Counterparty Credit Risk and CVA under Basel III

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The

- latest version of this document
- additonal resources
- examples

may be found on

`https://github.com/haenerconsulting/basel3cva`

Outline

Motivation

Definition (Credit Risk)

Risk to incur a loss due to counterparty's default or loss of creditworthiness.

Credit risk measures are introduced to

- Quantify the credit risk
- Help Mitigating that risk

Motivation

Risk measures are estimated by Credit Risk Models

Impact of Credit Risk Model

Trading activity limits set by *PE*

Capital charges regularity capital dependent of *EEPE*

P&L *EE* enters CVA/DVA

Motivation

Challenges

Measure	Asking the right question
Model	Picking right model to estimate measure
Act	Make descisions based on measurements

Overview

Likeliness vs Severity of Credit Events

Categories

Which dimensions to consider?

Severity	How much will we lose?
Likeliness	What's the chance that we lose?
Granularity	What does the measure refer to?

Granularity of Measure

Based on Defaults

- All Counterparties
- Single Counterparty

Other Aggregations

- Global/macro economic
- Sector, country
- Trade

Exposure and Recovery

How to measure severity? Need to value trade:

Definition (Exposure at Default)

$$\begin{aligned} \mathrm{EAD}(t) &= \max 0, p(t)|\tau = t \\ \tau &: \text{time at which CP defaults} \end{aligned}$$

Definition (Loss Given Default)

$$\text{Loss at time } t = \mathrm{LGD}(t)\mathrm{EAD}(t)$$

Definition (Recovery)

$$R(t) = 1 - \mathrm{LGD}(t)$$

Valuation Approaches

- **Accrual** Banking book; rarely adjust; illiquid assets
- **Mark to market** Trading book; frequently adjusted; traded assets
- **Mark to model** Trading book; frequently adjusted; complex structures

Example

`CreditRiskMeasures.xlsx`

Accrual

Loan to Acme Ltd

- value is face value
- maximal loss is notional of loan

Mark to market

Buy bond of Acme Ltd; assume liquid market

- value is mark to market of bond
- value lower than in risk-free valuation

Severity

Mark to model

Exotic interest rate swap with Acme Ltd. What is the value

- risk free: assuming Acme may never default
- risky: Acme may default
- risky with own risk: Acme and we may default

Forward Looking Measures

Assess exposure in future → model how state of the world evolves

Deterministic Evolution Scenario Analysis, Stress testing

Stochastic Evolution Model for risk factors

Scenario Analysis/Stress Test

Meanings of Stress

- change model parameters $\rightarrow$
- pick a single path $\rightarrow$ degenerate measure (Dirac measure)

Unified handling by Measure Transforms

Stochstic Process (Langevin Equation)

$$\dot{x} = f(\boldsymbol{x}) + \boldsymbol{\xi}$$

$\longleftrightarrow$

Probabiliy Measure (Path Integrals)

$$\begin{array}{rcll} \Gamma & : & t \to \boldsymbol{x}(t) & \text{Path} \\ \mu(\Gamma) & \sim & e^{-S(\Gamma)} & \\ S(\Gamma) & \equiv & \int_0^\tau L(\boldsymbol{x}, \dot{\boldsymbol{x}}) & \text{Action} \\ L(\boldsymbol{x}, \dot{\boldsymbol{x}}) & : & \text{Lagrangian} & \end{array}$$

Dual Model Representations

Types of Stress Tests

Approaches

- give economic scenario
- given loss (inverse stress)

Inverse stresses

Definition (Potential Future Exposure)

$$\begin{aligned} \mathrm{PFE}(t) &= \max 0, p(t)|\tau = t \\ \tau &: \text{time at which CP defaults} \end{aligned}$$

Definition (Expected Exposure (EE))

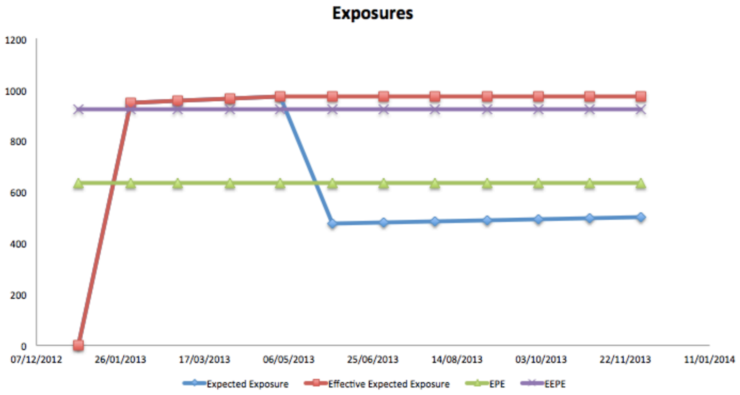
$$\mathrm{EE}(t) = \mathbb{E}[\mathrm{PFE}]$$

Definition (Expected Positive Exposure)

$$\mathrm{EPE}(T) = \frac{1}{T}\int_0^T \mathrm{EE}(t)\, dt$$

Definition (Effective Expected Exposure (EEE))

Maximum of EPE and past EEE: never decreasing.



Definition (Losses across Netting Sets)

$$L(t) = \sum_a \chi_{\tau_a \le t} \mathrm{LGD}_a \max 0, p_a(\tau_a)$$

a : Identifier of netting set

Meaningful risk measures for portfolios

Definition (Coherent Risk Measure)

Risk measure ρ: for portolio X:

Normalization $\rho(\emptyset) = 0$ empty portfolio has no risk

Monotonicity $X_1 \leq X_2 \rightarrow \rho(X_1) \geq \rho(X_2)$

Sub-additivity $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ diversification/netting

Homogeneity $\rho(\alpha X) = \alpha\rho(X) \quad \alpha > 0$

Translation invariance $\rho(X + a) = \rho(X) - a$ adding cash a reduces risk

Portfolio Measures

Quantile

q% quantile: value, for which q% of outcomes are smaller/larger. Quantiles are not coherent measures.

Expected Shortfall

Expected loss conditioned on the loss being larget than X. The Expected Shortfall (Mean Excess Loss) is a coherent measure.

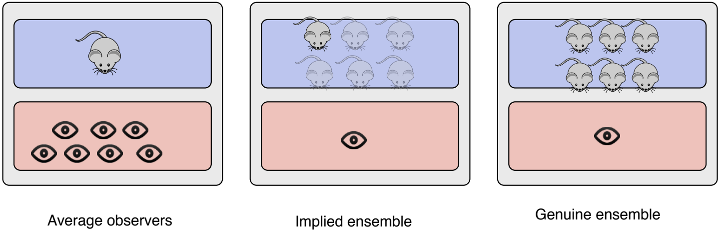
Example

`PortfolioMeasure.xlsx`

Example

`LikelihoodExperiment.xlsx`

What does probability mean?



Probability and Measurement

Need to define

- Ensemble
- Measurement process

Examples

Genuine Ensemble

- Mathematics
- Physics: Identically prepared experiment

Average observers

Consensus of observers:

- Market prices
- Betting quota

Implied Ensemble

Equivalence classes:

- Names with same rating
- Price returns in different time windows

Measures for Probability of Default

Definition (Survival/Default Probability, Default Intensity)

Let τ be time of default

$$
\begin{aligned}
S(t) &= p(\tau > t) \\
S &: \text{survival probability} \\
S(t) &= e^{-\lambda(t)t} \\
\lambda &: \text{term default intensity} \\
D(t) &= 1 - S(t) \\
D &: \text{default probability}
\end{aligned}
$$

Note: D is a CDF!

Forward Intensity

Forward default intensity

Probability $d(t)$ of defaulting between t and dt:

$$d(t) = \frac{dD(t)}{dt} \tag{1}$$

Estimating λ

Credit Rating Typically using historical data

Market Prices Current credit spreads from bonds or CDS

Implied Default Intensity

Let $s(t)$ be a credit spread

$$\begin{aligned} s(t) &= (1-R)\lambda(t) \\ R &: \text{recovery rate} \end{aligned}$$

Unifiying Severity and Frequency Measures

High Severity/Low Frequency vs. Low Severity High Frequency

How to compare

- Single large deal with good counterparty
- Set of small deals with bad counterparties

Answer

Pricing including credit risk allows comparing!

Approaches

Top-down vs Bottom-up

Top-down	Pricing from first principles
Bottom-up	Calculate price correction from building blocks: Exposure (EE) and PE, LGD

Bottom-Up approach

Assumptions

- Risk-free prices known
- Calculate EE
- Estimate PE, LGD
- Calculate correction to risk-free price

Riskiness of counterparty reduces the price:

Definition (CVA)

Risky price p_A^* as seen from counterparty A with counterparty B:

$$\begin{aligned} p^* &= p - \mathrm{CVA}_B \\ p &: \text{risk-free price} \\ CVA_B &: \text{Credit Valuation Adjustment for counterparty } B \end{aligned}$$

Does credit risk of counterparty A also affect price?

Definition (DVA)

Price p_A as seen from counterparty A with counterparty B:

$$\begin{aligned} p* &= p - \mathrm{CVA}_B + \mathrm{DVA}_A \\ p &: \text{risk-free price} \\ DVA_A &: \text{Debit Valuation Adjustment for counterparty } A \end{aligned}$$

DVA increases the price.

Accounting vs. Regulatory

DVA must be used for P&L but not for regulatory capital.

