```
print('hi')
```

```
hi
```

Question 1 of 4

Basic Questions for class (2): Logistic Regression

Setting this up as a logistic regression y = σ (β 1 x

- β 0) y=σ(β 1 x+β 0) (where σ (x) = e x 1
- e x σ(x)= 1+e x

e x

) problem:

What is the meaning of the input variable x here? What are the two parameters here? Why are they (and not the x values) parameters? What is the meaning of the output variable y here?

x represents the input feature or independent variable used to predict the outcome. It's the observed data you have for each instance.

The two parameters are β0 (the intercept) and β1 (the coefficient for x): β0: This is the intercept. It represents the odds of the outcome when x=0. β1: This is the weight or coefficient for the input variable x. It measures the change in the odds of the outcome for a one-unit change in x. They are parameters because they are the values the model learns from the data to best fit the relationship between x and y. The x values are not parameters; they are the input data you provide to the model.

y is the output or dependent variable that the model aims to predict. In logistic regression, y is usually binary (0 or 1), representing the two possible classes or outcomes.

Question 2 of 4: Likelihood

Remember the (y) value we get from logistic regression really is a conditional probability. Specifically, it is

$$P(\text{True} \mid x) = \sigma(\beta_1 x + \beta_0)$$

From this, we can also deduce:

$$P(\text{False} \mid x) = 1 - \sigma(\beta_1 x + \beta_0)$$

Time for an analogy:

If a coin has a probability (p) of landing heads up (and so, (1 - p) of landing tails up), and you observe heads, tails, heads, heads, tails, the likelihood is:

$$L(p) = p \cdot (1-p) \cdot p \cdot p \cdot (1-p)$$

And so the log likelihood is:

$$\ln L(p) = \ln(p) + \ln(1-p) + \ln(p) + \ln(p) + \ln(1-p)$$

Question: Fill in both expressions.

By analogy...the likelihood for our 100,000 observations (x_1, \dots, x_{100,000}) is:

$$L(\beta_0, \beta_1) = \prod_{i=1}^{100,000} \left[\sigma(\beta_1 x_i + \beta_0)\right)^{y_i} \left[1 - \sigma(\beta_1 x_i + \beta_0)\right)^{1-y_i}$$

and so our log likelihood is:

$$\ln L(\beta_0, \beta_1) = \sum_{i=1}^{100,000} \left[y_i \ln \sigma(\beta_1 x_i + \beta_0) + (1 - y_i) \ln\left(1 - \sigma(\beta_1 x_i + \beta_0)\right)\right)$$

