

# IE 519 Dynamic Programming Term Project: Restaurant Revenue Management

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In the term project of the Dynamic Programming course, I will work on yield management for restaurants of allocating customers to tables to maximize the revenue of the restaurants. Primarily, I am interested in combining dynamic programming-based yield management models with waiting-line models. Besides, I want to explore various statistical distributions and scenarios for arrivals and departures of customers. Analysis of multiple types of tables also will be considered.

*Keywords:* Revenue Management, Restaurant Revenue Management, Dynamic Programming, Guests Placement Scheme

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Current version is 12/06/2018, and the replication files are available on the author's Github account ([https://github.com/haedong31/dp\\_project](https://github.com/haedong31/dp_project)).

## 1. Introduction

The objective of revenue management, also known as yield management, is to maximize revenue by providing the right product to the right customers at the right price and time without compromising customer satisfaction (Kimes, 1989; Netessine and Shumsky, 2002). Although the main research area of revenue management is pricing, it is not only limited to pricing but also includes it also includes optimizing inventory capacity or duration of service time (Donaghy, McMahon and McDowell, 1995). Revenue management began at the U.S. airline industry since the deregulation that allows airline companies the flexibility at pricing seats and controlling seat inventory (Belobaba, 1987). Revenue management has brought enormous successes in the airline industry. It is reported that in the early 1990s that American Airline estimated the annual benefit of yield management at \$500 million (Smith, Leimkuhler and Darrow, 1992). After the successes of the airline industry, other industries have adopted revenue management techniques including but not limited to hotels, automobile rentals, and cruise lines (McGill and Van Ryzin, 1999).

In this study, we focus on the application of revenue management to restaurants. Restaurant revenue management, RRM for short, is pioneered by Kimes et al. (1998) and Kimes et al. (1999). Kimes et al. (1998) suggests the four characteristics that make revenue management more effective if some operation has them: relatively fixed capacity, predictable demand, perishable inventory, and appropriate cost and pricing structure. The authors argue that the restaurant industry has such features. Specifically, they claim that what perishable in restaurants is available time for tables to serve customers, instead of raw food. Therefore, they come up with the measure named revenue per available seat hour (RevPASH) to check the performance of restaurants. Recently, other measures taking into account margin of food and cost of raw food are presented (Heo, 2017). In Kimes et al. (1999), the five steps to implement RRM are described. A way to manage meal duration is also explored (Kimes, Wirtz and Noone, 2002). These methods were tested in real-world restaurant operations (Kimes and Wirtz, 2003; Kimes, 2004). While these studies provide general principles of RRM, they lack quantitatively rigorous analysis.

There are relatively few papers pertaining to quantitative RRM. The first research to mathematically model the restaurant operation has been done in Bertsimas and Shioda (2003). In Bertsimas and Shioda (2003), for seating policies, an approximate dynamic programming formulation based

on interger programming for restaurants do not accept booking is proposed, and for restaurants with reservation, a stochastic gradient algorithm is described. Seating considers how to place guests to the right table to maximize revenue. Another research field of RRM is table mix concerning how to constitute a restaurant with how many and what type of tables (Kimes and Beard, 2013). Table mix along with seating without reservation is quantitatively analyzed in Guerriero, Miglionico and Olivito (2014).

Both of Bertsimas and Shioda (2003) and Guerriero, Miglionico and Olivito (2014) use approximate dynamic programming to solve RRM problems. Since decisions in the restaurant operation are made sequentially by time, dynamic programming is a right choice to model it. However, because of the curse of dimensionality, the both studies leverage interger programming and linear programming respectively to approximate a value function in a dynamic programming formulation. In contrast, Kuo (2010) uses an exact dynamic programming model, so that accurately evaluates marginal utility of preserving a table by rejecting arrived party for a chance of bigger group arriving in the future. However, this study simplifies operation in many aspects. For example, the author assumes that there are only two sizes of parties and tables do not have a limitation so it can serve the both classes of guests. Therefore, in this paper, we propose an exact dynamic programming model that consider more realistic situations in restaurants. Specifically, we consider several sizes of parties and tables.

This paper is structured as follows. Section 2 introduces the specific problem statement including the dynamic programming formulations. Section 3 describes the solution procedure to generate seating policies. Section 4 shows a computational experiments using artificial data. Last, Section 5 presents the conclusions and suggestions for future research.