BCBS 189, paragraph 89:

$$CVA = (LGD_{MKT}) \cdot \sum_{i=1}^{T} Max\left(0; exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}\right)$$

Regulatory CVA

Similar to regulatory capital charge for default:
Assumes independence of exposure and default process.

$$\mathrm{CVA} = \int_0^T (1 - R) Df(t) EE(t) d(t)\, dt$$

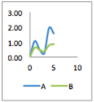
where d is the default probability from equation (1), Df discount factor

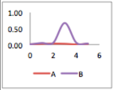
Example

`CVA.xlsx`

CVA

t	A		B	
	EE			
0	0.00		0.00	
1	1.07		0.63	
2	0.51		0.48	
3	0.25		0.39	
4	1.94		0.78	
5	1.58		0.87	
t	lambda	default prob	lambda	default prob
0	0.01	0.00	0.04	0.00
1	0.02	0.02	0.05	0.05
2	0.025	0.03	0.055	0.06
3	0.025	0.02	0.5	0.67
4	0.02	0.00	0.45	0.06
5	0.02	0.02	0.4	0.03
	CVA			
	0.41			0.06
	DVA			
	0.06			0.41





Regulatory vs Trading CVA

Regulatory	Historic measure for EE, implied for PD
Trading	Both EE and PD in implied measure

Pricing for Portfolio of Netting Sets

As for single netting sets: pricing combines severity and likeliness. Requires knowing

- prices of individual netting sets at default
- probability of default $P(\chi_{\tau_1 \leq t_1}, \chi_{\tau_2 \leq t_2}, \dots, \chi_{\tau_N \leq t_N})$

Additional useful quantity: in terms of total losses:

Definition (Loss distribution)

$$\mathcal{L}(l, t) = P(L(t) \geq l)$$

Granularity of Measure in Regulatory Context

Metrics used for Regulatory Purposes

Focus on measures for individual counterparties. No proper modelling of collective losses required.

Overview

- realized losses from default of counterparty
- P&L fluctuation from change in credit spreads of counterparty

Netting Agreement

Definition (Netting Set)

For all trades within a netting set long and short positions may be netted. The exposure is reduced due to

$$\max \sum_i p_i, 0 \leq \sum_i \max p_i, 0$$

Example

A short and a long position worth 10 Mios with counterparty acme yield an exposure of 10 Mio w/o netting agreement and 0 with.

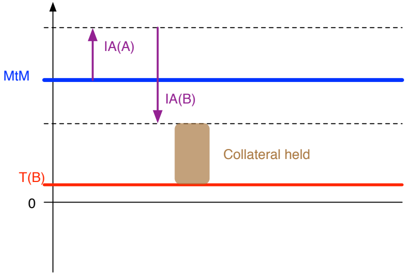
ISDA Credit Support Annex (CSA) : collateral that must be delivered between the parties

Definition

- the Secured Party's Exposure plus
- the aggregate of all Independent Amounts applicable to the Pledgor, minus
- all Independent Amounts applicable to the Secured Party, if any, minus
- the Pledgor's Threshold

The Secured Party is the party that is holding collateral; the Pledgor is the party that has delivered collateral

CSA



- At default losses offset by collateral
- Rebalancing needed
- Collateral price may move

Re-hypothecation

By Ruby Lian and Fayen Wong
SHANGHAI | Sun Sep 16, 2012 3:07pm EDT

(Reuters) - Chinese banks and companies looking to seize steel pledged as collateral by firms that have defaulted on loans are making an uncomfortable discovery: the metal was never in the warehouses in the first place.

- Counterparty may repo the collateral $\rightarrow$ when defaulting collateral gone
- Forbidding may increase costs
- Transferring collateral management to central counterparty

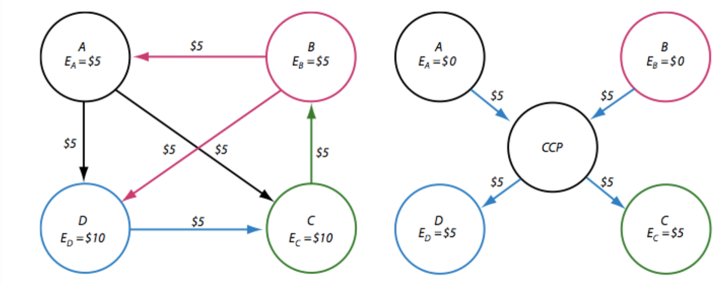
Gap Risk

Collateral process is discreet, e.g. daily.

- position value
- collateral value

may move

Central Counterparties



- Counterparty to every trade
- Multiateral netting $\rightarrow$ reducing risk
- Potentially mutualizing credit losses among participants
- Increased transparency

Spread vs Default Risk

Definition (Default Risk)

Risk of changes in price p induced by hitting default time τ:

$$\begin{aligned} t &\rightarrow \tau \\ p &\rightarrow p + \Delta_\tau p \\ \Delta_\tau p &= (R-1)p \end{aligned}$$

Definition (Spread Risk)

Risk of changes in price p induced by changes of counterparty credit spread s:

$$\begin{aligned} s &\rightarrow s + \Delta s \\ p &\rightarrow p + \Delta_s p \\ \Delta_s p &= \frac{\partial p}{\partial s}\Delta s \end{aligned}$$

Loans

Default Risk	Single name CDS or short bond	
Spread Risk	MtM accounting	Single name CDS/bond, index or proxy
	Accrual accounting	N/A

Hedging Instruments and Strategies

Static Hedges

Insurance for replacement of some asset on default:

- **Bond**: hedged by Credit Default Swap
- **Swap, CCYSwap**: Contingent Credit Default Swap (CCDS)
- **Netting sets**: insurance from CVA desk

Dynamic Hedges

Hedging default or spread risk: use instruments with price q. First order:

- **Default**: $\Delta_\tau q = -\Delta_\tau p$
- **Spread**: $\Delta_s q = -\Delta_s p$

Analogy Hedging Credit Risk for Bonds

Granularity	*Underlying*	*Hedge*
Single	Bond Netting Set	CDS CCDS
Portfolio	Bonds Netting Sest	CDO "CCDO"

Hedging losses across netting sets:

CDO on CVA

- bond↔exposure of netting set
- tranches like CDO

Issues

- notional of "bond"s fluctuating
- "bond" not transparent
- who rates the "bond"s and the CDO?

Systemic Risks

In distressed market environment correlations increase $\rightarrow$ less diversification.

- Macro hedges
- Buying insurance won't work ($\rightarrow$ Monolines)

Limit Monitoring

Limits

Manage size of exposure per

- counterparty
- industry
- region

Limits may typically expressed in terms of

Quantiles E.g. 95% quantile: 5% of losses are larger than that quantile

Mean Express Loss E.g. Average of the 5% biggest losses

A term structure of limits may be reflect risk appetite.

Mutualization

Central Counterparties

Losses distributed among participants $\rightarrow$ diversification

Systemic risks?

Internal CVA Desk

Trading CVA

Trading desk within the firm:

- Sells default protection to other desks
- Hedges counterparty credit spread risk

Costs/Benefits

⊖ High build and run costs

⊕ More accurate metrics for credit risk

⊕ Increased transparency on costs

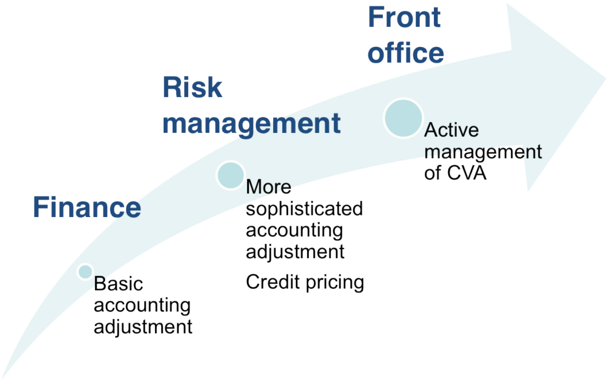
! Hegdges bought from CVA desk yield no regulatory capital relief

Mandate

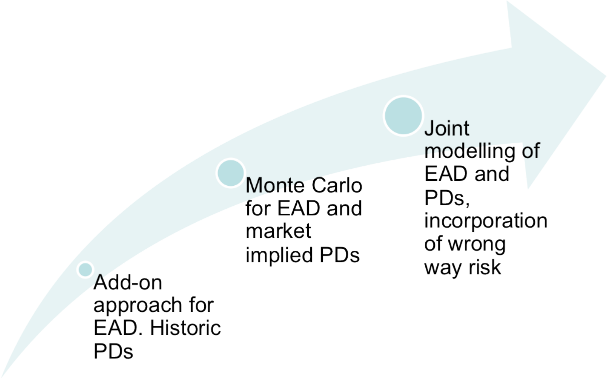
Depends on rationale for setting up

Risk management or front office function?

Responsibility often changes with sophistication . . .


Front office
Risk management
Finance
Active management of CVA
More sophisticated accounting adjustment
Credit pricing
Basic accounting adjustment

Approach	**Advantages**	**Disadvantages**
Profit Centre	• P&L enable performance to be measured • Easy to align renumeration with success and design incentives	• Potential conflicts of iterest • May lead to overactive position taking • Requires more infrastructure
Service Centre	• Requires less infrastructure • Less market activity • Less internal politics • Qualitative focus can lead to broader focus	• Difficult to measure performance • Harder to retain and incentivise appropriate staff

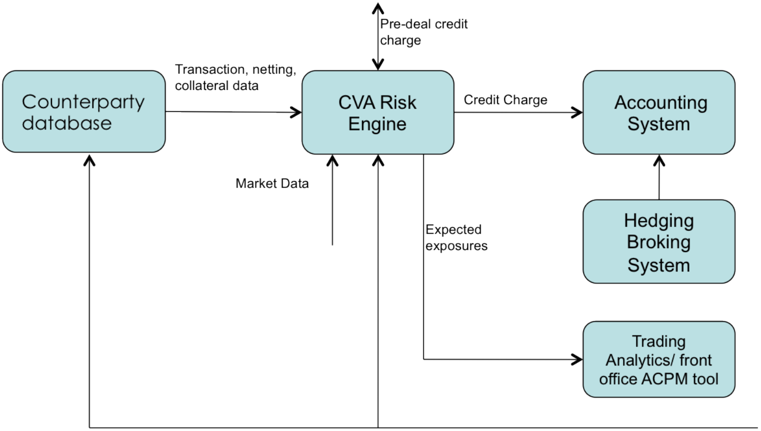


Add-on approach for EAD. Historic PDs
Monte Carlo for EAD and market implied PDs
Joint modelling of EAD and PDs, incorporation of wrong way risk

- Risk vs pricing models: Use same models?

