

# Model Risk and Model Control

Patrick Häner

Häner Consulting Berlin



Berlin, April 6, 2013

The

- latest version of this document
- additional resources
- examples

may be found on

<http://bitbucket.virtufin.com/modelrisk/>

# Outline

- 1 Model Classes
- 2 Credit Risk Measures
- 3 Model Implementation
- 4 Back Testing
- 5 Model Control

# Overview

- 1 Model Classes
- 2 Credit Risk Measures
- 3 Model Implementation
- 4 Back Testing
- 5 Model Control

# What is a Model?

## Definition (Model - Narrow)

A **Model** is a mathematical framework providing answers to a specific set of **Questions**.

- Refers only to mathematics
- No reference to implementation
- No relation to markets and trading activity of institution

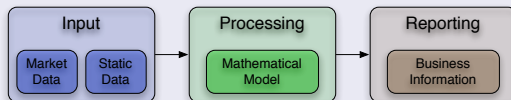
Not a useful definition!

# What is a Model?

## Definition (Model - Wider)

A **Model** provides answers to a specific set of **Questions**. It consists of

- Information input component
- Processing component, applying mathematical transformations
- Reporting component, creating business information



# Model - Wider Definition

Emphasis on **usage**

- Covers data, software and mathematics
- Context of institution, trading activity and market relevant

→ organizational impact for validation: roles and responsibilities

# What is the Question?

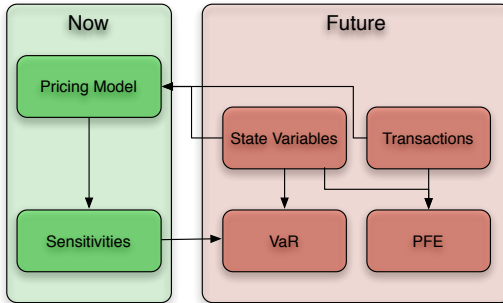
## Types of Questions

- How is the instrument defined
- What is the value  $\rightarrow$  pricing model
- What will the prices in the future be  $\rightarrow$  risk model



# What is the Question?

## Dependencies



# Categorization

## Types of Statements

Prescriptive Model independent, robust statements

Descriptive Explaining

Predictive Falsifiable

# Model Independent Replication

## Static Cashflow Replication

Floating leg of swap: replicate by long/short FRA.

## Model Dependent Static Replication

Barrier options: static replication dependent of model.

# Model Independent Relation

- Across trade parameters
- Across trade types

## Trade Parameters

Price of knock-out option increases with barrier height.

## Trade Types

- Barrier option is cheaper than a Plain-Vanilla
- Replicate Plain-Vanilla by in/out Barriers

# Trade Model

## Requirements

Contractual details as in term sheet need computer-readable representation:

- trade representation
- data exchange
- auditing

## Trade Repository

In US: Dodd-Frank regulation require DTCC data repository (DDR) as a multi-asset class repository.

# Pricing Model

## Usage

- No liquid prices (Mark to Model)
- Sensitivities
- Valuation under future/hypothetical scenarios

# Pricing Model

## Questions

- Implied price w/o credit risk
- Implied price w credit risk: CVA/DVA
- Implied price range: incomplete markets
- Bid/ask price: liquidity

# Pricing Model

## Sensitivities

Which sensitivities should be reported?

### Aggregation

How to aggregate vega sensitivities from two systems with different models?

- Lognormal model:  $\sigma_{BS}$
- Normal model:  $\sigma_{norm}$

### Model Independence

Report sensitivities wrt. to market observables, i.e. instead of  $\sigma_{BS}$ ,  $\sigma_{norm}$  use option prices.



# Overview

- 1 Model Classes
- 2 Credit Risk Measures**
- 3 Model Implementation
- 4 Back Testing
- 5 Model Control

# Likelihood vs Severity of Credit Events

## Categories

Which dimensions to consider?

**Severity** How much will we lose?

**Likelihood** What's the chance that we lose?

**Granularity** What does the measure refer to?

# Granularity of Measure

## Based on Defaults

- All Counterparties
- Single Counterparty

## Other Aggregations

- Global/macro economic
- Sector, country
- Trade

# Exposure and Recovery

How to measure severity? Need to value trade:

## Definition (Exposure at Default)

$$\text{EAD}(t) = \max(0, p(t) | \tau = t)$$

$\tau$  : time at which CP defaults

## Definition (Loss Given Default)

$$\text{Loss at time } t = \text{LGD}(t) \text{EAD}(t)$$

## Definition (Recovery)

$$R(t) = 1 - \text{LGD}(t)$$

# Severity


## Valuation Approaches

**Accrual** Banking book; rarely adjust; illiquid assets

**Mark to market** Trading book; frequently adjusted; traded assets

**Mark to model** Trading book; frequently adjusted; complex structures

## Example

CreditRiskMeasures.xlsx 

# Severity

## Accrual

Loan to Acme Ltd

- value is face value
- maximal loss is notional of loan

## Mark to market

Buy bond of Acme Ltd; assume liquid market

- value is mark to market of bond
- value lower than in risk-free valuation

# Severity

## Mark to model

Exotic interest rate swap with Acme Ltd. What is the value

- risk free: assuming Acme may never default
- risky: Acme may default
- risky with own risk: Acme and we may default

# Forward Looking Measures

Assess exposure in future → model how state of the world evolves

**Deterministic Evolution** Scenario Analysis, Stress testing

**Stochastic Evolution** Model for risk factors

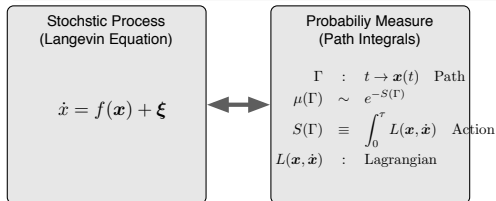


# Scenario Analysis/Stress Test

## Meanings of Stress

- change model parameters  $\rightarrow$
- pick a single path  $\rightarrow$  degenerate measure (Dirac measure)

Unified handling by **Measure Transforms**



Dual Model Representations

# Types of Stress Tests

## Approaches

- give economic scenario
- given loss (inverse stress)

Inverse stresses

# Statistical Measures

## Single Netting Set

### Definition (Potential Future Exposure)

$$\text{PFE}(t) = \max 0, p(t) | \tau = t$$

$\tau$  : time at which CP defaults

### Definition (Expected Exposure (EE))

$$\text{EE}(t) = \mathbb{E}[\text{PFE}]$$

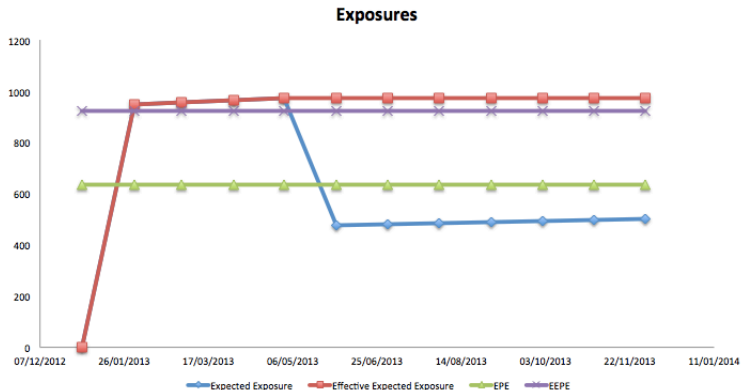
### Definition (Expected Positive Exposure)

$$\text{EPE}(T) = \frac{1}{T} \int_0^T \text{EE}(t) dt$$

# Regulatory Measures

## Definition (Effective Expected Exposure (EEE))

Maximum of EPE and past EEE: never decreasing.