## 2. Problem Statement

Usually, a restaurant consists of different dimensions of tables to serve various sizes of customers. During the operation of a restaurant, which has multiple sizes of tables, a floor manager has to make following decisions upon the arrival of a party. First, a manager has to decide whether to accept the party or turn down them. If she/he chooses to receive the arrived guests, now a manager needs to determine which table to serve the customers. Revenue may increase by

rejecting the arrived party to preserve a table for the chance of having a larger group in the future that generate bigger revenue than smaller parties. In addition, since there are multiple classes of tables, a manager must be careful to determine in which table to serve the party to maximize revenue. Therefore, this project develops a dynamic programming model that can be used to generate a seating scheme to increase revenue.

A set of assumptions are made to model the restaurant operation. It is assumed that table sizes are only even numbers for simplicity, and tables cannot be merged or divided. Similar to Kuo (2010), the time horizon is discretized by sufficiently small enough periods so that only one party arrives at a period. Arrivals of customers and the size of them for a given period are random events, so a modified Poisson distribution is used to model the arrivals. State space is represented as a tuple that has the length of the number of table sizes, so each element indicates the number of occupied tables of each size.

At the end of a period, the state will change according to the departures of parties have done their meals. The departures occur probabilistically, and a binomial distribution is used to model it. It is assumed that departure probability is the same across all sizes of parties, while it is possible to set up the different probability for sizes, such as a larger group has a lower probability than a smaller group for simplicity.

## *2.1. Assumptions*

- Operation hours are discretized into multiple small periods by some unit time.
- For a given period, customer arrivals occur at most once at the beginning of the period.
- Operation hours can be discretized by different unit times. For example, lunch and evening hours may be discretized finer than others so more parties arrive during that hours.
- Table sizes are even numbers.
- Overbooking is not allowed so that the number of occupied tables cannot exceed fixed capacity
- Tables cannot be merged or splitted.
- The probability of a table becoming available is same for all sizes of tables.
- The restaurant does not accept reservations.
- Queueing up is not allowed.

## 2.2. Data

- $n := \{1, 2, \dots, N\}$  period
- $p := \{0, 1, \dots, P\}$  party size.
- $t := \{2, 4, \dots, T\}$  table size, and  $T \geq P$ .
- $R_p := rp$  the revenue by a party of size  $p$ , which is a linear function of  $p$  with a coefficient  $r$ .
- $c_t :=$  the total number of tables (i.e. capacity) of size  $t$ .
- $q :=$  the probability of a table becoming available (i.e., a party leave the restaurant).
- $\alpha :=$  a penalty factor for wating table capacity
- $\lambda :=$  the average size of arriving parties.

## 2.3. Probability Distribution of Arrivals

Let  $A$  be the random variable of size of an arriving party for a period. If size is 0, it means there is no arrivals for that period. In this paper,  $A$  is modeled by a modified Poission distribution having  $p$  as a domain by adjusting the constant so that the probability distribution sums up to 1. Hence, its probability mass function is

$$P(A = p) = \frac{P!}{e^\lambda \Gamma(P+1, \lambda)} \frac{\lambda^p}{p!},$$

where  $\Gamma$  is the incomplete gamma function defined by

$$\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt.$$

We interpretate the number of arrivals for a given period as size of a party. The advantage of this approach is that we can control the tendency of arrival sizes by changing the parameter of  $A$ ,  $\lambda$ . It seems resonable to increase  $\lambda$  for the periods of evening hours to reflect that large parties more likely arrive in the evening.

## 2.4. State Space

For period  $n$ , let  $X_n$  be the tuple of the length of the number of table sizes. The first element of  $X_n$  is  $x_2$  that is the number of occuiped tables of size 2, the second element  $x_4$  is for the num-

ber of occupied tables of size 4, and so forth. Furthermore, each element of  $X_n$  is bounded by corresponding capacity. In other words,

$$X_n := (x_2, x_4, \dots, x_T) \leq (c_2, c_4, \dots, c_T).$$

## 2.5. State Transformation

At the end of each period, parties leave the restaurant with probability  $q$ , which causes the state transformation, so we need to model the departures of customers to express the state transformation.

