CSED232 Assignment 4

Due Saturday, April 12

Problem 1. The following program swaps the absolute values of two non-zero variables, x and y, while preserving their original signs. Answer the following questions.

```
1  {x = a ∧ y = b ∧ a ≠ 0 ∧ b ≠ 0}
2  if (x < 0)
3      t = - x;
4  else
5      t = x;
6  {t = |a| ∧ x = a ∧ y = b ∧ a ≠ 0 ∧ b ≠ 0}
7  if (y < 0) {
8      x = (x / t) * (- y);
9      y = - t;
10 } else {
11     x = (x / t) * y;
12     y = t;
13 }
14 {?}
```

(1) Write a postcondition that accurately captures the intended behavior of the program. (You may use the *sgn* function, where $sgn(z) = z/|z|$ for $z \neq 0$.)

(2) Write a Hoare logic proof (decorated program) to show that your specification is correct, with the precondition on line 1. (*hint*: use the assertion on line 6).

Problem 2. The following program calculates c^n, given two non-negative integers c and n, using an algorithm called *exponentiation by squaring.* Answer the following questions.

```
{c > 0 ∧ n ≥ 0}
int x = c;
int y = n;
int z = 1;
while (y > 0) {
    if (y % 2 == 0) {
        y = y / 2;
        x = x * x;
    } else {
        z = z * x;
        y = y - 1;
    }
}
{z = c^n}
```

(1) Identify a loop invariant that captures the relationship among the variables x, y, and z. Write a Hoare logic proof (decorated program) to prove the given specification.

(2) Identify a ranking function $\delta(x, y, z)$ that returns a non-negative integer. Write a Hoare logic proof (decorated program) to show that your ranking function is correct.

Problem 3. The following program finds the maximum value in an array of N integers. Write a Hoare logic proof (decorated program) to prove the given specification, where $\max_A(l, u)$ denote the maximum of the numbers $A[l], \ldots, A[u-1]$.

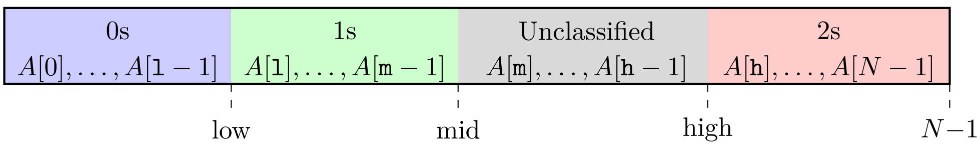
```
{0 < N}
int m = A[0];
int i = 1;
while (i < N) {
    if (A[i] > m)
        m = A[i];
    else
        skip;
    i = i + 1;
}
{m = max_A(0, N))}
```

Problem 4. Given an array of N elements consisting of 0s, 1s, and 2s, the following program reorders it in ascending order. This is known as Dijkstra's Dutch National Flag problem.

Let $v_A(c \mid j, k)$ denote that for each index $j \leq i < k$, $A[i] = c$. If $j \geq k$, $v_A(c \mid j, k)$ is true. For $j < k$, $v_A(c \mid j, k)$ iff $A[j] = c \wedge v_A(c \mid j+1, k)$ iff $v_A(c \mid j, k-1) \wedge A[k-1] = c$.

```
{0 ≤ N}
int m = 0;
int l = 0;
int h = N;
while (m < h) {
    if (A[m] == 0) {
        swap(A[l], A[m])
        l = l + 1;
        m = m + 1;
    } else if (A[m] == 2) {
        h = h - 1;
        swap(A[m], A[h]);
    } else
        m = m + 1;
}
{m = h ∧ v_A(0 | 0, l) ∧ v_A(1 | l, m) ∧ v_A(2 | h, N)}
```

(1) The invariant maintained by the loop is as follows: elements before `l` are 0s, between `l` and $\mathtt{m} - 1$ are 1s, and after `h` are 2s.



Write a Hoare logic proof (decorated program) to prove the given specification. For `swap`, use the following specification: $\{x = a \wedge y = b\}$ `swap`(x, y) $\{x = b \wedge y = a\}$.

(2) Identify a ranking function to prove termination. Write a Hoare logic proof (decorated program) to show that your ranking function is correct.