  Pricing model Fit market prices

  Risk model Predict future outcomes
- Bilateral vs unilateral calculation: Include products w/o PFE (e.g. notes)
- Define proxy spreads when no liquid CDS market
- Wrong way risk


Pre-deal credit charge
Transaction, netting, collateral data
Counterparty database
CVA Risk Engine
Credit Charge
Accounting System
Market Data
Expected exposures
Hedging Broking System
Trading Analytics/ front office ACPM tool

Basic requirements same as Monte Carlo Credit Risk system for Risk/Regulatory capital

Additional Requirements

- Calulation of sensitivities
- Include wrong way risk in price
- Simulate in risk neutral calibration
- Integration with FO infrastructure and processes
- Attribution ability

Pre-trade approval/marginal CVA/DVA calculation:

- Needs to be fast
- Avoid recalculating whole portfolio: add new trades in scenario-consitent manner

Overview

Building Blocks

Model Building Process

Business Analysis Materiality, specification

Model choice Find adequate model

Software implementation Develop and roll out

Materiality

What to Model?

Which risk factors material for current portfolio?
How can we assess materiality without exposure model in place?

Approach

Simple estimation of exposure assuming

- future portfolio prices normally distributed
- estimation of first two moments

Need to estimate $\mathbb{E}[p(T)]$, $\mathbb{E}[p^2(T_i)]$ at some future times T: Performing Taylor expansion for price p around expected risk factor:

$$\begin{aligned} p(\mathbf{x}(T), T) &\approx p(\mathbf{x}_0(T), T) + \sum_i \frac{\partial p(\mathbf{x_0}(T), T)}{\partial x_i} \Delta x_i(T) \\ &\quad + \frac{1}{2} \sum_{ij} \frac{\partial^2 p(\mathbf{x_0}(T), T)}{\partial x_i \partial x_j} \Delta x_i(T) \Delta x_j(T) \\ \mathbf{x}_0(T) &\equiv \mathbb{E}[\mathbf{x}(T)] \\ \Delta x_i(T) &\equiv x_i(T) - x_{0,i}(T) \end{aligned}$$

Gaussian Approximation

The expectation value M of the price is hence

$$
\begin{aligned}
M(T) &\approx p(\mathbf{x}_0(T), T) + \frac{1}{2}\sum_{ij}\gamma_{ij}(T)\Omega_{ij}(T) \\
M(T) &\equiv \mathbb{E}[p(\mathbf{x}(T), T)] \\
\gamma_{ij}(T) &\equiv \frac{\partial^2 p(\mathbf{x_0}(T), T)}{\partial x_i \partial x_j} \\
\Omega_{ij}(T) &\equiv \mathbb{E}[\Delta x_i(T)\Delta x_j(T)]
\end{aligned}
$$

For the variance V we obtain up to second order in $\Delta\mathbf{x}$:

$$
\begin{aligned}
V(T) &\approx \sum_{ij}\delta_i(T)\delta_j(T)\Omega_{ij}(T) \\
V(T) &\equiv \mathbb{E}[(p(\mathbf{x}(T), T) - \mathbb{E}[p(\mathbf{x}(T), T)])^2] \\
\delta_i(T) &\equiv \frac{\partial p(\mathbf{x_0}(T), T)}{\partial x_i}
\end{aligned}
$$

What can we learn?

Risk factor contributions

Matrix elements $\Psi_{ij} = \delta_i(T)\delta_j(T)\Omega_{ij}(T)$ indicate contribution of risk factors ij to total variance.

EE, PE

Knowning mean and variance of the Gaussian distribution, any statistical quantity may be evalued.

Caveat

Depending on specifics of portfolio this approximation may be more or less accurate: that is why we use Monte Carlo simulations after all.

Practical Implementation

- For $t = 0$: δ and γ from Market risk system. **But:** need netting set level aggregation $\rightarrow$ deal level granularity
- For $t > 0$ estimate future δ, γ by bumping

- Represent trades/products
- Standardize for interoperability

Product represented by parameters

FpML


```xml
▼<FpML xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance" xmlns="http://www.fpml.or
 xsi:schemaLocation="http://www.fpml.org/2003/FpML-4-0 fpml-main-4-0.xsd">
  ▼<trade>
    ▼<tradeHeader>
      ▼<partyTradeIdentifier>
          <partyReference href="CHASE"/>
          <tradeId tradeIdScheme="http://www.chase.com/swaps/trade-id">921934</tradeId>
        </partyTradeIdentifier>
      ▼<partyTradeIdentifier>
          <partyReference href="UBSW"/>
          <tradeId tradeIdScheme="http://www.ubsw.com/swaps/trade-id">204334</tradeId>
        </partyTradeIdentifier>
        <tradeDate>2000-04-03</tradeDate>
      </tradeHeader>
    ▼<swap>
      ▼<!--
           Chase pays the floating rate every 6 months, based on 6M EUR-EURIBOR-Telerate
                 + 10 basis points, on ACT/360 basis
        -->
      ▼<swapStream>
          <payerPartyReference href="CHASE"/>
          <receiverPartyReference href="UBSW"/>
        ▼<calculationPeriodDates id="floatingCalcPeriodDates">
          ▼<effectiveDate>
              <unadjustedDate>2000-04-05</unadjustedDate>
            ▼<dateAdjustments>
                <businessDayConvention>NONE</businessDayConvention>
              </dateAdjustments>
            </effectiveDate>
          ▼<terminationDate>
              <unadjustedDate>2005-01-05</unadjustedDate>
```

Pro/Con

- ⊕ standardized
- ⊖ logic in client

Product represented by casflows

Payoff macros

Notional	100
DCF	Act/Act

Date	**Libor fixing**	**Payoff**
02-Mar-13	1.34%	0.335
02-Jun-13	1.19%	0.2975
02-Sep-13	1.42%	0.355
02-Dec-13	1.39%	0.3475

Pro/Con

- ⊕ simple
- ⊖ not expressive enough (just cash is exchanged)
- ⊖ single product (no interations)

Approach

Multi agent simulation:

Time Map wall clock to simulation time

Market Events simulation time to events

Transactions events to transactions (e.g. cashflows)

Execution execute events

Pro/Con

- $\oplus$ general
- $\oplus$ all business logic in model $\rightarrow$ easy tooling
- $\ominus$ expensive

Categories

Independent of product Relate to Mathemathics or Physics

Dependent of product Specific to product type

Dependent of portfolio and market Context

Coordinate Sytems

From Physics we know: dynamics must not depend on choice of coordinates $\rightarrow$ dimension analysis.

Interpolation

How to interpolate r, σ. Interpolate dimension-less quantities: rt and $\sigma^2 t$.

State Variables vs. Parameters

- **Liquidity** Hedge frequency, transaction costs, close-out period
- **Completeness** Unhedgeable risk, uniqueness of price

Indicators of Model Quality

Parameter Dimensionality Avoid overparamerization

Stability of Parameters Frequent recalibration: indicator of poor model performance

GBM w termstructure vs Garch

	TS GBM	**Garch**
dimension	∞	3
recalibration	frequently for short end	less frequent
time-homogeneous	N	Y

Risk Model for Volatility surface

Directly modelling surface w/o arbitrage not trivial. Alternatively model option prices with HJM-like framework.

Liquidity & Completeness

Liquidity Hedge frequency, transaction costs, close-out period

Completeness Unhedgeable risk, uniqueness of price

Comparing Models

Assume state of the world evolves randomly:

Model as Process: Stochastic Differential Equation (Langevin Equation)

$$
\begin{aligned}
\frac{d\mathbf{x}}{dt} &= f(\mathbf{x}) + g(\mathbf{x})\xi(t) \quad \text{Physics Notation} \\
d\mathbf{x} &= f(\mathbf{x})dt + g(\mathbf{x})d\mathbf{W}(t) \quad \text{Finance Notation}
\end{aligned}
$$

Wiener Process (SDE)

$$
\begin{aligned}
dx &= dW(t) \\
W &: \text{Wiener Process}
\end{aligned}
$$

Model as Measure $\mathbb{P}$

$$\begin{aligned} \mathbb{P} \quad &: \quad \mathbf{\Gamma} \to \mu(\mathbf{\Gamma}) \quad \text{probability} \\ \mathbf{\Gamma} \quad &: t \quad \to \mathbf{x}(t) \quad \text{some path} \end{aligned}$$

Wiener Process (SDE)

$$\begin{aligned} \mathbf{\Gamma} \quad &\equiv \quad \{x_1, \dots x_N\} \\ \mu(\Gamma) \quad &\sim \quad \prod_i G(x_i, x_{i+1}) \\ G \quad &: \quad \text{Gaussian} \end{aligned}$$

Error Analysis

Infer from parameter uncertainty price/risk uncertainty.

Parameter Uncertainty E.g. such that hedging instrument prices still in bid-ask

Parameter Error Uncertainty of price/risk due to error in parameters

GBM with vol uncertainty

$$(\Delta p)^2 = \left(\frac{\partial p}{\partial \sigma} \Delta \sigma\right)^2$$

Benchmarking

Pricing/risk factor models $\mathbb{Q}, \mathbb{Q}'$, empirical measure $\mathbb{P}$

Comparing

Pricing Models $\mathbb{Q}$ vs $\mathbb{Q}'$

Risk Models $\mathbb{P}$ vs $\mathbb{Q}$

How far apart two models?