A party of an arbitrary size  $p$  finishes their meals and leave the restaurant with a probability  $q$ . It means that the probabilities of the departures of customers are the same as  $q$  regardless of part sizes, so the probabilities of arbitrary sized tables becoming available are also  $q$ . The Binomial distribution is a natural choice for the random variable  $D_t$  that is the number of the departures of parties have occupied size  $t$  tables out of  $x_t$  at the end a period. Its probability mass function is

$$P(D_t = d_t) = \binom{x_t}{d_t} q^{d_t} (1 - q)^{x_t - d_t}.$$

Since we deal with multiple sizes of tables, we need a joint probability

$$P(D_2 = d_2, D_4 = d_4, \dots, D_T = d_T)$$

to model all the becoming available tables at the end of a period. It is reasonable to assume that  $D_t$ 's are independent each other, because customers usually work on their meals independently and do not have effects on other's meal duration. Therefore, the joint probability is

$$P(D_2 = d_2, D_4 = d_4, \dots, D_T = d_T) = \prod_{t=\{2, \dots, T\}} P(D_t = d_t) = \prod_{t=\{2, \dots, T\}} \binom{x_t}{d_t} q^{d_t} (1 - q)^{x_t - d_t},$$

which is the probability of  $X_n$  becoming  $X_{n+1}$  such that

$$X_{n+1} = (x_2 - d_2, x_4 - d_4, \dots, x_T - d_T).$$

## 2.6. Action Space

Action space in the stated restaurant operations consists of two stages. First, it has to be decided whether to accept or refuse the customer request. Next, a floor manager needs to determine at which table to serve the party. These actions are only possible when there are available tables large enough to serve the arrived party, and if the arrival occurs.

If there are no tables large enough to serve the party, then only options for the manager is to reject the request of the party. In addition, if the arrival does not occur, the restaurant has no options to choose.

## 2.7. Optimality Equation

Crucial parts of a dynamic programming model, the optimal value function, recurrence relation, and boundary conditions, are presented in this section. Let  $V_n(X_n)$  be the maximum revenue from the beginning of period  $n$  to the end of the last period,  $N$ . Then the recurrence relation for the backward dynamic programming formulations is:

$$V_n(X_n) = \sum_{p=1}^P P(A=p) \max \left\{ \begin{aligned} &\max_u \{ R_p + \alpha(u-p) + \sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2=d_2, \dots, D_T=d_T) V_{n+1}(X'_{n+1}) \} \\ &\sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2=d_2, \dots, D_T=d_T) V_{n+1}(X''_{n+1}) \end{aligned} \right. \\ + P(A=0) \sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2=d_2, \dots, D_T=d_T) V_{n+1}(X''_{n+1}),$$

where

$$u \in \{t : t \geq p, x_t < c_t\},$$

$$X'_{n+1} = (x_2 - d_2, \dots, x_u + 1 - d_u, \dots, x_T - d_T), \text{ and}$$

$$X''_{n+1} = (x_2 - d_2, \dots, x_T - d_T).$$

Interpretation of the recurrence relation is as follow. At the beginning of a period, a floor manager first chose to whether accept or not the arrived party, and if he/she accept the party, then the follow-up decision is table size to serve the customers. At the end of the period, some guests leave the restaurant. The expected maximum revenue of the given period is the sum of imme-

diate revenue according to the decision structre stated earlier and the future expected maximum revenue.

Boundary conditions are need for the case when there is not available table for arrived party of size  $p$ .  $u = \emptyset$  in this case. Then, in the reccrence relation,

$$\begin{aligned} \max & \begin{cases} \max_u \{R_p + \alpha(u - p) + \sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2 = d_2, \dots, D_T = d_T) V_{n+1}(X'_{n+1})\} \\ \sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2 = d_2, \dots, D_T = d_T) V_{n+1}(X''_{n+1}) \end{cases} \\ &= \sum_{d_T=0}^{x_T} \dots \sum_{d_2=0}^{x_2} P(D_2 = d_2, \dots, D_T = d_T) V_{n+1}(X''_{n+1}). \end{aligned}$$

At the beginning of last period, the restaurant always accept a request of any party regardless of its size without the penalty of seating a party at a bigger table if there are available tables. Unless, the restaurant has no choice but to turn down the request. To model this senario, an indicator function  $\mathbb{I}(p, X_N)$  is introduced to make revenue 0 when there are no available tables for party size  $p$  at period  $N$ .  $\mathbb{I}(p, X_N)$  is 1 if there exist  $t \in \{2, 3, \dots, T\}$  such that  $t \geq p$  and  $x_t < c_t$  for  $x_t \in X_N$ , otherwise 0. Then,

$$V_N(X_N) = P(A = p) R_p \mathbb{I}(p, X_N).$$

### 3. Solution Procedure

#### 3.1. Seating Scheme

#### 3.2 Computational Effort

### 4. Computational Experiments

Written by Python 3.7.1

### 5. Conclusions and Future Research



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