Strassen’s Matrix Multiplication Algorithm

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< 목　차 >

1. 목표

행렬곱 과정에서 시간 복잡도가 3차보다 좋은 알고리즘인 쉬트라젠 알고리즘을 공부하고, 분할 정복법을 이용하여 크기가 큰 행렬곱의 성능을 향상시킨다. 쉬트라젠 알고리즘의 pseudo code를 충실하게 구현하고, 쉬트라젠의 알고리즘 계산과정을 직접 수행하며 이해도를 향상시킨다.

2. Problem & Input

* **Problem** : n이 2의 거듭제곱일 때 2개의 n X n 행렬의 곱을 구하시오

* **Input** : 2의 거듭제곱인 정수 n, 2개의 n X n 행렬 A와 B

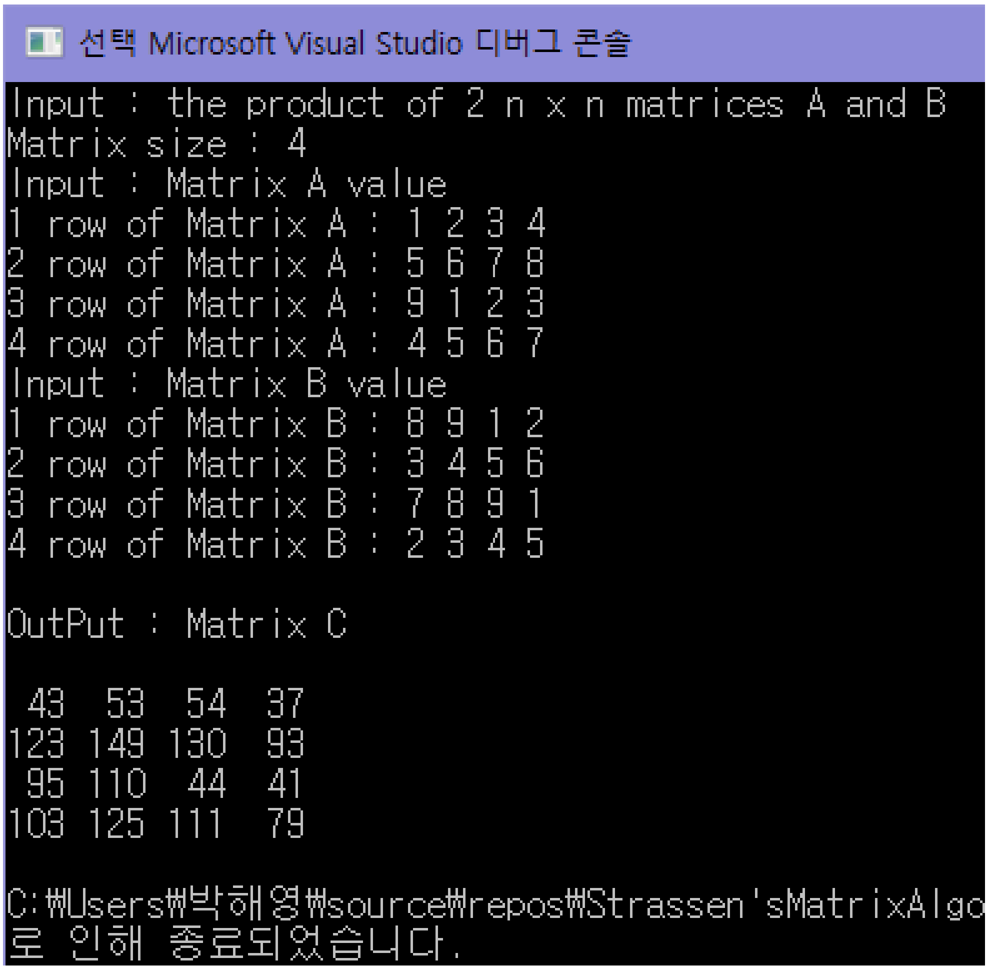
3. 구현 Language & 사용 Tool

* **구현 언어** : C language

* 사용 Tool : Visual Studio 2017

4. 교재의 입력 데이터 테스트

1) 4 X 4 행렬의 곱



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선택 Microsoft Visual Studio 디버그 콘솔
Input : the product of 2 n x n matrices A and B
Matrix size : 4
Input : Matrix A value
1 row of Matrix A : 1 2 3 4
2 row of Matrix A : 5 6 7 8
3 row of Matrix A : 9 1 2 3
4 row of Matrix A : 4 5 6 7
Input : Matrix B value
1 row of Matrix B : 8 9 1 2
2 row of Matrix B : 3 4 5 6
3 row of Matrix B : 7 8 9 1
4 row of Matrix B : 2 3 4 5

OutPut : Matrix C

 43  53  54  37
123 149 130  93
 95 110  44  41
103 125 111  79

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5. 자작 입력 데이터 생성 & 알고리즘의 과정 손계산

행렬곱 4x4 : n=4.