# Statistical Measures

## Multiple Netting Set

### Definition (Losses across Netting Sets)

$$L(t) = \sum_a \chi_{\tau_a \leq t} \text{LGD}_a \max(0, p_a(\tau_a))$$

$a$  : Identifier of netting set

# Portfolio Measures

Meaningful risk measures for portfolios

## Definition (Coherent Risk Measure)

Risk measure  $\rho$ : for portfolio  $X$ :

**Normalization**  $\rho(\emptyset) = 0$  empty portfolio has no risk

**Monotonicity**  $X_1 \leq X_2 \rightarrow \rho(X_1) \geq \rho(X_2)$

**Sub-additivity**  $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$  diversification/netting

**Homogeneity**  $\rho(\alpha X) = \alpha \rho(X) \quad \alpha > 0$

**Translation invariance**  $\rho(X + a) = \rho(X) - a$  adding cash  $a$  reduces risk

# Portfolio Measures


## Quantile

$q\%$  quantile: value, for which  $q\%$  of outcomes are smaller/larger.  
Quantiles are **not** coherent measures.

## Expected Shortfall


Expected loss conditioned on the loss being larger than  $X$ . The Expected Shortfall (Mean Excess Loss) is a coherent measure.

## Example

PortfolioMeasure.xlsx 

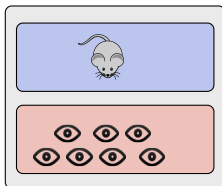
# Likelihood of Default

## Example

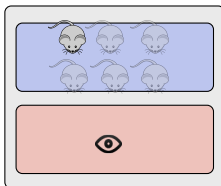
LikelihoodExperiment.xlsx 



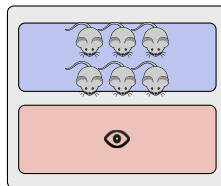
# What does probability mean?



Average observers



Implied ensemble



Genuine ensemble

## Probability and Measurement

Need to define

- Ensemble
- Measurement process

# Examples

## Genuine Ensemble

- Mathematics
- Physics: Identically prepared experiment

## Average observers

Consensus of observers:

- Market prices
- Betting quota

## Implied Ensemble

Equivalence classes:

- Names with same rating
- Price returns in different time windows

# Measures for Probability of Default

## Definition (Survival/Default Probability, Default Intensity)

Let  $\tau$  be time of default

$$S(t) = p(\tau > t)$$

$S$  : survival probability

$$S(t) = e^{-\lambda(t)t}$$

$\lambda$  : term default intensity

$$D(t) = 1 - S(t)$$

$D$  : default probability

Note:  $D$  is a CDF!

# Forward Intensity

## Forward default intensity

Probability  $d(t)$  of defaulting between  $t$  and  $dt$ :

$$d(t) = \frac{dD(t)}{dt} \quad (1)$$

# Estimating Probability of Default

Estimating  $\lambda$

**Credit Rating** Typically using historical data

**Market Prices** Current credit spreads from bonds or CDS

## Implied Default Intensity

Let  $s(t)$  be a credit spread

$$s(t) = (1 - R)\lambda(t)$$

$R$  : recovery rate

# Unifying Severity and Frequency Measures

## High Severity/Low Frequency vs. Low Severity High Frequency

How to compare

- Single large deal with good counterparty
- Set of small deals with bad counterparties

Answer

Pricing including credit risk allows comparing!

# Approaches

## Top-down vs Bottom-up

**Top-down** Pricing from first principles

**Bottom-up** Calculate **price correction** from building blocks:  
Exposure (EE) and PE, LGD

# Bottom-Up approach

## Assumptions

- Risk-free prices known
- Calculate EE
- Estimate PE, LGD
- Calculate correction to risk-free price



# Measuring the Corrections

Riskiness of counterparty reduces the price:

## Definition (CVA)

Risky price  $p_A^*$  as seen from counterparty  $A$  with counterparty  $B$ :

$$p^* = p - CVA_B$$

$p$  : risk-free price

$CVA_B$  : Credit Valuation Adjustment for counterparty  $B$

## Measuring the Corrections

Does credit risk of counterparty  $A$  also affect price?

### Definition (DVA)

Price  $p_A$  as seen from counterparty  $A$  with counterparty  $B$ :

$$p^* = p - CVA_B + DVA_A$$

$p$  : risk-free price

$DVA_A$  : Debit Valuation Adjustment for counterparty  $A$

DVA increases the price.

### Accounting vs. Regulatory

DVA must be used for P&L but not for regulatory capital.

# Regulatory CVA

BCBS 189, paragraph 89:

$$CVA = (LGD_{MKT}) \cdot \sum_{i=1}^T \text{Max} \left( 0; \exp \left( -\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}} \right) - \exp \left( -\frac{s_i \cdot t_i}{LGD_{MKT}} \right) \right) \cdot \left( \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2} \right)$$

## Regulatory CVA

Similar to regulatory capital charge for default:


Assumes independence of exposure and default process.

$$CVA = \int_0^T (1 - R) Df(t) EE(t) d(t) dt$$

where  $d$  is the default probability from equation (1),  $Df$  discount factor

# CVA

## Example

CVA.xlsx 

# Regulatory CVA

## Regulatory vs Trading CVA

**Regulatory** Historic measure for EE, implied for PD

**Trading** Both EE and PD in implied measure

# Top-down Approach

## Pricing w/o Credit Risk

From cashflows  $j$ :

$$p(t) = \mathbb{E}\left[\sum_{i:t_i \geq t} j_i(t_i) Df(t_i)\right]$$

How to introduce credit risk?

Condition cashflows on default time  $\tau_B$ , and if we consider also DVA on  $\tau_A$

# Top-down Approach

## Indicators

$$\chi_+(t, \tau) = \Theta(\tau - t)$$

$$\chi_-(t, \tau) = 1 - \chi_+$$

## Cashflows

Regular  $j_i(t_i)$  pays when neither defaulted

At default  $J_A(t), J_B(t)$

# Top-down Approach

## Pricing with Credit Risk

$$p(t) = \mathbb{E}[J_A(t)Df(\tau_A)\chi_+(t_i, \tau_B)\delta(t - \tau_A) + J_B(t)Df(\tau_B)\chi_+(t_i, \tau_A)\delta(t - \tau_B) + \sum_{i: t_i \geq t} j_i(t_i)\chi_+(t_i, \tau_A)\chi_+(t_i, \tau_B)]$$

Using this approach gives consistent bilateral CVA.



# Difficulties

## Practical

- Need know all cashflows
- Cashflows at default (recoveries) depend on price at forward date

## Pricing for Portfolio of Netting Sets

As for single netting sets: pricing combines severity and likeliness.  
Requires knowing

- prices of individual netting sets at default
- probability of default  $P(\chi_{\tau_1 \leq t_1}, \chi_{\tau_2 \leq t_2}, \dots, \chi_{\tau_N \leq t_N})$

Additional useful quantity: in terms of total losses:

Definition (Loss distribution)

$$\mathcal{L}(l, t) = P(L(t) \geq l)$$

# Granularity of Measure in Regulatory Context

## Metrics used for Regulatory Purposes

Focus on measures for individual counterparties. No proper modelling of collective losses required.

# Overview

- 1 Model Classes
- 2 Credit Risk Measures
- 3 Model Implementation**
- 4 Back Testing
- 5 Model Control

# Building Blocks

## Model Building Process

**Business Analysis** Materiality, specification

**Model choice** Find adequate model

**Software implementation** Develop and roll out

# Materiality

## What to Model?

Which risk factors material for current portfolio?

