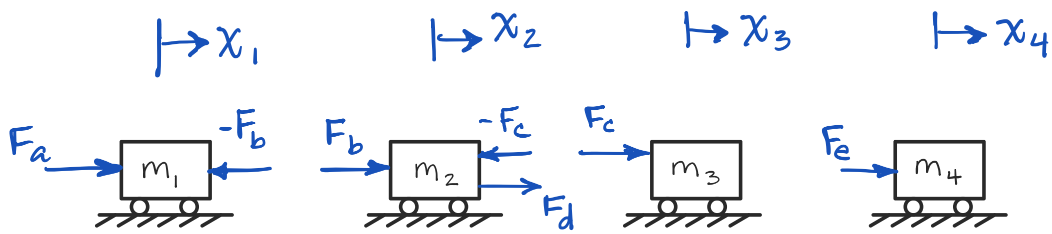
Homework 5 - Lagrangian equations of motions. See notes from Lectures 9, 10 and 11.

NO RECITATION PROBLEM due with HW 5. (Submit the “Maze” problem as part of HW 6.)

Problem 1. EOMs with relative versus absolute coordinates. (Similar systems have been analyzed during lecture.) No “Lagragians” are involved. Below is a system with four masses, each with various forces applied. Assume x_1 through x_4 are “absolute coordinates”, below.



a) Use Newton's 2nd Law to write the four equations of motion (EOMs) for the system using the following definition for the "generalized coordinates":

$\boxed{\xi_1 = x_1}$ $\boxed{\xi_2 = x_2}$ $\boxed{\xi_3 = x_3}$ $\boxed{\xi_4 = x_4}$

Above, each DOF is just the **<u>absolute coordinate</u>**. Each EOM is just "F=ma", so this is a VERY simple problem. For mass m_1, for example, the equation would just be: $F_a + -F_b = m_1\ddot{x}_1$.

<u>Relative coordinates:</u>

b) Next, assume that generalized coordinates (GC's) are instead defined as show below:

$\boxed{\xi_1 = x_1}$ $\boxed{\xi_2 = x_2 - x_1}$ $\boxed{\xi_3 = x_3 - x_2}$ $\boxed{\xi_4 = x_4 - x_1}$

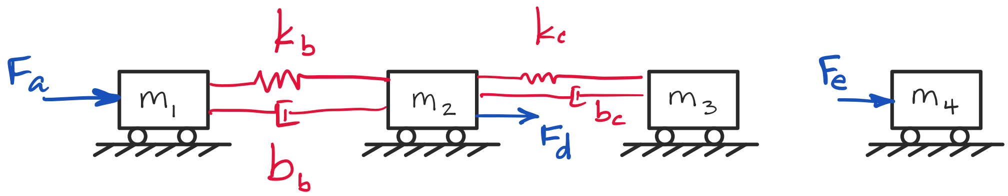
Use these equations to write expressions for the absolute coordinates, x_1 through x_4, in terms of the generalized coordinates. (For example, $x_1 = \xi_1, x_2 = \xi_2 - \xi_1$, etc., expressing each as some linear combination of ξ_1 through ξ_4.) **NOTE:** by doing step b here, the aim is to make it very easy to take partial derivatives to find "big Xi" terms (i.e., the non-conservative forces that appear in each EOM) <u>in part d</u>.

c) Using the generalized coordinates from part b, and referring to the Lecture 10 notes, write out the "rate of work" done, $\dot{W}$, as a sum of "force" times "velocity" terms. There are seven forces, so this just involves writing each force (e.g., F_a) times the velocity where the force occurs (here, that would be $dx_1/dt = \dot{\xi}_1$, using the GC’s).

