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Implementation of Kinematics and Dynamics of an Elephant Trunk like Robotic Arm

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Bachelor Thesis

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Motivation

proboscis

- simulation environment
- model based controller
- better understanding

State of the art

[1]

State of the art

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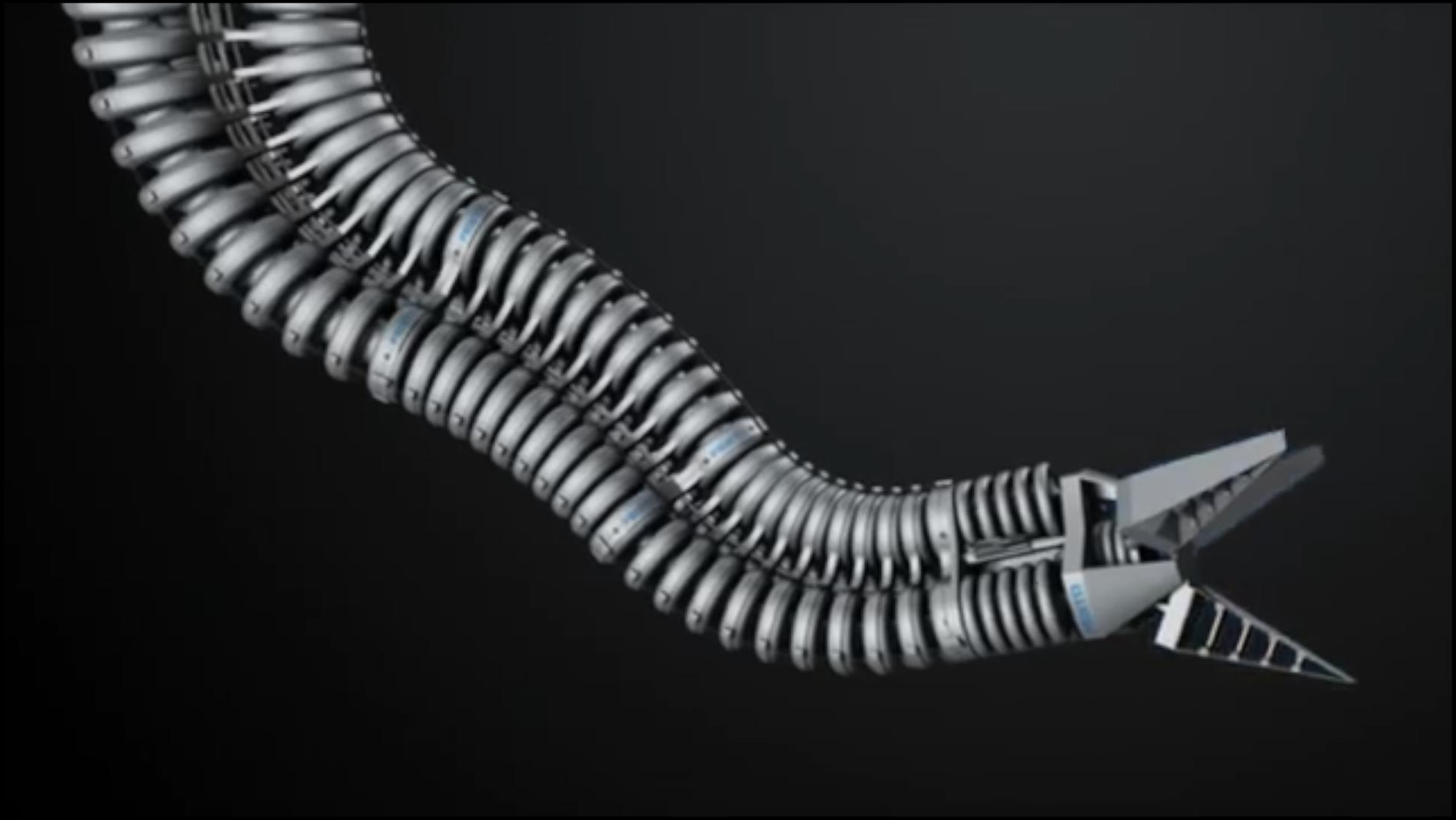
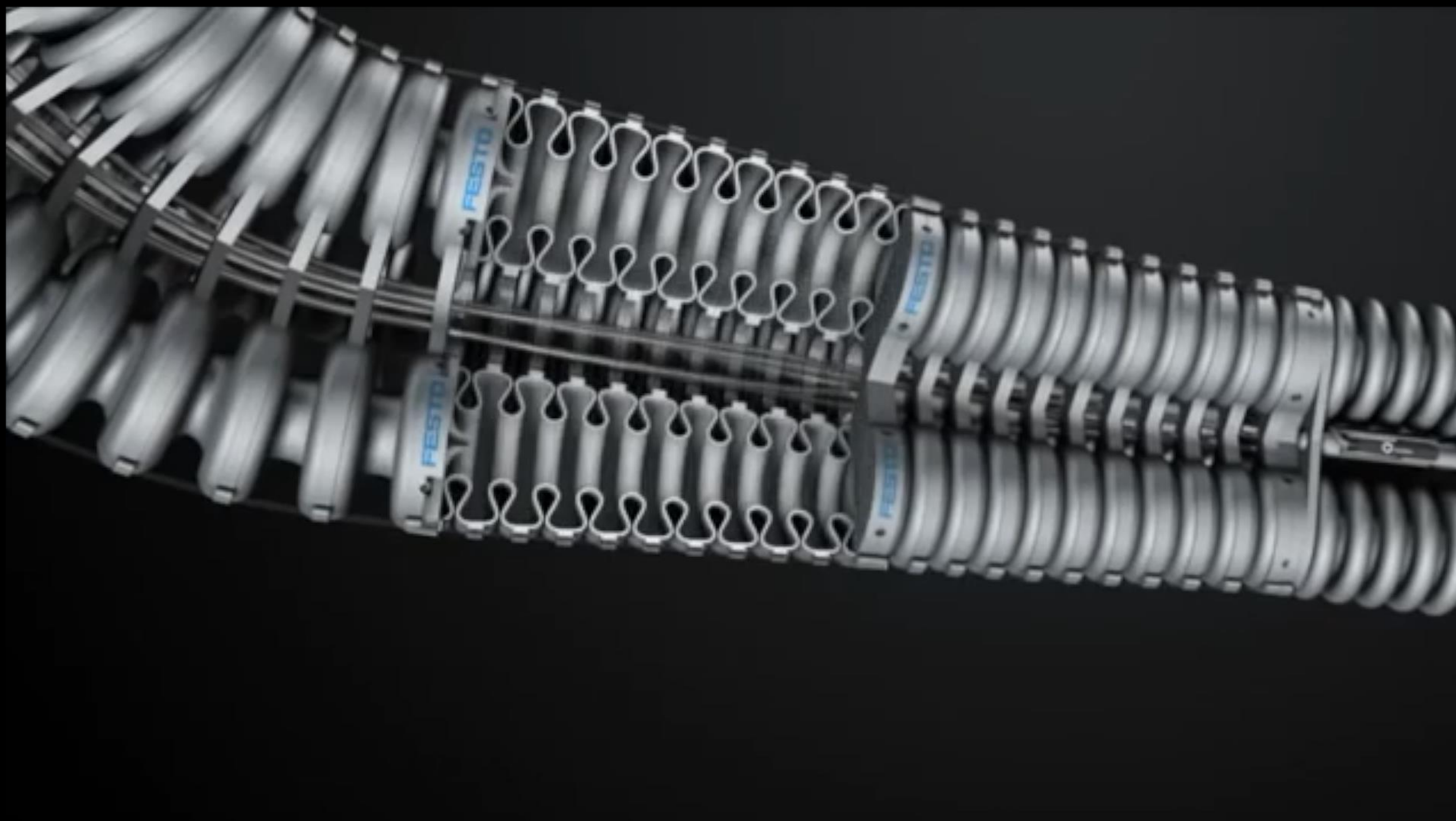
State of the art



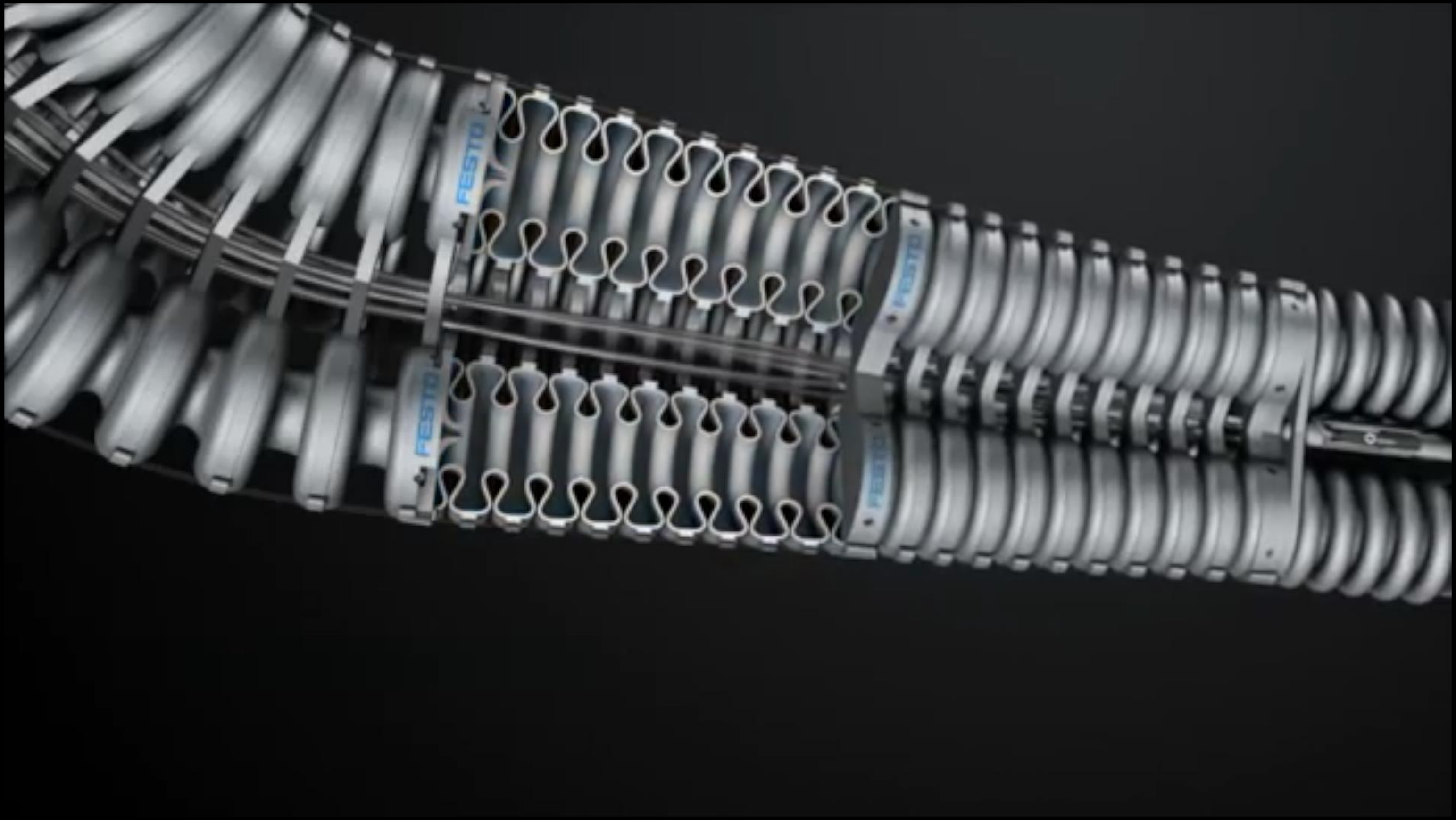
[1]

State of the art



[1]

State of the art



[1]