How can we assess materiality without exposure model in place?

## Approach

Simple estimation of exposure assuming

- future portfolio prices normally distributed
- estimation of first two moments

# Gaussian Approximation

Need to estimate  $\mathbb{E}[p(T)]$ ,  $\mathbb{E}[p^2(T_i)]$  at some future times  $T$ :  
Performing Taylor expansion for price  $p$  around expected risk factor:

$$p(\mathbf{x}(T), T) \approx p(\mathbf{x}_0(T), T) + \sum_i \frac{\partial p(\mathbf{x}_0(T), T)}{\partial x_i} \Delta x_i(T) + \frac{1}{2} \sum_{ij} \frac{\partial^2 p(\mathbf{x}_0(T), T)}{\partial x_i \partial x_j} \Delta x_i(T) \Delta x_j(T)$$

$$\mathbf{x}_0(T) \equiv \mathbb{E}[\mathbf{x}(T)]$$

$$\Delta x_i(T) \equiv x_i(T) - x_{0,i}(T)$$

# Gaussian Approximation

The expectation value  $M$  of the price is hence

$$M(T) \approx p(\mathbf{x}_0(T), T) + \frac{1}{2} \sum_{ij} \gamma_{ij}(T) \Omega_{ij}(T)$$

$$M(T) \equiv \mathbb{E}[p(\mathbf{x}(T), T)]$$

$$\gamma_{ij}(T) \equiv \frac{\partial^2 p(\mathbf{x}_0(T), T)}{\partial x_i \partial x_j}$$

$$\Omega_{ij}(T) \equiv \mathbb{E}[\Delta x_i(T) \Delta x_j(T)]$$

For the variance  $V$  we obtain up to second order in  $\Delta \mathbf{x}$ :

$$V(T) \approx \sum_{ij} \delta_i(T) \delta_j(T) \Omega_{ij}(T)$$

$$V(T) \equiv \mathbb{E}[(p(\mathbf{x}(T), T) - \mathbb{E}[p(\mathbf{x}(T), T)])^2]$$

$$\delta_i(T) \equiv \frac{\partial p(\mathbf{x}_0(T), T)}{\partial x_i}$$



# Gaussian Approximation

What can we learn?

## Risk factor contributions

Matrix elements  $\Psi_{ij} = \delta_i(T)\delta_j(T)\Omega_{ij}(T)$  indicate contribution of risk factors  $ij$  to total variance.

## EE, PE

Knowing mean and variance of the Gaussian distribution, any statistical quantity may be evaluated.

## Caveat

Depending on specifics of portfolio this approximation may be more or less accurate: that is why we use Monte Carlo simulations after all.

# Practical Implementation

- For  $t = 0$ :  $\delta$  and  $\gamma$  from Market risk system. **But:** need netting set level aggregation  $\rightarrow$  deal level granularity
- For  $t > 0$  estimate future  $\delta$ ,  $\gamma$  by bumping

# Trade Models

## Requirements

- Represent trades/products
- Standardize for interoperability

# Trade Models

## Trade Parameters

Product represented by parameters

### FpML

```
<?xml version="1.0" encoding="UTF-8"?>
<FpML xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance" xmlns="http://www.fpml.org"
xsi:schemaLocation="http://www.fpml.org/2003/FpML-4-0 fpml-main-4-0.xsd">
  <trade>
    <tradeHeader>
      <partyTradeIdentifier>
        <partyReference href="CHASE"/>
        <tradeId tradeIdScheme="http://www.chase.com/swaps/trade-id">921934</tradeId>
      </partyTradeIdentifier>
      <partyTradeIdentifier>
        <partyReference href="UBSW"/>
        <tradeId tradeIdScheme="http://www.ubsw.com/swaps/trade-id">204334</tradeId>
      </partyTradeIdentifier>
      <tradeDate>2000-04-03</tradeDate>
    </tradeHeader>
    <swap>
      <!--
        Chase pays the floating rate every 6 months, based on 6M EUR-EURIBOR-Telerate
        + 10 basis points, on ACT/360 basis
      -->
      <swapStream>
        <payerPartyReference href="CHASE"/>
        <receiverPartyReference href="UBSW"/>
        <calculationPeriodDates id="floatingCalcPeriodDates">
          <effectiveDate>
            <unadjustedDate>2000-04-05</unadjustedDate>
            <dateAdjustments>
              <businessDayConvention>NONE</businessDayConvention>
            </dateAdjustments>
          </effectiveDate>
          <terminationDate>
            <unadjustedDate>2005-01-05</unadjustedDate>
          </terminationDate>
        </calculationPeriodDates>
      </swapStream>
    </swap>
  </trade>
</FpML>
```

# Trade Models

## Trade Parameters

### Pro/Con

- ⊕ standardized
- ⊖ logic in client

# Trade Models

## Cashflows

Product represented by casflows

### Payoff macros

Notional	100
DCF	Act/Act

Date	Libor fixing	Payoff
02-Mar-13	1.34%	0.335
02-Jun-13	1.19%	0.2975
02-Sep-13	1.42%	0.355
02-Dec-13	1.39%	0.3475

# Trade Models

## Cashflows

### Pro/Con

- ⊕ simple
- ⊖ not expressive enough (just cash is exchanged)
- ⊖ single product (no interactions)

# Trade Models

## Transaction Model

### Approach

Multi agent simulation:

Time Map wall clock to simulation time

Market Events simulation time to events

Transactions events to transactions (e.g. cashflows)

Execution execute events

### Pro/Con

- ⊕ general
- ⊕ all business logic in model → easy tooling
- ⊖ expensive



# Pricing & Risk Models

## Criteria for Model Choice

### Categories

Independent of product Relate to Mathematics or Physics

Dependent of product Specific to product type

Dependent of portfolio and market Context

# Pricing & Risk Models

Independent of Product

## Coordinate Systems

From Physics we know: dynamics must not depend on choice of coordinates  $\rightarrow$  dimension analysis.

## Interpolation

How to interpolate  $r$ ,  $\sigma$ . Interpolate dimension-less quantities:  $rt$  and  $\sigma^2 t$ .

# Pricing & Risk Models

## Product Dependent

### State Variables vs. Parameters

**Liquidity** Hedge frequency, transaction costs, close-out period

**Completeness** Unhedgeable risk, uniqueness of price

# Pricing & Risk Models

## State Variables and Parameters

### Indicators of Model Quality

**Parameter Dimensionality** Avoid overparamerization

**Stability of Parameters** Frequent recalibration: indicator of poor model performance

### GBM w termstructure vs Garch

	<b>TS GBM</b>	<b>Garch</b>
<i>dimension</i>	$\infty$	3
<i>recalibration</i>	frequently for short end	less frequent
<i>time-homogeneous</i>	N	Y

# Pricing & Risk Models

## Arbitrage

### Risk Model for Volatility surface

Directly modelling surface w/o arbitrage not trivial. Alternatively model option prices with HJM-like framework.

