Model Risk and Model Control

Patrick Häner

Häner Consulting Berlin

Berlin, May 8, 2013

The

- latest version of this document
- additonal resources
- examples

may be found on

`https://github.com/haenerconsulting/modelrisk`

Outline

Overview

1. Model Classes
2. Credit Risk Measures
3. Model Implementation
4. Back Testing
5. Model Control

What is a Model?

Definition (Model - Narrow)

A Model is a mathematical framework providing answers to a specific set of Questions.

- Refers only to mathematics
- No reference to implementation
- No relation to markets and trading activity of institution

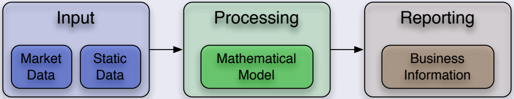
Not a useful definition!

What is a Model?

Definition (Model - Wider)

A Model provides answers to a specific set of Questions. It consists of

- Information input component
- Processing component, applying mathematical transformations
- Reporting component, creating business information



Model - Wider Definition

Emphasis on usage

- Covers data, software and mathematics
- Context of institution, trading activity and market relevant

$\rightarrow$ organizational impact for validation: roles and repsonsibilites

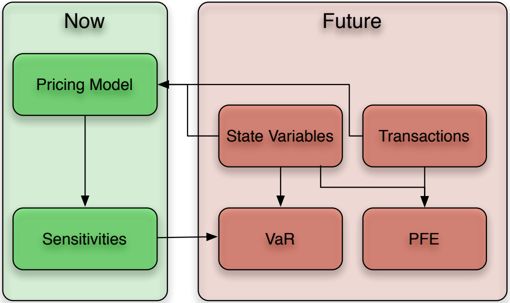
What is the Question?

Types of Questions

- How is the instrument defined
- What is the value $\rightarrow$ pricing model
- What will the prices in the future be $\rightarrow$ risk model

What is the Question?

Dependencies



Categorization

Types of Statements

Prescriptive	Model independent, robust statements
Descriptive	Explaining
Predictive	Falsifiable

Model Independent

Replication

Static Cashflow Replication

Floating leg of swap: replicate by long/short FRA.

Model Dependent Static Replication

Barrier options: static replication dependent of model.

Model Independent
Relation

- Across trade parameters
- Across trade types

Trade Parameters

Price of knock-out option increases with barrier height.

Trade Types

- Barrier option is cheaper than a Plain-Vanilla
- Replicate Plain-Vanilla by in/out Barriers

Trade Model

Requirements

Contractual details as in term sheet need computer-readable representation:

- trade representation
- data exchange
- auditing

Trade Repository

In US: Dodd-Frank regulation require DTCC data repository (DDR) as a multi-asset class repository.

Pricing Model

Usage

- No liquid prices (Mark to Model)
- Sensitivities
- Valuation under future/hypothetical scenarios

Pricing Model

Questions

- Implied price w/o credit risk
- Implied price w credit risk: CVA/DVA
- Implied price range: incomplete markets
- Bid/ask price: liquidity

Pricing Model

Sensitivities

Which sensitivities should be reported?

Aggregation

How to aggregate vega sensitivities from two systems with different models?

- Lognormal model: σ_{BS}
- Normal model: σ_{norm}

Model Independence

Report sensitivities wrt. to market observables, i.e. instead of $sigma_{\text{BS}}, \sigma_{\text{norm}}$ use option prices.

Overview

Likeliness vs Severity of Credit Events

Categories

Which dimensions to consider?

Severity	How much will we lose?
Likeliness	What's the chance that we lose?
Granularity	What does the measure refer to?

Granularity of Measure

Based on Defaults

- All Counterparties
- Single Counterparty

Other Aggregations

- Global/macro economic
- Sector, country
- Trade

Exposure and Recovery

How to measure severity? Need to value trade:

Definition (Exposure at Default)

$$
\begin{aligned}
\mathrm{EAD}(t) &= \max 0, p(t)|\tau = t \\
\tau &: \text{time at which CP defaults}
\end{aligned}
$$

Definition (Loss Given Default)

$$
\text{Loss at time } t = \mathrm{LGD}(t)\mathrm{EAD}(t)
$$

Definition (Recovery)

$$
R(t) = 1 - \mathrm{LGD}(t)
$$

Severity

Valuation Approaches

Accrual	Banking book; rarely adjust; illiquid assets
Mark to market	Trading book; frequently adjusted; traded assets
Mark to model	Trading book; frequently adjusted; complex structures

Example

`CreditRiskMeasures.xlsx`

Severity

Accrual

Loan to Acme Ltd

- value is face value
- maximal loss is notional of loan

Mark to market

Buy bond of Acme Ltd; assume liquid market

- value is mark to market of bond
- value lower than in risk-free valuation

Severity

Mark to model

Exotic interest rate swap with Acme Ltd. What is the value

- risk free: assuming Acme may never default
- risky: Acme may default
- risky with own risk: Acme and we may default

Forward Looking Measures

Assess exposure in future $\rightarrow$ model how state of the world evolves

Deterministic Evolution Scenario Analysis, Stress testing

Stochastic Evolution Model for risk factors

Scenario Analysis/Stress Test

Meanings of Stress

- change model parameters $\rightarrow$
- pick a single path $\rightarrow$ degenerate measure (Dirac measure)

Unified handling by Measure Transforms

Stochstic Process
(Langevin Equation)

$$\dot{x} = f(\boldsymbol{x}) + \boldsymbol{\xi}$$

$\longleftrightarrow$

Probabiliy Measure
(Path Integrals)

$$\begin{array}{rcll} \Gamma & : & t \rightarrow \boldsymbol{x}(t) & \text{Path} \\ \mu(\Gamma) & \sim & e^{-S(\Gamma)} & \\ S(\Gamma) & \equiv & \int_0^T L(\boldsymbol{x}, \dot{\boldsymbol{x}}) & \text{Action} \\ L(\boldsymbol{x}, \dot{\boldsymbol{x}}) & : & \text{Lagrangian} & \end{array}$$

Dual Model Representations

Types of Stress Tests

Approaches

- give economic scenario
- given loss (inverse stress)

Inverse stresses

Statistical Measures

Single Netting Set

Definition (Potential Future Exposure)

$$
\begin{aligned}
\mathrm{PFE}(t) &= \max 0, p(t)|\tau = t \\
\tau &: \text{time at which CP defaults}
\end{aligned}
$$

Definition (Expected Exposure (EE))

$$\mathrm{EE}(t) = \mathbb{E}[\mathrm{PFE}]$$

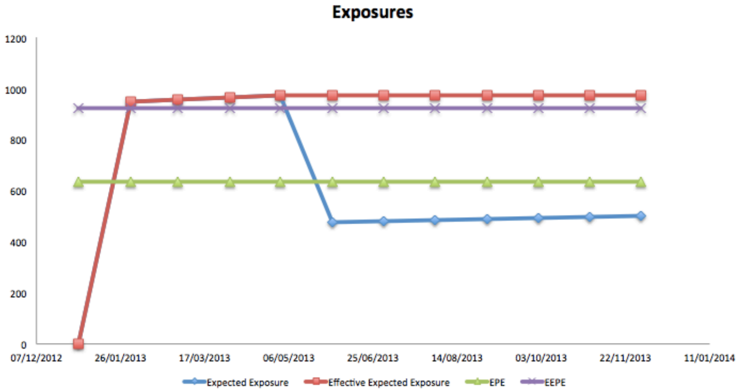
Definition (Expected Positive Exposure)

$$\mathrm{EPE}(T) = \frac{1}{T}\int_0^T \mathrm{EE}(t)\,dt$$

Regulatory Measures

Definition (Effective Expected Exposure (EEE))

Maximum of EPE and past EEE: never decreasing.