Need to define metric:

Expectation values E.g. differences of prices and EEs under different measures

Distributions E.g. Kullback-Leibler entropy $\int \frac{d\mathbb{P}'}{\mathbb{P}} \log \frac{d\mathbb{P}'}{\mathbb{P}} d\mathbb{P}$. Independent of quantity to average.

Model Uncertainty

Benchmarking giving limited answer:

Calibration-Consistent Measures

Define metric d to quantify goodness of calibration:

$$
\begin{aligned}
p_i^{\mathbb{P}} &: \quad \text{model price calibration instrument } i \\
p_i &: \quad \text{market price calibration instrument } i \\
d^{\mathbb{P}} &= \sum_i (p_i^{\mathbb{P}} - p_i)^2 \\
\mathcal{C}_\epsilon &= \{\mathbb{P} | d^{\mathbb{P}} \leq \epsilon\}
\end{aligned}
$$

Model Uncertainty

Non-uniqueness

For $d = 0$:

- Multiple measures
- For single parametric measure, multiple solutions for calibration $\rightarrow$ ill behaved
- Incomplete market

For $d > 0$:

- For single parametric measure: parameter risk

Pricing model descriptive:

- Replicates prices of hedging instruments
- Determines no-arbitrage price of illiquit product

How to asses quality of model?

There are implied predictions:

State variables vs parameters: Prediction: parameters are constant

Martingale: Total price of deal and self-financing hedges should be 0 at any point in time

State variables and parameters

State variables Temporal evolution or measure

Parameters Family of evolutions/measures

Analysis

- Choice of state variables: qualitative assessment
- Robustness of parameters: predicted are no changes

Hedge Performance

If perfectly hedged: pathwise replication $\rightarrow$ P&L distribution

- Unbiased
- Sharply peaked (Dirac)

Hedge Simulations

Self Consistency Use state variables simulated with pricing model

Performance Historical state variables

Exposure

Goal

Estimate Credit Risk measures $\rightarrow$ need to estimate exposure/price distributions in future.

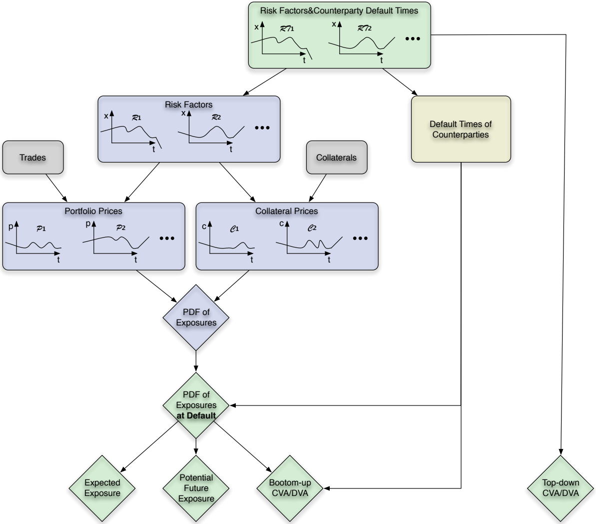
The exposure $e(t)$ at time t of a netting set is given by

$$e(t) = \max 0, \sum_i p_i(\mathbf{x}, t) - C(t) \tag{2}$$

where

- p_i price of trade i
- $\mathbf{x}$ risk factors
- $C(t)$ price of collateral

Calculating Exposure, CVA/DVA and Losses



Building Blocks

Components

Required for estimating risk measures for single and portfolios of netting sets:

- Pricing
- Risk-factor
- Collateral
- Netting
- Dependency

Pricing Models

Requirements

- Need to be fast!
- Ideally same as front office
- Perform well under stressed state variables

Dumb lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- define grid
- recaluclate for each gridpoint price
- interpolate

Smart lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- prices on grid are side effect of pricing at spot; e.g. pricing on tree or AMC
- interpolate

Purpose

Pricing Model Fit liquid market instruments; arbitrage-free

Risk Model Predict

Challenges for Risk Model

Dependency Simultaneously simulate all asset classes

Calibrationl Global calibration

Long term prediction a challenge:

- Reducing dimensionality
- Economic macro factors
- Co-integration

Arbitrage-free models used with risk calibration

- GBM
- HJM type of models

⊕ Well understood, tractable

⊖ Not intended for risk

Gaussian Dependency Modelling

Goal

Express random vector $\boldsymbol{\xi}$ with correlated ξ_i as

- linear combination of
- uncorrelated

random factors η_i:

$$\begin{aligned} \boldsymbol{\xi} &= \mathbf{M}\boldsymbol{\eta} \\ \mathbb{E}[\xi_i\xi_j] - \mathbb{E}[\xi_i]\mathbb{E}[\xi_j] &\equiv \Omega_{ij} \\ \mathbb{E}[\eta_i\eta_j] - \mathbb{E}[\eta_i]\mathbb{E}[\eta_j] &= \lambda_i^2\delta_{ij} \quad \text{diagonal, pos. sem. def.} \end{aligned}$$

What to consider?

- $\boldsymbol{\Omega}$?
- correlation matrix?

Dimensional Analysis

Risk factors ξ_i not dimension-less!

- interest rate :$[T^{-1}]$
- stock price :[Cash]
- volatility: $[T^{-\frac{1}{2}}]$

$\rightarrow \Omega_{ij}$ may have different dimensions,i.e. $\boldsymbol{\Omega}$ in general not a physically meaningful quantity!

Solution

Consider instead of $\mathbf{\Omega}$ following matrix $\mathbf{\Phi}$:

$$\begin{aligned}\Phi_{ij} &\equiv \frac{\partial f(\boldsymbol{\xi})}{\partial \xi_i}\frac{\partial f(\boldsymbol{\xi})}{\partial \xi_j}\Omega_{ij} \\ f &: \text{some function}\end{aligned}$$

For dimensionality $[\mathbf{\Phi}]$:

$$[\Phi_{ij}] = \frac{[f]}{[\xi_i]}\frac{[f]}{[\xi_j]}[\xi_i][\xi_j] = [f^2] \quad \forall i,j \qquad \checkmark \tag{3}$$

GBM Risk Factor Model

Multivariate GBM

$$
\begin{aligned}
X_i(t+\Delta t) &= X_i e^{(\mu_i - \frac{1}{2}\sigma_i)\Delta t + \sigma_i \sqrt{\Delta t}\xi_i(t)} \\
\mu &: \text{drift} \\
\sigma &\ \text{volatility} \\
\xi_i &: \text{Normal random} \\
\mathrm{Cov}(\ln X_i(t+\Delta t), \ln X_j(t+\Delta t)) &= \Omega_{ij}
\end{aligned}
$$

Dependent Gaussian Random Variables

Given uncorrelated Gaussian random number vector $\boldsymbol{\zeta}$. Need build $\boldsymbol{\eta}$:

$$\mathrm{Cov}(\eta_i, \eta_j) = \Omega_{ij}$$

Calibration

Definition

Calibration is the process to determine model parameters.

Approaches

Statistical	Using historical data
Implied	Market implied parameters
Economic	Macro economical relation between rates, infaltion

Asumptions

Statistical	Past is good predictor for future
Implied	Information in spot market predicts future
Economic	Some fundamental economic laws rule future

Statistical Calibration

For simple models: ad hoc parameter estimation

- averaging
- fitting

Example

`SimpleEstimation.xls`

Maximum Likelihood Estimation

Systematic way to calibrate

Approach

Parametric model with parameters $\boldsymbol{\alpha} \leftrightarrow$ parametric measure $\mu_{\boldsymbol{\alpha}}$:

$$\mu_{\boldsymbol{\alpha}}(\Gamma) = e^{-S_{\boldsymbol{\alpha}}(\Gamma)} \mathcal{D}[\Gamma]$$

Assume: historical path Γ_H is the most likely one. Find $\boldsymbol{\alpha}^*$ such that:

$$\mu_{\boldsymbol{\alpha}^*}(\Gamma_H) = \max_{\boldsymbol{\alpha}} \mu_{\boldsymbol{\alpha}}(\Gamma_H)$$

Implementation

Assuming iid:

$$\begin{aligned} \mu_{\boldsymbol{\alpha}}(\Gamma) &= \prod m(\mathbf{x}_i) \\ m(\mathbf{x}) &= e^{-s(\mathbf{x})} \\ \Gamma &= \{\mathbf{x}_1, \ldots, \mathbf{x}_n\} \end{aligned}$$

Maximizing $m \leftrightarrow$ minimizing

$$\sum_i s(\mathbf{x}_i) : \quad \text{log-likelihood}$$

Maximum Likelihood Estimation

Example

MLE.xls

Apply parameters used for pricing:

Drift and Volatility

- Drift μ from T forward price (Covered Parity)
- Volatility σ T years ATM implied volatility

Assumption

Risk neutral measure yield good predictor for real-world measure

Caveat

- Carry trades
- Supply/demand, risk premium

Perform analysis before using implied parameters!

Economic Calibration

Parities connect for instance

- FX rates
- Inflation rates
- Real interest rates
- Nominal interest rates
- Purchansing power

Example

Parities.xlsx

Parities

Example (Relative Purchasing Power Parity)

$$
\begin{aligned}
p_f(t_1)(1+i_f)X(t_2) &= p_d(t_2)(1+i_d) \\
p_{d/f} &: \text{domestic/foreign price} \\
i_{d/f} &: \text{domestic/foreign 1 yr inflation rate} \\
X &: \text{Exchange rate}
\end{aligned}
$$

Yields after averaging

$$
\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1+I_d}{1+I_f}
$$

where I is the expected inflation rate.