# Pricing & Risk Models

Market

## Liquidity & Completeness

**Liquidity** Hedge frequency, transaction costs, close-out period

**Completeness** Unhedgeable risk, uniqueness of price

# Comparing Models

Assume state of the world evolves randomly:

Model as Process: Stochastic Differential Equation (Langevin Equation)

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}) + g(\mathbf{x})\xi(t) \quad \text{Physics Notation}$$

$$d\mathbf{x} = f(\mathbf{x})dt + g(\mathbf{x})d\mathbf{W}(t) \quad \text{Finance Notation}$$

Wiener Process (SDE)

$$dx = dW(t)$$

$W$  : Wiener Process

# Comparing Models

## Model as Measure $\mathbb{P}$

$\mathbb{P} : \Gamma \rightarrow \mu(\Gamma)$  probability

$\Gamma : t \rightarrow \mathbf{x}(t)$  some path

## Wiener Process (SDE)

$\Gamma \equiv \{x_1, \dots, x_N\}$

$\mu(\Gamma) \sim \prod_i G(x_i, x_{i+1})$

$G : \text{Gaussian}$



# Pricing & Risk Models

## Parametric Models

### Error Analysis

Infer from parameter uncertainty price/risk uncertainty.

**Parameter Uncertainty** E.g. such that hedging instrument prices still in bid-ask

**Parameter Error** Uncertainty of price/risk due to error in parameters

### GBM with vol uncertainty

$$(\Delta p)^2 = \left( \frac{\partial p}{\partial \sigma} \Delta \sigma \right)^2$$

# Benchmarking

Pricing/risk factor models  $\mathbb{Q}, \mathbb{Q}'$ , empirical measure  $\mathbb{P}$

## Comparing

Pricing Models  $\mathbb{Q}$  vs  $\mathbb{Q}'$

Risk Models  $\mathbb{P}$  vs  $\mathbb{Q}$

# Benchmarking

## Distances

### How far apart two models?

Need to define metric:

**Expectation values** E.g. differences of prices and EEs under different measures

**Distributions** E.g. Kullback-Leibler entropy  $\int \frac{d\mathbb{P}'}{\mathbb{P}} \log \frac{d\mathbb{P}'}{\mathbb{P}} d\mathbb{P}$ .  
Independent of quantity to average.

# Model Uncertainty

Benchmarking giving limited answer:

## Calibration-Consistent Measures

Define metric  $d$  to quantify goodness of calibration:

$p_i^{\mathbb{P}}$  : model price calibration instrument  $i$

$p_i$  : market price calibration instrument  $i$

$$d^{\mathbb{P}} = \sum_i (p_i^{\mathbb{P}} - p_i)^2$$

$$\mathcal{C}_{\epsilon} = \{\mathbb{P} | d^{\mathbb{P}} \leq \epsilon\}$$

# Model Uncertainty

## Non-uniqueness

For  $d = 0$ :

- Multiple measures
- For single parametric measure, multiple solutions for calibration  $\rightarrow$  ill behaved
- Incomplete market

For  $d > 0$ :

- For single parametric measure: parameter risk

# Exposure

## Goal

Estimate Credit Risk measures → need to estimate exposure/price distributions in future.

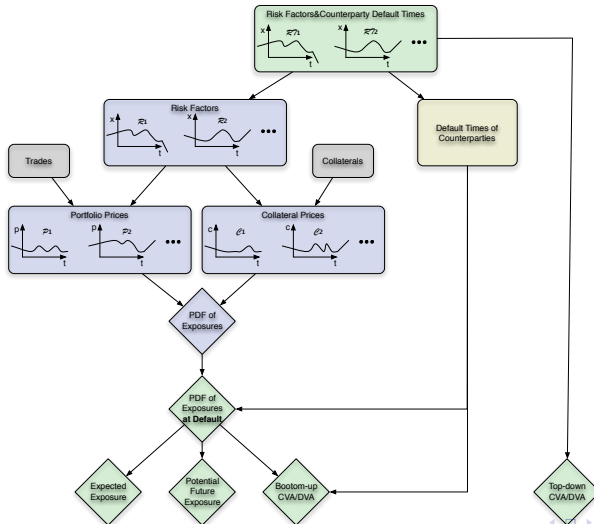
The exposure  $e(t)$  at time  $t$  of a netting set is given by

$$e(t) = \max 0, \sum_i p_i(\mathbf{x}, t) - C(t) \quad (2)$$

where

- $p_i$  price of trade  $i$
- $\mathbf{x}$  risk factors
- $C(t)$  price of collateral

# Calculating Exposure, CVA/DVA and Losses



# Building Blocks

## Components

Required for estimating risk measures for single and portfolios of netting sets:

- Pricing
- Risk-factor
- Collateral
- Netting
- Dependency



# Pricing Models

## Requirements

- Need to be fast!
- Ideally same as front office
- Perform well under stressed state variables

# Pricing Models

## Acceleration Techniques

### Dumb lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- define grid
- recalculate for each gridpoint price
- interpolate

# Pricing Models

## Acceleration Techniques

### Smart lookup

Approximate price as function of few variables

- define variables (e.g stock price)
- prices on grid are **side effect** of pricing at spot; e.g. pricing on tree or AMC
- interpolate

# Risk Factor Models

## Pricing vs Risk Models

### Purpose

**Pricing Model** Fit liquid market instruments; arbitrage-free

**Risk Model** Predict

### Challenges for Risk Model

**Dependency** Simultaneously simulate all asset classes

**Calibration** Global calibration

# Risk Factor Models

## Short vs Long term prediction

Long term prediction a challenge:

- Reducing dimensionality
- Economic macro factors
- Co-integration

# Risk Factor Models

## Pricing model Dynamics

Arbitrage-free models used with risk calibration

- GBM
- HJM type of models
- ⊕ Well understood, tractable
- ⊖ Not intended for risk

# Gaussian Dependency Modelling

## Goal

Express random vector  $\xi$  with correlated  $\xi_i$  as

- linear combination of
- uncorrelated

random factors  $\eta_i$ :

$$\xi = \mathbf{M}\eta$$

$$\mathbb{E}[\xi_i \xi_j] - \mathbb{E}[\xi_i] \mathbb{E}[\xi_j] \equiv \Omega_{ij}$$

$$\mathbb{E}[\eta_i \eta_j] - \mathbb{E}[\eta_i] \mathbb{E}[\eta_j] = \lambda_i^2 \delta_{ij} \quad \text{diagonal, pos. sem. def.}$$

## What to consider?

- $\Omega$ ?
- correlation matrix?

# Principal Component Analysis

## Dimensional Analysis

Risk factors  $\xi_i$  not dimension-less!

- interest rate :  $[T^{-1}]$
- stock price : [Cash]
- volatility:  $[T^{-\frac{1}{2}}]$

→  $\Omega_{ij}$  may have different dimensions, i.e.  $\Omega$  in general not a physically meaningful quantity!



# Principal Component Analysis

## Solution

Consider instead of  $\Omega$  following matrix  $\Phi$ :

$$\Phi_{ij} \equiv \frac{\partial f(\xi)}{\partial \xi_i} \frac{\partial f(\xi)}{\partial \xi_j} \Omega_{ij}$$

$f$  : some function

For dimensionality  $[\Phi]$ :

$$[\Phi_{ij}] = \frac{[f]}{[\xi_i]} \frac{[f]}{[\xi_j]} [\xi_i][\xi_j] = [f^2] \quad \forall i, j \quad \checkmark \quad (3)$$

# GBM Risk Factor Model

## Multivariate GBM

$$X_i(t + \Delta t) = X_i e^{(\mu_i - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i \sqrt{\Delta t} \xi_i(t)}$$