Statistical Measures
Multiple Netting Set

Definition (Losses across Netting Sets)

$$L(t) = \sum_a \chi_{\tau_a \le t} \mathrm{LGD}_a \max 0, p_a(\tau_a)$$

$$a \quad : \quad \text{Identifier of netting set}$$

Portfolio Measures

Meaningful risk measures for portfolios

Definition (Coherent Risk Measure)

Risk measure ρ: for portolio X:

Normalization $\rho(\emptyset) = 0$ empty portfolio has no risk

Monotonicity $X_1 \leq X_2 \rightarrow \rho(X_1) \geq \rho(X_2)$

Sub-additivity $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ diversification/netting

Homogeneity $\rho(\alpha X) = \alpha\rho(X) \quad \alpha > 0$

Translation invariance $\rho(X + a) = \rho(X) - a$ adding cash a reduces risk

Portfolio Measures

Quantile

q% quantile: value, for which q% of outcomes are smaller/larger. Quantiles are not coherent measures.

Expected Shortfall

Expected loss conditioned on the loss being larget than X. The Expected Shortfall (Mean Excess Loss) is a coherent measure.

Example

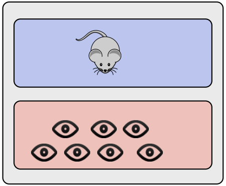
`PortfolioMeasure.xlsx`

Likeliness of Default

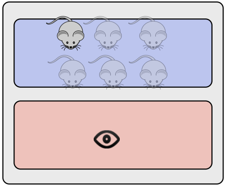
Example

`LikelihoodExperiment.xlsx`

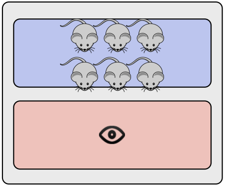
What does probability mean?



Average observers



Implied ensemble



Genuine ensemble

Probability and Measurement

Need to define

- Ensemble
- Measurement process

Examples

Genuine Ensemble

- Mathematics
- Physics: Identically prepared experiment

Average observers

Consensus of observers:

- Market prices
- Betting quota

Implied Ensemble

Equivalence classes:

- Names with same rating
- Price returns in different time windows

Measures for Probability of Default

Definition (Survival/Default Probability, Default Intensity)

Let τ be time of default

$$
\begin{aligned}
S(t) &= p(\tau > t) \\
S &: \text{survival probability} \\
S(t) &= e^{-\lambda(t)t} \\
\lambda &: \text{term default intensity} \\
D(t) &= 1 - S(t) \\
D &: \text{default probability}
\end{aligned}
$$

Note: D is a CDF!

Forward Intensity

Forward default intensity

Probability $d(t)$ of defaulting between t and dt:

$$d(t) = \frac{dD(t)}{dt} \tag{1}$$

Estimating Probability of Default

Estimating λ

Credit Rating Typically using historical data

Market Prices Current credit spreads from bonds or CDS

Implied Default Intensity

Let $s(t)$ be a credit spread

$$\begin{aligned} s(t) &= (1 - R)\lambda(t) \\ R &: \text{recovery rate} \end{aligned}$$

Unifiying Severity and Frequency Measures

High Severity/Low Frequency vs. Low Severity High Frequency

How to compare

- Single large deal with good counterparty
- Set of small deals with bad counterparties

Answer

Pricing including credit risk allows comparing!

Approaches

Top-down vs Bottom-up

Top-down Pricing from first principles

Bottom-up Calculate price correction from building blocks: Exposure (EE) and PE, LGD

Bottom-Up approach

Assumptions

- Risk-free prices known
- Calculate EE
- Estimate PE, LGD
- Calculate correction to risk-free price

Measuring the Corrections

Riskiness of counterparty reduces the price:

Definition (CVA)

Risky price p_A^* as seen from counterparty A with counterparty B:

$$\begin{aligned} p^* &= p - \mathrm{CVA}_B \\ p &: \text{risk-free price} \\ CVA_B &: \text{Credit Valuation Adjustment for counterparty } B \end{aligned}$$

Measuring the Corrections

Does credit risk of counterparty A also affect price?

Definition (DVA)

Price p_A as seen from counterparty A with counterparty B:

$$\begin{aligned} p* &= p - \mathrm{CVA}_B + \mathrm{DVA}_A \\ p &: \text{risk-free price} \\ DVA_A &: \text{Debit Valuation Adjustment for counterparty } A \end{aligned}$$

DVA increases the price.

Accounting vs. Regulatory

DVA must be used for P&L but not for regulatory capital.

Regulatory CVA

BCBS 189, paragraph 89:

$$CVA = (LGD_{MKT}) \cdot \sum_{i=1}^{T} Max\left(0; exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}\right)$$

Regulatory CVA

Similar to regulatory capital charge for default:
Assumes independence of exposure and default process.

$$\text{CVA} = \int_0^T (1 - R) Df(t) EE(t) d(t)\, dt$$

where d is the default probability from equation (1), Df discount factor

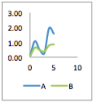
CVA

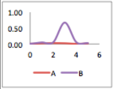
Example

CVA.xlsx

CVA

	A		B	
t		EE		
0	0.00		0.00	
1	1.07		0.63	
2	0.51		0.48	
3	0.25		0.39	
4	1.94		0.78	
5	1.58		0.87	
t	lambda	default prob	lambda	default prob
0	0.01	0.00	0.04	0.00
1	0.02	0.02	0.05	0.05
2	0.025	0.03	0.055	0.06
3	0.025	0.02	0.5	0.67
4	0.02	0.00	0.45	0.06
5	0.02	0.02	0.4	0.03
		CVA		
	0.41			0.06
		DVA		
	0.06			0.41





Regulatory CVA

Regulatory vs Trading CVA

Regulatory Historic measure for EE, implied for PD

Trading Both EE and PD in implied measure

Pricing for Portfolio of Netting Sets

As for single netting sets: pricing combines severity and likeliness. Requires knowing

- prices of individual netting sets at default
- probability of default $P(\chi_{\tau_1 \le t_1}, \chi_{\tau_2 \le t_2}, \ldots, \chi_{\tau_N \le t_N})$

Additional useful quantity: in terms of total losses:

Definition (Loss distribution)

$$\mathcal{L}(l, t) = P(L(t) \ge l)$$

Granularity of Measure in Regulatory Context

Metrics used for Regulatory Purposes

Focus on measures for individual counterparties. No proper modelling of collective losses required.

Overview

Building Blocks

Model Building Process

Business Analysis: Materiality, specification

Model choice: Find adequate model

Software implementation: Develop and roll out

Materiality

What to Model?

Which risk factors material for current portfolio?
How can we assess materiality without exposure model in place?