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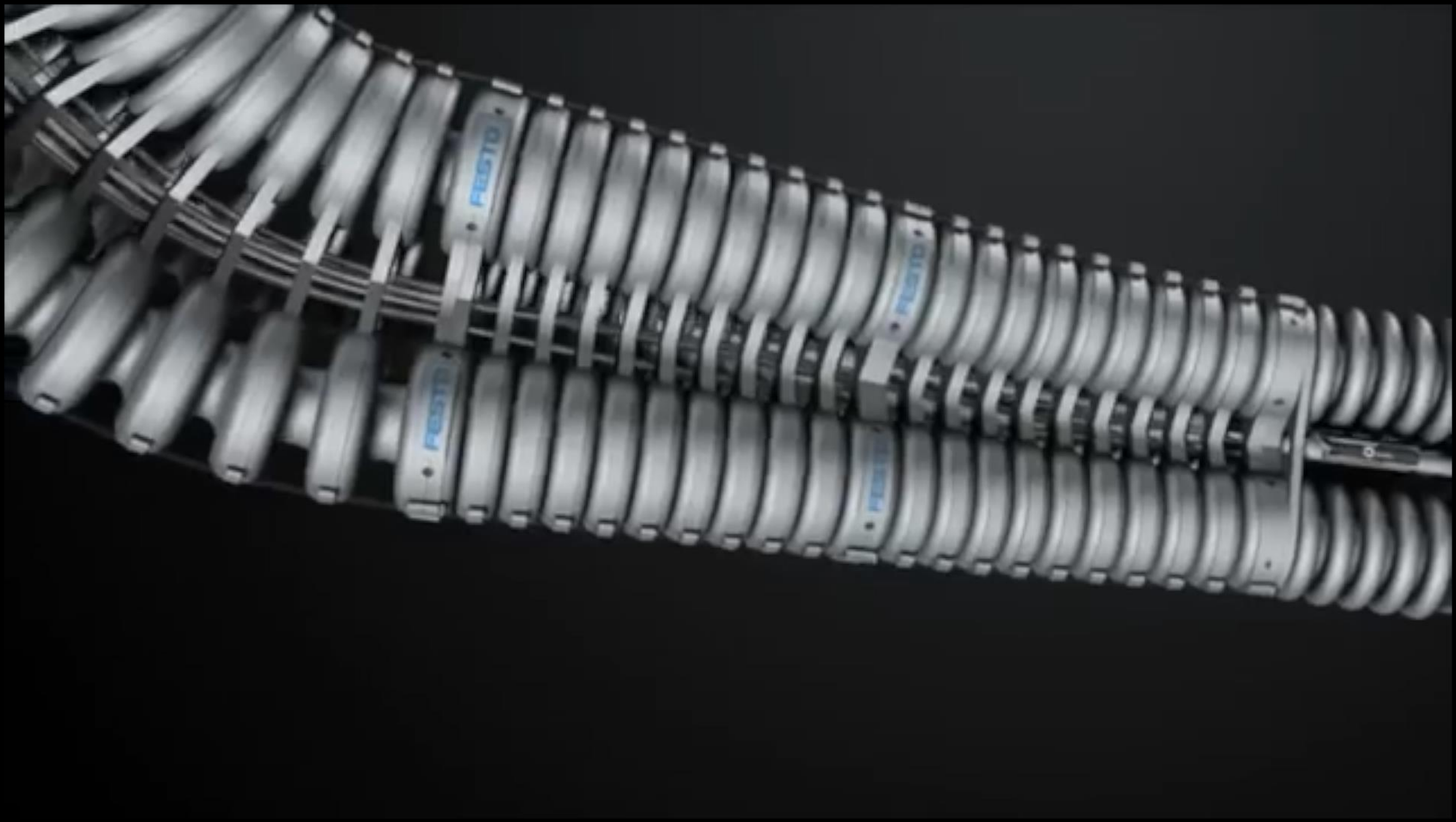
State of the art



[1]

State of the art

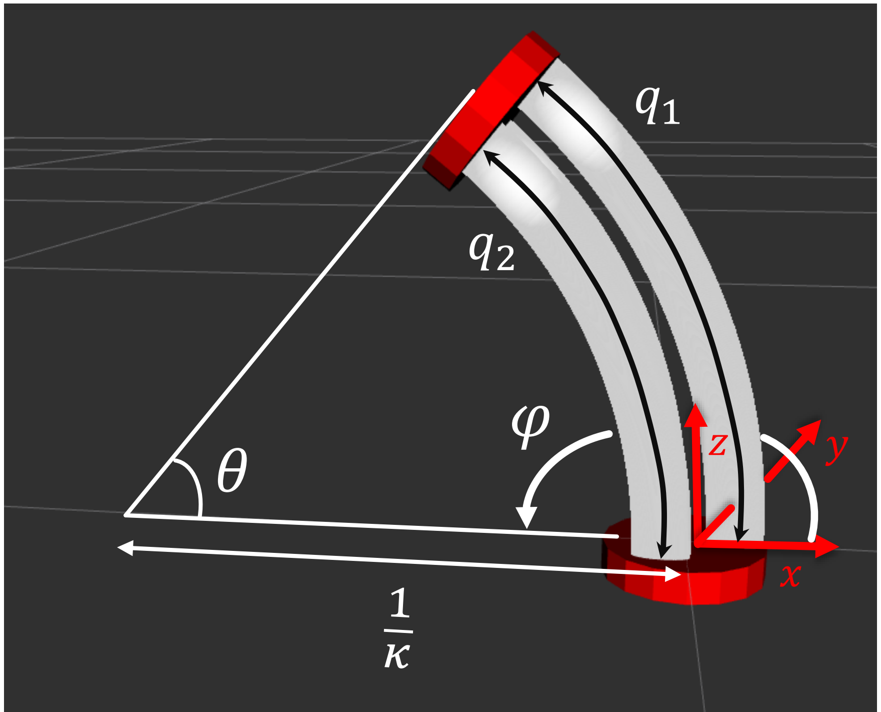
Doctoral thesis of Dr. Falkenhahn [2]

- derivation of dynamic model
- model simplifications
- validation
- model based path planning

Goal

- adaptation and implementation of
  - kinematic model
  - dynamic model
- parameter identification
- validation of both models

Kinematic model



key assumption: constant curvature [2]

coordinates

θ : curvature/bending angle

κ : curvature

φ : orientation angle

q_i : muscle lengths with $i = 1, 2, 3$

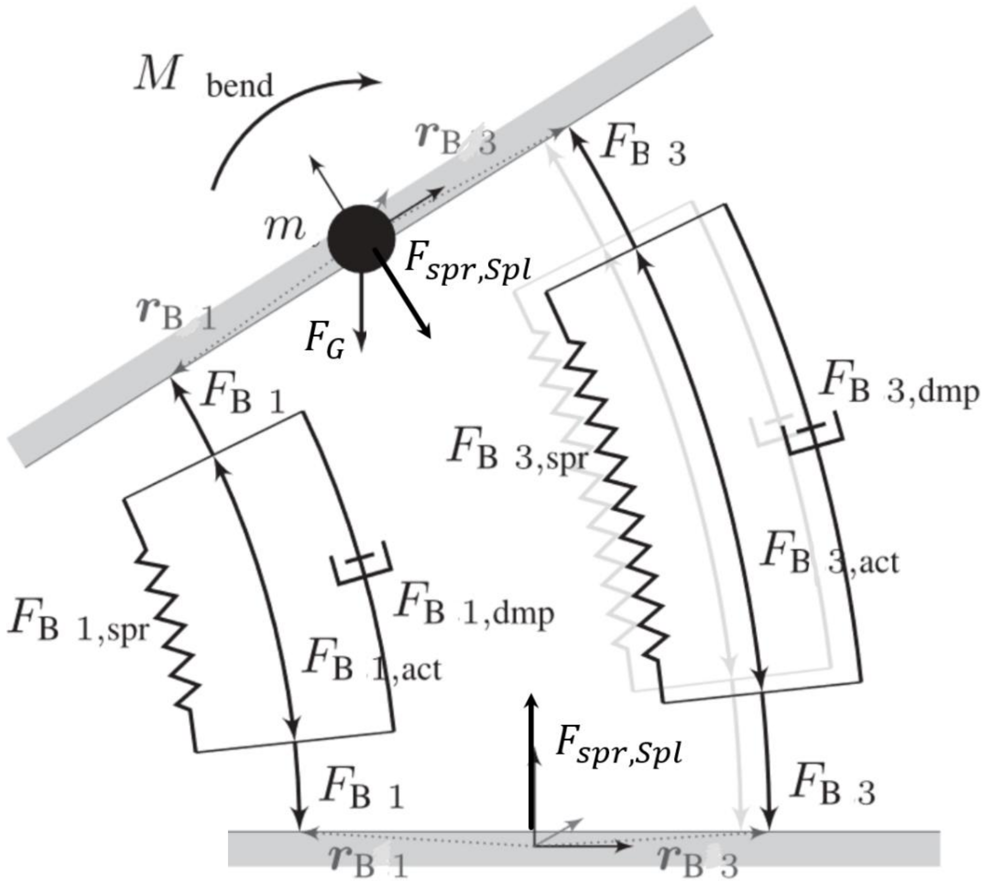
rotation matrix

displacement vector

$$
{}^{w}H_h = \begin{bmatrix} \cos^2(\varphi)\cdot(\cos(\theta)-1)+1 & \sin(\varphi)\cdot\cos(\varphi)\cdot(\cos(\theta)-1) & \cos(\varphi)\cdot\sin(\theta) & \frac{\cos(\varphi)\cdot(1-\cos(\theta))}{\kappa} \\ \sin(\varphi)\cdot\cos(\varphi)\cdot(\cos(\theta)-1) & \cos^2(\varphi)\cdot(1-\cos(\theta))+\cos(\theta) & \sin(\varphi)\cdot\sin(\theta) & \frac{\sin(\varphi)\cdot(1-\cos(\theta))}{\kappa} \\ -\cos(\varphi)\cdot\sin(\theta) & -\sin(\varphi)\cdot\sin(\theta) & \cos(\theta) & \frac{\sin(\theta)}{\kappa} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

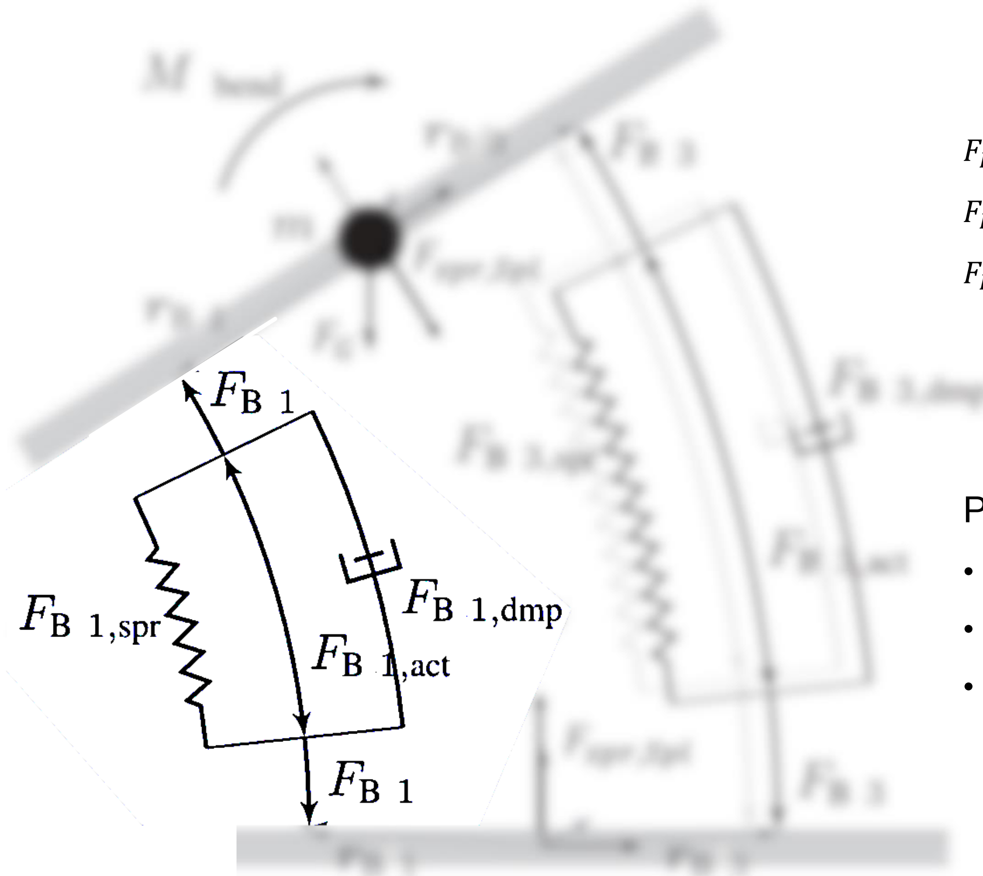
[2]