$\mu$  : drift

$\sigma$  volatility

$\xi_i$  : Normal random

$$\text{Cov}(\ln X_i(t + \Delta t), \ln X_j(t + \Delta t)) = \Omega_{ij}$$

# Dependent Gaussian Random Variables

Given uncorrelated Gaussian random number vector  $\zeta$ . Need build  $\eta$ :

$$\text{Cov}(\eta_i, \eta_j) = \Omega_{ij}$$

# Calibration

## Definition

**Calibration** is the process to determine model parameters.

## Approaches

**Statistical** Using historical data

**Implied** Market implied parameters

**Economic** Macro economical relation between rates, inflation

## Assumptions

**Statistical** Past is good predictor for future

**Implied** Information in spot market predicts future


**Economic** Some fundamental economic laws rule future

# Statistical Calibration

For simple models: ad hoc parameter estimation

- averaging
- fitting

## Example

SimpleEstimation.xls 

# Maximum Likelihood Estimation

Systematic way to calibrate

## Approach

Parametric model with parameters  $\alpha \leftrightarrow$  parametric measure  $\mu_\alpha$ :

$$\mu_\alpha(\Gamma) = e^{-S_\alpha(\Gamma)} \mathcal{D}[\Gamma]$$

Assume: historical path  $\Gamma_H$  is the most likely one. Find  $\alpha^*$  such that:

$$\mu_{\alpha^*}(\Gamma_H) = \max_{\alpha} \mu_{\alpha}(\Gamma_H)$$

# Maximum Likelihood Estimation

## Implementation

Assuming iid:


$$\begin{aligned}\mu_{\alpha}(\Gamma) &= \prod m(\mathbf{x}_i) \\ m(\mathbf{x}) &= e^{-s(\mathbf{x})} \\ \Gamma &= \{\mathbf{x}_1, \dots, \mathbf{x}_n\}\end{aligned}$$

Maximizing  $m \leftrightarrow$  minimizing

$$\sum_i s(\mathbf{x}_i) : \quad \text{log-likelihood}$$

# Maximum Likelihood Estimation

## Example

MLE.xls 



# Implied Parameters

Apply parameters used for pricing:

## Drift and Volatility

- Drift  $\mu$  from  $T$  forward price (Covered Parity)
- Volatility  $\sigma$   $T$  years ATM implied volatility

## Assumption

Risk neutral measure yield good predictor for real-world measure

## Caveat

- Carry trades
- Supply/demand, risk premium


Perform analysis before using implied parameters!

# Economic Calibration

Parities connect for instance

- FX rates
- Inflation rates
- Real interest rates
- Nominal interest rates
- Purchasing power

## Example

Parities.xlsx 

# Parities

## Example (Relative Purchasing Power Parity)

$$p_f(t_1)(1 + i_f)X(t_2) = p_d(t_2)(1 + i_d)$$

$p_{d/f}$  : domestic/foreign price

$i_{d/f}$  : domestic/foreign 1 yr inflation rate

$X$  : Exchange rate

Yields after averaging

$$\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1 + I_d}{1 + I_f}$$

where  $I$  is the expected inflation rate.

# Parities

## Example (International Fisher Effect (Uncovered Parity))

$$(1 + r_{d/f}) = (1 + \rho_{d/f})(1 + i_{d/f})$$

$r_{d/f}$  : domestic/foreign nominal 1 yr interest rate

$\rho_{d/f}$  : real domestic/foreign 1 yr interest rate

Assuming  $\rho_d = \rho_f$  gives

$$\frac{\mathbb{E}[X(t_2)]}{X(t_1)} = \frac{1 + r_d}{1 + r_f}$$

# Issues with standard GBM model

## Issues

- rigidity: calibration short vs long horizons → term structure of parameters
- dimensionality → factor models
- underestimation of rare events and bursts (clustering) → GARCH
- not suitable where spread stationary process → cointegration
- unable to capture some behaviour like regime-switches → parametric models (Nelson-Siegel)

# GBM with Term Structure

## Interpolation Principles

Interpolate dimension-less quantities

## Forward Drift/Covariance

Dimensionality analysis  $\rightarrow$  interpolate  $T\Omega$

# Factor Models

## Issues with general covariance matrix

$N$  risk factors  $\rightarrow \propto N^2$  parameters

- over-parametrization
- for empirical parameters: problems with positive definiteness

## Idea

Split return  $r$  of riskfactors into contributions from

Indices  $f_n$  shared by multiple risk factors

Idiosyncratic factors  $\epsilon$  unique to each risk factor

$$r = \alpha + \sum_n \beta_n f_n + \epsilon$$

and assume

- indices uncorrelated to idiosyncratics

# Types of Factor Models

## Classification

**Macroeconomic** Observables like changes in inflation, interest rate, unemployment rate

**Fundamental** Portfolios associated to security attributes like industry membership, book to market ratio, dividends

**Statistical** Factor analysis of covariance matrix



# Macroeconomic Factor Model

## Fast/Slow

**Slow variables** Macro-economic state of the economy: inflation, unemployment rate, GDP

**Fast** Asset prices

## Pros and Cons

- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing
- ⊖ Theories controversial

# Fundamental Factor Model

## Sector/Region

- 1 Define for each sector/region pair an index
- 2 Associate stock to sector/region
- 3 Regress stock return vs index return  $\rightarrow \alpha, \beta$

## Example

FactorModel.xls



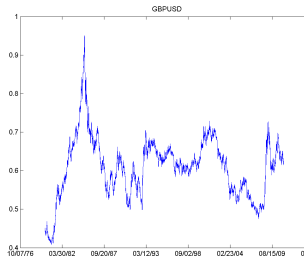
## Pros and Cons

- ⊕ Designed to predict long-term evolution
- ⊕ Able to reflect systemic macro risks
- ⊖ Empirical evidence not convincing

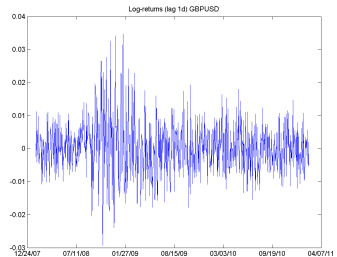
# Choice of Factors

How to know whether factors appropriate?  
Analyze variance explained by factors

# Volatility Clustering



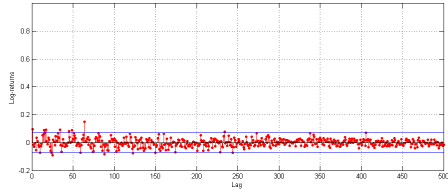
(a) Spot



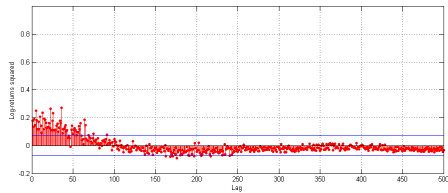
(b) Log-returns

Figure: GBPUSD spot

# Autocorrelation



(a) Autocorrelation: log-returns



(b) Autocorrelation: squared

log-returns

# Garch Model

Let  $X_n$  be the log-return of some foreign exchange rate  $f$  at time  $t_n$ :

$$X_n = \ln \frac{f_n}{f_{n-1}} \quad (4)$$

we may then express the foreign exchange rate  $f_N$  at some future sampling point time  $t_N$  by the initial value  $f_0$  at  $t_0$  and a series of returns:

$$f_N = f_0 e^{\sum_{i=1}^N X_i} \quad (5)$$

The observation points  $t_i$  are typically defined in terms of number of business days  $\Delta T$  between them. For short time horizon predictions we choose  $\Delta T = 1$  for larger horizon, we may choose a less granular time grid.