Approach

Simple estimation of exposure assuming

- future portfolio prices normally distributed
- estimation of first two moments

Gaussian Approximation

Need to estimate $\mathbb{E}[p(T)]$, $\mathbb{E}[p^2(T_i)]$ at some future times T:
Performing Taylor expansion for price p around expected risk factor:

$$\begin{aligned} p(\mathbf{x}(T), T) &\approx p(\mathbf{x}_0(T), T) + \sum_i \frac{\partial p(\mathbf{x_0}(T), T)}{\partial x_i} \Delta x_i(T) \\ &+ \frac{1}{2} \sum_{ij} \frac{\partial^2 p(\mathbf{x_0}(T), T)}{\partial x_i \partial x_j} \Delta x_i(T) \Delta x_j(T) \\ \mathbf{x}_0(T) &\equiv \mathbb{E}[\mathbf{x}(T)] \\ \Delta x_i(T) &\equiv x_i(T) - x_{0,i}(T) \end{aligned}$$

Gaussian Approximation

The expectation value M of the price is hence

$$
\begin{aligned}
M(T) &\approx p(\mathbf{x}_0(T), T) + \frac{1}{2}\sum_{ij}\gamma_{ij}(T)\Omega_{ij}(T) \\
M(T) &\equiv \mathbb{E}[p(\mathbf{x}(T), T)] \\
\gamma_{ij}(T) &\equiv \frac{\partial^2 p(\mathbf{x_0}(T), T)}{\partial x_i \partial x_j} \\
\Omega_{ij}(T) &\equiv \mathbb{E}[\Delta x_i(T)\Delta x_j(T)]
\end{aligned}
$$

For the variance V we obtain up to second order in $\Delta\mathbf{x}$:

$$
\begin{aligned}
V(T) &\approx \sum_{ij}\delta_i(T)\delta_j(T)\Omega_{ij}(T) \\
V(T) &\equiv \mathbb{E}[(p(\mathbf{x}(T), T) - \mathbb{E}[p(\mathbf{x}(T), T)])^2] \\
\delta_i(T) &\equiv \frac{\partial p(\mathbf{x_0}(T), T)}{\partial}
\end{aligned}
$$

Gaussian Approximation

What can we learn?

Risk factor contributions

Matrix elements $\Psi_{ij} = \delta_i(T)\delta_j(T)\Omega_{ij}(T)$ indicate contribution of risk factors ij to total variance.

EE, PE

Knowning mean and variance of the Gaussian distribution, any statistical quantity may be evalued.

Caveat

Depending on specifics of portfolio this approximation may be more or less accurate: that is why we use Monte Carlo simulations after all.

Practical Implementation

- For $t = 0$: δ and γ from Market risk system. **But:** need netting set level aggregation $\rightarrow$ deal level granularity
- For $t > 0$ estimate future δ, γ by bumping

Trade Models

Requirements

- Represent trades/products
- Standardize for interoperability

Trade Models

Trade Parameters

Product represented by parameters

FpML


```xml
▼<FpML xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance" xmlns="http://www.fpml.or
 xsi:schemaLocation="http://www.fpml.org/2003/FpML-4-0 fpml-main-4-0.xsd">
 ▼<trade>
  ▼<tradeHeader>
   ▼<partyTradeIdentifier>
      <partyReference href="CHASE"/>
      <tradeId tradeIdScheme="http://www.chase.com/swaps/trade-id">921934</tradeId>
    </partyTradeIdentifier>
   ▼<partyTradeIdentifier>
      <partyReference href="UBSW"/>
      <tradeId tradeIdScheme="http://www.ubsw.com/swaps/trade-id">204334</tradeId>
    </partyTradeIdentifier>
    <tradeDate>2000-04-03</tradeDate>
   </tradeHeader>
  ▼<swap>
   ▼<!--
       Chase pays the floating rate every 6 months, based on 6M EUR-EURIBOR-Telerate
             + 10 basis points, on ACT/360 basis
    -->
   ▼<swapStream>
      <payerPartyReference href="CHASE"/>
      <receiverPartyReference href="UBSW"/>
     ▼<calculationPeriodDates id="floatingCalcPeriodDates">
       ▼<effectiveDate>
          <unadjustedDate>2000-04-05</unadjustedDate>
         ▼<dateAdjustments>
            <businessDayConvention>NONE</businessDayConvention>
          </dateAdjustments>
        </effectiveDate>
       ▼<terminationDate>
          <unadjustedDate>2005-01-05</unadjustedDate>
```

Trade Models

Trade Parameters

Pro/Con

- ⊕ standardized
- ⊖ logic in client

Trade Models
Cashflows

Product represented by casflows

Payoff macros

Notional	100
DCF	Act/Act

Date	Libor fixing	Payoff
02-Mar-13	1.34%	0.335
02-Jun-13	1.19%	0.2975
02-Sep-13	1.42%	0.355
02-Dec-13	1.39%	0.3475

Trade Models

Cashflows

Pro/Con

- ⊕ simple
- ⊖ not expressive enough (just cash is exchanged)
- ⊖ single product (no interations)

Trade Models

Transaction Model

Approach

Multi agent simulation:

- **Time** Map wall clock to simulation time
- **Market Events** simulation time to events
- **Transactions** events to transactions (e.g. cashflows)
- **Execution** execute events

Pro/Con

- $\oplus$ general
- $\oplus$ all business logic in model $\rightarrow$ easy tooling
- $\ominus$ expensive

Pricing & Risk Models

Criteria for Model Choice

Categories

Independent of product Relate to Mathemathics or Physics

Dependent of product Specific to product type

Dependent of portfolio and market Context

Pricing & Risk Models

Independent of Product

Coordinate Sytems

From Physics we know: dynamics must not depend on choice of coordinates $\rightarrow$ dimension analysis.

Interpolation

How to interpolate r, σ. Interpolate dimension-less quantities: rt and $\sigma^2 t$.

Pricing & Risk Models

Product Dependent

State Variables vs. Parameters

Liquidity	Hedge frequency, transaction costs, close-out period
Completeness	Unhedgeable risk, uniqueness of price

Pricing & Risk Models

State Variables and Parameters

Indicators of Model Quality

Parameter Dimensionality Avoid overparameterization

Stability of Parameters Frequent recalibration: indicator of poor model performance

GBM w termstructure vs Garch

	TS GBM	**Garch**
dimension	∞	3
recalibration	frequently for short end	less frequent
time-homogeneous	N	Y

Pricing & Risk Models

Arbitrage

Risk Model for Volatility surface

Directly modelling surface w/o arbitrage not trivial. Alternatively model option prices with HJM-like framework.

Pricing & Risk Models

Market

Liquidity & Completeness

Liquidity Hedge frequency, transaction costs, close-out period

Completeness Unhedgeable risk, uniqueness of price

Comparing Models

Assume state of the world evolves randomly:

Model as Process: Stochastic Differential Equation (Langevin Equation)

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= f(\mathbf{x}) + g(\mathbf{x})\xi(t) \quad \text{Physics Notation} \\ d\mathbf{x} &= f(\mathbf{x})dt + g(\mathbf{x})d\mathbf{W}(t) \quad \text{Finance Notation} \end{aligned}$$

Wiener Process (SDE)

$$\begin{aligned} dx &= dW(t) \\ W &: \text{Wiener Process} \end{aligned}$$

Comparing Models

Model as Measure $\mathbb{P}$

$$\begin{aligned} \mathbb{P} &: \quad \mathbf{\Gamma} \to \mu(\mathbf{\Gamma}) \quad \text{probability} \\ \mathbf{\Gamma} &: t \quad \to \mathbf{x}(t) \quad \text{some path} \end{aligned}$$

Wiener Process (SDE)

$$\begin{aligned} \mathbf{\Gamma} &\equiv \{x_1, \ldots x_N\} \\ \mu(\Gamma) &\sim \prod_i G(x_i, x_{i+1}) \\ G &: \quad \text{Gaussian} \end{aligned}$$

Pricing & Risk Models

Parametric Models

Error Analysis

Infer from parameter uncertainty price/risk uncertainty.