$$A=\begin{bmatrix}1&2&3&4\\3&1&2&5\\4&2&1&1\\2&3&2&1\end{bmatrix} \quad B=\begin{bmatrix}2&0&4&3\\0&3&1&2\\1&2&3&1\\2&4&2&0\end{bmatrix}$$

행렬분할 4x4 → 2x2 : n=2.

$$A_{11}=\begin{bmatrix}1&2\\3&1\end{bmatrix}, A_{12}=\begin{bmatrix}3&4\\2&5\end{bmatrix}$$

$$A_{21}=\begin{bmatrix}4&2\\2&3\end{bmatrix}, A_{22}=\begin{bmatrix}1&1\\2&1\end{bmatrix}$$

$$B_{11}=\begin{bmatrix}2&0\\0&3\end{bmatrix}, B_{12}=\begin{bmatrix}4&3\\1&2\end{bmatrix}$$

$$B_{21}=\begin{bmatrix}1&2\\2&4\end{bmatrix}, B_{22}=\begin{bmatrix}3&1\\2&0\end{bmatrix}$$

① M1 계산 : $(A_{11}+A_{22})\times(B_{11}+B_{22})$

$$A_{11}+A_{22}=\begin{bmatrix}1&2\\3&1\end{bmatrix}+\begin{bmatrix}1&1\\2&1\end{bmatrix}=\begin{bmatrix}2&3\\5&2\end{bmatrix}$$

$$B_{11}+B_{22}=\begin{bmatrix}2&0\\0&3\end{bmatrix}+\begin{bmatrix}3&1\\2&0\end{bmatrix}=\begin{bmatrix}5&1\\2&3\end{bmatrix}$$

①-행렬곱 2x2 : n=2. (standard).

$$M1=\begin{bmatrix}2&3\\5&2\end{bmatrix}\times\begin{bmatrix}5&1\\2&3\end{bmatrix}=\begin{bmatrix}16&11\\29&11\end{bmatrix}$$

② M2 계산 : $(A_{21}+A_{22})\times B_{11}$

$$A_{21}+A_{22}=\begin{bmatrix}4&2\\2&3\end{bmatrix}+\begin{bmatrix}1&1\\2&1\end{bmatrix}=\begin{bmatrix}5&3\\4&4\end{bmatrix}$$

$$B_{11}=\begin{bmatrix}2&0\\0&3\end{bmatrix}$$

②-행렬곱 2x2 : n=2 (standard)

$$M2=\begin{bmatrix}5&3\\4&4\end{bmatrix}\times\begin{bmatrix}2&0\\0&3\end{bmatrix}=\begin{bmatrix}10&9\\8&12\end{bmatrix}$$

③ M3 계산 : $A_{11}\times(B_{12}-B_{22})$

$$A_{11}=\begin{bmatrix}1&2\\3&1\end{bmatrix}$$

$$B_{12}-B_{22}=\begin{bmatrix}4&3\\1&2\end{bmatrix}-\begin{bmatrix}3&1\\2&0\end{bmatrix}=\begin{bmatrix}1&2\\-1&2\end{bmatrix}$$

③-행렬곱 2x2 : n=2 (standard)

$$M3=\begin{bmatrix}1&2\\3&1\end{bmatrix}\times\begin{bmatrix}1&2\\-1&2\end{bmatrix}=\begin{bmatrix}-1&6\\2&8\end{bmatrix}$$

④ M4 계산 : $A_{22}\times(B_{21}-B_{11})$

$$A_{22}=\begin{bmatrix}1&1\\2&1\end{bmatrix}$$

$$B_{21}-B_{11}=\begin{bmatrix}1&2\\2&4\end{bmatrix}-\begin{bmatrix}2&0\\0&3\end{bmatrix}=\begin{bmatrix}-1&2\\2&1\end{bmatrix}$$

④-행렬곱 2x2 : n=4.

$$M4=\begin{bmatrix}1&1\\2&1\end{bmatrix}\times\begin{bmatrix}-1&2\\2&1\end{bmatrix}=\begin{bmatrix}1&3\\0&5\end{bmatrix}$$

⑤ M5 계산 : $(A_{11}+A_{12})\times B_{22}$

$$A_{11}+A_{12}=\begin{bmatrix}1&2\\3&1\end{bmatrix}+\begin{bmatrix}3&4\\2&5\end{bmatrix}=\begin{bmatrix}4&6\\5&6\end{bmatrix}$$

$$B_{22}=\begin{bmatrix}3&1\\2&0\end{bmatrix}$$

⑤-행렬곱 2x2 : n=2 (standard)

$$M_5=\begin{bmatrix}4&6\\5&6\end{bmatrix}\times\begin{bmatrix}3&1\\2&0\end{bmatrix}=\begin{bmatrix}24&4\\27&5\end{bmatrix}$$