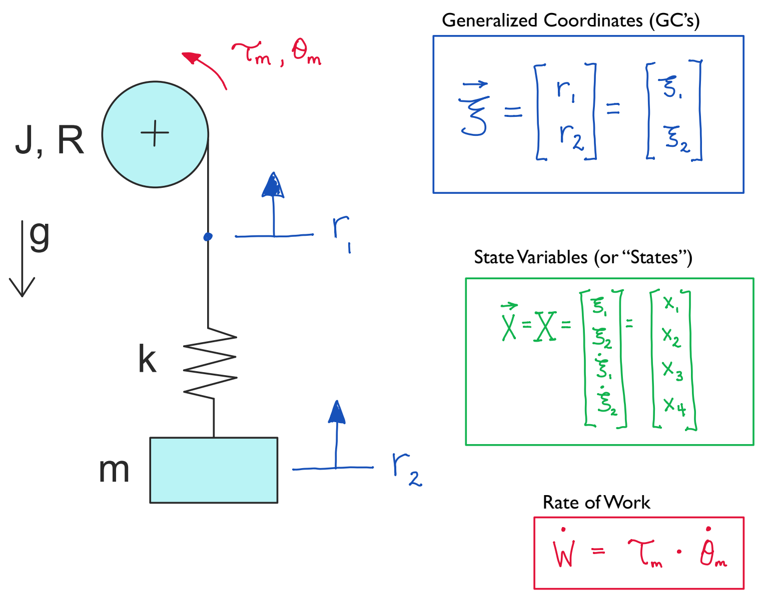
d) Write the four EOMs for the system using the generalized coordinates (GC's) as described in parts b and c, above. You can reason about how to draw free body diagrams around a “parent, plus all of its dependent children”, as described in Lectures 10 and 11, and/or you can use the typeset procedure in the notes for Lecture 10 to find non-conservative terms, Ξ_1 through Ξ_4, for each of the four equations of motion.

e) Show work to demonstrate that the equations of motion in part d <u>are identical</u> to the physics in the EOMs in part a. (See pages 13, 18, and 19 in the Lecture 9 notes, and/or pp. 4-6 in Lec11_coenergy_vs_energy.pdf) Specifically, you should be able to add together various equations in part a to create the equations you got in part d (or vice versa, if you prefer).

f) Previously, the masses were in NO WAY CONNECTED, and each force was just an external, non-conservative force. The forces in some equations were different in parts a and d exactly because the definitions of GCs were different. Below, now assume we have the following springs and dampers, which account for the forces F_b and F_c. Write expressions for F_b and F_c by comparing the previous diagrams with the version below:



Problem 2. Below is a "simple" system (<u>without</u> sin or cos functions are involved).



This system has a pulley with a fixed center, driven by a torque input. Its center of mass is at the rotational center, so that we do not need to consider what this mass is, in calculating the equations of motion. The generalized coordinates are the absolute vertical positions of a point above the spring (r_1) and at the center of mass "m" (r_2). Assume that the location of r_1 never "wraps" onto the pulley when deriving equations of motion. Also, we define the coordinates such that when $r_1=0$ and $r_2=0$, there is no potential energy stored in the spring.

a. Determine T^*, V and L. <u>You must write all terms with respect to the GCs.</u>
b. Determine the 2x1 vector Ξ of non-conservative forces.
c. Derive the equations of motion (EOMs) for the system BY HAND. (i.e., show work.)

Use r_1 and r_2 as generalized coordinates (GCs) within xi. With 2 GCs, there will be two EOMs.

Problem 3. Non-conservative terms ("big Xi").

This problem is similar to the example on pp. 5-8 of the Lecture 10 notes.

Below, F_p and F_c are forces applied from the external world onto the system, while the torques (1, 2, and 3) are [internal] motor torques, e.g., tau1 is applied between the cart and link 1.