Parameter Uncertainty E.g. such that hedging instrument prices still in bid-ask

Parameter Error Uncertainty of price/risk due to error in parameters

GBM with vol uncertainty

$$(\Delta p)^2 = \left(\frac{\partial p}{\partial \sigma}\Delta\sigma\right)^2$$

Benchmarking

Pricing/risk factor models $\mathbb{Q}, \mathbb{Q}'$, empirical measure $\mathbb{P}$

Comparing

Pricing Models $\mathbb{Q}$ vs $\mathbb{Q}'$

Risk Models $\mathbb{P}$ vs $\mathbb{Q}$

Benchmarking

Distances

How far apart two models?

Need to define metric:

- Expectation values: E.g. differences of prices and EEs under different measures
- Distributions: E.g. Kullback-Leibler entropy $\int \frac{d\mathbb{P}'}{\mathbb{P}} \log \frac{d\mathbb{P}'}{\mathbb{P}} d\mathbb{P}$. Independent of quantity to average.

Model Uncertainty

Benchmarking giving limited answer:

Calibration-Consistent Measures

Define metric d to quantify goodness of calibration:

$$\begin{aligned} p_i^{\mathbb{P}} &: \text{ model price calibration instrument } i \\ p_i &: \text{ market price calibration instrument } i \\ d^{\mathbb{P}} &= \sum_i (p_i^{\mathbb{P}} - p_i)^2 \\ \mathcal{C}_\epsilon &= \{\mathbb{P} | d^{\mathbb{P}} \leq \epsilon\} \end{aligned}$$

Model Uncertainty

Non-uniqueness

For $d = 0$:

- Multiple measures
- For single parametric measure, multiple solutions for calibration $\rightarrow$ ill behaved
- Incomplete market

For $d > 0$:

- For single parametric measure: parameter risk

Exposure

Goal

Estimate Credit Risk measures → need to estimate exposure/price distributions in future.

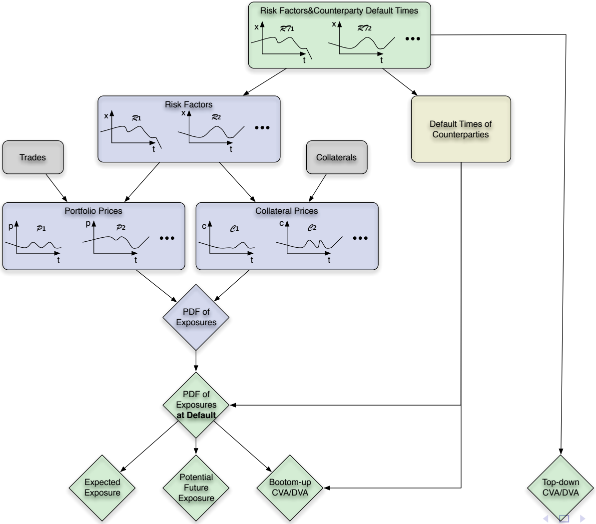
The exposure $e(t)$ at time t of a netting set is given by

$$e(t) = \max 0, \sum_i p_i(\mathbf{x}, t) - C(t) \qquad (2)$$

where

- p_i price of trade i
- $\mathbf{x}$ risk factors
- $C(t)$ price of collateral

Calculating Exposure, CVA/DVA and Losses



Building Blocks

Components

Required for estimating risk measures for single and portfolios of netting sets:

- Pricing
- Risk-factor
- Collateral
- Netting
- Dependency

Pricing Models

Requirements

- Need to be fast!
- Ideally same as front office
- Perform well under stressed state variables

Pricing Models

Acceleration Techniques

Dumb lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- define grid
- recaluclate for each gridpoint price
- interpolate

Pricing Models
Acceleration Techniques

Smart lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- prices on grid are side effect of pricing at spot; e.g. pricing on tree or AMC
- interpolate

Risk Factor Models

Pricing vs Risk Models

Purpose

Pricing Model Fit liquid market instruments; arbitrage-free

Risk Model Predict

Challenges for Risk Model

Dependency Simultaneously simulate all asset classes

Calibrationl Global calibration

Risk Factor Models
Short vs Long term prediction

Long term prediction a challenge:

- Reducing dimensionality
- Economic macro factors
- Co-integration

Risk Factor Models

Pricing model Dynamics

Arbitrage-free models used with risk calibration

- GBM
- HJM type of models

⊕ Well understood, tractable

⊖ Not intended for risk

Gaussian Dependency Modelling

Goal

Express random vector $\boldsymbol{\xi}$ with correlated ξ_i as

- linear combination of
- uncorrelated

random factors η_i:

$$
\begin{aligned}
\boldsymbol{\xi} &= \mathbf{M}\boldsymbol{\eta} \\
\mathbb{E}[\xi_i\xi_j] - \mathbb{E}[\xi_i]\mathbb{E}[\xi_j] &\equiv \Omega_{ij} \\
\mathbb{E}[\eta_i\eta_j] - \mathbb{E}[\eta_i]\mathbb{E}[\eta_j] &= \lambda_i^2\delta_{ij} \quad \text{diagonal, pos. sem. def.}
\end{aligned}
$$

What to consider?

- $\boldsymbol{\Omega}$?
- correlation matrix?

Principal Component Analysis

Dimensional Analysis

Risk factors ξ_i not dimension-less!

- interest rate :$[T^{-1}]$
- stock price :[Cash]
- volatility: $[T^{-\frac{1}{2}}]$

$\rightarrow \Omega_{ij}$ may have different dimensions,i.e. $\mathbf{\Omega}$ in general not a physically meaningful quantity!

Principal Component Analysis

Solution

Consider instead of $\mathbf{\Omega}$ following matrix $\mathbf{\Phi}$:

$$\begin{aligned} \Phi_{ij} &\equiv \frac{\partial f(\xi)}{\partial \xi_i}\frac{\partial f(\xi)}{\partial \xi_j}\Omega_{ij} \\ f &: \text{some function} \end{aligned}$$

For dimensionality $[\mathbf{\Phi}]$:

$$[\Phi_{ij}] = \frac{[f]}{[\xi_i]}\frac{[f]}{[\xi_j]}[\xi_i][\xi_j] = [f^2] \quad \forall i,j \qquad \checkmark \tag{3}$$

GBM Risk Factor Model

Multivariate GBM

$$
\begin{aligned}
X_i(t+\Delta t) &\quad = \quad X_i e^{(\mu_i - \frac{1}{2}\sigma_i)\Delta t + \sigma_i \sqrt{\Delta t}\xi_i(t)} \\
\mu &\quad : \quad \text{drift} \\
\sigma &\quad \text{volatility} \\
\xi_i &\quad : \quad \text{Normal random} \\
\mathrm{Cov}(\ln X_i(t+\Delta t), \ln X_j(t+\Delta t)) &\quad = \quad \Omega_{ij}
\end{aligned}
$$

Dependent Gaussian Random Variables

Given uncorrelated Gaussian random number vector $\boldsymbol{\zeta}$. Need build $\boldsymbol{\eta}$:

$$\mathrm{Cov}(\eta_i, \eta_j) = \Omega_{ij}$$

Calibration

Definition

Calibration is the process to determine model parameters.