Example (International Fisher Effect (Uncovered Parity))

$$\begin{aligned} (1 + r_{d/f}) &= (1 + \rho_{d/f})(1 + i_{d/f}) \\ r_{d/f} &: \text{domestic/foreign nominal 1 yr interest rate} \\ \rho_{d/f} &: \text{real rdomestic/foreign 1 yr interest rate} \end{aligned}$$

Assuming$\rho_d = \rho_f$ gives

$$\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1 + r_d}{1 + r_f}$$

Issues with standard GBM model

Issues

- rigidity: calibration short vs long horizons $\rightarrow$ term structure of parameters
- dimensionality $\rightarrow$ factor models
- underestimation of rare events and bursts (clustering) $\rightarrow$ GARCH
- not suitable where spread stationary process $\rightarrow$ cointegration
- unable to capture some behabiour like regime-switches $\rightarrow$ parametric models (Nelson-Siegel)

GBM with Term Structure

Interpolation Principles

Interpolate dimension-less quantities

Forward Drift/Covariance

Dimensionality analysis $\rightarrow$ interpolate $T\boldsymbol{\Omega}$

Factor Models

Issues with general covariance matrix

N risk factors $\rightarrow \propto N^2$ parameters

- over-parametrization
- for empirical parameters: problems with positive definiteness

Idea

Split return r of riskfactors into contributions from

Indices f_n shared by multiple risk factors

Idiosycratic factors ϵ unique to each risk factor

$$r = \alpha + \sum_n \beta_n f_n + \epsilon$$

and assume

- indices uncorrelated to indosyncratics
- idiosyncratics uncorrelated among each other

Types of Factor Models

Classification

Macroeconomic	Observables like changes in inflation, interest rate, unemployment rate
Fundamental	Portfolios associated to security attributes like industry membership, book to market ratio, dividends
Statistical	Factor analysis of covariance matrix

Fast/Slow

Slow variables Macro-economic state of the economy: inflation, unemployment rate, GDP

Fast Asset prices

Pros and Cons

- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing
- ⊖ Theories controversial

Sector/Region

1. Define for each sector/region pair an index
2. Associate stock to sector/region
3. Regress stock return vs index return $\rightarrow \alpha, \beta$

Example

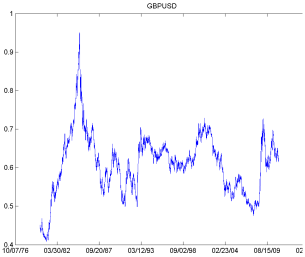
FactorModel.xls

Pros and Cons

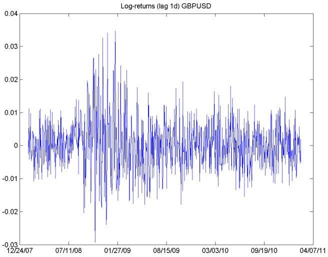
- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing
- ⊖ Theories controversial

Choice of Factors

How to know whether factors appropriate?
Analyze variance explained by factors



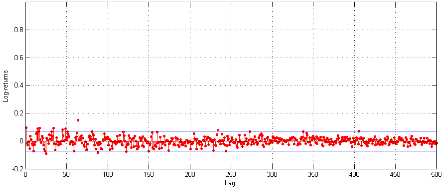
(a) Spot



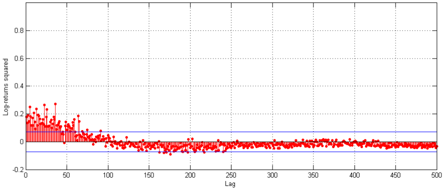
(b) Log-returns

Figure : GBPUSD spot

Autocorrelation



(a) Autocorrelation: log-returns



(b) Autocorrelation: squared log-returns

Figure : Autocorrelations GBPUSD log-returns

Let X_n be the log-return of some foreign exchange rate f at time t_n:

$$X_n = \ln \frac{f_n}{f_{n-1}} \tag{4}$$

we may then express the foreign exchange rate f_N at some future sampling point time t_N by the initial value f_0 at t_0 and a series of returns:

$$f_N = f_0 e^{\sum_{i=1}^{N} X_i} \tag{5}$$

The observation points t_i are typically defined in terms of number of business days ΔT between them. For short time horizon predictions we choose $\Delta T = 1$ for larger horizon, we may choose a less granular time grid.

Garch Model

The dynamics of the returns is then assumed to follow a Garch(1,1) process

$$
\begin{aligned}
X_n &= \mu + \epsilon_n \quad \epsilon_t \sim \mathrm{iid}(0, \sigma_n^2) && (6) \\
\sigma_{n+1}^2 &= \alpha + \beta \sigma_n^2 + \gamma \epsilon_n^2 && (7)
\end{aligned}
$$

The asymptotic value $\sigma_\infty^2 = \lim_{n \to \infty} \mathbb{E}[\sigma_n^2]$ is then obtained by equation (7) noting, that $\mathbb{E}[\epsilon^2] = \sigma^2$ and $\mathbb{E}[\sigma_{n+1}^2] \to \mathbb{E}[\sigma_n^2]$:

$$
\sigma_\infty = \frac{\alpha}{1 - \beta - \gamma} \tag{8}
$$

Weak limit:

- Stochastic variance
- Mean reverting variance

$$
\begin{aligned}
dX_t &= \mu X_t dt + \sqrt{v_t} X_t dW_t \\
dv_t &= \alpha(v_t) dt + \beta(v_t) dZ_t
\end{aligned}
$$

Copula

Dependence under Stress

In stressed markets correlations increase between

- downward price movements $\rightarrow$ systematic risk
- implied default probabilities $\rightarrow$ contagion

Definition (Copula)

Separate

- Marginal distributions from
- Dependency

Cointegration

Long-run Relationship

Variables moving together:

Macro-economic
- Consumption-Income
- Prices-Wages
- Domestic prices - fpreign prices

Exogeneous For instance managed currencies

- How to model processes. which stay close to each other?
- GBM with $\rho_{ij} \lessapprox 1$ **not**? No!
- Need dynamic, where difference is stationary

Definition

Stochastic processes x, y are cointegrated:

$$\begin{aligned} y(t) &= a + bx(t) + \xi(t) \\ \xi(t) &: \text{stationary stochastic process} \end{aligned}$$

1. find parameters a, b by regression
2. show residuals are stationary (e.g. Dickey-Fuller Test)

Example

`Cointegration.xlsx`

Nelson-Siegel model

$$
\begin{aligned}
r(T) &= r_\infty + a(T) r_0 + b(T) r_m \\
r_\infty &: \text{rate for long maturities} \\
r_0 &: \text{rate for short maturities} \\
r_m &: \text{rate for intermediate maturities} \\
a, b &: \text{decay functions}
\end{aligned}
$$

Nelson-Siegel model

- Normal/inverted curves
- But not arbitrage-free

How to introduce dynamics? E.g. PCA of (r_∞, r_0, r_m)

Example

NelsonSiegel.xlsm

Wrong Way Risk

Types

Specific Legal connection between underlying and counterparty

General Dependence between prob. of default of counterparty and exposure

SFT Transactions

Lend cash to counterparty A accepting their stock as collateral.

Emerging Market CCY swap

We are long strong currency. Weakening of emerging market currency, increased prob default $\rightarrow$ increase exposure

What is wrong with standard modelling?
p^+ is **not** conditioned on default.
Need to add in price function default state χ of counterparty:
extending state of the world

Approaches

Given a model for default times either

- Simulating counterparty's default
- Calculating price given default

Example

`WrongWayRisk.xls`

Margin Call Process Model margin calls with correct frequency and close-out period

Collateral Price E.g. model bond price if collateral is bond

Simplification

- Margin call process: just at spot $\rightarrow$ short-cut method
- All collateral as cash $\rightarrow$ haircuts

Definition (Basel II Short-Cut Method)

EE and PE of collateralized trades given by EE and PE for close-out period (5 days for SFT, 10d for OTC)

Benefits/Issues

- ⊕ Computationally cheap
- ⊕ No collateral exposure spikes at expity
- ⊖ Assumes exposures declining over time
- ⊖ Risk not accurately represented

Dependency Modelling

Among Risk Factors

Standard way to model dependence: Gaussian Copula.
Gaussian Copulas are Levy copulas. Replace Gaussian with other Levy coupula and obtain Levy model.

Between Defaults

Simulate either

Default times τ E.g. by Marshall-Olkin Copulas

Default state at t:$\chi_{\tau \leq t}$ E.g. structural models

Dependency Modelling

Between a Default and Risk Factors

To caputure Wrong Way risk need to model dependence between risk factor and default state

Example

`WrongWayRisk.xls`

Between a cross name Defaults and Risk Factors

Need modelling full state of the world $(\mathbf{x}(t), \{\chi_{\tau_1 \leq t}, \dots \chi_{\tau_1 \leq t}\})$.
$\rightarrow$ scenario consistency is system