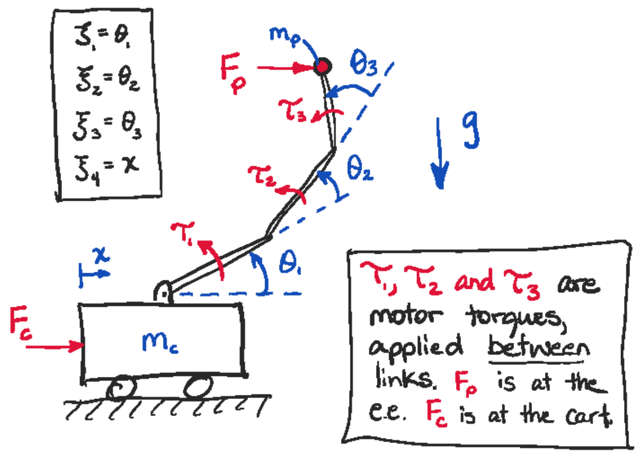
For the system below, just find Ξ_1 through Ξ_4 (each of the 4 non-conservative forces and/or torques that would go on the righthand side of the equations of motion; i.e., the "big Xi" terms).

The non-conservative forces and torques are:
tau$_1$, tau$_2$, tau$_3$, F_c and F_p.

Angles are relative, as shown.

Generalized coordinates are:
ξ_1 , ξ_2 , ξ_3 , and ξ_4
(as defined at right…)

The links are massless, with a point mass m_p at the very end, as shown. Cart mass is m_c. The link lengths are L_1, L_2, and L_3.



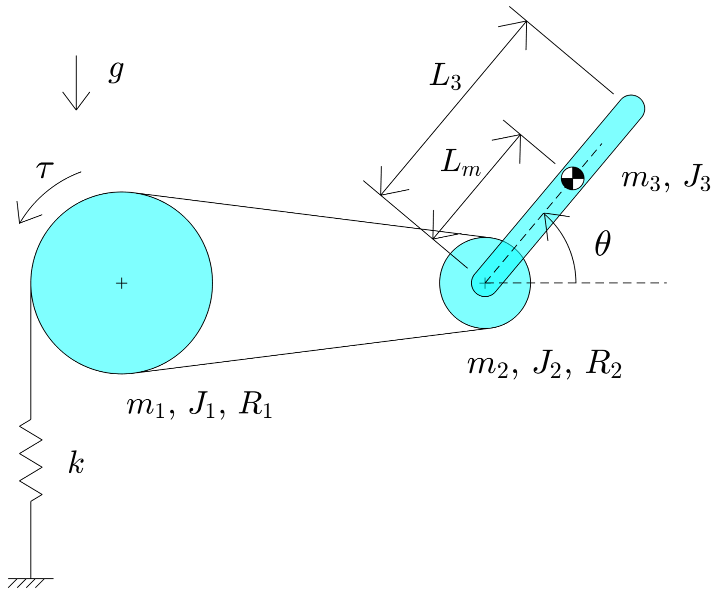
Hint: Follow the notes for Lecture 10 to write out "rate of work" done, $\dot{W}$, as a sum of "force" (or torque) times "velocity" (or angular velocity) terms. When you calculate each Ξ_i, you always leave each "force" unchanged and ONLY find the partial derivative of each respective "velocity" with respect to each GC velocity. The Lecture 10 notes go through a very similar example. Visually, what you want to do it to understand if any motion occurs in the direction of each force (or torque) when a given DOF "wiggles". For example, when only x changes, the cart moves horizontally, but none of the angles changes. So both F_c and F_p do work, but the torques do not. But what happens if a given angle moves?

Clearly, the trickiest part above is the "F_p" term. When any one of the angles changes (for the particular configuration shown) there will be some motion at the point where F_p is applied and – importantly – some motion in the x direction.

You need to provide general equations, keeping the GCs as open variables (as opposed to calculating things for *just* this configuration) for this problem. (i.e., this is not about the particular configuration shown, unlike some previous questions on Jacobians and/or wheeled systems.) i.e., your solution will involve some sin and/or cos type terms here.

Problem 4. Lagrangian EOMS. You are welcome/encouraged to use MATLAB to check your work here, if you wish. However, you need to write equations by hand for full credit.