⑥ M6 계산 : $(A_{21}-A_{11})\times(B_{11}+B_{12})$

$$A_{21}-A_{11}=\begin{bmatrix}4&2\\2&3\end{bmatrix}-\begin{bmatrix}1&2\\3&1\end{bmatrix}=\begin{bmatrix}3&0\\-1&2\end{bmatrix}$$

$$B_{11}+B_{12}=\begin{bmatrix}2&0\\0&3\end{bmatrix}+\begin{bmatrix}4&3\\1&2\end{bmatrix}=\begin{bmatrix}6&3\\1&5\end{bmatrix}$$

⑥ 행렬곱 2x2 : n=2 (standard)

$$M6=\begin{bmatrix}3&0\\-1&2\end{bmatrix}\times\begin{bmatrix}6&3\\1&5\end{bmatrix}=\begin{bmatrix}18&9\\-4&7\end{bmatrix}$$

⑦ M7 계산 : $(A_{12}-A_{22})\times(B_{21}+B_{22})$

$$A_{12}-A_{22}=\begin{bmatrix}3&4\\2&5\end{bmatrix}-\begin{bmatrix}1&1\\2&1\end{bmatrix}=\begin{bmatrix}2&3\\0&4\end{bmatrix}$$

$$B_{21}+B_{22}=\begin{bmatrix}1&2\\2&4\end{bmatrix}+\begin{bmatrix}3&1\\2&0\end{bmatrix}=\begin{bmatrix}4&3\\4&4\end{bmatrix}$$

⑦ - 행렬곱 2x2 : n=2 (standard)

$$M7=\begin{bmatrix}2&3\\0&4\end{bmatrix}\times\begin{bmatrix}4&3\\4&4\end{bmatrix}=\begin{bmatrix}20&18\\16&16\end{bmatrix}$$

[M1-M7 ⇒ result matrix C]

C = 4x4 행렬.

$$C=\begin{bmatrix}C_{11}&C_{12}\\C_{21}&C_{22}\end{bmatrix}$$

$$C_{11}=M1+M4-M5+M7$$

$$=\begin{bmatrix}16&11\\29&11\end{bmatrix}+\begin{bmatrix}1&3\\0&5\end{bmatrix}-\begin{bmatrix}24&4\\27&5\end{bmatrix}+\begin{bmatrix}20&18\\16&16\end{bmatrix}$$

$$=\begin{bmatrix}13&28\\18&27\end{bmatrix}$$

$$C_{12}=M3+M5$$

$$=\begin{bmatrix}-1&6\\2&8\end{bmatrix}+\begin{bmatrix}24&4\\27&5\end{bmatrix}=\begin{bmatrix}23&10\\29&13\end{bmatrix}$$

$$C_{21}=M2+M4$$

$$=\begin{bmatrix}10&9\\8&12\end{bmatrix}+\begin{bmatrix}1&3\\0&5\end{bmatrix}=\begin{bmatrix}11&12\\8&17\end{bmatrix}$$

$$C_{22}=M1+M3-M2+M6$$

$$=\begin{bmatrix}16&11\\29&11\end{bmatrix}+\begin{bmatrix}-1&6\\2&8\end{bmatrix}-\begin{bmatrix}10&9\\8&12\end{bmatrix}+\begin{bmatrix}18&9\\-4&7\end{bmatrix}$$

$$=\begin{bmatrix}23&17\\19&14\end{bmatrix}$$

[C_{11}, C_{12}, C_{21}, C_{22} → C : merge]

$$C=\begin{bmatrix}C_{11}&C_{12}\\C_{21}&C_{22}\end{bmatrix}$$

$$=\begin{bmatrix}\begin{bmatrix}13&28\\18&27\end{bmatrix}&\begin{bmatrix}23&10\\29&13\end{bmatrix}\\\begin{bmatrix}11&12\\8&17\end{bmatrix}&\begin{bmatrix}23&17\\19&14\end{bmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}13&28&23&10\\18&27&29&13\\11&12&23&17\\8&17&19&14\end{bmatrix}$$

: result

$$\begin{bmatrix}1&2&3&4\\3&1&2&5\\4&2&1&1\\2&3&2&1\end{bmatrix}\times\begin{bmatrix}2&0&4&3\\0&3&1&2\\1&2&3&1\\2&4&2&0\end{bmatrix}$$

$$=\begin{bmatrix}13&28&23&10\\18&27&29&13\\11&12&23&17\\8&17&19&14\end{bmatrix}$$

<정리>

- 재귀함수호출

- 재귀함수 return.

6. 자작 입력 데이터 테스트

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Microsoft Visual Studio 디버그 콘솔
Input : the product of 2 n x n matrices A and B
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2 row of Matrix A : 3 1 2 5
3 row of Matrix A : 4 2 1 1
4 row of Matrix A : 2 3 2 1
Input : Matrix B value
1 row of Matrix B : 2 0 4 3
2 row of Matrix B : 0 3 1 2
3 row of Matrix B : 1 2 3 1
4 row of Matrix B : 2 4 2 0

OutPut : Matrix C

 13  28  23  10
 18  27  29  13
 11  12  23  17
  8  17  19  14

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이 창을 닫으려면 아무 키나 누르세요.
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