problem_6

April 9, 2020

1 Problem 6, MCMC Sampling

1.1 Problem a)

[132]:
```
import numpy as np
np.random.seed(0)
```

[24]:
```
def gen_sample():
    r = np.random.rand(1)
    if r > 0.5:
        return np.random.normal(5, 1)
    else:
        return np.random.normal(-5, 1)

def gen_samples(n):
    samples = np.array([gen_sample() for i in range(n)])
    return samples
```

[27]:
```
num_x_samples = 100
x_samples = gen_samples(num_x_samples)
```

1.2 Problem b)

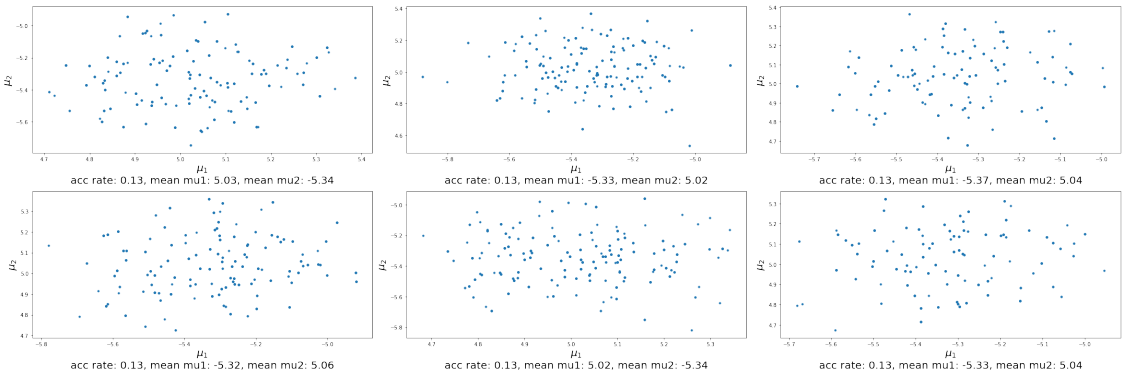
[34]:
```
def cal_acceptance_rate(x_samples, mu1, mu2, mu1_h, mu2_h, sigma):
    alpha = cal_log_posterior(x_samples, mu1_h, mu2_h) -␣
 ↪cal_log_posterior(x_samples, mu1, mu2)
    alpha = np.exp(alpha)

    # We do not need to calculate the transition probability,
    # since it is 1 always in this case.
    transition_prob = 1
    alpha = alpha * 1

    return alpha

def cal_log_posterior(x_samples, mu1, mu2):
    # log p(mu1, mu2 | x)
```

```
    log_prob_mus = -1 * (mu1 ** 2 + mu2 ** 2) / 200

    # p(x | mu1, mu2)
    prob_x_1 = 0.5 * np.exp(-0.5 * (x_samples - mu1) ** 2)
    prob_x_2 = 0.5 * np.exp(-0.5 * (x_samples - mu2) ** 2)
    prob_x = prob_x_1 + prob_x_2

    return np.sum(np.log(prob_x)) + log_prob_mus

def metropolis_hastings_sampling(x_samples, num_samples, sigma):
    # Initialize mu1 and mu2 be 0 and 0
    mu1, mu2 = 0, 0
    mu1s, mu2s = np.zeros(num_samples), np.zeros(num_samples)
    accept_count = 0

    # Run Metropolis Hastings Sampling
    for n in range(num_samples):
        mu1_h = np.random.normal(mu1, sigma)
        mu2_h = np.random.normal(mu2, sigma)
        alpha = max(0, min(1, cal_acceptance_rate(x_samples, mu1, mu2, mu1_h,␣
 ↪mu2_h, sigma)))
        if np.random.rand(1) < alpha:
            mu1 = mu1_h
            mu2 = mu2_h
            accept_count += 1
        mu1s[n] = mu1
        mu2s[n] = mu2

    acceptance_rate = accept_count / num_samples
    return mu1s, mu2s, acceptance_rate
```

[74]:
```
import matplotlib.pyplot as plt

num_samples = 11000
mean_mu1s = {}
mean_mu2s = {}
acc_rates = {}

for sigma in [0.5, 5]:
    print('sigma = {}'.format(sigma))

    mean_mu1s[sigma] = []
    mean_mu2s[sigma] = []
    acc_rates[sigma] = []

    plt.figure(figsize=(30, 10))
    for i in range(6):
```