(This is an old exam question, designed to doable by hand.)



$R_1 = 2R_2$

In the pulley system shown, the centers of J_1 and J_2 stay fixed, and the disks rotate with no slip. The pulley at J_2 is glued to a rod (of mass m_3, with moment of inertia about its center of mass J_3). There is also gravity, a spring, and an input torque, all as shown.

a) Derive an equation of motion (symbolically); use θ as the generalized coordinate.

Notice there is only a single degree of freedom (DOF), so only one GC.

You must show work by writing out T^*, V, and the Lagrangian (T^* - V), and you must use the Lagrangian approach, to get the equations.

b) Determine a control law (i.e., an equation) for τ so that this system behaves as a second-order linear system, with $\zeta = 0.7$ and $\omega_n = 5$ (rad/s), with theta oscillating about an equilibrium angle of pi/4 radians. [Hint: use "control law partitioning".]

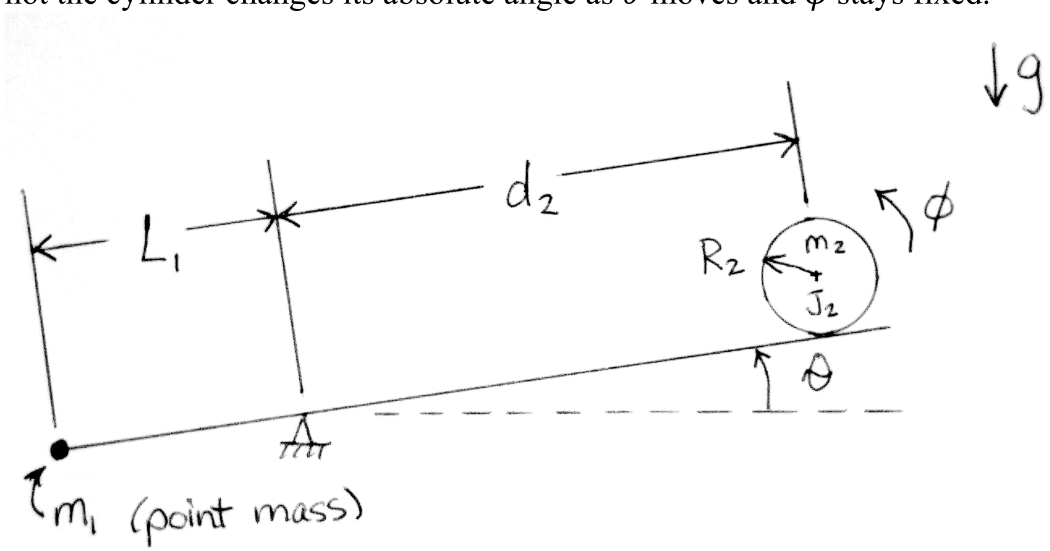
- Note: If you're unsure of part "a)", just make an approximate guess for the solution of "a)" to get a general form, and then proceed here...

Problem 5. Conservative system with two generalized coordinates.

(a) Solve for T*, V and the Lagrangian for the system below, using θ and ϕ as generalized coordinates. You must write this in terms of θ and ϕ! Since d_2 is actually a function of ϕ, d_2 MUST NOT appear in your equation, for example.

- $d_2 = 2.0 - R_2\phi$ [in meters], so the cylinder rolls on the see-saw without slipping.
- Mass m_1 is a point mass that is fixed at distance L_1, as shown.
- Total velocity of mass m_2 can be broken into two, orthogonal parts.

Does the absolute angle of the cylinder depend on both θ and ϕ? Visualize whether or not the cylinder changes its absolute angle as θ moves and ϕ stays fixed.



[Hint: "Yes, the cylinder angle depends on BOTH phi and theta."]

(b) Solve for the two equations of motion for the system, using θ and ϕ as generalized coordinates. (There are no non-conservative forces or torques, so $\Xi_1 = \Xi_2 = 0$.)