Approaches

Statistical	Using historical data
Implied	Market implied parameters
Economic	Macro economical relation between rates, infaltion

Asumptions

Statistical	Past is good predictor for future
Implied	Information in spot market predicts future
Economic	Some fundamental economic laws rule future

Statistical Calibration

For simple models: ad hoc parameter estimation

- averaging
- fitting

Example

SimpleEstimation.xls

Maximum Likelihood Estimation

Systematic way to calibrate

Approach

Parametric model with parameters $\boldsymbol{\alpha} \leftrightarrow$ parametric measure $\mu_{\boldsymbol{\alpha}}$:

$$\mu_{\boldsymbol{\alpha}}(\Gamma) = e^{-S_{\boldsymbol{\alpha}}(\Gamma)}\mathcal{D}[\Gamma]$$

Assume: historical path Γ_H is the most likely one. Find $\boldsymbol{\alpha}^*$ such that:

$$\mu_{\boldsymbol{\alpha}^*}(\Gamma_H) = \max_{\boldsymbol{\alpha}} \mu_{\boldsymbol{\alpha}}(\Gamma_H)$$

Maximum Likelihood Estimation

Implementation

Assuming iid:

$$\begin{aligned} \mu_{\boldsymbol{\alpha}}(\Gamma) &= \prod m(\mathbf{x}_i) \\ m(\mathbf{x}) &= e^{-s(\mathbf{x})} \\ \Gamma &= \{\mathbf{x}_1, \ldots, \mathbf{x}_n\} \end{aligned}$$

Maximizing $m \leftrightarrow$ minimizing

$$\sum_i s(\mathbf{x}_i) : \quad \text{log-likelihood}$$

Maximum Likelihood Estimation

Example

`MLE.xls`

Implied Parameters

Apply parameters used for pricing:

Drift and Volatility

- Drift μ from T forward price (Covered Parity)
- Volatility σ T years ATM implied volatility

Assumption

Risk neutral measure yield good predictor for real-world measure

Caveat

- Carry trades
- Supply/demand, risk premium

Perform analysis before using implied parameters!

Economic Calibration

Parities connect for instance

- FX rates
- Inflation rates
- Real interest rates
- Nominal interest rates
- Purchansing power

Example

`Parities.xlsx`

Parities

Example (Relative Purchasing Power Parity)

$$
\begin{aligned}
p_f(t_1)(1+i_f)X(t_2) &= p_d(t_2)(1+i_d) \\
p_{d/f} &: \text{domestic/foreign price} \\
i_{d/f} &: \text{domestic/foreign 1 yr inflation rate} \\
X &: \text{Exchange rate}
\end{aligned}
$$

Yields after averaging

$$
\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1+I_d}{1+I_f}
$$

where I is the expected inflation rate.

Parities

Example (International Fisher Effect (Uncovered Parity))

$$
\begin{aligned}
(1 + r_{d/f}) &= (1 + \rho_{d/f})(1 + i_{d/f}) \\
r_{d/f} &: \text{domestic/foreign nominal 1 yr interest rate} \\
\rho_{d/f} &: \text{real rdomestic/foreign 1 yr interest rate}
\end{aligned}
$$

Assuming$\rho_d = \rho_f$ gives

$$
\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1 + r_d}{1 + r_f}
$$

Issues with standard GBM model

Issues

- rigidity: calibration short vs long horizons $\rightarrow$ term structure of parameters
- dimensionality $\rightarrow$ factor models
- underestimation of rare events and bursts (clustering) $\rightarrow$ GARCH
- not suitable where spread stationary process $\rightarrow$ cointegration
- unable to capture some behabiour like regime-switches $\rightarrow$ parametric models (Nelson-Siegel)

GBM with Term Structure

Interpolation Principles

Interpolate dimension-less quantities

Forward Drift/Covariance

Dimensionality analysis $\rightarrow$ interpolate $T\mathbf{\Omega}$

Factor Models

Issues with general covariance matrix

N risk factors $\rightarrow \propto N^2$ parameters

- over-parametrization
- for empirical parameters: problems with positive definiteness

Idea

Split return r of riskfactors into contributions from

Indices f_n shared by multiple risk factors

Idiosycratic factors ϵ unique to each risk factor

$$r = \alpha + \sum_n \beta_n f_n + \epsilon$$

and assume

- indices uncorrelated to indosyncratics

Types of Factor Models

Classification

Macroeconomic Observables like changes in inflation, interest rate, unemployment rate

Fundamental Portfolios associated to security attributes like industry membership, book to market ratio, dividends

Statistical Factor analysis of covariance matrix

Macroeconomic Factor Model

Fast/Slow

Slow variables Macro-economic state of the economy: inflation, unemployment rate, GDP

Fast Asset prices

Pros and Cons

- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing
- ⊖ Theories controversial

Fundamental Factor Model

Sector/Region

1. Define for each sector/region pair an index
2. Associate stock to sector/region
3. Regress stock return vs index return $\rightarrow \alpha, \beta$

Example

FactorModel.xls

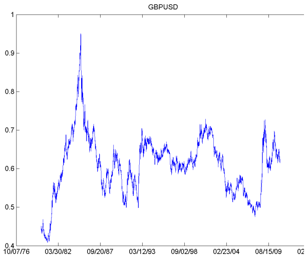
Pros and Cons

- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing

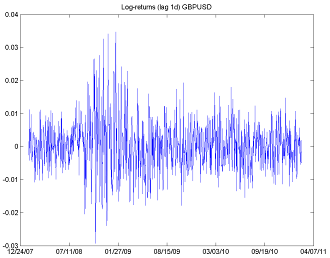
Choice of Factors

How to know whether factors appropriate?
Analyze variance explained by factors

Volatility Clustering



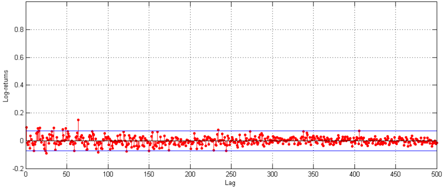
(a) Spot



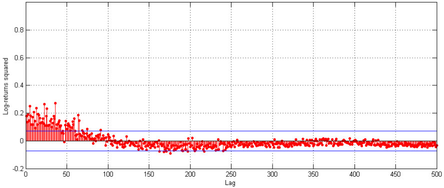
(b) Log-returns

Figure: GBPUSD spot

Autocorrelation



(a) Autocorrelation: log-returns



(b) Autocorrelation: squared log-returns

Garch Model

Let X_n be the log-return of some foreign exchange rate f at time t_n:

$$X_n = \ln \frac{f_n}{f_{n-1}} \tag{4}$$

we may then express the foreign exchange rate f_N at some future sampling point time t_N by the initial value f_0 at t_0 and a series of returns:

$$f_N = f_0 e^{\sum_{i=1}^{N} X_i} \tag{5}$$

The observation points t_i are typically defined in terms of number of business days ΔT between them. For short time horizon predictions we choose $\Delta T = 1$ for larger horizon, we may choose a less granular time grid.