Dynamic model - principle



adapted from [3]

Dynamic model – muscle forces



adapted from [3]

$F_{B,1,dmp}$: damping force of muscle 1

$F_{B,1,act}$: pressure force of muscle 1

$F_{B,1,spr}$: spring force of muscle 1

Parameters

- length stiffness coefficient
- damping coefficient
- pressure acting area

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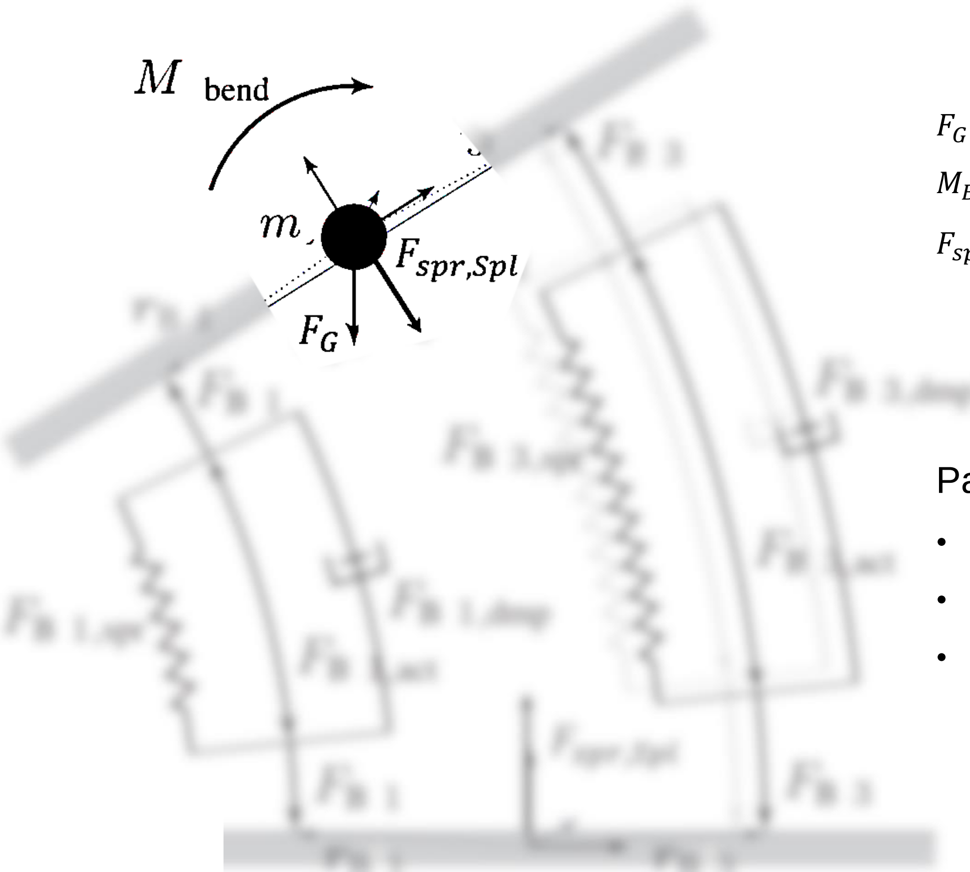
Dynamic model – tip forces



adapted from [3]

F_G: gravitational force

M_{Bend}: bending moment

$F_{spr,spline}$: spring force, center spline

Parameters

- bending stiffness
- spline spring coefficient
- mass

Dynamic model - approach

Lagrange equations: $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i, \quad i = \{1, 2, 3\}$

- q_i: generalized coordinates → 3 muscle lengths
- T: kinetic energy → $T = T_{translation} + \cancel{T_{rotation}} = \frac{1}{2}(m * {}^{w}\dot{r}_c^T * {}^{w}\dot{r}_c + \cancel{{}^{w}\omega_c^T * I_c * {}^{w}\omega_c})$
- U: potential energy → $U = -m * {}^{w}g^T * {}^{w}r_c + M_{bend}(\theta)$
- Q_i: generalized force acting on each muscle
- Assumption: $T_{rotation} \ll T_{translation} \rightarrow T_{rotation} \approx 0$ [2]

m: mass of segment

r_c: position vector of mass

I_c: inertia tensor

ω_c: angular velocity vector

g: gravity vector

M_{bend}: bending moment

Dynamic model - implementation

- The 3 Lagrange equations lead to [1]

$$M(q)\ddot{q} + \Omega(q,\dot{q})\dot{q} + N(q,\dot{q}) = \tau(p), \text{ where}$$

q: generalized coordinate vector

p: pressure vector.

- Reduction of order to 6 equations:

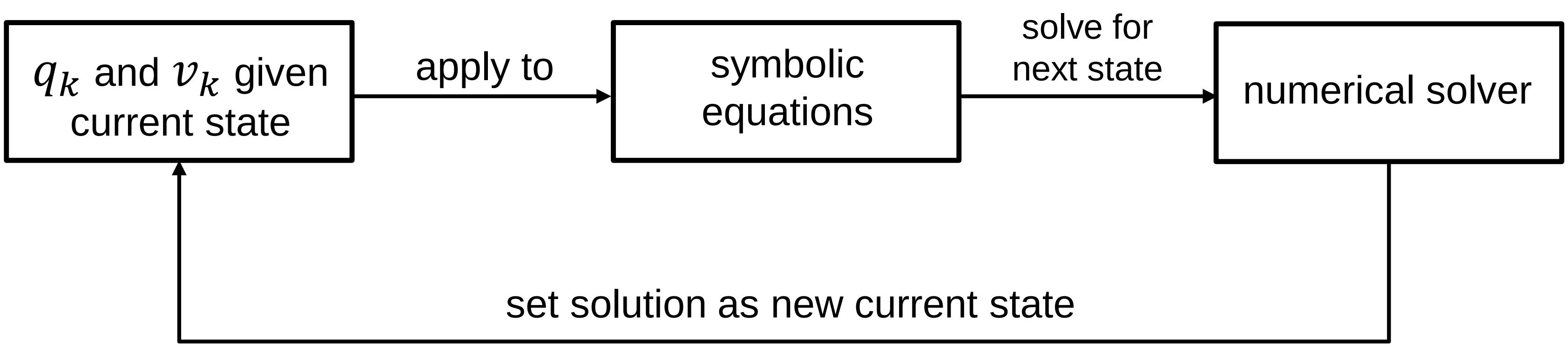
$$v := \dot{q}$$

$$M(q)\dot{v} + \Omega(q,v)v + N(q,v) = \tau(p)$$

Dynamic model - implementation

- Equations are symbolically derived once and implemented directly.

- Discretization of equation:



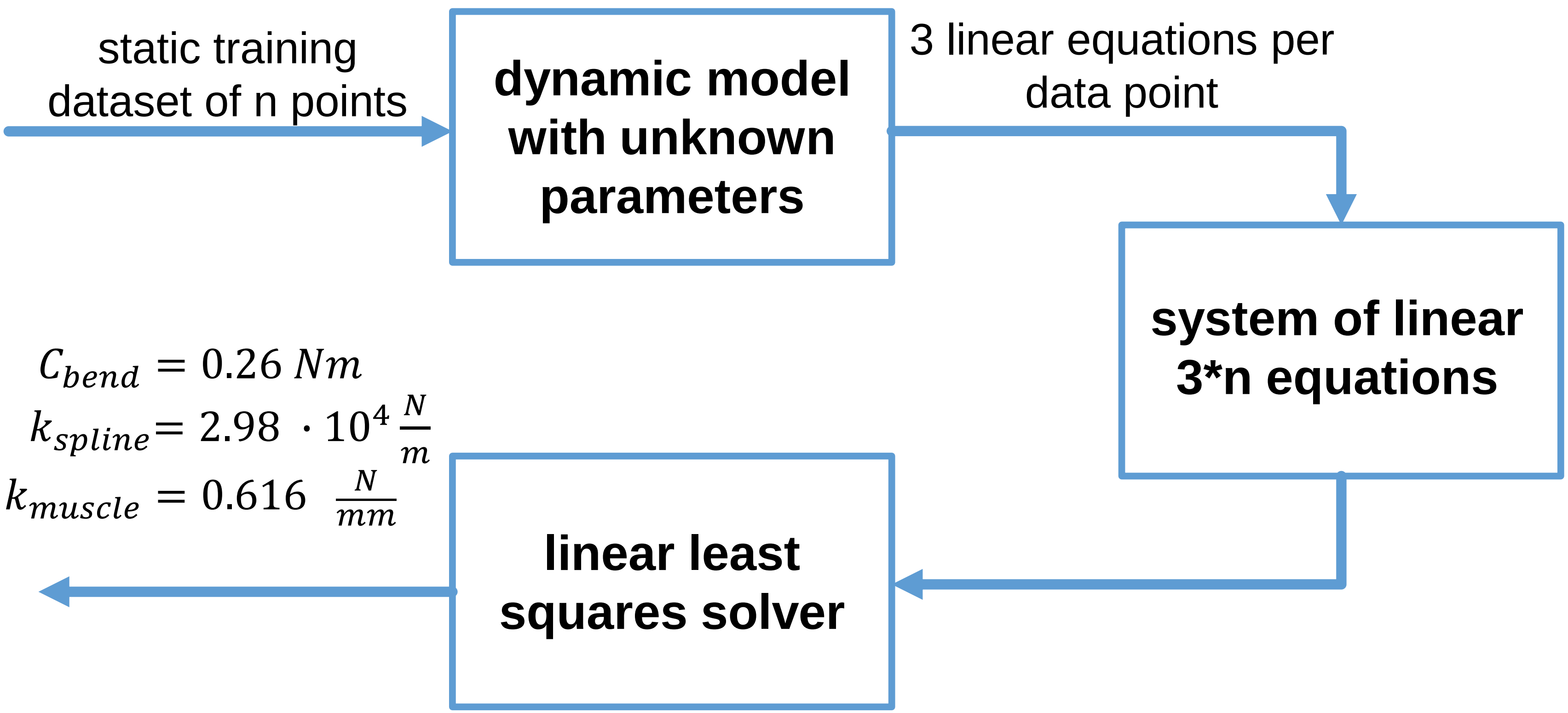
- Derivative is approximated with Euler Forward and 4th order Runge-Kutta method.

Implementation - challenges

- long terms to deal with
- singularities in equations
- complex calculus background
- prone to instability

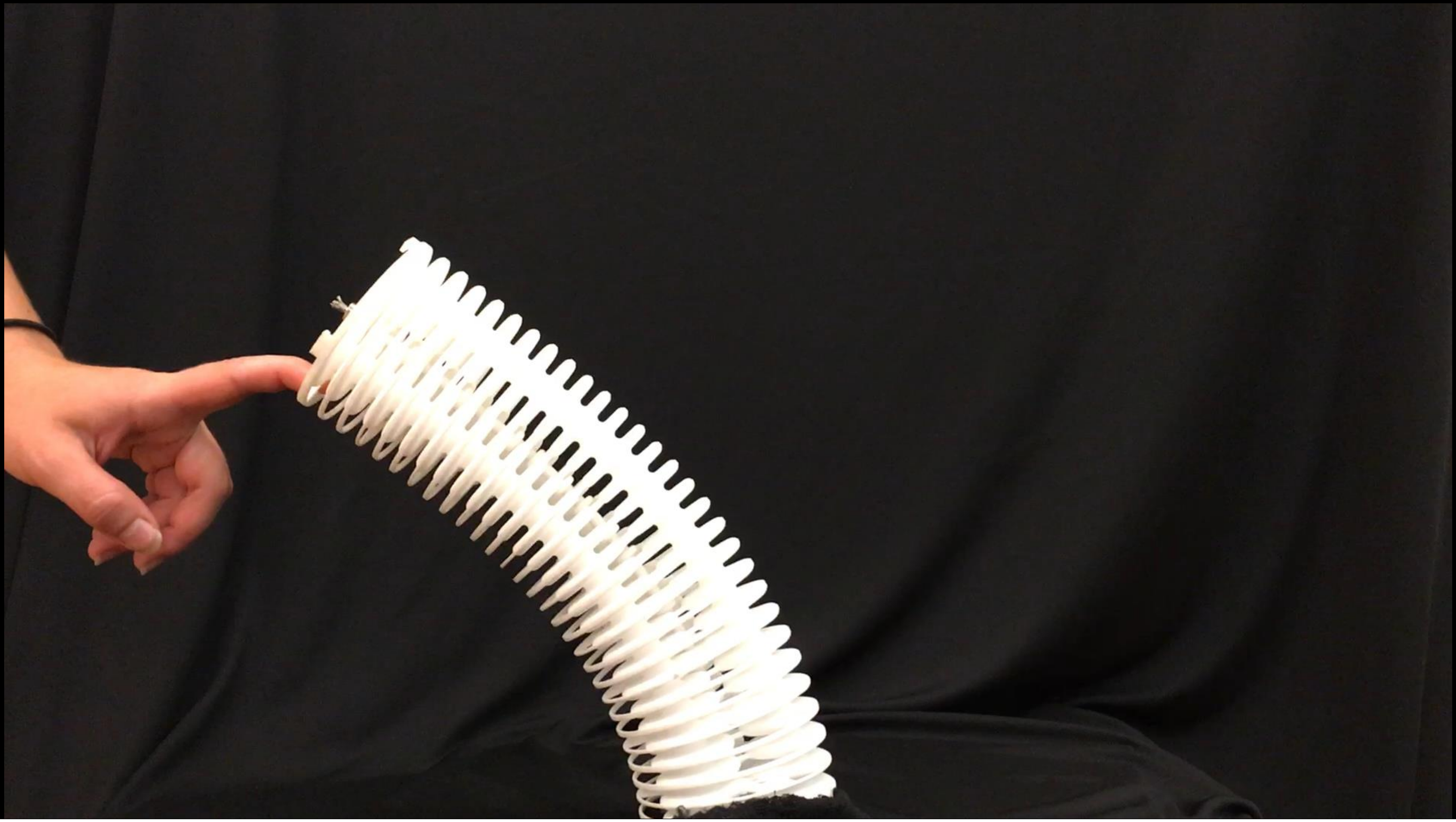
Parameter identification – stiffnesses

key idea: static points → $\dot{\boldsymbol{q}} = \ddot{\boldsymbol{q}} = \boldsymbol{0}$ → differential equations become linear



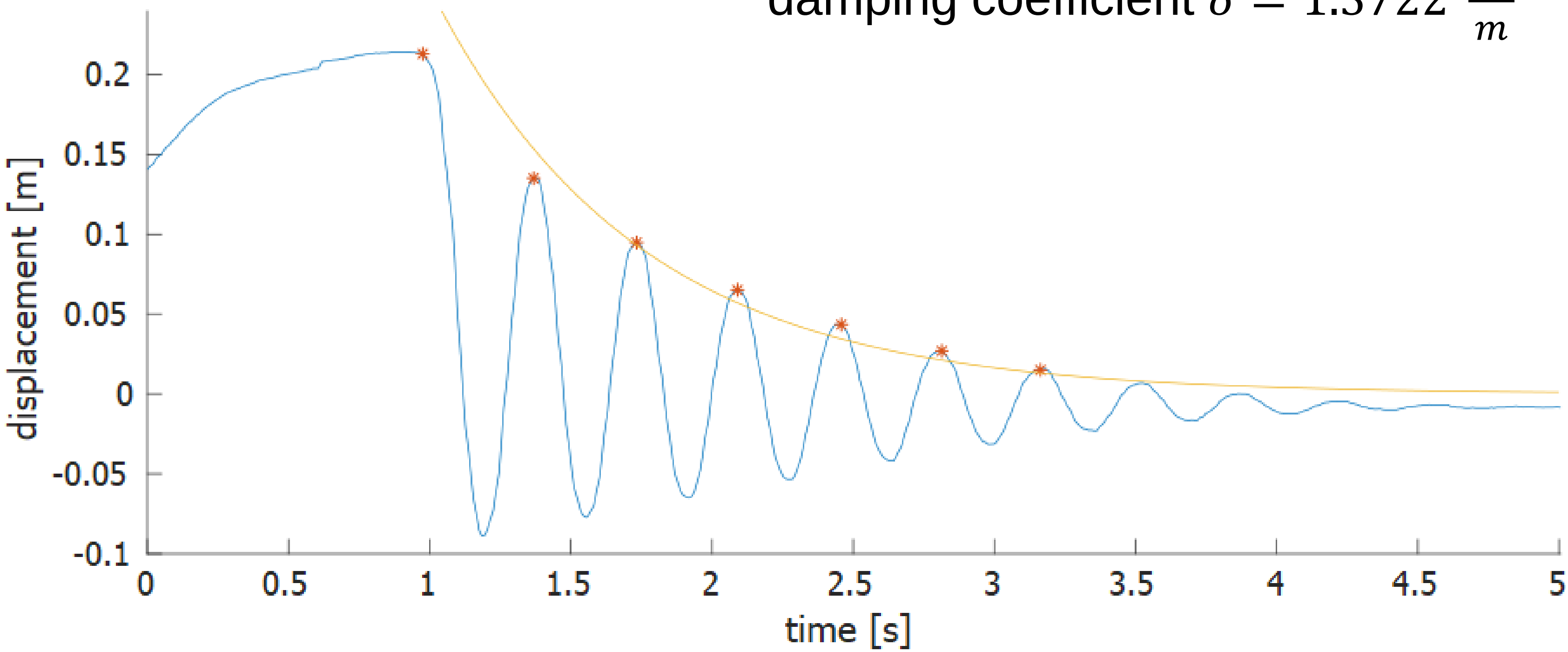
$$C_{bend} = 0.26\ Nm$$
$$k_{spline} = 2.98 \cdot 10^{4}\ \frac{N}{m}$$
$$k_{muscle} = 0.616\ \frac{N}{mm}$$

Parameter identification - damping coefficient

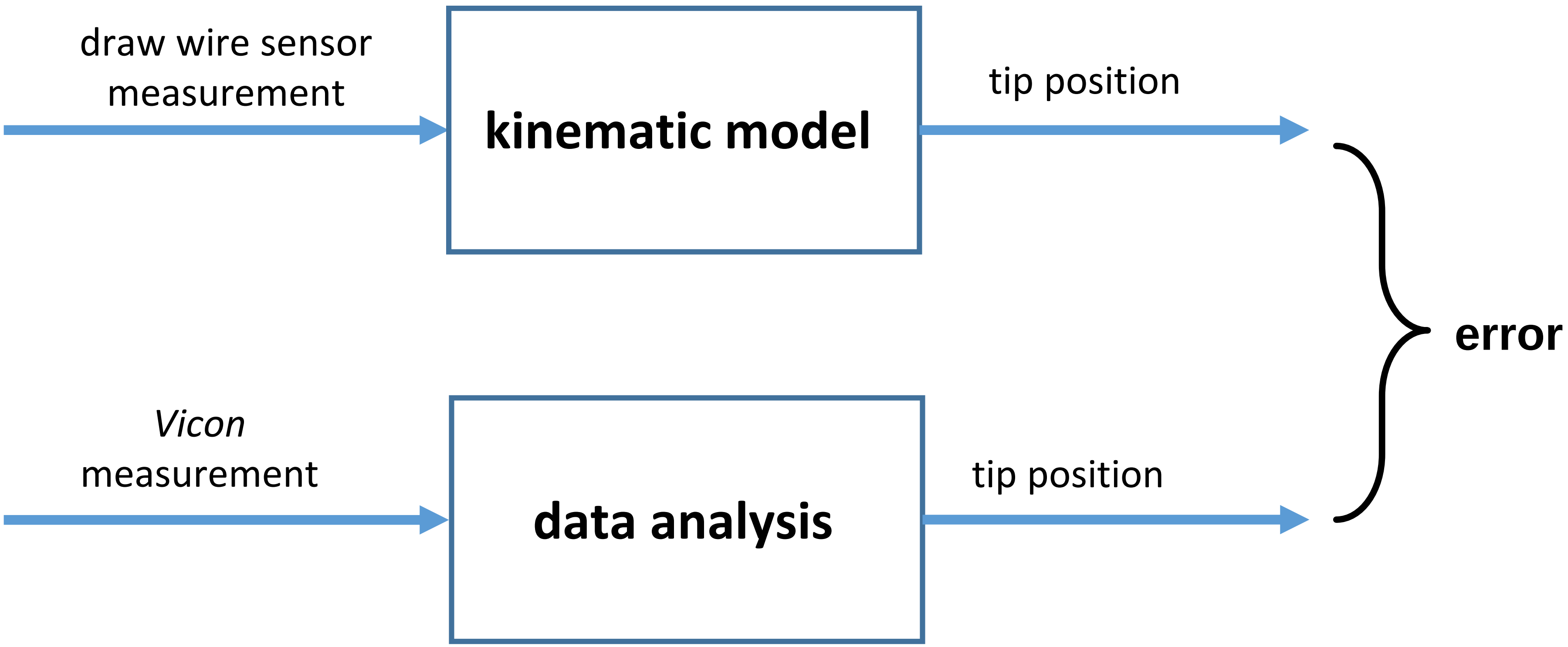


Parameter identification - damping coefficient

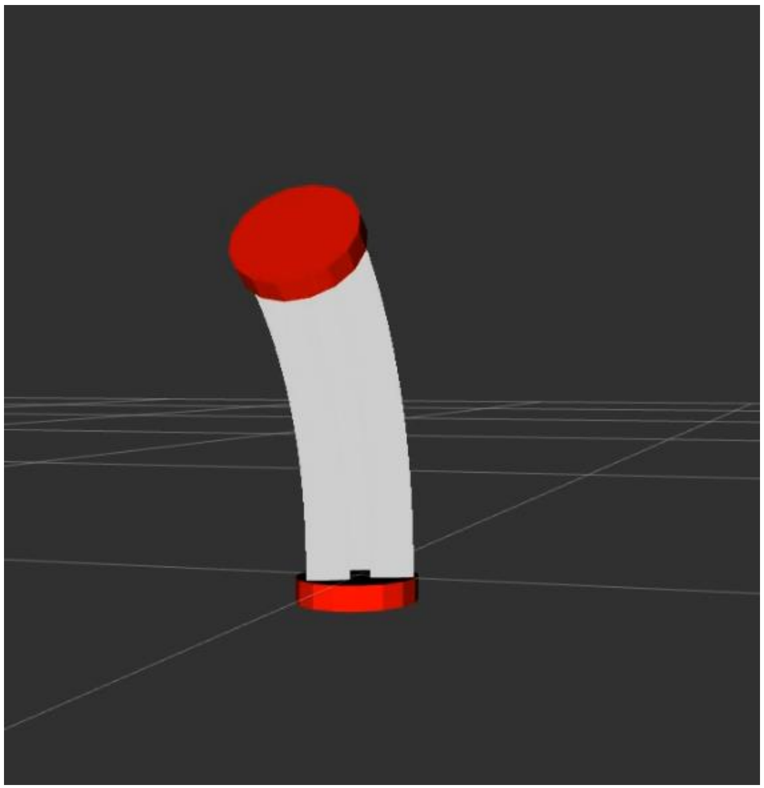
damping coefficient $\delta = 1.3722\ \frac{Ns}{m}$