# Garch Model

The dynamics of the returns is then assumed to follow a Garch(1,1) process

$$X_n = \mu + \epsilon_n \quad \epsilon_t \sim \text{iid}(0, \sigma_n^2) \quad (6)$$

$$\sigma_{n+1}^2 = \alpha + \beta \sigma_n^2 + \gamma \epsilon_n^2 \quad (7)$$

The asymptotic value  $\sigma_\infty^2 = \lim_{n \rightarrow \infty} \mathbb{E}[\sigma_n^2]$  is then obtained by equation (7) noting, that  $\mathbb{E}[\epsilon^2] = \sigma^2$  and  $\mathbb{E}[\sigma_{n+1}^2] \rightarrow \mathbb{E}[\sigma_n^2]$ :

$$\sigma_\infty = \frac{\alpha}{1 - \beta - \gamma} \quad (8)$$

# Garch Model: Limit

Weak limit:

- Stochastic variance
- Mean reverting variance

$$\begin{aligned}dX_t &= \mu X_t dt + \sqrt{v_t} X_t dW_t \\ dv_t &= \alpha(v_t) dt + \beta(v_t) dZ_t\end{aligned}$$



# Copula

## Dependence under Stress

In stressed markets correlations increase between

- downward price movements → systematic risk
- implied default probabilities → contagion

## Definition (Copula)

Separate

- Marginal distributions from
- Dependency

# Cointegration

## Long-run Relationship

Variables moving together:

- Macro-economic**
- Consumption-Income
  - Prices-Wages
  - Domestic prices - foreign prices

**Exogeneous** For instance managed currencies

- How to model processes. which stay close to each other?
- GBM with  $\rho_{ij} \approx 1$  **not**? No!
- Need dynamic, where difference is stationary

## Definition


Stochastic processes  $x, y$  are cointegrated:

$$y(t) = a + b x(t) + \epsilon(t)$$

# Implementation

- 1 find parameters  $a$ ,  $b$  by regression
- 2 show residuals are stationary (e.g. Dickey-Fuller Test)

## Example

Cointegration.xlsx 

# Risk Factor Models

## Empirical Models

### Nelson-Siegel model

$$r(T) = r_{\infty} + a(T)r_0 + b(T)r_m$$

$r_{\infty}$  : rate for long maturities

$r_0$  : rate for short maturities

$r_m$  : rate for intermediate maturities

$a, b$  : decay functions

# Risk Factor Models


## Empirical Models

### Nelson-Siegel model

- Normal/inverted curves
- **But** not arbitrage-free

How to introduce dynamics? E.g. PCA of  $(r_\infty, r_0, r_m)$

### Example

NelsonSiegel.xlsm 

# Wrong Way Risk

## Types

**Specific** Legal connection between underlying and counterparty

**General** Dependence between prob. of default of counterparty and exposure

## SFT Transactions

Lend cash to counterparty A accepting their stock as collateral.

## Emerging Market CCY swap

We are long strong currency. Weakening of emerging market currency, increased prob default → increase exposure

# Modelling Wrong Way Risk

What is wrong with standard modelling?

$p^+$  is **not** conditioned on default.

Need to add in price function default state  $\chi$  of counterparty:


**extending state of the world**

## Approaches

Given a model for default times either

- Simulating counterparty's default
- Calculating price given default

## Example

WrongWayRisk.xls 

# Collateral Modeling

## Components

**Margin Call Process** Model margin calls with correct frequency and close-out period

**Collateral Price** E.g. model bond price if collateral is bond

### Simplification

- Margin call process: just at spot → short-cut method
- All collateral as cash → haircuts



# Collateral Modeling

## Short-Cut Method

### Definition (Basel II Short-Cut Method)

EE and PE of collateralized trades given by EE and PE for close-out period (5 days for SFT, 10d for OTC)

### Benefits/Issues

- ⊕ Computationally cheap
- ⊕ No collateral exposure spikes at expiry
- ⊖ Assumes exposures declining over time
- ⊖ Risk not accurately represented

# Dependency Modelling

## Among Risk Factors

Standard way to model dependence: Gaussian Copula.  
Gaussian Copulas are Levy copulas. Replace Gaussian with other Levy coupula and obtain Levy model.

## Between Defaults

Simulate either

Default times  $\tau$  E.g. by Marshall-Olkin Copulas


Default state at t:  $\chi_{\tau \leq t}$  E.g. structural models

# Dependency Modelling

## Between a Default and Risk Factors

To capture Wrong Way risk need to model dependence between risk factor and default state

## Example

WrongWayRisk.xls 

## Between a cross name Defaults and Risk Factors

Need modelling full state of the world  $(\mathbf{x}(t), \{\chi_{\tau_1 \leq t}, \dots, \chi_{\tau_1 \leq t}\})$ .  
→ **scenario consistency** is system

# Model Lifecycle

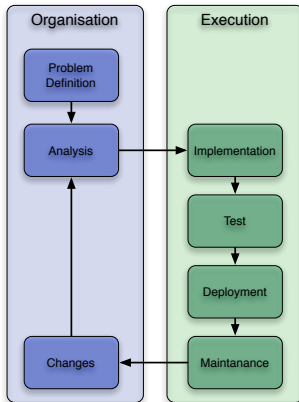


Figure: Model Development Lifecycle

# Specification

## Approaches

Human readable Business and functional specs

Machine readable Specification  $\sim$  test

# Specification Tools

## ScalaTest Code

```
class LookupSpec extends WordSpec with MustMatchers {
  def fixture = new {
    val i1 = Identifier[Double]("abc")
    val i2 = Identifier[Double]("xyz")
    val i3 = Identifier[Double]("ABC")
    val x1 = 12.3
    val x2 = 4.56
    val l = Lookup(i1 -> x1, i2 -> x2)
  }

  "A Lookup" when {
    "item exists" must {
      "retrieve with () the item" in {
        val f = fixture
        val y1 = f.l(f.i1)
        y1 must equal(f.x1)
        val y2 = f.l(f.i2)
        y2 must equal(f.x2)
      }
      "retrieve with get() an Option object containing the item" in {
        val f = fixture
        val y1 = f.l.get(f.i1)
        y1 must not be Option.empty
        y1.get must equal(f.x1)
        val y2 = f.l.get(f.i2)
        y2 must not be Option.empty
        y2.get must equal(f.x2)
      }
    }
  }
}
```

# Specification Tools

## ScalaTest Output

### Part of CI:

Dashboard Authors Reports Administration

Virtufin > Core > #13

✓ #13 Job: **Test** was successful

Stages & Jobs Job Summary Tests Changes Artifacts Logs Metadata Issues

Compile

✓ Compile

Test

✓ Test

Install

✓ Install

### Test Results

21 tests in total 1 second taken in total.