Model Lifecycle

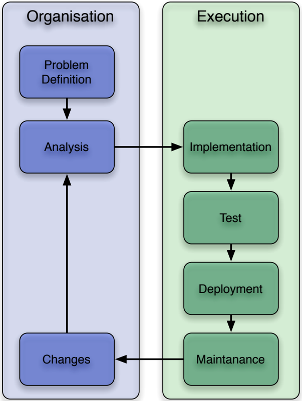


Figure : Model Development Lifcecyle

Specification

Approaches

Human readable Business and functional specs

Machine readable Specification $\sim$ test

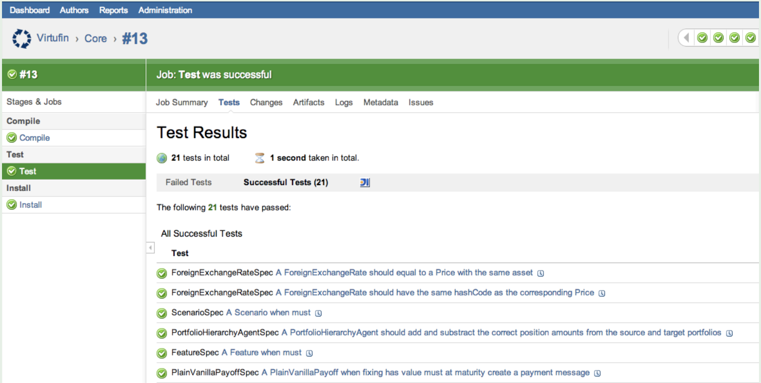
ScalaTest Code

```
class LookupSpec extends WordSpec with MustMatchers {
  def fixture = new {
    val i1 = Identifier[Double]("abc")
    val i2 = Identifier[Double]("xyz")
    val i3 = Identifier[Double]("ABC")
    val x1 = 12.3
    val x2 = 4.56
    val l = Lookup(i1 -> x1, i2 -> x2)
  }

  "A Lookup" when {
    "item exists" must {
      "retrieve with () the item" in {
        val f = fixture
        val y1 = f.l(f.i1)
        y1 must equal(f.x1)
        val y2 = f.l(f.i2)
        y2 must equal(f.x2)
      }
      "retrieve with get() an Option object containing the item" in {
        val f = fixture
        val y1 = f.l.get(f.i1)
        y1 must not be Option.empty
        y1.get must equal(f.x1)
        val y2 = f.l.get(f.i2)
        y2 must not be Option.empty
        y2.get must equal(f.x2)
      }
    }
```

ScalaTest Output

Part of CI:



Software

- in-house
- third-party

Require different validation strategies

Third Party

Strategies

- Black-box, no code review
- Reverse-engineering

Revision Control

Requirements

Audit	Who changed what/when
Resurrect	Roll-back to previous state
Collaborate	Merge contributions from different authors

Approaches

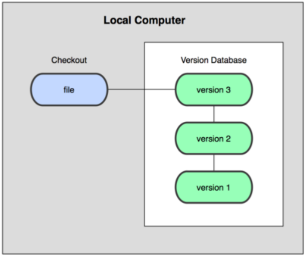
Plain files	Tag files/directories with version information
Local	Local database contains version information (e.g RCS)
Server	Database on server (e.g. SVN)
Distributed	Each developer has own databse with potentially central db (e.g. Git)

MyDirectoryV1.0

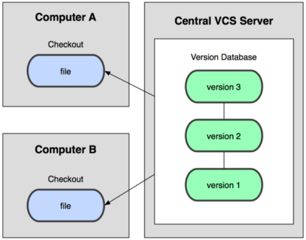
MyDirectoryV1.1

MyDirectoryV1.2-bugfix1

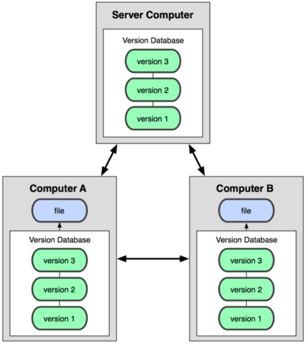
(a) File based



(b) Local VCS



(a) Centralized VCS



(b) Distributed VCS

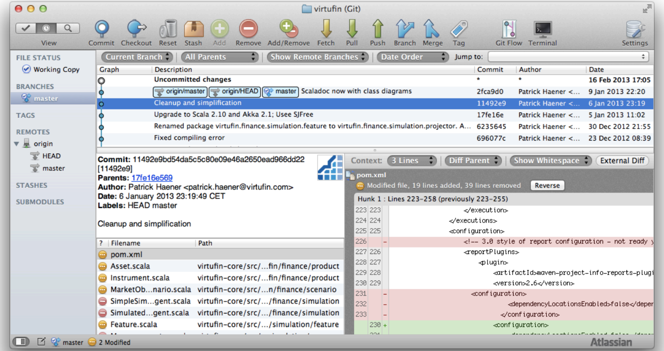


Figure : Git Gui (SourceTree)

Documentation

Requirement

Contain enough information to reverse-engineer.

Tools

- Automated API doc (*Doxygen*, *ScalaDoc*, ...)
- Internal wiki (e.g. *Confluence*)

Testing

Test Types

Unit	Library level
Integration	System level

```
class BlackScholesScenarioModelTest {
  @Test
  def test() {
    val marketObservable = Price(Stock("IBM"), Currency.USD)
    val modelDate = Time(2012, 10, 1)
    val t1 = Time(2013, 10, 1)
    val t2 = Time(2015, 10, 1)
    val t3 = Time(2022, 10, 1)
    val ts = List(t1, t2, t3)
    val scenarioObservables = ts.map(t => Index(t, marketObservable))
    val x0 = 100.0
    val scenario = Scenario(marketObservable, modelDate -> x0)
    val mu = 0.01
    val sigma = 0.4
    val blackParameters = BlackParameters(mu, sigma)
    val blackParametersId = BlackParameters.identifier(marketObservable)
    val parameters = Lookup(blackParametersId -> blackParameters)
    val request = ScenarioRequest(scenarioObservables, scenario, modelDate)
    val context = ModelContext(ModelDispatcher(ModelRegistry()), parameters)
    val model = new BlackScholesScenarioModel()
    val result = model.model(request, context)
    assertTrue(result.isSuccess)
    val iterable = result.toOption.get
    val n = 1000000
    var x = 0.0
    var x2 = 0.0
    val o = scenarioObservables.last
    Timing.timing(iterable, (s: Scenario) => {
      val a = s(o); x = x + a; x2 = x + a * a
    }, n)
    assertEquals(x0 * math.exp(mu * DateUtil.yearsBetween(modelDate, o.time)), x / n, 0.5)
  }
```

Release

Requirements

- Regression
- Impact analysis
- Sign-off
- Auditing
- Lock-down

Maintance

Bugs/Enhanements

- Tracking system
- Failing test cases
- Metrics: severity, resolution time

Robust system should have

Components

- Revsion Control system
- Build System
- Bug tracking system
- Wikin

Components integrated to workflow with high degree of automation

Overview

Motivation

Impact of Credit risk model

Trading activity limits set by *PE*

Capital charges regularity capital dependent of *EEPE*

P&L *EE* enters CVA/DVA

Model Risk

Back-testing should quantify model risk affecting these quantities.

Back-testing Process

Should provide

Definition of measure for model risk

Monitoring of metrics

Mitigating actions for model deficiencies

BCBS Guidance

G1

Guidance: Backtesting of forecast distributions produced by EPE models and market risk factor models needs to be performed on the entire forecast distribution.

G2

Guidance: The validation requirements as set out in Basel II for EPE Models should not make reference to VaR requirements and instead the qualitative standards set out in paragraph 718 (LXXIV) should be transposed into the validation requirements for EPE models and the language adapted where required.

G3

Guidance: The Validation of EPE models and all the relevant models that input into the calculation of EPE must be performed separately for a number of distinct time horizons.

BCBS Guidance

G4

Guidance: The performance of market risk factor models must be validated using backtesting. The validation must be able to identify poor performance in individual risk factors.

G5

Guidance: The validation of EPE models and all the relevant models that input into the calculation of EPE must be made using forecasts initialised on a number of historical dates.

G6

Guidance: Historical backtesting on representative counterparty portfolios and market risk factor models must be part of the validation process. At regular intervals as directed by its supervisor, a bank must conduct backtesting on a number of representative counterparty portfolios and its market risk factor models. The representative portfolios must be chosen based on their sensitivity to the material risk factors and correlations to which the bank is exposed.

BCBS Guidance

G7

Guidance: Backtesting of EPE and all the relevant models that input into the calculation of EPE must be based on recent performance.

G8

Guidance: The frequency with which the parameters of an EPE model are updated needs be assessed as part of the on-going validation process.

G9

Guidance: Firms need to unambiguously define what constitutes acceptable and unacceptable performance for their EPE models and the models that input into the calculation of EPE and have a written policy in place that describes how unacceptable performance will be remediated.

BCBS Guidance

G10

Guidance: Firms need to define what constitutes a representative counterparty portfolio for the purposes of carrying out EPE model backtesting.

G11

Guidance: IMM firms need to conduct hypothetical portfolio backtesting that is designed to test risk factor model assumptions, eg the relationship between tenors of the same risk factor, and the modelled relationships between risk factors.

G12

Guidance: Firms need to assess whether or not the firm level and netting set level exposure calculations are appropriate.

BCBS Guidance

G13

Guidance: Firms must backtest their EPE models and all relevant models that input into the calculation of EPE out to long time horizons of at least one year.

G14

Guidance: Firms must validate their EPE models and all relevant models that input into the calculation of EPE out to time horizons commensurate with the maturity of trades covered by the IMM waiver.

G15

Guidance: Prior to implementation of a new EPE model or new model that inputs into the calculation of EPE a firm must carry out backtesting of its EPE model and all the relevant models that input into the calculation of EPE at a number of distinct time horizons using historical data on movements in market risk factors for a range of historical periods covering a wide range of market conditions.

G16

Guidance: Under the internal model method, a measure that is more conservative than Effective EPE (eg a measure based on peak rather than average exposure) for every counterparty may be used in place of alpha times EEPE with the prior approval of the supervisor. The degree of relative conservatism will be assessed upon initial supervisory approval and at regular intervals in conjunction with other EPE models. The assessment needs to cover all counterparties. The firm must have an unambiguous definition of what constitutes acceptable performance for these models and a documented process in place for remediating poor performance.

What is the Question?

Types of Investigation

- Hypothesis testing (Answer in percentage or yes/no)
- Estimation of model uncertainty (Answer in cash terms)

Analysis at different levels: figure 7

Domains

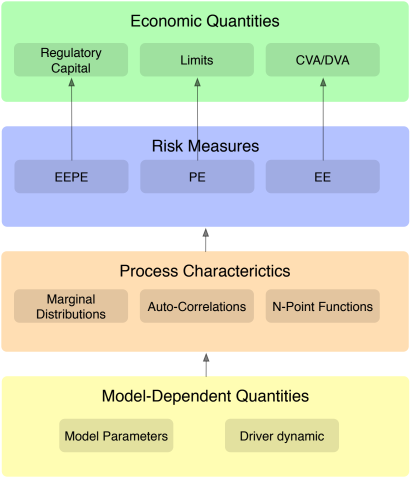


Figure : Domains

Definition

A model is represented by a measure $\mathbb{Q}$.
May be generated by a stochastic process.