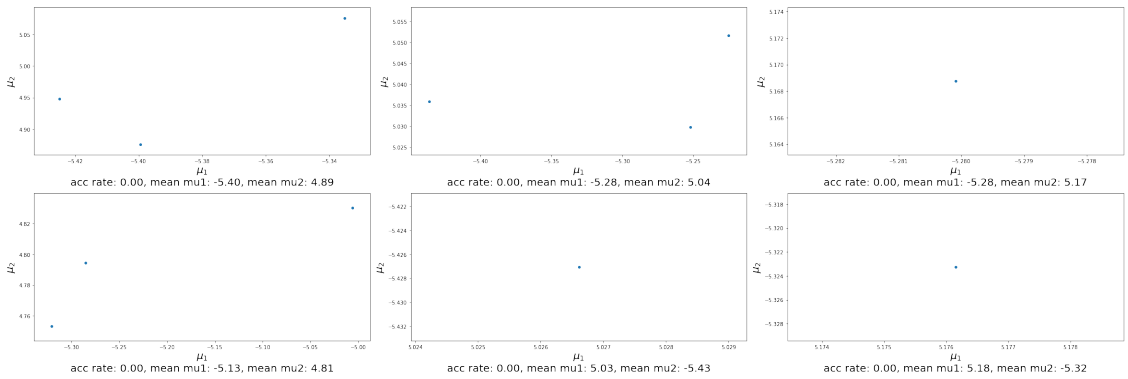
```
        # Run Metropolis Hastings Sampling
        mu1s, mu2s, acceptance_rate = metropolis_hastings_sampling(
                                        x_samples,
                                        num_samples,
                                        sigma
                                      )
        # Discard the first 10000 samples
        mu1s = mu1s[10000:]
        mu2s = mu2s[10000:]

        # Plot the next 1000 samples of mu1 and mu2
        plt.subplot(231 + i)
        plt.scatter(mu1s, mu2s, s=10)
        plt.xlabel(r'$\mu_1$' + '\nacc rate: %.2lf, mean mu1: %.2lf, mean mu2:␣
↪%.2lf' % (acceptance_rate, np.mean(mu1s), np.mean(mu2s)), fontsize=20)
        plt.ylabel(r'$\mu_2$', fontsize=20)
        plt.tight_layout()

        # Record the data
        mean_mu1s[sigma].append(np.mean(mu1s))
        mean_mu2s[sigma].append(np.mean(mu2s))
        acc_rates[sigma].append(acceptance_rate)

    plt.show()
```

sigma = 0.5



sigma = 5

```
/Users/haekyu/anaconda3/envs/massif/lib/python3.7/site-
packages/ipykernel_launcher.py:3: RuntimeWarning: overflow encountered in exp
  This is separate from the ipykernel package so we can avoid doing imports
until
```



[77]:
```
import pandas as pd

sigma = 0.5
df_1 = pd.DataFrame()
df_1['mean_mu1'] = mean_mu1s[sigma]
df_1['mean_mu2'] = mean_mu2s[sigma]
df_1['acceptance rate'] = acc_rates[sigma]

df_1
```

[77]:

	mean_mu1	mean_mu2	acceptance rate
0	5.025064	-5.340499	0.131000
1	-5.332442	5.021185	0.131091
2	-5.365892	5.035718	0.130727
3	-5.316841	5.056698	0.127909
4	5.019107	-5.335424	0.134545
5	-5.333710	5.039708	0.127909

[79]:
```
sigma = 5
df_2 = pd.DataFrame()
df_2['mean_mu1'] = mean_mu1s[sigma]
df_2['mean_mu2'] = mean_mu2s[sigma]
df_2['acceptance rate'] = acc_rates[sigma]

df_2
```

[79]:

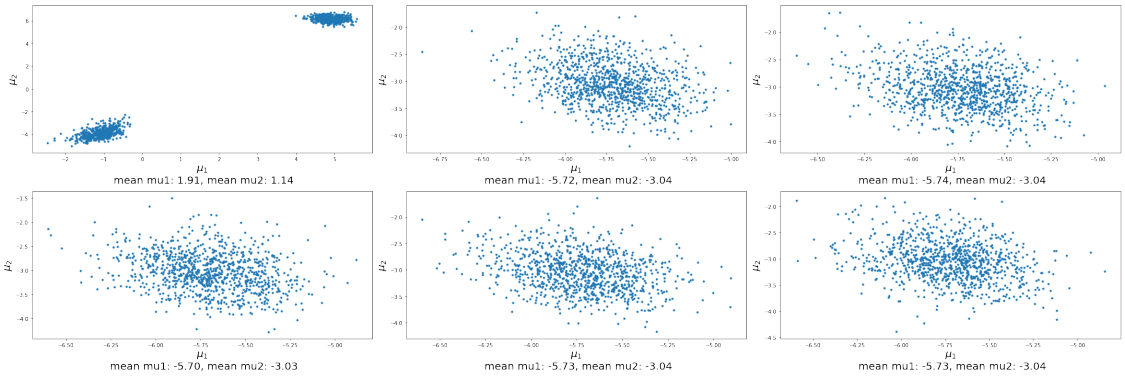
	mean_mu1	mean_mu2	acceptance rate
0	-5.401915	4.894947	0.002273
1	-5.277091	5.037582	0.001455
2	-5.280091	5.168751	0.002182
3	-5.128976	4.807459	0.003273
4	5.026610	-5.427057	0.001909
5	5.176153	-5.323255	0.002182

1.3 Problem c)

[156]:
```
def get_z_samples(x_samples, mu1, mu2):
    z = np.zeros(len(x_samples), dtype=np.int8)

    prob = np.exp(-0.5 * (x_samples - mu1) ** 2) / (np.exp(-0.5*(x_samples -␣
 ↪mu2) ** 2) + np.exp(-0.5 * (x_samples - mu1) ** 2))
    us = np.random.rand(len(z))

    # Mark that it represents data from mu1
    z[us < prob] = 1

    # Mark that it represents data from mu2
    z[us >= prob] = 2

    return z

def gibbs_sampling(x_samples, num_samples):
    # Initialize Gibbs sampling
    mu1, mu2 = 0, 0
    mu1s, mu2s = np.zeros(num_samples), np.zeros(num_samples)

    # Run Gibbs Sampling
    for n in range(num_samples):
        z = get_z_samples(x_samples, mu1, mu2).copy()

        mu1 = np.sum(z == 1)
        mu2 = np.sum(z == 2)

        mu1_h = 100 * np.sum((z == 1) * x_samples) / (100 * mu1 + 1)
        mu2_h = 100 * np.sum((z == 1) * x_samples) / (100 * mu2 + 1)

        mu1 = np.random.normal(mu1_h, np.sqrt(200 / (100 * mu1 + 1)))
        mu2 = np.random.normal(mu2_h, np.sqrt(200 / (100 * mu2 + 1)))

        mu1s[n] = mu1
        mu2s[n] = mu2

    return mu1s, mu2s
```

[162]:
```
num_samples = 11000
gibbs_mean_mu1s = []
gibbs_mean_mu2s = []

plt.figure(figsize=(30, 10))
for i in range(6):
    # Run Gibbs Sampling
    gibbs_mu1s, gibbs_mu2s = gibbs_sampling(x_samples, num_samples)
```

```
    # Discard the first 10000 samples
    gibbs_mu1s = gibbs_mu1s[10000:]
    gibbs_mu2s = gibbs_mu2s[10000:]

    # Plot the next 1000 samples of mu1 and mu2
    plt.subplot(231 + i)
    plt.scatter(gibbs_mu1s, gibbs_mu2s, s=10)
    plt.xlabel(r'$\mu_1$' + '\n mean mu1: %.2lf, mean mu2: %.2lf' % (np.
↪mean(gibbs_mu1s), np.mean(gibbs_mu2s)), fontsize=20)
    plt.ylabel(r'$\mu_2$', fontsize=20)
    plt.tight_layout()

    # Record the data
    gibbs_mean_mu1s.append(np.mean(gibbs_mu1s))
    gibbs_mean_mu2s.append(np.mean(gibbs_mu2s))

plt.show()
```



[163]:
```
df = pd.DataFrame()
df['mean_mu1'] = gibbs_mean_mu1s
df['mean_mu2'] = gibbs_mean_mu2s

df
```

[163]:

	mean_mu1	mean_mu2
0	1.912066	1.142613
1	-5.724099	-3.042464
2	-5.738514	-3.035543
3	-5.704386	-3.032615
4	-5.725242	-3.040966
5	-5.728205	-3.044541

[ ]: