1.) $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

$= \frac{1}{N}\left(\sum_{i=1}^{N}a + \sum_{i=1}^{N}bx_i\right)$

$= \frac{1}{N}\left(Na + b\sum_{i=1}^{N}x_i\right)$

$= a + b\left(\frac{1}{N}\sum_{i=1}^{N}x_i\right)$

$= a + b\,m(X)$

2.) $cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = s^2$

☆ by the def of sample variance

3.) $Z = a + bY$ where $z_i = a + by_i$

$m(Z) = m(a+bY) = a + b\,m(Y)$

$cov(X,Z) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(z_i - m(Z))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\left[(a+by_i) - (a+b\,m(Y))\right]$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\,b(y_i - m(Y))$

$= b\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

$= b\,cov(X,Y)$

4.) ☆ $U = a + bX$ $\quad V = a + bY$

$m(U) = a + b\,m(X)$ $\quad m(V) = a + b\,m(Y)$

$u_i - m(U) = (a + bx_i) - (a + b\,m(X)) = b(x_i - m(X))$

$v_i - m(V) = (a + by_i) - (a + b\,m(Y)) = b(y_i - m(Y))$

$cov(U,V) = \frac{1}{N}\sum_{i=1}^{N}(u_i - m(U))(v_i - m(V))$

$= \frac{1}{N}\sum_{i=1}^{N}\left[b(x_i - m(X))\right]\left[b(y_i - m(Y))\right]$

$= b^2\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

$= b^2\,cov(X,Y)$

5.) $median(a+bX) = a + b\,med(X)$

↳ yes this is true

$IQR(X) = Q_3(X) - Q_1(X)$

$Q_1(a+bX) = a + bQ(X)$

$Q_3(a+bX) = a + bQ_3(X)$

$IQR(a+bX) = (a + bQ_3(X)) - (a + bQ_1(X))$

$= b(Q_3(X) - Q_1(X))$

$= b(IQR(X))$

↳ no, not true for IQR

6.) $X = \{1, 2, 3\}$

$m(X) = \frac{1+2+3}{3} = 2$

$y = x^2$

$x^2 = \{1, 4, 9\}$

$m(x^2) = \frac{1+4+9}{3} = \frac{14}{3}$

$m(x)^2 = 2^2 = 4$

☆ $m(x^2) \neq m(x)^2$

$y = \sqrt{x}$

$\sqrt{x} = \{1, \sqrt{2}, \sqrt{3}\}$

$m(\sqrt{x}) = \frac{1+\sqrt{2}+\sqrt{3}}{3} = 1.382$

$\sqrt{m(x)} = \sqrt{2} = 1.414$

☆ $m(\sqrt{x}) \neq \sqrt{m(x)}$