Garch Model

The dynamics of the returns is then assumed to follow a Garch(1,1) process

$$\begin{aligned} X_n &= \mu + \epsilon_n \quad \epsilon_t \sim \text{iid}(0, \sigma_n^2) && (6) \\ \sigma_{n+1}^2 &= \alpha + \beta \sigma_n^2 + \gamma \epsilon_n^2 && (7) \end{aligned}$$

The asymptotic value $\sigma_\infty^2 = \lim_{n\to\infty} \mathbb{E}[\sigma_n^2]$ is then obtained by equation (7) noting, that $\mathbb{E}[\epsilon^2] = \sigma^2$ and $\mathbb{E}[\sigma_{n+1}^2] \to \mathbb{E}[\sigma_n^2]$:

$$\sigma_\infty = \frac{\alpha}{1 - \beta - \gamma} \qquad (8)$$

Garch Model: Limit

Weak limit:

- Stochastic variance
- Mean reverting variance

$$
\begin{aligned}
dX_t &= \mu X_t dt + \sqrt{v_t} X_t dW_t \\
dv_t &= \alpha(v_t) dt + \beta(v_t) dZ_t
\end{aligned}
$$

Copula

Dependence under Stress

In stressed markets correlations increase between

- downward price movements $\rightarrow$ systematic risk
- implied default probabilities $\rightarrow$ contagion

Definition (Copula)

Separate

- Marginal distributions from
- Dependency

Cointegration

Long-run Relationship

Variables moving together:

Macro-economic
- Consumption-Income
- Prices-Wages
- Domestic prices - fpreign prices

Exogeneous For instance managed currencies

- How to model processes. which stay close to each other?
- GBM with $\rho_{ij} \lessapprox 1$ **not**? No!
- Need dynamic, where difference is stationary

Definition

Stochastic processes x, y are cointegrated:

$$y(t) = a + bx(t) + \xi(t)$$

Implementation

1. find parameters a, b by regression
2. show residuals are stationary (e.g. Dickey-Fuller Test)

Example

`Cointegration.xlsx`

Risk Factor Models

Empirical Models

Nelson-Siegel model

$$
\begin{aligned}
r(T) &= r_\infty + a(T)r_0 + b(T)r_m \\
r_\infty &: \text{ rate for long maturities} \\
r_0 &: \text{ rate for short maturities} \\
r_m &: \text{ rate for intermediate maturities} \\
a, b &: \text{ decay functions}
\end{aligned}
$$

Risk Factor Models

Empirical Models

Nelson-Siegel model

- Normal/inverted curves
- But not arbitrage-free

How to introduce dynamics? E.g. PCA of (r_∞, r_0, r_m)

Example

`NelsonSiegel.xlsm`

Wrong Way Risk

Types

Specific Legal connection between underlying and counterparty

General Dependence between prob. of default of counterparty and exposure

SFT Transactions

Lend cash to counterparty A accepting their stock as collateral.

Emerging Market CCY swap

We are long strong currency. Weakening of emerging market currency, increased prob default $\rightarrow$ increase exposure

Modelling Wrong Way Risk

What is wrong with standard modelling?
p^+ is **not** conditioned on default.
Need to add in price function default state χ of counterparty:
extending state of the world

Approaches

Given a model for default times either

- Simulating counterparty's default
- Calculating price given default

Example

`WrongWayRisk.xls`

Collateral Modeling

Components

Margin Call Process Model margin calls with correct frequency and close-out period

Collateral Price E.g. model bond price if collateral is bond

Simplification

- Margin call process: just at spot $\rightarrow$ short-cut method
- All collateral as cash $\rightarrow$ haircuts

Collateral Modeling
Short-Cut Method

Definition (Basel II Short-Cut Method)

EE and PE of collateralized trades given by EE and PE for close-out period (5 days for SFT, 10d for OTC)

Benefits/Issues

- ⊕ Computationally cheap
- ⊕ No collateral exposure spikes at expity
- ⊖ Assumes exposures declining over time
- ⊖ Risk not accurately represented

Dependency Modelling

Among Risk Factors

Standard way to model dependence: Gaussian Copula.
Gaussian Copulas are Levy copulas. Replace Gaussian with other Levy coupula and obtain Levy model.

Between Defaults

Simulate either

Default times τ E.g. by Marshall-Olkin Copulas

Default state at t:$\chi_{\tau \leq t}$ E.g. structural models

Dependency Modelling

Between a Default and Risk Factors

To caputure Wrong Way risk need to model dependence between risk factor and default state

Example

`WrongWayRisk.xls`

Between a cross name Defaults and Risk Factors

Need modelling full state of the world $(\mathbf{x}(t), \{\chi_{\tau_1 \leq t}, \ldots \chi_{\tau_1 \leq t}\})$.
$\rightarrow$ scenario consistency is system

Model Lifecycle

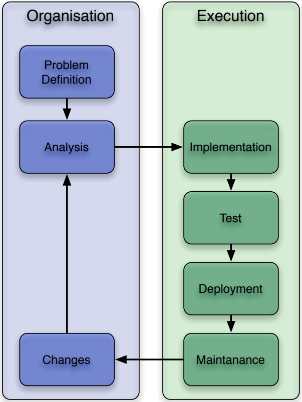


Figure: Model Development Lifcecyle

Specification

Approaches

Human readable Business and functional specs

Machine readable Specification $\sim$ test

Specification

Tools

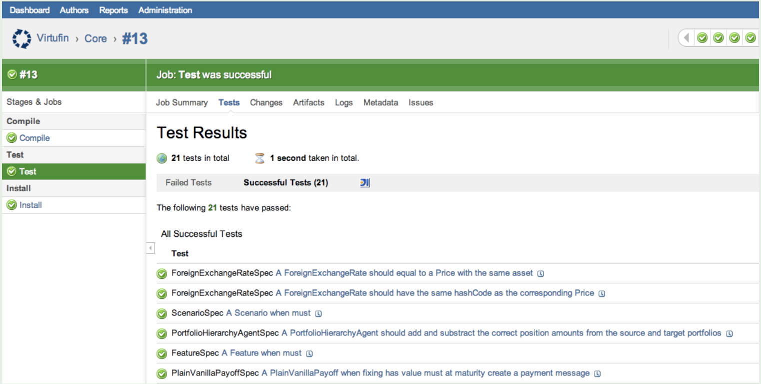
ScalaTest Code

```
class LookupSpec extends WordSpec with MustMatchers {
  def fixture = new {
    val i1 = Identifier[Double]("abc")
    val i2 = Identifier[Double]("xyz")
    val i3 = Identifier[Double]("ABC")
    val x1 = 12.3
    val x2 = 4.56
    val l = Lookup(i1 -> x1, i2 -> x2)
  }

  "A Lookup" when {
    "item exists" must {
      "retrieve with () the item" in {
        val f = fixture
        val y1 = f.l(f.i1)
        y1 must equal(f.x1)
        val y2 = f.l(f.i2)
        y2 must equal(f.x2)
      }
      "retrieve with get() an Option object containing the item" in {
        val f = fixture
        val y1 = f.l.get(f.i1)
        y1 must not be Option.empty
        y1.get must equal(f.x1)
        val y2 = f.l.get(f.i2)
        y2 must not be Option.empty
        y2.get must equal(f.x2)
      }
    }
```

