

150B/355B
Introduction to Machine Learning for Social Science
TA Section 4

Haemin Jee and Tongtong Zhang

February 2, 2018

1 Overfitting

Road Map

- 1 Overfitting
- 2 Lasso Regression

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 - Cross Validation

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 - Bias-Variance Tradeoff

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- 1 Overfitting
- 2 Lasso Regression
 - Cross Validation
 - Bias-Variance Tradeoff
- 3 R code in lectures this week

Document-Term Matrices

		Word1	Word2	Word3	...	WordP
$\mathbf{X} =$	Doc1	1	0	0	...	3
	Doc2	0	2	1	...	0
	\vdots	\vdots	\vdots	\ddots	\vdots	
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Let $p = (\text{Pr}(\text{Desk}_i = 1))$

$$\text{logit}(p) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_P X_P$$

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When overfitting happens, small perturbation in the training data can lead to substantial change in your model coefficients and thereby, lead to substantial change in your the predictions on the test set.

Overfitting

Reasons for overfitting:

Overfitting

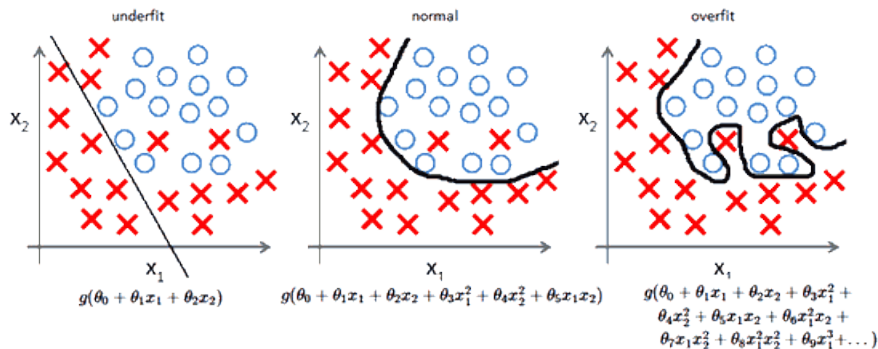
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Suppose we have two highly correlated predictors, X_1 and X_2 .

```
base<-1:100
set.seed(12345)
X1<-base+rnorm(100,0,0.01)
#both x1 and x2 have the same base with a little noise
X2<-base+rnorm(100,0,0.01)
cor(X1,X2)

## [1] 0.9999999
```

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```
set.seed(1234)
Y<-X1+X2+rnorm(100,0,1) #1st random sample
lm(Y~X1+X2)$coefficients
```

```
## (Intercept)          X1          X2
## -0.6004386    4.0852277  -2.0765643
```

```
set.seed(123456) # at a different seed, error term is different
Y<-X1+X2+rnorm(100,0,1) #2nd random sample with small changes in Y
lm(Y~X1+X2)$coefficients
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## (Intercept)          X1          X2
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With only small perturbation in the error term (Y), we got very different coefficients!

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Overcoming Overfitting \rightsquigarrow LASSO Regression

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Linear Regression: Choose β 's to minimize sum of squared residuals

$$\beta_{\text{OLS}} = \operatorname{argmin}_{\beta} \sum_{i=1}^N (Y_i - \beta \cdot \mathbf{x}_i)^2$$

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LASSO Regression: Choose β 's to minimize sum of squared residuals, **subject to a constraint on coefficients:**

$$\beta_{\text{LASSO}} = \operatorname{argmin}_{\beta} \sum_{i=1}^N (Y_i - \beta \cdot \mathbf{x}_i)^2, \text{ subject to } \sum_{p=1}^P |\beta_p| \leq t$$

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$$\beta_{\text{LASSO}} = \operatorname{argmin}_{\beta} \sum_{i=1}^N (Y_i - \beta \cdot \mathbf{x}_i)^2 + \lambda \sum_{p=1}^P |\beta_p|$$

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$$\begin{aligned}\hat{\beta}^{\lambda} &= \text{Coefficients at } \lambda \\ \hat{p}_{i,\lambda} &= \Pr(Y_i = 1 | \mathbf{X}_i, \hat{\beta}^{\lambda}) \rightarrow \text{Prediction}\end{aligned}$$

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In practice, we use cross-validation to complete these procedures

LASSO Regression in Practice

K-fold Cross Validation

Iteration	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
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Common K's: 5-fold, 10-fold, N (LOOCV)

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As we add more features to the model,

- Bias decreases \rightarrow we have perfect in-sample fit

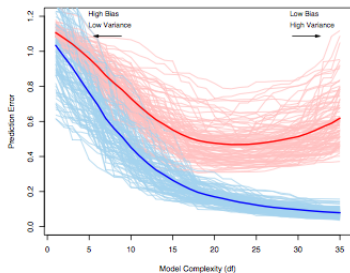


FIGURE 7.1. Behavior of test sample and training sample error as the model complexity is varied. The light blue curves show the training error \bar{err} , while the light red curves show the conditional test error Err_T for 100 training sets of size 50 each, as the model complexity is increased. The solid curves show the expected test error Err and the expected training error $E[\bar{err}]$.

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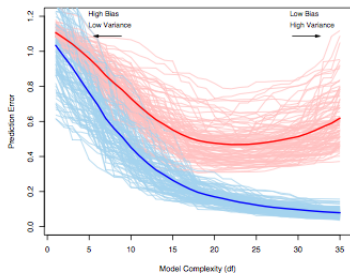


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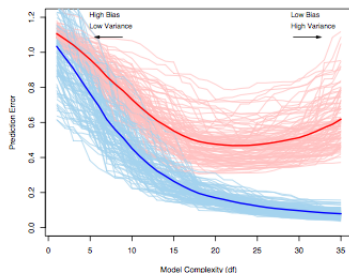


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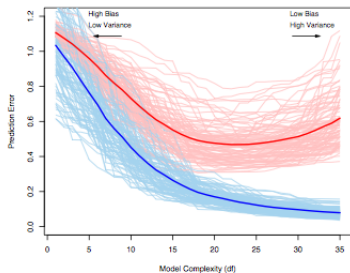


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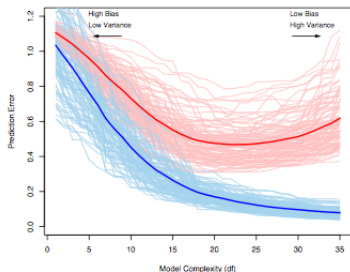


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 - Reduce variance by minimizing MSE on out-of-sample data.

R code!

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 - 3 We have our model!