Failed Tests Successful Tests (21) 21

The following 21 tests have passed:

All Successful Tests

Test

- ✓ ForeignExchangeRateSpec A ForeignExchangeRate should equal to a Price with the same asset ⓘ
- ✓ ForeignExchangeRateSpec A ForeignExchangeRate should have the same hashCode as the corresponding Price ⓘ
- ✓ ScenarioSpec A Scenario when must ⓘ
- ✓ PortfolioHierarchyAgentSpec A PortfolioHierarchyAgent should add and subtract the correct position amounts from the source and target portfolios ⓘ
- ✓ FeatureSpec A Feature when must ⓘ
- ✓ PlainVanillaPayoffSpec A PlainVanillaPayoff when fixing has value must at maturity create a payment message ⓘ

# Implementation

## Software

- in-house
- third-party

Require different validation strategies



# Third Party

## Strategies

- Black-box, no code review
- Reverse-engineering

# Revision Control

## Requirements

- Audit** Who changed what/when
- Resurrect** Roll-back to previous state
- Collaborate** Merge contributions from different authors

## Approaches

- Plain files** Tag files/directories with version information
  - Local** Local database contains version information (e.g. RCS)
  - Server** Database on server (e.g. SVN)
- Distributed** Each developer has own database with potentially central db (e.g. Git)

# Revision Control Tools

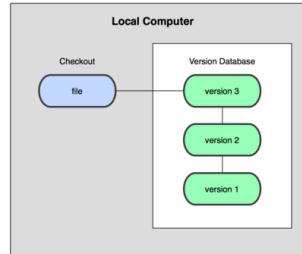
## Approaches

MyDirectoryV1.0

MyDirectoryV1.1

MyDirectoryV1.2-bugfix1

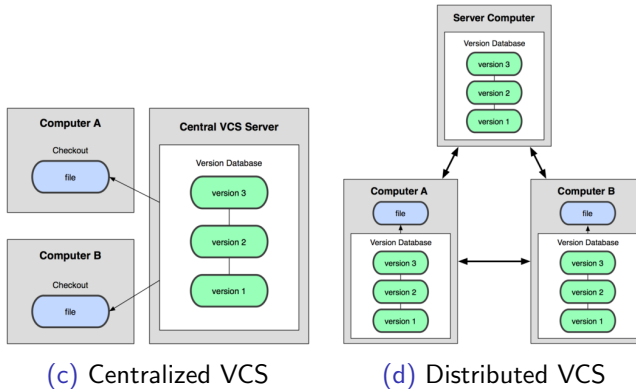
(a) File based



(b) Local VCS

# Revision Control Tools

## Approaches



# Revision Control Tools

## Git

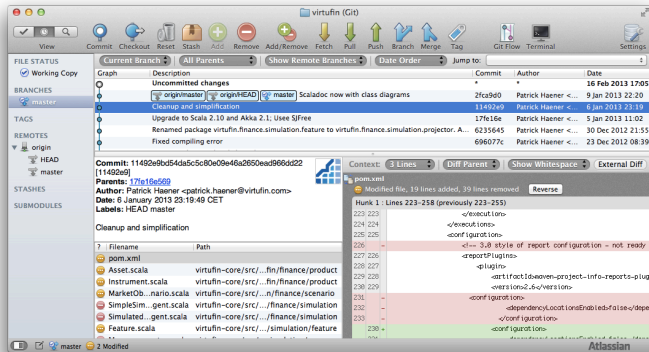


Figure: Git Gui (SourceTree)

# Documentation

## Requirement

Contain enough information to reverse-engineer.

## Tools

- Automated API doc (*Doxygen*, *ScalaDoc*, ...)
- Internal wiki (e.g. *Confluence*)

# Testing

## Test Types

**Unit** Library level

**Integration** System level

# Testing

## Unit Test

```
class BlackScholesScenarioModelTest {
  @Test
  def test() {
    val marketObservable = Price(Stock("IBM"), Currency.USD)
    val modelDate = Time(2012, 10, 1)
    val t1 = Time(2013, 10, 1)
    val t2 = Time(2015, 10, 1)
    val t3 = Time(2022, 10, 1)
    val ts = List(t1, t2, t3)
    val scenarioObservables = ts.map(t => Index(t, marketObservable))
    val x0 = 100.0
    val scenario = Scenario(marketObservable, modelDate -> x0)
    val mu = 0.01
    val sigma = 0.4
    val blackParameters = BlackParameters(mu, sigma)
    val blackParametersId = BlackParameters.identifier(marketObservable)
    val parameters = Lookup(blackParametersId -> blackParameters)
    val request = ScenarioRequest(scenarioObservables, scenario, modelDate)
    val context = ModelContext(ModelDispatcher(ModelRegistry()), parameters)
    val model = new BlackScholesScenarioModel()
    val result = model.model(request, context)
    assertTrue(result.isSuccess)
    val iterable = result.toOption.get
    val n = 1000000
    var x = 0.0
    var x2 = 0.0
    val o = scenarioObservables.last
    Timing.timing(iterable, (s: Scenario) => {
      val a = s(o); x = x + a; x2 = x + a * a
    }, n)
    assertEquals(x0 * math.exp(mu * DateUtil.yearsBetween(modelDate, o.time)), x / n, 0.5)
  }
}
```



# Release

## Requirements

- Regression
- Impact analysis
- Sign-off
- Auditing
- Lock-down

# Maintenance

## Bugs/Enhancements

- Tracking system
- Failing test cases
- Metrics: severity, resolution time

# Integrated Development Process

Robust system should have

## Components

- Revision Control system
- Build System
- Bug tracking system
- Wikin

Components integrated to workflow with high degree of  
**automation**

# Overview

- 1 Model Classes
- 2 Credit Risk Measures
- 3 Model Implementation
- 4 Back Testing**
- 5 Model Control

# Motivation

## Impact of Credit risk model

Trading activity limits set by  $PE$

Capital charges regularity capital dependent of  $EEPE$

P&L  $EE$  enters CVA/DVA

## Model Risk

Back-testing should quantify model risk affecting these quantities.

# Requirements

## Back-testing Process

Should provide

Definition of measure for model risk

Monitoring of metrics

Mitigating actions for model deficiencies

# BCBS Guidance

## G1

**Guidance:** Backtesting of forecast distributions produced by EPE models and market risk factor models needs to be performed on the entire forecast distribution.

## G2

**Guidance:** The validation requirements as set out in Basel II for EPE Models should not make reference to VaR requirements and instead the qualitative standards set out in paragraph 718 (LXXIV) should be transposed into the validation requirements for EPE models and the language adapted where required.

## G3

**Guidance:** The Validation of EPE models and all the relevant models that input into the calculation of EPE must be performed separately for a number of distinct time horizons.

## BCBS Guidance

### G4

**Guidance:** The performance of market risk factor models must be validated using backtesting. The validation must be able to identify poor performance in individual risk factors.

### G5

**Guidance:** The validation of EPE models and all the relevant models that input into the calculation of EPE must be made using forecasts initialised on a number of historical dates.

### G6

**Guidance:** Historical backtesting on representative counterparty portfolios and market risk factor models must be part of the validation process. At regular intervals as directed by its supervisor, a bank must conduct backtesting on a number of representative counterparty portfolios and its market risk factor models. The representative portfolios must be chosen based on their sensitivity to the material risk factors and correlations to which the bank is exposed.



# BCBS Guidance

## G7

**Guidance:** Backtesting of EPE and all the relevant models that input into the calculation of EPE must be based on recent performance.

## G8

**Guidance:** The frequency with which the parameters of an EPE model are updated needs be assessed as part of the on-going validation process.

## G9

**Guidance:** Firms need to unambiguously define what constitutes acceptable and unacceptable performance for their EPE models and the models that input into the calculation of EPE and have a written policy in place that describes how unacceptable performance will be remediated.

# BCBS Guidance

## G10

**Guidance:** Firms need to define what constitutes a representative counterparty portfolio for the purposes of carrying out EPE model backtesting.

## G11

**Guidance:** IMM firms need to conduct hypothetical portfolio backtesting that is designed to test risk factor model assumptions, eg the relationship between tenors of the same risk factor, and the modelled relationships between risk factors.

## G12

**Guidance:** Firms need to assess whether or not the firm level and netting set level exposure calculations are appropriate.

# BCBS Guidance

## G13

**Guidance:** Firms must backtest their EPE models and all relevant models that input into the calculation of EPE out to long time horizons of at least one year.

## G14

**Guidance:** Firms must validate their EPE models and all relevant models that input into the calculation of EPE out to time horizons commensurate with the maturity of trades covered by the IMM waiver.