Quantifying Difference of Models

- Comparing expectation values
- Comparing probability distributions

Note: PDFs and CDFs may be expressed as expectation values

Distance of model $\mathbb{Q}$ and end empirical measure $\mathbb{P}$ in terms of $\frac{d\mathbb{P}}{d\mathbb{Q}}$:

$$\mathbb{E}_{\mathbb{P}}[f] = \mathbb{E}_{\mathbb{Q}}[\frac{d\mathbb{P}}{d\mathbb{Q}} f] \tag{9}$$

Compare $\mathbb{P}$ and $\mathbb{Q}$

Direct $\frac{d\mathbb{P}}{d\mathbb{Q}} \approx id$?

Expectation values Empirical expectation measures in terms of model expectations

Relative Entropy Kullback-Leibler entropy $\rightarrow$ information geometry (see [?])

Radon-Nikodym Derivative

Let ξ be a scalar stochastic variable (e.g. portfolio price $\pi(t)$)

Definition

P empirical, Q model CDF

$$\Psi \quad : \quad [0,1] \to [0,1] \tag{10}$$

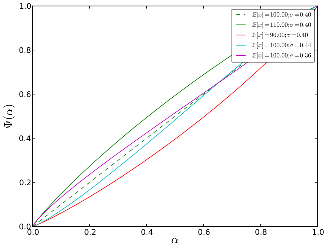
$$\Psi(\alpha) \quad = \quad P(Q^{-1}(\alpha)) \tag{11}$$

Radon-Nikodym derivative ψ

$$\mathbb{E}_{\mathbb{P}}[f] \quad = \quad \mathbb{E}_{\mathbb{Q}}[\psi(\alpha) f] \tag{12}$$

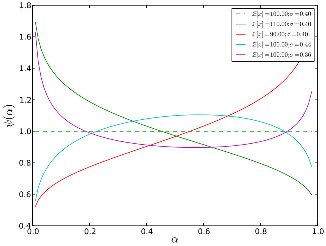
$$\psi(\alpha) \quad = \quad \frac{d\Psi(\alpha)}{d\alpha} \tag{13}$$

Example



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\Psi(\alpha)$
α
$\mathbb{E}[x]=100.00;\sigma=0.40$
$\mathbb{E}[x]=110.00;\sigma=0.40$
$\mathbb{E}[x]=90.00;\sigma=0.40$
$\mathbb{E}[x]=100.00;\sigma=0.44$
$\mathbb{E}[x]=100.00;\sigma=0.36$

(a)



1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.0
0.2
0.4
0.6
0.8
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$\mathbb{E}[x]=90.00;\sigma=0.40$
$\mathbb{E}[x]=100.00;\sigma=0.44$
$\mathbb{E}[x]=100.00;\sigma=0.36$

(b)

Cumulative Distribution Functions

Cumulative distribution function (CDF) for some state variable $\boldsymbol{\xi}$ expressed as expectation:

Definition

$$P(\boldsymbol{\xi}_0) = \mathbb{E}_{\mathbb{P}}[\Theta(\xi - \xi_0)] \tag{14}$$

where Θ is the Heaviside function.

Estimating

Ensemble averages $\mathbb{E}$ estimated well by time averages if

- ergodic
- stationary

CDF

$$P(\xi_0) \approx \frac{1}{N}\sum_{i=1}^{N}\Theta(\xi(t_i)-\xi_0) \tag{15}$$

Ψ

$$\Psi(\alpha) \approx \frac{1}{N}\sum_{i=1}^{N}\Theta(\xi(t_i)-Q^{-1}(\alpha)) \tag{16}$$

Requirements for Estimation

Process neeeds to be

- ergodic
- stationary

iid price process

If empirical price process is iid, the ergodic.

iid process of underlying

Even if underlying process the price return process of the deal may not be so, if deal not time homogeneus

Distances

Point Distance

$$d_i = |\Psi(q_i) - q_i| \tag{17}$$

Curve Distance

(Weighted) quadratic distance d between functions $q \to \Psi(q)$ and $q \to q$:

$$d(q, \Psi(q)) = \sum_i w_i(\Psi(q_i) - q_i)^2 \tag{18}$$

q_i e.g $(0.01, 0.05, 0.3, 0.5, 0.7, 0.95, 0.99)$

Null-Hypothesis

- Null-Hypothesis, is that distances are 0.
- Reject Null-Hypothesis p-values smaller than some threshold

Challenges estimating p-values

- Temporal dependence: overlap of time-windows
- Ensemble dependence: returns of netting sets not independent

Good p values get bigger

Bad Estimation tricky

Need some simplifications, like effective sample sizes

Issues using metrics for Ψ

- **Opaque** no cash denominated measure
- **Economics Product Dependent** with same distance different moments drive deviations in EE (see figure (**??**))
- **Limited usefulness** Passes test if not enough data available

Problems using metrics for Ψ

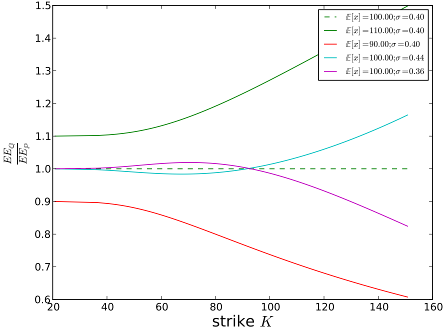


Figure : Comparing *EE*s for a forward using log-normal distributions with different parameters

Comparison using Cash denominated Quantities

Economically Relevant Model Dependent Quantities

Regulatory Capital depends on $EE(t)$ (through EEPE)

Limits impacted by CDF

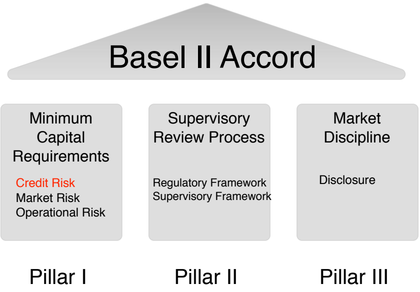
P&L impacted by $EE(t)$

Measure

These three quantities are functions of $\mathbb{E}_Q$.
Their value under empirical measure $\mathbb{P}$ estimated through equation (12) $\rightarrow$ difference in cash terms

Overview

Pillars of Basel II Framework



Basel II

Capital Charges

- Based on Expected Exposures (EE) of netting sets
- Charge for default risk
- No charge for credit spread risk

New in Basel III

In 2008 crisis:

- $\frac{2}{3}$ of losses not due to default but MtM changes due to credit spread widening
- Capture spread risk by VaR
- Introduction of new capital charge linked to VaR: CVA charge

Capital Charges

Basel II	Default charges
Basel III	Default and CVA charges

Advanced CVA Charge

VaR for credit spread for bond given by EE:

A. Banks with IMM approval and Specific Interest Rate Risk VaR model[34] approval for bonds: Advanced CVA risk capital charge

98. Banks with IMM approval for counterparty credit risk and approval to use the market risk internal models approach for the specific interest-rate risk of bonds must calculate this additional capital charge by modelling the impact of changes in the counterparties' credit spreads on the CVAs of all OTC derivative counterparties, together with eligible CVA hedges according to new paragraphs 102 and 103, using the bank's VaR model for bonds. This VaR model is restricted to changes in the counterparties' credit spreads and does not model the sensitivity of CVA to changes in other market factors, such as changes in the value of the reference asset, commodity, currency or interest rate of a derivative. Regardless of the accounting valuation method a bank uses for determining CVA, the CVA capital charge calculation must be based on the following formula for the CVA of each counterparty:

$$CVA = (LGD_{MKT}) \cdot \sum_{i=1}^{T} Max\left(0; exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}\right)$$

$$Regulatory\ CS01_i = 0.0001 \cdot t_i \cdot exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} - EE_{i+1} \cdot D_{i+1}}{2}\right)$$

Standardised CVA Charge

$$K = 2.33 \cdot \sqrt{h} \cdot \sqrt{\left(\sum_i 0.5 \cdot w_i \cdot \left(M_i \cdot EAD_i^{total} - M_i^{hedge} B_i\right) - \sum_{ind} w_{ind} \cdot M_{ind} \cdot B_{ind}\right)^2 + \sum_i 0.75 \cdot w_i^2 \cdot \left(M_i \cdot EAD_i^{total} - M_i^{hedge} B_i\right)^2}$$

Where

- h is the one-year risk horizon (in units of a year), $h = 1$.
- w_i is the weight applicable to counterparty 'i'. Counterparty 'i' must be mapped to one of the seven weights w_i based on its external rating, as shown in the table of this paragraph below. When a counterparty does not have an external rating, the bank must, subject to supervisory approval, map the internal rating of the counterparty to one of the external ratings.
- EAD_i^{total} is the exposure at default of counterparty 'i' (summed across its netting sets), including the effect of collateral as per the existing IMM, SM or CEM rules as applicable to the calculation of counterparty risk capital charges for such counterparty by the bank. For non-IMM banks the exposure should be discounted by applying the factor $(1-\exp(-0.05*M_i))/(0.05*M_i)$. For IMM banks, no such discount should be applied as the discount factor is already included in M_i.
- B_i is the notional of purchased single name CDS hedges (summed if more than one position) referencing counterparty 'i', and used to hedge CVA risk. This notional amount should be discounted by applying the factor $(1-\exp(-0.05*M_i^{hedge}))/(0.05* M_i^{hedge})$.
- B_{ind} is the full notional of one or more index CDS of purchased protection, used to hedge CVA risk. This notional amount should be discounted by applying the factor $(1-\exp(-0.05*M_{ind}))/(0.05* M_{ind})$.
- w_{ind} is the weight applicable to index hedges. The bank must map indices to one of the seven weights w_i based on the average spread of index 'ind'.
- M_i is the effective maturity of the transactions with counterparty 'i'. For IMM-banks, M_i is to be calculated as per Annex 4, paragraph 38 of the Basel Accord. For non-IMM banks, M_i is the notional weighted average maturity as referred to in the third bullet point of para 320. However, for this purpose, M_i should not be capped at 5 years.
- Mi^{hedge} is the maturity of the hedge instrument with notional B_i (the quantities $Mi^{hedge} \cdot B_i$ are to be summed if these are several positions).
- M^{ind} is the maturity of the index hedge 'ind'. In case of more than one index hedge position, it is the notional weighted average maturity.

