سوال ۳

$$H(z) = \frac{1 - z^{-1}}{1 - \frac{17}{6} z^{-1} - \frac{3}{2} z^{-2}}$$

الف) معادله‌ی تفاضلی ؟

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

$$-\frac{3}{2} y[n-2] - \frac{17}{6} y[n-1] + y[n] = x[n] - x[n-1]$$

ب) صفرها و قطب‌های سیستم ؟

صفرهای $H(z)$ ؟

$$\sum_{k=0}^{M} b_k z^{-k} = 0 \rightarrow 1 - z^{-1} = 0 \rightarrow \boxed{z = 1}$$

صفر

قطب‌های $H(z)$ ؟

$$-\frac{3}{2} z^{-2} - \frac{17}{6} z^{-1} + 1 = 0$$

ریشه‌های معادله‌ی درجه ۲ ؟ اگر $z^{-1} = s$ باشد $\leftarrow$

$$-\frac{3}{2} s^{2} - \frac{17}{6} s + 1 = 0 \rightarrow \Delta = \left(-\frac{17}{6}\right)^{2} - 4 \times -\frac{3}{2} = \frac{289}{36} + 6 = \frac{505}{36}$$

$$\Delta = 14 \rightarrow \sqrt{\Delta} \cong 3.7$$

$$x_1 = \frac{-b + 3.7}{2a} = \frac{\frac{17}{6} + 3.7}{-3} = \frac{2.8 + 3.7}{-3} \cong -2.2$$

$$x_2 = \frac{+\frac{17}{6} - 3.7}{-3} = \frac{-0.9}{-3} = +\boxed{0.3}$$

پس به طور تقریبی ریشه‌های فرض ؟ $\leftarrow$

$$z^{-1} = 0.3 \rightarrow \boxed{z = \frac{10}{3}} \quad , \quad z^{-1} = -2.2 \rightarrow z = \boxed{-\frac{5}{11}}$$

قطب ۲ — قطب ۱

ج) پاسخ سیستم به $x[n] = u[n]$ ؟

$$Z\{u[n]\} = \frac{1}{1 - z^{-1}}$$

$$\rightarrow Y(z) = X(z) \cdot H(z) = \frac{\cancel{(1 - z^{-1})}}{1 - \frac{17}{6} z^{-1} - \frac{3}{2} z^{-2}} \cdot \frac{1}{\cancel{1 - z^{-1}}} = \frac{1}{1 - \frac{17}{6} z^{-1} - \frac{3}{2} z^{-2}}$$

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ادامه سوال ۳

$$y[n] = Z^{-1}\{Y(z)\} = Z^{-1}\left\{ \frac{1}{-\frac{3}{2}z^{-2} - \frac{17}{6}z^{-1} + 1} \right\}$$

قطب‌های کسر را قبلاً بدست آورده بودیم

$$= Z^{-1}\left\{ \frac{1}{(z-\frac{10}{3})(z+\frac{5}{11})} \right\} = \frac{A}{z-\frac{10}{3}} + \frac{B}{z+\frac{5}{11}}$$

$$A: \ 1\Big|_{z=\frac{10}{3}} = \frac{1}{\frac{10}{3}+\frac{5}{11}} = \frac{1}{3,3 + 0,45} = \frac{1}{3,75} = \boxed{0,26}$$

$$B: \ 1\Big|_{z=-\frac{5}{11}} \rightarrow \frac{1}{-\frac{5}{11}-\frac{10}{3}} = \frac{1}{-0,45-3,3} = \frac{-1}{3,75} = \boxed{-0,26}$$

$$\rightarrow Z^{-1}\left\{ \frac{0,26}{z-\frac{10}{3}} + \frac{-0,26}{z+\frac{5}{11}} \right\} =$$

$$0,26\left(\frac{10}{3}\right)^n u[n] - 0,26\left(-\frac{5}{11}\right)^n u[n]$$

(ج)

$$x[n] = \cos\left(\frac{\pi n}{2}\right) = \frac{e^{j(\frac{\pi n}{2})} + e^{-j(\frac{\pi n}{2})}}{2}$$

$$X(z) =$$

طبق خاصیت scaling در حوزه Z

$$e^{j\omega_0 n} x[n] \overset{Z}{\longleftrightarrow} X(e^{-j\omega_0} z)$$

$$\alpha_1 = (e^{j\frac{\pi}{2}}) \rightarrow y_1[n] = e^{j\frac{\pi}{2}n} \cdot H(e^{j\frac{\pi}{2}})$$

$$y_2[n] = e^{j(-\frac{\pi}{2})n} \cdot H(e^{-j\frac{\pi}{2}})$$

$$\alpha_2 = (e^{-j\frac{\pi}{2}})$$

$$\rightarrow y[n] = y_1[n] + y_2[n]$$

$$H(z) = \frac{1 - z^{-1}}{1 - \frac{17}{6}z^{-1} - \frac{3}{2}z^{-2}} \rightarrow H(e^{j\frac{\pi}{2}}) = \frac{1 - e^{-j\frac{\pi}{2}}}{1 - \frac{17}{6}e^{-j\frac{\pi}{2}} - \frac{3}{2}e^{-j\pi}}$$

$$H(e^{-j\frac{\pi}{2}}) = \frac{1 - e^{j\frac{\pi}{2}}}{1 - \frac{17}{6}e^{j\frac{\pi}{2}} - \frac{3}{2}e^{j\pi}}$$

$$e^{j\frac{\pi}{۲}} = \cos(\tfrac{\pi}{۲}) + j\sin(\tfrac{\pi}{۲}) = j$$

$$e^{-j\frac{\pi}{۲}} = \cos(-\tfrac{\pi}{۲}) + j\sin(\tfrac{-\pi}{۲}) = -j$$

$$e^{-j\pi} = \cos(-\pi) + j\sin(-\pi) = -1$$

$$y_۱[n] = e^{j\frac{\pi}{۲}n} \cdot \left(\frac{۱-(-j)}{۱-\frac{۱۷}{۶}(-j)-\frac{۳}{۲}(-۱)}\right) =$$

$$(j)^n \cdot \frac{(۱+j)}{۱+\frac{۱۷}{۶}j+\frac{۳}{۲}} = \frac{(j)^n(۱+j)}{\frac{۱۷}{۶}j+\frac{۵}{۲}}$$

$$y_۲[n] = e^{-j\frac{\pi}{۲}n} \cdot \left(\frac{۱-j}{۱-\frac{۱۷}{۶}j+\frac{۳}{۲}}\right) = (-j)^n\left(\frac{۱-j}{-\frac{۱۷}{۶}j+\frac{۵}{۲}}\right)$$

$$\rightarrow y[n] = \frac{۱}{۲}\left(\frac{(j)^n(۱+j)}{\frac{۱۷}{۶}j+\frac{۵}{۲}} + \frac{(-j)^n(۱-j)}{-\frac{۱۷}{۶}j+\frac{۵}{۲}}\right)$$ جواب