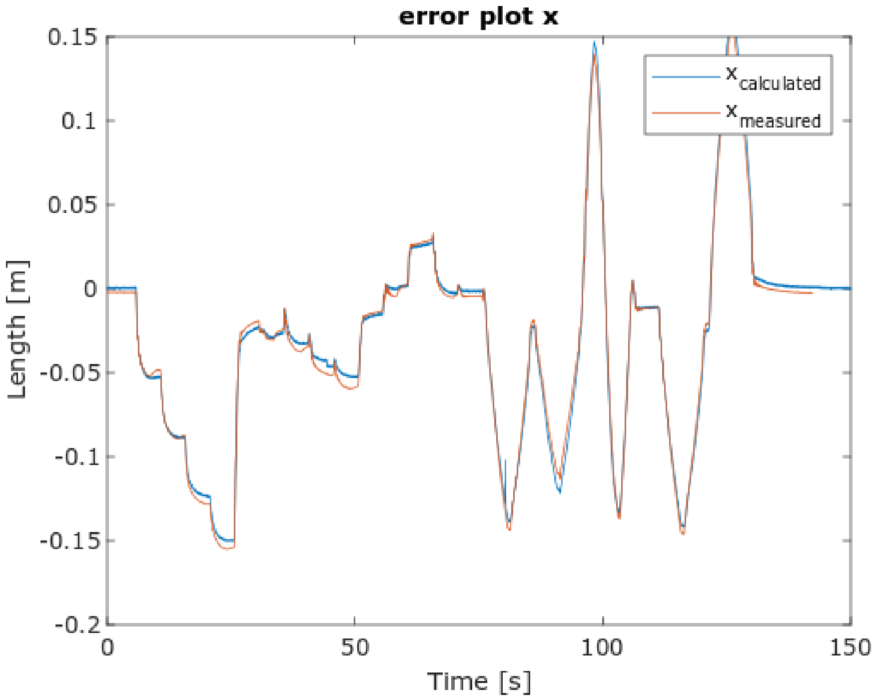
Validation - kinematic model

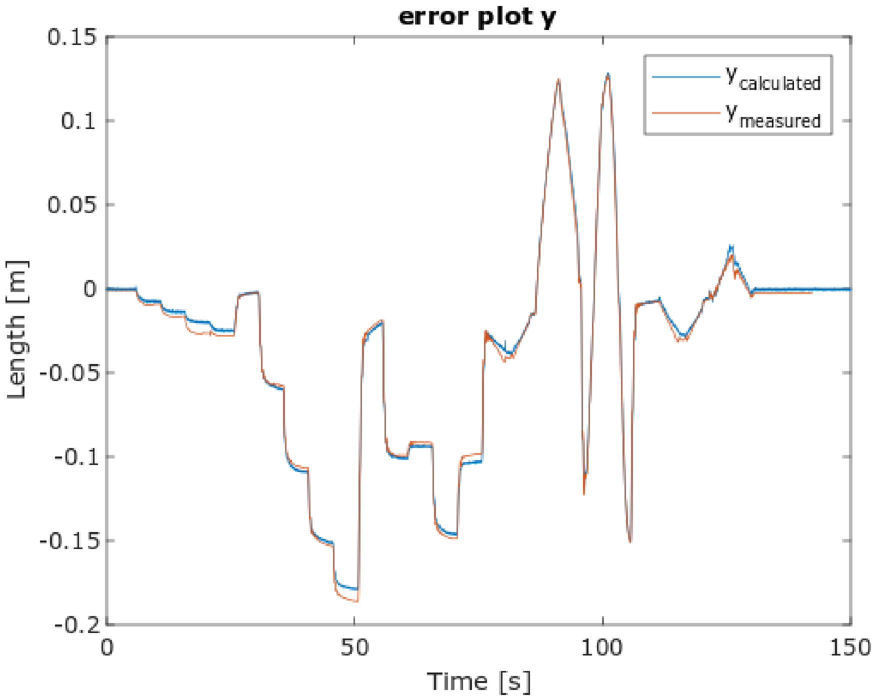


Visualization - kinematic model

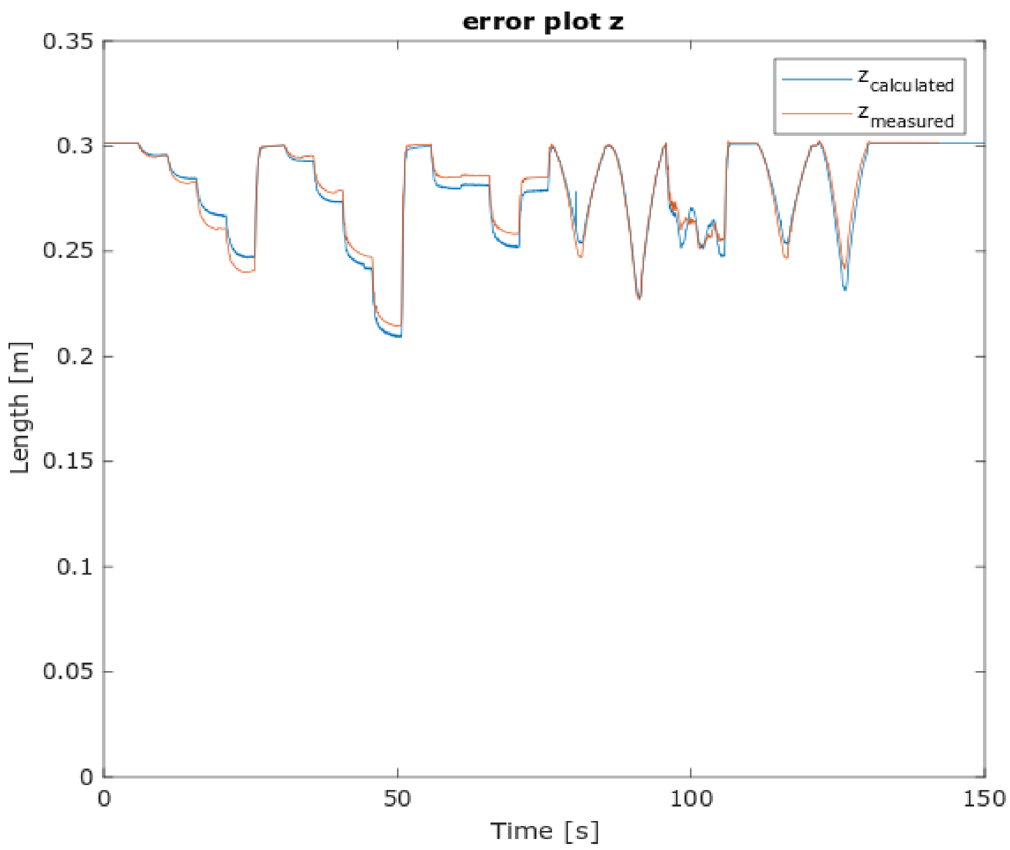


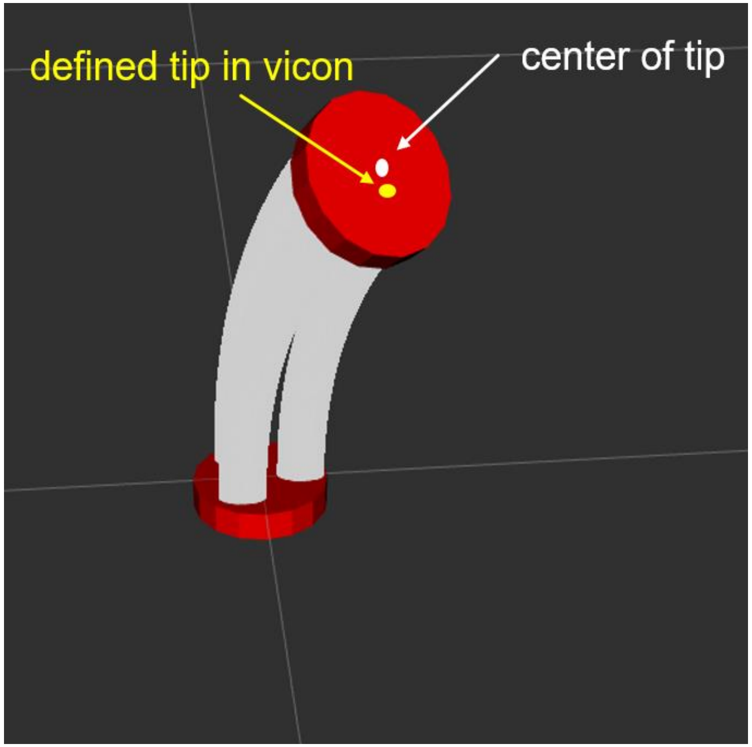
Results - kinematic model



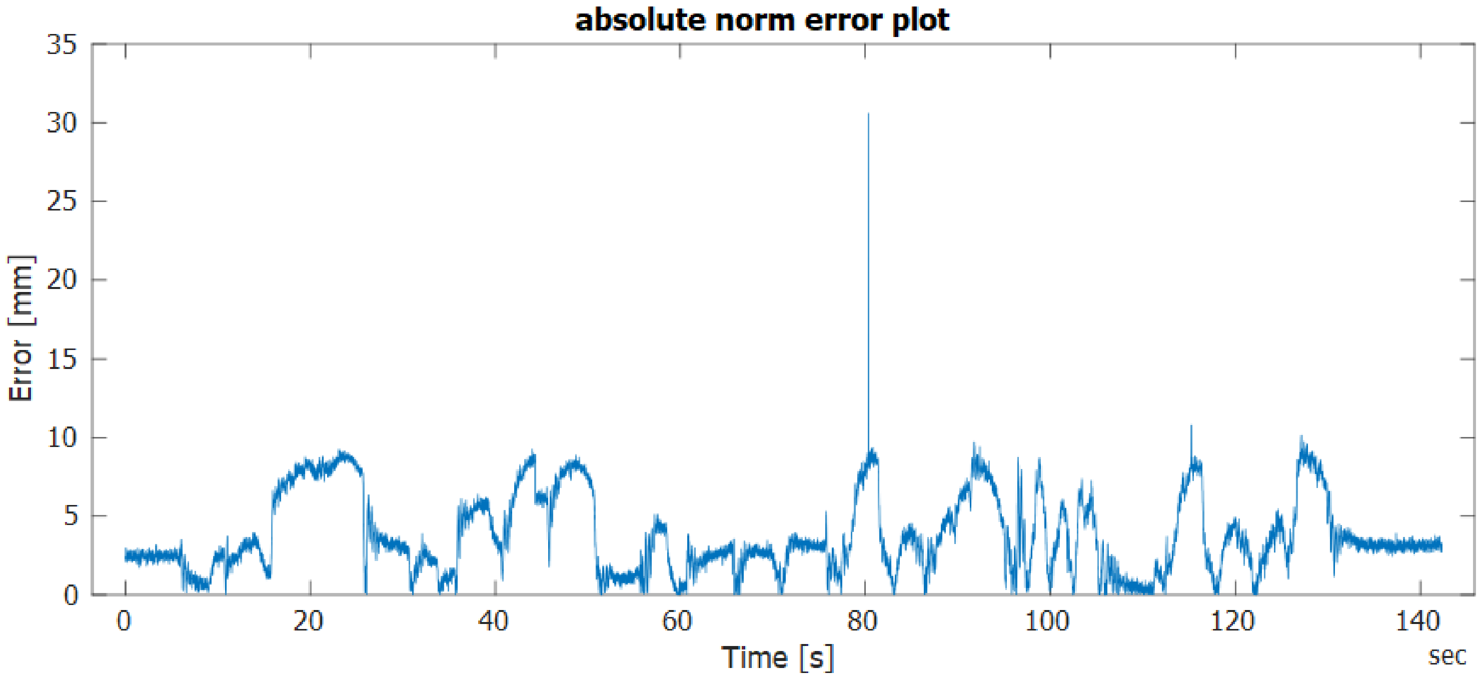


Results - kinematic model



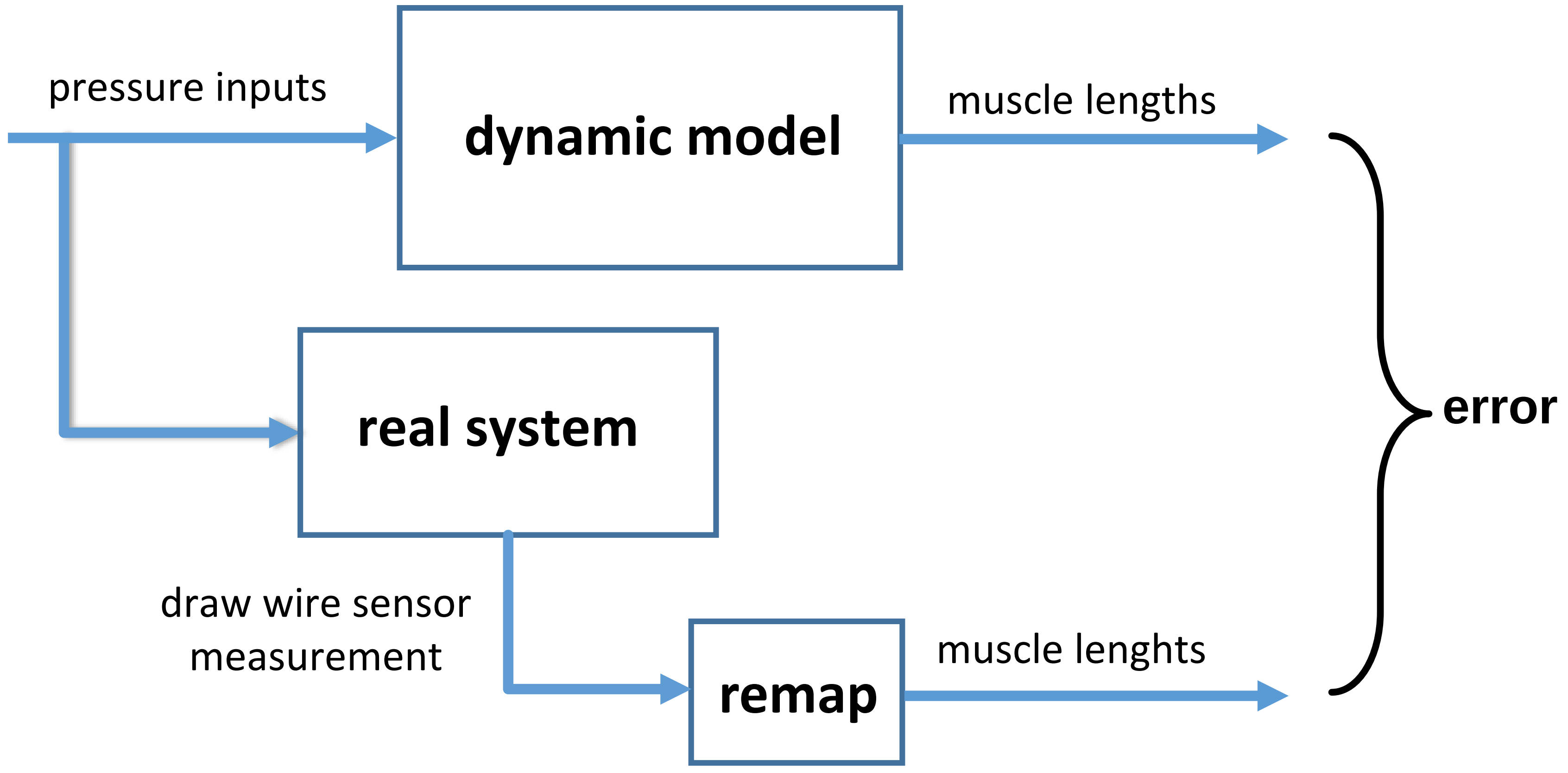


Results - kinematic model

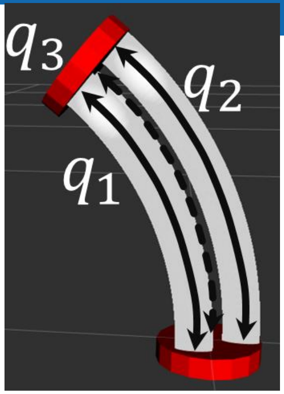


mean error = 6.3 mm $\pm$ 3.6 mm

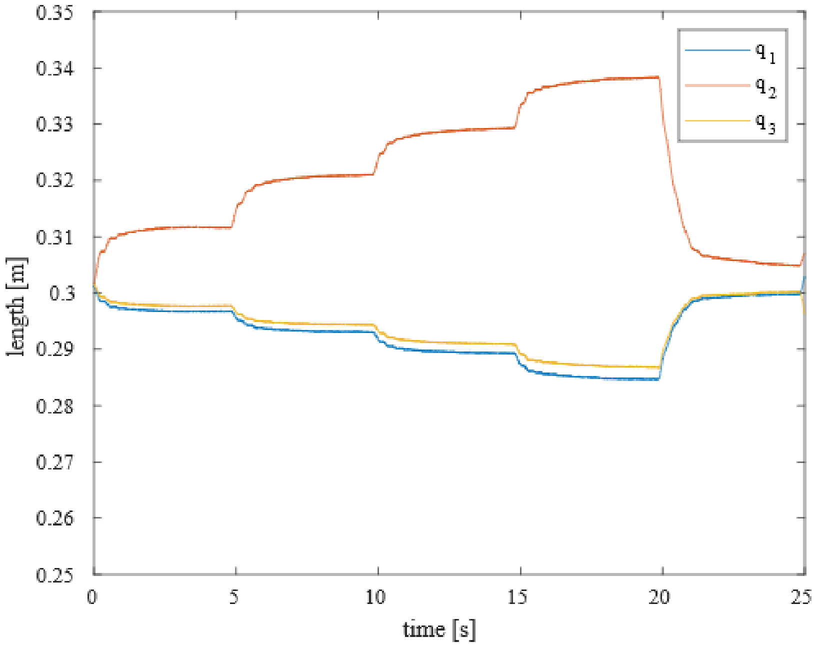
Validation - dynamic model



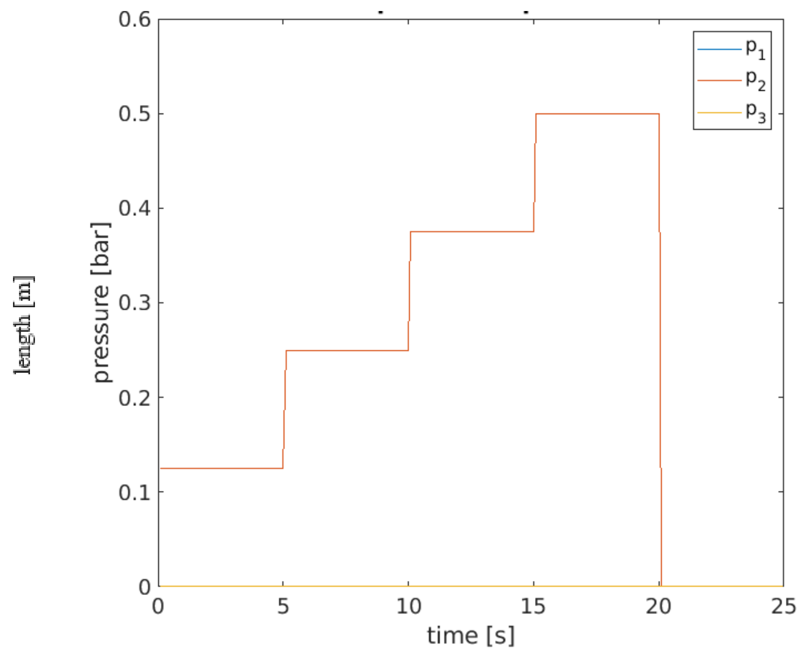
Results – dynamic model - steps



experimental muscle lengths

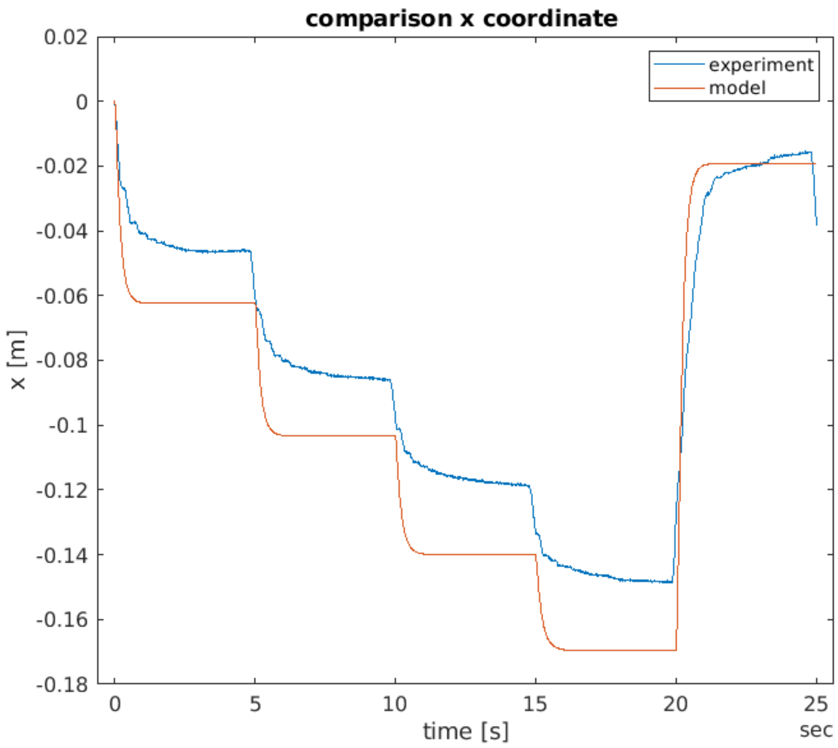


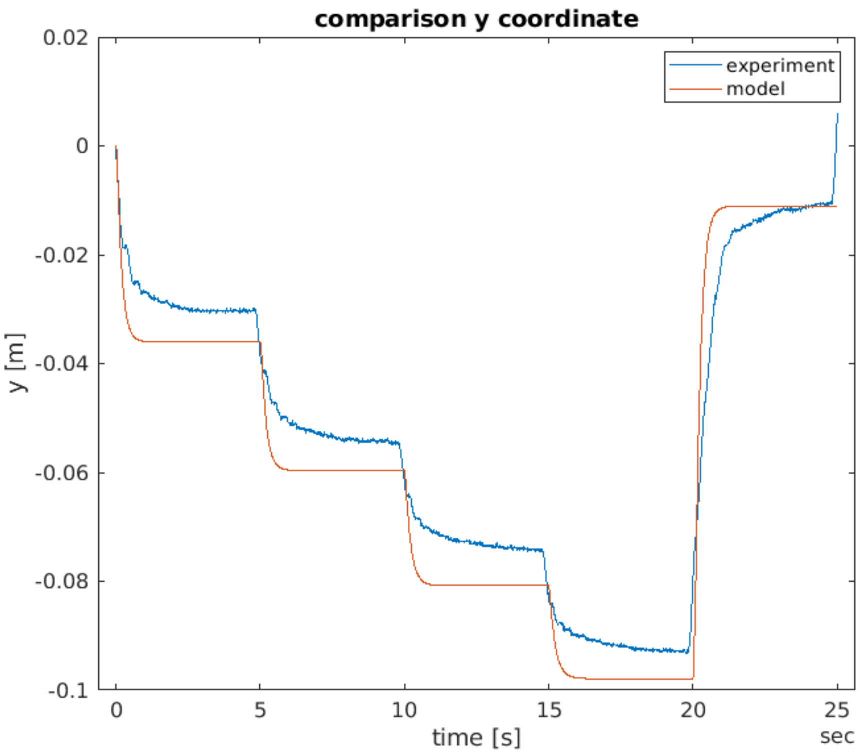
calculated muscle lengths
pressure input



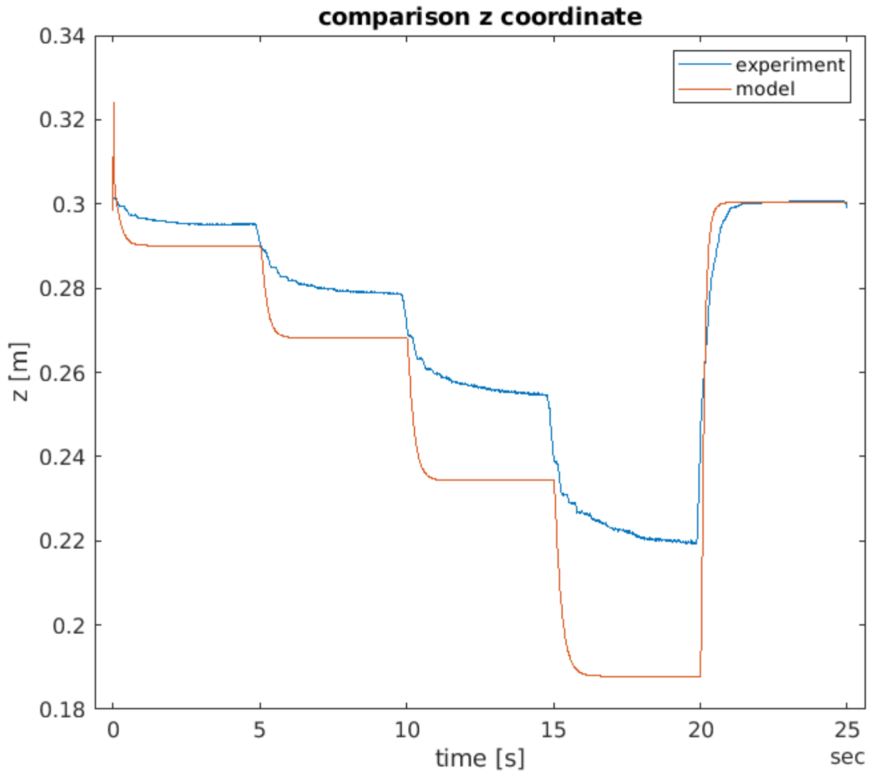
mean error = 5.6 $\pm 3\ mm$

Results – dynamic model - steps



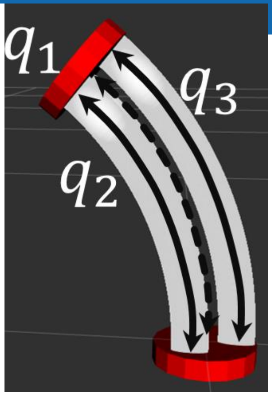


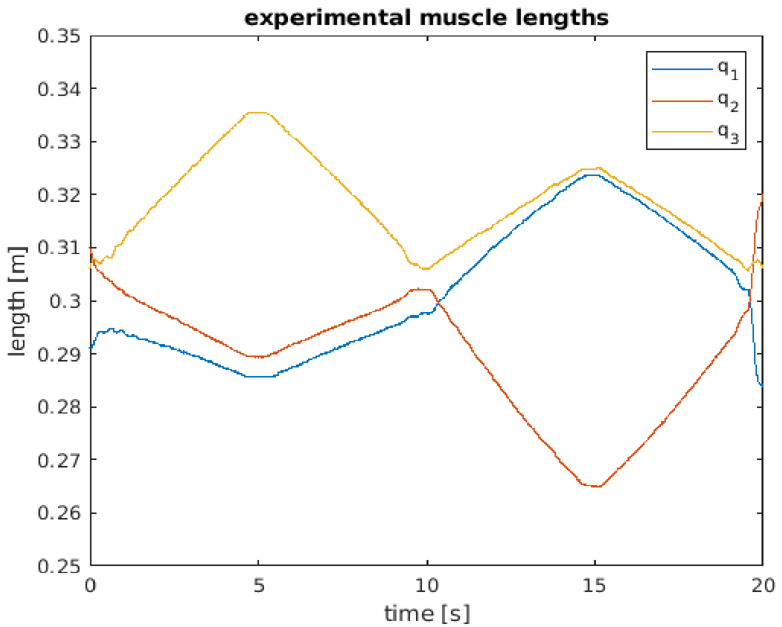
Results – dynamic model - steps

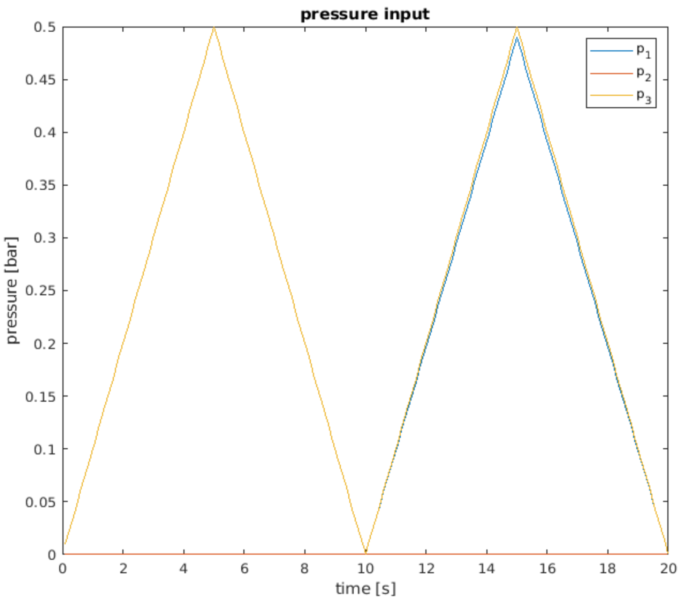


mean error = $25.3 \pm 13.2\ mm$

Results – dynamic model- linear pressure

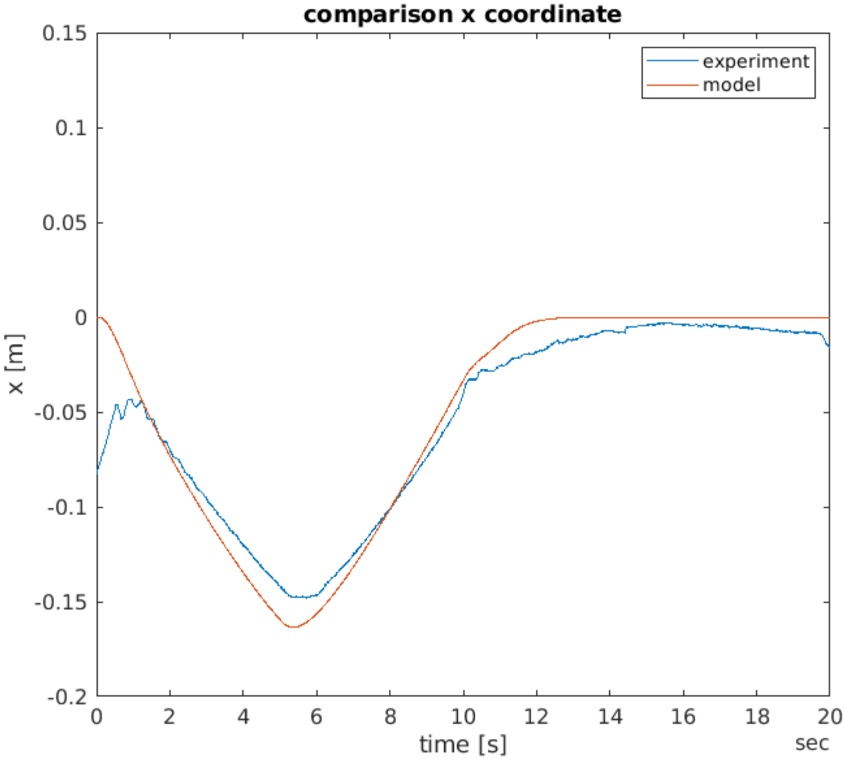


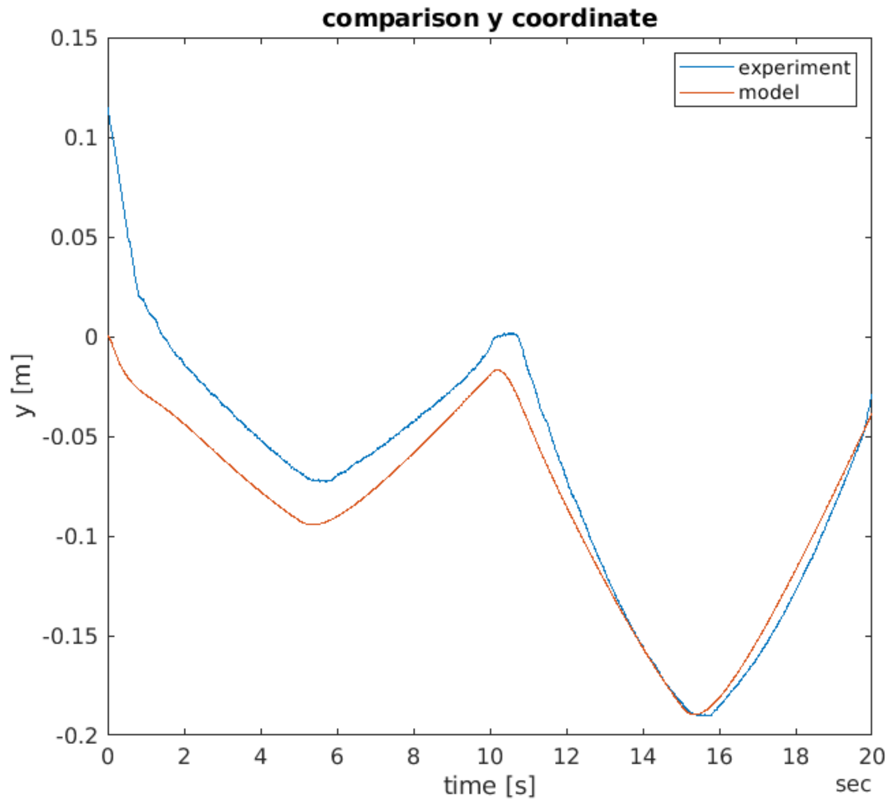




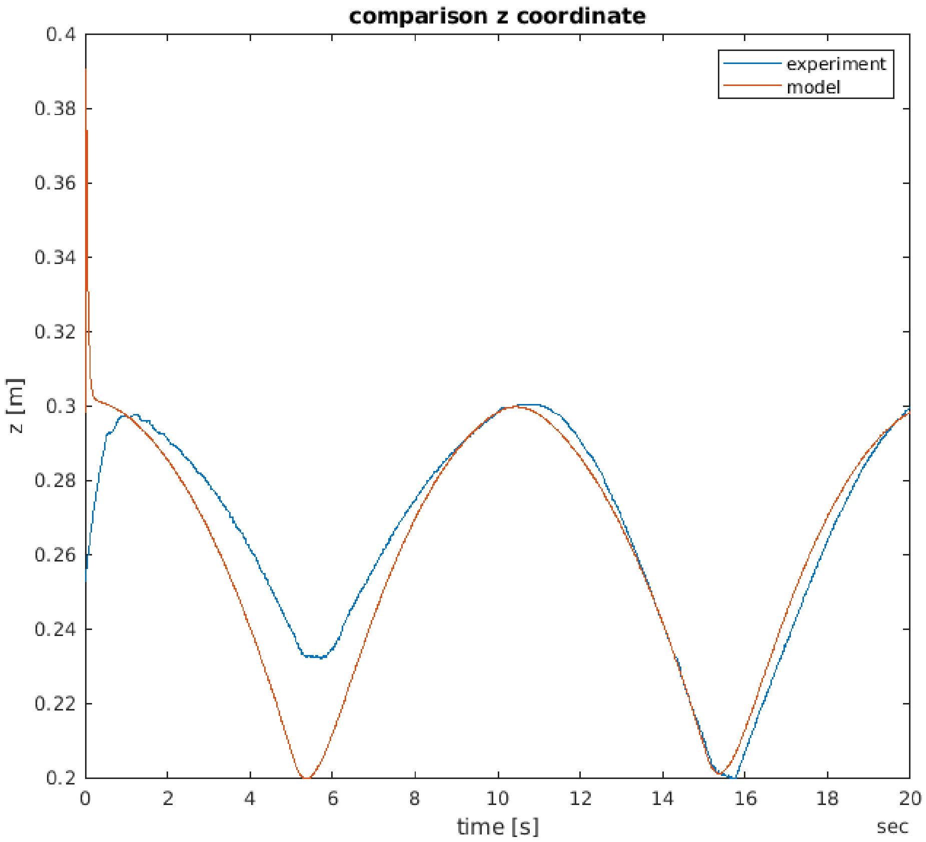
mean error = $5.9 \pm 3\,mm$

Results – dynamic model – linear pressure





Results – dynamic model – linear pressure



mean error = $25.7 \ \pm \ 23.4 \, mm$

Results – Euler vs. Runge-Kutta

- calculation time per time step:
  - Euler : 0.52 s
  - Runge-Kutta: 1.9 s
- Both are unstable for time steps > 0.0005 s.
- Even though Runge-Kutta is a higher order method accuracy is not improved.