Specification
Tools

ScalaTest Output

Part of CI:



Implementation

Software

- in-house
- third-party

Require different validation strategies

Third Party

Strategies

- Black-box, no code review
- Reverse-engineering

Revision Control

Requirements

Audit	Who changed what/when
Resurrect	Roll-back to previous state
Collaborate	Merge contributions from different authors

Approaches

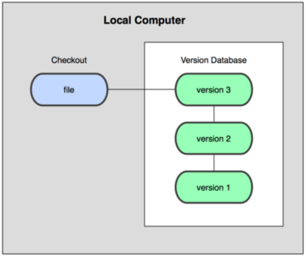
Plain files	Tag files/directories with version information
Local	Local database contains version information (e.g RCS)
Server	Database on server (e.g. SVN)
Distributed	Each developer has own databse with potentially central db (e.g. Git)

Revison Control Tools

Approaches

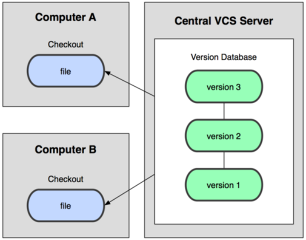
```
MyDirectoryV1.0

MyDirectoryV1.1

MyDirectoryV1.2-bugfix1
```

(a) File based

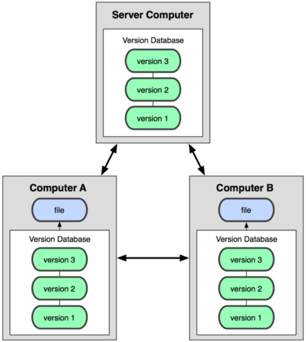


(b) Local VCS

Revison Control Tools
Approaches



(c) Centralized VCS



(d) Distributed VCS

Revision Control Tools
Git

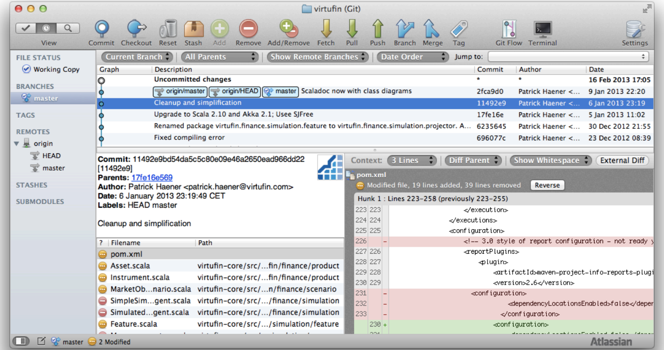


Figure: Git Gui (SourceTree)

Documentation

Requirement

Contain enough information to reverse-engineer.

Tools

- Automated API doc (*Doxygen*, *ScalaDoc*, ...)
- Internal wiki (e.g. *Confluence*)

Testing

Test Types

Unit	Library level
Integration	System level

Testing
Unit Test

```
class BlackScholesScenarioModelTest {
  @Test
  def test() {
    val marketObservable = Price(Stock("IBM"), Currency.USD)
    val modelDate = Time(2012, 10, 1)
    val t1 = Time(2013, 10, 1)
    val t2 = Time(2015, 10, 1)
    val t3 = Time(2022, 10, 1)
    val ts = List(t1, t2, t3)
    val scenarioObservables = ts.map(t => Index(t, marketObservable))
    val x0 = 100.0
    val scenario = Scenario(marketObservable, modelDate -> x0)
    val mu = 0.01
    val sigma = 0.4
    val blackParameters = BlackParameters(mu, sigma)
    val blackParametersId = BlackParameters.identifier(marketObservable)
    val parameters = Lookup(blackParametersId -> blackParameters)
    val request = ScenarioRequest(scenarioObservables, scenario, modelDate)
    val context = ModelContext(ModelDispatcher(ModelRegistry()), parameters)
    val model = new BlackScholesScenarioModel()
    val result = model.model(request, context)
    assertTrue(result.isSuccess)
    val iterable = result.toOption.get
    val n = 1000000
    var x = 0.0
    var x2 = 0.0
    val o = scenarioObservables.last
    Timing.timing(iterable, (s: Scenario) => {
      val a = s(o); x = x + a; x2 = x + a * a
    }, n)
    assertEquals(x0 * math.exp(mu * DateUtil.yearsBetween(modelDate, o.time)), x / n, 0.5)
  }
```

Release

Requirements

- Regression
- Impact analysis
- Sign-off
- Auditing
- Lock-down

Maintance

Bugs/Enhanements

- Tracking system
- Failing test cases
- Metrics: severity, resolution time

Integrated Development Process

Robust system should have

Components

- Revsion Control system
- Build System
- Bug tracking system
- Wikin

Components integrated to workflow with high degree of automation

Overview

Motivation

Impact of Credit risk model

Trading activity limits set by *PE*

Capital charges regularity capital dependent of *EEPE*

P&L *EE* enters CVA/DVA

Model Risk

Back-testing should quantify model risk affecting these quantities.

Requirements

Back-testing Process

Should provide

Definition of measure for model risk

Monitoring of metrics

Mitigating actions for model deficiencies

BCBS Guidance

G1

Guidance: Backtesting of forecast distributions produced by EPE models and market risk factor models needs to be performed on the entire forecast distribution.

G2

Guidance: The validation requirements as set out in Basel II for EPE Models should not make reference to VaR requirements and instead the qualitative standards set out in paragraph 718 (LXXIV) should be transposed into the validation requirements for EPE models and the language adapted where required.

G3

Guidance: The Validation of EPE models and all the relevant models that input into the calculation of EPE must be performed separately for a number of distinct time horizons.

BCBS Guidance

G4

Guidance: The performance of market risk factor models must be validated using backtesting. The validation must be able to identify poor performance in individual risk factors.

G5

Guidance: The validation of EPE models and all the relevant models that input into the calculation of EPE must be made using forecasts initialised on a number of historical dates.

G6

Guidance: Historical backtesting on representative counterparty portfolios and market risk factor models must be part of the validation process. At regular intervals as directed by its supervisor, a bank must conduct backtesting on a number of representative counterparty portfolios and its market risk factor models. The representative portfolios must be chosen based on their sensitivity to the material risk factors and correlations to which the bank is exposed.

BCBS Guidance

G7

Guidance: Backtesting of EPE and all the relevant models that input into the calculation of EPE must be based on recent performance.

G8

Guidance: The frequency with which the parameters of an EPE model are updated needs be assessed as part of the on-going validation process.

G9

Guidance: Firms need to unambiguously define what constitutes acceptable and unacceptable performance for their EPE models and the models that input into the calculation of EPE and have a written policy in place that describes how unacceptable performance will be remediated.

BCBS Guidance

G10

Guidance: Firms need to define what constitutes a representative counterparty portfolio for the purposes of carrying out EPE model backtesting.

G11

Guidance: IMM firms need to conduct hypothetical portfolio backtesting that is designed to test risk factor model assumptions, eg the relationship between tenors of the same risk factor, and the modelled relationships between risk factors.

G12

Guidance: Firms need to assess whether or not the firm level and netting set level exposure calculations are appropriate.

BCBS Guidance

G13

Guidance: Firms must backtest their EPE models and all relevant models that input into the calculation of EPE out to long time horizons of at least one year.