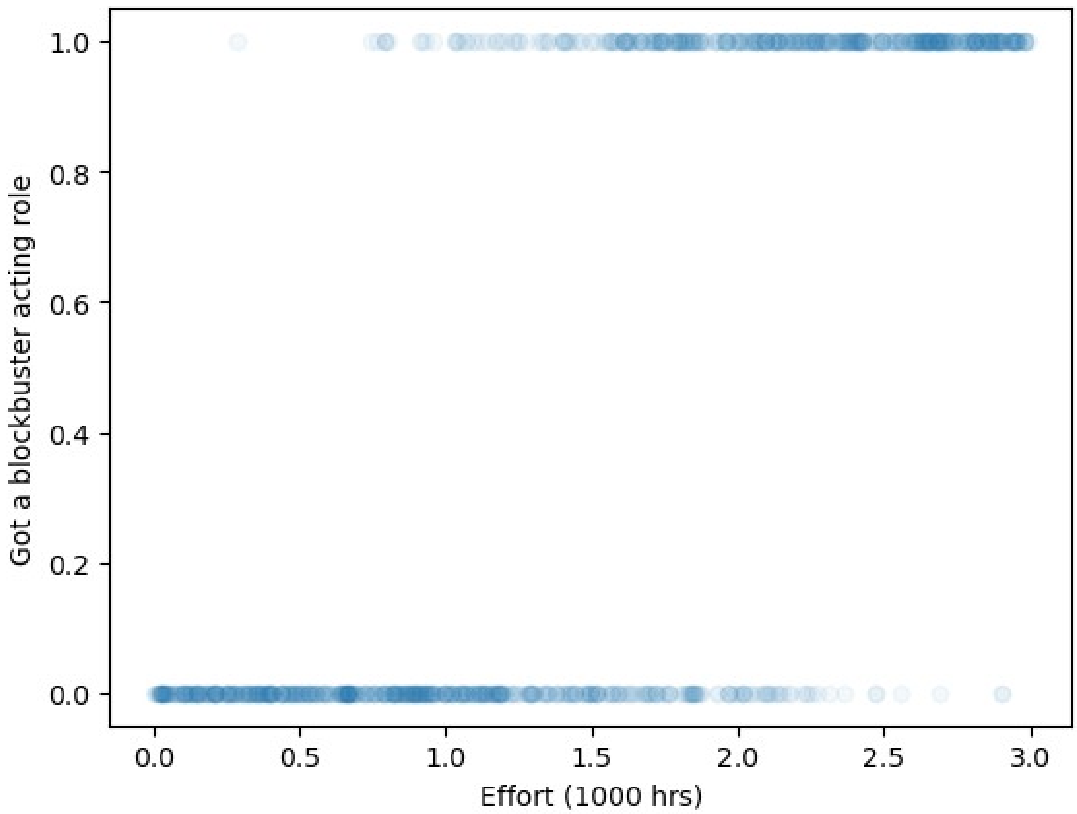
Core Question (3): Likelihood Calculation

Here's some code that "generates" some data, based on the probability distribution assumed by the logistic regression model. Specifically, the true values are β 0 =−5,β 1 =3 (but pretend you don't know that)

```
import numpy as np
from sklearn import metrics
import matplotlib.pyplot as plt

NUM_DATAPOINTS = 1000

def sigmoid(x):
    return np.exp(x) / (1 + np.exp(x))

def generate_data(x):
    a = 3  #beta_1
    b = -5  #beta_0
    prob = sigmoid(a*x+b)
    return np.random.uniform() < prob

x_data = np.random.uniform(high=3, size=NUM_DATAPOINTS)
y_data = [generate_data(x) for x in x_data]
```

Note that this simulation produces a dataset where, we go from a low probability of success, to a high probability of success at about ~1200 hours, but also a fair amount of variance around that mark:

```
plt.scatter(x_data, y_data, alpha=0.05)
plt.xlabel("Effort (1000 hrs)")
plt.ylabel("Got a blockbuster acting role")
plt.show()
```



Now, suppose your friend Aishwarya (who doesn't know the real values of) claims the values that best "fit" this data are . TA Irhum claims "well, actually..." the values are . Both are wrong, but who is more wrong?

Computing the log-likelihood helps us answer this question: higher is better! Fill out the function below, and comment: which estimate is better?

```
# Fix this function
# You might wish to use the sigmoid(x) function defined above!
def log_likelihood_per_point(beta_0, beta_1, x, y):
    prob = sigmoid(beta_0 + beta_1 * x)
    if y is True:
        return np.log(prob)
    else:
        return np.log(1 - prob)



# This one's complete!
def total_log_likelihood(beta_0, beta_1, xs, ys):
    total_ll = 0

    for (x, y) in zip(xs, ys):
        total_ll += log_likelihood_per_point(beta_0, beta_1, x, y)
```

```
    return total_ll

# Be careful, which is the *higher* value when two values are
negative?
print("Aishwarya's estimate: ", total_log_likelihood( 3, -3, x_data,
y_data))
print("TA Irhum's estimate: ", total_log_likelihood( 4, -1, x_data,
y_data))
```

```
Aishwarya's estimate:  -722.614375051468
TA Irhum's estimate:  -2643.5425603316207
```

