

$$x(e^{j\omega}) = r \cos^2(\omega) + \sin^2(r\omega) \quad m[n] = \delta$$

المطلوب

$$r \cos^2(\omega) = 1 + \cos(2\omega)$$

$$\sin^2(r\omega) = \frac{1 - \cos(2r\omega)}{2}$$

$$\rightarrow \sin^2(r\omega) = \frac{1 - \cos(2r\omega)}{2}$$

$$\rightarrow x(e^{j\omega}) = 1 + \cos(2\omega) + \frac{1 - \cos(4\omega)}{2} =$$

$$\frac{3}{2} + \cos(2\omega) - \frac{\cos(4\omega)}{2}$$

$$F^{-1}\left\{\frac{3}{2}\right\} = \frac{3}{2} \delta[n]$$

$$F^{-1}\{\cos(2\omega)\} = F^{-1}\left\{\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right\}$$

مبدأ التراكب:  $x[n-n_0] \leftrightarrow e^{-jn_0\omega} X(e^{j\omega})$  في زمان  $\omega$

$$F\{\delta[n]\} = 1$$

دالة الوحدة في  $\omega$  و  $\omega$  راسم

$$F^{-1}\{1\} = \delta[n]$$

بـ  $n_0 = -2$  و  $n_0 = 2$  و  $n_0 = 0$

$$\rightarrow F^{-1}\left\{\frac{1}{2} e^{j2\omega}\right\} + F^{-1}\left\{\frac{1}{2} e^{-j2\omega}\right\} =$$

$$\frac{1}{2} (\delta[n-2] + \delta[n+2])$$

$$F^{-1}\{\cos(4\omega)\} = F^{-1}\left\{\frac{e^{j4\omega} + e^{-j4\omega}}{2}\right\} =$$

$$\frac{1}{2} (\delta[n-4] + \delta[n+4])$$

$$\rightarrow F^{-1}\{x(e^{j\omega})\} = \frac{3}{2} \delta[n] + \frac{1}{2} (\delta[n-2] + \delta[n+2])$$

$$+ \frac{1}{2} (\delta[n-4] + \delta[n+4])$$

جواب الف

$$x[n] = (n-1) \left(\frac{1}{9}\right)^{n-1}$$

در رانج اولی  $x[n] = a^{n-1} \rightarrow |a| < 1 \rightarrow x(e^{j\omega}) = \frac{1-a^2}{1-2a\cos\omega+a^2}$  (ب)

است  $a = \frac{1}{9}$  است

$$x_1(e^{j\omega}) = \frac{1 - \frac{1}{9}}{1 - \frac{2}{9}\cos\omega + \frac{1}{9}} = \frac{\frac{8}{9}}{\frac{10}{9} - \frac{2}{9}\cos\omega}$$

اگر تغییر کنیم  $y[n] = n \left(\frac{1}{9}\right)^{n-1} - \left(\frac{1}{9}\right)^{n-1}$

$$\rightarrow y(e^{j\omega}) = F\left\{n \left(\frac{1}{9}\right)^{n-1}\right\} - F\left\{\left(\frac{1}{9}\right)^{n-1}\right\}$$

در رانج اولی، رانجی differentiation (فرقی) می‌کنیم

$$n x[n] \leftrightarrow j \frac{d}{d\omega} x(e^{j\omega})$$

پس به از  $x_1(e^{j\omega})$  مشتق می‌کنیم

$$j \frac{d}{d\omega} \left( \frac{\frac{8}{9}}{\frac{10}{9} - \frac{2}{9}\cos\omega} \right) = \frac{-\frac{2}{9}\sin\omega}{\left(\frac{10}{9} - \frac{2}{9}\cos\omega\right)^2}$$

$$\frac{8}{9} j \left( 0 \times \frac{10}{9} - \frac{2}{9}\cos\omega - 1 \times \left( -\frac{2}{9}\sin\omega \right) \right) =$$

$$\left( \frac{10}{9} - \frac{2}{9}\cos\omega \right)^2$$

$$\frac{\frac{8}{9} j \sin\omega - \frac{2}{9}\sin\omega}{\left( \frac{10}{9} - \frac{2}{9}\cos\omega \right)^2}$$

$$= \frac{-\frac{14}{9} j \sin\omega}{\left( \frac{10}{9} - \frac{2}{9}\cos\omega \right)^2} = x_2(e^{j\omega})$$

$$\rightarrow y(e^{j\omega}) = x_2(e^{j\omega}) - x_1(e^{j\omega}) =$$

$$\frac{-\frac{14}{9} j \sin\omega}{\left( \frac{10}{9} - \frac{2}{9}\cos\omega \right)^2} - \frac{8}{9} \cdot \frac{1}{\left( \frac{10}{9} - \frac{2}{9}\cos\omega \right)}$$

جواب: (ب)