G14

Guidance: Firms must validate their EPE models and all relevant models that input into the calculation of EPE out to time horizons commensurate with the maturity of trades covered by the IMM waiver.

G15

Guidance: Prior to implementation of a new EPE model or new model that inputs into the calculation of EPE a firm must carry out backtesting of its EPE model and all the relevant models that input into the calculation of EPE at a number of distinct time horizons using historical data on movements in market risk factors for a range of historical periods covering a wide range of market conditions.

BCBS Guidance

G16

Guidance: Under the internal model method, a measure that is more conservative than Effective EPE (eg a measure based on peak rather than average exposure) for every counterparty may be used in place of alpha times EEPE with the prior approval of the supervisor. The degree of relative conservatism will be assessed upon initial supervisory approval and at regular intervals in conjunction with other EPE models. The assessment needs to cover all counterparties. The firm must have an unambiguous definition of what constitutes acceptable performance for these models and a documented process in place for remediating poor performance.

What is the Question?

Types of Investigation

- Hypothesis testing (Answer in percentage or yes/no)
- Estimation of model uncertainty (Answer in cash terms)

Analysis at different levels: figure 5

Domains

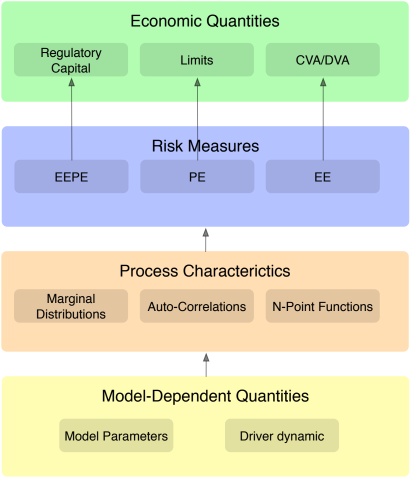


Figure: Domains

Definition

A model is represented by a measure $\mathbb{Q}$.
May be generated by a stochastic process.

Quantifying Difference of Models

- Comparing expectation values
- Comparing probability distributions

Note: PDFs and CDFs may be expressed as expectation values

Radon-Nikodym Derivative

Distance of model $\mathbb{Q}$ and end empirical measure $\mathbb{P}$ in terms of $\frac{d\mathbb{P}}{d\mathbb{Q}}$:

$$\mathbb{E}_{\mathbb{P}}[f] = \mathbb{E}_{\mathbb{Q}}[\frac{d\mathbb{P}}{d\mathbb{Q}} f] \tag{9}$$

Compare $\mathbb{P}$ and $\mathbb{Q}$

Direct $\frac{d\mathbb{P}}{d\mathbb{Q}} \approx id$?

Expectation values Empirical expectation measures in terms of model expectations

Relative Entropy Kullback-Leibler entropy $\rightarrow$ information geometry (see [?])

Radon-Nikodym Derivative

Let ξ be a scalar stochastic variable (e.g. portfolio price $\pi(t)$)

Definition

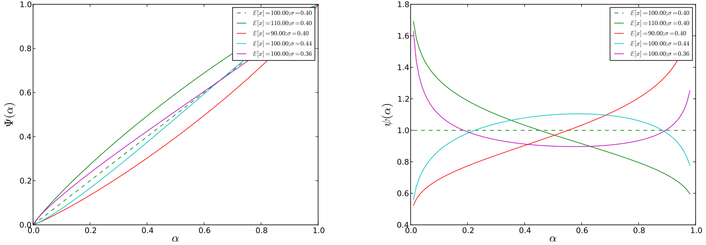
P empirical, Q model CDF

$$\Psi : [0,1] \to [0,1] \tag{10}$$
$$\Psi(\alpha) = P(Q^{-1}(\alpha)) \tag{11}$$

Radon-Nikodym derivative ψ

$$\mathbb{E}_{\mathbb{P}}[f] = \mathbb{E}_{\mathbb{Q}}[\psi(\alpha) f] \tag{12}$$
$$\psi(\alpha) = \frac{d\Psi(\alpha)}{d\alpha} \tag{13}$$

Example



(a) $\Psi(\alpha)$ (b) $\psi(\alpha)$

Figure: Comparison of log-normal distributions with different parameters

Cumulative Distribution Functions

Cumulative distribution function (CDF) for some state variable ξ expressed as expectation:

Definition

$$P(\xi_0) = \mathbb{E}_{\mathbb{P}}[\Theta(\xi - \xi_0)] \tag{14}$$

where Θ is the Heaviside function.

Estimating

Ensemble averages $\mathbb{E}$ estimated well by time averages if

- ergodic
- stationary

CDF

$$P(\xi_0) \approx \frac{1}{N} \sum_{i=1}^{N} \Theta(\xi(t_i) - \xi_0) \tag{15}$$

Ψ

$$\Psi(\alpha) \approx \frac{1}{N} \sum_{i=1}^{N} \Theta(\xi(t_i) - Q^{-1}(\alpha)) \tag{16}$$

Requirements for Estimation

Process neeeds to be

- ergodic
- stationary

iid price process

If empirical price process is iid, the ergodic.

iid process of underlying

Even if underlying process the price return process of the deal may not be so, if deal not time homogeneus

Distances

Point Distance

$$d_i = |\Psi(q_i) - q_i| \tag{17}$$

Curve Distance

(Weighted) quadratic distance d between functions $q \to \Psi(q)$ and $q \to q$:

$$d(q, \Psi(q)) = \sum_i w_i(\Psi(q_i) - q_i)^2 \tag{18}$$

q_i e.g $(0.01, 0.05, 0.3, 0.5, 0.7, 0.95, 0.99)$

Hypothesis Testing

Null-Hypothesis

- Null-Hypothesis, is that distances are 0.
- Reject Null-Hypothesis p-values smaller than some threshold

Challenges estimating p-values

- Temporal dependence: overlap of time-windows
- Ensemble dependence: returns of netting sets not independent

Good p values get bigger

Bad Estimation tricky

Need some simplifications, like effective sample sizes

Problems using metrics for Ψ

Issues using metrics for Ψ

Opaque no cash denominated measure

Economics Product Dependent with same distance different moments drive deviations in EE (see figure (7))

Limited usefulness Passes test if not enough data available

Problems using metrics for Ψ

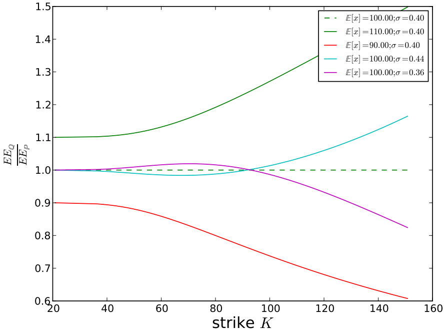


Figure: Comparing *EE*s for a forward using log-normal distributions with different parameters

Comparison using Cash denominated Quantities

Economically Relevant Model Dependent Quantities

Regulatory Capital depends on $EE(t)$ (through EEPE)

Limits impacted by CDF

P&L impacted by $EE(t)$

Measure

These three quantities are functions of $\mathbb{E}_Q$.
Their value under empirical measure $\mathbb{P}$ estimated through equation (12) $\rightarrow$ difference in cash terms

Overview

Requirements

Control Processes should be

- Complete
- Accurate
- Consistent
- Timely
- Appropriate and Relevant
- Auditable