## G15

**Guidance:** Prior to implementation of a new EPE model or new model that inputs into the calculation of EPE a firm must carry out backtesting of its EPE model and all the relevant models that input into the calculation of EPE at a number of distinct time horizons using historical data on movements in market risk factors for a range of historical periods covering a wide range of market conditions.

# BCBS Guidance

## G16

**Guidance:** Under the internal model method, a measure that is more conservative than Effective EPE (eg a measure based on peak rather than average exposure) for every counterparty may be used in place of alpha times EEPE with the prior approval of the supervisor. The degree of relative conservatism will be assessed upon initial supervisory approval and at regular intervals in conjunction with other EPE models. The assessment needs to cover all counterparties. The firm must have an unambiguous definition of what constitutes acceptable performance for these models and a documented process in place for remediating poor performance.

# What is the Question?

## Types of Investigation

- Hypothesis testing (Answer in percentage or yes/no)
- Estimation of model uncertainty (Answer in cash terms)

Analysis at different levels: figure 5

# Domains

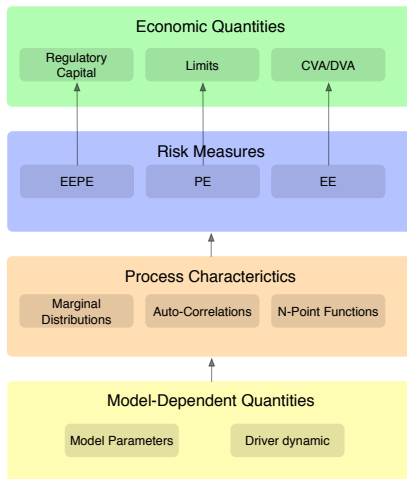


Figure: Domains

## Definition

A model is represented by a measure  $\mathbb{Q}$ .  
May be generated by a stochastic process.

## Quantifying Difference of Models

- Comparing expectation values
- Comparing probability distributions

Note: PDFs and CDFs may be expressed as expectation values

# Radon-Nikodym Derivative

Distance of model  $\mathbb{Q}$  and end empirical measure  $\mathbb{P}$  in terms of  $\frac{d\mathbb{P}}{d\mathbb{Q}}$ :

$$\mathbb{E}_{\mathbb{P}}[f] = \mathbb{E}_{\mathbb{Q}}\left[\frac{d\mathbb{P}}{d\mathbb{Q}} f\right] \quad (9)$$

## Compare $\mathbb{P}$ and $\mathbb{Q}$

Direct  $\frac{d\mathbb{P}}{d\mathbb{Q}} \approx id?$

**Expectation values** Empirical expectation measures in terms of model expectations

**Relative Entropy** Kullback-Leibler entropy  $\rightarrow$  information geometry (see [?])



# Radon-Nikodym Derivative

Let  $\xi$  be a scalar stochastic variable (e.g. portfolio price  $\pi(t)$ )

## Definition

$P$  empirical,  $Q$  model CDF

$$\Psi : [0, 1] \rightarrow [0, 1] \quad (10)$$

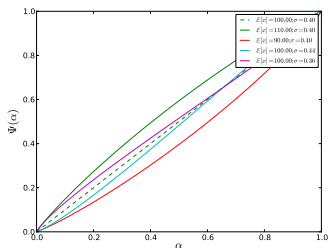
$$\Psi(\alpha) = P(Q^{-1}(\alpha)) \quad (11)$$

## Radon-Nikodym derivative $\psi$

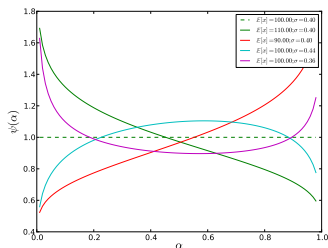
$$\mathbb{E}_P[f] = \mathbb{E}_Q[\psi(\alpha)f] \quad (12)$$

$$\psi(\alpha) = \frac{d\Psi(\alpha)}{d\alpha} \quad (13)$$

# Example



(a)  $\Psi(\alpha)$



(b)  $\psi(\alpha)$

Figure: Comparison of log-normal distributions with different parameters

# Cumulative Distribution Functions

Cumulative distribution function (CDF) for some state variable  $\xi$  expressed as expectation:

## Definition

$$P(\xi_0) = \mathbb{E}_{\mathbb{P}}[\Theta(\xi - \xi_0)] \quad (14)$$

where  $\Theta$  is the Heaviside function.

# Estimating

Ensemble averages  $\mathbb{E}$  estimated well by time averages if

- ergodic
- stationary

CDF

$$P(\xi_0) \approx \frac{1}{N} \sum_{i=1}^N \Theta(\xi(t_i) - \xi_0) \quad (15)$$

$\Psi$

$$\Psi(\alpha) \approx \frac{1}{N} \sum_{i=1}^N \Theta(\xi(t_i) - Q^{-1}(\alpha)) \quad (16)$$

# Requirements for Estimation

Process needs to be

- ergodic
- stationary

iid price process

If empirical price process is iid, the ergodic.

iid process of underlying

Even if underlying process the price return process of the deal may not be so, if deal not time homogeneous

# Distances

## Point Distance

$$d_i = |\Psi(q_i) - q_i| \quad (17)$$

## Curve Distance

(Weighted) quadratic distance  $d$  between functions  $q \rightarrow \Psi(q)$  and  $q \rightarrow q$ :

$$d(q, \Psi(q)) = \sum_i w_i (\Psi(q_i) - q_i)^2 \quad (18)$$

$q_i$  e.g. (0.01, 0.05, 0.3, 0.5, 0.7, 0.95, 0.99)

# Hypothesis Testing

## Null-Hypothesis

- Null-Hypothesis, is that distances are 0.
- Reject Null-Hypothesis  $p$ -values smaller than some threshold

## Challenges estimating $p$ -values

- Temporal dependence: overlap of time-windows
- Ensemble dependence: returns of netting sets not independent

Good  $p$  values get bigger

Bad Estimation tricky

Need some simplifications, like effective sample sizes

# Problems using metrics for $\Psi$

## Issues using metrics for $\Psi$

**Opaque** no cash denominated measure

**Economics Product Dependent** with same distance different moments drive deviations in EE (see figure (7))

**Limited usefulness** Passes test if not enough data available



# Problems using metrics for $\Psi$

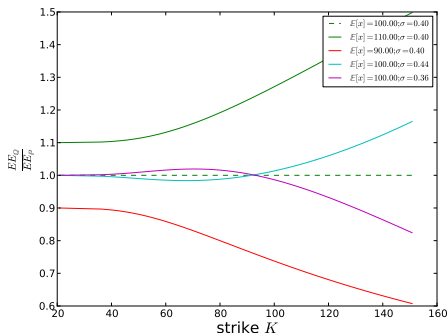


Figure: Comparing  $EEs$  for a forward using log-normal distributions with different parameters

# Comparison using Cash denominated Quantities

## Economically Relevant Model Dependent Quantities

Regulatory Capital depends on  $EE(t)$  (through EEPE)

Limits impacted by CDF

P&L impacted by  $EE(t)$

## Measure

These three quantities are functions of  $\mathbb{E}_Q$ .

Their value under empirical measure  $\mathbb{P}$  estimated through equation (12)  $\rightarrow$  difference in cash terms

# Overview

- 1 Model Classes
- 2 Credit Risk Measures
- 3 Model Implementation
- 4 Back Testing
- 5 Model Control

# Requirements

Control Processes should be

- Complete
- Accurate
- Consistent
- Timely
- Appropriate and Relevant
- Auditable