Aishwarya's estimate has a higher (less negative) log-likelihood than TA Irhum. Therefore, Aishwarya is less wrong, and her estimate better fits the data.

```
pip install jax
```

```
Collecting jax
  Using cached jax-0.4.33-py3-none-any.whl.metadata (22 kB)
Collecting jaxlib<=0.4.33,>=0.4.33 (from jax)
  Using cached jaxlib-0.4.33-cp312-cp312-
macosx_11_0_arm64.whl.metadata (983 bytes)
Collecting ml-dtypes>=0.2.0 (from jax)
  Using cached ml_dtypes-0.5.0-cp312-cp312-
macosx_10_9_universal2.whl.metadata (21 kB)
Requirement already satisfied: numpy>=1.24 in
/opt/anaconda3/lib/python3.12/site-packages (from jax) (1.26.4)
Collecting opt-einsum (from jax)
  Using cached opt_einsum-3.3.0-py3-none-any.whl.metadata (6.5 kB)
Requirement already satisfied: scipy>=1.10 in
/opt/anaconda3/lib/python3.12/site-packages (from jax) (1.13.1)
Using cached jax-0.4.33-py3-none-any.whl (2.1 MB)
Downloading jaxlib-0.4.33-cp312-cp312-macosx_11_0_arm64.whl (66.1 MB)
━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 66.1/66.1 MB 13.4 MB/s eta
0:00:0000:0100:01
l_dtypes-0.5.0-cp312-cp312-macosx_10_9_universal2.whl (750 kB)
━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 750.2/750.2 kB 17.0 MB/s eta
0:00:00a 0:00:01
-3.3.0-py3-none-any.whl (65 kB)
━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 65.5/65.5 kB 4.6 MB/s eta
0:00:00
, ml-dtypes, jaxlib, jax
Successfully installed jax-0.4.33 jaxlib-0.4.33 ml-dtypes-0.5.0 opt-
einsum-3.3.0
Note: you may need to restart the kernel to use updated packages.
```

```
import jax.numpy as jnp
from jax import grad
import numpy as np
```

```
# Sigmoid function using JAX
def sigmoid(x):
    return jnp.exp(x) / (1 + jnp.exp(x))

# Convert y_data from True/False to 0/1
y_data = np.array([int(y) for y in y_data])

# Function to compute the total log likelihood in JAX
def log_likelihood(beta_0, beta_1, x_data, y_data):
    linear_term = beta_1 * x_data + beta_0
    probs = sigmoid(linear_term)

    # Log likelihood: for y = 1, it's log(prob), for y = 0, it's log(1
- prob)
    return jnp.sum(y_data * jnp.log(probs) + (1 - y_data) * jnp.log(1
- probs))

# Compute gradient of the log likelihood w.r.t. beta_0 and beta_1
log_likelihood_grad = grad(log_likelihood, argnums=(0, 1))

# Aishwarya's initial estimate (for example, let's use these values)
initial_beta_0 = 3.0
initial_beta_1 = -3.0

# Compute the gradients at Aishwarya's initial estimate
grad_beta_0, grad_beta_1 = log_likelihood_grad(initial_beta_0,
initial_beta_1, x_data, y_data)

# Print the computed gradients
print(f"Gradient w.r.t. beta_0: {grad_beta_0}")
print(f"Gradient w.r.t. beta_1: {grad_beta_1}")

Gradient w.r.t. beta_0: 74.47683715820312
Gradient w.r.t. beta_1: 699.877197265625
```

Question 3 of 4

#MLDevelopment

Link your study Notion here and briefly describe the students you worked with and which materials you found particularly useful.

https://www.notion.so/b96dd2ab4cd245748285211b477ebbb3?v=d31b2c53bf9a413583a1d286eeb94b0b&pvs=4

I have worked with Hayoung to make this study Notion. https://rpsychologist.com/likelihood/ This visualization of MLE calculation helped me have a deeper intuition about the whole process.