	Advanced	**Standardized**
Reflect Diversification	$\oplus$	$\ominus$
Accurate Credit Spreads	$\oplus$	$\ominus$
Regulatory Capital	$\oplus$	$\ominus$
Build/Approval Costs	$\ominus$	$\oplus$
Running Costs	$\ominus$	$\oplus$
Synergies with Market Risk	$\oplus$	$\ominus$
Integration with CVA desk	$\oplus$	$\ominus$

Internal Model Method

Institutions who have IMM waiver may calculate their own regulatory capital for

OTC transactions Swaps, exotic deals, . . .

SFT transactions Bond repos, stock borrow/lending, . . .

- Reduced capital charges
- More accurate risk measures
- Consistent risk measures for
  - Regualtory capital
  - Limit monitoring
- Improved
  - Processes
  - Quality of information

Wrong Way Risk

Specific Wrong Way Risk

58. A bank is exposed to "specific wrong-way risk" if future exposure to a specific counterparty is highly correlated with the counterparty's probability of default. For example, a company writing put options on its own stock creates wrong-way exposures for the buyer that is specific to the counterparty. A bank must have procedures in place to identify, monitor and control cases of specific wrong way risk, beginning at the inception of a trade and continuing through the life of the trade. To calculate the CCR capital charge, the instruments for which there exists a legal connection between the counterparty and the underlying issuer, and for which specific wrong way risk has been identified, are not considered to be in the same netting set as other transactions with the counterparty. Furthermore, for single-name credit default swaps where there exists a legal connection between the counterparty and the underlying issuer, and where specific wrong way risk has been identified, EAD in respect of such swap counterparty exposure equals the full expected loss in the remaining fair value of the underlying instruments assuming the underlying issuer is in liquidation. The use of the full expected loss in remaining fair value of the

Stressed Calibration

1. Revised metric to better address counterparty credit risk, credit valuation adjustments and wrong-way risk

Effective EPE with stressed parameters to address general wrong-way risk

98. In order to implement these changes, a new paragraph 25(i) will be inserted in Section V (Internal Model Method: measuring exposure and minimum requirements), Annex 4, of the Basel II framework and the existing paragraph 61 of Annex 4 will be revised as follows for banks with permission to use the internal models method (IMM) to calculate counterparty credit risk (CCR) regulatory capital – hereafter referred to as "IMM banks":

> 25(i). To determine the default risk capital charge for counterparty credit risk as defined in paragraph 105, banks must use the greater of the portfolio-level capital charge (not including the CVA charge in paragraphs 97-104) based on Effective EPE using current market data and the portfolio-level capital charge based on Effective EPE using a stress calibration. The stress calibration should be a single consistent stress calibration for the whole portfolio of counterparties. The greater of Effective EPE using current market data and the stress calibration should not be applied on a counterparty by counterparty basis, but on a total portfolio level.

Stressed Calibration

Frquency of Comparison

- **How often is Effective EPE using current market data to be compared with Effective EPE using a stress calibration? and**
- **How this requirement is to be applied to the use test in the context of credit risk management and CVA (eg can a multiplier to the Effective EPE be used between comparisons)?**

The frequency of calculation should be discussed with your national supervisor.

The use test only applies to the Effective EPE calculated using current market data.

Hedges

CVA Charges

103. The only eligible hedges that can be included in the calculation of the CVA risk capital charge under paragraphs 98 or 104 are single-name CDSs, single-name contingent CDSs, other equivalent hedging instruments referencing the counterparty directly, and index CDSs. In case of index CDSs, the following restrictions apply:

- The basis between any individual counterparty spread and the spreads of index CDS hedges must be reflected in the VaR. This requirement also applies to cases where a proxy is used for the spread of a counterparty, since idiosyncratic basis still needs to be reflected in such situations. For all counterparties with no available spread, the bank must use reasonable basis time series out of a representative bucket of similar names for which a spread is available.
- If the basis is not reflected to the satisfaction of the supervisor, then the bank must reflect only 50% of the notional amount of index hedges in the VaR.

Default Charges

I. Default counterparty credit risk charge

1. With respect to identifying eligible hedges to the CVA risk capital charge, the Basel III provisions state that "tranched or nth-to-default CDSs are not eligible CVA hedges" (Basel III document, para 99 - inserting para 103 in Annex 4 of the Basel framework). Can the Basel Committee confirm that this does not refer to tranched CDS referencing a firm's actual counterparty exposures and refers only to tranched index CDS hedges?

Also, can the Committee clarify that Risk Protection Agreements, credit linked notes (CLN), short bond positions as credit valuation adjustment (CVA) hedges, and First Loss on single or baskets of entities can be included as eligible hedges?

All tranched or nth-to-default credit default swaps (CDS) are not eligible. In particular, credit linked notes and first loss are also not eligible. Single name short bond positions may be eligible hedges if the basis risk is captured. When further clarifications are needed, banks should consult with supervisors.

Non Cash Collateral for OTC

108. To implement the supervisory haircuts for non-cash OTC collateral, a new paragraph 61(i) would be incorporated in Annex 4 as follows:

> 61(i). For a bank to recognise in its EAD calculations for OTC derivatives the effect of collateral other than cash of the same currency as the exposure itself, if it is not able to model collateral jointly with the exposure then it must use either haircuts that meet the standards of the financial collateral comprehensive method with own haircut estimates or the standard supervisory haircuts.

Proxy,Index Hedges

14. The revised CCR rules in the Basel III document include a number of areas that have not previously received regulatory scrutiny. Does the Basel Committee consider that supervisory approvals will be required for Basel III, specifically in the areas of:

- Proxy models in respect of CDS spread used where no direct CDS available;
- Applicability of index hedges to obtain the base 50% offset of the new CVA charge;
- If the basis risk requirement for index hedges is sufficient to satisfy the supervisor, will this automatically enable a 100% offset or is it intended to be a sliding scale between 50% and 100%;
- Overall system and process infrastructure to deliver the Basel III changes, even if covered by existing approved models and processes;
- Choice of stress periods to ensure industry consistency. In this regard, for VaR calculation purposes how should the one year period within the three year stress period be identified;
- The fundamental review of the Trading Book will include further analysis of the new CVA volatility charge. Is there any indication as to implementation date and, in the meantime, should CVA market risk sensitivities be included in the firm's VAR calculation.

The use of an advanced or standardised CVA risk capital charge method depends on whether banks have existing regulatory approvals for both IMM and specific risk VaR model. Supervisors will review each element of banks' CVA risk capital charge framework based on each national supervisor's normal supervisory review process.

Elements of Application

A. High-level overview and implementation plans
B. Overview of your firm's own self assessment against relevant standards
C. Summary of your firm's approach in a number of key areas
D. Details of the IMM models being used
E. Sign-off

Overview/Implementation Plan

- Impact Analysis
- Scope
- Rollout Plan
- Orgchart

Self Assessment

- Description of self sssessment process
- Results: exceptions, remediation plan and status

- Roles of senior management, risk functions, audit functions, legal functions, collateral management functions, and the functions of any committees
- Governance of the model, covering the organisation charts and reporting lines of the model owner, developers, and other support functions; an overview of the management committee structure which approved the model and; how external vendor models, if any, are controlled;
- ad-hoc and on going stress testing.

- Show methodology used for the calculation of the IMM exposure is closely integrated into its day-to-day risk management processes.
- Management information where the IMM generated exposure and any other outputs from the methodology is presented
- Management information used by senior management to monitor and control counterparty and market risk including the composition / profile of the portfolios, concentration risk, wrong way risk and the results of stress testing

- Standards of data management; data architecture
- Process to ensure the accuracy, completeness and appropriateness
- Timeliness and robustness of the production systems
- Reconciliation finance and risk systems
- Business continuity

- Accountability, independence, scope, documentation and monitoring the effectiveness of the model on an ongoing basis
- Explanation of how senior management obtain comfort that the outputs from the IMM model are sufficiently robust for the business
- Summary of your approach to back testing, including the methodology and assessment of the results

- List of all the internal documents you hold that you consider relevant to the application, including a brief description of their contents. Relevant documentation would cover documentation specific to the IMM as well as the controls surrounding it

Details of the IMM models being used

- A description of the model coverage, in terms of businesses units, products and risk factors
- Documents relevant to the IMM, with dates when last updated. Including any pre-processing performed on transaction level information, features of the model, assumptions used, use of proxies, approximations, limitations of the output, modelling of risk factors, modelling of collateral and netting and, treatment of margins
- Use of market data
- Valuation analytics including assessment of how assumptions and approximations impact accuracy of the models

Details of the IMM models being used

- Assessment of the model's fitness for purpose in the light of the risks presented by the portfolio (e.g. correlation between counterparties' exposures, wrong way risk)
- Analysis performed on an ad hoc or on going basis to monitor model performance
- Materiality, of any relevant risk factors not covered by the IMM
- Validation reports
- Enhancement plans