Conclusion

- Kinematic model was implemented with a position error of 6.3 mm
- Dynamic model is tested with linear and step pressure inputs.
- Dynamic Model implemented with a muscle length error of 5.6-5.9 mm.
- Calculation time for 1 second simulation is 16.6 min.

Outlook

- to achieve real time calculation
  - model simplification → reduces calculation time
  - more stable approximation method → increases dt

- model valves

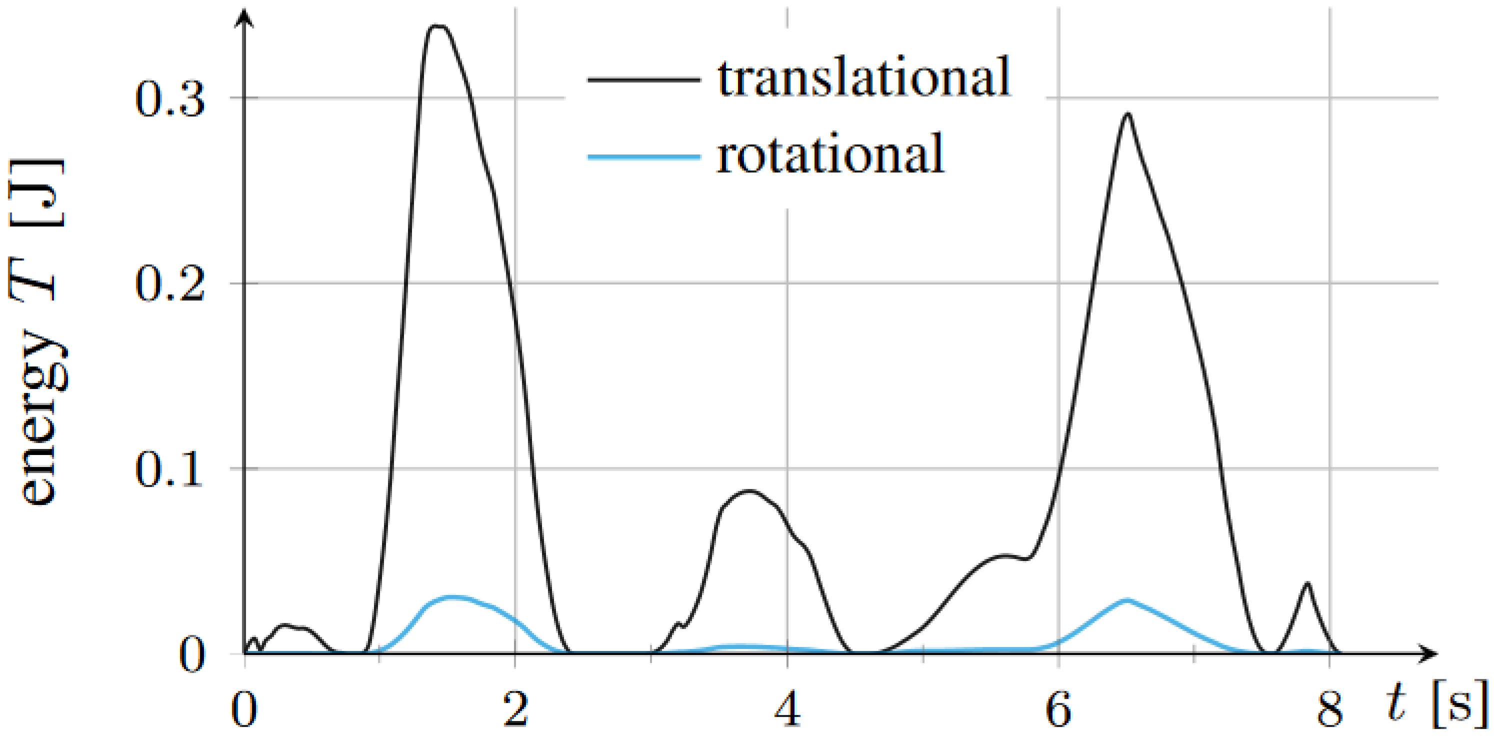
- parameter identification with optimization tools

- model mass in center of gravity

Bibliography

- [1] FestHQ. (2010,04,15). Festo – Bionic Handling Assistant- Animation, https://www.youtube.com/watch?v=EeSUPTAz2MM
- [2] V. Falkenhahn, 2017, Modellierung und modellbasierte Regelung von Kontinuum-Manipulatoren, DOI: http://dx.doi.org/10.18419/opus-9226
- [3] V. Falkenhahn, 2015, Dynamic Modeling of Bellows-Actuated Continuum Robots Using the Euler–Lagrange Formalism,DOI:10.1109/TRO.2015.2496826

Backup slide: rotational energy



Backup slide: dynamic model - kinetic energy

Lagrange Equations: $$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \cancel{\frac{\partial T}{\partial q_i}} + \frac{\partial U}{\partial q_i} = Q_i$$

$$T_{rot} \ll T_{trans} \rightarrow T_{rot} \approx 0$$

$$\frac{d}{dt}\frac{\partial T_{trans}}{\partial \dot{q}_i} = m({}^{w}\ddot{H}_h {}^{h}r_C)^T(\frac{\partial {}^{w}H_h}{\partial q_i} {}^{h}r_C) + \cancel{\frac{\partial T_{trans}}{\partial q_i}}$$

$${}^{w}\ddot{H}_h = \sum_{\chi=1}^{3}(\frac{\partial {}^{w}H_h}{\partial q_\chi}\ddot{q}_\chi + \sum_{\alpha=1}^{3}\frac{\partial^2 {}^{w}H_h}{\partial q_\alpha \partial q_\chi}\dot{q}_\chi \dot{q}_\alpha)$$

Backup slide: dynamic model - potential energy

Lagrange Equations: $$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i$$

Assuption: $M_{bend} \neq M_{bend}(\varphi)$

$$\frac{\partial U}{\partial q_i} = -m^w g^T \frac{\partial^w H_h}{\partial q_i} {}^h r_C + \frac{\partial M_{bend}}{\partial \theta} \frac{\partial \theta}{\partial q_i}$$

Backup slide: dynamic model – generalized force

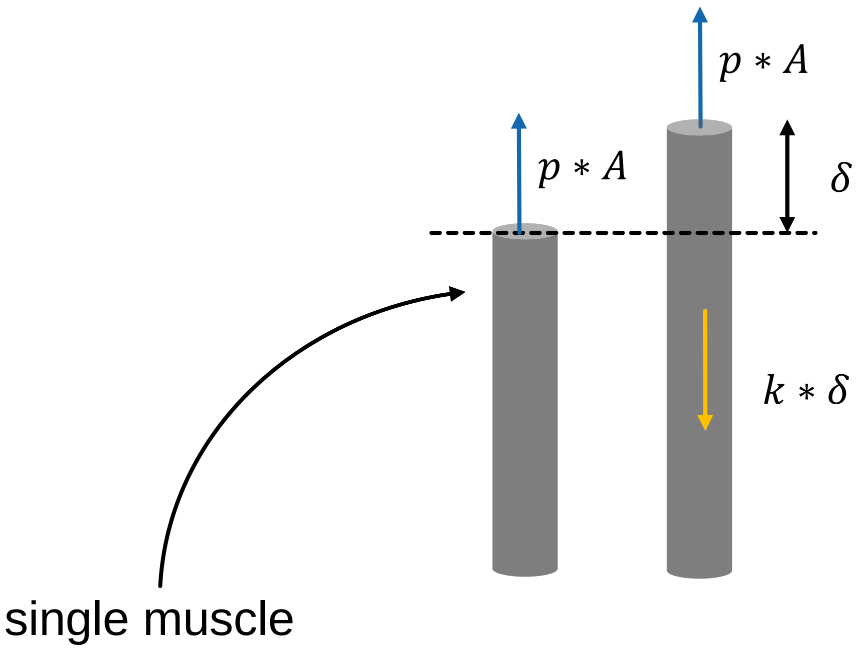
Lagrange Equations: $$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i$$

$$Q_i = {}^w F_C^T \frac{\partial {}^w H_h}{\partial q_i} {}^h r_C + \sum_{\chi=1}^{3} {}^w F_{e,M\chi}^T \frac{\partial {}^w H_h}{\partial q_i} {}^h r_{M,\chi}$$

$${}^w F_C = -{}^w H_h({}^h F_{e,C,spr}(l_{bar}) + {}^h F_{C,dmp}(\dot{l}_{bar}))$$

$${}^w F_{e,M,i} = {}^w H_h({}^h F_{e,M,i,akt}(p_i) - ({}^h F_{M,i,spr}(q_i) + {}^h F_{M,i,dmp}(\dot{q}_i)))$$

Parameter identification - length stiffness

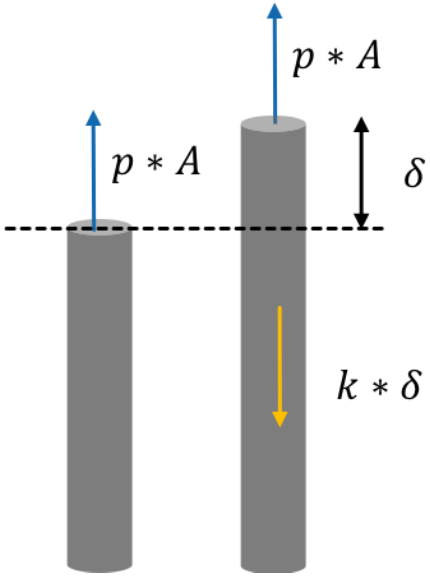


A: pressure surface

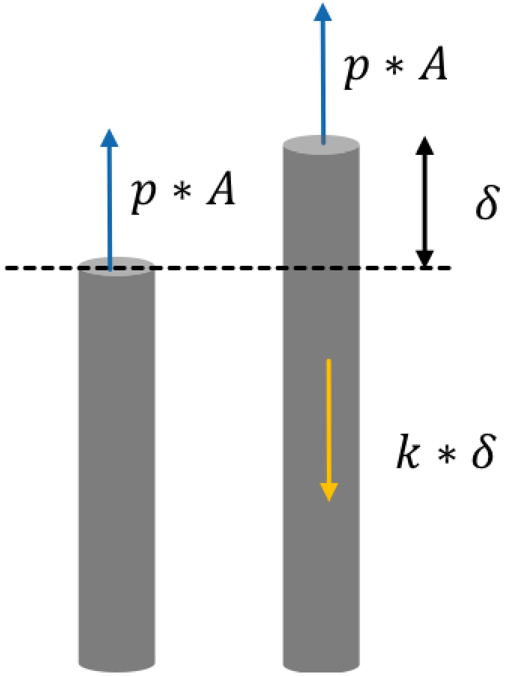
δ: displacement of muscle

k: length stiffness

Parameter identification - length stiffness



Parameter identification - length stiffness



Parameter identification - length stiffness