrow$ $\boxed{a=-6}$

(ج)

$a=-6 \rightarrow H(s) = \frac{s(s+1)}{s^2+5s+6} = \frac{s(s+1)}{(s+2)(s+3)}$

سیستم علی $ROC \in Re\{s\} < -3$

$H(s) = \frac{A}{s+2} + \frac{B}{s+3} = \frac{s(s+1)}{(s+2)(s+3)} \qquad A = \frac{s(s+1)}{s+3}\Big|_{s=-2} = \frac{-2(-2+1)}{-2+3} = 2$

$B = \frac{s(s+1)}{s+2}\Big|_{s=-3} = \frac{-3(-2)}{-1} = \boxed{-6}$

ارامون سوال ۴

ب)

$$H(s) = \frac{2}{s+2} - \frac{6}{s+4} \xrightarrow[\text{الف}]{\text{تبدیل لاپلاس}}$$

$$h(t) = -2e^{-2t}u(t) + 6e^{-4t}u(-t)$$

ج)

$$Y(s) = X(s)H(s) = \frac{2}{s+3} \times \frac{s(s+1)}{(s+2)(s+4)}$$

سیستم فیزیکی ← ROC_H شامل $\text{Re}\{s\} > -2$

ROC_X شامل $\text{Re}\{s\} > -3$ ← $x(t) = 0$ برای $t < 0$

$$\rightarrow \text{ROC}_Y = \text{ROC}_H \cap \text{ROC}_X \rightarrow \text{Re}\{s\} > -2$$

$$Y(s) = \frac{A}{s+3} + \frac{B}{s+2} + \frac{C}{s+4} = \frac{2s(s+1)}{(s+3)(s+2)(s+4)}$$

$$A = \left.\frac{2s(s+1)}{(s+2)(s+4)}\right|_{s=-3} = \frac{2(-3)(-2)}{(-1)(1)} = 12$$

$$B = \frac{2(-2)(-1)}{2} = 2 \qquad C = \frac{2(-4)(-3)}{(-2)(2)} = -12$$

$$\rightarrow Y(s) = \frac{12}{s+3} + \frac{2}{s+2} - \frac{12}{s+4}$$

↑ سمت راست — ↑ سمت چپ — ↑ سمت چپ — طبق ROC ها

$$\rightarrow y(t) = 12e^{-3t}u(t) - 2e^{-2t}u(-t) + 12e^{-4t}u(t)$$